Application of the Smooth Variable Structure Filter to a Multi-Target Tracking Problem

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ABSTRACT

The most popular and well-studied estimation method is the Kalman filter (KF), which was introduced in the 1960s. It yields a statistically optimal solution for linear estimation problems. The smooth variable structure filter (SVSF) is a relatively new estimation strategy based on sliding mode theory, and has been shown to be robust to modeling uncertainties. The SVSF makes use of an existence subspace and of a smoothing boundary layer to keep the estimates bounded within a region of the true state trajectory. This article discusses the application of two estimation strategies (the KF and the SVSF) on a multi-target tracking problem.

Keywords: Target tracking, estimation strategies, Kalman filter, smooth variable structure filter

1. INTRODUCTION

In target tracking applications, one may be concerned with surveillance, guidance, obstacle avoidance or tracking a target given some measurements [1]. In a typical scenario, sensors provide a signal that is processed and output as a measurement. These measurements are related to the target state, and are typically noise-corrupted observations [1]. The target state usually consists of kinematic information such as position, velocity, and acceleration. The measurements are processed in order to form and maintain tracks, which are a sequence of target state estimates that vary with time [1]. Multiple targets and measurements may yield multiple tracks. Gating and data association techniques help classify the source of measurements, and help associate measurements to the appropriate track [1]. Typically these gating techniques help to avoid extraneous measurements which would otherwise cause the estimation process to go unstable and fail. A tracking filter is used in a recursive manner to carry out the target state estimation.

State and parameter estimation techniques are quite useful for systems when not all of the dynamics are known. Estimation theory involves finding a value of some parameter of interest, which affects the output of the system, often in the presence of inaccurate or uncertain observations [2]. States are representative of the dynamics of a system. For example, for space vehicles, inertial measuring units may be used to calculate the acceleration. However, since their alignment deteriorates over time, calculating the acceleration by other means (i.e., state estimation) may be desirable [3]. The purpose of estimation, as described by Bar-Shalom et al. in [2], can be one of many reasons: determination of planet orbit parameters, statistical inference, aircraft traffic control system (i.e., tracking), use in control plants with uncertainties (i.e., parameter identification or state estimation), determination of model parameters (i.e., system identification), message retrieval from noisy signals (i.e., communication theory), and also signal and image processing. A filter may be used to estimate the state of a dynamic system, whether linear or nonlinear. The word filter is used because when finding the best estimate, one has to filter out the noisy signals or uncertain observations [3].

2. THE KALMAN FILTER

In the estimation world, even after 50 years, the Kalman filter (KF) method remains the most studied and one of the most popular tools used to estimate states from systems [2,4,5]. It may be applied on linear dynamic systems in the presence of Gaussian white noise, and provides an elegant and statistically optimal solution by minimizing the mean-squared estimation error. The following five equations form the core of the KF algorithm, and are used in an iterative fashion. Equations (2.1) and (2.2) define the a priori state estimate $\hat{x}_{k+1|k}$ based on knowledge of the system A and previous state estimate $\hat{x}_{k|k}$, and the corresponding state error covariance matrix $P_{k+1|k}$, respectively.

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1}$$

$$P_{k+1|k} = CP_{k|k}C^T + Q_k \tag{2.2}$$

The Kalman gain K_{k+1} is defined by (2.3), and is used to update the state estimate $\hat{x}_{k+1|k+1}$ as shown in (2.4). The gain makes use of an innovation covariance S_{k+1}, which is defined as the inverse term found in (2.3).

$$K_{k+1} = P_{k+1|k} C^T \left[C P_{k+1|k} C^T + R_{k+1} \right]^{-1} \tag{2.3}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \left[z_{k+1} - C \hat{x}_{k+1|k} \right] \tag{2.4}$$

The a posteriori state error covariance matrix $P_{k+1|k+1}$ is then calculated by (2.5), and is used iteratively, as per (2.2).

$$P_{k+1|k+1} = [I - K_{k+1} C] P_{k+1|k} \tag{2.5}$$

Equations (2.1) – (2.5) are used in an iterative fashion, and represent the KF estimation process for linear systems. A number of different methods have extended the classical KF to nonlinear systems, with the most popular and simplest method being the extended Kalman filter (EKF) [6,7]. The EKF is conceptually similar to the KF; however, the nonlinear system is linearized according to its Jacobian. Consider the following nonlinear system and measurement equations:

$$x_{k+1} = f(x_k, u_k) + w_k \tag{2.6}$$

$$z_{k+1} = h(x_{k+1}) + v_{k+1} \tag{2.7}$$

Where f and h represent the nonlinear system and measurement models, respectively. It is possible to use the nonlinear functions f and h to predict the state estimates and the measurements. However, these functions may not be directly used to calculate the covariance values. The partial derivatives are used to compute linearized system and measurement matrices F and H, respectively found as follows [8]:

$$F_k = \left. \frac{\partial f}{\partial x} \right|_{\hat{x}_{k|k}, u_k} \tag{2.8}$$

$$H_{k+1} = \left. \frac{\partial h}{\partial x} \right|_{\hat{x}_{k+1|k}} \tag{2.9}$$

Equations (2.8) and (2.9) essentially linearize the nonlinear system or measurement functions around the current state estimate [9]. This comes at a loss of optimality, as the KF gain is no longer considered to be the best solution to the estimation problem [10]. The EKF process may be summarized by the following seven equations. The state estimate $\hat{x}_{k+1|k}$ is predicted using the nonlinear system model (2.10), and the corresponding state error covariance matrix $P_{k+1|k}$ is found (2.11).

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}, u_k\right) \tag{2.10}$$

$$P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k \tag{2.11}$$

The measurement error (or innovation) $\tilde{y}_{k+1}$ is then found (2.12), based on the nonlinear measurement model h, followed by the measurement error (or innovation) covariance matrix S_{k+1} (2.13).

$$\tilde{y}_{k+1} = z_{k+1} - h\left(\hat{x}_{k+1|k}\right) \tag{2.12}$$

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \tag{2.13}$$

Next, the near-optimal KF gain K_{k+1} is calculated (2.14). This gain is then used in conjunction with the predicted state estimate $\hat{x}_{k+1}$ and the measurement error $\tilde{y}_{k+1}$ to update the state estimate (2.15).

$$K_{k+1} = P_{k+1|k} H_{k+1}^T S_{k+1}^{-1} \tag{2.14}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \tilde{y}_{k+1} \tag{2.15}$$

Finally, the state error covariance matrix is updated (2.16).

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} \tag{2.16}$$

Equations (2.10) – (2.16) form the EKF estimation process. The linearization process of (2.8) and (2.9) introduces uncertainties that can sometimes cause the filter to go unstable [7]. However, for mildly nonlinear systems, the EKF provides a very good estimate of the states, and is relatively easy to implement [6].

3. The Smooth Variable Structure Filter

A new form of predictor-corrector estimator based on sliding mode concepts referred to as the variable structure filter (VSF) was introduced in 2003 [11]. Essentially this method makes use of the variable structure theory and sliding mode concepts. It uses a switching gain to converge the estimates to within a boundary of the true state values (i.e., existence subspace). In 2007, the smooth variable structure filter (SVSF) was derived which makes use of a simpler and less complex gain calculation [12]. In its present form, the SVSF has been shown to be stable and robust to modeling uncertainties and noise, when given an upper bound on the level of un-modeled dynamics and noise [11,13]. The basic estimation concept of the SVSF is shown in Fig. 1.

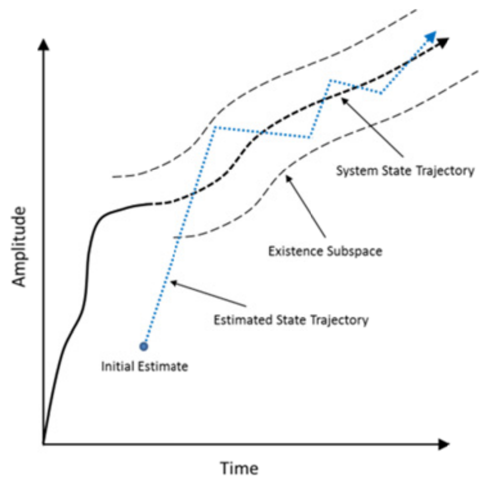


Figure 1. The above figure illustrates the SVSF estimation concept [12].

The SVSF method is model based and may be applied to differentiable linear or nonlinear dynamic equations. The estimation process is iterative and may be summarized by the following set of equations. The predicted state estimates $\hat{x}_{k+1|k}$ are first calculated respectively as follows:

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k) \tag{3.1}$$

Utilizing the predicted state estimates $\hat{x}_{k+1|k}$, the corresponding predicted measurements $\hat{z}_{k+1|k}$ and measurement errors $e_{z,k+1|k}$ may be calculated:

$$\hat{z}_{k+1|k} = H\hat{x}_{k+1|k} \tag{3.2}$$

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} \tag{3.3}$$

The SVSF process differs from the KF in how the gain is formulated. The SVSF gain is a function of: the a priori and the a posteriori measurement errors $e_{z,k+1|k}$ and $e_{z,k|k}$; the smoothing boundary layer widths ψ; and the ‘SVSF’ memory or convergence rate γ. It assumes that the measurement (3.2) is linear, as well as H is both positive and diagonal. Note that the assumptions pertaining to the measurement matrix are realistic since most applications use linear sensors as feedback in their operations. Moreover, these sensors are well calibrated and their structures are well-known [12]. The SVSF gain K_{k+1} is defined as follows [12]:

$$K_{k+1} = H^{+}\left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right|\right) \circ sat\left(\frac{e_{z_{k+1|k}}}{\psi}\right) \tag{3.4}$$

Where $\circ$ signifies Schur (or element-by-element) multiplication. Note that the saturation function of (3.4) is defined by:

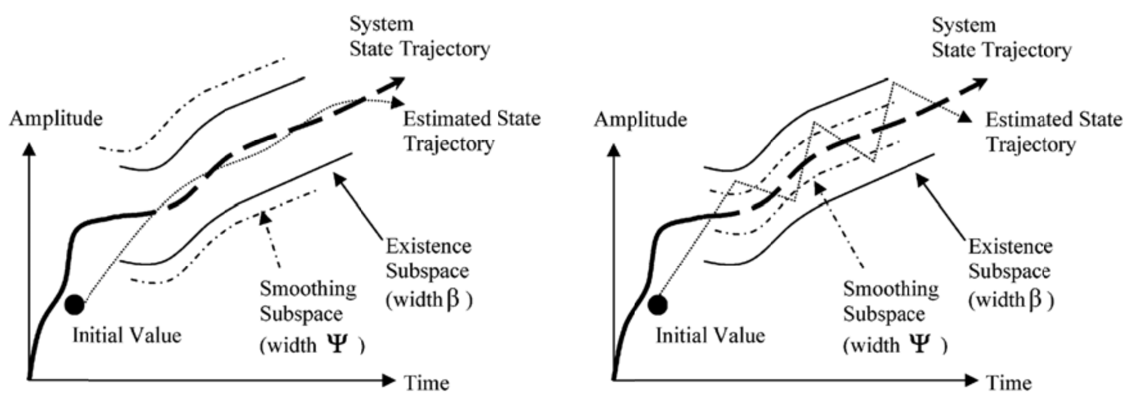
$$sat\left(\frac{e_{z_{k+1|k}}}{\psi}\right) = \begin{cases} 1, & e_{z_i,k+1|k}/\psi_i \geq 1 \\ e_{z_i,k+1|k}/\psi_i, & -1 < e_{z_i,k+1|k}/\psi_i < 1 \\ -1, & e_{z_i,k+1|k}/\psi_i \leq -1 \end{cases} \tag{3.5}$$

This gain (3.4) is used to calculate the updated state estimates $\hat{x}_{k+1|k+1}$ as follows:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \tag{3.6}$$

Finally, the updated measurement estimate $\hat{z}_{k+1|k+1}$ and measurement errors $e_{z,k+1|k+1}$ are calculated, and are used in later iterations:

$$\hat{z}_{k+1|k+1} = H\hat{x}_{k+1|k+1} \tag{3.7}$$
$$e_{z,k+1|k+1} = z_{k+1} - \hat{z}_{k+1|k+1} \tag{3.8}$$



(a) Smoothed estimated trajectory ($\psi \geq \beta$). (b) Presence of chattering effect ($\psi < \beta$).

Figure 2. Illustration of the smoothing boundary layer concept [12].

The existence subspace shown in Figs. 1 and 2 represents the amount of uncertainties present in the estimation process, in terms of modeling errors or the presence of noise. The width of the existence space β is a function of the uncertain dynamics associated with the inaccuracy of the internal model of the filter as well as the measurement model, and varies with time [12]. Typically this value is not exactly known but an upper bound may be selected based on a priori knowledge. Once within the existence boundary subspace, the estimated states are forced (by the SVSF gain) to switch back and forth along the true state trajectory. High-frequency switching caused by the SVSF gain is referred to as chattering, and in most cases, is undesirable for obtaining accurate estimates [12]. However, the effects of chattering may be minimized by the introduction of a smoothing boundary layer ψ. The selection of the smoothing boundary layer width reflects the level of uncertainties in the filter and the disturbances (i.e., system and measurement noise, and un-modeled dynamics). The effect of the smoothing boundary layer is shown in Fig. 2. When the smoothing boundary layer is defined larger than the existence subspace boundary, the estimated state trajectory is smoothed. However, when the smoothing term is too small, chattering remains due to the uncertainties being underestimated.

4. Multi-Target Tracking Scenario and Results

The scenario studied in this paper is similar to that presented in [14]. The measurements are obtained from an airborne sensor with varying revisit intervals. Let the m^{th} measurement in scan k be from the n^{th} target, and the true state of the n^{th} target at time t_{m_k} be defined by the following vector:

$$x^n(t_{m_k}) = \left[\xi^n(t_{m_k}) \quad \dot{\xi}^n(t_{m_k}) \quad \eta(t_{m_k}) \quad \dot{\eta}^n(t_{m_k})\right]^T \tag{4.1}$$

Where $\xi^n(t_{m_k})$ and $\eta(t_{m_k})$ are the distances of the target in the X and Y directions respectively from the origin. The corresponding velocities are $\dot{\xi}^n(t_{m_k})$ and $\dot{\eta}^n(t_{m_k})$, respectively. As presented in [14], the state of the sensor platform is known and is defined similarly by $x_p^n(t_{m_k})$. Only the scan index k is kept while the other indices m and n have been dropped for simplicity. The sensor-to-target range is defined as follows [14]:

$$r(x(t_k)) = \sqrt{r_\xi^2(x(t_k)) + r_\eta^2(x(t_k))} \tag{4.2}$$

Where $r_\xi(x(t_k))$ and $r_\eta(x(t_k))$ are the relative position components of the target at time t_k with respect to the platform in the X and Y directions, respectively. As per [14], the range rate $\dot{r}(x(t_k))$ and angle $\theta(x(t_k))$ of the target is given by the following two equations:

$$\dot{r}(x(t_k)) = \left(\dot{\xi}(t_k) - \dot{\xi}_p(t_k)\right)\cos\theta\left(x(t_k)\right) + \left(\dot{\eta}(t_k) - \dot{\eta}_p(t_k)\right)\sin\theta\left(x(t_k)\right) \tag{4.3}$$

$$\theta(x(t_k)) = \tan^{-1}\left(\frac{\eta(t_k) - \eta_p(t_k)}{\xi(t_k) - \xi_p(t_k)}\right) \tag{4.4}$$

Note that the measurements are in polar coordinates, whereas the target's motion is better modeled in Cartesian for the estimator [14]. As per [14], this necessitates the transformation of the received range-azimuth measurements into X and Y position measurements. As such, the converted measurement Kalman filter (CMKF) will be applied, and compared with the SVSF. In multi-target tracking, it is necessary to decide which measurement corresponds to which target. In other words, one needs a mechanism for measurement-to-track data association [2,14]. A number of methods exist for performing data association; however, it is the goal of this paper to only compare the estimation techniques.

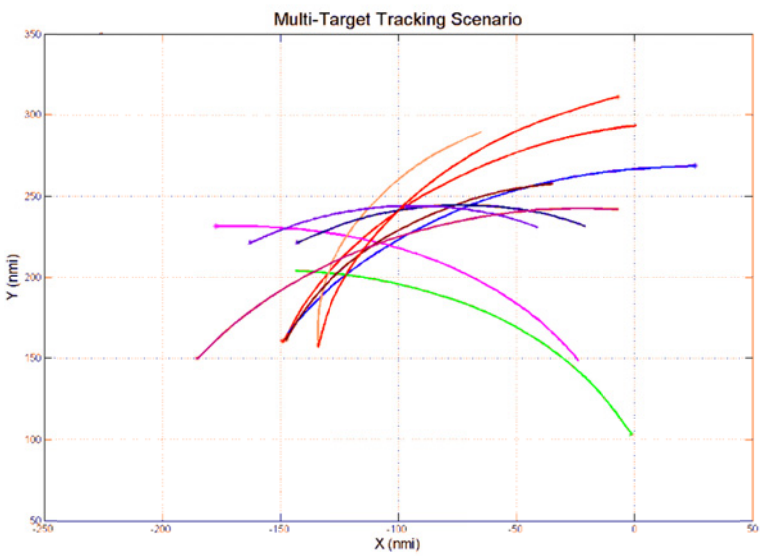


Figure 3. Multi-target tracking scenario (trajectory of 10 targets).

The target trajectories are shown in the previous figure. In total, there are 10 targets in the surveillance region. The targets undergo a wide variety of maneuver modes, including: constant velocity, constant acceleration, and coordinated turns [14]. These models are well established in literature, and may be found in [2,14]. A total of 100 Monte Carlo runs were performed, and the final results were averaged. The following table summarizes the results of the experiment, comparing the CMKF and the SVSF.

Table 1. Summary of the Simulation Results

Metric	**CMKF**	**SVSF**
Position RMSE	1.222	1.731
Velocity RMSE	0.089	0.052
Number of Missed Tracks	0.270	0.312

In this scenario, the CMKF provided the best result in terms of position RMSE. However, it is interesting to note that SVSF yielded the best result in terms of velocity RMSE. The following sets of figures help illustrate the simulation results. Figure 4 shows the position RMSE for the seventh target. The CMKF error was smaller than the SVSF; however, the SVSF still performed relatively well. Both methods demonstrate a more accurate estimate as time progresses. Figure 5 shows the velocity RMSE for the seventh target. This figure clearly shows that the SVSF velocity estimate was more accurate than the CMKF strategy. The final figure shows the number of missed tracks over the simulation period. Both filters performed essentially the same, with the CMKF having slightly less missed tracks. This paper compares the results of the SVSF strategy presented in [12]. It is important to note that advances have been made to improve the quality of the SVSF process, and may be found in [15,16]. It is the goal of this paper to simply demonstrate that the SVSF strategy functions properly in a multi-target tracking environment, and provides a reasonably good estimate.

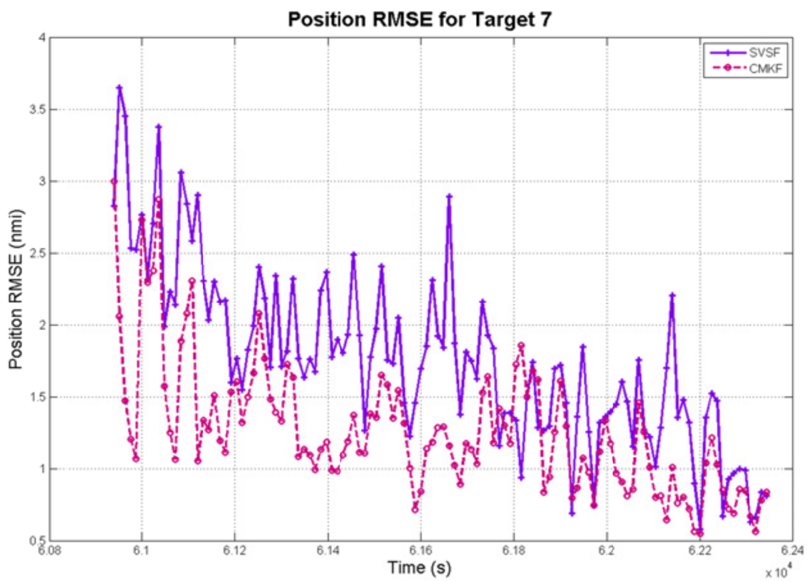


Figure 4. Position RMSE results for the 7th target.

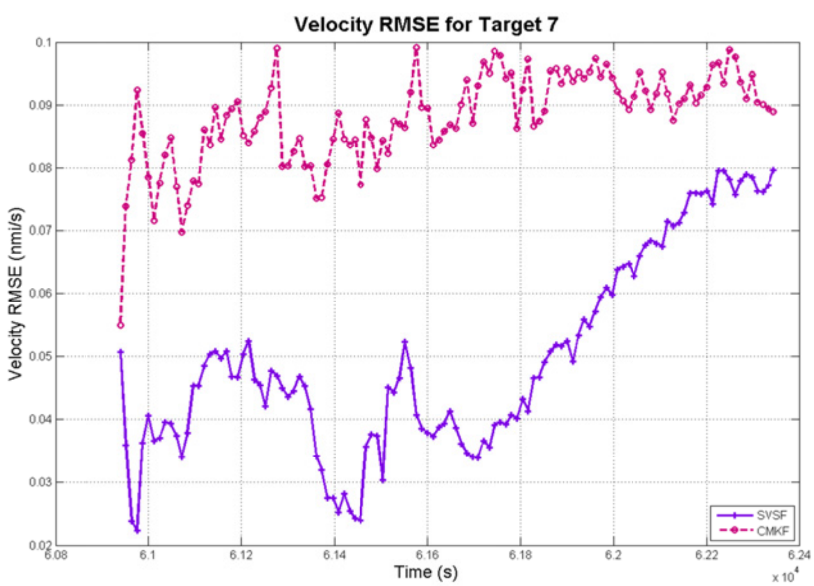


Figure 5. Velocity RMSE for the 7th target.

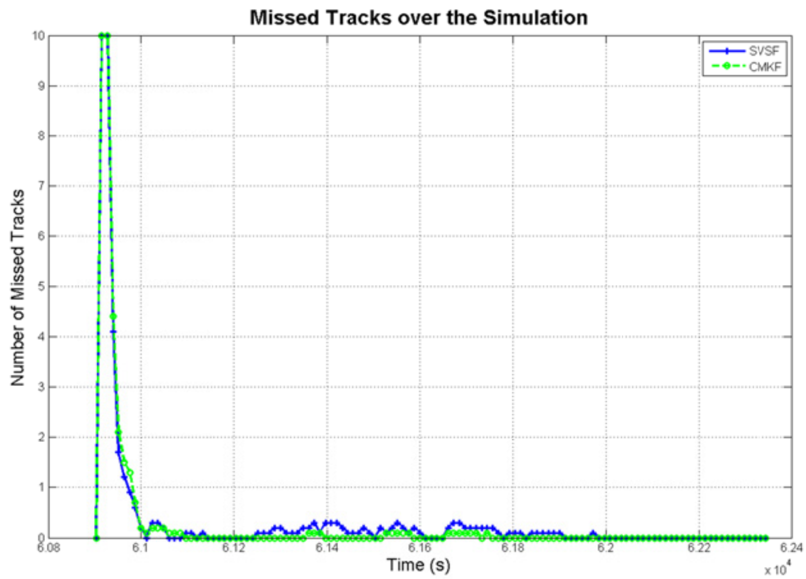


Figure 6. The number of missed tracks over the simulation period.

5. Conclusions

This article discussed the application of two estimation strategies (the CMKF and the SVSF) on a multi-target tracking problem. The smooth variable structure filter (SVSF) is a relatively new estimation strategy based on sliding mode theory, and has been shown to be robust to modeling uncertainties. The SVSF makes use of an existence subspace and of a smoothing boundary layer to keep the estimates bounded within a region of the true state trajectory. The results of the simulation demonstrated that the CMKF provided better position estimates, whereas the SVSF provided better velocity estimates. Future work involves implementing newer forms of the SVSF on multi-target tracking problems, and comparing it with the popular KF.

6. Appendix

The following table provides a list of the nomenclature used throughout this paper.

Table 2. List of Nomenclature

Parameter	Definition
x	State vector or values
z	Measurement (system output) vector or values
u	Input to the system
w	System noise vector
v	Measurement noise vector
A, F	Linear system transition matrix
B, G	Input gain matrix
C, H	Linear measurement (output) matrix
K	Filter gain matrix (i.e., KF or SVSF)
P	State error covariance matrix
Q	System noise covariance matrix
R	Measurement noise covariance matrix
S	Innovation covariance matrix
e	Measurement (output) error vector
f	Nonlinear system model
h	Nonlinear measurement model
$diag(a)$ or $\bar{a}$	Defines a diagonal matrix of some vector a

$sat(a)$	Defines a saturation of the term *a*
γ	SVSF 'convergence' or memory parameter
ψ	SVSF boundary layer width
$\lvert a \rvert$	Absolute value of some parameter *a*
$E\{\cdot\}$	Expectation of some vector or value
T	Transpose of some vector or matrix
^	Estimated vector or values
$k+1 \vert k$	A priori time step (i.e., before applied gain)
$k+1 \vert k+1$	A posteriori time step (i.e., after update)

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