

A Novel Interacting Multiple Model Method for Nonlinear Target Tracking

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Abstract – The state estimation of targets is a difficult task, particularly if the target exhibits nonlinear behaviour, which is often the case. Currently, the most popular filters used in target tracking are the Kalman filter (KF) and its various forms, as well as the particle filter (PF). Introduced in 2007, the smooth variable structure filter (SVSF) is a relatively new predictor-corrector method based on sliding mode estimation. In the past, this filter has been used successfully for the state and parameter estimation of mechanical and electrical systems for the purpose of control. This paper introduces a new interacting multiple model (IMM) method that makes use of the SVSF estimation strategy. An air traffic control (ATC) problem is used to compare the common EKF-IMM with the proposed SVSF-IMM in terms of tracking accuracy, robustness, and computational complexity. Furthermore, this paper demonstrates that the SVSF is an effective method for nonlinear target tracking.

Keywords: Target tracking, interacting multiple models, Kalman filtering, smooth variable structure filter, estimation.

1 Introduction

Nonlinear target tracking is of particular interest for both civilian and military applications. Both of these groups are concerned with the surveillance, guidance, obstacle avoidance, or tracking of a target which exhibits nonlinear behaviour given some measurements [1]. A sensor such as a radar is used within the environment to obtain information on the system of interest. Typically these measurements are noisy, such that a filter must be used to remove unwanted noise or uncertainties, and estimate the true state. The target state usually consists of kinematic information such as position, velocity, and acceleration. The ability to obtain accurate estimates of these states is critically important for civilian (i.e., air traffic control) and military (i.e., air defense) groups so that informed

decisions may be made based on knowledge of a targets' trajectory or future flight path. Estimation theory encompasses a variety of different methods and techniques that may be applied on linear or nonlinear systems in order to smooth, estimate, or predict the values of important states and parameters [2].

The most common and well-studied estimation technique is the Kalman filter (KF) [2,3,4]. In 1960, Kalman introduced a method that may be applied to linear dynamic systems in the presence of Gaussian noise [4]. It provides an elegant and statistically optimal solution by minimizing the mean-squared estimation error (i.e., difference between the true state values and the estimated values). However, in practice, most dynamic systems are in fact nonlinear. Since the introduction of the KF for linear systems, a variety of derivations have been created to handle nonlinear systems. The two most common are the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). The EKF makes use of the first-order linearization (i.e. Jacobian) of the nonlinear system and measurement equations. Although this method augments the standard KF to work for nonlinear systems, it comes at a cost of higher-order un-modelled uncertainties [2]. The UKF is able to handle a higher order of the nonlinearities by using a set of deterministically chosen sample points (i.e., sigma points) which are applied on a transformation such that it captures the true mean and covariance up to the second order of nonlinearity [1,5]. A great deal of research has been published on these filters, including on their robustness and numerical stability [6,7,8,9,10,11].

Another popular estimation technique that has also been well studied is the particle filter (PF). The PF takes the Bayesian approach to dynamic state estimation, in which one attempts to accurately represent the probability distribution function (PDF) of the values of interest [1]. Like the name suggests, the PF makes use of weighted particles or 'point masses' distributed in a manner that estimates the PDF, which contains all of the pertinent

statistical information, and may be considered as holding the solution to the estimation problem [3]. In the literature, it is shown that the PF is typically more accurate than the UKF, however at a cost of computation time, which is dependent on the number of particles [1]. Hence, a trade-off exists between the level of accuracy required by the problem, and the computational time available.

The smooth variable structure filter (SVSF) is a new predictor-corrector method used for state and parameter estimation [12,13,14]. It is based on sliding mode concepts, where one uses a sliding or switching surface along the state trajectory to predict the state estimates. A corrective term (based on a function of the measurement errors) is used to keep the estimate within a region of the true state values. In the past, this method has typically been used for the control of mechanical and electrical systems [14,15]. Although the SVSF method is not optimal, it is very robust and works extremely well for systems when not all of the dynamics are known. This paper demonstrates that the SVSF may be a viable estimation method for nonlinear target tracking applications.

It is quite common for target tracking applications to implement multiple model methods with estimation techniques to increase the overall accuracy. When tracking an aircraft, for example, it may be assumed that the system behaves according to one of a finite number of models (i.e., uniform motion, coordinated turn, etc.) [3]. In the interacting multiple model (IMM) method, the state estimate may be computed using each possible model, with each filter using a different combination of the previous model-conditioned estimates [3]. These filters then interact with each other based on some probability of the system model being correct. This paper introduces a new IMM method based on the SVSF (i.e., SVSF-IMM) and discusses the improvement that it brings when compared with the popular EKF-IMM.

2 Estimation Methods

The Kalman filter (KF) can quite easily be referred to as the ‘workhorse’ of estimation. It is the most studied and implemented method for target tracking and other estimation problems. For the purpose of this paper, and due to page constraints, only the results of the EKF-IMM will be compared with the SVSF-IMM.

2.1 Kalman Filter

As previously mentioned, the KF provides an elegant and statistically optimal solution for linear dynamic systems in the presence of Gaussian white noise. It is a method that utilizes measurements linearly related to the states, and error covariance matrices, to generate a corrective term referred to as the Kalman gain. This gain is applied to the a priori state estimate, thus creating an a posteriori estimate. The estimation process continues in a predictor-corrector fashion while maintaining a statistically minimal state error covariance matrix for linear systems.

The following two equations describe a linear system dynamic and measurement model used in general for state estimation:

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (1)$$

$$z_{k+1} = Hx_{k+1} + v_{k+1} \quad (2)$$

The next five equations form the core of the KF algorithm, and are used in an iterative fashion, in conjunction with the linear form of (1) and (2). Equation (3) extrapolates the a priori state estimate and (4) is the corresponding error covariance. The Kalman gain may be calculated by (5), and is used to update the state estimate and error covariance, described by (6) and (7), respectively.

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + \hat{B}u_k \quad (3)$$

$$P_{k+1|k} = HP_{k|k}H^T + Q_k \quad (4)$$

$$K_{k+1} = P_{k+1|k}H^T[HP_{k+1|k}H^T + R_{k+1}]^{-1} \quad (5)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}[z_{k+1} - H\hat{x}_{k+1|k}] \quad (6)$$

$$P_{k+1|k+1} = [I - K_{k+1}H]P_{k+1|k} \quad (7)$$

The effects due to mismodeling can be negative, as both the Kalman gain and covariance matrix calculations are dependent on the system and measurement matrices. Furthermore, the performance and stability of the KF may also be dependent on the definition of the process and measurement noise, made through covariance matrices [2,12]. Overlooked nonlinearities in the system may also cause the KF to become unstable. The EKF may be used for nonlinear systems (as described by the first two equations). It is conceptually similar to the iterative KF process, described above. The nonlinear system and measurement matrices are linearized according to its corresponding Jacobian, which is a first-order partial derivative. This linearization can sometimes cause instabilities when implementing the EKF [2].

2.2 Smooth Variable Structure Filter

Sliding mode control and estimation techniques have been around for quite a few decades, and are mainly popular due to their relative ease of implementation and robustness to modelling uncertainties [16,17]. In a typical sliding mode control scenario, one utilizes a discontinuous switching plane along some desired trajectory [18]. This plane is quite often referred to as a sliding surface. The purpose of the control method is to keep the state values along this surface by minimizing the state errors (i.e., difference between the desired trajectory and the estimated or actual values). If the value of the state is not on or away from the sliding surface, a corrective term is used to push the state towards the surface. The motion of the system as the states slide along the surface is called a sliding mode [18]. The switching term introduces an inherent amount of stability to the control or estimation strategy, while also introducing excessive chattering (i.e., high-frequency switching) which may be undesirable in

control since it may excite un-modelled dynamics. It is regular practice to introduce a boundary layer along the sliding surface in order to saturate or smooth-out the chattering within this region. These concepts are based on variable structure control, in which one alters the nonlinear dynamics of a system by high-frequency switching [16].

The original variable structure filter (VSF) was first proposed in 2003 [13]. It was a new type of estimator based on sliding mode concepts, used in a predictor-corrector fashion. Gain switching is incorporated to ensure that the estimates converge to within a boundary of the true state values. This boundary is referred to as the existence subspace. An estimate of the state is predicted using a linear or nonlinear model of the system. A corrective term (i.e., gain) is applied to update the estimate. This approach is similar to the KF, however the actual structure and development of the gain is very different. The smooth variable structure filter (SVSF) was later derived from the VSF, and uses a much simpler and less complex gain calculation [14]. In its present form, the SVSF is stable and robust to modelling uncertainties and noise, given an upper bound on the level of un-modeled dynamics or knowledge of the magnitude of noise [14,19]. The basic estimation concept of the SVSF is shown in the following figure.

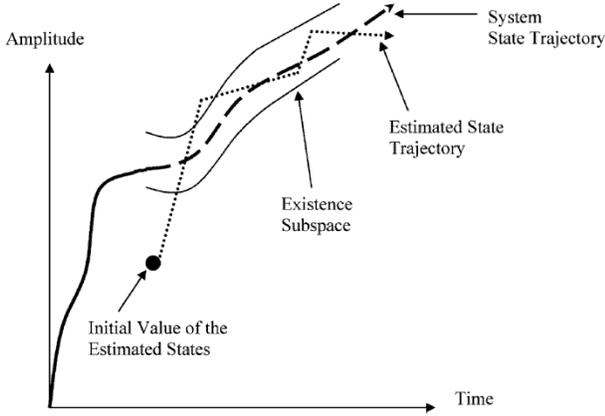


Figure 1. SVSF Estimation Concept [14]

An initial estimate of the state is made based on probability distributions or designer knowledge. An area around the true system state trajectory is defined as the existence subspace, and is a function of the uncertainties (i.e., level of noise) present in the system. Through the use of the SVSF gain, the estimated state will be forced to within this region. Once the value enters the existence subspace, the estimated state is forced into switching along the system state trajectory. A saturation term may be used in this region to reduce the magnitude of chattering or smooth-out the result. Typically the existence subspace or boundary layer width is defined larger than it should be (i.e., conservative). A variable boundary layer may be introduced such that a more accurate estimate of the state is made over time [18].

The SVSF method is model based and applies to smooth nonlinear dynamic equations. The main estimation process may be summarized by (8) to (12), and is repeated iteratively. An a priori state estimate is calculated using an estimated model of the system. This value is then used to calculate an a priori estimate of the measurement, defined by (9). A corrective term, referred to as the SVSF gain, is calculated as a function of the error in the predicted output, as well as a gain matrix and the smoothing boundary layer width. The corrective term calculated in (11) is then used in (12) to find the a posteriori state estimate.

$$\hat{x}_{k+1|k} = \hat{f}(\hat{x}_{k|k}, u_k) \quad (8)$$

$$\hat{z}_{k+1|k} = \hat{h}(\hat{x}_{k+1|k}) \quad (9)$$

$$e_{z_{k+1|k}} = z_{k+1} - \hat{z}_{k+1|k} \quad (10)$$

$$K_{k+1} = \left(|e_{z_{k+1|k}}| + \gamma |e_{z_{k|k}}| \right) \circ \text{sat} \left(\frac{e_{z_{k+1|k}}}{\psi} \right) \quad (11)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \quad (12)$$

$$e_{z_{k+1|k+1}} = z_{k+1} - \hat{h}(\hat{x}_{k+1|k+1}) \quad (13)$$

Two critical variables in this process are the a priori and a posteriori output error estimates, defined by (10) and (12), respectively [14]. Note that (13) is the updated output error estimate, and is used only in the gain calculation at the next time step.

3 Proposed SVSF-IMM Strategy

The SVSF provides an estimation process that is sub-optimal albeit stable. Therefore, utilizing a multiple model strategy which increases the overall accuracy of the estimation process is beneficial. However, it is important to note that the SVSF in its current form cannot be integrated with the IMM strategy. The standard IMM utilizes covariance outputs from the filter. However, the SVSF as introduced in [14] does not have a covariance derivation.

A revised form of the SVSF was introduced in [5] such that a simplified covariance matrix may be obtained. It was proposed that the SVSF gain and update estimate be modified respectively as follows:

$$K_{k+1} = \text{diag} \left[\left(|e_{z_{k+1|k}}| + \gamma |e_{z_{k|k}}| \right) \circ \text{sat} \left(\frac{e_{z_{k+1|k}}}{\psi} \right) \right] \left[\text{diag} (e_{z_{k+1|k}}) \right]^{-1} \quad (14)$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z_{k+1|k}} \quad (15)$$

The SVSF process essentially remains the same, as one divides the gain by the a priori output error, and then multiplies by it again in the a posteriori estimate equation. However, introducing the SVSF in this notation enables one to derive a much simpler covariance derivation. Furthermore, the proof of stability for the SVSF is not affected [5,14]. It is interesting to note that the a priori and a posteriori state error covariance matrices for the SVSF are similar to the KF for linear systems.

The corresponding covariance equations for the prediction and update stages are respectively defined as follows [5]:

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \quad (16)$$

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k}(I - K_{k+1}H)^T + K_{k+1}R_{k+1}K_{k+1}^T \quad (17)$$

Note that the linearized system (A) and measurement (H) matrices would be used for the above calculations.

In this paper, the IMM was implemented as per section 11.6 of [3]. The IMM concept is shown in the following figure. Essentially, both the EKF and SVSF estimation strategies may be applied on two models (M_1 and M_2). Prior to feeding the initial estimates and covariance's into the filter models, an interaction (mixing) stage takes place (as per (18) to (21)). Mixing probabilities are used like weights to determine the corresponding estimates and covariance. Using these values and the measurement as inputs, the filter calculates the corresponding estimates, covariance, and likelihood function. These likelihood functions (i.e., the probability density function of the measurements based on the innovation defined by (22)) are used to determine updates to the mode probability and mixing calculations ((23) and (24)). Furthermore, the two sets of state estimates and covariance's may be combined (for output purposes only) to determine the IMM estimate of the respective filter (i.e., EKF or SVSF), as per (25) and (26) [3].

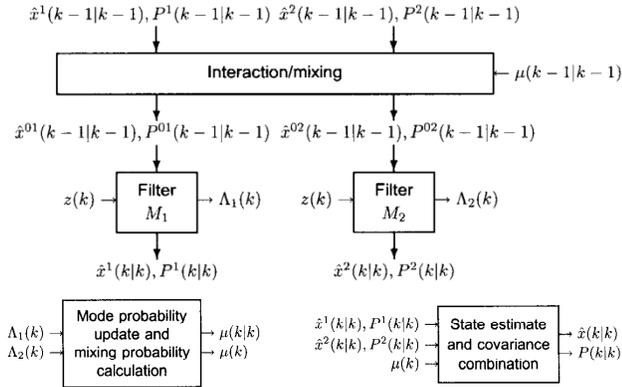


Figure 2. Interacting Multiple Model Concept [3]

The main IMM process and equations, as presented in [3], are defined as follows.

Mixing Stage:

$$\mu_{i|j|k|k} = \frac{1}{\bar{c}_j} p_{ij} \mu_{i_k} \quad i, j = 1, \dots, r \quad (18)$$

$$\bar{c}_j = \sum_{i=1}^r p_{ij} \mu_{i_k} \quad j = 1, \dots, r \quad (19)$$

$$\hat{x}_{k|k}^{0j} = \sum_{i=1}^r \hat{x}_{k|k}^i \mu_{i|j|k|k} \quad j = 1, \dots, r \quad (20)$$

$$P_{k|k}^{0j} = \sum_{i=1}^r \mu_{i|j|k|k} \left\{ P_{k|k}^i + [\hat{x}_{k|k}^i - \hat{x}_{k|k}^{0j}] [\hat{x}_{k|k}^i - \hat{x}_{k|k}^{0j}]^T \right\} \quad j = 1, \dots, r \quad (21)$$

Mode-Matched Filtering and Update (EKF or SVSF):

$$\Lambda_{j_{k+1}} = \mathcal{N} \left[z_{k+1}; \hat{z}_{k+1|k}^j \left(\hat{x}_{k|k}^{0j} \right), S_{k+1}^j \left(P_{k|k}^{0j} \right) \right] \quad j = 1, \dots, r \quad (22)$$

$$\mu_{j_{k+1}} = \frac{1}{c} \Lambda_{j_{k+1}} \bar{c}_j \quad j = 1, \dots, r \quad (23)$$

$$c = \sum_{j=1}^r \Lambda_{j_{k+1}} \bar{c}_j \quad (24)$$

Estimate and Covariance Combination:

$$\hat{x}_{k+1|k+1} = \sum_{j=1}^r \hat{x}_{k+1|k+1}^j \mu_{j_{k+1}} \quad (25)$$

$$P_{k+1|k+1} = \sum_{j=1}^r \mu_{j_{k+1}} \left\{ P_{k+1|k+1}^j + [\hat{x}_{k+1|k+1}^j - \hat{x}_{k+1|k+1}] [\hat{x}_{k+1|k+1}^j - \hat{x}_{k+1|k+1}]^T \right\} \quad (26)$$

The proposed SVSF-IMM estimation strategy utilizes (8) to (17) in conjunction with (18) to (26).

4 ATC Target Tracking Problem

The problem to be studied involves an air traffic control (ATC) situation, where one is interested in tracking the trajectory of an aircraft, as introduced in [3].

4.1 Simulation

A sensor (i.e., radar) stationed at the origin provides direct position only measurements, with a standard deviation of 100 m in each coordinate. As shown in the following figure, an aircraft starts from an initial position of [25,000 m, 10,000 m] at time $t = 0$ s, and flies westward at 120 m/s for 125 s. The aircraft then begins a coordinated turn for a period of 90 s at a rate of $1^\circ/s$. It then flies southward at 120 m/s for 125 s, followed by another coordinated turn for 30 s at $3^\circ/s$. The aircraft then continues to fly westward until it reaches its final destination.

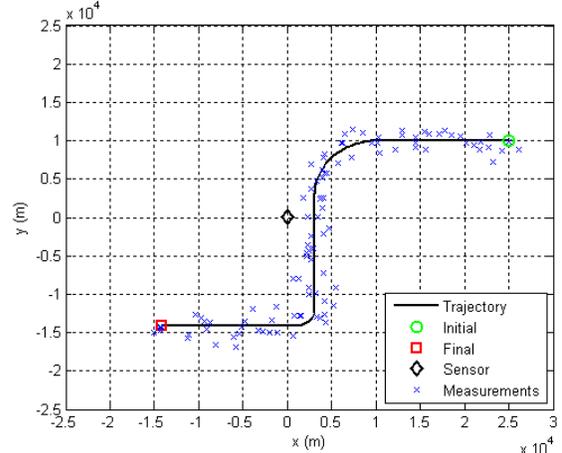


Figure 3. Aircraft Trajectory

In ATC scenarios, the behaviour of civilian aircraft may be modeled by two different modes: uniform motion (UM) which involves a straight flight path with a constant speed and course, and maneuvering which includes turning or climbing and descending [3]. In this case, maneuvering will refer to a coordinated turn (CT) model, where a turn is made at a constant turn rate and speed. The uniform motion model used for this target tracking problem is given as follows [3,20]:

$$x_{k+1} = \begin{bmatrix} 1 & 0 & \tau & 0 \\ 0 & 1 & 0 & \tau \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}\tau^2 & 0 \\ 0 & \frac{1}{2}\tau^2 \\ \tau & 0 \\ 0 & \tau \end{bmatrix} w_k \quad (27)$$

The state vector of the aircraft may be defined as follows:

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k]^T \quad (28)$$

The first two states refer to the position along the x and y-axis, respectively, and the last two states refer to the velocity along the x and y-axis, respectively. The sampling time used in this simulation was 5 seconds. When using the CT model, the state vector needs to be augmented to include the turn rate, as shown in (29). The CT model may be considered nonlinear if the turn rate of the aircraft is not known. Note that a left turn corresponds to a positive turn rate, and a right turn has a negative turn rate. This sign convention follows the commonly used trigonometric convention (the opposite is true for navigation convention) [3]. The CT model is then given as follows [3,20]:

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k \quad \Omega_k]^T \quad (29)$$

$$x_{k+1} = \begin{bmatrix} 1 & 0 & \frac{\sin \Omega_k \tau}{\Omega_k} & -\frac{1 - \cos \Omega_k \tau}{\Omega_k} & 0 \\ 0 & 1 & \frac{1 - \cos \Omega_k \tau}{\Omega_k} & \frac{\sin \Omega_k \tau}{\Omega_k} & 0 \\ 0 & 0 & \cos \Omega_k \tau & -\sin \Omega_k \tau & 0 \\ 0 & 0 & \sin \Omega_k \tau & \cos \Omega_k \tau & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}\tau^2 & 0 & 0 \\ 0 & \frac{1}{2}\tau^2 & 0 \\ \tau & 0 & 0 \\ 0 & \tau & 0 \\ 0 & 0 & \tau \end{bmatrix} w_k \quad (30)$$

Since the radar stationed at the origin provides direct position measurements only, the measurement equation may be formed linearly as follows:

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} x_k + v_k \quad (31)$$

Equations (27) to (31) were used to generate the true state values of the trajectory and the radar measurements for this target tracking scenario. As previously mentioned, the EKF uses a linearized form of the system and measurement matrices. In this case, the system defined above is nonlinear, such that the Jacobian of it yields a

linearized form as shown below. The terms in the last column of (32) are correspondingly defined in (33).

$$[\nabla_x A_{k,x}^T]_{x_k=\hat{x}_k}^T = \begin{bmatrix} 1 & 0 & \frac{\sin \hat{\Omega}_k \tau}{\hat{\Omega}_k} & -\frac{1 - \cos \hat{\Omega}_k \tau}{\hat{\Omega}_k} & A_{\hat{\Omega}_1} \\ 0 & 1 & \frac{1 - \cos \hat{\Omega}_k \tau}{\hat{\Omega}_k} & \frac{\sin \hat{\Omega}_k \tau}{\hat{\Omega}_k} & A_{\hat{\Omega}_2} \\ 0 & 0 & \cos \hat{\Omega}_k \tau & -\sin \hat{\Omega}_k \tau & A_{\hat{\Omega}_3} \\ 0 & 0 & \sin \hat{\Omega}_k \tau & \cos \hat{\Omega}_k \tau & A_{\hat{\Omega}_4} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} A_{\hat{\Omega}_1} \\ A_{\hat{\Omega}_2} \\ A_{\hat{\Omega}_3} \\ A_{\hat{\Omega}_4} \end{bmatrix} = \begin{bmatrix} \frac{(\cos \hat{\Omega}_k \tau) \tau \hat{\xi}_k}{\hat{\Omega}_k} - \frac{(\sin \hat{\Omega}_k \tau) \hat{\xi}_k}{\hat{\Omega}_k^2} - \frac{(\sin \hat{\Omega}_k \tau) \tau \hat{\eta}_k}{\hat{\Omega}_k} - \frac{(-1 + \cos \hat{\Omega}_k \tau) \hat{\eta}_k}{\hat{\Omega}_k^2} \\ \frac{(\sin \hat{\Omega}_k \tau) \tau \hat{\xi}_k}{\hat{\Omega}_k} - \frac{(1 - \cos \hat{\Omega}_k \tau) \hat{\xi}_k}{\hat{\Omega}_k^2} - \frac{(\cos \hat{\Omega}_k \tau) \tau \hat{\eta}_k}{\hat{\Omega}_k} - \frac{(\sin \hat{\Omega}_k \tau) \hat{\eta}_k}{\hat{\Omega}_k^2} \\ -(\sin \hat{\Omega}_k \tau) \tau \hat{\xi}_k - (\cos \hat{\Omega}_k \tau) \tau \hat{\eta}_k \\ (\cos \hat{\Omega}_k \tau) \tau \hat{\xi}_k - (\sin \hat{\Omega}_k \tau) \tau \hat{\eta}_k \end{bmatrix} \quad (33)$$

To generate the results for this section, the EKF used the following values for P , Q , and R :

$$P = \text{diag}([R_{11} \quad R_{22} \quad 100 \quad 100 \quad 1]) \quad (34)$$

$$Q = L_1 \begin{bmatrix} \frac{\tau^3}{3} & 0 & \frac{\tau^2}{2} & 0 & 0 \\ 0 & \frac{\tau^3}{3} & 0 & \frac{\tau^2}{2} & 0 \\ \frac{\tau^2}{2} & 0 & \tau & 0 & 0 \\ 0 & \frac{\tau^2}{2} & 0 & \tau & 0 \\ 0 & 0 & 0 & 0 & \tau \frac{L_2}{L_1} \end{bmatrix} \quad (35)$$

$$R = 100^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (36)$$

Note that L_1 and L_2 are the power spectral densities, and were defined as 0.16 and 0.01, respectively [20]. The system and measurement noise were generated using their respective covariance values. Furthermore, when obtaining the results for the SVSF, a ‘memory’ of $\gamma = 0.1$ was used. The boundary layer thickness (which is a measure of the uncertainties) was tuned by trial-and-error (with the goal of decreasing the estimation error), and was defined as $\psi = [20 \quad 2,500 \quad 20 \quad 2,500 \quad 1]^T$.

4.2 Results

The results of the simulation are shown in the following sets of figures. The first figure shows the general estimation results, with the EKF-IMM and SVSF-IMM estimates of the trajectory. Note that the estimates appear nearly concentric, however the EKF-IMM slightly deviates from the true trajectory at the second turn. The second figure shows the RMSE for the estimated position (i.e., x- and y-positions combined) over the simulation period. The final two figures display the operating mode probability (for the first model, i.e., uniform motion) for both the EKF-IMM and SVSF-IMM.

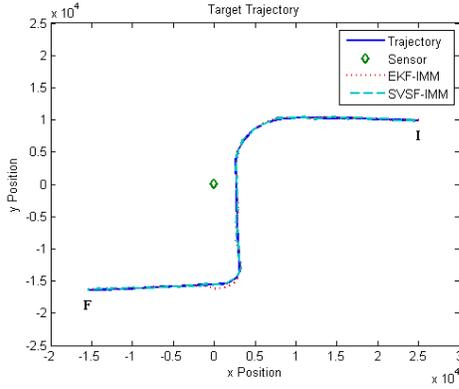


Figure 4. Estimation Results

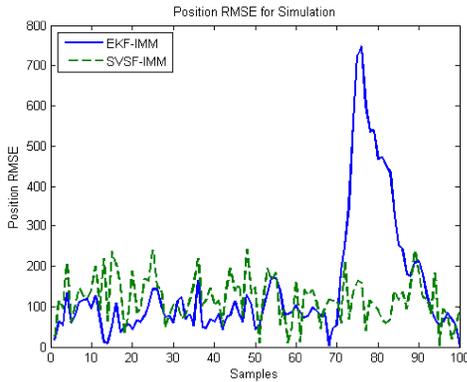


Figure 5. Position RMSE for Simulation

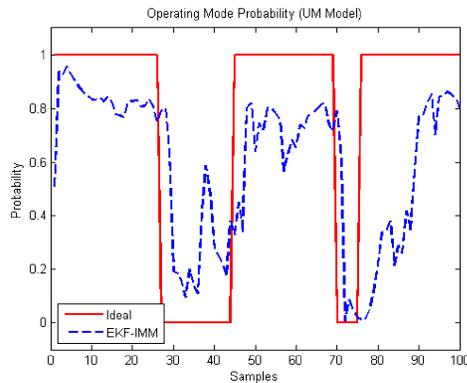


Figure 6. EKF-IMM Operating Mode (UM)

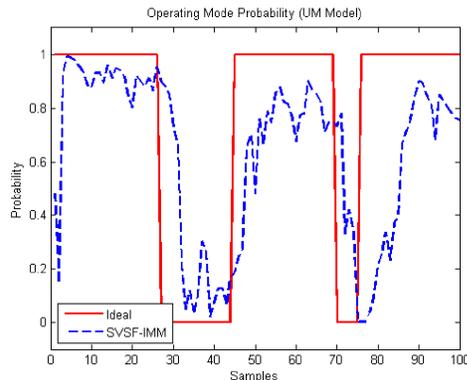


Figure 7. SVSF-IMM Operating Mode (UM)

The averaged RMSE results are shown in the following table. In terms of position RMSE, the SVSF-IMM performed the best, whereas the EKF-IMM performed better when estimating the velocity.

Table 1. RMSE Results

Strategy	Position	Velocity	Turn Rate
EKF-IMM	192.9	23.2	0.035
SVSF-IMM	131.2	28.9	0.004

5 Discussion

For this simulation, the results demonstrate that the proposed SVSF-IMM strategy may be used to obtain a better position estimate of the target (by roughly 30%), when compared with the popular EKF-IMM. It is interesting to note that the position error (shown in Fig. 5) for the combined method remained bounded, unlike the EKF-IMM strategy. Around the 70th to 85th sample interval, the EKF-IMM position error grew significantly before returning to within acceptable levels. This discrepancy can also be seen by looking carefully at the second turn of Fig. 4. The astute reader will observe that the EKF-IMM estimate slightly drifted away from the true trajectory during this period.

The reason why the SVSF-IMM method did not drift away from the true trajectory is due to the switching brought into the estimation process by the SVSF. As shown in Fig. 1, the gain formulation of (11) contains a boundary layer (defined by the existence subspace or ψ). This boundary layer brings about an inherent amount of stability, ensuring that the estimate stays within some region (defined by the amount of uncertainties within the system) close to the true state trajectory. Further to Fig. 5, although the SVSF in its current form is not optimal, it adds stability of the estimate (shown by the error being bounded). Note that the overall position RMSE still remains lower than the EKF-IMM method.

The operating mode probability for the uniform motion model is shown in Figs. 6 and 7. The EKF-IMM operating model followed the true probability relatively well, with the exception of a spike around the 38th sample. The operating mode for the SVSF-IMM also followed the true or ideal case much like the EKF-IMM. However, it appeared to follow it more closely. This most likely can also be attributed to the SVSF gain. The switching brings about a type of ‘chattering’ about the state trajectory. This chattering enables the filter to follow the true trajectory more closely.

In terms of estimation accuracy, the new SVSF-IMM method yielded an improvement of over 30% in the position estimate. This is a significant improvement, especially when considering the importance to most target tracking scenarios where one is primarily interested in the current and future position of the target. The new method is considered to be just as complex as the EKF-IMM method in terms of computational difficulty and simulation time, however provides a more accurate (overall) and stable estimate.

6 Conclusion

This paper introduced a new interacting multiple model (IMM) method that makes use of the SVSF estimation strategy. An air traffic control (ATC) problem was used to compare this new strategy with the popular EKF-IMM. For this simulation, it was found that the SVSF-IMM yielded a significant improvement in the position estimate (by nearly 30%). Furthermore, by virtue of the SVSF switching gain, it was found that the estimates of the new method were bounded to within a region of the true state trajectory, thereby demonstrating robust performance. Future research will compare this strategy with other IMM methods, applied on real ATC data. Furthermore, the SVSF-IMM method will be implemented on mechanical and electrical systems for fault detection and diagnosis.

Appendix

The following table defines the nomenclature used throughout this paper.

Table 2. List of Nomenclature

Parameter	Definition
x	State vector or values
z	Measurement (system output) vector
w	System noise vector
v	Measurement noise vector
A	Linear system transition matrix
B	Input gain matrix
H	Linear measurement (output) matrix
K	Filter gain matrix (i.e., KF or SVSF)
P	State error covariance matrix
Q	System noise covariance matrix
R	Measurement noise covariance matrix
S	Innovation covariance matrix
e	Measurement (output) error vector
$diag(a)$	Defines a diagonal matrix of some vector a
$sat(a)$	Defines a saturation of the term a
$\mathcal{N}(\mu, \sigma)$	Normal distribution with mean μ and variance σ
τ	Sample time (seconds)
Ω	Turn rate of the target (aircraft)
μ	Mixing weights
p_{ij}	Transition (model) probabilities
Λ	Likelihood function
γ	SVSF 'convergence' or memory parameter
ψ	SVSF boundary layer width
$ a $	Absolute value of some parameter a
\circ	Schur product (i.e., element-by-element multiplication)
\dot{a}	Denotes time derivative of a
T	Transpose of some vector or matrix
\sim	Denotes error or difference
$\hat{}$	Estimated vector or values
$k + 1 k$	A priori time step (i.e., before applied gain)
$k + 1 k + 1$	A posteriori time step (i.e., after update)

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