COUPLING EFFECTS OF INELASTIC SECONDARY SYSTEMS

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INELASTIC SECONDARY SYSTEMS

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ABSTRACT

The coupling effects of secondary inelastic systems on the primary and secondary seismic responses is the focus of this work. The interaction between a single-degree-of-freedom secondary system and a single-degree-of-freedom primary system is analyzed. The main objective is to determine the dynamic response of untuned and tuned systems when one or both systems behave inelastically. The Wilson-θ numerical integration method is used to determine the maximum response of the systems under coupled and uncoupled analyses. A total of 15 actual strong ground motion time histories are used in order to perform a statistical analysis. The influence of the A/V ratio (ratio of the peak ground acceleration to the peak ground velocity) of the earthquake records on the response using 3% and 5% damping ratio, is investigated.

The influence of various parameters on the system response are considered. The primary fundamental frequencies of 10.0, 5.0, 1.0, and 0.2 Hz are used with frequency ratios (ratio of the secondary fundamental frequency to the primary secondary fundamental frequency) of 0.1 to 5.0 to study the effect of untuned systems. Special attention is given to the closely-tuned systems. The degree of inelasticity is varied by using yield level factors of 1.0, 0.75, 0.50 and 0.25. The emphases is placed on the behaviour of elasto-plastic systems. Mass ratios of 0.1, 1.0, 2.0, 5.0 and 10.0% are used.

The results indicate that untuned systems can experience responses which are as important as tuned systems. The frequency content of the earthquake records is not an important factor. The effect of the mass ratio and yield level is mostly limited to systems with frequency ratios of 0.8 to 1.25.

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NOTATIONS

A, B	Constants
[C]	Damping matrix of total system
c	Subscript denotes "coupled"
c_{ij}	Damping constants of damping matrix [C]
c_p, c_s	Damping of single degree-of-freedom primary and secondary systems
d	Subscript denotes "decoupling"
d_i	Displacement of the i^{th} mass relative to the $(i-1)^{th}$ mass motion
d_c	Yield displacement of the i^{th} spring in compression
d_t	Yield displacement of the i^{th} spring in tension
e	Subscript denotes "elastic"
F	Force
$F_{y,i}$	Yield force of the i^{th} spring
fr	frequency ratio
g	Graviational acceleration (981 cm/sec^2) or subscript denotes "ground"
{I}	Unit vector
i,j	Subscript indices (i,j) = 1,2 denote primary and secondary subsystems in the coupled system or in decoupled systems.
[K]	Spring stiffness matrix
$[K]_n$	Spring stiffness matrix at the n^{th} integration point
$[K_e]_n$	Equivalent spring stiffness matrix at the n^{th} integration point

$k_{e,i}$	Spring constants of the i^{th} mass in the elastic range
$k_{i,j}$	Spring stiffness constants of stiffness matrix [K]
$k_{i,i}$	Spring constants of the i^{th} mass in the inelastic range
k_p, k_s	Spring constant of single-degree-of-freedom primary secondary systems (N/cm)
[M]	Mass matrix of total system
m	Subscript denotes "maximum"
m_{ij}	Mass constants of mass matrix [M]
m_p, m_s	Mass of single-degree-of-freedom primary and secondary systems (kg)
n	Subscript denotes the n^{th} integration point
p	Subscript denotes "plastic"
Ra	Ratio of acceleration of coupled system to acceleration of uncoupled system
R_c	Yielding force in compression
Rd	Ratio of displacement of coupled system to displacement of uncoupled system
R_i	Yield level of the i^{th} degree-of-freedom
R1	Yield level of the 1^{st} degree-of-freedom
R2	Yield level of the 2^{nd} degree-of-freedom
R_t	Yielding force in tension
Rv	Ratio of velocity of coupled system to velocity of uncoupled system
Rμ	Ratio of displacement ductility of coupled system to displacement ductility of uncoupled system
T_i	Undamped natural period of the i^{th} degree-of-freedom system (sec)

T1	Undamped natural period of the 1^{st} degree-of-freedom system (sec)
t	Time in seconds
u_i	Displacement of the i^{th} mass relative to the ground(cm)
$\dot{u}_i$	Relative velocity of the i^{th} mass (cm/sec)
$\ddot{u}_i$	Relative acceleration of the i^{th} mass (cm/sec^2)
$\{u\}$	Displacement vector relative to the ground
$\{\dot{u}\}$	Velocity vector relative to the ground
$\{\ddot{u}\}$	Relative acceleration vector
$\{u\}_n$	Displacement vector relative to the ground at the n^{th} time step
$\{\dot{u}\}_n$	Relative velocity vector at the n^{th} time step
$\{\ddot{u}\}_n$	Relative acceleration vector at the n^{th} time step
x_i	Absolute displacement of the i^{th} mass (cm)
$\dot{x}_i$	Absolute velocity of the i^{th} mass (cm/sec)
$\ddot{x}_i$	Absolute acceleration of the i^{th} mass (cm/sec^2)
y	Subscript denotes yield value of quantity to which it refers
y_g	Displacement of the ground or the base (cm)
$\dot{y}_g$	Velocity of the ground or the base (cm/sec)
$\ddot{y}_g$	Ground acceleration (cm/sec^2)
β	Damping ratio
β_i	Modal damping ratio of the i^{th} mode
μ	Mass ratio of secondary system to primary system
Δt	Integration time interval (sec)
δt	Time interval of record ground motion (sec)

$\delta\ddot{y}_{g\,n}$	Ground acceleration increment at integration point n over extended time step τ (cm/sec^2)
$\{\Delta u\}$	Increment of displacement vector relative to the ground in the normal integration time interval Δt
$\{\Delta\dot{u}\}$	Relative velocity increment vector in the normal integration time interval Δt
$\{\Delta\ddot{u}\}$	Relative acceleration increment vector in the normal integration time interval Δt
$\{\delta F\}_n$	Modified incremental load vector at the n^{th} integration point
$\{\delta u\}$	Relative displacement increment vector in the extended time step τ
$\{\delta\dot{u}\}$	Relative velocity increment vector in the extended time step τ
$\{\delta\ddot{u}\}$	Relative acceleration increment vector in the extended time step τ
μ_{di}	Displacement ductility ratio of mass i^{th}
τ	Extended integration time interval (sec)
θ	Constant in Wilson-θ integration method ($\theta = 1.4$)
ω_i	Modal circular frequency of the i^{th} mode
Ω_p, Ω_s	Circular frequency of single-degree-of-freedom primary and secondary systems

LIST OF ABBREVIATIONS

AVG.	Average
CPA	Coupled Primary Acceleration (absolute)
CPD	Coupled Primary Displacement (relative)
CPV	Coupled Primary Velocity (relative)
CSA	Coupled Secondary Acceleration (absolute)
CSD	Coupled Secondary Displacement (relative)
CSV	Coupled Secondary Velocity (relative)
DOF	degree-of-freedom
HIGH	High A/V ratio
INTER	Intermediate A/V ratio
LOW	Low A/V ratio
MDOF	Multi-degree-of-freedom
STD	Standard deviation
UPA	Uncoupled Primary Acceleration (absolute)
UPD	Uncoupled Primary Displacement (relative)
UPV	Uncoupled Primary Velocity (relative)
USA	Uncoupled Secondary Acceleration (absolute)
USD	Uncoupled Secondary Displacement (relative)
USV	Uncoupled Secondary Velocity (relative)

CHAPTER 1 -- INTRODUCTION

1.1 GENERAL

The design of structures and nonstructural components to resist earthquake forces can be a somewhat difficult and complicated task. The importance of seismic resistant design for equipment has been demonstrated by several authors (e.g. Jordan, 1978). In recent years, various authors have proposed different methods to approximate systems behavior in order to simplify and reduce the effort needed to perform dynamic analysis. These methods are particulary useful in the preliminary design stages. Most of these methods are developed for elastic systems. However, with the exception of systems such as nuclear power plants, most structural elements are expected to deform inelastically to various degrees under strong ground motion earthquakes. The inelastic behavior of structures and their nonstructural components needs to be investigated in more detail. A building (primary system) and a piece of equipment (secondary system) are two different systems with specific properties. In the early design stages, factors such as the mass, attachment points or location of the equipment in a structure may be unknown. A detailed computer model including all the nonstructural components would be time consuming and costly especially if numerous modifications are performed as part of the design process evolution. These factors lead to the introduction of various methods of analysis which neglect the interaction between the primary and secondary systems. Under specific conditions(decoupling conditions and criteria) the primary system and the secondary system can be analyzed separately. Once again very little work has been done to define these conditions for an inelastic system.

The present research will consider the effect of various parameters on the coupled system behavior (the secondary system is incorporated with the primary system in a single dynamic model) and decoupled system behavior (the primary and secondary systems are considered separately). The objective is to identify the conditions under which the uncoupled system will give a reasonable approximation to the response of the coupled system. Another objective is to define the level of forces and deformations which are expected in secondary and primary systems.

1.2 OBJECTIVE AND SCOPE

Most of the articles and research published in this field are based on elastic analysis. Alternate methods of analysis are typically proposed to replace the time consuming and costly numerical time-history method which is considered "exact". An "exact" method which is known to produce accurate results, is used. The numerical analysis utilizes the Wilson-θ method for various reasons which will be discussed further in later chapters. A computer program was developed in FORTRAN which can be used on a micro-computer or a main frame system.

Considering the complexity of the problem, the systems used by different authors vary considerably. Many articles dealing with linear elastic systems use multi-degree-of-freedom(MDOF) primary and/or MDOF secondary systems attached at one or more locations. Since the subject of inelastic response is much more involved, a simple configuration is used in the present work. The investigation is concerned with the response of the primary and secondary systems. Firstly, the response of a two-degree-of-freedom(2-DOF) coupled (primary-secondary) system subjected to a ground motion is studied. Secondly, a 1-DOF (primary) system is subjected to the same ground motion. The absolute acceleration response of this 1-DOF (primary) system is used as input to another 1-DOF (secondary) system. The responses of the primary and secondary component using these two different procedures of analysis are compared. The purpose is to determine the correlation between the two procedures and the various factors which can influence the behavior of the primary and secondary components. In this study specific parameters such as mass ratio, yield levels, frequency ratios and earthquake ground motion inputs, are covered. These parameters will be defined and discussed in later chapters.

The concept of design spectra is widely used in practice. This concept is well-established for elastic systems but for inelastic systems different interpretations are offered. The analysis in the current work is performed using actual strong ground motion earthquake records. As can be noticed in the references, most of the previous analyses were performed using artificial records, a design spectrum or a limited number of actual earthquake records. The present work uses a sufficient number of actual strong motion earthquakes to determine the effect of the frequency content of the records on the responses of the systems studied.

1.3 LITERATURE REVIEW

Many studies are available to analyze 2-DOF elastic systems. These studies can be divided in two major groups: methods where a modal analysis is performed to determine the eigenvalues or methods where a response analysis is performed.

Hadjian (1977) reviewed the existing decoupling criteria and proposed a more uniform approach based on mass and frequency ratios. Depending on the level of error deemed acceptable, practical curves can be plotted to identify the cases where uncoupling is possible. Hadjian states that a 15% error in frequency is considered acceptable. The author proposes an error based on 25% overestimation and 15% underestimation of the frequency as being acceptable when dealing with the response behavior.

Aziz and Duff (1978a) proposed a rational perturbation approach for decoupling based on limiting the changes in eigenvalues of the coupled system from those of the uncoupled systems. Their criteria is based on a maximum of 5% shift in frequencies which is believed to be consistent with the other aspects of the seismic design process. Their criteria are described by equations which are accurate and simple to apply.

Gupta and Tembulkar (1984b) developed algorithms to determine the change in the frequencies and the change in the response. The approach was limited to the primary system only. Curves representing various levels of accuracy for the frequency variation and the response variation, are presented. The authors suggest that both conditions should be met to allow uncoupling. Gupta and Tembulkar (1984a) extended their work to multiply connected MDOF secondary systems.

Hadjian and Ellison (1986) extended the investigation from a previous article (i.e. Hadjian (1977)) to include curves for various frequency error levels and curves for various errors in the response. In the study, the authors considered two types of models for their analysis. The "cascading model" where neither stiffness nor mass of the supported system are included in the supporting system model. The "lumped" model where the mass of the supported system is rigidly lumped into the mass of the supporting system. The lumped model is the preferred model for those cases where the supported system is stiffer that the supporting system. The cascading model is more appropriate for relatively softer supported system. The response errors are, in general, on the conservative side, and very large conservatisms are predicted for certain conditions.

In a paper by Sackman et al. (1983), closed-form expressions are derived to find the dynamic properties of the combined SDOF-equipment - MDOF primary system in terms of those of the structure alone and the equipment alone. It was noted that the errors in frequencies tend to increase as the equipment is tuned to higher structure modes. Suarez et al. (1987) used a similar approach and found the eigenvalues by using a standard Newton-Raphson method. This approach can be used effectively with light or heavy equipment.

Gerdes (1977) used standard time-history modal analyses techniques to calculate the response of a primary-secondary system. Realistic nuclear systems with multi-supported secondary system were analyzed.

Suzuki (1977) studied the uncertainty in the maximum response properties of the secondary systems. The analysis is performed by using a coupled SDOF primary - SDOF secondary system. The study is not concerned with coupled versus uncoupled analysis results. The analysis is performed numerically by using the Runge-Kutta-Gill method. A total of 19 strong ground motion records were used in this analysis. Various mass ratios, natural frequencies and damping ratios were investigated. It was found that the proposed method gives amplification factor for the secondary response of the same magnitude as the numerical analysis.

Ruzicka and Robinson (1980) used modal analysis and Fourier transforms to evaluate the response of a tuned secondary system. Nakhata et al. (1973) developed a method using modal analysis to determine the approximate dynamic response of light secondary systems by utilizing amplification factors.

Section 5.3 of the Design Procedures for Seismic Qualification of CANDU Nuclear Power Plants (1981) deals with the concept of decoupling criteria. It states that: "Decoupling is acceptable if the mass of the component is less than 1% of the mass of the structure."

For larger mass ratios, the method proposed by Aziz and Duff (1978a) is suggested.

Various authors used a response spectrum method. In many instances, amplification factors are derived and used to approximate the response of a secondary elastic system. Aziz and Duff (1978b) studied the effect of the mass ratio on the response using amplification factors and a floor response spectrum method. The secondary system response obtained by decoupling from the primary system was found to be always conservative.

Amin et al. (1971) presented a method to determine the response of a light secondary system which is connected at several points to a primary structure. A technique to obtain a modified spectrum to be used as input for a secondary system is compared with the results of a numerical solution scheme. Ignoring the interaction effects, the proposed method would be acceptable for a mass ratio of less than 0.01. When the frequencies of the two systems are not close, good results may be obtained even for higher values of this ratio.

Ishikura and Kajï (1985) used a model of a multi-supported reactor vessel and found that a spectrum analysis is extremely conservative compared to a time-history analysis.

Vanmarcke (1977) proposed a procedure to evaluate the secondary system response directly from the specified ground response spectra. The first step is to obtain the accelerogram at the support point of the secondary system. The second step is the derivation of amplification factor to evaluate the response of the secondary system. Villaverde and

Newmark (1980) proposed a simple approximate method to compute the maximum response of light secondary systems. The method is derived by considering that a secondary system and its supporting primary structure form a single assembled system, by applying a modified version of the response spectrum technique.

Sackman and Kelly (1978, 1979) studied a MDOF structure with a SDOF equipment. A simple analytical method was developed to determine the maximum acceleration and displacement of the equipment. The response spectrum for the equipment can be calculated by multiplying the design response spectrum by an amplification factor. An important variation in this approach is that the authors worked directly with the design spectrum. Kelly and Sackman (1978) described a similar method applicable to tuned systems only. Gupta and Jaw (1986a, 1986b) and Gupta (1984) presented a perturbation method which uses the response spectrum specified at the base of the primary system as the input. Comparisons are made between the time history analysis, conventional floor response spectrum method and this new procedure. It was shown that the response values from the present method were in good agreement with those from a coupled time history analysis.

Igusa et al. (1985) developed a perturbation method to take into account tuning, interaction and nonclassical damping for multiply supported MDOF secondary system. The emphasis is placed on deriving closed-form results that accurately characterise the dynamic behavior and are simple enough to facilitate their practical implementation. Der Kiureghian et al. (1983) presented a similar approach where a modesuperposition method is used to evaluate the dynamic response of light equipment subjected to stochastic excitations. Igusa (1985b) introduced another variable: spacial coupling, when dealing with multiply supported secondary systems.

Jeng (1985) in his thesis performed an extensive investigation of the maximum acceleration of a MDOF secondary system attached to a MDOF primary system. His research was based on a perturbation method presented by Hernried and Sackman (1984). A conventional Newmark time integration scheme was used for the combined system. Jeng (1985) states that: "In all instances, correlation between the results was quite favorable especially when the possibility of inaccuracy in the Newmark integration scheme due to ill-conditioning was considered. In addition, the methodology presented by Hernried and Sackman is significantly less costly than the Newmark integration scheme." The floor spectrum method was used to uncouple the systems where the effect of interaction between the structure and the equipment is neglected. Jeng (1985) found that the floor spectrum method is reasonably accurate for a completely detuned primary-secondary system but consistently fails for a tuned or nearly tuned primary-secondary system.

Hadjian (1971) suggested various methods of approach to the problems of obtaining accurate design spectrum curves at any location on the structure. In a response spectrum, the sharp peak corresponds to the fundamental frequency of the supporting structure. Since the frequencies of a structure cannot be precisely determined because of various factors such as: material properties, lumping of masses, stiffness and damping, the response spectrum curves should be modified by shifting of the peak response.

Penzien and Chopra (1965) presented an approximate method of analysis for appendages located on top of MDOF structure based on a response spectra input. A common practice in developing response spectra is to analyze an uncoupled model which only includes the weight of the heavy components in the building model. This procedure recognizes the mass effects but the coupling effects are neglected. The corresponding response spectra is considered conservative for the equipment in seismic design. Liu and Johnson (1985) presented a simplified heavy component model to be included in the building to account for the coupling effects.

Pal et al. (1977) considered that for preliminary seismic design, the design earthquake response spectrum method was the most appropriate approach. Most methods attempt to neglect the interaction between the equipment and the structure. However, in nuclear design applications, dynamic interaction effects are important due to safety requirements and complicated attachment configurations. The method presented is fairly involved and the author emphasize that this simplified coupled analysis procedure appears presently to be useful only for preliminary design purposes. The concept of decoupling criteria is reviewed and the authors believe that a design earthquake response spectra which is smoothed and widened to cover ± 15 % variation in estimation of the fundamental frequencies, is appropriate.

Hernried and Jeng (1987) used a perturbation method developed in an earlier paper (1984). MDOF primary and MDOF secondary systems are considered. Tuned and completely detuned systems with all frequencies of the subsystems well-spaced, are studied. The authors question the use of a numerical integration method because of the computational time, cost and the ill-conditioned nature (caused by the large differences in the mass and stiffness of the systems) of the property matrices involved.

Pickel (1972) was concerned with nuclear systems and studied different mathematical models to approximate the interaction between primary and secondary systems. A range of frequency ratios (secondary/primary) was considered from 0.5 to 2.0. Curves relating frequency ratios for a specific mass ratio and the error in eigenvalues, are presented. The maximum error occurs when the frequency ratio is between 0.9 and 1.0. The author suggest that a conservative 2 to 3% permissible error in eigenvalues should be used to uncouple the subsystems. These values were chosen on the basis of the limited number of cases studied. The following trends were observed: For cases where the supported subsystem is much more flexible than the supporting subsystem, the distortion of the supporting system has little effect, and the supported system can be analyzed as if it is supported directly on the ground. For cases where the supported system is stiff when compared with the supporting system, the motion of the supporting system is the predominant input to the supported system. Between these two conditions, resonant effects may result in a large amplification of the response of the supported subsystem.

The present investigation is concerned with the nonlinear behavior of primary and secondary system components. There are very little research results available on the response of 2-DOF nonlinear models.

One of the first attempts to study a 2-DOF elasto-plastic system was made by Anderson (1963). The Newmark numerical integration scheme was used to calculate the responses. A study is made of the response of a 2-DOF system subjected to two forms of base excitation; an instantaneous velocity change of the ground and two strong motion earthquake records. The results are presented in the form of deformation spectra. Comparisons were made between elastic and elasto-plastic systems. In the early seventies, Newmark (1972) addressed the problem of inelastic response to seismic loading by developing response spectra that would consider the ductility of inelastic systems.

König and Wörner (1985) presented the influence of local nonlinearities such as yielding, friction and gaps. They showed that the response amplification in the region close to the dominant excitation frequency is greatly reduced due to the hysteric behaviour. The results of investigations using SDOF systems are presented in a nonlinear response spectrum. In the region of higher eigen-frequencies even ductilities of the order of 10 cannot reduce the response acceleration. The reduction of the yielding force reduces the maximum displacement but after a certain level a greater reduction increases the response.

Ghobarah and Aziz (1987) studied the seismic behavior of tuned equipment-structure systems where one or both of the system components behave inelastically. The effect of the mass ratio and yield level of the system on the response is discussed. An elaborate study on the decoupling criteria of tuned inelastic systems was conducted by Nguyen (1986). The present research is a continuation of the work done by Nguyen covering a wider range of variables as well as the case of untuned systems.

1.4 UNIQUE CONTRIBUTION OF THE RESEARCH

The dynamic response of untuned secondary and primary inelastic systems is studied. A total of 15 actual strong ground time histories are used to perform a statistical analysis. The effect of the A/V ratio is investigated.

CHAPTER 2 -- METHOD OF ANALYSIS

2.1 INTRODUCTION

One of the objectives of this thesis is to gain knowledge of the performance of inelastic systems. In order to determine the behavior of such systems subjected to a ground motion, a simple 2-DOF model is used. The objective is to utilize a simple model to define the influence of a series of parameters on the system response. By working with a number of fixed parameters, the corresponding effects on the system response can be established. With the help of a statistical analysis, the conditions which allow decoupling of the 2-DOF system into 2 simple 1-DOF systems can be investigated.

As seen in chapter 1, numerous authors have developed methods and criteria for decoupling an elastic system. In general, the decoupling can be considered from two approaches: a variation in frequency or a variation in response. For an elastic system, parameters such as: mass ratio and frequency ratio are used. However, for inelastic systems, the variation in frequency is not acceptable because of the nonlinear behavior of the spring elements. The response variation and the mass ratio will be used as a means to deal with these systems.

An important feature of response variation is its dependence on the input. A statistical analysis is necessary in order to establish the correlation and validity of the results. Special attention will be given to the impact of various strong ground motion earthquake records on the response of the system. This is a point that has not been studied in great detail in past research. A series of analyses will be performed using records with differing frequency content.

The literature review reveals that most previous investigations attempt to find an approximate method that will give comparable results to the "exact" method. An exact method of analysis is used to calculate responses of the coupled and uncoupled systems. The purpose is to establish the influence of various parameters on the systems. According to the variation in the response, a decoupling criteria may be assumed based on an acceptable level of error. There are several numerical integration schemes available, among them: constant velocity method, Newmark-β method and Wilson-θ method. The Wilson-θ method will be used because it is simple, does not need any special starting procedure and is unconditionally stable (Bathe and Wilson, 1973). Commercial packages to calculate the dynamic response of a system are available. These programs are usually not flexible and are conceived for specific applications. A computer program in Fortran-77 compatible with a main frame or a micro-computer was developed by Nguyen (1986). The present program was modified to perform the analysis required for this research.

The use of a numerical step-by-step integration scheme is dictated because inelastic systems are examined. The yield force associated with the inelastic behavior is based on a percentage of the maximum elastic force. An uncoupled elastic response analysis is performed to obtain the maximum elastic force under specific conditions. A yield level factor (ratio of the yield force to the maximum elastic force) is used to calculate the yield forces needed in the inelastic analysis.

2.2 SYSTEMS CONSIDERED

Three different systems will be used throughout this work. A 1-DOF secondary subsystem attached to a 1-DOF primary subsystem will be used for the coupled analysis as illustrated by figure 2.1 . The input is applied at the base of the primary subsystem. For the uncoupled analysis, two separate systems will be considered: a) a 1-DOF primary system where the input is applied to the base of the primary system as seen in figure 2.2; b) a 1-DOF secondary system where the acceleration response of the 1-DOF primary system is applied at the base of the secondary system as demonstrated by figure 2.2.

The dynamic properties of the coupled 2-DOF secondary-primary system can be expressed as:

$$[M] = \begin{bmatrix} m_p & 0 \\ 0 & m_s \end{bmatrix} \quad [K] = \begin{bmatrix} k_p+k_s & -k_s \\ -k_s & k_s \end{bmatrix} \quad [C] = \begin{bmatrix} c_p+c_s & -c_s \\ -c_s & c_s \end{bmatrix} \qquad (2.1)$$

Where: [M] mass matrix of the total system

[K] stiffness matrix of the total system

[C] damping matrix of the total system

The subscripts p and s denote primary and secondary: m_p and m_s are lumped masses, k_p and k_s are weightless springs and c_p and c_s are dashpots representing viscous damping.

To consider the decoupling when the interaction is ignored, two 1-DOF systems (m_p, k_p, c_p) and (m_s, k_s, c_s) are used. The acceleration response of the primary system is used as input for the secondary system.

Many investigations, including Nguyen's (1986), considered only the case where the frequencies of both the primary and secondary systems are equal. This being the resonance case, the variation in response is expected to be the worst. It appears important also to investigate the response when both systems are detuned which is the non-resonance case. The intent is to obtain results for cases that are tuned or detuned and to evaluate the variation in response between the two types of problems.

The response values that are of interest here are the relative displacement, the relative velocity, the absolute acceleration and the ductility. The force-displacement relationship for each spring is assumed to be elasto-plastic or bilinear as illustrated in figure 2.3. For the bilinear case, the slope of resistance displacement curve is expressed as a percentage of the elastic stiffness.

2.3 PARAMETERS CONSIDERED

In order to perform an analysis of primary and secondary systems, certain parameters must be chosen. The mass ratio is the most obvious characteristic parameter that relates both systems. However, other parameters are important and can be set to specific values. The following parameters are used throughout this study: mass ratio, frequency ratio, period, damping ratio and yield level.

The computer program is structured according to the following procedure to determine the characteristics of each system. The input data is: mass of primary system (m_p), mass ratio (μ), period of primary system (T_1), frequency ratio (fr), damping ratios (β_p,β_s) and yield levels (R_1,R_2) for both primary and secondary systems. From these quantities, the mass (m_s) and period (T_2) of the secondary system, are calculated. The corresponding stiffness (k_p,k_s) for both systems, are calculated. An elastic analysis of the uncoupled systems is performed for a specific ground motion to determine the maximum elastic force ($F_{m,e}$) for both systems. The yield level (R) is used to calculate the corresponding maximum force that the system will sustain. Complete analyses using uncoupled and coupled models are performed to obtain the dynamic responses.

The different parameters are chosen to correspond as much as possible to practical physical characteristics. A limited number of parameters and values has to be identified. The objective is to limit the number of cases to a manageable level and at the same time obtain results that are general enough to be useful in practical applications.

One of the principal characteristics of a structure for dynamic analysis is its fundamental frequency (Ω) or natural period of vibration (T). The fundamental frequency of the primary or secondary system can be easily obtained and is unique to the system considered. The present analysis is limited to certain cases which are believed to be typical for such systems. The work of Nguyen (1986) was limited to a frequency ratio of one. The frequency ratio (fr) is defined as the ratio of the secondary frequency to the primary system frequency. The term frequency ratio always refers to the initial fundamental frequencies. In an inelastic analysis the frequency of a system will change because the spring stiffness varies. The present research covers both the resonance and the non-resonant cases. The frequency ratios are based on a number of chosen fundamental primary frequencies. For the primary system, values of 10 Hz, 5 Hz, 1.0 Hz and 0.2 Hz are selected. This range from 10 Hz to 0.2 Hz is considered to be adequate. Other frequencies higher or lower than this were considered to be of little practical significance. The selected frequencies cover a wide range of structures from low-rise and stiff systems to high-rise and flexible structures. The four chosen fundamental primary system frequencies should be sufficient to determine the impact of this parameter on the response of the systems.

It is recognized that a secondary system rarely has a frequency higher than 33 Hz. Heidebrecht et al. (1983) stated that it is a current practice in seismic qualification of nuclear power plants to consider a system whose fundamental frequency is above 33 Hz as rigid. A frequency of 1 Hz was selected as the lower practical limit for a secondary system. Extremely unusual pieces of equipment may have higher or lower frequencies. Those cases are not covered here. A frequency ratio of 1.0 indicates that both systems have the same frequency or in other words are in resonance. As mentioned earlier, the frequency of the primary system is selected from four different values. A series of ten frequency ratios were selected for the investigation. The ten frequency ratios are: 0.1, 0.5, 0.8, 0.9, 1.0, 1.1, 1.25, 1.5, 2.0 and 5.0. Special attention is given to the closely-detuned systems (frequency ratio around 1.0). However, for each primary system frequency, all 10 frequency ratios may not represent practical cases. For this reason, selected frequency ratios are considered. A requirement that the secondary system frequency should fall between 1 Hz to 33 Hz is also introduced. The following table indicates the frequency ratios studied for each primary frequency:

	Primary System Frequency			
	10 Hz	5 Hz	1.0 Hz	0.2 Hz
Frequency Ratio	0.1			
	0.5	0.5		
	0.8	0.8		
	0.9	0.9		
	1.0	1.0	1.0	
	1.1	1.1	1.1	
	1.2	1.25	1.25	
	1.5	1.5	1.5	
	2.0	2.0	2.0	
			5.0	5.0

The frequency ratios used depend on the primary frequency and the practical range for the secondary frequency. This explains the reason why the primary frequency of 0.2 Hz is used only with a frequency ratio of 5.0. Special attention is given to the region of frequency ratio around 1.0 where the response is expected to be more severe because of the resonance condition. Many researchers consider two main cases; the resonance and the non-resonance cases. The frequency ratios were chosen to give a detailed picture of the behavior at or near resonance. It is a case that deserves special attention. It is of interest to determine the zone which produces a response in the same range as the resonance case. In practical applications there is a variation between theoretical calculations and actual properties of the systems. For example, if it is theoretically predicted that a frequency ratio of 1.5 exists but, after construction the measured frequency was found to be only 1.25, will the design still be adequate? The answer would depend on the effect of these frequency ratios.

Another important parameter to consider in this type of investigation is the mass ratio. As presented in chapter 1, for mass ratios smaller than μ=0.1%, decoupling is acceptable. The objective is to determine the effect of this parameter on the response. Mass ratios of 0.1%, 1.0%, 2.0%, 5.0% and 10.0% will be used in this study. A maximum value of μ=10.0% is considered reasonable when the secondary system is a piece of equipment.

An important characteristic of any dynamic system is the damping. This value is somewhat difficult to establish precisely. Systems with damping ratios of 3% and 5% corresponding to steel and concrete structures, respectively, are typically used. In order to simplify the number of cases considered, the same percentage of the critical damping ratio (β) will be used for both the primary and secondary systems.

In order to consider various levels of inelasticity, a yield level factor (R) is introduced. Values of R equal to 0.25, 0.50, 0.75

and 1.0 of the elastic force case, are used. These yield levels will be used on the primary and secondary system in various combinations. The yield levels are factors applied to the maximum elastic force to obtain the maximum yield force that a specific inelastic system can sustain.

In order to limit the number of cases to a manageable level and obtain useful information, the following is a summary of the system parameters used in this study:

Mass ratios μ = 0.1, 1.0, 2.0, 5.0, 10.0 %
Primary periods T_1 = 0.1, 0.2, 1.0, 5.0 seconds
Frequency ratios fr= 0.1, 0.5, 0.8, 0.9, 1.0, 1.1, 1.25, 1.5, 2.0, 5.0
Damping ratios β = 3.0, 5.0 %
Yield levels R = 0.25, 0.50, 0.75, 1.0

The main part of this research is to evaluate the dynamic response of systems under all possible practical combinations of the above parameters. In order to compare the response variation of each system, a method of presenting the results must be established. For each specific system an uncoupled and a coupled analysis will be performed. The displacement, velocity and acceleration responses will be calculated. Throughout this thesis, the following abbreviations will be used to identify these responses: Uncoupled Primary relative Displacement, relative Velocity and absolute Acceleration (UPD, UPV, UPA); Uncoupled Secondary relative Displacement, relative Velocity and absolute Acceleration (USD, USV, USA); Coupled Primary relative

Displacement, relative Velocity and absolute Acceleration (CPD, CPV, CPA); and Coupled Secondary relative Displacement, relative Velocity and absolute Acceleration (CSD, CSV, CSA). Another approach to analyzing the results will be to present the data in the form of ratios between coupled and uncoupled response values. Four different response ratios will be used to determine the decoupling effect:

1) Ratio of absolute acceleration of coupled system to absolute acceleration of uncoupled system.

$$Ra_i = x_{c,i} / x_{d,i} \qquad i = p \text{ or } s$$

2) Ratio of relative velocity of coupled system to relative velocity of uncoupled system.

$$Rv_i = v_{c,i} / v_{d,i} \qquad i = p \text{ or } s$$

3) Ratio of relative displacement of coupled system to relative displacement of uncoupled system.

$$Rd_i = d_{c,i} / d_{d,i} \qquad i = p \text{ or } s$$

4) Ratio of displacement ductility of coupled system to displacement ductility of uncoupled system.

$$R\mu_i = \mu_{d\ c,i} / \mu_{d\ d,i} \qquad i = p \text{ or } s$$

All of the above ratios are calculated for both the primary and secondary systems.

2.4 GROUND MOTION INPUTS

In order to perform a statistical analysis, a certain and adequate number of strong earthquake ground motions should be used. A total of 15 earthquake records divided in three categories are selected. These strong earthquake records are listed in Table 2.1. Each earthquake record is unique. However, the frequency content of each earthquake can be used to classify the earthquake record. Naumoski et al. (1988) note that the peak ground acceleration to the peak ground velocity (A/V ratio) is a parameter that can be use to quantify the frequency content of a record. The records can be divided into three major categories: high A/V ratio (A/V > 1.2), intermediate A/V ratio (1.2 ≥ A/V ≥ 0.8) and low A/V ratio (A/V ≤ 0.8). According to Naumoski et al. (1988), records with high A/V ratios are normally associated with moderate earthquakes at close epicentral distance, while records with low A/V ratio are normally associated with large earthquakes at large epicentral distances. The plots of the acceleration time histories for each set of A/V ratio records are found in figures 2.4, 2.5 and 2.6. The records that are selected for this research are taken from the McMaster University Seismological Executive (MUSE) Database System. They are part of a set of 45 records that were compiled by Naumoski et al. (1988) for statistical studies of structural responses. The earthquake records have various durations. A preliminary analysis revealed that the peak maximum responses (i.e. displacement, velocity and acceleration) occur in the first 30 seconds. In order to limit the computing time, the analysis will be performed by using the first 30 seconds of each earthquake record.

In order to determine the effect of the A/V ratio on the dynamic response of the systems, a common base of comparison is established between the 15 earthquake records. In order to achieve this, the earthquake records are to be normalized. In practical terms, each record will be modified by a factor so that the maximum amplitude level of each record is the same. This normalization process may be performed on various parameters (i.e. displacement, velocity or acceleration). It is a common practice to use the acceleration as a normalizing parameter. In the current work, each record is normalized to a spectral acceleration of 1.0g at a specific primary system frequency.

The research work is based on 15 different earthquake records. This number was chosen with due consideration to the effort required to generate and process the data. Many projects have used far less earthquake records to demonstrate various theories or concepts. The objective in this particular research is to have enough data to allow a formal statistical analysis to be applied that will be useful in terms of practical applications. The concept of A/V ratio is typically used in the selection of a suitable earthquake record. The A/V ratio is a factor that can be easily calculated. Any earthquake record can be categorized to fall in one of three A/V ratio categories without knowing the specific nature of the earthquake. This is an important aspect to consider in the design process. It is somewhat feasible to determine for a specific site the most probable type of A/V ratio that could occur. It is practically much more difficult to determine the probable intensity, frequency content or any other specifics of a potential earthquake. A number of previous investigations were based on specific earthquake records. These investigations are useful to determine the effects of the motion on a structural component. However, all the data compiled is related exclusively to a specific set of earthquake records. It can be argued that the same structural component will react in a similar fashion to another earthquake but there is never any measure of the probability involved. In this research, by using 15 different earthquake records we are able to use statistical parameters to assess the validity of the trends found in the processed data.

The number of earthquake records that should be incorporated in a statistical analysis of this nature is an important consideration. However, three important constraints must be considered: a) the computing time required to perform the computations; b) the time and effort required to perform the statistical analysis of the data; 3) the degree of approximation that is required. The processing of the data obtained from numerical analysis is extensive when it comes to statistical analysis. The analysis does not need to be more precise than the uncertainty associated with typical design uncertainties in earthquake engineering. Taking all these factors into consideration, it was decided that the original research would concentrate on 15 earthquake records. At the final stage, depending on the correlations and observations of the results, recommendations regarding the number of earthquake records can be made.

2.5 NUMERICAL ANALYSIS

2.5.1 INTRODUCTION

In order to use numerical analysis techniques, a mathematical model is used. From dynamic equilibrium, the equations of motion for a 2-DOF system can be written as:

$$[M]\begin{Bmatrix}\ddot{u}_1\\ \ddot{u}_2\end{Bmatrix} + [C]\begin{Bmatrix}\dot{u}_1\\ \dot{u}_2\end{Bmatrix} + [K]\begin{Bmatrix}u_1\\ u_2\end{Bmatrix} = -\begin{Bmatrix}m_p\\ m_s\end{Bmatrix}\ddot{y}_g \qquad (2.2)$$

Where
$$[M] = \begin{bmatrix} m_p & 0 \\ 0 & m_s \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_p+c_s & -c_s \\ -c_s & c_s \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_p+k_s & -k_s \\ -k_s & k_s \end{bmatrix} \qquad (2.3)$$

$$u_1 = x_1 - y_g$$
$$u_2 = x_2 - y_g \qquad (2.4)$$

$$\dot{u}_1 = \dot{x}_1 - \dot{y}_g$$
$$\dot{u}_2 = \dot{x}_2 - \dot{y}_g \qquad (2.5)$$

$$\ddot{u}_1 = \ddot{x}_1 - \ddot{y}_g \longrightarrow \ddot{x}_1 = \ddot{u}_1 + \ddot{y}_g$$
$$\ddot{u}_2 = \ddot{x}_2 - \ddot{y}_g \longrightarrow \ddot{x}_2 = \ddot{u}_2 + \ddot{y}_g \qquad (2.6)$$

The subscripts p and s refer to the primary and secondary subsystems, respectively. The subscripts 1 and 2 denote the first degree-of-freedom and the second degree-of-freedom. Finally, the subscript g refers to the ground motion.

Equations 2.4 expresses the relative displacement in terms of the absolute displacements and the ground motion. Equations 2.5 and 2.6 are derived from equations 2.4 by differentiation. The dots denote the differentiation with respect to time. The ground displacement imposed at the foundation of the structure is defined by $\ddot{y}_g(t)$.

Equations 2.2 can be approximated for seismic design purpose by assuming the subsystems as uncoupled and that the supported system (secondary) does not affect the dynamic response of the supporting system (primary). Under this condition the following equations are derived:

$$m_p \ddot{u}_p + c_p \dot{u}_p + k_p u_p = -m_p \ddot{y}_g$$

$$m_s \ddot{u}_s + c_s \dot{u}_s + k_s u_s = -m_s \ddot{x}_p \qquad (2.7)$$

where the subscript p and s are used to denote the primary (structure) and secondary (equipment) responses.

$$u_p = u_1$$

$$u_s = u_2 - u_1 \qquad (2.8)$$

In equations 2.7, the $\ddot{x}_p$ represents the absolute acceleration response of the primary system and is used as the ground motion input for the secondary system, where

$$\ddot{x}_p = \ddot{u}_p + \ddot{y}_g \qquad (2.9)$$

2.5.2 THE WILSON-θ METHOD

The Wilson-θ method is an extension of the step-by-step linear acceleration method of numerical integration. The variation of acceleration of the system is assumed linear over the time interval considered. The Wilson-θ method is based on an extended time interval defined by $\tau = \theta \Delta t$ where Δt is the integration time interval (sec). The value of the factor θ is set to obtain the desired level of stability and accuracy. Various studies (Bathe and Wilson, 1973; Paz, 1980) have established that a θ value of at least 1.37 is required for unconditional convergence of the analysis.

In the current numerical solution approach, it is assumed that the properties of the system calculated at the beginning of each interval remain constant over the entire time interval.

The procedure and the method of analysis used are well established and discussed in many references such as by Paz (1980). The series of equations and steps to perform the analysis are listed in appendix A. The following is a summary of the steps and involved in this approach. The Wilson-θ method uses the known quantities of the system at a time t, to calculate the variations for the time interval $(t + \tau)$. The variation of the acceleration can be found for the extended time interval (τ). To obtain the variation of the acceleration for the time interval (Δt), a simple interpolation is performed between the time t and $(t + \tau)$. The variation in velocity and displacement for the time interval (Δt) are easily derived. To determine the actual displacement and velocity at the time $(t + \Delta t)$, the variations are added to the initial values at time t. The acceleration at time $(t + \Delta t)$ is found by considering the dynamic equilibrium using the equation of motion at the time $(t + \Delta t)$. The values calculated at the time $(t + \Delta t)$ will then become the initial conditions for the next step. The approach is repeated for the full duration of the earthquake ground motion record.

2.5.3 PROGRAM VERIFICATION

The program used in this research is an extension of the one developed by Nguyen (1986) with various modifications pertaining to the input and output of data. The core of the program remains unchanged. The modified version was verified by using the test cases found in Nguyen's (1986) thesis. The results were similar. Also, trial runs were performed using the three original earthquake records used by Nguyen. The results obtained were again satisfactory.

2.5.4 PROGRAM ACCURACY

In order to optimize the computing time available, the time intervals chosen were around 1/10 of the primary period. In a numerical analysis, the time interval is very important. From various papers (Bathe and Wilson, 1973; Biggs, 1964; Clough and Penzien, 1975) the 1/10 is an acceptable limit. Analyses performed with a smaller time interval may yield results which are slightly different. However, the trends will remain the same; especially when we consider the coupled over the uncoupled ratio. It must be understood that the objective is not to obtain the "exact" precise response values but, to determine the general behavior of structural components under various earthquake motions. Those earthquakes are highly variable in nature and the "exact" value for a specific earthquake will not provide any additional useful information.

The computing time required to perform the numerical analysis can be appreciable. For example, 3 minutes are needed on a VAX-11/785 to calculate the uncoupled and coupled responses when using 1 earthquake record, 1 mass ratio, 1 frequency ratio, 16 yield level combinations and a time interval of Δt equal to 0.01 second. The computing time is significant because hundreds of different parameter combinations are studied.

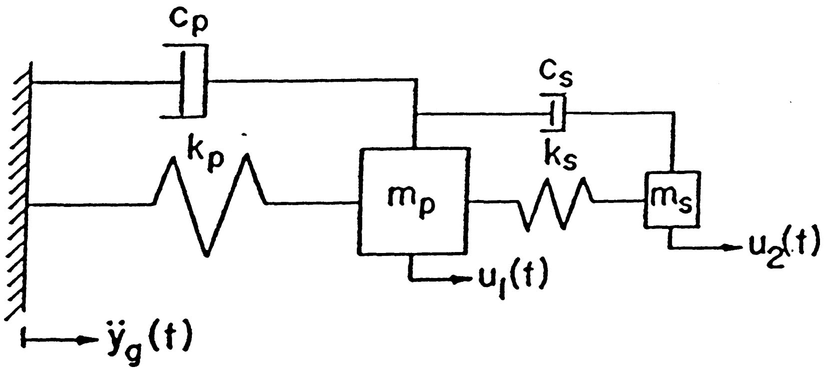


Figure 2.1 Two-degree-of-freedom coupled model.

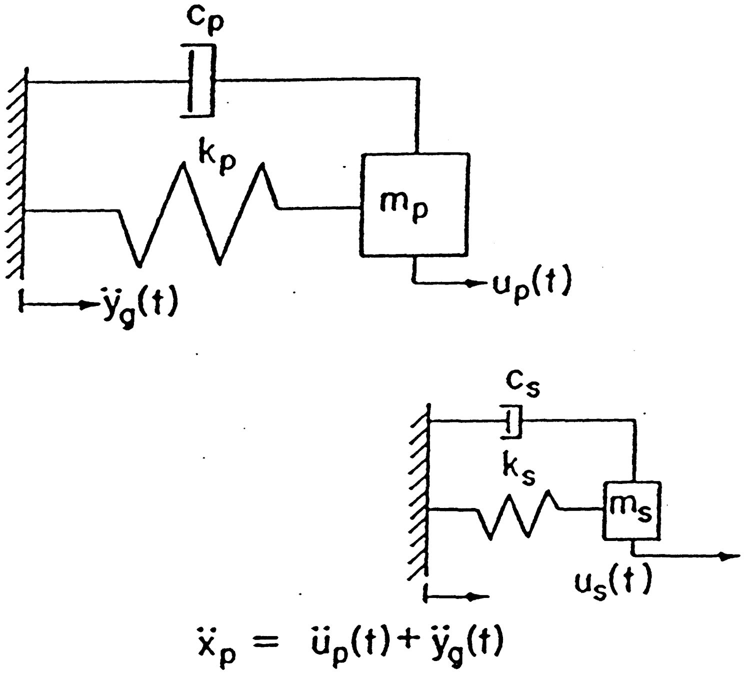


Figure 2.2 Decoupled models: a) 1-DOF primary system with ground motion input; b) 1-DOF secondary system with primary acceleration response input.

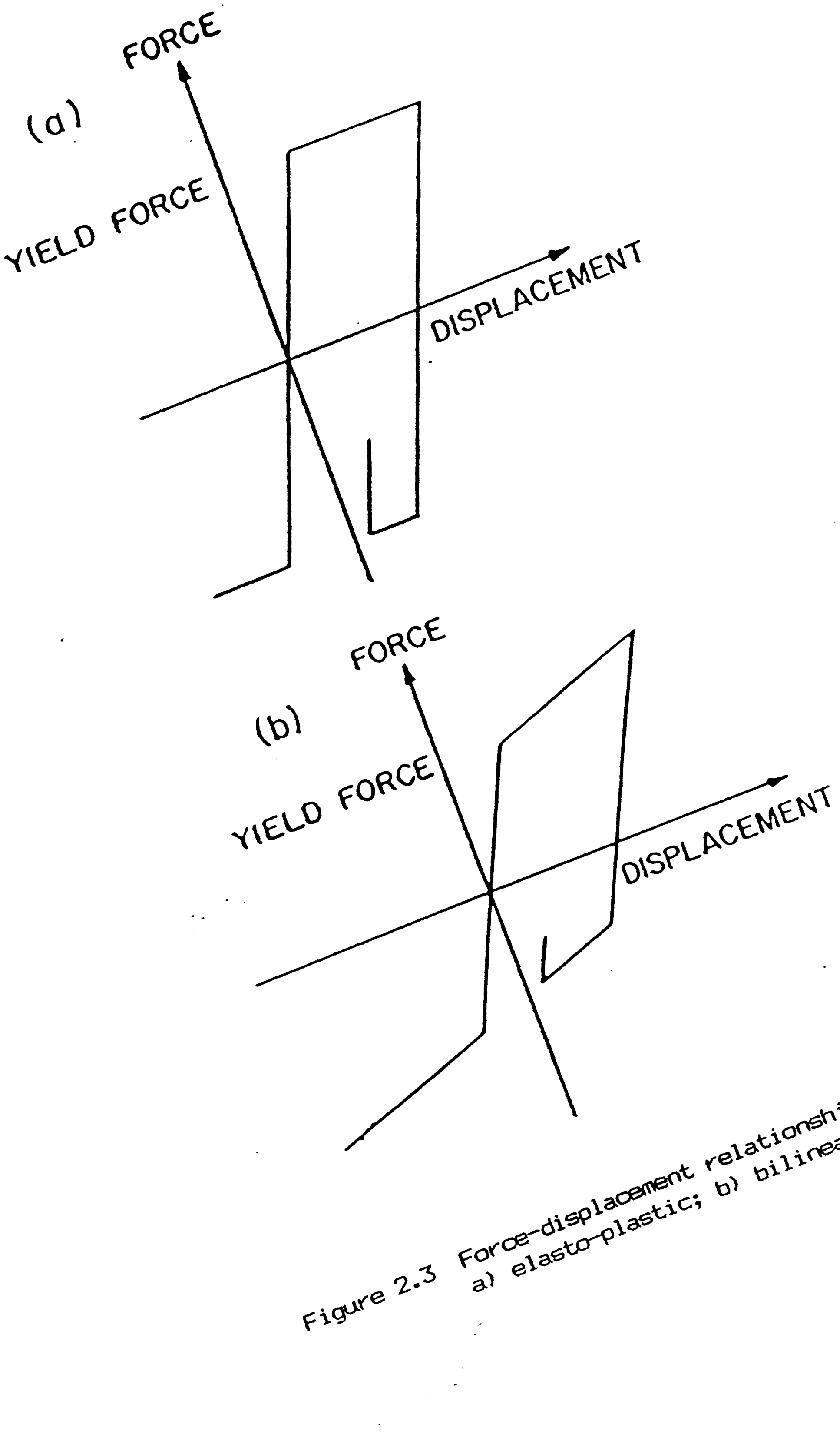
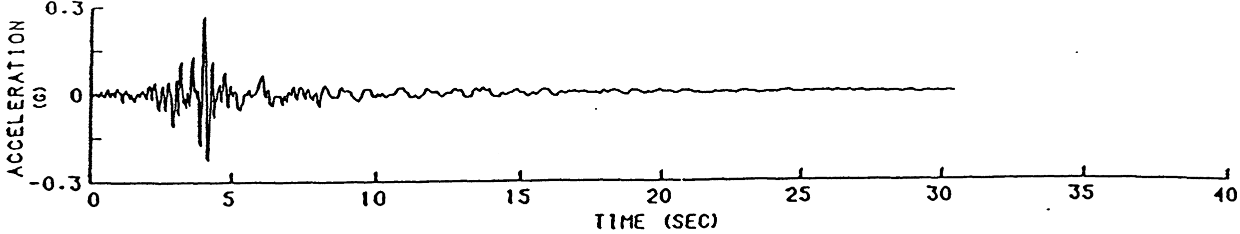
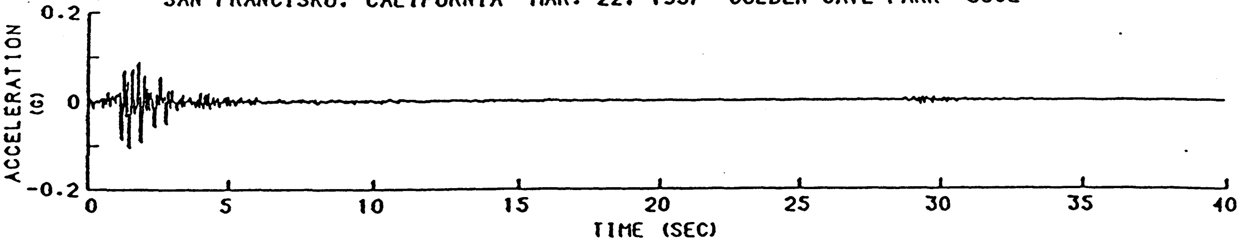


Figure 2.3 Force-displacement relationships:
a) elasto-plastic; b) bilinear.

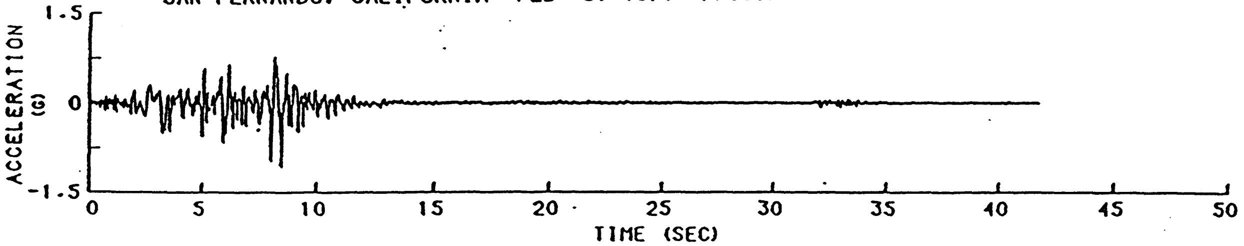
1 PARKFIELD. CALIFORNIA JUNE 27. 1966 TEMBLOR NO.2 N65W



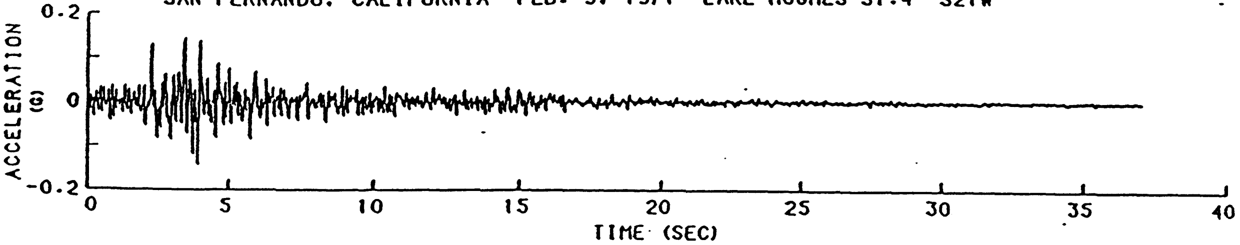
2 SAN FRANCISKO. CALIFORNIA MAR. 22. 1957 GOLDEN GATE PARK S80E



3 SAN FERNANDO. CALIFORNIA FEB. 9. 1971 PACOIMA DAM S74W



4 SAN FERNANDO. CALIFORNIA FEB. 9. 1971 LAKE HUGHES ST.4 S21W



5 BANJA LUKA. YUGOSLAVIA AUG. 13. 1981 SEISM. STATION N90W

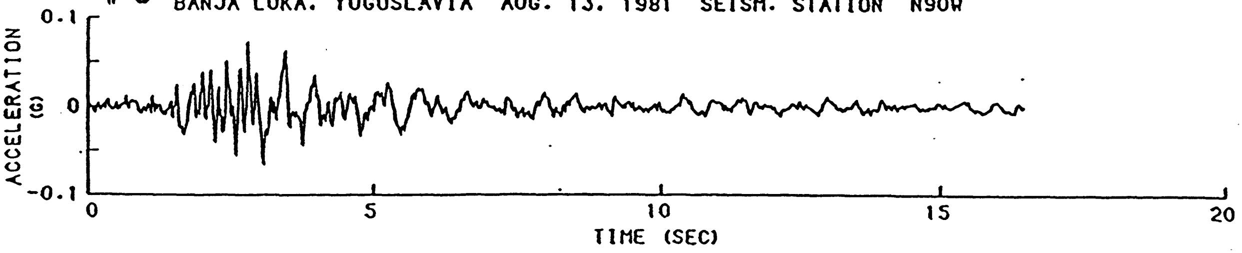
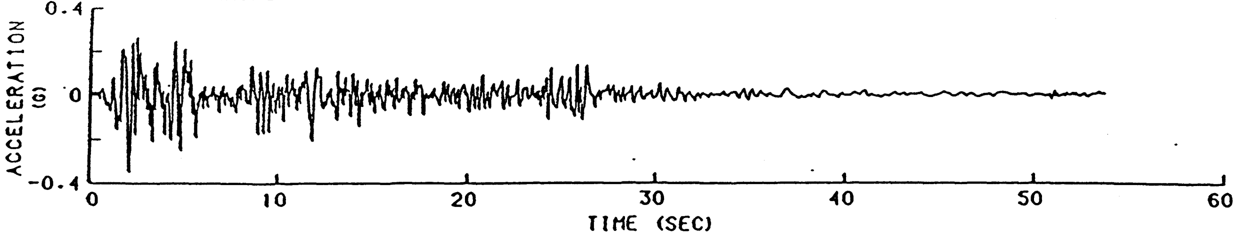
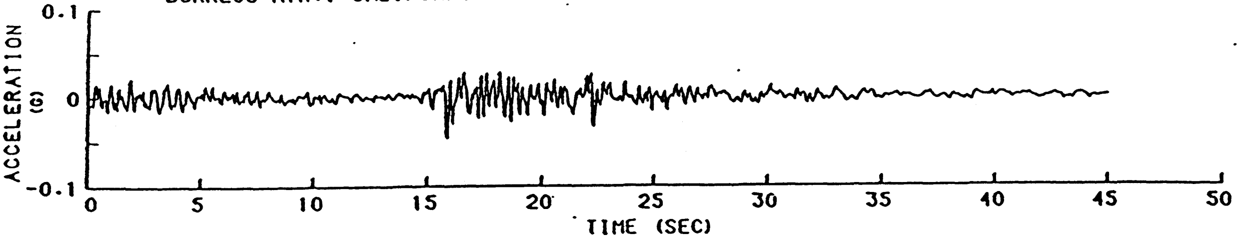


Figure 2.4 High A/V ratio earthquake records.

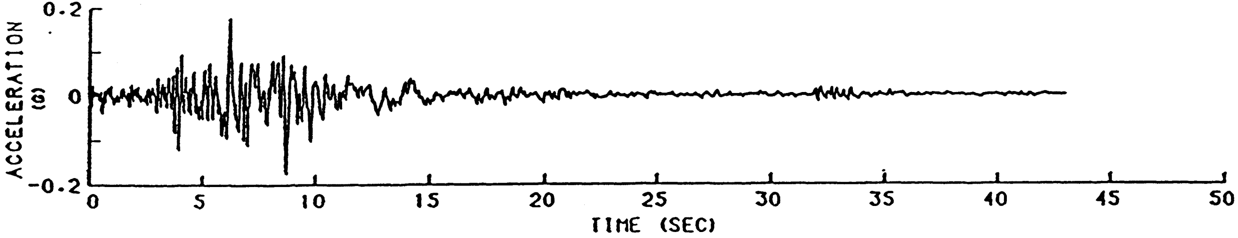
6 IMPERIAL VALLEY, CALIFORNIA MAY 18, 1940 EL CENTRO S00E



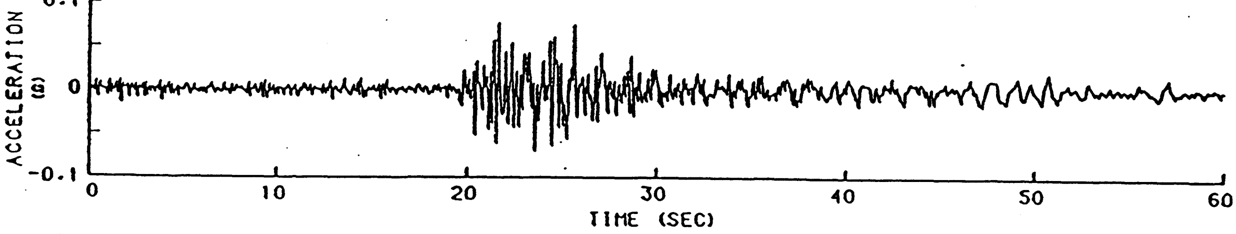
7 BORREGO MTN., CALIFORNIA APR. 8, 1968 SAN ONOFRE N57W



8 SAN FERNANDO, CALIFORNIA FEB. 9, 1971 GRIFFITH PARK OBS. S00W



9 NEAR E. COAST OF HONSHU, JAPAN AUG. 2, 1971 KUSHIRO CENTRAL WHARF N90E



#10 MEXICO EARTHQUAKE SEPT. 19, 1985 LA VILLITA N90E

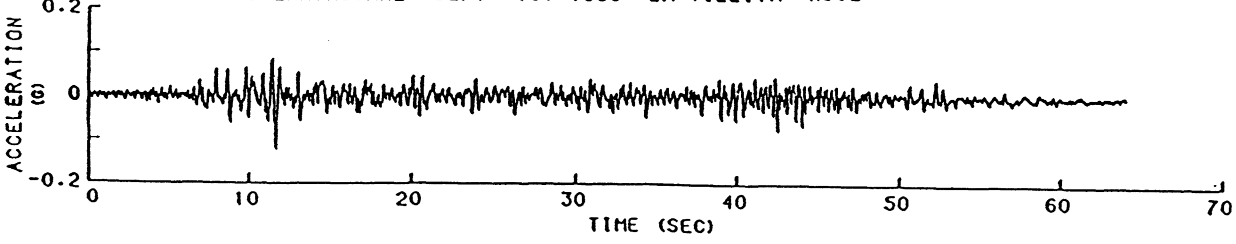
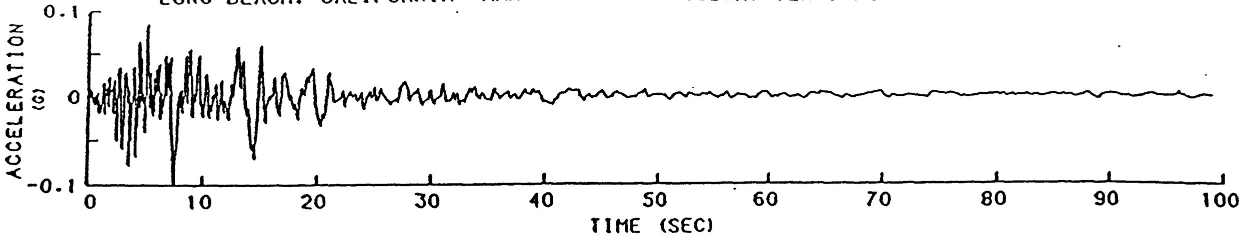
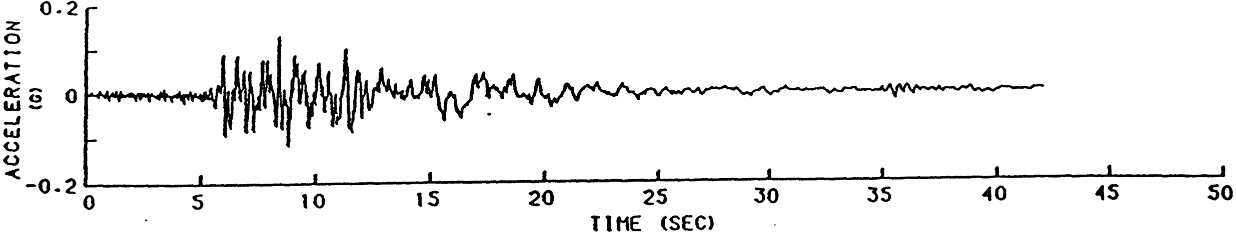


Figure 2.5 Intermediate A/V ratio earthquake records.

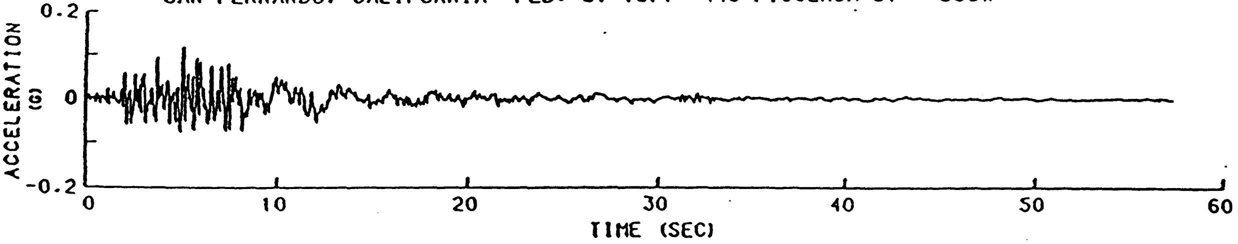
#11 LONG BEACH. CALIFORNIA MAR. 10. 1933 SUBWAY TERMINAL BLDG. N51W



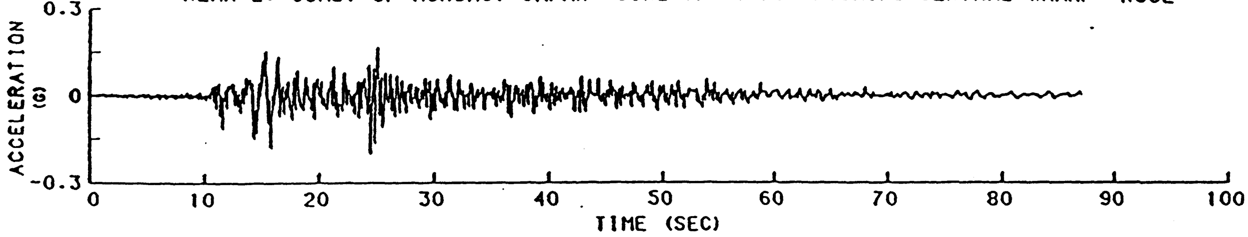
#12 SAN FERNANDO. CALIFORNIA FEB. 9. 1971 3550 WILSHIRE BLVD. WEST



#13 SAN FERNANDO. CALIFORNIA FEB. 9. 1971 445 FIGUEROA ST. S38W



#14 NEAR E. COAST OF HONSHU. JAPAN JUNE 17. 1973 KUSHIRO CENTRAL WHARF NOOE



#15 MEXICO EARTHOUAKE SEPT. 19. 1985 MESA VIBRADORA C.U.. MEXICO CITY N90W

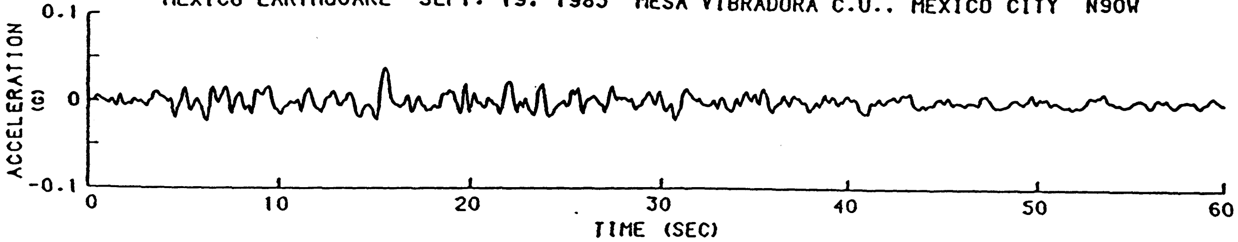


Figure 2.6 Low A/V ratio earthquake records.

Table 2.1 — Strong Ground Motion Earthquake Records

Record Number	Name	Date	Magn.	Epic. Dist. (km)	Comp.	Max. Acc. A(g)	Max. Vel. V(m/s)	A/V
HIGH A/V RATIOS								
1	Parkfield California	27/06/66	5.6	7	N65W	0.269	0.145	1.86
2	San Francisko California	22/03/57	5.25	11	S80E	0.105	0.046	2.28
3	San Fernando California	09/02/71	6.4	4	S74W	1.075	0.577	1.86
4	San Fernando California	09/02/71	6.4	26	S21W	0.146	0.085	1.72
5	Banja Luka Yugoslavia	13/08/81	6.1	8.5	N90W	0.074	0.032	2.31
INTERMEDIATE A/V RATIOS								
6	Imperial Valley California	18/05/40	6.6	8	S00E	0.348	0.334	1.04
7	Borrego Mtn. California	08/04/68	6.5	122	N57W	0.046	0.042	1.10
8	San Fernando California	09/02/71	6.4	31	S00W	0.180	0.205	0.88
9	Near South Coast of Honshu Japan	02/08/71	7.0	196	N90E	0.078	0.068	1.15
10	Mexico Earthquake	19/09/85	8.1	44	N90E	0.123	0.105	1.17
LOW A/V RATIOS								
11	Long Beach California	10/03/33	6.3	59	N51W	0.097	0.237	0.41
12	San Fernando California	09/02/71	6.4	39	WEST	0.132	0.216	0.61
13	San Fernando California	09/02/71	6.4	41	S38W	0.119	0.173	0.69
14	Near East Coast of Honshu Japan	17/06/73	7.4	112	N00E	0.205	0.275	0.75
15	Mexico Earthquake	19/09/85	8.1	379	N90W	0.040	0.110	0.36

CHAPTER 3 -- COUPLING EFFECTS OF ELASTIC SYSTEMS

3.1 INTRODUCTION

The response characteristics of elastic systems are analyzed in this chapter. The behavior and level of the dynamic response observed in this investigation will be established. The response of elastic systems is required to establish a base of comparison for non-linear systems. The effects of the mass ratio, frequency ratio and ground motion are quantified and discussed. Some comments regarding the decoupling of elastic systems, are formulated.

3.2 RESPONSE OF THE ELASTIC SYSTEM

The emphases in this work as discussed before is directed toward the response of the systems under a varied range of parameters. The study was performed using the range of parameters as specified in chapter 2. To illustrate the response, the results using a damping ratio of $\beta=3\%$ are presented in this chapter. The systems were subjected to a total of 15 earthquake records divided into three different A/V ratio categories as discussed before. Only elastic systems are considered in this chapter. For this purpose the yield level R was fixed at a value of one.

In order to interpret and compare the various response variations, the following scheme will be used to illustrate the various trends observed in the dynamic responses of the systems. Curves representing the various responses will be plotted against the frequency ratio as the main parameter. Each individual curve will represent one of the following: a) the average response of 5 earthquake records (either high A/V records, intermediate A/V records or low A/V records); or b) the average of all 15 earthquake records. Each curve has a legend that defines it according to the set of earthquake record used in the analysis.

In all cases, the analysis is performed by using ground motions which are normalized to a spectral acceleration of 1.0g at the primary system frequency of interest. The following quantities are used to plot the various dynamic responses: absolute acceleration (CPA, CSA, UPA and USA) as a percentage of (g); relative velocity (CPV, CSV, UPV and USV) in (cm/s); and relative displacement (CPD, CSD, UPD and USD) in (cm). These are always the maximum response value noted after an analysis with specific parameters is completed.

Throughout this chapter the coupled and uncoupled response are considered separately to determine their corresponding dynamic behavior under similar conditions.

Before analyzing the effect of various parameters, it is important to establish the general dynamic response in order to describe and quantify the effect of each parameter on the systems response. For this purpose, a set of coupled system responses for a primary period T_1=0.2 s and a mass ratio μ=0.1% was chosen. At this very low mass ratio μ=0.1%, the response of the coupled and uncoupled systems are for all practical purposes identical. Figures 3.1 to 3.6 demonstrate the characteristic behavior of each response value as a function of the frequency ratio. A few observations should be noted.

1. The primary system response is almost constant over the entire range of frequency ratios range.

2. The secondary system response has a very definite peak maximum response at a frequency ratio of 1.0. The values of the secondary response at any other frequency ratio are less than at the frequency ratio of fr=1.0.

The effect of the primary period on the dynamic response can be observed in figures 3.7 to 3.12 which represent the average of all 15 earthquake motion records. Figures 3.7, 3.9 and 3.11 represent the CSA response with a primary period of 0.1, 0.2 and 1.0 s respectively. Figures 3.8, 3.10 and 3.12 represent the CSD response under similar conditions. To gain further knowledge of the response variability involved; the following table gives the average response values for the uncoupled analysis at fr=1.0:

	T_1=0.1s	T_1=0.2s	T_1=1.0s	units
UPA	1.0	1.0	1.0	g
UPD	0.25	1.0	25	cm
UPV	11	27	160	cm/s
USA	7	8	8	g
USD	2	8	200	cm
USV	100	260	1200	cm/s

The primary period has a large influence on the response of the structural elements when the resonance case of fr=1.0 is considered.

There is a definite change in the response when the primary period is increased from 0.1 to 1.0 seconds. For instance, the USD changes by a factor of 100 between T_1=0.1 s and T_1=1.0 s and the USV varies by a factor of 10. However, the uncoupled acceleration response at fr=1.0 remains practically constant, independent of the primary period. The effects of the primary period on the CSA response are shown in figures 3.7, 3.9 and 3.11. Figures 3.8, 3.10 and 3.12 illustrate the change in CSD response according to the primary period.

Figure 3.8 reveals that the case of fr=0.1 and T_1=0.1 s produces relatively high response results for the secondary system. This behavior is present only for the secondary displacement and velocity component at that specific frequency ratio. The primary system has a frequency of 10 Hz and a secondary system has a frequency of 1 Hz. This represents a factor of 10 in variation. It should be noted that all the other frequency ratios have a factor of 5 only or less. The error may be attributed to the numerical analysis which may not be able to accommodate these large differences in properties.

3.3 EFFECT OF MASS RATIO ON SYSTEM RESPONSE

The mass ratio does not have any effect on the uncoupled response behavior.

For coupled systems, a very small mass ratio μ=0.1% produces response values that are practically identical to the uncoupled analysis. The effect of the mass ratio on the secondary system can be observed in figures 3.7 to 3.12 where the CSA and CSD response are plotted for each mass ratio studied. An increase in the mass ratio will generally decrease the response. The amount of decrease depends on the response quantity studied. Specific trends are observed when the mass ratio increases from μ=0.1% to μ=10% for coupled systems. The CSA, CSV and CSD response decrease for frequency ratio between fr=0.8 and fr=1.25. The CPA and CPD response can increase or decrease slightly depending on the frequency ratio. The CPV response usually decreases slightly between fr=0.8 and fr=1.5.

The main observation is that the mass ratio as a parameter affects, to a small extent, the primary system response. However, the magnitude of the secondary response is very dependant on the mass ratio. Another point that is revealed by the current analysis is that at low and high frequency ratio, the mass ratio does not have a significant effect on all the responses quantities.

It appears that the CSA and CSD responses are greatly influenced by the mass ratio. For example, the maximum response values are obtained for the case of a very small mass ratio μ=0.1%. At a μ=10%, the CSA response is approximately one third the value for μ=0.1% in the region of fr=1.0, as shown in figures 3.7, 3.9 and 3.11. It is observed that for high mass ratios (μ=5% or μ=10%), the peak maximum CSA response can occur at fr=0.8 or fr=0.9. Curves from figures 3.7 and 3.9 clearly demonstrate this behavior. Figure 3.10 shows that the peak maximum CSD response occurs at fr=0.5 for high mass ratios. This is an important observation because it demonstrates that the peak maximum response does not necessarily happen at fr=1.0. In this case, the heavy secondary system contributes to a shift in the modal frequencies and the resonance condition does not exist at fr=1.0.

3.4 EFFECT OF FREQUENCY RATIO ON RESPONSE VARIATION

For the USA response, the values vary according to the frequency ratios with the maximum occurring at about fr=1.0. The behavior shown by the curve is practically the same: at low and high frequency ratios (fr=0.1 or fr=2.0) the value of USA response is around 1.0g. Between these frequency ratios, the values are increasing towards a peak value at fr=1.0.

For USD, the behavior is usually consistent except for fr=0.1 where extremely large variations occur according to the A/V ratio. Apart from that point, the peak value occurs at fr=1.0. The values decrease to almost zero at fr=2.0. The behavior of USV is similar to the USD because the values at fr=0.1 are very different and are much higher than at fr=1.0.

The frequency ratio, fr=0.1 is only used when the primary system has a period of T_1=0.1 s. There appear to be some numerical instability at these lower limits. When we consider the standard deviation for the USD and USV, we find that the deviation seems higher for small frequency ratios (fr less than 1.0). This is especially true for fr=0.5 when the deviation is about the same as for fr=1.0.

For USA, USD and USV, we observe that the area of amplification (where the response is close or equal to the maximum) is very limited. The maximum response occurs between fr=0.8 and fr=1.25. When values outside these limits are considered, the response is low compare to the peak values at fr=1.0.

The UPD response is the most consistent value which is virtually independent of the earthquake record used. It remains constant over the range of frequency ratios studied.

Because of the normalization, the maximum uncoupled primary acceleration is constant at 1.0g. In the coupled analysis, for the low and high frequency ratios(fr=0.1, fr=2.0 or fr=5.0) the CPA response is equal to 1.0g. In the frequency ratio range from fr=0.8 to fr=1.25, the CPA response is less than 1.0g but usually not less than 0.8g depending on the mass ratio.

The trends for the CPV response are similar to CPA response. In general, the frequency ratio range from fr=0.8 to fr=1.25, the coupled response is less than the uncoupled analysis. The coupling reduces the response of the primary system, however, the reduction is minimal.

The behavior of the CSD response is similar to the uncoupled analysis where at fr=0.1 the values are extremely large. Also, for the frequency ratios smaller that fr=1.0, we notice that the A/V ratios affect the response, as shown in figure 3.5. It appears that a certain distinction can be made between low and high frequency ratios (for CSD and CSV). The configuration of the curve for CSD with mass ratio of μ=10% and μ=5% for T_1=0.1 s and T_1=0.2 s is very different compared to the other mass ratios. The difference is for frequency ratios less than 1.0. The response values are all greater than the one at fr=1.0. This is depicted in figure 3.10. This behavior appears to be limited to the

CSD and CSV responses. The combination of high mass ratio and low frequency ratio should be carefully considered, because the response is very unusual.

For the CSV response, the values are abnormally large at a fr=0.1. Also, for T_1=1.0 s and frequency ratios smaller that fr=1.0, sometimes the response is higher than that at fr=1.0. For T_1=0.2 s, trends are dependent on the mass ratio. The shape of the curves is similar but the region of the peak response is less definite except with μ=0.1%.

3.5 EFFECT OF GROUND MOTION ON SYSTEM RESPONSE

The effect of the ground motion can be best established by considering figures 3.1 to 3.6. Each curve in these figures, developed for a mass ratio of μ=0.1%, represents either an average of 5 high, intermediate or low A/V ratio records or either the average of the 15 earthquake records. As a general rule, the response values decrease from high to intermediate and to low A/V ratios. However, these response values remain in the same range and are not influenced greatly by the frequency content of the earthquake records. Usually, the degree of deviation in the response according to the A/V ratio will vary depending on the frequency ratio considered. However, it appears that the CPV response behaves differently. As figure 3.3 demonstrates, the difference between the various A/V ratios appear constant over the entire range of frequency ratios. Another aspect to consider is the behavior of the response at certain specific frequency ratios. This point can be illustrated be considering figure 3.5. It can be seen that at the frequency ratio, fr=0.8, all A/V ratio curves appear to coincide to a common CSD response value. For smaller frequency ratios, the usual tendency of response decrease when the A/V ratio decreases is now reversed. This behavior is observed mostly for secondary responses. It indicates that the frequency content of the earthquake does indeed have an effect on the secondary response. However, from a design point of view, figures 3.1 to 3.6 clearly demonstrate that the variations are negligible.

3.6 DISCUSSIONS ON POSSIBLE DECOUPLING CRITERIA

When we have a very small mass ratio, the correlation between the coupled and uncoupled response is extremely good. Practically, a coupled analysis with a very small secondary system has the same response as an uncoupled analysis. For the other mass ratios, the correlation is closer for frequency ratios larger than fr=1.0 compared to frequency ratios smaller than fr=1.0. However, these variations are still very small.

The shape of the uncoupled curves usually have a very definite peak. However, when considering the coupled analysis, we notice that the peak is not as definite. Also, in some instances, the maximum value does not occur at resonance where fr=1.0. This behavior can be associated to the shift in modal frequencies that takes place in a coupled analysis.

The ground motion frequency content does not influence significantly the uncoupled or coupled analysis.

The frequency ratio is another important aspect in terms of defining the area where the largest response will occur. As expected, in most cases, the peak occurs at the resonance case where fr=1.0. However, in the coupled analysis, this is not always true. The maximum peak may occur at a slightly lower frequency ratio. It rarely occurs at a frequency ratio higher than fr=1.0. The shift in modal frequencies of the systems in a coupled analysis can be associated with this behavior. The extreme maximum response values are for the most part contained in the region of fr=0.8 to fr=1.25.

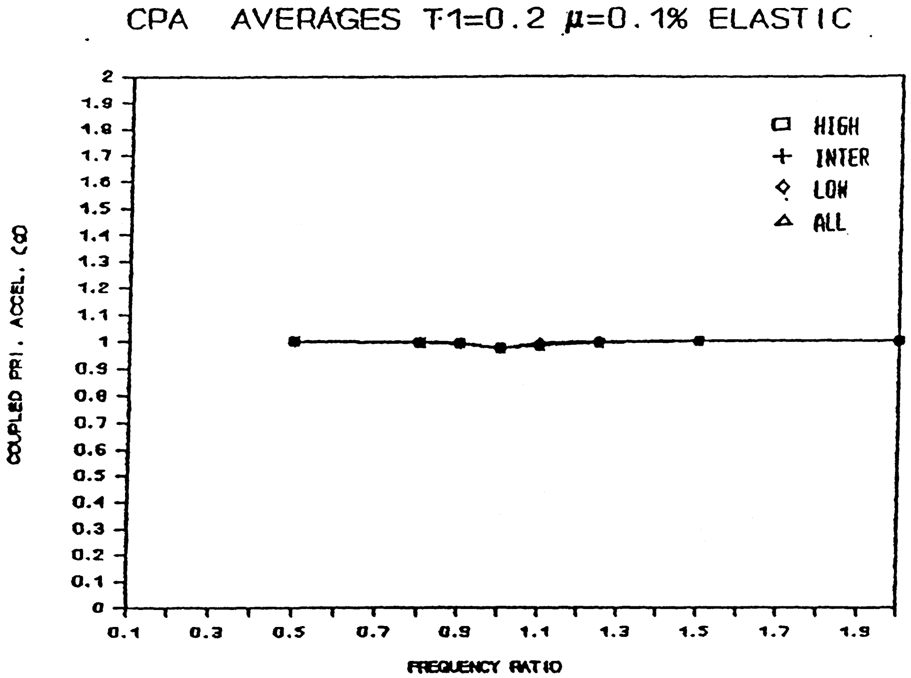


Figure 3.1 Coupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, averages for high, intermediate, low and all A/V earthquake records using β=3% and mass ratio of 0.1%.

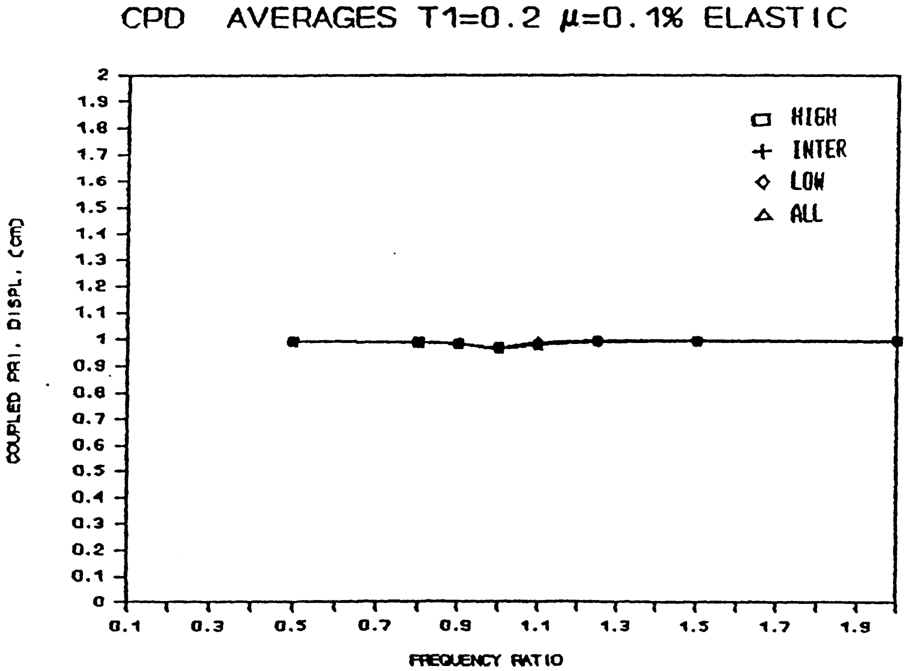


Figure 3.2 Coupled Primary Displacement response versus frequency ratio for T1=0.2 sec, averages for high, intermediate, low and all A/V earthquake records using β=3% and mass ratio of 0.1%.

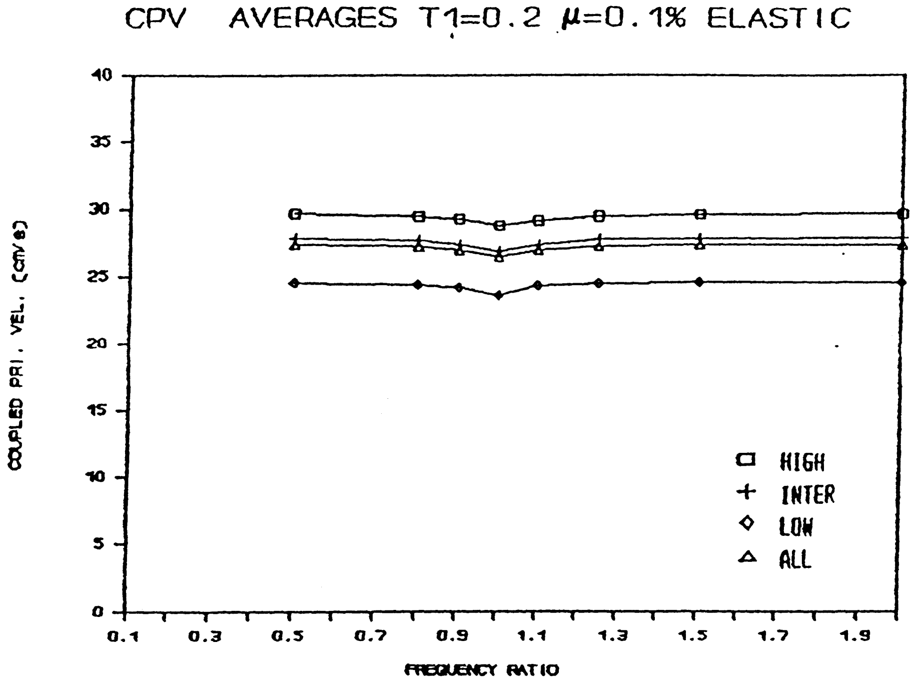


Figure 3.3 Coupled Primary Velocity response versus frequency ratio for T1=0.2 sec, averages for high, intermediate, low and all A/V earthquake records using β=3% and mass ratio of 0.1%.

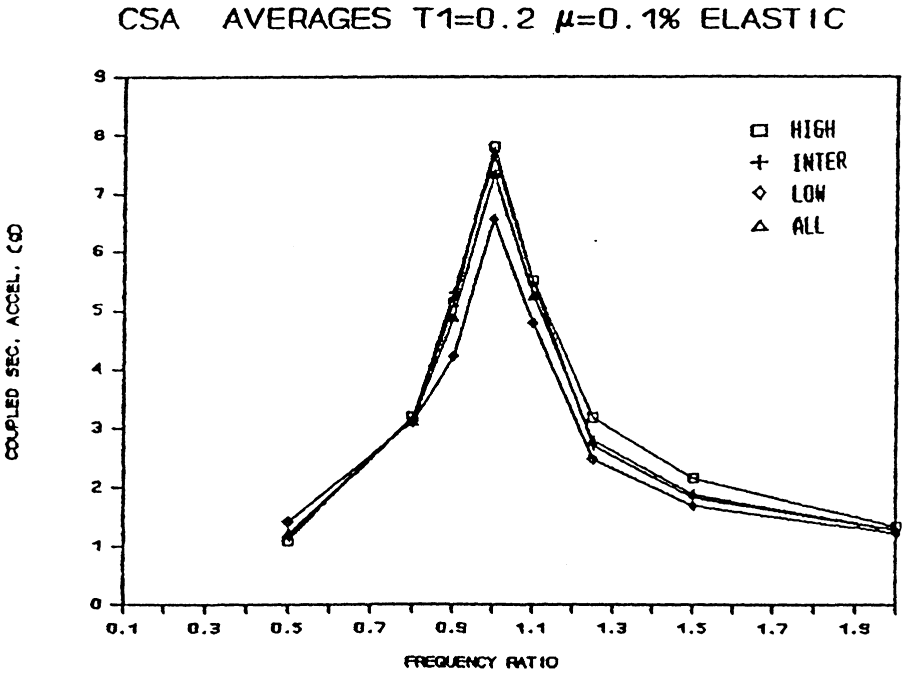


Figure 3.4 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, averages for high, intermediate, low and all A/V earthquake records using β=3% and mass ratio of 0.1%.

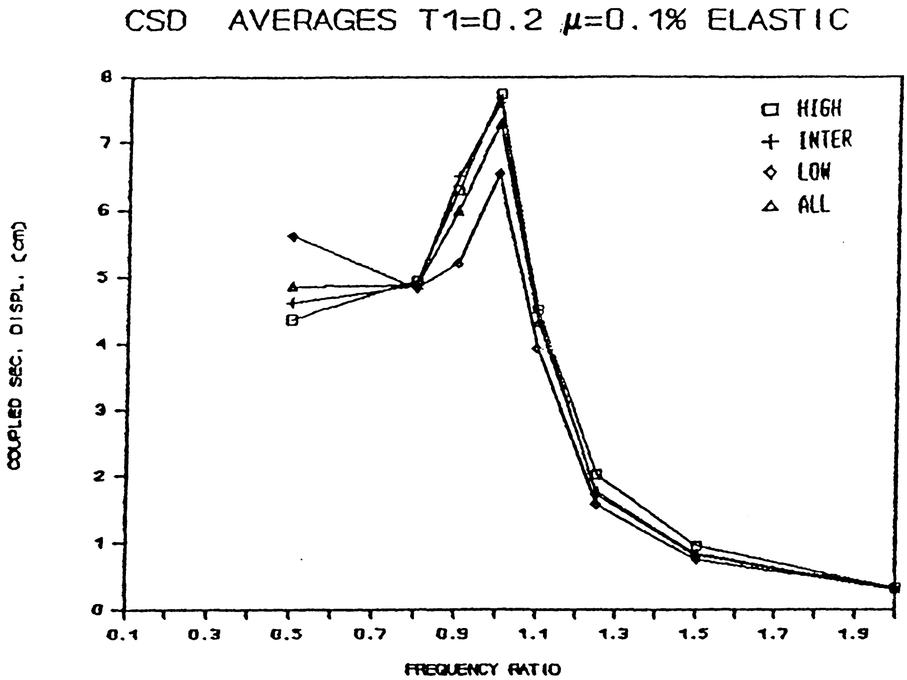


Figure 3.5 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, averages for high, intermediate, low and all A/V earthquake records using ß=3% and mass ratio of 0.1%.

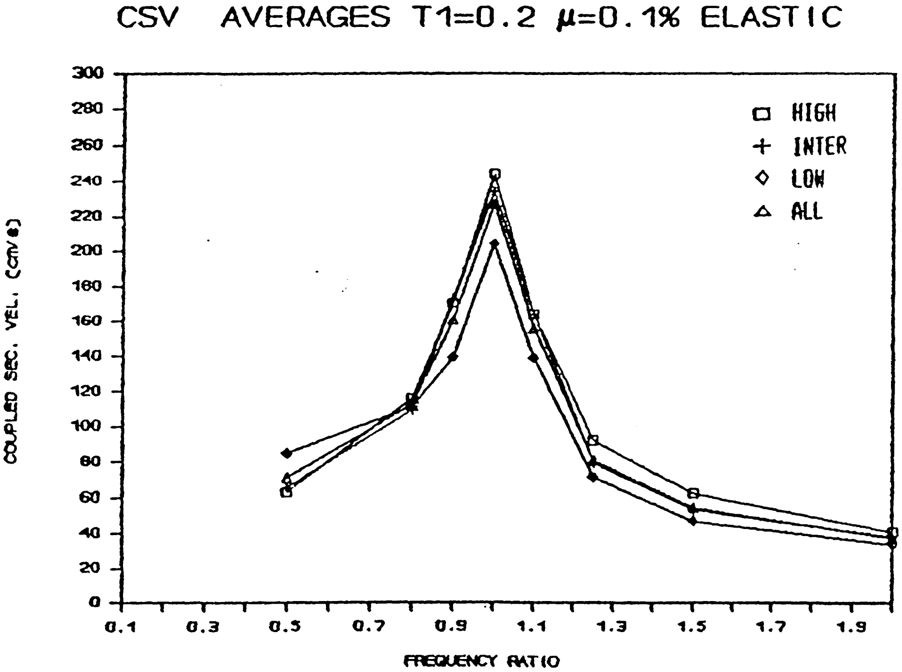


Figure 3.6 Coupled Secondary Velocity response versus frequency ratio for T1=0.2 sec, averages for high, intermediate, low and all A/V earthquake records using ß=3% and mass ratio of 0.1%.

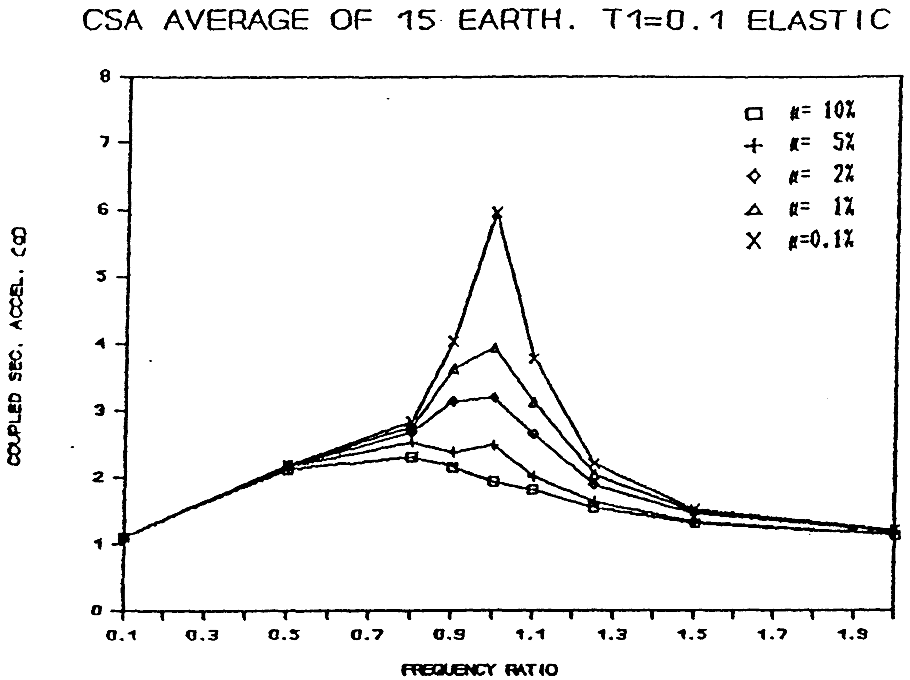


Figure 3.7 Coupled Secondary Acceleration response versus frequency ratio for T1=0.1 sec, average of 15 earthquake records using β=3% and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

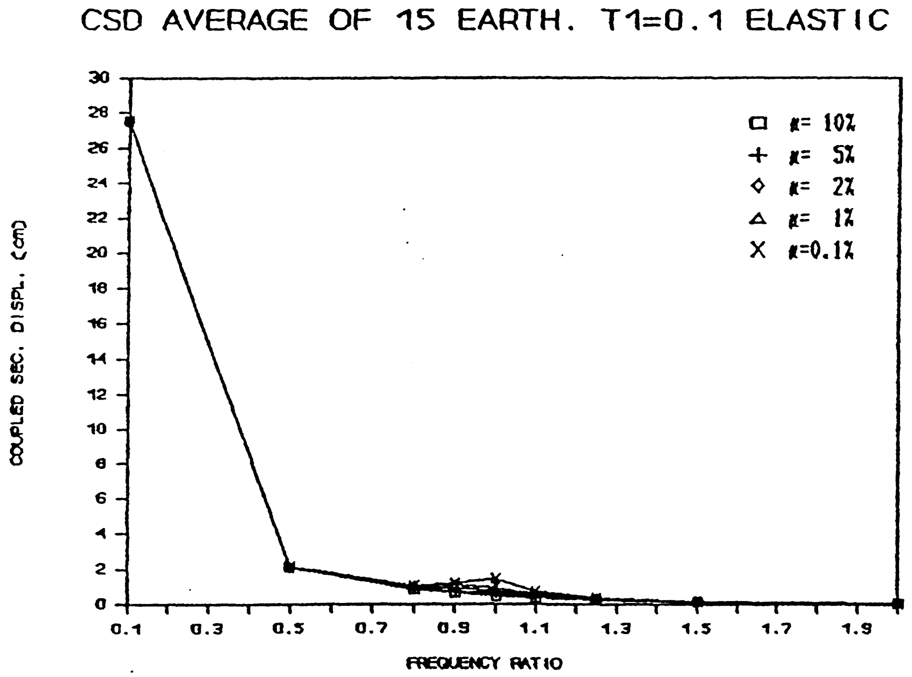


Figure 3.8 Coupled Secondary Displacement response versus frequency ratio for T1=0.1 sec, average of 15 earthquake records using β=3% and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

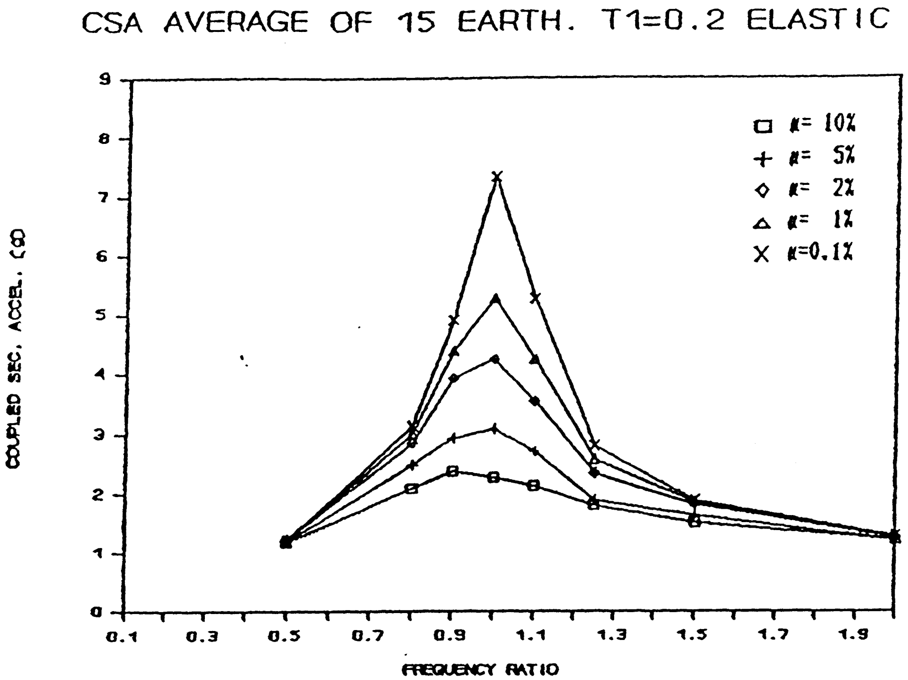


Figure 3.9 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 15 earthquake records using β=3% and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

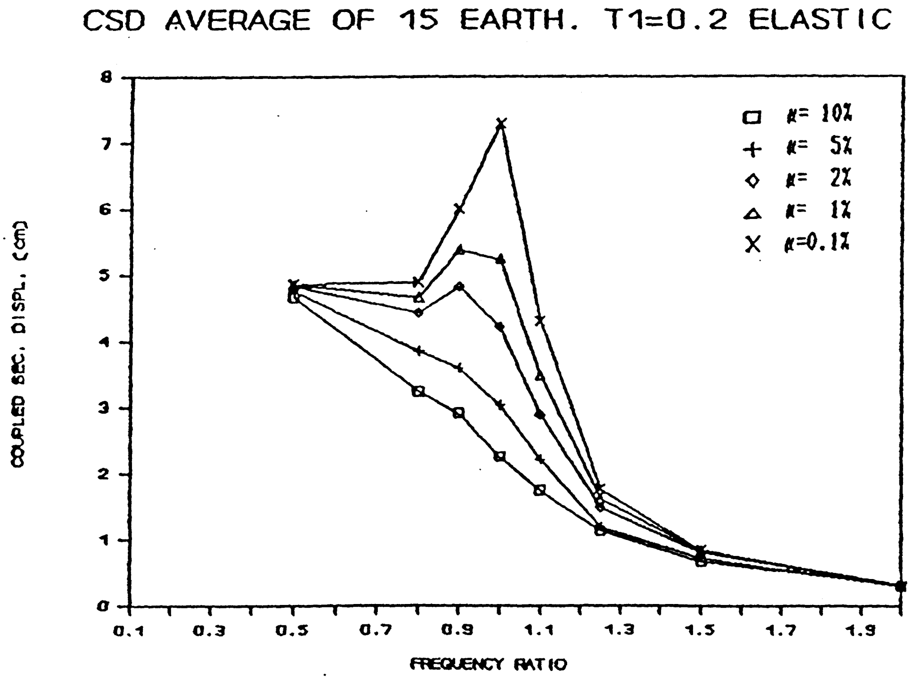


Figure 3.10 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 15 earthquake records using β=3% and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

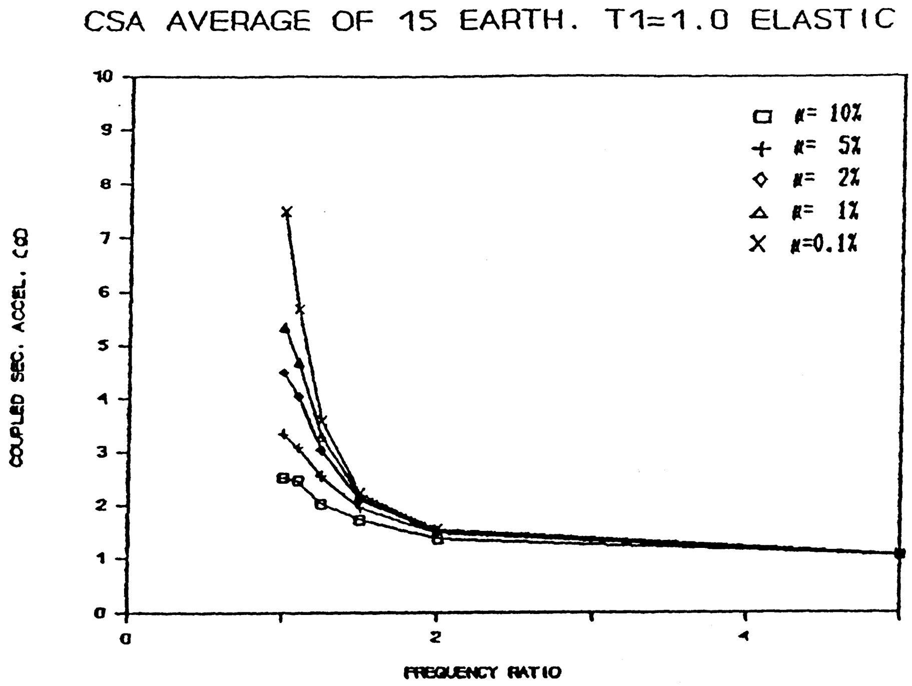


Figure 3.11 Coupled Secondary Acceleration response versus frequency ratio for T1=1.0 sec, average of 15 earthquake records using β=3% and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

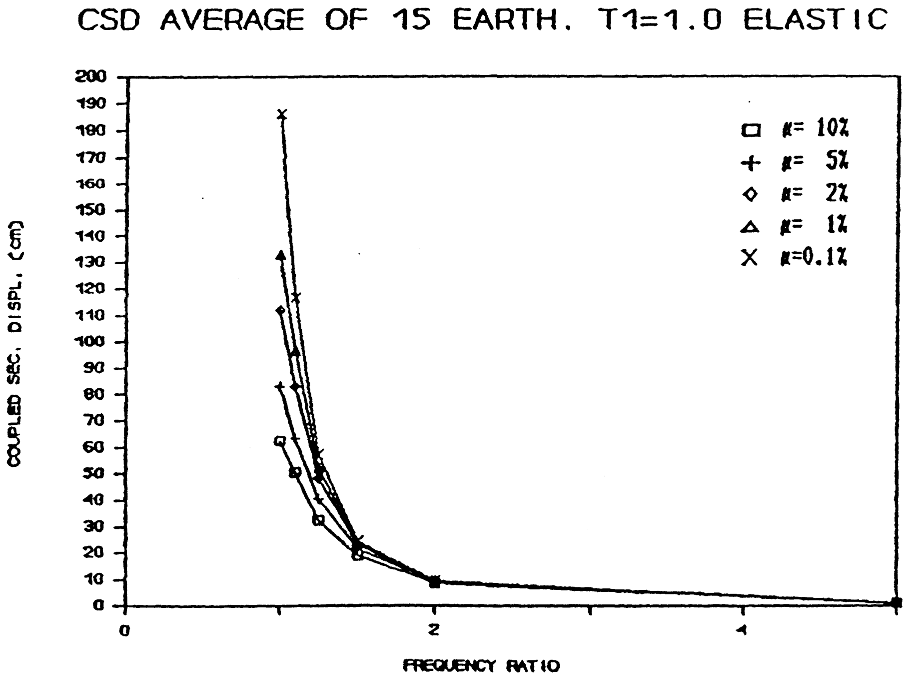


Figure 3.12 Coupled Secondary Displacement response versus frequency ratio for T1=1.0 sec, average of 15 earthquake records using β=3% and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

CHAPTER 4 -- COUPLING EFFECTS OF INELASTIC SYSTEMS

4.1 INTRODUCTION

The behavior of elasto-plastic systems is investigated in this chapter. The bilinear system is also discussed. The analysis of the results is performed in a similar manner to that of the elastic system. The main objective is to determine the effect of yield levels on the dynamic response of the systems. The effect of the mass ratio and ground motion characteristics on the system response are investigated. The decoupling of the subsystems is also addressed.

4.2 RESPONSE OF INELASTIC SYSTEMS

Inelastic systems play a major role in practical seismic design applications. In order to minimize materials, weights but, mostly cost, inelastic systems are usually used in design applications. The concepts of plasticity and inelasticity are complex subjects and can be developed or formulated by various approaches. In the present research, yield level factors based on the maximum elastic force are used. By limiting the available maximum force resistance, hysteretic loops will dissipate the input energy. Once again, various models can be used to represent the hysteretic loops. This study deals mainly with elasto-plastic systems. A bilinear model with a 10% slope of the inelastic stiffness curve is used to investigate the effect of the strain hardening. The emphasis is placed on the general interaction behavior and not the type of model used to include the inelastic effects in the system.

A comparison of the results obtained using damping ratios of 3 and 5% reveal that a 5% damping ratio usually produces smaller response values. The results presented in this chapter will be limited to a damping ratio of 3% which will result in large response values and more conservative observations.

The actual behavior of a bilinear system using ß=3% is compared to an elasto-plastic system using ß=5% and an elasto-plastic system using ß=3%. The purpose being to evaluate the effect of damping and the inelastic model on the system response. The Imperial Valley intermediate A/V ratio record #6 is used for this analysis. The uncoupled acceleration and displacement responses are presented in appendix B. It was found that the bilinear(ß=3%) and elasto-plastic(ß=3%) system give similar results for both the acceleration and displacement responses. The primary responses(UPA and UPD) obtained with an elasto-plastic(ß=5%) system are higher compared to the response of an elasto-plastic(ß=3%) system. The elasto-plastic(ß=5%) system produces secondary responses(USA and USA) which are lower than the ones obtained with an elasto-plastic(ß=3%) system. However, for frequency ratios of 1.25 and higher, both systems (elasto-plastic ß=3% or ß=5%) give almost identical results.

4.3 COUPLING EFFECT OF ELASTO-PLASTIC SYSTEM

The comments in the following sections are only applicable to the case of a primary period of T_1=0.2 s. In an attempt to demonstrate the dynamic behavior of both the primary and secondary systems, the comments will be limited to the acceleration and displacement components. For instance, the comments regarding the mass ratio effect on the CSA response and the CSV response are similar. In order to simplify the presentation, the velocity response analysis will not be included in this chapter.

Yield level factors of 0.25, 0.50, 0.75 and 1.0 are used for the primary and secondary systems. The yield level of 1.0 is used to analyze the response of elastic systems coupled with inelastic systems. A systematic study of 16 different yield level combinations is performed. The data is presented by plotting the response as a function of the frequency ratio (based on the initial fundamental frequencies) for specific primary and secondary yield level combinations. For instance, figures 4.1 and 4.8 illustrate the variation of the USA and the USD responses with different yield levels. In these graphs the secondary yield level is set to a specific value and the primary yield level is allowed to vary. The USA response is shown in figures 4.1 to 4.4. The USD response is shown in figures 4.5 to 4.8. A complete set of figures representing the CSA and the CSD inelastic response for mass ratios of 0.1, 1.0, 2.0, 5.0 and 10.0% is presented in appendix C.

4.3.1 EFFECT OF MASS RATIO ON SYSTEM RESPONSE

The uncoupled analysis is not influenced by the mass ratio due to the lack of interaction between the primary and secondary systems in the analysis procedure.

As a general observation, the secondary response quantities decrease as the mass ratio increases. However, figures 4.9 to 4.16 show that the response at high and low frequency ratios seems independent of the mass ratio.

The mass ratio has a similar effect on both the CSA and the CSD response even if these quantities are very different in terms of the type of response according to the frequency ratio. Figures 4.9 to 4.12 present the CSA response and figures 4.13 to 4.16 illustrate the CSD response. In these figures, the response is plotted against the frequency ratio according to the mass ratio. The curves(response vs frequency ratio) in figures 4.9 to 4.12 are presented for the yield level combinations: $R_1=R_2=1.0$, $R_1=R_2=0.75$, $R_1=R_2=0.50$ and $R_1=R_2=0.25$, respectively. The CSA and the CSD response generally decrease as the mass ratio increase. The mass ratio has a significant effect on the secondary responses when considering frequency ratios between fr=0.8 and fr=1.25. In all other cases, for low and high frequency ratios, the CSA and CSD responses are independent of the mass ratio. At a mass ratio of μ=0.1%, the magnitude and the shape of the curves(response vs frequency ratio) are practically the same as for the uncoupled response. For a mass ratio of μ=10%, the maximum response can be less than half to a third the peak response at μ=0.1% for frequency ratios in the range of fr=0.8 to fr=1.25. The decrease in the CSA or the CSD response is maximum when either the primary or the secondary system remain elastic. As the mass ratio decreases, all the curves(response vs frequency ratio) for all secondary yield levels tend to converge towards a common value. When the secondary yield level is equal to or less than R_2=0.50, the mass ratio does not have a significant effect on the CSA and CSD response. Similarly, a primary yield level of R_1=0.25 does not affect the secondary responses. The extent of the decrease in the CSA or CSD response is dependant on the available ductility of both the primary and secondary systems. The shape of the curves(CSA or CSD response vs frequency ratio) shown in figures 4.9 to 4.16 appears to be affected by the mass ratio. An important aspect is the location of the maximum response. In general, the curves do not have a definite peak maximum value for high mass ratios. For mass ratio of μ=2% and smaller, there is a definite peak at fr=1.0. However, for larger mass ratio the peak CSA response is in the range of fr=0.8 to fr=1.25 (except when R_2=0.25 where the peak remains at fr=1.0). The peak CSD maximum response for large mass ratios occurs at a very low frequency ratio, fr=0.5. This appears to suggest that when we have somewhat heavy secondary systems, the peak maximum CSA or CSD response will not automatically be at fr=1.0. In such a case, the response to a near field event must also be considered in practice.

The mass ratio has virtually no effect on the primary response quantities. Figures 4.17 to 4.20 demonstrate that the CPA and the CPD behave in a similar manner. These figures represent the CPA and the CPD responses for mass ratios of 0.1% and 10.0%.

4.3.2 EFFECT OF FREQUENCY RATIO ON SYSTEM RESPONSE

In general, the USA and CSA responses are close to or smaller than 1.0g at low and high frequency ratios(i.e. fr=0.5 and fr=2.0). Considering that the earthquake records are normalized to a spectral acceleration of 1.0g at a particular primary frequency, these frequency ratios do not contribute to an amplification of the secondary acceleration response.

The frequency ratios between 0.8 and 1.25 have a great influence on the USA response. A substantial increase in response reaching the maximum values occurs at the frequency ratio of fr=1.0. Figures 4.1 to 4.4 demonstrate that the combination of primary and secondary yield levels is a crucial factor in determining the magnitude of the USA response for each frequency ratio.

The peak maximum CSA response usually occurs at a frequency ratio of fr=1.0. Figures 4.9 to 4.12 represent the CSA response. For mass ratio of μ=2% or higher, the peak response may occur at a frequency ratio in the range of fr=0.8 to fr=0.9. This behavior appears to be limited mostly to systems with secondary yield levels of R_2=1.0 or R_2=0.75. It seems that the maximum response can take place at frequency ratio less than fr=1.0. However, there is no case where the peak response occurred at a frequency ratio higher than fr=1.0. The most sensitive region is between the frequency ratio of fr=0.8 and fr=1.25. Outside these limits, the CSA response is around 1.0g or less. From a practical perspective, it can be concluded that for mass ratios of μ=10% and μ=5%, the CSA response for the frequency ratios fr=0.9, 1.0 or 1.1 is similar except may be when the secondary yield level is very low R_2=0.25.

Figures 4.5 to 4.8 show that the USD response is sensitive to the combination of frequency ratio and yield level. There is a definite region from fr=0.8 to fr=1.1 where the peak maximum USD response occurs. The response at the low frequency ratio range is much higher than the response at the high frequency ratios. There is a great difference in magnitude depending on the frequency ratio considered. For yield levels of 0.50 and 0.25, the peak maximum response appears at a low frequency ratio, fr=0.5 (this is not always true for high A/V records). In the frequency ratio range from fr=0.8 to fr=1.1, the influence of resonance and near resonance is noticed by an amplification in the response. For the high frequency ratios, the response consistently diminishes toward a very small value except for the case of R_2=0.25, where there is an increase.

In most cases, the peak maximum CSD response does not occur at fr=1.0. This behavior is observed for all mass ratios except for μ=0.1%. The CSD response is shown in figures 4.13 to 4.16. The CSD response is maximum at a low frequency ratio (i.e. fr=0.5) then, the CSD response decreases almost linearly to values near zero for high frequency ratios. The curves (CSD response vs frequency ratio) for R_2=0.25 in combination with a primary yield level of 1.0, 0.75 or 0.50, behave differently: the CSD response is also high at high frequency ratios.

Figures 4.21 to 4.22 demonstrate that the primary uncoupled responses are not dependant on the frequency ratio because for a series of cases the primary frequency is fixed and only the secondary frequency varies in order to obtain various frequency ratios.

The frequency ratio has a minor effect on the CPA and CPD responses. The curves(response vs frequency ratio) are almost horizontal lines. Figures 4.17 to 4.20 illustrate the CPA and the CPD responses. We can identify a region from fr=0.8 to fr=1.25 where the magnitude of the response usually decreases slightly.

4.3.3 EFFECT OF YIELD LEVEL ON SYSTEM RESPONSE

The primary yield level has a significant influence on the USA response. Figures 4.1 and 4.4 illustrate the USA response. The maximum response values occur for an elastic primary system. When the primary yield level is reduced, the USA response also decreases. A yield level of R_1=0.75 does not reduce the response substantially. However, for yield levels of R_1=0.50 and R_1=0.25 the USA response is greatly reduced. The minimum response appears to occur when the primary and secondary yield levels are low. As the primary yield level decreases, the shape of the curves(response vs frequency ratio) in the vicinity of the peak value changes. The curves are much flatter and the peak response is not much higher than the response for frequency ratio from fr=0.8 to fr=1.25.

The secondary yield level is also an important parameter for the USA response. The response is maximum when the yield level is $R_2=1.0$ and minimum for $R_2=0.25$ and the difference in response is dramatic. An important phenomena to notice is the USA response at $R_2=0.25$. At this secondary yield level, the response is practically independent of the primary yield level. For $R_2=0.50$, the response is similar when using primary yield levels of 1.0, 0.75 or 0.50.

For a primary yield level of $R_1=0.25$, the USA response is not influence by the value of the secondary yield level. The curves are all of the same magnitude. It could be generalized as stating that at very low primary yield levels, the secondary yield level does not have much influence on the USA response. For other primary yield levels (i.e. 1.0, 0.75 or 0.50), the USA response is very dependent on the secondary yield level. By considering the level of reduction in USA response, we notice that the yield level is an important parameter in reducing the magnitude of the USA response. For the range of $R_1=1.0$ to $R_1=0.25$, the reduction in the USA response is more than two fold for $R_2=1.0$ and $R_2=0.75$. The reduction in response is less evident when $R_2=0.50$ or $R_2=0.25$. This points to the fact that ductility of the primary system as described by the primary yield level is more effective in reducing the secondary system response than the ductility of the secondary system itself as described by its yield level. On the other hand, if we can set the yield level of the secondary system to a low value (e.g. $R_2=0.25$), the yield level of the primary system does not appear as a factor in the USA response. These considerations can be very important from a design point of view where several constraints may limit the yield levels of the components.

Figures 4.9 to 4.12 show the CSA response when the primary and secondary yield levels are equal. A complete set of plots illustrating the CSA response is presented in appendix C. The general trend is that as the primary yield level decreases, the CSA response also decreases. For mass ratios μ=10%, 5% and 2%, the primary yield levels of R_1=1.0 and R_1=0.75 give practically the same response in terms of shape and magnitude. This is also true for mass ratios μ=1% or μ=0.1%, only when the secondary yield levels are R_2=0.50 or R_2=0.25. For the cases of R_1=0.50 and R_1=0.25, the secondary yield levels of R_2=1.0 or R_2=0.75 produce very similar results. For any primary yield level, the CSA response remains the same when the secondary yield level is R_2=0.25. As a general trend, the response decreases as the secondary yield level decreases. Throughout this analysis, the response for CSA is almost identical for systems with R_2=1.0 or R_2=0.75. For mass ratio of 2% or higher, the peak maximum response when R_2=0.50 is practically the same as for higher secondary yield levels. This suggests that to take advantage of the ductility of a secondary system, the system should have ample ductility. A yield level larger than R_2=0.50 does not significantly reduce the peak response.

The USD response appears to be one of the most variable responses in terms of frequency ratio and the types of earthquake considered. The USD response is illustrated in figures 4.5 and 4.8. In general, the USD response decreases with the decrease in the primary or secondary yield level. The case with a secondary yield level of 0.25 does not follow the trends of the other response cases for different values of R_2. With this in mind, the comments previously made are only applicable to R_2 values of 1.0, 0.75 and 0.50. The case with $R_2=0.25$ is considered special. It is somewhat difficult to explain the behavior of the USD response when the secondary yield level is at such low level (e.g. $R_2=0.25$). First, this response seems to be dependent on the type of earthquake record considered. In general, when considering high A/V ratio records, the response for the USD is "normal" which means that it follows the trends noticed with other response parameters. For example, the maximum peak response occurs at the same frequency ratio as the other responses with different values of R_2. The shape of the curves is similar to the curves with other R_2 values. For intermediate A/V ratios, the peak response values with $R_2=0.25$ is usually in the same range as for the case with $R_2=1.0$. Figures 4.5 and 4.8 illustrate the USD response for intermediate A/V ratios. This peak response occurs at a low frequency ratio, fr=0.5 and not at fr=1.0 as is the case with other R_2 values. The shape of the response is also very different. The lower magnitudes appear in the region where we noticed the peak maximum response for the other R_2 values. For low A/V ratios, the phenomena described for intermediate A/V ratios is even more noticeable. In fact, except for the values at the frequency ratios of fr=0.9, 1.0 and 1.1 all the other values can be considered "out of range" or unreasonable. At the frequency ratio of fr=1.0, the response values corresponding to the $R_2=0.25$ are very acceptable. However, especially with low A/V ratios, the response seems to be magnified erroneously. This phenomena appears to be linked to the frequency content of the earthquakes. For the high A/V ratio records, the response with R_2=0.25 appears "normal". With intermediate A/V ratios, we notice a distinct trend and at low A/V ratios, the response with R_2=0.25 is undoubtedly erroneous (except of course at fr=1.0). The secondary yield level may be a very sensitive factor that deserve special attention. The exact cause for these trends in the results remains unexplained. The accuracy of the program as been checked and established.

Figures 4.13 to 4.16 show the CSD response when the primary and secondary yield levels are equal. A detailed set of figures representing the CSD response with various yield level combinations is presented in appendix C. For all mass ratios, the cases where R_1=1.0 and R_1=0.75 produce practically identical CSD response values. For R_1=0.50 and R_1=0.25, the CSD response decreases, mostly for the region of low frequency ratios. For high frequency ratios, the decrease in CSD response only appears with R_2=0.50 and R_2=0.25.

The yield level is an important factor that influences greatly the UPA and the UPD responses, as demonstrated in figures 4.21 and 4.22. Similar effects are noted for the CPA and the CPD response as illustrated by figures 4.17 to 4.20. A decrease in the primary yield level will decrease the CPA response. Reducing the yield level factor from R_1=1.0 to R_1=0.25, the response values are decreased by two thirds. The secondary yield level has virtually no effect on the CPA response. A decrease in the primary yield level increases the CPD response. The increase in the response is significant when the yield level is reduced from $R_1=1.0$ to $R_1=0.50$. In this case, the increase in the response is a factor of five. The secondary yield level has minimal effect on the CPD response, all corresponding values are almost identical.

4.3.4 EFFECT OF GROUND MOTION ON SYSTEM RESPONSE

The USA and the CSA responses are practically not affected by the frequency content of the earthquake. For high, intermediate and low A/V ratios, the response remains practically identical. Only a very slight decrease is noticed when we compare low A/V ratios results to high and intermediate A/V ratios values. However, this small variation is negligible and all earthquake records could be grouped as a single sample.

The type of earthquake does not seem to be a factor affecting the USD response for the cases with $R_2=1.0$ or $R_2=0.75$. The corresponding values do not match exactly but their magnitude is of the same order. When $R_2=0.50$, the response appears a little more sensitive to the earthquake record for $R_1=1.0$ and $R_1=0.75$. More specifically, the results with the low A/V ratio records at a high frequency ratio differ from the trend observed with the intermediate and high A/V ratio records. With $R_1=0.50$ and $R_1=0.25$, the behavior with $R_2=0.50$ is similar for all the types of earthquake records.

The CSD response appear to be one of the most sensitive dynamic response to the ground motion. Figures 4.13 to 4.16 illustrate the CSD response for the intermediate A/V ratio. The A/V ratio plays a significant part in the CSD response when combined with the secondary yield level. For a secondary yield level of R_2=1.0 or R_2=0.75, the ground motion has virtually no influence on the CSD response. The cases with high A/V ratios are "normal" over the entire range of frequency ratio and for every secondary yield level combination. With the intermediate A/V ratio, the CSD response when R_2=0.25 is extremely large (except near fr=1.0). For low A/V ratios, the CSD response with R_2=0.50 is extremely large (except near fr=1.0) and the CSD response with R_2=0.25 is abnormally large (except near fr=1.0). As can be seen, the combination of A/V ratios and low secondary yield level values is critical. In general, the high, intermediate and low A/V ratio will produce a CSD response that is increasingly larger when R_2=0.25 or R_2=0.50 in that order.

The three sets of earthquake records give CPA responses which are all of the same magnitude. The responses are very similar. The strongest variations are in the case of R_1=1.0 where the shape of the curves may vary but the magnitudes are identical.

An earthquake with a low frequency content will produce a highly variable CSD response under certain conditions. This behavior is associated only with the specific response studied (i.e. CSD response) and the earthquake normalizing procedure. If the response values are globally considered, it is noted that the acceleration response is the most consistent, followed by the velocity response and finally the displacement response. All the earthquakes are normalized to a spectral ground acceleration of 1.0g. There is no limits on the corresponding velocity or displacement associated with these normalized earthquakes. Figure 4.23 taken from Naumoski et al. (1988) gives the mean response spectra for high, intermediate and low A/V records scaled to peak ground acceleration. Each curve of this tripartite plot of response spectra, represents an average of 15 earthquakes from the three different categories of A/V ratios. The curves are scaled to a peak ground acceleration of 1.0g and 5% damping ratio. For a period of 0.2 seconds and higher (i.e. a frequency of 5Hz and lower), large variations exist between the high, intermediate and low A/V ratios. In particular, for large periods (i.e. low frequencies) the spectral displacement for the high A/V ratio is almost constant. However, for the low A/V ratio, the spectral displacement is always increasing as the period increases. Figure 4.23 illustrates that the normalization process may significantly influence the dynamic response. A normalization with respect to the spectral displacement may produce a completely different set of response values in terms of magnitude. These comments confirm a basic aspect of all research projects: the findings have to be considered with due regard for the limitations, conditions and procedure used. In this work, the normalization was performed using the spectral acceleration associated with each primary frequency of interest.

4.3.5 EFFECT OF PRIMARY PERIOD ON SYSTEM RESPONSE

The primary system periods of 0.1, 0.2 and 1.0 seconds were studied using various frequency ratios. A frequency ratio of 5.0 was used with a primary system period of 5.0 seconds. Figures 4.24 to 4.29

show the USA and USD response for primary periods of 0.1, 0.2 and 1.0 seconds. These curves(response vs frequency ratio) are presented for the yield levels of $R_1=R_2=1.0$, $R_1=R_2=0.75$, $R_1=R_2=0.50$ and $R_1=R_2=0.25$, respectively. The secondary acceleration response, shown in figures 4.24, 4.26 and 4.28, is not greatly influenced by the period of the primary system. The secondary displacement response is very dependant on the primary system period. An increase in the primary period will generally increase the secondary displacement response. This behavior, as shown in figures 4.25, 4.27 and 4.29, is very noticeable in the range of frequency ratios from 0.8 to 1.25.

It is noted that frequency ratios of 2.0 and 5.0 produce similar effects on the system response.

4.3.6 DISCUSSIONS ON POSSIBLE DECOUPLING CRITERIA

The uncoupled analysis usually produces results which are slightly different from the coupled analysis. In order to establish if the values of the uncoupled analysis are sufficiently accurate, a level of error which is acceptable for practical purposes needs to be allowed. A permissible error of -15% or +25% in the response values is assumed.

This discussion is based on the average of 5 intermediate A/V ratio records. However, the range of the average response plus or minus one standard deviation is considered. This should ensure that the observations are fairly general and applicable to a system with similar characteristics. A sample of the ratio of coupled to uncoupled response values is presented in appendix D. In these figures, the average response and the average ± one standard deviation response is illustrated for a primary yield level of 0.50 and a secondary yield level which varies from 1.0 to 0.25. Mass ratios of 1.0, 2.0, 5.0, and 10.0% are used in these figures.

In general, the uncoupled secondary response is usually larger than the coupled secondary response.

The decoupling of secondary acceleration is always possible for μ=0.1%. When the secondary yield level is equal to 0.25, decoupling is usually possible even for high mass ratios. In the most extreme cases, the ratio of coupled to uncoupled secondary acceleration can be as low as 0.3 for fr=1.0. This implies that the uncoupled analysis is 3 times the value of the coupled analysis. For mass ratios of 1.0% and 2.0%, the decoupling is possible for high and low frequency ratios. Limitations arise when the secondary yield level is high in combination with near resonance properties. Decoupling of the secondary acceleration is usually not possible for mass ratios of 5.0% or higher.

Decoupling of the secondary displacement for μ=0.1% is usually possible except in a few instances involving a low primary yield level. Decoupling is in general not permitted for frequency ratios of 0.9 to 1.1 when using a mass ratio of 1.0%. At a mass ratio of 2.0%, the decoupling is only possible at low and high frequency ratios. For mass ratios of 5.0% or 10.0%, the decoupling is limited to a frequency ratio of 0.5. It is observed that the standard deviation of the secondary displacement is higher comparison to other responses especially when R_2=0.50 or R_2=0.25. This indicates that the secondary system response associated with each earthquake record is quite diverse.

The decoupling of the primary system responses is usually always possible for mass ratios of 0.1%, 1.0% and 2.0% except for frequency ratios of 1.0 and 1.1. For the primary acceleration system response, using μ=5% and μ=10%, the decoupling is always possible at high and low frequency ratios. Decoupling is also possible at these high mass ratios, whenever the primary yield level is equal to 0.75 or 0.50. The decoupling of the primary displacement is usually not permitted for mass ratios of 5.0% and 10.0%.

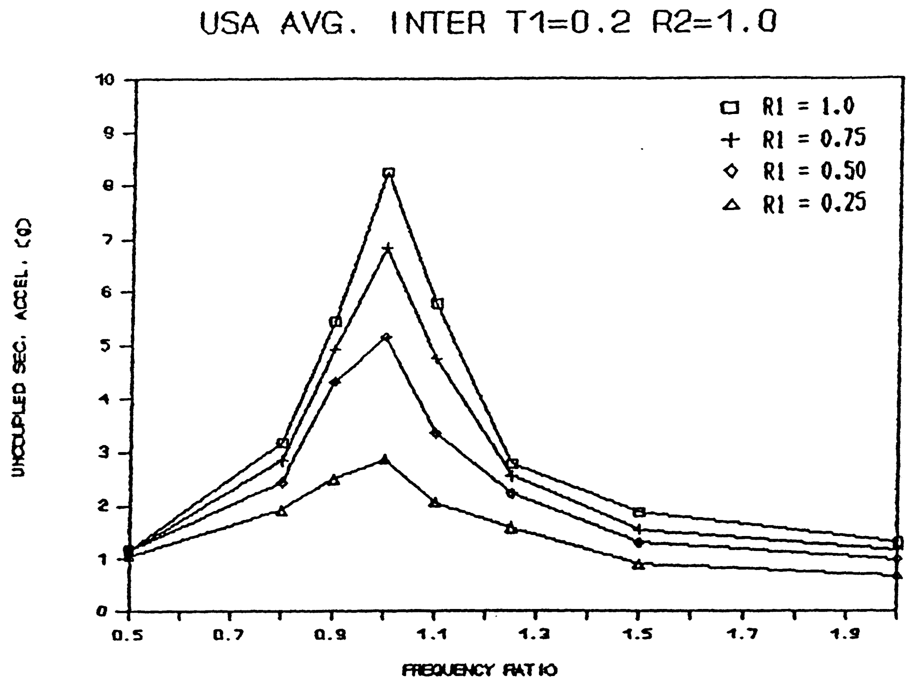


Figure 4.1 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R2=1.0 and R1 equal to 1.0, 0.75, 0.50 and 0.25.

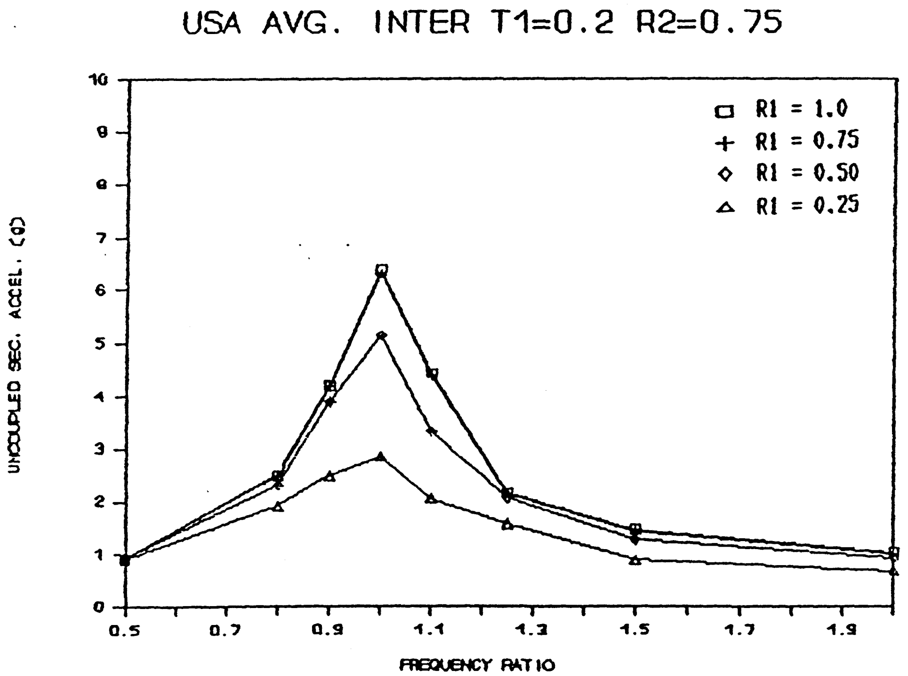


Figure 4.2 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R2=0.75 and R1 equal to 1.0, 0.75, 0.50 and 0.25.

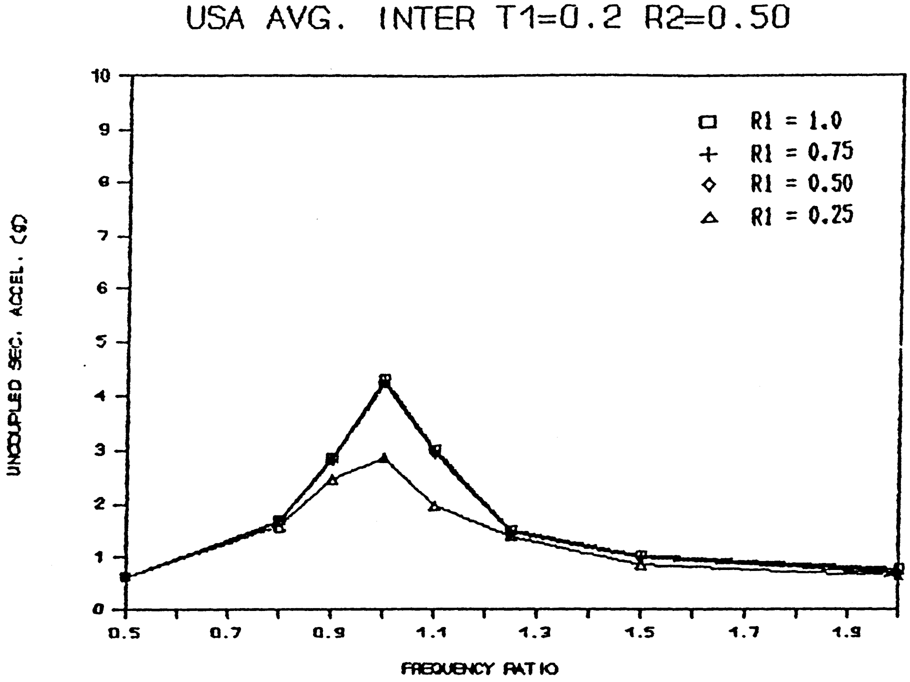


Figure 4.3 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R2=0.50 and R1= 1.0, 0.75, 0.50 and 0.25.

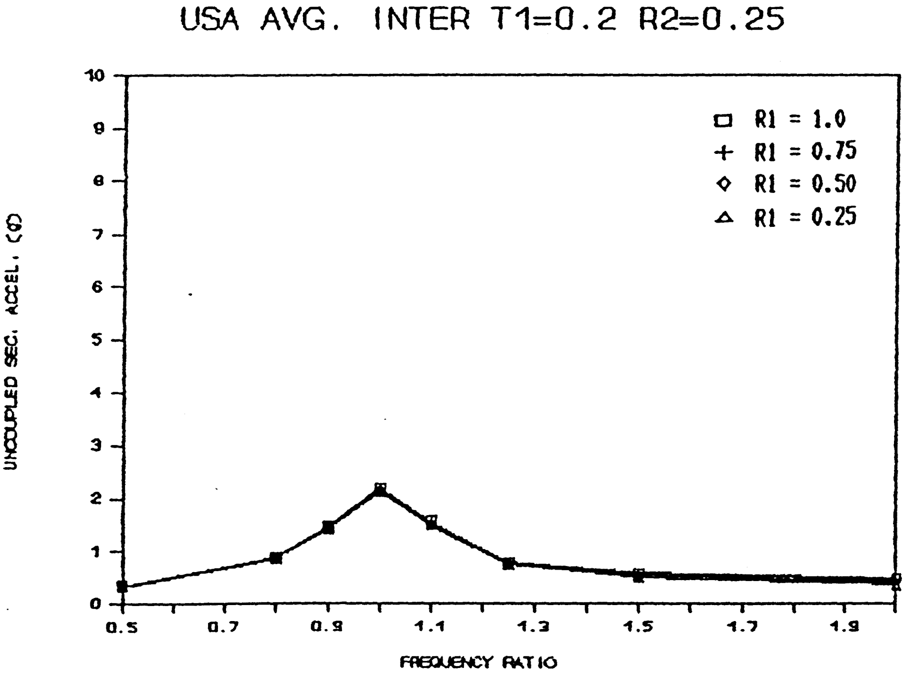


Figure 4.4 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R2=0.25 and R1= 1.0, 0.75, 0.50 and 0.25.

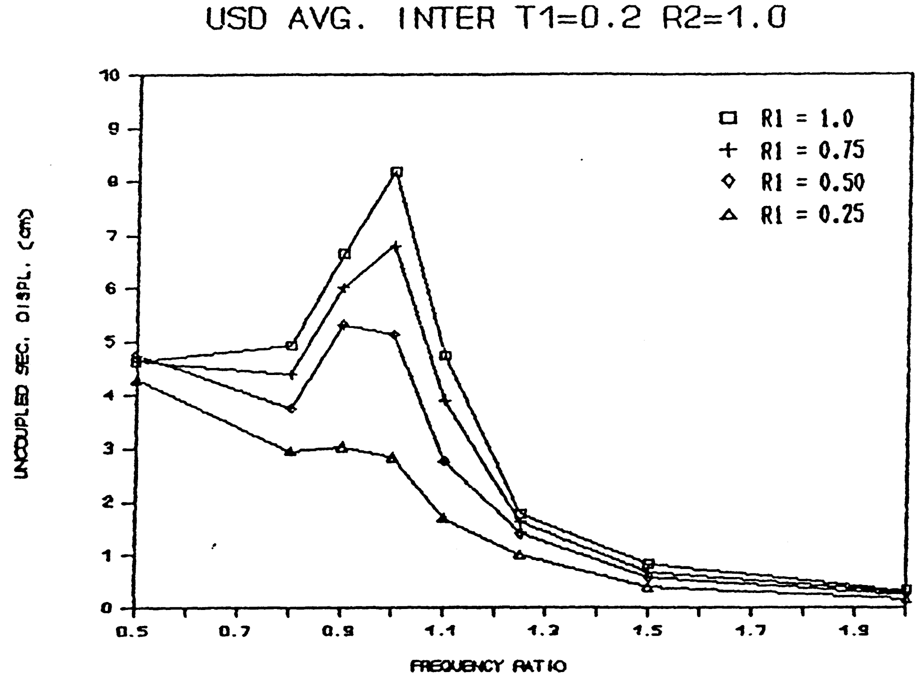


Figure 4.5 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R2=1.0 and R1= 1.0, 0.75, 0.50 and 0.25.

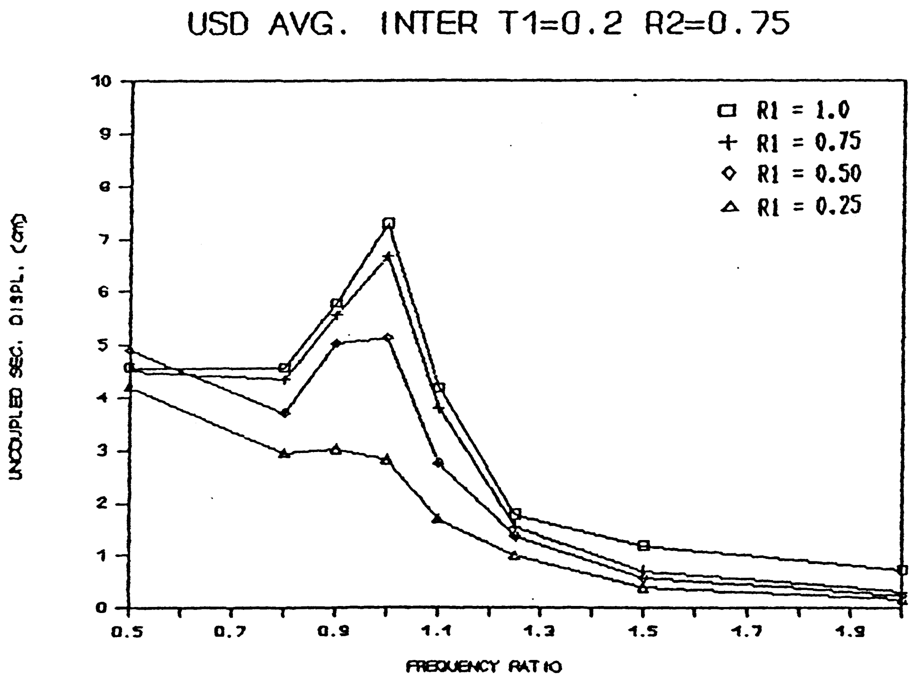


Figure 4.6 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R2=0.75 and R1= 1.0, 0.75, 0.50 and 0.25.

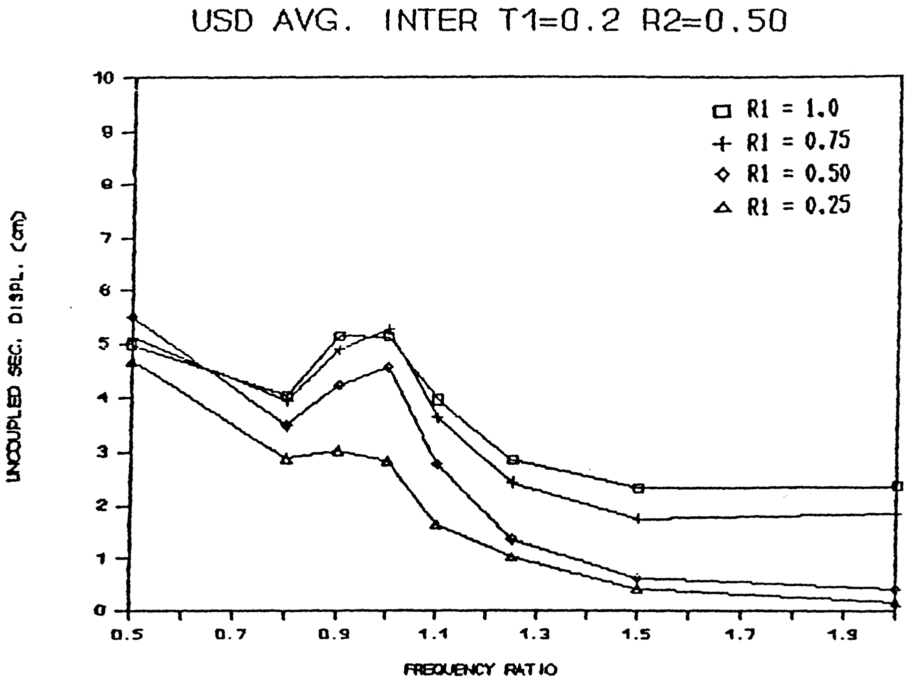


Figure 4.7 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with R2=0.50 and R1= 1.0, 0.75, 0.50 and 0.25.

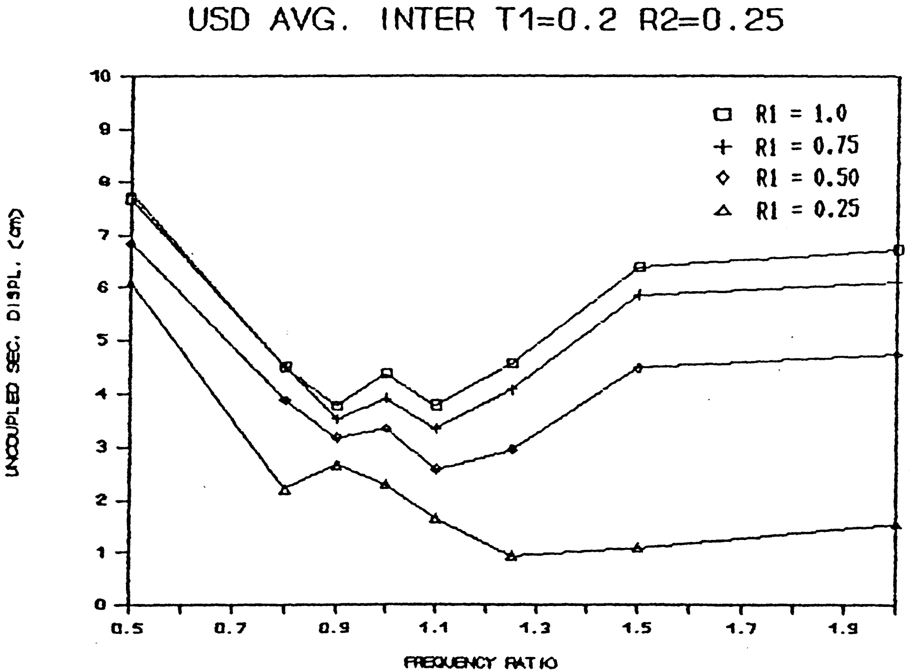


Figure 4.8 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with R2=0.25 and R1= 1.0, 0.75, 0.50 and 0.25.

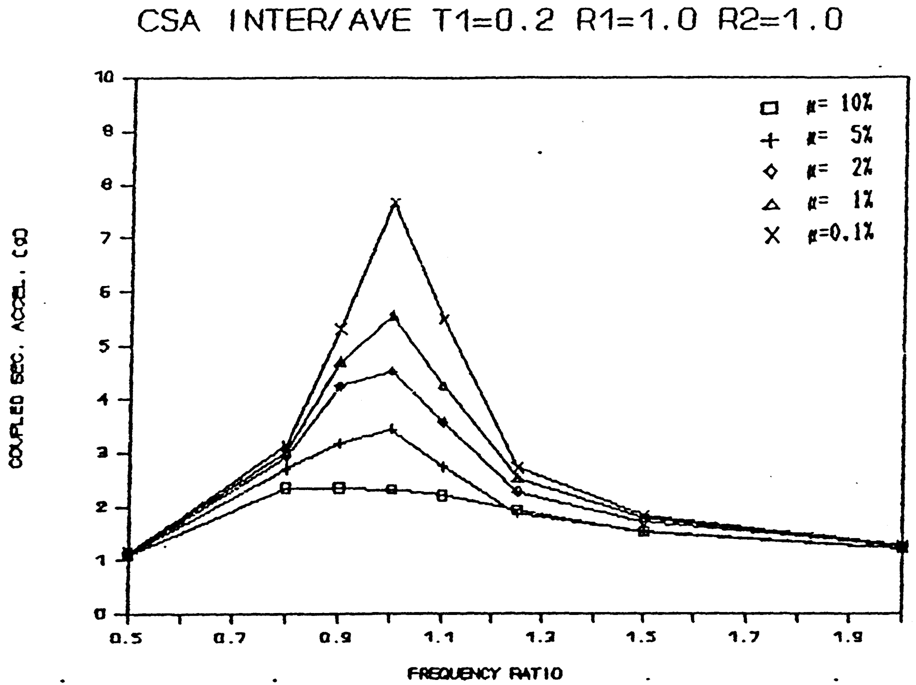


Figure 4.9 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1=R2=1.0 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

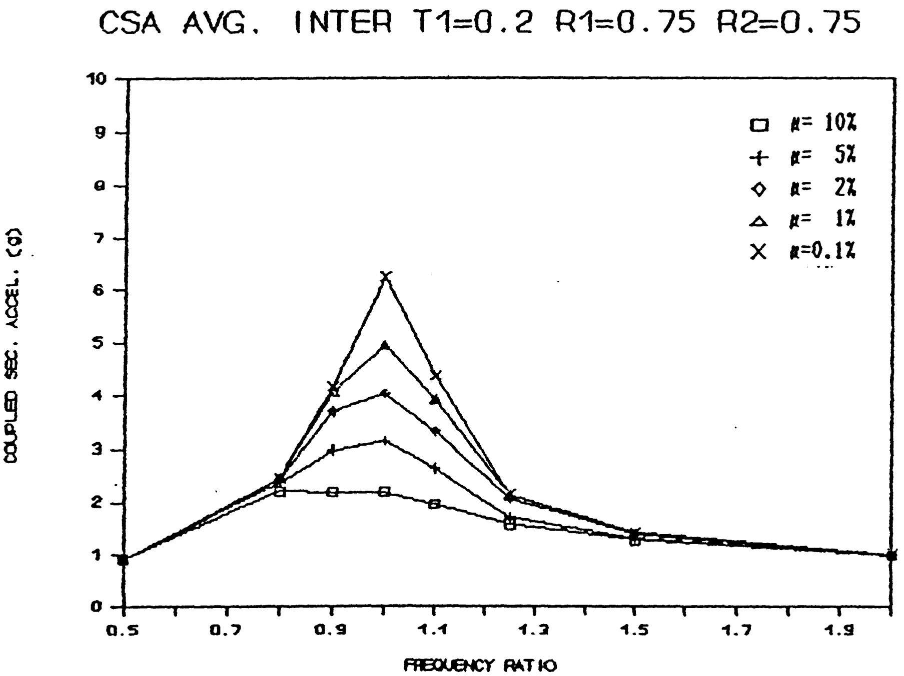


Figure 4.10 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1=R2=0.75 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

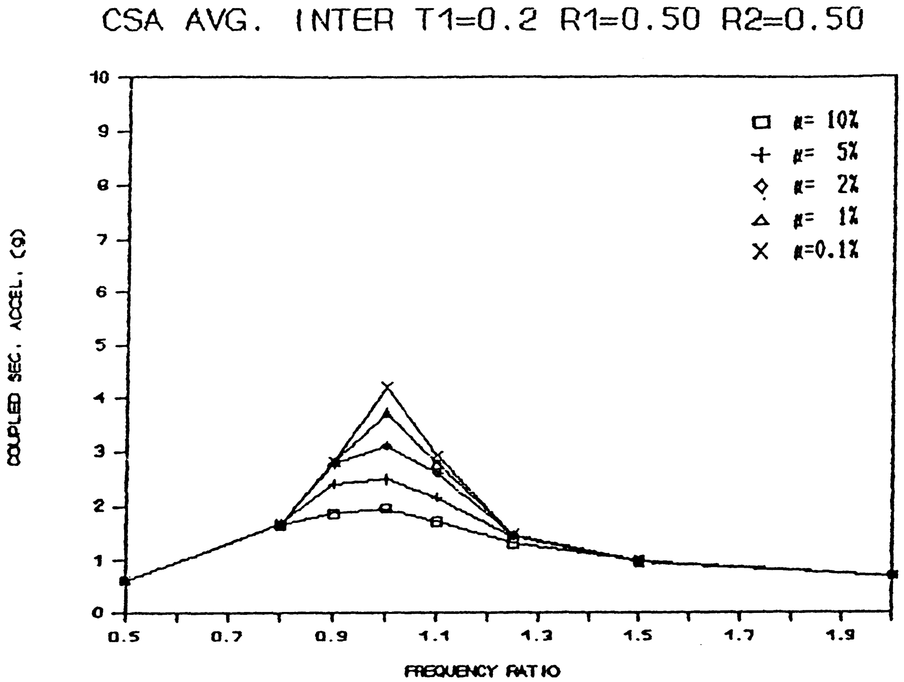


Figure 4.11 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1=R2=0.50 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

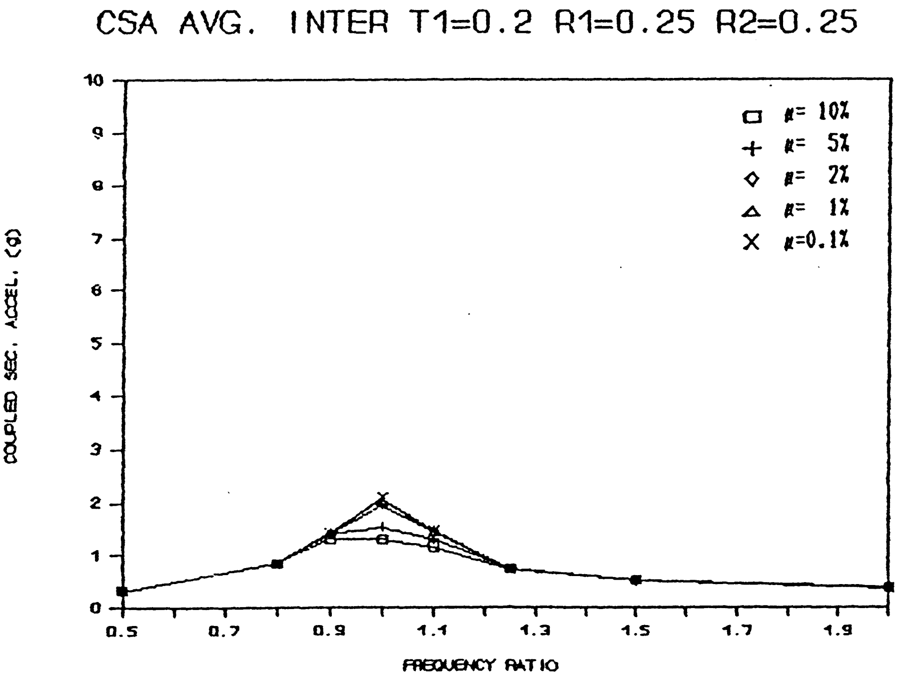


Figure 4.12 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1=R2=0.25 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

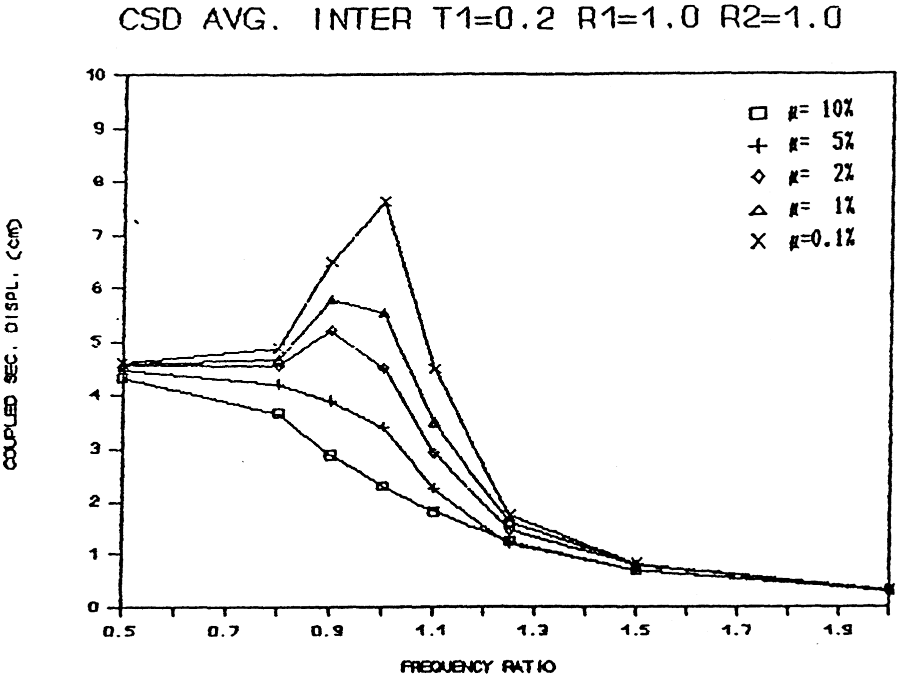


Figure 4.13 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1=R2=1.0 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

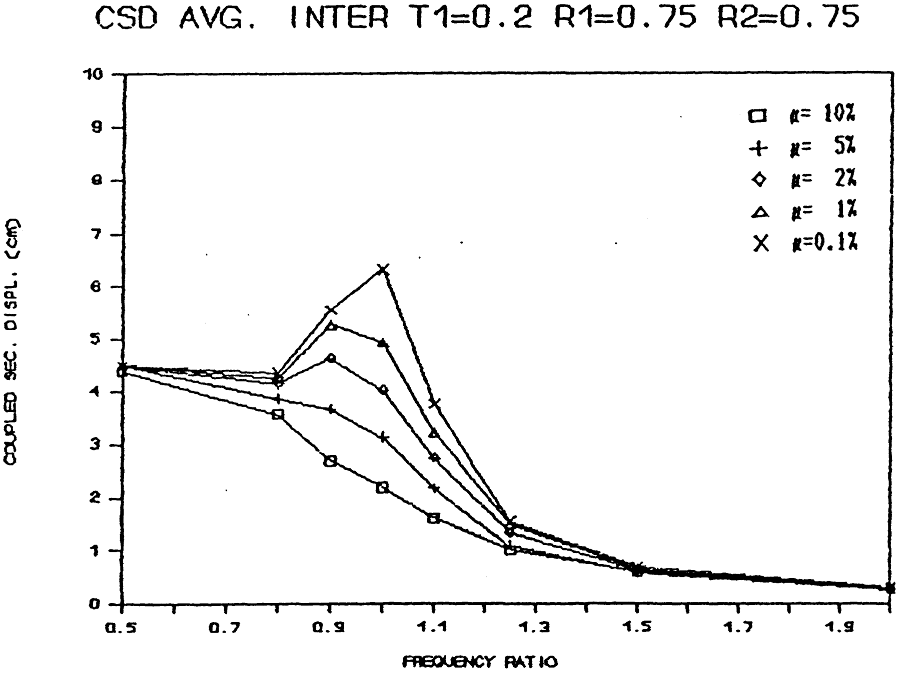


Figure 4.14 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1=R2=0.75 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

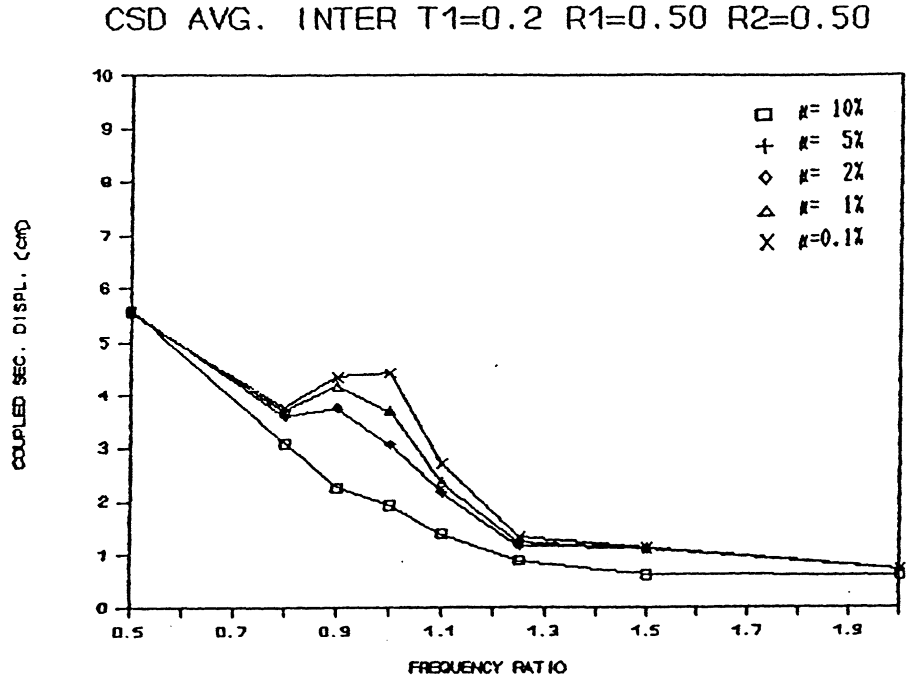


Figure 4.15 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with R1=R2=0.50 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

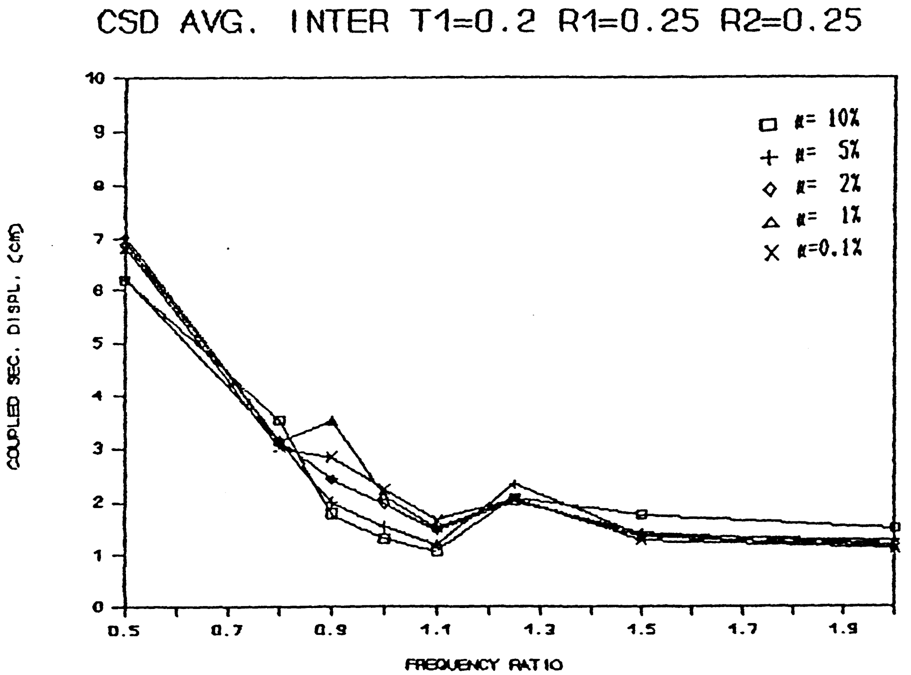


Figure 4.16 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with R1=R2=0.25 and mass ratio of 0.1, 1.0, 2.0, 5.0 and 10.0%.

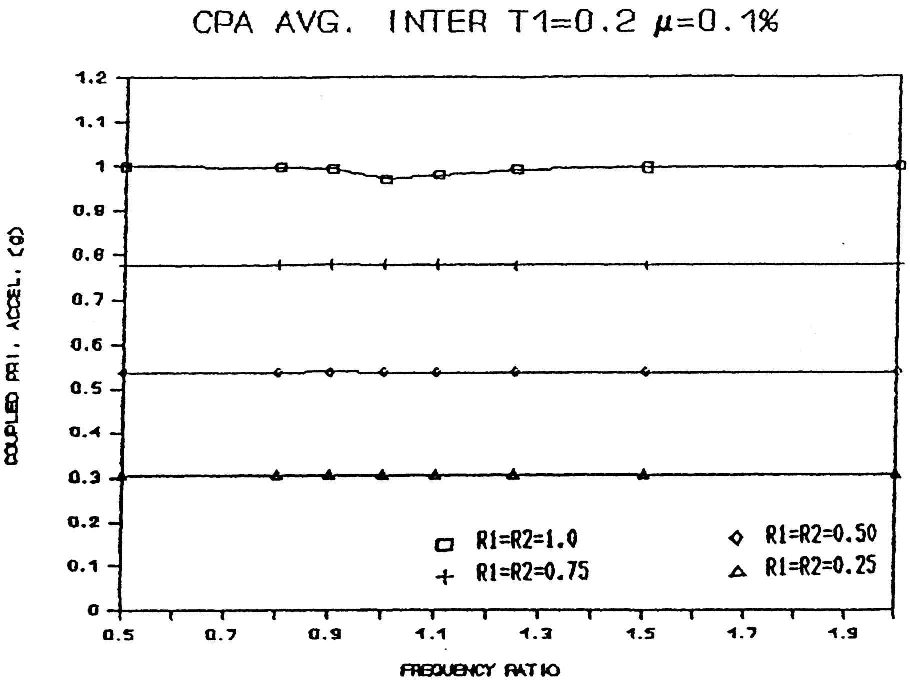


Figure 4.17 Coupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1 equal R2 and mass ratio of 0.1%.

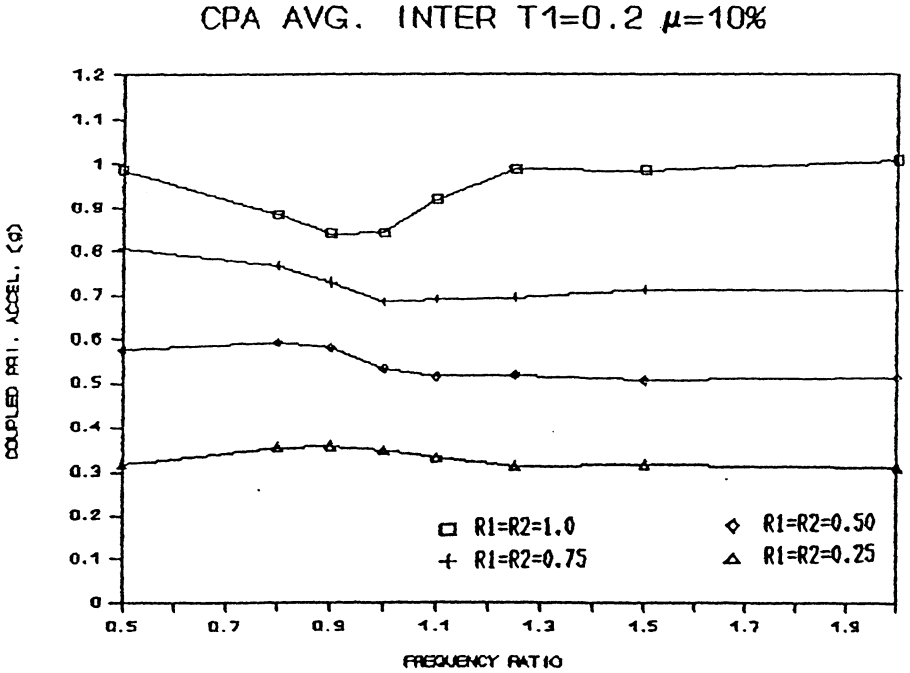


Figure 4.18 Coupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1 equal R2 and mass ratio of 10.0%.

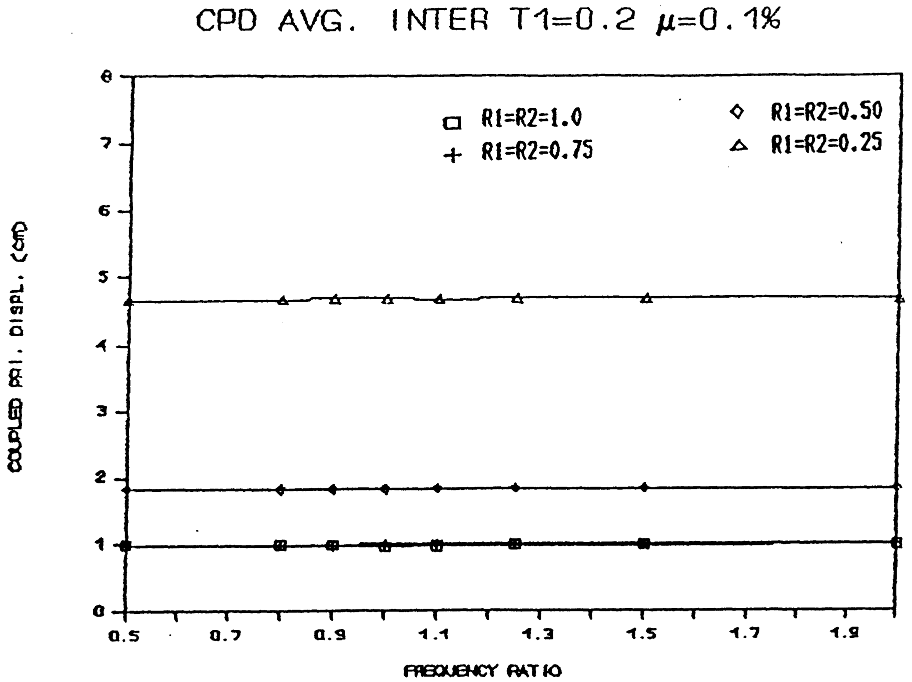


Figure 4.19 Coupled Primary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1 equal R2 and mass ratio of 0.1%.

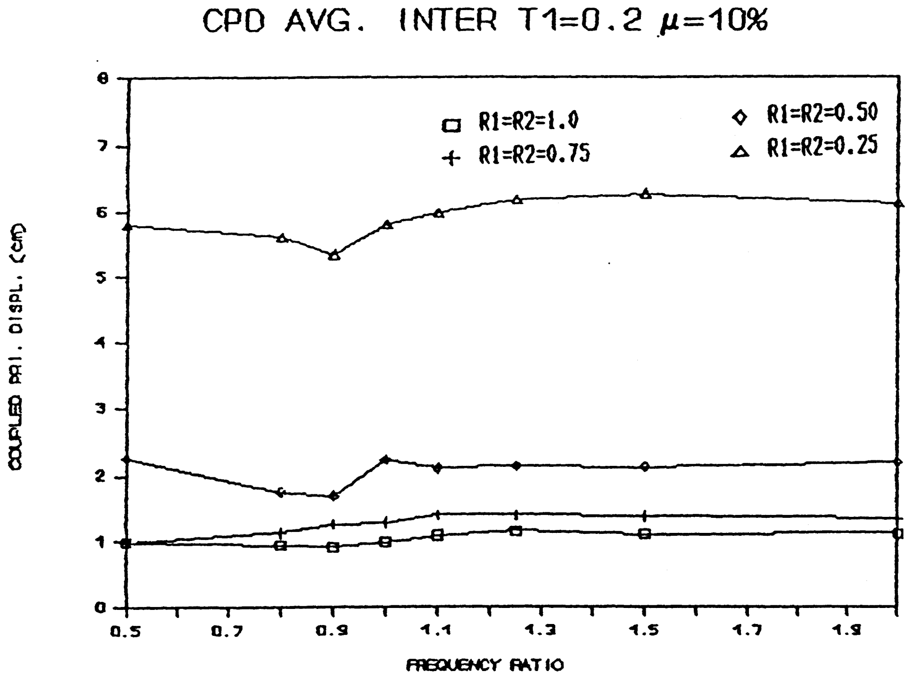


Figure 4.20 Coupled Primary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with R1 equal R2 and mass ratio of 10.0%.

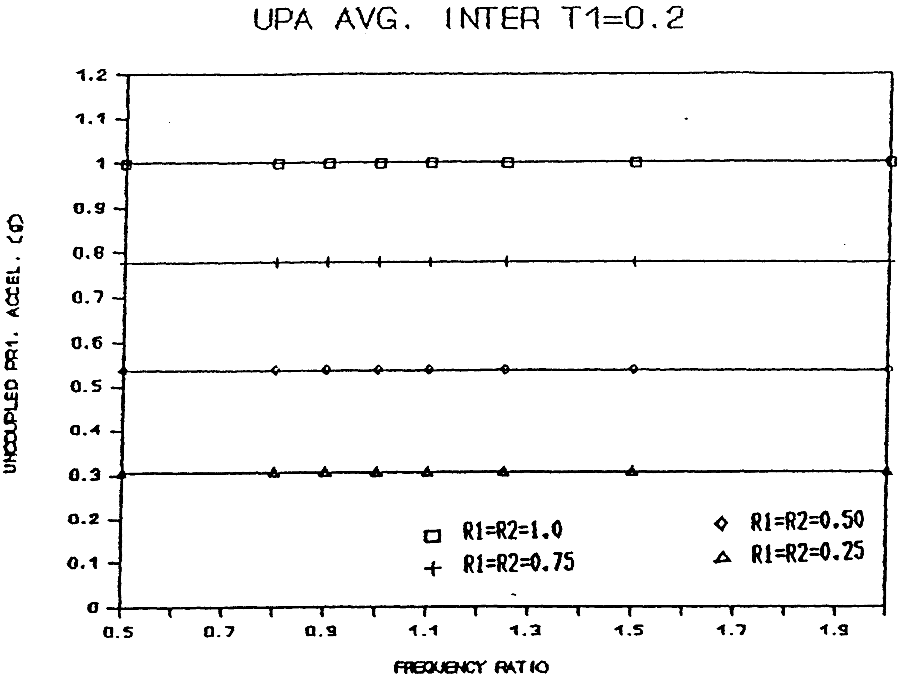


Figure 4.21 Uncoupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with R1 equal R2.

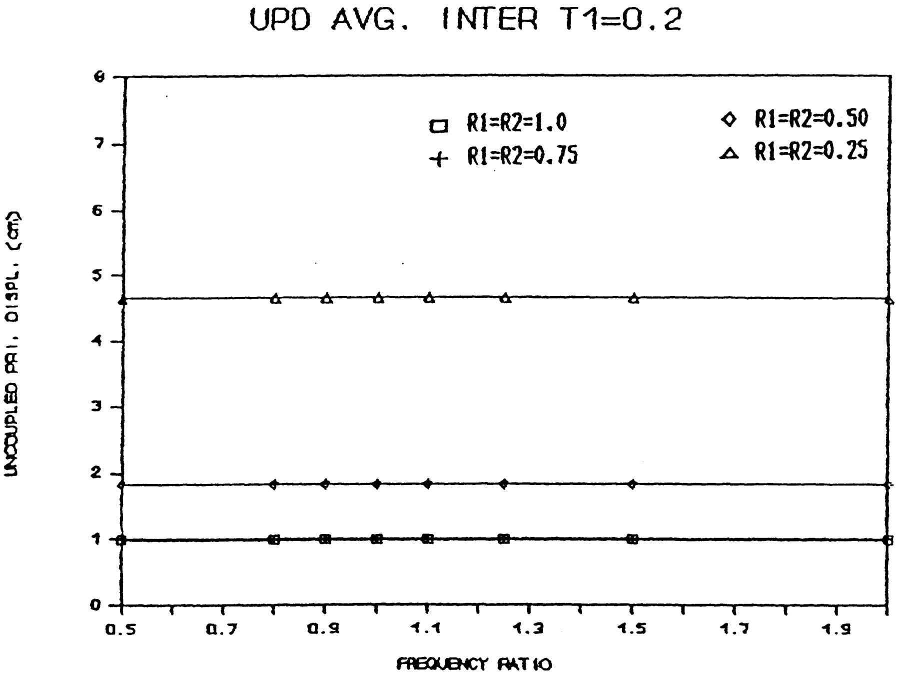


Figure 4.22 Uncoupled Primary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with R1 equal R2.

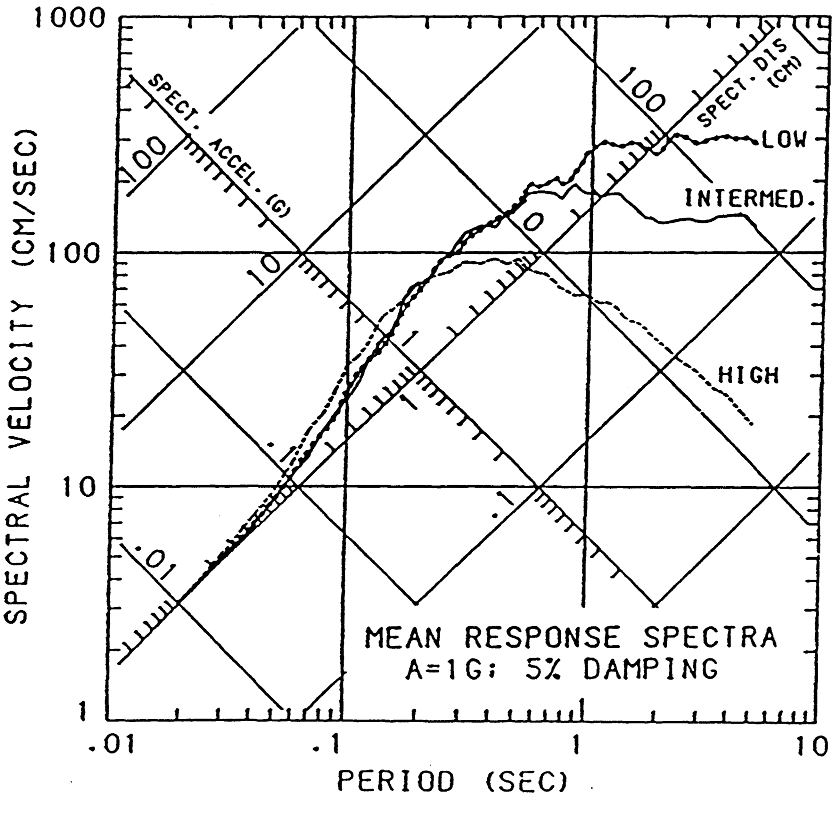


Figure 4.23 Mean Response Spectra for high, intermediate and low A/V records scaled to peak ground acceleration of 1g with 5% damping. (Naumoski et al., 1988)

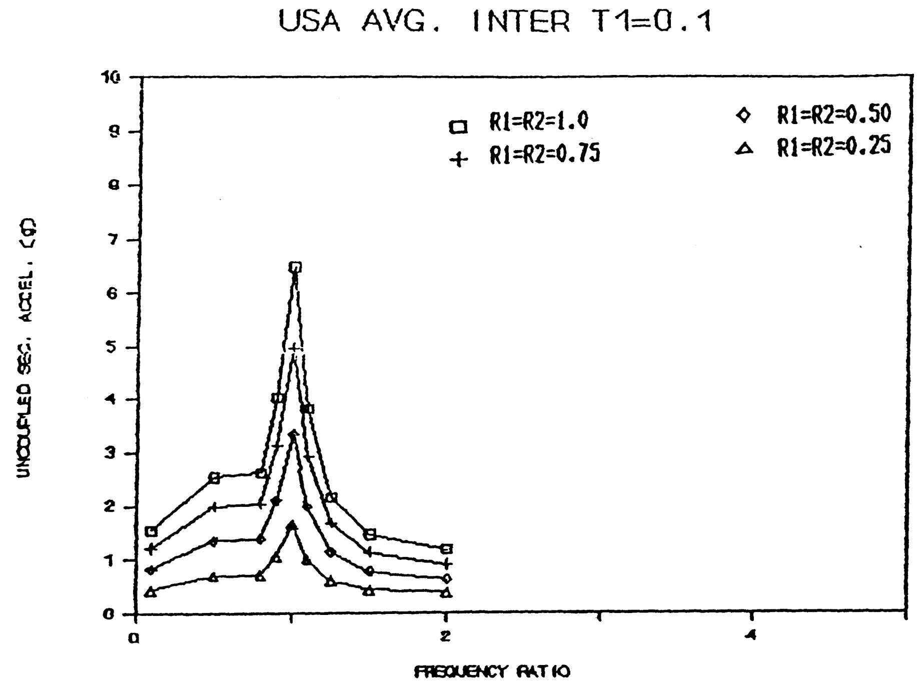


Figure 4.24 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.1 sec, average of 5 intermediate A/V ratio records using β=3%, with R1 equal R2.

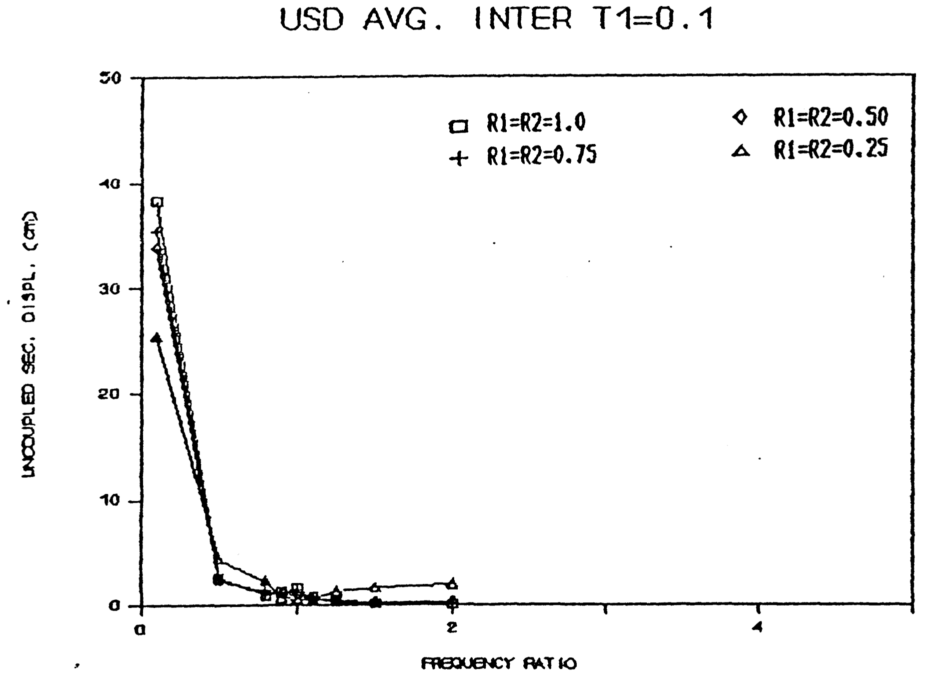


Figure 4.25 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.1 sec, average of 5 intermediate A/V ratio records using β=3%, with R1 equal R2.

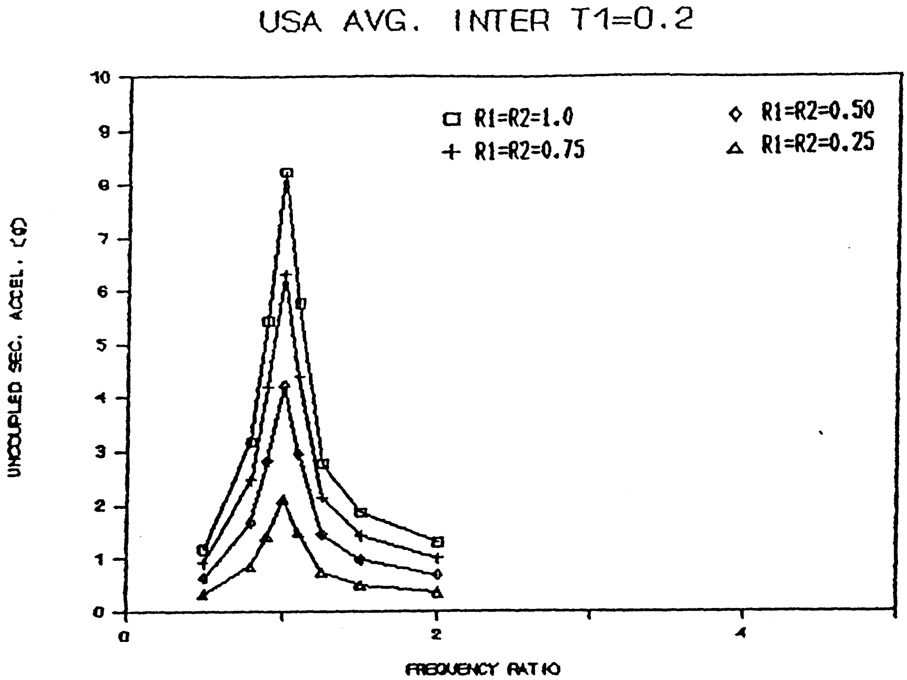


Figure 4.26 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3%, with R1 equal R2.

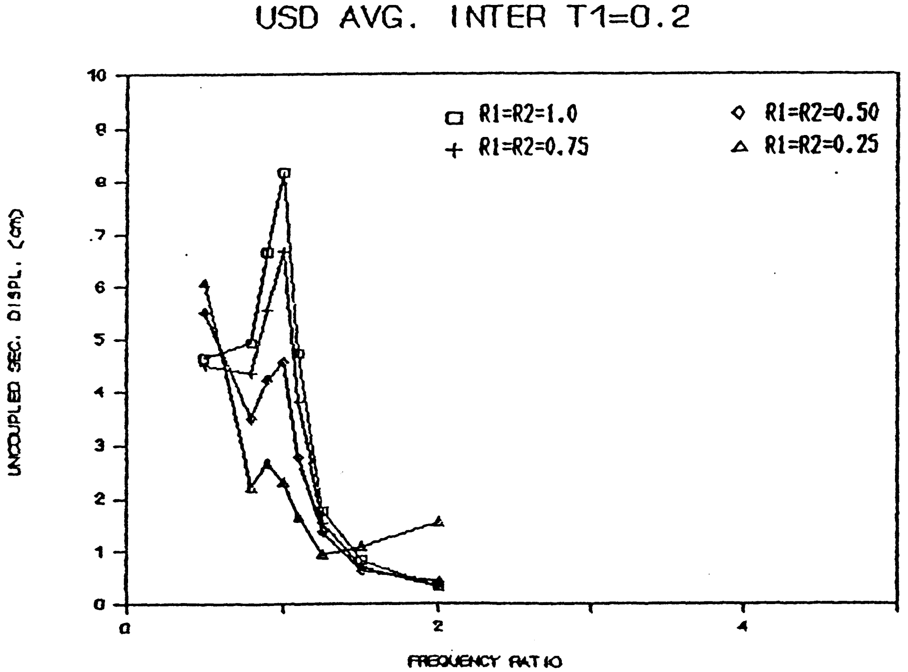


Figure 4.27 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3%, with R1 equal R2.

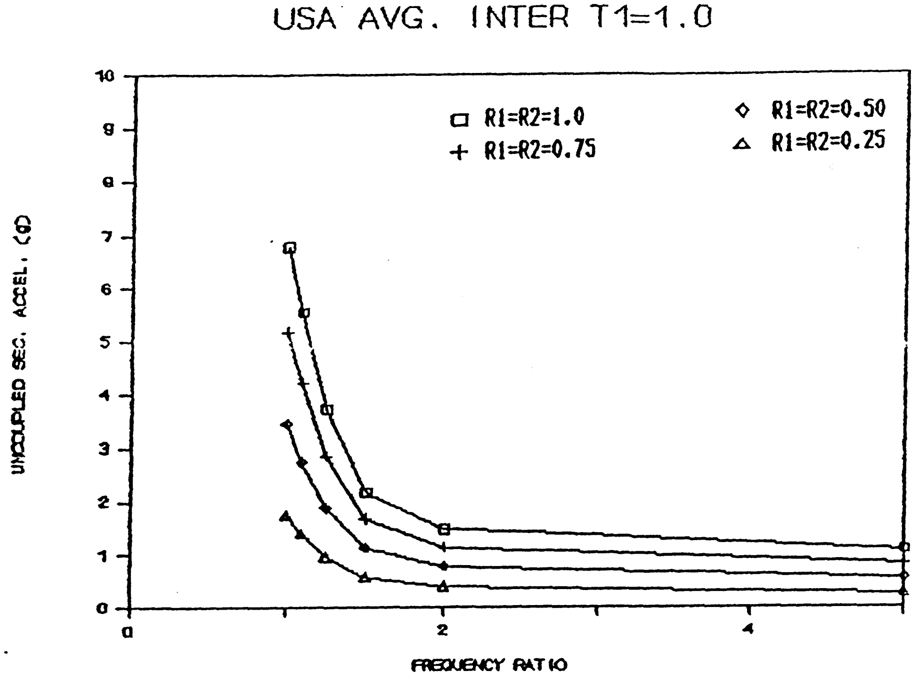


Figure 4.28 Uncoupled Secondary Acceleration response versus frequency ratio for T1=1.0 sec, average of 5 intermediate A/V ratio records using ß=3%, with R1 equal R2.

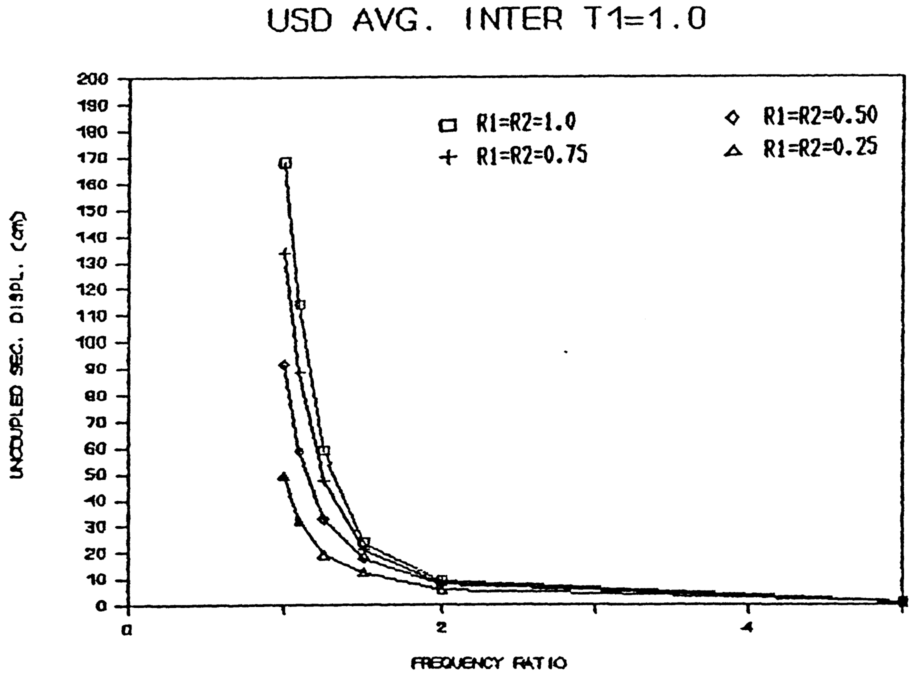


Figure 4.29 Uncoupled Secondary Displacement response versus frequency ratio for T1=1.0 sec, average of 5 intermediate A/V ratio records using ß=3%, with R1 equal R2.

CHAPTER 5 -- RESULTS AND DISCUSSIONS

5.1 ANALYSIS OF THE RESULTS

Throughout this work the response values studied represent the maximum response observed using numerical analysis for each earthquake. No consideration was given to the duration of the maximum response. Emphasis is given to actual uncoupled or coupled analysis results. The analyses were performed by normalizing the earthquake records to a spectral acceleration of 1.0g at a particular frequency. This implies that a given earthquake record needs to be typically amplified by a certain factor. The response values are associated with normalized earthquake records that are used to establish a comparative basis for the response.

At the beginning of this research the A/V ratio was thought to be an important factor to consider in the determination of structural response of secondary systems. This is a factor that has not been previously studied in great detail. By analyzing the data obtained, it seems that the A/V ratio is not as important as first believed. The response of the structural components are essentially not affected by the A/V ratios. For the most part, all the general trends are similar, independent of the A/V ratio. This observation is significant because the design process could be simplify when dealing with secondary systems. In other words, when secondary systems are considered, the A/V ratio of an earthquake is not a significant parameter. Factors such as mass ratios, frequency ratios and yield levels are much more important when dealing with secondary and primary systems. These parameters are easily available and can be controlled. The design process can be greatly facilitated and simple rules for earthquake engineering design can be formulated.

The mass ratio is a factor that does not affect the uncoupled response quantities under any condition. This behavior is easily explained by the nature of the analysis procedure used which is explained in chapter 2.

The type of system used, either elastic, elasto-plastic or bilinear will control the magnitude of the system responses.

The Wilson-θ numerical integration is performed using $\theta=1.4$ which assures satisfactory stability and accuracy. Another important parameter which influences the accuracy is the time interval chosen to perform the analysis. It is accepted that a time interval equal to or smaller than 1/10 of the period is sufficient to produce accurate results. This is the method used in this work to determine the time interval needed.

The results obtained in this work confirm the findings of Nguyen (1986) for the case of tuned primary and secondary systems. Nguyen used 3 strong motion earthquake records in his study.

The number of earthquake records used in the present research to perform the analysis appears sufficiant. The 15 strong ground motion time histories (divided into 3 A/V ratio categories) insure that the results can be presented and compared in a statistical manner.

5.1.1 ELASTIC SYSTEM

In all the analyses, the primary system responses are usually not greatly affected by a modification in the properties of the secondary system. However, the secondary system responses can be very sensitive to a change of properties in the primary or secondary systems.

An increase in the mass ratio, from μ=0.1% to μ=10%, will usually produce a decrease in the response quantities. However, only the mass ratio has a significant effect on values which are related to frequency ratio between fr=0.8 to fr=1.25. This observation demonstrates that at low and high frequency ratios, the mass ratio does not have a significant effect on all the response quantities. This is an important aspect that can be very useful from a seismic design point of view. In the event that a heavy secondary system is needed, the proper choice of frequency ratio could reduce the response under an earthquake.

The CSA response at high mass ratio reveals that the maximum peak response does not automatically occur at a frequency ratio of fr=1.0. In fact, for high mass ratios (μ=5% or μ=10%) the peak maximum response may occur at a frequency ratio of fr=0.8 or fr=0.9. This can be associated to the shift in modal frequencies. The importance of determining the response in the region around the resonance case is evident and should be considered when dealing with heavy secondary systems.

The frequency ratio plays a very important role in the dynamic response of the secondary system. From the various cases analyzed, a common factor emerges. The range from fr=0.8 to fr=1.25 is the most sensitive region among the frequency ratios considered.

The primary period has a large influence on the response of the structural elements. The response quantities(acceleration, velocity and displacement) are all affected differently. As a general rule, the acceleration response is practically independent of the primary period. As the primary period increases from T1=0.1 sec. to T1=1.0 sec., the displacement and velocity responses increase considerably. This behavior could be a result of the normalizing scheme adopted in this study.

The combination of low primary period, T1=0.1 sec, and low frequency ratio, fr=0.1, produces unusually large displacement and velocity responses. This behavior may be associated with the inability of numerical analysis to account for these extreme range of properties.

From an overall response point of view, the response components are not greatly influenced by the frequency content of the earthquake records. In general, the response values decrease when using high, intermediate to low A/V ratio records. It should be noted that the variability in a set a 5 earthquake records associated with each category is also quite pronounced. Basically, the only common factor between all 15 earthquake records is that they were all normalized to a spectral acceleration of 1.0g. The frequency content of an earthquake is a parameter that is not important in predicting the dynamic response of the systems studied.

5.1.2 ELASTO-PLASTIC SYSTEM

The comments offered in the pervious section regarding the general behavior of elastic systems are also appropriate for the case of elasto-plastic system. In this work, a factor named yield level was used to decrease the maximum elastic force required by an elastic system. The hysteretic behavior of the elasto-plastic system dissipates the input energy more efficiently. The most important difference between an elastic and inelastic analysis is the level of reduction in response. As a general rule, as the yield level decreases, the system response will also decrease.

The combination of mass ratio and frequency ratio usually control the behavior of the secondary system responses. The response at low and high frequency ratio is independent of the mass ratio. For a large mass ratio, the peak maximum CSA response occurs in the range of frequency ratios from 0.8 to 1.25. However, the peak maximum CSD response may occur at a lower frequency ratio. The mass ratio has virtually no effect on the primary response quantities.

The secondary system acceleration remains constant at low and high frequency ratios independent of other parameters. The maximum CSA response values occur in the vicinity of fr=1.0. The USD response is very dependant on the frequency ratio. The peak maximum USD response usually occurs between fr=0.8 and fr=1.1. In most cases, the peak maximum CSD response occurs at low frequency ratios. The frequency ratio has only a minor effect on the primary response quantities.

Systems experiencing inelastic behavior have lower response values than the elastic systems. However, a primary yield level of 0.75 does not reduce the response significantly. The minimum usually occur when both the primary and secondary yield levels equal 0.25. The various combinations of primary and secondary yield levels determine the level of response of the secondary acceleration or displacement. The primary acceleration or displacement system response is only influence by the primary yield level.

The frequency content of the earthquake does not significantly affect the system response. However, when the secondary yield level is equal to 0.50 or 0.25, the secondary displacement response can be affected by the A/V ratio. Under those conditions, the high, intermediate and low A/V ratio will produce a CSD response that is increasingly larger. This behavior can probably be associated with the normalization scheme which is based on the primary acceleration response.

5.2 PROPOSED DECOUPLING CRITERIA

This work confirms that the coupled analysis using a low mass ratio, μ=0.1% will produce the same results as the uncoupled analysis. In order to establish a decoupling criteria, it is required to determine the level of acceptable difference between the uncoupled and coupled analysis results. An error in the response variation based on 25% overestimation and 15% underestimation is deemed acceptable. The combination of the mass ratio, frequency ratio and yield level is extremely important. It has been demonstrated that the range of frequency ratio between fr=0.8 to fr=1.25 is very sensitive to the mass ratio. However, for low and high frequency ratios outside these limits, even a large mass ratio of μ=10% could be acceptable as a decoupling criteria. The yield level of the primary and secondary systems is another parameter of importance. For low yield levels, R_1=0.25 or R_2=0.25, the coupled system responses are virtually identical to the uncoupled system responses and are independent of the mass ratio.

The results show that decoupling is usually possible when the mass ratio is equal to or smaller than 2.0%. For higher mass ratio, decoupling is only acceptable for a limited number of cases with low or high frequency ratio.

CHAPTER 6 -- CONCLUSIONS AND RECOMMENDATIONS

6.1 SUMMARY

The coupling effects of various parameters on a one-degree-of-freedom primary system and a one-degree-of-freedom secondary system are investigated. Uncoupled and a coupled analyses of the systems are performed using the Wilson-θ numerical integration method. A total of fifteen strong ground motion earthquake records are used in this study. The properties of the systems are modified in order to determine the maximum peak response of the primary and secondary systems. Mass ratios of 0.1%, 1.0%, 2.0%, 5.0% and 10.0% are taken as parameters. In order to consider various levels of inelasticity, yield level factors of 0.25, 0.50, 0.75 and 1.0 of the elastic force case are used. In order to introduce the notion of frequency ratios between the primary and secondary systems, the primary fundamental frequency is fixed at 10 Hz, 5 Hz, 1.0 Hz or 0.2 Hz. The corresponding secondary fundamental frequencies are based on the following frequency ratios: 0.1, 0.5, 0.8, 0.9, 1.0, 1.1, 1.25, 1.5, 2.0 and 5.0. The elastic and elasto-plastic analyses are performed using a damping ratio of 3% and all fifteen earthquake records as input. The analysis using a damping ratio of 5% or the bilinear model is restricted to a few earthquake record inputs.

6.2 CONCLUSIONS

By analyzing the data and results obtained from the numerous cases considered in this work, the following conclusions and observations are arrived at:

* The frequency content of the earthquake records(high A/V ratio, intermediate A/V ratio or low A/V ratio) is not a significant factor in this type of analysis considering the normalization approach used.

* Under certain conditions, the maximum peak response does not occur at a frequency ratio of 1.0 (the resonance case). The system response for frequency ratios of fr=0.8 to fr=1.25 is of the same order of magnitude as for the resonance case.

* The effect of an increase in the mass ratio is usually a decrease in the secondary system response while the primary system response remains mostly unchanged. However, the effect of the mass ratio is limited to the range of frequency ratios of fr=0.8 to fr=1.25.

* The secondary system responses are very dependant on the yield levels of both the primary and secondary systems. There is no appreciable variation in the system response when using yield levels of 1.0 or 0.75. For a low primary yield level of 0.25, the secondary response is practically independent of the mass ratio.

* The primary system responses are in most cases, independent of the secondary system yield levels.

* Combination of low primary and secondary yield levels will significantly affect the response for low mass ratios.

* An elasto-plastic system will generally have a lower dynamic response as compared to an elastic system.

* There is no significant variation in the system response between the elasto-plastic or bilinear modelling when a reasonably small slope of the inelastic branch is used.

* An increase in the critical damping ratio from 3% to 5% for both the primary and secondary systems will increase the primary responses and decrease the secondary responses.

6.3 RECOMMENDATIONS FOR FUTURE RESEARCH

A single usable decoupling criteria for design purposes should be elaborated by synthesizing the results of this work. The combination of low frequency ratio and low primary period produces unusual responses. Some inconsistencies in the results are noted for cases with low yield levels and high mass ratios. Further investigation of these phenomena by considering for example, the numerical analysis and the normalizing scheme, is necessary. The characteristics of actual equipment-structure systems should be reflected in the analysis by using more realistic hysteretic models and damping parameters. A study of the problems associated with the analysis of basic multi-degree-of-freedom models should be attempted.

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APPENDIX A

WILSON-θ METHOD

The Wilson-θ method is described in reference such as Paz (1980), the steps and procedure of the method are the following:

An extended time step interval (τ) is obtained by using the Wilson-θ constant (θ) and the integration time interval (Δt), equation A.1.

$$\tau = \theta \Delta t \tag{A.1}$$

Equation A.2 represent the difference between the dynamic equilibrium conditions at time t_n and $t_{n+\tau}$.

$$[M]\{\delta\ddot{u}\}_n + [C]\{\delta\dot{u}\}_n + [K]\{\delta u\}_n = -[M]\{I\}\delta\ddot{y}_{g\,n} \tag{A.2}$$

The expression for the projected ground acceleration increment is:

$$\delta\ddot{y}_{g\,n} = \ddot{y}_g(t_n + \tau) - \ddot{y}_g(t_n) \tag{A.3}$$

The incremental responses at the n^{th} integration point for the extended time step τ are:

$$\{\delta u\}_n = \{u(t_n + \tau)\} - \{u(t_n)\} \tag{A.4a}$$

$$\{\delta\dot{u}\}_n = \{\dot{u}(t_n + \tau)\} - \{\dot{u}(t_n)\} \tag{A.4b}$$

$$\{\delta\ddot{u}\}_n = \{\ddot{u}(t_n + \tau)\} - \{\ddot{u}(t_n)\} \tag{A.4c}$$

The linear expression for the acceleration during the extended time step is:

$$\{\ddot{u}(t)\} = \{\ddot{u}\}_n + \frac{\{\delta\ddot{u}\}_n}{\tau} (t - t_n) \qquad (A.5)$$

By integration equation A.5, the velocity (A.6) and displacement (A.7) vectors are obtained:

$$\{\dot{u}(t)\} = \{\dot{u}\}_n + \{\ddot{u}\}_n(t - t_n) + \frac{1}{2} \frac{\{\delta\ddot{u}\}_n}{\tau} (t - t_n)^2 \qquad (A.6)$$

$$\{u(t)\} = \{u\}_n + \{\dot{u}\}_n(t - t_n) + \frac{1}{2} \{\ddot{u}\}_n(t - t_n)^2$$

$$+ \frac{1}{6} \frac{\{\delta\ddot{u}\}_n}{\tau} (t - t_n)^3 \qquad (A.7)$$

Evaluating equations A.6 and A.7 at the end of the extended time interval $t = t_n + \tau$ gives:

$$\{\delta\dot{u}\}_n = \{\ddot{u}\}_n\tau + \frac{1}{2}\{\delta\ddot{u}\}_n\tau \qquad (A.8)$$

$$\{\delta u\}_n = \{u\}_n\tau + \frac{1}{2}\{\ddot{u}\}_n\tau^2 + \frac{1}{6}\{\delta\ddot{u}\}_n\tau^2 \qquad (A.9)$$

Equation A.9 is solved for the incremental acceleration $\delta\ddot{u}_n$ and substitued in equation A.8. We obtain equations A.10 and A.11.

$$\{\delta\ddot{u}\}_n = \frac{6}{\tau^2} \{\delta u\}_n - \frac{6}{\tau} \{\dot{u})_n - 3\{\ddot{u}\}_n \qquad (A.10)$$

$$\{\delta\dot{u}\} = \frac{3}{\tau} \{\delta u\}_n - 3\{\dot{u}\}_n - \frac{\tau}{2} \{\ddot{u}\}_n \qquad (A.11)$$

Substituting equations A.10 and A.11 into the incremental equation of motion A.2 results in an equation for the incremental displacement $\{\delta u\}_n$:

$$[K_e]_n \{\delta u\}_n = \{\delta F\}_n \qquad (A.12)$$

where

$$[K_e]_n = [K]_n + \frac{6}{\tau^2}[M] + \frac{3}{\tau}[C] \tag{A.13}$$

and

$$\{\delta F\}_n = [M](-\{I\}\delta\ddot{y}_{g\ n} + (\frac{6}{\tau}\{\dot{u}\}_n + 3\{\ddot{u}\}_n)$$
$$+ [C](3\{\dot{u}\}_n + \frac{\tau}{2}\{\ddot{u}\}_n) \tag{A.14}$$

By a linear interpolation, the incremental acceleration $\{\Delta\ddot{u}\}_n$ is obtained:

$$\{\Delta\ddot{u}\}_n = \frac{\{\delta\ddot{u}\}}{\theta}n \tag{A.15}$$

The incremental velocity and displacement for the normal time interval Δt is calculated by using equations A.6 and A.7 where the extended time interval τ is substituted by Δt:

$$\{\Delta\dot{u}\}_n = \{\ddot{u}\}_n\Delta t + \frac{1}{2}\{\Delta\ddot{u}\}_n\Delta t \tag{A.16}$$

$$\{\Delta u\}_n = \{\dot{u}\}_n\Delta t + \frac{1}{2}\{\ddot{u}\}_n\Delta t + \frac{1}{6}\{\Delta\ddot{u}\}_n\Delta t^2 \tag{A.17}$$

Finally, the displacement and velocity at the end of the normal time interval are calculated:

$$\{u\}_{n+1} = \{u\}_n + \{\Delta u\}_n \tag{A.18}$$

$$\{\dot{u}\}_{n+1} = \{\dot{u}\}_n + \{\Delta\dot{u}\}_n \tag{A.19}$$

The initial acceleration for the next time step is calculated from the condition of dynamic equilibrium at the time $t_n + \Delta t$:

$$\{\ddot{u}\}_{n+1} = -\ \{I\}\ddot{y}_{g\ n+1} - [M]^{-1}(\ [C]\{\dot{u}\}_{n+1} + [K]_{n+1}\{u\}_{n+1}) \tag{A.20}$$

APPENDIX B

COMPARAISON OF ELASTO-PLASTIC SYSTEMS ß=3% AND ß=5% WITH BILINEAR SYSTEM ß=3%

The Imperial Valley intermediate A/V ratio record #6 is used in this analysis. The uncoupled primary and secondary responses are presented in figures B.1 to B.15. Each figure contains 3 plots: elasto-plastic system with a damping ratio of 3%; elasto-plastic system with a damping ratio of 5%; and bilinear system with a damping ratio of 3%.

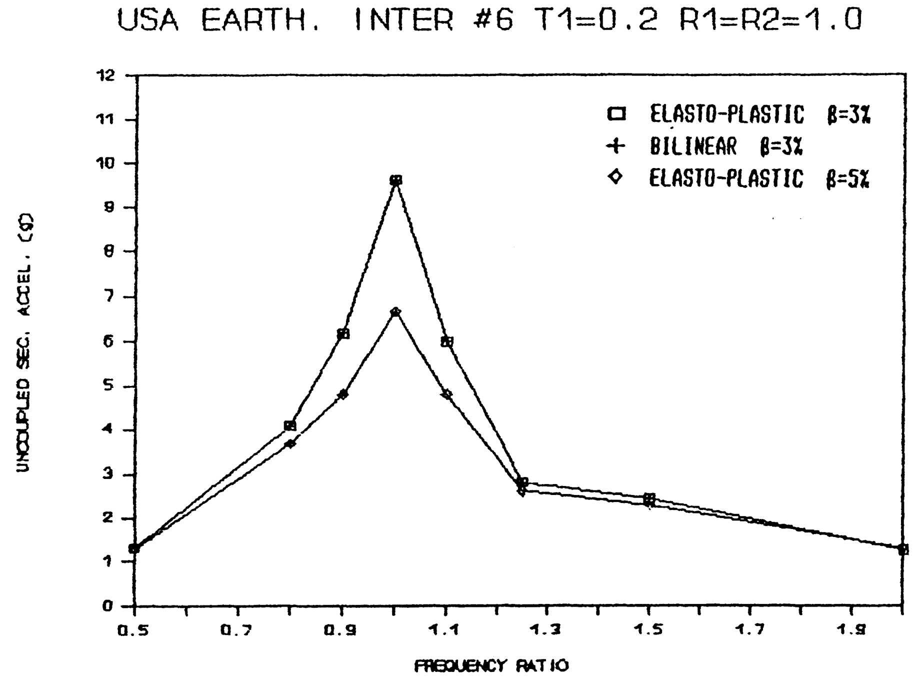


Figure B.1 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=1.0 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

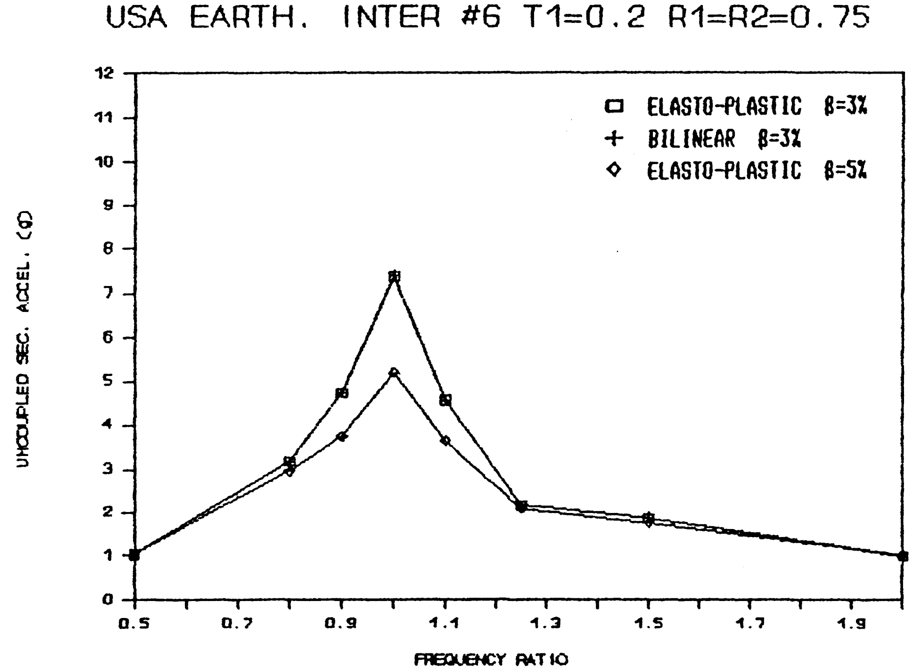


Figure B.2 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.75 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

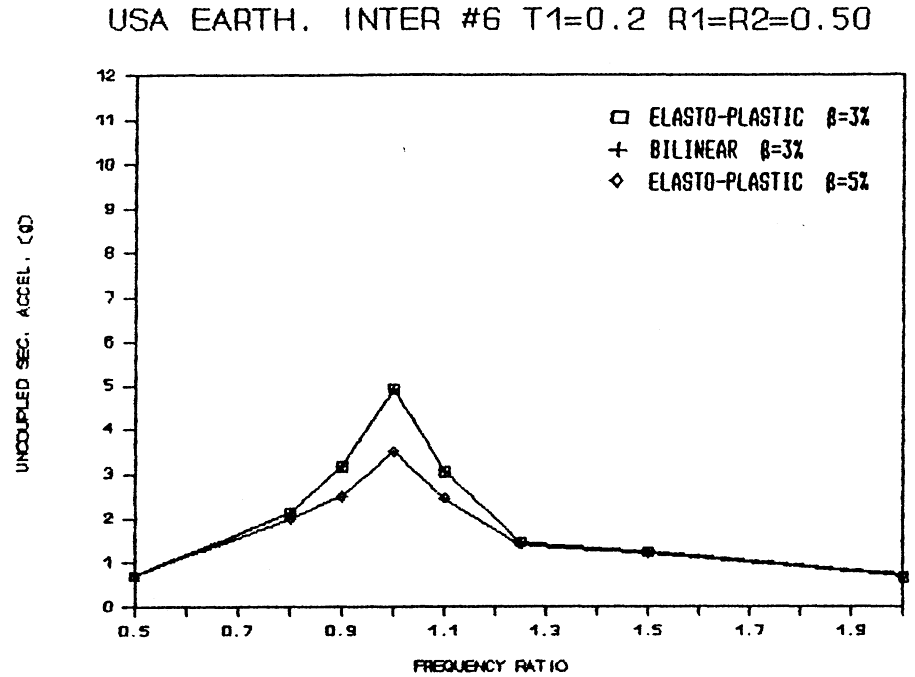


Figure B.3 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.50 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

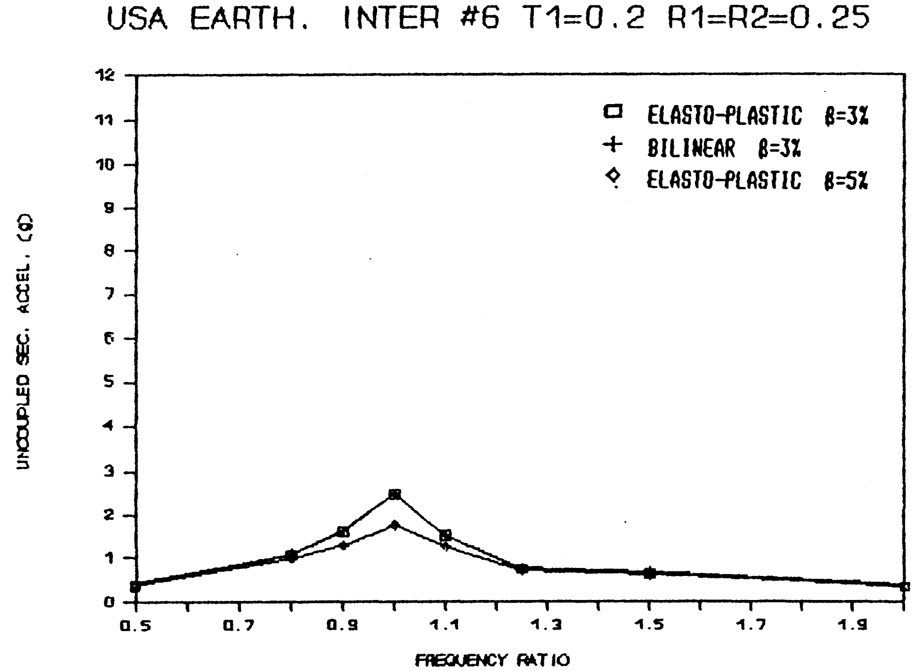


Figure B.4 Uncoupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.25 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

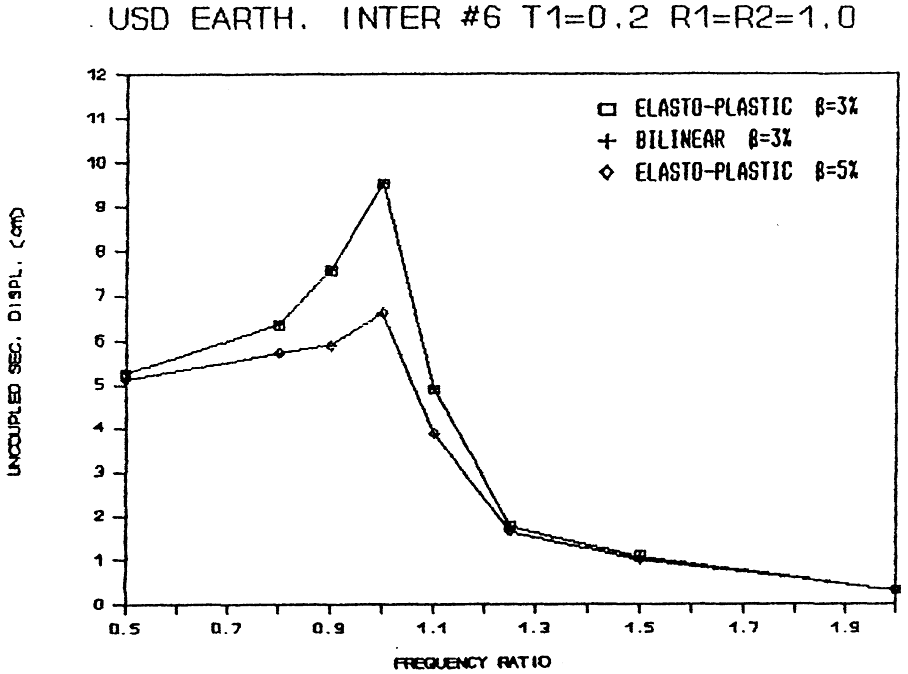


Figure B.5 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=1.0 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

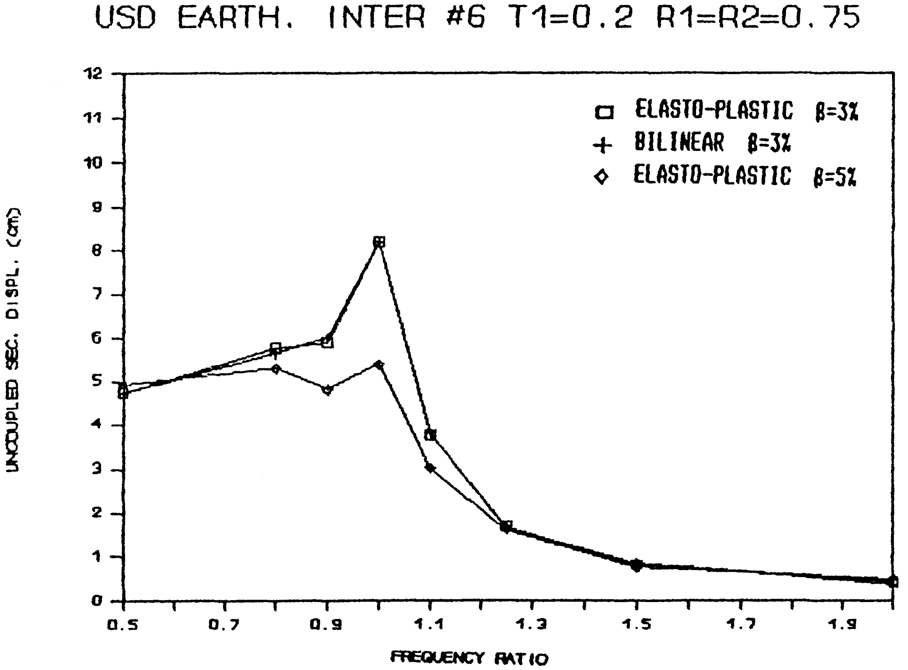


Figure B.6 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.75 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

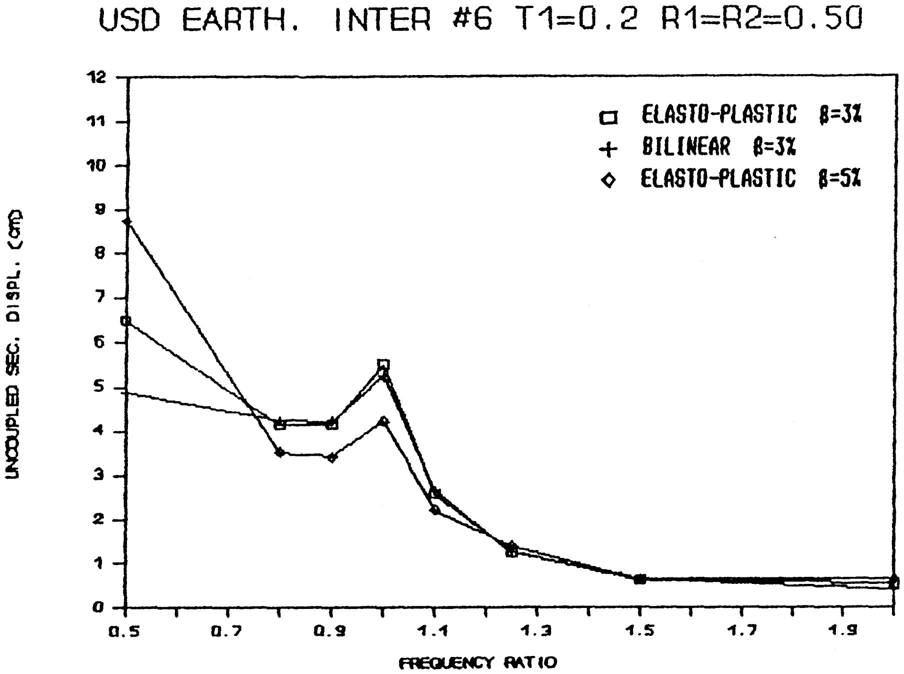


Figure B.7 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.50 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

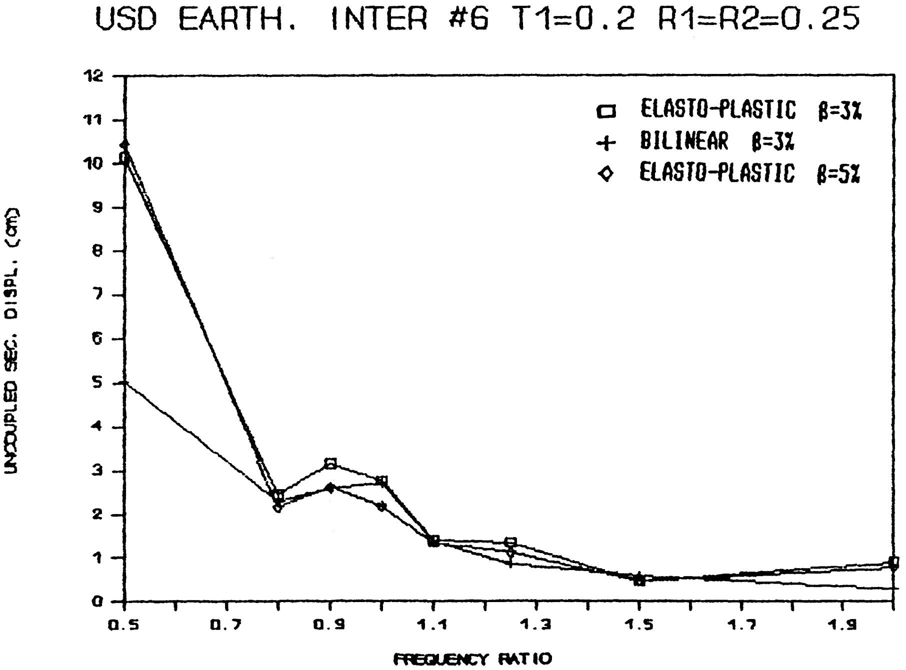


Figure B.8 Uncoupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.25 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

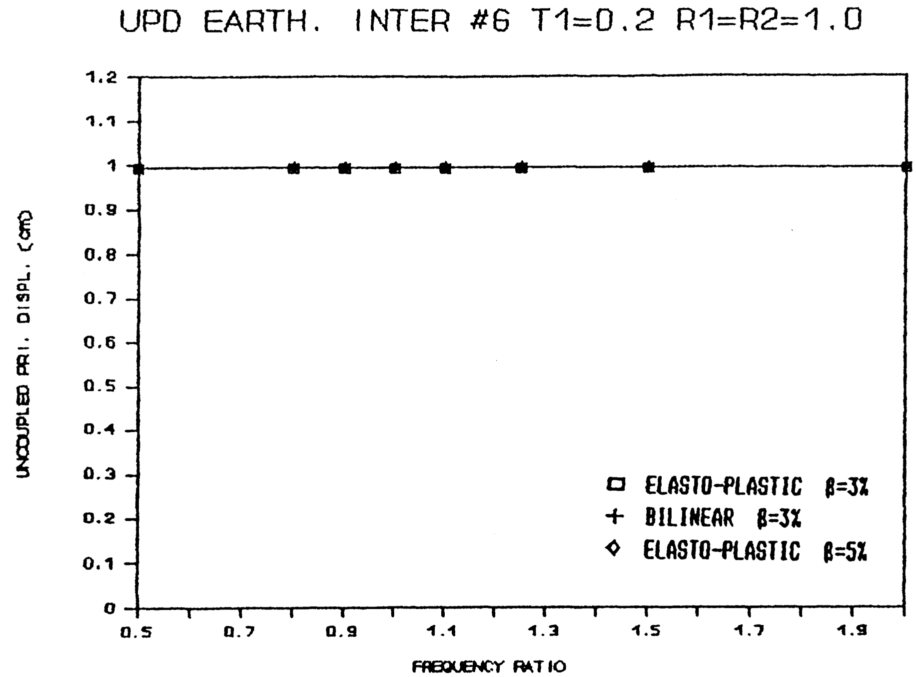


Figure B.9 Uncoupled Primary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=1.0 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

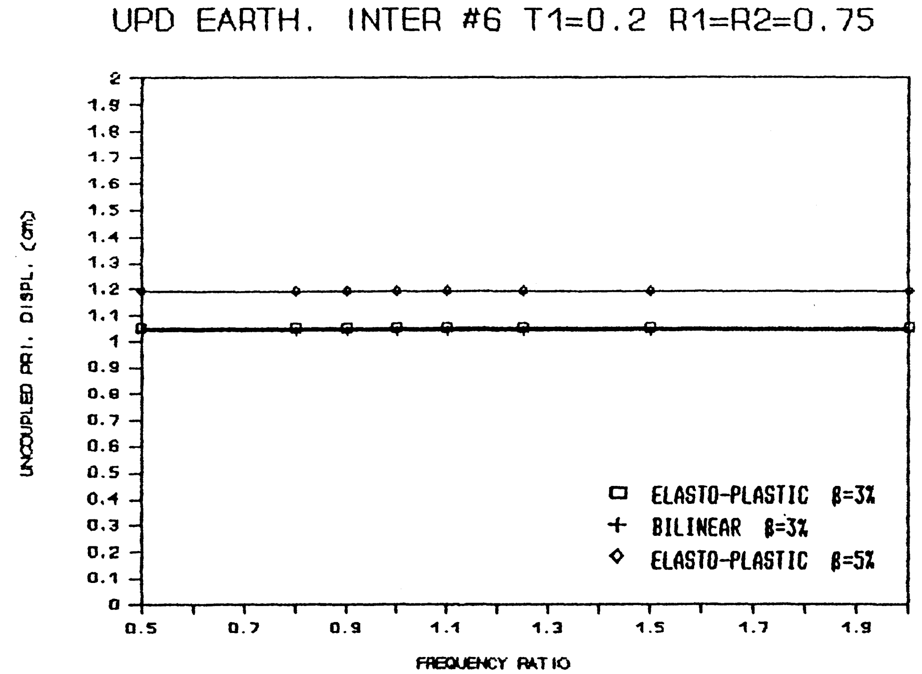


Figure B.10 Uncoupled Primary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.75 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

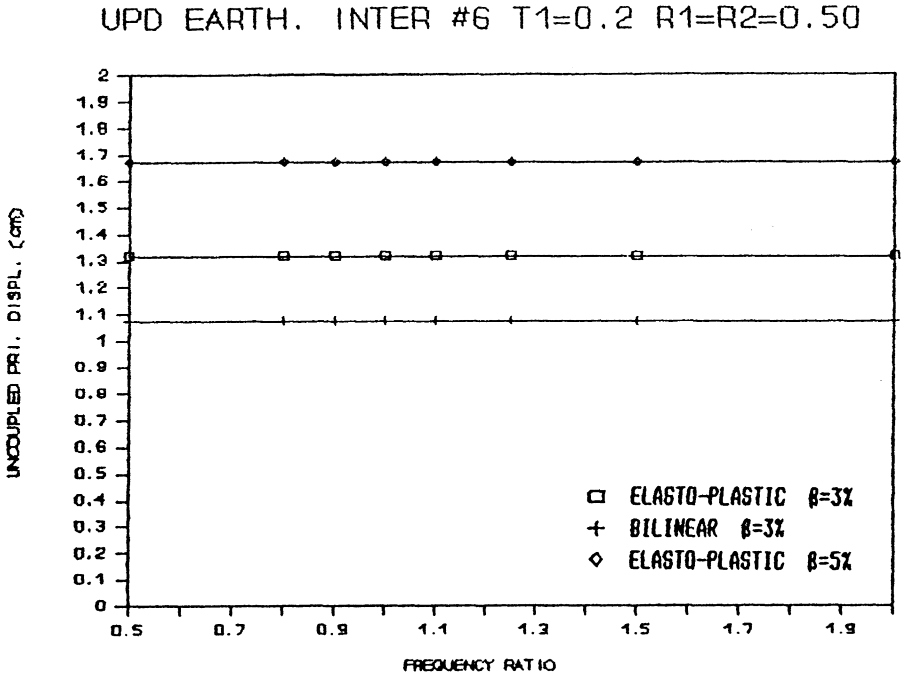


Figure B.11 Uncoupled Primary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.50 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

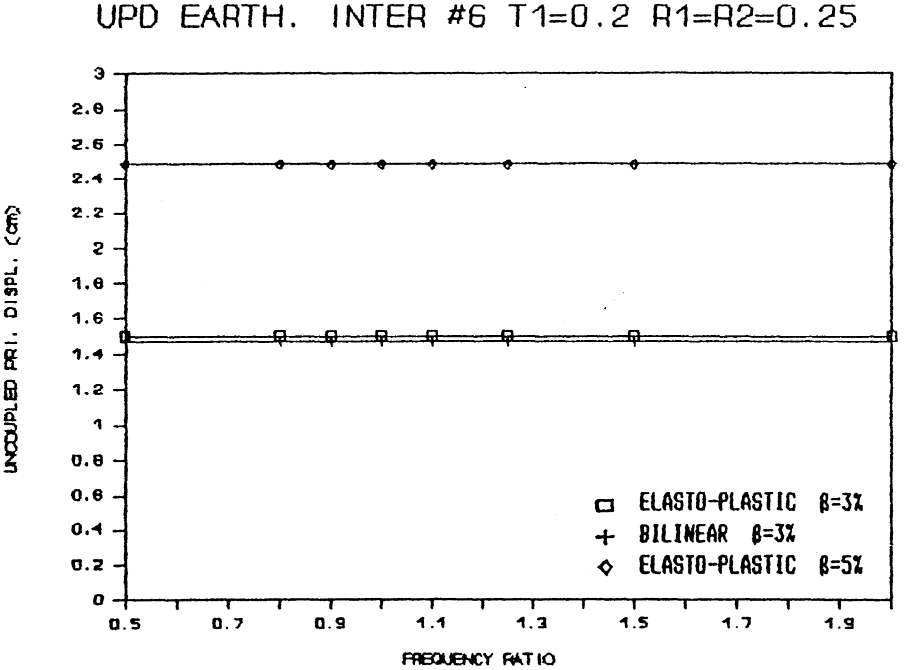


Figure B.12 Uncoupled Primary Displacement response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.25 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

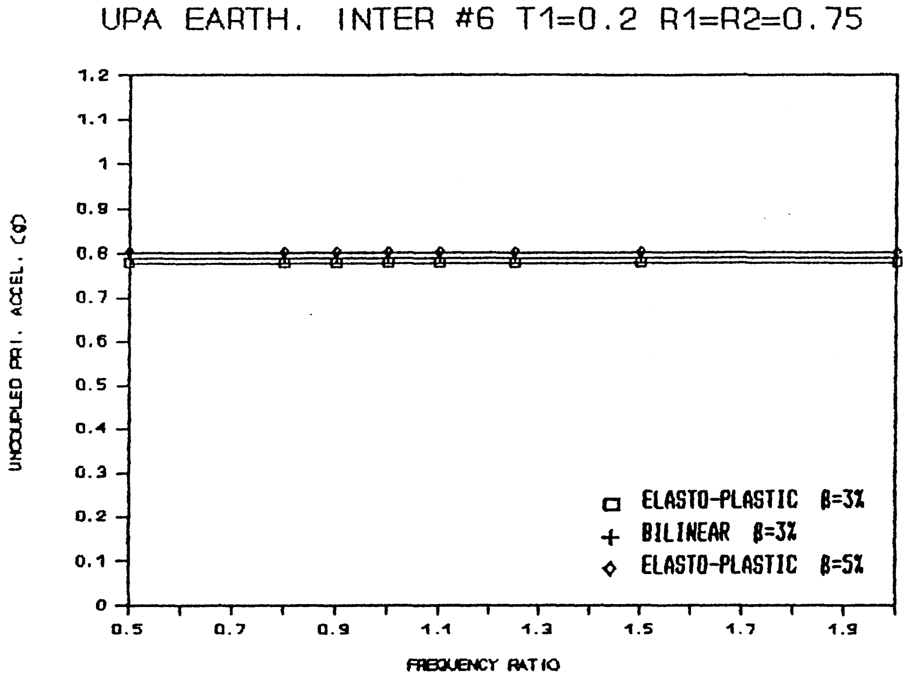


Figure B.13 Uncoupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.75 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

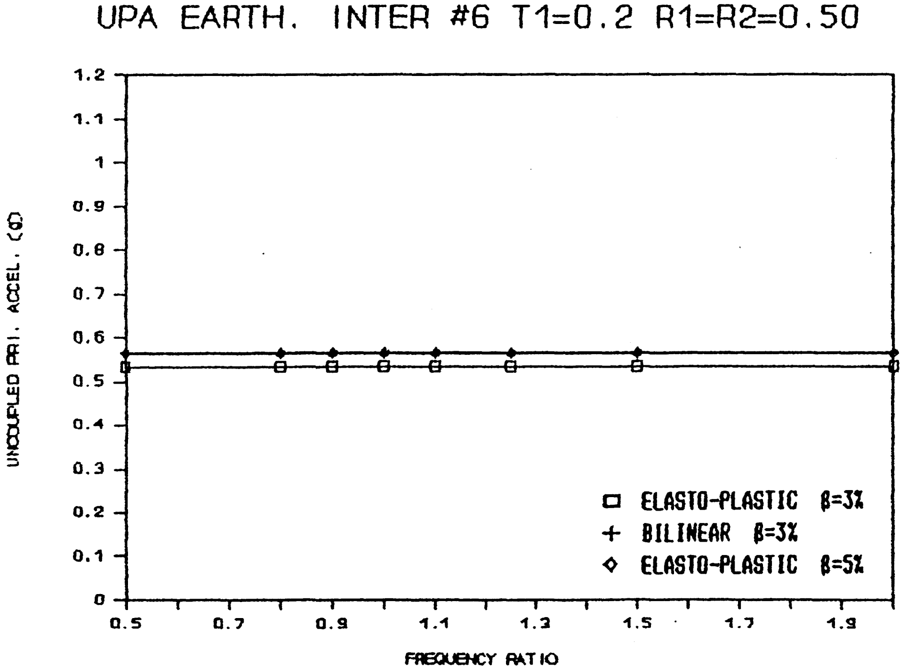


Figure B.14 Uncoupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.50 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

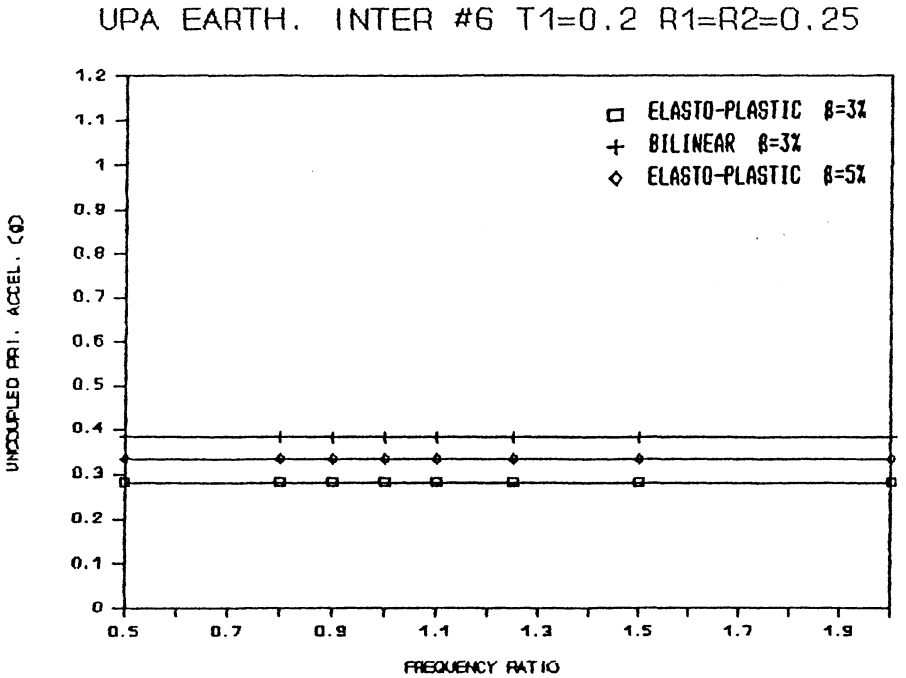


Figure B.15 Uncoupled Primary Acceleration response versus frequency ratio for T1=0.2 sec, intermediate A/V ratio record #6, with R1=R2=0.25 for elasto-plastic β=3%, elasto-plastic β=5% and bilinear β=3%.

APPENDIX C

EFFECT OF YIELD LEVELS ON THE SECONDARY RESPONSE

A complete set of response values for all 16 primary and secondary yield level combinations is presented in figures C.1 to C.40. The coupled secondary acceleration response is shown in figures C.1 to C.20. The coupled secondary displacement response is given is figures C.21 to C.40. Mass ratios of 0.1, 1.0, 2.0, 5.0 and 10.0% are used.

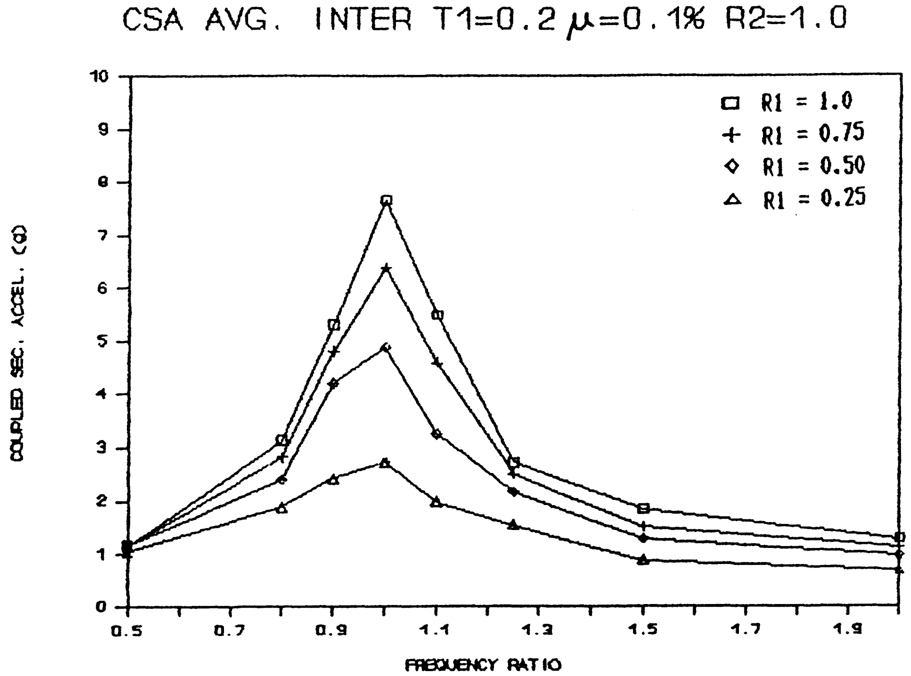

CSA AVG. INTER T1=0.2 μ=0.1% R2=1.0

Figure C.1 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 0.1%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

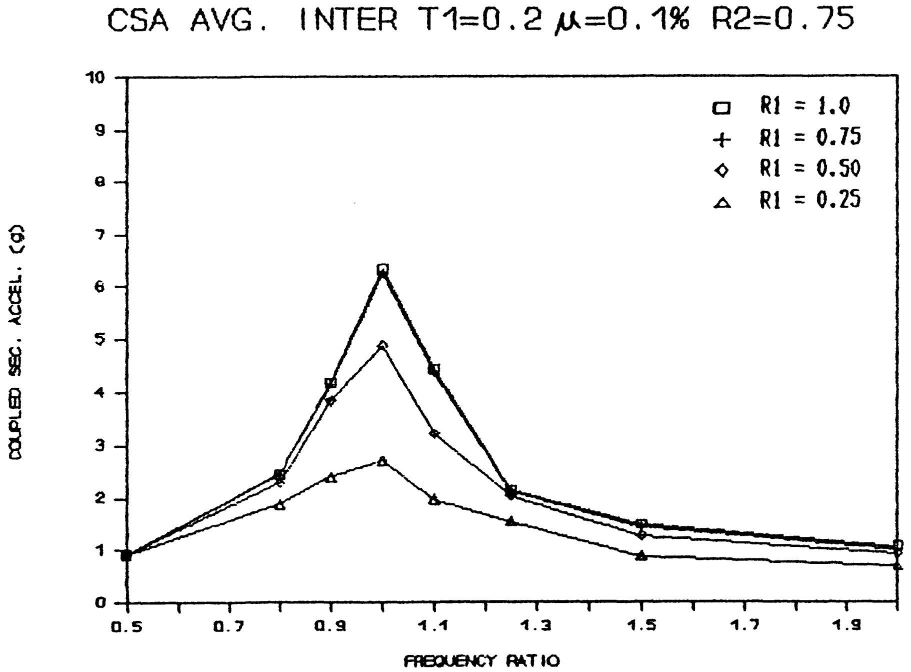

CSA AVG. INTER T1=0.2 μ=0.1% R2=0.75

Figure C.2 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 0.1%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

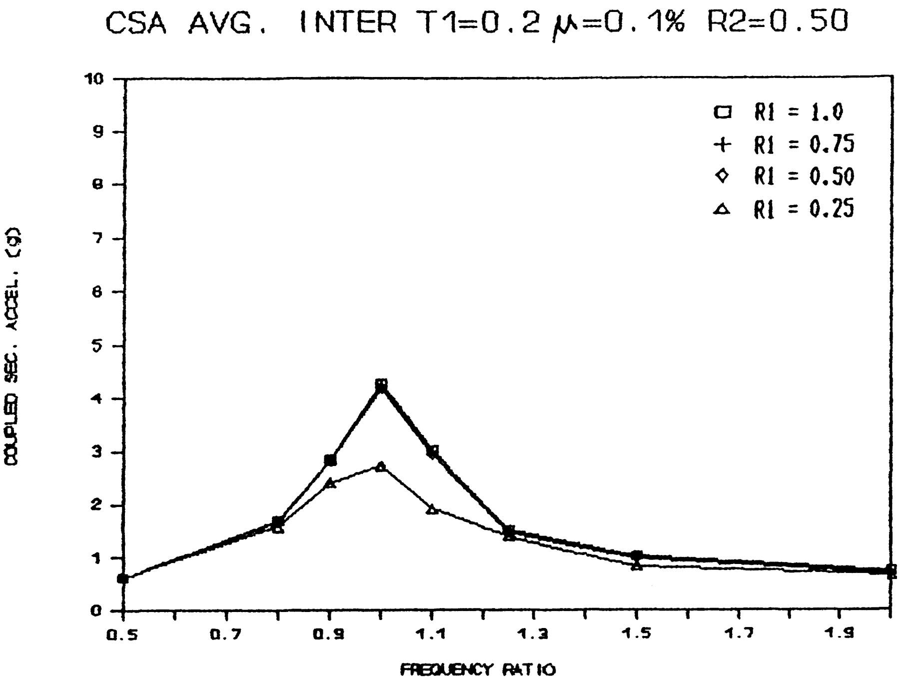


Figure C.3 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 0.1%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

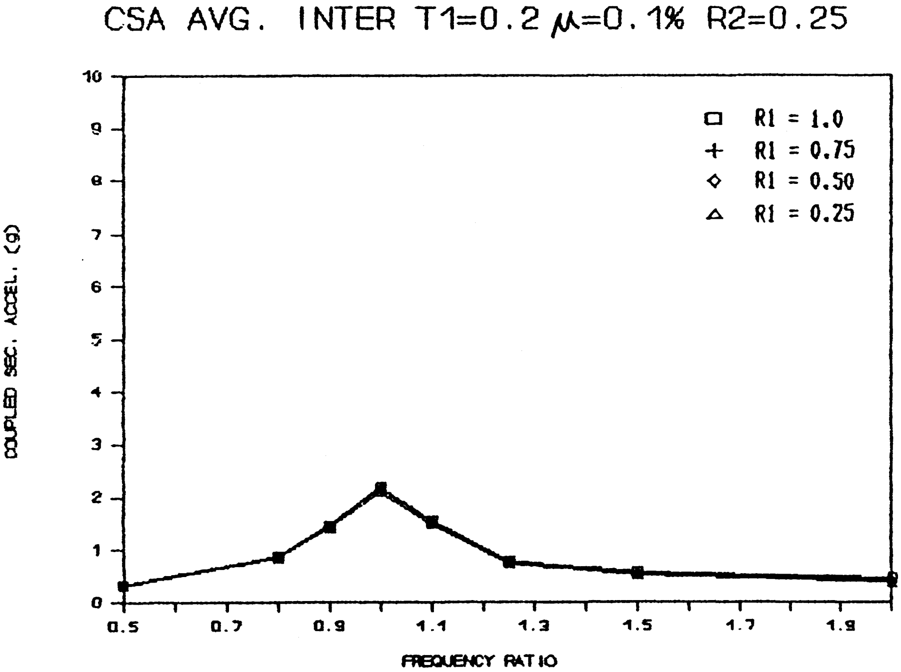


Figure C.4 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 0.1%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

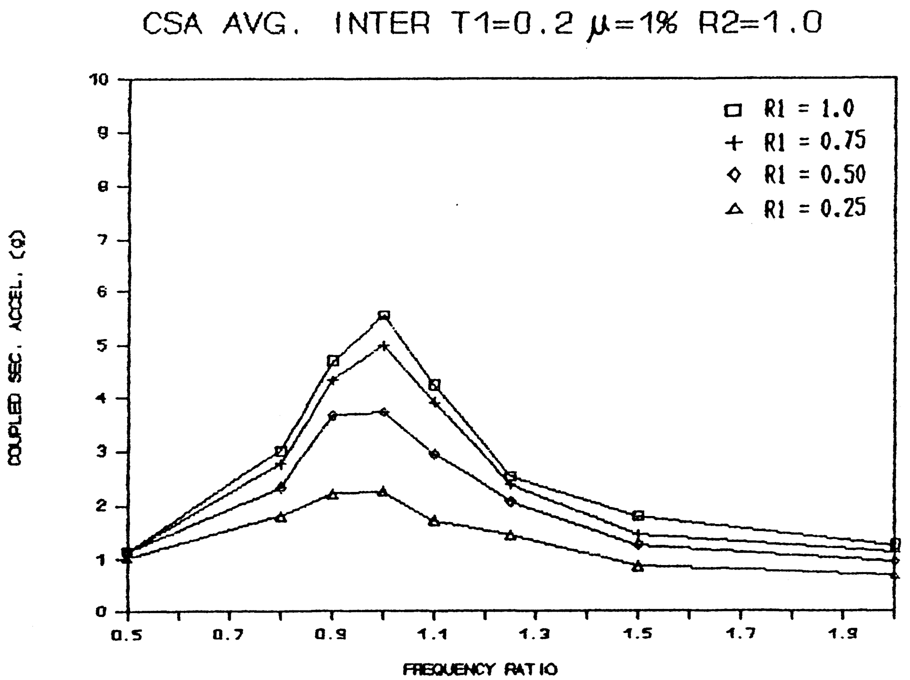


Figure C.5 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

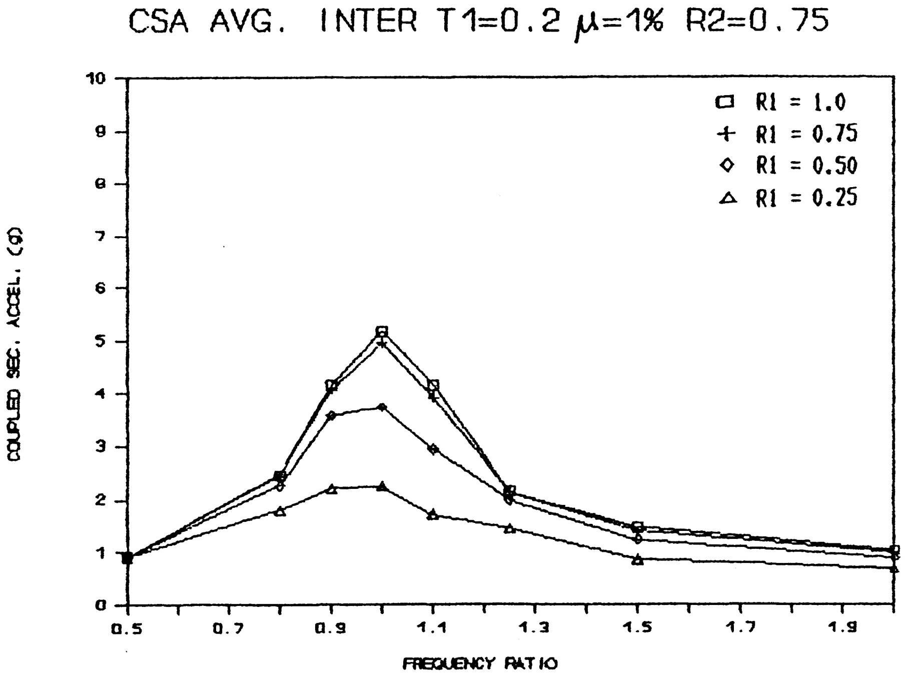


Figure C.6 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

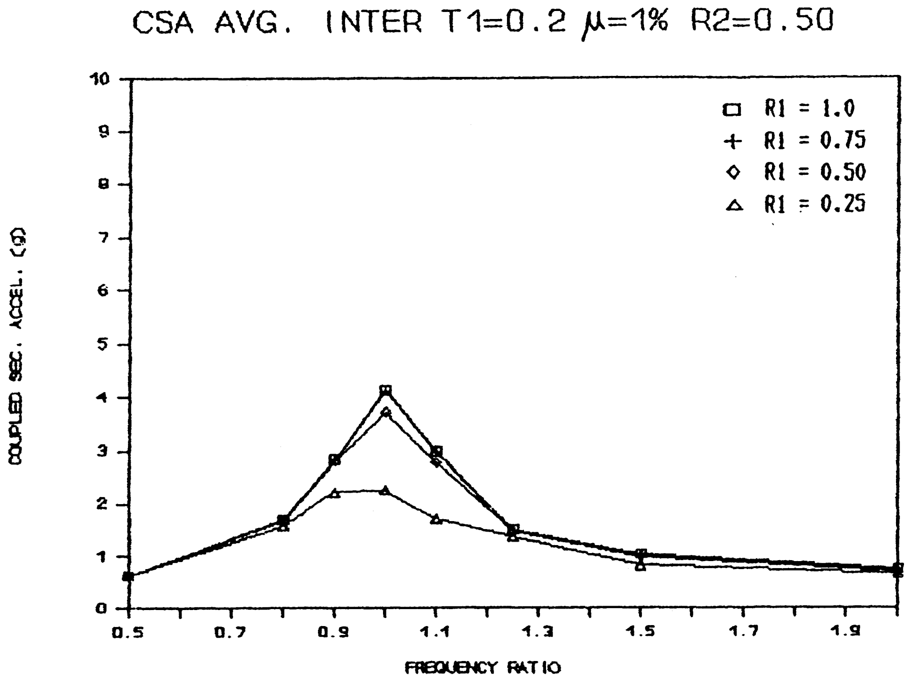


Figure C.7 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

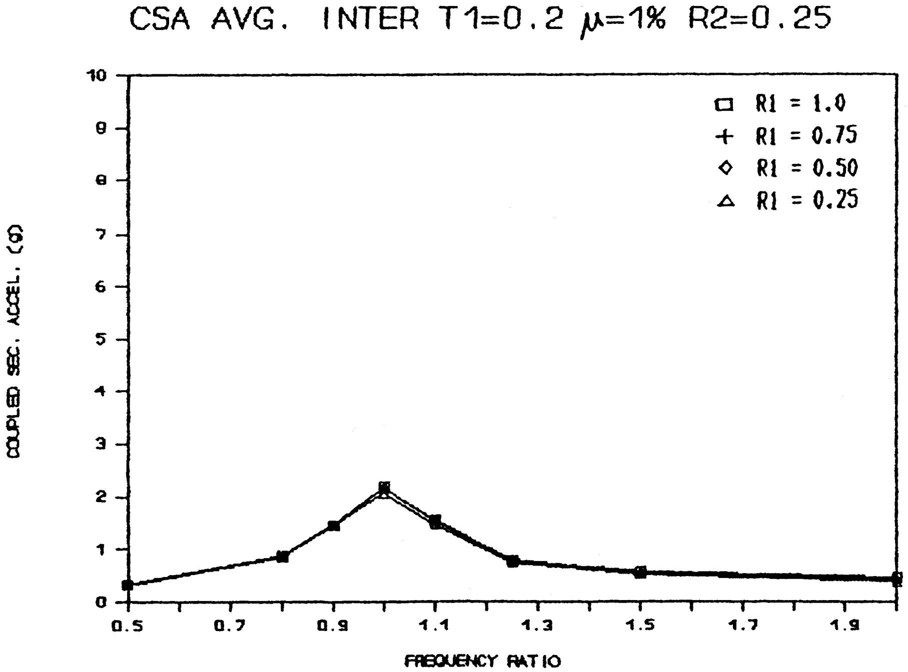


Figure C.8 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

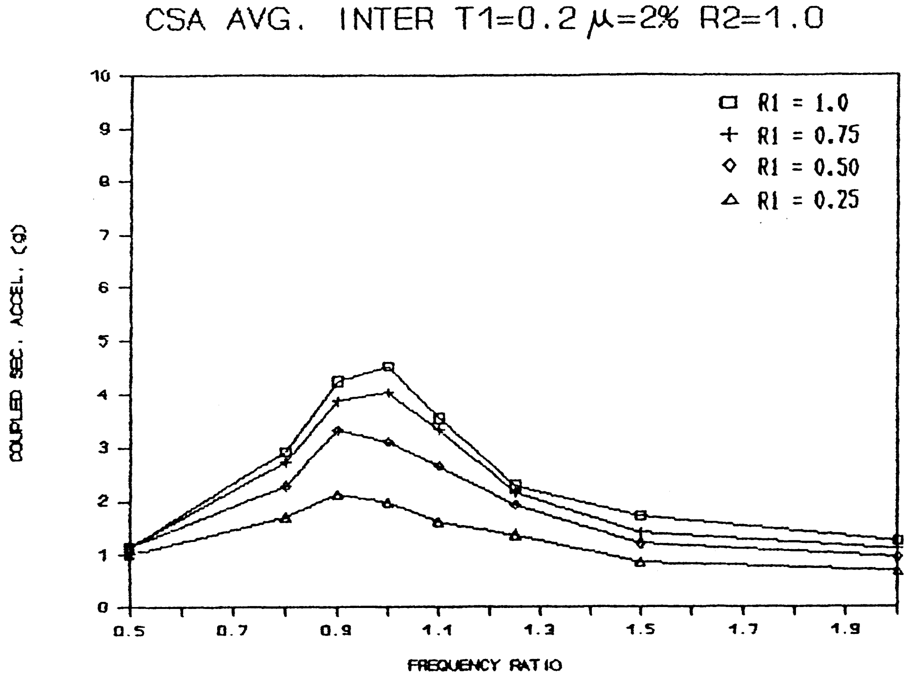


Figure C.9 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 2.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

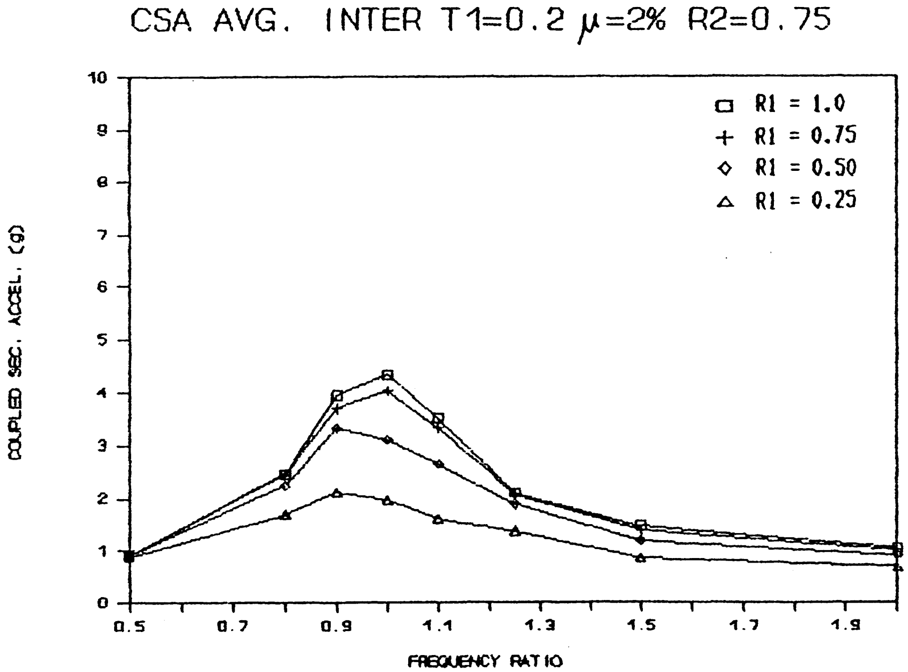


Figure C.10 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 2.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

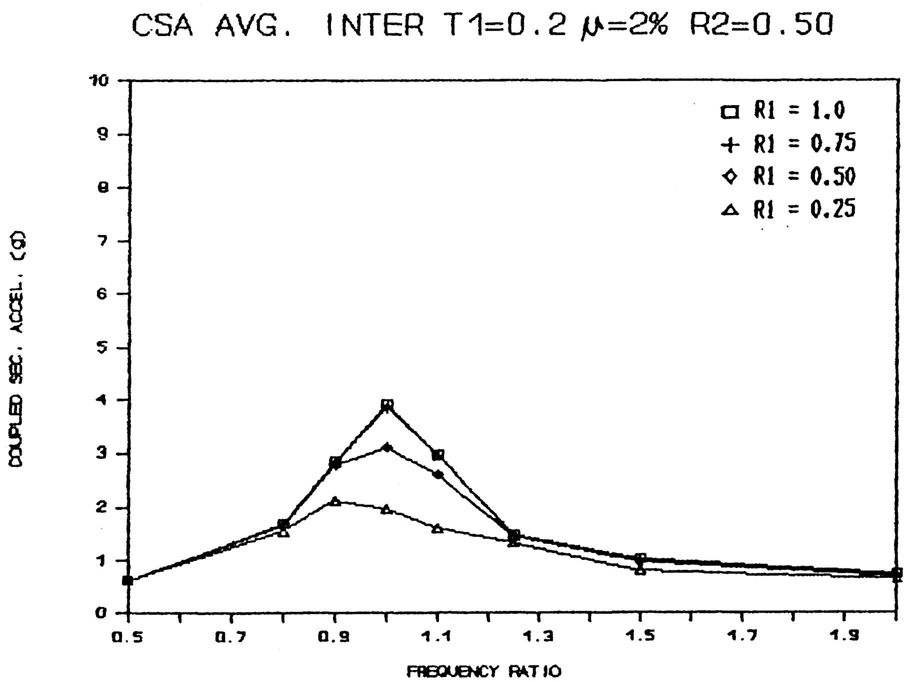


Figure C.11 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

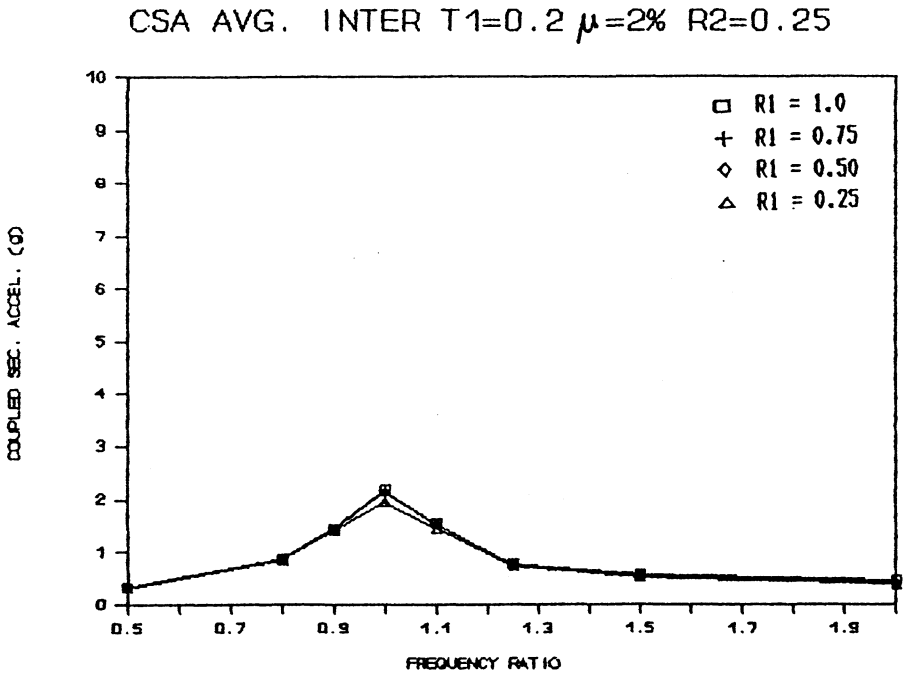


Figure C.12 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

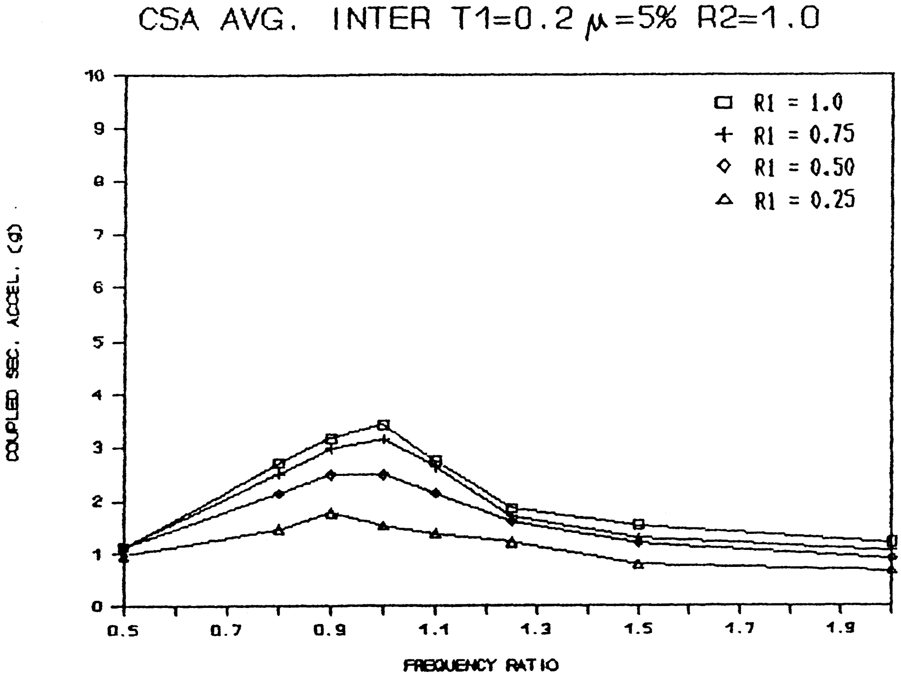


Figure C.13 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

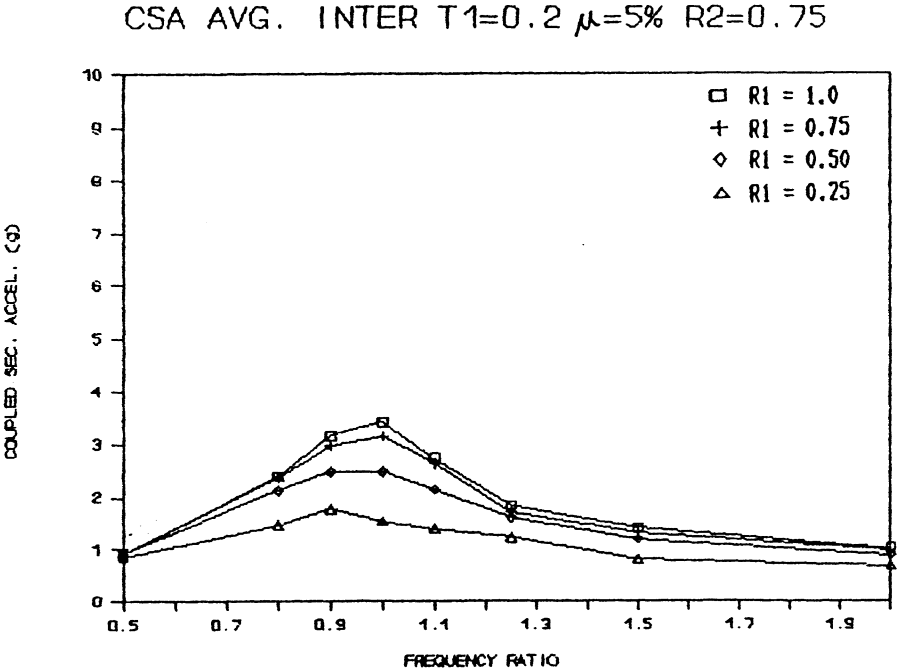


Figure C.14 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

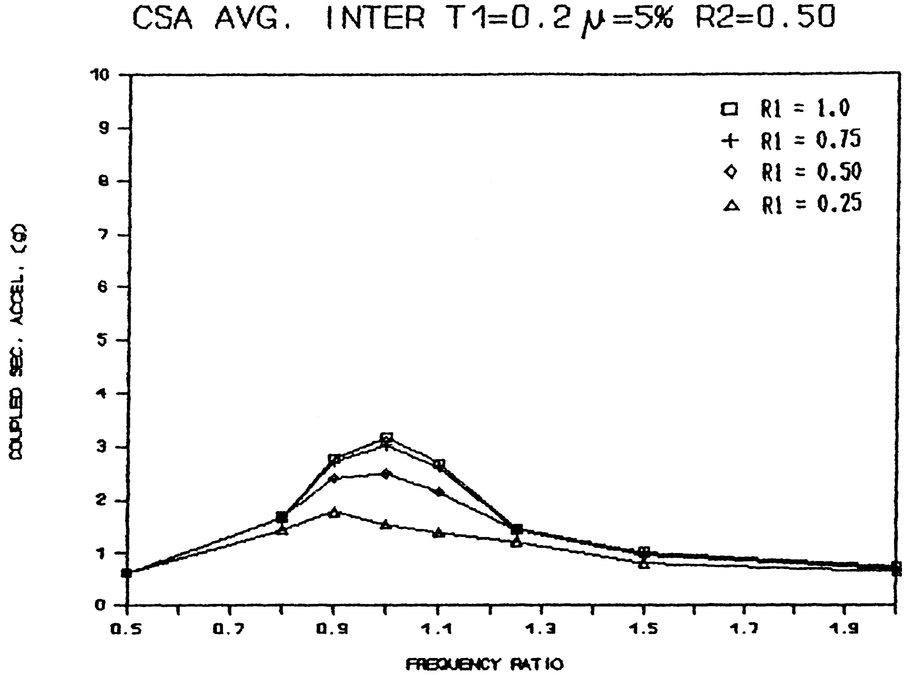

CSA AVG. INTER T1=0.2 μ=5% R2=0.50

Figure C.15 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 5.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

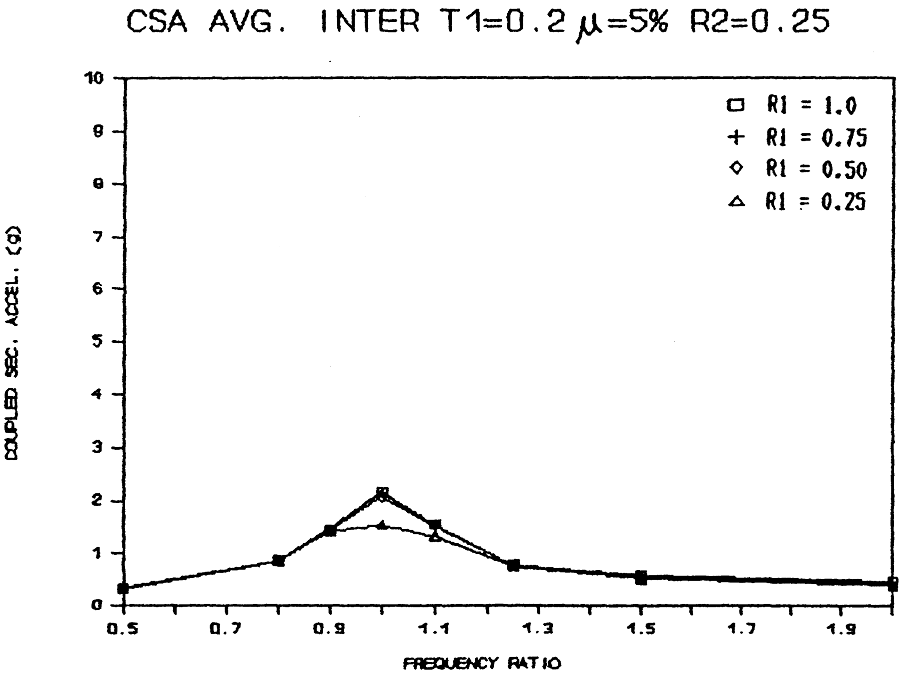

CSA AVG. INTER T1=0.2 μ=5% R2=0.25

Figure C.16 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 5.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

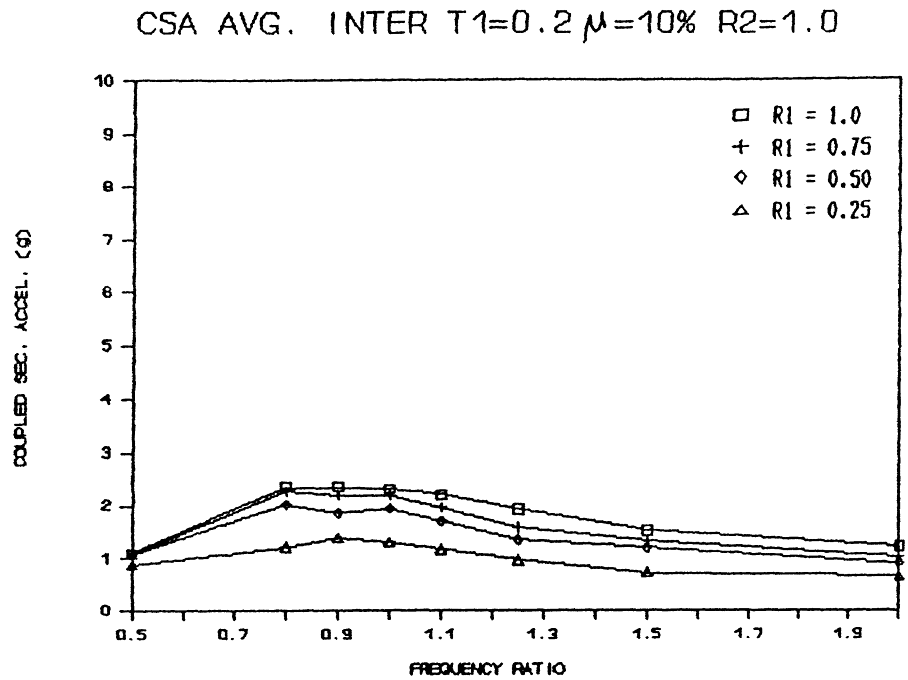


Figure C.17 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

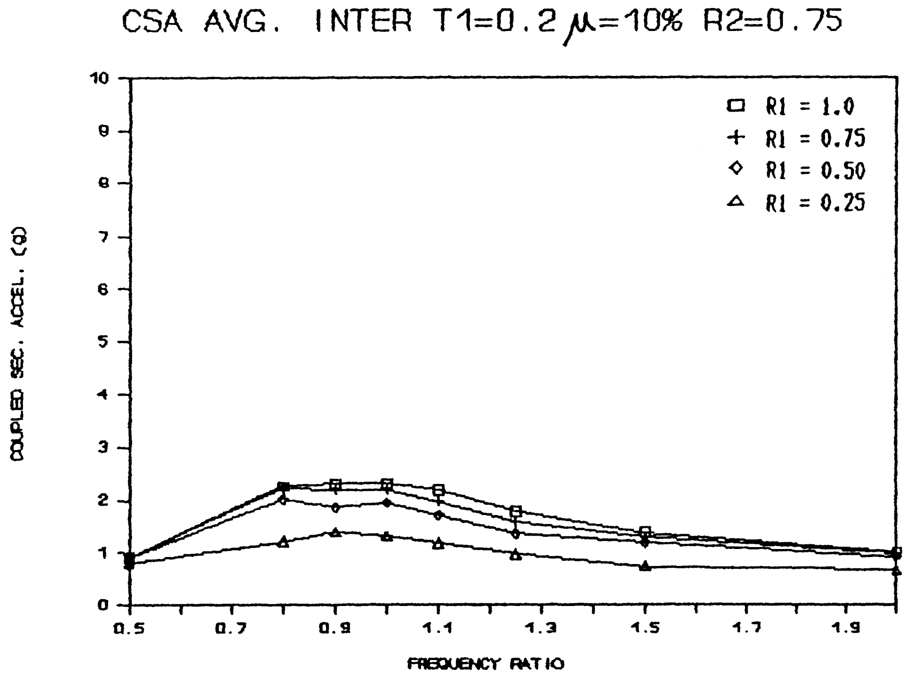


Figure C.18 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

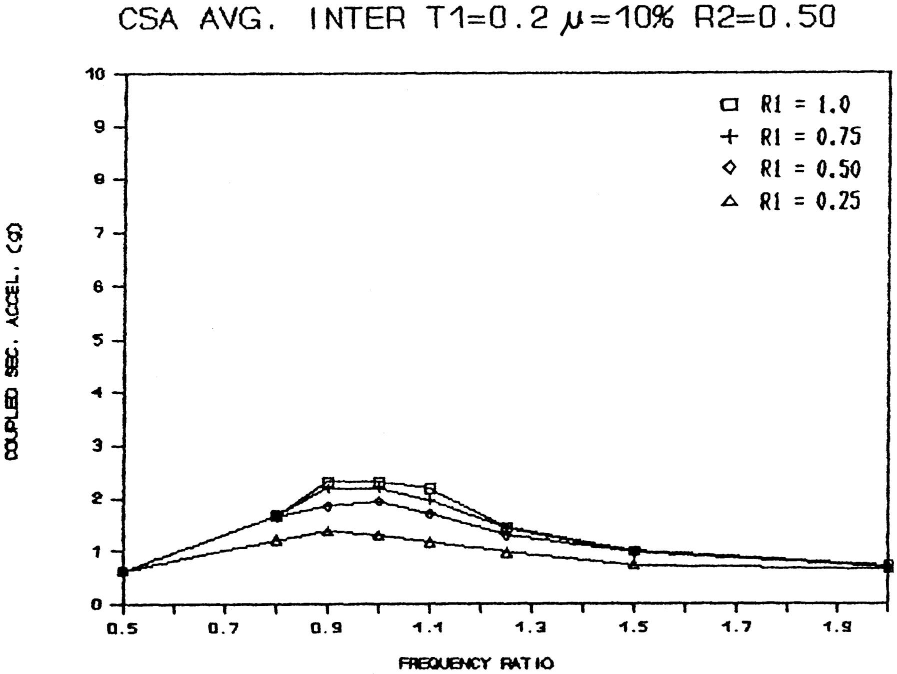


Figure C.19 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

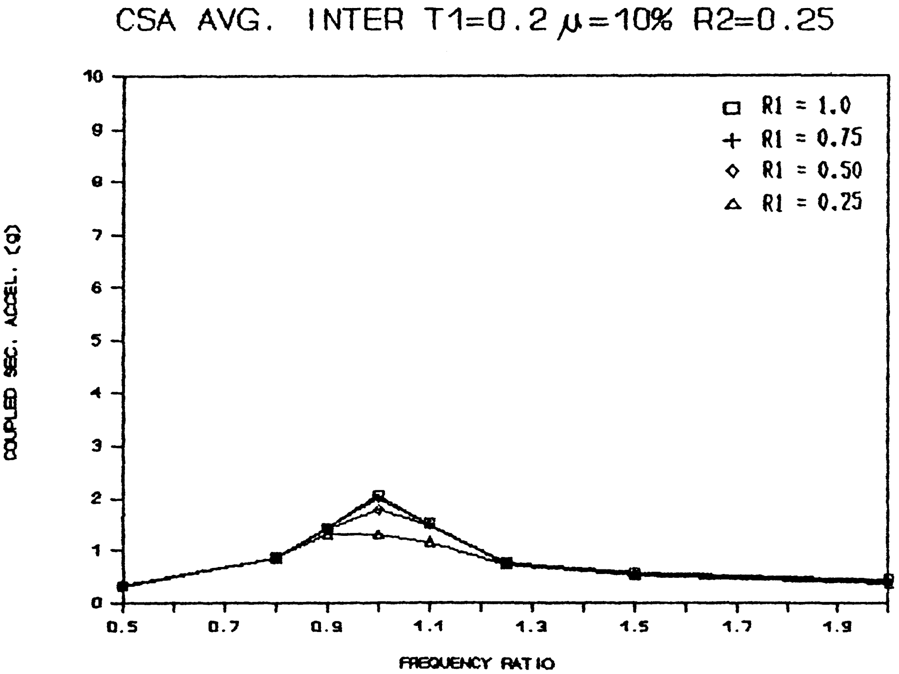


Figure C.20 Coupled Secondary Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

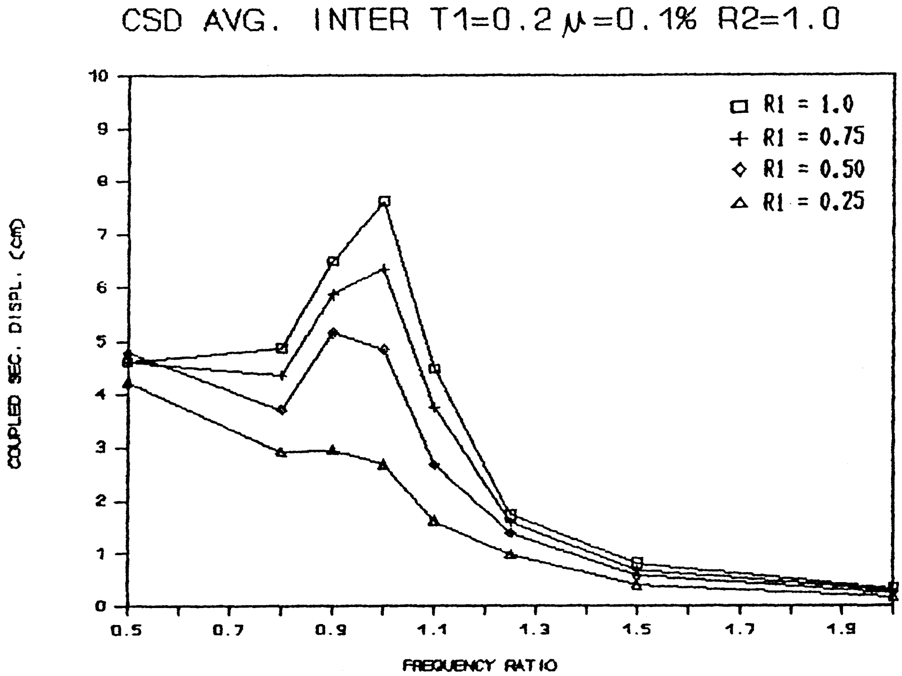


Figure C.21 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 0.1%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

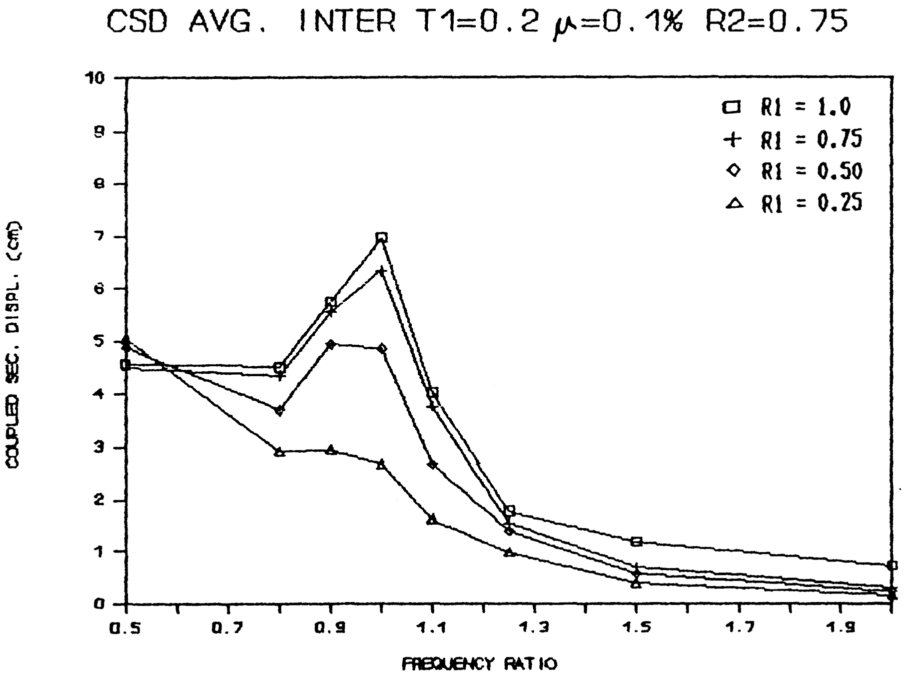


Figure C.22 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 0.1%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

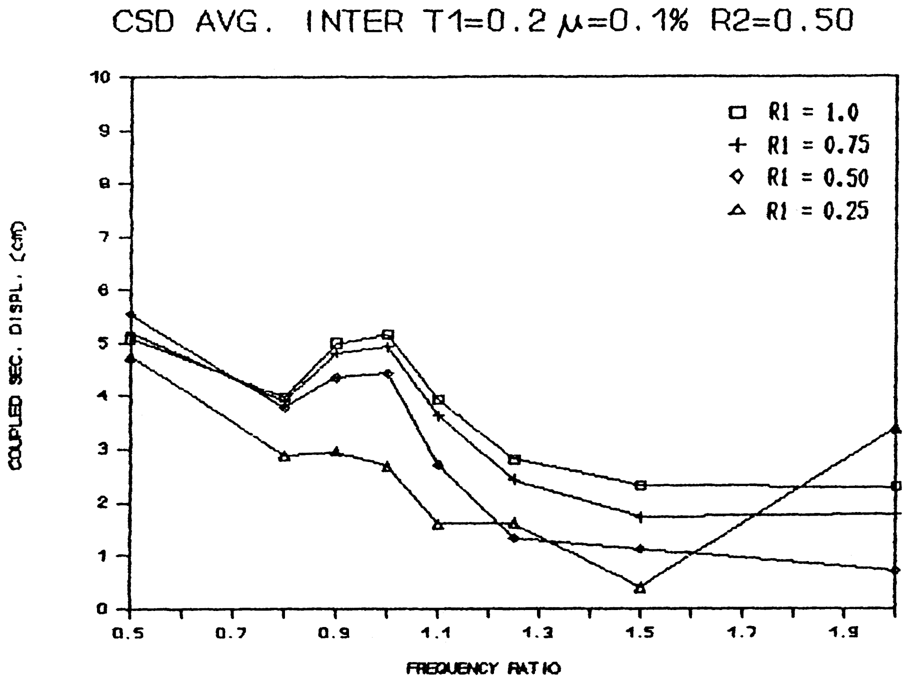


Figure C.23 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 0.1%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

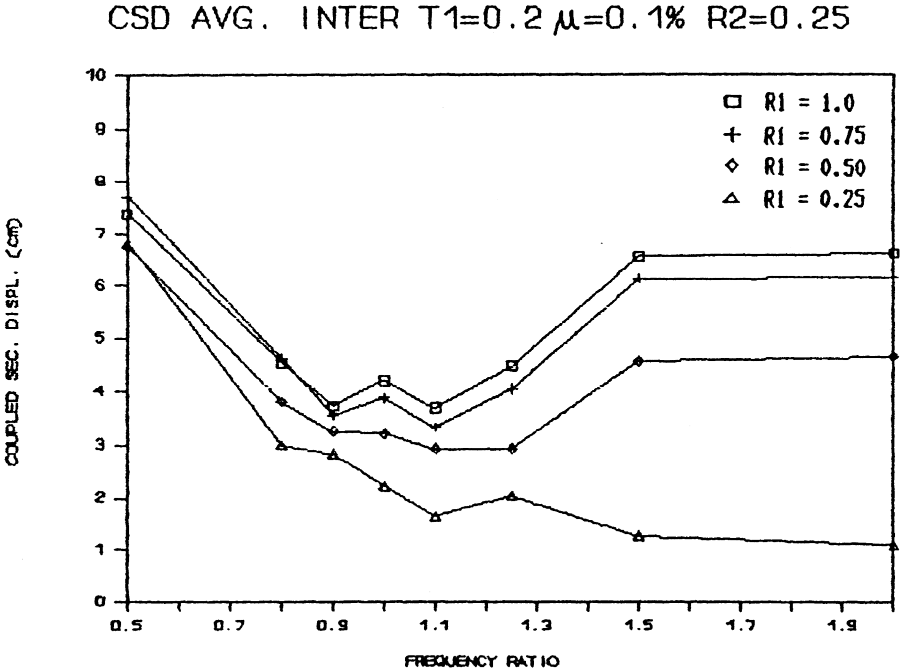


Figure C.24 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 0.1%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

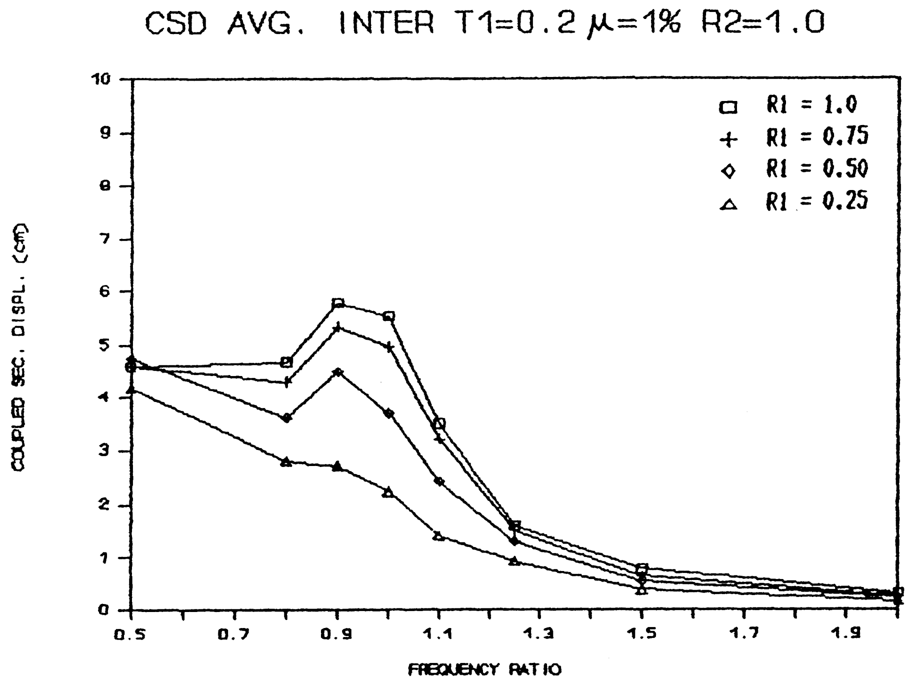


Figure C.25 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 1.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

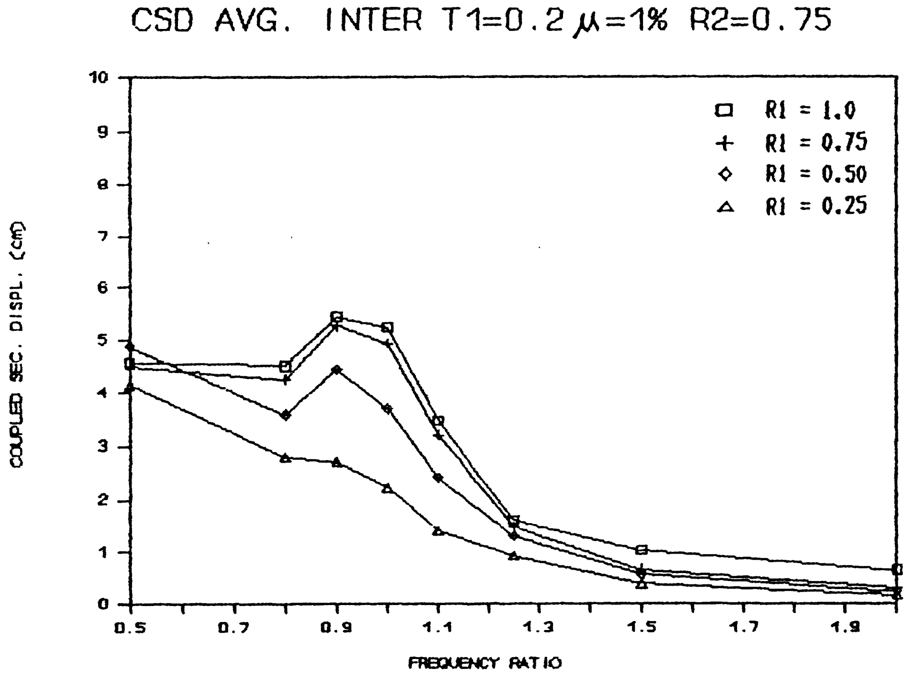


Figure C.26 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 1.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

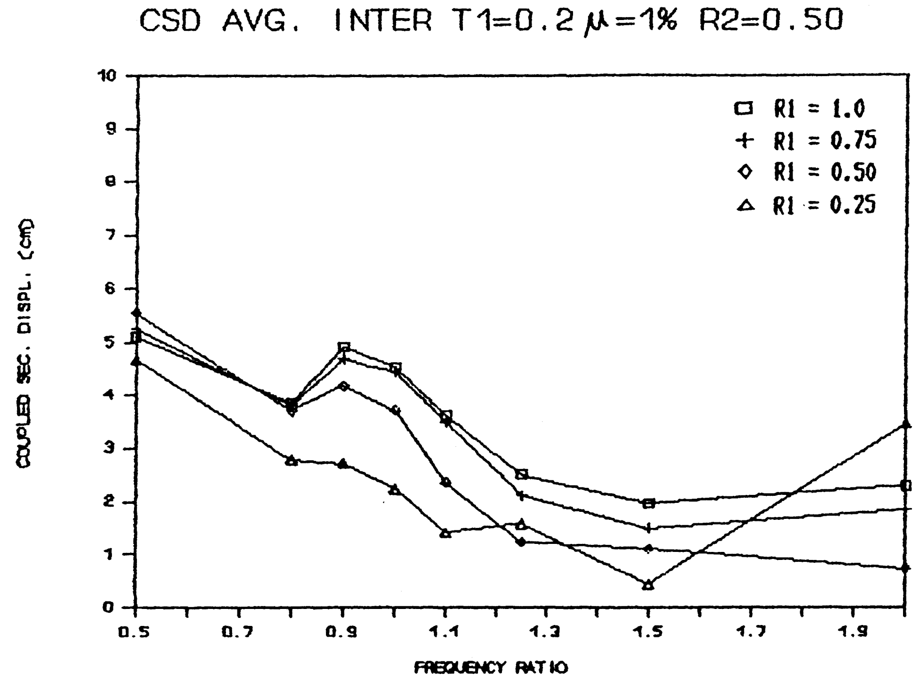


Figure C.27 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

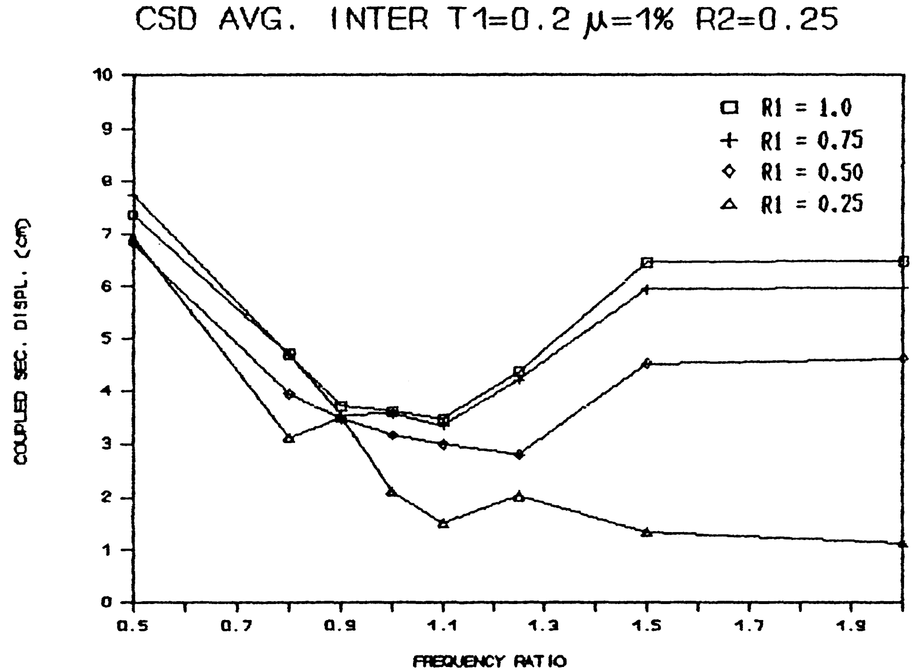


Figure C.28 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

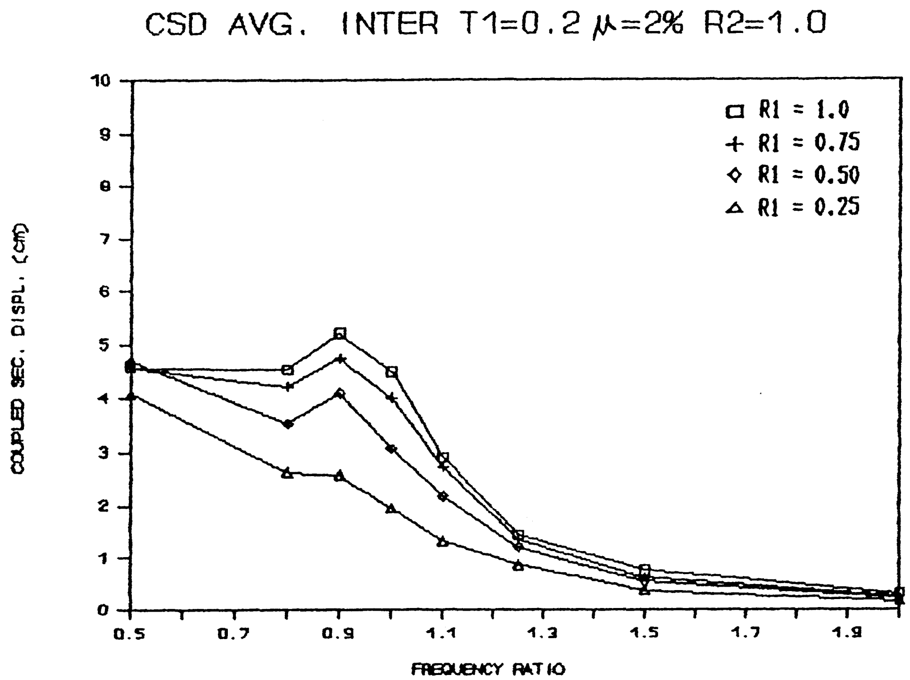


Figure C.29 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

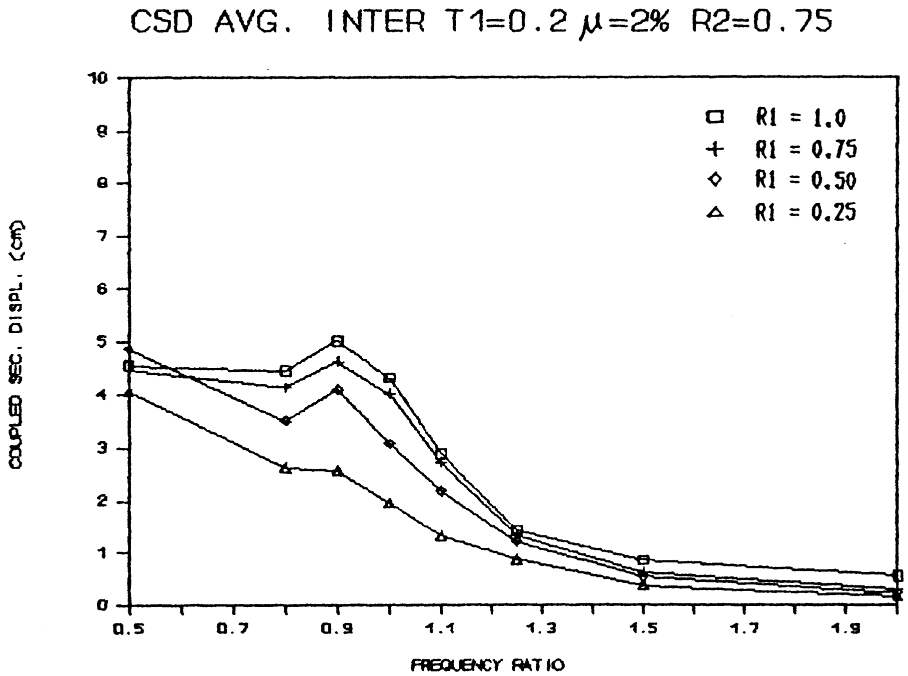


Figure C.30 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

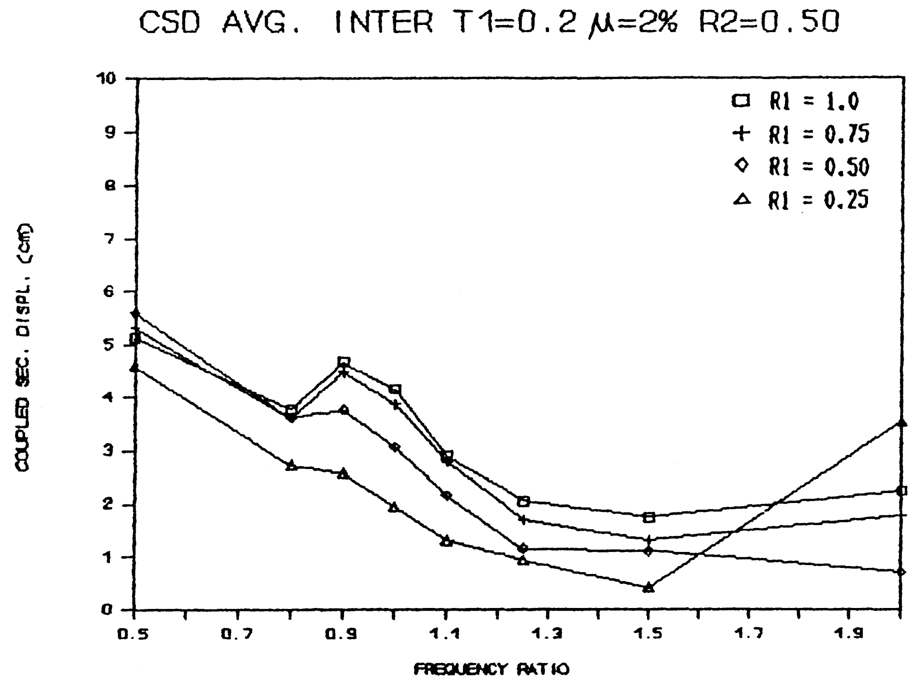


Figure C.31 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

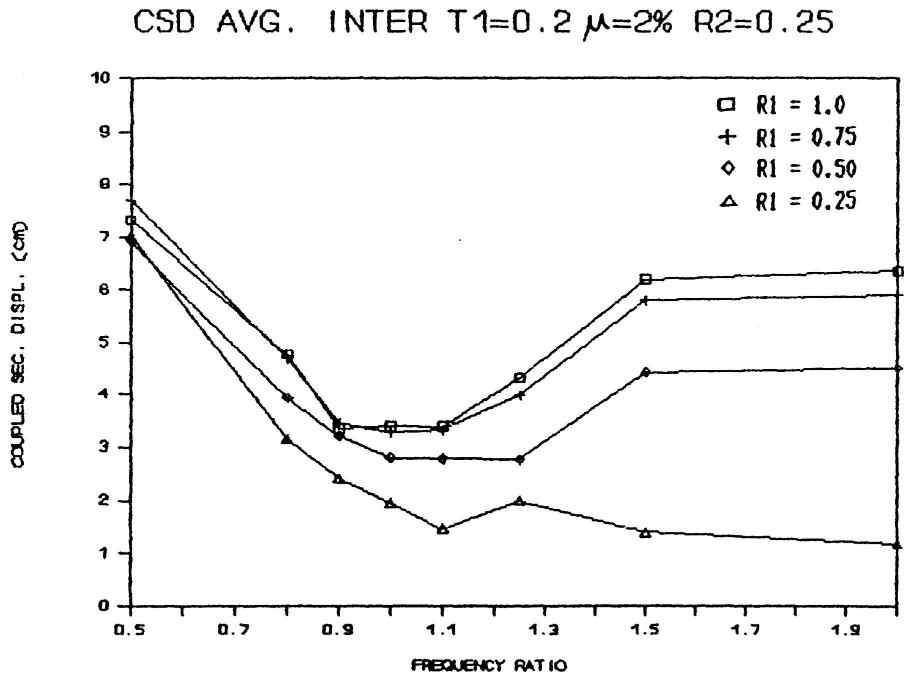


Figure C.32 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

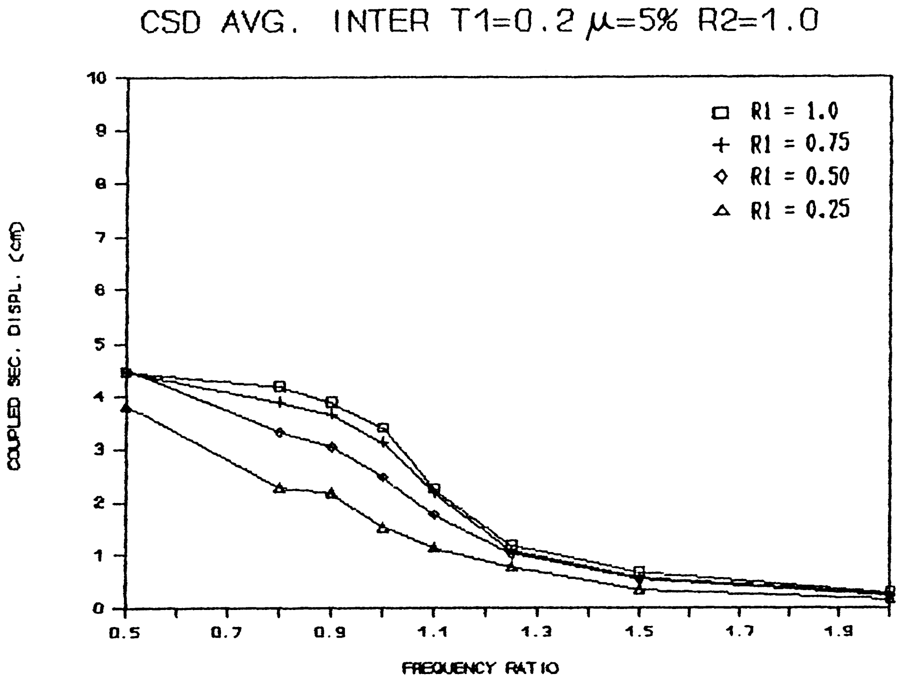


Figure C.33 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

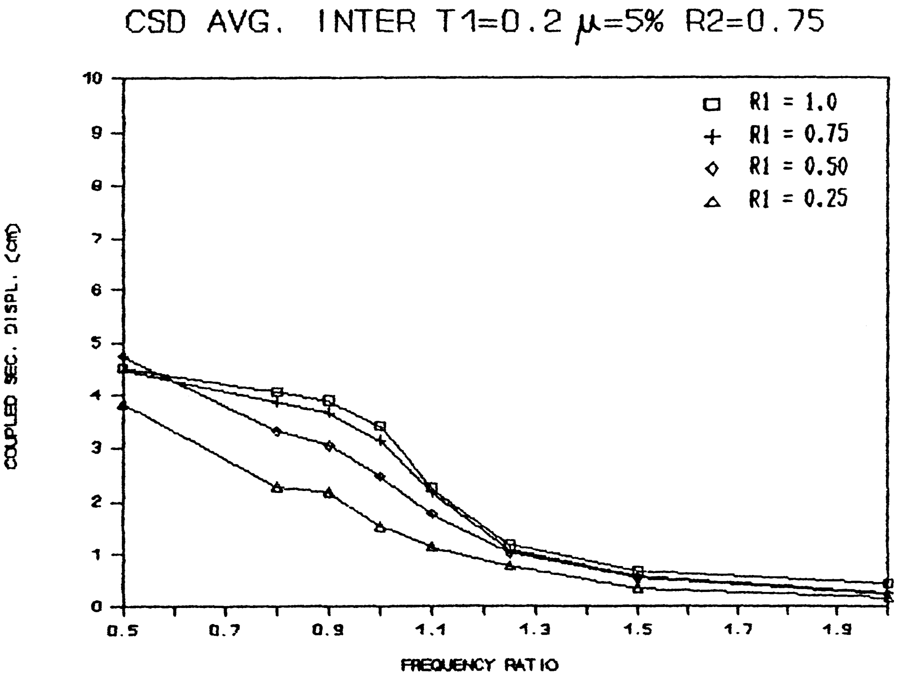


Figure C.34 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

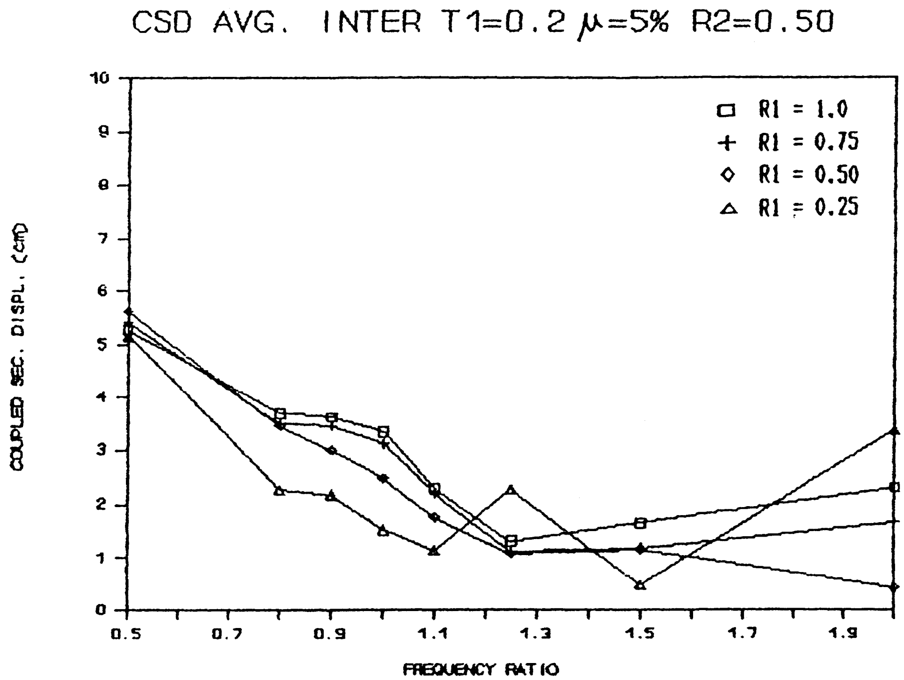


Figure C.35 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

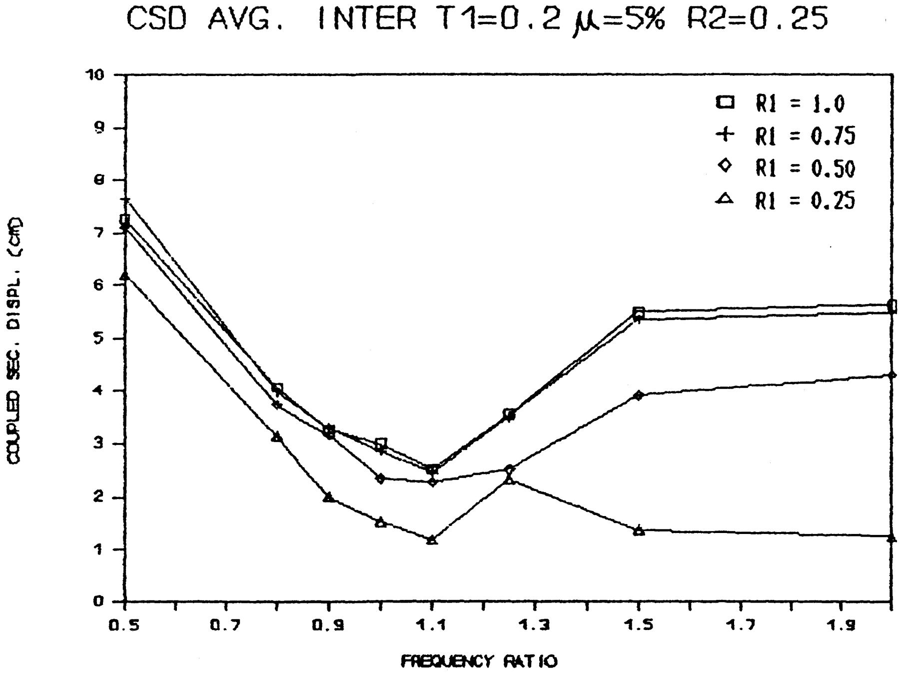


Figure C.36 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

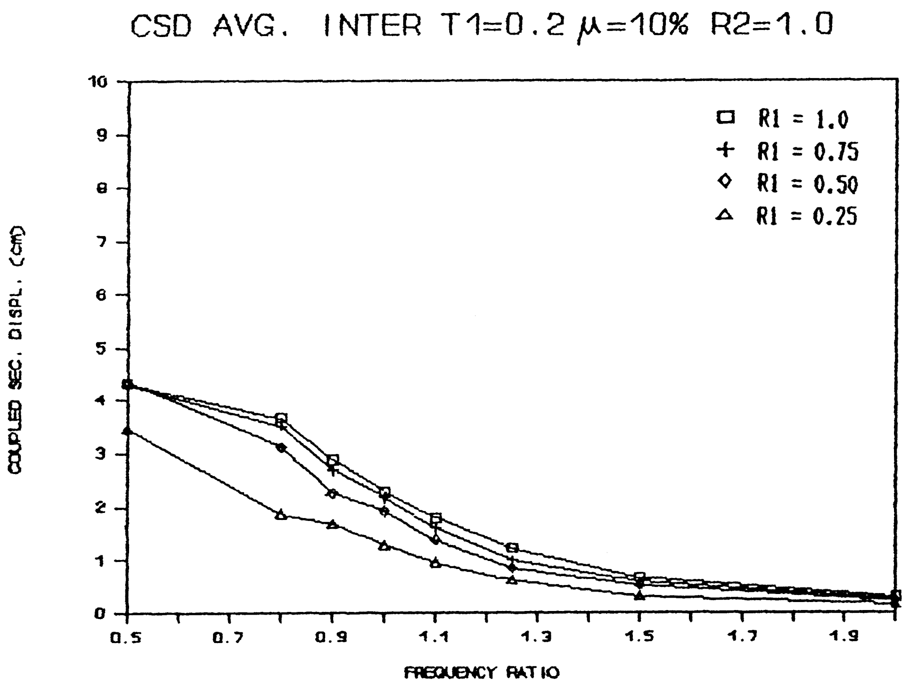


Figure C.37 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R2=1.0 and R1=1.0, 0.75, 0.50 or 0.25.

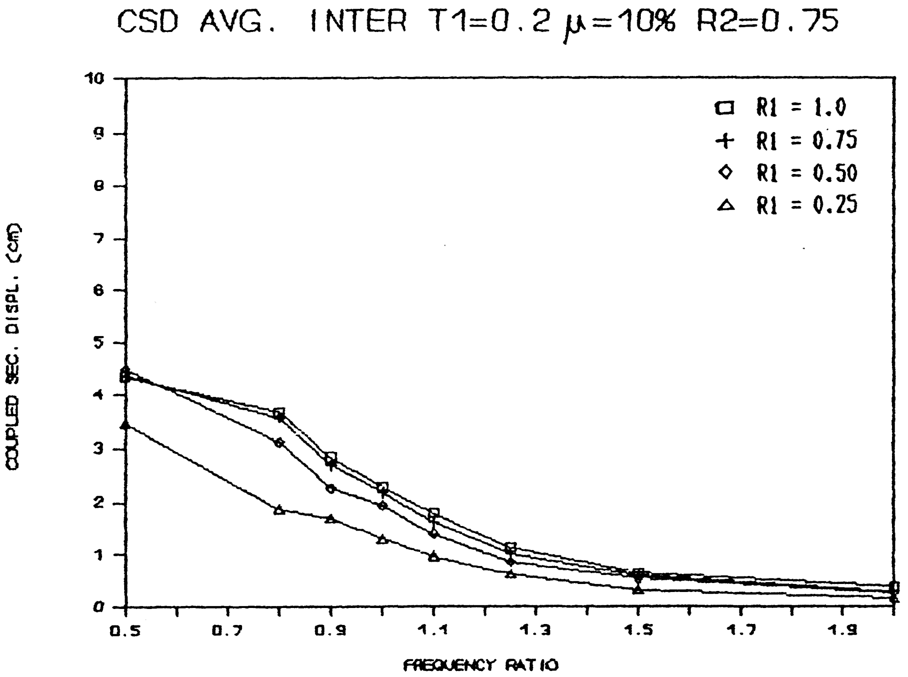


Figure C.38 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R2=0.75 and R1=1.0, 0.75, 0.50 or 0.25.

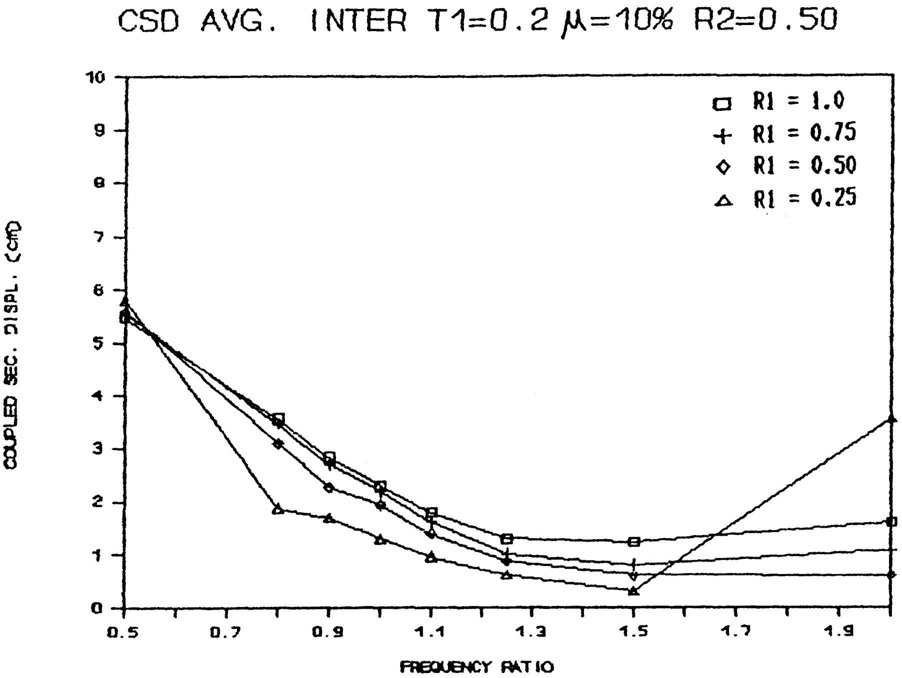


Figure C.39 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R2=0.50 and R1=1.0, 0.75, 0.50 or 0.25.

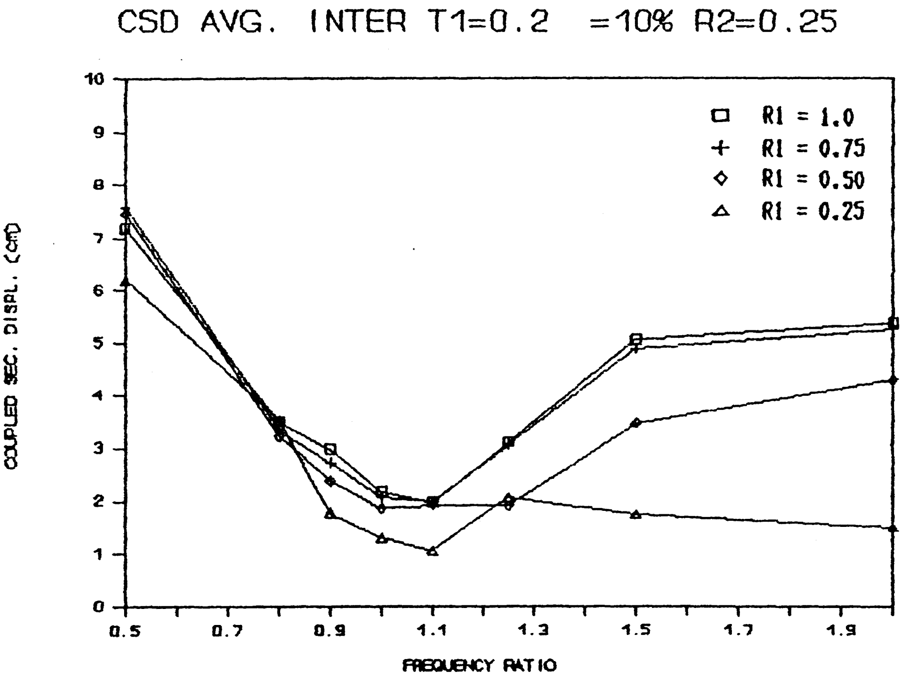


Figure C.40 Coupled Secondary Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R2=0.25 and R1=1.0, 0.75, 0.50 or 0.25.

APPENDIX D

RATIO OF COUPLED/UNCOUPLED RESPONSES

The ratio of coupled/uncoupled responses is shown in figures D.1 to D.32. Both the primary and secondary responses are given. The plots show the average and the average ± one standard deviation for 5 intermediate A/V ratio records. Figures D.1 to D.16 show the primary and secondary acceleration reponse ratios. Figures D.17 to D.32 show the primary and secondary displacement response ratios.

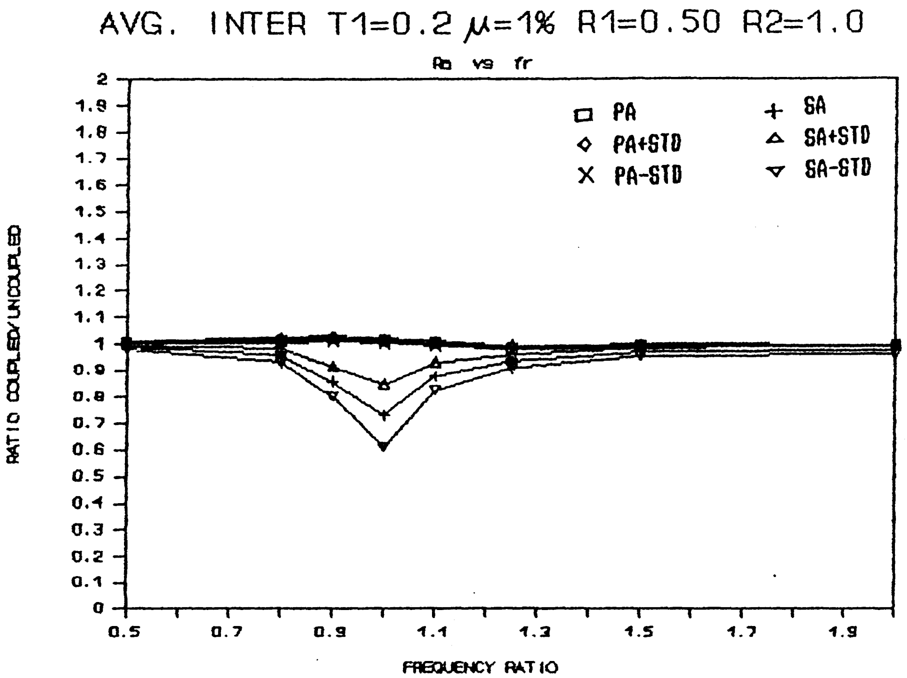


Figure D.1 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R1=0.50 and R2=1.0.

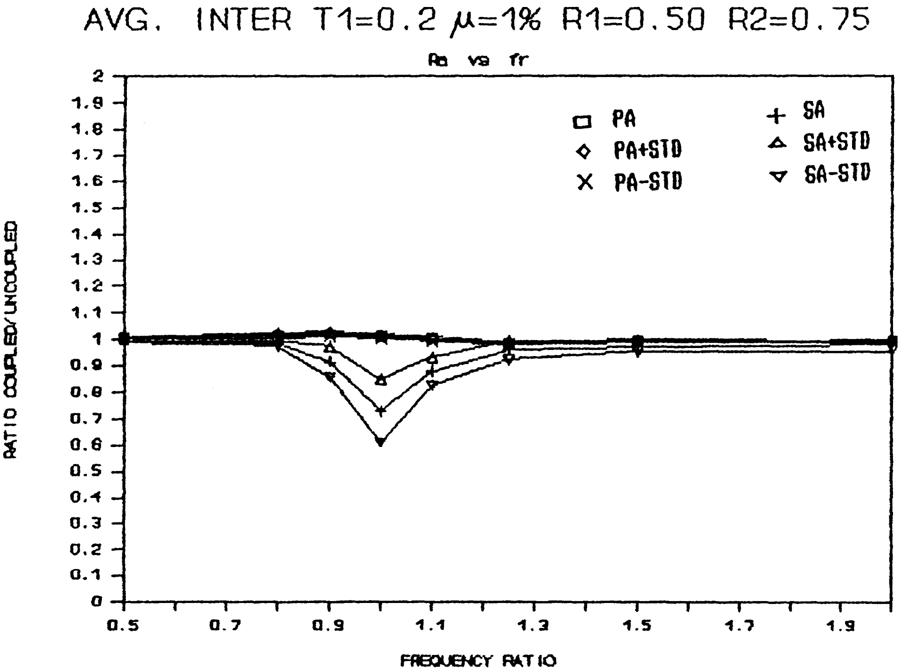


Figure D.2 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R1=0.50 and R2=0.75.

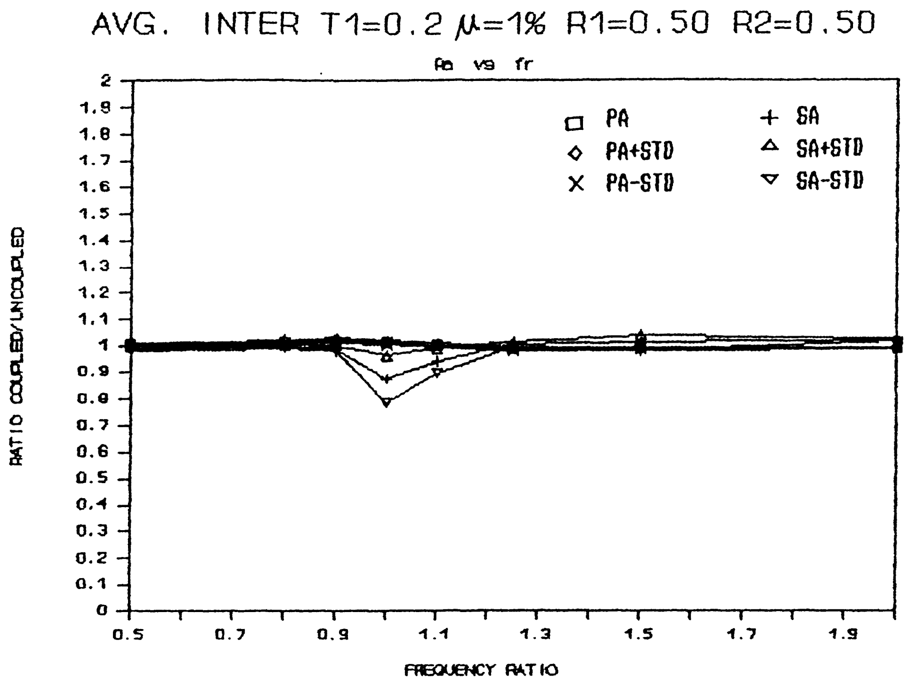


Figure D.3 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 1.0%, R1=0.50 and R2=0.50.

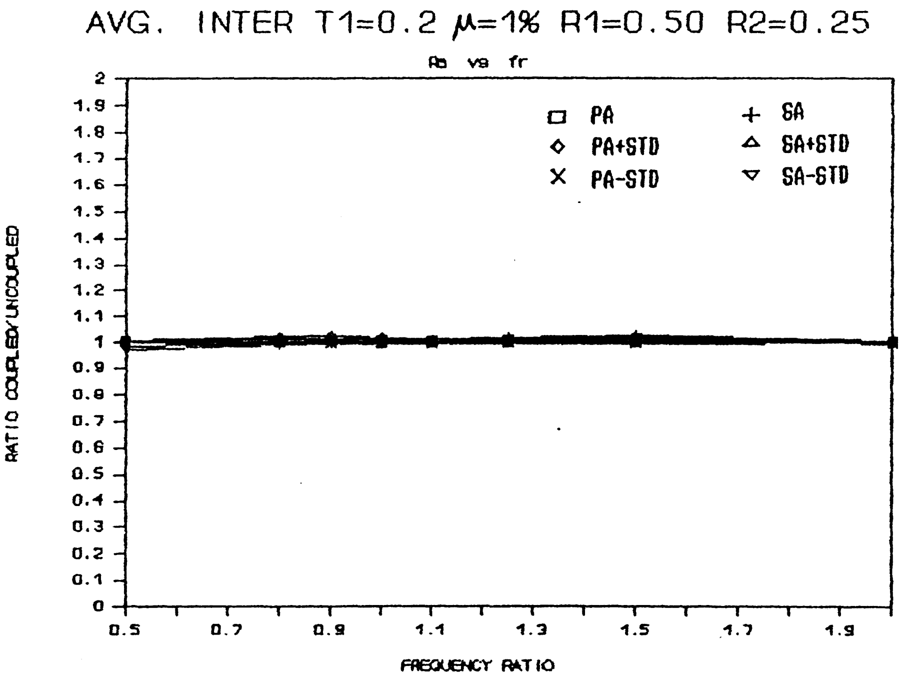


Figure D.4 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 1.0%, R1=0.50 and R2=0.25.

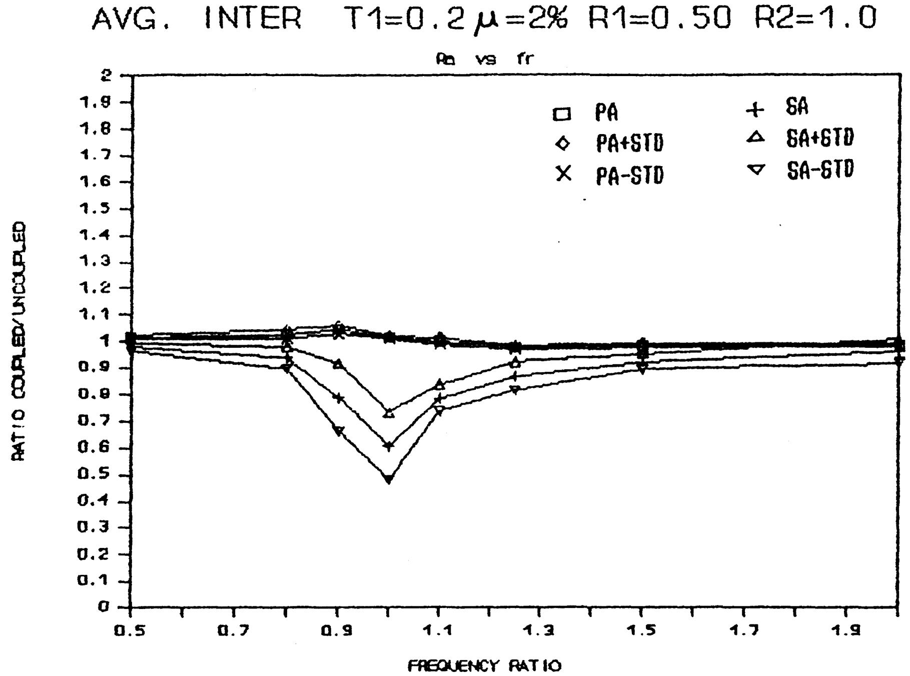


Figure D.5 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 2.0%, R1=0.50 and R2=1.0.

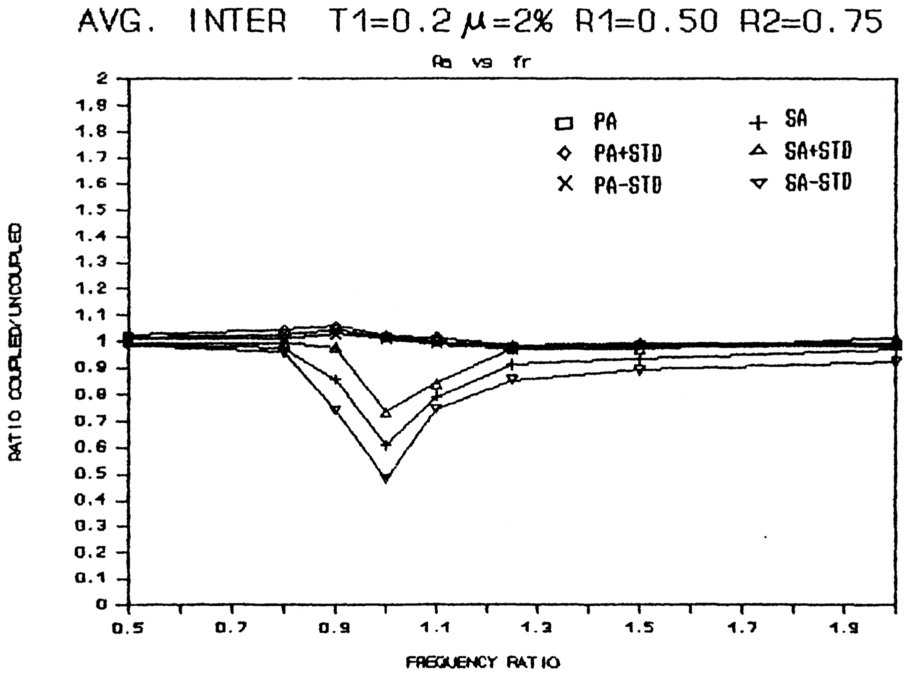


Figure D.6 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 2.0%, R1=0.50 and R2=0.75.

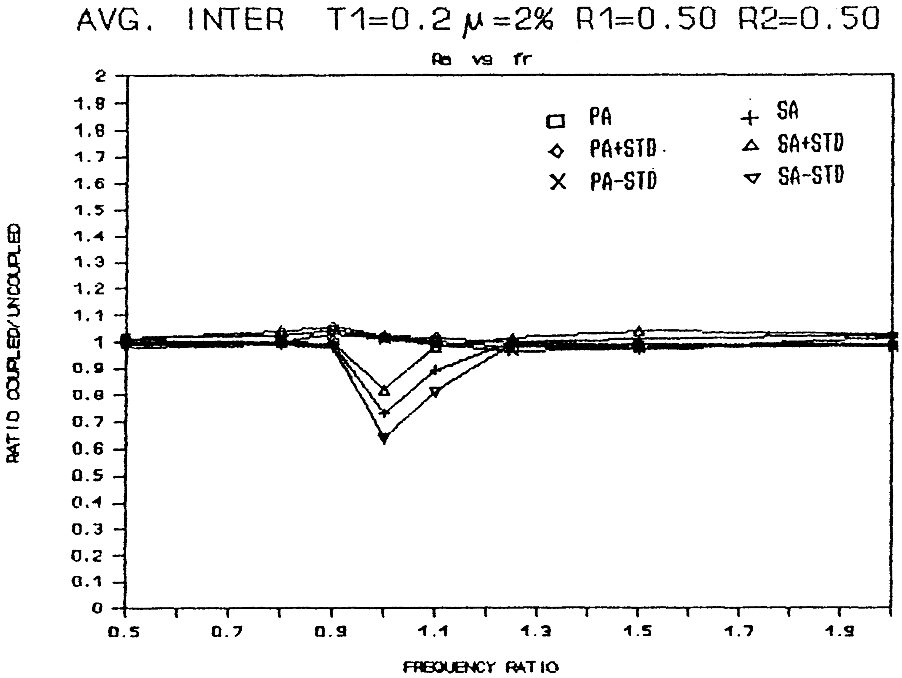


Figure D.7 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R1=0.50 and R2=0.50.

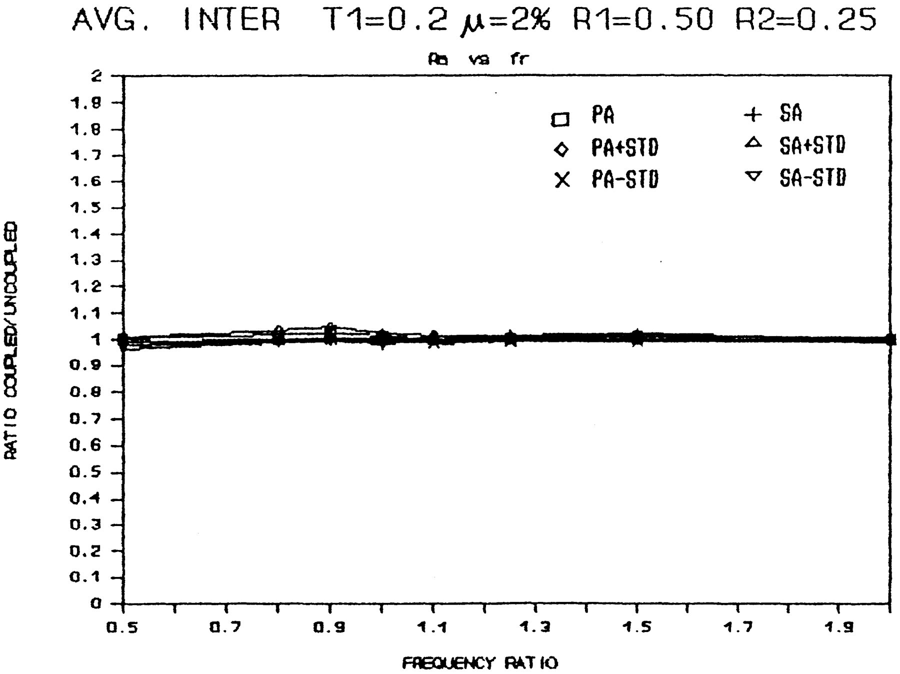


Figure D.8 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R1=0.50 and R2=0.25.

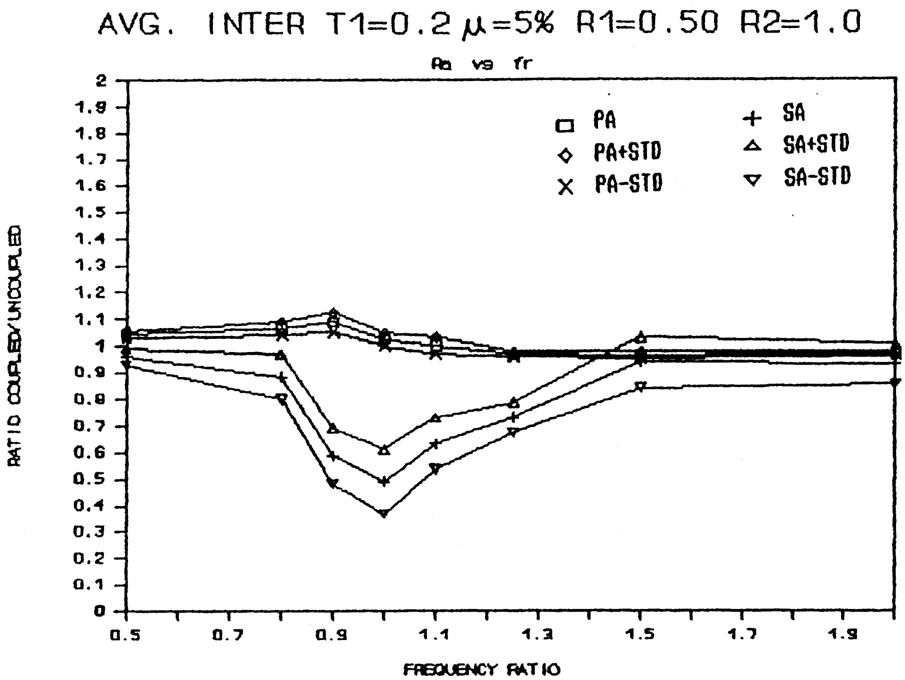


Figure D.9 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R1=0.50 and R2=1.0.

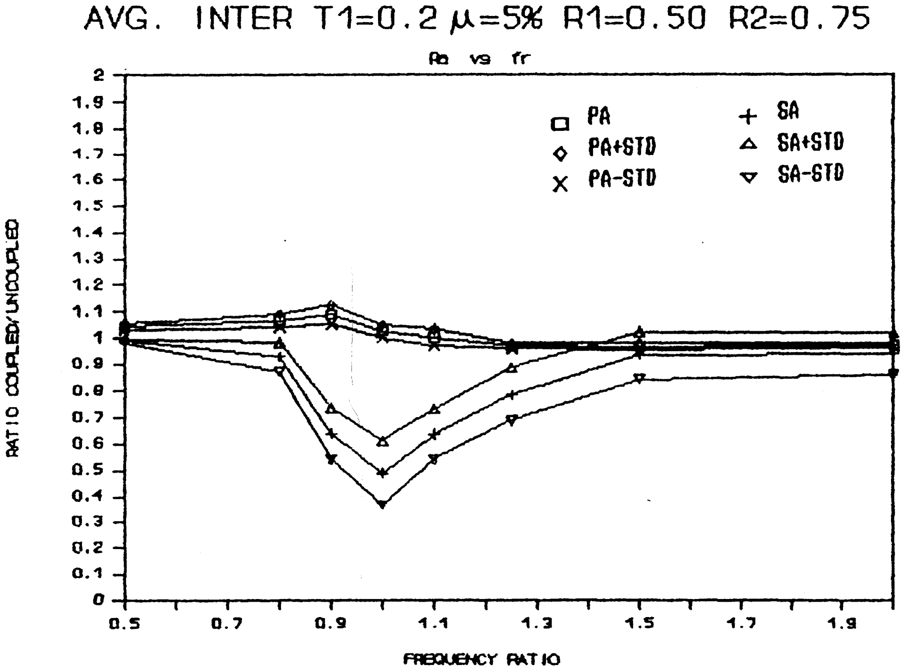


Figure D.10 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R1=0.50 and R2=0.75.

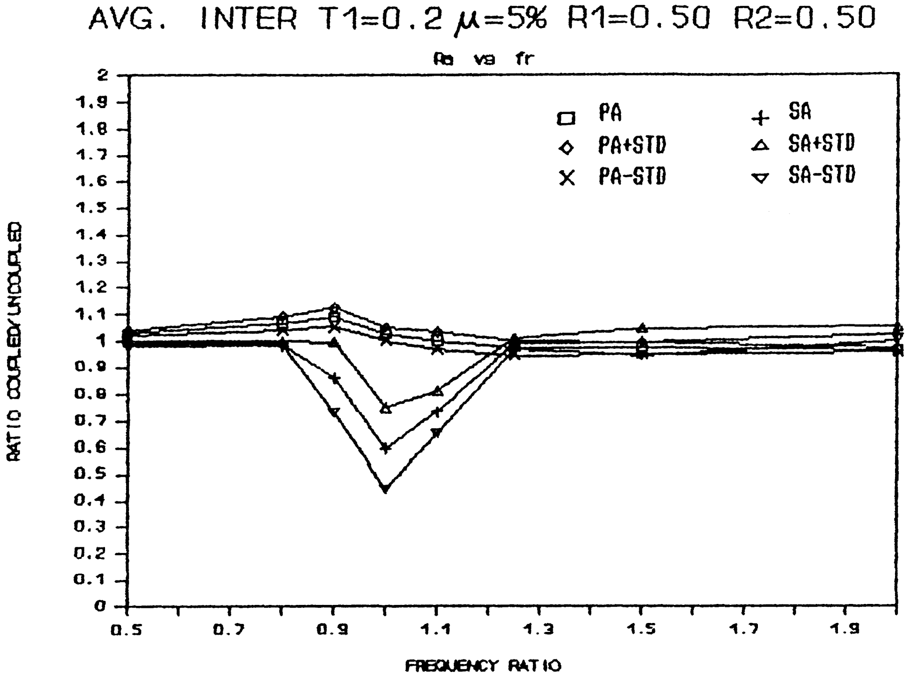


Figure D.11 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 5.0%, R1=0.50 and R2=0.50.

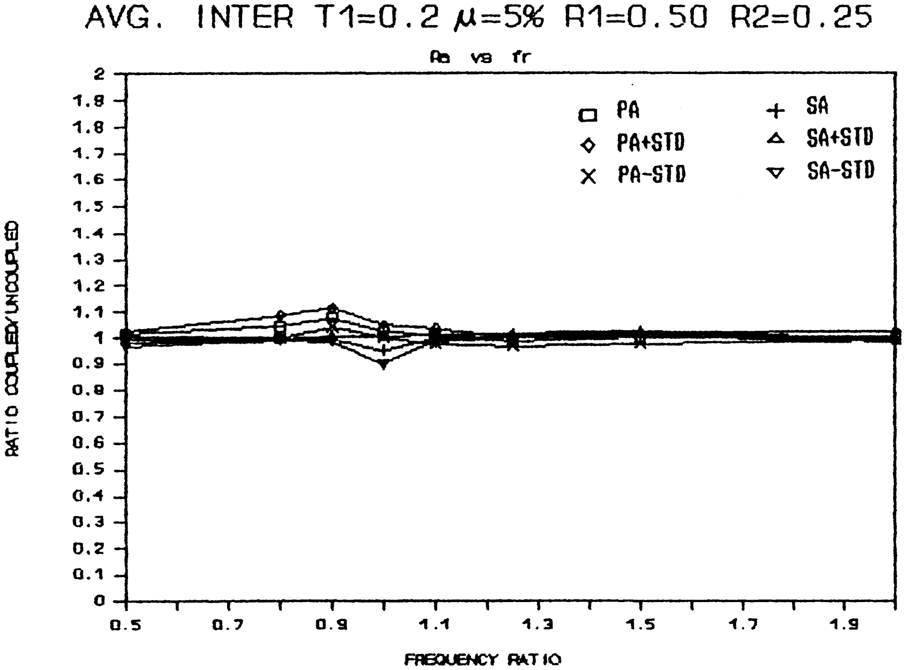


Figure D.12 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 5.0%, R1=0.50 and R2=0.25.

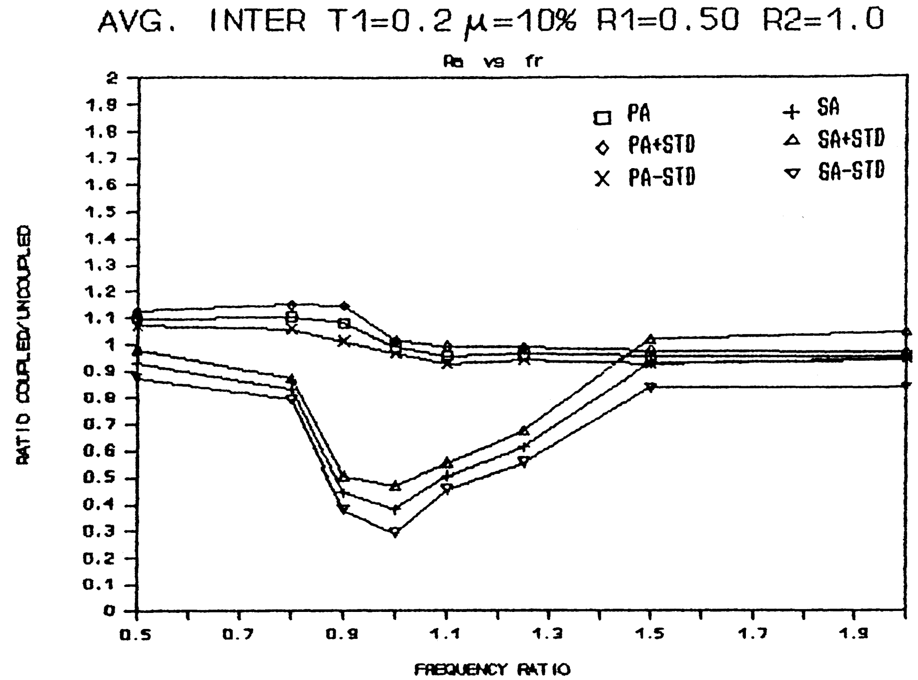


Figure D.13 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R1=0.50 and R2=1.0.

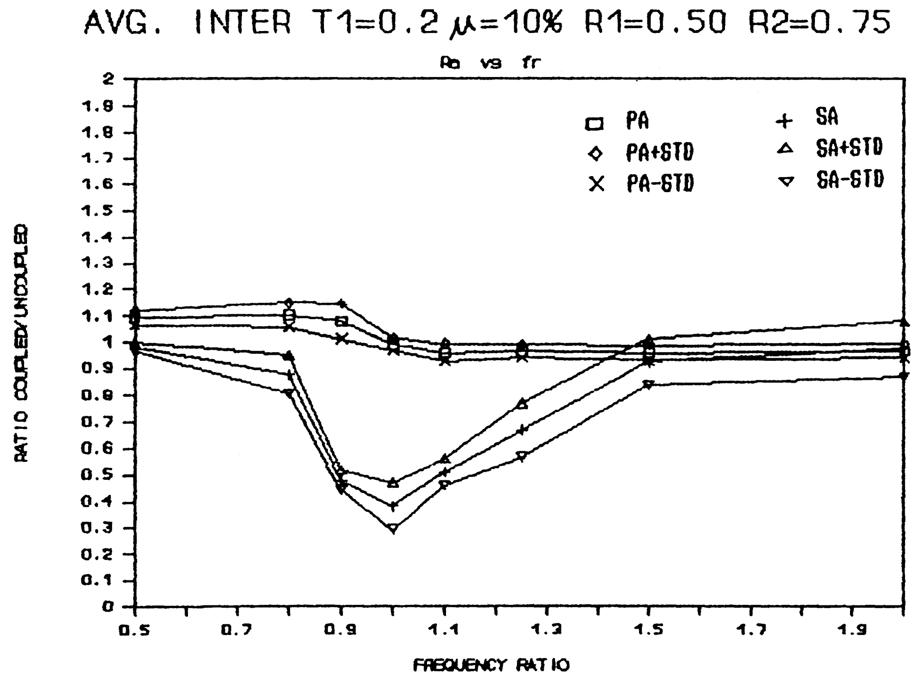
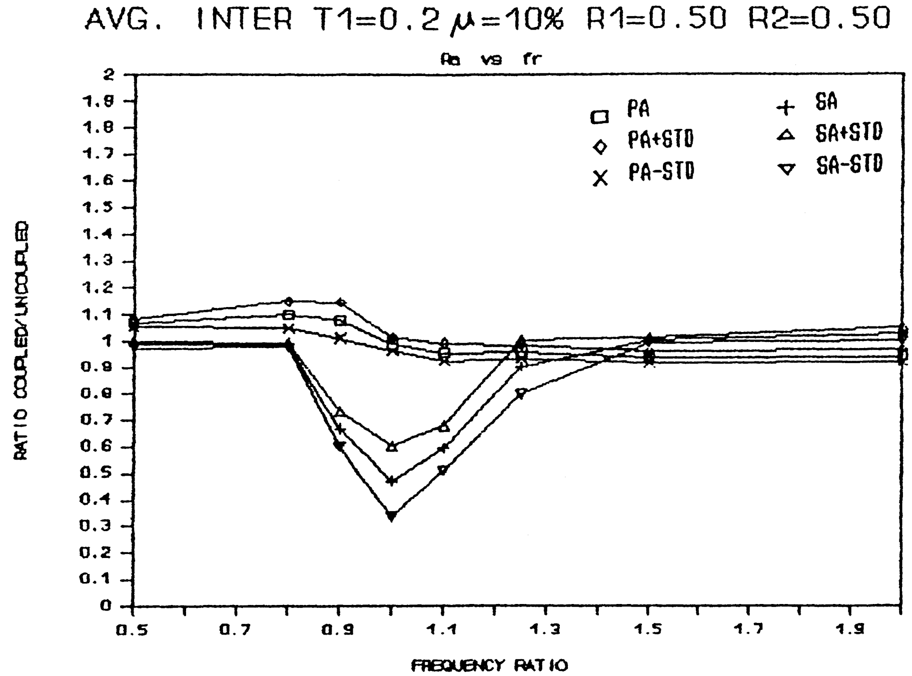
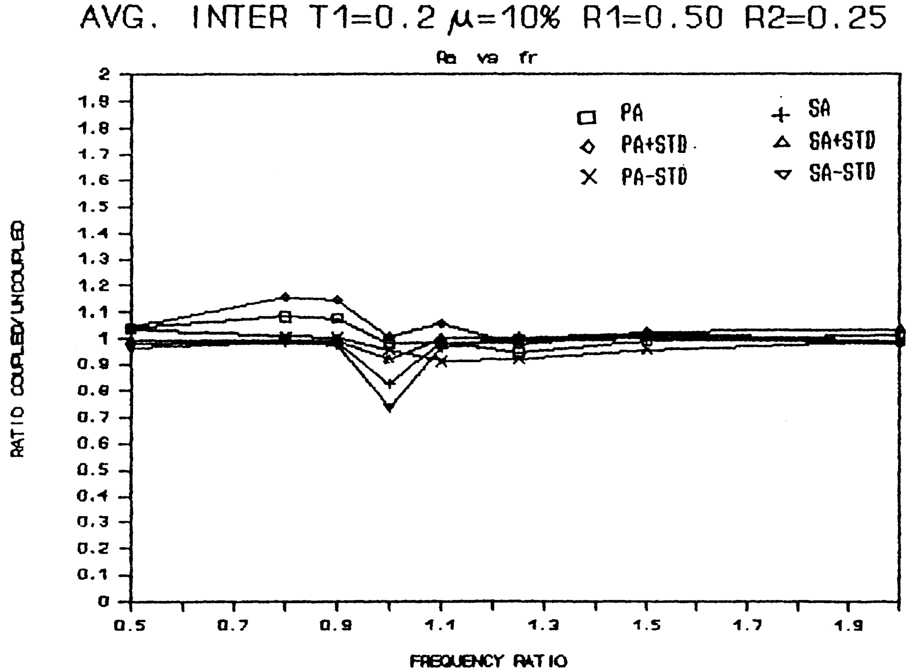


Figure D.14 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R1=0.50 and R2=0.75.



AVG. INTER T1=0.2 μ=10% R1=0.50 R2=0.50

Figure D.15 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R1=0.50 and R2=0.50.



AVG. INTER T1=0.2 μ=10% R1=0.50 R2=0.25

Figure D.16 Ratio Coupled/Uncoupled Acceleration response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R1=0.50 and R2=0.25.

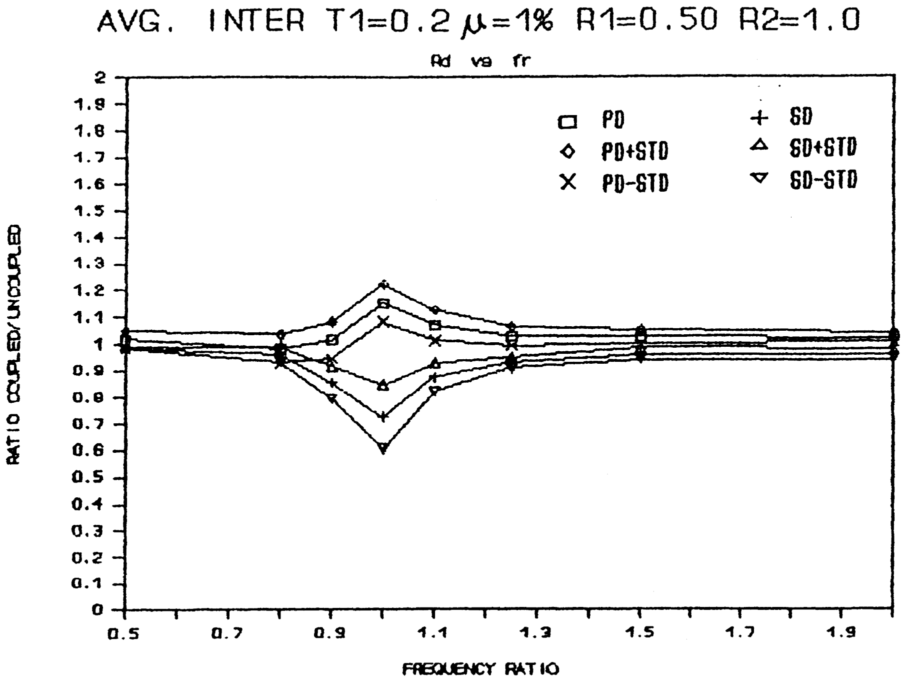


Figure D.17 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R1=0.50 and R2=1.0.

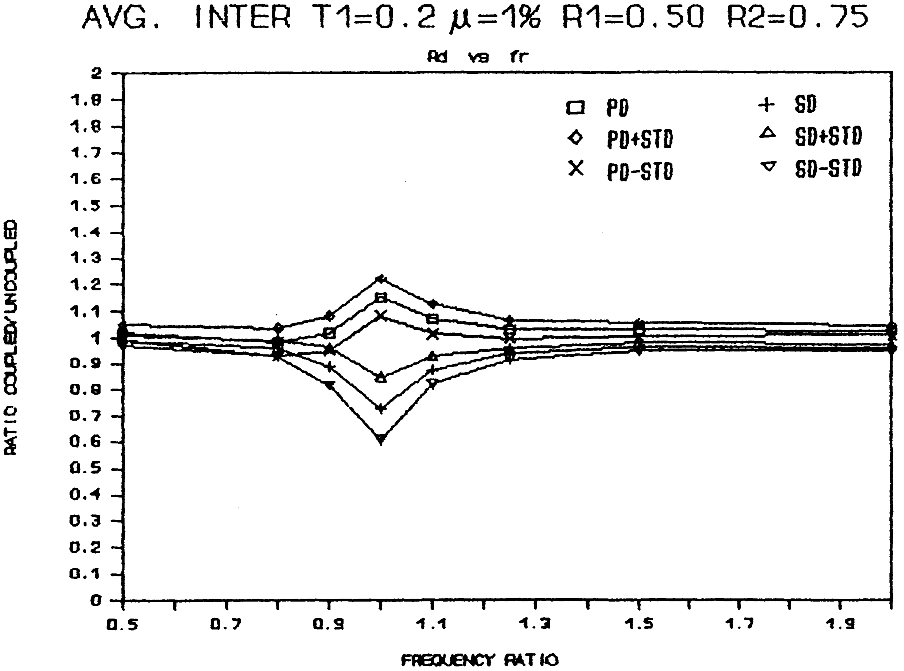


Figure D.18 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 1.0%, R1=0.50 and R2=0.75.

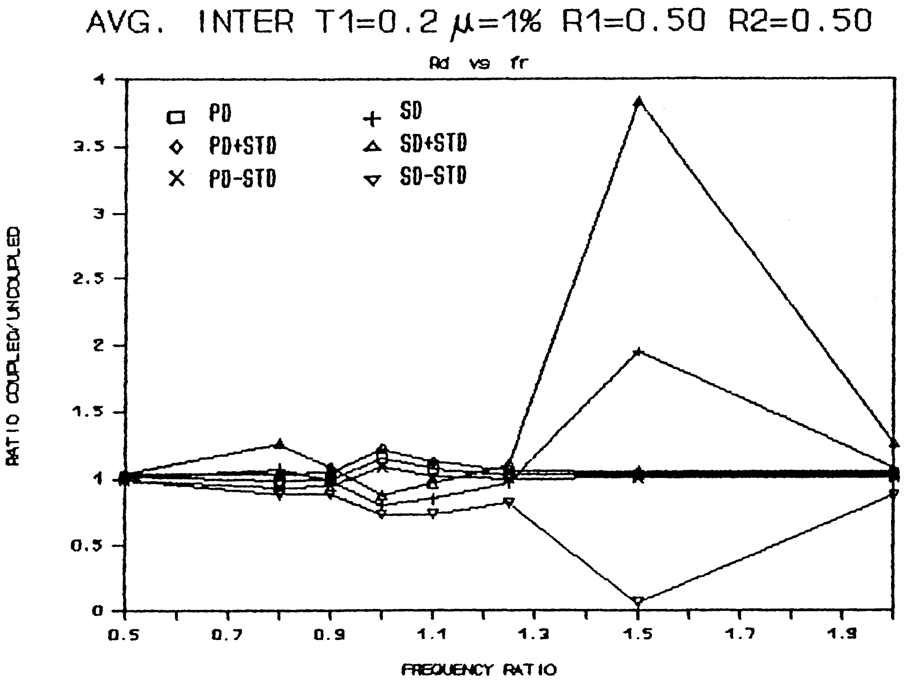


Figure D.19 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 1.0%, R1=0.50 and R2=0.50.

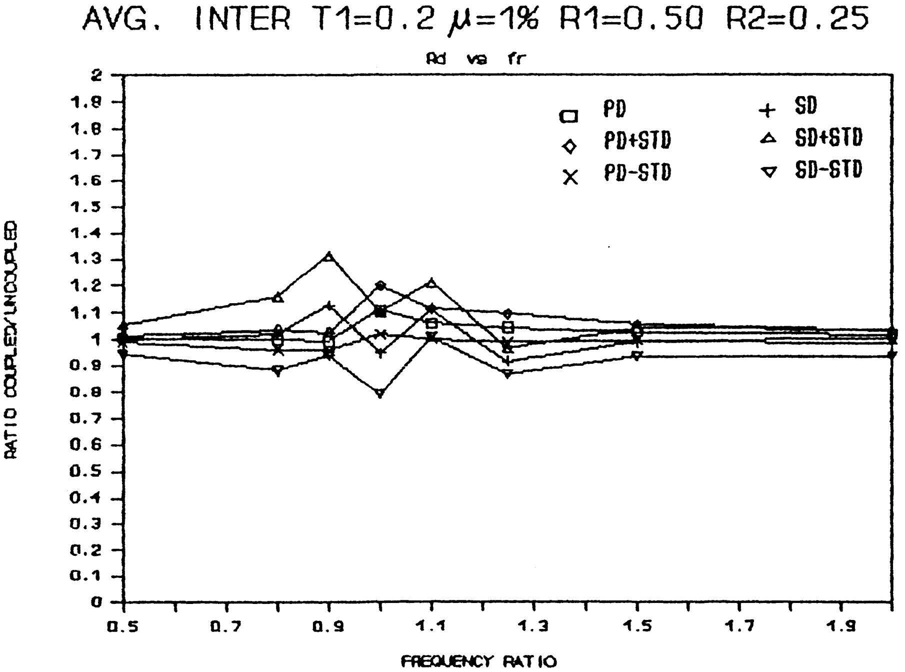


Figure D.20 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 1.0%, R1=0.50 and R2=0.25.

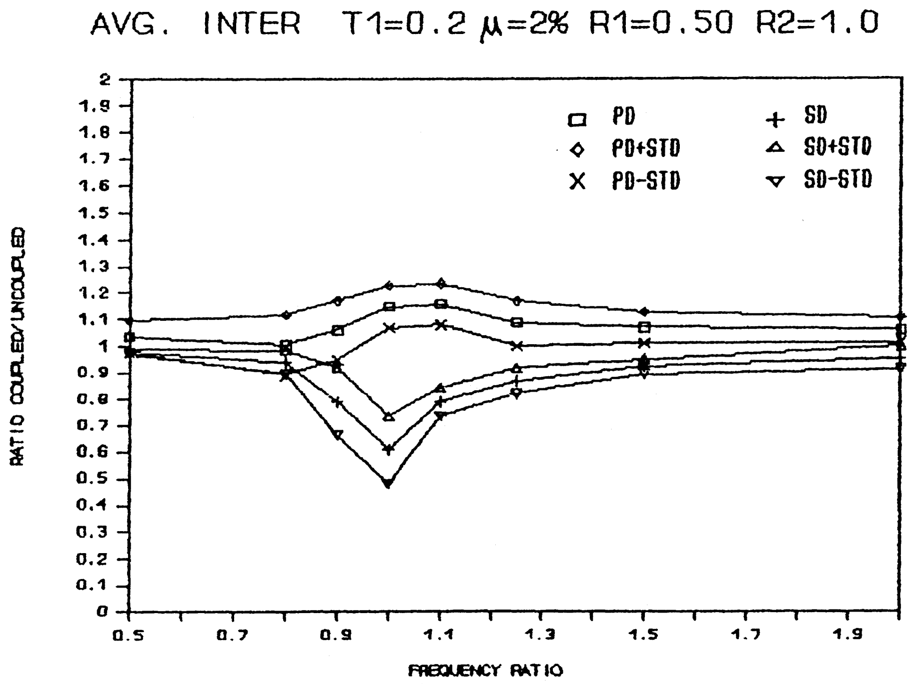


Figure D.21 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R1=0.50 and R2=1.0.

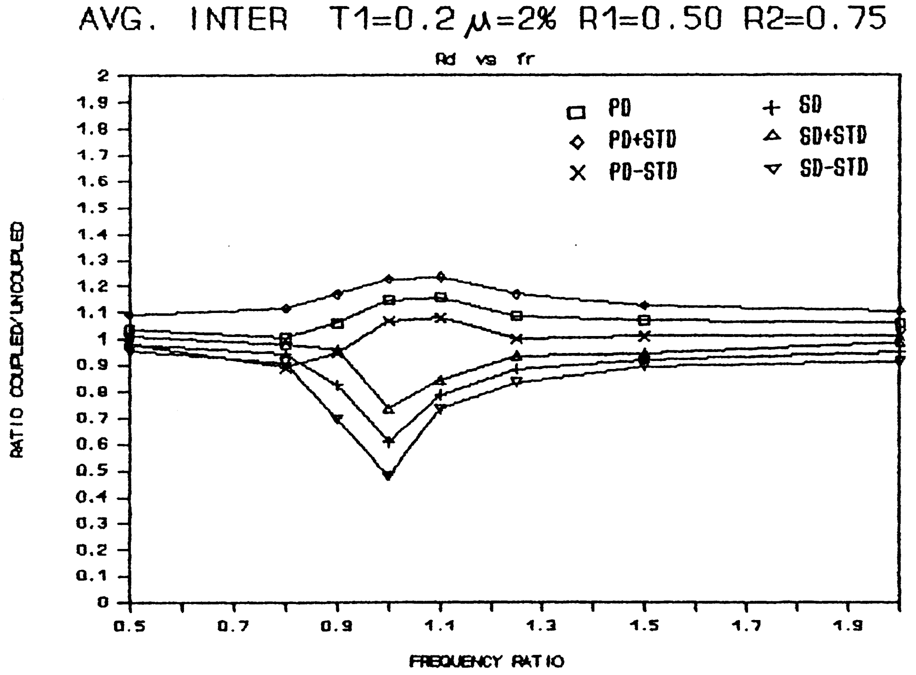


Figure D.22 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 2.0%, R1=0.50 and R2=0.75.

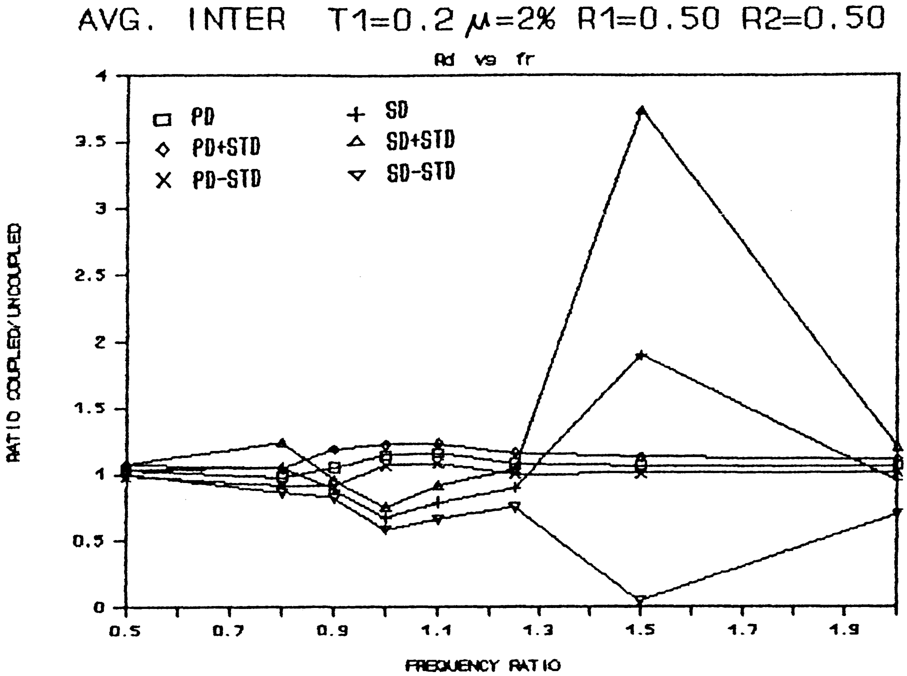


Figure D.23 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 2.0%, R1=0.50 and R2=0.50.

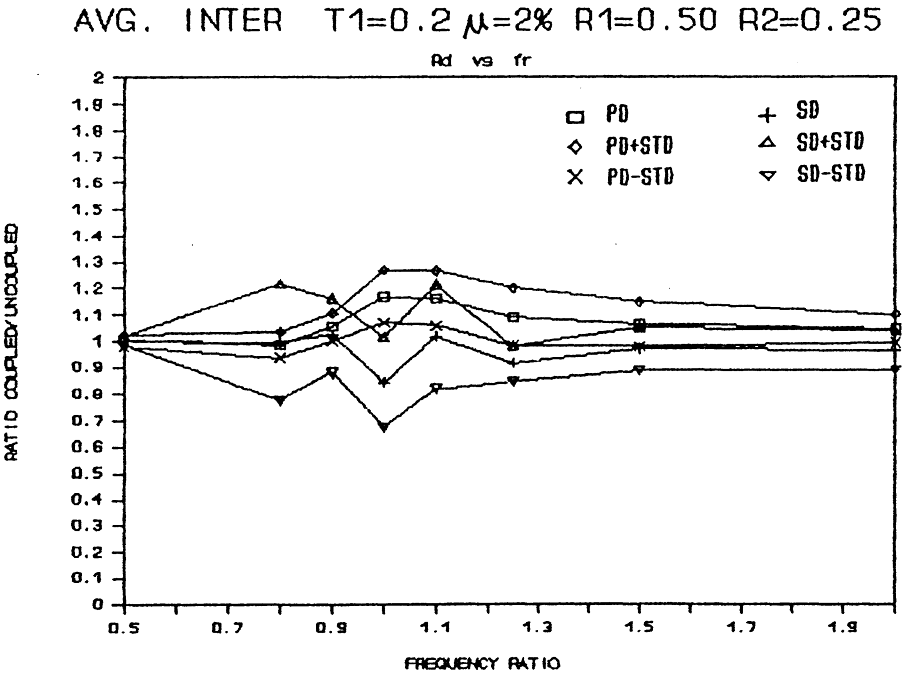


Figure D.24 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 2.0%, R1=0.50 and R2=0.25.

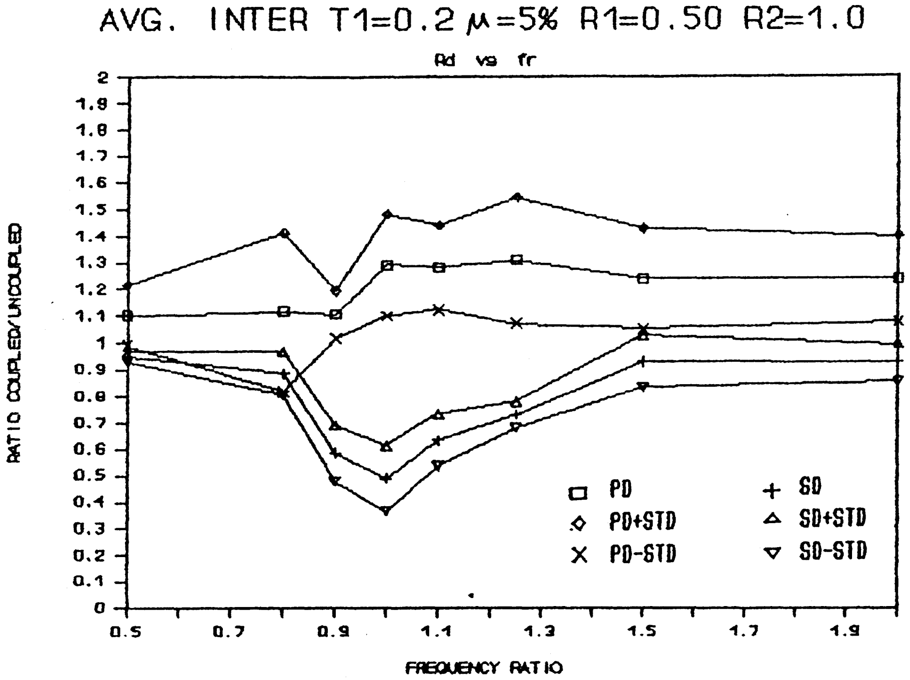


Figure D.25 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R1=0.50 and R2=1.0.

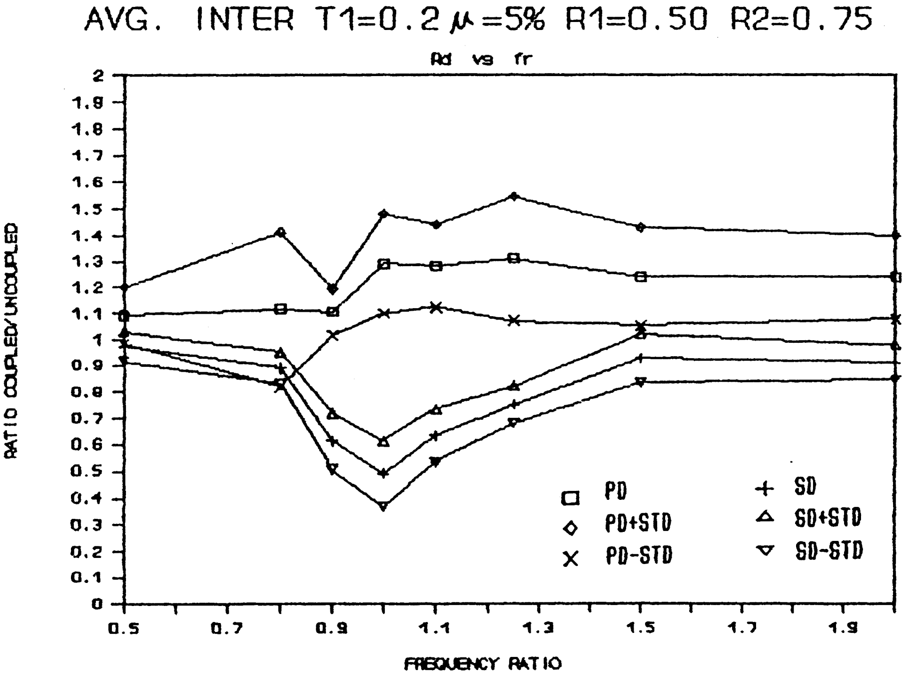


Figure D.26 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 5.0%, R1=0.50 and R2=0.75.

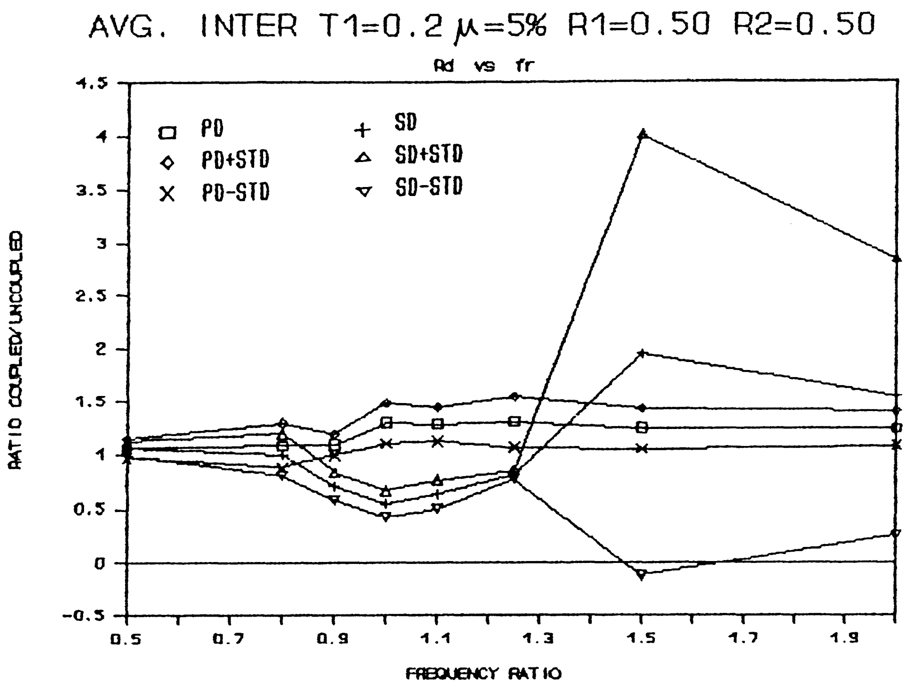


Figure D.27 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 5.0%, R1=0.50 and R2=0.50.

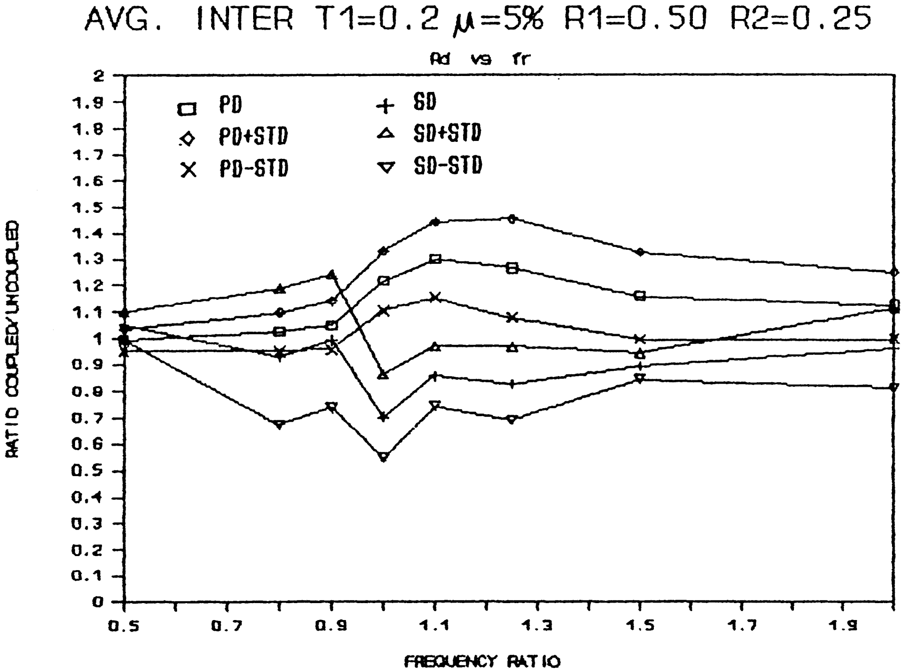


Figure D.28 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 5.0%, R1=0.50 and R2=0.25.

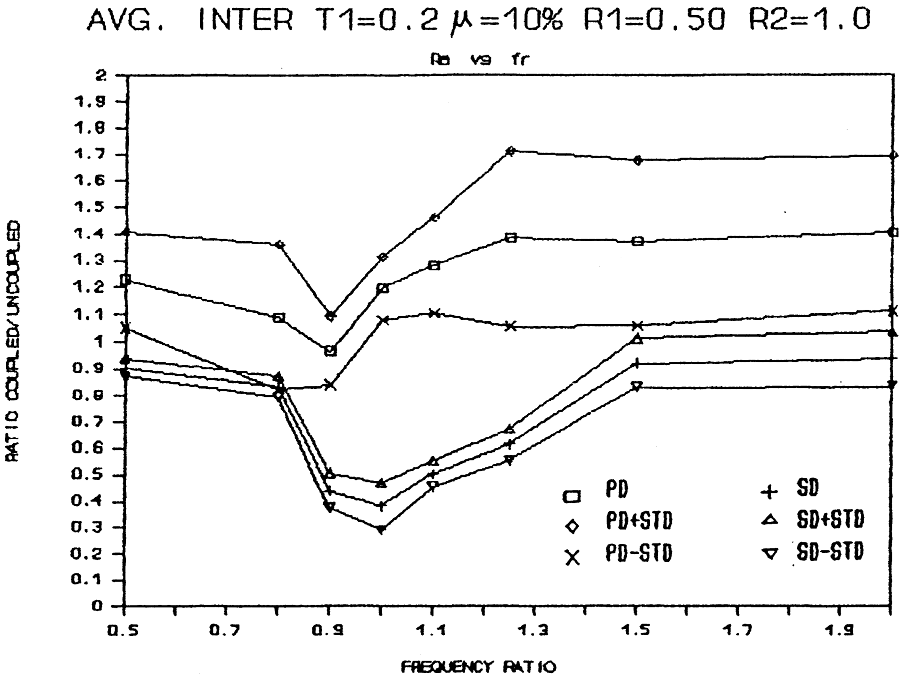


Figure D.29 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R1=0.50 and R2=1.0.

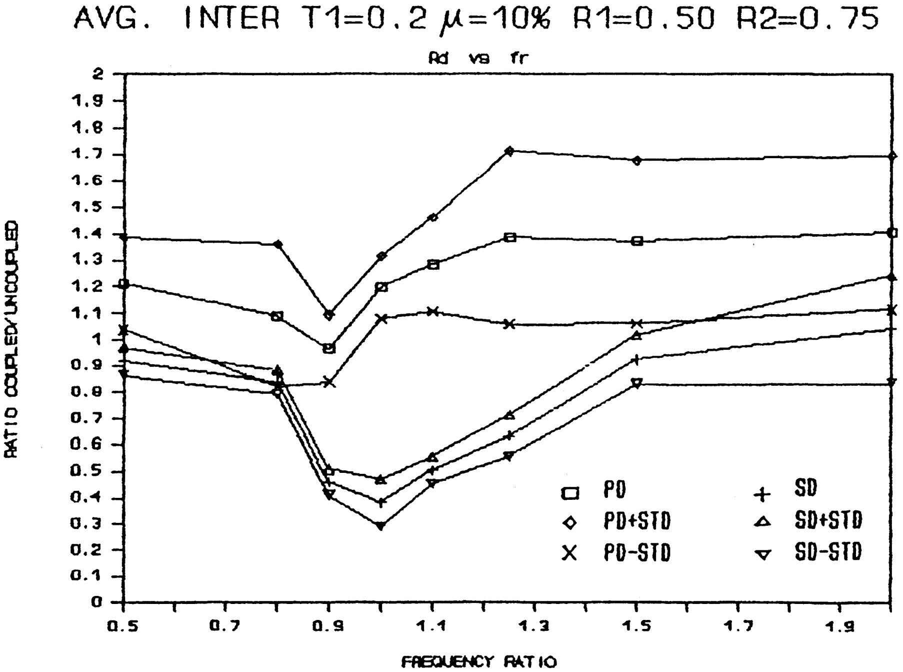


Figure D.30 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using ß=3% with mass ratio of 10.0%, R1=0.50 and R2=0.75.

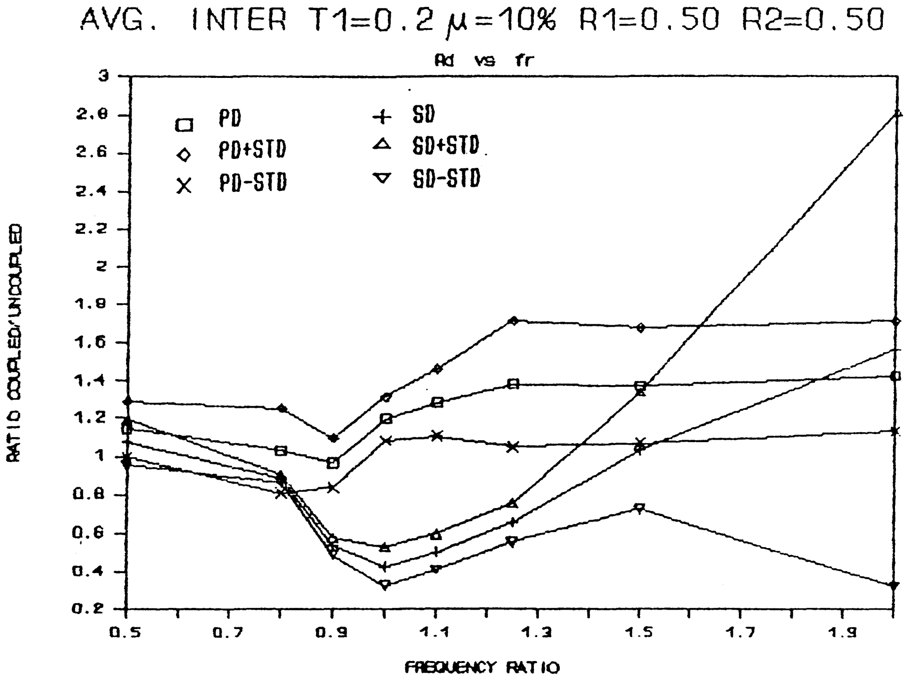


Figure D.31 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R1=0.50 and R2=0.50.

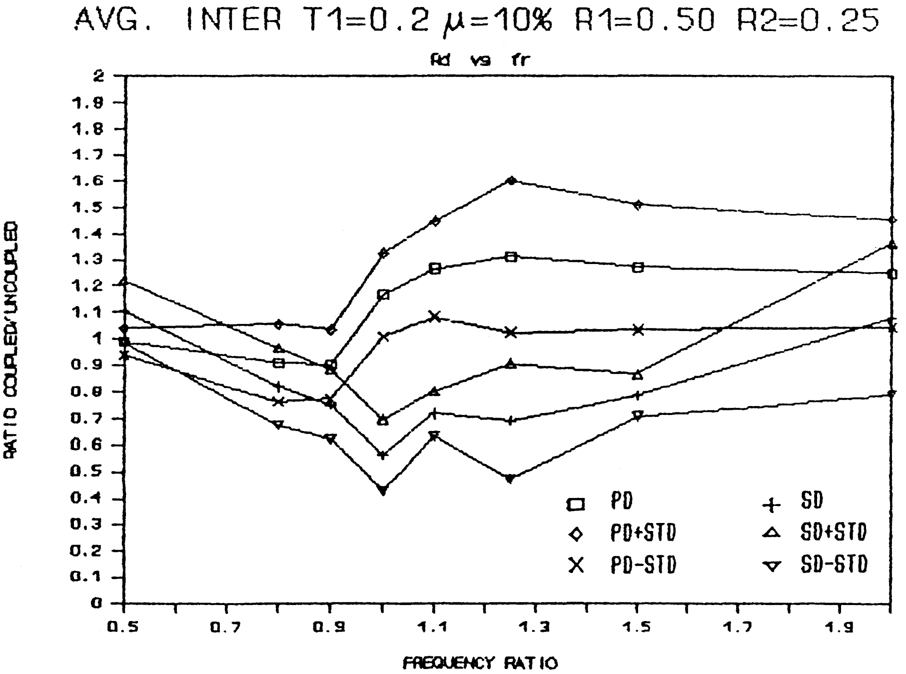


Figure D.32 Ratio Coupled/Uncoupled Displacement response versus frequency ratio for T1=0.2 sec, average of 5 intermediate A/V ratio records using β=3% with mass ratio of 10.0%, R1=0.50 and R2=0.25.