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Derivation of the sliding innovation information filter for target tracking

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ABSTRACT

An information filter is one that propagates the inverse of the state error covariance, which is used in the state and parameter estimation process. The term 'information' is based on the Cramer-Rao lower bound (CRLB), which states that the mean square error of an estimator cannot be smaller than an amount based on its corresponding likelihood function. The most common information filter (IF) is derived based on the inverse of the Kalman filter (KF) covariance. This paper introduces preliminary work completed on developing the information form of the sliding innovation filter. The SIF is a relatively new type of predictor-corrector estimator based on sliding mode concepts. In this brief paper, the recursive equations used in the sliding innovation information filter (SIIF) are derived and summarized. Preliminary results of application to a target tracking problem are also studied.

Keywords: Information filter, Kalman filter, sliding innovation filter, smooth variable structure filter, target tracking

1. BRIEF INTRODUCTION

The primary goal of estimation theory is to derive valuable information about the state of a system in the presence of system and measurement noise. The Kalman filter (KF) is one of the most well-studied methods and provides an optimal estimate for linear systems when dealing with known systems and white noise [1, 2, 3]. The KF has been applied to many areas and fields, including applications such as target tracking, signal processing, and fault detection [4, 5, 6]. When dealing with nonlinear systems, the KF can be modified to approximate the nonlinearities of the system, resulting in the extended Kalman filter (EKF). If the system is highly nonlinear, however, the EKF may diverge from the true state trajectory and result in numerical instabilities and poor estimation performance [7, 8, 9, 10, 11, 12]. In such cases, the unscented Kalman filter (UKF) is better equipped to deal with the nonlinearities by utilizing sigma points which formulate a weighted statistical linear regression approach to approximating the nonlinearities [13, 14, 15, 16]. While the UKF has demonstrated effectiveness in a number of signal processing applications, it can be resource-intensive and sensitive to modeling uncertainties and disturbances [17, 18, 19].

In target tracking applications where one may be concerned with the surveillance, guidance, obstacle avoidance or simply tracking of a target, sensors are responsible for providing a noise-corrupted signal or measurements [20, 21]. Typically, the signal from these sensors is processed by estimation techniques to result in and output uncorrupted measurements of the system's state, such as kinematic information regarding its position, velocity, and acceleration. These measurements are processed in a fashion which maintains tracks, which are a sequence of target state estimates that vary with time. Gating and data association techniques are often employed to help classify the source of measurements, as well as to associate the measurements to the appropriate track. These gating techniques help to avoid or minimize extraneous measurements which may result in the estimation process' instability or failure. Tracking filters are often used in recursive a fashion to carry out the estimation of the target states.

An information filter (IF) is a type of filter that propagates the inverse of the state error covariance instead of using the normal covariance in the gain calculation like with the KF [22]. The Cramer-Rao lower bound (CRLB) is the motivation behind the 'information 'term, whereby the Fisher information matrix (FIM) is computed as the inverse of the covariance matrix [23]. The CRLB dictates a theoretical limit on the squared error of an estimator, whereby it cannot be smaller than an amount based on its corresponding likelihood function [23]. If a filter's variance is equal to the CRLB, then it is considered to be an effective filter. The IF has certain advantages, such as the ability to initialize the information matrix to zero if no prior information is available, thus eliminating any bias in the a priori estimate [24]. In addition, the update of the information matrix after the observations is considered to be more robust than the covariance filter form, which is an especially attractive feature when round-off errors may pose an issue [24].

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Regardless, it is a well known fact that information filtering often presents with the problem of a lack of understanding or interpretability of the 'information states' [24]. However, the lack of interpretability may be remedied by inverting the information matrix.

Other types of estimation methods exist beyond the KF and IF, some of which are based on sliding mode theory. Sliding mode observers are examples of such methods, and are also based on variable structure theory [25, 26]. The gain of the observer is computed using the innovation and is implemented in a manner which forces the surface of the error to zero [27]. Sliding mode observer methods employ a hyperplane or sliding surface, and apply a discontinuous switching force upon the estimate to ensure it is bounded within the region of the hyperplane [25, 28]. As a result, these strategies result in estimates which offer more robustness to modeling uncertainties and external disturbances.

Based on the concepts of sliding mode observers, the smooth variable structure filter (SVSF) was proposed [4, 25, 29]. The SVSF demonstrates further improvements in robustness to modeling uncertainties, and provides a sub-optimal solution in terms of estimation accuracy [4, 30, 31]. The sliding innovation filter (SIF) was proposed more recently, and utilizes a simpler gain whilst also yielding improved estimate accuracy compared to the SVSF [32]. Similar to the EKF, the extended SIF (ESIF) was proposed to deal with nonlinear systems. Both the SIF and ESIF from [32] make use of a fixed-width siding boundary layer with the assumption of a constant upper limit on the modeling and measurement uncertainty [33]. An adaptive formulation of the SIF was presented in [34], which presented a time-varying sliding boundary layer that yielded optimal linear estimation results while maintaining robustness to uncertainties and disturbances.

In this paper, we introduce the derivation of the sliding innovation information filter (SIIF), which is effectively the SIF in an information form. We apply the SIIF to a target tracking problem, and compare the results with the IF method. The paper is organized as follows. The main estimation strategies used in the paper are summarized in Section 2. The SIIF is derived in Section 3, and the simulation setup and results are provided in Section 4. The paper is then concluded.

2. REVIEW OF ESTIMATION STRATEGIES

In this section, we provide a review of the main estimation strategies used in this paper. In particular, the Kalman filter, information filter, and sliding innovation filter are summarized.

2.1 Kalman Filter

Although it is the most popular method, the KF will be summarized here for completeness. The KF provides the optimal solution to the linear estimation problem which is described by (2.1.1) and (2.1.2). The goal of any estimator is to obtain the true state value x_{k+1} using noisy measurements z_{k+1}.

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{2.1.1}$$

$$z_{k+1} = Cx_{k+1} + v_{k+1} \tag{2.1.2}$$

where x represents the system state vector, A is the discretized linear system model matrix of differential equations, B is the input gain matrix, u is the input vector, w is the system noise, z is the measurement vector, H is the linear measurement matrix, v represents the measurement noise, and k represents the current timestep.

The Kalman filter (KF) works under the assumptions that the system model is relatively well-known, and the initial states are also known, and finally, that the system and measurement noise is normal and Gaussian meaning that it is white with zero mean and known respective covariance matrices [3]. The KF works as a predictor-corrector; the system model is used to obtain an *a priori* or predicted estimate of the states, whereupon measurements combined with the Kalman gain matrix are used to apply a correction term to create an *a posteriori* or updated state estimate [4], [5].

The prediction stage involves calculating the state estimates based on the previous state values and knowledge of the system, as per (2.1.3). The corresponding state error covariance matrix is calculated in (2.1.4) and is used in the update stage to calculate the KF gain in (2.1.5) and update the state error covariance as per (2.1.7).

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1.3}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.1.4}$$

The update stage is summarized by (2.1.5) through (2.1.7). The gain calculated in (2.1.5) is used to update the state estimates in (2.1.6) based on the measurement error (or innovation). The gain is also used along with the predicted state error covariance to update the state error covariance in (2.1.7).

$$K_{k+1} = P_{k+1|k}C^T\left(CP_{k+1|k}C^T + R_{k+1}\right)^{-1} \tag{2.1.5}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left(z_{k+1} - C\hat{x}_{k+1|k}\right) \tag{2.1.6}$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T \ldots$$
$$\ldots + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.1.7}$$

Note that k refers to the time step, $k|k$ refers to the updated values at the previous iteration, and $k+1|k$ refers to the predicted values at time $k+1$ based on information at time k. Equations (2.1.3) through (2.1.7) represent the KF estimation process for linear systems and measurements defined by (2.1.1) and (2.1.2), respectively. The process is iterative and repeats every time step k. Note that (2.1.7) is known as the Joseph covariance form, and is considered to be numerically stable. The basic nonlinear form of the KF, known as the extended Kalman filter (EKF), is based on linearizing the nonlinear system and/or measurement equations by first-order Taylor series expansions. This is described later at the end of Section 2.3.

2.2 Information Filter

The most common form of the information filter (IF) found in the literature is based on the KF derivations (Section 2.1), where the inverse of the state error covariance matrices is utilized. The 'information states' are functions of the covariance inverses and the true state vectors. The predicted and updated information states are defined by (2.2.1) and (2.2.1), respectively [35].

$$\hat{a}_{k+1|k} = P_{k+1|k}^{-1}\hat{x}_{k+1|k} \tag{2.2.1}$$

$$\hat{a}_{k+1|k+1} = P_{k+1|k+1}^{-1}\hat{x}_{k+1|k+1} \tag{2.2.2}$$

Applying the matrix inversion lemma to (2.2.4) and (2.2.7) yields the corresponding information matrices, as presented in [35]:

$$P_{k+1|k}^{-1} = \left[I - A_p\right](A^{-1})^T P_{k|k}^{-1} A^{-1} \tag{2.2.3}$$

$$P_{k+1|k+1}^{-1} = P_{k+1|k}^{-1} + CR_{k+1}^{-1}C^T \tag{2.2.4}$$

where A_p is defined by:

$$A_p = (A^{-1})^T P_{k|k}^{-1} A^{-1}\left[(A^{-1})^T P_{k|k}^{-1} A^{-1} + Q_{k+1}^{-1}\right]^{-1} \tag{2.2.5}$$

The gain associated with the IF (simplified) is defined in literature as follows [35, 22]:

$$K_{IF_{k+1}} = AP_{k+1|k+1}^{-1}CR_{k+1}^{-1} \tag{2.2.6}$$

Using the gain (2.2.6) and information matrices (2.2.3) and (2.2.4), the predicted and updated information vectors used by the information filter may be found respectively by [35, 22]:

$$\hat{a}_{k+1|k} = \left[I - A_p\right]A^{-T}\hat{a}_{k|k} \tag{2.2.7}$$

$$\hat{a}_{k+1|k+1} = \hat{a}_{k+1|k} + CR_{k+1}^{-1}z_k \tag{2.2.8}$$

Equations (2.2.3) through (2.2.8) constitute the main formulas used in the information filter. Furthermore, note that the actual inverses do not have to be calculated, as the states are solved in a recursive manner.

2.3 Sliding Innovation Filter

The sliding innovation filter (SIF) is a predictor-corrector estimator based on sliding mode concepts [32]. The difference between the KF and SIF is the structure of the corrective gain matrix. The SIF gain is calculated using the measurement matrix, innovation, and sliding boundary layer term. An initial estimate is pushed towards the sliding boundary layer which is based on the upper limit of uncertainties in the estimation process [32]. If the estimate is within the sliding boundary layer, the estimates are forced to switch about the true state trajectory by the SIF gain.

Figure 1 illustrates the SIF estimation concept. This section describes the linear SIF estimation process. The prediction stage is given by the following equations:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.3.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_{k+1} \tag{2.3.2}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{2.3.3}$$

where x refers to the state, $\hat{x}$ refers to the estimated state, u refers to the system input, z refers to the measurement, $\tilde{z}$ refers to the innovation (or measurement error), and k refers to the time step. In addition, A, B, C, P, Q, and R, are respectively defined as the system matrix, input gain matrix, measurement matrix, state error covariance matrix, system noise covariance, and measurement noise covariance. Note also that $k+1|k$ and $k+1|k+1$ refer to predicted and updated values, respectively.

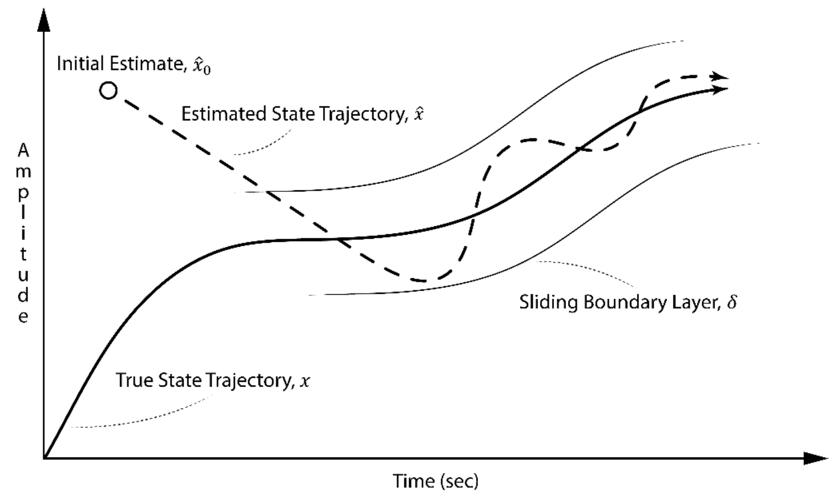

Figure 1. The sliding innovation filter (SIF) concept illustrating the effects of the switching gain and sliding boundary layer [32].

The states are predicted in (2.3.1) before being updated in (2.3.5) using the innovation defined in (2.3.3) which is also used in the gain formulation in (2.3.4). The state error covariance matrix is predicted in (2.3.2) before being updated in (2.3.6). Note that the gain (2.3.4) is also used to update the state error covariance (2.3.6). The <u>update state</u> is summarized by the following equations:

$$K_{k+1} = C^+\overline{sat}\left(\left|\tilde{z}_{k+1|k}\right|/\delta\right) \tag{2.3.4}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{2.3.5}$$

$$\begin{aligned} P_{k+1|k+1} = (I - K_{k+1}C^+)P_{k+1|k}(I - K_{k+1}C^+)^T \ldots \\ \ldots + K_{k+1}R_{k+1}K_{k+1}^T \end{aligned} \tag{2.3.6}$$

where C^+ refers to the pseudoinverse of the measurement matrix, $\left|\tilde{z}_{k+1|k}\right|$ refers to the absolute innovation value, T refers to transpose of a vector or matrix, δ refers to the fixed sliding boundary layer width, and $\overline{sat}$ refers to the diagonal matrix of the saturated vector values. The sliding boundary layer term may be tuned based on designer knowledge of the system (e.g., level of noise) in an effort to minimize the state estimation error.

Equations (2.3.1) through (2.3.6) represent the SIF estimation process for linear systems and measurements. The SIF proof of stability was discussed in detail in [32]. A Lyapunov function was defined based on the updated innovation, and was used to prove stability. Note that the nonlinear version of the SIF, the extended SIF (ESIF), is similar to the SIF with the main difference being the formulation of the gain [32]. Similar to the EKF, the ESIF uses Jacobian matrices to linearize the nonlinear system $f(\hat{x}_{k|k}, u_k)$ and nonlinear measurement $h(\hat{x}_{k+1|k})$ functions, respectively as follows:

$$F_k = \frac{\partial f}{\partial x}\bigg|_{(\hat{x}_{k|k}, u_k)} \tag{2.3.7}$$

$$H_{k+1} = \frac{\partial h}{\partial x}\bigg|_{(\hat{x}_{k+1|k})} \tag{2.3.8}$$

In its current formulation, the state error covariance matrix P defined in the SIF estimation process is not used to update the state estimates. However, as will be shown in Section III, it is used to derive a time-varying sliding boundary layer.

3. DERIVATION OF THE SLIDING INNOVATION INFORMATION FILTER

In this section, a new information filter based on the SIF is derived, and is referred to as the smooth innovation information filter (SIIF). To begin, consider the predicted and updated information vectors again, respectively as follows:

$$\hat{a}_{k+1|k} = P_{k+1|k}^{-1}\hat{x}_{k+1|k} \tag{3.1}$$

$$\hat{a}_{k+1|k+1} = P_{k+1|k+1}^{-1}\hat{x}_{k+1|k+1} \tag{3.2}$$

Next, the inverse of the covariances (or the information matrices) need to be solved. First, the predicted information matrix will be found. Consider the following definition:

$$A_q = A^{-T}P_{k|k}^{-1}A^{-1} \tag{3.3}$$

From the definition of the state error covariance matrix in (2.3.2), we then have:

$$P_{k+1|k}^{-1} = \left[A_q^{-1} + IQ_{k+1}I^T\right]^{-1} \tag{3.4}$$

Rewriting (3.4) allows the matrix inversion lemma to be applied [35]:

$$[a_1^{-1} + a_2a_3a_2^T]^{-1} = [I - a_1a_2(a_2^Ta_1a_2 + a_3^{-1})^{-1}a_2^T]a_1 \tag{3.5}$$

Which yields a more complete form of (3.4) as follows:

$$P_{k+1|k}^{-1} = \left[I - A_q\left(A_q + Q_{k+1}^{-1}\right)^{-1}\right]A_q \tag{3.6}$$

Alternatively, we have the following for the predicted information matrix:

$$P_{k+1|k}^{-1} = [I - A_r]A_q \tag{3.7}$$

$$A_r = A_q\left(A_q + Q_{k+1}^{-1}\right)^{-1} \tag{3.8}$$

Next, the updated information matrix needs to be solved. Consider the following, which is the inverse of (2.3.6):

$$P_{k+1|k+1}^{-1} = \left[(I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T + K_{k+1}R_{k+1}K_{k+1}^T\right]^{-1} \tag{3.9}$$

Note that K_{k+1} in this case is the SIF-based gain. Similar to before, consider the following definition:

$$A_s = \left[(I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T\right]^{-1} \tag{3.10}$$

Such that:

$$A_s = (I - K_{k+1}C)^{-T}P_{k+1|k}^{-1}(I - K_{k+1}C)^{-1} \tag{3.11}$$

The updated information matrix can then be rewritten as follows:

$$P_{k+1|k+1}^{-1} = [A_s^{-1} + K_{k+1}R_{k+1}K_{k+1}^T]^{-1} \tag{3.12}$$

Doing so allows the use of the matrix inversion lemma. This allows us to solve for the complete form of (3.9), as per:

$$P_{k+1|k+1}^{-1} = [I - A_s K_{k+1}(K_{k+1}^T A_s K_{k+1} + R_{k+1}^{-1})^{-1} K_{k+1}^T] A_s \tag{3.13}$$

Alternatively, we have the following for the updated information matrix:

$$P_{k+1|k+1}^{-1} = [I - A_t K_{k+1}^T] A_s \tag{3.14}$$

$$A_t = A_s K_{k+1}(K_{k+1}^T A_s K_{k+1} + R_{k+1}^{-1})^{-1} \tag{3.15}$$

Now that both the information matrices have been defined by (3.7) and (3.14), the next step in deriving the SIIF is to formulate the prediction and update equations for the information vectors. These were defined earlier by (3.1) and (3.2). Substitution of (3.7) into (3.1), and making use of our definition in (2.3.1), yields the following:

$$\hat{a}_{k+1|k} = [I - A_r] A_q \left(A\hat{x}_{k|k} + Bu_k\right) \tag{3.16}$$

Substitution of (3.3) into (3.1.6) yields:

$$\hat{a}_{k+1|k} = [I - A_r] A^{-T} P_{k|k}^{-1} A^{-1} A \hat{x}_{k|k} + [I - A_r] A^{-T} P_{k|k}^{-1} A^{-1} B u_k \tag{3.17}$$

Simplifying (3.1.7) and utilizing (3.2) yields the predicted information vector as follows:

$$\hat{a}_{k+1|k} = [I - A_r] A^{-T} \left(\hat{a}_{k|k} + P_{k|k}^{-1} A^{-1} B u_k\right) \tag{3.18}$$

The same approach may be used to solve for the updated information vector equation, starting with manipulating (3.2):

$$\hat{a}_{k+1|k+1} = \left[P_{k+1|k}^{-1} + K_{k+1}\right] \hat{x}_{k+1|k+1} \tag{3.19}$$

Substitution of (2.3.5) into (3.19) yields:

$$\hat{a}_{k+1|k+1} = \left[P_{k+1|k}^{-1} + K_{k+1}^{-1}\right]\left(\hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k}\right) \tag{3.20}$$

Expanding (3.20) gives:

$$\hat{a}_{k+1|k+1} = P_{k+1|k}^{-1}\hat{x}_{k+1|k} + P_{k+1|k}^{-1} K_{k+1}\tilde{z}_{k+1|k} + K_{k+1}^{-1}\hat{x}_{k+1|k} + \tilde{z}_{k+1|k} \tag{3.21}$$

Simplifying (3.21) further yields the following solution for the updated information vector:

$$\hat{a}_{k+1|k+1} = \hat{a}_{k+1|k} + K_{k+1}^{-1} z_{k+1} \tag{3.22}$$

Equations (3.1) through (3.22) are the derivation of the sliding innovation information filter. The following sets of equations summarize the iterative process for the SIIF.

$$\hat{a}_{k+1|k} = [I - A_r] A^{-T} \left(\hat{a}_{k|k} + P_{k|k}^{-1} A^{-1} B u_k\right) \tag{3.23}$$

$$P_{k+1|k}^{-1} = \left[I - A_q\left(A_q + Q_{k+1}^{-1}\right)^{-1}\right] A_q \tag{3.24}$$

$$\hat{a}_{k+1|k+1} = \hat{a}_{k+1|k} + K_{k+1}^{-1} z_{k+1} \tag{3.25}$$

$$P_{k+1|k+1}^{-1} = [I - A_s K_{k+1}(K_{k+1}^T A_s K_{k+1} + R_{k+1}^{-1})^{-1} K_{k+1}^T] A_s \tag{3.26}$$

Where the support equations are defined as follows:

$$A_q = A^{-T} P_{k|k}^{-1} A^{-1} \tag{3.27}$$

$$A_r = A_q\left(A_q + Q_{k+1}^{-1}\right)^{-1} \tag{3.28}$$

$$A_s = (I - K_{k+1}H)^{-T} P_{k+1|k}^{-1} (I - K_{k+1}H)^{-1} \tag{3.29}$$

$$A_t = A_s K_{k+1}(K_{k+1}^T A_s K_{k+1} + R_{k+1}^{-1})^{-1} \tag{3.30}$$

4. COMPUTER EXPERIMENTS

In this section, a simple target tracking example will be considered to demonstrate the application of the sliding innovation information filter.

4.1 Target Tracking Problem Setup

The target tracking problem involved in this study is based on a generic air traffic control (ATC) scenario found in [23] and is as described in [36] and [37]. A radar stationed at the origin provides direct position-only measurements, with a standard deviation of $50\ m$ in each coordinate. The average motion of the target of interest is illustrated in Figure 2 (next page).

As shown in Figure 2, an aircraft starts from an initial position of $[25{,}000\ m, 10{,}000\ m]$ at time $t = 0\ s$, and flies westward at $120\ m/s$ for $125\ s$. A coordinated turn is then performed by the aircraft for a period of $90\ s$ at a rate of $1^{\circ}/s$. Then, the aircraft proceeds to fly southward at $120\ m/s$ for $125\ s$, followed by another coordinated turn for $30\ s$ at $3^{\circ}/s$. Finally, the aircraft continues to fly westward until it reaches the ultimate points of its trajectory, indicating the end of the simulation.

The behaviour of civilian aircraft in ATC scenarios may be modeled by two different modes of operation: uniform motion (UM) which involves a straight flight path with a constant speed and course, and maneuvering which includes turning or climbing and descending [23]. In the case of this study, maneuvering refers to a coordinated turn (CT) model, where a turn is made at a constant turn rate and speed. The uniform motion model used for this target tracking problem is given by (4.1) [23, 38].

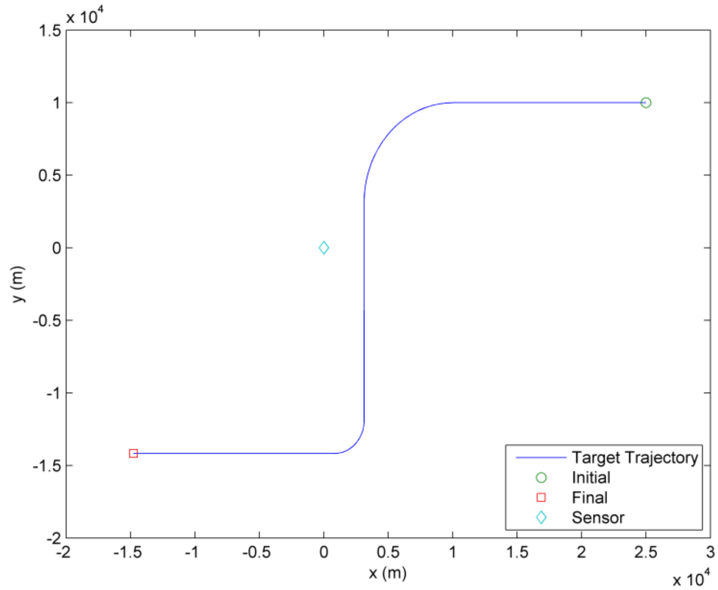

Figure 2. True target trajectory for the nonlinear estimation problem.

$$x_{k+1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}T^2 & 0 \\ 0 & \frac{1}{2}T^2 \\ T & 0 \\ 0 & T \end{bmatrix} w_k \tag{4.1}$$

The state vector of the aircraft may be defined as follows:

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k]^T \tag{4.2}$$

The first two states refer to the position along the x-axis and y-axis, respectively, and the last two states refer to the velocity along the x-axis and y-axis, respectively. The sampling time used in this simulation was 5 seconds. When using the CT model, the state vector needs to be augmented to include the turn rate, as shown in (4.3) [23]. The CT model may be considered nonlinear if the turn rate of the aircraft is not known. Note that a left turn corresponds to a positive turn rate, and a right turn has a negative turn rate. This sign convention follows the commonly used trigonometric convention (the opposite is true for navigation convention) [23]. As per [23, 38], the CT model is given by (4.4).

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k \quad \omega_k]^T \tag{4.3}$$

$$x_{k+1} = \begin{bmatrix} 1 & 0 & \dfrac{sin\omega_k T}{\omega_k} & -\dfrac{1-cos\omega_k T}{\omega_k} & 0 \\ 0 & 1 & \dfrac{1-cos\omega_k T}{\omega_k} & \dfrac{sin\omega_k T}{\omega_k} & 0 \\ 0 & 0 & cos\omega_k T & -sin\omega_k T & 0 \\ 0 & 0 & sin\omega_k T & cos\omega_k T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{2}T^2 & 0 \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} w_k \tag{4.4}$$

Since the radar stationed at the origin provides direct position measurements only, the measurement equation may be linearly formed as follows:

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} x_k + v_k \tag{4.5}$$

Equations (4.1) through (4.5) were used to generate the true state values of the trajectory and the radar measurements for this target tracking scenario. As previously mentioned, the EKF uses a linearized form of the system and measurement matrices. In this case, the system defined by (4.4) is nonlinear, such that the Jacobian of it yields a linearized form as shown in (4.6). The terms in the last column of (4.6) are correspondingly defined in (4.7) [23].

$$\left[\nabla_x F_{k,x}^T\right]^T\Big|_{x_k=\hat{x}_k} = \begin{bmatrix} 1 & 0 & \dfrac{sin\widehat{\omega}_k T}{\widehat{\omega}_k} & -\dfrac{1-cos\widehat{\omega}_k T}{\widehat{\omega}_k} & F_{\widehat{\omega}1} \\ 0 & 1 & \dfrac{1-cos\widehat{\omega}_k T}{\widehat{\omega}_k} & \dfrac{sin\widehat{\omega}_k T}{\widehat{\omega}_k} & F_{\widehat{\omega}2} \\ 0 & 0 & cos\widehat{\omega}_k T & -sin\widehat{\omega}_k T & F_{\widehat{\omega}3} \\ 0 & 0 & sin\widehat{\omega}_k T & cos\widehat{\omega}_k T & F_{\widehat{\omega}4} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.6}$$

$$\begin{bmatrix} F_{\widehat{\omega}1} \\ F_{\widehat{\omega}2} \\ F_{\widehat{\omega}3} \\ F_{\widehat{\omega}4} \end{bmatrix} = \begin{bmatrix} \dfrac{(cos\widehat{\omega}_k T)T}{\widehat{\omega}_k}\widehat{\dot{\xi}_k} - \dfrac{(sin\widehat{\omega}_k T)}{\widehat{\omega}_k^2}\widehat{\dot{\xi}_k} - \dfrac{(sin\widehat{\omega}_k T)T}{\widehat{\omega}_k}\widehat{\dot{\eta}_k} - \dfrac{(-1+cos\widehat{\omega}_k T)}{\widehat{\omega}_k^2}\widehat{\dot{\eta}_k} \\ \dfrac{(sin\widehat{\omega}_k T)T}{\widehat{\omega}_k}\widehat{\dot{\xi}_k} - \dfrac{(1-cos\widehat{\omega}_k T)}{\widehat{\omega}_k^2}\widehat{\dot{\xi}_k} - \dfrac{(cos\widehat{\omega}_k T)T}{\widehat{\omega}_k}\widehat{\dot{\eta}_k} - \dfrac{(sin\widehat{\omega}_k T)}{\widehat{\omega}_k^2}\widehat{\dot{\eta}_k} \\ -(sin\widehat{\omega}_k T)T\widehat{\dot{\xi}_k} - (cos\widehat{\omega}_k T)T\widehat{\dot{\eta}_k} \\ (cos\widehat{\omega}_k T)T\widehat{\dot{\xi}_k} - (sin\widehat{\omega}_k T)T\widehat{\dot{\eta}_k} \end{bmatrix} \tag{4.7}$$

To generate the results for this section, the following values were used for the initial state error covariance matrix $P_{0|0}$, the system noise matrix Q, and the measurement noise matrix R.

$$P_{0|0} = \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 \\ 0 & R_{22} & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.8}$$

$$Q = L_1 \begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ \frac{T^2}{2} & 0 & T & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 & T & 0 \\ 0 & 0 & 0 & 0 & \frac{L_2}{L_1}T \end{bmatrix} \tag{4.9}$$

$$R = 50^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.10}$$

Note that L_1 and L_2 are referred to as power spectral densities, and were defined as 0.16 and 0.01, respectively [38]. The system and measurement noise (w_k and v_k) were generated using their respective covariance values (Q and R). Also, when using the UM model, the fifth row and column of (4.8) and (4.9) were truncated. For the standalone SVSF estimation process, the limit on the sliding boundary layer widths were defined as $\delta = [500 \quad 1{,}000 \quad 500 \quad 1{,}000 \quad 1]^T$. These parameters were tuned based on some knowledge of the uncertainties (i.e., magnitude of noise) and with the goal of decreasing the estimation error. It is required to transform the measurement matrix into a square matrix (i.e., identity), such that an 'artificial' measurement is created. It is possible to derive 'artificial' velocity measurements based on the available position measurements. For example, consider the following artificial measurement vector y_k for the SIF:

$$y_k = \begin{bmatrix} z_{1,k} \\ z_{2,k} \\ (z_{1,k+1} - z_{1,k})/T \\ (z_{2,k+1} - z_{2,k})/T \\ 0 \end{bmatrix} \tag{4.11}$$

The accuracy of (4.11) depends on the sampling rate T. Applying the above type of transformation to non-measured states allows a measurement matrix equivalent to the identity matrix. The estimation process would continue as in the previous section, where $H = I$. Note however that the artificial velocity measurements would be delayed one time step. Furthermore, it is assumed that the artificial turn rate measurement is set to 0, since no artificial measurement could be created based on the available measurements. A total of 500 Monte Carlo runs were performed, and the results were averaged.

4.2 Results and Discussion

In this study's computer experimentation, both the information filter (IF) and the sliding innovation information filter (SIIF) were implemented and applied to the target tracking problem and setup described in earlier sections. Results of the target tracking are shown below in Figure 3. The IF and SIIF-based methods both were able to follow the target trajectory, however the SIIF-based methods followed more closely regardless of which flight model was used (e.g., uniform motion or coordinated turn). The IF-UM filter yielded good results when the aircraft was traveling straight, but fell off the true trajectory track when the aircraft turn. This was to be expected given that the filter dynamics did not match the target dynamics at this point. The EIF-CT performed well during the turn however it was unable to track the aircraft as well (the estimate kept jumping back and forth based on the turn dynamics in the filter). The absolute position error for each filter is shown in Figure 4. It is noticeable from this figure that the SIIF and ESIIF yielded relatively similar results, regardless of the type of model used by the filter (e.g., uniform motion or coordinated turn). This is primarily attributed to the robust estimation process inherent to the switching gain involved in the standard SIF (and now SIIF). A second case was studied, in which the measurement at 50 seconds was increased by one-hundred (multiplicator). In this case, the robustness of the SIIF was further demonstrated and reinforced by the faulty measurements. The IF-based methods were unable to overcome the measurement error, however the SIIF and ESIIF were able to maintain the true target state trajectory. The results of this case are shown in Figures 5 and 6.

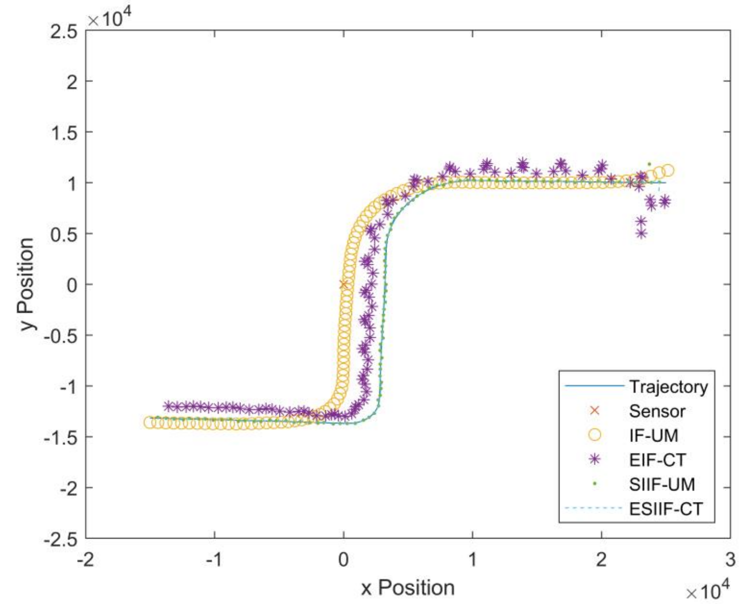

Figure 3. True and estimated target trajectories for the nonlinear estimation problem. Note the legend is defined as follows: IF-UM and SIIF-IM are the information filter and sliding innovation information filter with the uniform motion model, respectively; and, EIF-CT and ESIIF-CT are the extended IF and SIIF with the coordinated turn model, respectively.

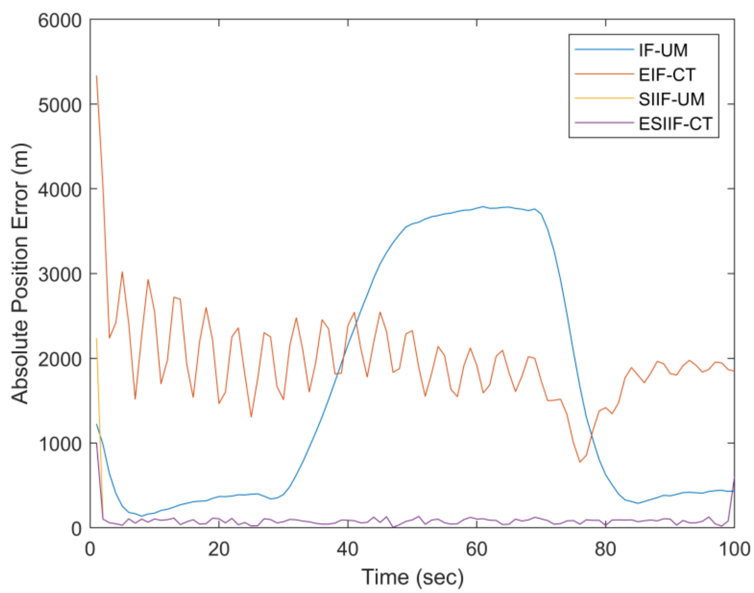

Figure 4. Absolute position estimation errors for the target tracking problem.

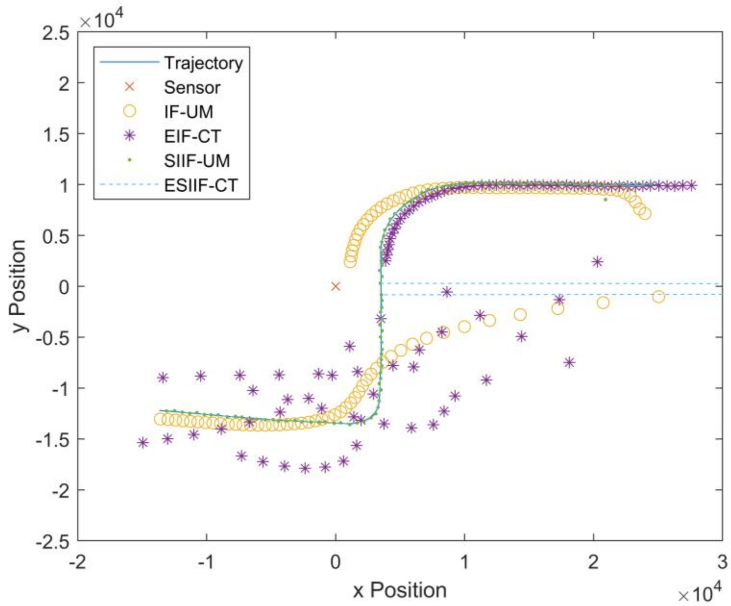

Figure 5. True and estimated target trajectories with the presence of measurement errors at half-way point.

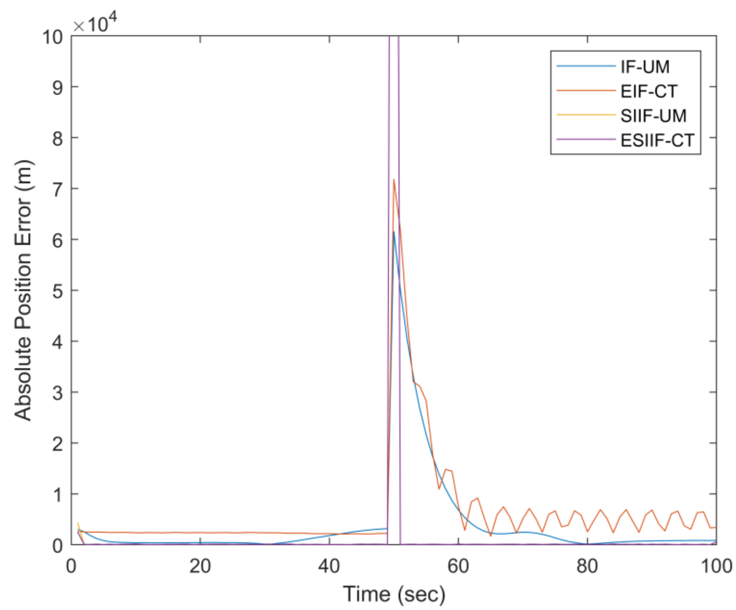

Figure 6. Absolute position errors for the filters with presence of measurement errors at half-way point.

5. CONCLUSIONS

In this paper, the relatively new sliding innovation filter (SIF) is reformulated as an 'information filter.' An information filter is one that propagates the inverse of the state error covariance, which is used in the state and parameter estimation process. This paper introduced preliminary work completed on developing the information form of the sliding innovation filter. The recursive equations used in the sliding innovation information filter (SIIF) are derived and summarized. Preliminary results of application to a target tracking problem are also studied. The results demonstrate the robustness of the SIF-based information filter as compared to the well known IF, which is based on the Kalman filter.

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