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The extended Luenberger sliding innovation filter

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ABSTRACT

The sliding innovation filter is a newly developed filter that was derived in 2020 to be a predictor-corrector filter. The filter uses the measurement as a hyperplane, and then applies a force that makes the estimates fluctuating around it. The filter works on systems with full ranked measurement matrix (all states are measured). However, once the rank becomes partial, the filter depends highly on the pseudo inverse of the measurement matrix. This means that if the measurement matrix does not have a direct link to the hidden states, then these states will not be correctly estimated. When the system is nonlinear, the problem becomes worse as the Jacobean matrix must be calculated for the measurement matrix before the pseudo inverse is applied. To solve this issue, this paper proposes a new formulation of the SIF that is based on the extended Luenberger filter. The proposed method is tested on extracting the damping ration for a third order system.

Keywords: Luenberger method, SIF, third order system.

1. INTRODUCTION

In a wide variety of estimation applications, filters play a critical part because of their ability to extract useful information from signals while at the same time mitigating the effects of disturbances, uncertainties, and noise [1-9]. The primary purpose of filters is to improve the overall dynamics performance of the system [10-19], which includes making the controller of the system of higher quality. Nevertheless, achieving optimal performance in the presence of a variety of challenges, such as limited measured signals, non-measured states (also known as hidden states) that are not directly linked to measurements, and the presence of disturbances and noise, can be quite a difficult task to accomplish.

The sliding innovation filter (SIF) is a common and widely utilized type of filter that is utilized in estimation applications [20-28]. The SIF is a model-based filter that is derived from the sliding mode theory. The sliding mode theory is well-known for its resistance to disturbances and uncertainties. An initial estimate is obtained by the SIF through the utilization of a system model, which is then stimulated by the system's input. Next, it utilizes a corrective gain that is derived from the Lyapunov stability theorem in order to further refine the estimate. As a consequence of this, the Smooth Image Filter (SIF) has been designated as a robust filter, which places it in the same category as smooth variable structure filters [30-45] and sliding mode observers [46-69].

It's possible that the SIF's performance won't always be at its best, even with how robust it is; this is especially true when there are disturbances and noise present. In addition, when there are fewer measured signals than there are states, the filter becomes highly dependent on the system and measurement matrices, which can lead to potential problems when attempting to extract the necessary information. This is especially the case in situations in which the non-measured states, also known as the hidden states, do not have a direct correlation with the measured states or measurements. As a consequence, there is a reduction in accuracy and efficiency.

Researchers have proposed a variety of solutions in order to overcome these limitations. One example of this is the combination of the SIF with other filters such as the Regular [70-84], Extended [85-90], and Sigma-point Kalman filters [91-134]. However, this type of combined approach frequently results in an increased level of complexity within the algorithm as well as an increase in the amount of time required for simulation, which makes them less practical for certain applications. In this paper, a new variant of the SIF is proposed with the intention of addressing these challenges in a straightforward while simultaneously effective manner [135-136]. The SIF is incorporated into the Luenberger method, which is well-known for its capacity to uncover latent states based on observable data using the measurements that are readily available [137-152]. The proposed method seeks to achieve a balance between simplicity and effectiveness while maintaining stability and robustness in the filtering process. This is done by combining the strengths of the SIF and the Luenberger method, which both have their own distinct advantages.

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2. METHODOLOGY

2.1. Nonlinear system model

One of most famous nonlinear system (f) with linear measurement matrix ($\mathbf{H}_k$) formulas has the shape of:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}, \mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \tag{1}$$

Where the subscript represents the time step, $\mathbf{x}$ and $\mathbf{z}$ are the state and measurement vectors, respectively, and $\mathbf{w}$ and $\mathbf{v}$ are noise vectors of the system and measurement, respectively. Some of the system's states are considered hidden when there are no direct measurements for them. In this case, estimation techniques, i.e. filters, are required to extract their information, while reducing the effect of the noise. This work is addressing a new formulation of the SIF that links the SIF to the Luenberger method. This work is different than [144, 152] as it addresses nonlinear systems.

2.2. SIF algorithm

The sliding Innovation filter for nonlinear systems consists of two steps:

1- <u>Prediction Stage</u>

The a priori estimate ($\hat{\mathbf{x}}_{k+1|k}$) and its measurement ($\hat{\mathbf{z}}_{k+1|k}$), are obtained by:

$$\hat{\mathbf{x}}_{k|k-1} = f\left(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}\right), \hat{\mathbf{z}}_{k|k-1} = \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \tag{2}$$

2- <u>Update/Correction Stage</u>, w

The a posteriori estimate ($\hat{\mathbf{x}}_{k|k}$) and its measurements ($\hat{\mathbf{z}}_{k|k}$) are obtained by:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \left[\mathbf{H}_k^{+}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right]^{\circ} sat\left(\left|\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right|, \mathbf{\Psi}_k\right), \hat{\mathbf{z}}_{k|k} = \mathbf{H}_k \hat{\mathbf{x}}_{k|k} \tag{3}$$

Where $\mathbf{H}_k^{+}$, $\mathbf{\Psi}_k$ and sat are the pseudoinverse of $\mathbf{H}_k$, is the boundary layer, and the saturated function. $A^{\circ}B$ is schur product.

According to [144, 152], the SIF performance becomes worse when hidden states do exist. To improve the performance, the filter is linked to the Luenberger method.

2.3. Luenberger/SIF algorithm

The Luenberger method is considered an observer as it assumes no uncertainties exist. If uncertainty exists, i.e. noise, the method needs to be merged with a filter to smooth out the noise effect [144, 152]. In this work, Luenberger method is combined with SIF as follows:

Assuming that two type of measurement signals exist, the actual one ($\mathbf{z}_k$) and imaginary one that is linked to the hidden states ($\mathbf{y}_k$), then the states are considered to be fully measured with the vector ($\mathbf{Z}_k$):

$$\mathbf{Z}_k = \begin{bmatrix} \mathbf{z}_k \\ \mathbf{y}_k \end{bmatrix} \tag{4}$$

where

$$\mathbf{Z}_k \cong \mathbf{x}_k \tag{5}$$

According to [144, 152], $\mathbf{y}_k$ is defined as

$$\mathbf{y}_k = f^{-1}(\mathbf{z}_k, \mathbf{z}_{k-2}, \dots, \mathbf{z}_{k-M}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1}, \mathbf{v}_k, \mathbf{w}_{k-1}) \tag{6}$$

By this, the modified SIF has the same equation (2), but equation (3) is modified to:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \left[\left(\mathbf{Z}_k - \hat{\mathbf{x}}_{k|k-1}\right)\right]^{\circ} sat\left(\left|\mathbf{Z}_k - \hat{\mathbf{x}}_{k|k-1}\right|, \mathbf{\Psi}_k\right), \hat{\mathbf{z}}_{k|k} = \hat{\mathbf{x}}_{k|k} \tag{7}$$

3. CASE STUDY

The Luenberger/SIF is tested on the following system:

$$f_k = \begin{bmatrix} x_{1,k} + \tau x_{2,k} \\ x_{2,k} + \tau x_{3,k} \\ -\omega_n^2 \tau x_{2,k} + x_{3,k}\left(1 - 2x_{4,k}\omega_n \tau\right) + b\tau u_k \\ 0 \end{bmatrix} + \mathbf{w}_{k-1}, z_k = x_{1,k} + v_k \tag{8}$$

Where ω_n , b, and τ have values of 10 $\mathrm{H_z}$, 3 $\frac{\mathrm{m}}{\mathrm{sec}\times\mathrm{rad}}$, and 0.001 sec, respectively [112]. The input is a multiple level random signal. The states include the position, velocity, acceleration and damping ratio ξ. The results are illustrated by Fig. 1 and the root mean squared error (RMSE) and the maximum absolute error (MAE) are calculated in tables 1 and 2, respectively using the equations:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{ns}\left(x_{Actual,i}-x_{Prediction,i}\right)^2}{ns}} \tag{9}$$

$$MAE = \max(|x_{Actual} - x_{Prediction}|) \tag{10}$$

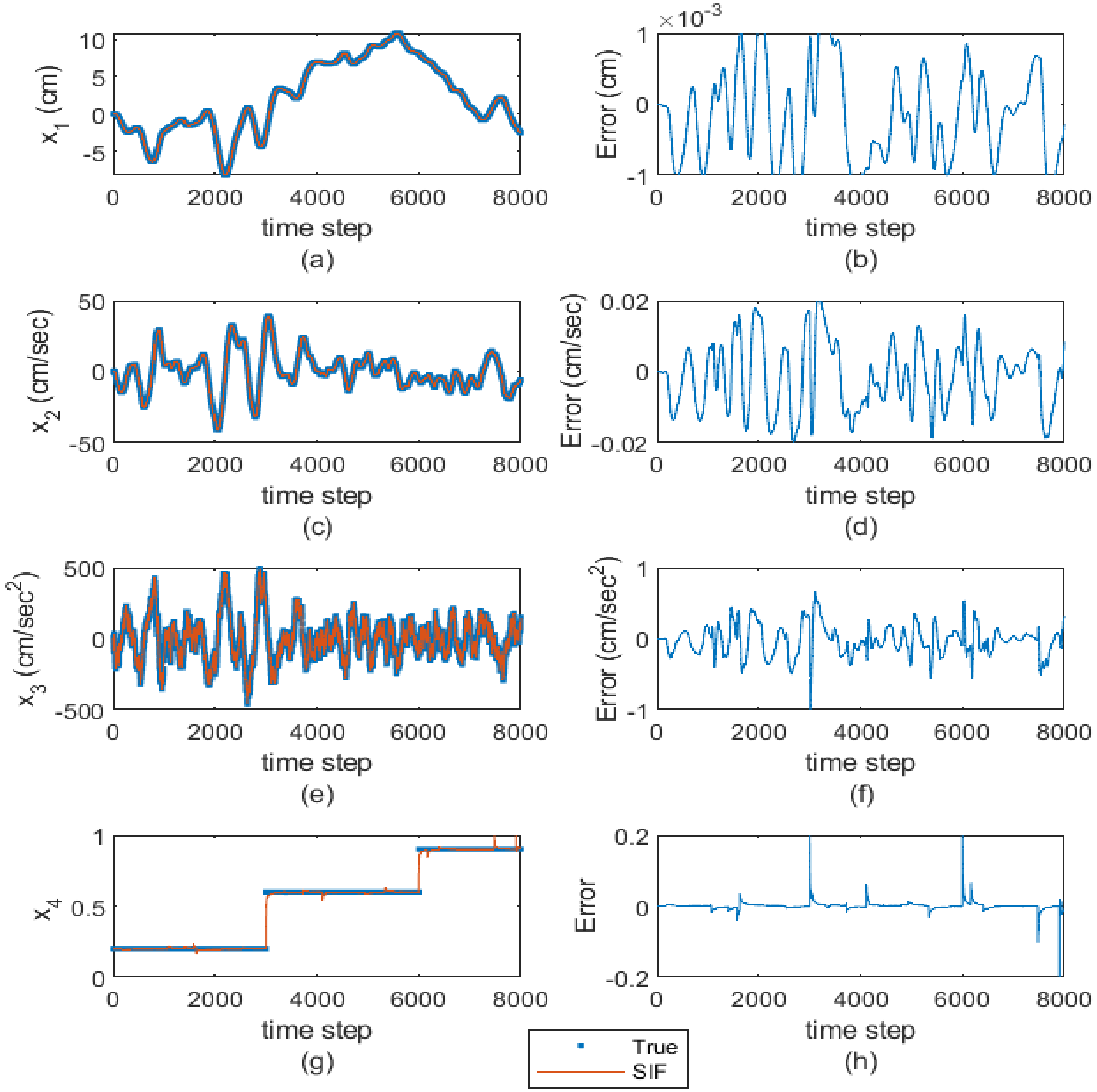

Figure 1. The results of the proposed method trajectories; (a) position, (b) error in position, (c) velocity, (d) error in velocity, (e) acceleration, (f) error in acceleration, (g) damping ratio, and (h) the error in damping ratio.

Table 1. RMSE of the SIF's results

	$RMSE$ in			
	$x_1\ (cm)$	$x_2(cm/s)$	$x_3(cm/s^2)$	x_4
SIF	6.8×10^{-05}	9.6×10^{-03}	2.1×10^{-01}	1.5×10^{-02}

Table 2. MAE of the SIF's results

	MAE in			
	$x_1\ (cm)$	$x_2(cm/s)$	$x_3(cm/s\hat{}2)$	x_4
SIF	1.3×10^{-03}	2.2×10^{-02}	1.6	1.1

The results show that the proposed method extract the hidden states and parameters with excellent accuracy; RMSEs have values of $6.8 \times 10^{-05}, 9.6 \times 10^{-03}, 2.1 \times 10^{-01}$, and 1.5×10^{-02} for position, velocity, acceleration and damping ratio coefficient, respectively. The errors are found to be less than 0.2% for more than 99% of the datasets. Only a few spikes appear in the damping ratio coefficient with amplitude of 20%. The results prove that the method has an excellent performance.

4. CONCLUSION

The proposed variant of the SIF formulated with the Luenberger method for nonlinear systems offers promising results in terms of performance and accuracy. The RMSE of the estimated states was found to be less than 0.2% for over 99% of the datasets, indicating high accuracy in estimating the system states. Moreover, the ability of the proposed method to extract three states from just one measurement signal in online processes highlights its potential for efficient state estimation with minimal sensor requirements. The excellent performance of the proposed method opens up avenues for further investigation and experimental verification in future work. One potential direction for future research is to conduct extensive experiments on real-world systems to validate the effectiveness and robustness of the proposed method in practical scenarios. Experimental verification can help uncover any potential limitations or challenges that may arise when implementing the algorithm in real-time applications and can provide insights into its performance under different operating conditions and system dynamics.

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