Introduction to the Space Mapping

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Purposes

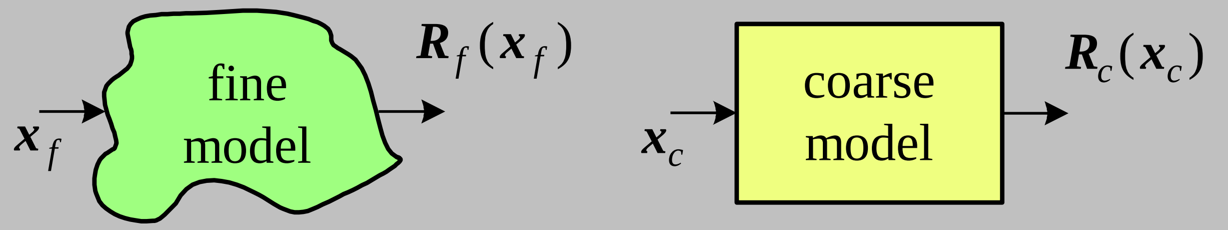
- Optimization of very expensive models
- Construct easy-to-calculate surrogate models

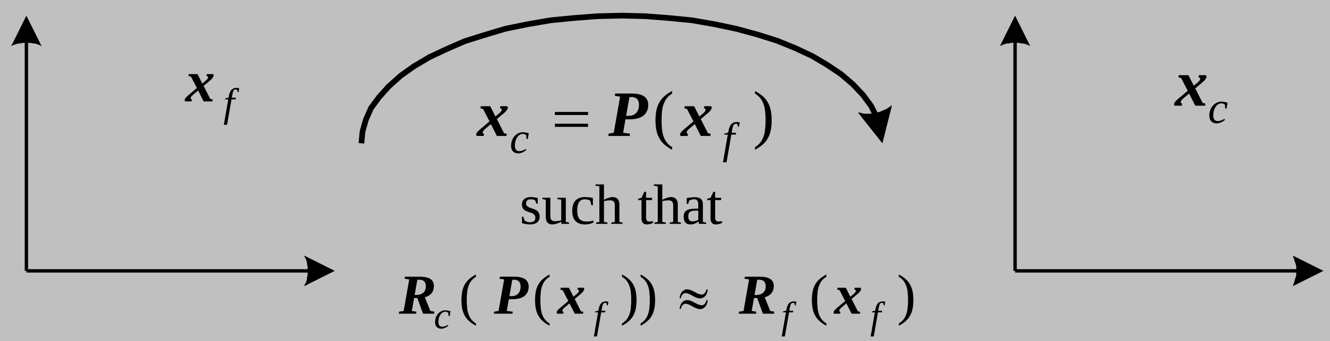
We assume two models of a physical object are available:

- an accurate fine model (expensive)
- a simpler coarse model (cheap)

The Space Mapping Concept

(*Bandler et al., 1994-*)





Applications

MS35:
K. Madsen, "*Introduction to Space Mapping*"
J.W. Bandler, "*Optimal Design of High-Fidelity Engineering Device Models Through Space Mapping*"
S. Koziel, "*On the Convergence of Space Mapping Optimization Algorithms*"
L. Nielsson, "*Optimization using Space Mapping, with Application on Contact and Impact Mechanical Problems*"

MS45:
D. Echeverria, "*Multi-Level Optimization with the Space-Mapping Technique*"
D. Lahaye, "*Space-Mapping Applied to Linear Actuator Design*"
F. Pedersen, "*Modeling Thermally Active Building Components Using Space Mapping*"
Q.J. Zhang, "*Neuro-Space Mapping for Nonlinear Electronic Device Modeling*"

Outline

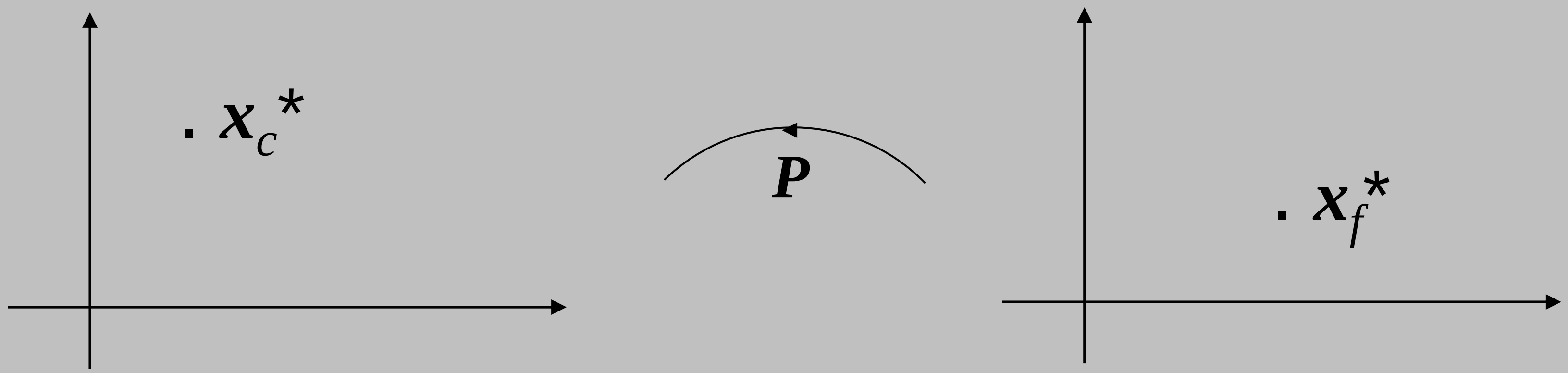
- Motivate the Space Mapping
- Define the Space Mapping
- Transmission-line example
- Compare with traditional methods

Space Mapping (Bandler, 1993)

Physical problem

$\boldsymbol{R}_c$
coarse model

$\boldsymbol{R}_f$
fine model



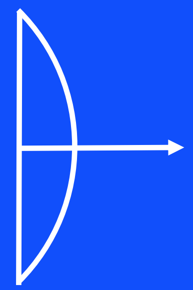
Connect similar responses

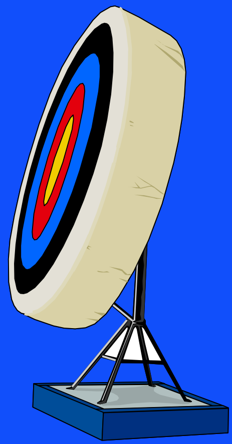
$$R_f(x_f) \approx R_c(P(x_f))$$

Archery example (Bandler (1995))

Coarse model

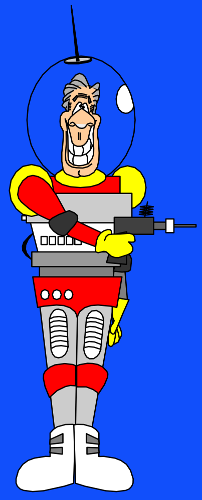
no wind, no gravity, etc.

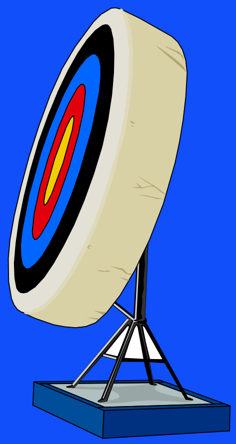




Fine model:

reality

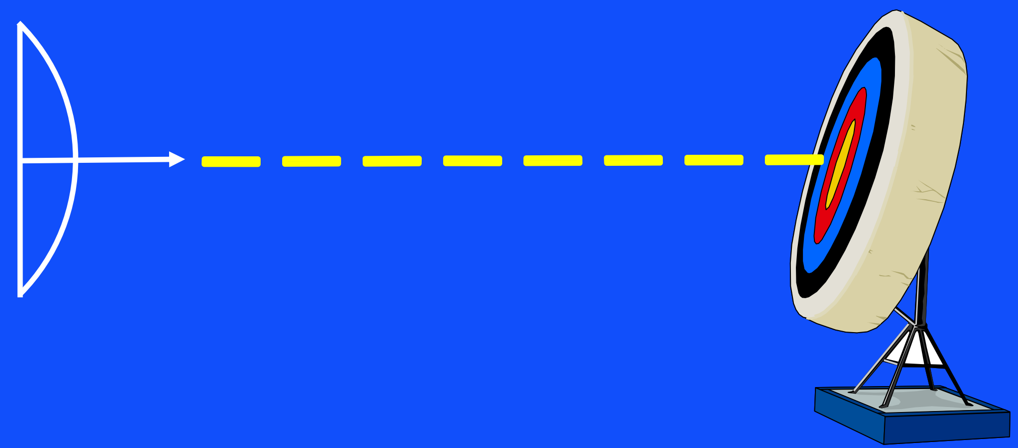




Archery example

First aim

(x_c* in coarse model)

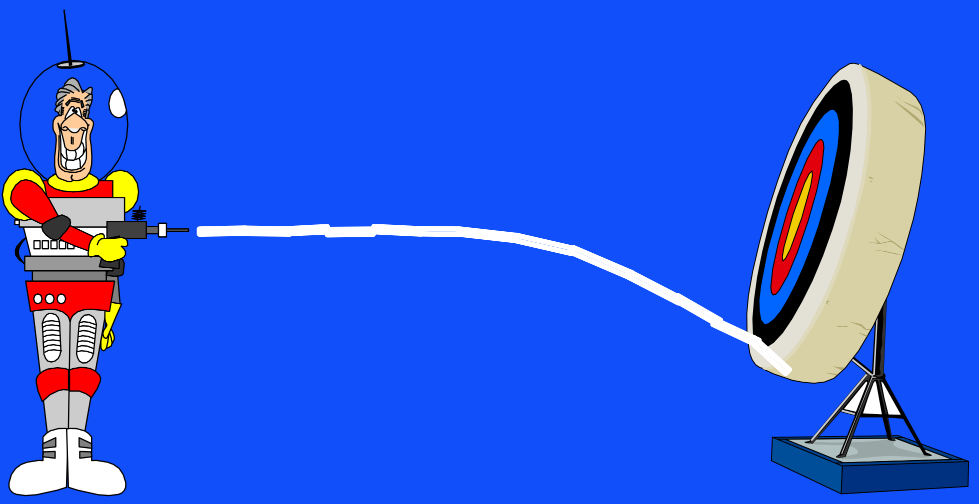


First shot:

"Calculate"

$R_f\,(x_f^{(0)})$

(fine model)

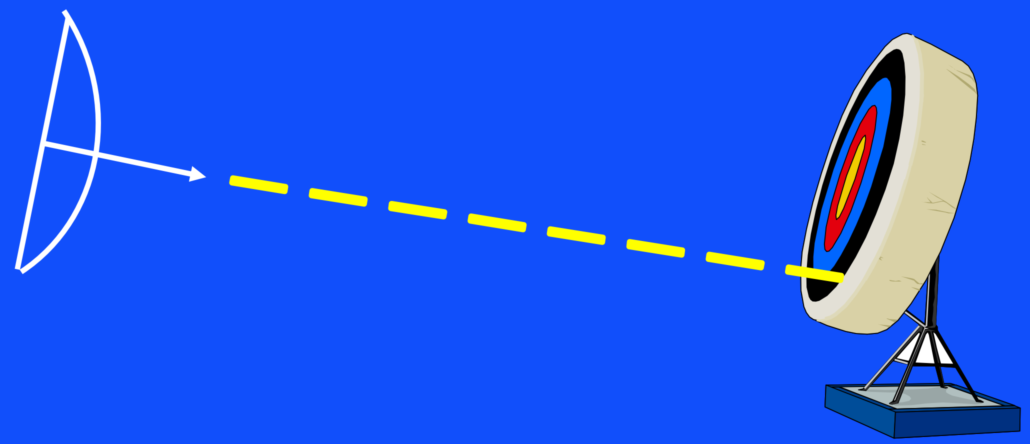


$$x_f^{(0)} = x_c^*$$

Parameter extraction

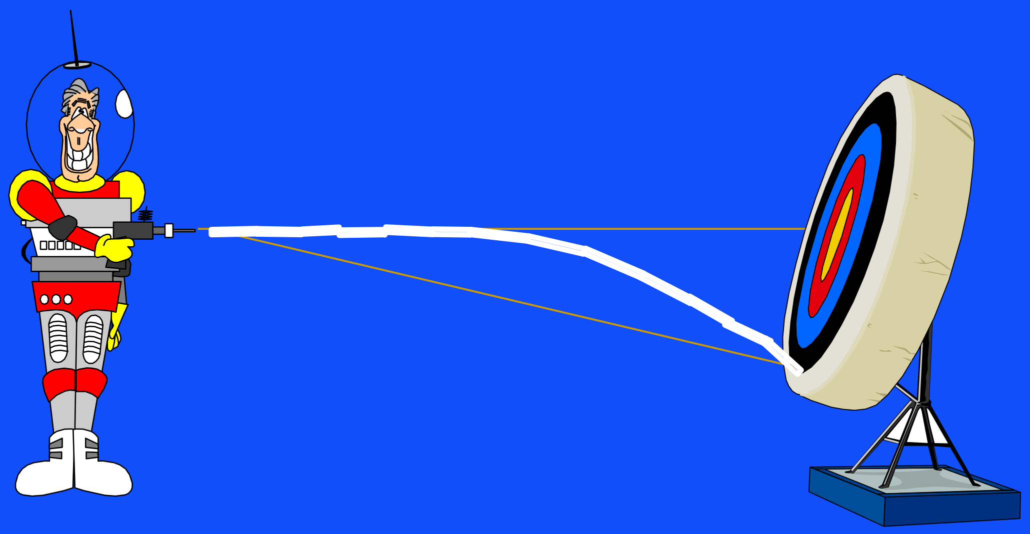
Match first shot: $x_c^{(0)}$

(coarse model)



First shot: $x_f^{(0)}$

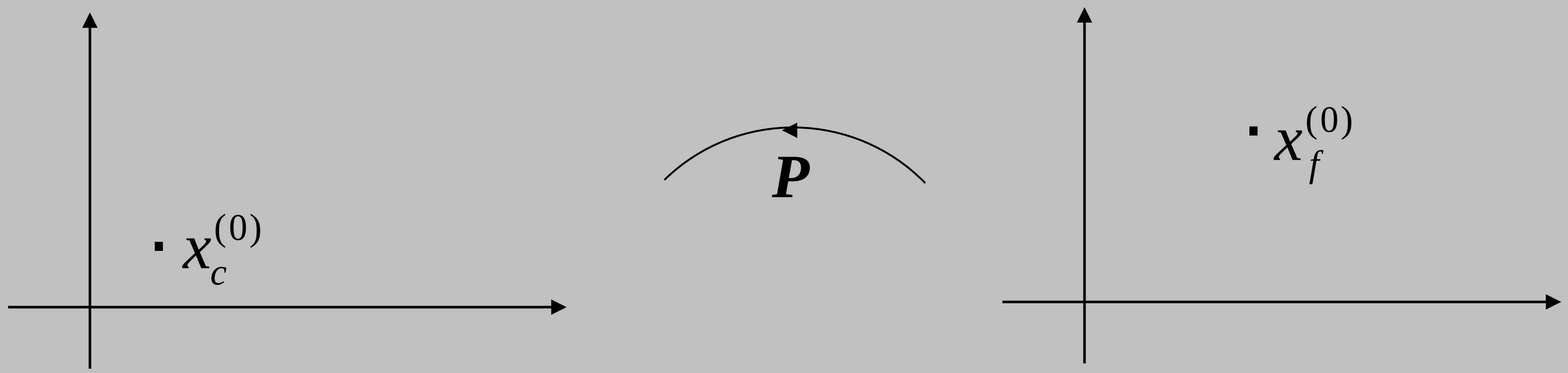
(fine model)



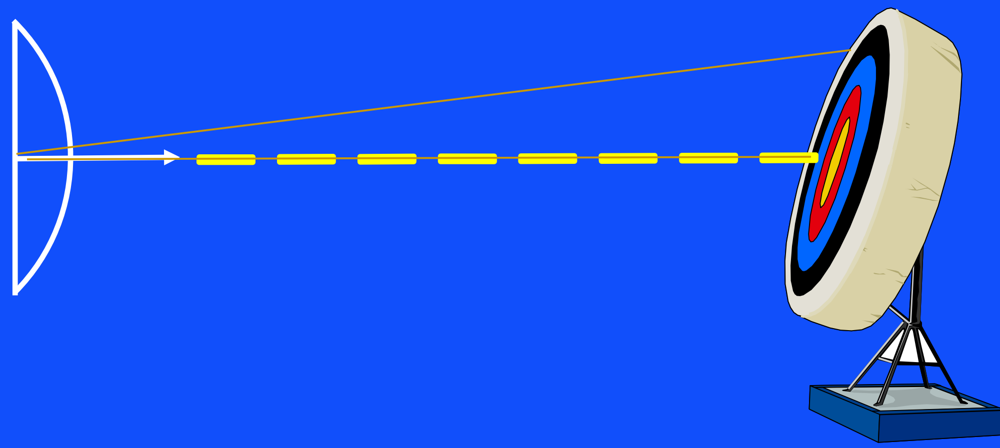
$$P(x_f^{(0)}) = x_c^{(0)}$$

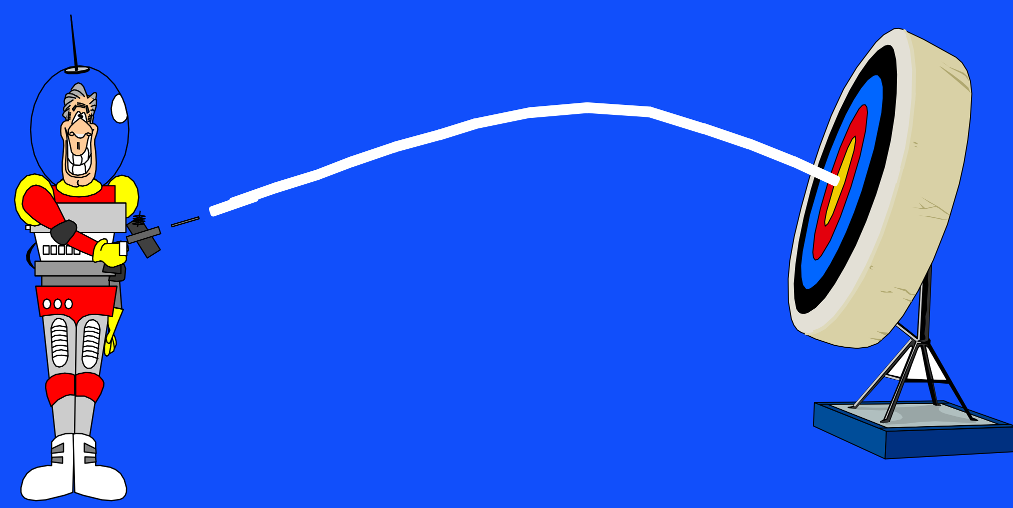
Parameter extraction

$$x_c^{(0)} = P(x_f^{(0)}) \equiv \arg\min_{x_c} \left\{ \left\| R_f(x_f^{(0)}) - R_c(x_c) \right\| \right\}$$



Better match to x_c^*





Adjusted aim $x_f^{(1)}$

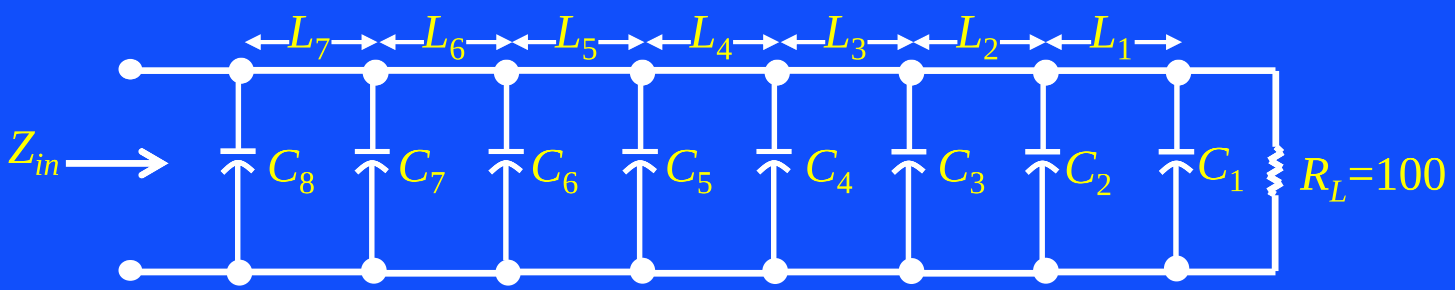
Another example

7 section capacitively loaded transmission-line transformer (TLT) to be optimized

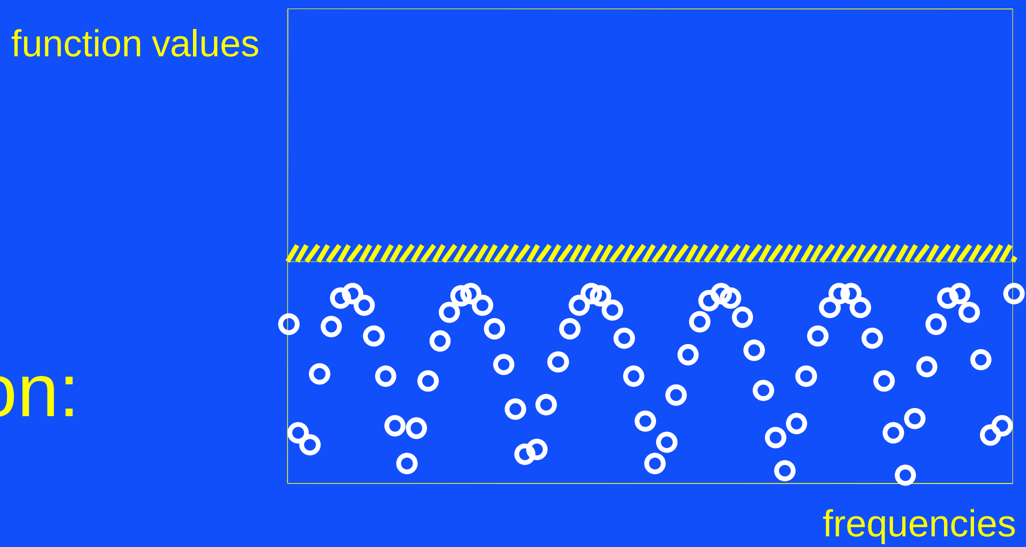
- capacitances are fixed at 0.025 pF
- characteristic impedances are kept fixed
- optimize only the lengths
- synthetic example

Bakr, Bandler, Madsen, Søndergaard (2002)

7 Section TLT



Optimal solution:



Type of problem considered

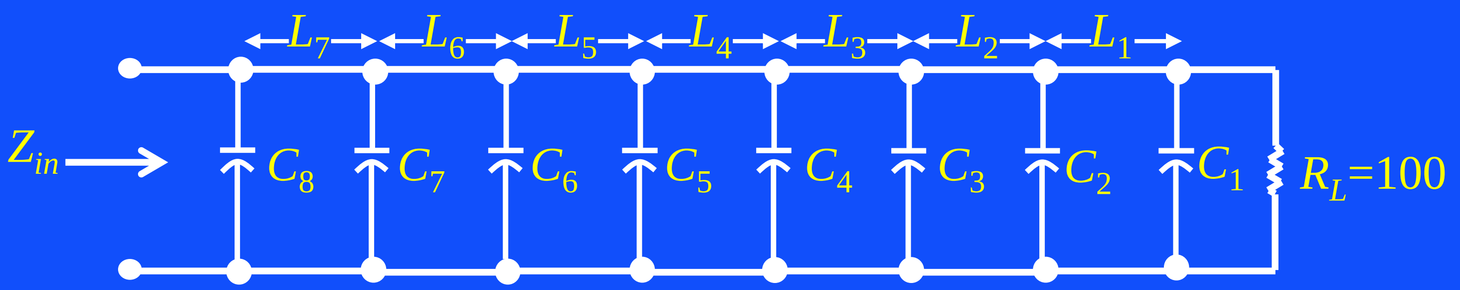
Minimize w.r.t. x_f the absolute values of the deviations between the set of function values $R_f\,(x_f\,;t_i)$ and some specifications y_i

$$f_i\,(x_f) = R_f\,(x_f\,;t_i\,) - y_i\,, \quad i = 1, \ldots, m$$

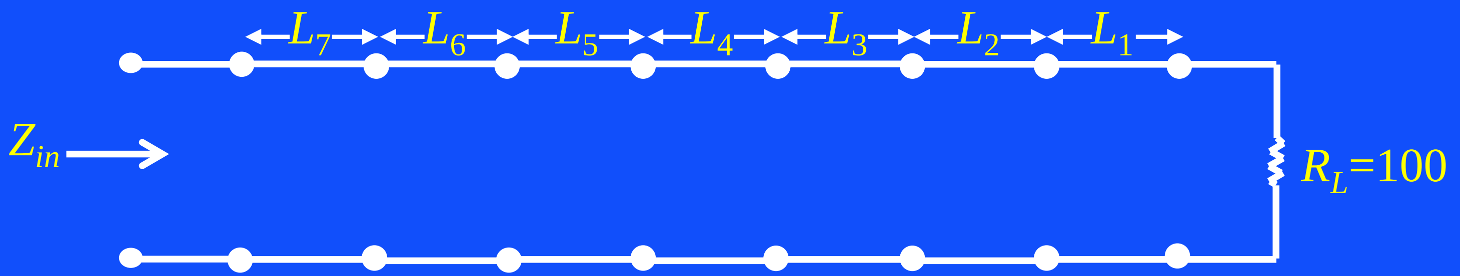
(In this example: $y_i = 0\,, \quad i = 1, \ldots, m$)

Space Mapping requirement:

Fine model



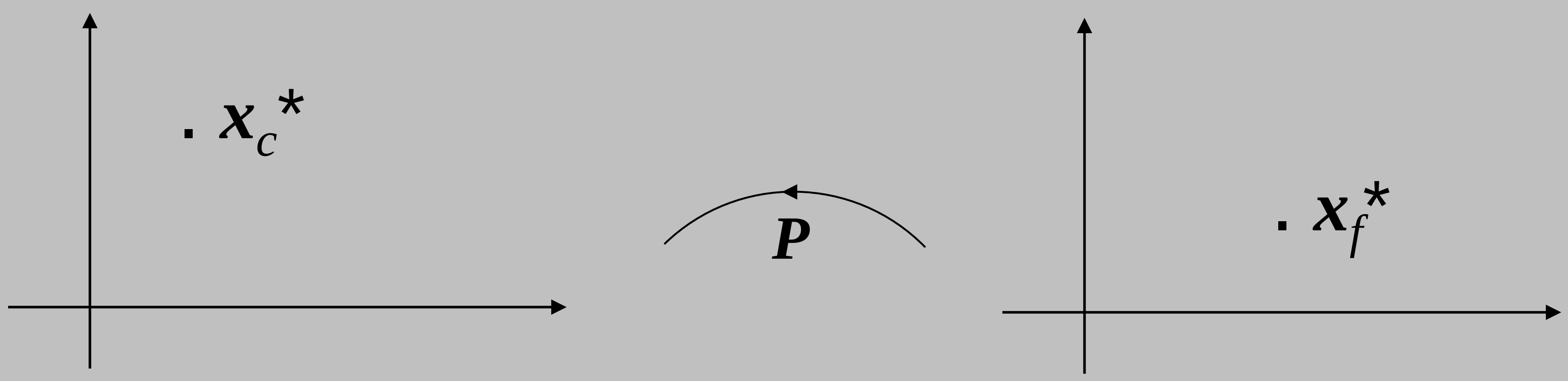
Coarse model



Find the Space Mapping P

$\boldsymbol{R}_c$
coarse model

$\boldsymbol{R}_f$
fine model



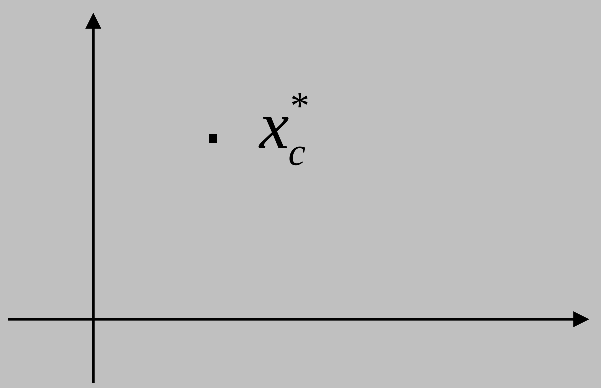
- by connecting similar responses

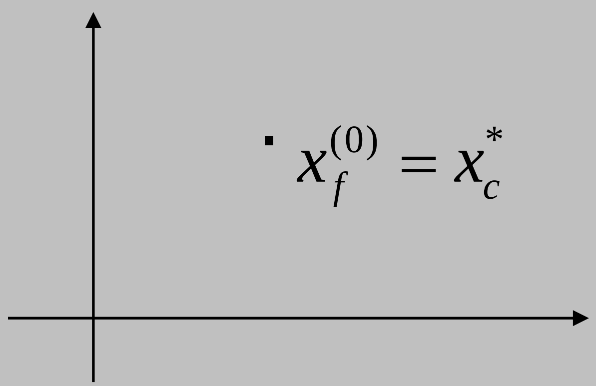
$$R_f(x_f) \approx R_c(P(x_f))$$

Initialization

$\boldsymbol{R}_c$
coarse model

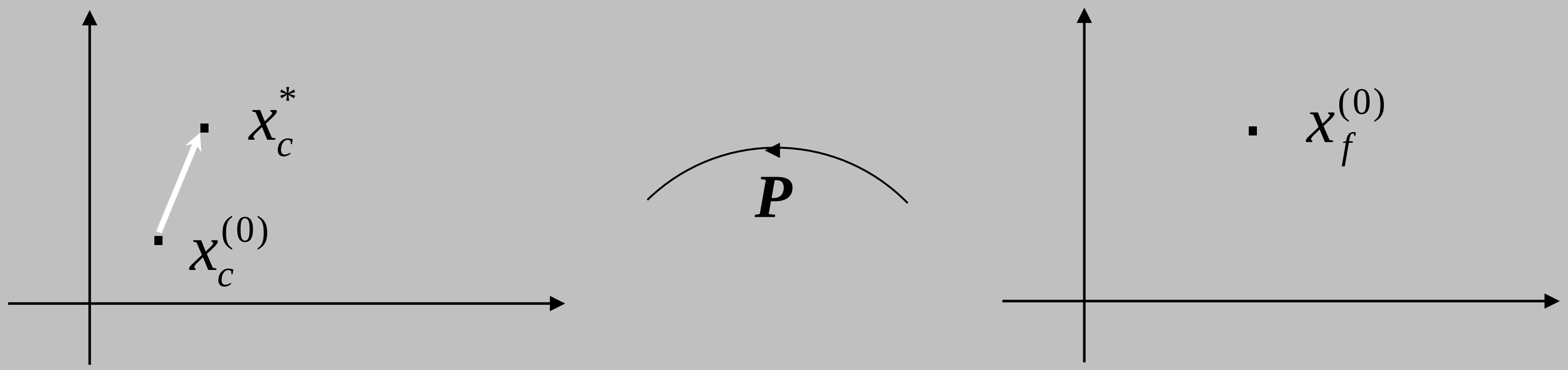
$\boldsymbol{R}_f$
fine model





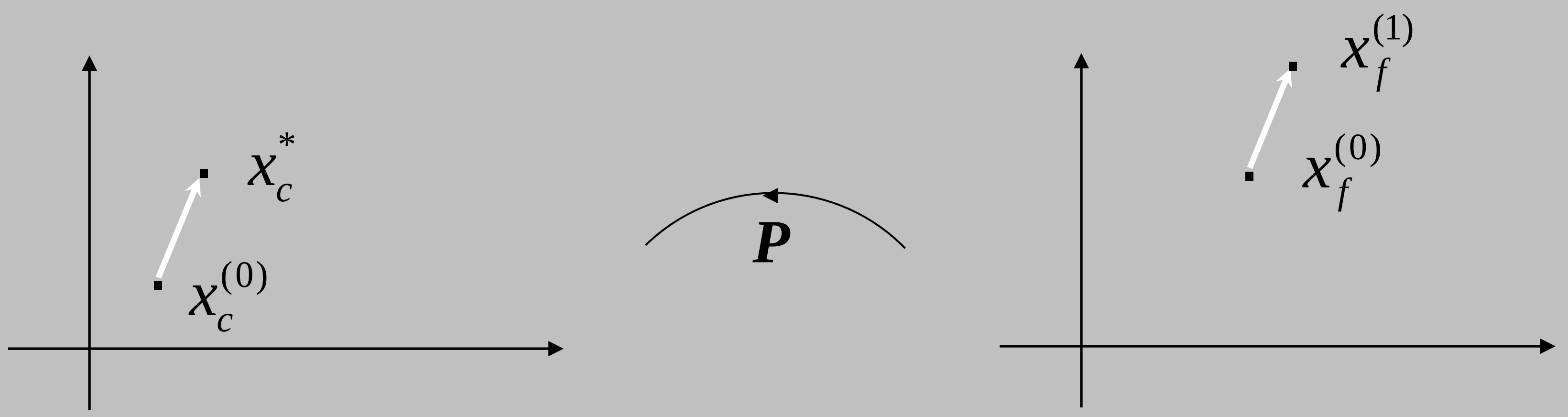
Find the coarse model solution x_c^*

Find $x_c^{(0)}$



$$x_c^{(0)} = P(x_f^{(0)}) \equiv \arg\min_{x_c} \left\{ \left\| R_f(x_f^{(0)}) - R_c(x_c) \right\| \right\}$$

Find $x_f^{(1)}$



Intuition: $$x_f^{(1)} = x_f^{(0)} + (x_c^* - x_c^{(0)})$$

We assume $R_f(x_f) \approx R_c(P(x_f))$

$$x_f^{(1)} = \arg\min_{x_f} \left\{ R_c(P(x_f)) \right\} \rightarrow P(x_f^{(1)}) = x_c^*$$

$$P(x_f) \approx P(x_f^{(0)}) + J_P(x_f^{(0)})(x_f - x_f^{(0)})$$

$$\approx P(x_f^{(0)}) + B_0(x_f - x_f^{(0)}) \ , \qquad B_0 = I$$

$$P(x_f^{(0)}) + (x_f^{(1)} - x_f^{(0)}) = x_c^* \ , \qquad P(x_f^{(0)}) = x_c^{(0)}$$

Intuition: $x_f^{(1)} = x_f^{(0)} + (x_c^* - x_c^{(0)})$

SM algorithm

$$x_f^{(0)} = x_c^*$$

for $k = 0, 1, 2, \ \ldots$ (while *not* *STOP*) do

calculate $R_f(x_f^{(k)})$

$$x_c^{(k)} = P(x_f^{(k)}) \equiv \arg\min_{x_c} \left\{ \left\| R_f(x_f^{(k)}) - R_c(x_c) \right\| \right\}$$

compute $P^{(k)}$ from $P(x_f^{(k)})$ and $B^{(k)}$

solve $P^{(k)}(x_f) = x_c^* \quad \rightarrow \quad x_f^{(k+1)}$

enddo

Bandler, Biernacki, Chen, Hemmers, Madsen (1995)

k'th iteration: Estimate P

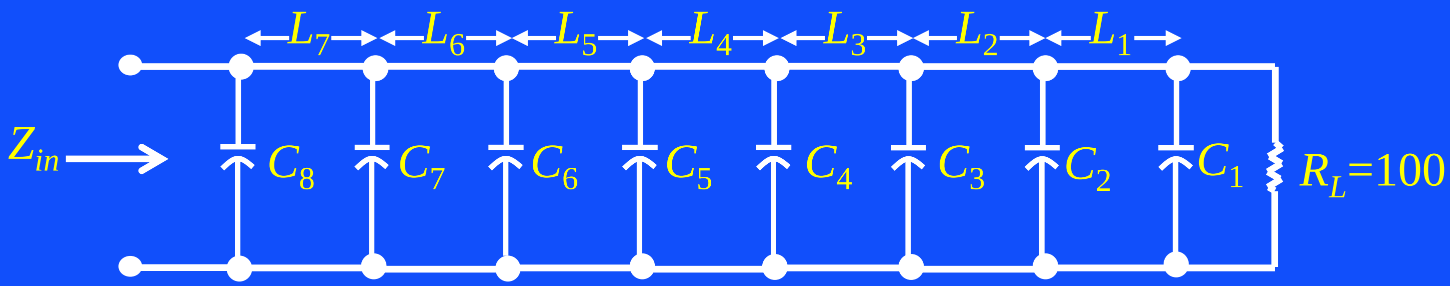
Assume P has been computed at $x_f^{(0)}, x_f^{(1)}, \ldots, x_f^{(k)}$

$$P(x_f) \approx P(x_f^{(k)}) + J_P(x_f^{(k)})(x_f - x_f^{(k)})$$
$$\approx P(x_f^{(k)}) + B_k(x_f - x_f^{(k)})$$
$$\equiv P^{(k)}(x_f)$$

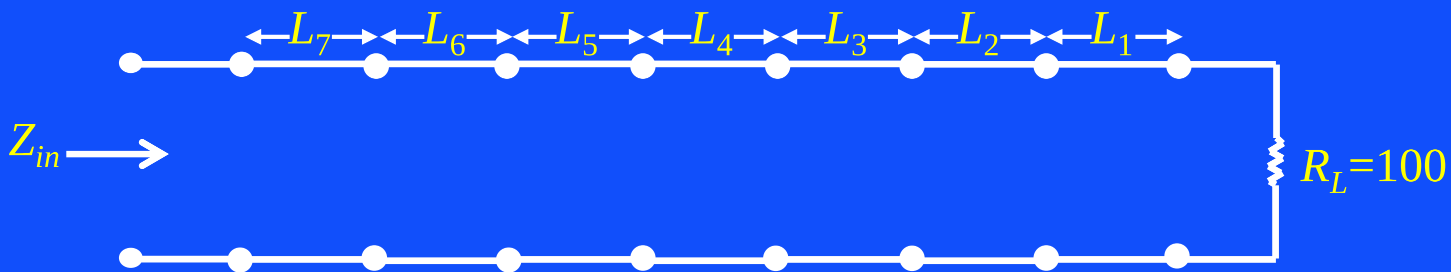
where $B^{(k)} \approx J_P(x_f^{(k)})$ is, e.g., a Broyden update

7 Section TLT

Fine model



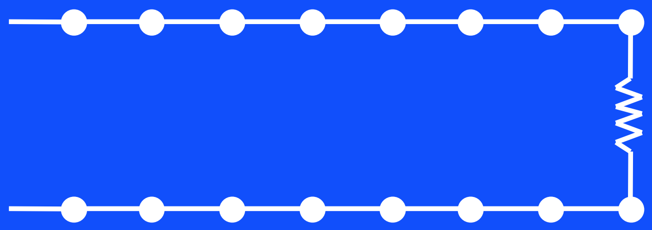
Coarse model

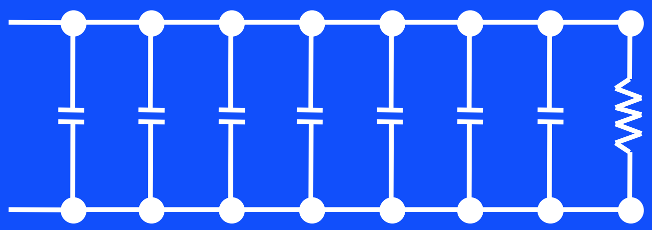


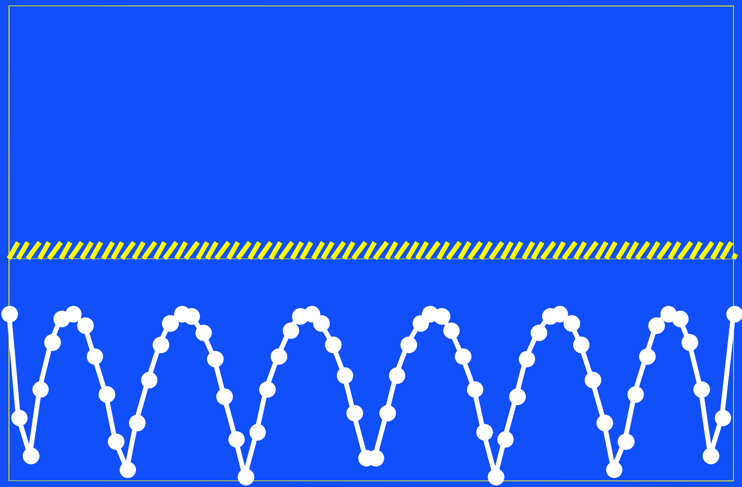
Initial value $x_f^{(0)}$

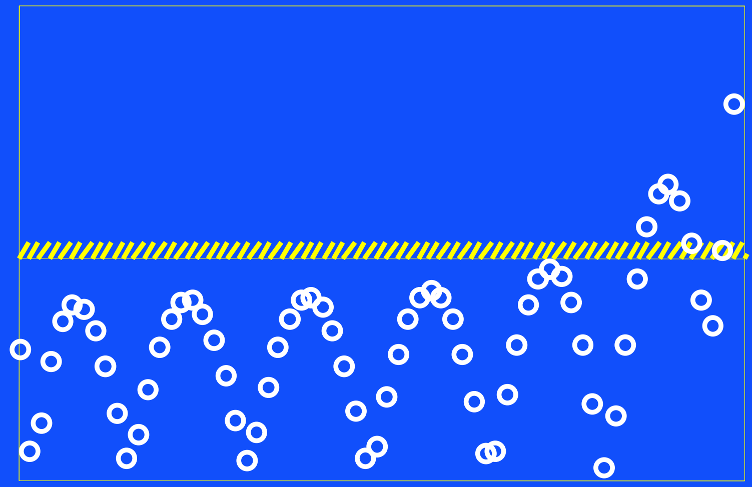
coarse model optimum: x_c^*

fine model at $x_f^{(0)} = x_c^*$







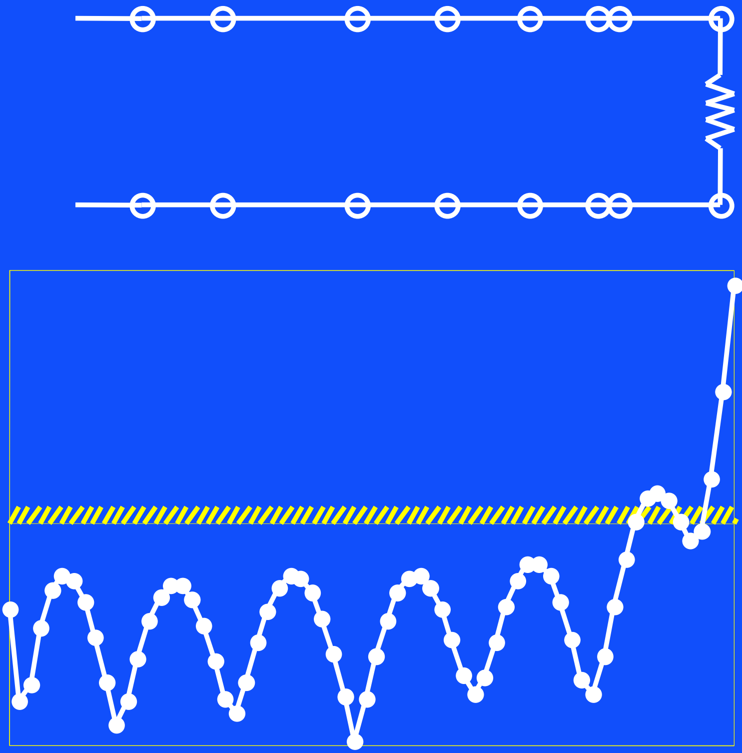


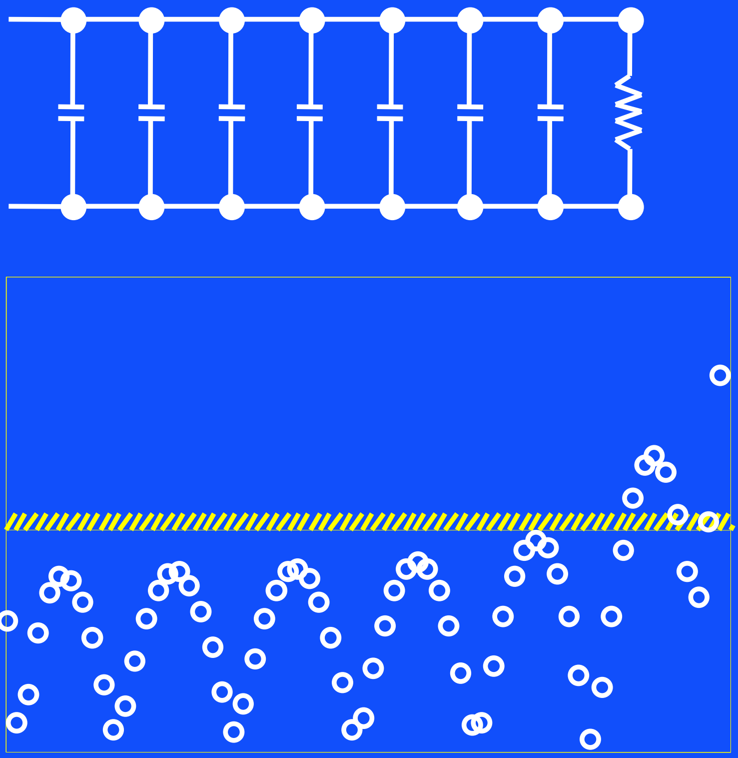
Parameter extraction: Find $P(x_f^{(0)})$

coarse model at

$$x_c^{(0)} = P(x_f^{(0)})$$

fine model at $x_f^{(0)} = x_c^*$

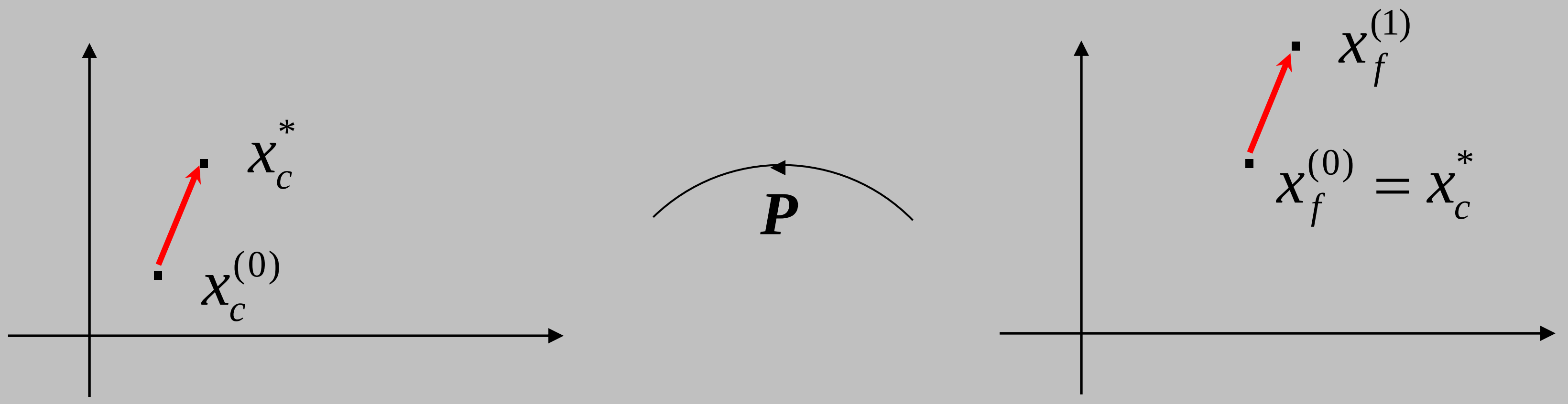




Find $x_f^{(1)}$

$$P^{(0)}(x_f) \equiv P(x_f^{(0)}) + B^{(0)}(x_f - x_f^{(0)}) \,, \qquad B^{(0)} = I$$

Solve $P^{(0)}(x_f) = x_c^{*} \quad \rightarrow \quad x_f^{(1)}$



x_c^*
$x_c^{(0)}$
P
$x_f^{(1)}$
$x_f^{(0)} = x_c^*$

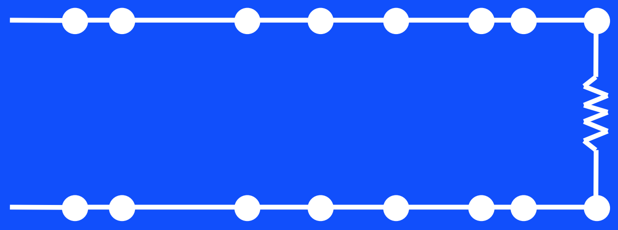
Find $x_f^{(1)}$ and $x_c^{(1)}$

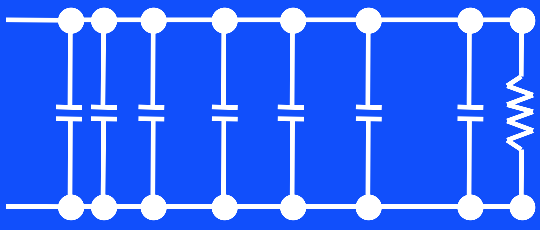
coarse model at

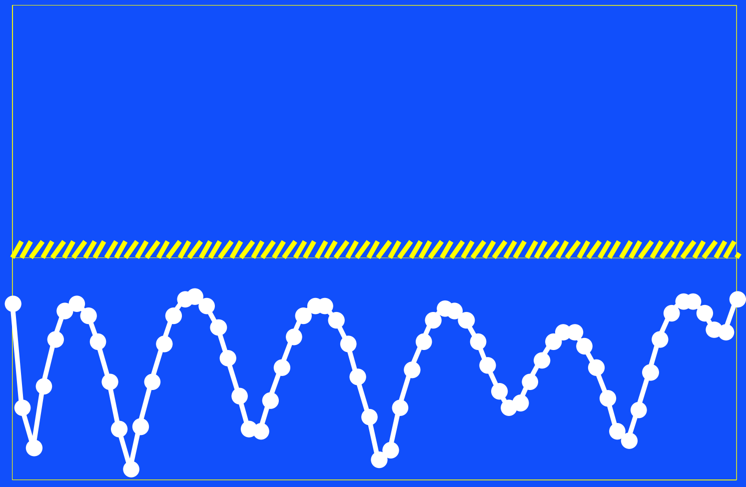
$$x_c^{(1)} = P(x_f^{(1)})$$

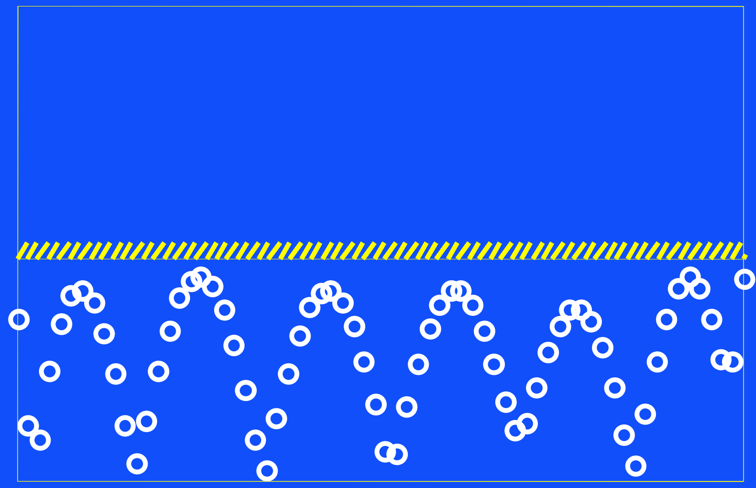
fine model at

$$x_f^{(1)}$$





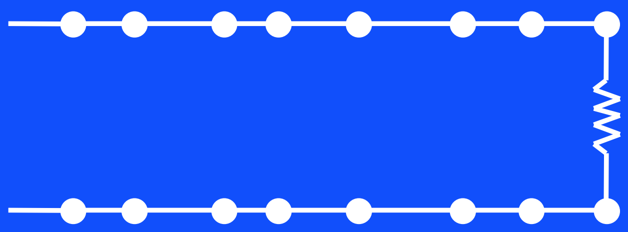


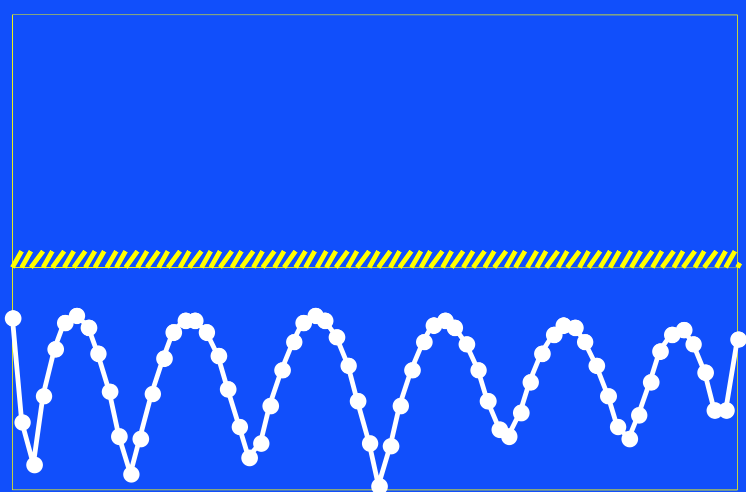


Find $x_f^{(2)}$ and $x_c^{(2)}$

coarse model at

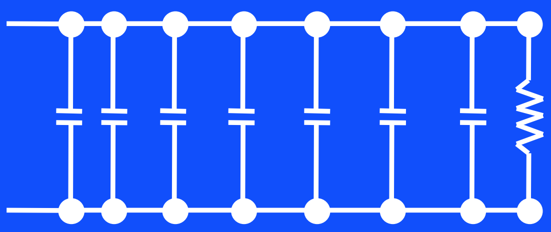
$$x_c^{(2)} = P(x_f^{(2)})$$

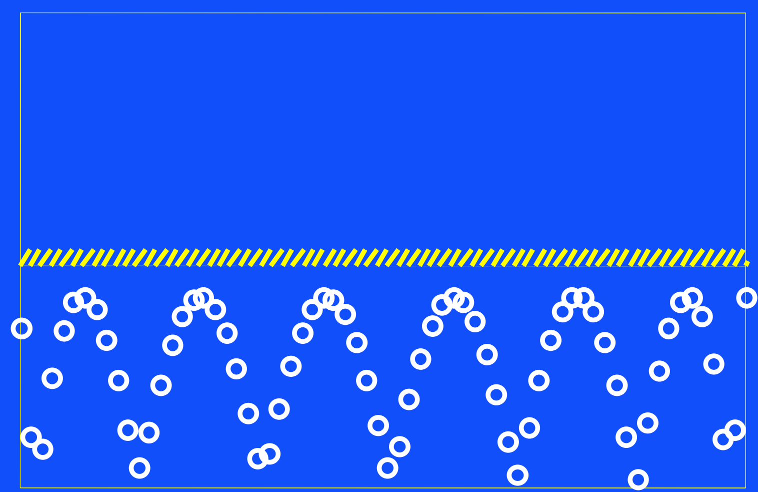




fine model at

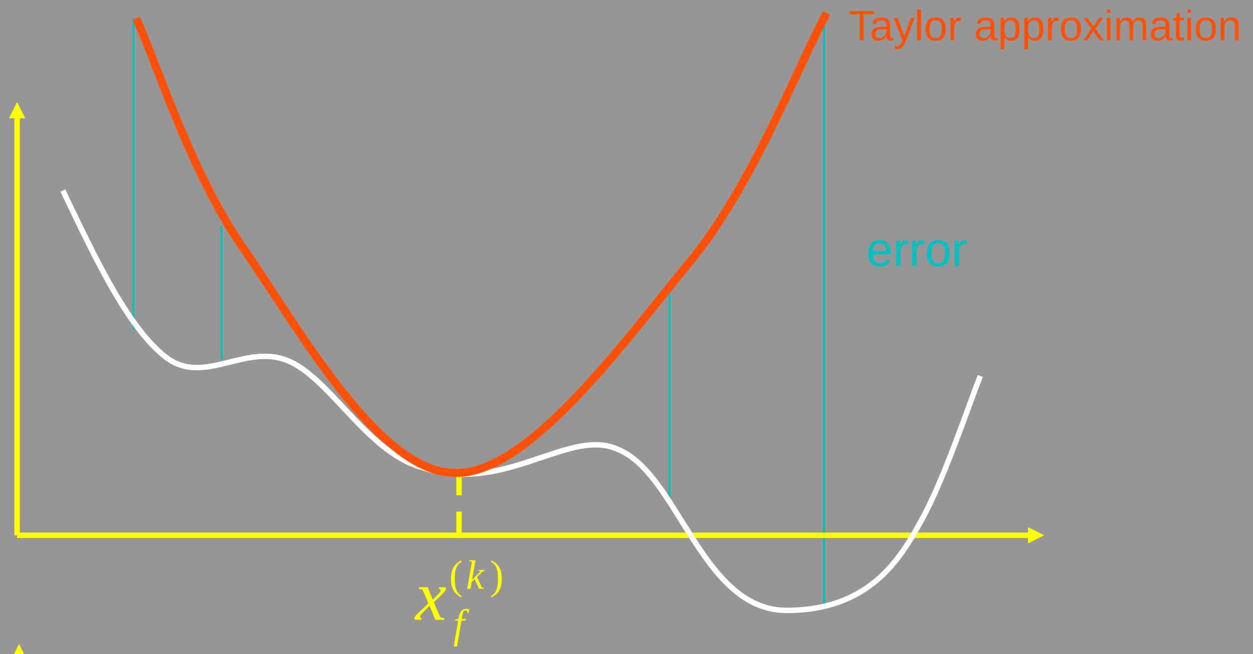
$$x_f^{(2)}$$



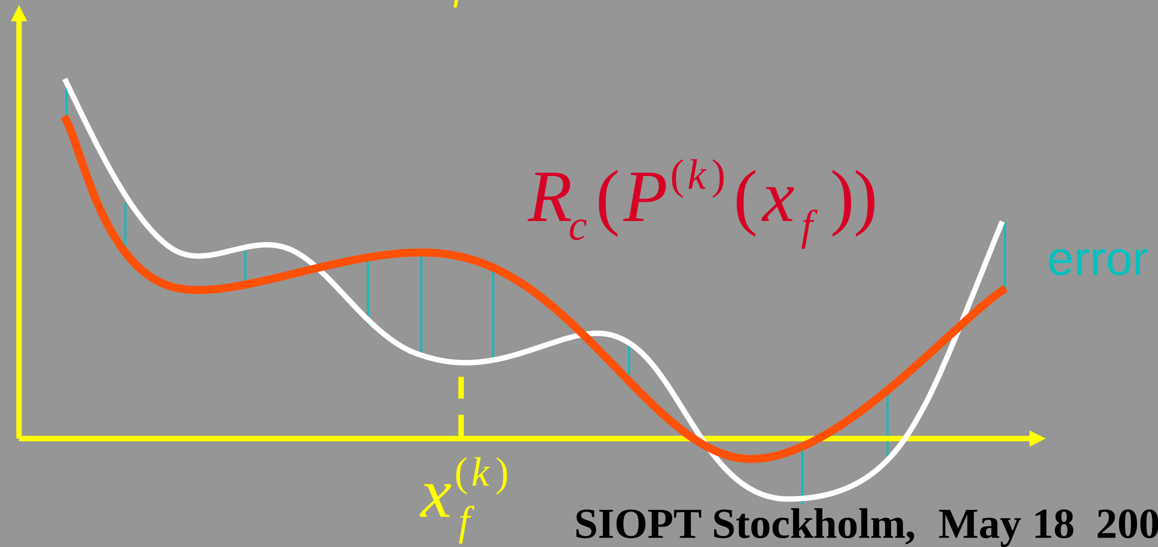


Optimization methodologies

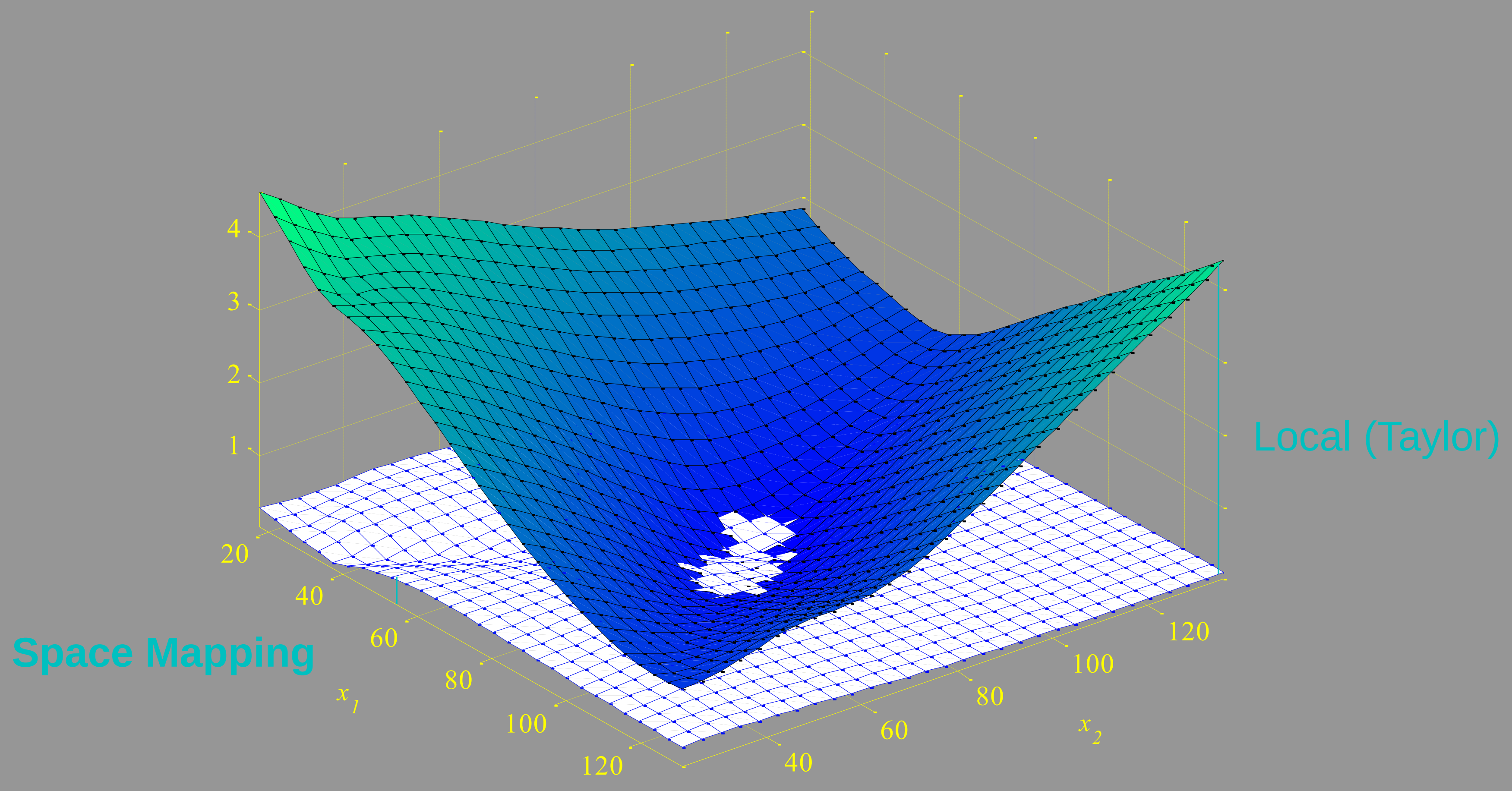
Local



Space Mapping



Approximation errors



Approximation errors

Taylor error at $x_f^{(k)}$

$$\left\| R_f(x_f) - R_f^{(k)}(x_f) \right\| \le C_{Taylor} * \left\| x_f - x_f^{(k)} \right\|^2$$

SM error at $x_f^{(k)}$

$$\left\| R_f(x_f) - R_c(P^{(k)}(x_f)) \right\| \le \varepsilon + \left\| J_c(P^{(k)}(x_f^{(k)})) \right\| * C_{SM} * \left\| x_f - x_f^{(k)} \right\|^2$$

Convergence theory

For a combination of the Space Mapping with a traditional algorithm convergence has been proved.

Traditional proof techniques can be applied.

Vicente (2003), for least squares objective.

Madsen, Søndergaard (2004), for general objective.

Convergence of Space Mapping without a safeguard.

Koziel, Bandler, Madsen (2005).

Summary

Space Mapping solves problems that have previously been considered unsolvable

Use of a coarse model may provide large iteration steps

Space Mapping may provide a good approximate solution in few iteration steps

Conditions for ultimate convergence are derived later in this session

Space Mapping for Modelling

Star Distribution for SM-based Modeling
(*Bandler et al., 2001*)

2*n*+1 points are used for a problem with *n* design parameters