NAG-014

69-TR-16

DEPARTMENT OF ELECTRICAL ENGINEERING

PUBLICATIONS

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NETWORKS AS OPTIMIZATION PROBLEMS

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June 17, 1969

Document No. NAG-014

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Abstract -- Some results of a study of the optimization of cascaded noncommensurate transmission lines acting as transformers between resistive terminations are presented and discussed. The objective function chosen for equal ripple performance is characterized by discontinuous partial derivatives.

Interest is growing both in computer-aided design and optimization of electrical networks generally, and in the synthesis of noncommensurate transmission-line networks in particular. Multivariable networks of the latter kind are, to date, still optimized in the real frequency domain [1 - 3]. To obtain equal-ripple responses directly involves the minimization of functions of several variables, the functions being characterized by discontinuous partial derivatives. Our experience indicates that virtually all available automatic optimization methods can be expected to fail, in general, to reach even a local optimum for such situations.

This communication presents and discusses the relevant results of a study of the optimization of cascaded noncommensurate transmission lines acting as transformers between resistive terminations. The discussion is illustrated by contour plots of those parts of the response

This work was carried out with financial assistance from the Faculty of Graduate Studies of the University of Manitoba and from the National Research Council of Canada.

hypersurface generated by varying two parameters of a typical network, the remaining parameters being fixed. The behaviour of some direct search strategies [1 - 8] on these contours, particularly pattern search [2, 4 - 6], is discussed.

An earlier publication [2] found that the optima for the problem of minimizing the maximum input reflection coefficient over specified bandwidths for networks of the type shown in Fig. 1(a) turn out to be the known quarterwave Chebyshev designs [9]. A formal proof of this does not yet appear to have been reported, however.

The present communication will restrict itself to the two-section transformer (4 variables) shown in Fig. 1(b). As shown, it has a load to source impedance ratio R of 10:1 and requires optimum performance over a 100% bandwidth. Formally, the problem is to minimize

$$U = \max_{i} |\rho(\underline{\phi}, f_i)| \qquad i = 1, 2, \ldots n \tag{1}$$

where

$$\underline{\phi} = \begin{bmatrix} \ell_1 \\ Z_1 \\ \ell_2 \\ Z_2 \end{bmatrix} \tag{2}$$

subject to

$$\left.\begin{array}{l} \ell_k \geq 0 \quad (3) \\ Z_k > 0 \quad (4) \end{array}\right\} \quad k = 1, 2$$

where $f_1 = .5$ and $f_n = 1.5$ (normalized to the center frequency f_o). The solution is [9]

$$U_{min} = 0.4286$$

$$\ell_1 = \ell_2 = \ell_q$$

$$Z_1 = 2.2361 \qquad Z_2 = 4.4721$$

where

$$\ell_q = c/4\ f_o$$

Any two parameters can be fixed and the others varied to produce contours of U. Perhaps the most interesting for our purpose are the three cases shown in Fig. 2. In Fig. 2(a) the lengths are fixed at their optimum values and the impedances are varied; in Fig. 2(b) the impedances are kept fixed at their optimum values and the lengths are varied; and in Fig. 2(c) the parameters of the second section are held at their optimum values while those of the first section are varied. The sharp points in the contours indicate the presence of the discontinuous partial derivatives of U. The discontinuities occur, of course, when U jumps from one test frequency to another.

The pattern search strategy[1] was started in each corner of the three diagrams in Fig. 2. Two runs per corner were made, one with

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[1] More recent modifications [5, 6] of the pattern search strategy of Hooke and Jeeves [4] were incorporated into the computer program.

parameter increments (for the exploratory moves) of one division on the Z scales and/or 2/3 of a division on the ℓ/ℓ_q scales, the other with double these increments. The increments were then reduced as necessary to keep the pattern search going in an effort to reach the optimum, either until the increments were less than 10^{-5} or the number of evaluations of U exceeded 1000.

The only situation which presented difficulties was that of Fig. 2(a). For point (1, 3) the optimization process terminated outside the bounded area. Convergence from the other corners onto points very close to the known optimum resulted when parameter increments of one division ($\Delta Z = .25$) were used. The corresponding paths are labelled B.[2] For $\Delta Z = .5$ (paths labelled A) the search failed to reach the optimum.

Similar difficulties also manifested themselves in the more general 4-variable optimization described by (1) to (4) and in other multivariable examples investigated [2].

A brief explanation of the failure is in order. Fig. 3 reproduces the contours of Fig. 2(a). It shows how pattern search[1] using $\Delta Z = .25$ and starting at a fails at f after 16 function evaluations. Thereafter, the parameter increments would be reduced and the process restarted at f. But it is clear that, unless a fortuitous choice of increments is struck,

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[2] The dashed lines labelled A and B in Fig. 2(a) are not, of course, the actual paths taken.

[1] More recent modifications [5, 6] of the pattern search strategy of Hooke and Jeeves [4] were incorporated into the computer program.

pattern search will repeatedly fail no matter how small the increments are made, and eventually terminate very close to f.

In those cases when the optimum was reached between 72 and 280 function evaluations were required taking from ¼ sec to 1 sec on the IBM 360/65. No attempt to minimize the number of function evaluations was made.

Although all the contours in Fig. 2 have discontinuous derivatives, only in Fig. 2(a) do the contours lie entirely in one quadrant. Once the search stops at a point of discontinuous derivatives exploration parallel to the coordinate axes will not yield any improvement. Wilde [10] and Wilde and Beightler [11] have discussed similar situations when methods which are usually good at following narrow valleys can fail. Powell's minimization method [7, 8] which does not restrict itself to exploring along the coordinate axes was also programmed, but failed in the same way as pattern search. O'Hagan's method [1] which relies on orthogonal exploratory moves in directions oblique to the coordinate axes chosen at random might eventually find the direction of the valley because of its reliance on randomness.

Little research appears to have been done to overcome this problem as contours involving discontinuous derivatives seem to be unpopular among exponents of optimization. It is usual to reformulate the problem by specifying an objective function which provides a nearly equal ripple response (see, for example, Temes and Calahan [12]).

It is felt, however, that optimization in the frequency domain is of sufficient importance to warrant a deeper investigation of methods for handling equal ripple response criteria directly. Even if the synthesis problem for multivariable transmission-line networks is solved, the relatively narrow range of practical characteristic impedance values (say 15 to 150 Ω), might still make optimization in the frequency domain more attractive. The example presented here might be found useful for testing new optimization methods [13].[3]

ACKNOWLEDGMENT

The authors are grateful to Dr. R. A. Rohrer of the Fairchild Semiconductor Research and Development Laboratory, Palo Alto, California, and Dr. A. Wexler of the Department of Electrical Engineering, University of Manitoba, for their interest and encouragement. The University of Manitoba's Institute for Computer Studies is acknowledged for its cooperation.

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[3] As described in a more recent paper [13], the authors have followed up this work by developing a direct search method based on pattern search which works reliably on the problems presented in this correspondence.

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REFERENCES

[1] F. E. Emery and M. O'Hagan, "Optimal design of matching networks for microwave transistor amplifiers, "IEEE Trans Microwave Theory and Techniques (Correspondence), vol. MTT-14, pp. 696 - 698, December 1966.

[2] J. W. Bandler, "Optimum noncommensurate stepped transmission-line transformers, "Electronics Letters, vol. 4, pp. 212 - 213, 31st May, 1968.

[3] J. W. Bandler, "Computer optimization of a stabilizing network for a tunnel-diode amplifier, "IEEE Trans Microwave Theory and Techniques, vol. MTT-16, pp. 326 - 333, June 1968.

[4] R. Hooke and T. A. Jeeves, " 'Direct search' solution of numerical and statistical problems," J. ACM, vol. 8, pp. 212 - 229, April 1961.

[5] A. F. Kaupe, Jr., "Algorithm 178: Direct search," Comm. ACM, vol. 6, pp. 313 - 314, June 1963.

[6] M. Bell and M. C. Pike, "Remark on Algorithm 178: Direct search," Comm. ACM, vol. 9, pp. 684 - 685, September 1966.

[7] M. J. D. Powell, "An efficient method for finding the minimum of a function of several variables without calculating derivatives," The Computer Journal, vol. 7, pp. 155 - 162, July 1964.

[8] W. I. Zangwill, "Minimizing a function without calculating derivatives," The Computer Journal, vol. 10, pp. 293 - 296, November 1967.

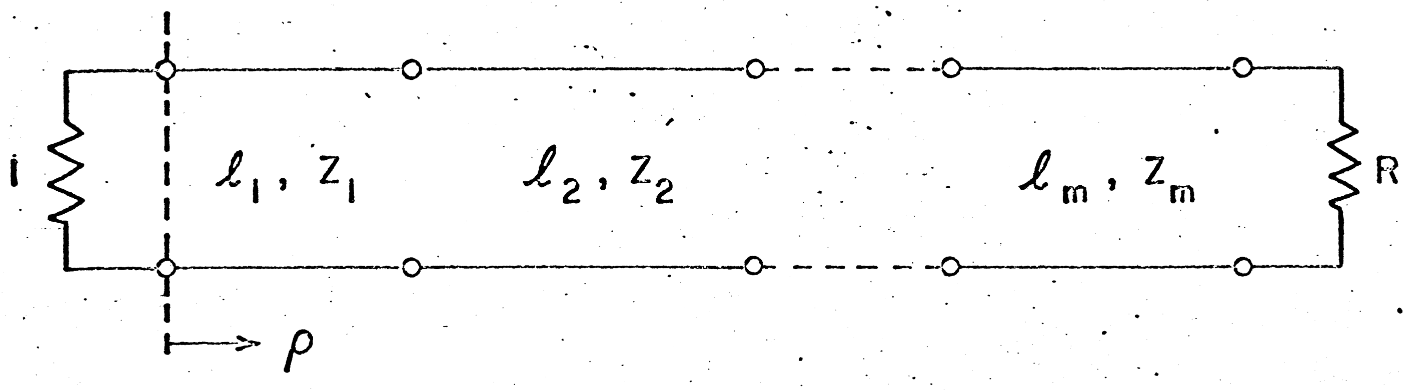
[9] G. L. Matthaei, L. Young and E. M. T. Jones, Microwave filters, impedance matching networks and coupling structures, New York: McGraw-Hill, 1964, chap. 6.

[10] D. J. Wilde, Optimum Seeking Methods, Englewood Cliffs, N. J.: Prentice-Hall, 1964, chap. 5.

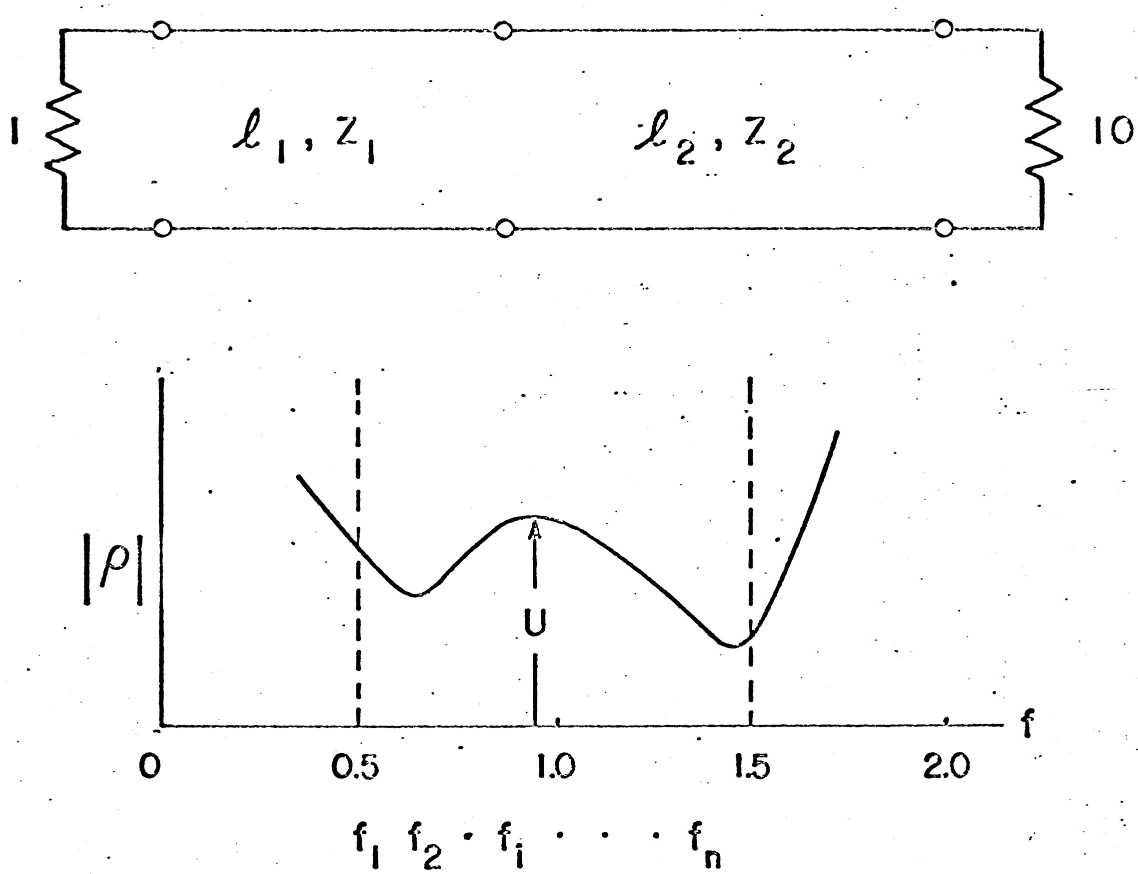
[11] D. J. Wilde and C. S. Beightler, Foundations of Optimization, Englewood Cliffs, N.J.: Prentice-Hall, 1967, chap. 7.

[12] G. C. Temes and D. A. Calahan, "Computer-aided network optimization the state-of-the-art," Proc. IEEE, vol. 55, pp. 1832 - 1863, November 1967.

[13] J. W. Bandler and P. A. Macdonald, "Optimization of microwave networks by Razor Search." To be published in IEEE Transactions on Microwave Theory and Techniques.



(a)



(b)

Fig. 1 Examples of multivariable cascaded transmission-line networks, (a) m-section resistively terminated noncommensurate transformer, (b) 2-section 10:1 transformer for optimum performance over

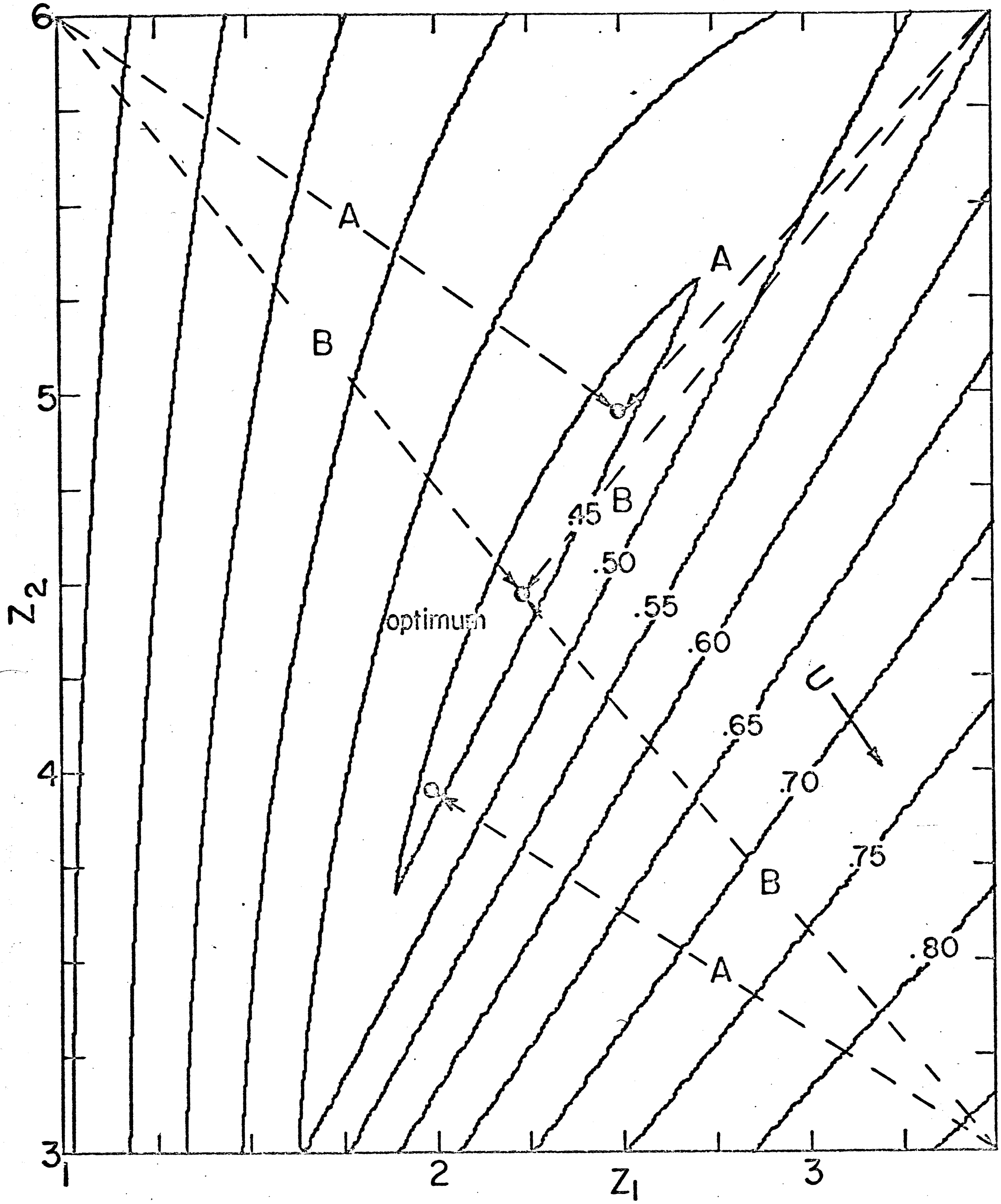


Fig. 2(a) Contours of U when $\ell_1 = \ell_2 = \ell_q$

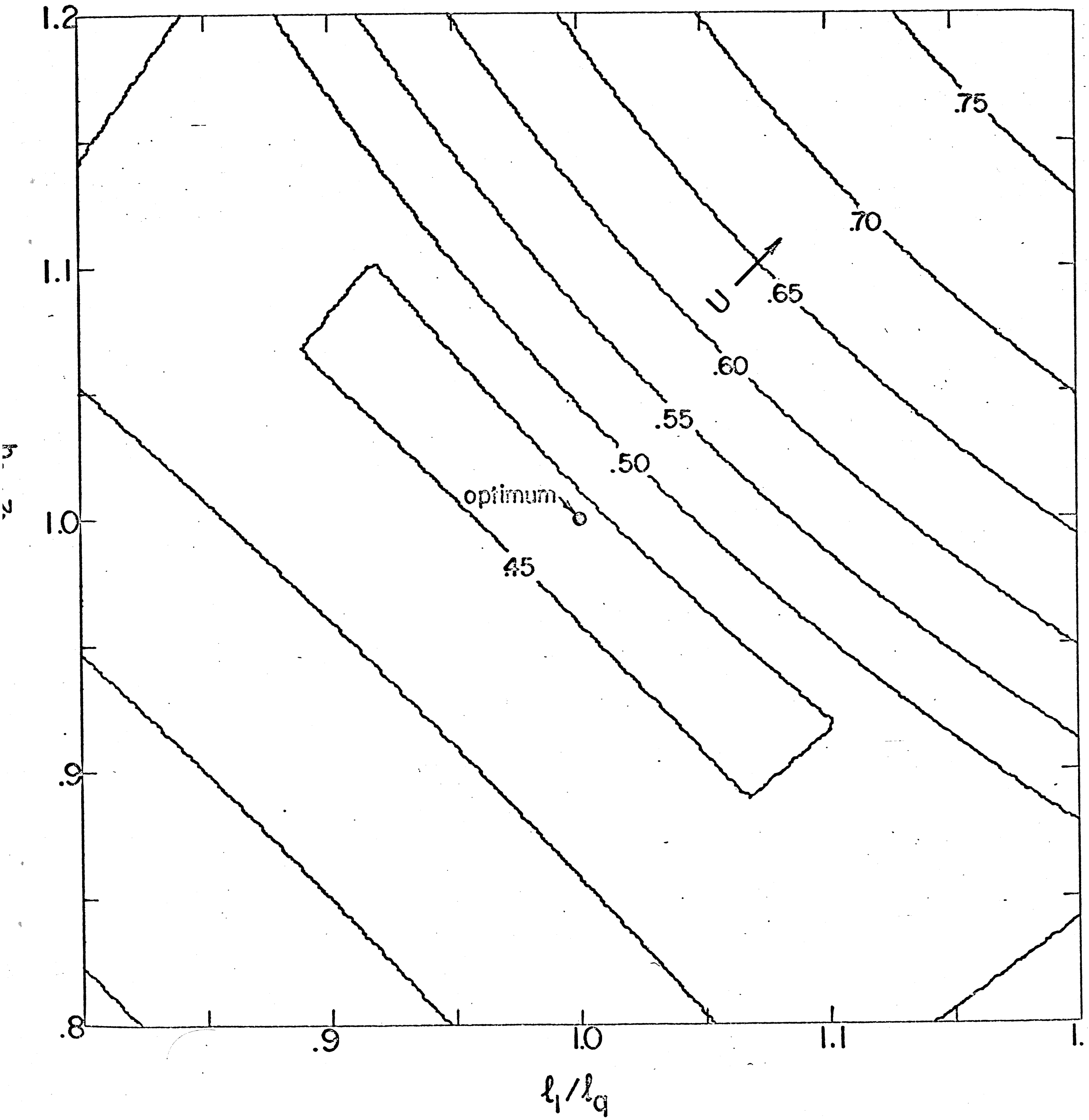


Fig. 2(b) Contours of U when Z_1 = 2.2361 and Z_2 = 4.4721

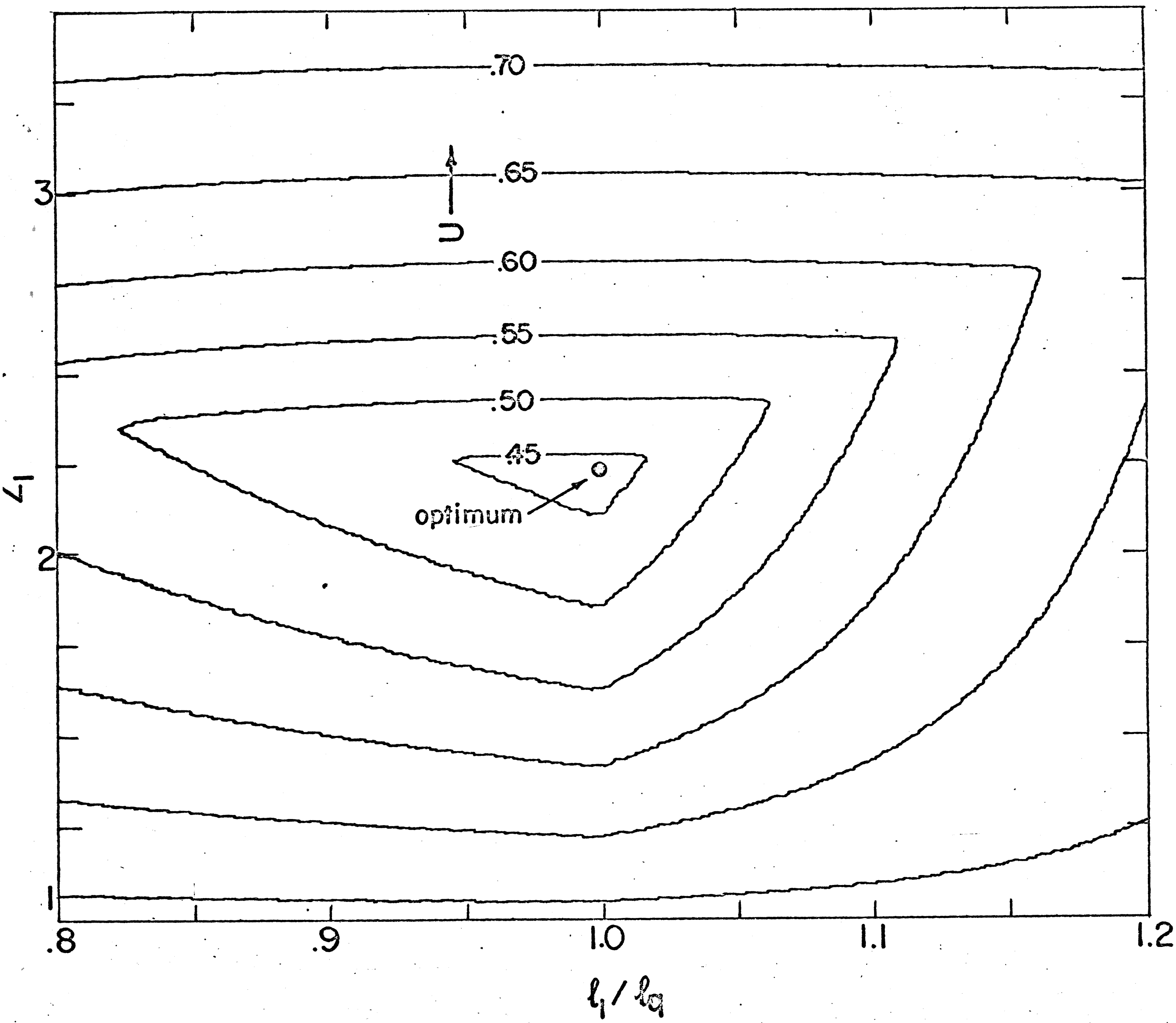


Fig. 2(c) Contours of U when $\ell_2 = \ell_q$ and $Z_2 = 4.4721$

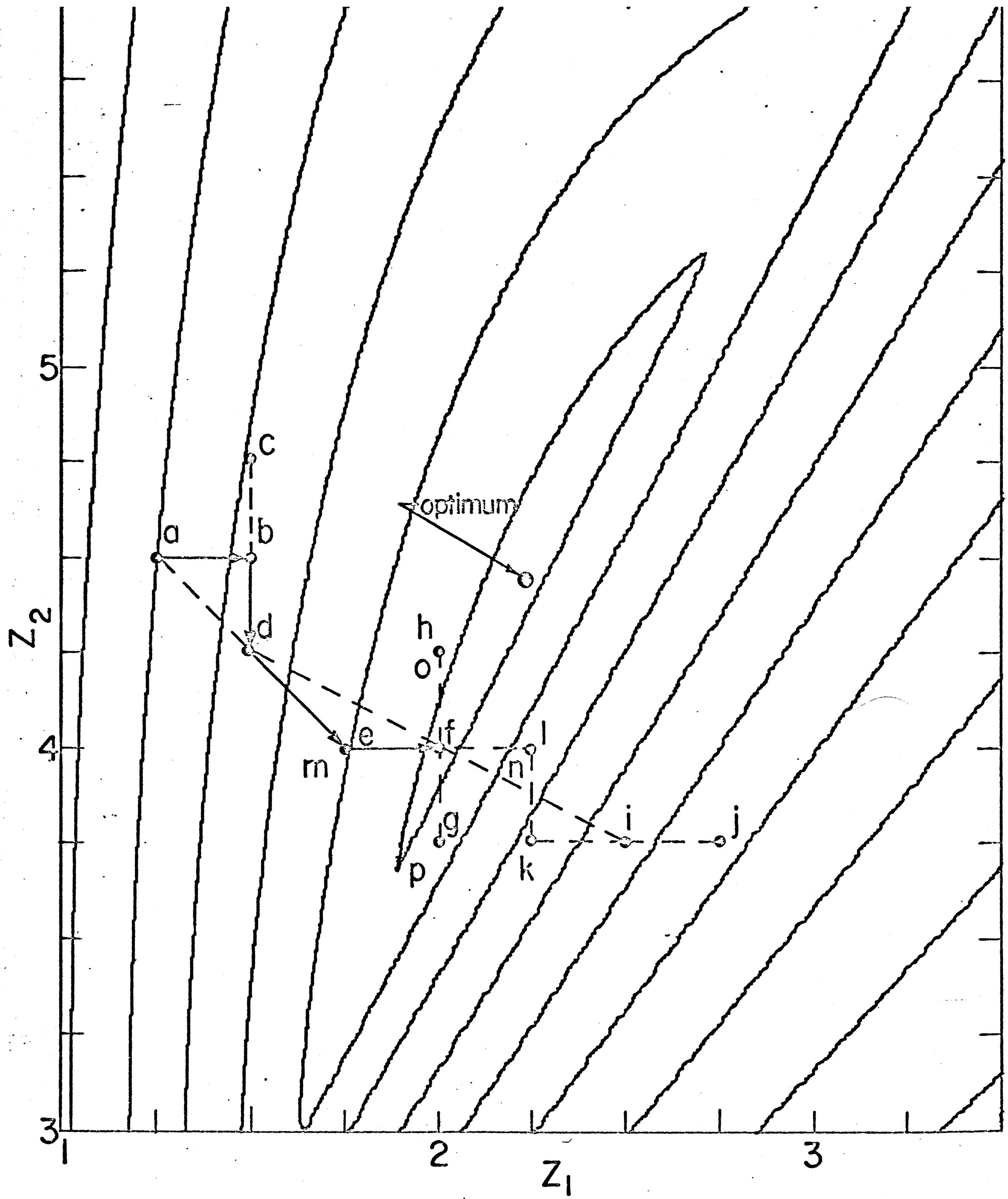


Fig. 3 The behaviour of pattern search[1] for the case $\ell_1 = \ell_2 = \ell_q$ starting from a typical point when $\Delta Z = .25$.

function evaluation				description of move
	a			initial base (or starting) point
b	c	d		exploratory moves from a
	e			pattern move in direction ad
f	g	h		exploratory moves from e
	i			pattern move in direction df
j	k	l		exploratory moves from i
m	n	o	p	exploratory moves from f