Space Mapping: A Sensible Concept For Engineering Optimization Exploiting Surrogates

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Outline

"Space Mapping" coined in 1993



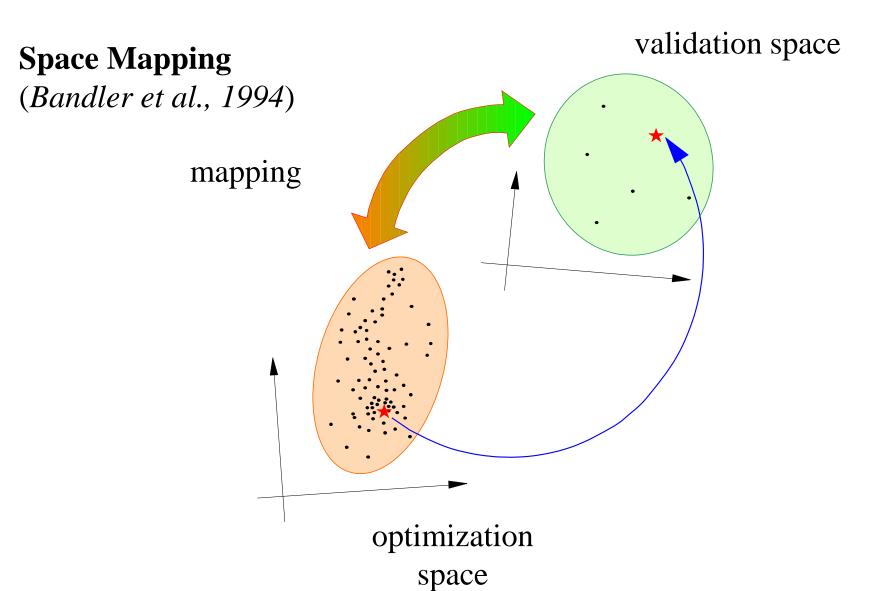
Space Mapping intelligently links companion "coarse" and "fine" models—full-wave electromagnetic (EM) simulations and empirical models

Space Mapping optimization follows traditional experience of designers

we discuss the 1993 concept and subsequent Aggressive Space Mapping











Space Mapping transformation, link, adjustment, correction,

shift (in parameters or responses)

Coarse Model simplification or convenient representation,

companion to the fine model,

auxiliary representation, cheap model

Fine Model accurate representation of system considered,

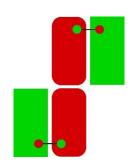
device under test, component to be optimized,

expensive model





Surrogate



model, approximation or representation to be used, or to act, in place of, or as a substitute for, the system under consideration

mapped or enhanced coarse model

Surrogate Model

alternative expression for Surrogate

Target Response

response the fine model should achieve, (usually) optimal response of a coarse model, enhanced coarse model, or surrogate





Companion coarse

Low Fidelity/

Resolution coarse

High Fidelity/

Resolution fine

Empirical coarse

Simplified Physics coarse

Physics-based coarse or fine

Device under Test fine

Electromagnetic fine or coarse

Simulation fine or coarse

Computational fine or coarse





Parameter (input) Space Mapping

mapping, transformation or correction of design variables

Response (output) Space Mapping

mapping, transformation or correction of responses

Response Surface Approximation

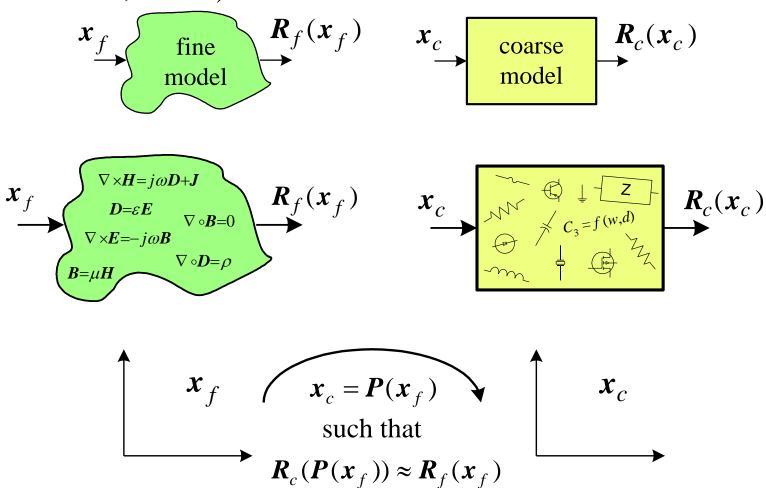
linear/quadratic/polynomial approximation of responses w.r.t. design variables





The Space Mapping Concept

(*Bandler et al., 1994-*)





Jacobian-Space Mapping Relationship

(Bakr et al., 1999)

through PE we match the responses

$$R_f(x_f) \approx R_c(P(x_f))$$

by differentiation

$$\left(\frac{\partial \mathbf{R}_f^T}{\partial \mathbf{x}_f}\right)^T \approx \left(\frac{\partial \mathbf{R}_c^T}{\partial \mathbf{x}_c}\right)^T \cdot \left(\frac{\partial \mathbf{x}_c^T}{\partial \mathbf{x}_f}\right)^T$$



Jacobian-Space Mapping Relationship

(Bakr et al., 1999)

given coarse model Jacobian J_c and space mapping matrix \boldsymbol{B} we estimate

$$\boldsymbol{J}_f(\boldsymbol{x}_f) \approx \boldsymbol{J}_c(\boldsymbol{x}_c)\boldsymbol{B}$$

given J_c and J_f we estimate (least squares)

$$\boldsymbol{B} \approx (\boldsymbol{J}_c^T \boldsymbol{J}_c)^{-1} \boldsymbol{J}_c^T \boldsymbol{J}_f$$





Space Mapping Notation

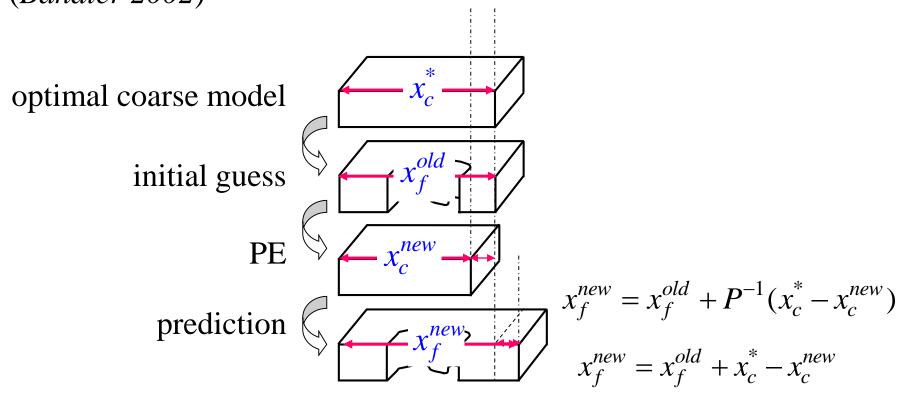
$$f^{(j)} = x_c^{(j)} - x_c^*$$
,
 $h^{(j)} = x_f^{(j+1)} - x_f^{(j)}$ and
 $B^{(j)}h^{(j)} = -f^{(j)}$





Space Mapping Practice—Cheese Cutting Problem

(*Bandler 2002*)





The Brain's Automatic Pilot

(Sandra Blakeslee, The New York Times, International Herald Tribune, February 21, 2002, p.7)

[certain brain] circuits are used by the human brain to assess social rewards ...

...findings [by neuroscientists] ...challenge the notion that people always make conscious choices about what they want and how to obtain it.

Gregory Berns (Emory University School of Medicine): ... most decisions are made subconsciously with many gradations of awareness.



The Brain's Automatic Pilot

(Sandra Blakeslee, The New York Times, International Herald Tribune, February 21, 2002, p.7)

P. Read Montague (Baylor College of Medicine): ... how did evolution create a brain that could make ... distinctions ... [about] ...what it must pay conscious attention to?

... the brain has evolved to shape itself, starting in infancy, according to what it encounters in the external world.

... much of the world is predictable: buildings usually stay in one place, gravity makes objects fall ...



The Brain's Automatic Pilot

(Sandra Blakeslee, The New York Times, International Herald Tribune, February 21, 2002, p.7)

As children grow, their brains build internal models of everything they encounter, gradually learning to identify objects ...

... as new information flows into it ... the brain automatically compares it with what it already knows.

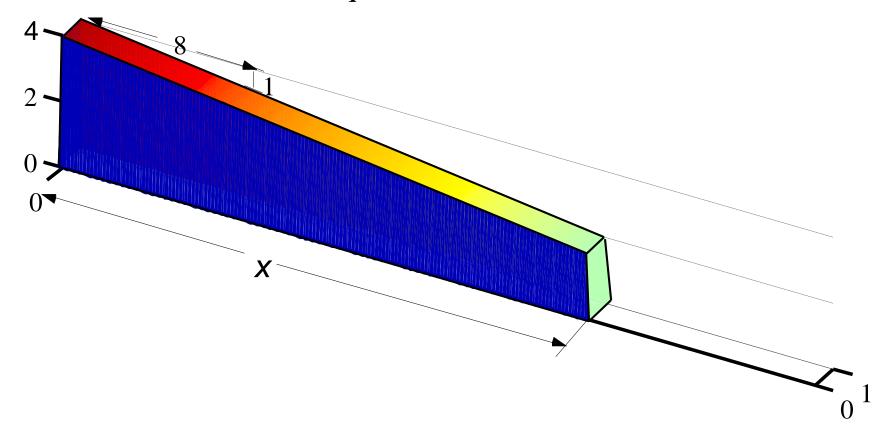
... if there is a surprise the mismatch ... instantly shifts the brain into a new state.

Drawing on past experience ... a decision is made ...



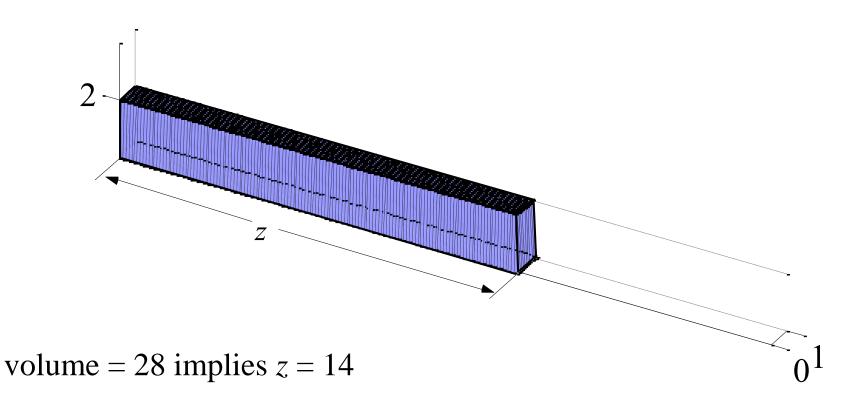
Wedge Cutting Problem (Bandler, 2002)

use space mapping to find the optimal position *x* of a cut such that the volume is equal to 28





Proposed Coarse Model







ASM Algorithm (Bandler et al., 1995)

Step 1 initialize
$$x^{(1)} = z^*$$
, $B^{(1)} = I$, $i = 1$

Step 2 extract
$$z^{(1)}$$
 such that $R_c(z^{(1)}) \approx R_f(x^{(1)})$

Step 3 evaluate
$$f^{(1)} = z^{(1)} - z^*$$
, if $||f^{(1)}|| \le \varepsilon$, stop

Step 4 solve
$$\mathbf{B}^{(i)}\mathbf{h}^{(i)} = -\mathbf{f}(\mathbf{x}^{(i)})$$
 for $\mathbf{h}^{(i)}$

Step 5 set
$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \mathbf{h}^{(i)}$$

Step 6 evaluate
$$\mathbf{R}_f(\mathbf{x}^{(i+1)})$$





ASM Algorithm (Bandler et al., 1995)

Step 7 extract
$$\mathbf{z}^{(i+1)}$$
 such that $\mathbf{R}_c(\mathbf{z}^{(i+1)}) \approx \mathbf{R}_f(\mathbf{x}^{(i+1)})$

Step 8 evaluate
$$f^{(i+1)} = z^{(i+1)} - z^*$$
, if $||f^{(i+1)}|| \le \varepsilon$, stop

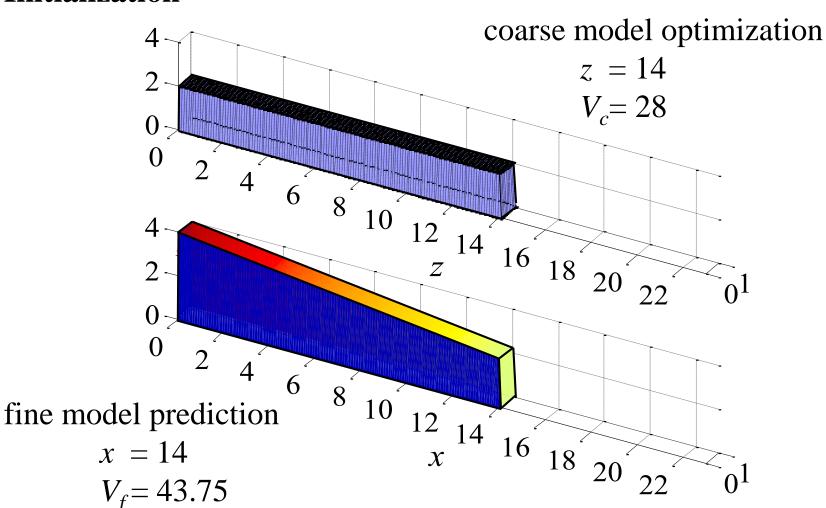
Step 9 update
$$\mathbf{B}^{(i+1)} = \mathbf{B}^{(i)} + \frac{\mathbf{f}^{(i+1)}\mathbf{h}^{(i)^T}}{\mathbf{h}^{(i)}}$$

Step 10 set
$$i = i + 1$$
 and go to Step 4

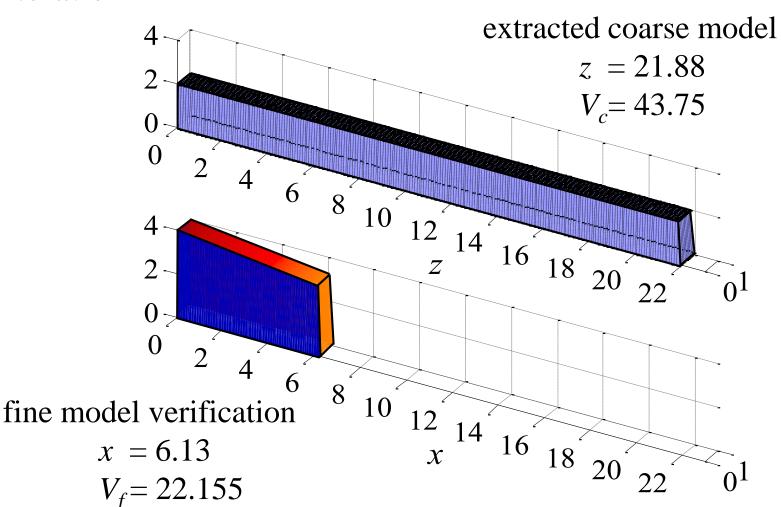




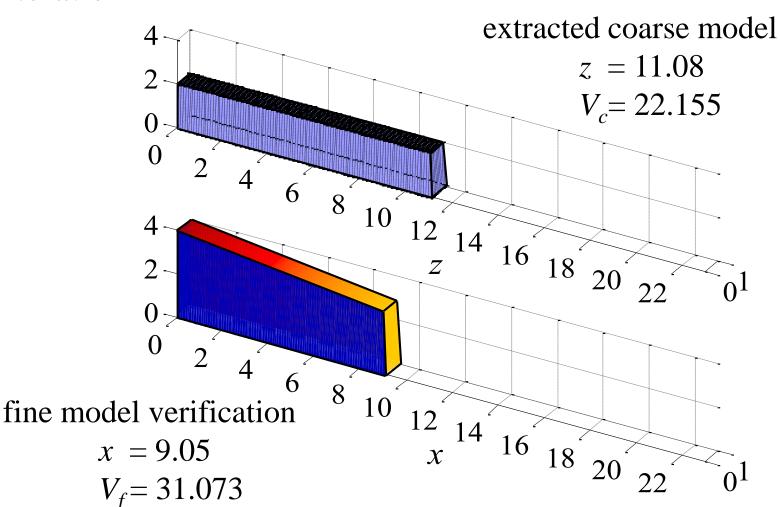
Initialization



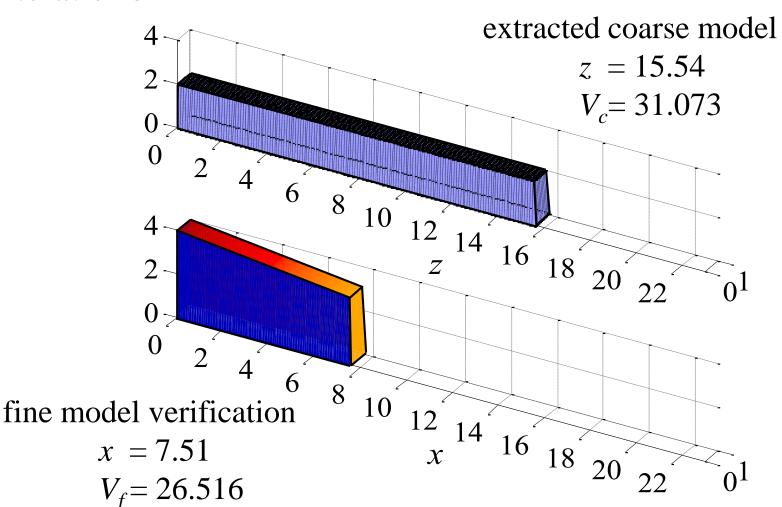




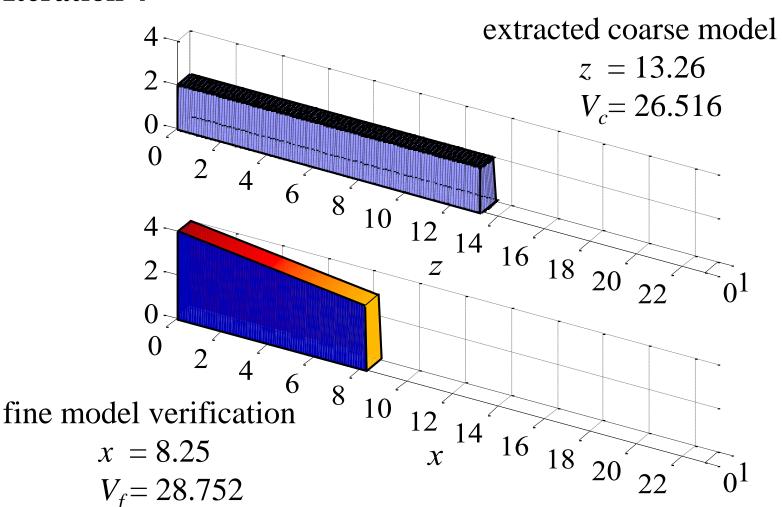




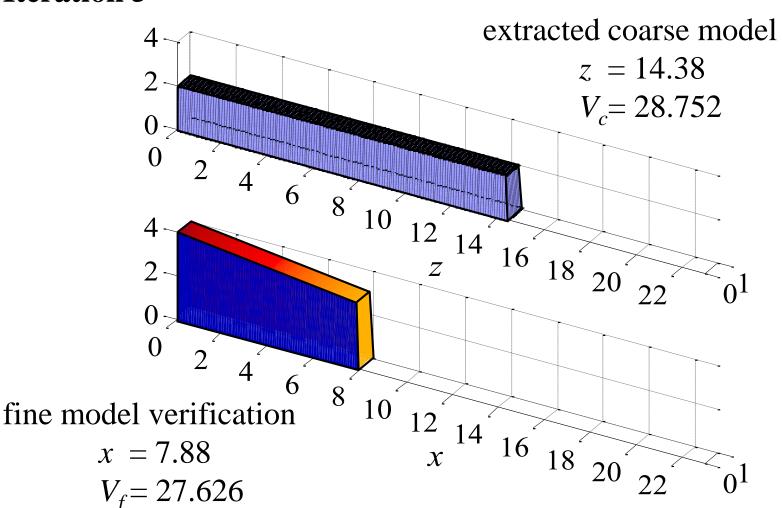




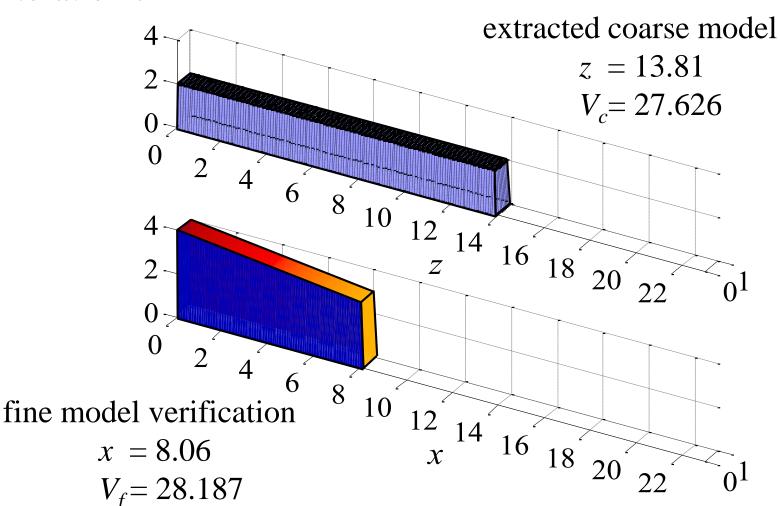




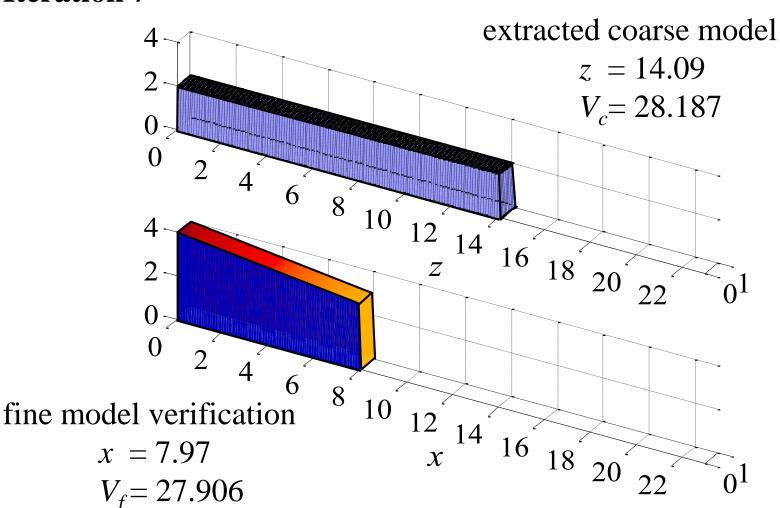














TRASM Algorithm (Bakr et al., 2000)

Step 1 initialize
$$x^{(1)} = z^*$$
, $B^{(1)} = I$, $i = 1$, $\delta^{(1)} = \delta_0$

Step 2 extract
$$z^{(1)}$$
 such that $R_c(z^{(1)}) \approx R_f(x^{(1)})$

Step 3 evaluate
$$f^{(1)} = z^{(1)} - z^*$$
, if $||f^{(1)}|| \le \varepsilon$, stop

Step 4 find the minimizer
$$\boldsymbol{h}^{(i)}$$
 of $\|\boldsymbol{f}^{(i)} + \boldsymbol{B}^{(i)}\boldsymbol{h}^{(i)}\|$ subject to $\|\boldsymbol{h}^{(i)}\| \le \delta^{(i)}$

Step 5 set
$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \mathbf{h}^{(i)}$$



TRASM Algorithm (Bakr et al., 2000)

Step 6 evaluate
$$\mathbf{R}_f(\mathbf{x}^{(i+1)})$$

Step 7 extract
$$\boldsymbol{z}^{(i+1)}$$
 such that $\boldsymbol{R}_c(\boldsymbol{z}^{(i+1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}^{(i+1)})$

Step 8 evaluate
$$f^{(i+1)} = z^{(i+1)} - z^*$$
, if $||f^{(i+1)}|| \le \varepsilon$ stop

Step 9 find
$$\rho^{(i)} = \frac{\|\boldsymbol{f}^{(i)}\| - \|\boldsymbol{f}^{(i+1)}\|}{\|\boldsymbol{f}^{(i)}\| - \|\boldsymbol{f}^{(i)} + \boldsymbol{B}^{(i)}\boldsymbol{h}^{(i)}\|}$$

Step 10 adjust the trust region size

if
$$\rho^{(i)} < \eta_1$$
 reject $\boldsymbol{x}^{(i+1)}$, take $\delta^{(i+1)} \in [\alpha_1 \delta^{(i)}, \alpha_2 \delta^{(i)}]$



TRASM Algorithm (Bakr et al., 2000)

else if
$$\eta_1 \leq \rho^{(i)} < \eta_2$$
, accept $\boldsymbol{x}^{(i+1)}$, $\delta^{(i+1)} \in [\alpha_2 \delta^{(i)}, \delta^{(i)}]$ else accept $\boldsymbol{x}^{(i+1)}$, take $\delta^{(i+1)} \geq \delta^{(i)}$

Comment
$$0 < \alpha_1 \le \alpha_2 < 1$$
, $0 < \eta_1 \le \eta_2 < 1$

(for example,
$$\eta_1 = 0.1$$
, $\eta_2 = 0.9$ and $\alpha_1 = \alpha_2 = 0.5$)

Step 11 update
$$\boldsymbol{B}^{(i+1)} = (\boldsymbol{J}_c^T \boldsymbol{J}_c)^{-1} \boldsymbol{J}_c^T \boldsymbol{J}_f$$

Comment J_f , J_c are evaluated at $\boldsymbol{x}^{(i+1)}$, $\boldsymbol{z}^{(i+1)}$

Step 12 set
$$i = i + 1$$
 and go to Step 4



Wedge Cutting Problem (Bandler et al., 2002)

Step 1
$$x^{(1)} = z^* = 14, B^{(1)} = 1, \delta^{(1)} = 2$$

Step 2
$$V_f(x^{(1)}) = 4x^{(1)} - \frac{(x^{(1)})^2}{16} = 43.75 \xrightarrow{PE} z^{(1)} = 21.875$$

Step 3
$$f^{(1)} = 21.875 - 14 = 7.875$$

Step 4
$$h^{(1)} = \arg\min_{h} ||7.875 + 1.h|| \text{ subject to } ||h|| < 2$$

 $h^{(1)} = -2$

Step 5
$$x^{(2)} = 14 + (-2) = 12$$



Wedge Cutting Problem (Bandler et al., 2002)

Step 6
$$V_f(x^{(2)}) = 39$$

Step 7 PE
$$z^{(2)} = 19.5$$

Step 8
$$f^{(2)} = 19.5 - 14 = 5.5$$

Step 9
$$\rho^{(1)} = \frac{7.875 - 5.5}{7.875 - (7.875 + 1(-2))} = 1.18$$

Step 10 adjust trust region size $\delta^{(2)} = 2\delta^{(1)} = 4$

Step 11
$$J_f = 4 - x^{(2)}/8$$
, $J_c = 2 \longrightarrow B^{(2)} = \frac{J_f}{J_c} = 1.25$



Wedge Cutting Problem (Bandler et al., 2002)

Step 4b
$$h^{(2)} = \arg\min_{h} ||5.5 + 1.25h|| \text{ subject to } ||h|| < 4$$

 $h^{(2)} = -4$

Step 5b set
$$x^{(3)} = 12 + (-4) = 8$$

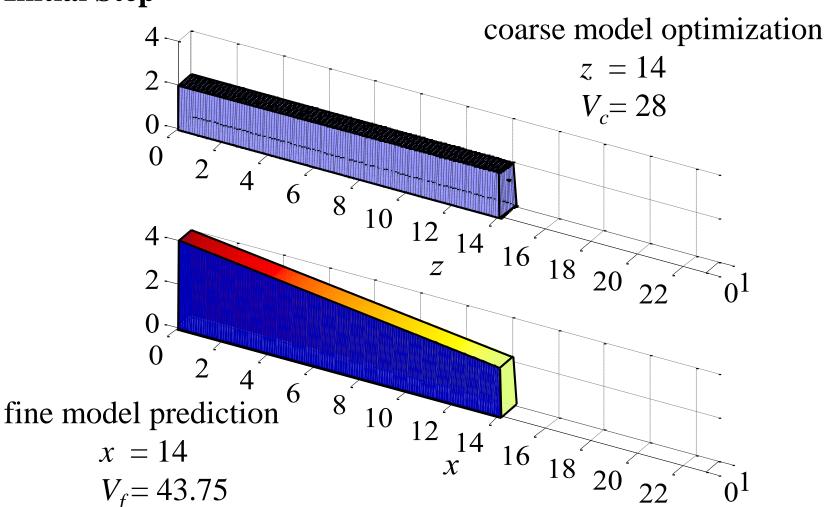
Step 6b
$$V_f(x^{(3)}) = 28$$

Step 7b PE
$$z^{(3)} = 14$$

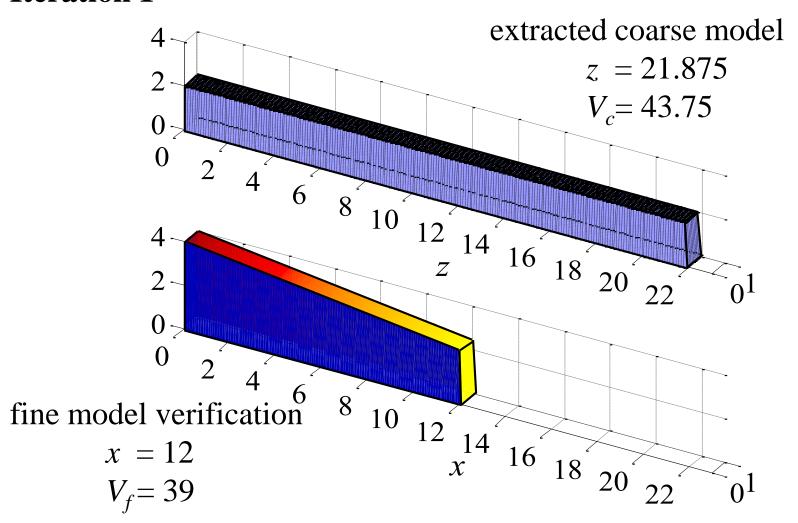
Step 8b
$$f^{(3)} = 14 - 14 = 0$$
 stop the algorithm



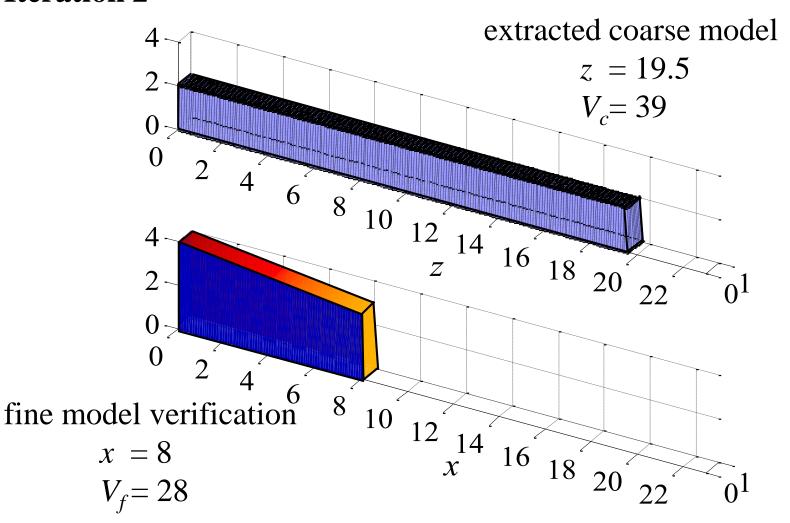
Initial Step















Change of Initial Trust Region Size

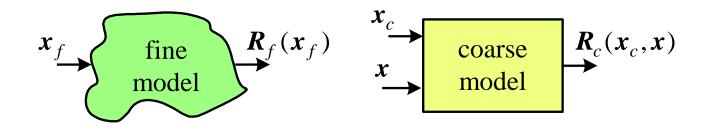
$\mathcal{S}^{(1)}$	χ^*	V_f	number of iterations
1	7.99905	27.99715	4
2	8	28	2
3	7.99905	27.99715	3
4	7.99983	27.99948	3

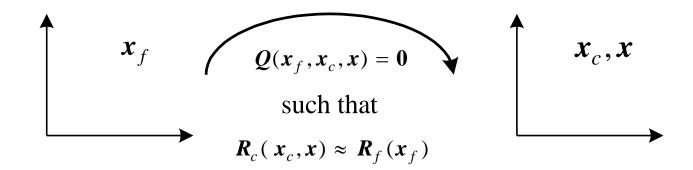




Implicit Space Mapping Theory

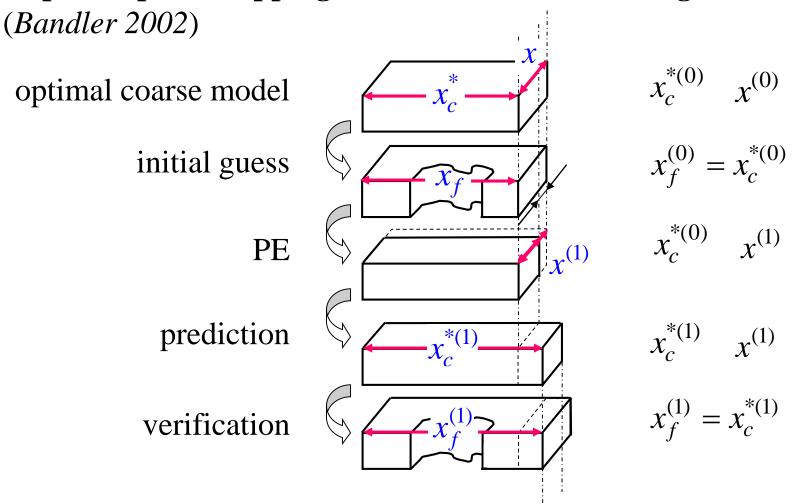
(*Bandler et al., 2002*)







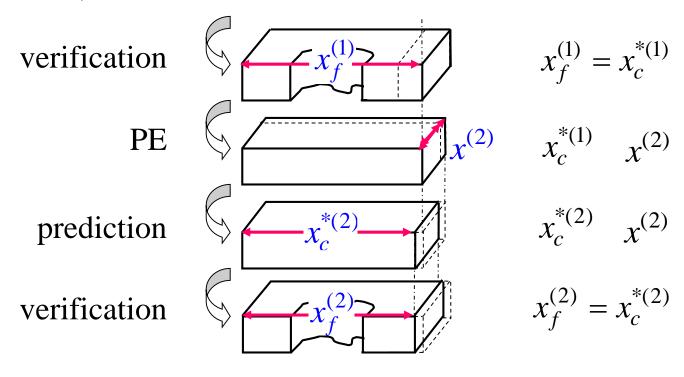
Implicit Space Mapping Practice—Cheese Cutting Problem





Implicit Space Mapping Practice—Cheese Cutting Problem

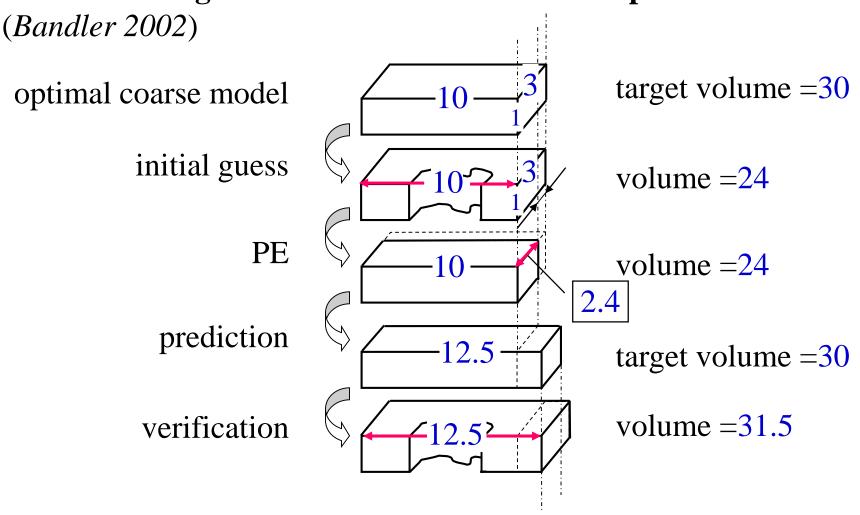
(*Bandler 2002*)



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Cheese Cutting Problem—A Numerical Example

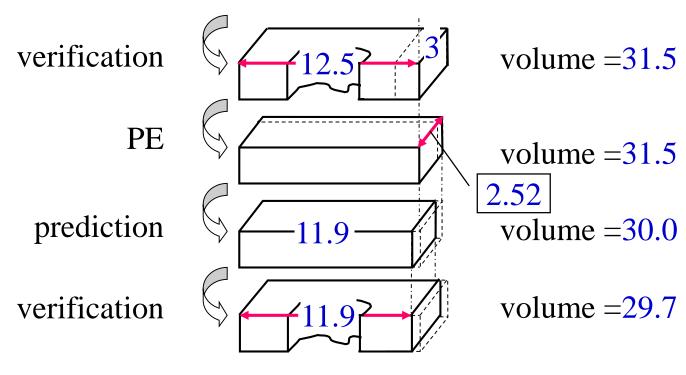






Cheese Cutting Problem—A Numerical Example

(*Bandler 2002*)





Implicit Space Mapping Practice

(*Bandler et al.*, 2002)

effective for EM-based microwave modeling and design

coarse model aligned with EM (fine) model through preassigned parameters

easy implementation

no explicit mapping involved

no matrices to keep track of





An Implicit Space Mapping Algorithm—Preassigned **Parameters**

Step 1 select candidate preassigned parameters x as in ESMDF or by experience

Step 2 set i = 0 and initialize $x^{(0)}$

Step 3 obtain optimal coarse model

$$\boldsymbol{x}_{c}^{*(i)} = \arg\min_{\boldsymbol{x}_{c}} U(\boldsymbol{R}_{c}(\boldsymbol{x}_{c}, \boldsymbol{x}^{(i)}))$$

Step 4 predict $x_f^{(i)}$ from

$$\boldsymbol{x}_f = \boldsymbol{x}_c^{*(i)}$$



An Implicit Space Mapping Algorithm—Preassigned Parameters (continued)

- Step 5 simulate the fine model at $\boldsymbol{x}_f^{(i)}$
- Step 6 terminate if a stopping criterion (e.g., response meets specifications) is satisfied
- Step 7 calibrate the coarse model by extracting the preassigned parameters x

$$\boldsymbol{x}^{(i+1)} = \arg \min_{\boldsymbol{x}} \|\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}) - \boldsymbol{R}_c(\boldsymbol{x}_f^{(i)}, \boldsymbol{x})\|$$

where we set

$$\boldsymbol{x}_c = \boldsymbol{x}_f^{(i)}$$



An Implicit Space Mapping Algorithm—Preassigned Parameters (continued)

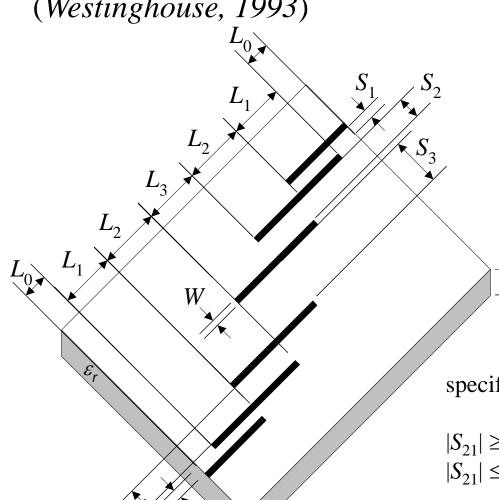
Step 8 increment i and go to Step 3





HTS Quarter-Wave Parallel Coupled-Line Microstrip Filter

(Westinghouse, 1993)



we take $L_0 = 50$ mil, H = 20 mil, $W = 7 \text{ mil}, \ \varepsilon_r = 23.425, \ \text{loss}$ tangent = 3×10^{-5} ; the metalization is considered lossless

the design parameters are

$$\mathbf{x}_f = [L_1 \ L_2 \ L_3 \ S_1 \ S_2 \ S_3]^T$$

specifications

 \uparrow H

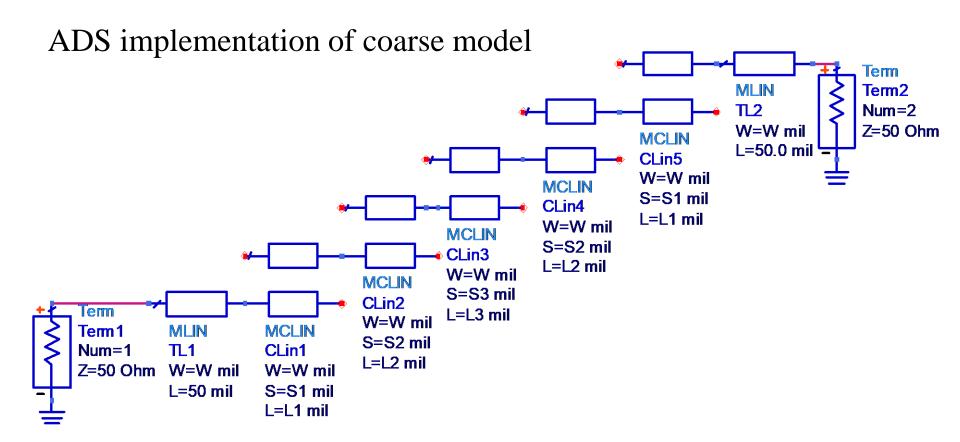
 $|S_{21}| \ge 0.95$ for 4.008 GHz $\le \omega \le 4.058$ GHz $|S_{21}| \le 0.05$ for $\omega \le 3.967$ GHz and $\omega \ge 4.099$ GHz





HTS Quarter-Wave Parallel Coupled-Line Microstrip Filter

(Westinghouse, 1993)





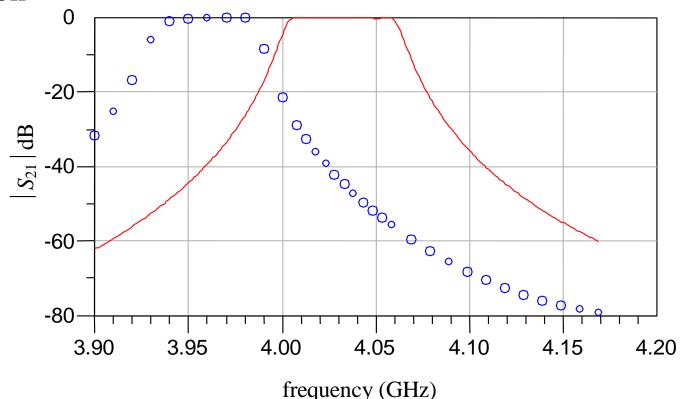
parameter	initial solution	solution reached by the algorithm
L_1	189.65	187.10
L_2	196.03	191.30
L_3	189.50	186.97
S_1	23.02	22.79
S_2	95.53	93.56
S_3	104.95	104.86
	all values are in	mils



preassigned original parameters values		final iteration	
$\frac{Parameters}{H_1}$	20 mil	19.80 mil	
H_2^{-}	20 mil	19.05 mil	
H_3	20 mil	19.00 mil	
\mathcal{E}_{r1}	23.425	24.404	
\mathcal{E}_{r2}	23.425	24.245	
\mathcal{E}_{r3}	23.425	24.334	

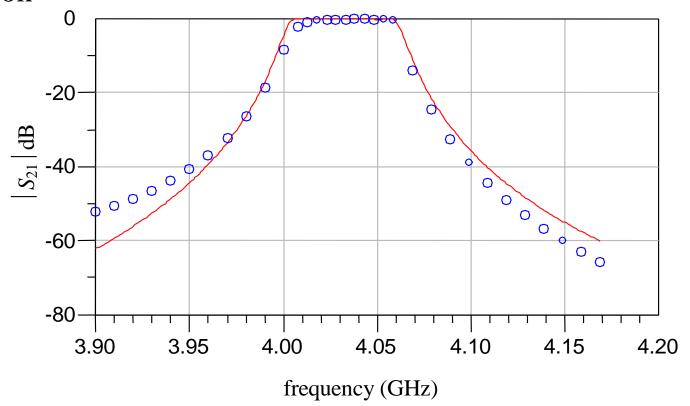


the fine (o) and optimal coarse model (—) responses at the initial solution





the fine (o) and optimal coarse model (—) responses at the final iteration





Bandler's Conjecture No. 1

Space Mapping is a natural mechanism for the brain to relate objects or images with other objects, images, reality, or experience

Bandler's Conjecture No. 2

brains of "clever", experienced or intuitive individuals employ a Broyden-like update in the Space Mapping process

Bandler's Conjecture No. 3

"experienced" engineering designers, knowingly or not, routinely employ Space Mapping to achieve complex designs



Kaj Madsen (Technical University of Denmark, 1993-) mapping updates, trust region methods

Pavio (Motorola, 1994-) companion model approach, filter design, LTCC circuits

Shen Ye (ComDev, 1997-) circuit calibration technique

Mansour (Com Dev, University of Waterloo, 1998-) Cauchy method and adaptive sampling

Stephane Bila (Limoges, France 1998-) space mapping, waveguide devices





Rayas-Sánchez (McMaster University; ITESO, Mexico 1998-) space mapping through artificial neural networks

Jacob Søndergaard (Technical University of Denmark, 1999-) space mapping: theory and algorithms

Qi-jun Zhang (Carleton University, 1999-) knowledge based neural networks, space mapping

Jan Snel (Philips Semiconductors, Netherlands, 2001) RF component design, library model enhancement

Dan Swanson (Bartley RF Systems, 2001) combline filter design





Steven Leary (University of Southampton, England, 2000-) constraint mapping, applications in civil engineering

Lehmensiek (University of Stellenbosch, South Africa, 2000, 2001) filter design, coupling structures

Frank Pedersen (Technical University of Denmark, 2001-) space mapping, neural networks

Ke-Li Wu (Chinese University of Hong Kong, 2001-) knowledge embedded space mapping, LTCC circuits

Pablo Soto (Polytechnic University of Valencia, Spain, 2001) aggressive space mapping, inductively coupled filters

Hong-Soon Choi (Seoul National University, Korea, 2001) aggressive space mapping, design of magnetic systems



Luis Vicente (University of Coimbra, Portugal, 2001-) mathematics of space mapping: models, sensitivities and trust regions

Marcus Redhe (Linköping University, Sweden, 2001) sheet metal forming and vehicle crashworthiness design

Dieter Peltz (Radio Frequency Systems, Australia, 2002) difference matrix approach, coupled resonator filters

Safavi-Naeini (University of Waterloo, 2002) multi-level generalized space mapping, multi-cavity microwave structures



Jan-Willem Lobeek (Philips Semiconductors, Netherlands, 2002) power amplifier design



Conclusions

Space Mapping intelligently links companion "coarse" or "surrogate" models with "fine" models—physical, empirical, electromagnetic

Space Mapping optimization follows traditional experience of designers

researchers and practitioners attracted to Aggressive Space Mapping

Space Mapping already used in the RF industry for enhanced (mapped) library (surrogate) models

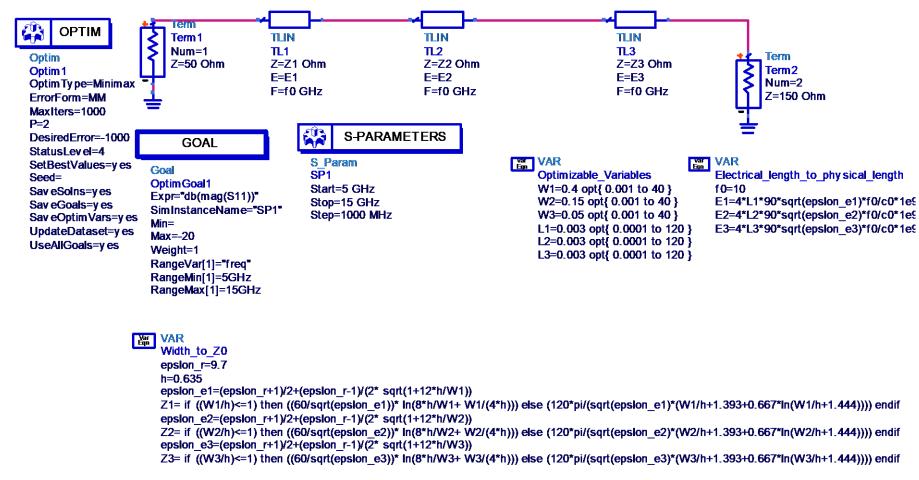
Implicit Space Mapping (ISM), where preassigned parameters change in coarse model—novel approach

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Implicit Space Mapping: Steps 1-3

optimize coarse model

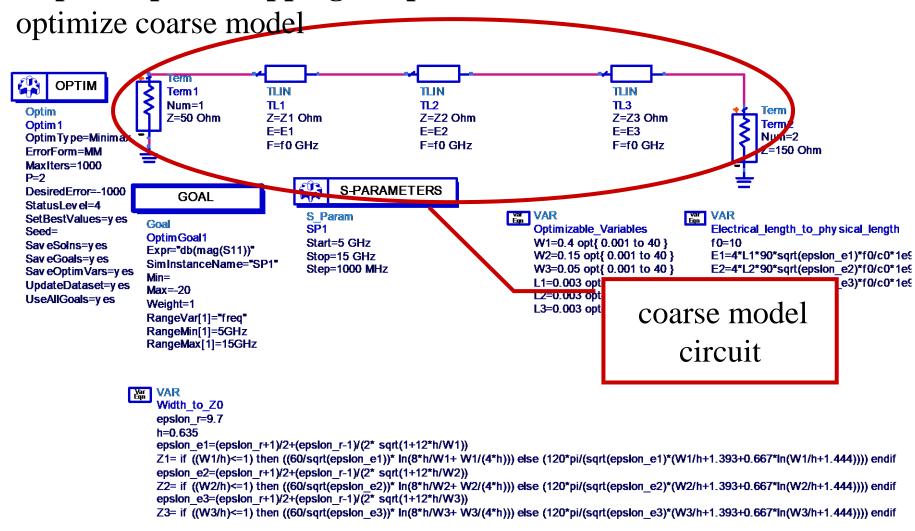




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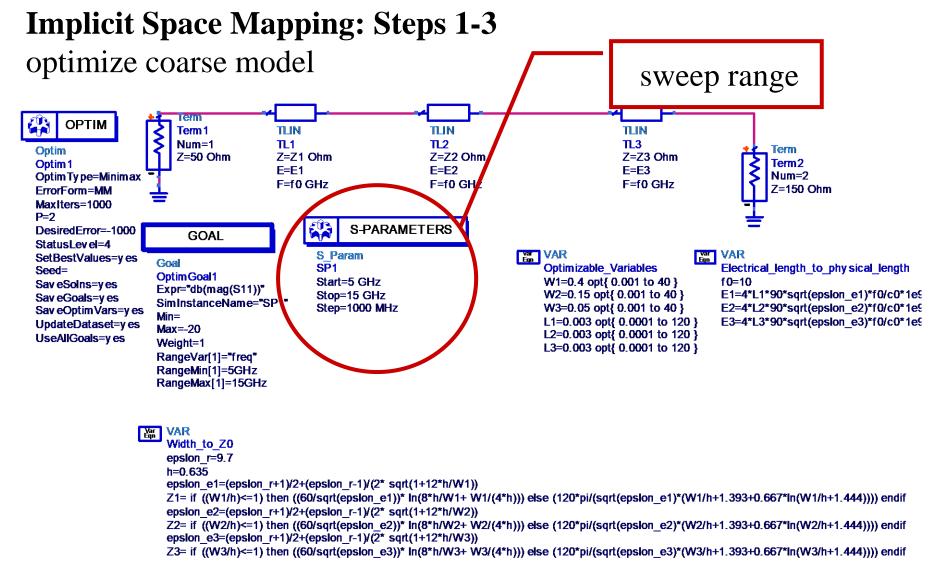


Implicit Space Mapping: Steps 1-3



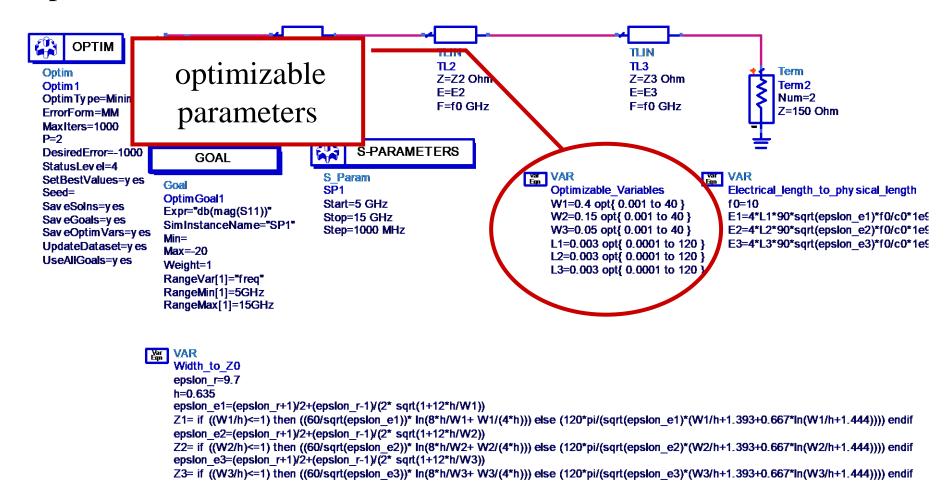






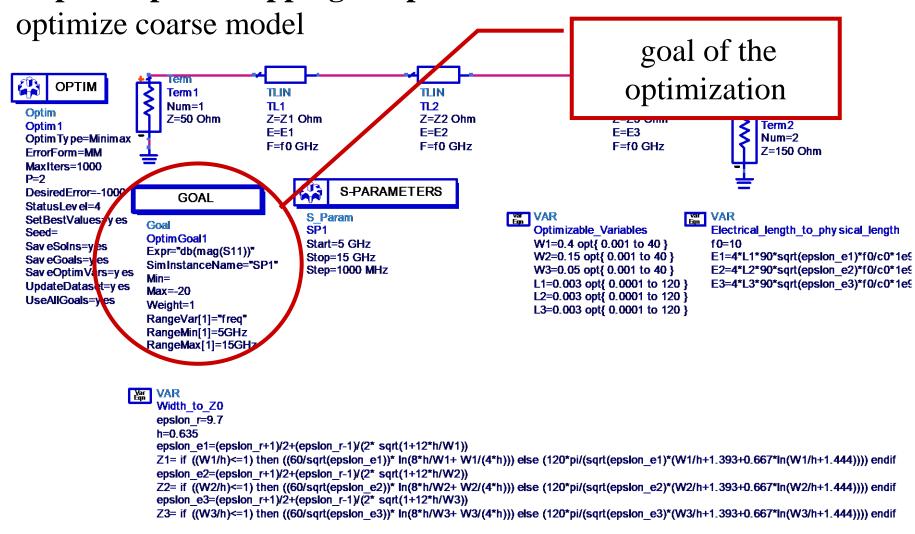


optimize coarse model







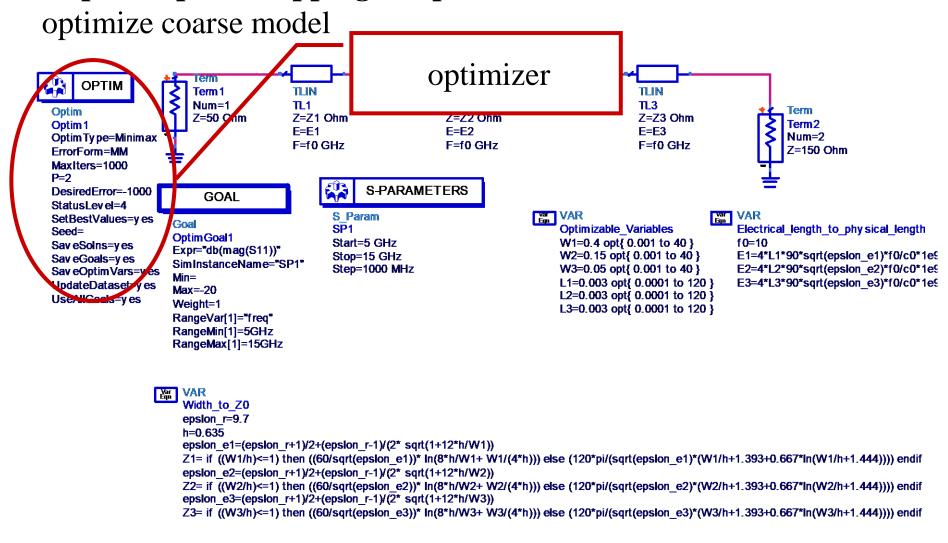




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Implicit Space Mapping: Steps 1-3

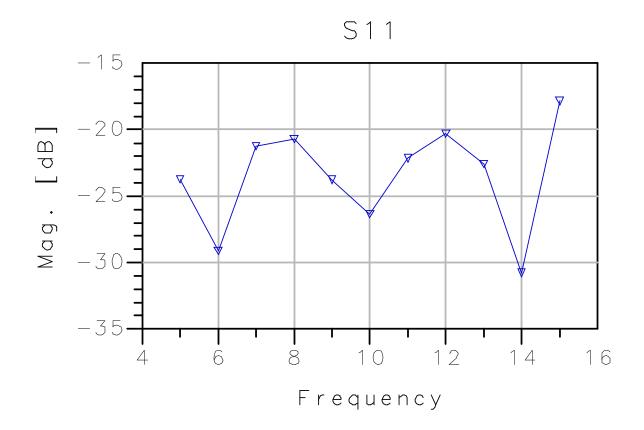




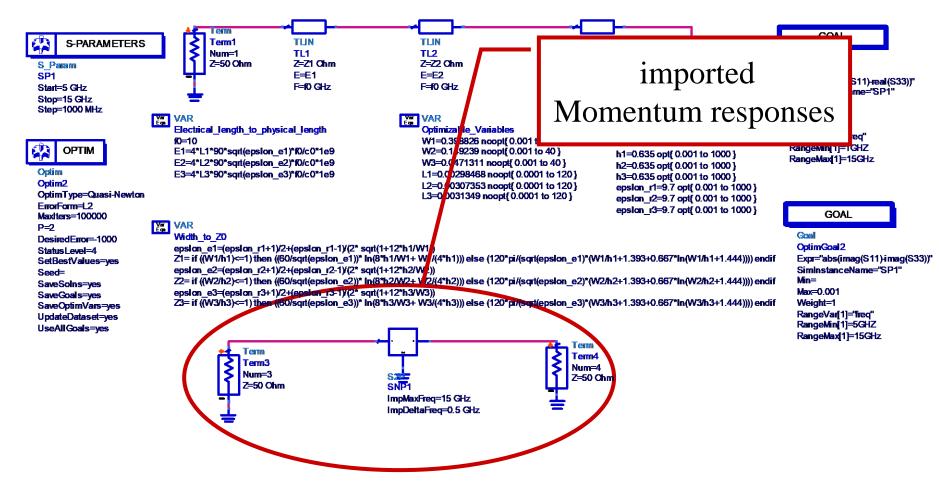
simulate fine model using Momentum



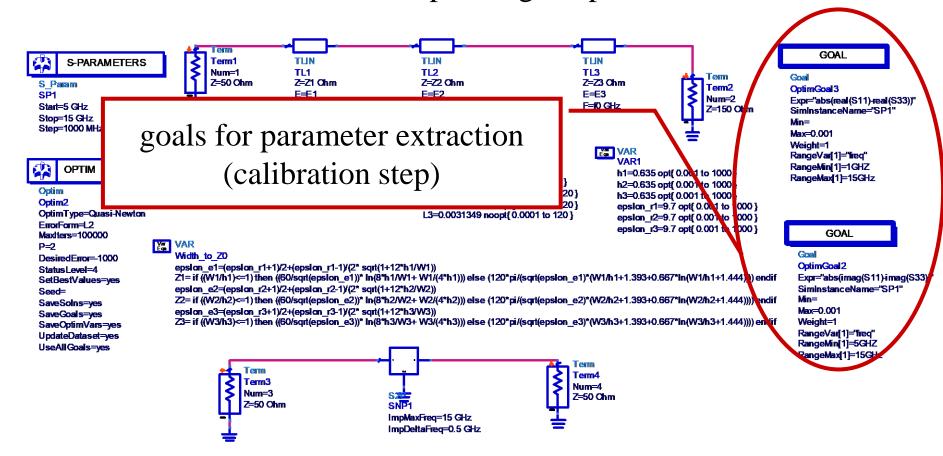
obtain the fine model result and check stopping criteria



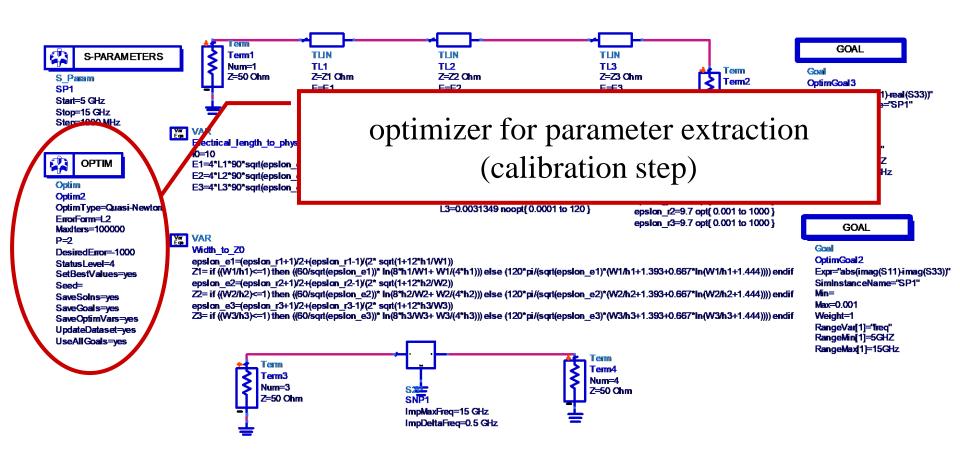




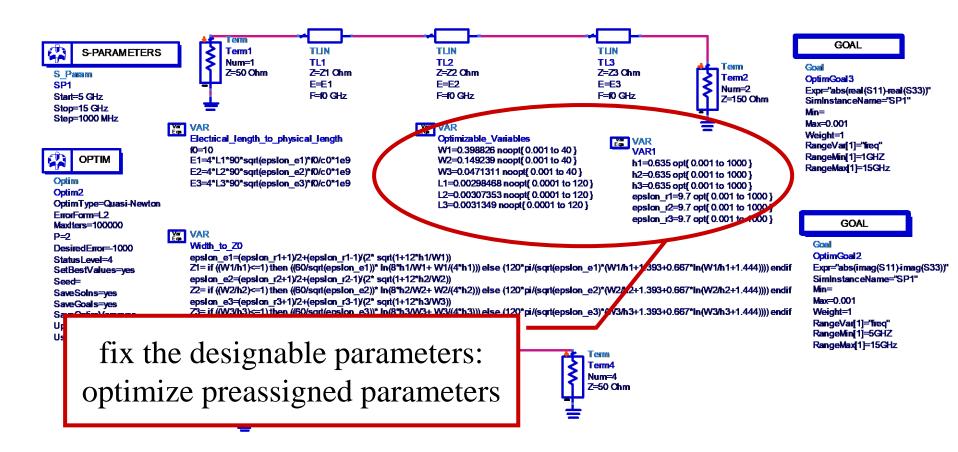






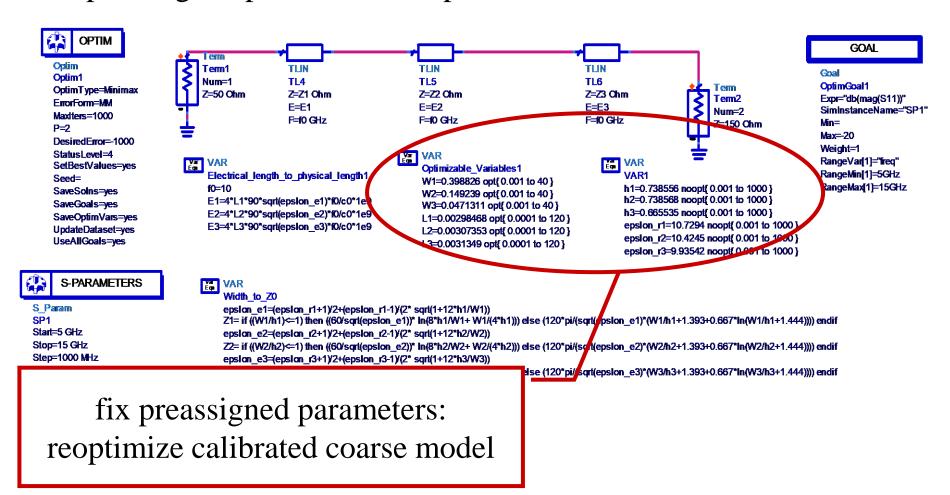








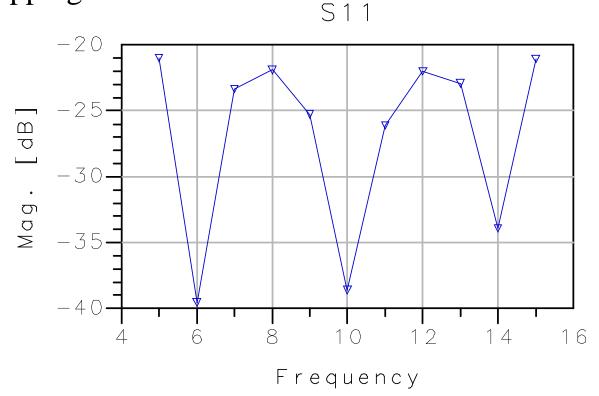
fix preassigned parameters: reoptimize calibrated coarse model





Implicit Space Mapping: Steps 4-6

simulate fine model using Momentum, satisfy stopping criteria







Space Mapping: a Glossary of Terms

Neuro implies use of artificial neural networks

Implicit Space Mapping space mapping when the mapping

is not obvious

Not Space Mapping (usually) space mapping

when not acknowledged

Parameter Transformation space mapping

Predistortion

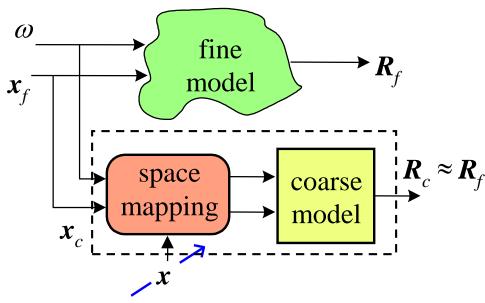


General Space Mapping Technology (Bandler et al., 1994-2002)

linearized: original and Aggressive Space Mapping

nonlinear: Neural Space Mapping, etc.

implicit: preassigned parameters (ISM)



parameters x: coarse space parameters, neuron weights mapping tableau, KPP (ISM)



Space Mapping - conceived as abstract concept by Bandler (1993), in collaboration with Biernacki, Chen and Madsen

Space Mapping - a fundamental new theory for design with CPU intensive simulators (1994)

EM design of high-temperature superconducting (HTS) microwave filters (1994)

Aggressive Space Mapping for EM design (1995)



Aggressive Space Mapping for EM design (1995)

IMS workshop on Automated Circuit Design Using Electromagnetic Simulators (Arndt, Bandler, Chen, Hoefer, Jain, Jansen, Pavio, Pucel, Sorrentino, Swanson, 1995)

fully-automated Space Mapping optimization of 3D structures (1996)





OSA's Empipe connection of OSA90/hope with Sonnet Software's *em* field simulator (1992)



OSA's Empipe3D connection of OSA90/hope with



Hewlett-Packard's HFSS 3D EM simulator (1996)



Ansoft's Maxwell Eminence 3D EM simulator (1996)



Space Mapping optimization with finite element (FEM) and mode matching (MM) EM simulators (1997)

further developments in Aggressive Space Mapping (1998-)

Generalized Space Mapping (GSM) tableau approach to device modeling (1999)

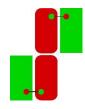
Neuro Space Mapping (NSM) device modeling (1999)





research begins on surrogate model/space mapping optimization algorithms (1999)

the SMX engineering optimization system (2000)



First International Workshop on Surrogate Modelling and Space Mapping for Engineering Optimization (2000)

Neural Inverse Space Mapping (NISM) optimization (2001)

Expanded Space Mapping Design Framework (ESMDF) (2001)

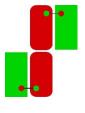


yield driven EM optimization using Space Mapping-based neuromodels (2001)

EM-based optimization exploiting Partial Space Mapping (PSM) and exact sensitivities (2002)

Implicit Space Mapping (ISM) EM-based modeling and design (2002)

introduction of Space Mapping to mathematicians (2002)



Special Issue of *Optimization and Engineering* on Surrogate Modelling and Space Mapping for Engineering Optimization (2002)



Original Rosenbrock Function (Coarse Model)

(*Bandler et al., 1999*)

$$R_c(\mathbf{x}_c) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

where
$$\mathbf{x}_c = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
 and $\mathbf{x}_c^* = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$

Shifted Rosenbrock Function (Fine Model)

(*Bandler et al., 1999*)

$$R_f(\mathbf{x}_f) = 100((x_2 + \alpha_2) - (x_1 + \alpha_1)^2)^2 + (1 - (x_1 + \alpha_1))^2$$

where
$$\mathbf{x}_f = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
, $\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$ hence $\mathbf{x}_f^* = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$



Gradient Parameter Extraction (GPE)

(*Bandler et al., 2002*)

at the jth iteration

$$\mathbf{x}_{c}^{(j)} = \arg\min_{\mathbf{X}_{c}} \| [\mathbf{e}_{0}^{T} \quad \lambda \mathbf{e}_{1}^{T} \quad \cdots \quad \lambda \mathbf{e}_{n}^{T}]^{T} \|_{2}^{2}, \quad \lambda \geq 0$$

where λ is a weighting factor and $E = [e_1 e_2 \dots e_n]$

$$\boldsymbol{e}_0 = \boldsymbol{R}_f(\boldsymbol{x}_f^{(j)}) - \boldsymbol{R}_c(\boldsymbol{x}_c)$$

$$\boldsymbol{E} = \boldsymbol{J}_f(\boldsymbol{x}_f^{(j)}) - \boldsymbol{J}_c(\boldsymbol{x}_c)\boldsymbol{B}$$



Shifted Rosenbrock Function Results

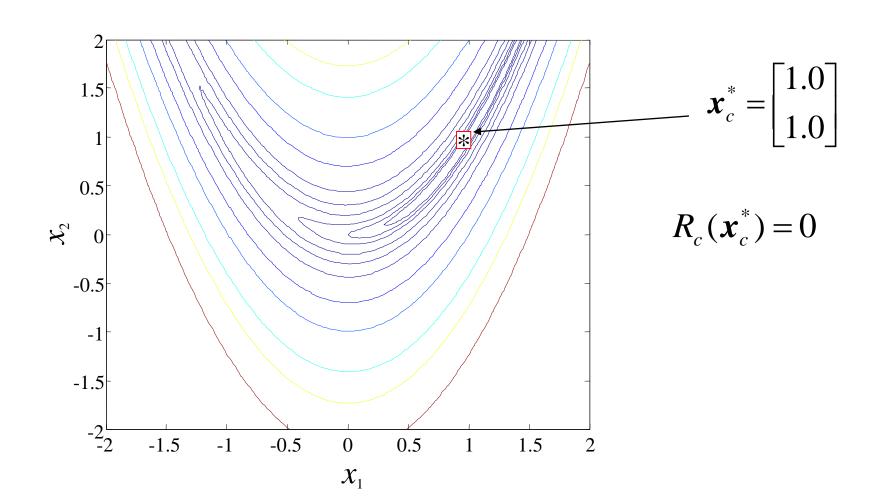
useful notation

$$oldsymbol{f}^{(j)} = oldsymbol{x}_c^{(j)} - oldsymbol{x}_c^*,$$
 $oldsymbol{h}^{(j)} = oldsymbol{x}_f^{(j+1)} - oldsymbol{x}_f^{(j)}$ and $oldsymbol{B}^{(j)} oldsymbol{h}^{(j)} = -oldsymbol{f}^{(j)}$



Original Rosenbrock Function (Coarse Model Contour Plot)

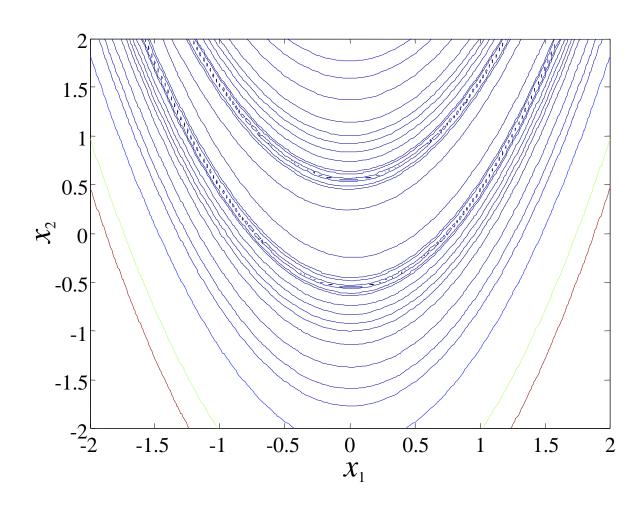
(*Bandler et al., 1999*)





Shifted Rosenbrock Function (Bandler et al., 2002)

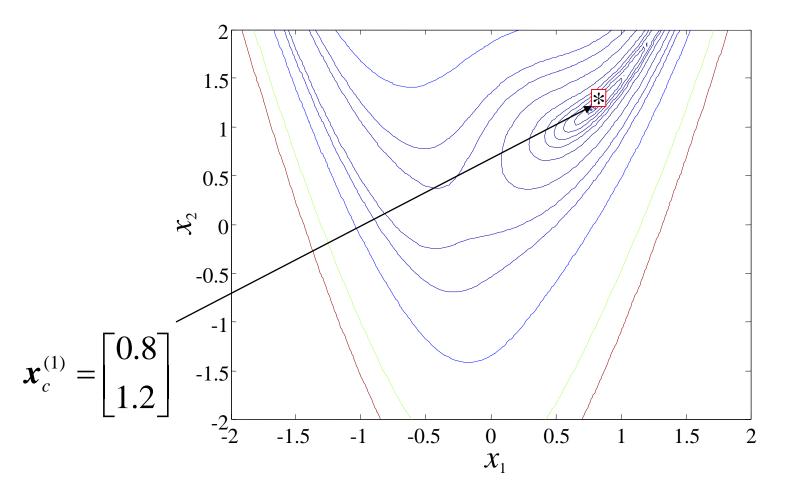
Single point PE (SPE): nonuniqueness exists





Shifted Rosenbrock Function (Bandler et al., 2002)

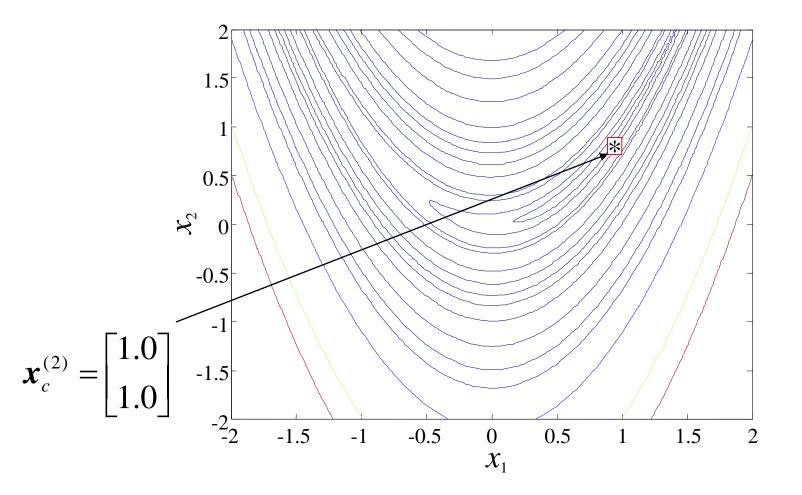
Gradient PE (1st iteration)





Shifted Rosenbrock Function (Bandler et al., 2002)

Gradient PE (2nd iteration)





Shifted Rosenbrock Function Results

(*Bandler et al.*, 2002)

iteration	$oldsymbol{x}_c^{(j)}$	$f^{(j)}$	$\boldsymbol{\mathit{B}}^{(j)}$	$\boldsymbol{h}^{(j)}$	$oldsymbol{x}_f^{(j)}$	R_f
0	$\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$				$\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$	31.4
1	$\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}$	$\begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$	$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}$	$\begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$	0
	$\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$				



Transformed Rosenbrock Function (Fine Model)

(*Bandler et al.*, 2002)

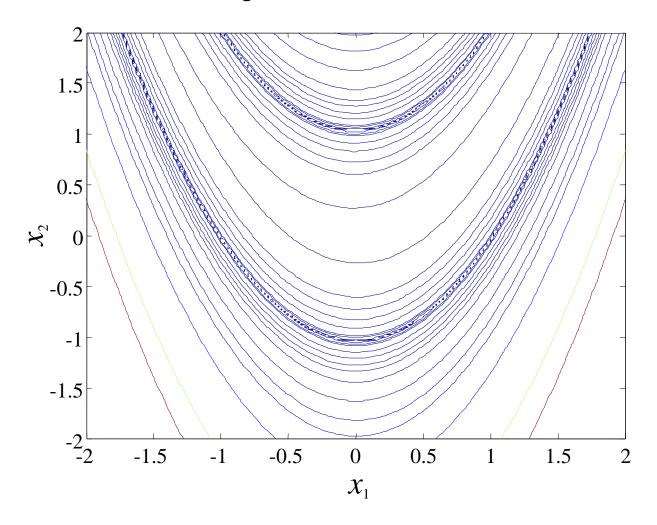
linear transformation of the original Rosenbrock function

$$R_{f}(\mathbf{x}_{f}) = 100(u_{2} - u_{1}^{2})^{2} + (1 - u_{1})^{2}$$
where $\mathbf{u} = \begin{bmatrix} u_{1} \\ u_{2} \end{bmatrix} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix} \mathbf{x}_{f} + \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$

$$\mathbf{x}_{f}^{*} = \begin{bmatrix} 1.2718447 \\ 0.4951456 \end{bmatrix}$$

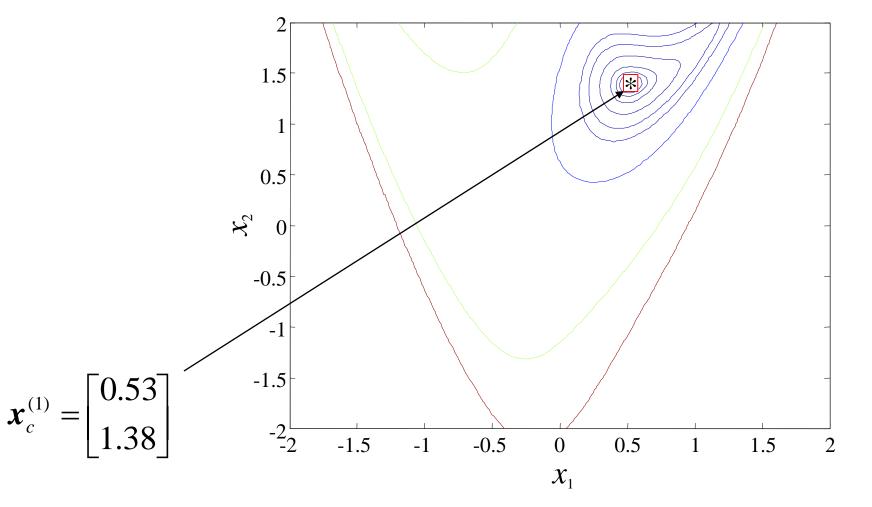


Single point PE (SPE): nonuniqueness exists



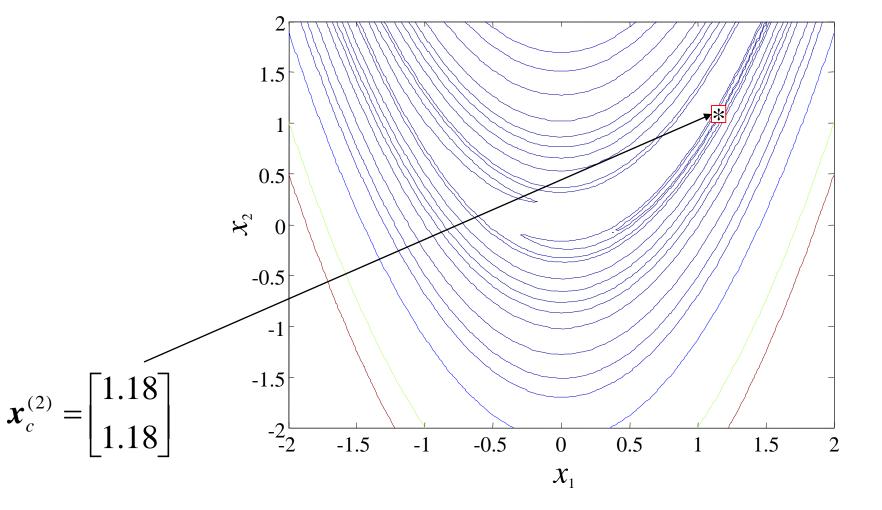


GPE (1st PE iteration)



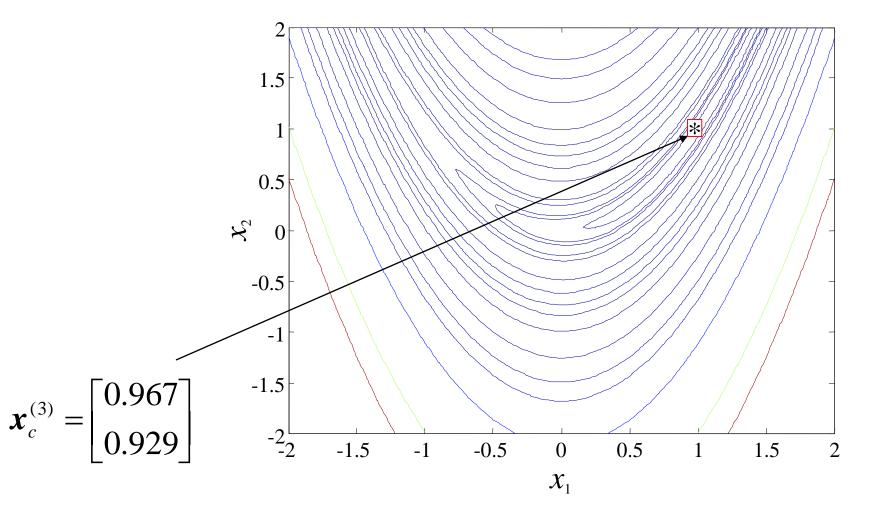


GPE (2nd PE iteration)



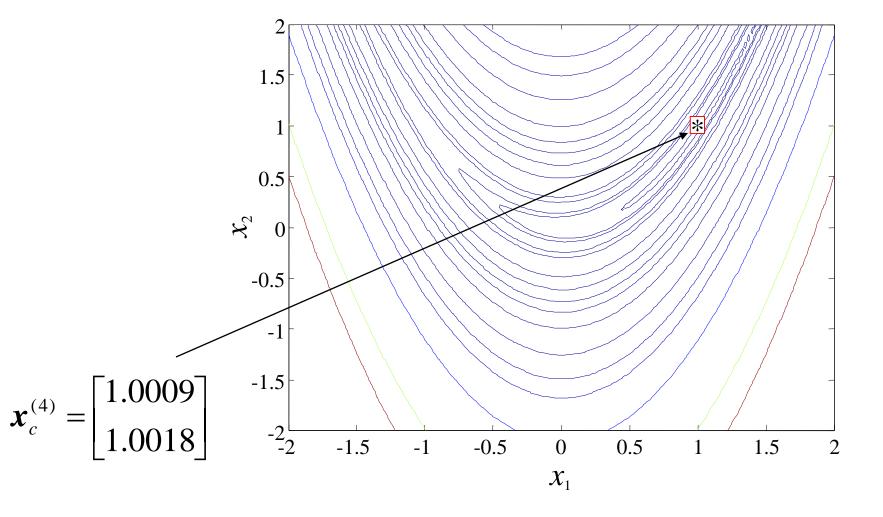


GPE (3rd PE iteration)



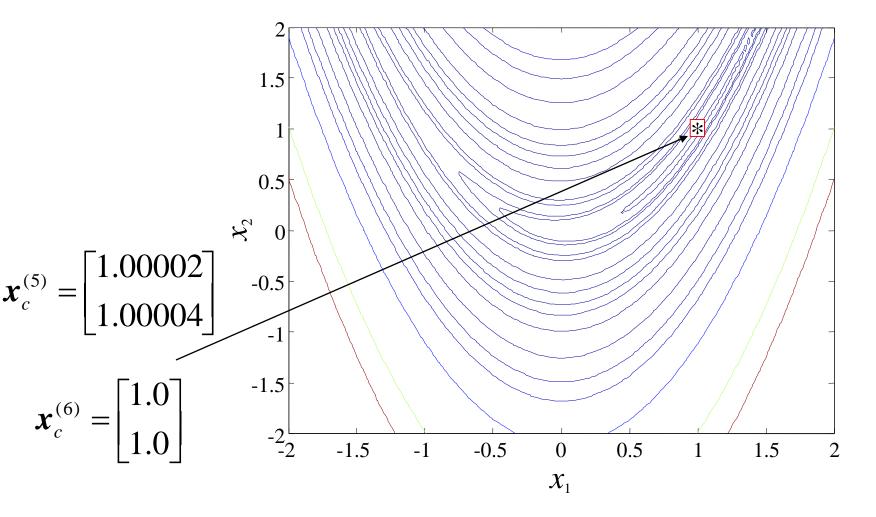


Transformed Rosenbrock Function (*Bandler et al., 2002*) GPE (4th PE iteration)





GPE (5th and 6th PE iteration)





Transformed Rosenbrock Results (Bandler et al., 2002)

iteration	$\boldsymbol{x}_{c}^{(j)}$	$oldsymbol{f}^{(j)}$	$\boldsymbol{B}^{(j)}$	$\boldsymbol{h}^{(j)}$	$oldsymbol{x}_f^{(j)}$	R_f
0	$\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$				$\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$	108.3
1	$\begin{bmatrix} 0.526 \\ 1.384 \end{bmatrix}$	$\begin{bmatrix} -0.474 \\ 0.384 \end{bmatrix}$	$\begin{bmatrix} 1.01 & -0.05 \\ 0.01 & 1.01 \end{bmatrix}$	$\begin{bmatrix} 0.447 \\ -0.385 \end{bmatrix}$	$\begin{bmatrix} 1.447 \\ 0.615 \end{bmatrix}$	5.119
2	[1.185] [1.178]	$\begin{bmatrix} 0.185 \\ 0.178 \end{bmatrix}$	$\begin{bmatrix} 0.96 & -0.12 \\ -0.096 & 1.06 \end{bmatrix}$	$\begin{bmatrix} -0.218 \\ -0.187 \end{bmatrix}$	$\begin{bmatrix} 1.23 \\ 0.427 \end{bmatrix}$	4.4E-3
3	$\begin{bmatrix} 0.967 \\ 0.929 \end{bmatrix}$	$\begin{bmatrix} -0.033 \\ -0.071 \end{bmatrix}$	$\begin{bmatrix} 1.09 & -0.19 \\ 0.168 & 0.92 \end{bmatrix}$	$\begin{bmatrix} 0.0429 \\ 0.0697 \end{bmatrix}$	$\begin{bmatrix} 1.273 \\ 0.4970 \end{bmatrix}$	1.8E–6
4	$\begin{bmatrix} 1.001 \\ 1.001 \end{bmatrix}$	$\begin{bmatrix} 0.001 \\ 0.001 \end{bmatrix} \begin{bmatrix} 0.001 \\ 0.001 \end{bmatrix}$	1.10001 -0.1999 0.1999 0.9001	$\begin{bmatrix} -0.001 \\ -0.002 \end{bmatrix}$	$\begin{bmatrix} 1.2719 \\ 0.4952 \end{bmatrix}$	5E-10





Transformed Rosenbrock Results (Bandler et al., 2002)

iteration	$oldsymbol{x}_c^{(j)}$	$oldsymbol{f}^{(j)}$	$oldsymbol{B}^{(j)}$	$oldsymbol{h}^{(j)}$	$oldsymbol{x}_f^{(j)}$	R_f
5	$\begin{bmatrix} 1.00002 \\ 1.00004 \end{bmatrix}$	$1E-4\times\begin{bmatrix}0.2\\0.4\end{bmatrix}$	$\begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}$	$1E-4 \times \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 1.2718 \\ 0.4951 \end{bmatrix}$	3E-17
6	$\begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$	$1E - 8 \times \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}$	$1E-8 \times \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}$	\boldsymbol{x}_f^*	9E–29

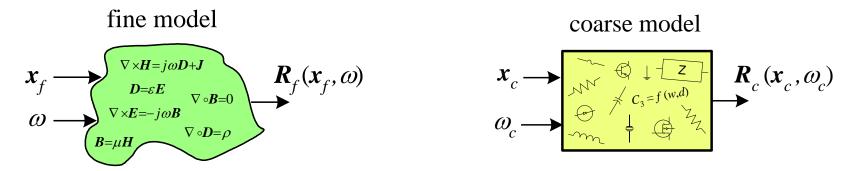
$$\boldsymbol{x}_{f}^{*} = \begin{bmatrix} 1.27184466 \\ 0.49514563 \end{bmatrix}$$





Conventional Space Mapping for Microwave Circuits

(*Bandler et al., 1994*)



find

$$\begin{bmatrix} \mathbf{x}_c \\ \mathbf{\omega}_c \end{bmatrix} = \mathbf{P}(\mathbf{x}_f, \mathbf{\omega})$$

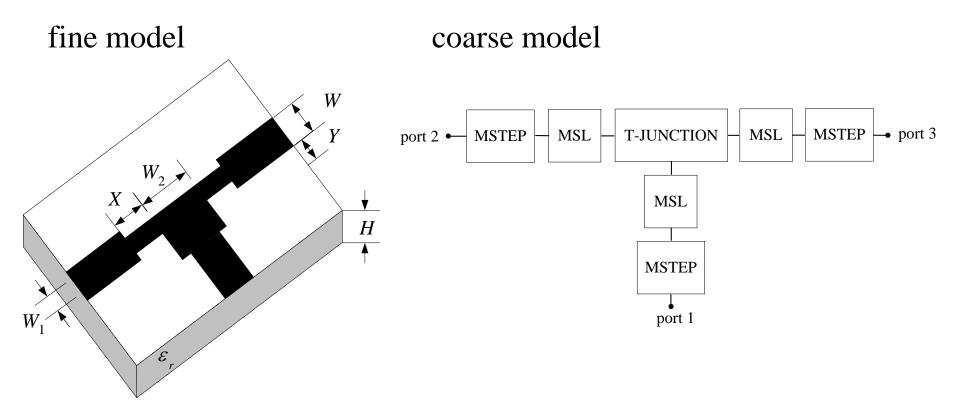
such that

$$\mathbf{R}_c(\mathbf{x}_c, \omega_c) \approx \mathbf{R}_f(\mathbf{x}_f, \omega)$$





Microstrip Shaped T-Junction (Bandler et al., 1999)





Microstrip Shaped T-Junction (Bandler et al., 1999)

the region of interest

15 mil
$$\leq H \leq$$
 25 mil
2 mil $\leq X \leq$ 10 mil
15 mil $\leq Y \leq$ 25 mil
 $8 \leq \varepsilon_r \leq$ 10

the frequency range is 2 GHz to 20 GHz with a step of 2 GHz the number of base points is 9, the number of test points is 50

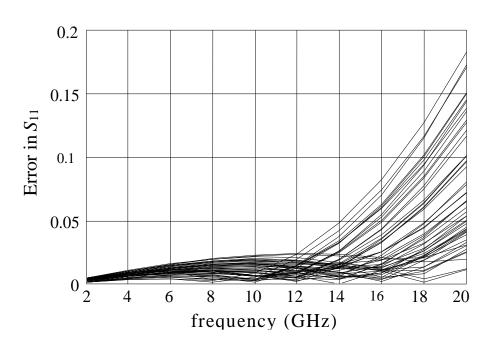
the widths W of the input lines track H so that their characteristic impedance is 50 ohm

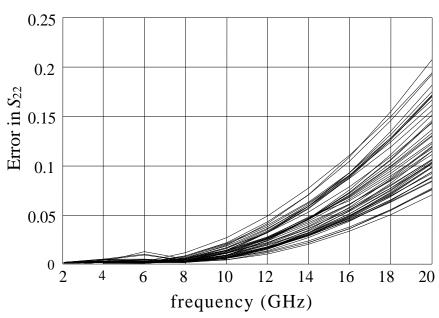
 $W_1 = W/3$, W_2 is suitably constrained



Microstrip Shaped T-Junction Coarse Model

errors w.r.t. Sonnet's em at the test points







Microstrip Shaped T-Junction Enhanced Coarse Model

errors w.r.t. Sonnet's em at the test points

