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# Advanced Optimization Techniques for Modern Filter Design—From Newton to Space Mapping

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# Traditional Gradient-based Optimization

minimize w.r.t. $\boldsymbol{x}$ a general, real-valued, non-linear function $F(\boldsymbol{x})$ in $n$ variables

traditional optimization algorithms are based on local information and Taylor's formula

early milestones in filter design by modern optimization methods
(*Temes and Calahan, 1967, the state of the art*)
(*Lasdon et al., 1966, 1967, linear arrays and filters*)
(*Bandler, 1969, the state of the art*)
(*Director and Rohrer, 1969, adjoint sensitivity evaluation*)

## Variable Metric Methods (Quasi-Newton Methods)

local approximation at $\hat{\boldsymbol{x}}$

$$q(\boldsymbol{x}) = F(\hat{\boldsymbol{x}}) + (\boldsymbol{x} - \hat{\boldsymbol{x}})^T F'(\hat{\boldsymbol{x}}) + \frac{1}{2}(\boldsymbol{x} - \hat{\boldsymbol{x}})^T \boldsymbol{B}(\boldsymbol{x} - \hat{\boldsymbol{x}})$$

where $\boldsymbol{B}$ is a positive definite approximation to the Hessian of $F$ at $\hat{\boldsymbol{x}}$

minimize $q$ and find the next iterate by a line search (*Davidon, 1959, Fletcher and Powell, 1963*), (*Broyden, Fletcher, Goldfarb and Shanno (BFGS), independently around 1970*)

trust regions were introduced by several authors in the early 1970s

## Sequential Quadratic Programming

minimize w.r.t. ***x*** a general, real-valued, non-linear function $F(\boldsymbol{x})$ in $n$ variables subject to a finite set of non-linear constraints

Han and Powell (1970s) developed a method similar to the variable metric method, with

- local quadratic approximation to the function
- constraints approximated by linear terms using
  first-order Taylor expansions
- the local subproblems solved by quadratic programming
- line search applied

# Type of Approximation/Optimization Problem Considered

minimize w.r.t. $\boldsymbol{x}$ the absolute values of the deviations between response $r(\boldsymbol{x}, t_i)$ and specifications $y_i$

$$f_i(\boldsymbol{x}) = r(\boldsymbol{x}, t_i) - y_i,\ i = 1, \ldots, m$$

traditional methods are based on local information and Taylor’s formula, including

- least-squares formulation
- minimax formulation
- $L_1$ formulation
- general formulation

## Traditional Least-Squares Formulation

(*Levenberg, 1944, Marquardt, 1963*)

$$F(\boldsymbol{x}) = \sum_{i=1}^{m} f_i^2(\boldsymbol{x})$$

local approximation at $\hat{\boldsymbol{x}}$:

$$\hat{L}(\boldsymbol{x}) = \sum_{i=1}^{m} \hat{l}_i^2(\boldsymbol{x})$$

$$\hat{l}_i(\boldsymbol{x}) = f_i(\hat{\boldsymbol{x}}) + f_i'(\hat{\boldsymbol{x}})^T(\boldsymbol{x} - \hat{\boldsymbol{x}})$$

minimize a damped version of $\hat{L}$

minimize $\hat{L}$ subject to some trust region (*Moré, 1983*)

## Traditional Minimax Formulation
(*Madsen, 1975*)

$$F(\boldsymbol{x}) = \max_i \left| f_i(\boldsymbol{x}) \right|$$

local approximation at $\hat{\boldsymbol{x}}$:

$$\hat{L}(\boldsymbol{x}) = \max_i \left| \hat{l}_i(\boldsymbol{x}) \right|$$

$$\hat{l}_i(\boldsymbol{x}) = f_i(\hat{\boldsymbol{x}}) + f_i'(\hat{\boldsymbol{x}})^T (\boldsymbol{x} - \hat{\boldsymbol{x}})$$

minimize $\hat{L}$ subject to some trust region

## Traditional $L_1$ Formulation

(*Hald and Madsen, 1985*)

$$F(\boldsymbol{x}) = \sum_{i=1}^{m} \left| f_i(\boldsymbol{x}) \right|$$

local approximation at $\hat{\boldsymbol{x}}$:

$$\hat{L}(\boldsymbol{x}) = \sum_{i=1}^{m} \left| \hat{l}_i(\boldsymbol{x}) \right|$$

$$\hat{l}_i(\boldsymbol{x}) = f_i(\hat{\boldsymbol{x}}) + f_i'(\hat{\boldsymbol{x}})^T (\boldsymbol{x} - \hat{\boldsymbol{x}})$$

minimize $\hat{L}$ subject to some trust region

## General Formulation

(*Madsen, 1986*)

$$\text{minimize } F(\boldsymbol{x}) = H(f(\boldsymbol{x}))$$

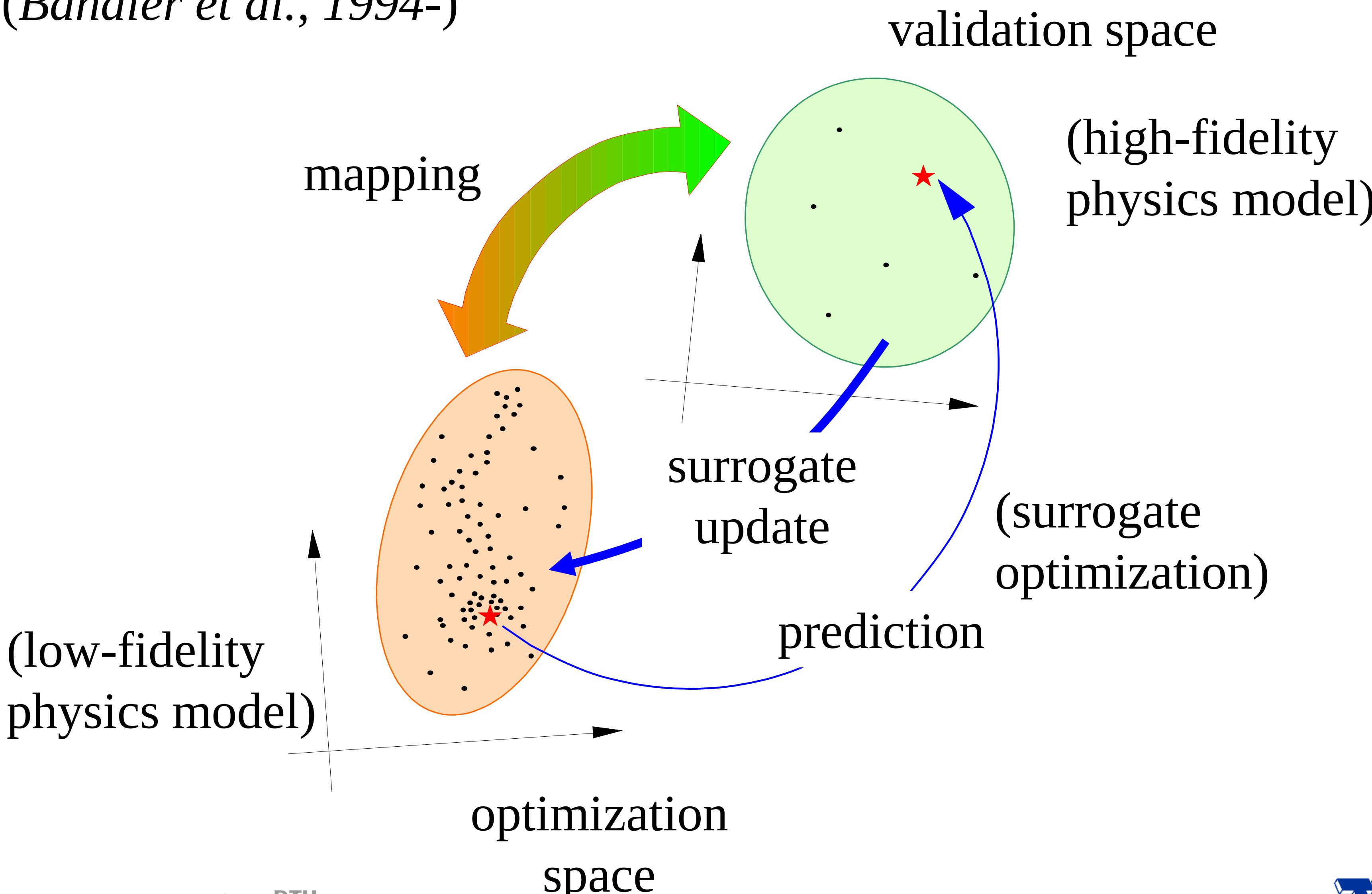
at the iteration $\hat{\boldsymbol{x}}$:

$$\hat{L}(\boldsymbol{x}) = H(\hat{l}(\boldsymbol{x}))$$

$$\hat{l}_i(\boldsymbol{x}) = f_i(\hat{\boldsymbol{x}}) + f_i'(\hat{\boldsymbol{x}})^T(\boldsymbol{x} - \hat{\boldsymbol{x}})$$

minimize $\hat{L}$ subject to some trust region

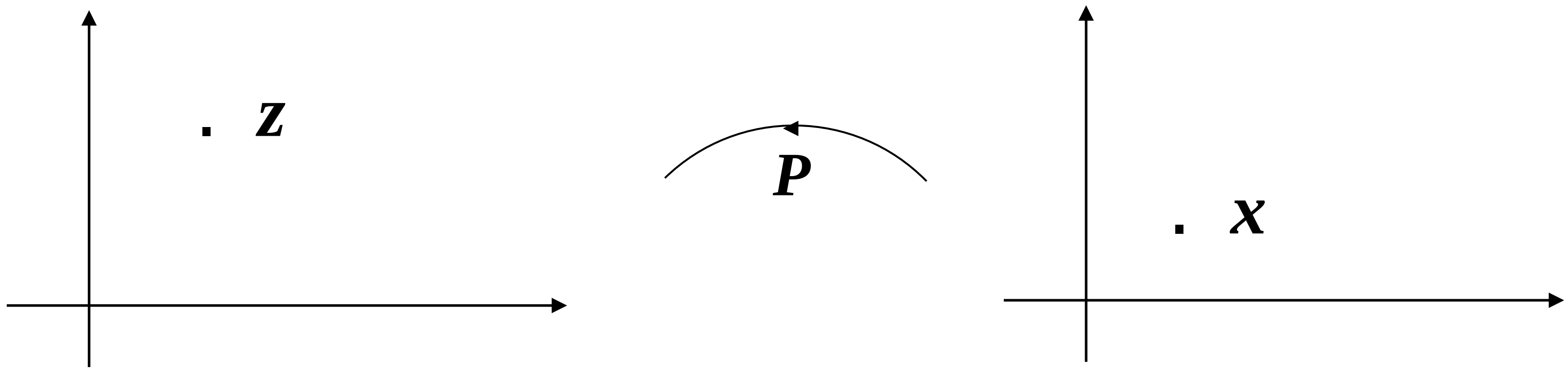
# The Space Mapping Concept
(*Bandler et al., 1994-*)



# **Original Space Mapping Optimization**

(*Bandler et al., 1994-*)

find mapping $\boldsymbol{P}(\boldsymbol{x})$ through parameter extraction




$c(\boldsymbol{z})$ $f(\boldsymbol{x})$

coarse model mapping fine model

$$\boldsymbol{z} = \boldsymbol{P}(\boldsymbol{x}) \equiv \arg\min_{\boldsymbol{z}} \left\{ \left\| f(\boldsymbol{x}) - c(\boldsymbol{z}) \right\| \right\}$$

# Aggressive Space Mapping Optimization
(*Bandler et al., 1995*)

estimate mapping $\boldsymbol{P}$ at the $k$th iteration

assume $\boldsymbol{P}$ has been computed at $\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_k$

$$
\begin{aligned}
\boldsymbol{P}(\boldsymbol{x}) &\approx \boldsymbol{P}(\boldsymbol{x}_k) + \boldsymbol{P}'(\boldsymbol{x}_k)(\boldsymbol{x} - \boldsymbol{x}_k) \\
&\approx \boldsymbol{P}(\boldsymbol{x}_k) + \boldsymbol{B}_k(\boldsymbol{x} - \boldsymbol{x}_k) \\
&\equiv \boldsymbol{P}_k(\boldsymbol{x})
\end{aligned}
$$

where $\boldsymbol{B}_k \approx \boldsymbol{P}'(\boldsymbol{x}_k)$ is, e.g., a Broyden (1970) update

approximate aim: $\boldsymbol{P}_k(\boldsymbol{x}) = \boldsymbol{z}^* \quad \rightarrow \quad \boldsymbol{x}_{k+1}$

# Aggressive Space Mapping Optimization
(*Bandler et al., 1995*)

first iteration

$$\boldsymbol{B}_0 = \boldsymbol{I}$$

let

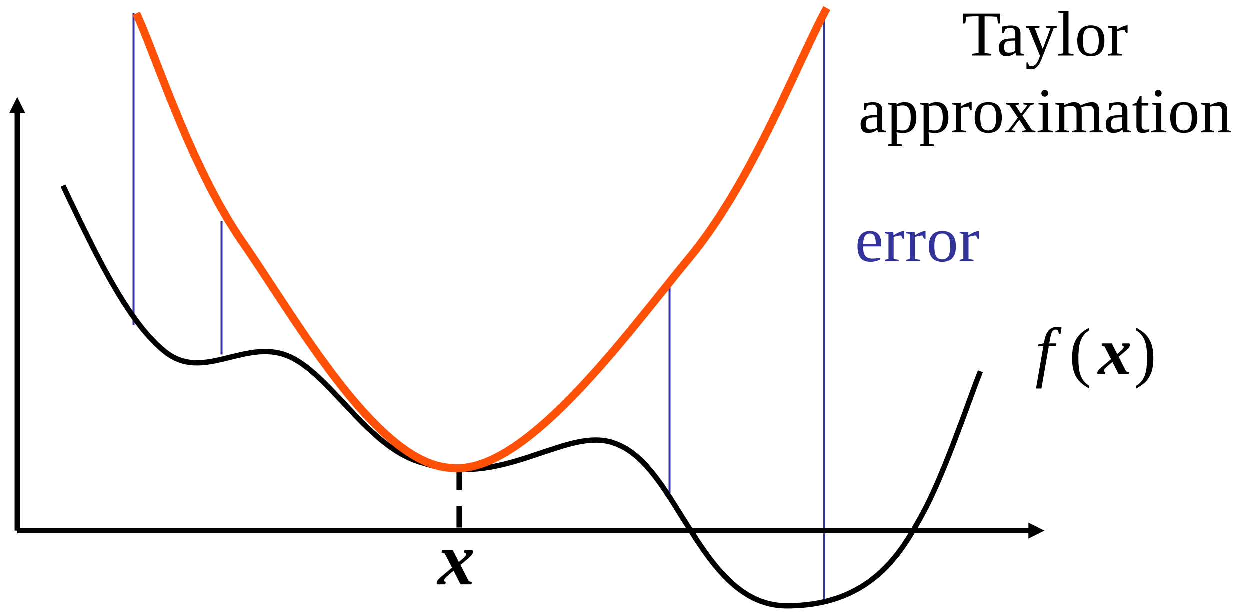
$$\boldsymbol{P}_0(\boldsymbol{x}) \equiv \boldsymbol{P}(\boldsymbol{x}_0) + \boldsymbol{B}_0(\boldsymbol{x} - \boldsymbol{x}_0)$$
$$\approx \boldsymbol{P}(\boldsymbol{x})$$

solve

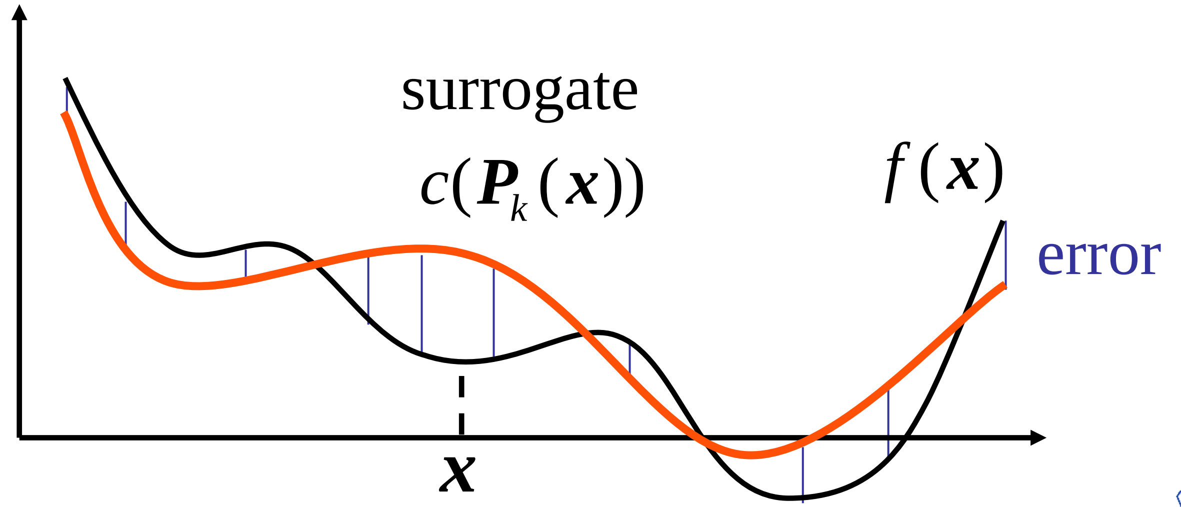
$$\boldsymbol{P}_0(\boldsymbol{x}) = \boldsymbol{z}^* \quad \rightarrow \quad \boldsymbol{x}_1$$

# Space Mapping Optimization Methodologies

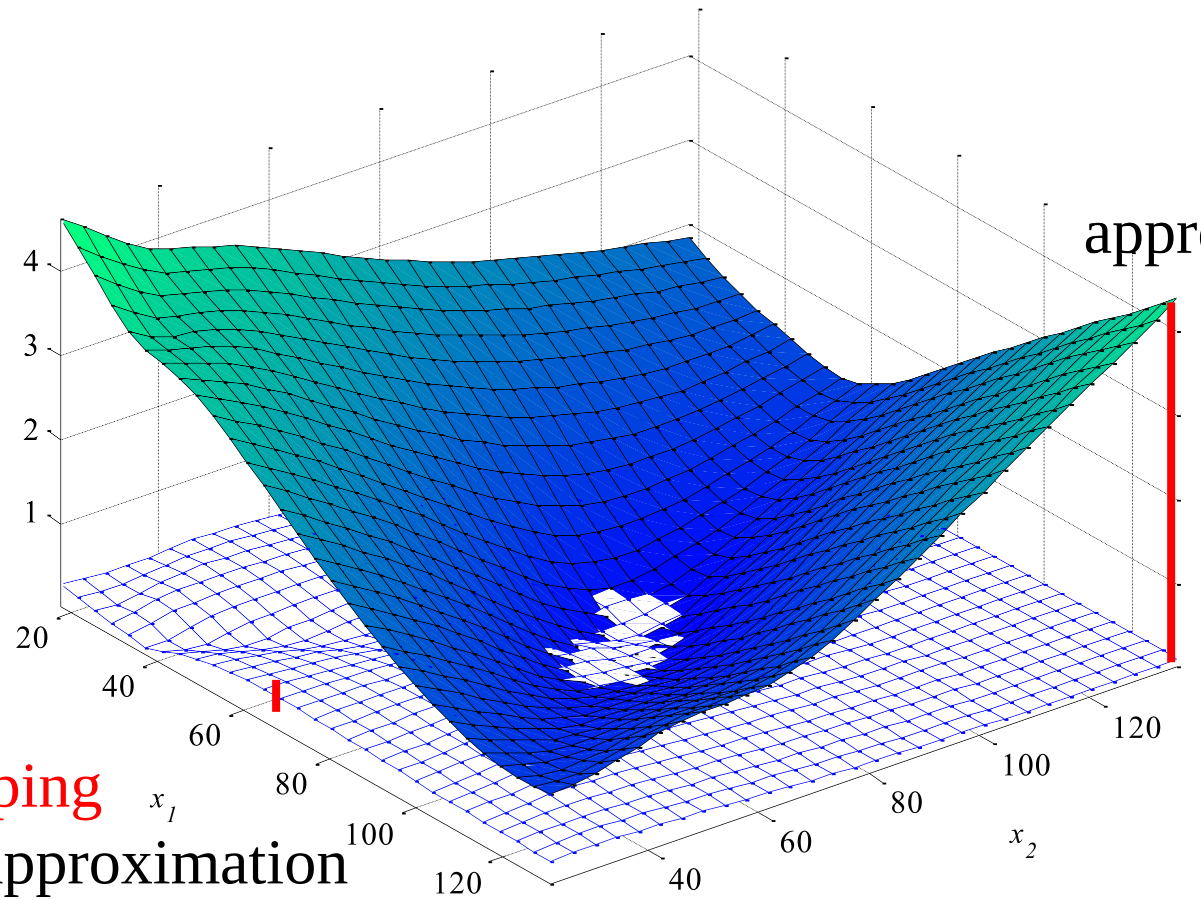
local information




space mapping



# Space Mapping Approximation Errors (*Bakr et al., 2001*)




local Taylor approximation error at $\boldsymbol{x}_k$

space mapping surrogate approximation error for *k*th mapping at $\boldsymbol{x}_k$

## Space Mapping vs. Taylor Approximation

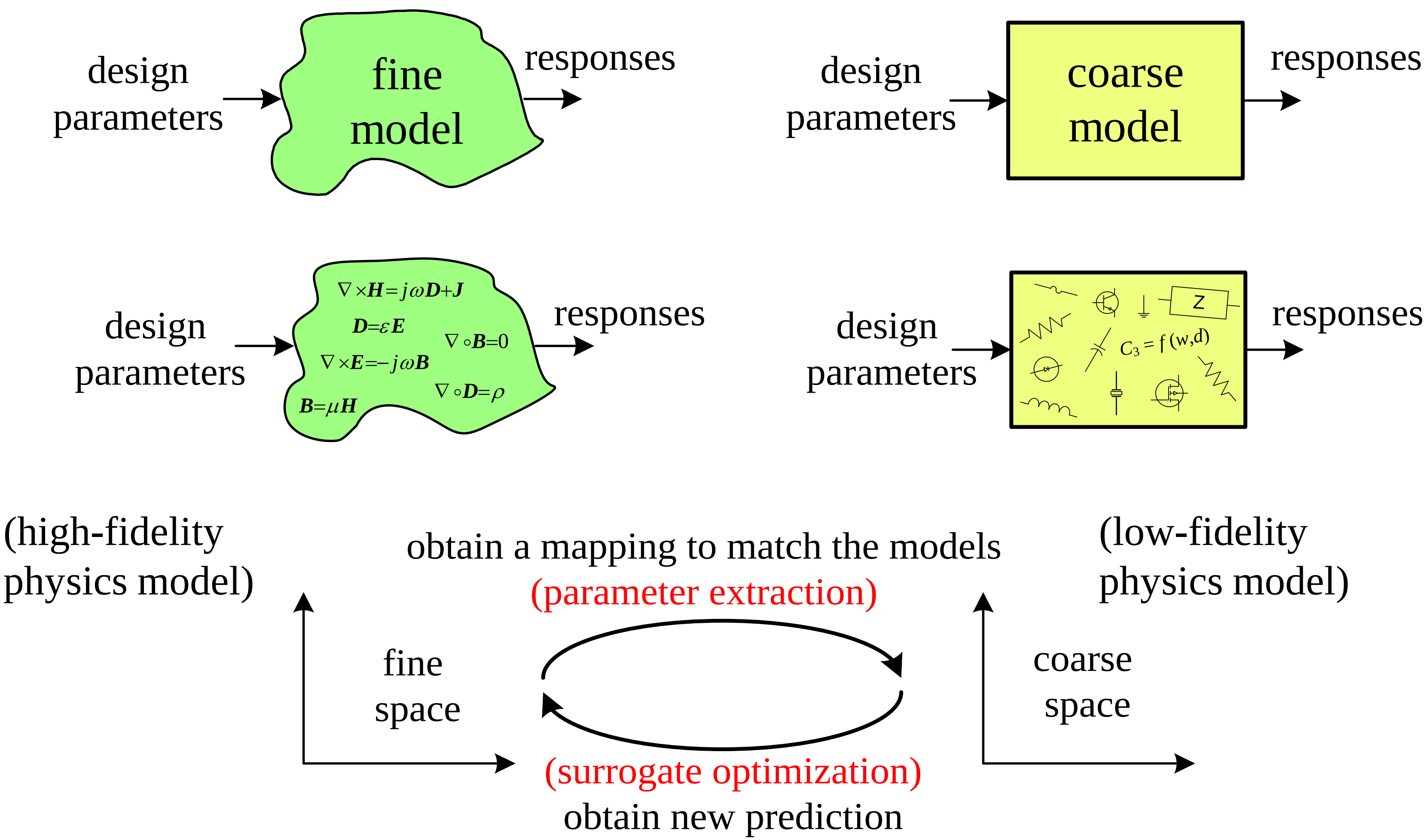
use of a suitable coarse (surrogate) model may provide large iteration steps

space mapping may provide a good approximate solution in a few iteration steps

large iteration steps: space mapping is best
small iteration steps: Taylor is best?

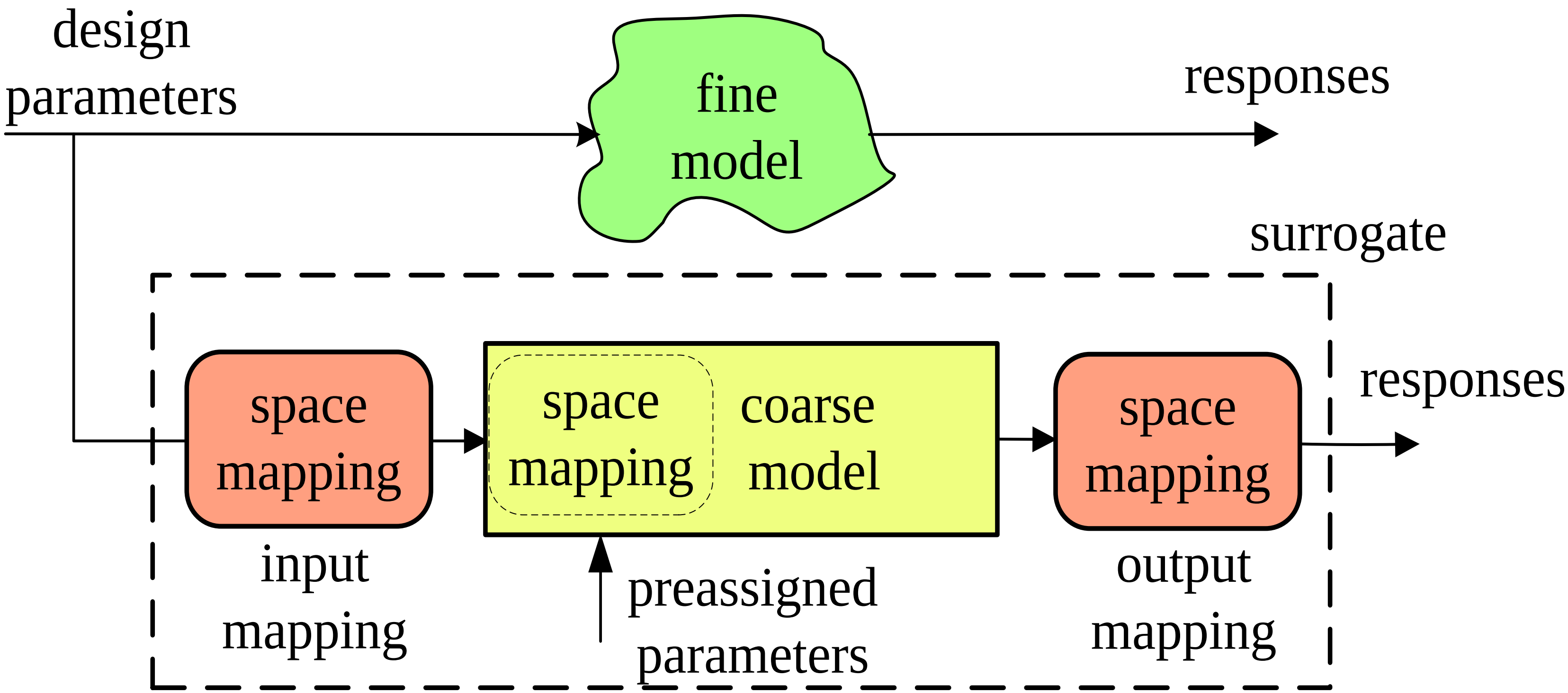
beyond aggressive space mapping: to enhance space mapping for all size steps

# Linking Companion Coarse (Empirical) and Fine (EM) Models Via Space Mapping (*Bandler et al., 1994-*)



# **Implicit, Input and Output Space Mappings**

(*Bandler et al., 2003*-)




expert engineering knowledge helpful (few designable variables)

expertise helpful in "tuning the surrogate" (many possibilities, e.g., dielectric constant)

engineering expertise perhaps less necessary (many output variables)

# The Novice-Expert Continuum

output space mapping: a "band-aid" solution for engineers and non-engineers; the parameter extraction step does not require coarse model re-analysis; good for final touch-ups

input space mapping: an engineering approach to find and cure the root-cause of a defect; but the parameter extraction step can be a difficult inverse optimization problem to solve w.r.t. the coarse model

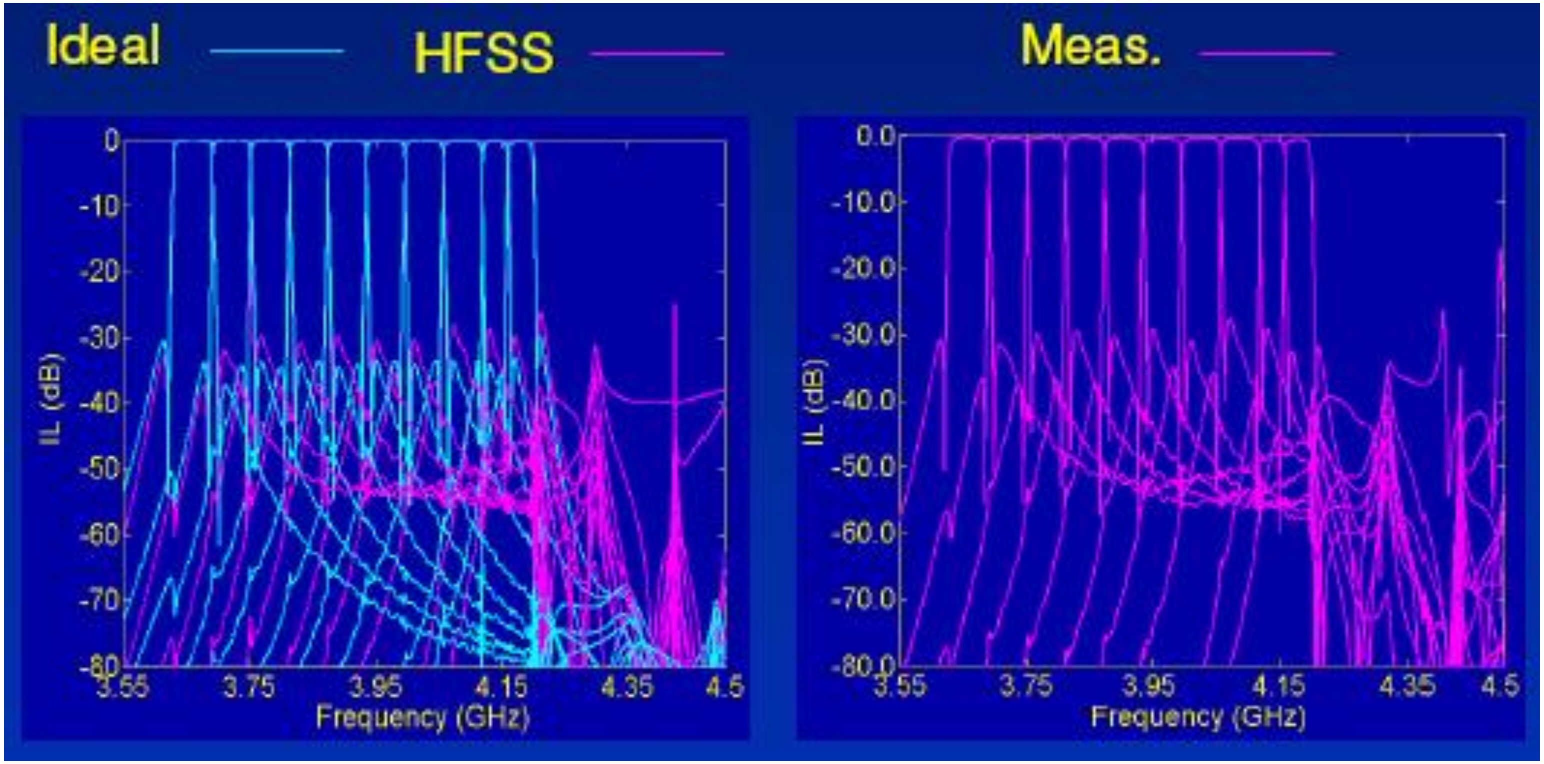
tuning space mapping (new): simulator-based expert approach

but all types of space mapping can be viewed as special cases of implicit space mapping

## Aggressive Space Mapping Design of Dielectric Resonator Multiplexers

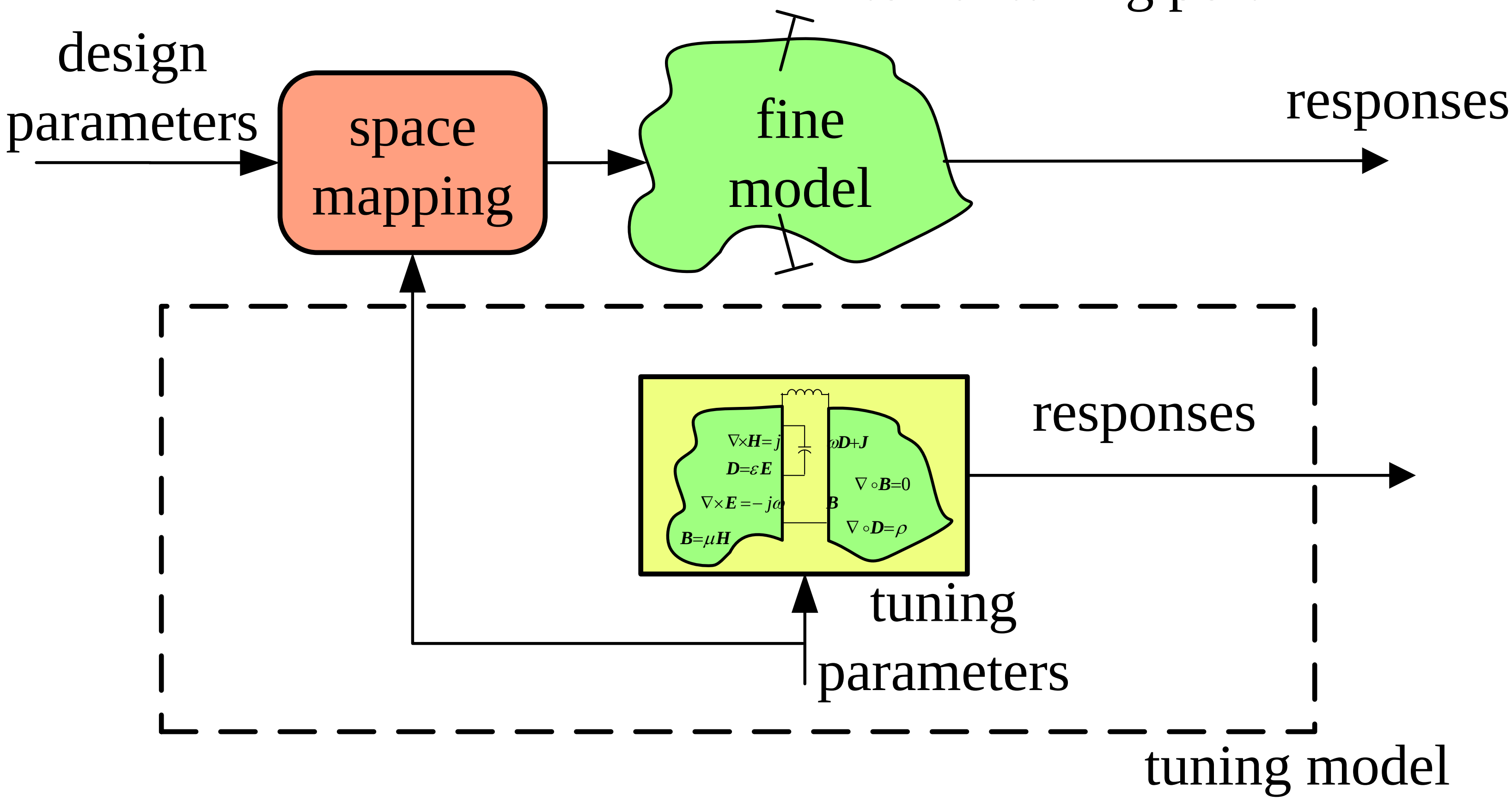
(*Ismail et al., 2003, Com Dev, Canada*)

10-channel output multiplexer, 140 variables



# For the Expert: Tuning Space Mapping (TSM)

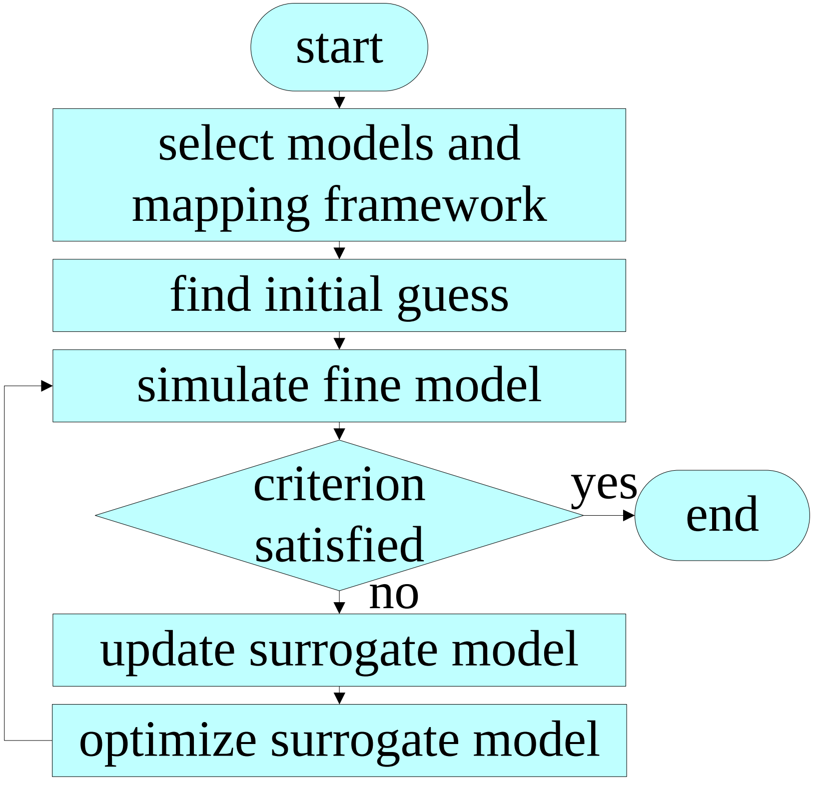
(*Bandler et al., 2006-*)



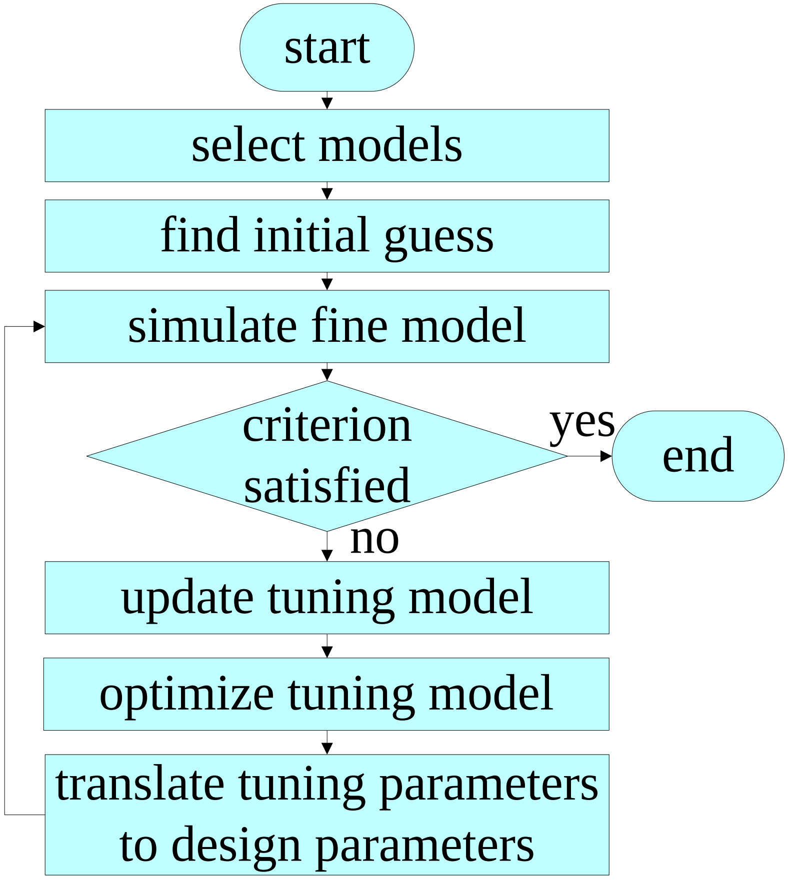

surrogate based on the fine model with internal tuning ports

# Tuning Space Mapping (TSM) Flowchart

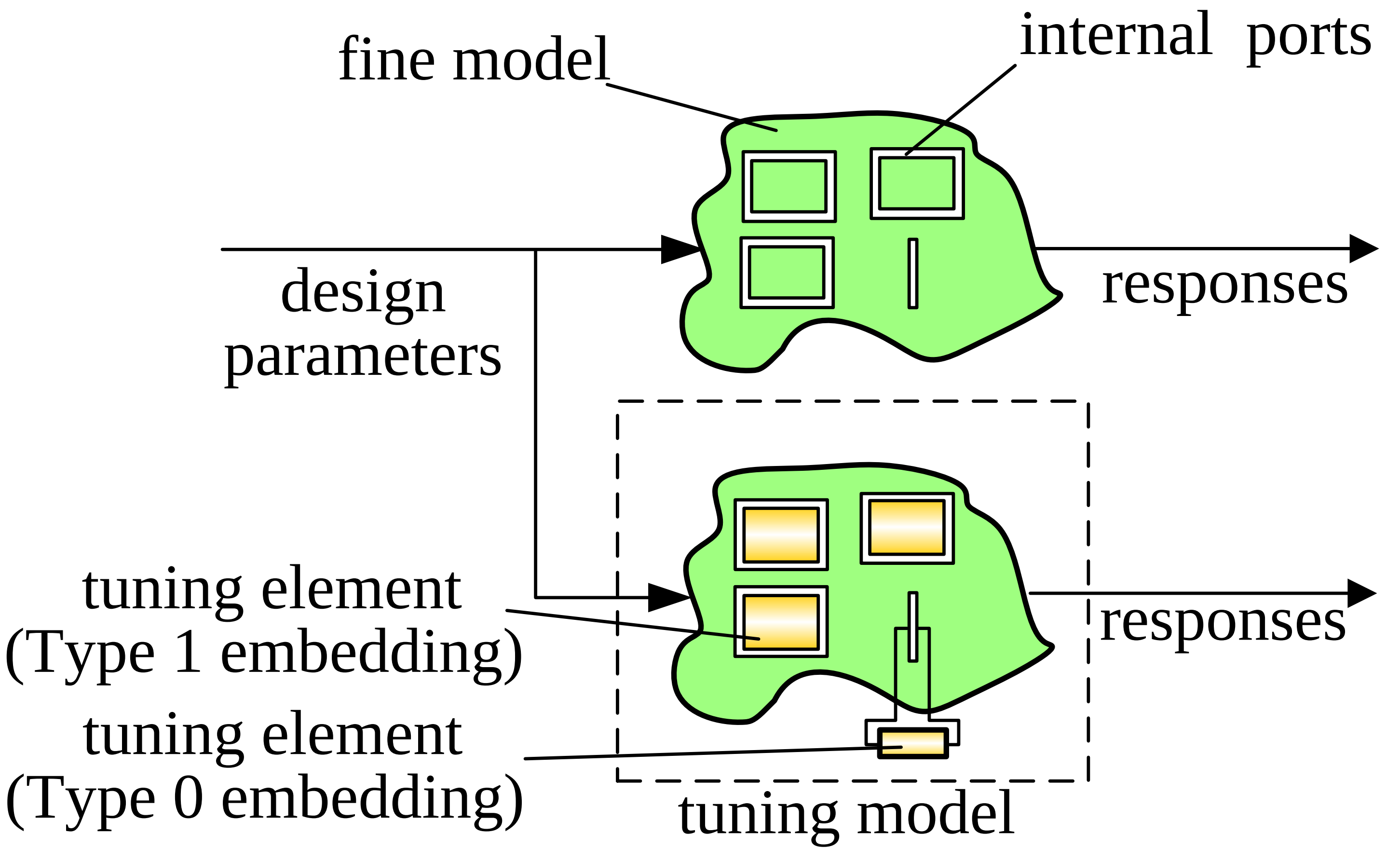
Classical Space Mapping
(*Bandler et al., 2004*)



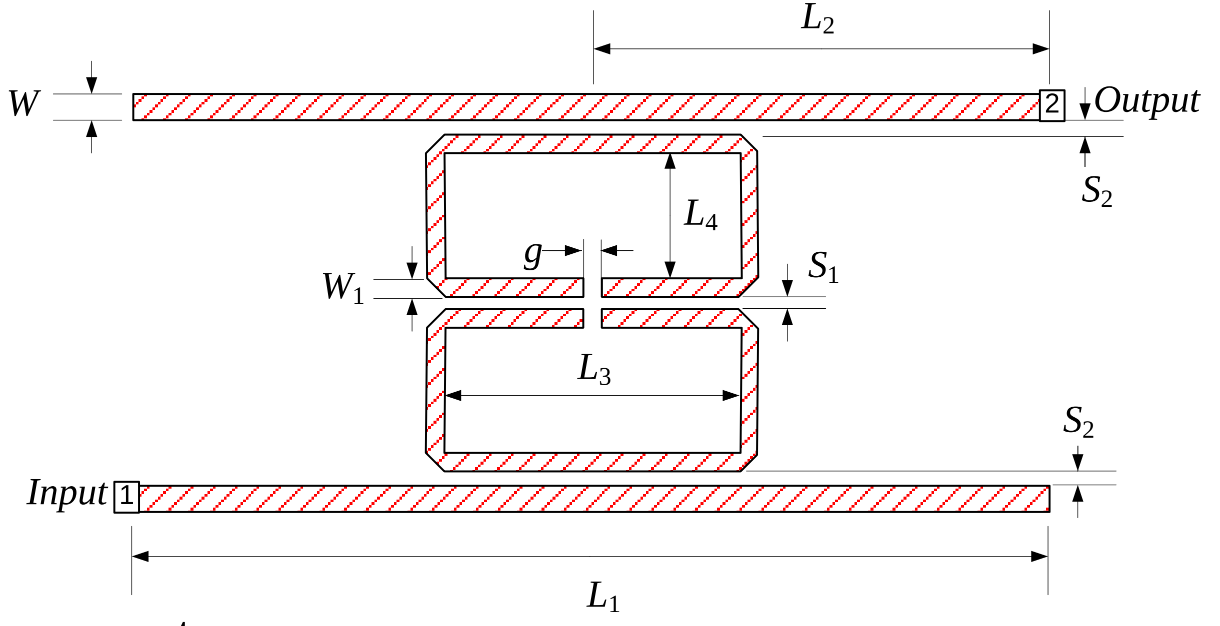

Tuning Space Mapping
(*Koziel et al., 2008*)



# Tuning Space Mapping (TSM): Type 1 and Type 0 Embedding



# **Open-loop Ring Resonator Bandpass Filter** (*Koziel et al., 2008*)




design parameters

$\boldsymbol{x} = [L_1 \;\; L_2 \;\; L_3 \;\; L_4 \;\; S_1 \;\; S_2 \;\; g]^T$ mm

specifications

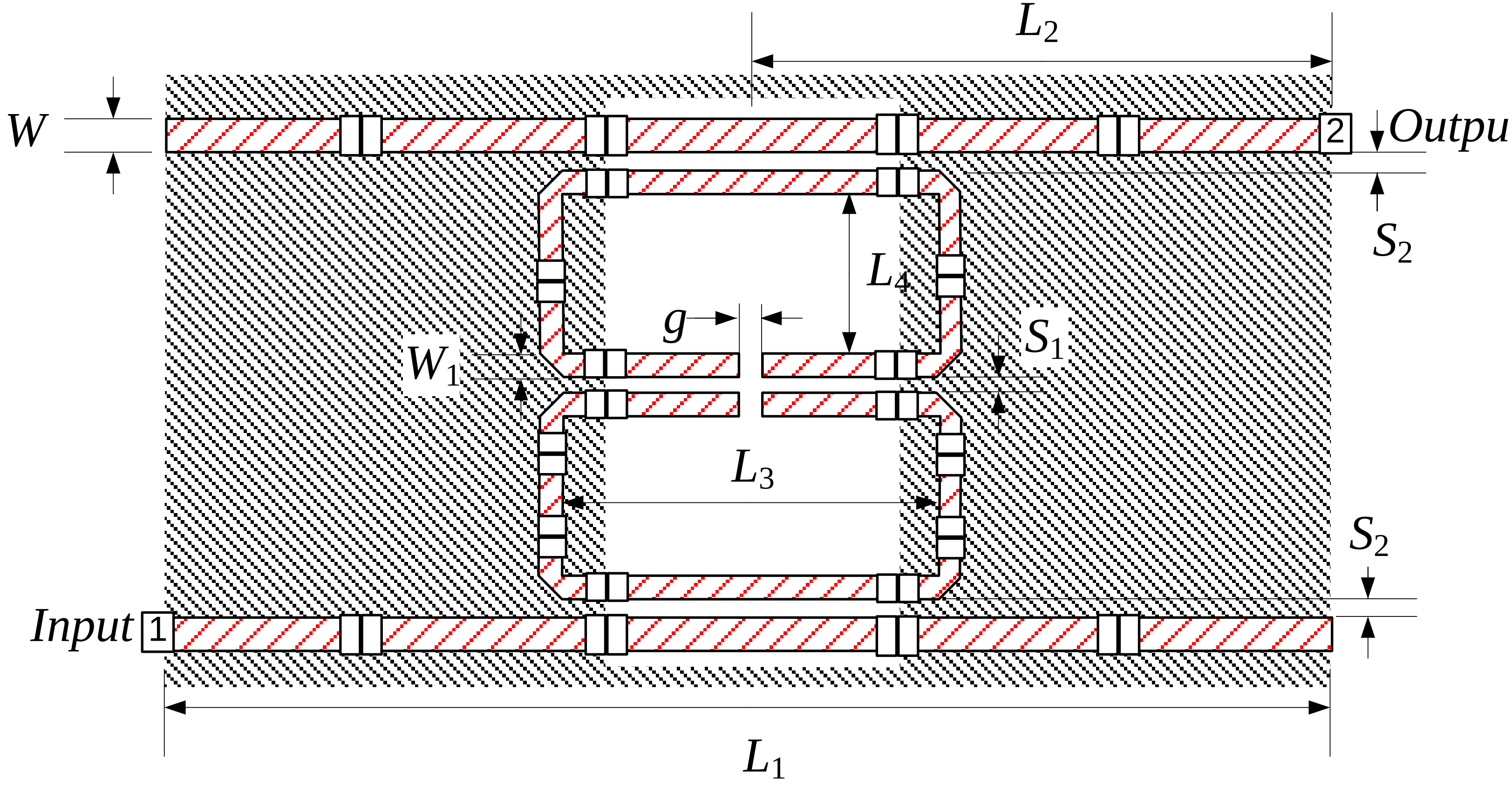
$|S_{21}| \geq -3$ dB for 2.8 GHz $\leq \omega \leq$ 3.2 GHz

$|S_{21}| \leq -20$ dB for 1.5 GHz $\leq \omega \leq$ 2.5 GHz

$|S_{21}| \leq -20$ dB for 3.5 GHz $\leq \omega \leq$ 4.5 GHz

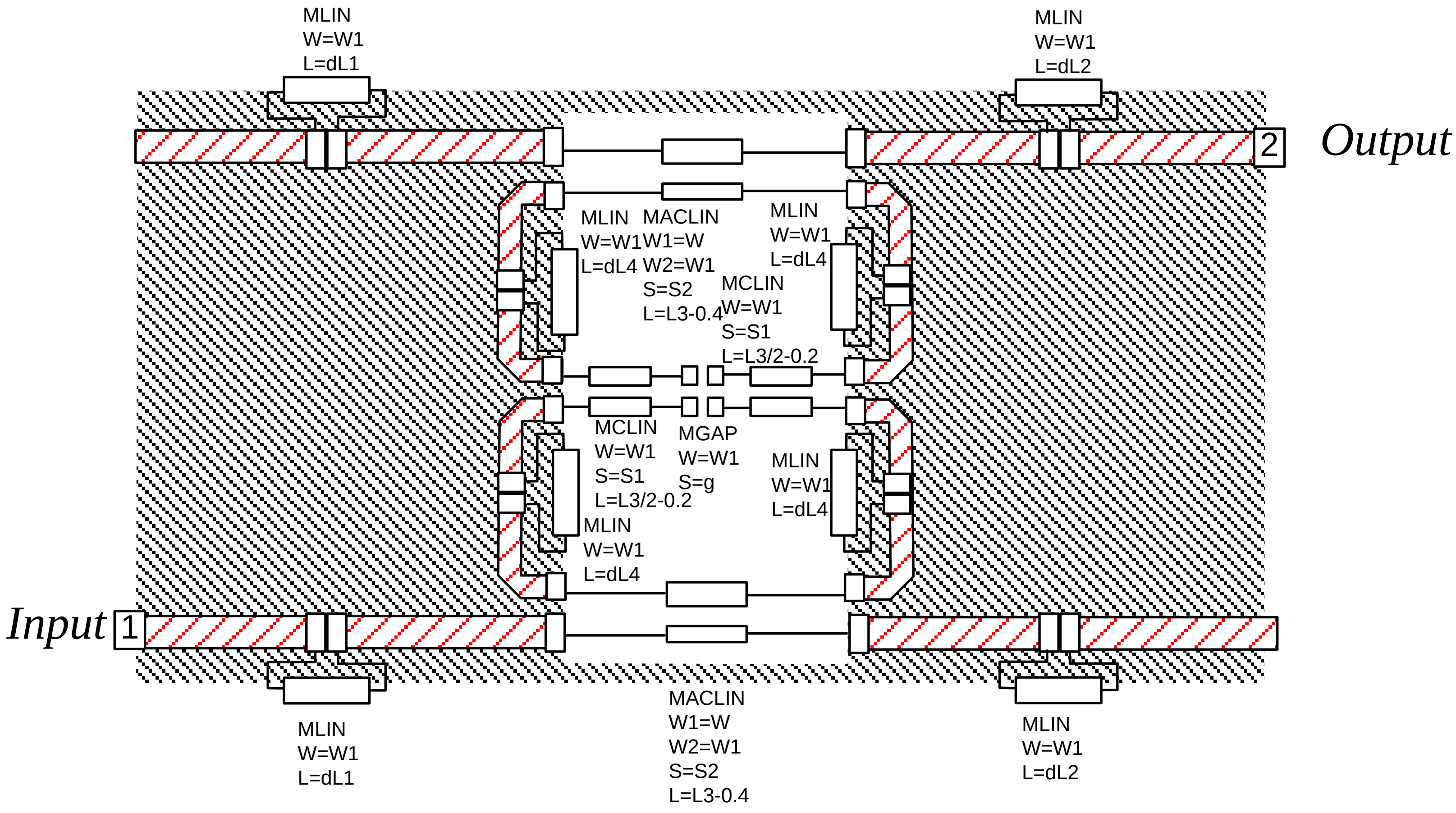
# Open-loop Ring Resonator Bandpass Filter (Type 1 and Type 0)

Sonnet ***em*** model with internal (co-calibrated) ports



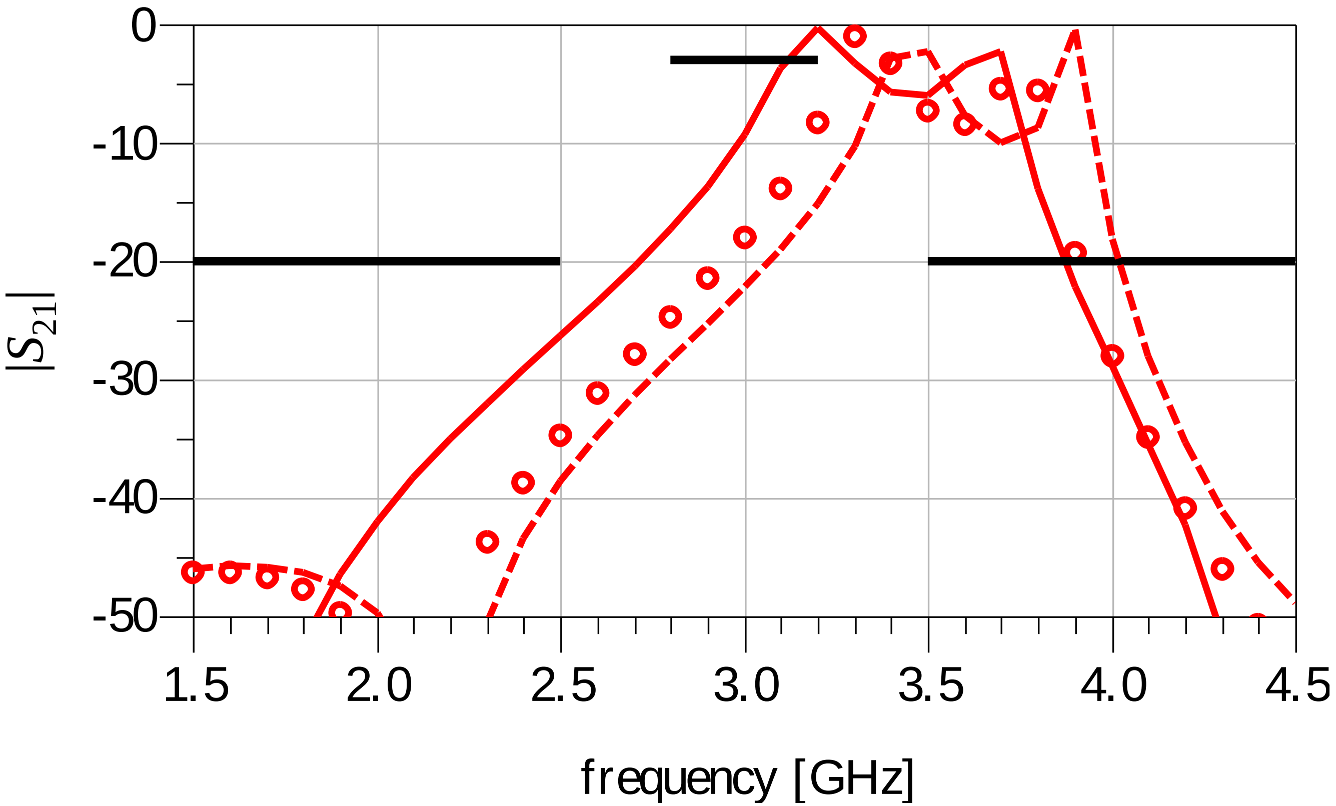
# Open-loop Ring Resonator Bandpass Filter (Type 1 and Type 0)

Sonnet ***em*** model with internal (co-calibrated) ports



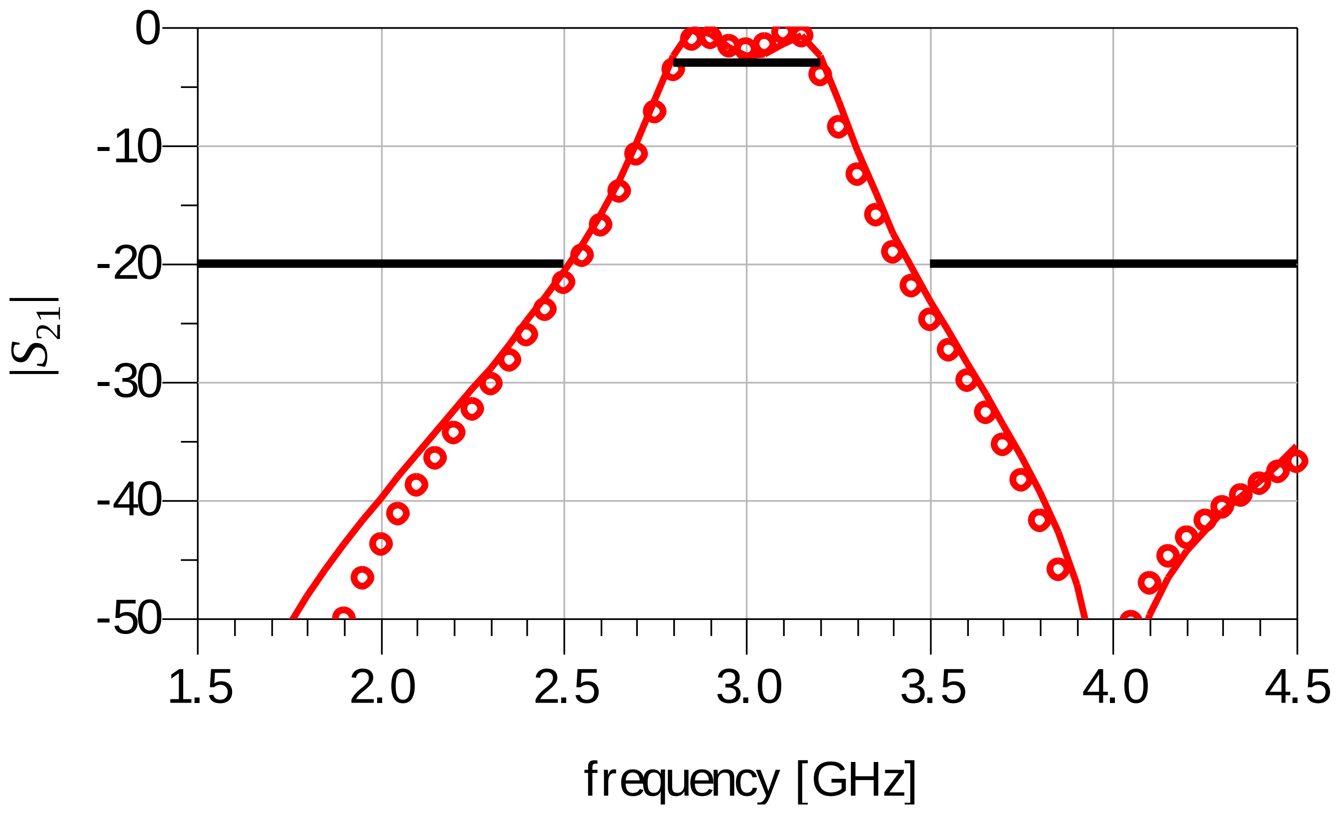
# Open-loop Ring Resonator Bandpass Filter (Type 1 and Type 0)

initial responses: tuning model (—), fine model (○), fine model with co-calibrated ports (---)



# Open-loop Ring Resonator Bandpass Filter (Type 1 and Type 0)

responses after two iterations: the tuning model (—), corresponding fine model (○)



# Tuning (Implicit) Space Mapping Algorithm (*Cheng et al., 2010*)

original problem

$$\boldsymbol{x}_f^* = \arg\min_{\boldsymbol{x}} U\left(\boldsymbol{R}_f(\boldsymbol{x})\right)$$

align the surrogate to match fine model

$$\boldsymbol{x}_p^{(i)} = \arg\min_{\boldsymbol{x}_p} \left\| \boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{R}_s^{(i)}(\boldsymbol{x}^{(i)}, \boldsymbol{x}_p) \right\|$$

design parameter value prediction

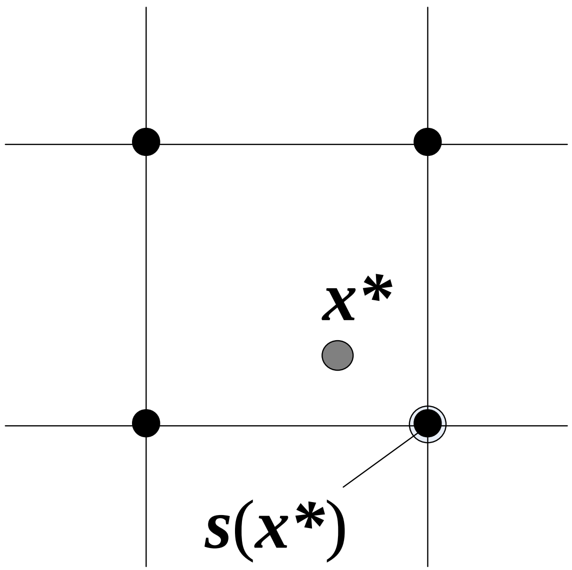
$$\boldsymbol{x}^{(i+1)} = \arg\min_{\boldsymbol{x}} U\left(\boldsymbol{R}_s^{(i)}(\boldsymbol{x}, \boldsymbol{x}_p^{(i)})\right)$$

## **Response-Corrected Tuning Space Mapping Algorithm**
(*Cheng et al., 2010*)

the response-corrected tuning model at optimum ***x****

$$\bar{\boldsymbol{R}}_s(\boldsymbol{x}) = \boldsymbol{R}_s(\boldsymbol{x}, \boldsymbol{x}_p^*) + \boldsymbol{R}_f(s(\boldsymbol{x}^*)) - \boldsymbol{R}_s(s(\boldsymbol{x}^*), \boldsymbol{x}_p^*)$$

*s* is a function that snaps a point to the nearest fine model grid point



## **Yield Analysis and Yield Optimization** (*Cheng et al., 2010*)

*Step* 1 Use tuning space mapping to obtain a nominal optimal design. A tuning model or surrogate is also obtained.

*Step* 2 Snap the optimal design to the nearest on-grid fine model point.

*Step* 3 Simulate the snapped design (EM fine model).

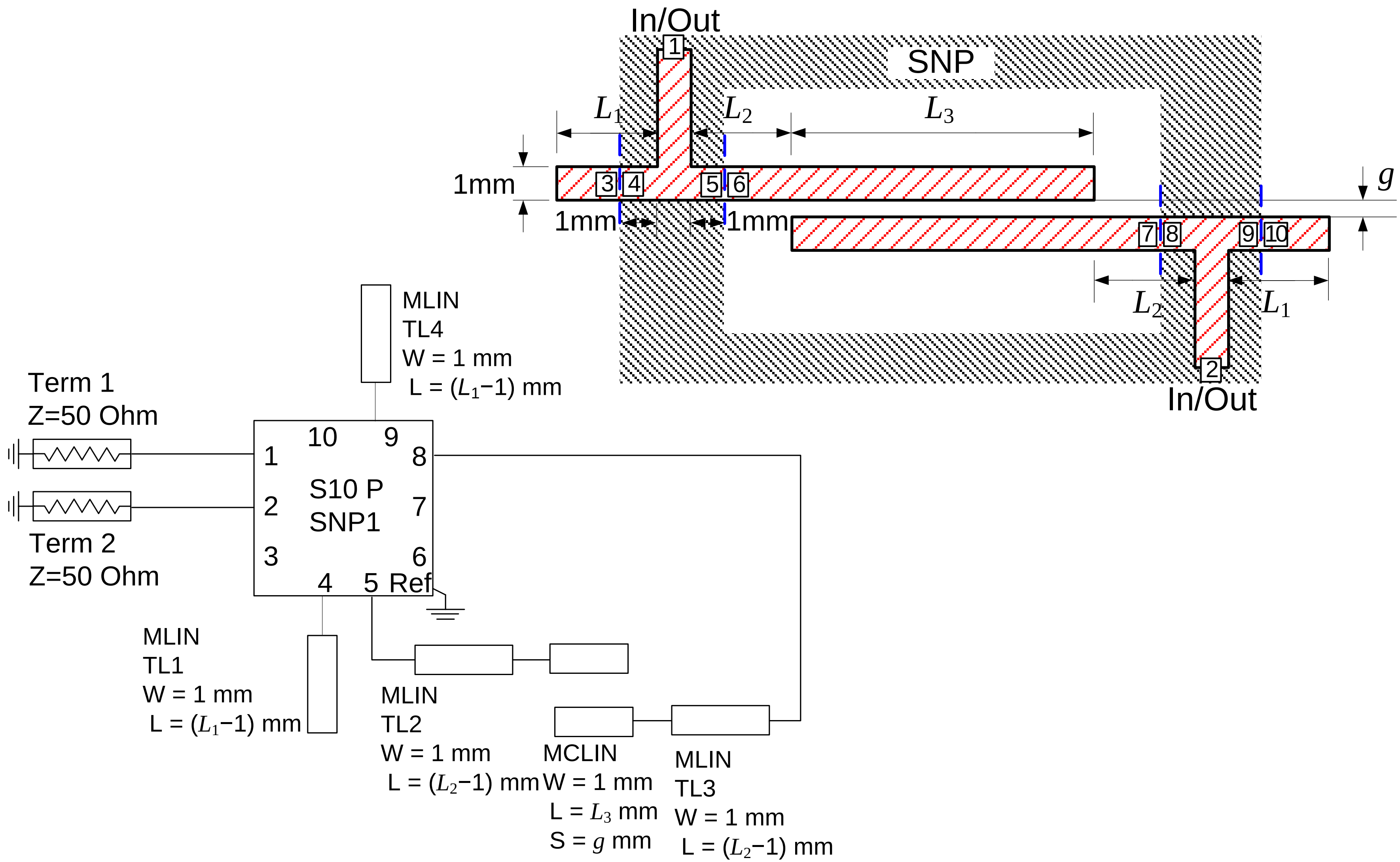
*Step* 4 Calculate the response difference between the fine model and the surrogate at the nearest on-grid point.

*Step* 5 Add the response difference to the surrogate to form a new surrogate: the response corrected surrogate.

*Step* 6 Perform yield analysis and yield optimization on the response-corrected surrogate.

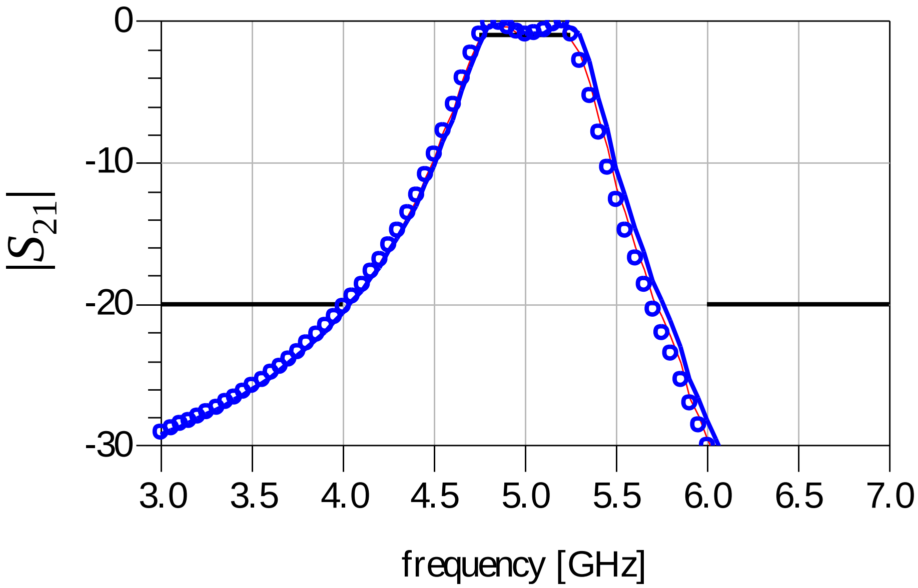
*Step* 7 Compare this response to that of the fine model.

# Second-order Tapped-line Microstrip Filter (Type 1)

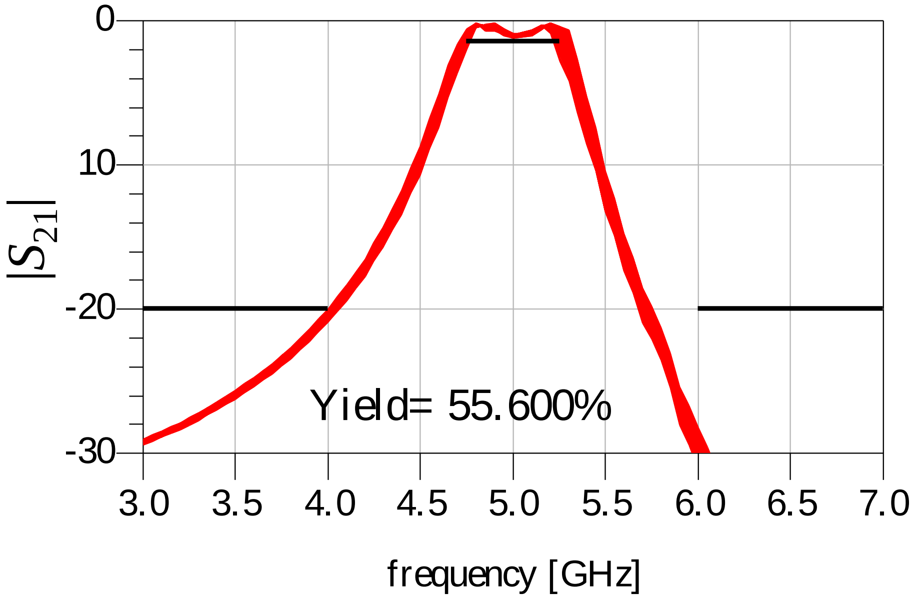


# Second-order Tapped-line Microstrip Filter (Type 1)

tuning model (—), fine model (○),
response corrected surrogate (—)




yield analysis (500 trials)



## Space Mapping with Constrained Parameter Extraction: Concept (*Koziel et al., 2010*)

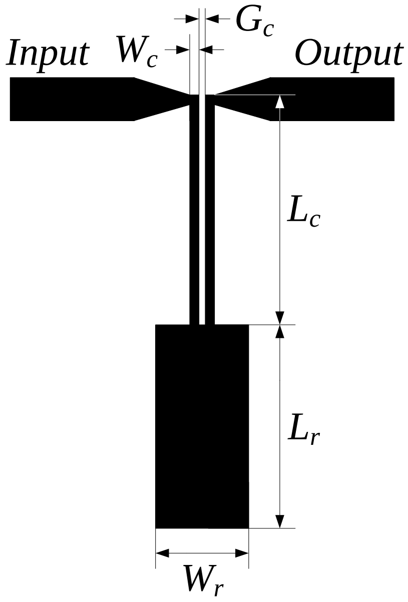
selection of surrogate is critical for space mapping performance

a novel technique replaces "manual" adjustment of the type and number of space mapping parameters, based on an adaptively constrained parameter extraction process

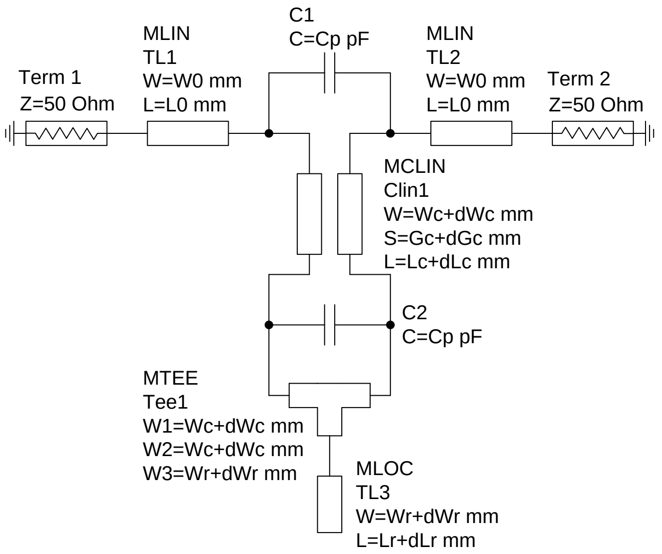
- construct initial, over-flexible surrogate with excellent approximation capability (so that $\varepsilon^{(i)} = ||\boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{R}_c(\boldsymbol{x}^{(i)},\boldsymbol{x}_p^{(i)})||$ can be brought to a very small value)
- adjust its generalization capability by constraining the parameter space

# Wideband Bandstop Microstrip Filter

fine model (FEKO)




coarse model (Agilent ADS)




design parameters: $\boldsymbol{x} = [L_r \; W_r \; L_c \; W_c \; G_c]^T$

initial surrogate model has 10 parameters

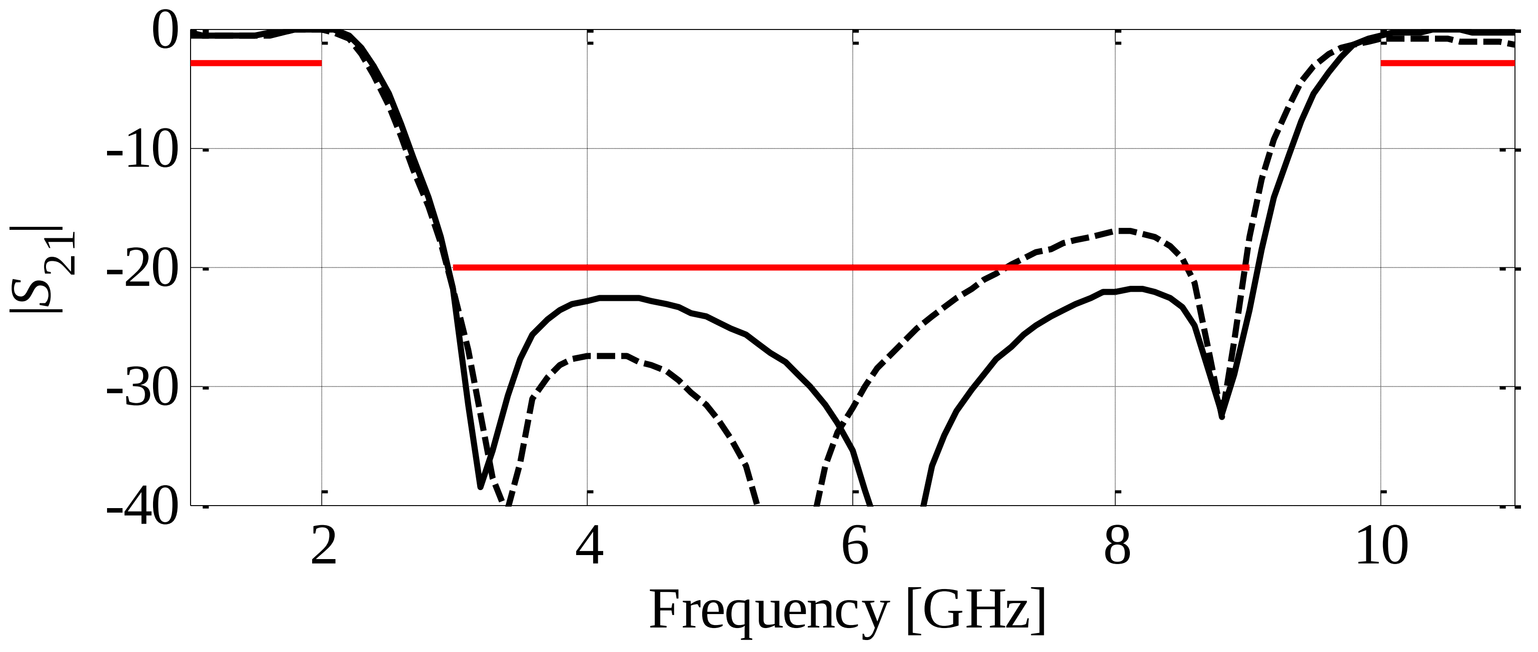
# Wideband Bandstop Microstrip Filter: Optimization Results

| Algorithm | Specification Error | | Number of Fine Model Evaluations |
|---|---|---|---|
| | Best Found | Final | |
| Standard SM | –1.8 dB | –1.7 dB | 21* |
| Constrained SM | –2.0 dB | –2.0 dB | 7 |

*algorithm terminated after 20 iterations without convergence

# Wideband Bandstop Microstrip Filter: Responses

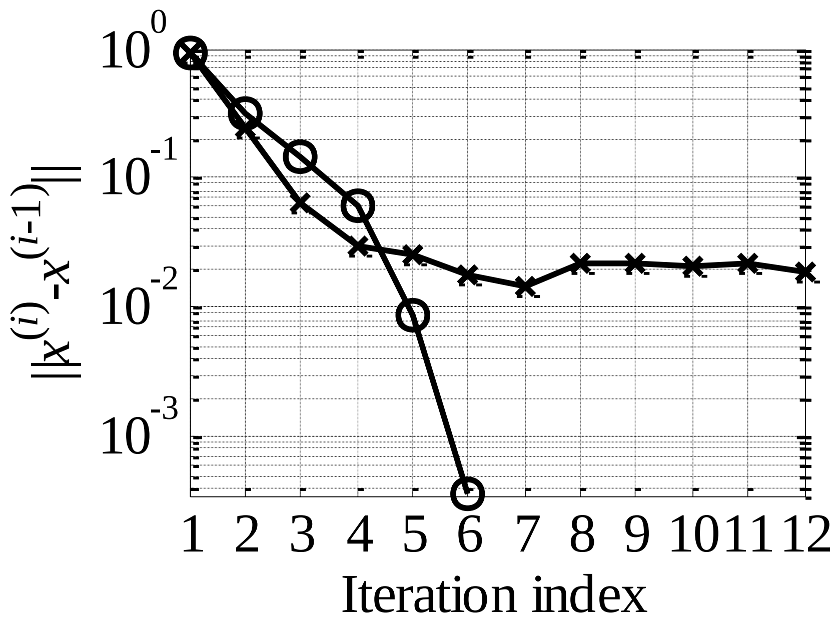
fine model at initial (dashed line) and final (solid line) designs obtained using the constrained space mapping algorithm



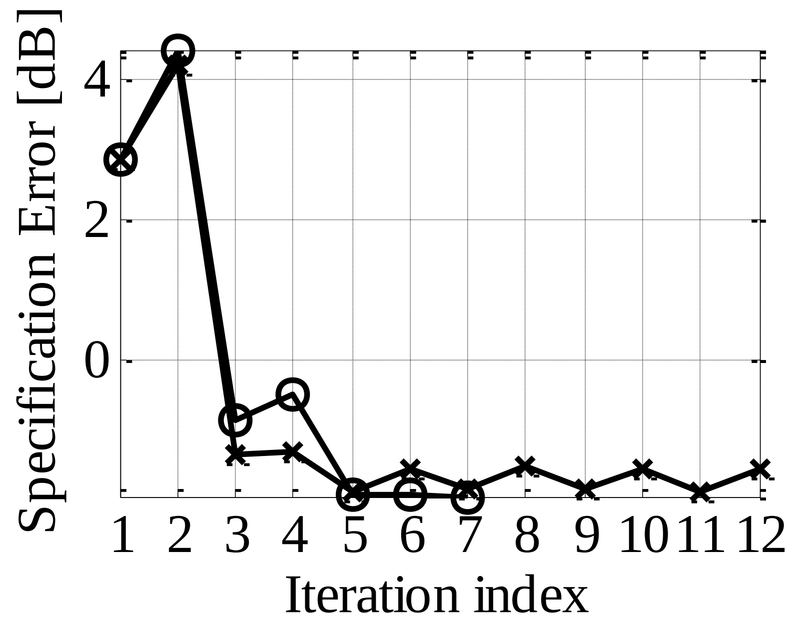
# Wideband Bandstop Microstrip Filter: Convergence

constrained SM algorithm (o) versus standard SM algorithm (×):

convergence plot




specification error evolution



## Conclusions

filters have been designed by modern optimization techniques for some 45 years

traditional Newton-based methods employ Taylor approximations

- required for "coarse" or "surrogate" optimizations
- required for model alignment (parameter extraction)

space mapping harnesses physics-based surrogates to remove expensive "fine" models from traditional optimization loops

space mapping facilitates full-wave EM-based, as well as multidisciplinary design optimization

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# Space Mapping with Constrained Parameter Extraction: Details

constrained parameter extraction process:

$$\boldsymbol{x}_p^{(i)} = \arg \min_{\boldsymbol{l}^{(i)} \le \boldsymbol{x}_p \le \boldsymbol{u}^{(i)}} \| \boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{R}_c(\boldsymbol{x}^{(i)}, \boldsymbol{x}_p) \| \qquad (*)$$

where $\boldsymbol{l}^{(i)} = \boldsymbol{x}_p^{(i-1)} - \boldsymbol{\delta}^{(i)}$ and $\boldsymbol{u}^{(i)} = \boldsymbol{x}_p^{(i-1)} + \boldsymbol{\delta}^{(i)}$

$\boldsymbol{x}_p^{(i-1)}$ model parameters at iteration $i-1$

$\boldsymbol{\delta}^{(i)}$ surrogate model parameter space size at iteration $i$

Updating $\boldsymbol{l}^{(i)}$ and $\boldsymbol{u}^{(i)}$ ($\boldsymbol{\delta}^{(i)}$, $\boldsymbol{x}_p^{(i-1)}$ and $\varepsilon_{max}$ are input arguments):

1. Calculate $\boldsymbol{l}^{(i)} = \boldsymbol{x}_p^{(i-1)} - \boldsymbol{\delta}^{(i)}$ and $\boldsymbol{u}^{(i)} = \boldsymbol{x}_p^{(i-1)} + \boldsymbol{\delta}^{(i)}$;
2. Find $\boldsymbol{x}_p^{(i)}$ using (*);
3. If $\varepsilon^{(i)} \le \alpha_{decr} \cdot \varepsilon_{max}$ then $\boldsymbol{\delta}^{(i+1)} = \boldsymbol{\delta}^{(i)}/\beta_{decr}$; Go to 5;
4. If $\varepsilon^{(i)} > \alpha_{incr} \cdot \varepsilon_{max}$ then $\boldsymbol{\delta}^{(i+1)} = \boldsymbol{\delta}^{(i)} \cdot \beta_{incr}$; Go to 5;
5. END;

Typically: $\alpha_{decr} = 1$, $\alpha_{incr} = 2$, $\beta_{decr} = 5$, $\beta_{incr} = 2$

## Space Mapping with Constrained Parameter Extraction: Interpretation

our algorithm tightens the constraints if the approximation error is sufficiently small, loosens them otherwise

constraint tightening improves the generalization capability of the surrogate: low error $\varepsilon^{(i-1)} = ||\boldsymbol{R}_f(\boldsymbol{x}^{(i-1)}) - \boldsymbol{R}_c(\boldsymbol{x}^{(i-1)}, \boldsymbol{x}_p^{(i-1)})||_p$ and $\varepsilon^{(i)} = ||\boldsymbol{R}_f(\boldsymbol{x}^{(i)}) - \boldsymbol{R}_c(\boldsymbol{x}^{(i)}, \boldsymbol{x}_p^{(i)})||_p$ makes it more likely to have $||\boldsymbol{R}_f(\boldsymbol{x}^{(i-1)}) - \boldsymbol{R}_c(\boldsymbol{x}^{(i-1)}, \boldsymbol{x}_p^{(i)})||_p$ small if $\boldsymbol{\delta}^{(i)}$ is reduced because a small $||\boldsymbol{x}_p^{(i)} - \boldsymbol{x}_p^{(i-1)})||_\infty \leq ||\boldsymbol{\delta}^{(i)}||_\infty$ implies similarity of the subsequent surrogate models

improved performance follows from rigorous convergence results for space mapping algorithms