# AN ADAPTIVE FORMULATION OF THE SMOOTH VARIABLE STRUCTURE FILTER BASED ON STATIC MULTIPLE MODELS

**Abstract**

The Kalman filter (KF) is the most well-known estimation strategy that yields the optimal solution to the linear quadratic estimation problem. The system in such applications shall be well modeled assuming the presence of Gaussian noise. While the KF is effective under the stated conditions, it lacks robustness to other type of disturbances. Therefore, numerous variants of the KF have been developed to accommodate its limitations. The smooth variable structure filter (SVSF) is as an alternative solution with improved robustness, especially in the case of modeling uncertainties. It is based on sliding mode technique that offers robustness at the cost of optimality. On the other hand, some algorithms and solutions involve with several possible operating modes and generates an estimation based on the output of these models, i.e. the static multiple models that obtains the estimates based on weighted statistical fusing of the outputs of the models depending on the likelihood of each mode. This paper introduces an adaptive formulation of the SVSF that is reformulated based on static multiple models. The proposed model is applied and tested on an electro-hydrostatic actuator (EHA). The proposed method takes the advantages of the SVSF's robustness and stability, while reducing the estimation error due to the use of adaptive modeling structure. The results show an improvement on the SVSF performance, where the root mean squared errors are reduced by 41%, 99% and 75% for the position, velocity and acceleration

estimated states. Therefore, the proposed method is a good candidate for parameter and state estimation problems.

**Keywords:** State and parameter estimation; Kalman filter; smooth variable structure filter; robustness; static multiple models

## Table of Nomenclature

| Symbols | Representation | Symbols | Representation |
|---|---|---|---|
| $\circ$ | Schur Product | $\gamma$ | SVSF Coefficient Matrix |
| $X^{+}$ | Pseudoinverse of $X$ | $\psi$ | Boundary layer vector |
| $X^{-1}$ | Inverse of $X$ | $z_k$ | measurement value at time $k$ |
| $\hat{X}$ | Estimated value of $X$ | $P$ | Error covariance matrix |
| $\lvert X \rvert$ | Absolute value of $X$ | $Q$ | System noise's covariance matrix |
| $A$ | System Matrix | $R$ | Measurement noise's covariance matrix |
| $B$ | Input Matrix | $x$ | State Vector |
| $C$ | Measurement Matrix | $z$ | Measurement Vector |
| $X_{k\mid k}$ | The a posteriori vale of $X$ at time $k$ | $e_z$ | Error in measurement. |
| $X_{k\mid k-1}$ | The a priori vale of $X$ at time $k$ | $K_k$ | Correction gain at time $k$ |
| $u_k$ | Input value at time $k$ | $M^j$ | Model $j$ structure. |
| $\mu_k^j$ | Weight at time $k$ for each model $M^j$ | $T$ | The time step, 1 msec. |
| $\sigma_j^2$ | The variance of model $\mathrm{M}^{\mathrm{j}}$ | $sat$ | Saturated function |
| $sgn$ | Sign function | $r$ | Number of models for SSM |
| $n, m$ | Number of states and measurements, respectively. | $diag$ | Convert the vector to a diagonal matrix where the elements of the vector are the diagonal elements of the matrix. |

## 1. Introduction

Estimating the dynamic behavior involves the extraction of important values known as states from noisy measurements [1, 2]. States change over time and are typically governed by equations that describe system dynamics [3]. The estimation process is referred to as Filter as it tries to minimize the noise effect. Most of filters try to minimize the error (difference between the actual and estimated state values) while simultaneously reducing the effects of noise. The other type of filters try being robust to disturbances [3]. Disturbances and noise are typically present in measurements, and may be caused by the sensor quality (system uncertainties) as well as environmental factors (Measurement uncertainties). System uncertainties may be caused by an inaccurate model and/or variations and nonlinearities in the physical system parameters. Reliable estimates of state and/or parameters are necessary for safely and accurately controlling a system in real-time. When system dynamics are changed abruptly in the presence of faults, adaptive estimation strategies that combined both types of filters can be used to mitigate inaccurate estimation. They maintain the stability of the filter during the fault, while reducing the error in the estimation.

Kalman expanded on the research of his predecessors and introduced a new solution to linear filtering and tracking problems [4]. He derived a filter that utilized linear models and measurements to yield an optimal estimation based on strict assumptions. This filter later became known as the Kalman filter (KF). Since the KF is applicable to linear Gaussian models, several works were conducted to modify the KF and make more applicable to nonlinear and/or non Gaussian models, i.e. Extended KF, and Unscented KF [4].

Another branch of estimation methods is still developing in parallel to the KF and its variants. This branch includes the well-known sliding mode observers (SMOs). These observers are based on variable structure (VS) and sliding mode (SM) techniques [5, 6, 7]. Both techniques consider the system has discontinuity in his structure. Therefore, they define discontinuations hyperplanes that divide the state space into different regions; within these regions, the equations used to describe the system are continuous [8, 9]. The name 'variable structure' is chosen since system dynamics may be mathematically described by a finite number of equations.

Variable structure theory provided the foundation for variable structure control (VSC). In VSC, the controller signal is formulated as a discontinuous state function, such that discontinuity hyperplanes are introduced [8, 9]. The most well-known type of VSC is the sliding mode controller

(SMC) [6, 10]. SMC makes use of a discontinuous switching plane along a desired state trajectory, which is referred to as the sliding surface. The primary objective for the SMC is to maintain the states within sliding surface neighbourhood. A switching gain is used to push the states towards the surface when they try to move away. Once the state values are on the surface, the states slides along the surface towards the desired values [10]. Although the switching effects bring robustness and stability to the control process, it also introduces high-frequency switching known as chattering [11]. Quite often a boundary layer is introduced in an effort to smooth out the control signal [10]. Prior to the 1980s, VSC and SMC methods were only considered in the continuous-time domain [12]. In 1985, a discrete-time formulation of SMC was presented [13]. A stability condition was provided shortly afterwards and is now typically used in the design of discrete controllers [14, 15].

SMOs, which was developed in 1980s [12, 16], reduces the error with the help of a switching function similar to VSC and SMC [17]. Observer gains are calculated based on the errors between the measurements and estimates [17]. Most SMOs apply a discontinuous signal to the estimates in order to keep them bounded to an area of the surface [12]. The motion consists of three phases: reachability, injection, and sliding [12, 18]. The reachability phase consists of forcing the estimates to the sliding surface from some initial conditions, in a finite period of time [12]. Once within a defined area of the surface (called an existence subspace), both the injection and sliding phases are present. The sliding phase forces the estimated errors to slide along a hyperplane towards the origin [12]. The injection phase consists of preventing the estimate from leaving the existence subspace; keeping it bounded within an area of the sliding surface [12]. According to [12, 16, 19], the action of the injection phase enables the observer to be robust enough to overcome uncertainties, modeling errors, and nonlinearities present in the system. A number of SMOs have been developed based on these principles. The most notable observers were introduced by Slotine et al. [9, 20], Walcott et al. [21, 22], Edwards et al. [19], and later by both Tan and Edwards [23]. SMOs have been applied to estimation problems, and fault detection and isolation [12].

Another filter called the smooth variable structure filter (SVSF) was presented in 2007, which was based on sliding mode and variable structure techniques [3, 12, 24]. The SVSF is formulated as a predictor-corrector estimator similar to the KF. However, it utilizes a gain structure based on sliding mode techniques. The filter's gain is calculated based on the error in measurements at the prediction stage of the current time (known as innovation), the error in measurements at the update stage from the previous time step, and a switching term [24]. Similar to SMOs, the switching gain structure improves stability and robustness of the estimation process by bounding the state estimates close to the true trajectory [25, 26]. The SVSF presented in [24] did not contain a state error covariance derivation, which is an important feature for optimal estimation strategies (it is another performance indicator). A state error covariance function was introduced and expanded in [25, 27, 28], which vastly improved the number of useful applications for the SVSF [29, 30, 31]. Other developments and improvements to the SVSF were conducted in the literature including fault detection using chattering, higher-order implementations, and tracking multiple targets [12, 32, 33, 34, 35]. The SVSF has demonstrated robust performance on a number of different estimation problems [4]. Most recently, a filter, which is referred to as the sliding innovation filter (SIF), was introduced in [36, 37, 38, 39]. The SIF is based on similar concepts to the SVSF, but offers a simpler formulation with improved results. An opportunity for improving the SVSF involves the development of an adaptive formulation. The ability for the SVSF to automatically modify its system and/or measurement models based on different operating modes offers significant room for improvement (e.g., in terms of both accuracy and robustness).

In this paper, a new adaptive formulation of the SVSF is presented and tested on an experimental setup. The novel method integrates the static multiple models estimator (SMM) with the SVSF predictor-corrector estimation strategy. The SMM consists of several possible operating modes where several possible estimates are obtained. The SMM then combines these estimates using some weights based on the likelihood of each mode. This strategy may be used for fault detection and diagnosis problems, and has demonstrated good accuracy and repeatability of results. The performance of the proposed method is evaluated using an electro-hydrostatic actuator (EHA) which was built for experimentation. The results are compared with the standard SVSF estimation method.

This paper is organized as follows. Section 2 summarizes the SVSF estimation process. Section 3 introduces the SMM estimator and the proposed SMM-SVSF or the adaptive SVSF algorithm. Section 4 describes the experimental setup as well as the equations of motion governing the EHA. Section 5 discusses the application of the standard SVSF and adaptive SVSF to the EHA system, followed by concluding remarks.

## 2. The Smooth Variable Structure Filter

The smooth variable structure filter (SVSF) is a predictor-corrector estimation strategy that offers solution with robustness and stability against disturbances and uncertainties. The SVSF uses a smoothing boundary layer with an upper bound that is defined based on the level of noise and unmodeled dynamics [40, 41]. The SVSF is model-based and may be applied to both linear or nonlinear systems and measurements [3, 12]. The SVSF's concepts are illustrated in Fig. 1.

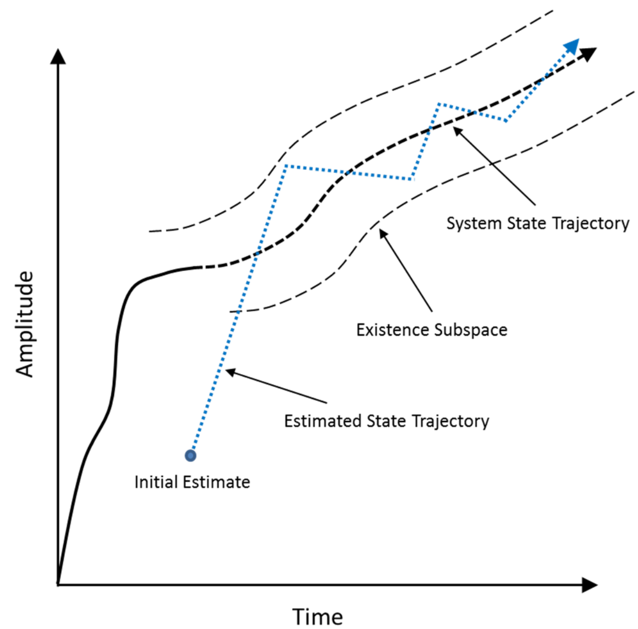



**Figure 1.** SVS's concepts with existence subspace boundary layer [3].

As described earlier, the SVSF strategy is structured similarly to the KF. However, it presents a novel way to calculate its gain. As per (2.1) and (2.2), $\hat{x}_{k+1|k}$ and $P_{k+1|k}$ are calculated.

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.2}$$

Then $\hat{z}_{k+1|k}$ and $e_{z,k+1|k}$ are calculated as per (2.3) and (2.4), respectively.

$$\hat{z}_{k+1|k} = C\hat{x}_{k+1|k} \tag{2.3}$$

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} \tag{2.4}$$

The gain used by the SVSF, $K_k$, is calculated with the use of the boundary layer widths, $\psi$, as follows [3]:

$$K_{k+1} = C_k^{+} diag\left[\left(\left|e_{z_{k+1|k}}\right| + \gamma \left|e_{z_{k|k}}\right|\right) \circ sat\left(\bar{\psi}^{-1} e_{z_{k+1|k}}\right)\right] diag\left(e_{z_{k+1|k}}\right)^{-1} \tag{2.5}$$

The saturation function is defined as follows:

$$sat\left(\bar{\psi}^{-1}e_{z_{k+1|k}}\right)=\begin{cases}1, & e_{z_i,k+1|k}/\psi_i \geq 1\\ \dfrac{e_{z_i,k+1|k}}{\psi_i}, & -1<\dfrac{e_{z_i,k+1|k}}{\psi_i}<1\\ -1, & e_{z_i,k+1|k}/\psi_i \leq -1\end{cases} \tag{2.6}$$

where $\bar{\psi}^{-1}$ is defined by (2.7) for $m$ number of measurements [3]:

$$\bar{\psi}^{-1}=\begin{bmatrix}\dfrac{1}{\psi_1} & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \dfrac{1}{\psi_m}\end{bmatrix} \tag{2.7}$$

The state vector and error covariance matrix are respectively updated as per (2.8) and (2.9).

$$\hat{x}_{k+1|k+1}=\hat{x}_{k+1|k}+K_{k+1}e_{z,k+1|k} \tag{2.8}$$

$$P_{k+1|k+1}=(I-K_{k+1}C)P_{k+1|k}(I-K_{k+1}C)^T+K_{k+1}R_{k+1}K_{k+1}^T \tag{2.9}$$

Finally, the updated measurement error, $e_{z,k+1|k+1}$, is found as per (2.10) and is used in the next iteration.

$$e_{z,k+1|k+1}=z_{k+1}-\hat{z}_{k+1|k+1} \tag{2.10}$$

The existence subspace, denoted by the dotted black line shown in Figure 1, refers to the level of uncertainty found in the estimation process. It is typically present due to the amount of noise and/or modeling uncertainties [3]. The existence space, $\beta$, is described mainly from the innovation signal [27, 34]. While the width is not precisely known, designer knowledge may be used to define the upper bound. When the smoothing boundary is defined larger than the existence subspace, the estimated states are smoothed. Likewise, if the smoothing term is set too small, chattering (high-frequency switching) may occur.

## 3. A Novel Adaptive Formulation of the Smooth Variable Structure Filter

The static multiple model (SMM) algorithm assumes that the system behaves according to a finite number of $r$ models $M^1$, $M^2$, … , $M^r$. The SMM uses variable weights, $\mu_k^j$, calculated at time $k$ to represent each model $M^j$. These weights represent a probability of the system behaving according to a corresponding operating mode (i.e., mathematical model). These weights are used to combine the corresponding model state estimates [42] which creates an overall estimate. The weights are initially uniformly distributed, and subsequent weights are calculated as follows:

$$\mu_k^j = \frac{p\left(z_k \middle| M^j\right)\mu_{k-1}^j}{\sum_{i=1}^r p\left(z_k \middle| M^i\right)\mu_{k-1}^i} \tag{3.1}$$

where $p\left(z_k \middle| M^j\right)$ is the likelihood value of measurement $z_k$ based on $M^j$ and is defined as follows:

$$p\left(z_k \middle| M^j\right) = \frac{1}{\sqrt{2\pi\sigma_j^2}} exp \frac{-\left(z_k - \hat{z}_{k|k-1}\right)^2}{2\sigma_j^2} \tag{3.2}$$

$$\sigma_j^2 = C_k^j P_{k|k-1}^j C_k^{j^T} + (\sigma_z^2)^j \tag{3.3}$$

where $\sigma_j^2$ refers to the variance of model $M^j$ based on the predicted measurement $\hat{z}_{k|k-1}$ for model $M^j$ [42]. Note that the parameter definitions may also be found in the Table of Nomenclature. Each model has its own likelihood value calculated from the filtering strategy (whether it is from a Kalman filter, smooth variable structure filter, or another type). The adaptive estimates are calculated using the weighted sum produced by the system models, as per (3.4).

$$\hat{x}_{k|k} = \sum_{j=1}^{r} \mu_k^j \, \hat{x}_{k|k}^j \tag{3.4}$$

The adaptive covariance is calculated in a similar fashion, as shown in (3.5).

$$P_{k|k} = \sum_{j=1}^{r} \mu_k^j \left[ P_{k|k}^j + \left( \hat{x}_{k|k}^j - \hat{x}_{k|k} \right) \left( \hat{x}_{k|k}^j - \hat{x}_{k|k} \right)^T \right] \tag{3.5}$$

The proposed SMM-SVSF (or adaptive SVSF) uses the model weights from the static multiple models estimator to generate a weighted prediction. The weighted state predictions are used to calculate the SVSF gain, which is used to generate an updated state estimate and state error covariance. Since the algorithm uses a weighted combination of system modes, the weights could be used to describe the mixing of different system modes. Figure 2 depicts the algorithm flow chart and Table 1 shows the corresponding pseudocode. Note that the initial mode weights can be defined by the user, provided that the sum of each value is 1 (so total probability is 100%).

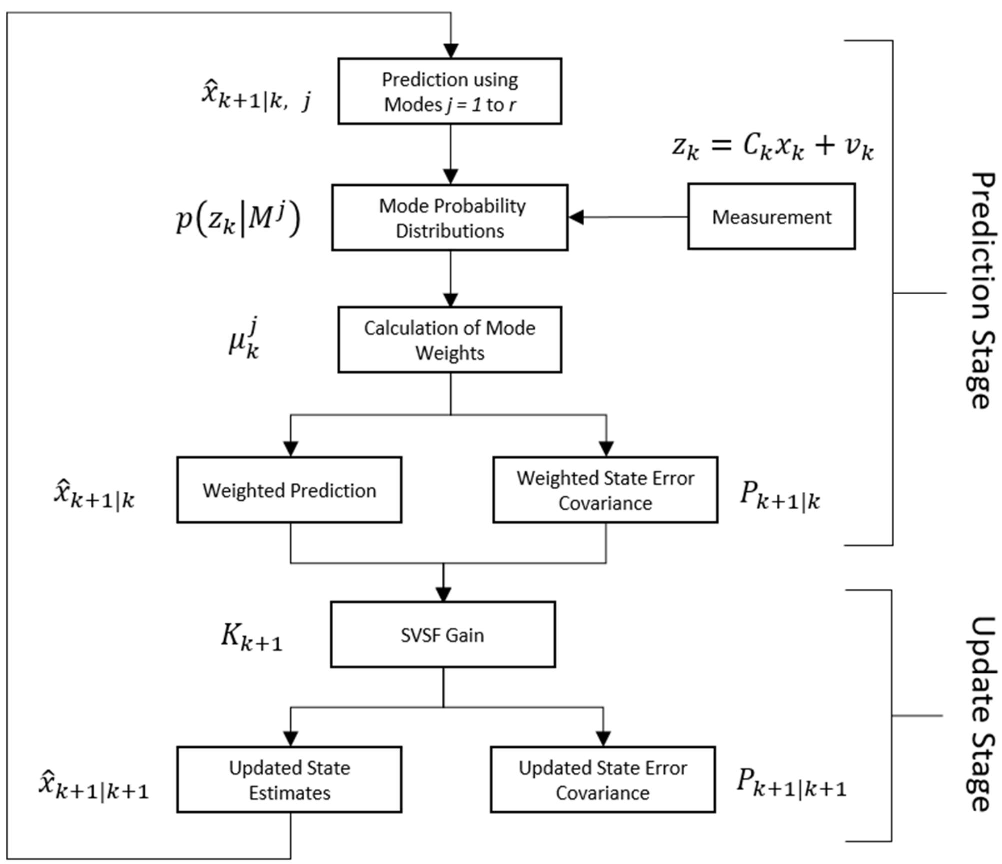



**Figure 2.** The proposed SMM-SVSF (or adaptive SVSF) flowchart.

**Table 1.** Pseudocode for the SMM-SVSF algorithm.

| | |
|---|---|
| 1: | For models $(M^j)$, $j = 1$ to $r$ |
| | $\hat{x}_{k+1\|k,\_j} \leftarrow (A_j, u)$ |
| 2: | For models $(M^j)$, $j = 1$ to $r$ |
| | $\sigma \leftarrow (Q, R, P_{k\|k})$ |
| | $p \leftarrow (\hat{x}_{k+1\|k,j}, z, \sigma)$ |
| 3: | $\mu_{k+1} \leftarrow (\hat{x}_{k+1\|k,j}, \mu_k)$ |
| 4a: | $\hat{x}_{k+1\|k} \leftarrow (\hat{x}_{k+1\|k,j}, \mu_{k+1})$ |
| 4b: | For operating modes $j = 1$ to $r$ |
| | $P_{k+1\|k,j} \leftarrow (A_j, \hat{x}_{k+1\|k})$ |
| 5: | $P_{k+1\|k} \leftarrow (P_{k+1\|k,j}, \mu_{k+1})$ |
| 6: | $K_{k+1} \leftarrow (C, \gamma,$ saturation$)$ |
| 7a: | $\hat{x}_{k+1\|k+1} \leftarrow (\hat{x}_{k+1\|k,}, z, C, K_{k+1})$ |
| 7b: | $P_{k+1\|k+1} \leftarrow (P_{k+1\|k}, C, K_{k+1}, R)$ |

After the SVSF boundary layer vector and convergence rate have been set and model weights have been initialized, a predicted state estimate for each system model is made. The standard deviation is calculated using three different covariance matrices: the state error, the system noise, and the measurement noise covariance matrices. Next, the updated estimates, standard deviations, and measurements are used to calculate the model probabilities. These probabilities are then used to update the model weights, which then are used to generate a weighted predicted state estimate and error covariance. This information is fed through the SVSF update stage as described in Section 2 using (2.8) through (2.10).

## 4. Experimental Setup

Electrohydrostatic actuators (EHAs) are a type of hydraulic and electrical actuator comprised of a linear or rotary actuator, a hydraulic circuit, and a bidirectional pump [43]. EHAs are used in automotive and aerospace industry due to their large force-to-weight ratios and their reliability. They are also used in various manufacturing applications such as metal forming, where control of the outlet pressure is required [44]. Electromechanical systems often function under different operating modes. In the case of the EHAs, faults such as internal leakage and increased friction may be present. Internal leakage is caused by wearing of the piston seal, which affects the overall actuation performance [45]. If the leakage remains undetected, then it cannot be repaired, which can deteriorate lifetime performance and increase maintenance costs [45]. Since detection of internal leakage in EHAs through disassembly of the cylinder and piston is costly, adaptive estimation strategies can be used to improve the overall estimation process in the presence of multiple operating modes.

The EHA model used in this paper was designed and manufactured at the Centre for Mechatronics and Hybrid Technology at McMaster University shown in Figure 3 [43]. The EHA used in this study is composed of several components, including: two linear actuators, a bi-directional external gear pump, a variable-speed servomotor, an accumulator, a pressure relief valve, and safety circuits [46]. A variable-speed brushless DC electric motor drives the pump and forces hydraulic oil into the cylinder, and modifies the actuation performance by varying the fluid flow rate. An accumulator is used to prevent cavitation and collect leakages from the gear pump. The EHA is controlled by modifying input voltage to the motor, which consequently changes the direction and speed of the pump. Controlling the fluid flow rate in the outer circuit adjusts the position of the piston, which could be used for aerospace applications such as changing flight surfaces.

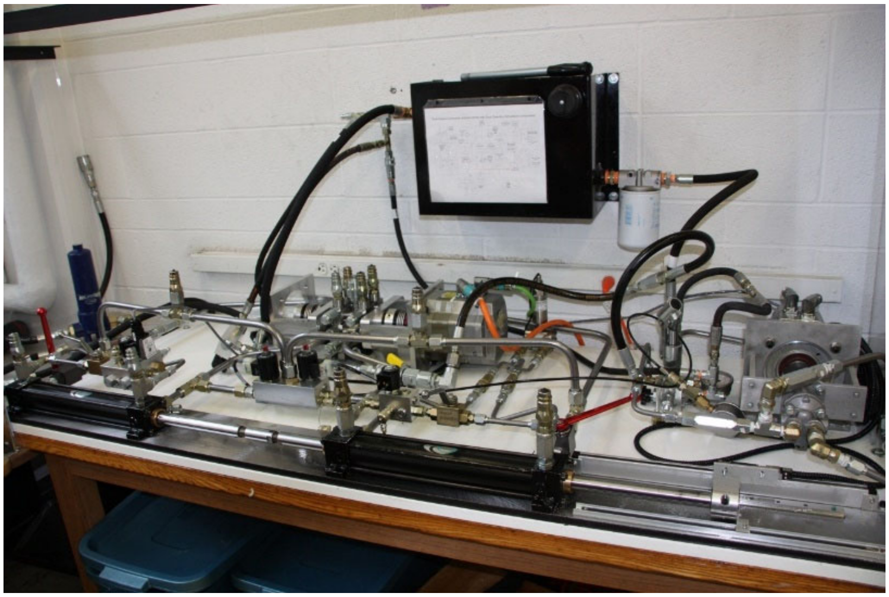


**Figure 3.** Prototype of the EHA used to collect experimental data [43].

The EHA was modelled using four states: the actuator position $x_1 = x$, velocity $x_2 = \dot{x}$, acceleration $x_3 = \ddot{x}$, and differential pressure across the actuator $x_4 = P_1 - P_2$. The physical modeling approach was used to obtain the nonlinear state-space equations in discrete-time described by [3, 47]:

$$x_{1,k+1} = x_{1,k} + Tx_{2,k} \tag{4.1}$$

$$x_{2,k+1} = x_{2,k} + Tx_{3,k} \tag{4.2}$$

$$\begin{aligned} x_{3,k+1} &= 1 - \left[T\frac{a_2 V_0 + M\beta_e L}{MV_0}\right] x_{3,k} - T\frac{\left(A_E{}^2 + a_2 L\right)\beta_e}{MV_0} x_{2,k} \dots \\ &\dots - T\frac{2a_1 V_0 x_{2,k} x_{3,k} + \beta_e L\left(a_1 x_{2,k}{}^2 + a_3\right)}{MV_0} sgn\left(x_{2,k}\right) + T\frac{A_E \beta_e}{MV_0}\, u \end{aligned} \tag{4.3}$$

$$x_{4,k+1} = \frac{a_2}{A_E} x_{2,k} + \frac{\left(a_1 x_{2,k}{}^2 + a_3\right)}{A_E} sgn\left(x_{2,k}\right) + \frac{M}{A_E} x_{3,k} \tag{4.4}$$

The system input is defined as follows:

$$u = D_p\omega_p - sgn(P_1 - P_2)Q_{L0} \tag{4.5}$$

where $\omega_p$ is the pump speed. Table 2 summarizes and defines the numeric values of the parameters in the equations (4.1) to (4.5).

**Table 2.** The EHA parameters, definitions, and values used in the experiment.

| Parameter | Description | Parameter Values |
|---|---|---|
| $A_E$ | Piston Area ($m^2$) | $1.52 \times 10^{-3}$ |
| $D_p$ | Pump Displacement ( $m^3$/rad ) | $5.57 \times 10^{-7}$ |
| $L$ | Leakage Coefficient ( $m^3/(s\times Pa)$) | $4.78 \times 10^{-12}$ |
| $M$ | Load Mass (kg) | $7.376$ kg |
| $Q_{L0}$ | Flow Rate Offset ( $m^3$/s) | $2.41 \times 10^{-6}$ |
| $V_0$ | Initial Cylinder Volume ( $m^3$) | $1.08 \times 10^{-3}$ |
| $\beta_e$ | Effective Bulk Modulus (Pa) | $2.07 \times 10^{8}$ |
| $a_1$ | Friction Coefficient | $6.589 \times 10^{4}$ |
| $a_2$ | Friction Coefficient | $2.144 \times 10^{3}$ |
| $a_3$ | Friction Coefficient | $436$ |

The friction was modeled using a quadratic function based on the actuator velocity. The friction coefficients were obtained by preforming experiments ranging from 15.6 to 109 radians per second with each data set containing four trials for repeatability [43].

## 5. Results and Discussion

The results of applying the proposed strategy on the EHA is discussed in this section. The state estimates were initialized to zero and the covariance matrices for system and measurement noises were defined respectively as $Q = 10^{-9}I_{4x4}$ and $R = 10^{-6}I_{4x4}$, where $I$ is an identity matrix. Furthermore, the state error covariance matrix $P$ was initialized as $10Q$.

Leakage faults were introduced to investigate the effects of parametric uncertainties in the system. The purpose of this study was to demonstrate the efficiency of the proposed strategy compared to the standard SVSF. The SMM-SVSF algorithm demonstrates robustness in the presence of multiple operating modes. Multiple system modes are introduced to the system in the form of leakage faults. In order to obtain the coefficients of the leakage values, the EHA was operated with a constant pump speed of 94.25 radians per second under a series of differential pressures. The differential pressure was modified using a throttling valve in the hydraulic system. To ensure repeatability, five sets of measurements were made. A linear regression was performed on each data set, and the slope and intercept were used to define $L$ and $Q_{L0}$, respectively. The leakage coefficients and flow rate offsets used for this study are presented in Table 3.

**Table 3.** Leakage coefficient values and flow rate offsets for varying operating conditions.

| **Condition** | **Leakage, $L$, ($m^3/(s\times Pa)$)** | **Flow Rate Offset, $Q_{L0}$, ($m^3/s$)** |
|---|---|---|
| Normal | $4.78 \times 10^{-12}$ | $2.41 \times 10^{-6}$ |
| Minor Leakage | $2.52 \times 10^{-11}$ | $1.38 \times 10^{-5}$ |
| Major Leakage | $6.01 \times 10^{-11}$ | $1.47 \times 10^{-5}$ |

A minor leakage is introduced to the system at $t = 3\ sec$ and a major leakage is introduced at $t = 6\ sec$. The effect on the input flow rate can be seen in Figure 4.

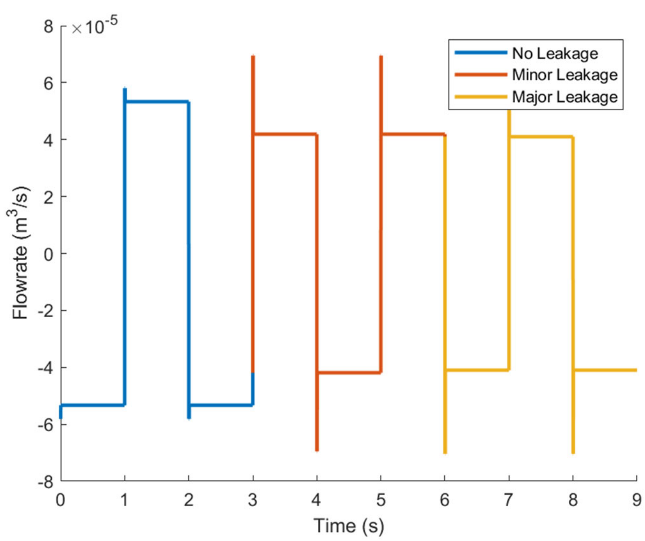
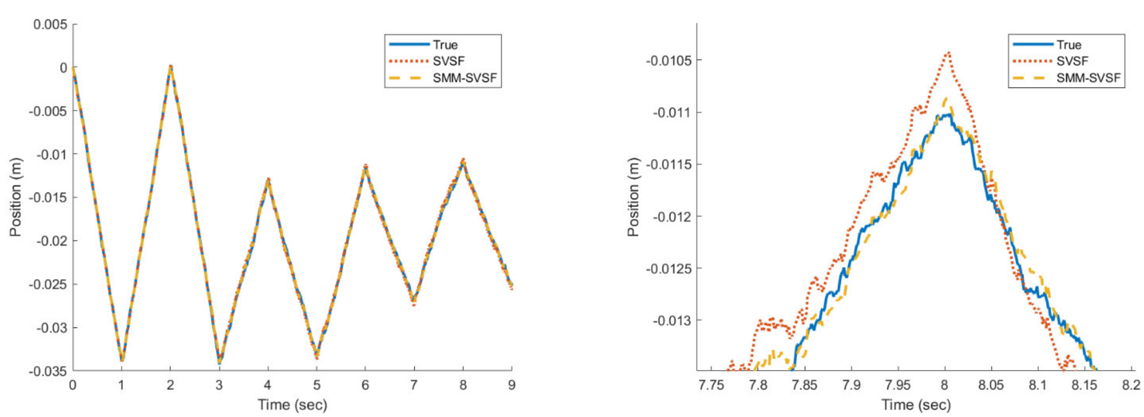



**Figure 4.** Input flow rate due to internal leakage faults.

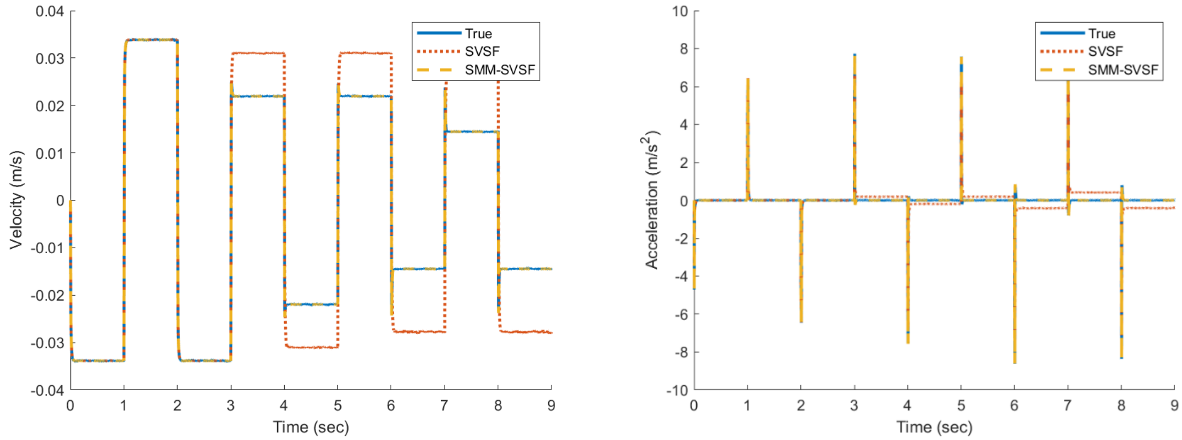
Once the EHA was modeled at all these operations and they were verified experimentally, we used these mathematical models and values in a Matlab Simulation to compare the performance between the proposed algorithm to the traditional one. The benefits of using the simulation can be summarized in two points; the values at certain point are known, i.e. the true state, and the prototype will not be damaged due to the fault when introduced. The results of the simulation are shown in Figure 5 to Figure 8 and table 5.

Figure 5 shows the position estimates, while the velocity and acceleration estimates are shown in Figure 6a and 6b, respectively. The SMM-SVSF performs slightly better than the classical SVSF when the major leakage fault is introduced as seen in Figures 5a and 5b. The SVSF filter shows a significant deviation from the true velocity when the minor leakage fault is introduced at 3 seconds, as shown in Figure 6a. The error becomes worse when the major leakage is introduced at 6 seconds as shown in Figure 6b. This error is caused by the modeling uncertainty of the acceleration state, particularly due to flow rate offset of the input. The greatest improvement can be seen in the velocity and acceleration estimates. Overall, the SMM-SVSF greatly outperforms the classical SVSF in the presence of modeling uncertainties such as leakage faults.




(a) (b)

**Figure 5.** Position estimates with leakage faults:
(a) 9 second simulation, (b) zoomed in at 8 seconds (major leakage).




(a) (b)

**Figure 6.** (a) Velocity estimates for EHA with leakage faults, (b) Acceleration estimates with leakage faults.

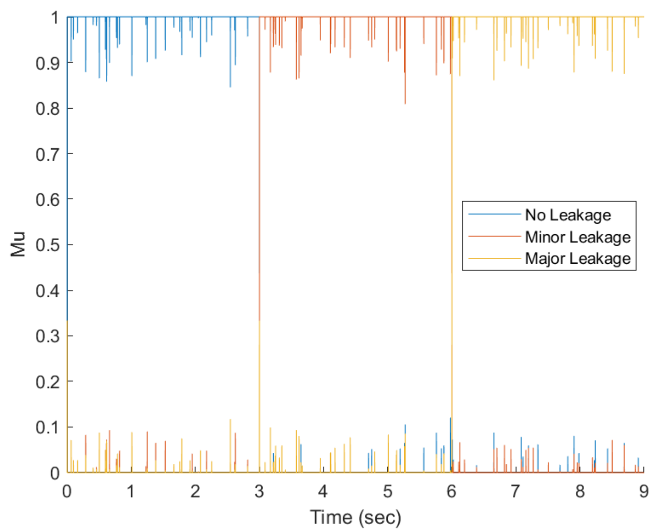
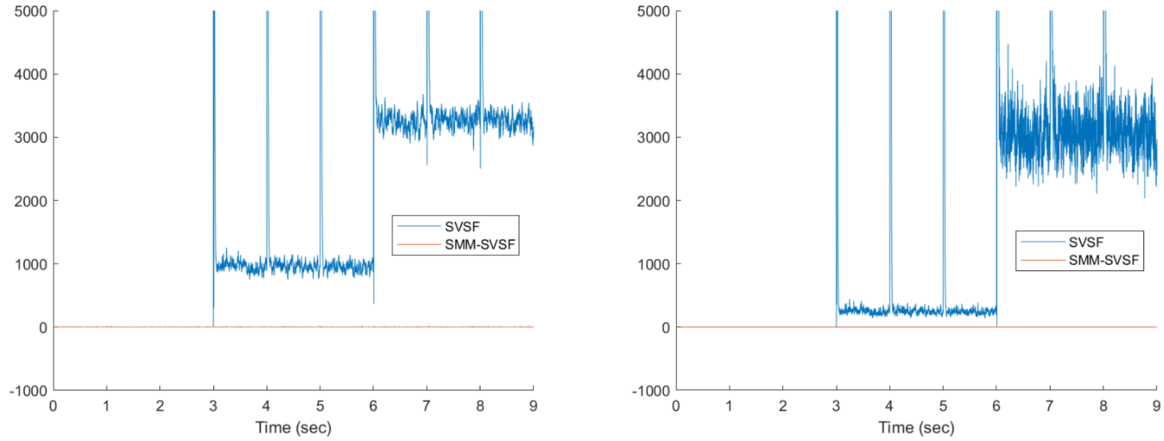



**Figure 7.** Model probability weights.




(a) (b)

**Figure 8.** The differences between SVSF and SMM-SVSF in terms of (a) Innovation squared, (b) Error squared.

**Table 5.** RMSE results for SVSF and SMM-SVSF for scenario with leakage faults.

| Filter | Position (m) | Velocity (m/s) | Acceleration ($m/s^2$) | Differential Pressure (Pa) |
|---|---|---|---|---|
| **SVSF** | 0.0003101 | 0.0091966 | 0.002810 | 0.001002 |
| **SMM-SVSF** | 0.0001828 | 0.0000799 | 0.000712 | 0.001002 |

The SMM-SVSF's ability to determine system modes can be seen in Figure 7, which shows the weights of each system mode used to calculate the estimate. Throughout the entire experiment, the SMM-SVSF filter calculates at least an 80% probability of the correct operating mode at every stage of operation. The figure shows clear transitions from normal operation, to minor leakage, to major leakage at 3 seconds and 6 seconds respectively. The innovation squared (IS), and error squared (ES) are calculated and shown in Figure 8. These two compare the a priori and the a posteriori squared errors between the two algorithms, respectively. Moreover, they show the existence subspaces around the estimates in both prediction and update steps. From the figure, it was easily obtained that the SMM-SVSF is more stable compared to the classical SVSF, and estimates are smoother with no chattering in SMM-SVSF compared to SVSF. The error of the classical SVSF in both steps increases due to introducing the faults, and it spikes when the actuator changes direction. In addition, the RMSE values in Table 5 show that the SMM-SVSF significantly reduces the errors in estimating the position, velocity, and acceleration.

## 6. Conclusions

This paper introduced the combination of the Smooth Variable Structure Filter and Static Multiple Mode estimation strategies to create an adaptive filtering method, SMM-SVSF, that can be used in fault and diagnosis applications. A brief background was provided on the estimation theory up to and including the sliding innovation filter. The SVSF was also included. The SMM-SVSF was tested on an Electro-Hydrostatic actuator. The filter performed well for this particular EHA model due to two main factors: the system parameters of the different leakage modes vary significantly enough for mode differentiation using the SMM method, and the system and measurement noise covariances are well-known. This paper demonstrates that the addition of SMM to the SVSF strategy improves overall estimation process for a system with multiple operating modes, and thereby creates an adaptive SVSF. This can be observed from the results, where the root mean squared errors were reduced by 41%, 99% and 75% for the position, velocity and acceleration estimated states when the SMM-SVSF is applied rather than SVSF. Potential future work will incorporate additional operating modes such as friction faults as well as the mixing of several different operating modes.

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