Financial networks: an agent-based model for the REPO market

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Abstract

Systemic risk is a complex topic, with a large number of variables and constraints. In this thesis we introduce an agent-based network to study the effects of financial shocks on the financial network. The model takes into consideration the repurchase agreement (repo) market and rehypothecation.

We introduce a financial network consisting of financial agents who are connected through direct channels (bilateral contracts) and indirect channels (markets). Each financial agent has a balance sheet with liquid assets (cash), collateral (bonds, shares), reverse repo assets, fixed assets (loans and mortgages) on the asset side and repo loans, deposits and equities on the liability side. Agents (i.e., banks) need to satisfy constraints on (i) liquidity, which deals with financial shocks, (ii) collateral, related to repo liabilities, rehypothecation, and (iii) solvency constraints, ensuring that equity is positive. Liquidity constrain can be broken by a financial shock (e.g., a bank run), while the collateral constraint can be broken by hoarding credit and collateral price reduction. When liquidity and collateral constraints are broken the financial agent will try to fix them through recalling reverse repos and firesale of fixed assets. Banks that fail to fix their constraints by the end of the day will be considered defaulted.

We introduce netting and novation techniques to deal with defaulted banks and lower the stress on the financial markets. In the netting step we lower the exposure of financial agents by removing cycles in the repo liabilities between banks, while in the novation we redistribute the ownership of bilateral contracts and settle any residuals that are left. We also establish that, under certain conditions on the set of defaulted banks, that the novation step is order indifferent.

Different network topologies and balance sheet compositions are tested under several financial shocks to check the robustness of the financial network under our framework.

Declaration of Authorship

I, Hassan Chehaitli, hereby declare that I am the **sole** author of this thesis. To the best of my knowledge this thesis contains no material previously published by any other person except where due acknowledgement has been made. Here is a decomposition chapter-by-chapter and the contribution of other people.

- Chapter 1: solely written by me.
- Chapter 2: Examples 1 and 2 were provided by Prof. Thomas Hurd.
- Chapter 3. Example 4 and the formulation of Theorem 3 were provided by Prof. Thomas Hurd, while the proof of Theorem 3 was done by me.
- Chapter 4: The effect of novation and the design of illiquidity shock were proposed by Prof. Matheus Grasselli.
- Chapter 5: solely done by myself.

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To my family , especially my son Okab, to whom I owe all my accomplishments.

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Abbreviations

ECB European Central Bank

EN Eisenberg-Noe

GHK Gai-Haldane-Kapadia

MBS mortgage backed securities

MtM mark-to-market

REPO repurchase agreement

RH rehypothecation

RSO REPO system operator

Chapter 1

Introduction

1.1 Motivation and Preliminary Definitions

The financial system is a marketplace where many institutions (e.g. banks, funds, insurance companies) conduct different types of activities. One type of activity consists of buying and selling of securities (e.g. stocks, bonds and options). This activity is indirect in the sense that different institutions are buying and selling the same securities in the market without interacting directly with each other. A second type of activity consists of direct bilateral business with other institutions, like taking a loan and posting collateral. The direct and indirect interaction between institutions leads to a financial network.

Studying the stability of the financial system is of great importance to the economy. One aspect of stability is the effect of stressed individual institutions on the overall health of the financial network and whether the stress will spread and cause the failure of the network. Figure 1.1 shows the time series for the percentage of distressed banks in the US and Europe, together with markers for major recessions as referenced in Montagna et al. 2020. The figure shows a threshold of 1.5%, beyond which a systemic event is deemed to have occurred.

There is no precise or agreed upon definition regarding what is a systemic event or what constitutes systemic risk. Some of the definitions are as follows.

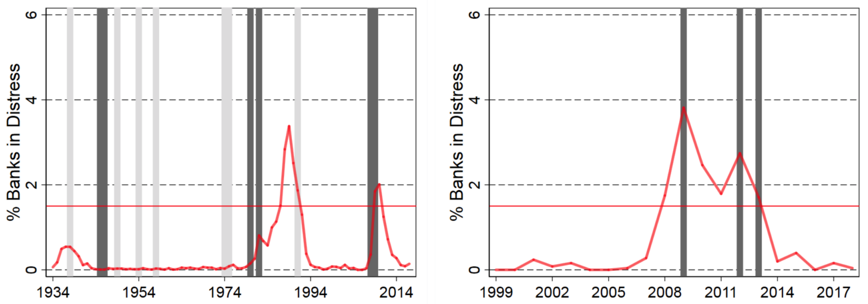


FIGURE 1.1: Left side represents USA banks and right side represents banks in the Euro region. The vertical bars represents economic recession and the color indicates its strength. The red line represents 1.5% level where a systemic event is declared. Source: Lang et al. 2018

Benoit et al. 2017 states that:

> "Systemic risk is often seen as a "hard-to-define-but-you-know-it-when-you-see-it" concept"

In Diebold and Yilmaz 2015 it is mentioned that an "often cited definition" for systemic risk is found in De Bandt and Hartmann 2000:

> "A systemic crisis can be defined as a systemic event that affects a considerable number of financial institutions or markets in a strong sense, thereby severely impairing the general well-functioning of the financial system . . . Systemic risk goes beyond the traditional view of single banks' vulnerability to depositor runs. At the heart of the concept is the notion of . . . particularly strong propagation of failures from one institution, market or system to another"

Another definition is given by European Central Bank (ECB) Bank 2009:

> "There is no commonly accepted definition of systemic risk at present. One perspective is to describe it as the risk of experiencing a strong systemic event. Such an event adversely affects a number of systemically important intermediaries or markets. There is no clear definition what is a systemic risk, multiple definitions have been proposed . . . The trigger of the event could be an exogenous shock which means from outside the financial system. Alternatively,the event could emerge endogenously from within the financial system or from within the economy at large"

In Silva et al. 2017 it is stated that Abdymomunov 2013 defines systemic risk as follows:

> "In general, systemic risk is perceived as the risk of a negative shock, severely affecting the entire financial system and the real economy. This shock can have different causes and triggers, such as a macroeconomic shock, a shock caused by the failure of an individual market participant that affects the entire system due to tight interconnections in the system, or a shock caused by information disruption in financial markets"

Other definitions for systemic risk are also available. Silva et al. 2017 has a collection of definitions that are associated to the problem at hand. As we will see in the next section, the definition of systemic risk also varies according to the criterion used to collect and classify papers in the literature.

1.2 Financial Agents and Dynamics

The financial system can be seen as an interconnected web, both on the local and global scale, and the same is also true for the economies in which the financial system operates. An economic crisis or a financial distress in a certain part of the world will travel through this web to other parts of the world, contributing to systemic risk.

The subprime mortgage crisis of 2008-09 and the recent COVID-19 pandemic and the ensuing financial crisis had global financial implications that illustrate the nature of systemic risk in the financial system. One of the main lessons learned in these crises is that regulating financial institutions is a complex task since a financial institution posseses a dynamic nature. Macro and micro level regulations should be considered with extra care as contagion channels interact in complex and sometimes unpredictable ways. Since the subprime crisis, researchers from different disciplines are developing new models to monitor, measure and manage systemic risk.

There are two main approaches for modelling the financial system. The first approach is known as top-down, where we have a bird's eye view of the system and much of the interactions of its individual components is hidden. The second approach is known as bottom-up, where we look at the individual components, observe their interactions and then we can understand the aggregate effect of theses agents on the system as a whole.

1.2.1 Financial Agents and Complex Interactions

Different types of agents exist in the financial system. Some of these agents are simple in nature while other agents are complex and composed of other sub-agents. We will take a look at these agents with some brief description about each of them.

We have the following agents as described by Bookstaber 2019:

- **Bank**: Banks can be thought as the middlemen in the economy. A bank is made of multiple departments (sub-agents).

- **Prime Broker**: this department works as a connection point between hedge funds and cash providers/security lenders (see below). It collects collateral from different hedge funds and tries to find a funding source (cash provider), as well as arranges unsecured funding directly from cash providers.
- **Finance Desk**: this department is responsible for secured funding for both the bank and its hedge fund clients. In order to fund bank operations, collateral is passed from the bank to the cash provider. As for hedge funds, collateral received by the prime broker is passed to the finance desk, and the finance desk will rehypothecate that collateral in order to raise funds from cash providers. Secured funding is usually done with repurchase agreements (REPOs), the central topic of this thesis.
- **Trading Desk**: this department works as a security broker by filling orders for the bank and clients as well as by having long positions and short positions in the market. The trading desk also manages inventory of securities that it holds on its own behalf and for its clients, and is therefore exposed to market risk.
- **Treasury**: this department is responsible for funding bank operations, in particular unsecured funding. Unsecured funding is used in the case it is hard to find secured funding for certain operations

- **Hedge Funds**: Hedge funds are leveraged financial institution. They look for funding sources in order to support their leverage positions. Usually, this is done by posting the security bought on margin as a collateral to the prime broker in return for funding to purchase the security. Hedge funds also deal with prime brokers when they want to short-sell securities.
- **Cash Provider**: Cash providers encompass a wide range of agents but the important point to keep in mind is that cash providers are short-term, secured-funding

lenders. Since the money that is lent is short term, cash providers can recall the money almost immediately which can affect funding liquidity to financial agents.

- **Asset Markets**: these include markets for different traded financial instruments, such as options, equity and foreign exchange. In normal times these markets are supposed to be liquid in the sense that orders that are placed in market will not have severe effect on the asset price. During stressed times asset prices will be affected severely due to liquidation to meet regulations and policies.

It can be seen from the financial system described above that banks play an important role due to their diverse departments and their interaction with other financial agents through different paths by the exchange of funds, securities and collateral. Although we have limited types of financial agents in this thesis, the financial model is still complicated with all these heterogeneous agents that have different heuristics and different goals.

We have mentioned earlier that banks work as the middlemen between other agents that constitute the financial system. Their interactions with other agents is a complex process. A bank's job is not only to bring and pass assets around, but also introduce transformations to assets in order to meet its client needs, which in return introduce nonlinear relations to the system and thus leads to heterogeneous interactions of agents. Some of the transformations cited in Bookstaber et al. 2018 are discussed below:

- **Maturity transformation**: this happens when a bank takes a short-term funding from depositors and money markets and turn it into a long-term funding for its clients. This is usually referred to as maturity mismatch between assets and liabilities. Maturity transformation is done for a fee paid by the agent who wants to borrow funds. Maturity transformation introduce risk to the bank especially in stressed times where short-term depositors redeem their money back and are not willing to roll forward the funding given to the bank. On the other side, the long-term maturity is not redeemable by the bank which affects bank liquidity and leverage.

- **Liquidity transformation**: this refers to the easiness of converting an asset into cash in a short amount of time without incurring any significant losses due to the short notice. Banks usually take highly liquid assets and transform them into less liquid long-term investment options. For example, banks can take deposits in the form of cash (a highly liquid asset) and invest in real estate property which is fairly illiquid compared to cash as it takes time to sell the property. Selling real estate property on a short notice can incur significant loss to the investor. Another example consists of a trading desk structuring debt instruments into different tranches of collateralized debit obligation products. Just like maturity transformation, liquidity transformation introduces risk to the bank, especially in stressed times.

- **Credit transformation**: this happens when funds are transferred from the cash provider to hedge funds. Under a typical REPO contract, to be analyzed in detail throughout this thesis, a cash provider asks for collateral in exchange of cash. This lowers, and ideally removes, the risk of loss when the counterparty defaults, which is why REPOs are an example of what is called secured funding, but introduces funding costs for the party that is posting the collateral. For this reason, not all funding is secured: sometimes a financial institution can seek funding without posting collateral, for example through interbank loans. When a hedge fund asks for unsecured funding from the bank and the bank has to post collateral to the cash provider this introduces credit mismatch between the bank, the hedge fund and cash provider. If the hedge fund defaults, the bank might not receive any money from it, but still has to pay the cash provider.

The transformations above show the complexity introduced by the bank by interacting with different agents and by catering for their different needs. This leads to the well-known phenomena called the fallacy of composition, namely the false belief that the behaviour of the system can be explained in terms of the individual agents that constitute the system. Rather, when we have a collection of heterogeneous agents interacting in a complex way, we observe what is called emergent phenomena, that is, the system behaves as a whole in an unexpected way.

1.2.2 Contagion Channels

Shocks move in the financial system through contagion channels. Contagion channels connect agents in different ways and induce different magnitudes of stress on the financial system. Here are the contagion channels discussed in Aymanns et al. 2018a:

- **Counterparty loss**: financial institutions can be related to each other through their balance sheets by entering into bilateral contracts. A bilateral contract connects financial institutions by being an asset in the balance sheet for one bank while at the same time being a liability in the balance sheet for another bank. Multiple banks can be linked together as a chain through their balance sheets by bilateral contracts. A shock to the market that affects one bank and causes it to default will make the default propagate through the chain due to the interconnectedness of these balance sheets.

- **Overlapping portfolios**: in this channel, financial agents are connected through traded securities. When different financial agents invest in the same security and when one of the agents receives a shock and is obligated to liquidate in order to meet some regulations and rules, the shock propagates to the other agents. Security liquidation has to happen fast regardless of the loss that can be incurred. Liquidation will cause the price of the security to fall which will affect other financial agents holding the same security. This price drop in one security, might push the financial agents to liquidate even other securities they hold, which can potentially lead to the spread of the initial shock to all securities and all agents in the securities market.

- **Funding liquidity**: this is an indirect contagion channel that is created by the effect of counterparty loss and overlapping portfolios. At the end of every predetermined period the bank has to decide whether to roll forward the loans given to other banks or not. Suppose we have two banks i and j, where bank i has lent money to bank j. If bank j incurs a credit loss, or if its credit rating deteriorates, then bank i might decide not to roll the loan for bank j. Bank j now will need money and it will stop rolling its loans to other banks and this action will cascade through the network. Conversely, when bank i is under stress itself, it will also stop rolling loans so that it can collect money and pay its liabilities. The chain of not rolling loans will cascade through the financial network.

The effects of different contagion channels are often combined. When a bank incurs counterparty loss it might be required to liquidate some of it securities quickly which will have an effect on other securities and financial agents due to portfolio overlapping. If a security receives a shock it will lead the leverage of banks holding that security to increase, thus forcing them to liquidate this security and other securities due to portfolio overlapping. This might create a liquidation spiral in the assets market affecting all agents in the financial system. Finally, even when banks stop rolling loans in order to collect money to meet liabilities, they might still be short on money, which will require them to liquidate some securities and, due to the portfolio overlapping channel, also lead to a liquidation spiral.

1.2.3 Dynamics of Financial Crises

Financial crises are rare and unpredictable. There are important factors that contribute to the crisis, like shocks (i.e. through funds and assets), and there are catalysts (leverage and liquidity) that allow the crisis to spread fast in the financial network. The financial system is a highly dynamical system where feedback loops exist and actions are magnified due to the interaction of the agents among themselves and with the environment (see Bookstaber et al. 2018). The following leverage and liquidity catalysts are described in Aymanns et al. 2018a:

- **Leverage**: leveraged investment is the use of borrowed funds in order to expand banks' investments. Borrowing can be either secured, where a collateral is posted, or unsecured. Leverage is used to increase the potential return on an investment. Nonetheless, leverage is a two-edged sword, as it can magnify both profits and losses. The higher the leverage, the riskier it becomes for a financial agent. Highly leveraged financial institutions are highly sensitive to small fluctuations in the markets. Usually leverage is determined by the haircut that is required by the cash provider (see Section 2.2 for a precise definition) and this is known as maximum leverage available. On the other hand, there is a regulatory leverage that is imposed on financial agents. Whenever the regulatory limit is exceeded the financial agent is expected to lower their leverage through a liquidation process. Leverage can be measured in different ways; frequently used form is the ratio of assets to equity, i.e. $\lambda = A/E$, where equity is defined as the difference between assets and liabilities, that is $E = A - L$. When assets approach the level of liabilities, equity approaches zero, leverage tends to infinity, and the bank is deemed to be *insolvent*.

- **Liquidity**: liquidity is an ambiguous term that can have different meaning depending on the context. In this thesis we will define three types of liquidity. The first is *asset liquidity*, defined as how fast a bank can convert an asset to cash. Usually assets are divided into different liquidity categories according to how easy it is to convert them to cash. For example, real estate, mortgages and loans are highly illiquid assets, in the sense that you can not demand borrowers to pay these long term assets immediately, whereas the most liquid asset is of course cash. The second type is *funding liquidity*, defined as the ease of borrowing from cash providers with low funding costs. Changes in funding liquidity can impose high risk on the financial agent. A bank that cannot meet its short-term liabilities because of either asset or funding illiquidity is known as an *illiquid bank*. An illiquid bank is not necessarily insolvent: a bank can have illiquid assets that cover more than the short-term liabilities but these illiquid assets cannot be converted to cash fast enough to meet its obligations. The last type is *market liquidity* and refers to how quickly and efficiently securities can be sold in the asset market without incurring significant costs or losses due to price fluctuations. Though related to each other, asset and market liquidity are distinct. For example, a long-maturity Treasury bond has low asset liquidity according to the definition above, because the principal can only be received if the bond is held to maturity, but typically has high market liquidity, as it can be sold in secondary markets relatively easily, without any significant effect on its price. Conversely, an overnight reverse-REPO has high asset liquidity, as it can be converted into cash very quickly simply by its holder not rolling it over, but has low market liquidity, as there are no secondary markets where it can be sold.

Leverage and liquidity (all three types) play an important role in any financial crisis. A Financial crisis can be started by having a shock in different layers (assets or funds) and each layer involves different players. Even though the financial crisis starts at one layer (assets/funds) it will eventually move on to the other layer (funds/assets) since banks have multiple layers.

The first place for a shock to happen is at the assets layer. A shock at the asset layer will involve three agents: prime brokers (bank department), hedge funds and assets in the market. As we have explained earlier, highly leveraged institutions are highly sensitive to market fluctuations. If a shock is given into one of the assets in the market (decreasing asset value), a moderately leveraged institution can become highly leveraged. As equity decreases, by definition of leverage, leverage will increase most probably beyond regulatory limits. Exceeding regulatory leverage limits will make financial institution liquidate assets thus depressing asset values more. Less leveraged institutions will eventually be affected and forced to liquidate assets as well. This will then be transferred through contagion channels thus leading to forced selling in all markets due to the fact of overlapping portfolios, asset liquidity limitations and the need to meet regulatory limits. This will lead to decrease in collateral value as well, since collateral usually consists of assets bought from the market, leading to a funding shock cycle of the sort we explain next.

The second place for a shock to happen is at the funding layer. Two agents are involved in this cycle: the finance desk (bank department) and the cash provider. A Funding crisis can start due to deposits being redeemed by investors or due to trust issues where a higher funding cost is imposed. Less trustworthy financial agents will face hardship to meet their short-term obligation and this might cause default. Financial institutions are connected through contagion channels thus leading to reduction of funding to all institutions due to uncertainty. As institutions start holding up their money, funding sources will dry up and the only way for banks to meet their short-term obligations is to liquidate assets. The fast liquidation process this will introduce a shock to the assets market and this will lead to the cycle described above.

Not only are contagion channels important in the financial crisis, the catalysts play an important role as well. If markets are liquid and they can process large orders without significant disruption of prices, then the markets will absorb the shock and nothing will be transferred from one agent to the other through contagion channels. On the other hand, leverage plays an important role as well. If financial institutions are not leveraged, they will not be forced to deleverage which will lead to asset firesale, price depreciation and further deleveraging. Though leveraging worsens financial crises, market liquidity plays a more critical role than leveraging. If leverage companies are forced to liquidate into highly liquid markets the shock will be absorbed without being transferred or affecting other agents.

1.3 Literature Review

As we have just explained, the financial system is composed of different interacting components with each other and with the outer world. Interdisciplinary approaches, techniques and tools have emerged to deal with the complexity of the subject. In this section, before introducing in more detail the models that form the basis for this thesis, we will look at the different classification schemes for the large and growing literature on financial systemic risk proposed in the following papers (with the main classifying criterion in brackets): Benoit et al. 2017 (sources of systemic risk), Accornero et al. 2020 (collateral re-use) , Aymanns et al. 2018b (contagion channels), Iori and Mantegna 2018 (network structure), Silva et al. 2017 (method or object). In addition to providing an opportunity to highlight the main topics of interest in this area, this literature review will allow us to better contextualize the contributions of this thesis.

1.3.1 Classification by Sources of Systemic Risk

Benoit et al. 2017 encompasses a wide umbrella of 220 papers for classification and adopted the following definition for systemic risk:

> "We define it as the risk that many market participants are simultaneously affected by severe losses, which then spread through the system"

Figure 1.2 shows how Benoit et al. 2017 classified the surveyed papers into four categories according to the source of systemic risk as follows:

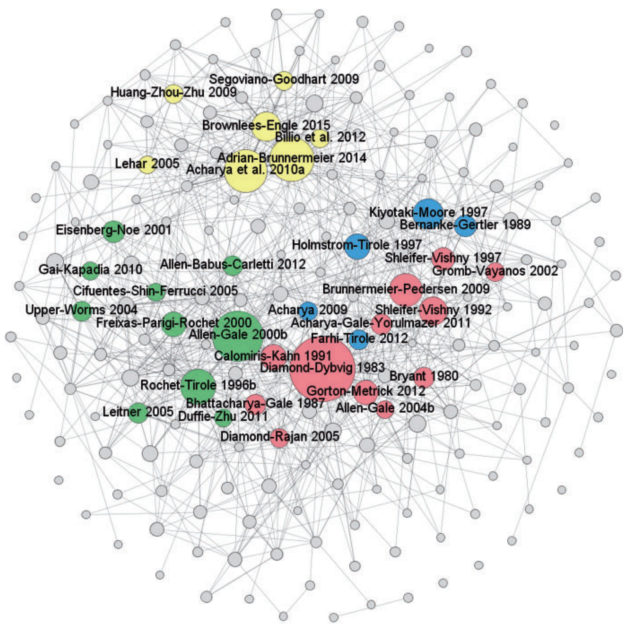


FIGURE 1.2: The classification of papers in Benoit et al. 2017 according to the source of systemic risk: (a) systemic risk-taking (blue/right), (b) contagion (green/left), (c) amplification mechanisms (red/bottom). A fourth category, dedicated to systemic risk measures is also shown (yellow/top). The size of the circle indicates its popularity within the surveyed literature, while the edges shows citations.

(a) Systemic Risk-Taking

Systemic risk-taking studies why financial institutions choose to have correlated exposures and how this affects their own stability and default probability, which in turn leads to the contagion of distress associated with such event. Defaulted banks face costs and impose stress and costs on non-defaulted banks. In order to lower such costs and distress, for both defaulted and non-defaulted banks, it is in their best interest to either survive or default together. The impact of too many defaulted banks is disastrous to the economy, which leads to government intervention in the form of bailouts. Benoit et al. 2017 recognizes the following channels for systemic risk-taking behavior (with representative papers in brackets):

- **Correlated investments:** institutions have overlapping portfolios through investing in same assets (Acharya 2009; Acharya and Yorulmazer 2008b; Acharya and Yorulmazer 2008a; Farhi and Tirole 2012).
- **Liquidity risk:** liquidity risk arises when banks invest in illiquid assets or there is a mismatch, in the term length, between liquidity of assets and liabilities (Bhattacharya et al. 1985; Brunnermeier and Oehmke 2013).
- **Tail risk:** tail risk indicates how the size of the the exposure of a bank. When banks have a big tail risk combined with correlated investments this leads to a large number of banksbeing stressed and in turn to the amplification and contagion of the event (Gennaioli et al. 2013; Biais et al. 2010; Freixas and Rochet 2013).
- **Leverage cycles and bubbles:** leverage cycles happen between good times and bad times in financial markets. In good times, financial institutions and investors expand on their investing and take bigger risk. In bad times, losses happen and liquidation is inevitable (Adrian and Shin 2014; Bhattacharya et al. 2011); bubbles happen when big investments are concentrated in a specific asset. Leverage cycles and bubbles are linked (Allen and Gale 2000a).

(b) Contagion

Contagion in financial markets is similar to disease contagion in humans. A human gets infected by a virus, which distresses the immune system of that person. Upon contact between an infected person and non-infected, depending on the immunity condition of the non-infected person, the infection will spread. A similar analogy can be used for when a shock (virus) hits a financial institutions (human). If the financial institution is weak (i.e over leveraged, maturity missmatch) it will become distressed/default and distress will be passed to other institutions through direct (bilateral exposure) and indirect (financial markets) channels. Benoit et al. 2017 recognizes the following channels for contagion:

- **Balance sheet:** When banks have direct contact with each other also known as bilateral connections. An example of a bilateral interaction is a loan from bank i to bank j These bilateral links between banks create a complex network of exposures between banks where a set of distressed banks can cause other bank to be distressed. The network can be a double-edged sword: just as it can behave as a contagious medium, it can also reduce the risk of banks default. Allen and Gale 2000b show that a complete network (i.e. one in which each bank is connected to all others) is more robust than an incomplete network, whereas Freixas et al. 2000 show that a circular chain is less stable than a complete network. Allen et al. 2012 show that a network with disconnected clusters prevents contagion when extreme events occur.

- **Payment and clearing infrastructures:** Banks can have indirect links because of their customers. Customer operations can lead different banks to owe money to each other. Clearing these payments can be a smooth operation in good times, but in times of stress these operations can freeze.

- **Information:** As we discussed earlier, financial institutions are correlated in investments and tail risk; this correlation allows investors and depositors to draw conclusions about others banks given some information regarding a specific bank. This is known as contagion of information. If a well-known bank has some issues regarding meeting its liabilities, investors and depositors will assume the same thing regarding other banks (Chen 1999; Acharya and Thakor 2016; Cespa and Foucault 2014).

(c) Amplification Mechanisms

Amplification deals with the reasons and mechanisms of why financial shocks on certain banks spill over to other banks and end up stressing a large portion of the financial network. Benoit et al. 2017 recognizes the following channels for amplification:

- **Liquidity-driven crises:** liquidity refers to a bank having enough cash or highly liquid assets to meet short term obligations like interest payments/margin calls. When a bank has a liquidity crisis the bank is forced to liquidate assets which are not highly liquid to cover up for its obligations. This liquidation process will cause the asset price to drop down, thus more liquidation is required by the banks to cover their required obligations (Allen and Gale 2004; Shleifer and Vishny 1992; Gromb and Vayanos 2002).

- **Market freezes:** a simplified definition of a market freeze is that financial transactions stop despite the benefits (e.g. profits, buying or paying obligations) that result from such transactions. One example of a market freeze is observed in REPO markets, a central topic in this thesis: in good times, investors do not acquire information about the collateral that is posted, but as market conditions change investors start questioning the value of the collateral and information gathering starts (see information contagion above). The asymmetry of information problems start and investors will try to avoid certain types of collateral. This in turn leads full chains of collateral to freeze (Heider et al. 2015; Gorton and Ordonez 2014a; Gorton and Ordonez 2014b).

- **Coordination failures and runs:** In bad times, bank creditors tend to have herd behavior. The problem is amplified by term mismatch between assets and liabilities, especially if banks depend on short-term funding. As creditors rush to call back their money in case of distress, this will leave the bank with big exposure and liquidity crises (Calomiris and Kahn 1991; Martin et al. 2014; Bernardo and Welch 2004). As we see, the sources of contagion do not behave independently but rather interact which leads to amplification of the contagion problem.

1.3.2 Classification by Collateral Re-use

As we will see repeatedly throughout this thesis, the main idea of repurchase agreement (REPO) is to secure financial transactions. A REPO is accomplished when a borrower gives the lender collateral. The collateral will be held by the lender in case the borrower defaults. If the borrower defaults, then lender has the right to liquidate the collateral and cover its loss. Collateral re-use (rehypothecation) happens when the lender borrows money on a collateral that it received from a previous REPO transaction and posts it to another lender, as shown in Figure 1.3.

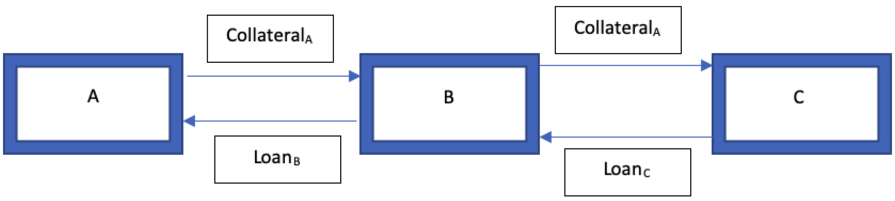


FIGURE 1.3: This figure shows two REPOs. In the first REPO, between banks A and B, bank A gives bank B a collateral and bank A gives a loan to bank B. In the second REPO, between banks B and bank C, bank B posts to bank C the same collateral it received from bank A.

According to Accornero et al. 2020, there has been a debate on whether REPOs stabilize financial markets or not. The following ideas are noted for collateral use and re-use:

- Collateral use links securities and credit markets thus forming a channel to transfer systemic risk.
- Collateral re-use allows the same security to be used by multiple financial institutions thus linking the financial institutions together.
- Collateral re-use allows multiple financial institutiosn to borrow on the same security thus increasing leverage of the financial system.

On the one hand, Accornero et al. 2020 classifies papers based on "collateral re-use in connection with systemic fragility" according to the following mechanisms:

(a) REPO runs (coordination failure)

A REPO run is similar to bank run in the sense that financial institutions and investors stop REPO rollover. Stopping rollover can take different forms: rejecting entering into a new REPO transaction, increasing hair cut, modifying margin call specification, rejecting specific collateral type and depressed collateral price (Gorton and Metrick 2012a; Martin et al. 2010; Kuong 2015).

(b) Costly information and declining collateral quality

Costly information comes in the form that one side of the financial transaction has valuable information that the other side lacks. For example, in the 2007-2008 mortgage backed securities (MBS) crisis, financial institutions packaged the MBS and sold it as a high quality security to investors. Investors did not know that many of the individual mortgages that were packaged were owned by struggling home owners. This information asymmetry led investors to be stuck and the financial markets to collapse. Collateral quality is also an important aspect in a REPO market. The better the quality of a collateral the easier is the transaction between highly rated and lowly rated financial institutions. In good times, the requirement for high quality collateral is eased and the quality of posted collateral starts to decline in the financial markets. In bad times, low quality collateral will lose a large amount of its value when it is liquidated and will result in huge losses or not being able to cover losses by defaulting banks (Gorton and Ordonez 2014c).

(c) Margin/leverage cycles/leverage spirals

Leverage happens when financial institutions borrow money to invest. In good times, credit is available for cheap interest and financial institutions borrow large amounts of money to invest in financial assets. This will lead to an increase in leverage, and consequently, because of higher demand, security prices go up. As bad times arrive and security prices drop, this will lead to big losses for financial institutions and will further lead to fire sales to close financial positions. After closing financial positions and losses are realized, the leverage of the financial institution will decrease (Geanakoplos 2003; Geanakoplos 2010; Geanakoplos and Zame 2014; Brunnermeier 2009; Brunnermeier and Pedersen 2009; Mancini et al. 2016; Ranaldo et al. 2016; Ranaldo, Wrampelmeyer, et al. 2016).

(d) Collateral scarcity

Having high quality collateral is important in order to preserve collateral value in bad financial times. Not having enough high quality collateral will make institutions substitute lower quality collateral and subject them to fire sale exposure. Another possibility is that many of the financial institutions will be excluded from financial transactions (Domanski, Neumann, et al. 2001; International Settlements 2013; Heider and Hoerova 2009; Gorton and Ordonez 2022).

Additionally, Accornero et al. 2020 classifies papers based on “collateral re-use and for the assessment of its interactions with the rest of the financial system”:

(e) Re-use, liquidity, and leverage

Collateral re-use allows more financial institutions in the financial markets if collateral quality is an issue. More over, re-use eases market prices for collateral by stopping competition for collateral. On the other hand, collateral re-use allows leverage to build up fast especially if hair cuts are low for high quality securities (Bottazzi et al. 2012; Andolfatto et al. 2017a; Gottardi et al. 2019).

(g) Re-use, inflation, and liquidity

If cash and collateral are both available, collateral will be preferred. Inflation lowers the value of cash with time. By limiting the number of times a collateral can be used this will create a balance between cash and collateral demand (Andolfatto et al. 2017b).

(h) Re-use, leverage, and volatility

As we discussed earlier, collateral haircut has an inverse relation with leverage. The lower the haircut the higher the leverage a security offers. for example, if a security requires 1% hair cut that means the investor only has to put 1% of the value of the security as a down payment. When collateral can be used multiple times leverage is increased as a whole in the financial system. In a leveraged financial system small fluctuations in security price can lead to big wins or losses thus affecting security volatility as investors react to price fluctuations by either taking profits or cutting losses (Grill et al. 2017).

1.3.3 Classification by Contagion Channels

Aymanns et al. 2018b introduces agent-based models as an alternative to the classical models approach(equilibrium models). To encompass a behavioural approach model (compared to classical approach) they adopted the following definition for systemic risk:

> "Systemic risk occurs when the decisions of individuals, which might be prudent if considered in isolation, combine to create risks at the level of the whole system that may be qualitatively different from the simple combination of their individual risks. By its very nature systemic risk is an emergent phenomenon that comes about due to the nonlinear interaction of individual agents. To understand systemic risk we need to understand the collective dynamics of the system that gives rise to it."

The classical approach deals with equilibrium models. In equilibrium models there are many simplifications and assumptions. One of the main assumption is the rational behaviour of the agents. One of the main criticisms of the classical approach that during stressed times the rational behaviour and equilibrium are unattainable.

Aymanns et al. 2018b defines a contagion channel as:

> "A channel of contagion is a mechanism by which distress can spread from one financial institution to another. Often the channel of contagion is such that distress can only spread from one institution to a subset of all institutions in the system. These susceptible institutions are said to be linked to the stressed institution. The set of all links then forms a financial network associated with the channel of contagion."

Aymanns et al. 2018b classifies papers according to the following contagion channels:

(a) Counterparty loss

Financial institutions balance sheets are linked together through direct bilateral contracts. A direct loan is an example of a bilateral contract. Retrieving the loan from a counter party depends on the default probability and recovery rate of the borrowing financial institution. When a financial institution is hit with a shock (endogenous or exogenous), this increases the probability of default and might affect recovery rate as well. The change in default probability and recovery rate of the borrower will affect the market price of the loan that is given by the lender. Lowering the loan value puts stress on the lending financial institution and increases the lender default probability. Contagion starts in the network as there are bilateral links between financial institutions in the network (Eisenberg and Noe 2001; Gai and Kapadia 2010; May and Arinaminpathy 2010; Elliott et al. 2014; Acemoglu et al. 2015; Battiston et al. 2012; Capponi et al. 2016).

(b) Overlapping portfolios

As there are direct links (bilateral contracts), financial institutions can also be linked indirectly. When two or more financial institution hold the same security in their balance sheets, balance sheets will be linked together through the fire sale mechanism. Assume we have banks A and B. Both banks hold security S. When bank A is stressed, it will try to cover its losses by liquidating security S in the market. As the fire sale starts the security price drops, which in turn will affect bank B's balance sheet. Now bank B faces losses due to bank A's stressed situation. This stress effect will affect the entire network through market connection (Caccioli et al. 2015; Cont and Schaanning 2017; Eisenbach, Duarte, et al. 2014; Greenwood et al. 2015; Cont and Wagalath 2016).

(c) Funding liquidity

This happens when the lender decides to stop rolling over its loans to its counter parties, which can happen for many reasons: (1) the lender bank has experienced a shock and need liquidity to cover its losses; (2) the default probability of the borrower bank has increased which raises concerns regarding its liquidity; (3) collateral value decreased and a new collateral type is needed (in case of secured loan); (4) interest rate increases; (5) in stressed times, financial institution tend to hoard liquidity in case something unexpected happens. It is worth mentioning that there is market liquidity. Market liquidity refers to how efficiently securities can be liquidated without losing value in the market. Fire-sales take advantage of illiquid markets. In illiquid markets selling a large amount of securities tend to depress the security price and thus incurs losses on financial institution balance sheets (Gai et al. 2011; Diamond and Dybvig 1983; Morris and Shin 2001; Anand et al. 2015).

(d) Contagion channel interactions

Contagion channels are not independent but rather spill-over to each other. When spill-over happens this tend to happen in a nonlinear fashion which in turn leads to amplifying the effects of the shocks (Chen 1999; Acharya and Thakor 2016; Cespa and Foucault 2014; Arinaminpathy et al. 2012).

1.3.4 Classification by Network Structure

Financial institutions form links between each other for many reasons. These links on the macro scale form a financial network. Financial networks topology has been studied to check which structures make the network weak and vulnerable to contagion, and which structures make the network robust and resilient to financial shocks transfer between financial institutions. Studying the location of the shock in the financial network and its effect on the spillover has been of great importance as well. Iori and Mantegna 2018 adopted the following classification based on network structure:

(a) Interbank Networks Connectivity and Contagion

There is a debate between academics whether highly dense networks are robust and resilient against financial shocks. Allen and Gale 2000b argues that highly dense networks allows the losses occurred by a bank to be distributed over to many banks in small portions. On the other hand, other academics argue that highly connected networks will increase interaction between agents and have spillover effects (Anand et al. 2012; Lenzu and Tedeschi 2012; Georg 2013; Roukny et al. 2013). Haldane 2013; Glasserman and Young 2015 show that highly connected networks can be robust till a certain point where robustness breaks down, after which spillover and other mechanisms (e.g. bankruptcy costs, mark-to-market (MtM) losses) can amplify the financial shock on the network.

(b) Empirical Interbank Networks

Studies of interbank financial networks have been done on multiple countries. These studies highlight the following points regarding the properties of the networks: (1) Financial networks are not dense/complete, they rather have a fat tailed degree distribution, namely one in which most nodes have few connections while few nodes have the majority of connections (Bech and Atalay 2010; Peter 2010; Langfield et al. 2014); (2) some studies identify the "power-law, scale-free" distribution of the form $k^{-\alpha}$ (Boss et al. 2004; Inaoka et al. 2004; Soramäki et al. 2007); (3) other studies identify structure as "core-periphery" network (Martinez-Jaramillo et al. 2014; Fricke and Lux 2015; Van Lelyveld et al. 2014). Observe that there is a difficulty in classifying the distribution in empirical networks between scale-free and core periphery networks. The reason for this is that scale-free almost surely contains a dense core and sparse peripheries which is similar to core periphery network structure. See figure 1.4 for details.

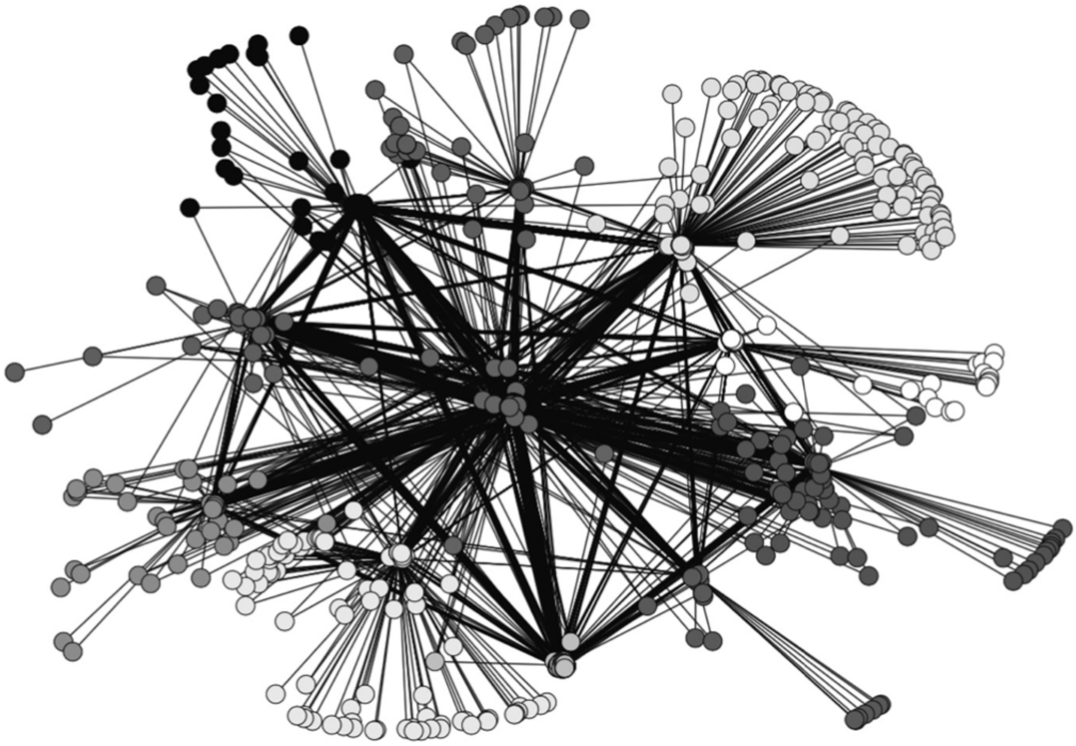


Figure 1.4: Interbank network structure of Austria. Different colors are used for nodes to indicate grouping of financial institutions. Source:Boss et al. 2004

(c) Multilayer Networks

So far our discussion was revolving around single layer networks. The financial network describes connection of banks with respect to a certain contract (e.g. a loan). In real life banks are connected to each other through different types of financial contracts (e.g. loans, options, swaps), and it makes sense to have a network layer for each financial contract. Multilayer networks can lead to a different conclusions regarding financial stability compared to single layer network (Boccaletti et al. 2014; Kivelä et al. 2014). For exmaple, a single layer scale-free network is more robust than a single layer Poisson network, while a multilayer Poisson network can be more robust than a multilayer scale-free under certain conditions (Kenett et al. 2014).

1.3.5 Other Classification Criterions

Silva et al. 2017 uses ten categories to classify financial systemic risk. We will introduce only two of these categories that are relevant to our research, namely:

(a) Methods used

It is evident in this classification category, from table 1.1, that computer simulations, whether used in isolation or combined with other methods, still represent a new area of study, one which allows us to relax some of the assumption and opens new opportunities to understand systemic risk.

Method Used	Number of Articles
Econometric/Statistical/Multivariate Analysis	134
Computational/Simulation	1
Mathematical Modelling	40
Econom./Stat./ Multivariate An. & Comput./Simulation	9
Econom./Stat./ Multivariate An. & Mathematical Model	22
Comput./Simulation & Mathematical Model	15
Econom./Stat./ Multivariate An. & Comput./Simulation & Mathematical Model	5
Not applicable	40

TABLE 1.1: Methods used to study financial systemic risk. Source: Silva et al. 2017

(b) Object of Study

In this classification, articles are grouped according to channels that affect financial systemic risk. Table 1.2 shows that these factors each have been studied thoroughly but the table also shows that research of the effect of multiple channels is not well represented. Table 1.2 shows these channels/factors:

Other classification categories that are used by Silva et al. 2017 are: type of study (theoretical and empirical), approach (quantitative and qualitative), comprehensiveness in geographic terms (one vs multiple country), context (developed vs undeveloped country), focus (bank, insurance or hedge fund), period studied (number of years used), type

Object	Number of Articles
Regulation	71
Market risk	53
Credit Risk/Default/Counterparty/Sovereign	46
Liquidity risk.	23
Contagion	55
Size of institutions.	14
Interconnectivity/Interdependence	54
Concentration/Diversification/Competition	15
Others	56

TABLE 1.2: Object of study for systemic risk. Source:Silva et al. 2017

of data analysed (market, balance sheet or macroeconomic) and results (comparative study and consistency with previous literature).

1.4 Discussion of Network Models

In this section, the Eisenberg and Noe 2001 and Gai et al. 2011 models will be reviewed and discussed, in particular with respect to their assumptions and limitations. These models serve as the building block for the agent-based model discussed in this thesis. We use notation that is consistent with the remainder of this thesis, which in a few instances differs from the notation used in the original papers.

1.4.1 Eisenberg–Noe Model

This section will discuss the Eisenberg-Noe (EN) model, considered one of the pioneer model in contagion modelling of financial networks.

Model Components: The model is made of the following parts:

- A set $[N] = \{1, 2, 3, ..., N\}$ of N financial institutions.

- An $N \times N$ liability matrix with components L_{ij} representing the amount that i owes j, for $i, j \in [N]$, satisfying two conditions: (1) $L_{ij} \geq 0$, as a negative entry would be considered as liability from j to i instead, and (2) $L_{ii} = 0$, meaning that no financial institution has liabilities to itself.

- An $n \times 1$ vector e, where $e_i \geq 0$ represents the cash-flow received by institution i from outside the financial network.

The total liability $\bar{p}_i$ owed by bank i defined as the row sum:

$$\bar{p}_i := \sum_{j=1}^{n} L_{ij}, \tag{1.1}$$

from which we define the *total obligation vector* $\bar{p} = (\bar{p}_1, \bar{p}_2, ..., \bar{p}_n)^T$, as well as the *total payments vector* $p = (p_1, p_2, ..., p_n)^T$ with components $p_i \in [0, \bar{p}_i]$ to be determined.

Define the *relative liabilities matrix* Π as:

$$\Pi_{ij} = \begin{cases} \frac{L_{ij}}{\bar{p}_i}, & \text{if } \bar{p}_i > 0 \\ 0, & \text{otherwise} \end{cases}, \tag{1.2}$$

so that we have $\forall i \in [N], \sum\limits_{j=1}^{n} \Pi_{ij} = 1$. Thus the financial system in EN can be equivalently defined by $(\Pi, \bar{p}, e)$.

Assumptions: The following additional assumptions are listed by Eisenberg and Noe 2001:

- Interbank debit of equal priority: allows the use of proportionality of nominal liabilities to pay creditors, that is, the total cash flow received by a financial institution i is

$$A_i := \sum_{j=1}^{n} \Pi^T_{ij} p_j + e_i, \tag{1.3}$$

where $0 \leq p_j \leq \bar{p}_j$ is the total amount paid by institution j.

- Limited liability: The financial institution will never pay its creditors more than the assets value it receives:

$$p_i \leq A_i \tag{1.4}$$

- Absolute Priority: all available cashflows must be used to pay creditors, that is, in the situation when $A_i < \bar{p}_i$, we must have

$$p_i = A_i \tag{1.5}$$

Combining the three above assumptions Eisenberg and Noe 2001 reaches the following formula for the *clearing payment vector* :

$$p_i = \min\left[\bar{p}_i, A_i\right] = \min\left[\bar{p}_i, \sum_{j=1}^{n} \Pi_{ij}^T p_j + e_i\right]. \tag{1.6}$$

In words, if the total cash flow A_i received by institution i is strictly greater than its total obligations $\bar{p}_i$, then the payment made p_i is equal to the original obligation $\bar{p}_i$ and the institution is deemed not to have defaulted. Conversely, if the total cash flow received is less than or equal to its total obligations, then the payment made equals the total cash flow received, and the institution is deemed to have defaulted.

If we interpret the total cash flow received as the only assets of the financial institution, we can rephrase the above in terms of the capital (or equity) for financial institution i, defined as

$$E_i := A_i - p_i = \sum_{j=1}^{n} \Pi_{ij}^T p_j + e_i - p_i, \tag{1.7}$$

in the sense that the institution defaults if and only if $E_i = 0$, which is equivalent to $p_i = \bar{p}_i$. This simplified balance sheet structure is represented in Figure 1.5, from which it can be seen directly that the balance sheet of EN is rather too simple and it does not represent the real life complexity that financial institution face.

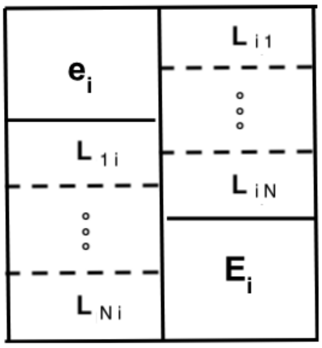


FIGURE 1.5: Bank i Balance Sheet as in EN.

It is clear from (1.6) that the payment made by institution i depends on the payments made by all other institutions, so that the *clearing payment* for the financial network is obtained as the solution to the fixed point equation

$$p = \bar{p} \wedge (\Pi^T p + e) = \Phi(p), \tag{1.8}$$

where $x \wedge y = (\min[x_1, y_1], ..., \min[x_n, y_n])$ for $x, y \in IR^n$. Under certain technical conditions, Eisenberg and Noe 2001 prove the existence and uniqueness of the clearing vector p and use the following *fictitious algorithm* in order to find the fixed point in 1.8.

Clearing mechanism (fictitious algorithm): The aim of the clearing mechanism is to determine how much each bank should pay its neighbor banks after a financial shock and update the balance sheet accordingly. The steps in clearing process are as follows:

- in the first round, compute the payment of each bank given that it will receive its assets value in full from other banks.
- if no bank defaults, terminate the algorithm.
- if some banks default, another round of computation starts. Rerun the algorithm by changing the payment values of the defaulted banks. In this round some banks who are connected to the defaulted banks will not receive full value for their assets.

- invoke a new round of computation until the set of defaulted banks stops changing.

It is worth mentioning that since the system has N nodes, the algorithm will take at most N iterations to terminate. On the other hand, the time point at which a bank defaults while solving the fictitious algorithm will determine the systemic risk exposure of the banks. Banks that default at the beginning have a higher systemic risk than banks defaulting at the end of the fictitious algorithm.

Problems with EN model: The following issues are observed with EN model:

- Overly simplistic balance sheet: as stated above, the only liabilities in the balance sheet of a financial institution i are the payments L_{ij} owed to other financial institutions in the network, whereas the only assets are payments owed from other institutions plus the single external cash flow e_i.

- $e \geq 0$ is a strict condition: Elsinger et al. 2009 provides an example where the algorithm breaks down if this condition is not satisfied. Consider the following financial network $(\Pi, \bar{p}, e)$ and the output:
$$e = \begin{bmatrix} 1 \\ \frac{3}{4} \\ -\frac{9}{8} \end{bmatrix}, \Pi = \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} & 0 \end{bmatrix}, \bar{p} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \xRightarrow{\text{fictitious algorithm}} p^1 = f(p^0) = \begin{bmatrix} 1 \\ -\frac{3}{20} \\ -\frac{6}{5} \end{bmatrix}$$
Which clearly shows the solution is wrong since all entries of vector p should be positive to represent liabilities. The correct answer, according to Elsinger et al. 2009, is $p = (1, \frac{3}{4}, 0)^T$.

- No default costs: (1) assuming a defaulted bank i will receives the entire value of its external assets e_i after liquidating, and thus firesale has no effects in the market on their e_i cashflow. (2) Assuming upon default that the defaulted bank will be able to retrieve their entire loans, represented by $\sum_{j=1}^{n} \Pi^T_{ij} p_j$ without a loss, immediately. Rogers and Veraart 2013 suggested the following extension to EN model:

$$p_i = \begin{cases} \bar{p}_i, & \text{if } \bar{p}_i < \sum_{j=1}^{n} \Pi_{ij}^T p_j + e_i \\ \beta \sum_{j=1}^{n} \Pi_{ij}^T p_j + \alpha e_i, & \text{otherwise} \end{cases} \tag{1.9}$$

Where $\alpha, \beta \in [0, 1]$ and they represent recovery rate. If $\alpha = \beta = 1$ this will retrieve EN model.

- Deterministic and static model: (1) EN is a deterministic in the sense that, once the financial model $(\Pi, \bar{p}, e)$ is initiated, it has no stochastic components during the life time of the model. In particular, there are no fluctuations in market prices or fluctuations in cashflow e. (2) It is static in the sense that when the fixed point problem is solved the model stops. The model starts by assuming that there is a healthy financial network, the network receives a shock and after the shock the fictitious algorithm finds the new value of the liabilities in the network then the model stops. That is to say, it is a one-period model.

- Conservation of losses: Visentin et al. 2016 states that EN suffers from conservation of losses, in the sense that the final cumulative loss cleared by the system equals the initial cumulative financial shock received by the network. Thus EN model lacks amplification mechanisms, and therefore low contagion levels, by design.

- Implicit assumptions: according to Visentin et al. 2016 some of the implicit assumptions that exist in EN model are: (1) removal of uncertainty and stochasticity by full knowledge of exogenous shocks, financial institution balance sheets and financial network. (2) All financial institutions agree to pay what the clearing algorithm dictates. There is no uncertainty in payments and thus there is no amplification of losses due to further rejection of payments. (3) contagion of financial shock does not happen when a financial institution capital reaches zero value, it only happens when the required payments exceed the capital value.

1.4.2 Gai-Haldane-Kapadia Model

Gai et al. 2011 is a threshold cascade model. The main idea of Gai-Haldane-Kapadia (GHK) is adapted from the Watts 2002 model, which is a basis for many cascade models.

In Watts 2002, a randomly connected network has nodes that have binary state 0 or 1. Each node is initialized with a state, with a given probability, and a threshold fraction ϕ, where ϕ is taken from a probability distribution on the unit interval. If a node i has a state of 0, it checks the status of its neighbors. If a fraction of its neighbors larger than the threshold ϕ have status 1 then node i changes its status to 1; otherwise its status stays 0. Let p_k be the degree distribution of the network, z be the average degree of the graph and $\rho_0 \ll 1$ be the portion of the graph that has state 1 at $t = 0$. Under certain technical conditions, a seed which is a node with initial state 1 can grow if one of its neighbors has $\phi < \frac{1}{k}$, where k is number of neighbors, which is known as the vulnerability condition. The main result of the model is that a very small fraction of seeds in an infinite network can lead to activating a considerable fraction of the network, which is known as a global cascade, provided z is in a range of values that indicates the network is neither sparse nor dense. The final fraction ρ of activated nodes in the graph can be computed through two ways: (1) tree-based method or (2) generating function method. Figure 1.6 shows an example of a final activated fraction of the network for different values of z, using tree-based method:

In the remainder of this section, we will discuss Gai et al. 2011 model in more detail. Although the Watts 2002 model uses undirected graph, Gai et al. 2011 is a directed graph and the arguments of Watts 2002 can be extended by introducing in-degree distribution which represents the distribution of edges pointing toward a node from other nodes and an out-degree distribution which represents the distribution of number of edges pointing out from a node toward other nodes.

Model Components: GHK model encompasses a more sophisticated balance sheet

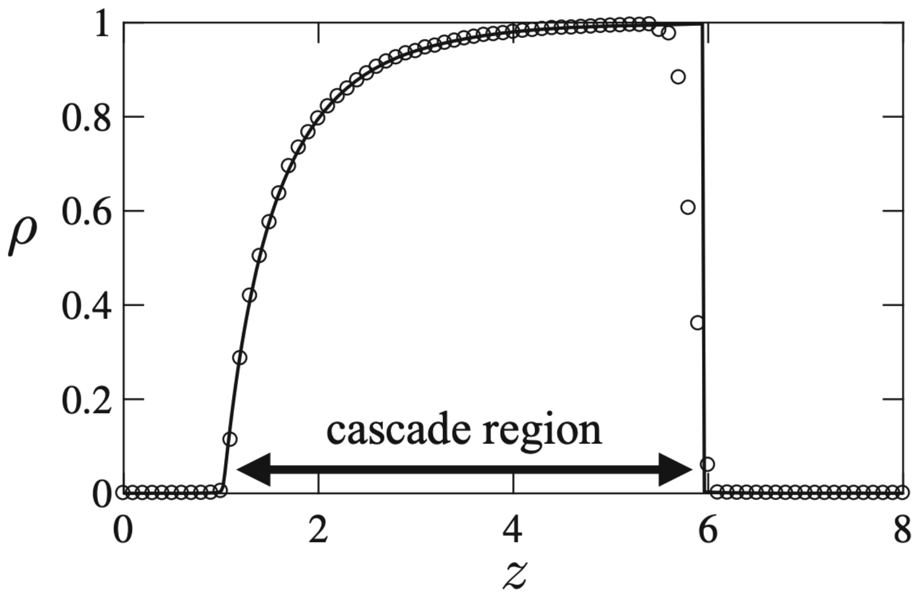


FIGURE 1.6: This graph is generated using $N = 10^5$, $\phi = 0.18$, a seed fraction $\rho = 10^{-4}$, and Poisson degree distribution, z is the mean of the Poisson distribution. Circles represent average cascade size over 1000 runs. It can be seen that cascade fraction develops when the network is not too sparse and not too dense. Source: Caccioli et al. 2018.

and it has more model components compared to EN model as shown in Figures 1.7 and 1.8:

Accordingly, the assets, liabilities and capital (or equity) in the GHK model are given by[1]:

$$A_i = A_i^L + A_i^C + A_i^{RR} + A_i^{IB} + A_i^F, \tag{1.10}$$

$$L_i = L_i^R + L_i^{IB} + L_i^D \tag{1.11}$$

$$E_i = A_i - L_i \tag{1.12}$$

It can be seen that GHK model has a more realistic representation of aggregate

[1]In the remainder of the thesis we use A_i^R to denote the reverse-REPO assets of bank i, but decided to leave this as A_i^{RR} in this subsection to be consistent with Figure 1.7, reproduced from Gai et al. 2011. Similarly, for the remainder of the thesis we use A_i^U and L_i^U to denote the unsecured interbank assets and liabilities denoted by A_i^{IB} and L_i^{IB} in Gai et al. 2011.

Parameter	Description
n	Number of banks
j_i	Number of bilateral unsecured interbank lending links for bank i
k_i	Number of bilateral unsecured interbank borrowing links for bank i
z	Average degree or connectivity
L_i^{IB}	Unsecured interbank liabilities
L_i^R	Repo liabilities (i.e. borrowing secured with collateral)
L_i^D	Retail deposits
K_i	Capital
L_i^N	New unsecured interbank borrowing raised after a shock
A_i^{IB}	Unsecured interbank assets
A_i^F	Fixed assets (e.g. individual corporate loans or mortgages)
A_i^C	'Collateral' assets (assets which may be used as collateral in repo transactions)
A_i^{RR}	Reverse repo assets (i.e. collateralised lending)
A_i^L	Unencumbered fully liquid assets
h	Aggregate haircut applied to collateral used to obtain repo funding
h_i	Bank-specific haircut applied to collateral used to obtain repo funding
$-\varepsilon_i$	Idiosyncratic liquidity shock
μ_i	Fraction of banks linked to bank i which hoard (withdraw deposits from bank i)
λ	Proportion of deposits withdrawn by hoarding banks

FIGURE 1.7: Model parameters used in GHK model. Source: Gai et al. 2011.

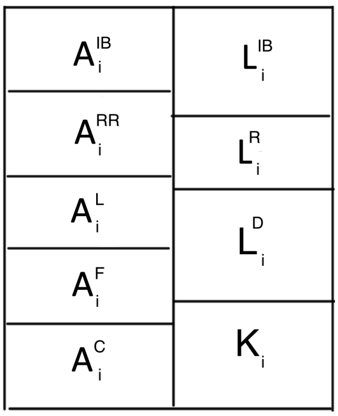


FIGURE 1.8: Bank i Balance Sheet as in GHK.

balance sheet of financial institutions than the EN model, which corresponds to the special case where $A_i = A_i^{IB}$ and $L_i = L_i^{IB}$.

Assumptions: The following assumptions are listed by GHK:

- In-degree distribution equals out-degree distribution: In a directed graph, a joint bivariate in-out-degree distribution is needed. But since every lender should have a borrower this leads to the observation that the average degree distribution of lenders should equal the average degree distribution of borrowers. In this model, z represent the average degree of connectivity of a node.

- Division of interbank liability: L^{IB} of a bank will be divided equally on its counter-parties. That means a bank will have the same exposure to its counter-parties, and concentration of risk will be avoided. It should be noted that although the aggregate value of assets in the network should equal the aggregate value of liabilities in the network, a specific bank can still have a surplus or deficit in its inter-bank assets/liabilities.

- Collateral type: fully liquid assets A^L can be used as collateral but fixed assets A^F and inter-bank assets A^{IB} cannot be used as collateral.

- Hair cuts (h and h_i): the parameter $h \in [0,1]$ represents the risk associated with collateral as perceived by the market and is a function of illiquidity, asymmetric information, and increase in probability of default of underlying instrument. Any change in these factors will lead to a change in h value. In the event of default of the collateral giver, the collateral receiver will liquidate the collateral but might not be able to collect the full value. Thus, h is used as a defence mechanism in case the value of the collateral drops. On the other hand, we have h_i that represent the risk associated with a particular bank defaulting on its obligations. The maximum value a collateral can raise in case of a secured loan is $(1-h-h_i)A^C$.

- Collateral Rehypothecation: if a bank enters in a secured lending (reverse-REPO) it will receive collateral that the bank can re-use in another secured lending to collect money for itself. The maximum refunding a bank will receive in this rehypothecation scenario is $\frac{(1-h-h_i)A_i^{RR}}{(1-h)}$. For example, assume that bank A borrows from bank B using Apple shares in the amount \$100 as collateral with a common haircut $h = 0.1$ and idiosyncratic haircut $h_A = 0$. Bank B will lend A the amount $(1-h)\times\$100 = \$90$, thus B will have $A^R = \$90$ on its balance sheet, corresponding to \$100 in Apple shares pledged as collateral. Assuming that $h_B = 0.05$, Bank B can then use Apple shares as collateral for another REPO with bank C, for which it will receive $(1 - h - h_i) \times \$100 = \$85$.

- Other assumptions: (1) No inflow or outflow of deposits; (2) new capital raising is not allowed; (3) no favourable collateral treatment, all banks get what the market offers; (4) are withdrawn is done in equal proportions from counterparties in order to distribute the pressure of liquidity hoarding.

Contagion dynamics: For contagion to happen, certain conditions have to be met, so that liquidity shocks propagate from one bank to another. Namely:

- Liquidity condition: At each period, bank i needs to have access to funding to pay for a financial shock ϵ_i, REPO borrowing L_i^R, and a proportion μ_i of its counterparties for unsecured interbank liabilities L_i^{IB}, each recalling an average fraction λ, leading to the following liquidity condition:

$$A_i^L + (1 - h - h_i)A_i^C + \frac{(1 - h - h_i)}{(1 - h)}A_i^{RR} + L_i^N - L_i^R - \lambda\mu_i L_i^{IB} - \epsilon_i > 0, \quad (1.13)$$

  where the first four terms correspond to

  - A_i^L: cash assets;
  - $(1 - h - h_i)A_i^C$: maximum REPO funding from collateral;
  - $\frac{(1-h-h_i)}{(1-h)}A_i^{RR}$: maximum REPO funding from rehypothecated collateral;
  - L_i^N: new loans.

- Liquidity hoarding: when a bank recalls loans from its counterparties, this will put pressure on the counterparties to continue their normal operation. In turn, these counterparties will be forced to recall loans from their counterparties and thus the pressure moves through the network in the form of waves from one set of banks to another. The wave of distressed banks will keep expanding until it reaches banks that do not suffer distress, thus the wave stops expanding or all the banks will become distressed as a result of liquidity hoarding. Assume that bank i has k_i counterparties, one of which (represented by $\mu_i = 1/k_i$) hoards liquidity and recalls its interbank loans. Then the liquidity condition in 1.13 will become negative and contagion will start to spread provided we have:

$$\frac{A_i^L + (1-h-h_i)A_i^C + \frac{(1-h-h_i)}{(1-h)}A_i^{RR} + L_i^N - L_i^R - \epsilon_i}{\lambda L_i^{IB}} < \frac{1}{k_i} \tag{1.14}$$

- Stark assumptions: to have a better understanding of the dynamics of the contagion and in order to attain an analytical result, extra assumptions have to be made: (1) assume $j_i = k_i = z$ for all banks, which means rather than generating the network randomly with z as average edge connection, each bank shall have exactly z connections as a lender and z connections as a borrower; (2) no haircut based on individual bank specifics ($h_i = 0$) and no external financial shock ($\epsilon_i = 0$); (3) full recall of loan ($\lambda = 1$); (4) new unsecured lending is not allowed ($L_i^N = 0$); (5) all balance sheets are identical in banks. The last assumption will allow the removal of subscript i from the formula. All these assumptions will lead for formula 1.14 to be written as:

$$z < z^* = \frac{A^{IB}}{A^L + (1-h)A^C} \tag{1.15}$$

Formula 1.15 shows the "tipping point" condition that will lead to the financial shock spreading across the financial network. If $1 \leq z < z^*$, which means there is connectivity in the network and condition 1.15 is met, and any bank recalls loans, all its neighbors will be forced to recall loans and liquidity hoarding happens. Liquidity hoarding will travel the network in waves from closer neighbors to further neighbors since all banks now have the same "tipping point"condition. If the condition in formula 1.15 is not met, a single financial institution hoarding liquidity will not affect the network. Moreover, the numerator and the denominator in equation 1.15 tell us how the composition of the balance sheet affects the spread of the contagion and which components should be increased or decreased. Namely, in order to decrease the likelihood of contagion due to liquidity hoarding, one should decrease the amount of interbank lending (i.e smaller A^{IB}) or increase the amount of liquid assets and collateral (i.e larger A^L and A^C).

As mentioned before, it can be seen that GHK model has a better aggregate balance sheet compared to EN model. But GHK still share some drawbacks with EN model, namely: (1) It is deterministic and static; (2) a financial shock wave is resolved in one time step; (3) the simplifications of 1.15 require too many unreasonable assumptions; (4) default resolution, in particular what happens to rehyptothecated collateral upon default of multiple banks, is not discussed.

1.5 Scope of the thesis

As we have seen, both the Eisenberg-Noe (EN) and Gai-Haldane-Kapadia (GHK) models have strict assumptions and they have some drawbacks and limitations. The main purpose of this thesis is to introduce and analyze a model that addresses these limitations.

We generalize the EN model assumptions in the following way. First we overcome the simple balance sheet in EN by adopting a GHK-type balance sheet. As for $e \geq 0$ condition, we allow A^L in GHK to be negative to present the possibility of a credit line. Although we do not have default costs in our model, they can be easily added by imposing a penalty on the assets values. As for deterministic and static features, we have a stochastic component in our model in the form of security prices, and we can incorporate more stochasticity by adding it to liquidation and fire sale channels. As for conservation of losses, our model overcomes this shortcoming by having a liquidity function that suppresses security prices as the fire sale channel is triggered.

Similarly, we generalize the remaining GHK model assumptions, that is, beyond those that are shared with the EN model, in the following way. The one-time-step financial shock resolution is replaced in our model by a multi-step resolution. Moreover, agents not only have to deal with initial financial shock, but also have to deal with market price fluctuations (stochastic price) as well, whereas market price fluctuations are not present in the GHK model. Moreover, whereas the GHK model does not present default resolution for failed banks, especially for rehypothecated collateral, our model introduces an explicit algorithm called *novation* that not only allows removing defaulted banks with minimal effect to the financial network, it also allows us to tell how much each bank owns of the rehypothecated collateral.

This thesis consists of this introduction and three other chapters. Chapter 2 introduces financial agents, financial contracts and the REPO market in greater detail, as well as the constraints and rules associated with this market. In this chapter we shall discuss important topics that affect financial agent survival while keeping the stability of the financial system a priority. We shall also discuss liability matrix initialization as this process is not trivial . Chapter 3 deals with our proposed novation and auction mechanisms. Novation is a technique used in a default situation, whereby the ownership of contracts that are held by defaulted banks is transferred to other, non-defaulting banks. As we will see, provided there are no liability cycles among the banks to be removed, novation is order-indifferent, meaning that the order in which we remove banks from the network will have no effect on the final result of the balance sheet of the remaining banks. When cycles are present, a netting technique is introduced as well, which is used to lower the total exposure in multilateral contracts between counterparties, and we will observe that netting, unlike novation, depends on the order in which we pick pairs of banks, although the effects of this are reduced when followed by novation. As for the auction part, we will explain how a bank in our model might encounter a financial shock and what the response should be to these shocks. In chapter 4 we introduce simulations in three parts: the first part checks different netting setups and the effect they have on the network; the second part checks different novation setups and confirms that the order of novation does not affect the final result of removing banks from the system; the third part deals with financial shocks and their effects on different network topologies and balance sheet composition. We will notice that using the a hoarding metric only is not enough and that by introducing an additional default metric we will gain a clearer picture of the effect of the financial shocks and a better understanding of the financial agent's behaviour.

Chapter 2

An Agent-Based Model for the REPO Market

2.1 Introduction

Nowadays, banks hold intense connections worldwide. Since the beginning of 21$^{\text{st}}$ century, researchers started to build models for the whole financial system, instead of focusing on single or few companies. For example, in 2001, Eisenberg and Noe 2001 introduce the network model described in the previous chapter to analyze the effect of one default to the whole financial market. As we have seen, they use the uncollateralized liabilities between two firms to construct relative liability matrices describing the connections between firms. Based on this description, they compute the payouts of firms to their counterparties. This model is one of the fundamental models for many subsequent papers, such as Liu and Staum 2010 and Feinstein et al. 2018, which conduct a a sensitivity analysis of the EN model.

Instead of borrowing or lending in the regular loan market, increasingly banks borrow or lend their money through a collateralized loan market – the sale and repurchase (REPO) market. As briefly described in the previous chapter, a REPO agreement is the sale of a security combined with an agreement to repurchase the same security at a specified price at the end of the contract[1]. This transaction can be also viewed as a collateralized loan. Over the last 40 years, the size of the REPO market increased dramatically, doubling in size from 2002 to 2007 alone.

As a result, many researchers include the collateralized loan into their research. For example, Gai et al. 2011 construct the network model described in the previous Chapter, which includes borrowing from a REPO market and withdrawing of the deposits. They focus on an analysis of a shock in haircut of the collateral and its effect to the whole network, using a mean-field approximation. As another example, Luu et al. 2021 build a rehypothecation network model and focus on the effect of collateral hoarding in different network topologies. Relatedly, Paddrik et al. 2018 use a fixed-point method to analyze credit default swaps (CDS) network through central counterparties (CCP) and liquidity buffer. They use a matrix to describe the collateral relationship and introduce a soft default system, that is, one with delayed or partial payments.

During the 2008 financial crisis, one major channel for the spread of the subprime financial shock was the REPO market. Severe bankruptcies of companies led to low confidence in many securities. Because the trades in the REPO market use these securities as collateral, low confidence in these securities caused adverse effects to this market. As the REPO market froze, companies could not use it to finance their positions, which caused liquidity issues that aggravated the ongoing crisis.

In this chapter, we want to introduce the financial concepts related to a REPO contract and the REPO market, as well as an agent-based model that can be used to analyze it. The structure of this chapter is as follows. Section 2.2 fully specifies the type of overnight REPO that will be traded across the network of banks, including the stock-flow consistent treatment of bank balance sheets. Section 2.3 describes the need for an RSO in such a network, that plays the roles of auditor, provider of daylight

[1] For more information regarding regulation and usage of collateral and repurchase agreement (REPO) check details in Gorton and Metrick 2010; Gorton and Metrick 2012b

overdraft to banks, and oversees the removal of any banks from the REPO network when they are identified as insolvent. In Section 2.4, we explain balance sheets of banks, including collateralized liabilities accounts. In Section 2.4.4, we study the source of liquidity pressure. In Section 2.5 we introduce a priority of debt resolution and priority of liquid assets and liquidation process to meet policy constraints. In section 2.6, the full agent-based model is described, together with a discussion regarding the initialization of the liability matrix and balance sheets.

2.2 Repurchase Agreement (REPO) Basics

A contract in the REPO market specifies two transactions. At the beginning of the contract, one party sells a specific security to its counterparty at a given price. At the end of the contract, the party repurchases the same security from its counterparty at an agreed price, which was decided by two parties during the contract's negotiation. Here, the specific security can be seen as a collateral in this "collateralized" borrowing transaction, with the collateral provider seen as a borrower (or cash receiver) and the collateral receiver seen as a lender (or cash provider). The relative increment from the initial price to the repurchase price of the collaterl can be seen as an "interest rate", called the REPO rate. In order to cover the potential loss of a cash lender, the market value of a collateral is typically larger than the cash transaction, with the relative difference being called the haircut, which can vary from 0.5% to over 8% based on different quality of the collateral.

More explicitly, the fundamental parameters of a REPO are:

1. the period $[t_1, t_2]$, $t_2 = t_1 + \delta t$;
2. the market value of the collateral A^C at time t_1;
3. the original sale price L^R at time t_1, or equivalently the haircut h defined by $(1-h)A^C = L^R$;

4. the repurchase price $(1 + r_R \delta t)L^R$ at time t_2, or equivalently the REPO rate r_R.

Sometimes, the margin m, defined by $1+m = (1-h)^{-1}$, is quoted instead of the haircut, so that $A^C = (1 + m)L^R$.

Drilling deeper, one finds a wide diversity across different countries and markets of REPO specifications, the allowed collateral, the regulatory, legal and accounting rules that are applied, and how they are used.

For example, in the most common form, REPOs are initiated by a party that seeks cash and posts collateral. In such cases, the purchase price is less than the market value, in other words, the haircut h is positive, and the REPO rate should be slightly higher than the risk free rate. Moreover, the collateral, called “general collateral”, is some form of a highly liquid, low-risk security such as a Treasury bill or government bond. The alternative form of REPO, called a “special REPO”, is initiated by a party that seeks a particular security, such as a stock, for the purpose of short-selling. Here, the purchase price paid by the collateral buyer will typically be higher than both the market value and the repurchase price, which means the haircut h is negative and the REPO rate r_R is less than the risk free rate.

Another dimension of variation is the duration of the REPO, that may range from overnight (24 hours), to months or even years. Some REPOs are “open”, meaning they roll over at fixed dates until one party opts to close out.

Many different REPO resolution mechanisms are used to deal with events when one party defaults. An important design feature in some REPOs is called “safe harbour” that exempts REPO creditors from the “automatic stay” provisions that prevent other creditors from seizing defaulted assets. This means when the borrower defaults, the REPO lender has priority over the collateral ahead of other creditors. This is important to emphasize becasue, as Maclachlan 2014 explains, although the REPO conveys legal ownership of the collateral to the collateral buyer (lender), accounting rules are such that the collateral seller (borrower) keeps the asset on their balance sheet and receives any dividends or coupons delivered by the collateral.

Often special entities such as a clearing bank (intermediating in triparty REPOs) or the central bank (acting as a liquidity provider) intermediate in the core of the REPO market. Figures 2.1 & 2.2 shows some summary facts about the roles and size of REPO markets in several countries.

Size of repo markets

Jurisdiction	Repo and reverse repo transactions against government bonds (mid-2016)		
	Amounts outstanding (in USD billions)	As a share of global total (in %)	As a share of outstanding government debt securities in jurisdiction (in %)
Euro area	2,800	32	32
United States	2,700	30	16
Japan[1]	2,200	23	21
United Kingdom	900	10	33
Canada	211	2	18
Australia	106	1	18
Mexico	79	1	21
Sweden	74	1	44
Switzerland[2]	10	0.1	11
Total	8,800	100	

Only repos against securities issued by the central government are included. Euro area repos include those backed by the central governments of Austria, Belgium, Finland, France, Germany, Italy, the Netherlands and Spain. The global total is defined as the total of the jurisdictions in the table. The numbers may not add up due to rounding.

[1] Includes transactions against non-government bonds; however, most repo transactions in Japan are made against government bond collateral. [2] Comprises only repo transactions denominated in Swiss francs against high-quality liquidity asset (HQLA) collateral (which does not exclusively consist of government debt) conducted in Switzerland.

Sources: Bank of England Sterling Money Market Survey (United Kingdom); other national central banks; ICMA Repo Survey (euro area); Tokyo Money Market Survey (Japan); SIFMA (United States); BIS debt securities statistics.

FIGURE 2.1: Size of REPO market in different countries. Source: CGFS 2017.

Economic functions (EF) and users of repo

Economic functions of repo	Users of repo							
	Banks	Hedge funds	Money market funds	Insurers, pension funds	Long-only asset managers	Corporates	Public agencies	Central banks	CCPs
EF1. Low-risk option for cash investment	✓		✓	✓	✓	✓	✓	✓	✓
EF2. Transformation of collateral	✓	✓		✓				✓	
EF3. Supporting cash market efficiency and liquidity	✓	✓		✓				✓	
EF4. Facilitating hedging of risk	✓	✓		✓				✓	
EF5. Enabling monetisation of liquid assets	✓							✓	✓

FIGURE 2.2: Functions of REPO contract. EF1:having a REPO contract with high quality collateral reduces risk significantly. EF2:REPO allows institution to acquires certain security that is needed in some transactions. EF3:REPO allows market participants to engage in arbitrage opportunities when mispricing is present. This gives liquidity to the markets and supports market efficiency. EF4:REPO allows financial institutions to buy securities in a cheap manner to hedge their portfolio. EF5:Stressed financial institution can engage in REPO transaction to secure liquidity. Source: CGFS 2017.

As we will see more explicitly in the next section, for the remainder of this thesis, we consider a market in which a single type of REPO is traded, with open roll over and duration of 24 hours.

2.3 REPO Network with Rehypothecation

We now introduce a REPO market with full rehypothecation (RH), designed to retain essential characteristics of the network of overnight REPOs at the core of real world financial systems, while essentially eliminating counterparty risk. The model presented here, henceforth called the *RH REPO network*, is based on use of a single standardized type of REPO, traded across a network of N financial institutions (henceforth called "banks", but might also include some government agencies and large funds), subject

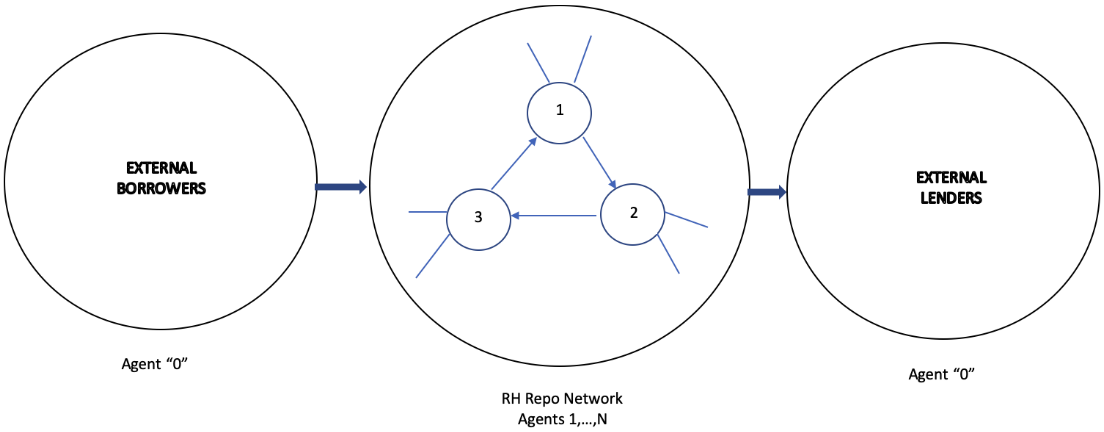


FIGURE 2.3: The full REPO market consists of the RH REPO market at its core, with nodes called banks and labelled by $[N] := \{1, 2, \ldots, N\}$, plus the external agents collectively labelled by "0".

to oversight by a REPO system operator (RSO). Critically, the RSO is endowed with the authority to organize the liquidation of REPO contracts whenever a bank's REPO privileges must be revoked.

The real world context is illustrated in Figure 2.3, showing that the RH REPO system is central to a much larger network, called the *full REPO market* that includes money market funds (REPO lenders), hedge funds etc. Such external agents are henceforth denoted by the label "0". In the full REPO market, REPOs may have a diversity of types (special and general collateral), maturities and other characteristics not applicable to the RH REPO system considered in this thesis. Moreover, the RSO has no direct role or responsibility in the wider network. The focus here is exclusively on the core RH REPO network. In particular, the question of rehypothecation in the wider market will be ignored.

This chapter considers the state of the RH REPO market at a single moment in time when a number of banks are suddenly determined to be insolvent. It is important, however, to place this study into the context of a dynamical model, whose schematics are illustrated in Figure 2.4.

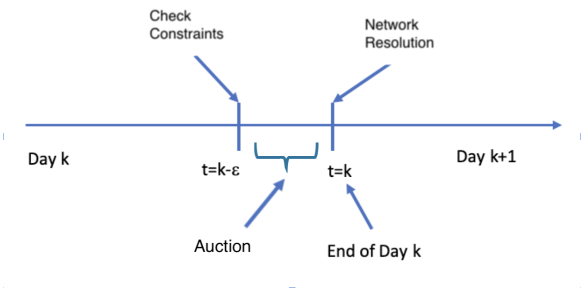


FIGURE 2.4: This figure shows the two scheduled daily events that the RSO manages, with the aim to keep all banks compliant with regulatory REPO constraints and, when necessary, to remove (or "resolve") failed banks. For network resolution refer to section 3.2 and for network auction refer to section 3.3.

It can be seen from this that the RSO's actions are focused on three events that occur at the end of each day.The first event, called *check constraints*, the RSO identifies banks that has broke the collateral or the liquidity constraints, this includes defaulted and non-defaulted banks. Non-defaulted banks will have the chance to go to the second step known as the *auction* step to fix their situation buy calling back repos, selling/buying collateral and selling/buying fixed assets. During the auction step the collateral is considered liquid while fixed assets have liquidity constraints.. Then, comes the last step which is known as the *network resolution* step, in this step the banks that defaulted at the *check constraints* step and the banks that defaulted during the *auction* step because they could not fix their constraints will be removed from the financial system.

2.3.1 Standard REPOs

We define a *standard REPO with general collateral* through the following properties:

1. Overnight nature: the contract can either be rolled-over or recalled each day. If it is recalled during the day, it will be closed and paid at the end of the day.

2. General collateral: we assume the vector A^C represents the amounts of a single collateral asset held by each bank; the analysis extends to the case of multiple collateral securities, but will not be pursued here.

3. Rehypothecation: the collateral is allowed to be reused. The reuse is allowed to happen multiple times, leading to the existence of collateral chains.

4. Haircuts: h represents haircut for a security that is posted as a collateral for the first time, while h_R is a haircut used for the security each time it is rehypothecated.

5. The original collateral holder keeps it on its balance sheet, while the lender records a reverse-repo asset A^R in its balance sheet. However, the lender retains the right, guaranteed by the REPO System Operator, of receiving the collateral instantaneously (i.e. prior to bankruptcy proceedings) in any event the borrower is removed from the network by the RSO.

We assume that all REPOs in the RH REPO market network have the same specification above. Since as we shall see, these REPOs theoretically have no counterparty risk, counterparties will be considered "fungible", and hence it is natural to require that haircuts are counterparty independent.

2.4 Financial Agent Characteristics

In this section we will introduce the characteristics of financial agents, the regulator and shocks, as well as the corresponding balance sheet items and collateral constraints. Crucially, we describe the REPO system operator, which has the role of monitoring the network and resolving defaults. At the end of this section we provide examples to highlight certain characteristics of the RH REPO network.

2.4.1 Bank Balance Sheets

In this thesis, we assume that there are N banks in the financial system, labeled by $i = 1, 2, \cdots, N$, and denote $[N] = \{1, 2, \cdots, N\}$, each with a balance sheet as shown in Figure 2.5. For the assets side, there are five accounts and their symbols are listed in Table 2.1

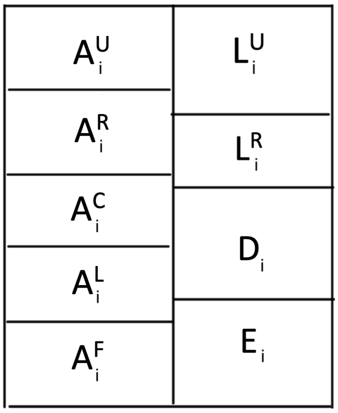


FIGURE 2.5: Bank i Balance Sheet

- Based on the notation in Table 2.1, the assets and liability from collateralized and uncollateralized contracts are denoted as $N \times N$ matrices L^R and L^U, with L_{ij} represents what bank i owes bank j. Furthermore, we have $A^R = (L^R)'$ and $A^U = (L^U)'$, where B' is the transpose of the matrix B.

- Because banks cannot borrow or lend cash from themselves, the diagonal elements in matrix L^U, L^R are zero.

- The collateral account A_i^C of bank i includes both used collateral and unused collateral. As mentioned before, we follow the convention that when a bank i enters a REPO contract and provides collateral, this bank still keeps the corresponding collateral in its balance sheet, but labels it as used collateral A_i^{RC}.

- Considering the definition of a REPO transaction and a reverse REPO transaction, we have the equation $A_{ij}^R = L_{ji}^R, \forall i, j \in [N]$.

- Following the convention in Gai et al. 2011, A_i^F includes loans and mortgages. Similarly, A_i^L includes all liquid assets, such as cash and central bank reserves.

- Similarly to the setting in Hurd 2017, the value of the equity account is the difference between the sum of all accounts on the asset side and the sum of all other

TABLE 2.1: Notation. We list the symbols of counts in asset side and liability side in a balance sheet of i^{th} bank in the financial network.

$(\bar{A}_i^U)' = (A_{i1}^U, A_{i2}^U, \cdots, A_{iN}^U)$	assets of uncollateralized lending (N dimensional vector)
$\bar{(}A_i^R)' = (A_{i1}^R, A_{i2}^R, \cdots, A_{iN}^R)$	Reverse REPO account (N dimensional vector)
A_i^C	Collateral account (include unused collateral)
A_i^L	Liquid assets
A_i^F	Fixed assets
$\bar{(}L_i^R)' = (L_{i1}^R, L_{i2}^R, \cdots, L_{iN}^R)$	Collateralized liability account
$\bar{(}L_i^U)' = (L_{i1}^U, L_{i2}^U, \cdots, L_{iN}^U)$	Uncollateralized liability account
D_i	Deposit
E_i	Equity

accounts in liability side, namely

$$\begin{aligned} E_i &= \sum_{j=1}^{N} \left(A_{ij}^U + A_{ij}^R \right) + A_i^C + A_i^L + A_i^F - \sum_{j=1}^{N} \left(L_{ij}^U + L_{ij}^R \right) - D_i \\ &= \left(A_i^U + A_i^R + A_i^C + A_i^L + A_i^F \right) - \left(L_i^U + L_i^R + D_i \right), \end{aligned} \tag{2.1}$$

where we have used x_i to denote the sum of the components of the row vector $\bar{x}_i'$, for example:

$$A_i^R = \sum_{j=1}^{N} A_{ij}^R, \qquad L_i^R = \sum_{j=1}^{N} L_{ij}^R. \tag{2.2}$$

Besides these N banks with this balance sheet setting, which represent the RH REPO network, one special node exists in this financial system, denoted with index 0. As mentioned before, this node stands for all economic agents outside of the N banks that comprise the RH REPO network, including other financial institutions, depositors, firms, and government agencies. In particular, this special node includes a central bank that provides liquidity to financial institutions through "open market operations and direct credit extension through standing lending facilities" (Bernanke 2008).

2.4.2 Collateral, Liquidity, and Solvency Constraints

The main concern for market design in a RH REPO network is to ensure that the rehypothecated collateral is sufficient to provide fair compensation to lenders under any conceivable market dislocation. Fundamental to controlling this issue is that all banks must satisfy their collateral constraint. We pay particular attention to rehypothecation, which means that reverse-REPOs on a bank's balance sheet can be used as additional collateral to increase the total amount of REPO funding. In the following we focus on "full rehypothecation", which allows collateral chains of any length.

Consider bank i holding A_i^C in general collateral, plus A_i^R in reverse-REPO assets, which means they have lent this amount to other banks who in turn pledged $(1-h)^{-1}A_i^R$ in general collateral to bank i. Under full rehypothecation, bank i can then borrow in the REPO market by applying a haircut h to its own collateral and a haircut h_R to the collateral associated with its reverse-REPO assets. Accordingly, the *collateral constraint* associated with the L_i^R in REPO liabilities is

$$\mathcal{C} := (1-h)A_i^C + \frac{1-h_R}{1-h}A_i^R - L_i^R \geq 0 \; . \tag{2.3}$$

In the special case when $h = h_R$, this reduces to

$$\mathcal{C} := (1-h)A_i^C + A_i^R \geq L_i^R \; , \tag{2.4}$$

which coincides with the collateral constraint in the GHK model described in the previous chapter under the assumption that $h_i = 0$, that is, the haircut on general collateral is equal for all banks. The condition $\frac{1-h_R}{1-h} < 1$, or equivalently $h < h_R < 1$, will make it impossible for a collateral to be rehypothecated unlimited times and artificially inflate balance sheets, whereas the additional assumption $\frac{1-h_R}{1-h} \leq (1-h)$ makes it at least as hard to meet the collateral constraint by entering new reverse-REPOs as to purchasing general collateral. The limit $h_R = 1$ corresponds to the case when rehypothecation is forbidden.

In the following, it is useful to introduce θ_i, the fraction of bank i's collateral that has been pledged to cover its REPO liabilities, defined as

$$\theta_i := \frac{L_i^R}{(1-h)A_i^C + A_i^R}.$$

It follows that an equivalent way to express the collateral constraint (2.4) is to require that $\theta_i \in [0,1]$. Another constraint, the liquidity constraint that measures how much cash we have, that we will see in Chapter 3 is defined as follows:

$$\mathcal{L} := A_i^L \geq 0 \tag{2.5}$$

Finally, we impose that all banks need to be solvent, in the sense that the value of their assets exceed the value of their liabilities. In view of (2.1), this is equivalent to

$$\mathcal{E} := E_i \geq 0. \tag{2.6}$$

2.4.3 REPO System Operator

We assume that the RH REPO network has a RSO that:

1. Certifies that stock-flow consistency is maintained, that is, that all payments between banks result in correct changes in their balance sheet entries.

2. Monitors and enforces the collateral constraint (2.3).

3. Provides overdraft protection to all market agents. This means that any agent may borrow cash from the RSO, to be repaid at the next reset time, in order to instantaneously pay any REPO close-out prior to the reset time.

4. Conducts the network resolution: this algorithm, described in Section 3.2, enforces a certain type of netting and novation of REPOs for banks being removed in the event of a default.

5. Oversees the network auction mechanism described in Section 3.3.3

2.4.4 Financial Shocks

We now list all potential financial shocks to a bank in a financial system that are relevant to our model:

1. Liquidity shock $\Delta A^L < 0$: where the bank has tapped into its credit line and needs to repay it. Among other reasons, this could be caused by a sudden withdraw in deposits $\Delta D < 0$, that is to say, a traditional bank run.

2. Collateral haircut shock $\Delta h > 0$: this puts pressure on the financial institution to post more collateral.

3. Withdrawal of collateralized loans from counterparties $\Delta A^R < 0$: when a counterparty refuses to continue a day-by-day REPO contract.

4. Defaulted banks shock: we will create defaulted banks , by giving banks large shocks, banks that has no ability to repay any money back upon default for default resolution.

5. Fixed assets market liquidity shock $\alpha > 0$: this is the liquidity parameter, which will cause any selling of fixed assets to suppress the price of this asset (see (3.25)).

The following shocks will not be discussed in this thesis:

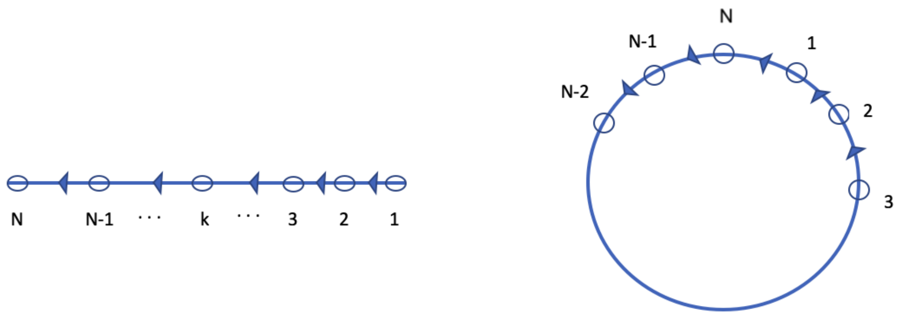


FIGURE 2.6: two types of collateral chains. One is open ended and the other one is circular. Unless the collateral rehypothecation haircut is set to $h_R > 0$, the collateral can be reused infinitely many times in a circular chain.

1. Withdraw of uncollateralized liability from a counterparties $\Delta L^U < 0$: refusal to roll over a contract.
2. An increase of REPO rate r_R during a rollover of a day-by-day REPO contract.
3. Requirement to increase the percentage of liquid assets, for example to a proportion of deposits, leading to a more stringent liquidity condition than (2.5).

2.4.5 Collateral Chains and Rehypothecation

At this point it is useful to illustrate the concepts introduced thus far with a couple of examples highlighting some of the most salient issues in a RH REPO network.

Luu et al. 2021 provide a theoretical background for such networks that emphasizes the creation of collateral chains and studies their potential role in generating financial systemic risk. The core assumptions of their model are similar to ours, but the interpretation differs, as we now discuss. First, we illustrate how chains and cycles may appear in the RH REPO network.

Example 1 (RH chain)**.** Figure 2.6 illustrates the chains and cycles that arise in RH REPO networks. Here we introduce N banks and connect them either by an open chain, or a closed cycle.

In the chain example, we suppose that bank 1 originally has collateral worth $A_1^C = (1-h)^{-1}$ and the other banks have an empty balance sheet. Bank 1 then uses the collateral to borrow $L_1^R = L_{12} = 1$ from bank 2 and keeps the funds received in the form of liquid assets (i.e. cash) $A_1^L = 1$ as shown in the first row of Table 2.2. Notice that both before and after the REPO transaction the equity (or capital) of bank 1 is equal to $(1-h)^{-1}$. In order to lend to bank 1, bank 2 initially borrows 1 from outside the network (or, equivalently, from the RSO), but once it has the reverse-REPO $A_2^R = L_{12} = 1$ as its asset, it can use it as collateral in order to borrow the amount $L_2^R = L_{23} = 1 - h_R$ from bank 3, so that its net position in the liquid asset is $A_2^L = (1-h_R) - 1 = -h_R$ (for example, owed to the RSO), as shown in the second row of Table 2.2. In its turn, bank 3 initially borrows $(1-h_R)$ from outside the network, but then uses the reverse-REPO $A_3^R = L_{23} = 1-h_R$ in order to borrow $L_3^R = L_{34} = (1-h_R)^2$ from bank 4, so that its net position in the liquid asset is $A_3^L = (1-h_R)^2 - (1-h_R) = -h_R(1-h_R)$, as shown in the third row of Table 2.2. The process continues in this manner until bank N, which needs to borrow the entire amount lent to bank $N-1$ from outside the network, resulting in a net position in the liquid asset of $A_N^L = -(1-h_R)^{N-2}$.

We can observe from Table 2.2 that, similar to bank 1, the capital of every other bank remains the same after the REPO chain as it was in the beginning, namely zero. Moreover, we see that all banks except bank N saturate their collateral constraint. One also sees that the total REPO loading is

$$X = \sum_{i=1}^{N-1} L_i^R = \frac{1}{h_R}(1-(1-h_R)^{N-1}),$$

which for N large is approximately $\frac{1-h}{h_R}$ times A_1^C. The original collateral is kept on the books of bank 1. However, its value can be divided amongst the lending banks in proportion to the net amount lent. Upon simultaneous default of banks, retrieval and ownership of collateral will be a problem, as we will see below in Section 2.4.6.

	A^L_i	A^C_i	$A^R_i = L_{i-1,i}$	$L^R_i = L_{i,i+1}$
$i = 1$	1	$(1-h)^{-1}$	0	1
$i = 2$	$-h_R$	0	1	$1 - h_R$
$i = 3$	$-h_R(1-h_R)$	0	$1 - h_R$	$(1-h_R)^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i = k$	$-h_R(1-h_R)^{k-2}$	0	$(1-h_R)^{k-2}$	$(1-h_R)^{k-1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i = N-1$	$-h_R(1-h_R)^{N-3}$	0	$(1-h_R)^{N-3}$	$(1-h_R)^{N-2}$
$i = N$	$-(1-h_R)^{N-2}$	0	$(1-h_R)^{N-2}$	0

TABLE 2.2: Balance sheets after the open chain has formed, when bank 1 starts with $A^C_i = (1-h)^{-1}$ and all other banks start with an empty balance sheet.

One can now ask how robust this example is in the event of a single default. Suppose bank 2 fails for any reason. Naive liquidation of its positions would close out both its REPO assets and liabilities, having a gross impact of $A^R_2 + L^R_2 = 2 - h_R$ on other banks. However, there is a natural "resolution" of bank 2 with much less impact. Replace liabilities $L_{12} = 1, L_{13} = 0, L_{23} = 1 - h_R$ by new values $\widetilde{L}_{12} = h_R, \widetilde{L}_{13} = 1 - h_R, \widetilde{L}_{23} = 0$. Then close out the single residual REPO involving bank 2, namely $\widetilde{L}_{12} = h_R$. This action is an example of "novation", whereby two REPOs through bank 2 are rerouted to banks 1 and 3, leaving the total REPO assets and liabilities of bank 3 unchanged and total REPO liabilities of bank 1 reduced by only h_R. We will return to this topic in the next chapter, where the novation procedure will be analyzed in more detail.

Example 2 (RH cycle)**.** In the cycle example, if $A^C_i = 1$ and $L^R_i = A^R_i = x$ for all i, then all banks will satisfy the collateral constraint if $(1-h) + (1-h_R)x \geq x$. The maximal cycle where $\theta_i = 1$ has $x = \frac{1-h}{h_R}$. Note that this maximal cycle generates a total REPO loading $X = \frac{1-h}{h_R}N$ which is $\frac{1-h}{h_R}$ times the total collateral owned by the network. When dealing with cycles, as we will see in Chapter 3, we will use a netting technique, when applied to this example, allow us to open the circle and change its shape to the linear shape like the RH chain of the previous example.

2.4.6 REPO Settlement Examples

As further illustration of the subtleties of REPO network, consider the examples below. The aim of these examples is twofold: first, they show that defaulted banks impose stress on the financial network if their REPO contracts need to be resolved in the regular way, namely by closing the contracts and recalling collateral; secondly, they show that instantaneous defaults of multiple banks who share a collateral through rehypothecation introduce difficulties in collateral resolution and how the collateral to split between the defaulted banks. We start the examples with a simple setup with no rehypothecation, then move on to a new setup that allows rehypothecation. These examples show that upon default, there many cases to be considered, and non-trivial complications arise.

1. First scenario (Figure 2.7): Consider the setting with banks A and B, where bank A pledges collateral to bank B and bank B gives cash to bank A. We assume that bank A does not give the right of rehypothication of the collateral to bank B, so that bank B has to place the collateral in a segregate account. We have two possibilities:

   (a) If bank A defaults and fails to return the cash to bank B, bank B becomes the legal owner of the collateral and it will liquidate the collateral to offset its loss.

   (b) If bank B defaults (say because of other liabilities with other banks), the collateral is retrieved from bank B's segregate account and given back to bank A, while bank A still has to fullfill its repayment duties.

2. Second scenario (Figure 2.8): assume the same setting as in the first scenario above, but with bank B rehypothecating the collateral to bank C, while bank C does not conduct any rehypothecate. Consider first the following cases involving the default of one bank only:

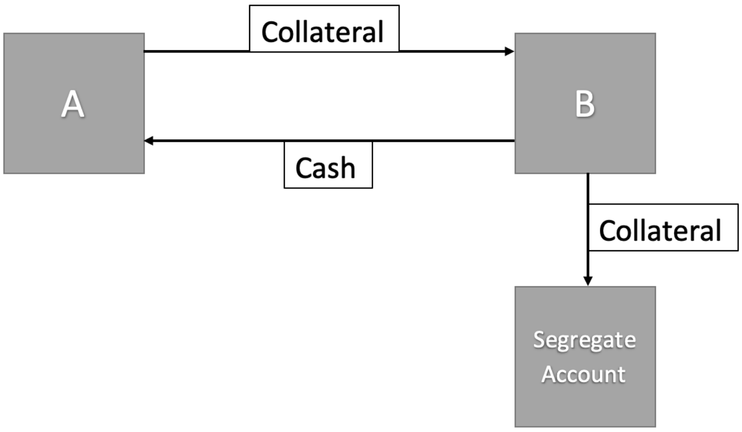


FIGURE 2.7: Contract with no rehypothication allowed

(a) if bank A defaults and fails to return cash, Bank B becomes the legal owner of the collateral as before. Now, if bank B is under liquidity stress, Bank B will have to stop rolling over the REPO contract with bank C and thus will ask for the collateral back. Bank B will seize the collateral and liquidate it to offset its losses. This will cause stress on bank C.

(b) if bank B defaults, now the collateral (Col^O) is owned by bank C as Col^{RH}. Bank C will liquidate the collateral to offset its losses as bank C is now the legal owner of the collateral. Bank A will then list the excess collateral value ($Col^O - Cash^O$) (assuming the most common situation of a positive haircut $h > 0$) as a loss on its capital.

(c) if Bank C defaults, this reduces to the first scenario above.

In the context of this second scenario, we can see that when more than one bank defaults, closing REPO contracts the regular way will impose stress on counterparties of the defaulted banks, no matter in which order defaults happen. For example, consider the situation in which bank B defaults followed by bank C, and assume further that the collateral value is larger than $Cash^0$. When bank B defaults, the usual steps to remove it from the REPO network are:

- bank A returns cash ($Cash^0$) to bank B, and gets collateral from segregate account;
- bank B gets cash ($Cash^0$) from bank A, then returns cash ($Cash^{RH}$) to bank C and keeps the difference ($Cash^0$ - $Cash^{RH}$).
- bank C gets cash ($Cash^{RH}$) from bank B.
- a new REPO contract is created between bank A and bank C, namely bank A borrows cash ($Cash^{RH}$) from bank C and provides collateral (Col^{RH}) to bank C, which is kept in segregate account. This step might cause problems to the network, as bank A and bank C may not know each other in the REPO market, and their tri-party (intermediary) can be different. In addition, because $Cash^{RH} < Cash^0$, bank C maybe short of enough cash to sign a REPO contract ($Cash^0$) with bank A.

When bank C defaults, the usual steps to settle the REPO account between bank A and bank C are:

- bank A returns cash ($Cash^{RH}$) to bank C, and gets collateral (Col^{RH}) from the segregate account.
- bank C gets the cash ($Cash^{RH}$) from bank A.

Conversely, consider the situation in which bank C defaults followed by bank B. When bank C defaults, we try to remove this bank from the banking system and REPO market. We try to stop all contracts in the REPO market and settle all of them. Then we can have the similar results with previous default order. However, this has a confusing financial meaning. When bank C defaults, only their counterparties should be affected. Based on our setting, bank A and even other banks will be forced to settle their REPO market, when the REPO chain has fourth, fifth and more banks as components. Namely:

- bank A returns cash ($Cash^0$) to bank B, and gets collateral from segregate account.
- bank B gets cash ($Cash^0$) from bank A, then returns cash ($Cash^{RH}$) to bank C and keeps the difference ($Cash^0$ - $Cash^{RH}$).
- bank C gets cash ($Cash^{RH}$) from bank B.
- a new REPO contract is created between bank A and bank B: bank A borrows cash ($Cash^0$) from bank B and provides collateral (Col^0) to bank B. Bank B keeps the received collateral in segregate account.

When bank B defaults, we try to settle the REPO account between bank A and bank B

- bank A: return cash ($Cash^0$) to bank B, and get collateral (Col^0) from the segregate account.
- bank B: get the cash ($Cash^0$) from bank A.

Similar steps can be taken for other default cases when at most two banks default, namely only banks A and B, or only banks A and C. However, if banks A, B and C fail, then there is no direct mechanism for clearing the obligations of the defaulted banks in the form of retrieving collateral from one another, and typically all banks end up in court with individual claims on the collateral.

2.5 Debt, Funding Options, and Liquidation

This section discusses options in dealing with debt, funding and liquidation situations. These situations arise when dealing with illiquidity or insolvency. The order in which

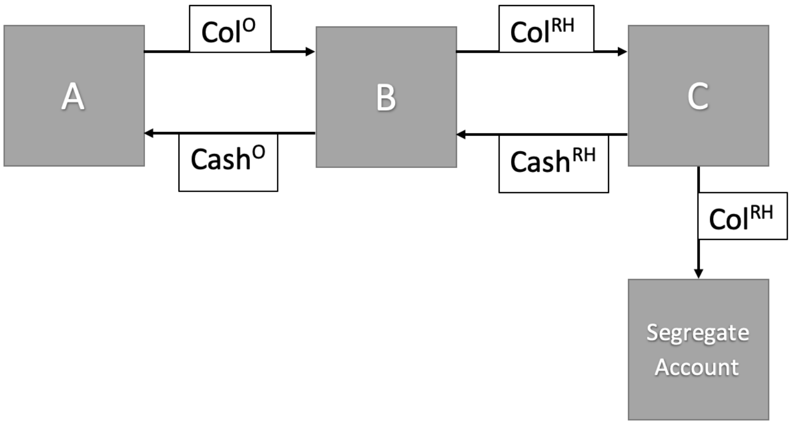


FIGURE 2.8: Contract allowing rehypothecation

assets are liquidated or liabilities are paid will have great impact on markets and investors.

2.5.1 Priority of Debt

When a bank experiences a shortage of funding to pay out its liability, we set the following priority for liabilities to be paid out to debtors. First, deposits D have the highest priority in this process, since this form of debt is not meant for investment, but rather the depositors are trusting the bank with their money to keep it safe. Second, because a collateralized borrower can provide the collateral to cover the loss of their counterparties, the borrower would prefer to pay out their uncollateralized liability in cash. As a result, the priority of debt during a funding pressure can be summarized in this order:

1. Collateralized liability L^R.

2. Deposits D.

3. Uncollateralized liability L^U.

2.5.2 Funding Options

In this section we discuss funding options that a bank can use to raise cash in order to meet its liability requirements, namely accessing cash, liquidating assets, recalling loans, or raising more debt. The list below specify different actions that are available to banks

1. Access liquid assets A^L (e.g cash).
2. Sell part of unused collateral $A^C - A^{RC}$.
3. Sell used collateral A^{RC}.
4. Recall collateralized or uncollateralized loans A^R and A^U.
5. Sell fixed asset (sharp drop of the value).
6. Borrow money from REPO market with unused collateral asset.
7. Borrow money from REPO market with used collateral asset (rehypothecation).
8. Borrow money by uncollateral contract.
9. Borrow money from a compartment outside of this ($[N]$) financial system .

It is clear that using liquid assets (say cash) should be the first choice of any bank, so that (1) above is the preferred funding option, whereas borrowing from outside the financial system should be the last, so that (9) above is the least preferred option. It should also be clear that (2) is preferred to (3), as pledging collateral introduces restrictions on the ability of the owner to sell it. It is also clear that borrowing using unused collateral should happen before borrowing with used collateral, which in turn should be easier than borrowing without collateral, so (6), (7), and (8) above should be executed in this order. Apart from these obvious relations, however, there is no clear ordering among the groups of actions above, which should instead be based on the choice of individual banks and underlying assumptions the bank works within. In our framework, as we will see in Chapter 3, we have certain assumptions regarding the markets, that is why we have such a list with the shown structure. It is important to note that the numbering does not describe the hierarchy of importance but rather to list and group things that are similar close to each other.

For example, let us compare two funding options: (2) selling unused collateral assets or (6) use these collateral assets to borrow money from REPO market. When we borrow money at REPO market, we can get $(1-h)A^C$ cash at most. In the future, the bank can decide to pay out these collateralized loans using liquid assets and get their collateral back. Another point to mention, it is mostly desirable to stay away from market operations. Involving in the markets in order to buy or sell large number of shares of security will affect market price and thus in turn will affect balance sheet constraints of the bank that initiated the transaction and other banks investing in the same market.

It is important to stay away from the markets in time of stress, as in stressed times markets becomes too sensitive to firesale. Thus, it is wise to meet default consequences, by focusing on the elements from the previous list of actions that involve the sale of assets and considering the following priority of liquidation actions:

1. Sell liquid assets A^L: these are equivalent to cash and experience no fluctuation in value.
2. Sell part of unused collateral $A^C - A^{RC}$: because of assumed high quality of common collateral, this should lead to minimal change in value due to sale, which we assume to be negligible.
3. Sell fixed asset A^F: we assume that this is subject to firesale effects, namemly a sharp drop in value when attempted to be sold quickly.

2.6 Agent-based Model

In this section we will explain the details of the agent-based model, using the definitions and concepts introduced so far. The agent-based model we are using is rule based, where every bank has a set of rules that it follows in order to keep its balance sheet healthy.

Notation and Nomenclature: The N banks of a network are labelled by $i \in [N] := \{1, 2, \ldots, N\}$ and we use standard matrix notation for $\mathbb{R}^N$ consistent with MATLAB: $N \times 1$ column vectors are denoted $\bar{x}, \bar{y}, \ldots$ and $N \times N$ square matrices by $A, B, \ldots$; their transposes are $\bar{x}', A', \ldots$ and satisfy the identity $(AB)' = B'A'$, the identity matrix is denoted by $\mathbb{I}$, whereas $\bar{e}_i$ denotes the ith basis column vector with jth component given by the Kronecker δ_{ij}, and $\mathbb{1} = [1, 1, \ldots, 1]'$.

2.6.1 Agent-based Model Flow Chart

This section presents Figure 2.9 as flow chart for the agent-based model. Figure 2.9 presents a bird's eye view of the model, some of the boxes in the flow chart are submodels which are made of different components or a set of rules. Figure 2.9 shows the main components of the model and how they interact.

The algorithm starts by setting the time variable $k = 0$, the prices of collateral and fixed assets at $S_k^C = S_k^F = 1$, an empty set of defaulted banks, and the initial values of the liability matrix L, as described below in Section 2.6.2. Next one needs to define the remaining terms of the balance sheet taking the liability matrix L into account as explained in Section 2.6.3.

As shown in Figure 2.9, the algorithm continues to run until one of the following four halting conditions is met. First, if there is no balance sheet violating the constraints at initialization phase, then the program will stop. Second, if the market conditions did not change or there are no financial shocks, or more generally if market fluctuations and financial shocks did not lead to any constraints to be broken by any bank, then the banks do not need to be involved in the market and the program halts. Third, some of the banks have healthy balances sheet where no constraints are broken or constraints has been fixed and other banks have defaulted. Lastly, the program shall stop if all the bank have defaulted, meaning that all the banks have been removed from the system and all obligations have been settled.

The functions $f(\cdot)$ and $g(\cdot)$ represent, respectively, the time evolution of the prices S^C and S^F for collateral and fixed assets from one day to the next, as shown in the fifth box in Figure 2.9, and can correspond to any stochastic or deterministic process for price. In this thesis we will use geometric Brownian motion, whose defining dynamics and explicit solution are provided in equations 2.7 and 2.8 below:

$$\frac{dS_t}{S_t} = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t \tag{2.7}$$

$$S_t = S_{t-1}e^{(\mu - \frac{\sigma^2}{2})\Delta t + \sigma(\Delta W_t)}. \tag{2.8}$$

Here W_t is a Wiener process (in particular $W_t \sim \mathcal{N}(0, t)$), μ is the mean of the compounded returns and σ is the standard deviation of the compounded returns, and $\Delta W_t = W_t - W_{t-1}$. Recall that geometric Brownian motion has the following properties:

$$E(S_t) = S_{t-1}e^{\mu \Delta t} \tag{2.9}$$

$$Var(S_t) = S_{t-1}^2 e^{2\mu \Delta t}(e^{\sigma^2 \Delta t} - 1) \tag{2.10}$$

The geometric Brownian motion is a Markov process; that is to say, its future values depend only on its present state and not depend on previous historic states.

After asset prices evolve from one day to the next, one checks if either the collateral, liquidity, or solvency constraints were violated as a result of the shocks or changes in asset prices. In case this happens, it means that some of the banks will need to participate in the Market Operations and the Network Resolution algorithms, which are described in detail in Chapter 3.

After that, provided not all banks have defaulted, the algorithm proceeds by moving to the next time step.

2.6.2 Initialization of the Liability Matrix L

The liability matrix L can be seen as a random graph. Random graphs can be generated in different ways. Some of these ways are probability distributions, generating processes and pre-given degree sequence. Generating a liability matrix is an important step in the program as having a liability matrix that does not represent real world networks or scenarios can lead to wrong conclusions. In this section we will be looking at some of the popular models of random graphs, how they are built and what are the pros and cons of such models.

Edgar-Gilbert Model: Also known as $G(n, p)$ model, where n is number of nodes (or vertices) in a graph, and $0 < p < 1$ is the probability of connecting two nodes. The $G(n, p)$ model is constructed in the following way: choose two random nodes from the n nodes in the graph and connect them with an edge with a probability p independent of all other edges. The number of possible edges in a graph with n nodes is $\binom{n}{2} = \frac{n(n-1)}{2}$. The probability to generate a graph with n nodes and M edges is given by:

$$P(G(n, p) = G) = p^M (1-p)^{\binom{n}{2}-M} \tag{2.11}$$

As p increases, the number of edges in the graph increases and vice versa. For $p = \frac{1}{2}$ equation 2.11 reduces to:

$$P(G(n, p) = G) = p^{\binom{n}{2}} \tag{2.12}$$

Equation 2.12 means that any graph with n nodes can be be generated with equal probability independent of the number of edges M.

The degree distribution of a node v from the graph is given by:

$$P(deg(v) = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k} \tag{2.13}$$

As the network becomes bigger and the number of nodes n tends to infinity, Newman et al. 2001 states that equation 2.13 tends to Poisson distribution:

$$P(deg(v) = k) \to \frac{(np)^k e^{-np}}{k!} \text{ as } n \to \infty \tag{2.14}$$

Erdős–Rényi Model: Also known as $G(n, M)$ model, where n is number of nodes in a graph and $0 \leq M \leq \binom{n}{2}$ is the number of edges in the graph. The number of graphs that can be generated with M edges from n nodes is

$$\binom{\binom{n}{2}}{M}.$$

from which we pick up graph uniformly, or in other words all the graphs in the set have been assigned equal probability. Thus the probability of picking up a graph from the set of n nodes and M edges is

$$\binom{\binom{n}{2}}{M}^{-1}.$$

Consider the example of $G(3, 2)$, where we have 3 nodes, $\{1, 2, 3\}$, and 2 edges. The following graphs will be generated:

- G1: node 1 will be connected to nodes $\{2, 3\}$ through an edge to each node.
- G2: node 2 will be connected to nodes $\{1, 3\}$ through an edge to each node.
- G3: node 3 will be connected to nodes $\{1, 2\}$ through an edge to each node.

Thus, $G(3, 2)$ will generate 3 graphs in total. We will pick up one these graphs randomly with equal probability of 1/3.

These two models include the following two assumption:

- The probability of an edge connecting two nodes is equal among all nodes.
- The probability of of connecting, p two nodes with an edge is independent of other edges, in other words the number of edges that already exist has no effect on p.

The above two assumptions will lead the two models to lack the following characteristics which occur in real world networks:

- No local clustering or triadic closure: roughly speaking, local clustering measure the degree of the connection among the neighbors of a node. Triadic closure is a property where is a node is connected to two neighbors with strong ties, these two neighbors should also be connected between each other with strong ties.A way to measure triadic closure is through local clustering coefficient; as local clustering increases, the number of triads also increases in the graph.
- No hub formation: real world networks have hubs, where a large number of nodes should have a low number of connections, some of the nodes should have a medium number of connections and a very small number of nodes should have a large number of connection.

To solve the problem with local clustering and triadic closure the Watts-Strogatz model was developed. In order to solve the hub problem, the Barabasi-Albert and the Configuration models were developed.

In addition to the deficiencies noted above, Barabási and Pósfai 2016 states that there are two different assumptions between random networks and real world networks:

- **Growth**: in real world networks, the network starts small and then grows up. There is a growth process that takes place in building the real world network. On the other hand, random networks start with the final number of nodes the network reached.

- **Preferential Attachment:** in real world networks, when a new node is introduced to the network the new nodes prefers to get attached to the most popular node in the network. In other words the new node prefers to get attached to node with most connections. In random networks, the new node attaches randomly to any node, sometimes with equal probability.

Barabasi-Albert Model: This model uses the preferential attachment method as follows:

1. start network with m_0 connected nodes, that represents graph G
2. introduce new node v to graph G
3. for node $i \in [N]$ connect node i to node v with $P(i) = \frac{degree(i)}{\sum_{x \in [N]} degree(x)}$
4. go to step 2

Step 3 in the algorithm shows that if a node i has as twice connections as node j then $P(i)$ shall be twice the value of $P(j)$ and most probably node v will connect to node i. Barabasi-Albert model allow us to create scale-free networks by generating power law degree distributions.

Configuration Model: Given a degree sequence, i.e a set of nodes [N] and each node is preassigned a number of half-links known as stubs. The configuration model is used to generate a graph the following way:

1. generate a degree sequence $\{k_i | i \in [N]\}$ such that $\sum_{i \in [N]} k_i = 2L$. The sum of the degree sequence should be even.
2. uniformly draw two unconnected stubs and connect them. The probability of a connection between node i and node j is given by $P_{ij} = \frac{k_i k_j}{2L-1}$.

3. if unconnected stubs still exist, go to step 2.

Given a degree sequence, the algorithm above will generate different graphs if the order of drawing the stubs to be connected is changed. Thus, multiple graphs can be generated from the same degree sequence and can be compared with the characteristics that arise with them. Moreover; the algorithm allows self loops and multi-links to appear in the generated graph. If self loops and multi-links are stopped this means stubs are not drawn uniformly and the probability of connection is not equal among all connections. As the number of nodes increases in a graph, self loops and multi-links will become more difficult to form. Figure 2.10 gives an idea how the algorithm works and what type of graphs can be generated.

Watts-Strogatz Model: This model generates random graphs that possess small-world properties. It helps in generating high clustering and short average path lengths. Constructing a graph following the Watts-Strogatz model by using the rewiring method as follows:

1. N nodes are required.

2. k represents the mean degree of the graph required, it should be an even integer and satisfying a certain technical condition.

3. create a regular ring, regular means each node has same pattern of connectivity as other nodes. One way to achieve regular ring is to connect a node to $k/2$ of the neighbors on the left side and the other $k/2$ neighbors should be on the right side of the node.

4. rewire each $edge(v_i, v_j)$ in the network with probability p with a node v_m uniformly chosen from the set of nodes forming the $edge(v_i, v_m)$. Two conditions apply, 1)no self looping: $v_i \neq v_m$ and 2) no link duplication: $v_j \neq v_m$.

Figure 2.11 shows how the value of p affects rewiring. When $p = 0$ we get the left graph, if $p = 1$ we get a graph similar to the right graph. As p gets closer to one, the generated graph becomes more random and it becomes similar to a graph generated by Erdős–Rényi.

2.6.3 Initialization of the Balance Sheet

The initialization of balance sheet depends on the initialization of the liability matrix L. For the remainder of the Thesis we will assume $L^U = 0$ (and consequently $A^U = 0$). Accordingly, the liability matrix L represents REPO liabilities L^R, as well as the correspooding REPO assets A^R, since $A^R = (L^R)'$ (i.e. the transposed matrix). For a bank $i \in [N]$ recall the following identity:

$$A_i = L_i + E_i$$

Expanding the above identity with our balance sheet entries we shall have:

$$A_i^L + A_i^C + A_i^R + A_i^F = L_i^R + D + E_i$$

Recall as well the following identities and constraints:

$L^R = L * \mathbb{1}$ (represents L_i^R in vector form for $i \in [N]$)

$L_i^R = \bar{e}_i{}' * L * \mathbb{1}$

$L_i^R = (1 - h)A_i^C$ (collateral constraint without novation)

$A^R = \mathbb{1}' * L$ (represents A_i^R in vector form for $i \in [N]$)

$A_i^R = \mathbb{1}' * L * \bar{e}_i$

Example 3 (Balance Sheet Building)**.** We are going to build a balance sheet step-by-step, this example is needed to make things clear. The balance sheet can be built starting from the assets side or the liabilities side. In this example we are building the balance sheet from the liabilities side. Assume our balance sheet value is \$100. Therefore $A_i = 100$. We need more variables to define liabilities side. Assume $D_i = 66\%$ and $L_i^R = 24\%$. In this example we will have all banks have the same balance sheet details on the liabilities side. Moreover, for simplicity assume $h = 0$. All balance sheets will be created to meet constraints and without haircut. Assume we have 5 banks, that is $[N] = \{1, 2, 3, 4, 5\}$, and we have the following variables:

$$L = \begin{bmatrix} 0 & 6 & 6 & 6 & 6 \\ 8 & 0 & 0 & 8 & 8 \\ 12 & 0 & 0 & 12 & 0 \\ 6 & 6 & 6 & 0 & 6 \\ 12 & 0 & 12 & 0 & 0 \end{bmatrix},$$

and

$$A_i = \$100,\ L_i^R = \$24,\ D_i = \$66 \text{ and } E_i = \$10 \text{ for } i \in [N]$$

The value for A_i^R (column sum of L) and A_i^C (collateral constraint without haircut and no rehypothecation, that is $A^C = L^R$ with $h = 0$, $h_R = 0$) are given as

$$A^R = \begin{bmatrix} 38 & 12 & 24 & 26 & 20 \end{bmatrix}$$

and

$$A^C = \begin{bmatrix} 24 & 24 & 24 & 24 & 24 \end{bmatrix}$$

We can see that although all banks started with the same value of L_i^R, each one of them ended with a different A_i^R, which in turn will give different values on assets side to these banks in most entries. The remaining amount for each bank on the assets side is given by the formula $R = 100 - A^C - A^R$:

$$R = \begin{bmatrix} 38 & 64 & 52 & 50 & 56 \end{bmatrix}$$

Now in the last step we need two more variables to determine the share of A_i^L and A_i^F as a percentage of R_i, call them A_p^L and A_p^F respectively. All banks will have the same percentages.

2.7 Conclusion

In this chapter we introduced the financial market, financial agents and balance sheets. In the financial market we introduced REPO contracts, rehypothecation chains and financial agents. We looked at the characteristics of the REPO network, REPO contracts, financial agents and how they are all linked together from a financial network prespective and balance sheet prospective. We discussed liquidity characteristics of assets in the balance sheet and financial shocks that can happen to both asset and liability sides. As for balance sheets and network initialization, we saw that from a simulation point of view it can be easily initialized but having an initialization that represents real life financial networks characteristics is not a trivial task.

We introduced two examples that deal with REPO rehypothecated chains and the issues that arise with them. The process of solving such issues is not trivial and it can reach the point of ambiguity when it comes to deciding who owns how much in a rehypothecated collateral.

In Chapter 3, we introduce a novation algorithm that deals with ambiguity that arises from defaulting banks which gives a clear answer to which banks owns what portion of the rehypothecated collateral. Moreover, we introduce decision rules that banks use in order to fix their balance sheet such that it meets all the constraints required by regulations. These rules give priority to save the collateral and pay the REPO liability, which is in contrast with what Gai et al. 2011 did. Gai et al. 2011 did not give any priority in debt resolving for REPO liability. Our resolving algorithm for constraints gives clear priority for liquidation of assets and clear priority of REPO liability payment.

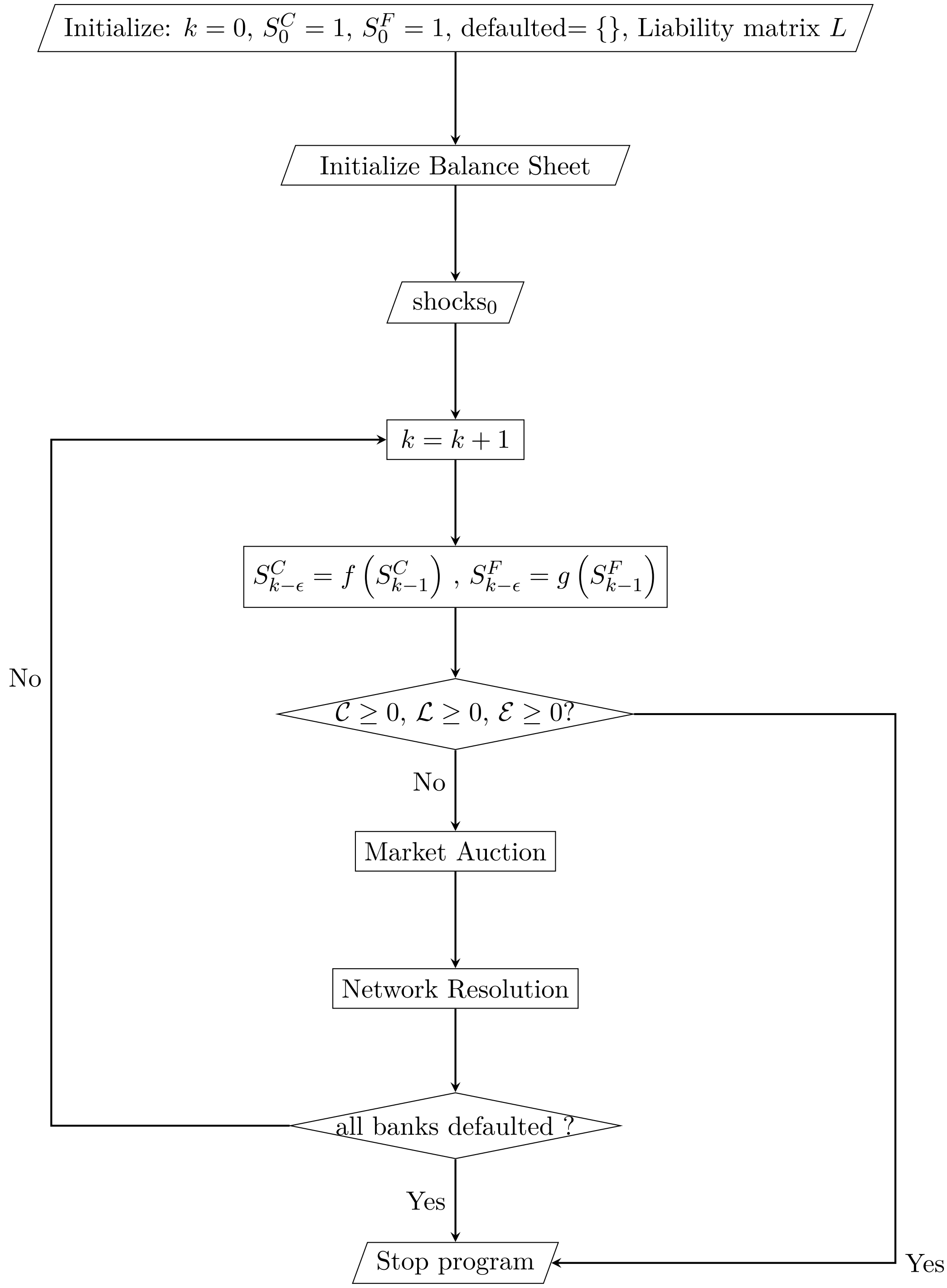


FIGURE 2.9: Flow chart of agent-based model. Here $\mathcal{C}$ is collateral constraint, $\mathcal{L}$ is liquidity constraint, and $\mathcal{E}$ is the solvency constraint. The functions $f(\cdot)$ and the $g(\cdot)$ represent the evolution of the prices S^C and S^F for one share of general collateral and fixed asset according to a geometric Brownian motion processes. The set "defaulted" represents the defaulted banks as the program is running.

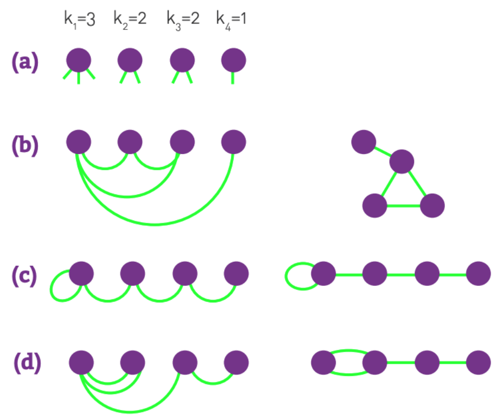


FIGURE 2.10: Configuration model building steps:
In step (a) we have 4 nodes and their associated stubs/half-links. Step (b) shows a generated graph with no self-loops and no multi-links. step (c) show a generated graph with self loop. While, step (d) shows a generated graph with multi-links. The graphs on the left shows how the stubs are connected according to the algorithm, while the graphs on the right show the final shape of the generated graphs. Source: Barabási and Pósfai 2016.

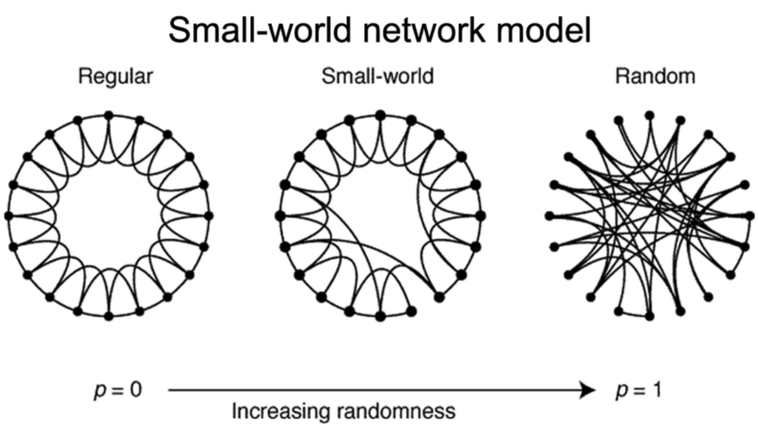


FIGURE 2.11: Watts-Strogatz Model:
As p (probability of rewiring) increases, the generated graph moves from the left side ring in the figure to the right side ring of the figure. Source: Liao et al. 2017.

Chapter 3

Novation and Auction

3.1 Introduction

As discussed in the previous chapter, REPOs are a class of contracts that function like collateralized lending and are used extensively for funding by primary banks at the core of modern financial systems. The single allowed collateral type in this thesis is a highly liquid, low risk asset such as a Treasury bill or government bond, called "general collateral". In this market, the collateral purchaser is free to re-use the collateral, or a fraction of it, to borrow cash with a second REPO, which is known as *rehypothecation*.

This chapter shows that counterparty risk is zero in a "perfect" repurchase agreement (REPO) market that allows fully rehypothecated general collateral, which implies in particular that chains of rehypothecated collateral are not a source of systemic risk. The chapter presents a theoretical market design for a network of banks exchanging REPOs that resets every 24 hours and permits full re-use (rehypothecation) of a general collateral, taken to be a highly liquid government debt security. Assumptions for a "perfect" REPO market include stock-flow consistent accounting, daily overdraft provisions and rules for a REPO system operator. It is proved that such a REPO market possesses a number of key properties. First, provided mandated collateral constraints hold, default risk does not arise in the REPO network. Second, if any subset of market participants defaults simultaneously, there is a consistent instantaneous network-wide resolution of all defaulted REPO contracts that can be enforced fairly and transparently immediately prior to these banks moving into bankruptcy proceedings. Finally, this chapter explores the degree to which real world REPO markets share these ideal properties.

The main theoretical innovation in the chapter is an algorithm called "network resolution" that optimally resolves the entire network at a moment when an arbitrary subset of agents must be removed. The key features of the network resolution algorithm are: (i) removed banks are treated symmetrically (i.e. fairly), as are the remaining banks; (ii) remaining banks experience a minimal impact to their balance sheets and, under ideal conditions, they recover the full value from removed counterparties; (iii) the entire process, by design, occurs prior to any legal proceeding, such as bankruptcy, and provides the all-important "safe harbour" protection giving priority of claims by REPO creditors over non-REPO creditors. We argue that, in the real world, a REPO market designed with such a resolution algorithm would be more transparent, resilient and efficient than traditional REPO markets.

The finance literature devoted to the properties and dangers of collateral rehypothecation (RH) is extensive and made difficult by the intrinsic complexity of such a system and the broad diversity of kinds of REPO and market agents. As the literature emphasizes, RH creates overlapping chains of collateral that need to be understood and then disentangled. Finally, the lack of clarity in REPO markets can severely undermine market sentiment during times of financial crisis. As we demonstrate, these difficulties and ambiguities can be understood and effectively eliminated within the theoretical network framework developed in this chapter. Consequently, the core of this chapter focuses on assumptions and properties of an "ideal" REPO market, with the main goal to prove that counterparty risk is zero in such networks.

The aim of this chapter is to build a model that deals with defaults that include rehypothecation of collateral. It is important to create a default resolution algorithm that removes defaulted banks without imposing stress effects on the network regardless of the topology of the financial network, whether or not lending and borrowing creates cyclic structures, and the length of the rehypothecation chain. The topology of the network can be a sparse one that is characterized by a Poisson distribution, a dense topology where the network is complete or near to complete graph, or a concentrated network, like in core-periphery networks, characterized by a geometric distribution. Cyclical rehypothecation structure refers to areas in the financial network where banks involved in lending and borrowing form a simple loop. The length of rehypothecation refers to to the number of times a single collateral has been passed from one bank to the other in a series of rehypothecation contracts. This chapter also considers whether rehypothecated collateral - a feature desired by market participants because of the dramatic efficiency gains it creates - can always be relied upon by creditors at moments of crisis when one or more market participants fail. Such complex scenarios represent systemic risk in perhaps its most extreme form. The aim is to demonstrate that a well-designed REPO market with rehypothecation is fundamentally resilient to such events. Moreover, this chapter also addresses the "static" question of network resolution that might be triggered at any moment. The focus is exclusively on defining the network resolution algorithm, and exploring its fundamental properties. This chapter will also explore the systemic risk effects that may arise dynamically in an RH REPO network. Network resolution being a static problem, henceforth in this chapter ignores a large amount of detail, including cash flows such as interest and REPO rates, collateral flows and asset prices.

The structure of this chapter is as follows: Section 3.2 proves how the RSO can impose three steps - netting, novation and clearing - to simultaneously remove any number of default banks from the REPO network in a fair and consistent manner. Section 3.2.5 provides answers to the question regarding the ownership of a rehypothecated collateral.

In Section 3.3 we introduce all the necessary components to build a model and the order they fit into the model. Decision regions for financial agents are described and labeled. Section 3.4 returns to the real world of REPO and discusses how the theoretical properties of the RH network can justify market participants' trust in real world REPO contracts.

This chapter adopts the following terminology for REPO counterparties: *borrower* is the receiver of cash and provider of collateral; *lender* is a cash provider and collateral receiver. A *pure borrower* is a bank with no borrowing counterparties; a *pure lender* is a bank no lending counterparties. A *net borrower* is a bank whose REPO liabilities exceed their (reverse) REPO assets; a *net lender* is a bank whose REPO assets exceed their REPO liabilities.

3.2 Network Resolution

In view of its 24 hour cycle as illustrated in Figure 2.4, a potentially devastating scenario for the RH REPO market might start with the failure of a number of banks to successfully manage the REPO auction at the end of a day. In such a scenario, many banks may need to be removed simultaneously. As we now demonstrate, the REPO system operator (RSO) can apply a clear cut algorithm to remove this subset of failed banks.

The essential characteristic of our idealized RH REPO Network is the existence of a natural algorithm that the RSO can implement at any moment to simultaneously removes any *resolution subset* $\sigma \subset [N]$ of failed banks from the network, for whatever reason, with minimal impact for the remainder of the network. The algorithm has certain desirable properties that are guaranteed, provided each node satisfies the collateral constraint at the moment of resolution. In the real world, where some banks may sometimes fail to satisfy the collateral constraint by a small amount, for example when the collateral price drops rapidly, the damage to the network should be proportionally small when such defaulted banks are removed. In this sense we say that RH REPO networks are "robust".

The resolution algorithm consists of the following general steps:

1. Identification: select the set of removed banks (the resolution subset), either for reasons or insolvency, illiquidity, or others.

2. Netting: removal of cycles in the resolution subset.

3. Novation: redistribution of REPO lending and borrowing from the resolution subset to a maximal number of counterparties in the remaining network.

4. Close-out: remove the remaining liabilities to and from the resolution subset.

Note that banks might be removed by the RSO for a variety of reasons apart from strict insolvency. For example, they might lose their overdraft privileges due to ongoing liquidity difficulties. Or a bank might simply decide to exit the REPO market by liquidating all their REPOs at the same time. As we will see, provided the collateral constraints were satisfied prior to novation, the remaining bank do not experience any loss. In the general case, however, the resolution algorithm might result in losses to the remaining banks.

In order to understand the rationale for the proposed resolution algorithm, it is helpful to consider a simple network in which a single bank must be removed.

Example 4. Figure 3.1 shows how the algorithm works in two steps when a single net-borrowing bank, in this case Bank 5, is removed from the network. In the novation step, 15 units of the assets of Bank 5 to be received from Bank 2 are rewired to Banks 1 and 4 in proportion to Bank 5's liabilities to these banks, while simultaneously 30 units of Bank 5's assets to be received from Bank 3 are rewired to Banks 1 and 4, again in proportion to Bank 5's liabilities to these banks. As a result, Bank 5 has no remaining REPO assets and only 15 units of REPO liabilities: it has become a pure-borrowing bank. This

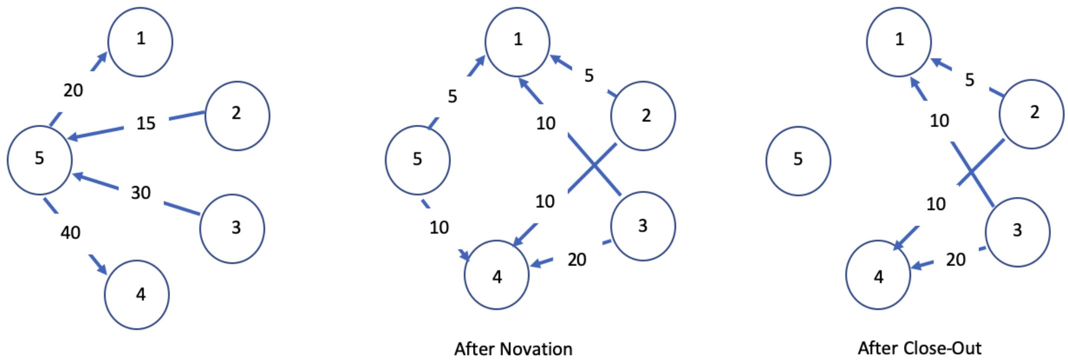


FIGURE 3.1: Bank 5, a net-borrowing bank, is removed from the network.

means it does not stand in the middle of any collateral chains, and as we will now see, their remaining REPO liabilities can be unambiguously closed out. To understand the close-out step, for simplicity we suppose haircut is zero. Then, the collateral constraint 2.3 for Bank 5 before novation was $A^c_5 + 45 \geq 60$, so that $A^C_5 \geq 15$. That is, since the existing collateral exceeds the residual REPO liabilities, Bank 5 can close them out fully using collateral if it cannot find cash.

The novation step when Bank 5 is a net-borrowing bank redistributes all its REPO assets to other counterparties j in proportion to L_{5j}, and leaves the total REPO assets and liabilities of bank j unchanged. One can check directly that network novation leads to the new liability matrix with ij components given by

$$\widetilde{L}_{ij} = L_{ij} + \zeta_5[L_{i5}L_{5j} - A^R_5\delta_{i5}L_{5j} - L^R_5 L_{i5}\delta_{5j}] \tag{3.1}$$

where $\zeta_5 = (A^R_5)^{-1}$ and δ_{ij} is the Kronecker delta function.The second term gives new REPOs between remaining banks. The third term erases corresponding REPO liabilities of Bank 5 and the fourth term erases corresponding REPO assets of Bank 5. The novation is such that the total REPO assets and liabilities of remaining banks are unchanged.

Figure 3.2 shows the opposite situation of a net-lending bank. After novation, Bank

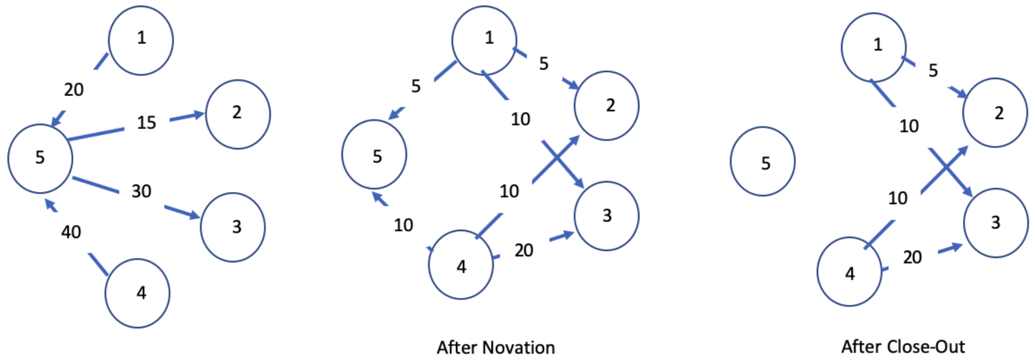


FIGURE 3.2: Bank 5, a net lending bank, is removed from the network.

5 is a pure-lending bank. The close-out of the remaining REPO assets is guaranteed because the initial collateral constraint for banks 1 and 4, namely $A^c_4 \geq 40$ and $A^c_1 \geq 20$, can now be reduced to $A^c_4 \geq 30$ and $A^c_1 \geq 15$, with 15 units of cash used to pay Bank 5.

One can now check that both the case when Bank 5 is a net-borrowing bank or the case when it is a net-lending bank are described by the same formula (3.1) if we define, in general

$$\zeta_5 = \min\left(\frac{1}{A^R_5}, \frac{1}{L^R_5}\right). \tag{3.2}$$

We emphasize that the removed Bank 5 in both these examples is not assumed to be liquid or solvent, but rather only that it satisfies the collateral constraint (2.3). Thus we see in these examples that the REPO market is stable to removal of a single bank, with no impact on the total REPO assets and liabilities of the remaining banks.

In the next section we develop the full algorithm for resolving the network at a moment when an audit of the network reveals that a subset $\sigma \subset [N]$ of banks of any size $1 \leq |\sigma| = m \leq N$ should be simultaneously removed. The algorithm takes as input the instantaneous matrix L of REPO notional amounts and the vector A^C of collateral amounts. This algorithm can be implemented by the RSO to completely remove this subset of banks, leading to a new liability matrix $\tilde{L}$ for the remaining banks, and corresponding adjustments to their cash and collateral amounts.

The key idea when several banks need to be removed is to apply the novation step one bank at a time.

3.2.1 Multilateral Novation

First consider the novation step for a single bank i, following example 4. With careful attention to matrix algebra using the notation summarized at the end of Section 1, one can verify that equation 3.1 can be written as $\widetilde{L} = f_5(L)$, where for general i, the non-linear function f_i of the matrix L is given by

$$f_i(L) = L + \zeta_i Q_i(L, L) \tag{3.3}$$

with

$$\zeta_i = \min\left(\frac{1}{\bar{e}_i' L \mathbb{1}}, \frac{1}{\mathbb{1}' L \bar{e}_i}\right) \tag{3.4}$$

and Q_i the following matrix-valued bilinear function of two square matrices:

$$Q_i(A, B) = A\bar{e}_i\bar{e}_i'B - \bar{e}_i\mathbb{1}'A\bar{e}_i\bar{e}_i'B - A\bar{e}_i\bar{e}_i'B\mathbb{1}\bar{e}_i' \tag{3.5}$$

It is convenient to note that (3.5) can be written two other ways:

$$\begin{aligned} Q_i(A, B) &= A\bar{e}_i\bar{e}_i'B(I - \mathbb{1}\bar{e}_i') - \bar{e}_i\mathbb{1}'A\bar{e}_i\bar{e}_i'B & (3.6) \\ &= (I - \bar{e}_i\mathbb{1}')A\bar{e}_i\bar{e}_i'B - A\bar{e}_i\bar{e}_i'B\mathbb{1}\bar{e}_i' & (3.7) \end{aligned}$$

We being with a result establishing that, after novation, the bank to be removed is either a pure lender or a pure borrower, depending on whether it was a net lender or net borrower prior to novation, whereas the total REPO assets and liabilities of the other banks remain unchanged.

Proposition 1. *Let $i \in \sigma$ be the bank to be removed and let $\tilde{L}$ be the liability matrix obtained after the novation map (3.3). Provided $L_{ii} = 0$, we have that*

$$\tilde{A}^R_{ij} = \max\left[A^R_{ij}\left(1 - \frac{L^R_i}{A^R_i}\right), 0\right] \tag{3.8}$$

$$\tilde{L}^R_{ij} = \max\left[L^R_{ij}\left(1 - \frac{A^R_i}{L^R_i}\right), 0\right], \tag{3.9}$$

and consequently

$$\tilde{A}^R_i = \max(A^R_i - L^R_i, 0)\ , \quad \tilde{L}^R_i = \max(L^R_i - A^R_i, 0). \tag{3.10}$$

Moreover, let $k \neq i$ be any of the remaining banks. Then provided $L_{kk} = 0$ we have that $\tilde{A}^R_k = A^R_k$ and $\tilde{L}^R_k = L^R_k$.

Proof. Using the fact that $\tilde{L} = f_i(L)$ and $L_{ii} = 0$, we have:

$$\begin{aligned}
\tilde{A}^R_{ij} &= \bar{e}'_j \tilde{L} \bar{e}_i = \bar{e}'_j (L + \zeta_i Q_i(L, L))\bar{e}_i \\
&= (\bar{e}'_j L \bar{e}_i) + \zeta_i[(\bar{e}'_j L \bar{e}_i)(\bar{e}'_i L \bar{e}_i) - \\
&\quad \bar{e}'_j \bar{e}_i (\mathbb{1}' L \bar{e}_i)(\bar{e}'_i L \bar{e}_i) - (\bar{e}'_j L \bar{e}_i)(\bar{e}'_i L \mathbb{1})(\bar{e}'_i \bar{e}_i)] \\
&= A^R_{ij} + \zeta_i[A^R_{ij} L_{ii} - \bar{e}'_j \bar{e}_i A^R_i L_{ii} - A^R_{ij} L^R_i \bar{e}'_i \bar{e}_i] \\
&= A^R_{ij}(1 - \zeta_i L^R_i)
\end{aligned}$$

We have two cases to consider, namely:

1. If $A^R_i \geq L^R_i$, then $\zeta_i = 1/A^R_i$ and

$$\tilde{A}^R_{ij} = A^R_{ij}\left(1 - \frac{L^R_i}{A^R_i}\right) \geq 0 \tag{3.11}$$

2. If $A_i^R < L_i^R$, then $\zeta_i = 1/L_i^R$ and

$$\tilde{A}_{ij}^R = A_{ij}^R \left(1 - \frac{L_i^R}{L_i^R}\right) = 0. \tag{3.12}$$

Similar calculations establish the result of $\tilde{L}_{ij}^R$. For any of the remaining banks $k \neq i$, using the fact that $L_{kk} = 0$ we obtain

$$\begin{aligned}
\tilde{A}_k^R &= \mathbb{1}'\tilde{L}\bar{e}_k = \mathbb{1}'(L + \zeta_i Q_i(L, L))\bar{e}_k \\
&= (\mathbb{1}'L\bar{e}_k) + \zeta_i[(\mathbb{1}'L\bar{e}_i)(\bar{e}_i'L\bar{e}_k) - \\
&\quad \mathbb{1}'\bar{e}_i(\mathbb{1}'L\bar{e}_i)(\bar{e}_i'L\bar{e}_k) - (\mathbb{1}'L\bar{e}_i)(\bar{e}_i'L\mathbb{1})(\bar{e}_i'\bar{e}_k)] \\
&= A_k^R + \zeta_i[A_i^R L_{ik} - \underbrace{\mathbb{1}'\bar{e}_i}_{1} A_i^R L_{ik} - A_i^R L_i^R \underbrace{\bar{e}_i'\bar{e}_k}_{0}] = A_k^R,
\end{aligned}$$

with similar calculations establishing the result for $\tilde{L}_k^R$. □

We illustrate this result using example shown in Figure 3.1, for which a direct calculation shows that

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & 0 & 30 \\ 0 & 0 & 0 & 0 & 0 \\ 20 & 0 & 0 & 40 & 0 \end{bmatrix}, \quad \tilde{L} := f_5(L) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 10 & 0 \\ 10 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 10 & 0 \end{bmatrix}. \tag{3.13}$$

Consider next the case where two banks i, j need to be removed from the network. The most important fact about the novation map (3.3) is that, provided there is no cycle between these two banks, that is to say, provided $L_{ij}L_{ji} = 0$, the order in which they are novated does not affect the resulting liability matrix. We first need the following technical lemma:

Lemma 2. *For any pair of banks i, j and liability matrix L satisfying $L_{ii} = 0$, we have:*

1. $\bar{e}_i' Q_j(L, L) = L_{ij} \bar{e}_j' L[I - \mathbb{1} e_j']$
2. $Q_i(L, L)\bar{e}_j = [I - \bar{e}_i \mathbb{1}'] L \bar{e}_i L_{ij}$
3. $\mathbb{1}' Q_i(L, L)\bar{e}_j = 0$
4. $\bar{e}_j' Q_i(L, L)\mathbb{1} = 0$
5. $Q_j(Q_i(L, L), L) = Q_i(L, Q_j(L, L))$
6. *If* $L_{ij} L_{ji} = 0$ *then* $Q_j(Q_i(L, L), Q_i(L, L)) = 0$

Proof. Using the definitions 3.3 and 3.5 , we have:

1. $$\bar{e}_i' Q_j(L, L) = \underbrace{\bar{e}_i' L \bar{e}_j}_{L_{ij}} \bar{e}_j' L - \underbrace{\bar{e}_i' \bar{e}_j}_{0} \mathbb{1}' L \bar{e}_j \bar{e}_j' L - \underbrace{\bar{e}_i' L \bar{e}_j}_{L_{ij}} \bar{e}_j' L \mathbb{1} \bar{e}_j' = L_{ij} \bar{e}_j' L[I - \mathbb{1} \bar{e}_j']$$

2. $$Q_i(L, L)\bar{e}_j = L \bar{e}_i \underbrace{\bar{e}_i' L \bar{e}_j}_{L_{ij}} - \bar{e}_i \mathbb{1}' L \bar{e}_i \underbrace{\bar{e}_i' L \bar{e}_j}_{L_{ij}} - L \bar{e}_i \bar{e}_i' L \mathbb{1} \underbrace{\bar{e}_i' \bar{e}_j}_{0} = [I - \bar{e}_i \mathbb{1}'] L \bar{e}_i L_{ij}$$

3. $$\mathbb{1}' Q_i(L, L)\bar{e}_j = \mathbb{1}' L \bar{e}_i \underbrace{\bar{e}_i' L \bar{e}_j}_{L_{ij}} - \underbrace{\mathbb{1}' \bar{e}_i}_{1} \mathbb{1}' L \bar{e}_i \underbrace{\bar{e}_i' L \bar{e}_j}_{L_{ij}} - \mathbb{1}' L \bar{e}_i \bar{e}_i' L \mathbb{1} \underbrace{\bar{e}_i' \bar{e}_j}_{0} = 0$$

4. $$\bar{e}_j' Q_i(L, L)\mathbb{1} = \underbrace{\bar{e}_j' L \bar{e}_i}_{L_{ji}} \bar{e}_i' L \mathbb{1} - \underbrace{\bar{e}_j' \bar{e}_i}_{0} \mathbb{1}' L \bar{e}_i \bar{e}_i' L \mathbb{1} - \underbrace{\bar{e}_j' L \bar{e}_i}_{L_{ji}} \bar{e}_i' L \mathbb{1} \underbrace{\bar{e}_i' \mathbb{1}}_{1} = 0$$

5.
$$\begin{aligned}
Q_j(Q_i(L,L),L) &= Q_i(L,L)\bar{e}_j\bar{e}_j'L - \bar{e}_j\underbrace{\mathbb{1}'Q_i(L,L)\bar{e}_j}_{=0 \text{ by item } 3}\bar{e}_j'L - Q_i(L,L)\bar{e}_j\bar{e}_j'L\mathbb{1}\bar{e}_j' \\
&= \underbrace{Q_i(L,L)\bar{e}_j}_{\text{by item } 2}\bar{e}_j'L[I-\mathbb{1}\bar{e}_j'] \\
&= \overbrace{[I-\bar{e}_i\mathbb{1}']L\bar{e}_iL_{ij}}\bar{e}_j'L[I-\mathbb{1}\bar{e}_j'] \\
&= [I-\bar{e}_i\mathbb{1}']L\bar{e}_i\underbrace{L_{ij}\bar{e}_j'L[I-\mathbb{1}\bar{e}_j']}_{\text{by item } 1} \\
&= [I-\bar{e}_i\mathbb{1}']L\bar{e}_i\overbrace{\bar{e}_i'Q_j(L,L)} \\
&= L\bar{e}_i\bar{e}_i'Q_j(L,L) - \bar{e}_i\mathbb{1}'L\bar{e}_i\bar{e}_i'Q_j(L,L) - L\bar{e}_i\underbrace{\bar{e}_i'Q_j(L,L)\mathbb{1}}_{=0 \text{ by item } 4}\bar{e}_i' \\
&= Q_i(L,Q_j(L,L))
\end{aligned}$$

6.
$$\begin{aligned}
Q_j(Q_i(L,L),Q_i(L,L)) &= \underbrace{Q_i(L,L)\bar{e}_j}_{\text{by item } 2}\underbrace{\bar{e}_j'Q_i(L,L)}_{\text{by item } 1} - \bar{e}_j\underbrace{\mathbb{1}'Q_i(L,L)\bar{e}_j}_{=0 \text{ by item } 3}\bar{e}_j'Q_i(L,L) \\
&\quad - Q_i(L,L)\bar{e}_j\underbrace{\bar{e}_j'Q_i(L,L)\mathbb{1}}_{=0 \text{ by item } 4}\bar{e}_j' \\
&= \overbrace{[I-\bar{e}_i\mathbb{1}']L\bar{e}_iL_{ij}}\overbrace{L_{ji}\bar{e}_i'L[I-\mathbb{1}e_i']} = 0
\end{aligned}$$

where the last step uses the fact that $L_{ij}L_{ji} = 0$.

□

We are now ready to prove our main theorem.

Theorem 3. *Let $L = (L_{ij})$ be a REPO liability matrix satisfying $L_{ii} = 0$ for all $i \in [N]$. If, in addition, for a given pair $i, j \in [N]$ the liability matrix satisfies $L_{ij}L_{ji} = 0$, then $f_i(f_j(L)) = f_j(f_i(L))$.*

Proof. Equation 3.3 is a bilinear function, thus we have:

$$
\begin{aligned}
f_j(f_i(L)) &= f_i(L) + \zeta_j Q_j(f_i(L), f_i(L)) \\
&= L + \zeta_i Q_i(L,L) + \zeta_j Q_j\left(L + \zeta_i Q_i(L,L), L + \zeta_i Q_i(L,L)\right) \\
&= L + \zeta_i Q_i(L,L) + \zeta_j Q_j(L,L) \\
&\quad + \zeta_i\zeta_j[\underbrace{Q_j(Q_i(L,L),L)}_{=Q_i(L,Q_j(L,L)) \text{ by Lemma 2 (5)}} + \underbrace{Q_j(L,Q_i(L,L))}_{=Q_i(Q_j(L,L),L) \text{ by Lemma 2 (5)}}] \\
&\quad + \zeta_i^2\zeta_j \underbrace{Q_j(Q_i(L,L),Q_i(L,L))}_{=0 \text{ by Lemma 2 (6)}} \\
&= f_i(f_j(L)),
\end{aligned}
$$

□

Consider now an arbitrary resolution subset $\sigma \subset [N]$ with $1 \leq |\sigma| = m \leq N$. We say that a liability matrix L is *acyclic* on a subset $\sigma = \{i_1, \ldots, i_m\} \in [N]$ if for each $1 \leq l \leq m = |\sigma|$ and any set of nodes $\{i_1, i_2, i_3, \ldots, i_l\} \in \sigma$ we have that

$$L_{i_l i_1} \prod_{k=2}^{l} L_{i_{k-1} i_k} = 0. \tag{3.14}$$

In other words, the liability matrix L is acyclic on σ if there are no closed loops within the subgraph defined by the nodes in σ. As we can see, (3.14) generalizes the condition $L_{ij}L_{ji} = 0$ for the case of a resolution subset with more than two banks.

Denoting by P_σ the projection onto the subspace spanned by $\{\bar{e}_i, i \in \sigma\}$, observe that the condition that L is acyclic on the subset σ, as stated in (3.14), is equivalent to $M = P_\sigma L P'_\sigma$ being acyclic on the entire network $[N]$, that is to say, for each $1 \leq l \leq N$ and any set of nodes $\{i_1, i_2, i_3, \ldots, i_l\} \in [N]$ we have that

$$M_{i_l i_1} \prod_{k=2}^{l} M_{i_{k-1} i_k} = 0. \tag{3.15}$$

Our next result shows that the novation map (3.3) does not introduce cycles in an otherwise acyclic liability matrix on a given subset of nodes.

Theorem 4. *Let L be a REPO liability matrix with $L_{ii} = 0$ and assume that L is acyclic on a subset of nodes $\sigma = \{i_1, i_2, \ldots, i_m\} \in [N]$. Then $f_i(L)$ is acyclic on σ, for each $i \in \sigma$.*

Proof. For ease of notation let $P = P_\sigma$ and note that P is an orthogonal projection matrix, that is $P^2 = P = P'$, and that we also have $P\bar{e}_i = \bar{e}_i$. Then using definition (3.3) we find

$$Pf_i(L))P' = PLP' + \zeta_i PQ_i(L,L)P' := M + \zeta_i PQ_i(L,L)P' \tag{3.16}$$

For the second term above, definition (3.5) gives the following:

$$\begin{aligned} PQ_i(L,L)P' &= PL\bar{e}_i\bar{e}_i'LP' - (P\bar{e}_i)\mathbb{1}'L\bar{e}_i\bar{e}_i'LP' - PL\bar{e}_i\bar{e}_i'L\mathbb{1}(\bar{e}_i'P') \\ &= PL(P\bar{e}_i)(P\bar{e}_i)'LP' - \bar{e}_i(\mathbb{1}'L\bar{e}_i)\bar{e}_i'LP' - PL\bar{e}_i(\bar{e}_i'L\mathbb{1})\bar{e}_i' \\ &= PLP'\bar{e}_i\bar{e}_i'PLP' - A_i^R\bar{e}_i\bar{e}_i'PLP' - L_i^R PLP'\bar{e}_i\bar{e}_i' \\ &= M\bar{e}_i\bar{e}_i'M - A_i^R\bar{e}_i\bar{e}_i'M - L_i^R M\bar{e}_i\bar{e}_i' := T, \end{aligned}$$

so that the components of the matrix T are $t_{iy} = -A_i^R m_{iy}$, $t_{xi} = -L_i^R m_{xi}$ and $t_{xy} = m_{xi}m_{iy}$ if $x, y \neq i$. We therefore need to show that $U := M + \zeta_i T$ with components $u_{xy} = m_{xy} + \zeta_i t_{xy}$ is acyclic, which is a trivial task, considering that $M = PLP'$ is acyclic by assumption. For example, for $x, y \neq i$ we have

$$\begin{aligned} u_{xy}u_{yx} &= (m_{xy} + \zeta_i t_{xy})(m_{yx} + \zeta_i t_{yx}) \\ &= m_{xy}m_{yx} + \zeta_i m_{xy}t_{yx} + \zeta_i t_{xy}m_{yx} + \zeta_i^2 t_{xy}t_{yx} \\ &= m_{xy}m_{yx} + \zeta_i m_{xy}m_{yi}m_{ix} + \zeta_i m_{xi}m_{iy}m_{yx} + \zeta_i^2 m_{xi}m_{iy}m_{yi}m_{ix} \\ &= 0 \end{aligned}$$

since M is acyclic. Similarly,

$$\begin{aligned} u_{xi}u_{ix} &= (m_{xi} + \zeta_i t_{xi})(m_{ix} + \zeta_i t_{ix}) \\ &= m_{xi}m_{ix} + \zeta_i m_{xi}t_{ix} + \zeta_i t_{xi}m_{ix} + \zeta_i^2 t_{xi}t_{ix} \\ &= m_{xy}m_{yx} - \zeta_i A_i^R m_{xi}m_{ix} - \zeta_i L_i^R m_{xi}m_{ix} + \zeta_i^2 A_i^R L_i^R m_{xi}m_{ix} \\ &= 0 \end{aligned}$$

since M is acyclic. Similarly calculations show the products of entries of U of the form (3.14) for arbitrary lengths all vanish. □

We now define multilateral novation of a REPO liability matrix L that is acyclic an an arbitrary resolution subset $\sigma = \{i_1, \ldots, i_m\} \subset [N]$ with $1 \leq |\sigma| = m \leq N$ to be the composition of single-bank novations:

$$\widetilde{L} := \tilde{L}^{i_1, i_2, \ldots, i_m} = (f_{i_m} \circ f_{i_{m-1}} \cdots \circ f_{i_1})(L) \ . \tag{3.17}$$

Thus, provided the original liability matrix has no cycles on the resolution subset σ, the multilateral novation defined above can be done one bank at a time in any given order, as Theorem 4 guarantees that each step does not introduce any cycles, so that the conditions of Theorem 3 are satisfied and the order of any two steps does not alter the result. Moreover, because each individual step satisfies Proposition 1, we have the following result establishing that all resolved banks will be either pure lenders or pure borrowers, while the total REPO assets and liabilities of the remaining banks are unchanged.

Proposition 5. *Under the conditions Theorem 4, we have that the total REPO assets and liabilities of any bank j after novation are given by:*

$$\tilde{A}_j^R = \begin{cases} A_j^R & j \notin \sigma \\ \max(A_j^R - L_j^R, 0) & j \in \sigma \end{cases}$$

and

$$\tilde{L}_j^R = \begin{cases} L_j^R & j \notin \sigma \\ \max(L_j^R - A_j^R, 0) & j \in \sigma \end{cases}.$$

Proof. Let $\tilde{L}$ be given by (3.17). If $j \notin \{i_i, \ldots, i_m\}$, then Proposition 1 gives that

$$\begin{aligned} \tilde{A}_j^R &:= \mathbb{1}'\tilde{L}^{i_1,\ldots,i_m}\bar{e}_j \\ &= \mathbb{1}'\tilde{L}^{i_1,\ldots,i_{m-1}}\bar{e}_j \\ &\vdots \\ &= \mathbb{1}'\tilde{L}^{i_1}\bar{e}_j \\ &= \mathbb{1}'L\bar{e}_j = A_j^R, \end{aligned}$$

with a similar argument establishing the result for $\tilde{L}_j^R$ if $j \neq \sigma$. On the other hand, if $j = i_l \in \{i_i, \ldots, i_m\}$, then the same argument shows that, up to i_{l-1} we have

$$\mathbb{1}'\tilde{L}^{i_1,\ldots,i_{l-1}}\bar{e}_j = A_j^R, \tag{3.18}$$

since $j \notin \{i_1, \ldots, i_{l-1}\}$, whereas using Proposition 1 for step $i_l = j$ gives

$$\mathbb{1}'\tilde{L}^{i_1,\ldots,i_{l-1},j}\bar{e}_j = \max(A_j^R - L_j^R, 0). \tag{3.19}$$

Therefore, since $j \notin \{i_{l+1}, \ldots, i_m\}$ we find

$$\begin{aligned}
\tilde{A}^R_j &:= \mathbb{1}'\tilde{L}^{i_1,\ldots,j,i_{l+1}\ldots i_m}\bar{e}_j \\
&= \mathbb{1}'\tilde{L}^{i_1,\ldots,j,i_{l+1},\ldots i_{m-1}}\bar{e}_j \\
&\vdots \\
&= \mathbb{1}'\tilde{L}^{i_1,\ldots,j,i_{l+1}}\bar{e}_j \\
&= \mathbb{1}'\tilde{L}^{i_1,\ldots,j}\bar{e}_j \\
&= \max(A^R_j - L^R_j, 0),
\end{aligned}$$

with a similar argument establishing the result for $\tilde{L}^R_j$ when $j \in \sigma$. □

Thus, provided there are no cycles in the resolution subset σ, multilateral novation can be done one bank at a time in any given order and all resolved banks will be either pure lenders or pure borrowers, while the total REPO assets and liabilities of the remaining banks are unchanged. Finally, the actual novations defined by the functions $Q_i, i \in \sigma$ depend only on the rows and columns of L for the removed banks. This means that an important confidentiality is preserved: the remaining banks need only reveal their exposures to removed banks.

3.2.2 Close-out Step

After multilateral novation, resolved banks will be completely removed from the REPO network when their remaining REPOs are closed out. However, since they are either pure lenders or pure borrowers, provided they satisfy their collateral constraint, we will now show that closing out has no negative impact one their counterparties. We recall that, by assumption, each resolved bank $i \in \sigma$ has lost their overdraft protection. They may have positive or negative cash A^L_i.

In case of a pure borrowing bank $i \in \sigma$, that is, with $\widetilde{A}_i^R = 0$, we note that the collateral constraint implies $(1 - h_i)A_i^C \geq \widetilde{L}_i^R$. All of this banks' REPO lenders can be instantly "closed-out" by return of any allowed combination of cash and collateral A_i^C, following the normal REPO rules for a defaulted bank. More precisely, since i prefers to deliver cash as much as possible, the best i can do to settle its REPO debts is to deliver $\min((A_i^L)^+, \widetilde{L}_i^R)$ in cash, and any remainder in collateral, worth $\max((1 - h_i)^{-1}(\widetilde{L}_i^R - (A_i^L)^+), 0)$.

In case of a pure lending bank, that is, with $\widetilde{L}_i^R = 0$, the borrowing counterparties may be either remaining banks or removed banks. In the first case, the remaining bank can either use its overdraft protection with the REPO System Operator or sell any collateral above its revised collateral constraint and close-out their REPO in cash. In the latter case, the removed borrowing bank settles with cash and/or collateral as described in the previous paragraph.

3.2.3 Multilateral Netting

The combination of multilateral novation followed by the close-out is sufficient to resolve any subset $\sigma \subset [N]$ provided $P_\sigma L P'_\sigma$ is an *acyclic matrix*. When this condition is not true, one applies a preliminary step called *multilateral netting* that removes all existing cycles. It can be argued that such REPO cycles are inefficient and economically undesirable, and their removal by netting typically improves the REPO network. Certainly, no bank's collateral constraint gets worse under netting.

Let us consider first an example where the subset of removed banks is $\sigma = \{1, 2, 3\}$, and $L_{12}L_{23}L_{31} > 0$ so there is a cycle. Suppose also that $L_{12} \leq \min(L_{23}, L_{31})$. Then the cycle can be removed by triparty netting which subtracts L_{12} from each of the edges of the cycle, leading to a new acyclic matrix with $\tilde{L}_{12} = 0, \tilde{L}_{23} = L_{23} - L_{12}, \tilde{L}_{31} = L_{31} - L_{12}$. Note that this triparty netting creates no additional balance sheet changes, and improves all the banks' collateral constraints provided haircuts are positive.

To remove all cycles from the general liability subgraph $P_\sigma L P'_\sigma$ with nonoverlapping cycles, one can simply remove cycles one at a time. However, if the subgraph has overlapping cycles, the result is order-dependent. In such instances, the algorithm needs to pick the order of removal, for example a prespecified lexicographical order.

3.2.4 Resolution Algorithm

To summarize, the following algorithm removes the specified banks from the REPO network:

1. Input the liability matrix L and collateral vector A^C and the resolution subset $\sigma \subset [N]$, with elements listed in lexicographical order.
2. Multilateral Netting Step: If $P_\sigma L P'_\sigma$ is not acyclic, list all its cycles. Apply sequential netting in order of increasing cycle length, and lexicographically amongst all cycles of the same size, leading to a new liability matrix L^{net} which satisfies the acyclic condition.
3. Multilateral Novation Step: Apply the mapping $\widetilde{L} = (f_{i_m} \circ f_{i_{m-1}} \cdots \circ f_{i_1})(L^{net})$ to the result of Step 2.
4. Close-Out Step: Close out all remaining REPO assets of pure lending resolved banks and all remaining REPO liabilities of pure borrowing resolved banks, in any order.

Observe that, although the netting step when $P_\sigma L P'_\sigma$ has overlapping cycles has the possibility of dependence on the ordering of the banks, in practice the novation step tends to minimize any resultant differences, as we will see in the simulations in the next chapter.

3.2.5 Application: Ownership of Collateral

The REPO literature (for example Maclachlan 2014) makes it clear that REPO contracts typically assign legal ownership of collateral to the REPO lender for the period of the contract, even though for accounting purposes the collateral is kept as an asset on the balance sheet of the borrower, whereas the lender records a reverse-REPO as an asset. If the REPO lender uses the reverse-REPO to rehypothecate the original collateral, thereby transferring ownership to a third party (while still being kept on the balance sheet of the original borrower), the resultant chain of ownership becomes an additional thread in a confusing web of potentially contradictory claims on the same original collateral.

We now show that the resolution algorithm of Section 3.2.4 provides a definitive answer to the important question: given the exposure matrix L and collateral vector A^C how does one determine the percentage of the collateral posted by bank i that is owned by bank j?

The key is to apply formulas for the resolution algorithm taking the resolution set to be the entire network $\sigma = [N]$. In general, the full matrix L will not be acyclic, so the first step will be multilateral netting as defined in Section 3.2.3 leading to an acyclic matrix L^{net}. Next, the multilateral novation is applied to the full network, to produce the matrix

$$\widetilde{L} = (f_N \circ f_{N-1} \cdots \circ f_1)(L^{net}) \tag{3.20}$$

The resultant REPO matrix has no collateral chains. Since all banks are either pure lenders or pure borrowers, and we continue to assume all banks satisfy the collateral constraint, every remaining REPO contract $\widetilde{L}_{ij} \neq 0$ is collateralized by assets A_i^C whose total value exceeds $(1-h)^{-1} \sum_j \widetilde{L}_{ij}$.

This observation leads to the following collateral ownership rule for the original REPO network with exposure matrix L and collateral vector A^C:

Collateral Ownership Rule: Each bank j is assigned ownership of collateral from any other bank i with value $(1-h)^{-1}\widetilde{L}_{ij}$, where $\widetilde{L}$ is given by 3.20 with L^{net} given by applying multilateral netting to L. Every borrowing bank i has non-negative remaining total collateral $A^C - (1-h)^{-1}\sum_j \widetilde{L}_{ij} \geq 0$.

As discussed in Section 3.2.3, in case the matrix L has overlapping cycles, there is the potential for multilateral netting to depend on the order that cycles are removed. Hence, the result may be non-unique. However, in every example we have investigated, the effects of this nonuniqueness are minimized after the application of the novation step. We conjecture, but have not proven, that the proposed Collateral Ownership Rule algorithm always leads to a *unique* assignment of collateral ownership across the REPO network.

To illustrate, consider the REPO chain of length N given as Example 2.2. Since in this example the matrix L is acyclic, only the multilateral novation step (which can be performed one bank at a time in any order) is needed. This produces the matrix $\widetilde{L} = (f_N \circ f_{N-1} \cdots \circ f_1)(L)$ whose only non-zero entries are given by

$$\widetilde{L}_{1j} = \begin{cases} h_R(1-h_R)^{j-2} , & 2 \leq j < N \\ (1-h_R)^{N-2} , & j = N \end{cases} \tag{3.21}$$

It is apparent in this example that, after novation, bank 1 becomes the only pure borrowing bank. It owes $\widetilde{L}_{1j}$ cash to the other banks, all pure lenders, which sum to $A_1^L = \$1$. This means that the Collateral Ownership Rule assigns to each bank j for $j \geq 2$ ownership of collateral with value $(1-h)^{-1}\widetilde{L}_{1j}$ originally owned by bank 1, while bank 1 itself retains ownership of zero collateral.

3.3 Market Operations

This section discusses what operations have happen at the end of a market day in order to deal with troubled banks.

Let day k for $k \geq 1$ denote the period $(k-1, k]$ depicted in Figure 2.4. For $k = 1$, we are assuming that some of the balance sheet constraints (2.5), (2.3) and (2.6) are violated by at least one bank as a result of an initial shock as described in Section 2.4.4 and Figure 2.9. For the remaining days, as explained in Section 2.4.3, we assume that the RSO provides overdraft protection to any bank that needs to close-out any REPO contracts prior to the end of the day, so that it is possible that some banks do not satisfy the liquidity constraint (2.5) at the end of day k. In addition, either as a result of asset price movements or the actions of other banks, it is possible that either the solvency constraint (2.6) or the collateral constraint (2.3), or both, are not satisfied by some banks at the end of day k.

If a bank is insolvent at the end of day k, then it is automatically identified as a bank to be removed from the network, as no amount of trading with its assets can increase its total value. Among the solvent banks, however, those that fail to satisfy either the liquidity constraint (2.5) or the collateral constraint (2.3), or both, are then given the opportunity to trade general collateral and fixed assets in the market, as well as to recall reverse-REPOs, in an attempt to restore these constraints, according to the auction step described below. Any bank that fails to do so after being given the opportunity to trade in the market is identified to be removed from the network.

To summarize, the market operations at the end of period k involve the following steps:

1. The RSO identifies all the banks that do not satisfy either one of the liquidity, collateral or solvency constraints (2.5), (2.3), (2.6).

2. Auction: among the banks identified above, those that satisfy the solvency constraint are allowed to act in the market to address collateral and liquidity constraints as described in Sections 3.3.3 and 3.3.3.

3. Shock propagation: the result of the actions of each bank in the previous step propagates to other banks through reduction in value of fixed assets A^F and hoarding of reverse-REPOs A^R. If this causes any bank to violate the constraints (2.5), (2.3), (2.6), such violations are only taken into account by the RSO in the next period.

4. Network Resolution: the banks identified in Step 1 that either: (i) were insolvent and therefore not allowed to participate in the auction in Step 2, or (ii) did not meet the collateral constraint in the Collateral sub-step of the auction, or (iii) did not meet the liquidity constraint in the Liquidity sub-step of the auction become the subset $\sigma \in [N]$ of banks that are then removed from the network according to the resolution algorithm of Section 3.2.4.

Once the market operations are concluded, the system moves to day $k+1$, that is, the period $(k, k+1]$, where fixed assets and general collateral undergo a Geometric Brownian motion (GBM) as described in Section 2.6 until the end of the day, when the steps above are repeated.

3.3.1 Pre-auction setup

As mentioned in Section 2.6.3, in this thesis we assume for simplicity that all interbank lending and borrowing is collateralized, that is, $L_i^U = A_i^U = 0$ for all i. The aim of this section and the next is to figure out how a bank i will act after having been identified as a solvent bank that does not meet either the collateral or liquidity constraint at the end of day $k-1$. Let $(A_i^{L-}, A_i^{C-}, A_i^{R-}, L_i^{R-})$ denote the value of the liquid asset, general collateral, reverse-REPO and REPO accounts at the start of the auction, namely

$t = k - \epsilon$, and let $[N_k^a]$ be the subset of solvent banks (i.e satisfying (2.6)) that are either illiquid or insufficient collateralized, that is, either $A_i^{L-} < 0$ or

$$(1-h)A_i^{C-} + \frac{1-h_R}{1-h}A_i^{R-} - L_i^{R-} < 0 \ . \tag{3.22}$$

Observe now that the change in collateral that is needed for the bank to satisfy this constraint as a strict equality is given by

$$\widetilde{\delta A_i}^C = -A_i^{C-} + \frac{L_i^{R-}}{1-h} - \frac{(1-h_R)A_i^{R-}}{(1-h)^2}, \tag{3.23}$$

so that a bank finds itself insufficiently collateralized provided $\widetilde{\delta A_i}^C > 0$.

As mentioned in Section 2.2, general collateral is typically a highly liquid asset, so we assume that it is not subject to any price impact due to market operations. It is therefore reasonable for a bank facing illiquidity to sell any excess collateral before attempting to sell the fixed asset, for which there is a price impact as described below. Conversely, because collateral is essential to guarantee loans between banks in the REPO network, it also makes sense for a bank to attempt to restore the violation (3.22) by selling fixed asset before addressing illiquidity issues. These adjustments are achieved in the *collateral sub-step* of the auction. At the end of this sub-step, banks will either have satisfied the collateral constraint (2.3) or else be identified to be removed from the network.

After the Collateral sub-step is concluded, we move to the Liquidity sub-step, whereby banks that still owe money to the RSO attempt to restore the liquidity constraint (2.5) by selling any remaining fixed assets, by recalling reverse-REPOs, or both. At the end of this sub-step, banks will either have satisfied the collateral constraint (2.5) or else be identified to be removed from the network.

We denote the total change in value of asset A_i^X, for $X = L, F, C$ (i.e liquid, fixed, collateral), resulting from auction (either through the Collateral sub-step, the Liquidity sub-step, or both) by δA_i^X, so that immediately after the auction (namely at $t = k$), we have

$$A_i^X = A_i^{X-} + \delta A_i^X, \qquad X \in \{L, F, C, R\}. \tag{3.24}$$

When needed for clarity, we use the notation $\delta_c A_i^X$ and $\delta_\ell A_i^X$ to denote the changes in value of asset A_i^X in each individual sub-step separately.

3.3.2 Price Impact

All banks in $[N_k^a]$ will come to the auction at the same time and they will execute buy or sell orders. In our algorithm, the order in which banks access the market during the auction period is random, as this guarantees that no bank will get a better price deal, since due to illiquidity in the market, the earlier the buying or the selling happens the better the price.

When a bank liquidates assets to address illiquidity and collateral constraints, it will get the prevailing market price for the required security/asset, which will in turn be affected by such "fire sale", in the sense that the market price will drop in relation to the amount of shares that have been liquidated. If the security is highly liquid, as is the case with general collateral A^C and cash-like assets A^L, we assume there shall be no price impact. On the other hand, fixed assets (e.g. long maturity bonds) are assumed to be illiquid, so that fire sale will leave an impact on their price for the next transaction. Specifically, the following formula is used to determine the price after liquidation, but other types of relations can be used as well:

$$\text{price}_{\text{new}} = e^{-\alpha q}\text{price}_{\text{old}}, \tag{3.25}$$

where $q \geq 0$ represents the number of shares to be liquidated and $\alpha \geq 0$ represents the liquidity of the asset. If $\alpha = 0$, this represents highly liquid market, where fire sales have no effect on the market price. As the value of α increases the impact of the fire sale will become more and more noticeable.

Let us denote the moment that bank $i \in [N_k^a]$ enters the market as a randomized agent by $t_{k,i}$. The following equations represent the value of fixed assets for bank i before and after liquidation:

$$A_i^{F-} = q_i^- S_{t_{k,i}^-} \tag{3.26}$$

$$A_i^F = q_i S_{t_{k,i}} \tag{3.27}$$

where q_i^- and q_i represent the quantity of fixed asset that bank i has before and after liquidation, whereas $S_{t_{k,i}}^-$ and $S_{t_{k,i}}$ represents the market price bank i sees before and after liquidating a given quantity of fixed assets, that is,

$$S_{t_{k,i}} = e^{-\alpha(q_i^- - q_i)} S_{t_{k,i}^-}. \tag{3.28}$$

Observe that the total change in value of the fixed asset account can be decomposed as

$$\begin{aligned} \delta A_i^F = A_i^F - A_i^{F-} &= q_i S_{t_{k,i}} - q_i^- S_{t_{k,i}^-} \\ &= q_i (S_{t_{k,i}} - S_{t_{k,i}^-}) + (q_i - q_i^-) S_{t_{k,i}^-} \end{aligned} \tag{3.29}$$

where both terms are negative, since $q_i < q_i^-$ and consequently $S_{t_{k,i}} < S_{t_{k,i}^-}$ on account of (3.28). Notice further that the first term in (3.29) is a pure re-evaluation term, arising from the change in value of the fixed asset resulting from the sale, whereas the second term, namely

$$\widetilde{\delta A_i}^F := (q_i - q_i^-) S_{t_{k,i}^-} \tag{3.30}$$

is the actual cash that is raised from the sale of the fixed asset at price $S_{t_{k,i}^-}$ and that can then be used to purchase either liquid assets or general collateral. Because we are assuming that the other assets are not subject to any liquidity effects, we obtain that the following self-financing condition holds at each step of the auction:

$$\delta A_i^L + \delta A_i^C + \delta A_i^R + \widetilde{\delta A_i}^F = 0 \tag{3.31}$$

Moreover, the liquidation of fixed assets by bank i at time $t_{k,i}$ causes the value of the fixed assets of any other bank j in the entire network to change by

$$\delta A_j^F(t_i) = q_j(t_i)(S_{t_{k,i}} - S_{t_{k,i}^-}) < 0, \tag{3.32}$$

where $q_j(t_i)$ is the quantity of the fixed asset held by bank j at time $t_{k,i}$. Finally, if bank $j \in [N_k^a]$ is then chosen randomly to enter the market auction to liquidate fixed assets at moment $t_{k,j}$ immediately after bank i, it observes $S_{t_{k,j}^-} = S_{t_{k,i}}$ as the price of the fixed asset.

3.3.3 The Collateral sub-step

For this sub-step we assume that $\delta A_i^R = 0$, that is, there are no changes in the reverse-REPO accounts of any banks. Recall that the set $[N_k^a]$ consists of banks for which either illiquid ($A_i^{L-} < 0$) or insufficiently collateralized ($\widetilde{\delta A_i}^C > 0$). We now consider the following alternatives for each bank $i \in [N_k^a]$, where we drop the label i for expedience of notation.

1. **Selling collateral**: suppose that $A^{L-} < 0$ and $\widetilde{\delta A}^C < 0$ that is, the bank is illiquid but has more collateral than is needed to fulfill condition (2.3). Then the bank sells all or part of its excess collateral in order to repay the loan to the RSO, but will not attempt to use its fixed asset until the next sub-step in the auction.

We can formalize this by setting

$$\delta A^C = \max\left(A^{L-}, \widetilde{\delta A}^C\right) \tag{3.33}$$

$$\delta_c A^L = -\delta A^C \tag{3.34}$$

$$\delta_c A^F = 0. \tag{3.35}$$

which corresponds to the following cases, listed from most to least favourable, depending on the size of the liquidity shock $A^{L-} < 0$:

(a) If $\widetilde{\delta A}^C \leq A^{L-} < 0$, then $\delta A^C = A^{L-}$, $\delta A^L = \delta_c A^L = -A^{L-}$ and $\delta A^F = \delta_c A^F = 0$, so that

$$A^L = A^{L-} + \delta A^L = 0$$
$$A^F = A^{F-} + \delta A^F = A^{F-}$$

and

$$\begin{aligned}(1 - h_c)A^C &= (1 - h_c)(A^{C-} + \delta A^C) \\ &= (1 - h_c)(A^{C-} + A^{L-}) \\ &\geq (1 - h_c)(A^{C-} + \widetilde{\delta A}^C) \\ &= L^R - \frac{1 - h_R}{1 - h_c} A^R,\end{aligned}$$

that is, the bank clears its debt with the RSO, does not change its fixed asset position, and still has excess collateral, in the sense that (2.3) is satisfied as an inequality. This concludes the market operations for the bank, which does not need to proceed to the Liquidity sub-step.

(b) If $A^{L-} < \widetilde{\delta A}^C < 0$ then $\delta A^C = \widetilde{\delta A}^C$, $\delta_c A^L = -\widetilde{\delta A}^C$ and $\delta_c A^F = 0$, so that

$$\begin{aligned} A^{L_c} &= A^{L-} + \delta_c A^L < 0 \\ A^{F_c} &= A^{F-} + \delta_c A^F = A^{F-} \end{aligned}$$

and

$$\begin{aligned} (1-h_c)A^C &= (1-h_c)(A^{C-} + \delta A^C) \\ &= (1-h_c)(A^{C-} + \widetilde{\delta A}^C) \\ &= L^R - \frac{1-h_R}{1-h_c} A^R. \end{aligned}$$

That is, the bank sells all of its extra collateral assets, so that the collateral constraint (2.3) is satisfied as an equality, but still has some debt $A^{L_c} < 0$ with the RSO, which will be dealt with in the Liquidity sub-step of the auction. Here we used the notation A^{L_c} and A^{F_c} to indicate that these are the positions in liquid and fixed assets immediately after the Collateral sub-step, but prior to the Liquidity sub-step.

2. **Buying collateral**: conversely, suppose that the bank has insufficient collateral, that is, $\widetilde{\delta A}^C > 0$. In this case, it will first use any cash A^{L-} that it holds with the RSO in order to purchase the necessary collateral. If that is not enough (including the situation when $A^{L-} < 0$), the bank will sell some or all of its fixed assets in the market. We assume that such "fire sale" of these assets only affects the price after the bank has sold them, thereby impacting all banks in the network as described in Section 3.3.2. If even after using all its cash and fixed assets the bank is still not able to satisfy the collateral constraint, then it is identified to be removed from

the network. We formalize these steps by setting

$$\delta_c A^L = -\max\left[0, \min\left(A^{L-}, \widetilde{\delta A}^C\right)\right] \tag{3.36}$$

$$\widetilde{\delta_c A}^F = -\max\left[0, \min\left(\widetilde{\delta A}^C + \delta A^L, A^{F-}\right)\right] \tag{3.37}$$

$$\delta A^C = -\delta_c A^L - \widetilde{\delta_c A}^F. \tag{3.38}$$

In the expressions above, recall that, according to (3.26)-(3.28), the value of fixed assets depend on the order in which the bank accesses the market during the auction, that is,

$$A^{F-} := A_i^{F-} = a_i^- * S_{t_{k,i}^-},$$

whereas A^{C-} and A^{L-} are set for each bank immediately prior to the start of the auction.

This results in the following cases:

(a) If $0 < \widetilde{\delta A}^C \leq A^{L-}$ then $\delta A^L = \delta_c A^L = -\widetilde{\delta A}^C$, $\delta A^F = \delta_c A^F = 0$ and $\delta A^C = \widetilde{\delta A}^C$, so that

$$A^L = A^{L-} + \delta A^L = A^{L-} - \widetilde{\delta A}^C \geq 0,$$
$$A^F = A^{F-} + \delta A^F = A^{F-}$$

and

$$\begin{aligned}(1-h_c)A^C &= (1-h_c)(A^{C-} + \delta A^C) \\ &= (1-h_c)(A^{C-} + \widetilde{\delta A}^C) \\ &= L^R - \frac{1-h_R}{1-h_c}A^R.\end{aligned}$$

That is, the bank has more than enough liquid assets A^L to purchase collateral so that (2.3) is satisfied as an equality, without the need to liquidate any fixed assets. Such bank does not need to proceed to the Liquidity sub-step.

(b) If $0 < A^{L-} < \widetilde{\delta A}^C \leq A^{L-} + A^{F-}$ then $\delta A^L = \delta_c A^L = -A^{L-}$,

$$-A^{F-} \leq \widetilde{\delta A}^F = \widetilde{\delta_c A}^F = -\widetilde{\delta A}^C + A^L < 0$$

and $\delta A^C = \widetilde{\delta A}^C$ so that

$$A^L = A^{L-} + \delta A^L = 0$$
$$0 \leq A^F = A^{F-} + \delta A^F < A^{F-}$$

and

$$\begin{aligned}(1 - h_c)A^C &= (1 - h_c)(A^{C-} + \delta A^C) \\ &= (1 - h_c)(A^{C-} + \widetilde{\delta A}^C) \\ &= L^R - \frac{1 - h_R}{1 - h_c} A^R,\end{aligned}$$

That is, the bank uses all of its liquid assets A^L plus a portion of its fixed assets to purchase collateral so that (2.3) is satisfied as an equality. Such bank does not need to proceed to the Liquidity sub-step.

(c) If $A^{L-} > 0$ and $0 < (A^{L-} + A^{F-}) < \widetilde{\delta A}^C$ then $\delta A^L = -A^{L-}$, $\delta A^F = -A^{F-}$ and

$$\delta A^C = A^{L-} + A^{F-} < A^{C-} + \delta A^C < \widetilde{\delta A}^C$$

so that $A^L = A^F = 0$ and

$$\begin{aligned}(1-h_c)A^C &= (1-h_c)(A^{C-} + \delta A^C) \\ &= (1-h_c)(A^{C-} + A^{L-} + A^{F-}) \\ &< L^R - \frac{1-h_R}{1-h_c}A^R,\end{aligned}$$

That is, the bank uses all of its liquid and fixed assets to purchase collateral, but is still not able to satisfy the collateral constraint. In this case, the bank is considered defaulted and is removed from the network.

(d) If $A^{L-} \leq 0$ and $0 < \widetilde{\delta A}^C \leq A^{F-}$ then $\delta A^L = 0$, $\widetilde{\delta_c A}^F = -\widetilde{\delta A}^C$ and $\delta A^C = \widetilde{\delta A}^C$ so that

$$\begin{aligned}A^{L_c} &= A^{L-} + \delta A^L = A^{L-} \leq 0 \\ 0 \leq A^F_c &= A^{F-} + \delta A^F < A^{F-}\end{aligned}$$

and

$$\begin{aligned}(1-h_c)A^C &= (1-h_c)(A^{C-} + \delta A^C) \\ &= (1-h_c)(A^{C-} + \widetilde{\delta A}^C) \\ &= L^R - \frac{1-h_R}{1-h_c}A^R.\end{aligned}$$

That is, the bank uses a portion of its fixed assets and is able to satisfy the collateral constraint as an equality, but still owes money $A^{L_c} \leq 0$ to the RSO, which it will attempt to pay in the Liquidity sub-step of the auction.

(e) If $A^{L-} \leq 0$ and $0 < A^{F-} < \widetilde{\delta A}^C$ then $\delta A^L = 0$, $\delta A^F = -A^{F-}$ and

$$\delta A^C = A^{F-} < \widetilde{\delta A}^C$$

so that

$$A^{L_c} = A^{L-} \leq 0$$
$$A^F = 0$$

and

$$\begin{aligned}(1-h_c)A^C &= (1-h_c)(A^{C-} + \delta A^C) \\ &= (1-h_c)(A^{C-} + A^{F-}_{t,i}) \\ &< L^R - \frac{1-h_R}{1-h_c}A^R,\end{aligned}$$

That is, the bank sells all of its fixed assets and is still not able to meet the collateral constraint, in addition to owing money to the RSO. In this case, the bank is considered defaulted and is removed from the network.

As a result of the collateral sub-step all banks have sufficient collateral, since those that did not meet the collateral constraint were removed from the network, although some banks might have negative cash positions.

The Liquidity sub-step

In this section, we assume that $\delta A_i^C = 0$ for all banks, that is to say, only reverse-REPOs, fixed assets, and liquid assets A_i^R, A_i^F, A_i^L are allowed to be adjusted, as any adjustments to general collateral A_i^C would have been done in the Collateral sub-step explained above.

As we have just seen, a bank will arrive at this sub-step as the result of either case 1b or case 2d of the collateral sub-step above. In either case, the collateral constraint (2.3) is satisfied as an equality but $A^{L_c} < 0$, meaning that the bank did not satisfy the liquidity constraint at the end of the collateral sub-step. The only difference between the cases is that, if arriving at the liquidity sub-step from case 1b in the collateral sub-step, a bank would not have liquidated any of its fixed assets yet. In both cases, however, the value of the fixed asset account would have been impacted by any liquidation made by other banks, so we continue to use

$$A^{F-} := A_i^{F-} = q_i^- S_{t_{k,i,\ell}^-} \tag{3.39}$$

to denote the value of fixed assets at the time $t_{k,i,\ell}$ when bank i enters the liquidity sub-step of the action in period k.

At this point each bank faces a behavioural choice: continue to liquidate fixed assets prior to recalling any reverse-REPO or the other way around. As we have discussed before, either choice has the potential to propagate the shock to other banks by putting pressure on their balance sheets. In the first case, the bank will further depress the price of the fixed asset, potentially causing some banks in the network to violate the solvency constraint (2.6). In the second case, the bank will find itself in violation of the collateral constraint (2.3), which was satisfied as an equality prior to any reduction in A^R, as well as forcing other banks to repay their REPO liabilities with liquid assets, potentially causing some of them to violate the liquidity constraint (2.5). The simulations in Chapter 4 will address the effects of these actions for the network as a whole.

In all cases, as mentioned before, we assume that banks that fail to satisfy the solvency, liquidity and collateral constraints as the result of market operations of other banks at the end of period k are allowed by the RSO to proceed to the next period. In particular, the RSO will continue to extend overdraft protection to any bank that needs to close-out their REPO liabilities because of recalls from banks in the liquidity sub-step. Accordingly, it is possible that, by the random time in which bank i enters the liquidity sub-step, its liquid asset account will be further overdrawn than it was at the end of the collateral sub-step. In this case, we make the additional assumption that this extra amount can also be dealt with in the next period. In other words, the value of the liquid assets for bank i that needs to be repaid in the liquidity sub-step is what it was at the end of the collateral sub-step, namely $A_i^{L_c}$.

As before, we omit the subscript i for expedience.

1. For a bank that prefers to recall reverse-REPOs prior to liquidating fixed assets, we adopt the following procedure:

$$\delta A^R = -\min\left(A^{R-}, -A^{L_c}\right) \tag{3.40}$$

$$\widetilde{\delta_\ell A}^F = -\max\left[\min\left(\delta A^R - A^{L_c}, A^{F-}\right), 0\right] \tag{3.41}$$

$$\delta_\ell A^L = -\delta A^R - \widetilde{\delta_\ell A}^F \tag{3.42}$$

This results in the following possibilities:

(a) If $0 < -A^{L_c} \leq A^{R-}$, then $\delta A^R = A^{L_c}$, $\widetilde{\delta_\ell A}^F = 0$ and $\delta_\ell A^L = -A^{L_c}$ so that

$$A^R = A^{R-} + A^{L_c} \geq 0$$
$$A^F = A^{F-}$$
$$A^L = 0.$$

That is, the bank is able to repay all it owed to the RSO at the end of the collateral sub-step by recalling a portion of its reverse-REPOs and does not need to further liquidate fixed assets.

(b) If $0 < A^{R-} < -A^{L_c} \leq A^{R-} + A^{F-}$, then $\delta A^R = -A^{R-}$, $\widetilde{\delta_\ell A}^F = A^{R-} + A^{L-}$ and $\delta_\ell A^L = -A^{L_c}$ so that

$$
\begin{aligned}
A^R &= 0 \\
0 \leq A^F &= A^{F-} + \delta A^F < A^{F-} \\
A^L &= 0.
\end{aligned}
$$

That is, the bank is also able to repay all it owed to the RSO at the end of the collateral sub-step, this time by recalling all of its reverse-REPOs and liquidating a portion of its fixed assets.

(c) If $0 < A^{R-} + A^{F-} < -A^{L_c}$, then $\delta A^R = -A^{R-}$, $\widetilde{\delta_\ell A}^F = -A^{F-}$ and $\delta_\ell A^L = A^{R-} + A^{F-}$ so that

$$
\begin{aligned}
A^R &= 0 \\
A^F &= 0 \\
A^L &= A^{L_c} + A^{R-} + A^{F-} < 0
\end{aligned}
$$

That is, the bank uses all of its reverse-REPOs and liquidates all of its fixed assets but still cannot repay all it owed to the RSO at the end of the collateral sub-step. Such bank is deemed to have defaulted and is identified to be removed from the network.

2. For a bank that prefers to liquidate fixed assets prior to recalling reverse-REPOs, we adopt the following procedure:

$$\widetilde{\delta_\ell A}^F = -\min\left(A^{F-}, -A^{L_c}\right) \tag{3.43}$$

$$\delta A^R = -\max\left[\min\left(\widetilde{\delta_\ell A}^F - A^{L_c}, A^{R-}\right), 0\right] \tag{3.44}$$

$$\delta_\ell A^L = -\delta A^R - \widetilde{\delta_\ell A}^F \tag{3.45}$$

This results in the following possibilities:

(a) If $0 < -A^{L_c} \leq A^{F-}$, then $\widetilde{\delta_\ell A}^F = A^{L_c}$, $\delta A^R = 0$, and $\delta_\ell A^L = -A^{L_c}$ so that

$$\begin{aligned} 0 \leq A^F &= A^{F-} + \delta A^F < A^{F-} \\ A^R &= A^{R-} \\ A^L &= 0. \end{aligned}$$

That is, the bank is able to repay all it owed to the RSO at the end of the collateral sub-step by liquidating a portion of its fixed assets and does not need to recall any of its reverse-REPOs.

(b) If $0 < A^{F-} < -A^{L_c} \leq A^{R-} + A^{F-}$, then $\widetilde{\delta_\ell A}^F = -A^{F-}$, $\delta A^R = A^{F-} + A^{L_c}$, and $\delta_\ell A^L = -A^{L_c}$ so that

$$\begin{aligned} A^F &= 0 \\ 0 \leq A^R &= A^{R-} + A^{F-} + A^{L_c} < A^{R-} \\ A^L &= 0. \end{aligned}$$

That is, the bank is also able to repay all it owed to the RSO at the end of the collateral sub-step, this time by liquidating all of its fixed assets and recalling a portion of its reverse-REPOs.

(c) If $0 < A^{R-} + A^{F-} < -A^{L_c}$, then $\widetilde{\delta_\ell A}^F = -A^{F-}$, $\delta A^R = -A^{R-}$, and $\delta_\ell A^L = A^{R-} + A^{F-}$ so that

$$A^R = 0$$
$$A^F = 0$$
$$A^L = A^{L_c} + A^{R-} + A^{F-} < 0$$

That is, the bank uses all of its reverse-REPOs and liquidates all of its fixed assets but still cannot repay all it owed to the RSO at the end of the collateral sub-step. Such bank is deemed to have defaulted and is identified to be removed from the network.

3.4 Discussion

The theoretical properties of the RH REPO network provide understanding and confidence about how it can remain intact during any type of real crisis of the financial system. For example, when a number of banks fail simultaneously, the RSO can step in at the earliest moment (i.e. within 24 hours) and compute the new matrix L with this collection of banks removed. The only instantaneous losses experienced by the remaining banks occur when the received collateral from removed banks is actually worth less than the notional loan amount: such losses will be small even during a crisis.

The second point is that netting and novation reduce the total notional REPO amounts of the remaining banks, but do not change the net amount borrowed/lent $|L_i^R - A_i^R|$. The only funding shock (total amount of cash to be paid to close-out) to remaining banks is equivalent to $max(0, L_i^R - A_i^R)$.

A third point is to note that the only time the collateral constraint is needed is when a removed borrowing bank i is closing-out the residual amount $L_i^R - A_i^R \geq 0$, and has insufficient cash. At such a time, this bank must deliver cash plus collateral whose market value is not less than this residual amount, and is able to do so provided the collateral constraint was satisfied prior to novation. In this case, all its counterparties are redeemed in full. For this reason, we say that a sufficiently collateralized RH REPO network has no counterparty risk.

A fourth point is that remaining banks are required to deliver cash not exceeding A_i^R to all removed lending banks. Since such banks retain their overdraft protection until the end of the day, in a dynamical model they will transmit funding shocks by borrowing from the REPO Systems Operators, or alternatively liquidate excess collateral, as they seek to refund their positions in the REPO market. This is the true spillover effect, leading to liquidity hoarding and other systemic risk effects. The next chapter will present a dynamical REPO market model based on the RH REPO network we have introduced here, with the aim of understanding the systemic risks arising from the intertwining of funding and market liquidity.

A fifth point is that timing issues may create additional stress on market agents. Since all agents have the right to close out a REPO at the end of the day, there is a question about whether this right conflicts with the novation step that apparently occurs at the same moment. We have not explored this issue in detail, but clearly, the novation step should precede by at least an instant the close-out moment at the end of the day. The close-out right attached to the novated REPOs could then be exercised at the end of the day.

A sixth point is the role of the haircut. Our main results work for any haircut, even $h = 0$. In hindsight the role of the haircut is to limit the losses to lenders when the removed bank fails the collateral constraint. If the haircut is too small these losses can be large.

Chapter 4

Simulations

4.1 Introduction

Simulations play an important part in any model or framework. Simulations can be used to fulfill different objectives. Some of these objectives are projection into the future, understanding dynamics and performing tests on a system. This chapter is divided into three sections: novation, netting, and framework. In each of these sections, will develop simulation cases to bring forward some interesting insights into our REPO model.

In the novation section we will put the claim that novation is order indifferent under numerical simulation test. We will also how novation changes the configuration of networks under certain toplogies. In the netting section we will consider different network configurations and the effect of netting on these configurations. In the Framework section we will try to replicate some of the test cases in Gai et al. 2011 paper and introduce some other tests we see fit for the systemic risk problem we are considering at hand.

4.2 Novation

In this section we will discuss some of the properties of the novation procedure introduced in the previous chapter. In particular, we want to verify that the order in which these banks are removed and have their balance sheet items redistributed does not affect the final result.

In the first two tests that follow, we consider the situation in which all of N banks in the network are deemed to have defaulted and need to be removed.

We are fixing the topology of the graph, meaning that if there is an edge between two nodes the edge will be maintained in all examples, but the weight of the edge can change, and if there is no edge between two nodes, no edge will be introduced. In other words, we will keep the same non-zero entries in the liability matrix L, but with varying values, whereas the zero-valued entries in L will remain the same.

The aim of the simulation to is to create different weighted graphs for the same graph topology and novate nodes on the same graph in all possible permutations. For each simulation the nodes will be removed in different orders given by all possible $N!$ permutations of the nodes. In each simulation, for each weighted graph the final novated matrix for each permutation order of nodes will be compared against all other novated matrices in that simulation (each novated matrix represent a permutation case) in the same weighted graph.

4.2.1 Novation Test 1

Consider a network in the form of the linear chain shown in Figure 4.2[1].

Since $N = 6$, there are $6! = 720$ possible ways for the banks to be removed. We run the simulation for 10,000 times to generate different weighted graphs of the same topology. For each simulation, we will generate weights for the edges of our fixed graph topology, which represents our liability matrix. The liability matrix in each simulation

[1]The reason the graph has an arbitrary shape though it is a linear chain is that graphs are generated automatically by Python package NetworkX.

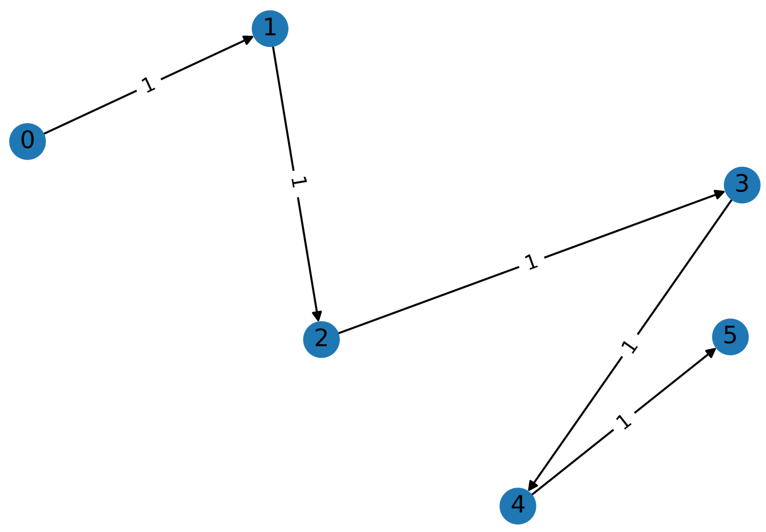


FIGURE 4.1: Novation test 1: linear chain.

was then novated in 720 different ways. In this case we verified that novation is order indifferent: for each of the 10,000 simulations, all 720 different ways to remove the banks resulted in the same final liability matrix.

4.2.2 Novation Test 2

Consider the more complicated network topology represented by the graph in Figure 4.2. We again ran the simulation for 10,000 times to generate different weighted graphs of the same topology and novated the liability matrix in each simulationin 720 different ways, in each case finding the exact same final liability matrix, thus supporting that novation is order-indifferent.

4.2.3 Novation Test 3

This test will show the effect of novation on a single graph by considering its topological properties, namely the in-degree and out-degree distributions, before and after novation.

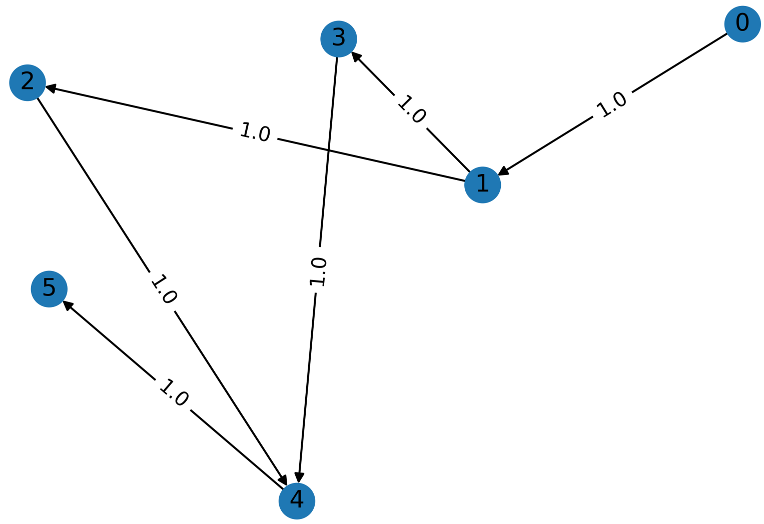


FIGURE 4.2: Novation test 2: non-trivial graph with no cycles.

Consider the network represented by the graph in Figure 4.3

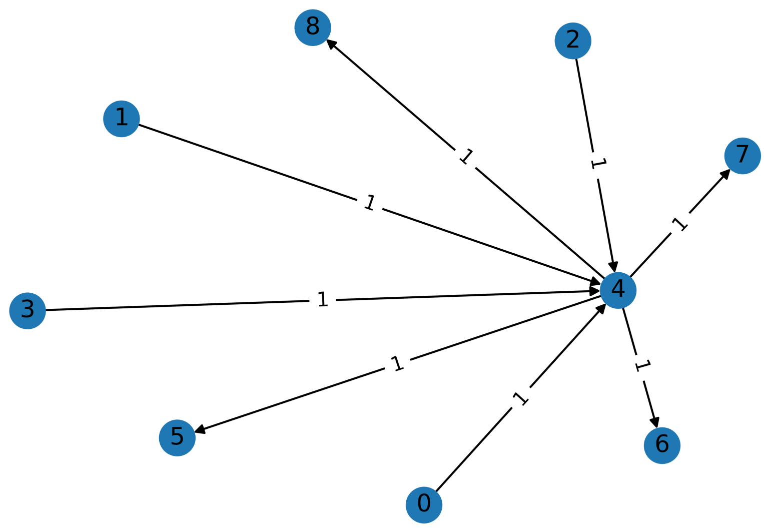


FIGURE 4.3: Novation test 3: star-shaped graph.

The graph in Figure 4.3 has the in-degree and out-degree distributions as shown in

Figure 4.4, namely out of the 5 nodes with nonzero incoming links (nodes 4,5,6,7,8), node 4 has in-degree equal to 4 and all others have in-degree 1, whereas out of the 5 nodes with nonzero outgoing links (nodes 0,1,2,3,4), node 4 has out-degree equal to 4 and all others have out-degree 1.

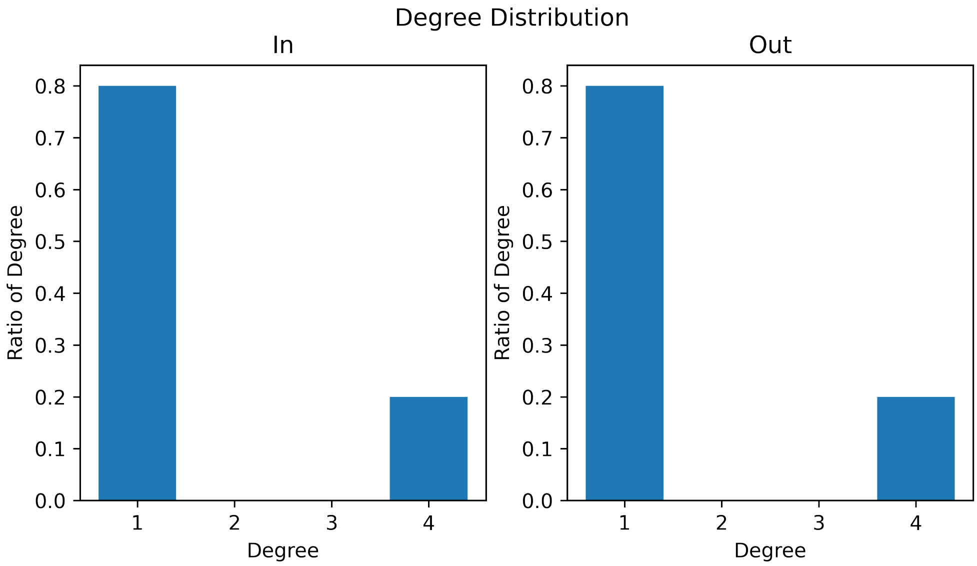


FIGURE 4.4: Degree distribution for the graph in Figure 4.3.

After applying novation on node 4 and clearing the residuals ($|A^R - L^R|$) from the graph, we obtain the updated graph and degree distributions as shown in Figures 4.5 and 4.6, namely nodes 5,6,7,8 each have in-degree 4, whereas nodes 0,1,2,3 each have out-degree 4.

We therefore conclude that novation can change the distribution of edges in a network, especially if we have a fat tailed distribution network and we remove a node with in-out degree of connections.

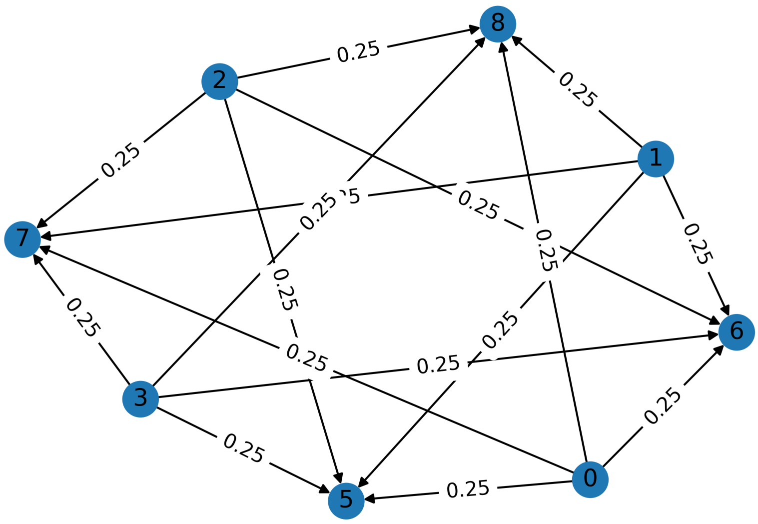


FIGURE 4.5: New assets and liabilities after novation is applied to the network in Figure 4.3.

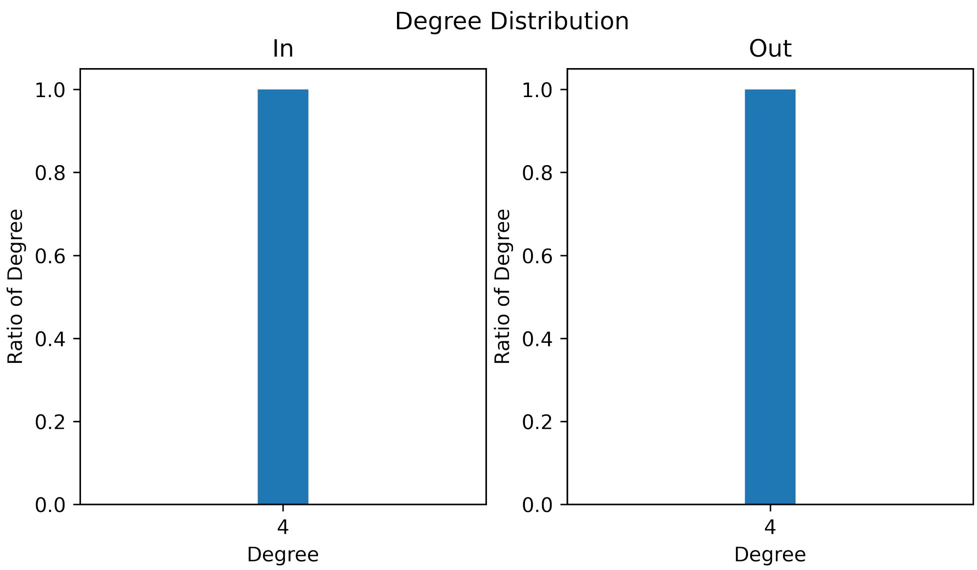


FIGURE 4.6: Degree distribution for the graph in Figure 4.5.

4.3 Netting

In this section we will be looking at the type of cycles we might encounter in our agent based REPO model that arise due to different topologies and the corresponding effect of the neeting procedure.

4.3.1 No-cycles

Consider the case where there are no loops given by the liability matrix L_{NE1}, corresponding to the graph shown in Figure 4.7. As there are no cycles in this graph, the netting function will have no effect on the liability matrix. That is to say, neither the matrix L_{NE1} nor the graph in Figure 4.7 will change as a result of netting.

$$L_{NE1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

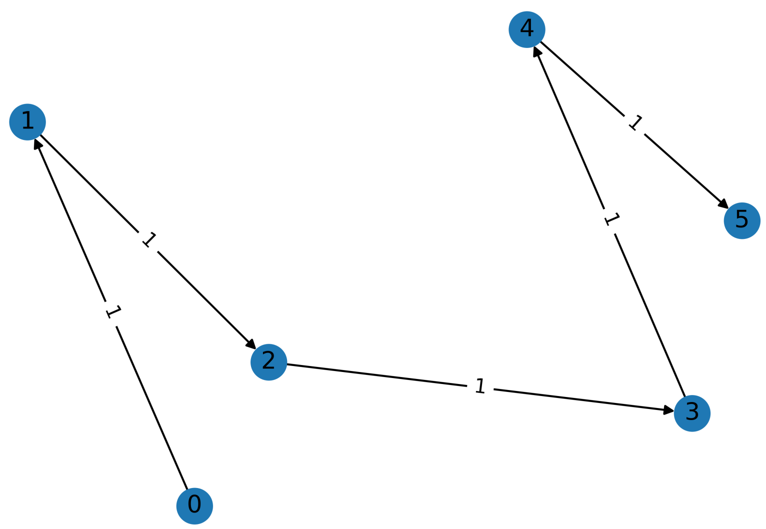


FIGURE 4.7: Netting Example 1: the set of cycles in the NE1 graph is empty and netting has no effect.

4.3.2 Simple-cycle

Consider the case where we have one cycle given by the liability matrix L_{NE2}, corresponding to the graph shown in Figure 4.8, where we can observe a single cycle connecting all the nodes. In this test, the netting procedure will subtract the least weighted edge from all the edges on the graph and this will break the cycle. The updated result of L_{NE2} and Figure 4.8 will be reflected in L_{NE2}^{netted} and Figure 4.9.

$$L_{NE2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \\ 6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

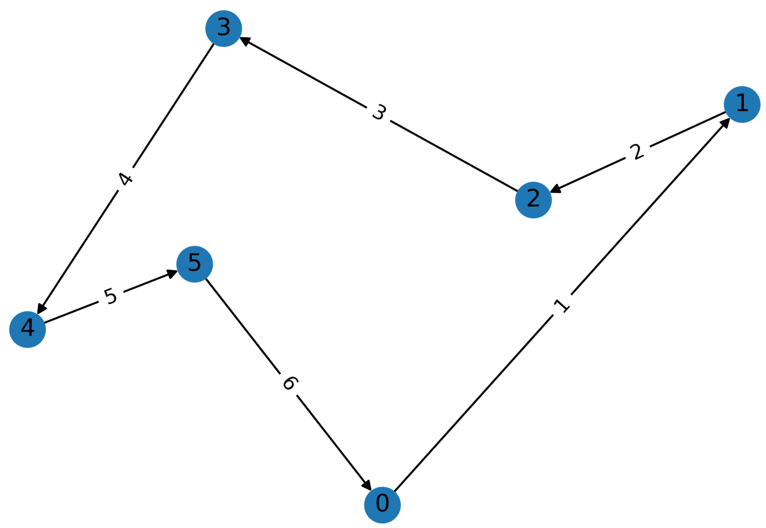


FIGURE 4.8: Netting Example 2: the set of cycles in the NE2 graph is $cycles_{NE2} = [\ [0, 1, 2, 3, 4, 5]\ ]$, which is removed by netting.

$$L_{NE2}^{netted} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

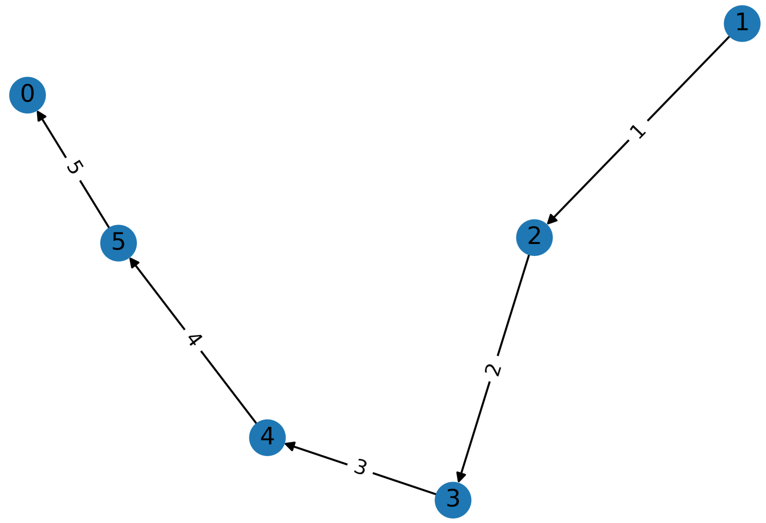


FIGURE 4.9: Netting Example 2, Netted: as the netting step removed the only cycle in the graph, the set of cycles in the liability matrix after netting is empty.

4.3.3 Two simple, disjoint cycles

From this test case forward we will omit the liability matrix L, unless it is needed. Consider the graph in Figure 4.10. In this case we have two cycles, each with a different number of nodes and a different total weight (sum of weights of edges of the cycle). We need an extra criterion on how to process the cycles. In our model we have three options.

Maximum option:

1. Given a graph G, find cycles and sort them according to number of nodes.

2. Remove the cycle with the largest number of nodes.

3. Update graph.

4. Repeat steps number 1, 2 and 3 until there are no cycles left.

Minimum option:

1. Given a graph G, find cycles and sort them according to number of nodes.

2. Remove the cycle with the smallest number of nodes.

3. Update graph.

4. Repeat steps number 1, 2 and 3 until there are no cycles left.

Random option:

1. Given a graph G, find cycles.

2. Randomly pick a cycle, with equal probability, and remove it.

3. Update graph.

4. Repeat steps number 1, 2 and 3 until there are no cycles left.

In this test case, the option that is chosen to process the cycles is not important as these cycles are disjoint. The final result of the cycle removal is shown in Figure 4.11

4.3.4 Two simple cycles with common node

Consider the graph in Figure 4.14. We again find that different approaches (maximum, minimum, or random) in removing cycles from the graph in Figure 4.14 have no effect on the final result, as netting only affects the edges of the cycle to which it is applied,

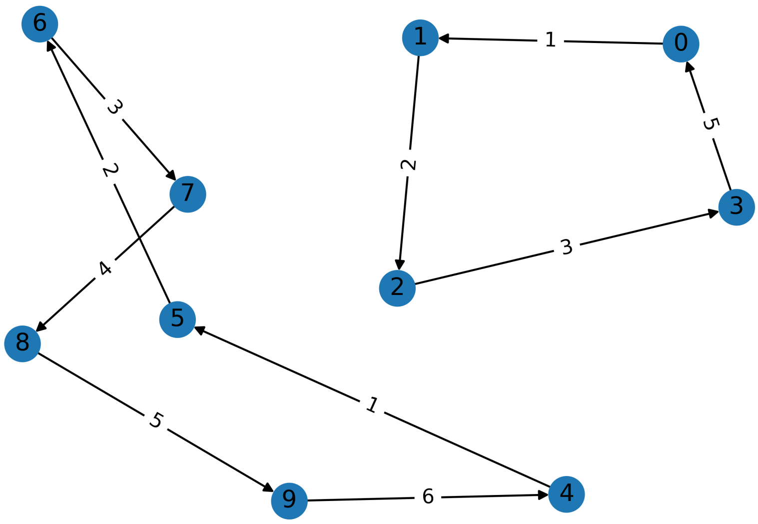


FIGURE 4.10: Netting Example 3: the set of cycles in the NE3 graph is $cycles_{NE3}$ = [[4, 5, 6, 7, 8, 9] , [0, 1, 2, 3]], that is, the cycles are disjoint.

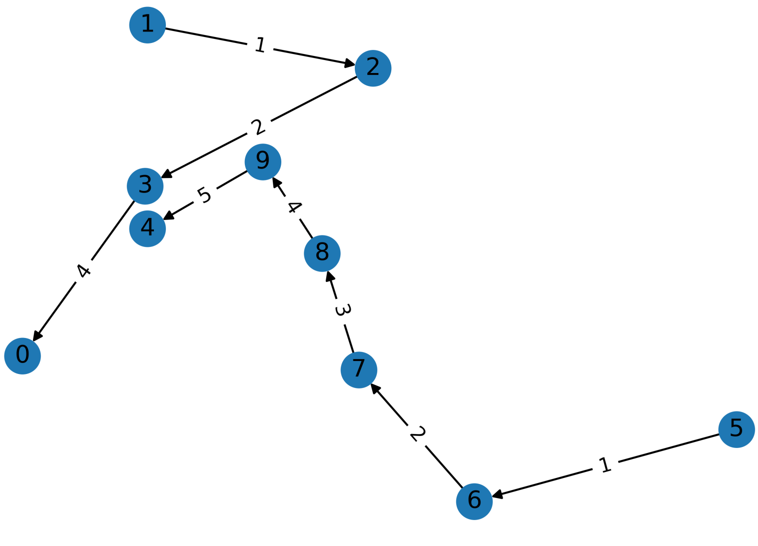


FIGURE 4.11: Netting Example 3, netted: the set of cycles in the NE3 graph after netting is empty.

and in this case the cycles have no common edges, just a common node. The netted graph is represented in Figure 4.13.

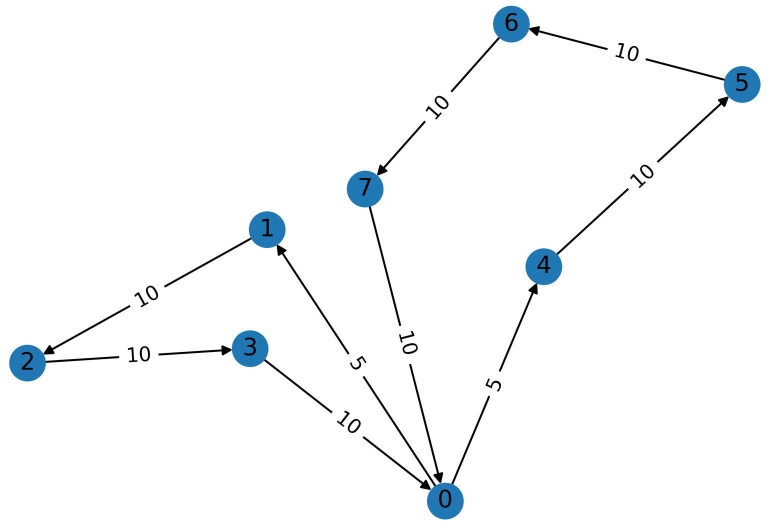


Figure 4.12: Netting Example 4: the set of cycles in the NE4 graph is $cycles_{NE4} = [\ [0,4,5,6,7]\ ,\ \ [0,1,2,3]\ ]$, that is, they have 0 as a common node.

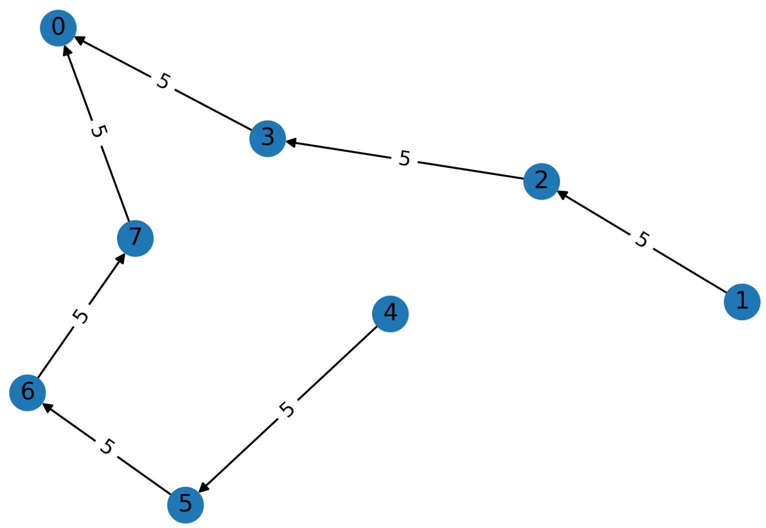


Figure 4.13: Netting Example 4, netted using either the maximum or minimum approaches, resulting in the same graph with no cycles.

4.3.5 Two simple cycles with a common edge

Consider the graph in Figure 4.14. This graph is similar to graph in Figure 4.10 with one difference:the two cycles have a common edge. Figures 4.15 and 4.16 represent different approaches (namely maximum or minimum approach) in removing cycles from the graph in Figure 4.14. As it is obvious from the two figures, each approach leads to a different result. In Figure 4.15, it can be seen that all the nodes are connected to each other, and if we start from node number one, all the other nodes can be reached. On the contrary, Figure 4.16 shows that there are no nodes in the graph from which we can reach all other nodes in the graph.

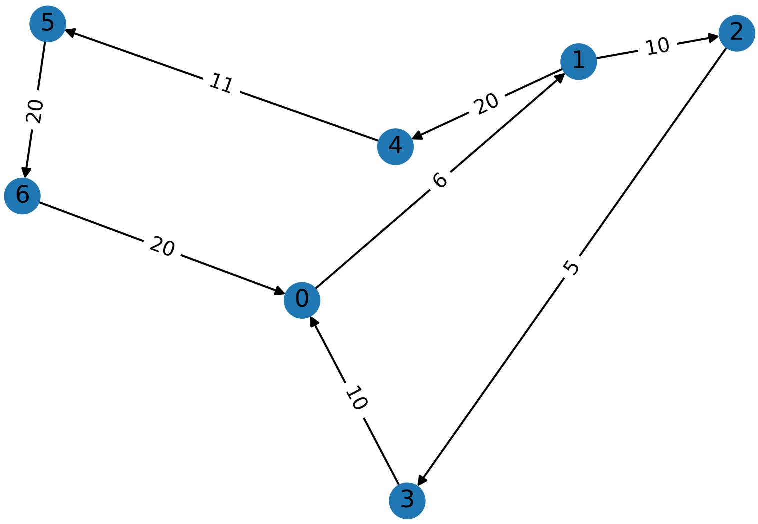


FIGURE 4.14: Netting Example 5: the set of cycles in the NE5 graph is $cycles_{NE5}$ = [[0, 1, 4, 5, 6] , [0, 1, 2, 3]], that is, they have the edge between nodes 0 and 1 in common.

This leads to the following interesting test. Suppose we build two nearly identical

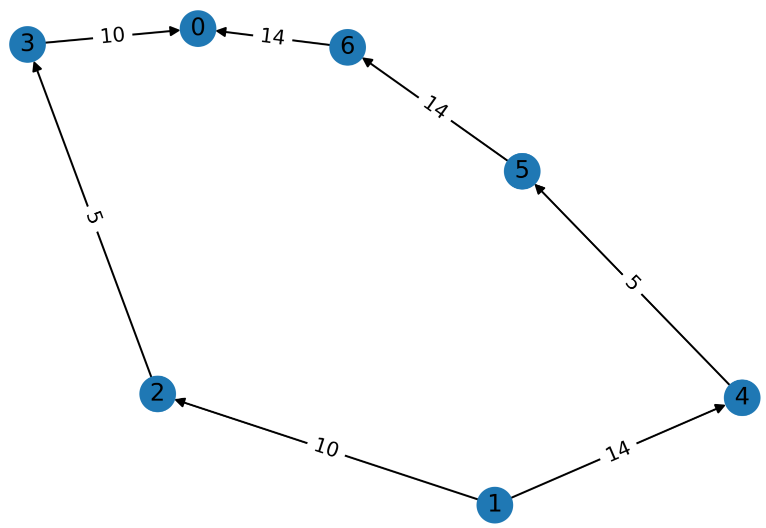


FIGURE 4.15: Netting Example 5, netted using maximum approach: the set of cycles in the NE5 graph after netting is empty, but all the nodes are still connected. In particular, all nodes are reached from node 1.

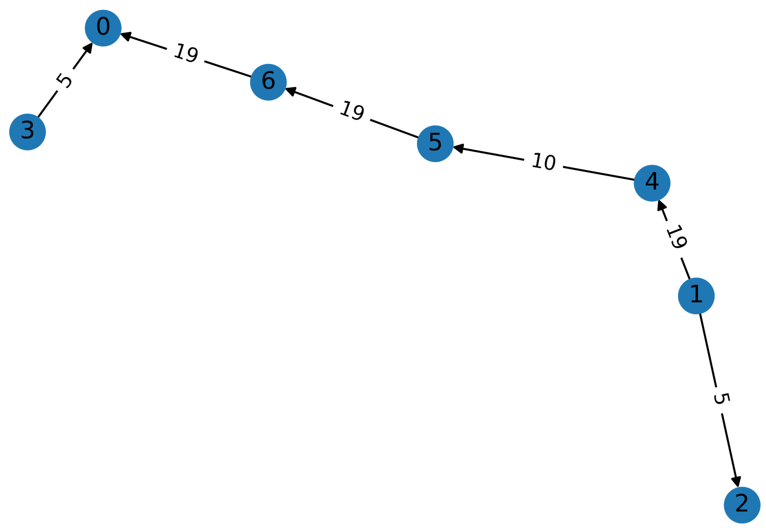


FIGURE 4.16: Netting Example 5, netted using the minimum approach: the set of cycles n the NE5 graph after netting is also empty, but there are no nodes from which all other nodes can be reached.

graphs for a large network, with the only difference between being what is shown in Figures 4.15 and 4.16 for a subset of banks. We would like to see the effect of novation on the final result of the liability matrix starting from these two cases. Consider the following example:

- let the liability matrix L_{EX5M} represent a large network built on top of the core graph in Figure 4.15, with entries shown in Figure 4.17[2].
- let the liability matrix L_{EX5m} represent a large network built on top of the core graph in Figure 4.16, with entries shown in Figure 4.18.
- assume that all entries in the liability matrices (L_{EX5M} and L_{EX5m}) are the same, except for the core nodes, where the entries are proportional to the weights in Figures 4.15 and 4.16, as shown in Figure 4.19, showin the difference between the two networks prior to novation.
- let the liability matrices L^{nov}_{EX5M} and L^{nov}_{EX5m} be the novated versions of the liability matrices L_{EX5M} and L_{EX5m}, respectively.

We would like to check whether the liability matrices L^{nov}_{EX5M} and L^{nov}_{EX5m} are close to each other, in other words if the resulting graphs are very close in shape and weights. As we can see in Figure 4.22, the difference between the entries in the balance sheets is very small, and we can also see in Figures 4.21 and 4.20 that the networks have a very similar structure. In other words, the novation step smoothes and washes away the effect of non-uniqueness introduced by netting.

[2] we fixed the core that is made of 7 nodes in figure 4.15, and to each node we added 100 peripheral nodes where the in-out arrows are randomly assigned.

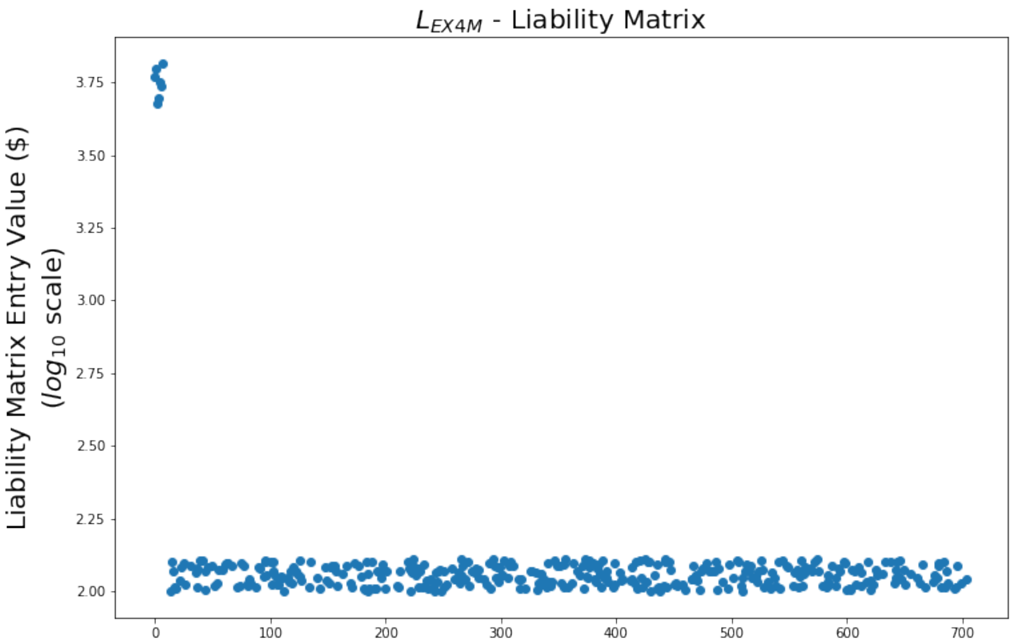


FIGURE 4.17: Large financial network built on core graph of Figure 4.15, corresponding to netting of the graph in Figure 4.14 using the maximal weight removal approach.

4.4 Framework

This section will have different types of test cases. The first subsection will explain in details what are the parameters involved in each test case. The second subsection will introduce a benchmark test case taken from Gai et al. 2011. The third subsection will introduce a bank run as a shock. The fourth subsection will have a bank run and increased hair cut as a shock. The fifth subsection will introduce illiquidity to fixed assets and rerun the third subsection test case.

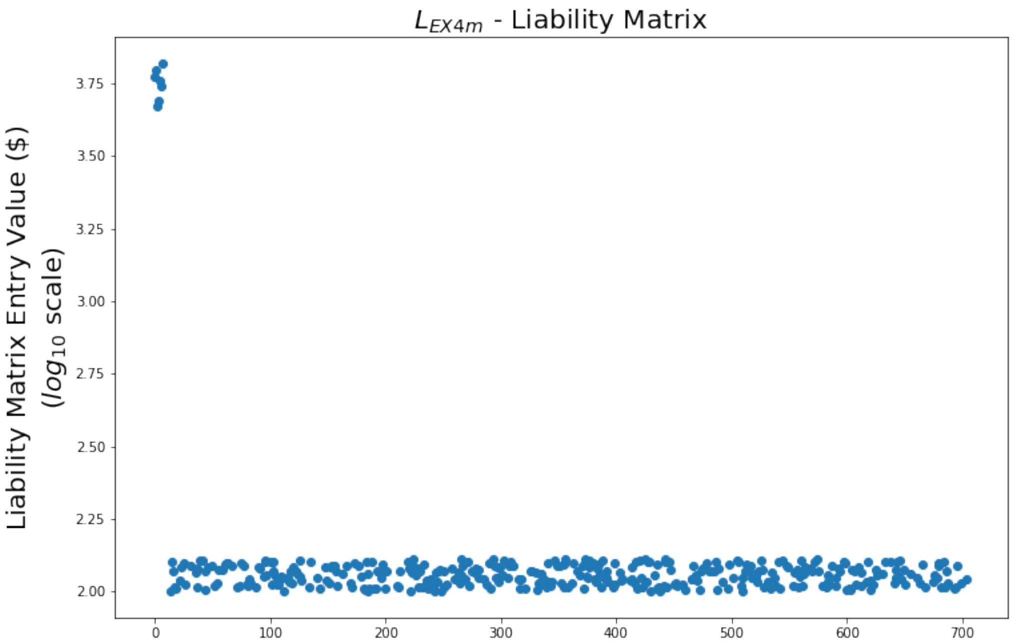


FIGURE 4.18: Large financial network built on core graph of Figure 4.16, corresponding to netting of the graph in Figure 4.14 using the minimal weight removal approach.

4.4.1 Test Case Types

In this section we will be replicating test cases from Gai et al. 2011. Table 4.1 describes the general parameters for financial networks where different types of test cases will be considered.

In all test cases we are going to enforce the following collateral condition:

$$\mathcal{C} :(1-h)A_i^C + \frac{1-h_R}{1-h}A_i^R - L_i^R = 0 \tag{4.1}$$

We would like to remind the reader of the definition of liquidity constraint:

$$\mathcal{L} : A_i^L \geq 0. \tag{4.2}$$

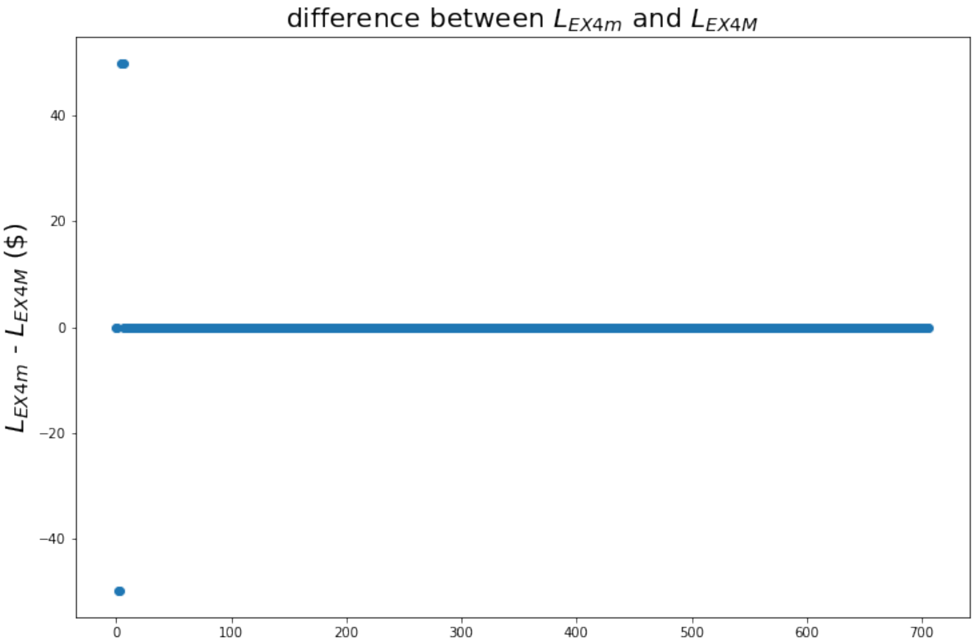


FIGURE 4.19: Difference Matrix for $(L_{EX5m} - L_{EX5M})$.

In each test case type we are going to vary the average connectivity of the financial network. The average financial connectivity is represented by variable z. For every value of z, 1000 simulations will be run and in each simulation certain part of the balance sheet will be shocked. The shock will propagate and statistics will be collected regarding hoarding banks (i.e. banks that have recalled part or all of their reverse-REPO contracts and defaulting banks (i.e. banks that have been removed from the network). Recall that, in our framework, the crisis ends when all banks have resolved their financial conditions, namely when either the collateral, liquidity, and solvency constraints are satisfied by all remaining banks or all banks have been removed from the network.

According to Gai et al. 2011, a systemic hoarding event is said to have occurred when at least 10% of the network hoards liquidity, which in their case means to recall

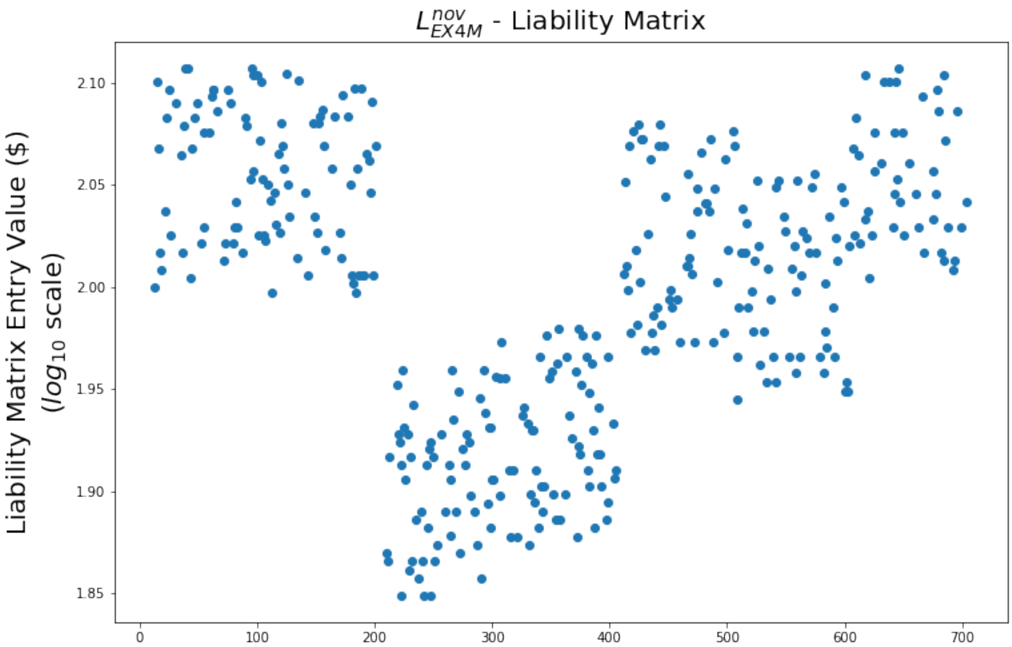


FIGURE 4.20: Novated version of L_{EX5M}, which resulted from netting the core cycle showing in Figure 4.14 using maximum weight removal approach

unsecured loans A_i^U made to other banks, whereas in our framework it means recalling reverse-repurchase agreement (REPO) contracts A_i^R made with other banks. More specifically, two metrics are introduced by Gai et al. 2011 as follows: SHF (systemic hoarding frequency) defined as the fraction of simulations that experience a systemic hoarding event and SHE (systemic hoarding extent), defined as the average fraction of hoarding banks conditional on a systemic hoarding event. In our framework, we introduce the analogous definition for a systemic default event (namely at least 10% of banks being removed from the network), SDF (systemic default frequency: the fraction of simulations with a systemic default event) and SDE (systemic default extent: average fraction of removed banks, conditional on a systemic default event).

In this section we have five types of shocks:

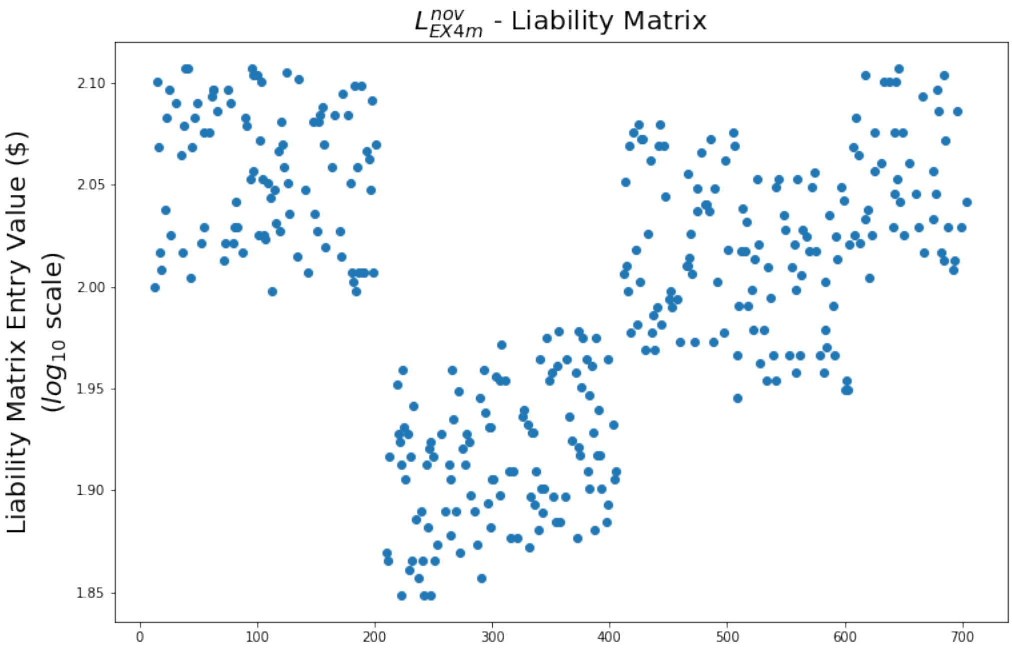


FIGURE 4.21: Novated version of L_{EX5m}, which resulted from netting the core cycle showing in Figure 4.14 using minimum weight removal approach.

1. Liquidity shock: where $\mathcal{L} \leq 0$ which is equivalently $A_i^L \leq 0$.
2. Collateral shock: where $\mathcal{C} \leq 0$ which we can achieve by increasing h from 0.1 to 0.2.
3. Reverse REPO shock: by recalling reverse REPO contracts from counterparties.
4. Fixed assets shock: by decreasing liquidity of fixed assets and use a shock from items 1,2, or 3.
5. Default shock: by forcing banks to default in the network.

In each of the shock types we can use some variations below and check the final result of the contagion on the network. More specifically, we consider the following:

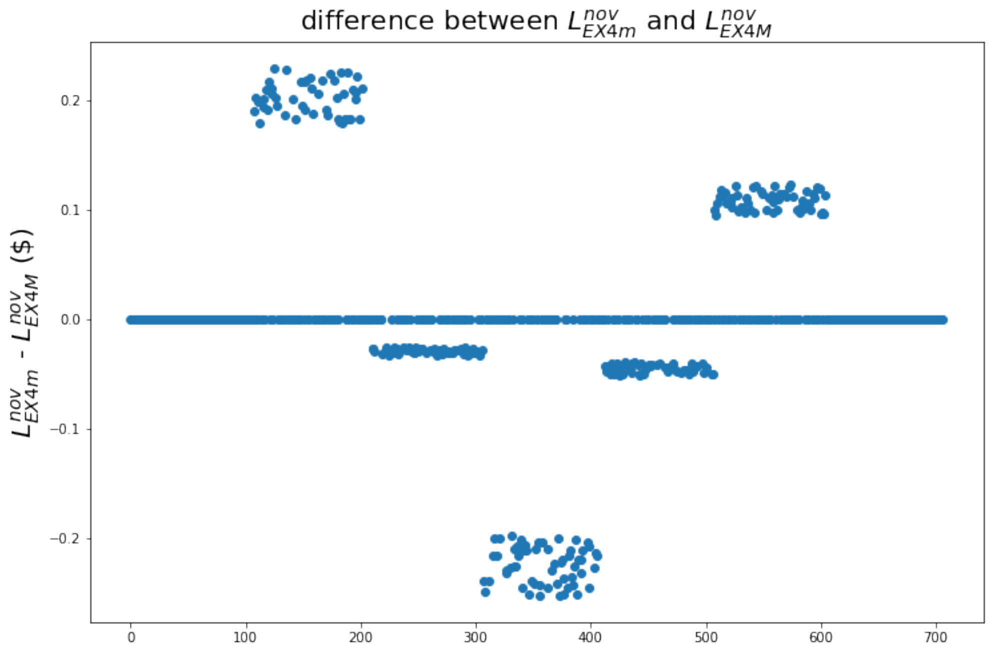


FIGURE 4.22: Difference Matrix for $(L^{nov}_{EX4m} - L^{nov}_{EX4M})$.

Node degree distribution variation: We will use two node degree distributions. The first distribution is Poisson. In Poisson distribution every node is connected to another node with probability p. The second distribution is geometric where most banks in a financial network have low number of counterparties and a few nodes have a high number of connections. As we can see in Figures 4.23 and 4.24, Poisson distribution shows that most nodes have connectivity close to the average connectivity z, while for Geometric distribution we can see that most nodes have very low node connectivity and very few nodes have very high node connectivity.

Non-targeted vs targeted shocks: We can shock banks in the financial network in

Parameter	Description	Value
n	number of banks	250
i	bank identity	varies, $i \in [0, 250)$
A_i^L	cash	2% of balance sheet
A_i^C	collateral assets	varies, depends on in-out degree distribution
A_i^R	reverse repo assets	11% of balance sheet
A_i^F	fixed assets	rest of balance sheet
hc	collateral hair cut	0.1
L_i^R	repo liabilities	varies, depends on in-out degree distribution
P_c	collateral price	1
μ_c	mean of $GBM_{collateral}$	0
σ_c	standard deviation of $GBM_{collateral}$	0
P_f	fixed assets price	1
μ_f	mean of $GBM_{(fixedassets)}$	0
σ_f	standard deviation of $GBM_{(fixedassets)}$	0
α	fixed assets liquidity	0 (infinite liquidity)

TABLE 4.1: Fixed parameters in test cases that represent financial network.

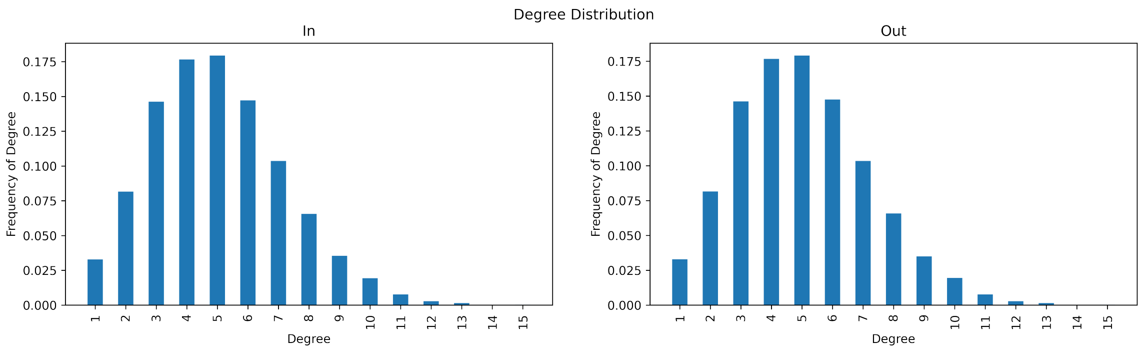


FIGURE 4.23: Poisson Distribution with $z = 5$.

two ways. The first way is a non-targeted shock, where some banks are chosen randomly in a uniform way and suffer a financial shock. The second way is targeted shock, in which we choose the lender with highest number of connections.

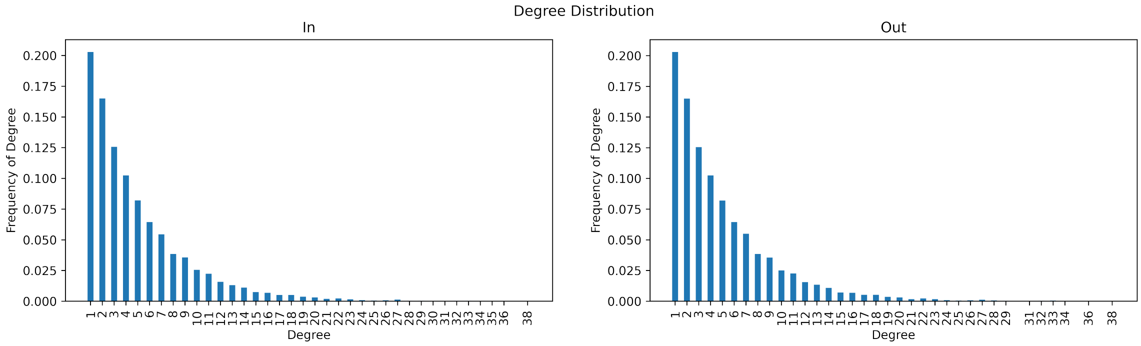


FIGURE 4.24: Geometric Distribution with $z = 5$.

4.4.2 GHK Shock

The benchmark test case in Gai et al. 2011 involves a large increase in the systematic haircut h_i for a single bank i, which causes it to violate the liquidity condition (1.13). This bank then is forced to hoard liquidity by recalling a portion of its interbank loans A_i^{IB} (corresponding to A_i^U in our framework) from other banks, potentially causing a different bank j to violate (1.13) through an increase in the term $-\lambda\mu_j L_j^U$, thereby propagating the initial shock through the network.

First we will show the results that Gai et al. 2011 had and revisit them in the later sections when we will introduce shocks and results to our own system.

Figures 4.25 and 4.26 show the result of simulations. In each z simulation only one bank was shocked.

In Figure 4.25 we have the following 3 test cases and results as stated by Gai et al. 2011 for Poisson distributed nodes:

1. **Poisson baseline:** In this test case, systemic hoarding happens for value of z between 0 and 20. The tipping point for z is 7.5. The frequency of systemic hoarding relative to z increase first then drops around the tipping point of z. The reason that small values of z have low frequency of systemic hoarding is because

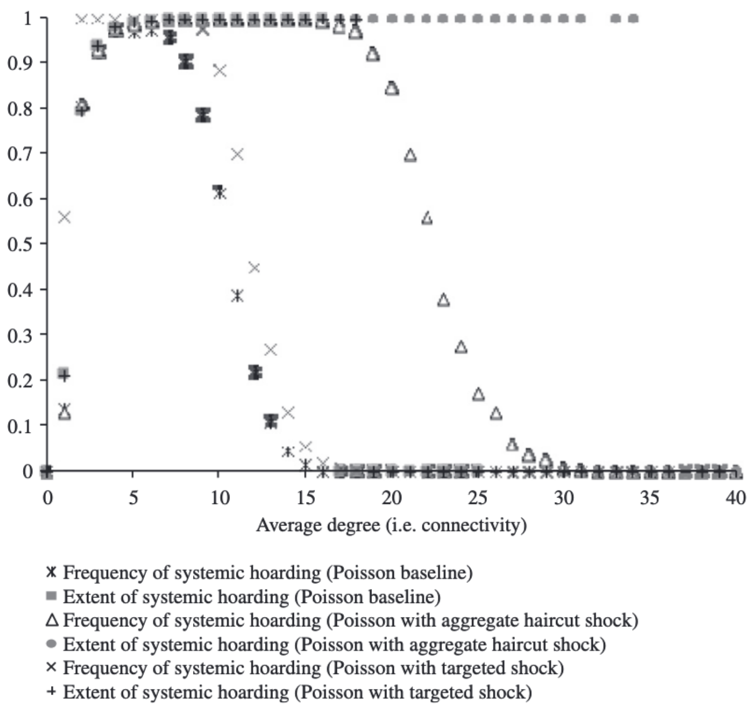


FIGURE 4.25: GHK Poisson distribution test. Source Gai et al. 2011.

the financial institutions are isolated. Although high values of z offers diversity of counterparties and lowers shock magnitude in general, some configurations due to randomness will still lead systemic hoarding breakout. As it can be seen in Figure 4.25 when systemic hoarding happens it affects most of the financial network, as can be seen from the extent of systemic hoarding being close to 1 for z between 5 and 17, after which it drops to zero as the frequency itself is negligible.

2. **Poisson with aggregate haircut shock:** In this test case, the aggregate haircut shock is achieved by increasing the haircut h_c from 0.1 to 0.2. Increasing the haircut requires banks to increase the collateral posted in order to satisfy the collateral condition and achieve the equality in equation 4.1 which will put stress on their liquid assets A^L. This leads to the shift of the tipping point from the baseline at value $z = 7.5$ to value 15.

3. **Poisson with targeted shock:** In this test case the authors shock the bank with highest number of out going connections. In Poisson network targeting highly connected nodes do not make much difference, since the highly connected node in not much different from the rest of the nodes in the network, so the result is very close to the baseline.

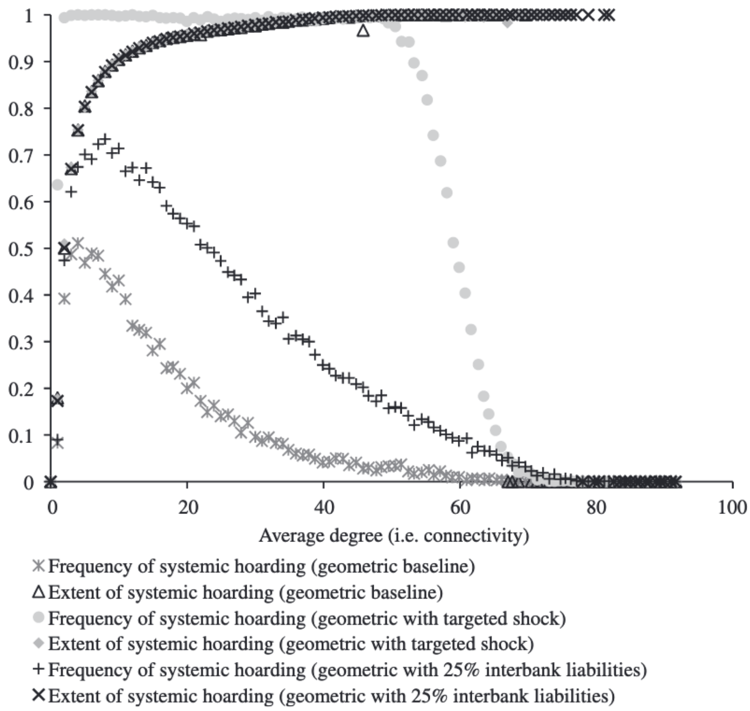


FIGURE 4.26: GHK Geometric distribution test. Source Gai et al. 2011.

In Figure 4.26 we have the following 3 test cases and results as stated by Gai et al. 2011 for geometric distributed nodes:

1. **Geometric baseline:** Because the geometric networks are more sparse than in the Poisson case, they turn out to be more resilient for low or high values of z. There are a few dense nodes and if the shock misses them the shock will end up hitting a sparse part of the network. For example, in geometric baseline, Figure

4.26 the frequency peaks at 0.5 with a corresponding extent of roughly 0.65; While in the Poisson base line, Figure 4.25, we have the frequency peaks at almost one with a corresponding extent of roughly one as well .

2. **Geometric with targeted shock:** Unlike Poisson targeted shock, Geometric targeted shock has devastating consequences, as can be seen in Figures 4.26 and 4.25, the frequency and extent of geometric reaches 1 faster than frequency and extent of possion. The range of z for which systemic hoarding happens in geometric is wider than Poisson targeted shocks, for Poisson systemic hoarding happens for $1 \leq z \leq 16$ while for geometric it happens for $1 \leq z \leq 70$.

3. **Geometric with Liabilities increase:** In this test case the authors increase interbank liabilities from 15% to 25%. This shows that contagion frequency and magnitude become bigger and more serious compared to the baseline test case. The results associated with the 25% is due to the fact that having large liabilities will lead to bigger amounts to be withdrawn from counterparties.

4.4.3 Liquidity Shock (A^L-T1)

In this section we will be shocking deposits D in the balance sheet which will lead to A^L being negative, check table 4.2. Making A^L negative means that liquidity condition is broken and the bank has tapped into the credit line at a previous time step to fix its financial position. Gai et al. 2011 has no test strictly equivalent to this, since idiosyncratic haircut h_i is set to zero in this test case, but their shock to h_i can be seen as a proxy for the more direct liquidity shock that we use here. In other words, we can replicate something similar to experiment 1 in Gai et al. 2011 by having a bank run (deposit run). For this test, number of banks is 100 and number of simulations is 1000.

For this test case and the following ones we will be adopting the following notation: systemic hoarding frequency (SHF), systemic hoarding extent (SHE), systemic default frequency (SDF) and systemic default extent (SDE). We also refer to line A and B as G(geometric) lines while C and D are referred as P(poisson) lines.

Test	Network Distribution	Targeted	Shock Value	#Banks Shocked
A	Geometric	False	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
B	Geometric	True	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
C	Poisson	False	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
D	Poisson	True	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1

TABLE 4.2: Liquidity Shock Parameters.

For T1 the result for all sub-graphs SHF, SHE, SDF and SDE for $A^R(priority = 0)$ will be zero lines. First, liquid and fixed assets will compensate for the deposit shock. Fixed asset is a big portion of the balance sheet we can see that fixed assets and liquid assets combined in our test can cover the deposit shock without the need to use A^R please refer to graph 4.32 for details regarding balance sheet composition for different values of z. As for sub-graphs of $A^R(priority = 100)$, we can see there is some hoarding but there are no defaults. We forced the network to use reverse repo's as first line of defense, and when reverse repo's were depleted fixed assets compensated for the shortage. Even when hoarding happned it was still close to 10%. The reason that systemic hoarding and extent is low is z. When z is low that means the mean value of connection is low and thus there will be islands in the graph which will stop systemic hoarding; on other side, when z is high there will be many counterparties for every bank that receives a shock and thus in return the shock shall be divided equally among the counterparties. For example, assume each bank has on average 10 counterparties in their A^R, if a bank receives a\$600 shock in their deposits. The shock will be \$60 per bank. Each bank already has \$20 in their A^L which lead that each bank has to pass a \$40 shock to its 10 counterparties. Thus the second level of hoarding will be \$4 per bank, which can be covered again from A^L. As for the systemic default, there is no systemic default since the amount of funds available in A^R and A^F will cover any deposit shock(bank run).

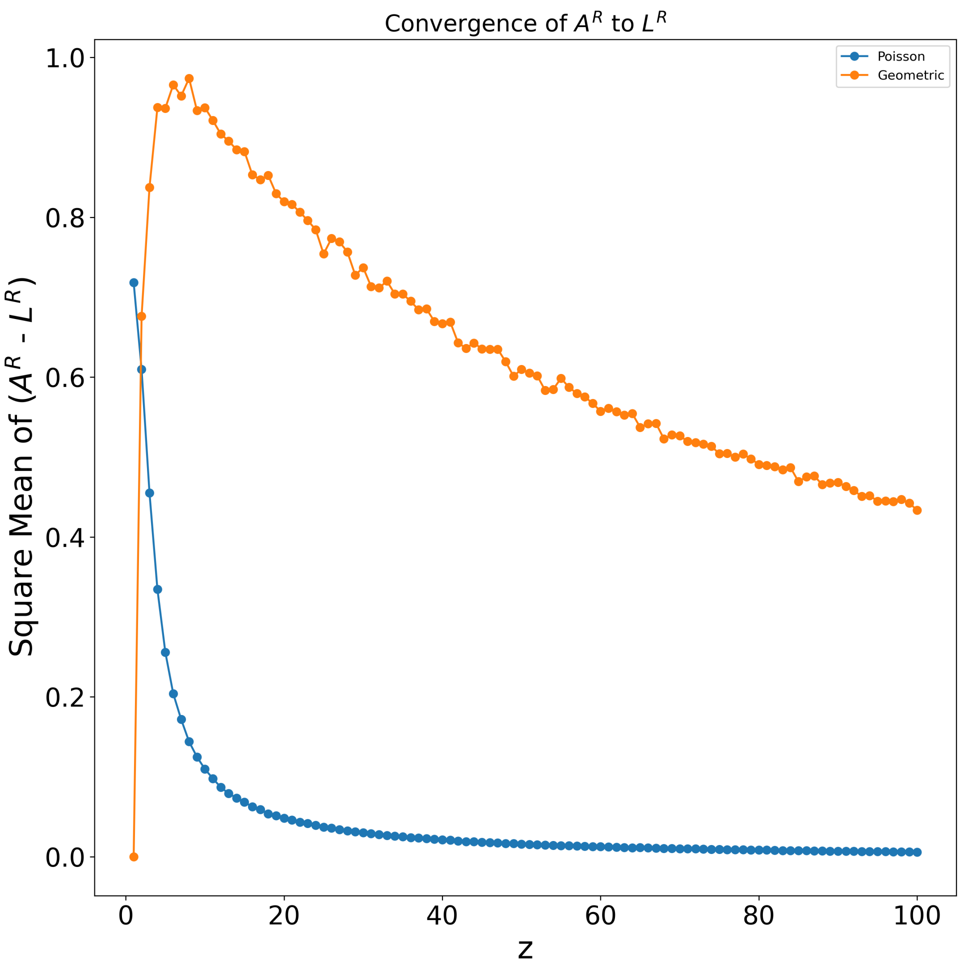


FIGURE 4.27: This Figure shows how A^R value approaches L^R value as z value increases for Poisson and Geometric distributions.

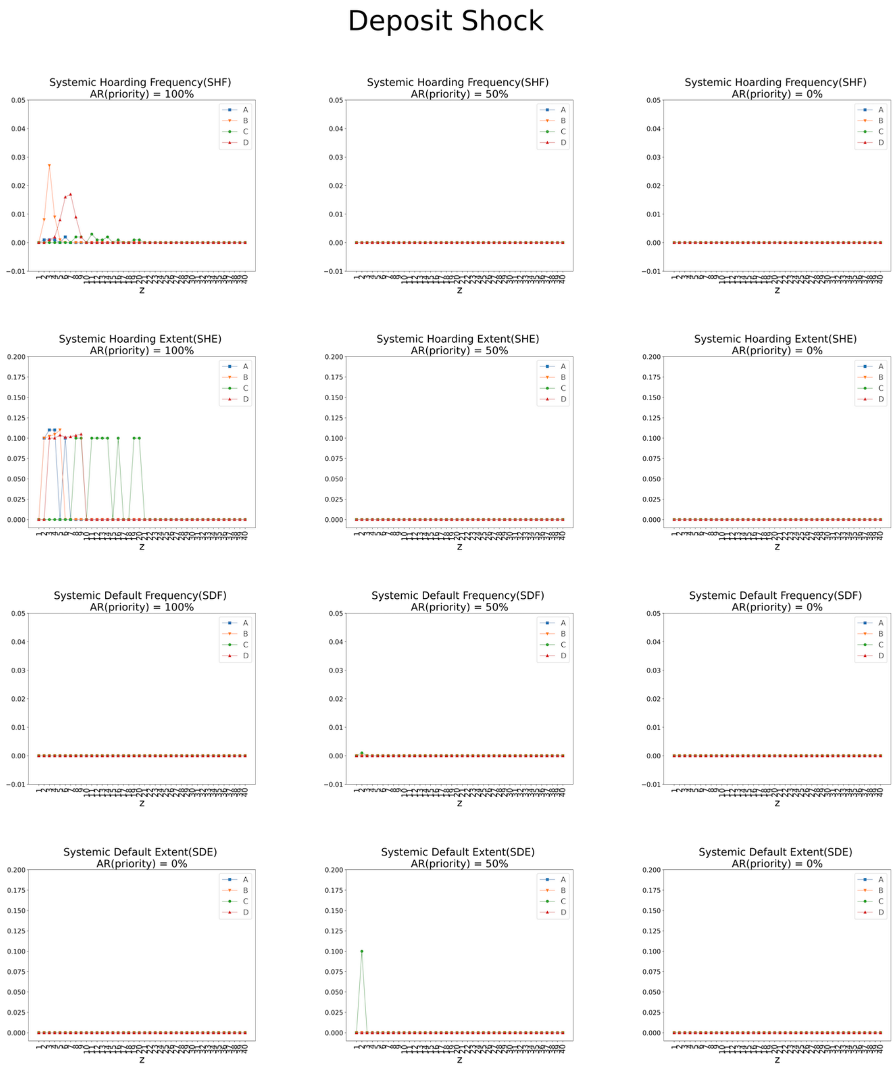


Figure 4.28: T1(Liquidity Shock): $\alpha = 0, D_{new} = 0.15D_{old}, A^L = -0.85D_{old}$. A and B stand for Geometric non-targeted and Geometric targeted shocks respectively while C and D stand for Poisson non-targeted and Poisson targeted respectively.

4.4.4 Collateral Shock (A^C-T2)

In this section we will be shocking A^C in the balance sheet,check table 4.3, by increasing required collateral haircut h on top of the deposit shock, which means banks need to post extra collateral plus fix liquidity constraint. For this test, number of banks is 100 and number of simulations is 1000. This test is similar in spirit to experiment 2 in Gai et al. 2011, which adds a systematic haircut increase to the setup of their experiment 1.

Test	Network Distribution	Targeted	Shock Value1	Shock Value2	#Banks Shocked
A	Geometric	False	$h = 0.2$	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
B	Geometric	True	$h = 0.2$	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
C	Poisson	False	$h = 0.2$	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
D	Poisson	True	$h = 0.2$	$D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1

TABLE 4.3: Collateral Shock Parameters.

In this test, refer to Figure 4.29, we build the balance sheets by using $h_c = 0.1$ then we broke collateral constraint by raising the value of collateral hair cut and we made it $h_c = 0.2$. We break as well the liquidity constraint as we did in T1. The break in collateral constraint will force financial institutions to buy more collateral. Looking into figure 4.29 we can see the following: (1) $A^R(priority = 100)$ case, where we see a decrease in SHF compared to T1 but this decrease is accompanied by the presence of defaults as it is shown in SDF and SDE. (2) $A^R(priority = 50)$ case, we see we have some hoarding in SHF and SHE compared to T1 as the shock is bigger than fixed and liquidity assets available to cover the shock very small number of cases can use hoarding to cover the shock through using reverse repo. (3) $A^R(priority = 0)$ case, we see SDF and SDE are zero and the reason for this we forced the agents to use fixed assets as first line of defence, and as the required money left to cover the shock is not enough the agent defaulted without recalling any reverse repo's in order not to increase the stress in the network.

We can see through cases (1), (2) and (3) that SDF and SDE is consistent which shows the extent of defaults is independent of the approach used as the shock is bigger than the available assets to cover it.

The shares required for collateral can not be covered by A^L, thus tapping into A^F is required. As discussed in T1 the difference between A^R and A^L is too big and most banks will firesale A^F and have final value $A^F = 0$. SHF and SHE should be zero through out the simulations and that makes perfect sense. We already broke the collateral constraint and we need to fix it. By hoarding more liquidity and recalling A^R we will only make the collateral constraint go further away form the required value. The only way to fix the constraint is to keep A^R fixed in value and then we shall increase the value of A^C in the balance sheet until $\mathcal{C} = 0$. The reason why SHF and SHE are not zero is because of the default. When default happens non-defaulted banks still have to pay residuals to defaulted banks and thus will tap into reverse repo.

The same line of reasoning shall be used in order to interpret as to why we see improvement in P lines in graph 4.29 while G lines do not benefit from the increase in z variable. Let's perform an experiment, we will fix the value of $L^R = 1$ for all banks, where L^R is uniformly divided among its counterparties. Then, we will create financial networks with different values of z, for each value of z, we are going to compute mean squared(MS) as the following:

$$MS = \frac{1}{n}\sum_{i=1}^{n}(L_i^R - A_i^R)^2 \tag{4.3}$$

As we can see in the Figure 4.27, for Possion $L^R = A^R$ is reached way earlier than the Geometric. As $L^R = A^R$ the residual will be zero.

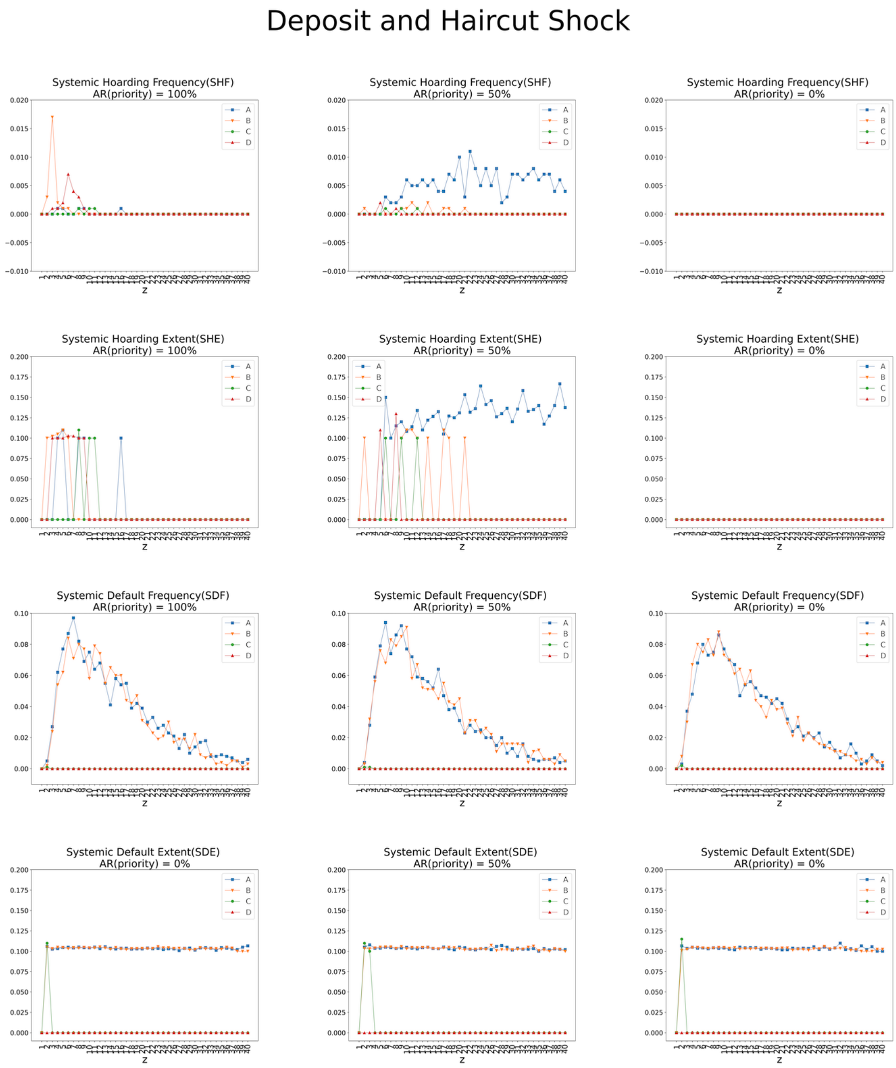


FIGURE 4.29: T2(Collateral Shock): $\alpha = 0, h = 0.2, D_{new} = 0.15D_{old}, A^L = -0.85D_{old}$. A and B stand for Geometric non-targeted and Geometric targeted shocks respectively while C and D stand for Poisson non-targeted and Poisson targeted respectively.

4.4.5 Fixed Assets Illiquidity Shock (A^F-T3)

In this section we will be shocking the network by making fixed assets A^F illiquid. By introducing illiquidity for fixed assets, banks will not be able to collect full value of this assets as it is recorded on their balance sheet. Every time a financial institution liquidates part of A^F, the price of the fixed asset will decrease and all banks in the network will be effected by the new price. In this section we run different tests with different parameter value and check the effect of these parameters on the financial netowrk. Gai et al. 2011 has no test case similar to the one we are performing in this section. In this section we run three different test cases under same initial shock conditions. First case when all agents choose to liquidated their A^R first to meet shock requirements $A^R(priority) = 100\%$. Second case is when half agents give priority to A^R liquidation and the other half chooses to liquidate A^F first, this corresponds to $A^R(priority) = 50\%$. Third case is when all agents give priority to liquidate A^F first and this corresponds to $A^R(priority) = 0\%$. For this test, number of banks is 250 and number of simulations is 3000.

Extreme Deposit Shock

Table 4.4 shows the values for this shock. In this test we use extreme values and this is represented by full bank run (full deposit run) and extreme illiquidity in fixed assets.

Observations that we can see in this test case through figure 4.30 that as $A^R(priority)$ goes from 100% to 50% then 0%, SHF and SHE decreases in value until it becomes nill. The reason for that is the firesale channel(check explanation below for more details). Second observation that can be seen through the mentioned graphs and priority levels is the similarity of results in SDF and SDE. That tells us when illiquidity reaches a certain level hoarding and non-hoarding does not play a role in the default of the banks, as the fire sale channel will have the final say on who is going to default and this becomes a balance sheet composition problem. Banks with high exposure to illiquid assets will end up defaulting.

Test	Network Distribution	Targeted	Shock Value	#Banks Shocked
A	Geometric	False	q=0.00075 $D_{new} = 0$ $A^L = -D_{old}$	1
B	Geometric	True	q=0.00075 $D_{new} = 0$ $A^L = -D_{old}$	1
C	Poisson	False	q=0.00075 $D_{new} = 0$ $A^L = -D_{old}$	1
D	Poisson	True	q=0.00075 $D_{new} = 0$ $A^L = -D_{old}$	1

TABLE 4.4: Fixed Assets Illiquidity Shock Parameters: N/A stands for not-applicable

Discussion of these observations and results:

- for $A^R(priority) = 100\%$ in figure 4.30 : We have the following observations:
  - We can see that SDF and SDE for Poisson distribution(lines C and D) is higher than Geometric distribution(lines A and B). For SDF and SDE, it can be seen in figure 4.32 the Poisson distribution has $A^C > 0$ for most values of z, this means the collateral condition $\mathcal{C} = 0$, when an agent recalls A^R we will have $\mathcal{C} < 0$ thus this will lead to more usage of A^F to bring $\mathcal{C}$ back to zero as done by collateral sub-step. Geometric distribution show $A^C = 0$ for most values of z that means $\mathcal{C} > 0$ and any repo calling will either result in no A^F usage or only a minor sale of fixed assets.
  - We can see that SHF and SHE show activity for Geometric distribution(lines A and B) while Poisson distribution(lines C and D) shows no activity. One reason is that the Poisson has $A^C > 0$ as shown in 4.32. When A^R is recalled that causes the collateral constraint $\mathcal{C}$ to be negative for the agent calling A^R and $\mathcal{C}$ to be positive for the agent paying A^R. Since Poisson has $A^C > 0$ that allows the agents paying the A^R to use their $\mathcal{C}$ and sell A^C to cover

their A^L negative credit, while this is not possible in Geometric. Another reason, Poisson distribution does not show hoarding that is because it reaches 100% SDF and SDE very fast as explained previously. Another factor the contributes to hoarding is the default of banks and the resolution of residuals, as it can be seen in figure 4.32, in the L^R graph, banks in Poisson distribution will have close A^R and L^R while banks in Geometric distribution will have pure lending banks $A^R > L^R$. If a bank defaults in Geometric distribution and is removed then its counterparties will have to pay big residuals and this will make these counterparties recall A^R.

- for $A^R(priority) = 50\%$ in figure 4.30: similar remarks can be taken in consideration when interpreting the results of this figure as $A^R(priority) = 100\%$.
- for $A^R(priority) = 0\%$ in figure 4.30: similar remarks can be taken in consideration when interpreting the results of this figure as $A^R(priority) = 100\%$.

Varying deposit shock and illiquidity parameter

As we have seen in T1,T2 and extreme deposit shock we have systemic hoarding and systemic default are varying but they are taking extreme values. We decided to further investigate what should be a good combination of α(illiquidity for fixed assets) and $\Delta D_i = -d * D_i$(deposit shock) to get an intermediate joint occurrence for systemic hoarding and systemic default for fixed $z = 10$ and $AR(priority) = 100\%$.

Figure 4.33(a) shows that for $\alpha = 0.00075$ there is no significant systemic hoarding, regardless of the size of the initial shock for deposits, whereas SDF and SDE start to increase at 15% initial shock all the way to 50% as they reach values of one or very close to one. We can see that the Poisson lines have higher value than the Geometric lines, for explanation check extreme deposit shock. As for figure 4.33(b) for initial deposit shock of 100% we notice systemic hoarding is close to zero regardless of the value of α but for

SDF and SDE we can see a noticeable increase in the graphs as soon alpha is not having a value of zero.

Moving into an intermediate value for $\alpha = 0.00015$, figure 4.34(a) show us that SDF and SDE starts to increase around initial deposit shock of 70%. We can see as well a noticable increase in SHE reaching to 30% A line and SHE of 20% for most lines. Figure 4.34(b) shows us a noticeable SHF around $\alpha = 0.00015$. As a consequence of the results in figure 4.34, we decided to fix $\alpha = 0.00015$, $d = 0.85$ and run a test for varying values of connectivity z as it will be discussed in

Intermediate Deposit Shock

Initial shock values for this test are presented in table 4.5.As discussed in section 4.4.5 and as expected we see a good amount of liquidity hoarding in figure 4.31.

Test	Network Distribution	Targeted	Shock Value	#Banks Shocked
A	Geometric	False	$q = 0.00015$ $D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
B	Geometric	True	$q = 0.00015$ $D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
C	Poisson	False	$q = 0.00015$ $D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1
D	Poisson	True	$q = 0.00015$ $D_{new} = 0.15D_{old}$ $A^L = -0.85D_{old}$	1

TABLE 4.5: Fixed Assets Illiquidity Shock Parameters: N/A stands for not-applicable

We will discuss the results of figure 4.31.The following points should be noted before we start the discussion:

- each agent has an initial $A^L = \$20$

- the effect of z on the propagation of the shock. When z is small we have separated islands of banks, as z increases the connectivity in the graph increases. When z is small the bank will have less counterparties and thus bigger shocks will be passed to the counterparties. If z is large then most of the shocks will be absorbed by A^L in these counterparties. For example assume a bank gets \$400 shock in deposits. If the bank has 2 counterparties each counterparty will have \$200 shock, each of these counterparties has \$20 in A^L thus each of these counterparties will have to deal with \$180 shock. On the other hand, if a bank has 20 counterparties, for the same deposit shock each counterparty will recieve \$20 as a shock which can be nullfied by the \$20 that is found in A^L.

- the effect of the distribution on the residual $A^R - L^R$. As we can see in figure 4.32 that L^R in possion distribution converges to A^R, while the geometric distribution has a residual of \$100.

Keeping the three upper points in mind and looking at the figures 4.31 (G1) and 4.30 (G2) the following points can be noted.

- the shock that causes systemic hoarding is the residual from defaulting banks. As we discussed earlier when z is small we have islands and when z is large we many counterparties that will absorb a big part of the shock by their A^L. The only option left when illiquidity hits is that if a bank fails it has either to receive money ($A^R > L^R$) or pay money ($A^R < L^R$).As we have seen for possion A^R and L^R get close to each other fast, but most banks have $A^R > L^R$ for all z values for geometric. Illiquidity makes many banks to default, defaulted banks have to be compensated for the residual and thus systemic hoarding is forced if the first option to deal with shock is with liquidating A^R. This explains why geometric shows hoarding while poisson does not.

- by comparing SDF,SDE, SHF and SHE in G1 and G2, we see when SDE is high and

SHF/SHE are low and vice versa. The reason for this is that when illiquidity hits, it affects all banks at the same time, if illiquidty factor is too high most banks(90% to 100%) will fail at the same time and thus the shock will be absorbed(in form of residual) by whatever banks that are left (they are very low in number, which will not cause systemic hoarding). If there is default(60%) then there is enough banks to absorb the shock in form of residual.

- as we change $AR(prioterty)$ from 100% to 50% then to 0% in G1 and G2 we can see that SHF decreases and this is because SDE is increasing as illiquidity is having more effect on our financial network since we are liquidating more and more shares.
- for G1, we can see that SDF curves converge as we lower the $AR(priority)$ the reason for that banks are failing fast due to illiquidity since we are liquidating more shares.
- for G1 and G2 we see that SHE has positive values for geometric distribution that is due to he residual affect of the failed banks.

4.5 Conclusion

Many variables control systemic risk contagion and its effect on financial institutions in the financial network. Two important variables that have significant impact in systemic risk have been investigated. These two variables are network topology and balance sheet composition vs risk factors(bank run and fixed assets illiquidity). Two techniques in resolving defaulted banks have been included in our framework. These techniques are

FIGURE 4.30: T5(Fixed Assets Illiquidity Shock): alpha is 0.00075 and deposit shock is 100%. A and B stand for Geometric non-targeted and Geometric targeted shocks respectively while C and D stand for Poisson non-targeted and Poisson targeted respectively.

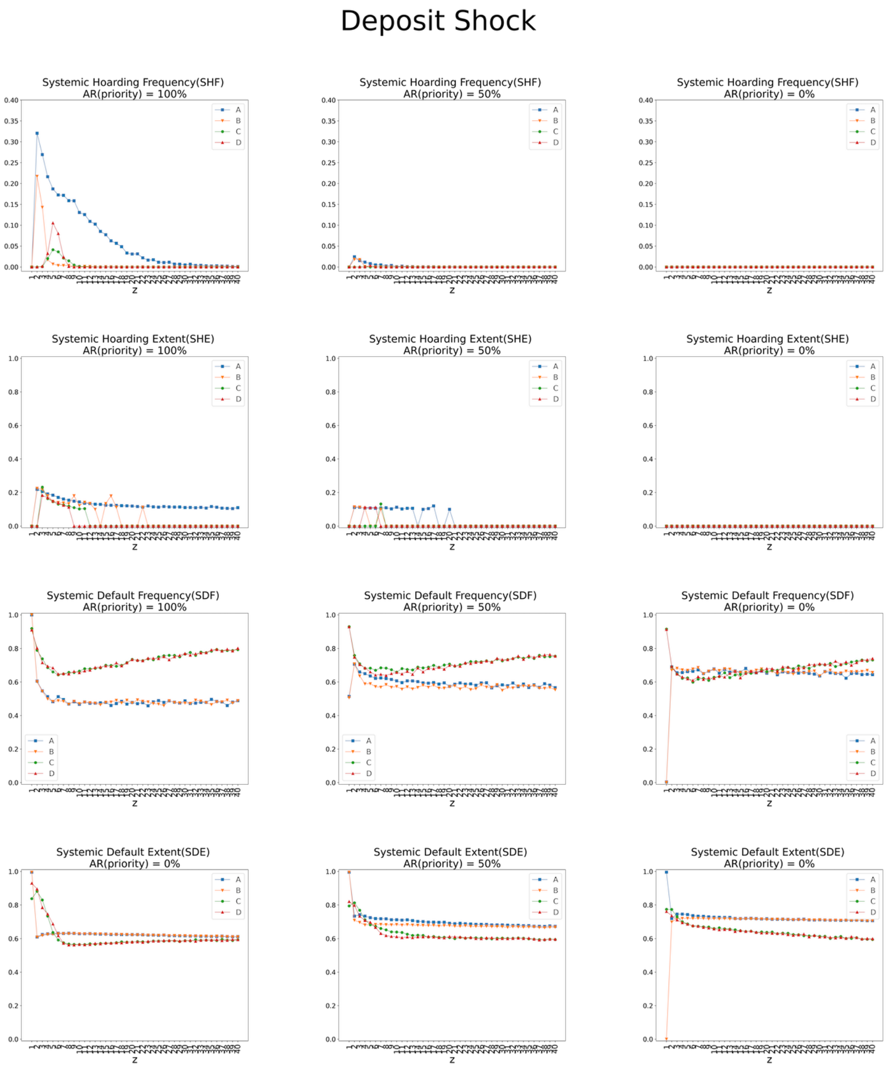


FIGURE 4.31: T5(Fixed Assets Illiquidity Shock): alpha is 0.00015 and deposit shock is 85%. A and B stand for Geometric non-targeted and Geometric targeted shocks respectively while C and D stand for Poisson non-targeted and Poisson targeted respectively.

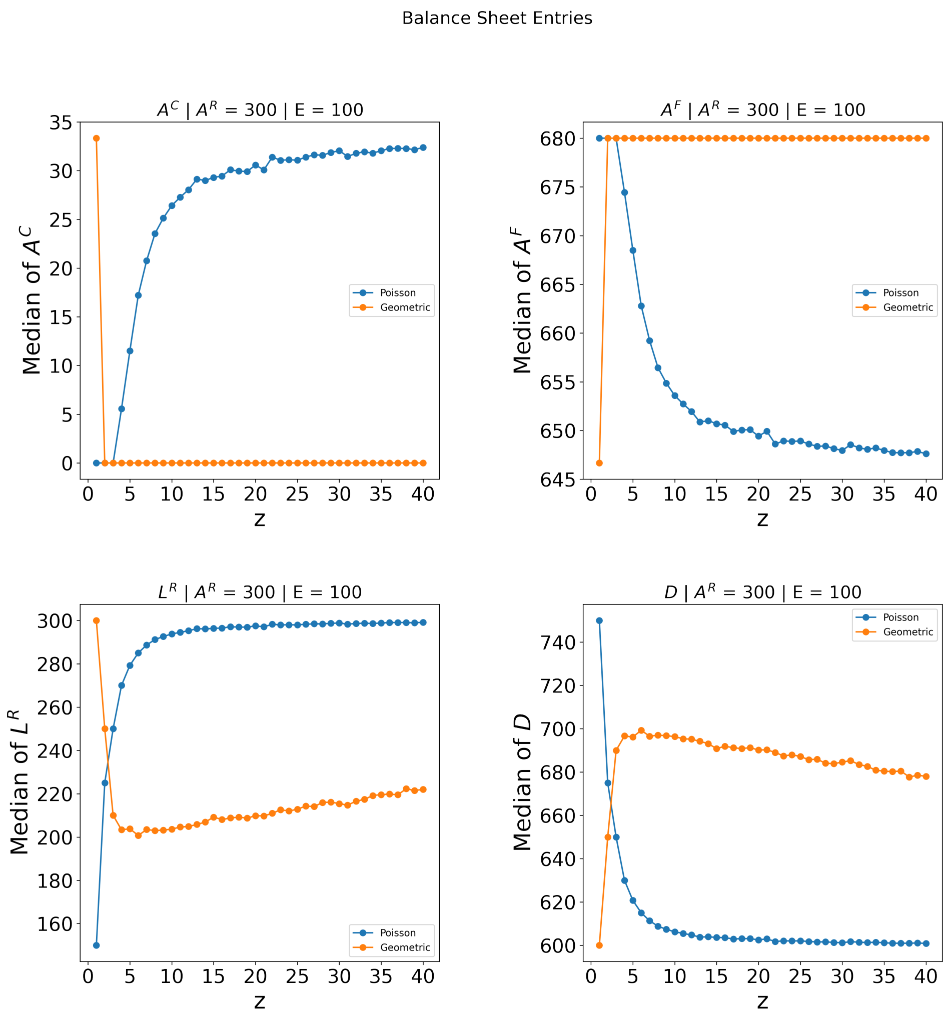
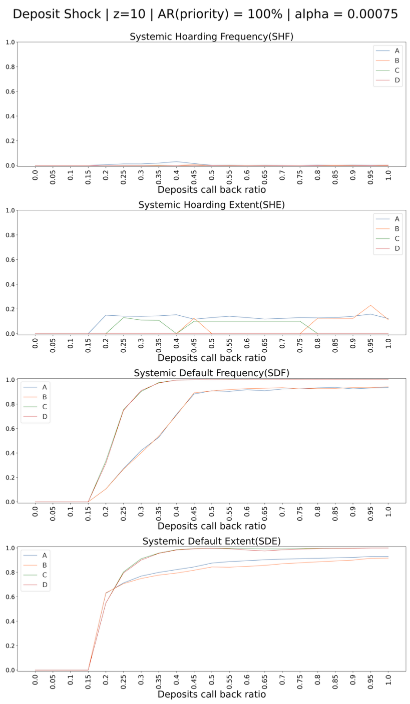
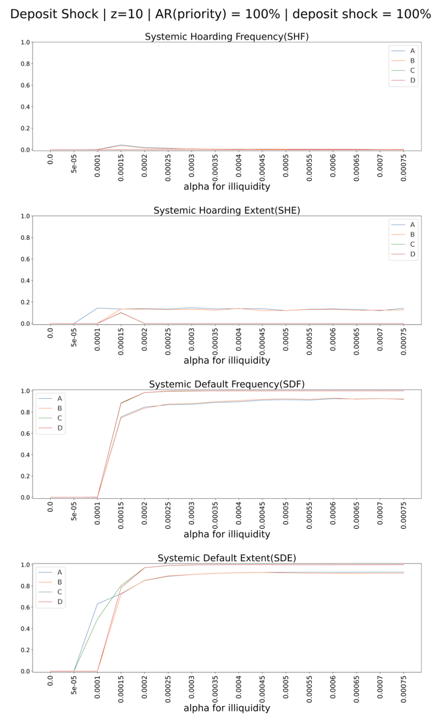


FIGURE 4.32: Balance sheet entries: comparison of balance sheet entries under different network distributions.

netting and novation. In the framework, priority of debt was given to repo liabilities in order to insure the integrity of secured lending. It was shown through simulations that network topology plays a role in systemic risk contagion when we have a financial shock, but has a mild effect when it comes to defaulting banks. As the graph becomes closer and closer to complete the balance sheets start to become similar across banks. Balance

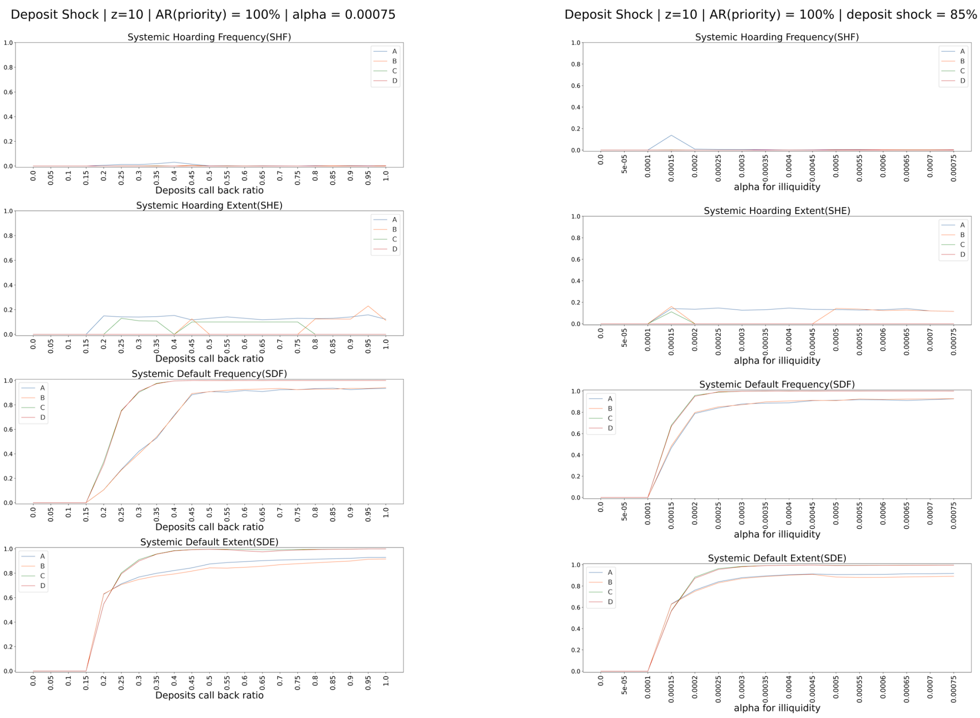


(A) Varying deposit shock, $\alpha = 0.00075$



(B) Varying fixed asset illiquidity, d=1

FIGURE 4.33: Varying deposit call back ratio d and illiquidity of fixed assets parameter α, extreme cases.



(A) Varying deposit shock, $\alpha = 0.00015$

(B) Varying fixed asset illiquidity, d=0.85

FIGURE 4.34: Varying deposit call back ratio d and illiquidity of fixed assets parameter α, intermediate cases.

sheet composition played a role when there is a big residual between repo liabilities and reverse repo assets. As the residuals subsidize the balance sheet, loss that incurred due to default will be less severe to repo contract holders and it will be easier with other entries in the balance sheet. Netting played a role in lowering the exposure of banks by bucketing financial transaction between two banks and leaving only the difference in these transaction to be dealt with, thus lowering the stress on the network. Novation, on the other hand, lowered the impact of a defaulted bank by stopping the close or recall of repo contracts and just changing the ownership of these contracts. Adding defaulted banks to the framework and interpreting the effect of the financial shocks gave a better explanation to what is happening in the financial network. We saw that lower hoarding frequency and/or extent does not necessarily mean better results as this lower hoarding rate is accompanied by higher default frequency. In this thesis we tried to make sure that the bank uses its resources in the most efficient way for both the bank itself and the financial network. The use of the resources tried to keep a balance between bank survival, meeting constraints and having the lowest impact of stress in the financial network.

Chapter 5

Conclusion

In this thesis we study financial agents. Financial agents do not live in vacuum but rather these agents affect each other through direct and indirect channels. Ensuring that the financial network is stable under stressed situation is of vast importance to the economy. Financial models allow us to understand the dynamics of interaction between financial agents, decision making process for each financial agent and the effect of the interaction between the financial agents and the markets. These models introduce metrics that measure the the final effect stressed individual banks impose on the financial network as a whole and what portion of this financial network will be affected/ruined.

In chapter 1 we introduced the two models that we used as the basis upon which to build our own model, namely the Eisenberg-Noe (EN) model from Eisenberg and Noe 2001 and the Gai-Haldane-Kapadia (GHK) model from Gai et al. 2011, and explain their main limitations. For the EN model, the main issues where: (1) an overly simplistic balance sheet, (2) the strict positive liquid assets condition, (3) absence of default cost, (4) a deterministic and static model, and (5) conservation of losses. For GHK, we have the following issues: (1) a deterministic and static model, (2) a financial shock that is resolved in one time step, (3) too many restrictive assumptions (e.g 100% recall of reverse repo, same in and out degrees of nodes) needed to justify the "tipping point" condition 1.15, and (4) an unclear default resolution mechanism (in particular what happens to rehyptothecated collateral upon default of multiple banks). The purpose of our model is to address many of these issues. Specifically, our model has: (1) a more realistic balance sheet than EN, (2) the possibility of negative liquid assets, (3) a stochastic and dynamic structure, (4) non-conservation of losses, (5) financial shocks resolved over multiple time steps, and lastly (6) fully specified default and collateral resolutions.

In chapter 2 we introduced our model that adopts many parts from GHK. GHK has a good representation of a balance sheet, which solves problem (1) in EN. We switched the model type from threshold model to agent-based model. Agent-based model allow us to relax many of the assumptions and issues that are discussed in GHK. For example, it allowed us to introduce priority in debt (to ensure the priority of repo contracts in default resolution) and priority in liquidation (to keep the markets stable by assessing markets as a last resort). By introducing stochastic prices for collateral A^C and fixed assets A^F we were able to deal with issue of deterministic and static nature of EN and GHK and by introducing illiquidity for fixed assets A^F where able to solve conservation of losses problem, namely by introducing firesale effects. The decision to switch to agent based model allows us also to solve the financial shock in multiple time steps and it also allows us to use any network topology as reaching an analytical answer is not a goal in our model. We introduced examples that show issues that arise with REPO rehypothecated chains. The main issue is how to redistribute the rehypothecated chain collateral in the case of simultaneous defaults of multiple banks.

In chapter 3 we introduce a novation algorithm that specifies which defaulted bank gets what part in rehypothecated collateral chain, which solves problem (4) in GHK. Moreover, we proved that the order of novation is irrelevant. Novation makes sure it keeps the repo market calm by stopping the recalling of repos and reverse repo that would otherwise lead to market instability. Instead of closing down repo contracts, novation rewires the ownership of the contracts. We introduced netting as well which lowers the exposure of banks to each other in a loop configuration. Moreover, just like GHK, we have liquidity and collateral constraints that govern the balance sheet. The constraints need to be met during the life of the financial agents, who make decisions according to well-define regions in the $A^R \times A^F$ plane. These constraints guarantees the safety of the collateral, the payment of the REPO liability and minimizes the interaction of financial agents with the markets.

Finally in chapter 4 we run different tests and simulation for representative cases. In the tests we run, we show that, unlike novation, netting does not always produce a unique solution. Nevertheless, we gave an example that shows the non-uniqueness of netting will be washed away by the effects of novation. The effect takes place since novation changes the distribution of connection in a network. For example, novating a star topology at the star will change the topology into Poission distribution. We also gave examples that confirm our theorem for uniqueness of novation. Lastly, we had test cases with benchmark test cases in Gai et al. 2011. We added another metric to the framework which is the defaulted banks. It has been shown that lower hoarding frequency and/or extent does not mean the financial network is healthy. Lower hoarding frequency/extent is usually accompanied by higher default frequency.

Bank decisions does not only effect itself regarding profit and loss but it rather effect the entire financial network through contagion channels that we discussed in previous chapters. Our framework took three objectives into account. First objective is financial survival, second objective is meeting balance sheet constraints and the third objective lowering impact of stressed banks on the financial network. These objectives are met by minimizing interaction with markets and agents.

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