CONOPT

A PACKAGE FOR SOLVING NONLINEAR PROGRAMMING PROBLEMS USING A NEW (MINIMAX) APPROACH WITH EFFICIENT GRADIENT METHODS

PURPOSE

CONOPT is a modified version of GRADMIN to be used for solving constrained optimization problems. A new technique proposed by Bandler and Charalambous [1,2] is used to transform the constrained optimization problem into the minimization of an unconstrained objective function. Practical least pth approximation [3] is used to solve the resulting minimax problem. Equality constraints treated as two inequality constraints, e.g., $\psi(x_1, x_2, \ldots, x_n) = 0$ will be treated as $\psi(x_1, x_2, \ldots, x_n) \geq 0$ and $-\psi(x_1, x_2, \ldots, x_n) \geq 0$. The program is currently limited to 100 inequality constraints.

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PROGRAMMERS

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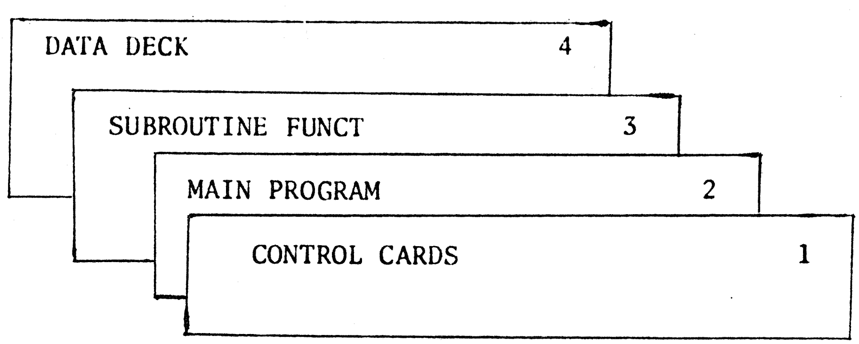
HOW TO USE

Set up the input deck as follows (McMaster CDC 6400)



CONTROL CARDS   Use the following set of control cards.

```
AAAA                                        USER NAME
ATTACH(TAPE, CONOPT, ID=*********,MR=1)†
RUN(S,,,,,,,,X)
LOAD(TAPE)
LGO.
END OF RECORD
PROGRAM TST(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)
```

MAIN PROGRAM   Write the main program as indicated below.

(a) Dimension the following arrays

X(NN), G(NN), X1(NN), X2(NN), G2(NN), PY(NN), ALFA(NN), P(NN,NN), Y(NN), PE(NN), BIGV(NN), XSTRT(NN), DUM1(NN), DUM2(NN), EPS(N), H(K)

where N = The number of independent variables

NN = N + 2

K = N*(N+7)/2

(b) Supply the values of the following parameters

N = The number of independent variables

NN = N + 2

KR = 1 if data deck is to be read

= 0 if data deck is not to be read; always set it equal to 1 for the first call of CONOPT.

---

†Appropriate identification parameter ID should be inserted in *********.

(c) Call subroutine CONOPT as follows

```
CALL CONOPT(N, X, XSTRT, X2, X1, G, G2, ALFA, H, P, PY, PE,
            BIGV, EPS, NN, Y, DUM1, DUM2, KR)
```

(d) Add STOP and END cards.

<u>SUBROUTINE FUNCT</u> This subroutine defines the actual objective function, the inequality constraints, and the gradients of the objective function and the constraints. A subroutine FMINMAX is then called which combines the objective function and the constraints in a suitable manner to give the unconstrained objective function F and its gradients. The subroutine FUNCT should be written as follows. The actual numerical values of N and NCONS should be substituted in the dimension statements. NCONS is the number of inequality constraints.

SUBROUTINE FUNCT(N, X, F, G)

DIMENSION X(1), G(1), PHI(NCONS), GU(N), GPHI(N, NCONS), A(NCONS+1),
  T1(NCONS+1)

U = $f(x_1, x_2, \ldots, x_n)$ [The actual objective function]

PHI(1) = $\phi_1(x_1, x_2, \ldots, x_n)$ [The first inequality constraint]

PHI(2) = $\phi_2(x_1, x_2, \ldots, x_n)$

PHI(NCONS) = $\phi_{NCONS}(x_1, x_2, \ldots, x_n)$

GU(1) = $\frac{\partial f}{\partial x_1}$

GU(2) = $\frac{\partial f}{\partial x_2}$

⋮

GU(N) = $\frac{\partial f}{\partial x_n}$

GPHI(1,1) $= \frac{\partial \phi_1}{\partial x_1}$

GPHI(2,1) $= \frac{\partial \phi_1}{\partial x_2}$

$\vdots$

GPHI(N,1) $= \frac{\partial \phi_1}{\partial x_n}$

$\vdots$

GPHI(N,2) $= \frac{\partial \phi_2}{\partial x_n}$

$\vdots$

GPHI(N,NCONS) $= \frac{\partial \phi_{NCONS}}{\partial x_n}$

NCONS = Number of inequality constraints

P = The power to be used in the least pth approximation (set = 1.E5 if no information is available)

EPSPHI = A small number used to determine the margin by which constraints may be violated (set =1.E-5 if no information is available)

```
CALL FMINMAX (U, GU, PHI, GPHI, N, NCONS, F, G, P, EPSPHI, A, TI)
RETURN
END
```

If any other statements are necessary to define the actual objective function and the constraints they may be added to this subroutine, e.g., function U may be defined in another subprogram which may then be called by subroutine FUNCT.

<u>DATA DECK</u> Parameters to be supplied as data are defined below

MET1 First method to be called by CONOPT

	if MET1=1 Fletcher method will be called
	if MET1=2 Jacobson-Oksman method will be called
	if MET1=3 Fletcher-Powell method will be called
	if MET1=0 No first method will be called
MET2	Second method to be called by CONOPT
	if MET2=1 Fletcher method will be called
	if MET2=2 Jacobson-Oksman method will be called
	if MET2=3 Fletcher-Powell method will be called
	if MET2=0 No second method will be called
MET3	Third method to be called by CONOPT
	if MET3=1 Fletcher method will be called
	if MET3=2 Jacobson-Oksman method will be called
	if MET3=3 Fletcher-Powell method will be called
	if MET3=0 No third method will be called
M	A parameter to select the starting point; if M=1 the same starting point is used by all the methods called; for any other value of M each method starts with the optimum left by the last method
MAX	Maximum number of permissible iterations
MODE	A parameter to choose the stopping criterion for the Jacobson-Oksman method
	if MODE=1 criterion will be $\Delta F \leq$ a small number
	if MODE=2 criterion will be $\|gradients\| \leq$ a small number
IPRINT	Intermediate output is printed out every IPRINT

iterations; it should be set = 0 if no intermediate output is desired.

IDATA	Input data is printed out if IDATA=1; it should be set = 0 if input data is not to be printed out
EST	Minimum estimated value of the objective function
EPS1	Small test quantity used by the Fletcher-Powell method
ETA(I),I=1,4	Test quantities used by the Jacobson-Oksman method
EPS(I),I=1,N	Test quantities used by the Fletcher method
XSTRT(I),I=1,NN	Starting values for variables x_1, x_2, ..., x_n and x_{n+1}, x_{n+2}. Two extra variables x_{n+1} and x_{n+2} are required by the Jacobson-Oksman method. Suggested starting values for x_{n+1} and x_{n+2} are the estimated order of the objective function and the minimum estimated value of the function, respectively.
AO	Initial value of the positive parameter α used in the formulation of the unconstrained objective function. It should be set = 1.0, if no information is available.

Recommended values for some of the parameters are

MAX = 100

EPS1 = 1.E-6

ETA(1) = 1.E-4

ETA(2) = 1.E-8

ETA(3) = ETA(4) = 1.E-16

EPS(I), I=1,N   Each = 1.E-6

EST   A lower bound on the minimum value of the objective function may be obtained from physical reasons if the true minimum is not known, e.g., for approximation problems 0.0 is convenient.

Setting up the data deck

Card No.	Format	Parameters
1	8I5	MET1, MET2, MET3, M, MAX, MODE, IPRINT, IDATA
2	5E16.8	EPS1, (ETA(I), I=1,4)
As many as required	5E16.8	EST, (EPS(I), I=1,N)
As many as required	5E16.8	(XSTRT(I), I=1,NN)
Last	E16.8	AO

COMMENTS

As explained above any number one, two, or all the three methods and in any order may be called depending upon the values of MET1, MET2, and MET3 chosen by the user. Similarly by choosing the appropriate values of IPRINT and IDATA the user may or may not print out the input data and the intermediate output. Results for some of the problems solved using this package along with the results obtained using the Fiacco-McCormick approach have been included in Appendix A and B. Appendix C shows the general structure of the package. Though some of the subroutines in this package have exactly the same names as the ones in GRADMIN and GRNLP2 they are not written exactly the same. For this reason the user must treat the packages separately and should not try to mix subroutines from one package to another package.

REFERENCES

[1] J.W. Bandler and C. Charalambous, "A new approach to nonlinear programming", Proc. 5th Hawaii Int. Conf. on Systems Science (Honolulu, Jan. 1972), pp. 127-129.

[2] J.W. Bandler and C. Charalambous, "Nonlinear programming using minimax techniques", J. Optimization Theory and Applications, accepted for publication.

[3] J.W. Bandler and C. Charalambous, "Practical least pth optimization of networks", IEEE Trans. Microwave Theory Tech., vol. MTT-20, pp. 834-840, December 1972.

[4] R. Fletcher and M.J.D. Powell, "A rapidly convergent descent method for minimization", Computer J., vol. 6, pp. 163-168, June. 1963.

[5] R. Fletcher, "A new approach to variable metric algorithms", Computer J., vol. 13, pp. 317-322, August 1970.

[6] D.H. Jacobson and W. Oksman, "An algorithm that minimizes homogeneous functions of n variables in n+2 iterations and rapidly minimizes general functions", Division of Engineering and Applied Physics, Harvard University, Cambridge, Mass., Technical Report No. 618, October 1970.

ACKNOWLEDGEMENTS

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APPENDIX A

To minimize (starting points $x_1=0$, $x_2=0$, $x_3=0$, $x_4=0$)

$$U = x_1^2 + x_2^2 + 2x_3^2 + x_4^2 - 5x_1 - 5x_2 - 21x_3 + 7x_4,$$

subject to

$$- x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 - x_3 + x_4 + 8 \geq 0$$

$$- x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 + 10 \geq 0$$

$$- 2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 + 5 \geq 0$$

A suitable listing of the input deck, and printouts of the input data and some final results for the Fletcher method are given in Figs 1-3.

```
GSGA.                                                                CHEN
ATTACH,TAPE,CONOPT,ID=GSGABNLER,MR=1.
RUN(S,,,,,,,,X)
LOAD(TAPE)
LGO.
'        6400 END OF RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION X(6),G(6),X1(6),X2(6),G2(6),PY(6),ALFA(6),P(6,6),
     1Y(6),PE(6),BIGV(6),XSTRT(6),DUM1(6),DUM2(6),EPS(4),H(22)
      N=4
      NN=N+2
      KR=1
      CALL CONOPT  (N,X,XSTRT,X2,X1,G,G2,ALFA,H,P,PY,PE,BIGV,EPS,NN,
     1Y,DUM1,DUM2,KR)
      STOP
      END
      SUBROUTINE FUNCT (N,X,F,G)
      DIMENSION X(1),G(1),PHI(3),GU(4),A(4),T1(4),GPHI(4,3)
       U=X(1)**2+X(2)**2+2.*X(3)**2+X(4)**2-5.*X(1)-5.*X(2)
     1 -21.*X(3)+7.*X(4)
       PHI(1)=-X(1)**2-X(2)**2-X(3)**2-X(4)**2-X(1)+X(2)-X(3)+X(4)+8.
       PHI(2)=-X(1)**2-2.*X(2)**2-X(3)**2-2.*X(4)**2+X(1)+X(4)+10.
       PHI(3)=-2.*X(1)**2-X(2)**2-X(3)**2-2.*X(1)+X(2)+X(4)+5.
       GU(1)=2.*X(1)-5.
       GU(2)=2.*X(2)-5.
       GU(3)=4.*X(3)-21.
       GU(4)=2.*X(4)+7.
       GPHI(1,1) =-2.*X(1)-1.
       GPHI(2,1)=-2.*X(2)+1.
       GPHI(3,1) =-2.*X(3)-1.
       GPHI(4,1)=-2.*X(4)+1.
       GPHI(1,2)=-2.*X(1)+1.
       GPHI(2,2)=-4.*X(2)
       GPHI(3,2)=-2.*X(3)
       GPHI(4,2)=-4.*X(4)+1.
       GPHI(1,3)=-4.*X(1)-2.
       GPHI(2,3)=-2.*X(2)+1.
       GPHI(3,3)=-2.*X(3)
       GPHI(4,3)=1.
      NCONS=3
      P=1.E5
      EPSPHI=1.E-5
      CALL EMINMAX(U,GU,PHI,GPHI,N,NCONS,F,G,P,EPSPHI,A,T1)
      RETURN
      END
'        6400 END OF RECORD
   1     0     0    1  100     2     1     1
 1.00000000E-09  1.00000000E-06  1.00000000E-08  1.00000000E-16  1.00000000E-16
     -100.       1.00000000E-05  1.00000000E-05  1.00000000E-05  1.00000000E-05
      0.              0.              0.              0.

     10.
'           END OF FILE
```

Fig. 1

CD TOT   0053

```
INPUT DATA
----------

FOLLOWING METHODS HAVE BEEN CALLED
FLETCHER METHOD
NUMBER OF INDEPENDENT VARIABLES..................................N=      4
MAXIMUM NUMBER OF ALLOWABLE ITERATIONS.........................MAX=    100
INTERMEDIATE OUTPUT TO BE PRINTED EVERY IPRINT ITERATIONS.....IPRINT=     1
STARTING POINT TO BE SAME FOR ALL THE METHODS IF M=1...............M=      1
STARTING VALUE FOR VECTOR X(I).............................XSTRT( 1)=   0.
                                                           XSTRT( 2)=   0.
                                                           XSTRT( 3)=   0.
                                                           XSTRT( 4)=   0.
TEST QUANTITIES TO BE USED IN JACOBSON-OKSMAN METHOD.........FTA( 1)=   1.00000000E-06
                                                             FTA( 2)=   1.00000000E-08
                                                             FTA( 3)=   1.00000000E-16
                                                             FTA( 4)=   1.00000000E-16
TEST QUANTITIES TO BE USED IN FLETCHER METHOD................FPS( 1)=   1.00000000E-05
                                                             FPS( 2)=   1.00000000E-05
                                                             FPS( 3)=   1.00000000E-05
                                                             FPS( 4)=   1.00000000E-05
TEST QUANTITY TO BE USED IN FLETCHER-POWELL METHOD..............FPS1=   1.00000000E-09
ESTIMATE OF LOWER BOUND ON FUNCTION TO BE MINIMIZED..............EST=  -1.00000000E+02
INITIAL VALUE OF THE PARAMETER ALPHA..............................A0=   1.00000000E+01
```

Fig. 2

```
                    FOLLOWING IS THE OPTIMUM SOLUTION
                    ---------------------------------

          ARTIFICIAL UNCONSTRAINED FUNCTION F = -4.39996472E+01
               ACTUAL OBJECTIVE FUNCTION U = -4.39998039E+01

                                   X( 1)= -2.08680081E-06
                                   X( 2)=  9.99992876E-01
                                   X( 3)=  1.99999213E+00
                                   X( 4)= -9.99987587E-01

           INEQUALITY CONSTRAINTS

                                 PHI( 1)=  8.57603558E-05
                                 PHI( 2)=  1.00011994E+00
                                 PHI( 3)=  5.51750139E-05
            NUMBER OF FUNCTION EVALUATIONS =  100
        FINAL VALUE OF THE PARAMETER ALPHA =  1.00000000E+01

                 EXECUTION TIME IN SECONDS=   1.96300
```

Fig. 3

APPENDIX B

To minimize (starting points $x_1=0.5$, $x_2=0.5$, $x_3=0.5$)

$$U = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$$

subject to $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$ and $x_1 + x_2 + 2x_3 \leq 3$

A suitable listing of the input deck, and printouts of the input data and some final results for the Fletcher-Powell method are given in Figs. 4-6.

```
GSGA.                                                                CHEN
ATTACH,TAPE,CONOPT,ID=GSGABNLER,MR=1.
RUN(S,,,,,,,,X)
LOAD(TAPE)
LGO.
'        6400 END OF RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION X(5),G(5),X1(5),X2(5),G2(5),PY(5),ALFA(5),P(5,5),
     1Y(5),PE(5),BIGV(5),XSTRT(5),DUM1(5),DUM2(5),EPS(3),H(15)
      N=3
      NN=N+2
      KR=1
      CALL CONOPT (N,X,XSTRT,X2,X1,G,G2,ALFA,H,P,PY,PE,BIGV,EPS,NN,Y,
     1DUM1,DUM2,KR)
      STOP
      END
      SUBROUTINE FUNCT (N,X,F,G)
      DIMENSION X(1),G(1),PHI(4),GU(3),GPHI(3,4),A(5),T1(5)
      U=9.-8.*X(1)-6.*X(2)-4.*X(3)+2.*X(1)**2+2.*X(2)**2+X(3)**2
     1 +2.*X(1)*X(2)+2.*X(1)*X(3)
      PHI(1)=X(1)
      PHI(2)=X(2)
      PHI(3)=X(3)
      PHI(4)=3.-X(1)-X(2)-2.*X(3)
      GU(1)=-8.+4.*X(1)+2.*X(2)+2.*X(3)
      GU(2)=-6.+4.*X(2)+2.*X(1)
      GU(3)=-4.+2.*X(3)+2.*X(1)
      GPHI(1,1)= 1.
      GPHI(2,1)= 0.
      GPHI(3,1)= 0.
      GPHI(1,2)= 0.
      GPHI(2,2)= 1.
      GPHI(3,2)=0.
      GPHI(1,3)= 0.
      GPHI(2,3)= 0.
      GPHI(3,3)= 1.
      GPHI(1,4)=-1.
      GPHI(2,4)=-1.
      GPHI(3,4)=-2.
      NCONS=4
      P=1.E5
      EPSPHI=1.E-5
      CALL EMINMAX (U,GU,PHI,GPHI,N,NCONS,F,G,P,EPSPHI,A,T1)
      RETURN
      END
'        6400 END OF RECORD
   3     0     0    1  100     2     1     1
 1.00000000E-06  1.00000000E-06  1.00000000E-08  1.00000000E-16  1.00000000E-16
    0.               1.00000000E-05  1.00000000E-05  1.00000000E-05
       0.5                  0.5                  0.5
       1.
'         END OF FILE
```

Fig. 4   CD TOT   0052

```
INPUT DATA
----------

FOLLOWING METHODS HAVE BEEN CALLED
FLETCHER-POWELL METHOD
NUMBER OF INDEPENDENT VARIABLES..................................N=     3
MAXIMUM NUMBER OF ALLOWABLE ITERATIONS.........................MAX=   100
INTERMEDIATE OUTPUT TO BE PRINTED EVERY IPRINT ITERATIONS.....IPRINT=     1
STARTING POINT TO BE SAME FOR ALL THE METHODS IF M=1...............M=     1
STARTING VALUE FOR VECTOR X(I).........................XSTRT( 1)=  5.00000000E-01
                                                       XSTRT( 2)=  5.00000000E-01
                                                       XSTRT( 3)=  5.00000000E-01
TEST QUANTITIES TO BE USED IN JACOBSON-OKSMAN METHOD.........FTA( 1)=  1.00000000E-06
                                                             FTA( 2)=  1.00000000E-08
                                                             FTA( 3)=  1.00000000E-16
                                                             FTA( 4)=  1.00000000E-16
TEST QUANTITIES TO BE USED IN FLETCHER METHOD................FPS( 1)=  1.00000000E-05
                                                             FPS( 2)=  1.00000000E-05
                                                             FPS( 3)=  1.00000000E-05
TEST QUANTITY TO BE USED IN FLETCHER-POWELL METHOD...............FPS1=  1.00000000E-06
ESTIMATE OF LOWER BOUND ON FUNCTION TO BE MINIMIZED...............FST=  0.
INITIAL VALUE OF THE PARAMETER ALPHA...............................A0=  1.00000000E+00
```

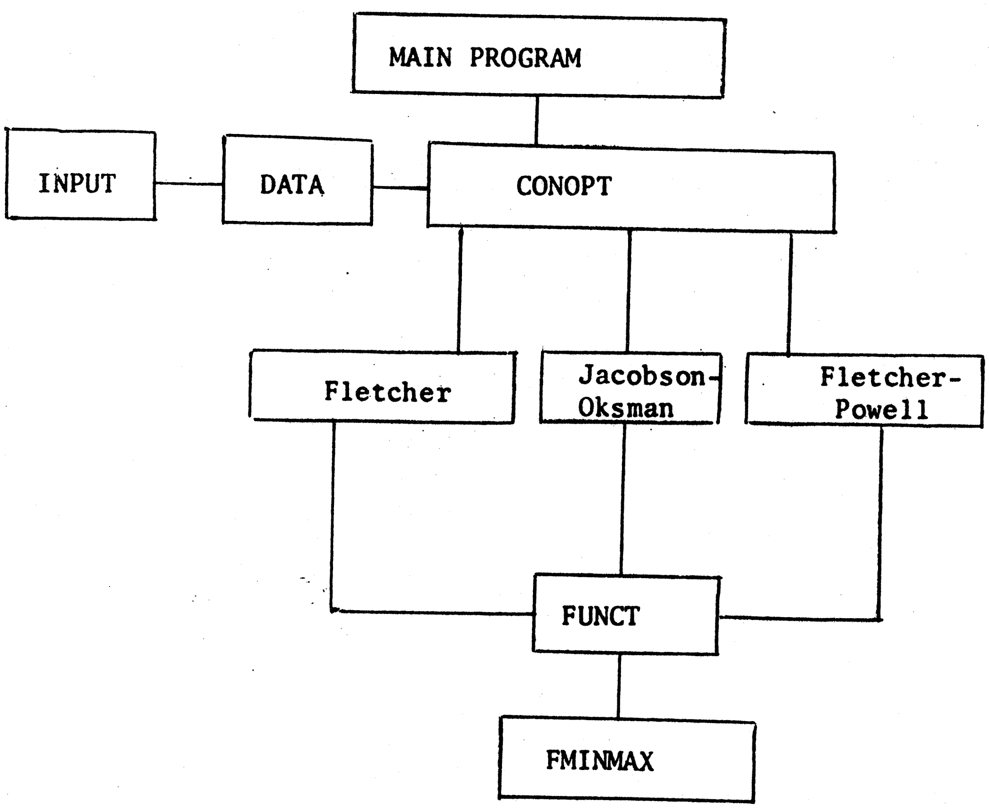
Fig. 5

```
FOLLOWING IS THE OPTIMUM SOLUTION
---------------------------------

ARTIFICIAL UNCONSTRAINED FUNCTION F =   1.11111700E-01
      ACTUAL OBJECTIVE FUNCTION U =   1.11111420E-01
                            X( 1)=   1.33333380E+00
                            X( 2)=   7.77777469E-01
                            X( 3)=   4.44443671E-01

INEQUALITY CONSTRAINTS
                          PHI( 1)=   1.33333380E+00
                          PHI( 2)=   7.77777469E-01
                          PHI( 3)=   4.44443671E-01
                          PHI( 4)=   1.39196652E-06
   NUMBER OF FUNCTION EVALUATIONS =    68
FINAL VALUE OF THE PARAMETER ALPHA =   1.00000000E+00

        EXECUTION TIME IN SECONDS=     .66800
```

Fig. 6

APPENDIX C

The general structure of the package is



Attached is the complete FORTRAN listing of the package.

```
      SUBROUTINE FMINMAX (U,GU,PHI,GPHI,N,NCONS,F,G,P,EPSPHI,A,T1)
      DIMENSION GU(1),PHI(1),GPHI(N,1),G(1),A(1),T1(1)
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
      COMMON /BLK2/ UR,NC,PPHI(100)
      AE=0.
      UR=U
      NC=NCONS
      IFLAGA=0
      IF (NCONS.EQ.0.OR.ALPHA.EQ.0.) GO TO 15
      DO 1 I=1,NCONS
      A(I)=U-ALPHA*PHI(I)
 1    CONTINUE
      NT=NCONS+1
      A(NT)=U
      AM=A(1)
      DO 2 I=2,NT
      AM=AMAX1(AM,A(I))
 2    CONTINUE
      IF (AM.LE.0.) P=-P
      SUM1=0.
      DO 7 I=1,NT
      IF (AM) 6,3,5
 3    AE=1.E-8
      GO TO 6
 5    IF (A(I).LE.0.) GO TO 7
 6    T1(I)=(A(I)-AE)/(AM-AE)
      T1P=T1(I)**P
      SUM1=SUM1+T1P
 7    CONTINUE
      F=(AM-AE)*SUM1**(1./P)
      DO 14 I=1,N
      SUM2=0.
      DO 13 J=1,NT
      IF (AM) 11,11,10
 10   IF (A(J).LE.0.) GO TO 13
 11   IF (J.EQ.NT) GO TO 12
      SUM2=SUM2+T1(J)**(P-1.)*(GU(I)-ALPHA*GPHI(I,J))
      GO TO 13
 12   SUM2=SUM2+T1(J)**(P-1.)*GU(I)
 13   CONTINUE
      G(I)=SUM1**(1./P-1.)*SUM2
 14   CONTINUE
      GO TO 17
 15   F=U
      DO 16 I=1,N
      G(I)=GU(I)
 16   CONTINUE
 17   IF (ICHEK.EQ.0) GO TO 19
      DO 18 I=1,NCONS
      PPHI(I)=PHI(I)
      PHIT=PHI(I)+EPSPHI
      IF (PHIT.LT.0.) IFLAGA=1
 18   CONTINUE
 19   CONTINUE
      RETURN
      END
```

CD TOT 0056

```
      SUBROUTINE CONOPT  (N,X,XSTRT,X2,X1,G,G2,ALFA,H,P,PY,PE,BIGV,EPS,N
     1N,Y,DUM1,DUM2,KR)
      COMMON /BLK/ KO
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
C
      EXTERNAL FUNCT
      LOGICAL CONV,UNITH
      DIMENSION DUM1(1), DUM2(1)
      DIMENSION X(1), XSTRT(1), X2(1), X1(1), G(1), G2(1), ALFA(1), H(1)
     1, P(NN,1), Y(1), PY(1), PE(1), BIGV(1), EPS(1), ETA(4)
      UNITH=.TRUE.
      ICHEK=0
      IF (KR.EQ.0) GO TO 1
      CALL DATA (N,NN,XSTRT,EST,EPS1,ETA,MET1,MET2,MET3,M,MAX,MODE,IPRIN
     1T,EPS,AO)
    1 DO 2 I=1,NN
      X(I)=XSTRT(I)
    2 CONTINUE
      IF (MET1.EQ.0) MET1=4
      IF (MET2.EQ.0) MET2=4
      IF (MET3.EQ.0) MET3=4
      INDEX=0
      GO TO (5,12,20,27), MET1
    3 GO TO (5,12,20,27), MET2
    4 GO TO (5,12,20,27), MET3
    5 IF (IPRINT.EQ.0) GO TO 6
      CALL WRITE1 (1)
    6 CONTINUE
      KKK=0
      CALL SECOND (T1)
      IF (KR.NE.0) GO TO 8
      DO 7 I=1,NN
      X(I)=DUM1(I)
    7 CONTINUE
    8 CONTINUE
      ICHEK=0
      ALPHA=AO
    9 CONTINUE
      KKK=KKK+1
      CALL VMM01 (N,X,F,G,H,UNITH,EST,EPS,MAXFN,IPRINT,IEXIT,PE)
      ICHEK=1
      CALL FUNCT (N,X,F,G)
      IF (IFLAGA.EQ.0) GO TO 10
      ALPHA=ALPHA*10.
      GO TO 9
   10 CONTINUE
      DO 11 I=1,NN
      DUM1(I)=X(I)
   11 CONTINUE
      CALL SECOND (T2)
      CALL FINAL (X,F,N)
      T=T2-T1
      WRITE (6,22) T
      GO TO 27
   12 IF (IPRINT.EQ.0) GO TO 13
      CALL WRITE1 (2)
   13 CONTINUE
      KKK=0
      CALL SECOND (T1)
```

```
      IF (KR.NE.0) GO TO 15
      DO 14 I=1,NN
      X(I)=X2(I)
14    CONTINUE
15    CONTINUE
      ICHEK=0
      ALPHA=A0
16    CONTINUE
      KKK=KKK+1
      CALL THETA (X,N,ETA,EST,MAX,MODE,X2,X1,G,G2,ALFA,H,P,Y,PY,PE,BIGV,
     1EPS1,NN,IPRINT,F2)
      ICHEK=1
      CALL FUNCT (N,X,F,G)
      IF (IFLAGA.EQ.0) GO TO 19
      ALPHA=ALPHA*10.
17    DO 18 I=1,NN
      X(I)=X2(I)
18    CONTINUE
      GO TO 16
19    CONTINUE
      CALL SECOND (T2)
      CALL FINAL (X2,F2,N)
      T=T2-T1
      WRITE (6,32) T
      GO TO 27
20    IF (IPRINT.EQ.0) GO TO 21
      CALL WRITE1 (3)
21    CONTINUE
      KKK=0
      CALL SECOND (T1)
      IF (KR.NE.0) GO TO 23
      DO 22 I=1,NN
      X(I)=DUM2(I)
22    CONTINUE
23    CONTINUE
      ICHEK=0
      ALPHA=A0
24    CONTINUE
      KKK=KKK+1
      CALL EMER (FUNCT,N,X,F,G,EST,EPS1,MAX,IER,H,IPRINT)
      ICHEK=1
      CALL FUNCT (N,X,F,G)
      IF (IFLAGA.EQ.0) GO TO 25
      ALPHA=ALPHA*10.
      GO TO 24
25    CONTINUE
      DO 26 I=1,NN
      DUM2(I)=X(I)
26    CONTINUE
      CALL SECOND (T2)
      CALL FINAL (X,F,N)
      T=T2-T1
      WRITE (6,32) T
27    INDEX=INDEX+1
      IF (M.EQ.1) GO TO 28
      GO TO 30
28    DO 29 I=1,NN
      X(I)=XSTRT(I)
```

```
29    CONTINUE
30    CONTINUE
      GO TO (3,4,31), INDEX
31    CONTINUE
      RETURN
C
C
C
32    FORMAT (1H0,//25X,*EXECUTION TIME IN SECONDS=*,F10.5)
      END
```

CD TOT 0127

```
      SUBROUTINE DATA (N,NN,XSTRT,EST,EPS1,ETA,MET1,MET2,MET3,M,MAX,MODE
     1,IPRINT,EPS,AO)
      DIMENSION XSTRT(1), EPS(1), ETA(4)
      READ (5,1) MET1,MET2,MET3,M,MAX,MODE,IPRINT,IDATA
      READ (5,2) EPS1,(ETA(I),I=1,4)
      READ (5,2) EST,(EPS(I),I=1,N)
      READ (5,2) (XSTRT(I),I=1,NN)
      READ (5,2) AO
      IF (IDATA.EQ.0) RETURN
      CALL INPUT (MET1,MET2,MET3,M,MAX,MODE,N,IPRINT,IDATA,EPS1,ETA,EST,
     1EPS,XSTRT,AO)
      RETURN
C
C
C
    1 FORMAT (8I5)
    2 FORMAT (5F16.8)
      END
```

CD TOT   0018

```
      SUBROUTINE FINAL (X,F,N)
      DIMENSION X(1)
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
      COMMON /BLK/ KO
      COMMON /BLK1/ NUMF
      COMMON /BLK2/ UR,NC,PPHI(100)
      WRITE (6,9)
      IF (KO.EQ.0) GO TO 1
      WRITE (6,10)
      GO TO 2
1     WRITE (6,11)
2     CONTINUE
      WRITE (6,12) F
      WRITE (6,3) UR
      WRITE (6,13) (I,X(I),I=1,N)
      WRITE (6,4)
      WRITE (6,5) (I,PPHI(I),I=1,NC)
      WRITE (6,6) NUMF
      WRITE (6,7) ALPHA
      RETURN
C
C
C
3     FORMAT (1H0,21X,*ACTUAL OBJECTIVE FUNCTION U =*,F16.8,/)
4     FORMAT (1H0,//,21X,*INEQUALITY CONSTRAINTS*,/)
5     FORMAT (43X,*PHI(*,I2,*)=*,E16.8)
6     FORMAT (1H0,/,19X,*NUMBER OF FUNCTION EVALUATIONS =*,I5)
7     FORMAT (1H0,/,15X,*FINAL VALUE OF THE PARAMETER ALPHA =*,F16.8)
9     FORMAT (1H1)
10    FORMAT (1H0,25X,*FOLLOWING IS THE OPTIMUM SOLUTION*,/,26X,*-------
     1---------------------------*)
11    FORMAT (1H0,29X,*RESULTS AT LAST ITERATION*/,30X,*----------------
     1---------*)
12    FORMAT (1H0,//,14X,*ARTIFICIAL UNCONSTRAINED FUNCTION F =*,E16.8,/
     1)
13    FORMAT (1H0,44X,*X(*,I2,*)=*,E16.8)
      END
```

CD TOT   0037

```
      SUBROUTINE VMM01 (N,X,F,G,H,UNITH,FEST,EPS,MAXFN,IPRINT,IEXIT,PE)
      DIMENSION PE(1)
      DIMENSION X(1), G(1), H(1), EPS(1)
      COMMON /BLK1/ NFENS
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
      LOGICAL CONV,UNITH
      COMMON /BLK/ KO
      KO=0
      CALL FUNCT (N,X,F,G)
      IF (F.LT.FEST) GO TO 26
      IF (KKK.NE.1) GO TO 1
      NFENS=1
      ITN=0
      CALL SECOND (T3)
1     CONTINUE
      STEP=1.
      IDX=N
      IDG=N+N
      IH=IDG+N
      IF (.NOT.UNITH) GO TO 3
      IJ=IH+1
      DO 2 I=1,N
      DO 2 J=I,N
      H(IJ)=0.
      IF (I.EQ.J) H(IJ)=1.0
2     IJ=IJ+1
3     CONV=.TRUE.
      GDX=0.
      DO 7 I=1,N
      Z=0.
      IJ=IH+I
      IF (I.EQ.1) GO TO 5
      II=I-1
      DO 4 J=1,II
      Z=Z-H(IJ)*G(J)
      IJ=IJ+N-J
4     CONTINUE
5     DO 6 J=I,N
      Z=Z-H(IJ)*G(J)
      IJ=IJ+1
6     CONTINUE
      IF (ABS(Z).GT.EPS(I)) CONV=.FALSE.
      H(IDX+I)=Z
      GDX=GDX+G(I)*Z
7     CONTINUE
      DO 8 I=1,N
      PE(I)=X(I)
8     CONTINUE
C
      IF (IPRINT.EQ.0) GO TO 10
      IF (MOD(ITN,IPRINT).NE.0) GO TO 10
      CALL SECOND (T4)
      TIME=T4-T3
      IF (KKK.NE.1) GO TO 9
      CALL WRITE2 (X,N,G,F,NFENS,ITN,TIME)
9     KKK=1
10    IEXIT=1
      IF (CONV) GO TO 27
      IEXIT=2
```

```
      IF (GDX.GE.0.) GO TO 27
      Z=1.
      IF (ITN.LT.N.AND.UNITH) Z=STEP
      W=2.*(FEST-F)/GDX
      IF (W.LT.Z) Z=W
      STEP=Z
11    GDX=GDX*Z
      DO 12 I=1,N
      H(IDX+I)=H(IDX+I)*Z
      X(I)=X(I)+H(IDX+I)
12    CONTINUE
      CALL FUNCT (N,X,FP,H)
      IF (FP.LT.FEST) GO TO 26
      NFNS=NFNS+1
      IEXIT=3
      IF (ITN.EQ.MAXFN) GO TO 27
      GPDX=0.
      DO 13 I=1,N
      H(IDG+I)=H(I)-G(I)
      GPDX=GPDX+H(I)*H(IDX+I)
13    CONTINUE
      DGDX=GPDX-GDX
      IF (F.GT.FP-.0001*GDX) GO TO 15
      IEXIT=4
      IF (GPDX.LT.0..AND.ITN.GT.N) GO TO 27
      Z=3.*(F-FP)+GPDX+GDX
      W=SQRT(1.-GDX/Z*GPDX/Z)*ABS(Z)
      Z=1.-(GPDX+W-Z)/(DGDX+2.*W)
      IF (Z.LT.0.1) Z=0.1
      DO 14 I=1,N
      X(I)=X(I)-H(IDX+I)
14    CONTINUE
      GO TO 17
15    F=FP
      DO 16 I=1,N
      G(I)=H(I)
16    CONTINUE
      IF (DGDX.GT.0.) GO TO 18
      GDX=GPDX
      Z=4.
17    STEP=Z*STEP
      GO TO 11
18    IF (GPDX.LT.0.5*GDX) STEP=2.*STEP
      DGHDG=0.
      DO 22 I=1,N
      Z=0.
      IJ=IH+I
      IF (I.EQ.1) GO TO 20
      II=I-1
      DO 19 J=1,II
      Z=Z+H(IJ)*H(IDG+J)
      IJ=IJ+N-J
19    CONTINUE
20    DO 21 J=I,N
      Z=Z+H(IJ)*H(IDG+J)
      IJ=IJ+1
21    CONTINUE
      DGHDG=DGHDG+Z*H(IDG+I)
```

```
      H(I)=Z
22    CONTINUE
      IF (DGHDG.LT.0.0) DGHDG=DGDX*0.01
      IF (DGDX.LT.DGHDG) GO TO 24
      W=1.0+DGHDG/DGDX
      DO 23 I=1,N
      H(IDX+I)=W*H(IDX+I)-H(I)
23    CONTINUE
      DGDX=DGDX+DGHDG
      DGHDG=DGDX
24    IJ=IH
      DO 25 I=1,N
      W=H(IDX+I)/DGDX
      Z=H(I)/DGHDG
      DO 25 J=I,N
      IJ=IJ+1
25    H(IJ)=H(IJ)+W*H(IDX+J)-Z*H(J)
      ITN=ITN+1
      GO TO 3
26    IEXIT=5
27    IF (IEXIT.EQ.1) KO=1
      IF (IEXIT.NE.4) GO TO 29
      DO 28 I=1,N
      X(I)=PE(I)
28    CONTINUE
29    CONTINUE
      IF (IPRINT.EQ.0) RETURN
      GO TO (30,31,32,31,33), IEXIT
30    WRITE (6,35) IEXIT
      GO TO 34
31    WRITE (6,36) IEXIT
      GO TO 34
32    WRITE (6,37) IEXIT
      GO TO 34
33    WRITE (6,38) IEXIT
34    CONTINUE
      RETURN
C
C
C
35    FORMAT (/,1H0,*IEXIT=*,I2,*CRITERION FOR OPTIMUM (CHANGE IN VECTOR
     1 X .LT.EPS) HAS BEEN SATISFIED*)
36    FORMAT (/,1H0,*IEXIT=*,I2,*EITHER OF THE FOLLOWING THINGS HAS HAPP
     1ENED*,/,9X,*1. EPS CHOSEN IS TOO SMALL*,/,9X,*2. GRADIENTS ARE NOT
     2 CORRECT*,/,9X,*3. MATRIX H GOES SINGULAR*)
37    FORMAT (/,1H0,*IEXIT=*,I2,*MAXIMUM NUMBER OF ALLOWABLE ITERATION H
     1AS BEEN EXCEEDED*)
38    FORMAT (/,1H0,*IEXIT=*,I2,*FUNCTION VALUE LESS THAN MINIMUM ESTIMA
     1TED HAS BEEN DETECTED*)
      END
```

```
      SUBROUTINE THETA (X,NDIM,ETA,EST,MAX,MODE,X2,X1,G1,G2,ALFA,H,P,Y,P
     1Y,PE,BIGV,EPS1,NN,IPRINT,F2)
      DIMENSION X(1), X1(1), ETA(1), X2(1), G1(1), G2(1), ALFA(1), H(1),
     1 P(NN,1), Y(1), PY(1), PE(1), BIGV(1)
      COMMON /BLK1/ NUMF
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
      COMMON /BLK/ KO
      KO=0
      IFLAG=0
      KK=0
      N2=NDIM+1
      N1=NDIM+2
      IF (KKK.NE.1) GO TO 1
      CALL SECOND (T3)
      M=0
      NUMF=0
    1 CONTINUE
      IER=0
      DO 2 I=1,N1
      X1(I)=X(I)
    2 CONTINUE
      CALL FUNCT (NDIM,X1,F1,G1)
      NUMF=NUMF+1
      CALL SECOND (T4)
      TIME=T4-T3
      IF (IPRINT.EQ.0) GO TO 4
      IF (KKK.NE.1) GO TO 3
      CALL WRITE2 (X1,NDIM,G1,F1,NUMF,M,TIME)
    3 KKK=1
    4 CONTINUE
      DO 5 I=1,NDIM
      X2(I)=X1(I)
      G2(I)=G1(I)
      H(I)=-G1(I)
    5 CONTINUE
      F2=F1
      X2(N2)=X1(N2)
      X2(N1)=X1(N1)
    6 CONTINUE
      KOUNT=0
      EPS=EPS1
      CALL MIN1D (FUNCT,X2,H,RO,NDIM,F2,G2,NUMF,IER,EPS,EST)
      IF (IER.NE.0) GO TO 32
      DO 7 I=1,N1
      BIGV(I)=X2(I)
      ALFA(I)=X2(I)
    7 CONTINUE
      RO=-RO
      GG=0.
      DO 8 I=1,NDIM
      GG=GG+G2(I)*G2(I)
    8 CONTINUE
      GG=SQRT(GG)
      IF (IPRINT.EQ.0) GO TO 9
      IF (KK.NE.IPRINT) GO TO 9
      CALL SECOND (T4)
      TIME=T4-T3
      KK=0
      CALL WRITE2 (X2,NDIM,G2,F2,NUMF,M,TIME)
```

```
9       CONTINUE
        DO 11 I=1,N1
        DO 10 J=1,N1
        P(I,J)=0.
10      CONTINUE
        P(I,I)=1.
11      CONTINUE
12      CONTINUE
        KOUNT=0
        KOUNT=KOUNT+1
13      DO 14 I=1,NDIM
        Y(I)=G2(I)
14      CONTINUE
        Y(N2)=F2
        Y(N1)=ETA(1)
        V=0.
        DO 15 I=1,NDIM
        V=V+X2(I)*G2(I)
15      CONTINUE
        YA=0.
        DO 16 I=1,N1
        YA=YA+Y(I)*ALFA(I)
16      CONTINUE
        VYA=V-YA
        BIGV(KOUNT)=V
        DO 17 I=1,N1
        PY(I)=0.
        PE(I)=P(I,KOUNT)
        DO 17 J=1,N1
17      PY(I)=PY(I)+P(J,I)*Y(J)
        EPY=PY(KOUNT)
        IF (ABS(EPY).LT.ETA(3)) GO TO 33
        PY(KOUNT)=PY(KOUNT)-1.
        DO 18 I=1,N1
        DO 18 J=1,N1
18      P(I,J)=P(I,J)-PE(I)*PY(J)/EPY
        DO 19 I=1,N1
        ALFA(I)=0.
        DO 19 J=1,N1
19      ALFA(I)=ALFA(I)+P(I,J)*BIGV(J)
        DEL=0.
        DO 20 I=1,NDIM
        DEL=DEL+G2(I)*(X2(I)-ALFA(I))
20      CONTINUE
        IF (ABS(DEL).GT.ETA(4)) GO TO 21
        IF (IFLAG.EQ.1) GO TO 31
        IFLAG=1
        GO TO 33
21      IFLAG=0
        DO 22 I=1,N1
        H(I)=X2(I)-ALFA(I)
        IF (DEL.GT.0) H(I)=-H(I)
22      CONTINUE
        DO 23 I=1,NDIM
        X1(I)=X2(I)
        G1(I)=G2(I)
23      CONTINUE
        F1=F2
```

```
      X1(N2)=X2(N2)
      X1(N1)=X2(N1)
      X2(N2)=ALFA(N2)
      X2(N1)=ALFA(N1)
      CALL MIN1D (FUNCT,X2,H,RO,NDIM,F2,G2,NUMF,IER,EPS,EST)
      IF (IER.NE.0) GO TO 32
      IF (DEL.GT.0) RO=-RO
      GG=0.
      DO 24 I=1,NDIM
      GG=GG+G2(I)*G2(I)
24    CONTINUE
      GG=SQRT(GG)
      KOUNT=KOUNT+1
      M=M+1
      KK=KK+1
      IF (KK.NE.IPRINT) GO TO 25
      KK=0
      CALL SECOND (T4)
      TIME=T4-T3
      CALL WRITE2 (X2,NDIM,G2,F2,NUMF,M,TIME)
25    CONTINUE
      IF (M.GT.MAX) GO TO 28
      IF (MODE.EQ.2) GO TO 26
      IF (((F1-F2).LE.ETA(2))) GO TO 29
      GO TO 27
26    IF ((GG.LT.ETA(1))) GO TO 30
27    CONTINUE
      IF (KOUNT.LE.N1) GO TO 13
      GO TO 12
28    WRITE (6,37)
      GO TO 32
29    KO=1
      IF (IPRINT.EQ.0) RETURN
      WRITE (6,38)
      GO TO 32
30    KO=1
      IF (IPRINT.EQ.0) RETURN
      WRITE (6,39)
      GO TO 32
31    WRITE (6,36)
      KO=1
32    RETURN
33    IF (IPRINT.EQ.0) GO TO 34
      PRINT 40
34    CONTINUE
      DO 35 I=1,NDIM
      X1(I)=X2(I)
      G1(I)=G2(I)
      H(I)=-G1(I)
35    CONTINUE
      F1=F2
      X1(N2)=X(N2)
      X1(N1)=X(N1)
      X2(N2)=X(N2)
      X2(N1)=X(N1)
      GO TO 6
C
C
```

```
C
36    FORMAT (1HO,*RESTART COULD NOT YIELD SIGNIFICANT IMPROVEMENT,OPTIM
     1UM HAS BEEN REACHED*)
37    FORMAT (1HO,*MAXIMUM NUMBER OF ALLOWABLE ITERATIONS HAS BEEN EXCEE
     1DED*)
38    FORMAT (1HO,*CRITERION FOR OPTIMUM (FUNCTION VALUE DOES NOT CHANGE
     1  SIGNIFICANTLY ,MODE=1) HAS BEEN SATISFIED*)
39    FORMAT (1HO,*CRITERION FOR OPTIMUM (GRADIENTS HAVE BECOME TOO SMAL
     1L MODE=2) HAS BEEN SATISFIED*)
40    FORMAT (///20X,*A RESTART HAS OCCURRED*///)
      END
```

CD TOT 0186

```
      SUBROUTINE INPUT (MET1,MET2,MET3,M,MAX,MODE,N,IPRINT,IDATA,EPS1,ET
     1A,EST,EPS,XSTRT,AO)
      DIMENSION XSTRT(1), ETA(1), EPS(1)
      WRITE (6,7)
      IF (MET1.EQ.0) MET1=4
      IF (MET2.EQ.0) MET2=4
      IF (MET3.EQ.0) MET3=4
      INDEX=0
      GO TO (3,4,5,6), MET1
1     GO TO (3,4,5,6), MET2
2     GO TO (3,4,5,6), MET3
3     WRITE (6,8)
      GO TO 6
4     WRITE (6,9)
      GO TO 6
5     WRITE (6,10)
6     CONTINUE
      INDEX=INDEX+1
      IF (INDEX.EQ.1) GO TO 1
      IF (INDEX.EQ.2) GO TO 2
      WRITE (6,11) N
      WRITE (6,12) MAX
      WRITE (6,13) IPRINT
      WRITE (6,14) M
      WRITE (6,15) XSTRT(1)
      WRITE (6,16) (I,XSTRT(I),I=2,N)
      WRITE (6,17) ETA(1),ETA(2),ETA(3),ETA(4)
      WRITE (6,18) EPS(1)
      WRITE (6,19) (I,EPS(I),I=2,N)
      WRITE (6,20) EPS1
      WRITE (6,21) EST
      WRITE (6,22) AO
      RETURN
C
C
7     FORMAT (1HO,*INPUT DATA*,/,1X,10(*-*),//,1X,*FOLLOWING METHODS HAV
     1E BEEN CALLED*,/)
8     FORMAT (1HO,*FLETCHER METHOD*)
9     FORMAT (1HO,*JACOBSON-OKSMAN METHOD*)
10    FORMAT (1HO,*FLETCHER-POWELL METHOD*)
11    FORMAT (1HO,/1X,*NUMBER OF INDEPENDENT VARIABLES*,36(*.*),*N=*,I5,
     1/)
12    FORMAT (1HO,*MAXIMUM NUMBER OF ALLOWABLE ITERATIONS*,27(*.*),*MAX=
     1*,I5,/)
13    FORMAT (1HO,*INTERMEDIATE OUTPUT TO BE PRINTED EVERY IPRINT ITERAT
     1IONS*,5(*.*),*IPRINT=*,I5,/)
14    FORMAT (1HO,*STARTING POINT TO BE SAME FOR ALL THE METHODS IF M=1*
     1,15(*.*),*M=*,I5,/)
15    FORMAT (1HO,*STARTING VALUE FOR VECTOR X(I)*,29(*.*),*XSTRT( 1)=*,
     1E16.8)
16    FORMAT (1HO,59X,*XSTRT(*,I2,*)=*,E16.8)
17    FORMAT (1HO,/,1HO,*TEST QUANTITIES TO BE USED IN JACOBSON-OKSMAN M
     1ETHOD*,9(*.*),*ETA( 1)=*,E16.8,/,62X,*ETA( 2)=*,E16.8,/,62X,*ETA(
     23)=*,E16.8,/62X,*ETA( 4)=*,E16.8)
18    FORMAT (1HO,/,1HO,*TEST QUANTITIES TO BE USED IN FLETCHER METHOD*,
     116(*.*),*EPS( 1)=*,E16.8)
19    FORMAT (1HO,61X,*EPS(*,I2,*)=*,E16.8)
20    FORMAT (1HO,/,1HO,*TEST QUANTITY TO BE USED IN FLETCHER-POWELL MET
     1HOD*,14(*.*),*EPS1=*,E16.8)
```

```
21     FORMAT (1HO,/,1HO,*ESTIMATE OF LOWER BOUND ON FUNCTION TO BE MINIM
      1IZED*,14(*.*),*FST=*,E16.8)
22     FORMAT (1HO,/,1HO,*INITIAL VALUE OF THE PARAMETER ALPHA*,30(*.*),*
      1AO=*,E16.8)
       END
```

CD TOT   0064

```
      SUBROUTINE MIN1D (FUNCT,X,H,AMBDA,N,F,G,NUMF,IER,EPS,EST)
      DIMENSION H(1), X(1), G(1)
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
      IER=0
      DY=0.
      HNRM=0.
      GNRM=0.
      DO 1 J=1,N
      HNRM=HNRM+ABS(H(J))
      GNRM=GNRM+ABS(G(J))
      DY=DY+H(J)*G(J)
1     CONTINUE
      IF (DY) 2,31,31
2     IF (HNRM/GNRM-EPS) 31,31,3
3     FY=F
      ALFA=2.*(EST-F)/DY
      IF (X(N+1).GT.0.) ALFA=X(N+1)*ALFA/2.
      AMBDA=1.
      IF (ALFA) 6,6,4
4     IF (ALFA-AMBDA) 5,6,6
5     AMBDA=ALFA
6     ALFA=0.
7     FX=FY
      DX=DY
      DO 8 I=1,N
      X(I)=X(I)+AMBDA*H(I)
8     CONTINUE
      CALL FUNCT (N,X,F,G)
      NUMF=NUMF+1
      IF (F.LT.FX) RETURN
      FY=F
      DY=0.
      DO 9 I=1,N
      DY=DY+G(I)*H(I)
9     CONTINUE
      IF (DY) 10,30,13
10    IF (FY-FX) 11,13,13
11    AMBDA=AMBDA+ALFA
      ALFA=AMBDA
      IF (HNRM*AMBDA-1.E10) 7,7,12
12    IER=2
      GO TO 31
13    T=0.
14    IF (AMBDA) 15,30,15
15    Z=3.*(FX-FY)/AMBDA+DX+DY
      ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))
      DALFA=Z/ALFA
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
      IF (DALFA) 31,16,16
16    W=ALFA*SQRT(DALFA)
      ALFA=DY-DX+W+W
      IF (ALFA) 17,18,17
17    ALFA=(DY-Z+W)/ALFA
      GO TO 19
18    ALFA=(Z+DY-W)/(Z+DX+Z+DY)
19    ALFA=ALFA*AMBDA
      DO 20 I=1,N
      X(I)=X(I)+(T-ALFA)*H(I)
20    CONTINUE
```

```
      CALL FUNCT (N,X,F,G)
      NUMF=NUMF+1
      IF (F.LT.FX) GO TO 30
      IF (F-FX) 21,21,22
21    IF (F-FY) 30,30,22
22    DALFA=0.
      DO 23 I=1,N
      DALFA=DALFA+G(I)*H(I)
23    CONTINUE
      IF (DALFA) 24,27,27
24    IF (F-FX) 26,25,27
25    IF (DX-DALFA) 26,30,26
26    FX=F
      DX=DALFA
      T=ALFA
      AMBDA=ALFA
      GO TO 14
27    IF (FY-F) 29,28,29
28    IF (DY-DALFA) 29,30,29
29    FY=F
      DY=DALFA
      AMBDA=AMBDA-ALFA
      GO TO 13
30    AMBDA=AMBDA-ALFA
      RETURN
31    CONTINUE
      IF (DY.GE.0.) IER=-2
      IF (HNRM/GNRM.LE.EPS) IER=-3
      IF (DALFA.LT.0.) IER=-1
      II=IABS(IER)
      GO TO (32,33,34), II
32    WRITE (6,36) IER
      GO TO 35
33    WRITE (6,37) IER
      GO TO 35
34    WRITE (6,38) IER
35    RETURN
C
C
C
36    FORMAT (1H0,*IER=*,I2,*THERE IS AN ERROR IN GRADIENTS CALCULATION*
     1)
37    FORMAT (1H0,*IER=*,I2,*ERROR HAS OCCURED, SEARCH DIRECTION IS NOT
     1A DESCENT DIRECTION*)
38    FORMAT (1H0,*IER=*,I2,*ERROR HAS OCCURED, SEARCH DIRECTION VECTOR
     1 IS TOO SMALL IN COMPARISON TO GRADIENT VECTOR*)
      END
```

CD TOT 0106

```
      SUBROUTINE WRITE1 (N)
      WRITE (6,5)
      GO TO (1,2,3), N
1     WRITE (6,6)
      GO TO 4
      WRITE (6,7)
      GO TO 4
3     WRITE (6,8)
4     CONTINUE
      WRITE (6,9)
      RETURN
C
C
C
5     FORMAT (1H1)
6     FORMAT (1HO,*OPTIMIZATION BY FLETCHER METHOD*,/,1HO,*-------------
     1------------------*)
7     FORMAT (1HO,*OPTIMIZATION BY JACOBSON-OKSMAN METHOD*,/,1HO,*------
     1-------------------------------*)
8     FORMAT (1HO,*OPTIMIZATION BY FLETCHER-POWELL METHOD*,/,1HO,*------
     1-------------------------------*)
9     FORMAT (1HO,*ITERATION*,2X,*FUNCTION*,6X,*TIME ELAPSED*,8X,*OBJECT
     1IVE*,9X,*    ALPHA      *,5X,*VARIABLE VECTOR X(I)*,5X,*GRADIENT VE
     2CTOR G(I)*,/1HO,*NUMBER*,5X,*EVALUATIONS*,3X,*(SECONDS)*,11X,*FUNC
     3TION*,/)
      END
```

```
      SUBROUTINE WRITE2 (X,N,G,F,NUMF,ITER,TIME)
      DIMENSION X(1), G(1)
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
      WRITE (6,1) ITER,NUMF,TIME,F,ALPHA,X(1),G(1),     ((X(I),G(I)),I=2
     1,N)
      RETURN
C
C
C
    1 FORMAT (1H0,I5,7X,I5,5X,E16.8,3X,E16.8,4X,E16.8,6X,F16.8,9X,F16.8,
     1/,84X,          F16.8,9X,F16.8,/,98(84X,E16.8,9X,F16.8,/))
      END
```

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```
      SUBROUTINE FMFP (FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H,IPRINT)
      COMMON /BLK1/ NUMF
      COMMON /BLK/ KO
      COMMON /SPL/ ALPHA,IFLAGA,ICHEK,KKK
C        SUBROUTINE FMFP
C
C        PURPOSE
C           TO FIND A LOCAL MINIMUM OF A FUNCTION OF SEVERAL VARIABLES
C           BY THE METHOD OF FLETCHER AND POWELL
C
C        USAGE
C           CALL FMFP(FUNCT,N,X,F,G,EST,EPS,LIMIT,IER,H)
C
C        DESCRIPTION OF PARAMETERS
C           FUNCT  - USER-WRITTEN SUBROUTINE CONCERNING THE FUNCTION TO
C                    BE MINIMIZED. IT MUST BE OF THE FORM
C                    SUBROUTINE FUNCT(N,ARG,VAL,GRAD)
C                    AND MUST SERVE THE FOLLOWING PURPOSE
C                    FOR EACH N-DIMENSIONAL ARGUMENT VECTOR  ARG,
C                    FUNCTION VALUE AND GRADIENT VECTOR MUST BE COMPUTED
C                    AND, ON RETURN, STORED IN VAL AND GRAD RESPECTIVELY
C           N      - NUMBER OF VARIABLES
C           X      - VECTOR OF DIMENSION N CONTAINING THE INITIAL
C                    ARGUMENT WHERE THE ITERATION STARTS. ON RETURN,
C                    X HOLDS THE ARGUMENT CORRESPONDING TO THE
C                    COMPUTED MINIMUM FUNCTION VALUE
C           F      - SINGLE VARIABLE CONTAINING THE MINIMUM FUNCTION
C                    VALUE ON RETURN, I.E. F=F(X).
C           G      - VECTOR OF DIMENSION N CONTAINING THE GRADIENT
C                    VECTOR CORRESPONDING TO THE MINIMUM ON RETURN,
C                    I.E. G=G(X).
C           EST    - IS AN ESTIMATE OF THE MINIMUM FUNCTION VALUE.
C           EPS    - TESTVALUE REPRESENTING THE EXPECTED ABSOLUTE ERROR.
C                    A REASONABLE CHOICE IS 10**(-6), I.E.
C                    SOMEWHAT GREATER THAN 10**(-D), WHERE D IS THE
C                    NUMBER OF SIGNIFICANT DIGITS IN FLOATING POINT
C                    REPRESENTATION.
C           LIMIT  - MAXIMUM NUMBER OF ITERATIONS.
C           IER    - ERROR PARAMETER
C                    IER = 0 MEANS CONVERGENCE WAS OBTAINED
C                    IER = 1 MEANS NO CONVERGENCE IN LIMIT ITERATIONS
C                    IER =-1 MEANS ERRORS IN GRADIENT CALCULATION
C                    IER = 2 MEANS LINEAR SEARCH TECHNIQUE INDICATES
C                    IT IS LIKELY THAT THERE EXISTS NO MINIMUM.
C           H      - WORKING STORAGE OF DIMENSION N*(N+7)/2.
C
C        REMARKS
C            I) THE SUBROUTINE NAME REPLACING THE DUMMY ARGUMENT  FUNCT
C               MUST BE DECLARED AS EXTERNAL IN THE CALLING PROGRAM.
C           II) IER IS SET TO 2 IF , STEPPING IN ONE OF THE COMPUTED
C               DIRECTIONS, THE FUNCTION WILL NEVER INCREASE WITHIN
C               A TOLERABLE RANGE OF ARGUMENT.
C               IER = 2 MAY OCCUR ALSO IF THE INTERVAL WHERE F
C               INCREASES IS SMALL AND THE INITIAL ARGUMENT WAS
C               RELATIVELY FAR AWAY FROM THE MINIMUM SUCH THAT THE
C               MINIMUM WAS OVERLEAPED. THIS IS DUE TO THE SEARCH
C               TECHNIQUE WHICH DOUBLES THE STEPSIZE UNTIL A POINT
C               IS FOUND WHERE THE FUNCTION INCREASES.
C
```

```
C         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C            FUNCT
C
C         METHOD
C            THE METHOD IS DESCRIBED IN THE FOLLOWING ARTICLE
C            R. FLETCHER AND M.J.D. POWELL, A RAPID DESCENT METHOD FOR
C            MINIMIZATION,
C            COMPUTER JOURNAL VOL.6, ISS. 2, 1963, PP.163-168.
C
C     ..................................................................
C
C
C         DIMENSIONED DUMMY VARIABLES
      DIMENSION H(1), X(1), G(1)
      KO=0
      KAML=0
C
C         COMPUTE FUNCTION VALUE AND GRADIENT VECTOR FOR INITIAL ARGUMENT
      KO=0
      CALL SECOND (T3)
      CALL FUNCT (N,X,F,G)
      IF (KKK.NE.1) GO TO 1
      KOUNT=0
      NUMF=1
1     CONTINUE
      CALL SECOND (T4)
      TIME=T4-T3
      IF (IPRINT.EQ.0) GO TO 2
      CALL WRITE2 (X,N,G,F,NUMF,KOUNT,TIME)
2     CONTINUE
C
C         RESET ITERATION COUNTER AND GENERATE IDENTITY MATRIX
      IER=0
      KK=0
      N2=N+N
      N3=N2+N
      N31=N3+1
3     K=N31
      DO 6 J=1,N
      H(K)=1.
      NJ=N-J
      IF (NJ) 7,7,4
4     DO 5 L=1,NJ
      KL=K+L
      H(KL)=0.
5     CONTINUE
      K=KL+1
6     CONTINUE
C
C         START ITERATION LOOP
7     IF (KOUNT.EQ.0) GO TO 8
      IF (KK.NE.IPRINT) GO TO 8
      KK=0
      CALL SECOND (T4)
      TIME=T4-T3
      CALL WRITE2 (X,N,G,F,NUMF,KOUNT,TIME)
8     CONTINUE
      IF (KAML.EQ.0) GO TO 11
```

```
      FTE=ABS(F-OLDF)
      EOF=EPS*ABS(F)
      IF (FTE.LT.EOF) GO TO 9
      KQ=0
      GO TO 10
9     KQ=KQ+1
10    CONTINUE
11    KAML=KAML+1
      IF (KQ.EQ.4) GO TO 71
      KOUNT=KOUNT+1
      KK=KK+1
C
C        SAVE FUNCTION VALUE, ARGUMENT VECTOR AND GRADIENT VECTOR
      OLDF=F
      DO 15 J=1,N
      K=N+J
      H(K)=G(J)
      K=K+N
      H(K)=X(J)
C
C        DETERMINE DIRECTION VECTOR H
      K=J+N3
      T=0.
      DO 14 L=1,N
      T=T-G(L)*H(K)
      IF (L-J) 12,13,13
12    K=K+N-L
      GO TO 14
13    K=K+1
14    CONTINUE
      H(J)=T
15    CONTINUE
C
C        CHECK WHETHER FUNCTION WILL DECREASE STEPPING ALONG H.
      DY=0.
      HNRM=0.
      GNRM=0.
C
C        CALCULATE DIRECTIONAL DERIVATIVE AND TESTVALUES FOR DIRECTION
C        VECTOR H AND GRADIENT VECTOR G.
      DO 16 J=1,N
      HNRM=HNRM+ABS(H(J))
      GNRM=GNRM+ABS(G(J))
      DY=DY+H(J)*G(J)
16    CONTINUE
C
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTIONAL
C        DERIVATIVE APPEARS TO BE POSITIVE OR ZERO.
      IF (DY) 17,61,61
C
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTION
C        VECTOR H IS SMALL COMPARED TO GRADIENT VECTOR G.
17    IF (HNRM/GNRM-EPS) 61,61,18
C
C        SEARCH MINIMUM ALONG DIRECTION H
C
C        SEARCH ALONG H FOR POSITIVE DIRECTIONAL DERIVATIVE
18    FY=F
```

```
      ALFA=2.*(FST-F)/DY
      AMBDA=1.
C
C        USE ESTIMATE FOR STEPSIZE ONLY IF IT IS POSITIVE AND LESS THAN
C        1. OTHERWISE TAKE 1. AS STEPSIZE
      IF (ALFA) 21,21,19
19    IF (ALFA-AMBDA) 20,21,21
20    AMBDA=ALFA
21    ALFA=0.
C
C        SAVE FUNCTION AND DERIVATIVE VALUES FOR OLD ARGUMENT
22    FX=FY
      DX=DY
C
C        STEP ARGUMENT ALONG H
      DO 23 I=1,N
      X(I)=X(I)+AMBDA*H(I)
23    CONTINUE
C
C        COMPUTE FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT
      CALL FUNCT (N,X,F,G)
      NUMF=NUMF+1
      FY=F
C
C        COMPUTE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT.  TERMINATE
C        SEARCH, IF DY IS POSITIVE. IF DY IS ZERO THE MINIMUM IS FOUND
      DY=0.
      DO 24 I=1,N
      DY=DY+G(I)*H(I)
24    CONTINUE
      IF (DY) 25,45,28
C
C        TERMINATE SEARCH ALSO IF THE FUNCTION VALUE INDICATES THAT
C        A MINIMUM HAS BEEN PASSED
25    IF (FY-FX) 26,28,28
C
C        REPEAT SEARCH AND DOUBLE STEPSIZE FOR FURTHER SEARCHES
26    AMBDA=AMBDA+ALFA
      ALFA=AMBDA
C        END OF SEARCH LOOP
C
C        TERMINATE IF THE CHANGE IN ARGUMENT GETS VERY LARGE
      IF (HNRM*AMBDA-1.E10) 22,22,27
C
C        LINEAR SEARCH TECHNIQUE INDICATES THAT NO MINIMUM EXISTS
27    IER=2
      GO TO 66
C
C        INTERPOLATE CUBICALLY IN THE INTERVAL DEFINED BY THE SEARCH
C        ABOVE AND COMPUTE THE ARGUMENT X FOR WHICH THE INTERPOLATION
C        POLYNOMIAL IS MINIMIZED
28    T=0.
29    IF (AMBDA) 30,45,30
30    Z=3.*(FX-FY)/AMBDA+DX+DY
      ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))
      DALFA=Z/ALFA
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
      IF (DALFA) 61,31,31
```

```
31    W=ALFA*SQRT(DALFA)
      ALFA=DY-DX+W+W
      IF (ALFA) 32,33,32
32    ALFA=(DY-Z+W)/ALFA
      GO TO 34
33    ALFA=(Z+DY-W)/(Z+DX+Z+DY)
34    ALFA=ALFA*AMBDA
      DO 35 I=1,N
      X(I)=X(I)+(T-ALFA)*H(I)
35    CONTINUE
C
C        TERMINATE, IF THE VALUE OF THE ACTUAL FUNCTION AT X IS LESS
C        THAN THE FUNCTION VALUES AT THE INTERVAL ENDS. OTHERWISE REDUCE
C        THE INTERVAL BY CHOOSING ONE END-POINT EQUAL TO X AND REPEAT
C        THE INTERPOLATION.  WHICH END-POINT IS CHOOSEN DEPENDS ON THE
C        VALUE OF THE FUNCTION AND ITS GRADIENT AT X
C
      NUMF=NUMF+1
      CALL FUNCT (N,X,F,G)
      IF (F-FX) 36,36,37
36    IF (F-FY) 45,45,37
37    DALFA=0.
      DO 38 I=1,N
      DALFA=DALFA+G(I)*H(I)
38    CONTINUE
      IF (DALFA) 39,42,42
39    IF (F-FX) 41,40,42
40    IF (DX-DALFA) 41,45,41
41    FX=F
      DX=DALFA
      T=ALFA
      AMBDA=ALFA
      GO TO 29
42    IF (FY-F) 44,43,44
43    IF (DY-DALFA) 44,45,44
44    FY=F
      DY=DALFA
      AMBDA=AMBDA-ALFA
      GO TO 28
C
C        TERMINATE, IF FUNCTION HAS NOT DECREASED DURING LAST ITERATION
45    IF (OLDF-F+EPS) 61,46,46
C
C        COMPUTE DIFFERENCE VECTORS OF ARGUMENT AND GRADIENT FROM
C        TWO CONSECUTIVE ITERATIONS
46    DO 47 J=1,N
      K=N+J
      H(K)=G(J)-H(K)
      K=N+K
      H(K)=X(J)-H(K)
47    CONTINUE
C
C        TEST LENGTH OF ARGUMENT DIFFERENCE VECTOR AND DIRECTION VECTOR
C        IF AT LEAST N ITERATIONS HAVE BEEN EXECUTED. TERMINATE, IF
C        BOTH ARE LESS THAN  EPS
      IFR=0
      IF (KOUNT-N) 51,48,48
48    T=0.
```

```
      Z=0.
      DO 49 J=1,N
      K=N+J
      W=H(K)
      K=K+N
      T=T+ABS(H(K))
      Z=Z+W*H(K)
49    CONTINUE
      IF (HNRM-EPS) 50,50,51
50    IF (T-EPS) 66,66,51
C
C        TERMINATE, IF NUMBER OF ITERATIONS WOULD EXCEED  LIMIT
51    IF (KOUNT-LIMIT) 52,59,59
C
C        PREPARE UPDATING OF MATRIX H
52    ALFA=0.
      DO 56 J=1,N
      K=J+N3
      W=0.
      DO 55 L=1,N
      KL=N+L
      W=W+H(KL)*H(K)
      IF (L-J) 53,54,54
53    K=K+N-L
      GO TO 55
54    K=K+1
55    CONTINUE
      K=N+J
      ALFA=ALFA+W*H(K)
      H(J)=W
56    CONTINUE
C
C        REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF RESULTS
C        ARE NOT SATISFACTORY
      IF (Z*ALFA) 57,3,57
C
C        UPDATE MATRIX H
57    K=N31
      DO 58 L=1,N
      KL=N2+L
      DO 58 J=L,N
      NJ=N2+J
      H(K)=H(K)+H(KL)*H(NJ)/Z-H(L)*H(J)/ALFA
58    K=K+1
      GO TO 7
C        END OF ITERATION LOOP
C
C        NO CONVERGENCE AFTER  LIMIT  ITERATIONS
59    IER=1
      IF (KK.NE.IPRINT) GO TO 60
      CALL WRITE2 (X,N,G,F,NUMF,KOUNT)
60    CONTINUE
      GO TO 66
C
C        RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS
61    DO 62 J=1,N
      K=N2+J
      X(J)=H(K)
```

```
62      CONTINUE
        CALL FUNCT (N,X,F,G)
        NUMF=NUMF+1
C
C          REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DERIVATIVE
C          FAILS TO BE SUFFICIENTLY SMALL
        IF (GNRM-EPS) 65,65,63
C
C          TEST FOR REPEATED FAILURE OF ITERATION
63      IF (IER) 66,64,64
64      IER=-1
        GO TO 3
65      IER=0
66      II=IER+2
        IF (II.EQ.2) KO=1
        IF (IPRINT.EQ.0) RETURN
        GO TO (67,68,69,70), II
67      WRITE (6,74) IER
        GO TO 72
68      WRITE (6,75) IER
        GO TO 72
69      WRITE (6,76) IER
        GO TO 72
70      WRITE (6,77) IER
71      WRITE (6,73)
        KO=1
72      RETURN
C
C
C
73      FORMAT (1H0,*THERE IS NO SIGNIFICANT DECREASE IN THE FUNCTION VALU
     1E OPTIMUM IS ASSUMED TO HAVE BEEN REACHED*)
74      FORMAT (1H0,*IER=*,I2,* ERROR IN GRADIENTS CALCULATIONS*)
75      FORMAT (1H0,*IER=*,I2,* CRITERION FOR OPTIMUM HAS BEEN SATISFIED*)
76      FORMAT (1H0,*IER=*,I2,* MAXIMUM NUMBER OF ALLOWABLE ITERATIONS HAS
     1 BEEN EXCEEDED*)
77      FORMAT (1H0,*IER=*,I2,* CHANGE IN ARGUMENTS GETS TOO LARGE, LINEAR
     1 SEARCH INDICATES THAT NO MINIMUM EXISTS*)
        END
```