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ESTIMATION OF AN ELECTROHYDROSTATIC ACTUATOR USING A COMBINED CUBATURE KALMAN AND SMOOTH VARIABLE STRUCTURE FILTER

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ABSTRACT

Electrohydrostatic actuators (EHAs) are an emerging type of actuator typically used in the aerospace industry, and are self-contained units comprised of their own pump, hydraulic circuit, and actuating cylinder. Recently, a novel EHA system has been developed. A particularly important parameter of this system is the bulk modulus. This value cannot be measured directly and must be estimated using state and parameter estimation techniques such as the Kalman filter (KF), particle filter (PF), or the smooth variable structure filter (SVSF). The cubature Kalman filter (CKF), recently proposed in 2009, is an estimation method that makes use of the standard derivation of the Kalman filter, as well as defined cubature points which are used to draw the probability distribution of the states with greater accuracy. The SVSF is a relatively new estimation strategy which is very robust and stable to modeling errors and uncertainties. The combination of these two methods (CKF-SVSF) yields an accurate and stable estimation method which has been applied on an EHA. The results of this combination are compared against the standard KF in terms of accuracy, robustness to errors and uncertainties, and filter complexity.

INTRODUCTION

For the successful control of many mechanical and electrical systems, knowledge of the current states and parameters is critical. Sensors are used to obtain measurements from the environment, most commonly taking kinematic readings (i.e., position, velocity, and acceleration). It is quite common for measurements to be corrupted by sensor noise. Whether it is from the measurements or the system, noise represents unwanted signals that reduce the quality of the information available for the controller **[1]**. To minimize the effects of noise, one may apply a filter, which essentially attempts to estimate the true state or parameter value by removing (or filtering) the unwanted signals **[2]**. Filters belong in the domain of estimation theory, which encompasses mathematical techniques and algorithms used to determine the true values of system states. In space vehicles, for example, inertial measuring units may be used to calculate the acceleration. However, since their alignment deteriorates over time, calculating the acceleration by other means (i.e., state estimation) may be desirable **[3]**.

For linear systems, the most well-known and studied filter is the Kalman filter (KF). The KF was introduced in 1960, and provides an elegant solution to linear estimation problems **[4]**. It provides a statistically optimal solution in the sense that it minimizes the state estimation error for linear systems with Gaussian distributed noise **[5,6]**. Although the KF is an effective estimation strategy, the accuracy of the filter comes at a trade-off with stability and robustness. The very strong assumptions (i.e., linearity, Gaussian noise, no modeling uncertainties) are seldom held in practice, particularly in the area of control or target tracking. If one of the assumptions is violated, the performance of the KF can become degraded which increases the chance of numerical instability **[7,1]**. Nonlinear estimation problems introduce another level of uncertainty through un-modeled dynamics.

In the presence of nonlinear systems or measurements, suboptimal techniques are required to tackle the estimation problem. Popular nonlinear estimation techniques include the extended Kalman filter (EKF), the unscented Kalman filter (UKF), and the particle filter (PF) **[2]**. The EKF utilizes the first-order Taylor series expansion (i.e., Jacobian) of the nonlinear equations to create linearized system and measurement matrices. One major drawback of this method is that it comes at a cost of higher-order un-modeled uncertainties **[7]**. The UKF is able to capture a higher order of the nonlinearities by using a set of deterministically chosen sample points (typically referred to as sigma points) which, after a transformation, captures the true mean and covariance up to the second order of nonlinearity **[8]**. The PF has recently become a very popular method for solving nonlinear estimation problems. Like the name suggests, the PF makes use of weighted particles or 'point masses' distributed in a manner that estimates the probability distribution function (PDF) of the values of interest **[8]**. The PDF contains all of the pertinent statistical information, and may be considered as holding the solution to the estimation problem **[2]**.

Most recently, the cubature Kalman filter (CKF) and smooth variable structure filter (SVSF) have been proposed. The CKF makes use of a third-degree cubature rule to numerically compute Gaussian-weighted integrals, which represent the joint state-measurement predictive density **[9]**. This enables a very close approximation of the nonlinear estimation problem. The SVSF is a relatively new predictor-correct estimation method based on sliding mode theory **[10]**. It yields suboptimal estimates, however is extremely robust and stable to modeling uncertainties and errors. This paper discusses a combined estimation method (referred to as the CKF-SVSF) which makes use of the accuracy of the CKF and the stability of the SVSF. For demonstration purposes, the filter is applied on a parameter estimation problem involving an electrohydrostatic actuator (EHA). The results (CKF-SVSF) of which are compared against the EKF, CKF, and SVSF. Note that the results of the UKF were nearly identical to the EKF, such that they were omitted.

REVIEW OF ESTIMATION METHODS

a) Kalman Filter

As previously mentioned, the KF provides an elegant and statistically optimal solution for linear dynamic systems in the presence of Gaussian white noise. It is an estimation method that utilizes measurements linearly related to the states or parameters of the systems, and error covariance matrices, to generate a gain referred to as the Kalman gain. This gain is applied to the a priori state estimate, thus creating an a posteriori (i.e., updated) estimate of the states. The estimation process is iterative and continues in a predictor-corrector fashion while maintaining a statistically minimal state error covariance matrix (for linear systems).

A typical linear dynamic system and measurement model are defined by using the following two equations, respectively:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

$$z_{k+1} = Hx_{k+1} + v_{k+1} \tag{2}$$

Please refer to the end of the paper for a list of pertinent nomenclature and variable definitions. The following five equations form the core of the KF algorithm, and are used in an iterative fashion. Equation (3) defines the a priori estimate based on the system definition, and (4) is the corresponding state error covariance matrix. The Kalman gain is defined by (5), and is used to update the state estimate shown in (6). The a posteriori state error covariance matrix is calculated by (7).

$$\hat{x}_{k+1|k} = \hat{A}\hat{x}_{k|k} + \hat{B}u_k \tag{3}$$

$$P_{k+1|k} = HP_{k|k}H^T + Q_k \tag{4}$$

$$K_{k+1} = P_{k+1|k}H^T\left[HP_{k+1|k}H^T + R_{k+1}\right]^{-1} \tag{5}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left[z_{k+1} - H\hat{x}_{k+1|k}\right] \tag{6}$$

$$P_{k+1|k+1} = [I - K_{k+1}H]P_{k+1|k} \tag{7}$$

b) Cubature Kalman Filter

As presented in **[11]**, the CKF is the closest known approximate filter in the sense of completely preserving second-order information due to the maximum entropy principle **[9]**. According to the maximum entropy principle, given the first two order statistics of a hidden process, it is Gaussian that maximizes the information entropy criterion of that process. In deriving the CKF, it is assumed that the predictive density of the joint state-measurement random variable is Gaussian. Under this assumption, the Bayesian filter reduces to the problem of how to compute integrals in which the integrands are all of the following form:

$$nonlinear\ function\ \times Gaussian \tag{8}$$

The CKF uses a third-degree cubature rule to numerically compute the above Gaussian-weighted integrals. For example, the cubature rule approximates an n-dimensional Gaussian weighted integral as follows:

$$\int_{\mathbb{R}^n} f(x)\,\mathcal{N}(x;\mu,\Sigma)dx \approx \frac{1}{2n}\sum_{i=1}^{2n} f\left(\mu + \sqrt{\Sigma}\xi_i\right) \tag{9}$$

Where a square-root factor of the covariance Σ satisfies the relationship $\Sigma = \sqrt{\Sigma}\sqrt{\Sigma}^T$ and the set of $2n$ cubature points are given by:

$$\xi_i = \begin{cases} \sqrt{n}e_i, & i = 1, 2, \ldots, n \\ -\sqrt{n}e_{i-n}, & i = n+1, n+2, \ldots, 2n \end{cases} \tag{10}$$

With e_i denoting the i^{th} elementary column vector. The third-degree cubature rule is exact for polynomial integrands up to the third degree or for any odd-degree polynomial. For a detailed exposition of how the cubature points were derived, the reader may consult **[9]**. For improved numerical stability, the CKF can be restructured to propagate the square-roots of the error covariances. The following equations represent the iterative square-root CKF estimation process **[9]**:

$$X_{i_{k|k}} = S_{k|k}\xi_i + \hat{x}_{k|k} \quad i = 1, 2, \ldots, 2n \tag{11}$$

$$X^*_{i_{k+1|k}} = f\left(X_{i_{k|k}}\right) \quad i = 1, 2, \dots, 2n \tag{12}$$

$$\hat{x}_{k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} X^*_{i_{k+1|k}} \tag{13}$$

$$\mathcal{X}^*_{i_{k+1|k}} = \frac{1}{\sqrt{2n}}\left[\left(X^*_{1_{k+1|k}} - \hat{x}_{k+1|k}\right) \quad \left(X^*_{2_{k+1|k}} - \hat{x}_{k+1|k}\right) \quad \dots \quad \left(X^*_{2n_{k+1|k}} - \hat{x}_{k+1|k}\right)\right] \tag{14}$$

$$S_{k+1|k} = tria\left(\left[\mathcal{X}^*_{i_{k+1|k}} \quad S_{Q_k}\right]\right) \tag{15}$$

$$X_{i_{k+1|k}} = S_{k+1|k}\xi_i + \hat{x}_{k+1|k} \quad i = 1, 2, \dots, 2n \tag{16}$$

$$Z_{i_{k+1|k}} = h\left(X_{i_{k+1|k}}\right) \quad i = 1, 2, \dots, 2n \tag{17}$$

$$\hat{z}_{k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} Z_{i_{k+1|k}} \tag{18}$$

$$\mathcal{X}_{k+1|k} = \frac{1}{\sqrt{2n}}\left[\left(X_{1_{k+1|k}} - \hat{x}_{k+1|k}\right) \quad \left(X_{2_{k+1|k}} - \hat{x}_{k+1|k}\right) \quad \dots \quad \left(X_{2n_{k+1|k}} - \hat{x}_{k+1|k}\right)\right] \tag{19}$$

$$\mathcal{Z}_{k+1|k} = \frac{1}{\sqrt{2n}}\left[\left(Z_{1_{k+1|k}} - \hat{z}_{k+1|k}\right) \quad \left(Z_{2_{k+1|k}} - \hat{z}_{k+1|k}\right) \quad \dots \quad \left(Z_{2n_{k+1|k}} - \hat{z}_{k+1|k}\right)\right] \tag{20}$$

$$\begin{pmatrix} A & 0 \\ B & C \end{pmatrix} = tria\begin{pmatrix} \mathcal{Z}_{k+1|k} & S_{R_{k+1}} \\ \mathcal{X}_{k+1|k} & 0 \end{pmatrix} \tag{21}$$

$$W_{k+1} = B/A \tag{22}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + W_{k+1}\left(z_{k+1} - \hat{z}_{k+1|k}\right) \tag{23}$$

$$S_{k+1|k+1} = C \tag{24}$$

c) Smooth Variable Structure Filter

As presented in **[12]**, sliding mode control and estimation techniques have been around for quite a few decades, and are mainly popular due to their relative ease of implementation and robustness to modeling uncertainties **[13,14]**. In a typical sliding mode control scenario, one utilizes a discontinuous switching plane along some desired trajectory **[15]**. This plane is quite often referred to as a sliding surface, in which the purpose is to keep the state values along this surface by minimizing the state errors (between the desired trajectory and the estimated or actual values). Ideally, if the state value is off or away from the surface, a switching gain would be used to push the state towards the sliding surface. Once upon the surface, the motion of the system as the states slide along the surface is called a sliding mode **[15]**. The discontinuous switching brings an inherent amount of stability to the control or estimation strategy, while also introducing excessive chattering (i.e., high-frequency switching) which may be undesirable in control since it may excite un-modeled dynamics. A boundary layer may be introduced along the sliding surface in order to saturate and smooth out the chattering within the boundary region. These sliding mode concepts are based on variable structure control, in which one alters the nonlinear dynamics of a system by the introduction of high-frequency switching **[13]**.

The variable structure filter (VSF) was first proposed in 2003, and was introduced as a new type of predictor-corrector estimator based on sliding mode concepts **[16]**. It is a type of sliding mode estimator, where gain switching is used to ensure that the estimates converge to within a boundary of the true state values (i.e., existence subspace). An internal model of the system, either linear or nonlinear, is used to predict an a priori state estimate. A corrective term (i.e., gain) is then applied to calculate the a posteriori state estimate, and the estimation process is repeated iteratively. The smooth variable structure filter (SVSF) was later derived from the VSF, and uses a much simpler and less complex gain calculation **[10]**. In its present form, the SVSF is stable and robust to modeling uncertainties and noise, given an upper bound on the level of un-modeled dynamics or knowledge of the magnitude of noise. The basic estimation concept of the SVSF is shown in the following figure. Some initial values of the estimated states are made based on probability distributions or designer knowledge. An area around the true system state trajectory is defined as the existence subspace. Through the use of the SVSF gain, the estimated state will be forced to within this region. Once the value enters the existence subspace, the estimated state is forced into switching along the system state trajectory. A saturation term may be used in this region to reduce the magnitude of chattering or smooth-out the result. As previously mentioned, the SVSF gain introduces a certain amount of chattering which brings an inherent amount of stability. This makes the estimation strategy an attractive method for control problems when not all of the dynamics are well known or defined correctly.

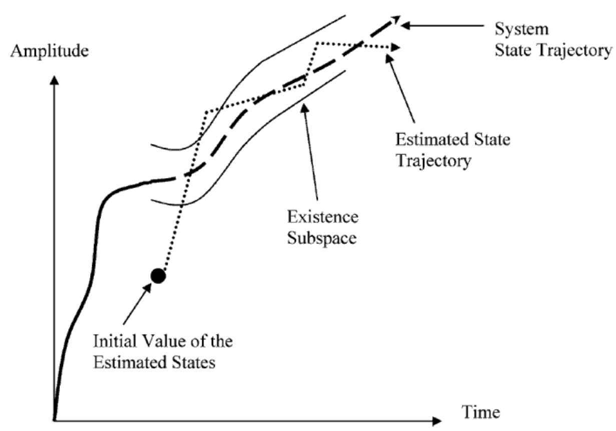

Fig. 1. The smooth variable structure filter estimation concept is shown in the above figure **[10]**.

The SVSF method is model based and applies to smooth nonlinear dynamic equations. The estimation process is iterative and may be summarized by the following set of equations (for a linear control or estimation problem). Like the KF, the system model is used to calculate a priori state and measurement estimates. A corrective term, referred to as the SVSF gain, is calculated as a function of the error in the predicted output and a smoothing boundary layer. This gain is then used to update the state estimate. The estimation process is stable due to the gain calculation of (28). Furthermore, the switching found within the existence subspace is smoothed out by using the saturation term of (28), which is defined by the a priori output error and some predetermined boundary layer width.

$$\hat{x}_{k+1|k} = \hat{A}\hat{x}_{k|k} + \hat{B}u_k \tag{25}$$

$$\hat{z}_{k+1|k} = \hat{H}\hat{x}_{k+1|k} \tag{26}$$

$$e_{z_{k+1|k}} = z_{k+1} - \hat{z}_{k+1|k} \tag{27}$$

$$K_{k+1} = \left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right|\right) \circ sat\left(\frac{e_{z_{k+1|k}}}{\psi}\right) \tag{28}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \tag{29}$$

$$e_{z_{k+1|k+1}} = z_{k+1} - \hat{H}\hat{x}_{k+1|k+1} \tag{30}$$

Two critical variables in the SVSF estimation process are the a priori and a posteriori output error estimates, defined by (27) and (30), respectively. The estimation process is stable and convergent if the following lemma is satisfied:

$$|e_k| > |e_{k+1}| \tag{31}$$

The proof, as defined in **[10]**, is such that if one defines η_{k+1} as a random but bounded amplitude such that $\eta_{k+1} \le \beta$, then $|e_k| > |e_{k+1}|$ applies to the phase where $|e_k| > \beta$ that is defined by the reachability phase. Let a Lyapunov function be defined such that $v_{k+1} = e_{k+1}^2$. Hence, the estimation process is stable if $(\Delta v_{k+1} = e_{k+1}^2 - e_k^2) < 0$. Furthermore, note that this stability condition is satisfied by (31) **[10,17]**.

COMBINED CUBATURE KALMAN AND SMOOTH VARIABLE STRUCTURE FILTERING STRATEGY

In this section, a new estimation method (referred to as the CKF-SVSF) is discussed, as presented in **[11]**. In 2008, it was proposed that the EKF may be combined with the SVSF for an improved estimation process **[18,19]**. The CKF-SVSF method takes a similar approach to the method of combination. The SVSF provides an estimation process that is sub-optimal albeit stable. It is hence beneficial to be able to combine the accurate performances of the CKF with the stability of the SVSF. As such, the SVSF strategy can be used to force the estimated states to within a boundary and then on, the corrective action being principally transferred to the CKF.

To combine these two methods, the gain of the SVSF needs to be further modified. The new form takes advantage of the orthogonality of the Kalman gain with respect to the actual system states **[7,1]**. The augmented gain includes the addition of a constant positive vector (Π), which is forced in a direction determined by a vector (d_{x_k}). The vector d_{x_k} originates from the a priori state estimates and is orthogonal to the trajectory traced by the actual system states. This vector points towards the state values and the switching surface. The orthogonality condition is satisfied by setting $d_{x_k} = W_{k+1}e_{z_{k+1|k}}$, by virtue of the Kalman derivation **[7]**. Furthermore, one may combine the CKF with the SVSF by setting the boundary layer widths (ψ) to one, and by making the additive term (Π) relatively large with respect to γ. The CKF plays a greater role in the estimation process with a larger additive term with respect to γ **[18]**. It is important to note that when outside of the smoothing boundary layer, the corrective gain is essentially the same as the SVSF gain. While inside the boundary layer, the corrective term is primarily composed of the CKF gain.

The following is a summary of the CKF-SVSF estimation process for nonlinear systems, broken up into two stages (prediction and update).

Prediction equations:

$$\hat{x}_{k+1|k} = \int_{\mathbb{R}^{n_x}} \hat{f}(x_k, u_k)\mathcal{N}(x_k; \hat{x}_{k|k}, P_{k|k})\,dx_k \tag{32}$$

$$P_{k+1|k} = \int_{\mathbb{R}^{n_x}} \hat{f}(x_k, u_k)\hat{f}^T(x_k, u_k)\mathcal{N}(x_k; \hat{x}_{k|k}, P_{k|k})\,dx_k - \hat{x}_{k+1|k}\hat{x}_{k+1|k}^T + Q_k \tag{33}$$

$$\hat{z}_{k+1|k} = \int_{\mathbb{R}^{n_x}} \hat{h}(x_{k+1}, u_{k+1})\mathcal{N}(x_{k+1}; \hat{x}_{k+1|k}, P_{k+1|k})\,dx_{k+1} \tag{34}$$

$$e_{z_{k+1|k}} = z_{k+1} - \hat{z}_{k+1|k} \tag{35}$$

Update equations:

$$P_{xz_{k+1|k}} = \int_{\mathbb{R}^{n_x}} x_{k+1}\hat{h}^T(x_{k+1}, u_{k+1})\mathcal{N}(x_{k+1}; \hat{x}_{k+1|k}, P_{k+1|k})\,dx_{k+1} - \hat{x}_{k+1|k}\hat{z}_{k+1|k}^T \tag{36}$$

$$P_{zz_{k+1|k}} = \int_{\mathbb{R}^{n_x}} \hat{h}(x_{k+1}, u_{k+1})\hat{h}^T(x_{k+1}, u_{k+1})\mathcal{N}(x_{k+1}; \hat{x}_{k+1|k}, P_{k+1|k})\,dx_{k+1} - \hat{z}_{k+1|k}\hat{z}_{k+1|k}^T + R_{k+1} \tag{37}$$

$$W_{k+1} = P_{xz_{k+1|k}}P_{zz_{k+1|k}}^{-1} \tag{38}$$

$$K_{k+1} = diag\left[\left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right| + \Pi\right) \circ sat\left(\frac{W_{k+1}e_{z_{k+1|k}}}{\psi}\right)\right]\left(diag\left[e_{z_{k+1|k}}\right]\right)^{-1} \tag{39}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}e_{z_{k+1|k}} \tag{40}$$

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1}P_{zz_{k+1|k}}K_{k+1}^T \tag{41}$$

$$\hat{z}_{k+1|k+1} = \int_{\mathbb{R}^{n_x}} \hat{h}(x_{k+1}, u_{k+1})\mathcal{N}(x_{k+1}; \hat{x}_{k+1|k+1}, P_{k+1|k+1})\,dx_{k+1} \tag{42}$$

$$e_{z_{k+1|k+1}} = z_{k+1} - \hat{z}_{k+1|k+1} \tag{43}$$

Note that the integrals found within the CKF-SVSF estimation method represent the summation of cubature points used to approximate the state distribution, as suggested by (9).

APPLICATION TO AN EHA SYSTEM

In this section, an electrohydrostatic actuator (EHA) is simulated. The EHA to be simulated is based on an actual prototype built for experimentation **[10,20]**. The purpose of this simulation is to demonstrate that the combined estimation process (CKF-SVSF) yields a very accurate estimate, without negatively impacting its stability to modeling errors or uncertainties.

The EHA is a third order (typically linear) system with state variables related to its position, velocity, and acceleration. It is assumed that all three states have measurements associated with them (i.e., full measurement matrix). The input to the system is a random normal distribution with magnitude 1. The sample time of the system is 0.001 seconds. The entire EHA system description may be found in **[20]**. However, for the purpose of this paper, three states (kinematic information) and one parameter (bulk modulus) will be estimated. The estimation of the parameter creates a nonlinear estimation problem. The system model equations are defined as follows:

$$\begin{gathered} x_{1_{k+1}} = x_{1_k} + T x_{2_k} \\ x_{2_{k+1}} = x_{2_k} + T x_{3_k} \\ x_{3_{k+1}} = \left(1 - T\varphi_3 - T\varphi_2 x_{4_k}\right) x_{3_k} - T\varphi_1 x_{2_k} \\ + G_E T x_{4_k} u_k \\ x_{4_{k+1}} = x_{4_k} \end{gathered} \tag{44}$$

Where:

$$\begin{gathered} G_E = 2 D_p A_E / M V_0 \\ \varphi_1 = 2 A_E^2 / M V_0 \\ \varphi_2 = L / V_0 \\ \varphi_3 = B_E / M \end{gathered} \tag{45}$$

The initial state values are set to zero. The initial true bulk modulus is set to $2 \times 10^8\ Pa$, whereas the corresponding initial estimate is $1.5 \times 10^8\ Pa$. Half-way through the simulation the true bulk modulus is changed. The system and measurement noises are considered to be Gaussian with maximum amplitude corresponding to $W_{Max} = [0.0001 \quad 0.001 \quad 0.01]^T$ and $V_{Max} = [0.01 \quad 0.01 \quad 0.01]^T$ for the states. The initial state error covariance, system noise covariance, and measurement noise covariance are defined respectively as follows:

$$P_{0|0} = 10Q \tag{46}$$

$$Q = 5 W_{Max} W_{Max}^T \tag{47}$$

$$R = 5 V_{Max} V_{Max}^T \tag{48}$$

For the SVSF estimation process, the 'memory' or convergence rate was set to $\gamma = 0.1$, and the boundary layer widths were defined as $\psi = [0.1 \quad 10 \quad 50]^T$. These parameters were tuned by trial-and-error, with the goal of decreasing the estimation error. The main results of applying the EKF, CKF, SVSF, and the CKF-SVSF on the EHA problem are shown in the following sets of figures. The first figure shows the true position of the EHA with the corresponding estimates. For the position, the estimation results of both filters are relatively the same. The velocity and acceleration estimates were relatively the same as those shown in the following figure (and were thus omitted for space constraints).

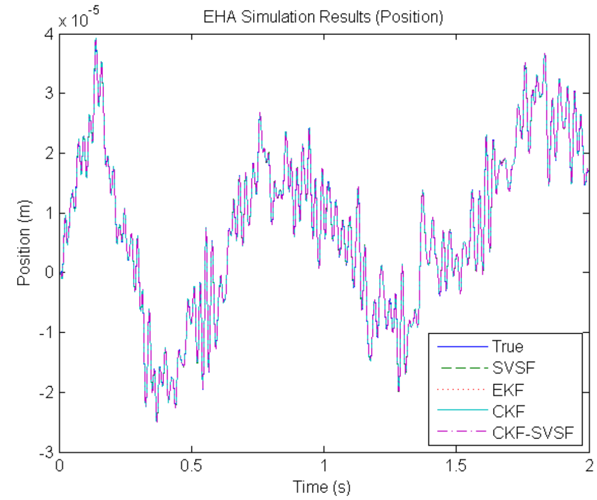

Fig. 2. The position estimates for the EHA simulation are shown in the above figure. Note that the lines are nearly concentric and thus are relatively difficult to distinguish.

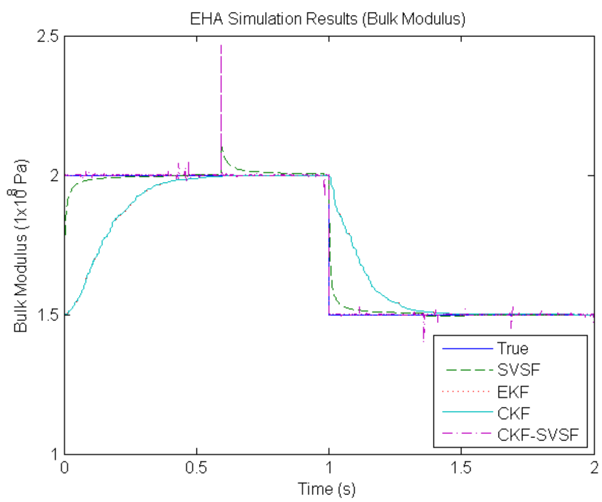

Fig. 3. The bulk modulus estimates for the EHA simulation are shown in the above figure. Note that the EKF and CKF estimates are nearly the same and thus are relatively difficult to distinguish.

The RMSE results of running the simulation are as follows:

Table 1. RMSE Simulation Results

Filter	Position (m)	Velocity (m/s)	Accel. (m/s^2)	Bulk Modulus (Pa)
SVSF	1.095×10^{-7}	1.626×10^{-5}	4.742×10^{-4}	0.0314
EKF	1.304×10^{-7}	5.348×10^{-5}	4.524×10^{-6}	0.1457
CKF	1.301×10^{-7}	5.317×10^{-5}	4.497×10^{-6}	0.1451
CKF-SVSF	7.931×10^{-8}	4.827×10^{-6}	5.558×10^{-5}	0.0194

As shown in the above table, the CKF-SVSF provides the best overall result in terms of estimation accuracy and rate of convergence. The three state estimates were relatively the same for all of the filters. A notable difference occurs when attempting to estimate the parameter. The convergence rate is clearly shown, where the EKF and CKF are the slowest, and the combined method is the fastest. Note that the EKF and CKF yield the same solution for this estimation problem, as it is only mildly nonlinear. The robustness of the combined method is shown when modeling errors are introduced into the simulation. The results of increasing the load mass by 10 times after half-a-second are shown in the following sets of figures.

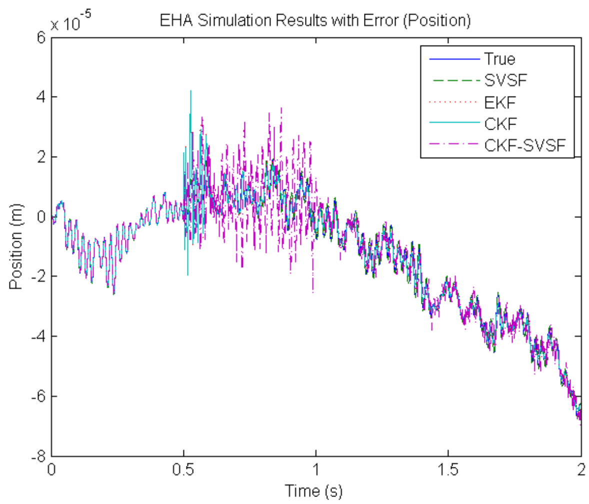

Fig. 4. The position estimates for the EHA simulation with modeling error are shown in the above figure.

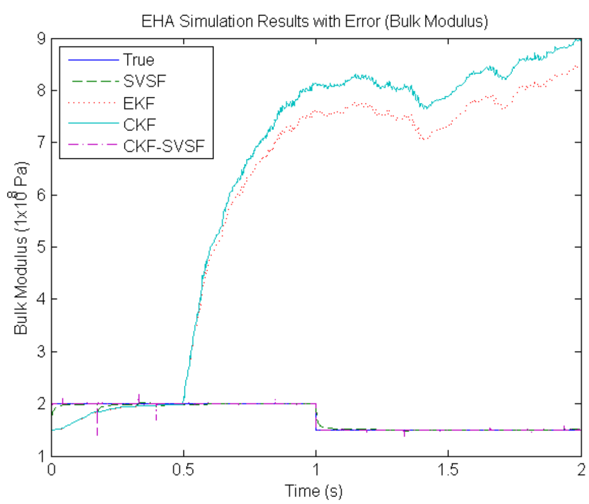

Fig. 5. The bulk modulus estimates for the EHA simulation with modeling error are shown in the above figure.

The effect of changing the mass used by the filters in the estimation process is clearly shown at 0.5 seconds. Figure 5 shows that the EKF and CKF both fail to provide a good estimate of the bulk modulus. However, the SVSF and CKF-SVSF still provide very good estimates of the parameter. This example demonstrates the robustness of the combined method due to the boundary layer concepts of the SVSF. The SVSF remains bounded to within a region of the true state trajectory, thus minimizing the effects of the modeling error. Furthermore, the addition of the CKF increases the estimation accuracy, which makes for an attractive filter (CKF-SVSF).

CONCLUSIONS

This paper discussed the results of applying a variety of filters for state and parameter estimation on an EHA. The combined estimation strategy was applied and compared with the standard EKF, CKF, and SVSF. For the normal case, it was found that all of the filters performed relatively the same in terms of state estimation (the system was only mildly nonlinear). However, both the SVSF and CKF-SVSF were able to determine the parameter with higher estimation accuracy. When a modeling error was introduced, the EKF and CKF methods both failed to accurately represent the bulk modulus of the system. The boundary layers introduced by the SVSF created a more robust estimation process. Typically a trade-off exists between estimation accuracy and robustness to modeling uncertainties and errors. However, the combined method (CKF-SVSF) was able to maintain a high-level of accuracy while remaining robust to errors.

NOMENCLATURE

The following is a list of the nomenclature used throughout this paper.

Table 2. List of Nomenclature

Parameter	Definition
x	State vector or values
z	Measurement (system output) vector or values
w	System noise vector
v	Measurement noise vector
A	Linear system transition matrix
B	Input gain matrix
H	Linear measurement (output) matrix
K	Filter gain matrix (i.e., KF or SVSF)
P	State error covariance matrix
P_{xz}	Covariance matrix between x and z
P_{zz}	Covariance matrix between z and z
Q	System noise covariance matrix
R	Measurement noise covariance matrix
S	Innovation covariance matrix
W	CKF update weight or gain
X	Array of cubature points
e	Measurement (output) error vector
$diag(a)$	Defines a diagonal matrix of some vector a
$sat(a)$	Defines a saturation of the term a
$\mathcal{N}(\mu, \sigma)$	Normal distribution with mean μ and variance σ
ξ	Defined as a cubature point
γ	SVSF 'convergence' or memory parameter

ψ	SVSF boundary layer width
$\|a\|$	Absolute value of some parameter a
T	Transpose of some vector or matrix
$\sim$	Denotes error or difference
^	Estimated vector or values
$k+1\|k$	A priori time step (i.e., before applied gain)
$k+1\|k+1$	A posteriori time step (i.e., after update)
A_E	Piston area ($5.05 \times 10^{-4}\ m^2$)
B_E	Load friction ($760\ Ns/m$)
D_p	Pump displacement ($1.69 \times 10^{-7}\ m^3/rad$)
K_a	Back EMF constant ($6980\ rad \cdot s/m$)
L	Leakage coefficient ($2.5 \times 10^{-11}\ m^3/Pa \cdot s$)
M	Load mass ($20\ kg$)
V_0	Chamber volume ($6.85 \times 10^{-5}\ m^3$)
β_e	Effective bulk modulus ($2 \times 10^8\ Pa$)

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