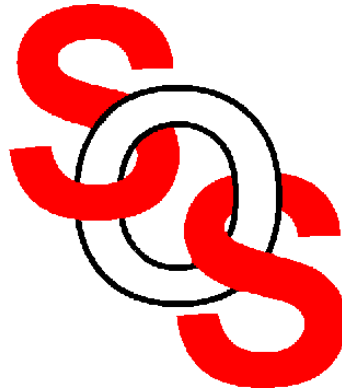


# Space Mapping: Engineering Modeling And Optimization Exploiting Surrogates

John W. Bandler

Simulation Optimization Systems Research Laboratory  
McMaster University



Bandler Corporation, [www.bandler.com](http://www.bandler.com)  
[john@bandler.com](mailto:john@bandler.com)

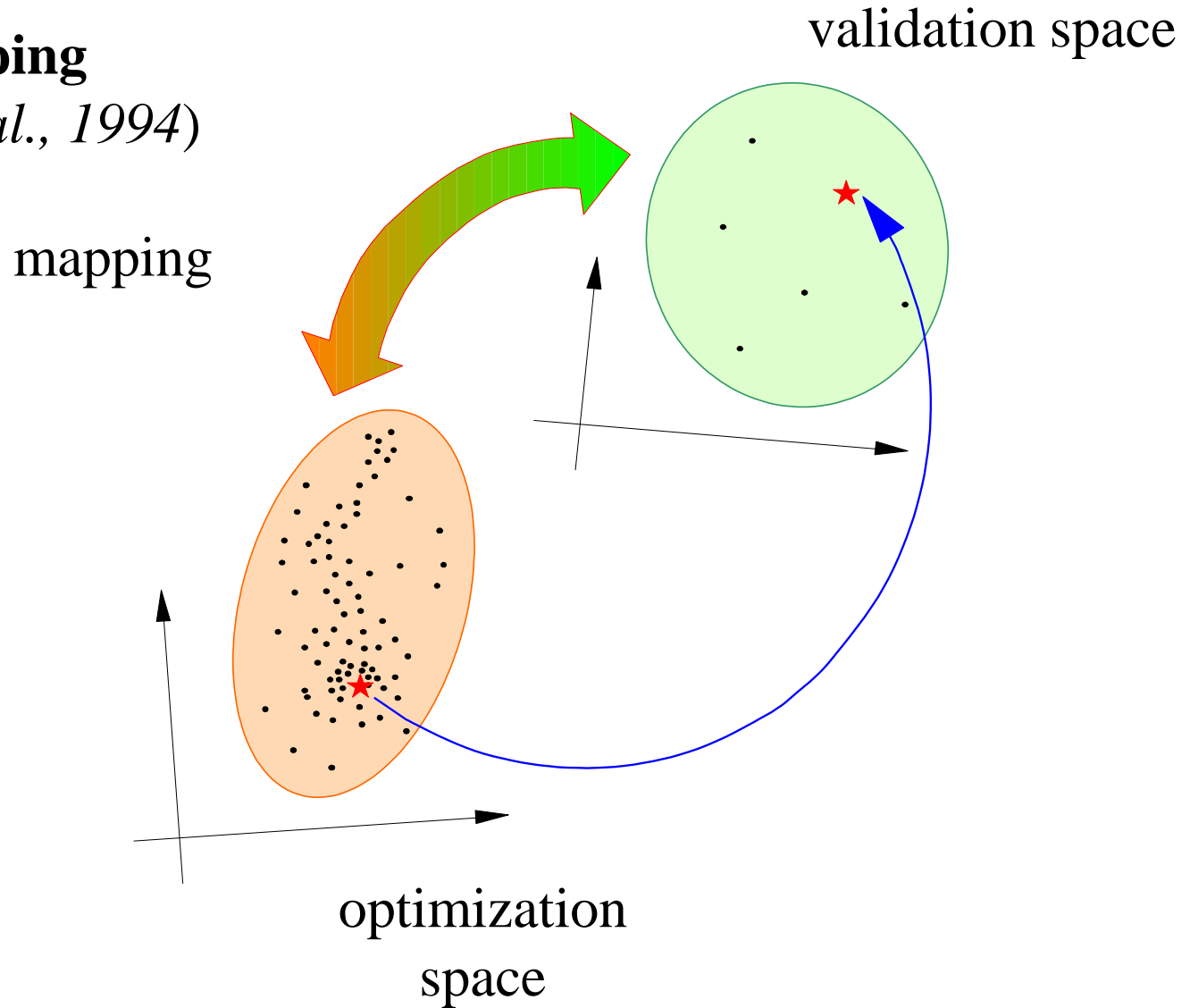


presented at



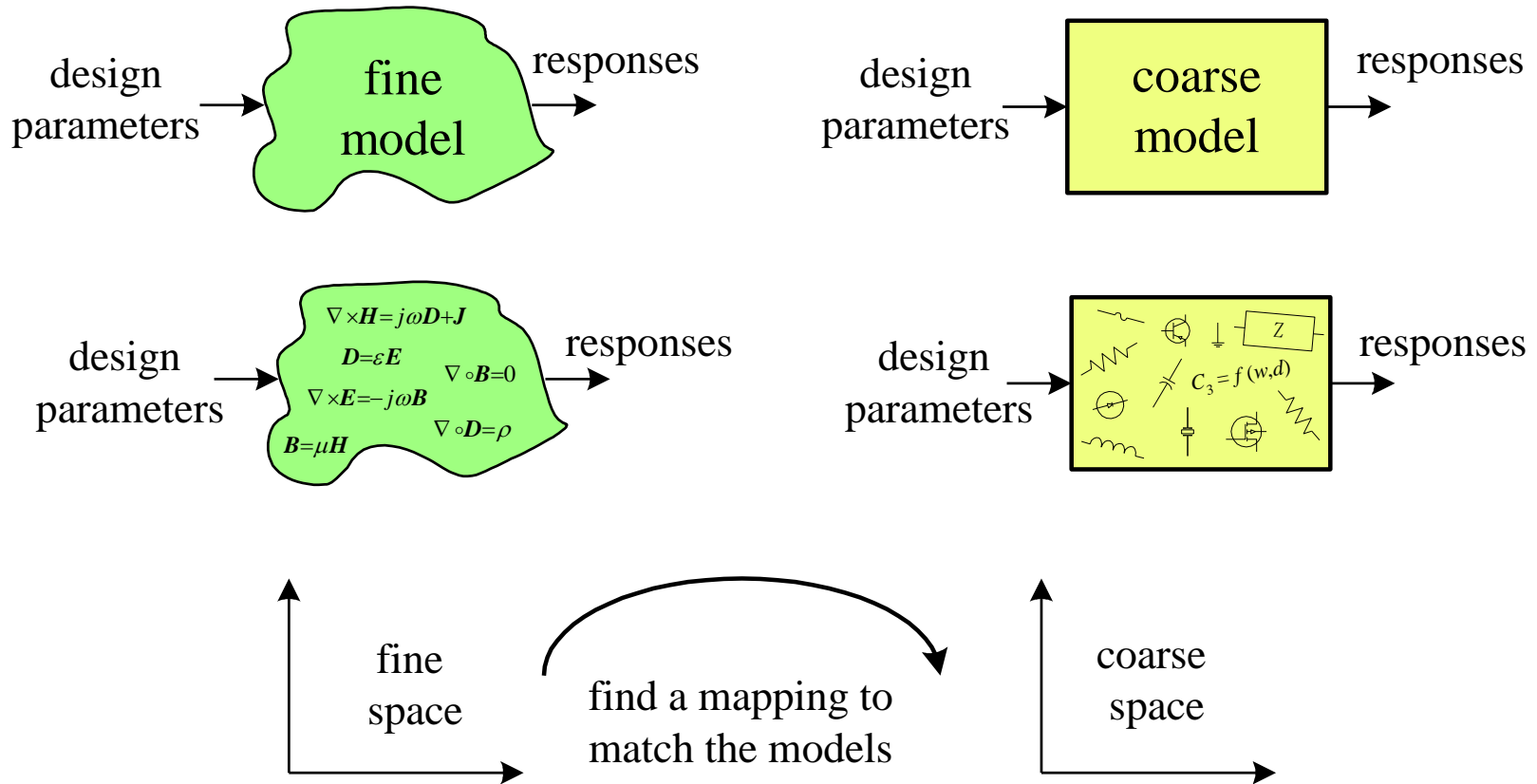
# Space Mapping

(Bandler et al., 1994)





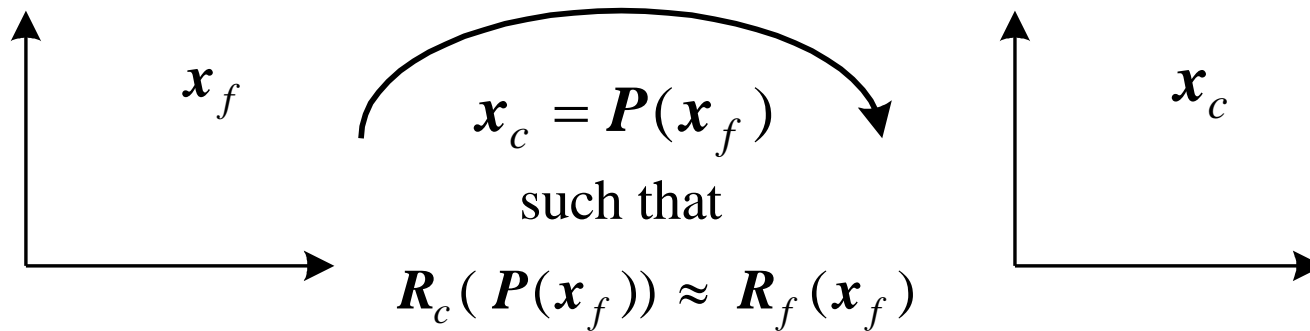
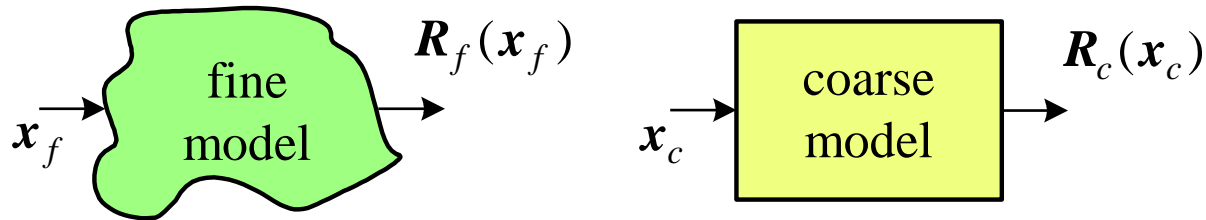
# Linking Companion Coarse (Empirical) and Fine (EM) Models





# The Space Mapping Concept

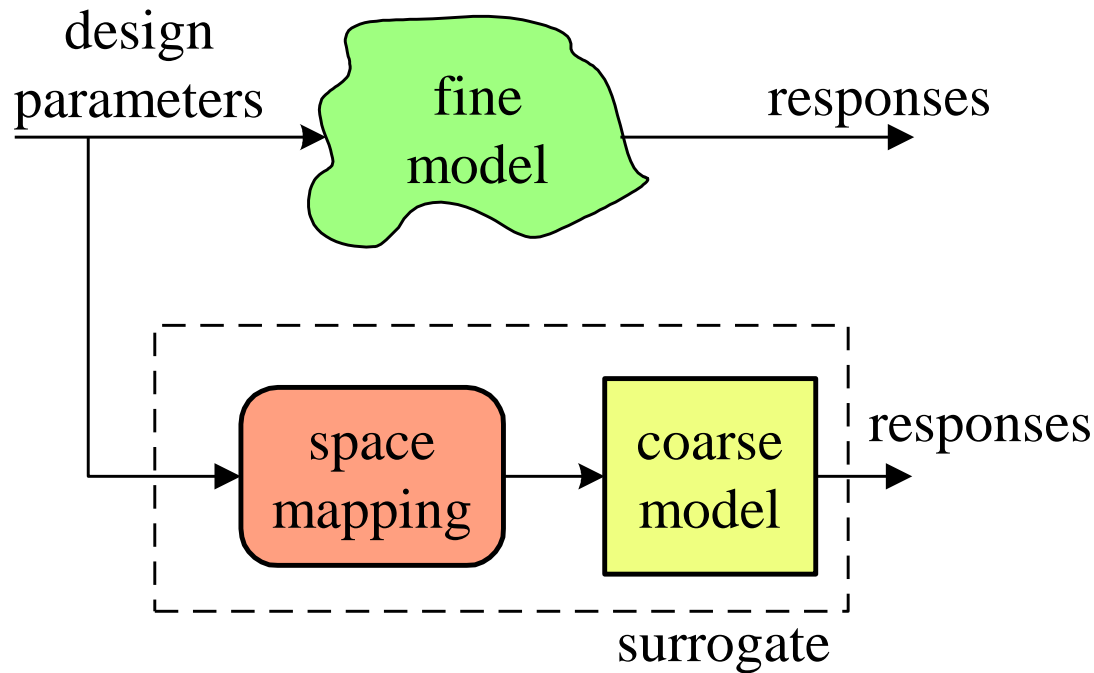
(Bandler et al., 1994-)





# Explicit **Space Mapping** Concept

(Bandler et al., 1994-)





## Space Mapping: a Glossary of Terms

### Space Mapping

transformation, link, adjustment, correction, shift (in parameters or responses)

### Coarse Model

simplification or convenient representation, companion to the fine model, auxiliary representation, cheap model

### Fine Model

accurate representation of system considered, device under test, component to be optimized, expensive model



## Space Mapping: a Glossary of Terms

<b>Surrogate</b>	model, approximation or representation to be used, or to act, in place of, or as a substitute for, the system under consideration
	mapped or enhanced coarse model
<b>Surrogate Model</b>	alternative expression for <b>Surrogate</b>
<b>Target Response</b>	response the fine model should achieve, (usually) optimal response of a coarse model, enhanced coarse model, or surrogate



## Space Mapping: a Glossary of Terms

Companion	coarse
Low Fidelity/ Resolution	coarse
High Fidelity/ Resolution	fine
Empirical	coarse
Simplified Physics	coarse
Physics-based	coarse or fine
Device under Test	fine
Electromagnetic Simulation	fine or coarse
Computational	fine or coarse





## Space Mapping: a Glossary of Terms

<b>Parameter (input) Space Mapping</b>	mapping, transformation or correction of design variables
<b>Response (output) Space Mapping</b>	mapping, transformation or correction of responses
<b>Response Surface Approximation</b>	linear/quadratic/polynomial approximation of responses w.r.t. design variables



## Space Mapping: a Glossary of Terms

Neuro	implies use of artificial neural networks
Implicit Space Mapping	space mapping when the mapping is not obvious
Not Space Mapping	(usually) space mapping when not acknowledged
Parameter Transformation	space mapping
Predistortion	?



## Jacobian-Space Mapping Relationship

*(Bakr et al., 1999)*

through PE we match the responses

$$\mathbf{R}_f(\mathbf{x}_f) \approx \mathbf{R}_c(\mathbf{P}(\mathbf{x}_f))$$

by differentiation

$$\left( \frac{\partial \mathbf{R}_f^T}{\partial \mathbf{x}_f} \right)^T \approx \left( \frac{\partial \mathbf{R}_c^T}{\partial \mathbf{x}_c} \right)^T \cdot \left( \frac{\partial \mathbf{x}_c^T}{\partial \mathbf{x}_f} \right)^T$$



## Jacobian-Space Mapping Relationship

*(Bakr et al., 1999)*

given coarse model Jacobian  $\mathbf{J}_c$  and space mapping matrix  $\mathbf{B}$   
we estimate

$$\mathbf{J}_f(\mathbf{x}_f) \approx \mathbf{J}_c(\mathbf{x}_c)\mathbf{B}$$

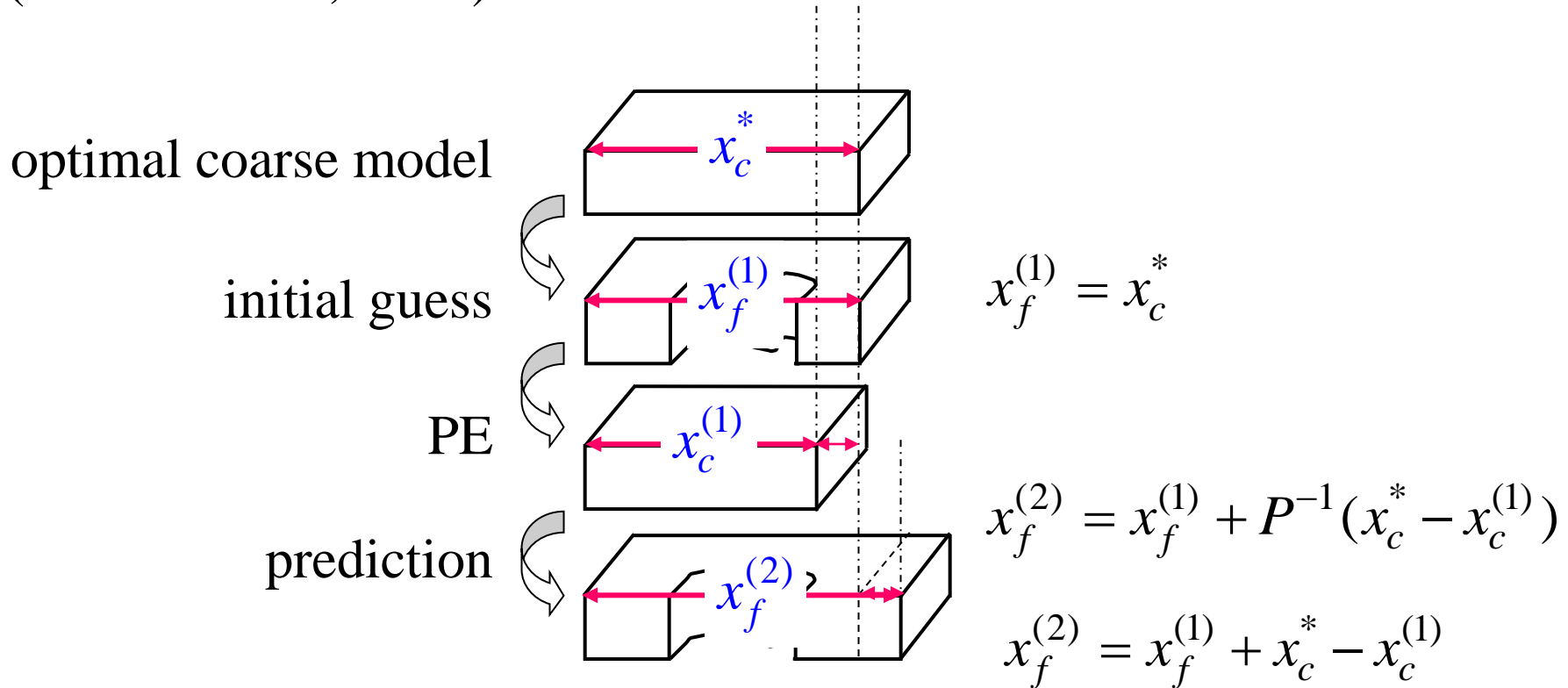
given  $\mathbf{J}_c$  and  $\mathbf{J}_f$  we estimate (least squares)

$$\mathbf{B} \approx (\mathbf{J}_c^T \mathbf{J}_c)^{-1} \mathbf{J}_c^T \mathbf{J}_f$$



# Aggressive Space Mapping Practice—Cheese Cutting Problem

(Bandler et al., 2002)





## Interpretation of Space Mapping Optimization

the original optimization problem

$$\mathbf{x}^* \square \arg \min_{\mathbf{x}} U(\mathbf{R}(\mathbf{x}))$$

consider  $\mathbf{R}_c(\mathbf{P}(\mathbf{x}_f))$  as an “enhanced” coarse model or “surrogate,”  
then

$$\bar{\mathbf{x}}_f = \arg \min_{\mathbf{x}_f} U(\mathbf{R}_c(\mathbf{P}(\mathbf{x}_f)))$$

is equivalent to

$$\mathbf{f}(\mathbf{x}_f) \square \mathbf{P}(\mathbf{x}_f) - \mathbf{x}_c^*, \quad \mathbf{f} \rightarrow \mathbf{0}$$



## Aggressive Space Mapping Approach

*(Bandler et al., 1995)*

iteratively solves the nonlinear system

$$\mathbf{f}(\mathbf{x}_f) = \mathbf{0}$$

the quasi-Newton step  $\mathbf{h}^{(j)}$  in the fine space is given by

$$\mathbf{B}^{(j)}\mathbf{h}^{(j)} = -\mathbf{f}^{(j)}$$

the next iterate

$$\mathbf{x}_f^{(j+1)} = \mathbf{x}_f^{(j)} + \mathbf{h}^{(j)}$$



## Aggressive Space Mapping Approach (continued)

Broyden-like updates (*Bandler et al., 1995*)

$$\mathbf{B}^{(j+1)} = \mathbf{B}^{(j)} + \frac{\mathbf{f}^{(j+1)} - \mathbf{f}^{(j)} - \mathbf{B}^{(j)} \mathbf{h}^{(j)}}{\mathbf{h}^{(j)T} \mathbf{h}^{(j)}} \mathbf{h}^{(j)T}$$

Jacobian based updates (*Bandler et al., 1999, 2002*)

$$\mathbf{B} = (\mathbf{J}_c^T \mathbf{J}_c)^{-1} \mathbf{J}_c^T \mathbf{J}_f \quad \mathbf{E} = \mathbf{J}_f - \mathbf{J}_c \mathbf{B}$$

constrained update (*Bakr et al., 2000*)

$$\Delta \mathbf{B} = \mathbf{B} - \mathbf{I}$$

$$\mathbf{B} = \arg \min_B \left\| \begin{bmatrix} \mathbf{e}_1^T & \cdots & \mathbf{e}_n^T & \eta \Delta \mathbf{b}_1^T & \cdots & \eta \Delta \mathbf{b}_n^T \end{bmatrix}^T \right\|_2^2$$

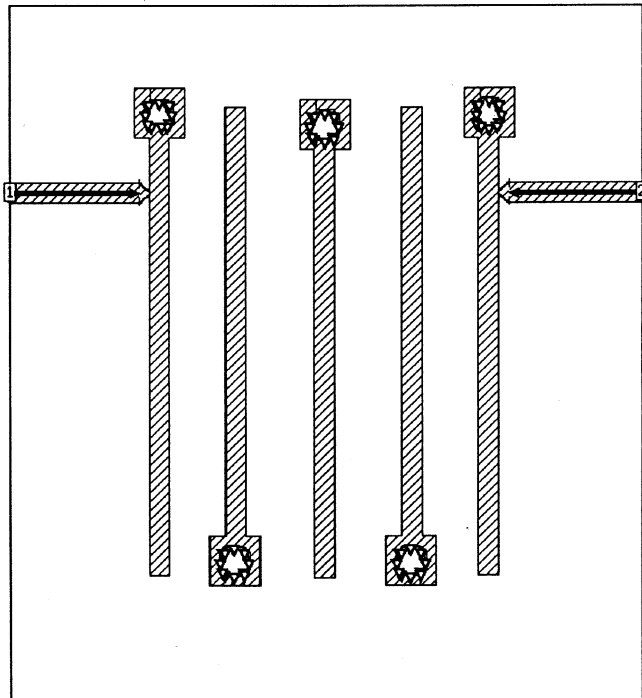




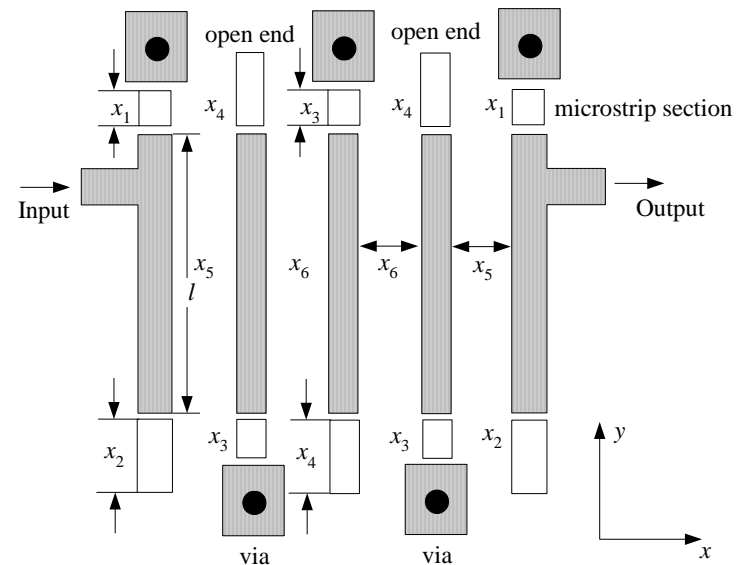
# A Five-pole Interdigital Filter Design

(Bandler et al., 1997)

Sonnet's *em* model



decomposed coarse model

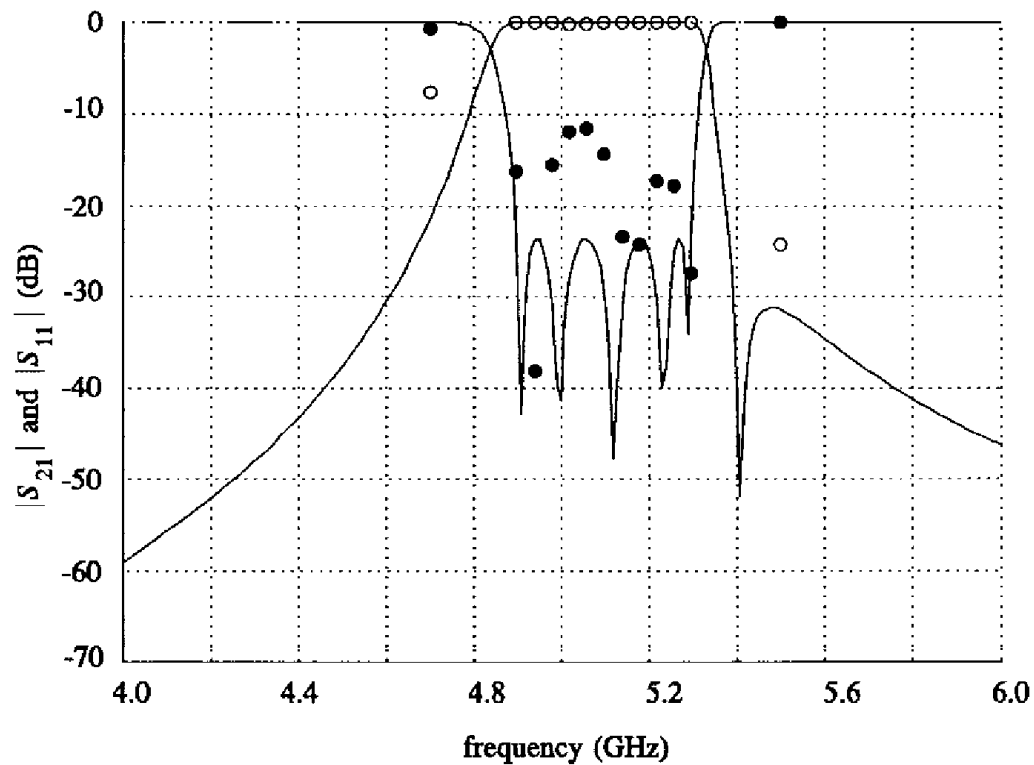


passband ripple  $\leq 0.1$  dB for  $4.9 \text{ GHz} \leq \omega \leq 5.3 \text{ GHz}$   
isolation: 30 dB, isolation bandwidth: 0.95 GHz



# A Five-pole Interdigital Filter Design (continued)

(Bandler et al., 1997)

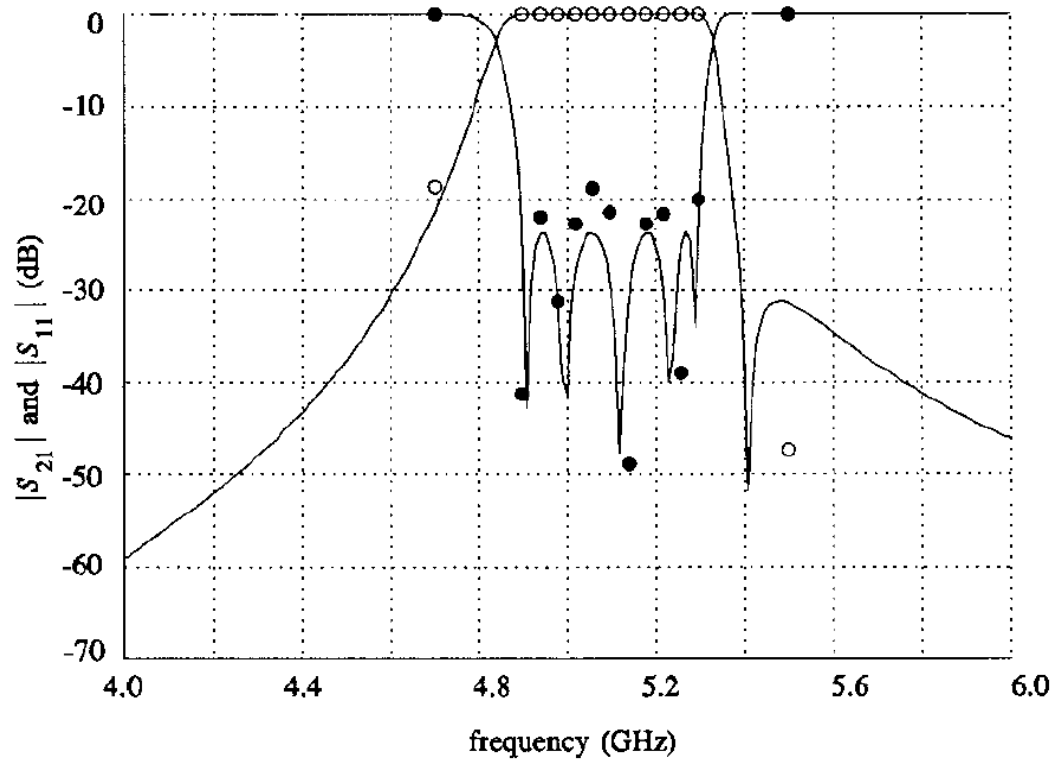


starting point



# A Five-pole Interdigital Filter Design (continued)

(Bandler et al., 1997)

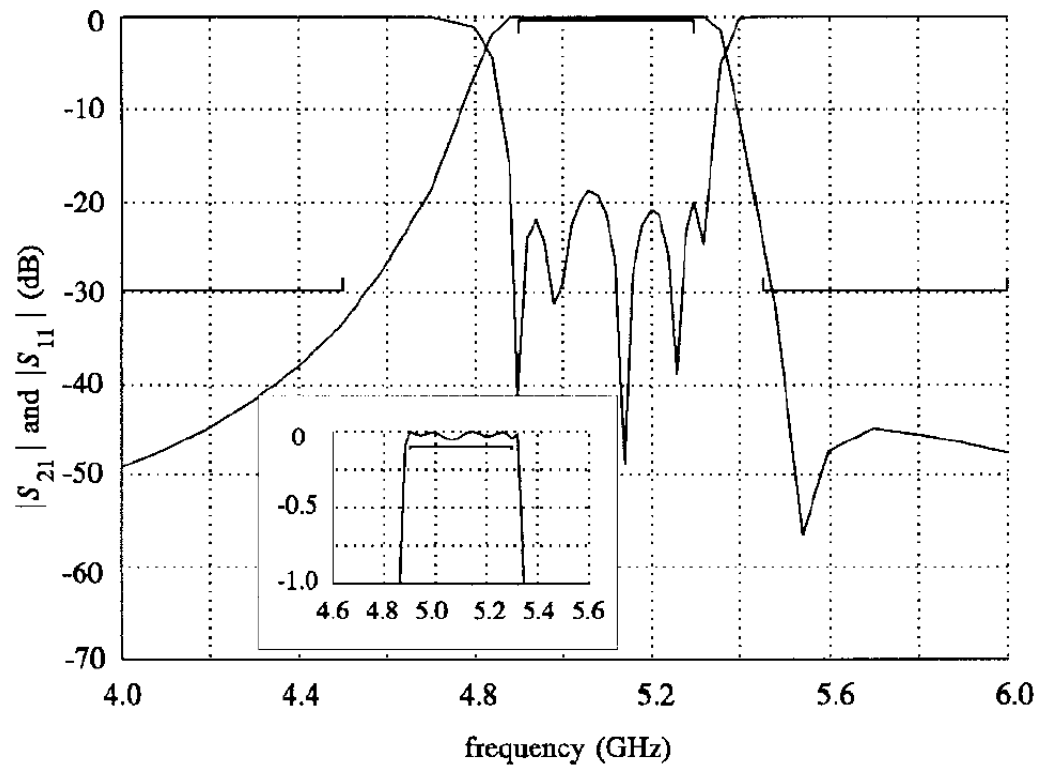


second iteration



## A Five-pole Interdigital Filter Design (continued)

(Bandler et al., 1997)



second iteration, fine frequency sweep



## Mathematical Motivation for Space Mapping

at the current iterate, a first-order Taylor model

$$\mathbf{L}_f(\mathbf{x}_f^{(j)} + \mathbf{h}) \approx \mathbf{R}_f(\mathbf{x}_f^{(j)}) + \mathbf{J}_f(\mathbf{x}_f^{(j)})\mathbf{h}$$

the deviation of this model from  $\mathbf{R}_f$  can be bounded as

$$\left\| \mathbf{R}_f(\mathbf{x}_f^{(j)} + \mathbf{h}) - \mathbf{L}_f(\mathbf{x}_f^{(j)} + \mathbf{h}) \right\| \leq C_T \|\mathbf{h}\|^2$$

Taylor approximation to  $P$

$$\mathbf{L}_P(\mathbf{x}_f^{(j)} + \mathbf{h}) \approx \mathbf{P}(\mathbf{x}_f^{(j)}) + \mathbf{J}_P(\mathbf{x}_f^{(j)})\mathbf{h}$$

the approximation bound

$$\left\| \mathbf{P}(\mathbf{x}_f^{(j)} + \mathbf{h}) - \mathbf{L}_P(\mathbf{x}_f^{(j)} + \mathbf{h}) \right\| \leq C_P \|\mathbf{h}\|^2$$

constant



## Mathematical Motivation for Space Mapping

the difference between  $\mathbf{R}_f$  and the mapped coarse model can be bounded as follows

$$\begin{aligned} & \left\| \mathbf{R}_f(\mathbf{x}_f^{(j)} + \mathbf{h}) - \mathbf{R}_c(\mathbf{P}(\mathbf{x}_f^{(j)} + \mathbf{h})) \right\| \\ & \leq \varepsilon + C_P \cdot \left\| \mathbf{J}_c(\mathbf{P}(\mathbf{x}_f^{(j)})) \right\| \cdot \|\mathbf{h}\|^2 \end{aligned}$$

$C_T$  and  $C_P$  are problem specific constants

$\varepsilon$  is a constant independent of  $\mathbf{x}_f$



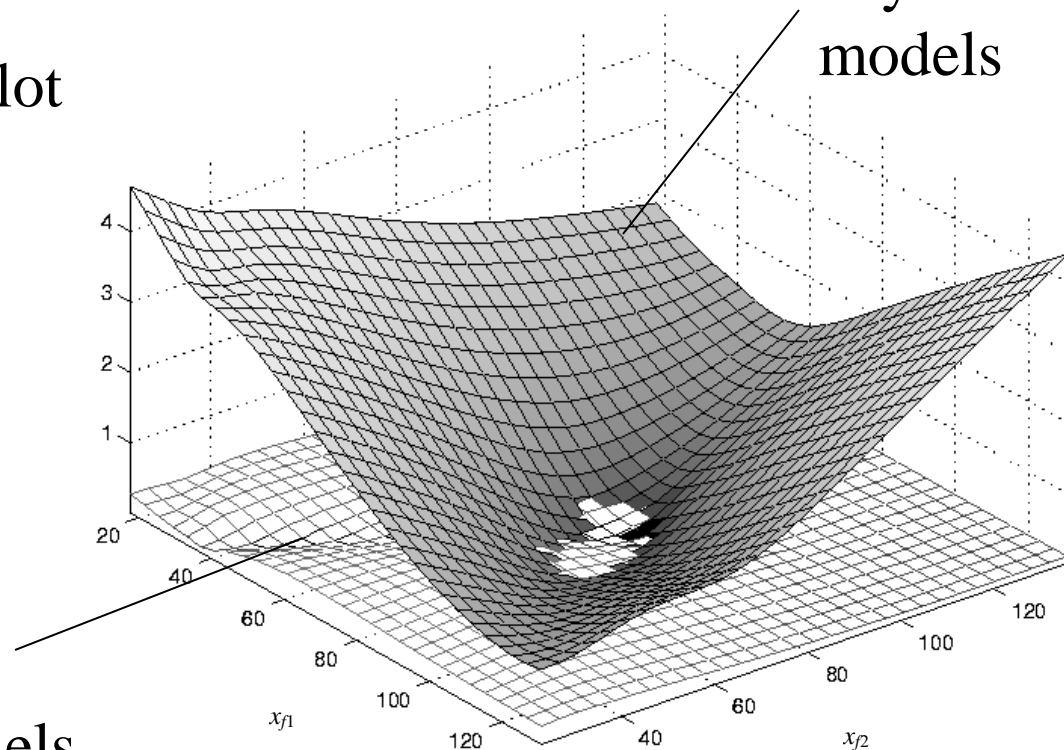
## Mathematical Motivation for Space Mapping

the **Space Mapped** model is a good approximation far from  $\mathbf{x}_f^*$

the first-order Taylor model is best close to  $\mathbf{x}_f^*$

error between  
Taylor and fine  
models

model effectiveness plot  
for a two-section  
capacitively  
loaded impedance  
transformer  
(*Bakr et al. 2001*)



error between **Space Mapped** and fine models



## Trust Regions and **Aggressive Space Mapping**

solving equivalent problem

$$(\mathbf{B}^{(j)T} \mathbf{B}^{(j)} + \lambda \mathbf{I}) \mathbf{h}^{(j)} = -\mathbf{B}^{(j)T} \mathbf{f}^{(j)}$$

where  $\mathbf{B}^{(j)}$  is an approximation to the Jacobian of mapping  $\mathbf{P}$  and is updated using Broyden's formula

$\lambda$  and  $\delta$  (trust region size) are related

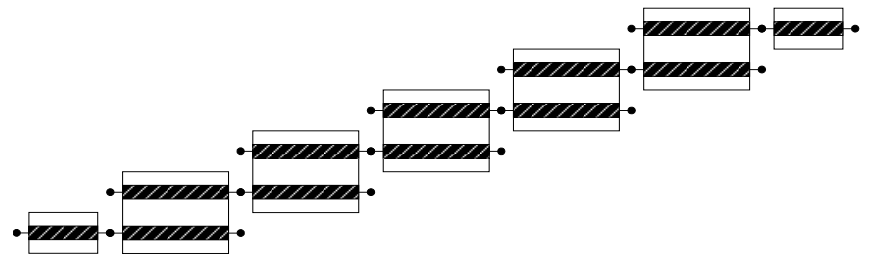
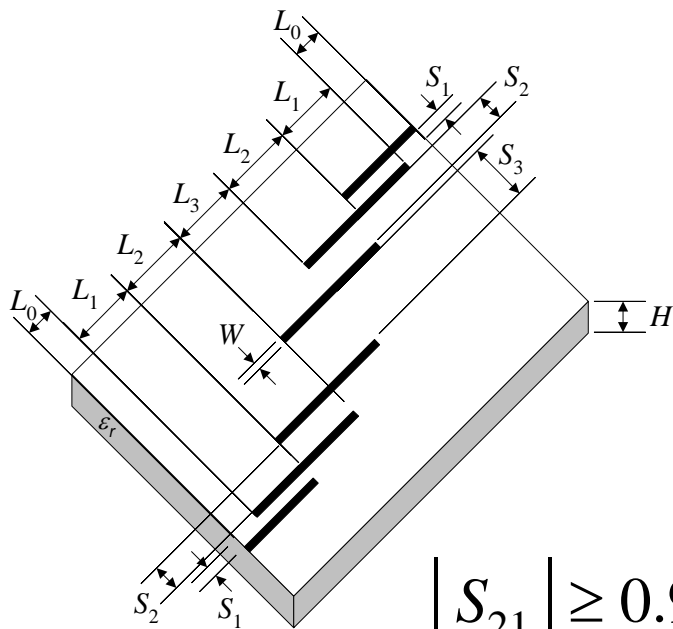
$\lambda$  is the Lagrange multiplier for the trust region constraint



## Trust Regions and **Aggressive Space Mapping**

(Bandler et al., 1993-2003)

HTS quarter-wave parallel coupled-line microstrip filter  
(Westinghouse, 1993)



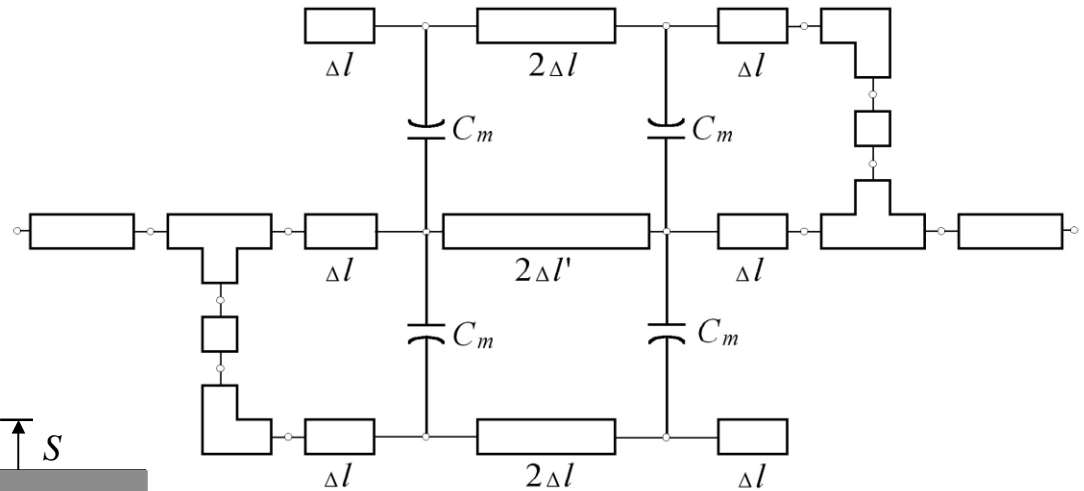
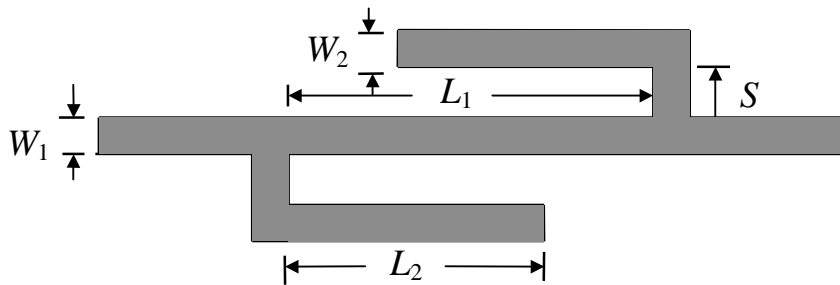
$$\begin{cases} |S_{21}| \geq 0.95 & \text{for } 4.008 \text{ GHz} \leq \omega \leq 4.058 \text{ GHz} \\ |S_{21}| \leq 0.05 & \text{for } \omega \leq 3.967 \text{ GHz and } \omega \geq 4.099 \text{ GHz} \end{cases}$$

# Hybrid Aggressive Space Mapping Optimization

(Bakr et al., 1999)

double-folded stub filter

(Rautio, 1992)

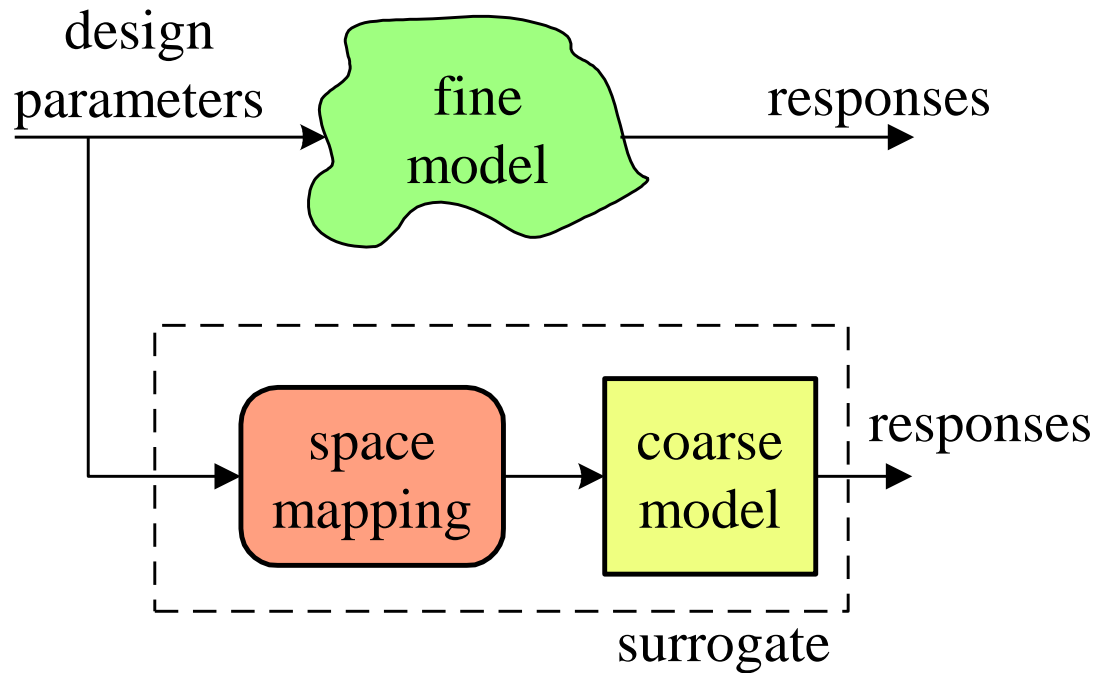


$$\begin{aligned} |S_{21}| &\geq -3 \text{ dB for } \omega \leq 9.5 \text{ GHz and } \omega \leq 16.5 \text{ GHz} \\ |S_{21}| &\leq -30 \text{ dB for } 12 \text{ GHz} \leq \omega \leq 14 \text{ GHz} \end{aligned}$$



# Explicit **Space Mapping** Concept

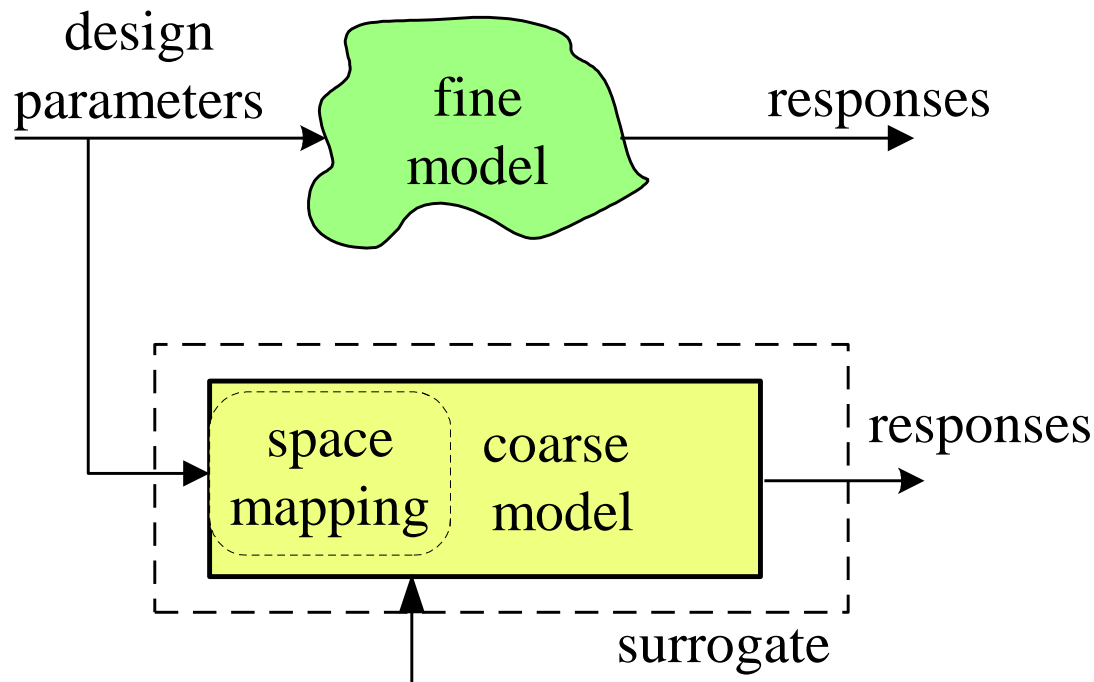
(Bandler et al., 1994-)





# Implicit **Space Mapping** Concept

(Bandler et al., 2004)

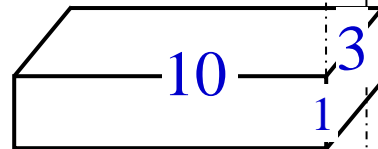




# Implicit Space Mapping Practice—Cheese Cutting Problem

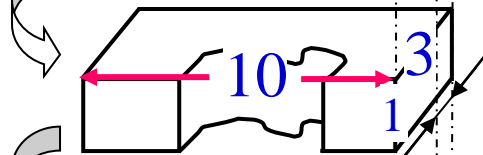
(Bandler, 2002)

optimal coarse model



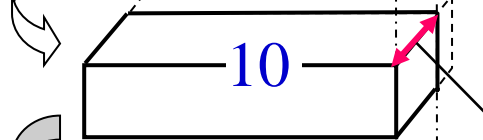
target volume = 30

initial guess



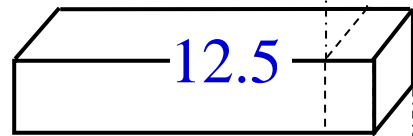
volume = 24

PE



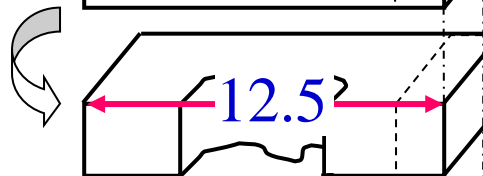
volume = 24

prediction



target volume = 30

verification

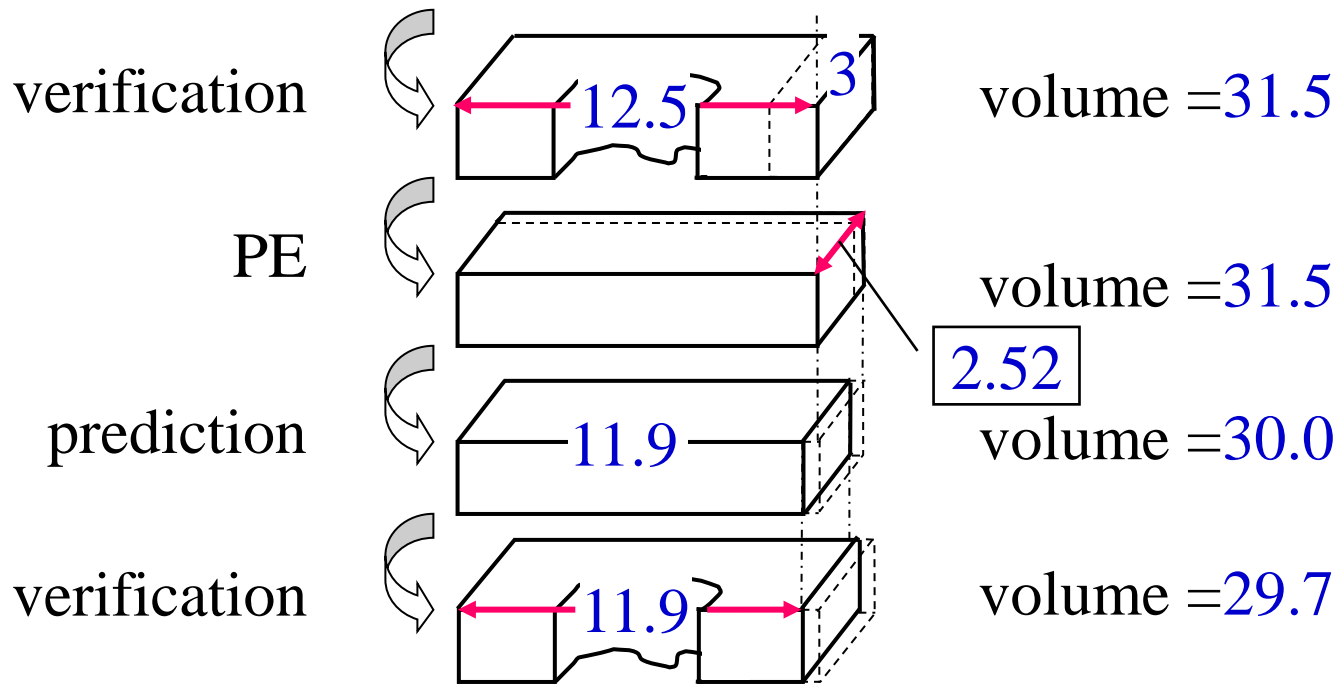


volume = 31.5



# Implicit Space Mapping Practice—Cheese Cutting Problem

(Bandler, 2002)

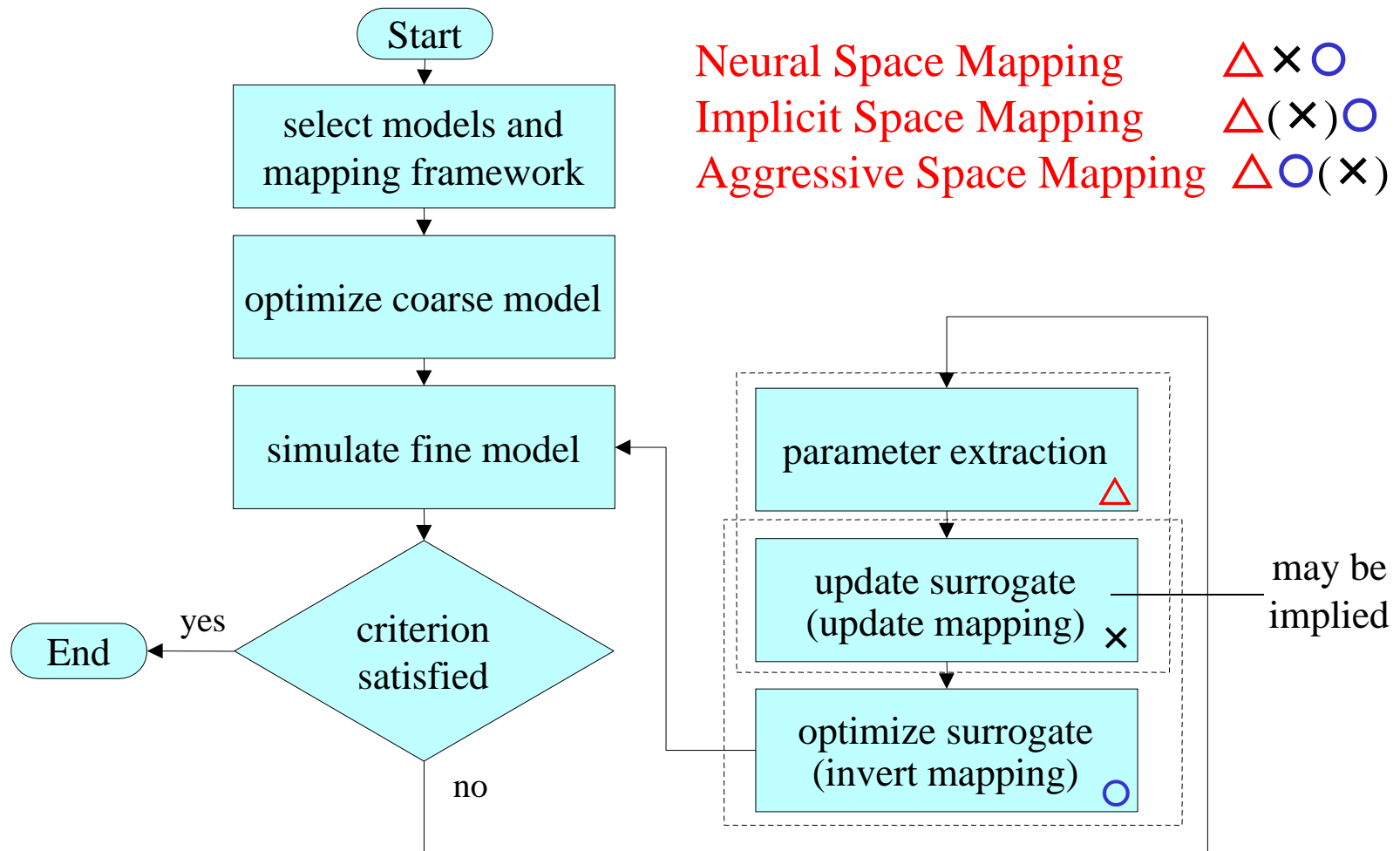


$$\begin{aligned} & \vdots \quad \text{error} = (30 - 29.7) / 30 \times 100\% \\ & \bullet \quad \quad = 1\% \end{aligned}$$



# Space Mapping Framework

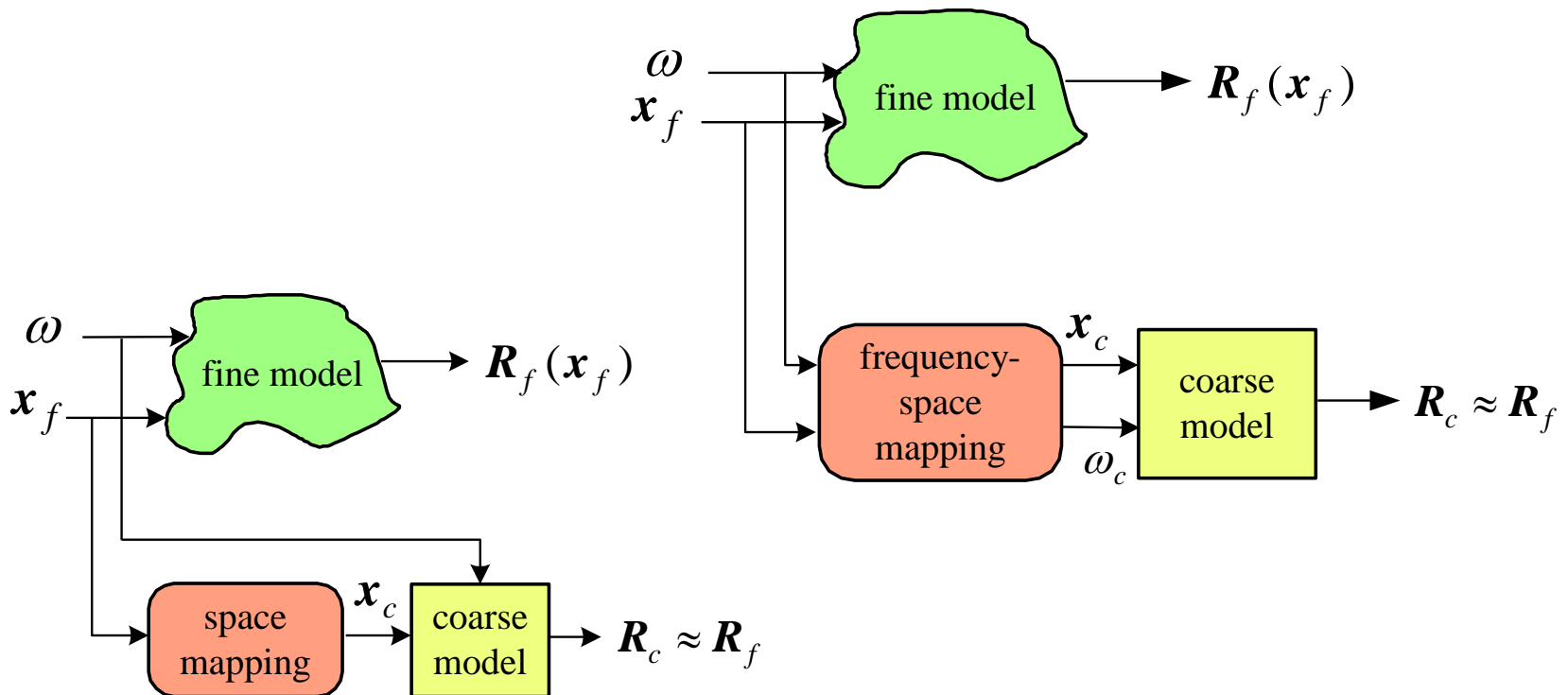
(Bandler et al., 2004)





## Model Enhancement—the SM Tableau Approach (Bandler et al., 2001)

already used in the RF industry for new library models (Snel, 2001)







## Space Mapping Implementation and Applications 1

RF and microwave implementation (*Bandler et al., 1994-2003*)

civil engineering structural design (*Leary et al., 2000*)

automobile crashworthiness design (*Redhe et al., 2001-2002*)

generating microwave neural models (*Devabhaktuni et al., 2002*)

compline filter design (*Swanson and Wenzel, 2001*)

microwave filter design (*Harscher, et al., 2002, 2003*)

CAD of integrated passive elements on PCBs (*Draxler, 2002*)



## Space Mapping Implementation and Applications 2

CAD technique for microstrip filter design

*(Ye and Mansour, 1997)*

**SM** models (model enhancement) for RF components *(Snel, 2001)*

multilayer microwave circuits (LTCC) *(Pavio et al., 2002)*

cellular power amplifier output matching circuit *(Lobeek, 2002)*

multilevel **ASM** strategy applied to filter optimization

*(Safavi-Naeini et al., 2002)*

coupled resonator filter *(Pelz, 2002)*



## Space Mapping Implementation and Applications 3

LTCC RF passive circuit design (*Wu et al., 2002*)

waveguide filter design (*Steyn et al., 2001*)

inductively coupled filters (*Soto et al., 2000*)

magnetic systems (*Choi et al., 2001*)

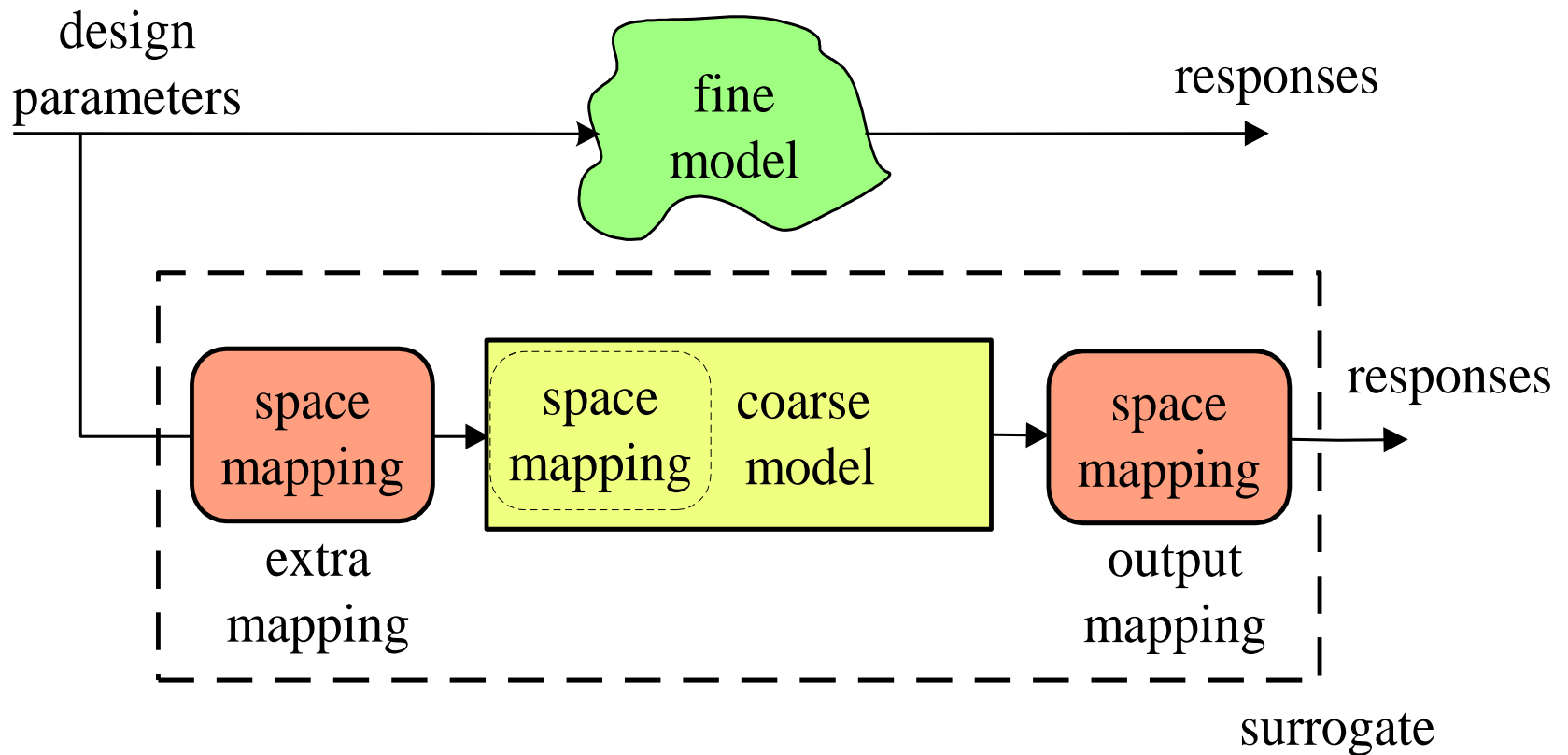
**Implicit Space Mapping** optimization with preassigned parameters  
(*Bandler et al., 2002*)

**Output Space Mapping** optimization (*Bandler et al., 2003*)



# Implicit, Extra and Output Space Mappings

(Bandler et al., 2003)





## Space Mapping Implementation and Applications 4

EM-based optimization of microwave oscillators  
(*Rizzoli et al., 2003*)

circuit level, neuro-**SM** modeling of nonlinear devices  
(*Zhang et al., 2003*)

optimization of dielectric resonator filters and multiplexers  
(*Ismail et al., 2003*)

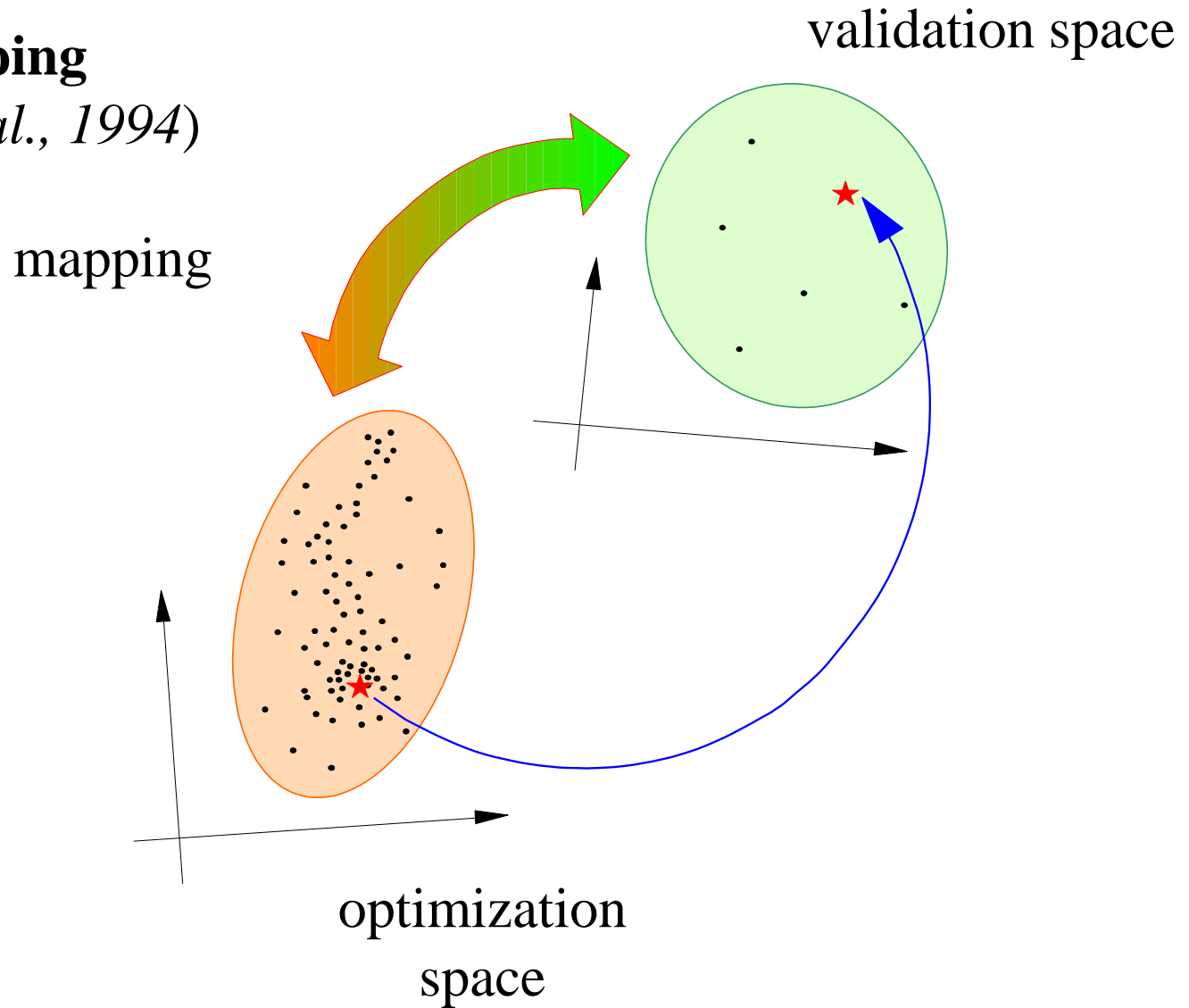
waveguide filter design (*Morro et al., 2003*)

optimal control of partial differential equations  
(*Hintermueller and Vicente, 2003*)



# Space Mapping

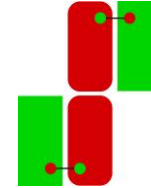
(Bandler et al., 1994)





## Conclusions

mathematical motivation for **Space Mapping**



**Space Mapping** optimization: original (1993)

**Aggressive Space Mapping** optimization: Broyden-based (1995),  
trust region, hybrid . . .

parameter extraction

**Space Mapping** surrogate model based optimization

interesting implementations and applications



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J.W. Bandler, R.M. Biernacki, S.H. Chen, R.H. Hemmers and K. Madsen, “Electromagnetic optimization exploiting aggressive space mapping,” *IEEE Trans. Microwave Theory Tech.*, vol. 43, 1995, pp. 2874–2882.

J.W. Bandler, R.M. Biernacki, S.H. Chen and Y.F. Huang, “Design optimization of interdigital filters using aggressive space mapping and decomposition,” *IEEE Trans. Microwave Theory Tech.*, vol. 45, 1997, pp. 761–769.

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