TARGET TRACKING USING THE SMOOTH VARIABLE STRUCTURE FILTER

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ABSTRACT

This article discusses the application of the smooth variable structure filter (SVSF) on a target tracking problem. The SVSF is a relatively new predictor-corrector method used for state and parameter estimation. It is a sliding mode estimator, where gain switching is used to ensure that the estimates converge to true state values. An internal model of the system, either linear or nonlinear, is used to predict an a priori state estimate. A corrective term is then applied to calculate the a posteriori state estimate, and the estimation process is repeated iteratively. The results of applying this filter on a target tracking problem demonstrate its stability and robustness. Both of these attributes make using the SVSF advantageous over the well-known Kalman and extended Kalman filters. The performances of these algorithms are quantified in terms of robustness, resilience to poor initial conditions and measurement outliers, tracking accuracy and computational complexity.

NOMENCLATURE

A System matrix.
B Input matrix.
C Output matrix.
e State estimation error.
k Time step index.
m Number of measurements.
n Number of states.
P Error covariance matrix.
Q System noise covariance matrix.
R Measurement noise covariance matrix.
Sat Saturation function.
t Simulation time.
u Input.
v Measurement noise.
w System noise.
x System states.
z Measurement output.
ξ Cartesian coordinate (position) along the x-axis.
γ Constant diagonal gain matrix with elements having values between 0 and 1.
η Cartesian coordinate (position) along the y-axis.
Ω Turn rate of the target.
Ψ Smoothing boundary layer.
τ Sampling time.
^ Denotes an estimated value.
~ Denotes an error value.
· On top of a parameter denotes a time derivative.

Furthermore, note that subscript $k+1|k$ refers to an a priori time step and the subscript $k+1|k+1$ refers an a posteriori time step. A superscript of T denotes a matrix transpose.

INTRODUCTION

In the estimation world, even after 50 years, the Kalman filter (KF) method remains the most studied and one of the most popular tools used to estimate states from systems [1-3]. It may be applied on linear dynamic systems in the presence of Gaussian white noise, and provides an elegant and statistically optimal solution by minimizing the mean-squared estimation error. The impact that the KF has had on estimation and control problems is considered to be one of the greatest achievement in estimation theory [2]. For example, the KF may be used to precisely track spacecraft through the solar system, and according to many, was one of the enabling technologies for the modern Space Age [2].

In practice, many systems are in fact nonlinear, such that linear estimation techniques may not be used to provide optimal solutions. However, suboptimal techniques may be applied to handle the nonlinearities. Such techniques include the extended Kalman filter (EKF) and the smooth variable structure filter (SVSF). The EKF is a popular extension of the KF, and is commonly used in target tracking [4]. It uses partial derivatives of the nonlinearities in the state dynamic and measurement models, such that linearized approximations are obtained and then used in the estimation process [2, 4]. The SVSF is a new predictor-corrector method used for state and parameter estimation [5, 6]. It is a type of sliding mode estimator, where gain switching is used to ensure that the estimates converge to true state values.

In target tracking applications, one may be concerned with surveillance, guidance, obstacle avoidance or tracking a target given some measurements [4]. In a typical scenario, sensors provide a signal that is processed and output as a measurement. These measurements are related to the target state, and are typically noise-corrupted observations [4]. The target state usually consists of kinematic information such as position, velocity, and acceleration. The measurements are processed in order to form and maintain tracks, which are a sequence of target state estimates that vary with time [4]. Multiple targets and measurements may yield multiple tracks. Gating and data association techniques help classify the source of measurements, and help associate measurements to the appropriate track [4]. A tracking filter is used in a recursive manner to carry out the target state estimation.

STATE ESTIMATION

State and parameter estimation techniques are quite useful for systems when not all of the dynamics are known. Estimation theory involves finding a value of some parameter of interest, which affects the output of the system, often in the presence of inaccurate or uncertain observations [3]. States are representative of the dynamics of a system. For example, for space vehicles, inertial measuring units may be used to calculate the acceleration. However, since their alignment deteriorates over time, calculating the acceleration by other means (i.e. state estimation) may be desirable [7].

The purpose of estimation, as described by Bar-Shalom et al in [1], can be one of many reasons: determination of planet orbit parameters, statistical inference, aircraft traffic control system (i.e. tracking), use in control plants with uncertainties (i.e. parameter identification or state estimation), determination of model parameters (i.e. system identification), message retrieval from noisy signals (i.e. communication theory), and also signal and image processing. A filter may be used to estimate the state of a dynamic system, whether linear or nonlinear. The word filter is used because when finding the best estimate, one has to filter out the noisy signals or uncertain observations [3]. In this paper, two filters (the commonly used KF/EKF and the relatively new SVSF) are applied to a target tracking problem, and the performances in terms of robustness, stability, and accuracy are compared.

Kalman and Extended Kalman Filters

As previously mentioned, the KF provides an elegant and statistically optimal solution for linear dynamic systems in the presence of Gaussian white noise. It is a method that utilizes measurements linearly related to the states, and error covariance matrices, to generate a gain referred to as the Kalman gain. This gain is applied to the a priori state estimate, thus creating an a posteriori estimate. The estimation process continues in a predictor-corrector fashion while maintaining a statistically minimal state error covariance matrix for linear systems.

The following two equations describe the system dynamic model and the measurement model used in general for (linear) state estimation.

$$x_{k+1} = A_k x_k + w_k \tag{1}$$

$$z_{k+1} = C_{k+1} x_{k+1} + v_{k+1} \tag{2}$$

The next five equations form the KF algorithm, and are used in an iterative fashion, in conjunction with Eqs. (1) and (2). Equation 3 extrapolates the a priori state estimate, and Eq. (4) is the corresponding error covariance. The Kalman gain may be calculated by Eq. (5), and is used to update the state estimate and error covariance, described by Eqs. (6) and (7), respectively.

$$\hat{x}_{k+1|k} = A_k \hat{x}_{k|k} \tag{3}$$

$$P_{k+1|k} = A_k P_{k|k} A_k^T + Q_k \tag{4}$$

$$K_k = P_{k+1|k} C_k^T [C_k P_{k+1|k} C_k^T + R_k]^{-1} \tag{5}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_k [z_k - C_k x_{k+1|k}] \tag{6}$$

$$P_{k+1|k+1} = [I - K_k C_k] P_{k+1|k} \tag{7}$$

The effects due to mismodeling can be negative, as both the Kalman gain and covariance matrix calculations are dependent on the system and measurement matrices. Furthermore, the performance and stability of the KF may also be dependent on the definition of the process and measurement noise, made through covariance matrices [2, 5]. Overlooked nonlinearities in the system may also cause the KF to become unstable. The EKF may be used for nonlinear systems. It is conceptually similar to the iterative KF process. The nonlinear system and measurement matrices are linearized according to its corresponding Jacobian, which is a first-order partial derivative. This linearization can sometimes cause instabilities when implementing the EKF [2].

Smooth Variable Structure Filter

In 2002, the variable structure filter (VSF) was introduced as a new predictor-corrector method used for state and parameter estimation [5, 6]. It is a type of sliding mode estimator, where gain switching is used to ensure that the estimates converge to true state values. An internal model of the system, either linear or nonlinear, is used to predict an a priori state estimate. A corrective term is then applied to calculate the a posteriori state estimate, and the estimation process is repeated iteratively. The SVSF was later derived from the VSF, and uses a simpler and less complex gain calculation [8]. In its present form, the SVSF is stable and robust to modeling uncertainties and noise, given an upper bound [8]. The basic concept of the SVSF is shown in Fig. 1. Assume that the solid line in Fig. 1 is a trajectory of some state (amplitude versus time). An initial value is selected for the state estimate. The estimated state is pushed towards the true value. Once the value enters the existence subspace, the estimated state is forced into switching along the system state trajectory [8].

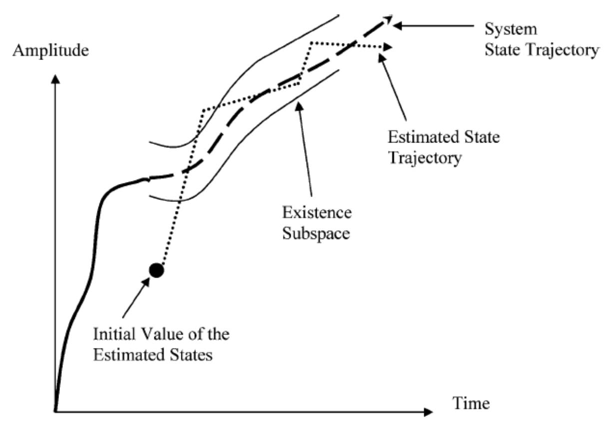

FIGURE 1. SVSF ESTIMATION CONCEPT [8]

The SVSF method is model based and applies to smooth nonlinear dynamic equations. The estimation process may be summarized by Eqs. (8) to (11), and is repeated iteratively. An a priori state estimate is calculated using an estimated model of the system. This value is then used to calculate an a priori estimate of the measurement, defined by Eq. (9). A corrective term, referred to as the SVSF gain, is calculated as a function of the error in the predicted output, as well as a gain matrix and the smoothing boundary layer width. The corrective term calculated in Eq. (10) is then used in Eq. (11) to find the a posteriori state estimate.

$$\hat{x}_{k+1|k} = \hat{F}(\hat{x}_{k|k}, u_k) \tag{8}$$

$$\hat{z}_{k+1|k} = \hat{C}\hat{x}_{k+1|k} \tag{9}$$

$$K_{k+1} = \hat{C}^{+}\left|\left(\left|e_{z_{k+1|k}}\right|_{ABS} + \gamma\left|e_{z_{k|k}}\right|_{ABS}\right)\right|_{ABS} \circ sat(e_{z_{k+1|k}}, \Psi) \tag{10}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \tag{11}$$

$$e_{z_{k|k}} = z_k - \hat{z}_{k|k} \tag{12}$$

$$e_{z_{k+1|k}} = z_{k+1} - \hat{z}_{k+1|k} \tag{13}$$

Two critical variables in this process are the a priori and a posteriori output error estimates, defined by Eqs. (12) and (13), respectively [8]. Note that Eq. (12) is the output error estimate from the previous time step, and is used only in the gain calculation.

TARGET TRACKING SCENARIO

A generic air traffic control (ATC) scenario is described in this section. The target tracking problem is based on the scenario found in section 11.7 of [3]. A radar stationed at the origin provides direct position only measurements, with a very large standard deviation of 1,000 m in each coordinate. As shown in Fig. 2, an aircraft starts from an initial position of [25,000 m, 10,000 m] at time $t = 0$ s, and flies westward at 120 m/s for 125 s. The aircraft then begins a coordinated turn for a period of 90 s at a rate of 1˚/s. It then flies southward at 120 m/s for 125 s, followed by another coordinated turn for 30 s at 3˚/s. The aircraft then continues to fly westward until it reaches its final destination.

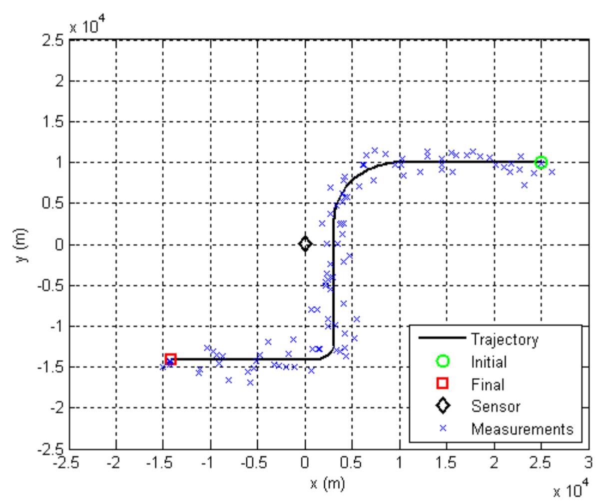

FIGURE 2. AIRCRAFT TRAJECTORY

In ATC scenarios, the behaviour of civilian aircraft may be modeled by two different modes: uniform motion (UM) which involves a straight flight path with a constant speed and course, and maneuvering which includes turning or climbing and descending [3]. In this case, maneuvering will refer to a coordinated turn (CT) model, where a turn is made at a constant turn rate and speed. The uniform motion model used for this target tracking problem is given by Eq. (14) [3, 9].

$$x_{k+1} = \begin{bmatrix} 1 & 0 & \tau & 0 \\ 0 & 1 & 0 & \tau \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}\tau^2 & 0 \\ 0 & \frac{1}{2}\tau^2 \\ \tau & 0 \\ 0 & \tau \end{bmatrix} w_k \tag{14}$$

The state of the aircraft may be defined as follows:

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k]^T \tag{15}$$

The first two states refer to the position along the x and y-axis, respectively, and the last two states refer to the velocity along the x and y-axis, respectively. The sampling time used in this simulation was 5 seconds. When using the CT model, the state vector needs to be augmented to include the turn rate, as shown in Eq. (16). The CT model may be considered nonlinear if the turn rate of the aircraft is not known. Note that a left turn corresponds to a positive turn rate, and a right turn has a negative turn rate. This sign convention follows the commonly used trigonometric convention (the opposite is true for navigation convention) [3]. The CT model is then given by Eq. (17) [3, 9].

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k \quad \Omega_k]^T \tag{16}$$

$$x_{k+1} = \begin{bmatrix} 1 & 0 & \frac{\sin\Omega_k\tau}{\Omega_k} & -\frac{1-\cos\Omega_k\tau}{\Omega_k} & 0 \\ 0 & 1 & \frac{1-\cos\Omega_k\tau}{\Omega_k} & \frac{\sin\Omega_k\tau}{\Omega_k} & 0 \\ 0 & 0 & \cos\Omega_k\tau & -\sin\Omega_k\tau & 0 \\ 0 & 0 & \sin\Omega_k\tau & \cos\Omega_k\tau & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}\tau^2 & 0 & 0 \\ 0 & \frac{1}{2}\tau^2 & 0 \\ \tau & 0 & 0 \\ 0 & \tau & 0 \\ 0 & 0 & \tau \end{bmatrix} w_k \tag{17}$$

Since the radar stationed at the origin provides direct position measurements only, the measurement equation may be formed linearly as follows:

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} x_k + v_k \tag{18}$$

Equations (14) to (18) were used to generate the true state values of the trajectory and the radar measurements for this target tracking scenario.

SIMULATION RESULTS

This section provides the results of using both the EKF and the SVSF for tracking the target trajectory. Both the UM and the CT models were used by each filter. Note that in the following figures: EKF 1 or SVSF 1 refers to the UM model, and EKF 2 or SVSF 2 refers to the CT model. For each simulation, a total of 500 Monte Carlo runs were generated to obtain the results.

As previously mentioned, the EKF uses a linearized form of the system and measurement matrices. In this case, the system defined in Eq. (17) is nonlinear, such that the Jacobian of it yields a linearized form as shown in Eq. (19). The terms in the last column of Eq. (19) are defined in Eq. (20).

$$[\nabla_x A_{k,x}^T]^T\Big|_{x_k=\hat{x}_k} = \begin{bmatrix} 1 & 0 & \frac{\sin\hat{\Omega}_k\tau}{\hat{\Omega}_k} & -\frac{1-\cos\hat{\Omega}_k\tau}{\hat{\Omega}_k} & A_{\hat{\Omega}1} \\ 0 & 1 & \frac{1-\cos\hat{\Omega}_k\tau}{\hat{\Omega}_k} & \frac{\sin\hat{\Omega}_k\tau}{\hat{\Omega}_k} & A_{\hat{\Omega}2} \\ 0 & 0 & \cos\hat{\Omega}_k\tau & -\sin\hat{\Omega}_k\tau & A_{\hat{\Omega}3} \\ 0 & 0 & \sin\hat{\Omega}_k\tau & \cos\hat{\Omega}_k\tau & A_{\hat{\Omega}4} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} A_{\hat{\Omega}1} \\ A_{\hat{\Omega}2} \\ A_{\hat{\Omega}3} \\ A_{\hat{\Omega}4} \end{bmatrix} = \begin{bmatrix} \frac{(\cos\hat{\Omega}_k\tau)\tau\hat{\dot{\xi}}_k}{\hat{\Omega}_k} - \frac{(\sin\hat{\Omega}_k\tau)\hat{\dot{\xi}}_k}{\hat{\Omega}_k^2} - \frac{(\sin\hat{\Omega}_k\tau)\tau\hat{\dot{\eta}}_k}{\hat{\Omega}_k} - \frac{(-1+\cos\hat{\Omega}_k\tau)\hat{\dot{\eta}}_k}{\hat{\Omega}_k^2} \\ \frac{(\sin\hat{\Omega}_k\tau)\tau\hat{\dot{\xi}}_k}{\hat{\Omega}_k} - \frac{(1-\cos\hat{\Omega}_k\tau)\hat{\dot{\xi}}_k}{\hat{\Omega}_k^2} - \frac{(\cos\hat{\Omega}_k\tau)\tau\hat{\dot{\eta}}_k}{\hat{\Omega}_k} - \frac{(\sin\hat{\Omega}_k\tau)\hat{\dot{\eta}}_k}{\hat{\Omega}_k^2} \\ -(\sin\hat{\Omega}_k\tau)\tau\hat{\dot{\xi}}_k - (\cos\hat{\Omega}_k\tau)\tau\hat{\dot{\eta}}_k \\ (\cos\hat{\Omega}_k\tau)\tau\hat{\dot{\xi}}_k - (\sin\hat{\Omega}_k\tau)\tau\hat{\dot{\eta}}_k \end{bmatrix} \tag{20}$$

To generate the results for this section, the EKF used the following values of P, Q, and R:

$$P = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 \\ 0 & R_{22} & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

$$Q = L_1 \begin{bmatrix} \frac{\tau^3}{3} & 0 & \frac{\tau^2}{2} & 0 & 0 \\ 0 & \frac{\tau^3}{3} & 0 & \frac{\tau^2}{2} & 0 \\ \frac{\tau^2}{2} & 0 & \tau & 0 & 0 \\ 0 & \frac{\tau^2}{2} & 0 & \tau & 0 \\ 0 & 0 & 0 & 0 & \tau\frac{L_2}{L_1} \end{bmatrix} \tag{22}$$

$$R = 1{,}000^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{23}$$

Note that L_1 and L_2 are the power spectral densities, and were defined as 0.16 and 0.01, respectively [9]. The system and measurement noise were generated using their respective covariance values. Also, when using the UM model, the fifth row and column of Eqs. (21) and (22) were truncated. Furthermore, when obtaining the results for the SVSF, the following parameters were defined: γ = 0.2, Ψ = 5,000 for the first, second, and fifth states, and Ψ = 45,000 for the third and fourth states. These values were obtained by trial and error, based on minimizing error.

(I) Normal Conditions

This case involved normal conditions, without poor initial conditions and the presence of outliers. The initial state estimates were set to the true initial state values. The SVSF performed significantly better than the EKF for both models.

EKF. The simulation results obtained using the EKF are shown in Fig. 3. While the EKF was using the UM model, the trajectory was not tracked well. After the first turn, the EKF was unable to recover and performed poorly for the remainder of the tracking. The EKF did perform better using the second model; however, there was a significant amount of chattering across the target trajectory resulting in a higher RMSE.

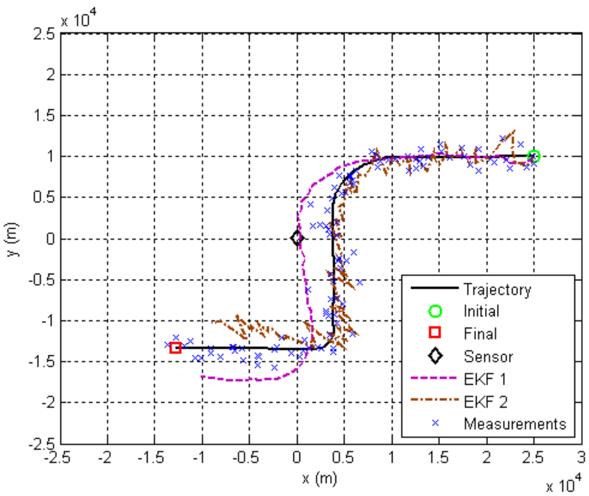

FIGURE 3. EKF RESULTS FOR CASE (I)

SVSF. The simulation results obtained using the SVSF are shown in Fig. 4. Note how the SVSF was able to track the trajectory and measurements relatively well with each model. The SVSF appeared to be impartial to both models, with the exception of a higher velocity RMSE for the second model.

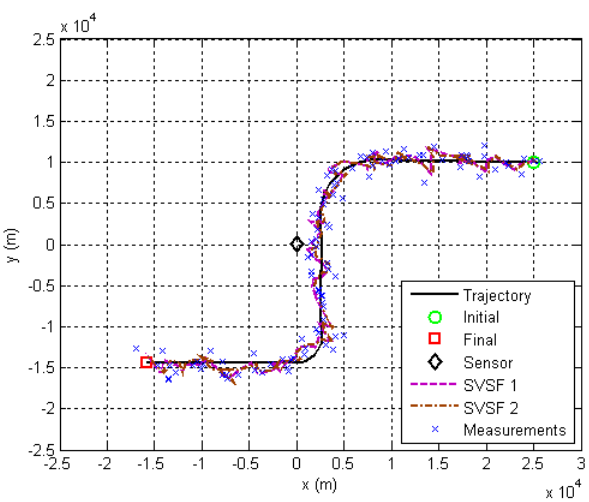

FIGURE 4. SVSF RESULTS FOR CASE (I)

The following two tables summarize the RMSE for both filtering strategies.

TABLE 1. SUMMARY OF RMSE FOR BOTH STRATEGIES UNDER NORMAL CONDITIONS (UM MODEL)

RMSE per State	**EKF**	**SVSF**
Position (States 1, 2)	3,140 m	1,086 m
Velocity (States 3, 4)	86.0 m/s	84.0 m/s

TABLE 2. SUMMARY OF RMSE FOR BOTH STRATEGIES UNDER NORMAL CONDITIONS (CT MODEL)

RMSE per State	**EKF**	**SVSF**
Position (States 1, 2)	2,565 m	1,162 m
Velocity (States 3, 4)	40,985 m/s	737.5 m/s
Omega (State 5)	1.84 rad/s	1.78 rad/s

(II) Poor Initial Conditions

This case involved poor initial conditions (the starting estimates were increased by a factor of 100).

EKF. Changing the initial estimates by a factor of 100 greatly affected the quality of the results obtained by the EKF, as demonstrated in Fig. 5. The EKF was unable to recover using any of the two models, thus resulting in an unstable estimate.

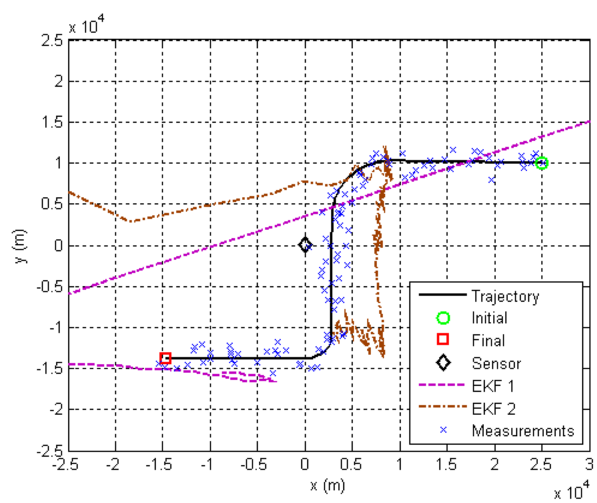

FIGURE 5. EKF RESULTS FOR CASE (II)

SVSF. Changing the initial conditions did not greatly affect the behaviour of the SVSF, as shown in Fig. 6. The SVSF recovered after only a few time steps, and was stable for the remainder of the simulation.

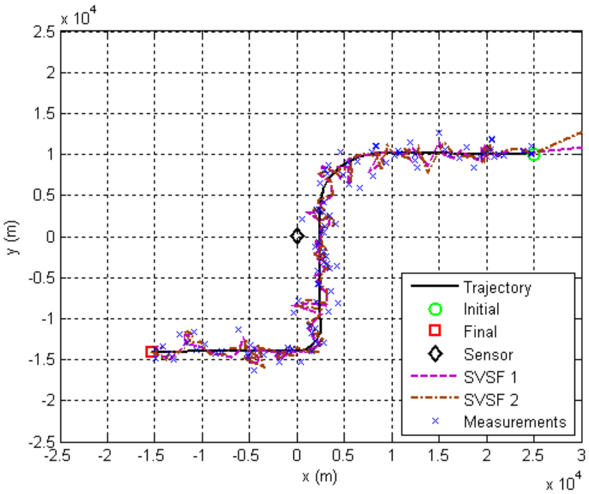

FIGURE 6. SVSF RESULTS FOR CASE (II)

(III) Presence of an Outlier

This case involved the presence of an outlier among the measurements (the middle measurement (scan 50 of 100) was multiplied by a factor of 500), without poor initial conditions.

EKF. The presence of an outlier greatly affected the results of the EKF, as shown in Fig. 7. After the onset of the outlier (about half-way through tracking the target), the EKF became unstable and was unable to accurately continue with the estimation.

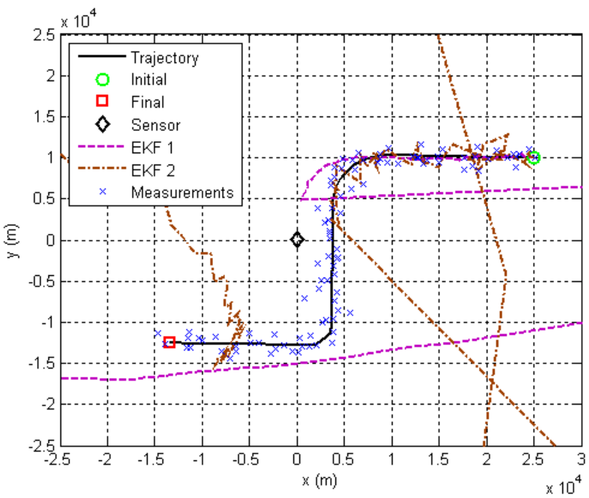

FIGURE 7. EKF RESULTS FOR CASE (III)

SVSF. The presence of an outlier did have some effect on the SVSF. As shown in Fig. 8, chattering began at the onset of the outlier. However, the presence of the chatter was beneficial as it allowed the SVSF to remain stable and bounded to within the target trajectory. The remainder of the estimation continued as in the normal case.

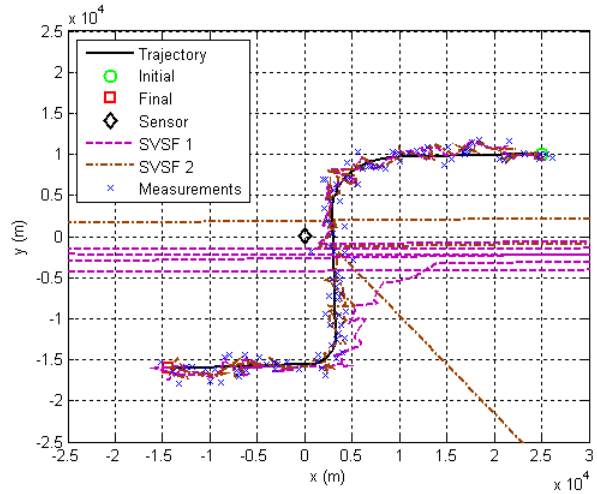

FIGURE 8. SVSF RESULTS FOR CASE (III)

(IV) Poor Initial Conditions and Presence of an Outlier

This case involved both the poor initial conditions (the starting estimates were multiplied by a factor of 100) and the presence of an outlier among the measurements (the middle measurement was multiplied by a factor of 500).

EKF. In this case, the EKF did not perform well at all. In fact, as shown in Fig. 9, the EKF was completely unstable and was unable to provide any sort of estimate.

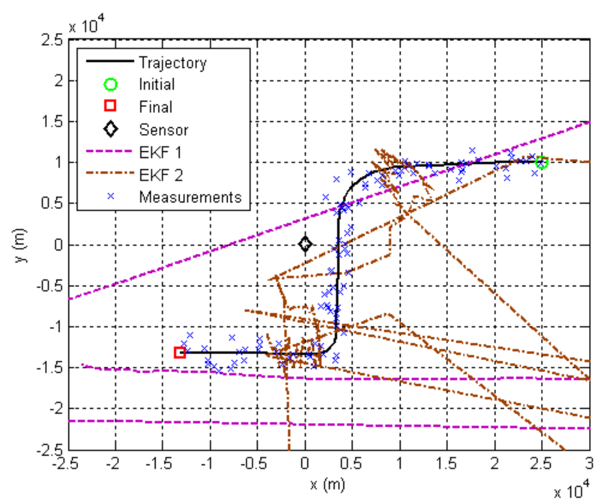

FIGURE 9. EKF RESULTS FOR CASE (IV)

SVSF. Unlike the EKF, the SVSF was able to overcome the poor initial conditions and the presence of an outlier. Fig. 10 demonstrates the stability and robustness of this filter.

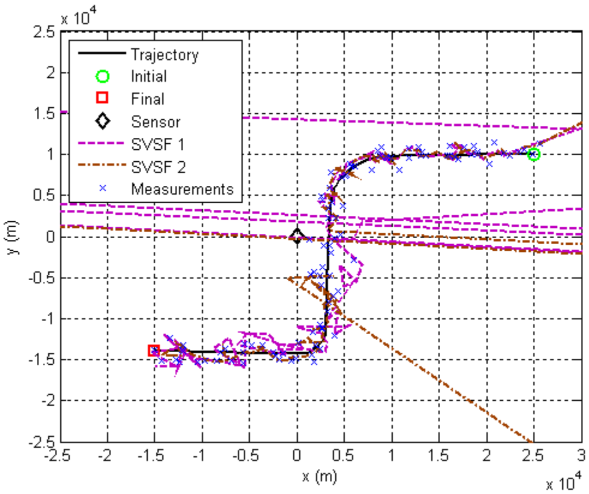

FIGURE 10. SVSF RESULTS FOR CASE (IV)

DISCUSSION

A generic air traffic control problem was studied under four different cases: normal, poor initial conditions, presence of an outlier, and a combination of the latter two. It was demonstrated throughout that the EKF was less robust to modeling uncertainties, poor initial conditions, and the presence of outliers. The chattering that is present in the SVSF, caused by the gain switching, brings an inherent amount of stability and robustness to the filter. This is clearly demonstrated in the third case where the EKF failed due to the outlier; however the SVSF was able to chatter about the trajectory until recovering and providing an estimate. The performance of these algorithms was ranked in terms of robustness, resilience to poor initial conditions and measurement outliers, tracking accuracy and computational complexity. Table 3 shows the ranking of the estimation strategies for the results of the simulation. The EKF is thought to be slightly more complex since it requires the computation of a matrix inverse at each correction step, as well as a linearization of the nonlinear matrices. The SVSF was also shown to be more robust, stable and accurate.

TABLE 3. PERFORMANCE RANKING OF THE ESTIMATION STRATEGIES

Performance Characteristic	EKF	SVSF
Robustness	2	1
Stability	2	1
Accuracy	2	1
Complexity	2	1

CONCLUSIONS

The results of applying the SVSF on a target tracking problem demonstrate its stability and robustness. It is shown that the EKF performs poorly in the presence of bad initial conditions and measurement outliers. However, the SVSF is able to overcome these difficulties, and provide a stable estimate of the states. Furthermore, the EKF appears to be sensitive to model mismatch, as demonstrated by the different estimates of the same target, which was calculated using two different target motion models. The SVSF was not as affected, and yielded relatively the same estimate for both models. Its stability to model mismatch and robustness to poor initial conditions and outliers make using the SVSF advantageous over the well-known Kalman and extended Kalman filters.

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