Pricing, Returns, and Donations in Single- and Dual-Channel Retailing

PRICING, RETURNS, AND DONATIONS IN SINGLE- AND DUAL-CHANNEL RETAILING

BY

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To Nevichan and my parents...

Abstract

In this dissertation, we study the operational planning problem of a retailer under single- and dual-channel settings with product returns and donations considerations. It is composed of 6 chapters. Having provided the overview and motivation of this work in Chapter 1, we present a structured literature review of the bricks-and-clicks dual-channels in Chapter 2. Next, we propose a quality-dependent newsvendor problem, which models a socially responsible food-retailer's operational planning problem for a continuously deteriorating inventory over two periods with the consideration of donation and quality-sensitive customers in Chapter 3. The retailer's operational planning comprises of inventory and pricing decisions, where she plans not only for the purchase of the goods but also for donating them. We assume each unit of donation generates a constant reward derived from a blend of government incentives and the improved public image of the company due to its corporate social responsibility effort. Our results reveal that charitable donations can enhance the profit while at the same time mitigate the waste and the retailer's optimal donation volume is increasing (decreasing) in the donation reward (quality of the goods). We extend this model in Chapter 4 to incorporate a tax deduction policy into the retailer's problem and examine the impacts of quality and tax subsidy parameters on the retailer's optimal decisions. Although the retailer is still better off engaging in donations, we observe

that a larger tax subsidy (higher quality) does not always bring in more (less) donations. In Chapter 5, we develop an analytical model to help a bricks-and-clicks dual channel retailer determine the optimal price in each channel and whether to welcome the cross-channel returns to her physical facility. We find that the cross-channel returns are likely to cannibalize the physical channel sales and may hurt the retailer's profit when customer returns are highly sensitive to refund. Finally, Chapter 6 summarizes our main contributions and proposes future research directions.

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Chapter 1

Introduction

1.1 Overview and Motivation

The beginning of e-commerce can be traced back to the late nineties. Despite the dot-com collapse in 2000, brick-and-mortar retailers realized the benefits of online sales and started adopting it. Since then there has been a gradual and continuous increase in e-commerce adoption. The number of online shoppers has increased in 2020 by 9.5% to reach 3.4 billion. E-commerce revenue has increased by 25% to reach US $2.43 trillion in 2020. Revenues and users of e-commerce are expected to continue to increase in 2021 to reach US $2.7 trillion and 3.8 billion users, respectively (Skeldon, 2021). In the United States, the online retail market showed 14.9% growth while total retail sales increased only by 3.4% during the fourth quarter of 2019 compared to the same period in 2018 (U.S. Department of Commerce, 2020b). The growth in 2020 has almost tripled to reach 44% (Ali, 2021), the largest increase in the last two decades, which is largely due to the COVID-19 pandemic lockdown conditions. Moreover, e-commerce business constituted 14.3% of total sales in 2020 which was only 6.4% in 2015 (U.S. Department of Commerce, 2016, 2020a)

The proliferation of e-commerce has reshaped the retail market in most industries. As a result of intensified competition, many traditional bricks-and-mortar retailers expanded their operations to the online market and have become bricks-and-clicks. On the other hand, some e-tailers responded by establishing a conventional channel or forming a partnership with traditional retailers. For example, in August 2017, Amazon bought Whole Foods to extend its operations through physical stores to strengthen its position in the market. The increasing prevalence of corner pick-up and mobile stores are making the competition even tougher among retailers (Brynjolfsson *et al.*, 2013). These developments have meant that operating in both e-commerce and conventional retail markets in tandem has its unique challenges.

With this new era of commerce, dual-channel businesses not only have faced new challenges but also have discovered new synergies in operating both online and offline channels in tandem. For example, many dual-channel firms nowadays utilize their physical facilities to enhance delivery/return operations in their web store. However, such cross-channel activities necessitate a partial integration of the channels, which is likely to disturb one or both of the channels.

The mainstream dual-channels literature can be divided into two categories: (1) a novel competition between a manufacturer (or a supplier) and a traditional retailer where either of the players (mostly the manufacturer) considers establishing an online channel (see, for example, Chiang *et al.*, 2003; Tsay and Agrawal, 2004; Huang *et al.*, 2018b), and (2) inventory management and/or demand fulfillment policies (see, for example, Mahar *et al.*, 2009; Liang *et al.*, 2014; Ishfaq and Bajwa, 2019). In fact, the operational planning problem of a dual-channel retailer has received little attention from the field until recently (see, for example, Yan *et al.*, 2018a; Radhi and Zhang, 2019; Nageswaran *et al.*, 2020). Interestingly, despite the two decades of accumulated research, there are no recent comprehensive literature reviews. The extant reviews offer only a fragmented view of the literature as they merely focus on the distribution logistics (Melacini *et al.*, 2018; Kembro *et al.*, 2018). Hence, in this thesis, we address these research gaps by offering an analytical model that analyzes a dual-channel retailer's optimal pricing and return policies, and a structured literature review of the bricks-and-clicks dual-channels.

On the other hand, global demand for food, which has been gradually growing at an average rate of 3.6% per year since 2012, generated a total revenue of US $7 trillion in 2019 (Frimpong, 2020). Nielsen (2013) reported that fresh food accounts for about 30–60% of all food, grocery and personal care sales in the world. Unfortunately, a significant portion of the food supply chain goods are wasted. The Food and Agriculture Organization of the United Nations (FAO) indicated that, approximately 1.3 billion tons of food (one-third of the total produced) is wasted globally every year (Gustavsson *et al.*, 2011). According to the U.S. Department of Agriculture (2017), 30-40% of the total food supply is thrown away each year. In Canada, retailers were responsible for 10% of the total annual food loss of $3.1 billion Canadian dollars in 2014 (Gooch and Felfel, 2014).

Sadly, at the same time there are millions of people who are struggling to maintain a healthy diet and rely on food-banks. For example, United Nations estimated that 690 million suffered from hunger in 2018 (FAO, 2020). To make matters worse, any excess food that ends up in a landfill, rather than a food-bank, leaves a carbon footprint and wastes valuable natural resources, such as freshwater and energy, that are used in production (Hall *et al.*, 2009). The environmental impact of landfills is so vast that, if food waste was a country it would be the third-largest greenhouse gases (GHGs) emissions generating country, coming after China and the U.S. (FAO, 2015). Thus, beside its potential for an economic advantage, better management of perishable goods can also contribute to social and environmental welfare. While there are multiple causes of food waste, 64% of store waste is due to operational practices (Food Marketing Institute and Retail Control Group, 2012).

To address these challenges, some governments offer tax relief to retailers who donate food. However, the retailers often tend to see donation as an opportunity to salvage low-quality inventory, whereas, the food-banks prefer goods that will stay fresh during the distribution, since they do not have the resources and capabilities to manage inventories. Besides, there is a lack of research on charitable donations (Alexander and Smaje, 2008; Giuseppe *et al.*, 2014; Chu *et al.*, 2018). Thus, this thesis proposes two analytical models to contribute to this body of the literature.

1.2 Summary of Contributions and Organization of the Thesis

This thesis studies operational planning optimization under single- and dual-channel frameworks with a special emphasis on returns and inventory. With the issue of inventory waste, we consider the opportunity of donating the products to charities as a mitigation strategy.

Given the growth in the literature and the absence of a comprehensive literature review on bricks-and-clicks dual channels, we devoted Chapter 2 to present a structured literature review of the topic. When necessary additional relevant literature will be reviewed in other chapters. Our review covers more than 260 published contributions, develops a comprehensive outlook of the field, provides a systematic discussion of common demand modeling functions, and analyzes the reviewed literature, identifies recent research trends and opportunities, and illustrates how existing research can be used to address up-to-date challenges in the industry.

In Chapter 3, we study a socially responsible food-retailer's operational planning problem for a continuously deteriorating inventory over two periods with the consideration of donation and quality-sensitive customers. We develop an optimization model that incorporates a retailer's corporate social responsibility effort, in the form of charitable donations, and makes use of the internet of things (IoT)-enabled condition tracking technologies, such as time-temperature indicators data, to accurately estimate the effective (true) quality of the goods and its impacts on consumer demand. We formulate a quality-dependent newsvendor problem (QDNP) to determine the stocking quantity and the first period price of the goods at the beginning of the selling season, and the second period price and donation policy at the end of the first period. The optimal donation policy at the end of the first period depends on the quality (time to expiration), on-hand inventory, and the per unit reward of donation. Specifically, the retailer is more willing to donate when the due date is near, on-hand inventory is high, and/or per unit reward is large. Moreover, for a given inventory level, expected food waste is always greater in the absence of donations. QDNP outperforms the no-donation model, particularly when the uncertainty is high and/or the length of the second period is short. Interestingly, the two models react to an increase in uncertainty oppositely: QDNP orders more to avoid possible shortage expenses in the future as a part of the inventory can be donated if the first period demand turns out to be low, whereas, the no-donation policy orders less to avoid possible disposal costs at the end of the selling season.

In Chapter 4, we analyze a similar problem to the one in Chapter 3. However, this time, we incorporate the actual U.S. government tax deduction policy for food donations into the retailer's after-tax profit function and analyze the impact of the tax subsidy parameters on the retailer's optimal decisions. Our analysis revealed that as opposed to the conventional wisdom, the retailer's optimal donation volume may decline with respect to the amount of leftovers at the end of period 1, their effective quality and the tax incentive coefficients. Such unorthodox findings arise as a result of the government's tax deduction being tied to the retailer's second period price. Moreover, we observe that the enhanced tax deduction benefits the retailer most when the degree of uncertainty is high. Finally, donations trigger only a slight increase in price while significantly increasing the stocking quantity.

In Chapter 5, we develop a stylized model where a dual-channel retailer, who sells a single product through an online channel as well as a bricks-and-mortar store, wants to determine the optimal price in each channel and whether to welcome the cross-channel returns to her physical facility. We explicitly model the demand and returns as a function of channel prices and investigate the effects of cross-channel returns on individual channel-prices, -demand, and -profits as well as the firm's overall profit. In particular, unexpectedly, the retailer may still mark up the bricks-and-mortar (B&M) price, despite a drop in the in-store demand due to the cannibalization effect. Although the cross-channel returns may increase the overall demand, it is not assured the firm is better off offering this service. Unless the growth in online customers compensate the retailer for the enhanced returns and cannibalized B&M sales, she should not accept cross-channel returns.

Finally, we highlight the major contributions of this dissertation and suggest directions for future research in Chapter 6.

Chapter 2

A Review of Bricks-and-Clicks Dual-Channels Literature: Trends and Opportunities

2.1 Introduction

Not surprisingly, the ongoing developments in the industry have been reflected in the academic literature, with increasing interest during the last two decades. However, two recent reviews (Kembro *et al.*, 2018; Melacini *et al.*, 2018) are limited in scope and depth: they focus on fulfillment and distribution issues only and they cover a limited portion of the literature, less than 60 journal papers versus more than 260 papers in this review. There is a lack of a structured review that provides a comprehensive overview of the bricks-and-clicks dual-channels literature. Therefore, our goal in this chapter is to review the extant literature, classify it, consolidate its findings, and identify a future research agenda. In doing so we incorporate studies that have strategic as well operational perspectives, recognize recent research trends, and identify the gaps between theory and practice.

We have found two prominent inquiries driving the dual-channels literature: (1) channel competition and (2) inventory management and demand fulfillment. The former accounts for more than 80% of the literature. We further classify the first stream of research based on demand functions due to the abundance of the contributions, but also briefly mention the salient papers extending the basic quantitative models and/or studying a unique dual-channel phenomenon, such as free-riding. As the second stream of research includes much fewer contributions, we are able to present an in-depth analysis, where we classify the studies based on the key characteristics, such as the decision(s) to be made, the methodology used to model the problem, the number of products considered, etc., as well as the demand function formulations.

Unlike in the industry, the term dual-channel has been used interchangeably with other multiple channel concepts to define different channel structures. Thus, one of our other objectives in this survey is to offer an ontology for the literature on dual-channel supply chains in Section 2.2. The remainder of this chapter is organized as follows: We discuss our survey methodology and present an overview of the literature in Section 2.3. In Sections 2.4 and 2.5, we summarize the major research questions and methodologies used. Finally, in Section 2.6, we identify major literature gaps and recent research trends, suggest future research themes, and give an example on how the existing research can be used to address those gaps/trends.

2.2 Concepts of Multiple-Channel Retailing

There does not seem to be a consensus on the definitions of the concepts multi-, cross-, and omni-channel. These concepts and some others (such as mixed-channel) have been used interchangeably by OM scholars, though they, in fact, refer to different operational structures. In line with Beck and Rygl (2015), we shall use the term multiple-channel as the most general definition of reaching out to the end-consumers using at least two alternate ways.

Dual-channel is one of the most popular multiple-channel concepts along with multi-, cross-, and omni-channel. To define these concepts, we revisit the basic definition of a 'channel'. Neslin *et al.* (2006) define a channel as *"a customer contact point, or a medium through which the firm and the customer interact."* The term 'interact' implies a two-way communication and thus excludes one-way communication channels such as TV advertisement. According to this definition, physical stores, catalogues, telephone, TV, online, and mobile shopping are examples of the most popular channels (Beck and Rygl, 2015).

In a multi-channel setting, all channels are managed separately. This separation is not only in the products and/or services provided and prices but also in the data flow and management such as that of inventory and consumer relationship management (CRM) data. For example, an online shopping coupon cannot be redeemed in traditional stores and a product bought from one channel cannot be returned to another (Beck and Rygl, 2015). In contrast, in a cross-channel environment either more than one, but not all channels, are partially or fully integrated, or all channels are partially integrated. An example of a cross-channel practice is when an in-store purchase provides a coupon that can only be redeemed online.

Finally, omni-channel retailing aims to create a seamless retail environment in which consumers can shop across all channels anywhere and at any time. The creation of this business model has been facilitated by the new retailing landscape in which borders between channels have been blurred due to easier and faster access to information (Brynjolfsson *et al.*, 2013). Within this context, a channel can involve every intermediary that retailers use to sell their products or to reach customers, e.g., online catalogues, flyers, television and mobile marketing (Verhoef *et al.*, 2015; Bertulli, 2014). With this extended meaning, customer-to-customer interaction points, such as social media, can also be referred to as channels since they contribute to brand management. Such integrated channels help retailers engage in nascent buying behaviors of consumers such as free-riding, also called research shopping, where a customer searches for a product in one channel and buys it from another. Retailers can track their customers more closely by consolidating CRM information coming from all channels and thereby better manage their demand and sales. For instance, a retailer can use its mobile application to offer a discount, that is valid for 24-hours in all channels, to the customers who are close to a specific physical store. Similarly, real-time inventory and price information of nearby store locations can be shared with customers through a mobile application. In an omni-channel setting, a customer can return her product to any store regardless of where she bought it (Beck and Rygl, 2015). This is not the case in classical multi-channel settings.

It is important to distinguish between omni-channel and cross-channel concepts as they are often treated as if they were referring to the same channel setup. For example, all O2O operations, such as in-store pickup option of online orders from the company website or mobile application are a part of either cross-channel or omni-channel practices. However, although many cross-channel companies provide O2O services, they do not price every product the same across all channels, or they offer more customization/wider assortment options in one channel. Thus, it depends on the degree of integration among channels to determine whether a firm adopts a cross-channel or an omni-channel strategy. Table 2.1 briefly summarizes the multiple-channel concepts.

Multi-Channel	independent channels
Cross-Channel	partial (or full) integration among up to n (or $n-1$) channels
Omni-Channel	fully integrated channels

Table 2.1: Multiple-channel retailing concepts.

Even though dual-channel retailing can be placed under any of the aforementioned concepts, it is often used to refer to a multi-channel structure in the Operations Management (OM) literature as the majority of the papers in this area attempt to optimize vertical competition between a manufacturer (or a supplier) and a traditional retailer with the manufacturer introducing a direct Internet channel. Moreover, it is worthwhile to note that the term 'channel' also includes composite media. For instance, Levi's is selling its products through many department stores, such as Hudson's Bay and Sears, all of which are considered as one channel to reach the end-customers. Meanwhile, Levi's is also operating individually-owned outlet stores, all of which can be considered as another channel.

2.3 Research Methodology

We followed the structured literature review (SLR) paradigm to locate the relevant contributions. It suggests a systematic, transparent, and reproducible approach to conduct a review and thereby minimizes bias (Tranfield *et al.*, 2003; Denyer and Tranfield, 2009). SLRs have been widely published in many fields to develop/consolidate knowledge on emerging areas and the recent literature surveys on multiple-channel supply chain logistics are representative examples (see, e.g., Melacini *et al.*, 2018). The present study follows a common 5-step guideline, which is also suggested by Denyer and Tranfield (2009): (1) formulating research questions, (2) locating studies, (3) study selection and evaluation, (4) analysis and synthesis, (5) reporting findings.

2.3.1 Formulating Research Questions

As already mentioned, existing dual-channel reviews focus on e-fulfillment logistics and thereby offer a fragmented overview of the literature. The present review, however, follows a more holistic approach to analyze the bricks-and-clicks dual-channel supply chains phenomenon. Hence, we aim to address the following research questions in this survey:

RQ1. What are the mainstream research themes in bricks-and-clicks dual-channel supply chains literature?

Our goal is to cover both strategic (such as competition studies) and operational studies (such as inventory and demand fulfillment).

RQ2. What is a future research agenda for bricks-and-clicks dual-channel supply chains literature?

We develop a research agenda based on studying gaps that have been identified in the reviewed literature as well as from our own comprehensive analysis of the same.

2.3.2 Locating Studies

An imperative part of conducting a SLR is to locate and retrieve the studies pertinent to the research questions (Denyer and Tranfield, 2009). We selected Web of Science (WoS) Core Collection as our research database due to its vast repository, rigorous journal selection process, and commonality in such reviews. Moreover, WoS Core Collection ensures a certain level of quality due to its journal selection criteria and periodic update of the list (Web of Science, 2020).

While determining the keywords, we relied on the results of an initial research, in which the keywords 'dual channel supply chain' and 'dual channel retailing' were searched for years from 2000 to 2019 within Operations Research-Management Science category. The choice of the start of study period corresponds with the time when bricks-and-clicks dual channel started being used in practice. Such pilot reviews are suggested for SLRs to improve the quality of the findings (Thomé *et al.*, 2016; Kembro *et al.*, 2018). As reported in Section 2.2, we realized an absence of consensus on definitions of the multiple-channel concepts. Therefore, we decided to include the keywords: "multiple channel", "multi channel", "cross channel", "omni channel" , and "omnichannel". It is worth noting that we did not use too specific keywords, such as "closed-loop dual channels", "O2O logistics", or "returns in omni channels" as our methodology has proven to be sufficient in locating relevant studies.

2.3.3 Study Selection and Evaluation

Because the initial research retrieved a large pool of contributions, we narrowed our focus to published articles that used a quantitative approach, which are sufficient to address the research questions. We decided on which papers to include by reading the abstracts and conclusions. In particular, we looked for publications that fit in the bricks-and-clicks dual-channel context, in which at least one conventional channel and one online channel operate in tandem. Other dual-channel settings, such as dual reverse supply chains (Feng *et al.*, 2017) and dual sales channels without an e-channel (Chen and Bell, 2012; Feng *et al.*, 2018; Niu *et al.*, 2019b), are not within the scope of this study. Although some modeling frameworks in bricks-and-clicks dual-channels may align with those in other dual-channel or multiple-channel contexts, operating in both e-commerce and conventional retail market in tandem has its unique characteristics and challenges. Table 2.2 depicts the keywords and the Boolean-logic used to retrieve relevant studies.

Keywords and Boolean-logic
dual channel supply chain **OR** dual channel retailing **OR** dual channel competition **OR** dual channel demand fulfillment **OR** multiple channel supply chain **OR** multiple channel competition **OR** multiple channel demand fulfillment **OR** multi channel supply chain **OR** multi channel competition **OR** multi channel demand fulfillment **OR** cross channel supply chain **OR** cross channel competition **OR** cross channel demand fulfillment **OR** omni channel supply chain **OR** omni channel competition **OR** omni channel demand fulfillment **OR** omnichannel supply chain **OR** omnichannel competition **OR** omnichannel demand fulfillment

Table 2.2: Keyword search results on WoS Core Collection from 2000 to 2019 within the OR-MS category.

Our search resulted in 319 contributions; after eliminating the irrelevant studies, 234 papers were found from WoS and through cross-referencing we retrieved 29 more papers. Thus, in total, we have reviewed 263 articles published between 1998 and 2019. As can be seen from Figure 2.1, the number of publications in this field has been increasing significantly in the last decade.

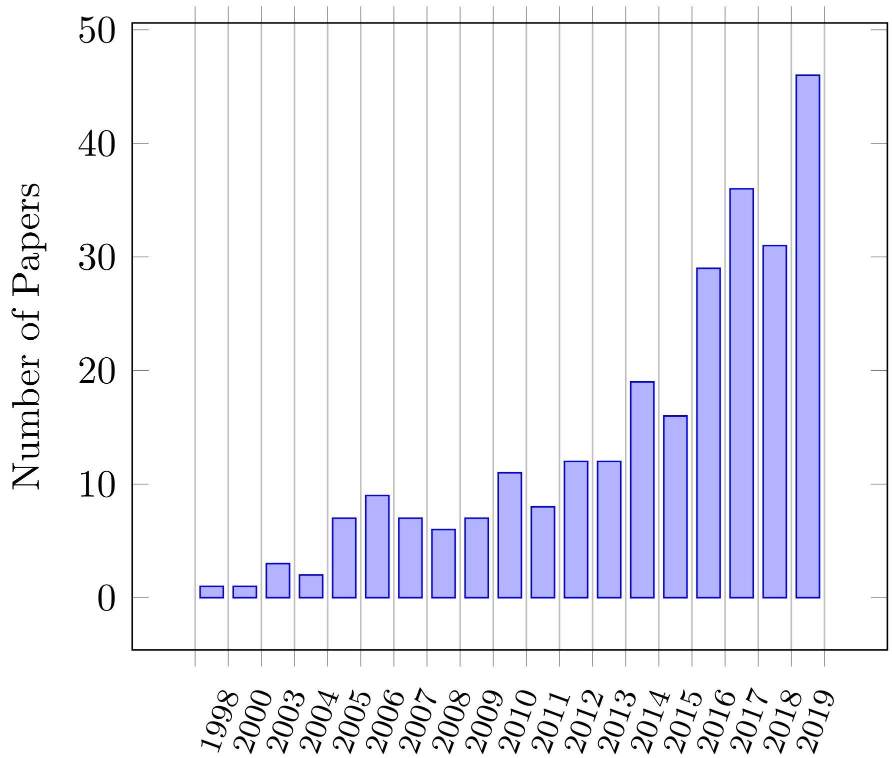

Figure 2.1: Time distribution of reviewed papers.

2.3.4 Analysis and Synthesis

At this point, all papers included in this study were reviewed thoroughly and all relevant data were entered into an Excel spreadsheet. In the spreadsheet, the articles were grouped by their research theme, problem dynamics (number of periods, products, channels, etc.), demand formulation, solution methodology, and future research suggestions as well as their basic characteristics (publication year, journal name, etc.). The initial review had been very useful for determining the defining characteristics of the papers as well as categorizing them on different relevant dimensions.

2.3.5 Reporting Findings

Once all contributions were analyzed, findings were consolidated to address the research questions. We present an overview of our findings here and elaborate on them in the subsequent sections.

Overall, 81% of the papers study a single-period problem while 65% develop a deterministic model. Moreover, 79% of all articles use game theory models of which 80% are two-player game models. These statistics indicate the presence of common research trends. We have found two prominent inquiries in bricks-and-clicks supply chain literature. The first stream of research investigates a novel competition between a manufacturer (or a supplier) and a traditional retailer where either of the players (mostly the manufacturer) considers establishing an online channel. The second line of research investigates inventory management and/or demand fulfillment policies.

Based on our literature review, the first stream includes more than 80% of the reviewed papers. One reason for the prominence of this topic is that channel competition has been attracting many researchers from the marketing literature as well. In fact, research on channel distribution and coordination lies at the interface of OM and Marketing, and one of the earliest papers was published in a marketing journal (Balasubramanian, 1998). Another reason for the commonality of this subject is its richness and broadness, so that it is possible to examine a variety of scenarios under different assumptions. For instance, the interaction between the channels can be examined from diverse angles by assuming different decision variables, such as price, ordering quantity, service effort, low carbon emission investment, or a combination thereof. In the next two sections we delve deeper into each topic and recognize overlapping frameworks and research trends.

2.4 Channel Competition in Dual-Channel Supply Chains

Channel competition is one of the main research themes in the bricks-and-clicks supply chain literature. In the most common modeling framework, a manufacturer/supplier decides to introduce an internet-enabled direct channel while there is already an existing channel between the same manufacturer/supplier and a retailer. Because the decision makers not only compete over the wholesale price, but also over the end customers' demand, an unconventional supply chain competition (both horizontal and vertical) arises (see, e.g., Chiang *et al.*, 2003; Tsay and Agrawal, 2004; Mukhopadhyay *et al.*, 2008a; Xiao *et al.*, 2014; Huang *et al.*, 2018b). The main goal is to develop a better understanding of the benefits and costs to the manufacturer's e-commerce initiative for both parties as well as customers. However, we also include papers investigating *bricks versus clicks* channel competition without a game theory perspective. Such papers are sparse in numbers and often model a dual-channel retailer (see, e.g., Yao and Liu, 2003; Hu and Li, 2012; Carrillo *et al.*, 2014; Zhang and Wang, 2017; Du *et al.*, 2019).

Alternative supply chain structures, such as a supplier with a direct channel and multiple retailers (Lei *et al.*, 2014), a wholesale manufacturer with a retailer having an online presence (Hsiao and Chen, 2014), and the competition between two dual-channel supply chains (Jamali and Rasti-Barzoki, 2018) were studied under various assumptions. In the basic model, supply chain members play a Stackelberg or a Nash game to make their respective decisions which can be one or a combination of price, order quantity, and service effort. Moreover, players are assumed to have symmetric information and to be risk-neutral. The main focus of these papers is to study the impacts of the online channel on the ongoing business.

The basic model is often extended in two ways: by relaxing the assumptions of symmetric information or risk-neutrality of players (Cao *et al.*, 2013; Liu *et al.*, 2016; Huang *et al.*, 2018a). In addition to theoretical extensions, there have also been some application extensions in the areas of sustainability, remanufacturing and coordination (Cao, 2014; Carrillo *et al.*, 2014; Yan *et al.*, 2015; David and Adida, 2015; Li *et al.*, 2016a; Xie *et al.*, 2017; Xu *et al.*, 2018). One interesting marketing application area is cross-selling, or free-riding, where a customer finds a product in one channel, but purchases it from the other (Bernstein *et al.*, 2009; Balakrishnan *et al.*, 2014; Dan *et al.*, 2014; He *et al.*, 2016; Pu *et al.*, 2017; Zhou *et al.*, 2018). In a novel work, Salmani *et al.* (2018) studies the optimal investment allocations, where channels compete over a limited funding, instead of end-customers.

We group this body of the literature by demand formulations due to the vast number of publications. It is also worth noting that certain demand modeling approaches often lead to similar conclusions. As the conventional supply chain structure is complemented by a new (online) channel, traditional demand functions fall short of capturing the true nature of retail operations in a dual-channel context. To cope with this issue, two main approaches have been adopted in the literature: extending the existing demand functions or establishing novel ones. Due to the additional complexity imposed by the introduction of an e-channel, about 74% of the studies reviewed use deterministic demand models while the rest uses either fuzzy, probabilistic, or stochastic approaches. The focus on deterministic demand may be explained by the nascence of the field where pioneer studies often start with simple settings. In the remainder of this section we discuss the prevailing demand models that were used to model channel competition. We focus on deterministic formulation schemes as they significantly outnumber the stochastic ones and also as many studies consider demand uncertainty by extending a deterministic model.

2.4.1 Extensions of a Simple Linear Demand Function

One of the most widely used and arguably the simplest form of demand (in the traditional bricks-and-mortar context) is defined as $D(p) = a - bp$ where $a > 0$ is the potential market size, $b > 0$ is the price sensitivity parameter and $0 \leq p \leq a/b$ is the price (Mills, 1959; Petruzzi and Dada, 1999). For a detailed survey of demand functions used in a conventional supply-chain settings, we refer to Huang *et al.* (2013a).

To account for the addition of an e-channel, the linear demand function has been extended to $D_i(p_i, p_j) = a_i - \hat{b}_i p_i + \beta_i(p_j - p_i)$, where $i, j \in \{r, e\}$ and $i \neq j$ and the subscripts r and e denote traditional retail and online channels, respectively. As in the single-channel model, the terms $a_i > 0$, $\hat{b}_i > 0$ and $0 \leq p_i \leq (a_i + \beta_i p_j)/(\hat{b}_i + \beta_i)$ stand for market share, self-price sensitivity and price in channel i, respectively. The cross-price sensitivity parameter of channel $i \in \{r, e\}$ is denoted as β_i. It is assumed that self-price sensitivities are always prominent, i.e., $0 \leq \beta_i \leq \min_{i \in \{r,e\}} \hat{b}_i$, $i \in \{r, e\}$. This assumption also plays an important role for proving the concavity of the objective function. One can write a more compact version of the linear demand function for dual-channels with some simple algebraic manipulations as,

$$D_i(p_i, p_j) = \theta_i a - b_i p_i + \beta_i p_j, \quad i, j \in \{r, e\}, \quad i \neq j, \tag{2.1}$$

where $a = a_i + a_j$ is the aggregate market size and $0 \leq \theta_i \leq 1$ denotes the market share of channel i (which means $\theta_i = a_i/a$). Also, we have an updated self-price sensitivity parameter $b_i = \hat{b}_i + \beta_i$. Equation (2.1) is often considered with symmetric self- and/or cross-price sensitivity parameters. In Table 2.3 we list the papers that used demand form (2.1) and indicate what factors are considered in demand modeling.

Note that the above formulation is also used to model demand in other supply chain frameworks employing two distribution channels. For example, Tsay and Agrawal (2000) use it to model a situation where a manufacturer distributes its products through two independent retailers. On the other hand, price is often not the only driver of demand. Depending on the product's characteristics, other factors, such as service level (offered either in the offline channel or in both channels) and advertisement investment can be considered with their associated self- and cross-sensitivity parameters. In our classifications, we include modifications that may deviate slightly from demand form (2.1). For example, Tsay and Agrawal (2004) fractionate aggregate demand rather than the market size while considering service effort as the only driver of demand in both channels.

Given the commonality of this classical demand-price functional form, it is not surprising that the majority of the studies adopted a demand function that depends solely on price. A hybrid function where demand is influenced not only by price but also by service received significant attention as well. In this context, the term service includes a variety of bargains offered by companies such as in-store assistance, warranty, and refund policy. Delivery lead time (in the internet-enabled channel) and eco-friendliness issues have been the focus of some recent studies. In another recent work, Zhou and Ye (2018) model price competition as in (2.1) and incorporate low

Papers	Price	Service	Lead Time	Eco-friendliness
Park and Keh (2003); Yao and Liu (2005); Yue and Liu (2006); Kurata *et al.* (2007); Cai *et al.* (2009); Huang and Swaminathan (2009); Yan (2011); Chen *et al.* (2012); Ryan *et al.* (2012); Huang *et al.* (2012); Niu *et al.* (2012); Huang *et al.* (2013b); Hong *et al.* (2013); Cao *et al.* (2013); Carrillo *et al.* (2014); Hsieh *et al.* (2014); Feng and Geunes (2014); Xu *et al.* (2014); Lei *et al.* (2014); Panda *et al.* (2015); Shang and Yang (2015); Lu and Liu (2015); Zhang *et al.* (2015); Soleimani *et al.* (2016); Soleimani (2016); Chen *et al.* (2016b); Saha (2016); Saha *et al.* (2016); Yan *et al.* (2016); Ding *et al.* (2016); Li *et al.* (2016b); Wang *et al.* (2016a); Giri *et al.* (2017); Zhang and Wang (2017); Matsui (2017); Zhao *et al.* (2017); Dennis *et al.* (2017); Chen *et al.* (2017b); Jafari *et al.* (2017); Saha *et al.* (2018b); Lu *et al.* (2018); Moon *et al.* (2018); Kim and Chun (2018); Zhou *et al.* (2019); Raza and Govindaluri (2019); Wang *et al.* (2019b)	X			
Tsay and Agrawal (2004); Dan *et al.* (2014); Li *et al.* (2017a); Dan *et al.* (2018); Yang *et al.* (2018b)		X		
Kumar and Ruan (2006); Mukhopadhyay *et al.* (2008b,a); Dan *et al.* (2012); Tsao and Su (2012); Lu and Liu (2013); Chen *et al.* (2013); Chen (2015); Giri and Roy (2016); Li and Li (2016); Liu *et al.* (2016); Pu *et al.* (2017); Wang *et al.* (2017); Xie *et al.* (2017); Jiang *et al.* (2017); Zhou *et al.* (2018); Li *et al.* (2019c,b); Noori-daryan *et al.* (2020); Pathak *et al.* (2020); Li *et al.* (2019e)	X	X		
Hua *et al.* (2010); Saha *et al.* (2018a); Modak and Kelle (2019)	X		X	
Li *et al.* (2016a); Jamali and Rasti-Barzoki (2018); Heydari *et al.* (2019); Wang *et al.* (2020)	X			X
Ji *et al.* (2017b)	X	X		X

Table 2.3: Papers using an extension to the simple linear demand model.

carbon emission efforts in a multiplicative manner.

2.4.2 Consumer Valuation Models

In this section we review utility-based demand functions. The overlapping formulations can be categorized into three groups: 1) Vertical differentiation models, 2) Horizontal differentiation models, 3) A quadratic utility function based models. However, hybrid (both horizontal and vertical) or some sophisticated utility structures are also used, albeit less commonly. In Table 2.4, we present the list of papers reviewed for each category.

Model	**Papers**
Vertical differentiation	Chiang *et al.* (2003); Fruchter and Tapiero (2005); Yan (2008); Dumrongsiri *et al.* (2008); Yan and Pei (2009); Bernstein *et al.* (2009); Moon *et al.* (2010); Jiang *et al.* (2010); Yan and Ghose (2010); Yan and Pei (2011); Ma *et al.* (2013); Moon and Yao (2013); Ren *et al.* (2014); Hsiao and Chen (2014); Yan *et al.* (2015); Wang *et al.* (2016b); Luo *et al.* (2016); Ha *et al.* (2016); Niu *et al.* (2017); Zhang *et al.* (2017b); Ji *et al.* (2017a); Chen and Chen (2017); Gan *et al.* (2017); Zhang *et al.* (2017c); Li *et al.* (2018c,a); Wang *et al.* (2018a); Ji *et al.* (2018); Yang *et al.* (2018c); Li *et al.* (2018b); Wang *et al.* (2018b); He *et al.* (2019); Li *et al.* (2019a); Yang and Tang (2019); Xu *et al.* (2021); Yan *et al.* (2020a); Liu *et al.* (2019b); Wang *et al.* (2019a); Shao (2020); Liu *et al.* (2019a)
Horizontal differentiation	Balasubramanian (1998); Chun and Kim (2005); Cattani *et al.* (2006); Liu *et al.* (2006); Liu and Zhang (2006); Yoo and Lee (2011); Ofek *et al.* (2011); Chun *et al.* (2011); Jeffers and Nault (2011); Xiao *et al.* (2014); Li *et al.* (2015a); Cao *et al.* (2016); Xiao and Shi (2016); He *et al.* (2016); Xia *et al.* (2017); Zhang *et al.* (2019a); Li *et al.* (2019d); Nault and Rahman (2019); Fan *et al.* (2021)
Hybrid models (both vertical and horizontal)	Shi *et al.* (2013); Cao *et al.* (2016)
Quadratic utility or inverse demand formulation	Arya *et al.* (2007); Cai (2010); Zhang *et al.* (2012); Xiong *et al.* (2012); Hsiao and Chen (2013); Pei and Yan (2013); Arya and Mittendorf (2013); Cao (2014); Li *et al.* (2014b, 2015d); David and Adida (2015); Yang *et al.* (2015); Li *et al.* (2015c); Amrouche and Yan (2016); Matsui (2016); Li *et al.* (2016c); Yoon (2016); Abhishek *et al.* (2016); Chen *et al.* (2017a); Zheng *et al.* (2017); Qing *et al.* (2017); Liu *et al.* (2017); Yan *et al.* (2018b); Tang *et al.* (2018); Huang *et al.* (2018a); Xu *et al.* (2018); Lai *et al.* (2018); Yang *et al.* (2018a); Huang *et al.* (2018b); Yang *et al.* (2018d); Zhang *et al.* (2019b); Feng *et al.* (2019); Guan *et al.* (2020); Chen *et al.* (2019); Nie *et al.* (2019)
Other valuation models	Chen *et al.* (2008); Bernstein *et al.* (2008); Khouja *et al.* (2010); Khouja and Wang (2010); Balakrishnan *et al.* (2014); Luo and Sun (2016); Huang *et al.* (2017); Cai and Chen (2017); Du *et al.* (2019); Niu *et al.* (2019a)

Table 2.4: Papers using a consumer valuation model.

2.4.2.1 Vertical Differentiation

Various variants of vertically differentiated utility functions have been extensively used to model channel competition in the literature. In the most common framework, Chiang *et al.* (2003) introduced a simple valuation model where reservation price, V, is uniformly distributed over $[0,1]$. A representative consumer gets the net utility $U_r = V - p_r$ by shopping from a traditional retailer and $U_e = \theta V - p_e$ by shopping from an e-channel, where $\theta \in (0,1)$ is defined as the channel acceptance level or web-fit of the product, and, p_r and p_e are the traditional retailer and e-channel prices, respectively. This structure gives rise to the following demand functions (see Appendix A.1).

$$D_r = \begin{cases} 1 - \dfrac{p_r - p_e}{1-\theta} & \text{if } \dfrac{p_e}{\theta} \leq p_r, \\ 1 - p_r & \text{otherwise,} \end{cases} \tag{2.2}$$

$$D_e = \begin{cases} \dfrac{\theta p_r - p_e}{\theta(1-\theta)} & \text{if } \dfrac{p_e}{\theta} \leq p_r, \\ 0 & \text{otherwise,} \end{cases} \tag{2.3}$$

As can be seen from (2.2)-(2.3), dual-channel structure arises only for $p_e/\theta \leq p_r$ as the online channel would not be active otherwise. This model is the so-called *vertical differentiation* model with market size normalized to 1. When V is uniformly distributed over $\left[0, \bar{V}\right]$, where $\bar{V} > 0$ is a constant upper-bound, a similar structure arises with mild adjustments.

Many researchers adopt such a demand formulation as it is (e.g., see Arya *et al.*, 2007 and Chen and Bell, 2012), while others implement some modifications. Besides the adjustments mentioned in the linear demand case (in Section 2.4.1), such as service assistance level in the physical store (e.g., Yan and Pei, 2009) and delivery lead time (e.g., Xu *et al.*, 2012), return considerations have been a popular theme lately (Ma *et al.*, 2013; Chen and Chen, 2017; Li *et al.*, 2018a,c) due to the flexibility that consumer utility models offer.

One can also derive the inverse demand functions as $p_r = \theta(1 - D_r - D_e)$ and $p_e = 1 - D_r - \theta D_e$ with the condition $p_e/\theta \leq p_r$. This system of inverse demand functions is, in fact, also used to differentiate the quality of the products in the OM literature (Ferguson and Toktay, 2006; Ferguson and Koenigsberg, 2007; Yan *et al.*, 2018b). Even in the dual-channel context, some papers assume that there are two types of products (each sold in either of the channels) and differentiate consumer valuations based on quality instead of channel preference (e.g., see Yan *et al.*, 2015; Ha *et al.*, 2016).

2.4.2.2 Horizontal Differentiation

Horizontally differentiated utility structures have not received as much attention as vertical differentiation models have. This is partially because the basic valuation model, i.e., the Hotelling (1929) line, aims to imitate a market with two competing bricks-and-mortars retailers and has a limited capacity in explaining the competition between online- and offline-channel. Moreover, the traditional channel had been considered to be superior during the first decade of e-commerce and vertical differentiation models are appropriate tools to model such situations. However, with an increasing popularity of cross- and omni-channel practices, the number of papers using horizontally differentiated utility functions has recently grown as it can be seen from Table 2.4.

In the basic bricks-and-clicks model, a retail store is assumed to be located at the left-end of a Hotelling (1929) line of length one. The consumers are assumed to be uniformly distributed over this line and a representative consumer derives the net utility $U_r = v - p_r - tX$, where v is called the reservation price, t is the unit transportation cost and X is a uniform random variable with support $[0, 1]$. In other words, the utility consumers gain diminishes through the right-end of the city. On the other hand, when a consumer shops from the online channel, she derives the utility $U_e = v - p_e - s$, where $s < t$ is a constant inconvenience/delivery cost of purchasing online.

Assuming $U_e \geq 0$ (online channel is always active), $p_r \leq p_e + s$ (neither of the channels dominates the other), and $v > 2t$ (making sure that market can be fully covered) one can develop demand function of each channel as (see Appendix A.2),

$$D_r = \frac{p_e - p_r + s}{t}, \tag{2.4}$$

$$D_e = 1 - \frac{p_e - p_r + s}{t}. \tag{2.5}$$

This basic model is proposed by Liu *et al.* (2006) and many papers under this category adopt a mildly modified version of this framework (see, e.g., Chun and Kim, 2005; Xiao and Shi, 2016). In a salient work, Cattani *et al.* (2006) follow a slightly different formulation by assuming uniformly distributed inconvenience costs in both channels. The simplicity of horizontally differentiated consumer valuation formulations enables the researchers to model novel dual-channel phenomena. For example, He *et al.* (2016) consider free-riding behavior (browsing the product in a physical store, but shopping it from the online channel) of the consumers while Zhang *et al.* (2019a) study a buy online, pickup-in-store situation.

Moreover, as an extension of the Hotelling line, Salop (1979) proposes a spatial circular market model. Other studies have followed a similar approach to model a bricks-and-clicks market (Balasubramanian, 1998; Jeffers and Nault, 2011; Xiao *et al.*, 2014; Nault and Rahman, 2019).

2.4.2.3 A Quadratic Utility Formulation

The third consumer valuation model we discuss stemmed from the economics literature where it has been used to model a duopoly market condition (Singh and Vives, 1984). Later, the marketing literature advocated a slightly modified version to model channel competition (Ingene and Parry, 2004, 2007). Accordingly, the net utility of a representative consumer can be described as

$$U = \sum_{i \in \{r,e\}} (A_i D_i - B_i D_i^2/2) - \gamma D_i D_j - \sum_{i \in \{r,e\}} p_i D_i, \tag{2.6}$$

where the last expression represents the cost of purchasing from both channels while the rest represents a concave utility function with A_i, B_i, $i \in \{r, e\}$ and γ being positive constants such that $B_r B_e - \gamma^2 > 0$ and $A_i B_j - A_j \gamma > 0$, $i, j \in \{r, e\}$, $i \neq j$.

A representative consumer maximizes her net utility given in (2.6) with respect to quantities D_r and D_e. In the sequel, we use the following linear demand structure (see Appendix A.3):

$$D_i = \frac{B_j(A_i - p_i) - \gamma(A_j - p_j)}{B_r B_e - \gamma^2} \quad \text{or,} \tag{2.7}$$

$$p_i = A_i - B_i D_i - \gamma D_j, \quad i, j \in \{r, e\}, \quad i \neq j. \tag{2.8}$$

Following the relevant literature, by letting $\delta = B_r B_e - \gamma^2$, $a_i = (A_i B_j - A_j \gamma)/\delta$,

$b_i = B_j/\delta$ $i, j \in \{r, e\}$, $i \neq j$ and $\beta = \gamma/\delta$, one can rewrite direct demands in (2.7) as,

$$D_i = a_i - b_i p_i + \beta p_j, \quad i, j \in \{r, e\}, \quad i \neq j,$$

which is, in fact, analogous to the linear demand form given in (2.1) for $\beta_r = \beta_e = \beta$.

The studies modeling demand as in (2.1) are already outlined in Section 2.4.1. Therefore, in Table 2.4, we only present the papers deriving demand from the utility function (2.6) or adopting an inverse demand scheme similar to (2.8). Such demand formulations have been adopted by many scholars from different fields, including economics, marketing, and operations management, to model a dual-channel framework.

A popular approach is to model demand as in (2.7) for $B_r = B_e = 1$ (see, e.g., Cai, 2010; Zhang *et al.*, 2012; Lai *et al.*, 2018; Tang *et al.*, 2018). The same simplification is assumed by many papers adopting an inverse demand formulation as in (2.8) as well. Among those, for example, Yang *et al.* (2015) consider channel-specific γ values while Arya and Mittendorf (2013); Yoon (2016) and Yang *et al.* (2018a) assume identical market conditions for both channels, i.e., $A_r = A_e = A$. Moreover, Huang *et al.* (2018a) allow A to be random over a two-point distribution (high or low) whereas, Huang *et al.* (2018b) consider an additive, uniformly distributed random component in their formulation. In a similar vein, some papers establish a Cournot competition-like framework by further letting $B_r = B_e = \gamma = B$, where B is mostly taken as 1 (see, e.g., Arya *et al.*, 2007; Xiong *et al.*, 2012; Li *et al.*, 2016c, 2017b).

Finally, some studies formulate the utility function in (2.6) with some modifications. For example, Matsui (2016) uses a slightly different notation while Chen *et al.* (2017a) and Xu *et al.* (2018) add new terms to consider quality and carbon emission abatement, respectively.

2.4.3 Uncertain Demand and Miscellaneous Formulations

In this section, we outline demand modeling approaches besides linear formulations and consumer valuation models.

Most papers under this category incorporate uncertainty into demand by extending the formulations presented in Sections 2.4.1 and 2.4.2 (Fruchter and Tapiero, 2005; Yue and Liu, 2006; Hendershott and Zhang, 2006; Geng and Mallik, 2007; Chen *et al.*, 2008; Dumrongsiri *et al.*, 2008; Bernstein *et al.*, 2008; Jiang *et al.*, 2010; Yan and Ghose, 2010; Khouja and Wang, 2010; Yan and Pei, 2011; Hu and Li, 2012; Ryan *et al.*, 2012; Chen *et al.*, 2013; Feng and Geunes, 2014; Carrillo *et al.*, 2014; Li *et al.*, 2014a; Liu *et al.*, 2014; Xu *et al.*, 2014; Lei *et al.*, 2014; Chen, 2015; Li *et al.*, 2015b, 2016b; Liu *et al.*, 2016; Ha *et al.*, 2016; Huang *et al.*, 2017; Yang *et al.*, 2017; Li *et al.*, 2017b; Liu *et al.*, 2017; Modak and Kelle, 2019; Zhang *et al.*, 2019b; Zhou *et al.*, 2019; Raza and Govindaluri, 2019; Wang *et al.*, 2019b; Hsieh *et al.*, 2014; Guan *et al.*, 2020). Several papers consider demand as a random variable. The common approach is to assume that demand in different channels are uncorrelated and follow a general or known probability distribution (Yao *et al.*, 2005; Netessine and Rudi, 2006; Seifert *et al.*, 2006; Yao *et al.*, 2009; Liu *et al.*, 2010a; Shao, 2012; He *et al.*, 2014; Xie *et al.*, 2014; Yang *et al.*, 2016; Zhao *et al.*, 2016; Liu *et al.*, 2018; Zhang *et al.*, 2019c; Liao *et al.*, 2019; Yu *et al.*, 2019). Some papers also consider spillovers from the competing channel in the case of stock-outs (Boyaci, 2005; Geng and Mallik, 2007; Yang *et al.*, 2017).

We gather differential demand models and unique frameworks under miscellaneous formulations. We have found four papers studying channel competition by using a differential demand formulation scheme (Yao and Liu, 2003; Berger *et al.*, 2006;

Hendershott and Zhang, 2006; Sayadi and Makui, 2014). Moreover, Chiang (2010) model demand as a Poisson process while Liu *et al.* (2014) and Rodríguez and Aydın (2015) adopt a multiplicative and a nested-logit formulation, respectively.

2.5 Inventory Management and Demand Fulfillment Decisions in Dual-Channel Supply Chains

For more detailed surveys of e-fulfillment logistics in the era of multiple-channel retailing, we refer the reader to Kembro *et al.* (2018) and Melacini *et al.* (2018). As the present study aims to provide a literature review of a broader topic, our paper-selection criteria and analysis are different than those of past surveys.

This stream of research constitutes around 18% of the whole bricks-and-clicks dual-channel literature. In Table 2.5, we portray this body of the literature by listing some key features of the published works. Unlike the competition-based models, this line of research has incorporated lead time and demand uncertainty (we abbreviate 'random variable' as 'RV' in the Demand column) leading to complex problems. Seeking analytical tractability, researchers have opted for imposing strict assumptions on demand probability distributions and used heuristic procedures to tackle large-sized problems. Moreover, multi-period frameworks outnumber single-period ones. Decision column shows the main tool(s) used to tackle/analyze the problem: inventory refers to any sort of stock acquisition decisions such as order-up-to levels while fulfillment refers to the assignment of inventories to satisfy incoming demand from either channel. The situation where the retailer offers O2O services, such as an in-store pickup option, as well as last-mile delivery, is denoted as hybrid delivery in the table.

Papers	No. of periods	No. of products	Demand	Methodology	Decision	Lead Time	Hybrid Delivery	Heuristic	Return
Bendoly (2004)	Multi	1	RV-Normal	Simulation	Inventory, Fulfillment	X			
Chiang and Monahan (2005); Takahashi *et al.* (2011)	Infinite	1	Poisson process	Markov Chains	Inventory	X			
Alptekinoğlu and Tang (2005)	Multi	1	RV-Normal	NLP	Fulfillment	X		X	
Huang *et al.* (2007)	Infinite	1	RV	DP	Inventory	X			
Bendoly *et al.* (2007)	Multi	1	RV-Normal	NLP	Inventory, Fulfillment	X			
Hovelaque *et al.* (2007)	1	1	RV-Normal	Newsvendor	Inventory		X		
Mahar *et al.* (2009); Mahar and Wright (2009)	Multi	1	RV-Normal	MIP	Inventory, Fulfillment	X		X	
Bretthauer *et al.* (2010)	Multi	1	RV-Normal	MIP	Inventory, Fulfillment	X			
Liu *et al.* (2010b)	1	1	RV-Normal	MIP	Fulfillment	X		X	
Wu and Chiang (2011)	2	1	RV	Stochastic Programming	Inventory, Transship.			X	
Mahar *et al.* (2012)	Multi	1	RV-Normal	IP, Simulation	Inventory, Delivery	X	X		
Schneider and Klabjan (2013)	Multi	1	Poisson process	Markov Chains	Inventory				
Liang *et al.* (2014)	1	1	RV-3 Point Dist.	Analytic	Inventory, Transship.				

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Table 2.5 – continued from previous page

Papers	No. of periods	No. of products	Demand	Methodology	Decision	Lead Time	Hybrid Delivery	Heuristic	Return
Tetteh *et al.* (2014)	Infinite	1	Poisson process	Markov Chains	Inventory				
Li *et al.* (2015b)	Multi	1	Linear	Stochastic DP	Inventory				
Zhang (2015)	Multi	Multi	Linear	MIP, DP	Inventory	X			
Zhao *et al.* (2015)	Multi	1	AR(1)	Analytic	Inventory	X			
Widodo (2015)	2	1	Linear	EOQ	Inventory, Price				
Batarfi *et al.* (2016)	Multi	2	Linear	EPQ	Inventory	X			
Zhang *et al.* (2016)	1	1	Scalar	Multi-obj	Inventory			X	
Batarfi *et al.* (2017)	Infinite	2	Linear	Analytic	Inventory				X
Liao *et al.* (2017)	1	1	RV-Normal	Multi-obj NLP	Inventory, Fulfillment	X			
Mahar and Wright (2017)	Infinite	1	RV-Normal	Nonlinear IP	Fulfillment, Return	X	X		X
Yu and Deng (2017)	1	1	RV	Robust Opt.	Inventory				
Zhang *et al.* (2017a)	1	1	RV	Simulation	Channel Timing			X	
Gao *et al.* (2017)	Multi	1	AR(1)	Analytic	Inventory				
Gao and Su (2017a)	1	1	RV	Analytic	Inventory		X		
Gao and Su (2017b)	1	1	RV	Analytic	Inventory				
Alawneh and Zhang (2018)	Multi	Multi	RV	NLP	Inventory	X			
Jiang *et al.* (2018)	Multi	1	Linear	MIP	Price, Promotion			X	

Continued on next page

Table 2.5 – continued from previous page

Papers	No. of periods	No. of products	Demand	Methodology	Decision	Lead Time	Hybrid Delivery	Heuristic	Return
Wei *et al.* (2020)	1	1	Linear	Analytic	Price				
Arikan and Silbermayr (2018)	1	1	RV	Game Theory	Inventory				
Geunes and Su (2020)	1	Multi	RV	Stochastic Programming	Price, Inventory			X	
Nekoiemehr *et al.* (2019)	Multi	1	RV	Algorithmic	Scheduling	X	X	X	
Radhi and Zhang (2019)	1	1	RV	Newsvendor	Inventory, Return				X
Fan *et al.* (2019)	Infinite	1	Poisson process	Markov Chains	Inventory				
Ovezmyradov and Kurata (2019)	1	2	RV	Analytic	Inventory				
Gupta *et al.* (2019)	Multi	1	Logit	Multi-obj	Price, Inventory			X	
Ma *et al.* (2019)	Multi	1	AR(1)	Game Theory	Price				
Mutlu and Bish (2019)	Multi	1	Utility	Analytic	Service, Marketing				
MacCarthy *et al.* (2019)	1	1	Scalar	Analytic	Fulfillment		X		
Ishfaq and Bajwa (2019)	Multi	Multi	Deterministic	NLP	Fulfillment, Price			X	
Dijkstra *et al.* (2019)	Multi	1	RV	MDP	Transship.			X	X
He *et al.* (2020)	1	2	RV	Game Theory	Inventory				X

Table 2.5: Papers studying inventory management and/or demand fulfillment problem.

A common problem is to optimize inventory management practices in a multiple-channel environment, where fulfillment policies are assumed to be given. Some papers use Markov chains models in search of order-up-to levels for warehouses of a manufacturer and a traditional retailer in a two-echelon supply chain (Chiang and Monahan, 2005; Takahashi *et al.*, 2011; Schneider and Klabjan, 2013; Fan *et al.*, 2019). Despite their elegance and usefulness in decomposing the problem, Markov chains often lead to complex formulations where it is not possible to obtain closed-form or well-structured results. It is also popular to consider demand coming from each channel as a random variable (see, e.g., Huang *et al.*, 2007; Yu and Deng, 2017; Arikan and Silbermayr, 2018; Geunes and Su, 2020) or develop a multi-period version of linear demand function given in equation (2.1) to study this problem (see, e.g., Li *et al.*, 2015b; Zhang, 2015; Widodo, 2015; Jiang *et al.*, 2018). However, the number of papers examining transshipment decisions is surprisingly sparse (Wu and Chiang, 2011; Liang *et al.*, 2014).

Another widely studied problem is the assignment of distribution centers to online and offline demand sites (see, e.g., Bendoly, 2004; Mahar *et al.*, 2009; Mahar and Wright, 2009; Bretthauer *et al.*, 2010; Liu *et al.*, 2010b). For example, the retailer can establish a separate distribution center, along with the existing ones, to meet online orders exclusively from that facility. This would be a more responsive yet costly supply chain design as otherwise the retailer could have pooled the inventory and reduced its safety stock. Such trade-offs have been analyzed extensively to develop the optimal order fulfillment policy which can be static (always use the same facility to meet the demand coming from certain sites) or dynamic (make a real-time assignment by assessing the standing position of the inventory and predicted future demand).

Although most papers only consider last-mile delivery, hybrid delivery offerings are also discussed by few studies (Hovelaque *et al.*, 2007; Mahar *et al.*, 2012; Mahar and Wright, 2017; Nekoiemehr *et al.*, 2019). Incorporating a return policy appears to be another theme that has not received much attention. The lack of attention to these topics is largely due to the overwhelming complexity imposed by consideration of such practices. For example, there is only one paper studying returns with lead time consideration (Mahar and Wright, 2017) while others simply ignore it (Batarfi *et al.*, 2017; Radhi and Zhang, 2019).

There are also some sophisticated topics analyzed in this part of the literature. The impact of the demand distribution shape on the optimal transshipment and service level decisions are studied by Liang *et al.* (2014) and Mutlu and Bish (2019), respectively. Nekoiemehr *et al.* (2019) propose an algorithm to develop an order fulfillment schedule while Zhao *et al.* (2015), Gao *et al.* (2017) and Ma *et al.* (2019) analyze the bullwhip effect under a dual-channel framework by using time-series demand formulations. Zhang *et al.* (2017a) study a fast-fashion retailer's channel switching behavior. Finally, in their qualitative research paper, Peinkofer *et al.* (2019) study the impact of drop-shipping fulfillment operations.

2.6 Research Agenda

A research agenda is proposed by carefully gathering the common future research suggestions of the extant literature as well as using current practice and industry challenges to propose areas that have scarcely or never been addressed. Moreover, in Section 2.6.1, we point out that two research inquiries mentioned in this review have started to intertwine, which signals a paradigm shift in the literature. Finally, in Section 2.6.3, we propose a basic model to illustrate how the existing research can be expanded to address this new direction.

2.6.1 Future Research Directions Inspired from the Literature

In this part, we thoroughly gather the overlapping future research themes suggested by the reviewed studies. Because the publications under *channel competition* vastly outnumber the ones listed under *inventory management and demand fulfillment*, we analyze each stream of research separately.

2.6.1.1 Channel Competition

Despite the vast number of studies on *channel competition*, there are still some research gaps and areas that may benefit from further inquiry. Figure 2.2 presents the most frequently indicated future research directions in this part of bricks-and-clicks dual-channel literature. Accordingly, information asymmetry and demand uncertainty appear to be the prevailing themes. They are followed by multiple retailer and multi-period considerations.

The literature assumes information asymmetry either in demand (Li *et al.*, 2014b, 2015d, 2017b; Zhou *et al.*, 2019) or in cost (Mukhopadhyay *et al.*, 2008a; Cao *et al.*, 2013) parameters. However, the extant works are limited to two-player games in the context of manufacturer-retailer competition. We believe this topic can be extended in two directions: 1) studying more sophisticated supply chain scenarios, such as a manufacturer selling her products through an e-tailer and a retailer, or 2) considering information asymmetry in an attribute of the product, such as quality or delivery

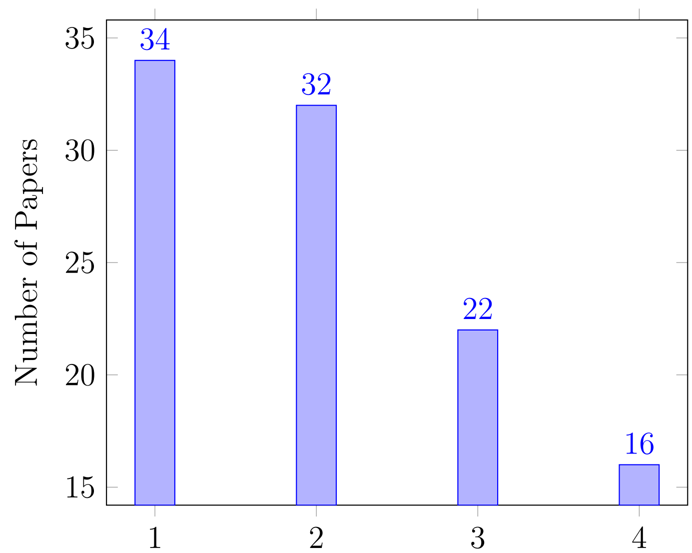


Figure 2.2: Top four most suggested future research themes: 1) asymmetric information, 2) demand uncertainty, 3) multiple retailer, 4) multi-period.

lead time.

Not surprisingly, demand uncertainty is another venue for future inquiry as non-deterministic (includes random and fuzzy models) formulations constitute only about 27% of the papers in this research stream. The consideration of returns, hybrid delivery methods (e.g., in-store or corner pick up points for online orders), and product customization are barely studied under a stochastic demand assumption. Moreover, except Yao *et al.* (2005) and Hu and Li (2012), the existing studies ignore the correlation of demand in different channels.

Next, we have the extension of the supply chain structure to include more than one retailer. As described in Section 2.4, the most common dual-channel supply chain framework consists of a manufacturer/supplier and a retailer. Nevertheless, the competition between independent dual-channel retailers (Ofek *et al.*, 2011; Nault and Rahman, 2019), a manufacturer and independent retailers (Lei *et al.*, 2014; David and Adida, 2015), and two dual-channel supply chains (Yang *et al.*, 2015; Jamali and Rasti-Barzoki, 2018) have not received much attention. Such channel structures seem to be worthy of probing further.

Relaxation of the single-period assumption is the final research direction repeatedly suggested in the literature. As it follows from the demand formulations discussed in Section 2.4, the vast majority of the papers, around 90%, have a one-period horizon. Most single-period models focus on a supply chain member's e-commerce initiative on the whole system. In a multi-period context, however, it would be more practical to consider a more specific problem, such as returns management in the apparel or high-tech industry.

2.6.1.2 Inventory Management and Demand Fulfillment

We find no topic that prevails in future research suggestions of the reviewed papers. However, we can make use of Table 2.5 to identify the gaps in this part of the literature.

The assignment of the warehouses/distribution centers to demand sites is already a persistent problem in the context of e-commerce, and yet it does become even a more challenging one in a dual-channel environment. Thus, we believe, there is still room for further research in this area. Specifically, trending practices in the industry, including but not limited to hybrid delivery methods (in-store and corner pick-up points), return and refund policies, have been barely studied.

Moreover, the literature appears to be solely focusing on stocking decisions while designing a demand fulfillment policy. In practice, however, companies may implement varying assortment and pricing decisions in different channels. Hence, another line of future research is the consideration of product assortment and/or pricing while designing the optimal fulfillment policy.

2.6.2 Future Research Inspired by Current Industry Trends and Challenges

We offer two research avenues based on our readings of the current trends and challenges of dual channels in the presence of online shopping. One can argue that demand modeling approaches of channel competition are not so different from those of product competition. Therefore, an empirical analysis shall be useful to verify the respective formulations that explain the channel- and product-competition better.

With the COVID-19 outbreak, the degree of digitalization in many industries has leaped up. For instance, the growth of e-grocery market has paced-up while the maximum number of shoppers physical stores can accommodate has declined (Mecatus and INCIVIS, 2020). Therefore, the operational planning of online grocery sales has become even more challenging. Similarly, with the lockdown conditions, many shopping malls and physical stores have been forced to shut down their doors which stimulated companies to offer extended return policies and, if possible, digital showrooms for fashion products (Abdulla, 2020). Given the new strains of the Corona virus and its widespread across the globe, it is expected that some of the current conditions, including online shopping, will become part of the “new normal” (Yang, 2020). To date, very few papers have addressed the disruptions in a bricks-and-clicks environment (Huang *et al.*, 2012, 2013b; Cao, 2014; Zhang *et al.*, 2015; Soleimani *et al.*, 2016; Tang *et al.*, 2018), all of which studied the problem under a manufacturer/supplier-retailer competition context. We believe research that addresses industry-specific issues will bring in new modeling challenges and aid firms in operating efficiently while accommodating the drastic changes in their customers shopping behaviors.

2.6.3 Paradigm Shift and an Illustrative Model

We see a paradigm shift in the literature as online shopping become more and more ubiquitous. With the drastic changes brought in by the COVID-19 pandemic, e-commerce has received a boost that was predicted to take at least 3 years. Companies that have been considering the addition of an online channel have now found it the only way to stay relevant in the "new normal." The firms now opt to determine the optimal dual-channel operating policy. This has opened new research venues for the OM academics and we see that the line between the research inquiries reported in this review has been blurred.

The competition between online and offline channels is still a perpetual issue, however, it is yet to be analyzed under a different context than manufacturer/supplier-retailer competition. Following the developments in practice, the OM field has turned its attention to the challenges faced by bricks-and-clicks companies. We see the projection of this new trend in recent publications (e.g., Dijkstra *et al.*, 2019; MacCarthy *et al.*, 2019; Nie *et al.*, 2019; Chai *et al.*, 2020). Indeed there are several questions that provide good starting points in this direction: To what extent, and how, the retailer should integrate the channels? Which multiple-channel setting should she adopt (multi-, cross-, or omni-channel)? When she should price a product the same across all channels? How should she manage the impact of free riding (reviewing the product in one channel, but making purchase from the other)? Should she offer in-store/curbside pick-up option for online orders? Should she welcome the return of online purchases to her physical facilities? If so, how does this affects its pricing and shipping fees policies and how the inventory management practices should be altered to comply with the new policies? Does the dual-channel setup provide more flexibility and resilience under supply-chain disruptions, if so, how? What are the idiosyncrasies of each industry (grocery, high-tech, fashion, etc.) in this new era of commerce? We believe these are important practical research questions that require the attention of the OM community.

To illustrate how the existing research can be extended to address this growing trend, we introduce a basic model, where a dual-channel firm considers offering hybrid delivery and return options with pricing.

Following the previous discussions and equation (2.1), we assume a dual-channel retailer, who is operating a physical store as well as an online shop, has the following demand function,

$$D_i^K(p_i^K, p_j^K) = \theta_i^K a^K - b^K p_i^K + \beta^K p_j^K, \quad i, j \in \{r, e\}, \quad i \neq j, \tag{2.9}$$

where the superscript $K \in \{M, C_1, C_2, O\}$ denotes the channel strategy which is either a multi-, cross-, or omni-channel strategy. A cross-channel strategy can be implemented by either consistent pricing (C_1) or hybrid delivery and return policy (C_2). For now, we assume that demand is deterministic. The company incurs a marginal selling cost, $c_i > 0$ in channel $i \in \{r, e\}$, and she wants to find out which dual-channel structure is optimal by changing price. Let $0 < \xi_i < 1$ and $0 < \alpha_i < c_i$, denote return rate and its associated cost in channel $i \in \{r, e\}$, respectively. The retailer offers O2O services in both directions: delivery and return. For cross-returns from online channel to physical store (under C_2 and O), parameters ξ_{er} and $\alpha_{er} > 0$

are defined. We also set $\xi_e + \xi_{er} < 1$ for obvious reasons (less returns than sales). We assume that returned products can be successfully recycled and sold as new. For online-to-store pick-up option, we define $0 < \delta < 1$ and $0 < \gamma < \min_{i\in\{r,e\}}\{c_i\}$ to denote the rate of online shoppers that prefer in-store/curbside pick-up and its associated expense, respectively.

Because this model is proposed for illustrative purposes, we depict the profit functions (π^K) only for the multi- and omni-channel structures, where the channels are managed separately in the former and are fully integrated in the latter.

$$\pi^M(p_r^M, p_e^M) = \sum_{\substack{i,j\in\{r,e\},\\ i\neq j}} (p_i^M - c_i - \alpha_i\xi_i) D_i^M(p_i^M, p_j^M),$$
$$\pi^O(p^O) = \sum_{i\in\{r,e\}} (p^O - c_i - \alpha_i\xi_i) D_i^O(p^O) - (\xi_{re}\alpha_{re} + \delta\gamma) D_e(p^O).$$

In the multi-channel system, the retailer differentiates the selling price across the channels and her overall profit is simply the sum of individual channel profits as the channels are managed independently. In each channel, the retailer incurs a unit purchasing cost, c_i, plus self-channel return expenses, $\alpha_i\xi_i D_i^M$, $i \in \{r, e\}$. On the other hand, the retailer adopts a consistent pricing strategy in the omni-channel system and as well as self-channel return costs she considers expenses associated with the cross-channel deliveries, $\delta\gamma D_e(p^O)$, and returns, $\xi_{er}\alpha_{er} D_e(p^O)$.

A couple of observations are in order. First, the relative performance of a policy with respect to the other depends on parameters $\theta_i^K, a^K, b^K, \beta^K$. For example, consistent pricing may alleviate the price searching behavior of the customers and thereby result in a weaker price sensitivity to compensate the retailer's reduced control over channels. On the other hand, offering hybrid delivery and return options may expand the market size and/or alter the market share of both channels while, at the same time, introducing new expenses. Depending on the product type, one may determine the varying parameters and carry out a comparative statics analysis to find out the optimal channel setup for different combinations of them. It is also possible to modify the model to account for randomness either in demand or in return rates.

Next, the model can easily be extended to analyze different aspects of the problem. For instance, based on similar assumptions, we can analyze the performance of each channel structure under demand disruptions in the bricks-and-mortar store, as it happened during the COVID-19 outbreak. Let $0 \leq \eta^K + \zeta \leq 1$, where $\eta^K, \zeta \in [0, 1]$, be the fraction of in-store demand that can be transferred to the internet channel at cost $\mathcal{C}(\zeta)$. We assume that η^K, $K \in \{M, C_1, C_2, O\}$ represents the part of cross-channel demand flow which depends on the channel structure and that ζ (to be determined) represents the part which depends on the retailer's efforts to boost online sales, such as, introducing an online showroom and/or a same-day-delivery option. It is plausible to expect $\eta^M < \eta^O$ for two reasons: 1) the firm may offer the customers the opportunity of returning the product to a physical store when the quarantine ends, and 2) as the product is priced the same across both channels, the customers would be less concerned about switching channels. Such a model gives rise to the following profit functions (assuming a constant p_r):

$$\pi^M(p_e^M, \zeta) = (p_e^M - c_e - \alpha_e \xi_e)[D_e^M(p_e^M, p_r^M) + (\eta^M + \zeta) D_r^M(p_r^M, p_e^M)] - \mathcal{C}(\zeta),$$

$$\pi^O(p^O, \zeta) = (p^O - c_e - \alpha_e \xi_e)[D_e^O(p^O) + (\eta^O + \zeta) D_r^O(p^O)] - \delta\gamma D_e^O(p^O) - \mathcal{C}(\zeta).$$

Here, the online demand increases by $(\eta^K + \zeta) D_r^K$ and the retailer incurs $\mathcal{C}(\zeta)$ to transfer ζD_r^K units of demand from the offline channel to the online channel in setting $K \in \{M, O\}$. Similar to the previous model, the omni-channel setting follows a consistent pricing policy and the retailer incurs an additional O2O delivery cost, $\delta\gamma D_e^O$.

As mentioned earlier, this problem is introduced for illustrative purposes and it could have been framed by a completely different approach. The model can also be extended in two important ways. Firstly, to consider the cannibalization impact of online channels, especially in states of emergency and lockdown during a pandemic, and incorporate the decision of possibly shutting down the bricks-and-mortar channel where stores are used only as mini-warehouses or may be shut completely. Secondly, to consider competition between different retailers, where intuitively first online movers and those that offer loyalty programs, or subscription models, may lock their customers.

2.7 Conclusion

The steady growth of e-commerce sales has reshaped the retail industry. Nowadays, most retailers operate in a bricks-and-clicks mode, where they reach the end customers through an online channel as well as traditional bricks-and-mortar stores. In this chapter, we presented a literature review of this dual-channel phenomenon. An ontology for multiple-channel retailing concepts was also presented.

Although we mainly focus on the OM literature, we reviewed 263 articles published between 1998 – 2019 that also included articles in the interface of OM and Marketing or Economics fields. We find two prominent themes driving the literature: 1) channel competition between a manufacturer/supplier and a traditional retailer where either of the players establishes an e-channel, 2) inventory management and demand fulfillment policies in a dual-channel environment. We carry out an in-depth analysis of both research themes, reveal the gaps yet to be addressed and areas that seek further inquiry.

The first stream of research, which constitutes around 82% of the whole literature, studies a novel competition between a manufacturer/supplier and a retailer, where either of the players expands their operations to the online market. We identify and discuss the overlapping demand formulations used to quantify the channel competition and show the links between them and multi-product demand formulations used in traditional bricks-and-mortar models. The research agenda of this theme includes the considerations of demand uncertainty, multi-periodicity, and information asymmetry.

The other theme includes fewer articles, but it is concerned with a more structured problem of determining the optimal inventory management policy for a firm receiving demand from both online and offline channels. A detailed illustration of this research area is presented in Table 2.5, where the papers are categorized according to their key features. Our analysis reveals that there is a limited number of studies addressing the hybrid delivery methods, return and refund policy making. Moreover, price and assortment decisions in each channel are often ignored while determining the optimal fulfillment policy.

Finally, we see a paradigm shift in the literature, which blurs the line between these two research themes and opens up new research venues for the field. Following the industry, the OM literature has changed the lens from which the channel competition is analyzed: the issue of whether or not to open an online channel under manufacturer/supplier-retailer competition has been replaced by the challenges faced by dual-channel firms. In particular, we believe that industry-specific channel management issues (high-tech, grocery, fashion) and the disruptions caused by the COVID-19 pandemic present promising research opportunities. We also illustrate how existing research can be used to address this new trend.

We note that as the bricks-and-clicks dual-channel topic lies at the interface of the OM, Marketing, and Economics literature, our research methodology may have omitted relevant works. However, we believe that this review adequately serves the purpose of portraying the dual-channel literature and identifying the research trends and gaps.

Chapter 3

Donate More to Earn More

3.1 Introduction

Temperature-controlled food supply chain management practices have been playing an increasingly important role in improving the economic performance of food-retailers (Smith and Sparks, 2004). This is not only because customers have become more sensitive to freshness (Soysal *et al.*, 2012), but also because supermarkets can increase their profit margins by reducing food waste. This trend has been aided by the wide availability of the internet of things (IoT)-enabled condition tracking technologies, such as time-temperature indicators (TTIs) due to their miniaturization and price decline (Dada and Thiesse, 2008). However, despite the economic significance of grocery retail business and the global food-waste problem, there is a lack of studies on the integration of such technologies into quantitative models that aim to improve operational efficiency.

Although organic waste is detrimental to food-retailers as well, particularly when they are charged according to a pay as you throw (PAYT) system which is a common policy implemented by many municipalities in North America and Europe, a retailer may be willing to bear any expense associated with waste as long as it is offset by a revenue. Food-retailers are profit-driven organizations nonetheless. They often desire to maintain fresher offerings with full shelves to escalate customer traffic and enhance the earnings. As an attempt to mitigate waste and challenge food insecurity, some governments motivate corporate social responsibility (CSR) activities by offering tax deductions to donors and/or provide funding to food-rescue organizations (Alexander and Smaje, 2008; Giuseppe *et al.*, 2014; Arya and Mittendorf, 2015; Chu *et al.*, 2018).

Such incentive policies may indeed benefit retailers as well as the society. However, donors and charities often have conflicting interests: food-banks seek items that will stay fresh during the transfer and distribution process, whereas supermarkets are more willing to donate food items that are closer to their expiry date and thus of low-quality that would largely be disposed of otherwise. In fact our research has been motivated by one of the author's experience with a local food bank. While the food bank is in desperate need of fresh food, they indicated that they do not wish to accept food that supermarkets donate close to its expiry date as they do not have the resources to store and ship it to their customers before it is expired. Unlike consumer packaged goods, fresh/frozen food items very often have tentative due dates. Fortunately, TTIs can be used to accurately estimate the true (effective) quality, which may be different than the visible quality, and thereby may impact the due date estimation. This improves firms' ability to plan for the future and alleviates charities' food-safety concerns.

In this part, we study a food-retailer's operational planning problem for a continuously deteriorating inventory. We propose a quality-dependent newsvendor problem (QDNP) which takes the effective quality of the products into account to help food-retail chains reduce food spoilage while retaining the earnings. We consider a two-period model, where the retailer jointly determines purchasing quantity and first period price at the beginning of the planning horizon, and reevaluates the price and decides on her donation policy at the end of the first period. We develop the necessary and sufficient conditions, and analyze the impact of quality on the decision variables as well as the expected profit. Moreover, we provide insights about the donation decision. Our findings reveal that in contrast to a common belief, food-retailers can improve their profits while alleviating waste by donating more of their inventory under certain conditions. To the best of our knowledge this study presents the first stochastic donation model that considers freshness.

The rest of this chapter is organized as follows. In the next section we present the literature review. In Section 3 we outline the assumptions, define the demand functions and formulate our models. We present managerial insights derived from the model in Section 4. Numerical experiments are conducted to evaluate the performance of the proposed models and to examine their behavior with respect to several parameters in Section 5. Finally, the last section summarizes the chapter and points out possible future directions.

3.2 Literature Survey

This study is linked to three main research streams: deteriorating inventory management, quality loss models, and food donation models. In this section we survey the relevant literature for each stream.

3.2.1 Deteriorating Inventory Management

We focus on joint ordering and pricing and/or repricing decisions of perishables, particularly when quality of the products is under consideration. Karaesmen *et al.* (2011) and Pahl and Voß (2014) provide more detailed surveys of perishable inventory management practices. Pahl and Voß (2014) point out the scarcity of studies in lifetime modeling of deteriorating goods to mitigate waste. In this research, one of our goals is to address this gap in the literature by focusing on the retailer's operations.

The newsvendor-pricing type settings have been used extensively to study the joint ordering and pricing problem. Petruzzi and Dada (1999) published one of the milestone papers in this area by proposing a reformulation of the newsvendor problem under joint ordering and pricing. In this study, we follow their reformulation approach with a quality-sensitive demand function. For further articles studying newsvendor-pricing problems, see, for example, Kocabıyıkoğlu and Popescu (2011); Yang *et al.* (2011); Baron *et al.* (2015). There are also some game-theoretical models of the newsvendor problem that consider quality as an influencer of the demand (Cai *et al.*, 2010; Xu *et al.*, 2011). However, the aging process of the on-shelf perishables is ignored by these studies.

Freshness-dependent demand schemes have recently been popular in deterministic settings. The demand formulation in these papers is an extension of the work of Wang and Li (2012), who perform price markdowns with a deterministic time-dependent demand function. The majority of the papers use an EOQ model (see, for example, Wu *et al.*, 2016; Chen *et al.*, 2016a; Dobson *et al.*, 2017; Hua *et al.*, 2016) to analyze the impacts of freshness on the order cycle. We extend this stream of research by considering a stochastic demand and find optimal joint pricing and ordering in a newsvendor setting.

There is also a substantial number of papers which study the competition between new and old inventory by considering the old units as substitutes of the new units. In this context, the quality level of the old items is often assumed to be lower than the quality of the new ones and thereby operational decisions are taken accordingly (Li *et al.*, 2012; Ferguson and Koenigsberg, 2007; Sainathan, 2013; Chen *et al.*, 2014). These studies do not consider donation and often regard the disposal of edible/usable items as a viable decision alternative. For example, according to Ferguson and Koenigsberg (2007) and Sainathan (2013), the depletion of the old inventory, which is assumed to generate no cost to the newsvendor, can be the optimal decision under certain circumstances. In contrast to such studies, in this chapter, we suggest adjusting the price and donating surplus inventory to food-banks in order to reduce the number of outdated products at the end of the selling period.

3.2.2 Quality Loss Functions

Quality loss functions originated from the food chemistry literature, but have lately become popular in the operations management (OM) literature as well. In fact, condition monitoring has always been an issue for researchers studying the distribution of perishable products. Akkerman *et al.* (2010) and Soysal *et al.* (2012) provide reviews of quantitative quality models in food logistics.

In line with the previous works, timely temperature data is assumed to be provided by TTIs placed into batches of the products, transportation trucks and warehouses.

We rely on that data to make a judgment on products' immediate condition. There are several studies testing the reliability of TTI labels and indicating the potential improvements they offer in temperature-controlled supply chains by examining various inventory rationing policies (Sahin *et al.*, 2007; Bowman *et al.*, 2009; Dada and Thiesse, 2008; Giannakourou and Taoukis, 2002, 2003). However, these studies use simulation-based approaches and their main goal is to draw attention to the functionality of TTIs.

Osvald and Stirn (2008) and Rong *et al.* (2011) exhibit more complex models that focus on the transportation issues of the perishables; and tackle the problems by using mixed integer programming. We extend the approach of Osvald and Stirn (2008) to model the linear quality deterioration process of vegetables. We provide a general modelling framework that incorporates different quality deterioration schemes depending on the product type (linear or exponential) as well as the possibility of donations.

Wang and Li (2012) introduced a time-dependent deterministic demand model that incorporates the output of TTIs in order to identify the optimal markdown policy that increases the profit and reduces food waste. One shortcoming of the Wang and Li (2012) model is that it ignores demand uncertainty. Furthermore, their work is limited to markdown pricing situations when the product has an exponentially decaying quality. In our models, we propose continuous quality deterioration and a stochastic demand function. We investigate the optimal ordering and pricing, and optimal repricing and donation decisions.

3.2.3 Food Donation Models

There is a limited number of papers that study food donation in the retail industry context. Alexander and Smaje (2008) point out social and economical reasons for food recovery and qualitatively analyze a case study. Giuseppe *et al.* (2014) develop a deterministic mathematical model to optimize retailers' food recovery policy when government supports the industry with several incentives. They apply their model to Italian food chains and demonstrate the economic benefits of food donation. Their model is more suited for customer packaged goods than fresh foods and meat products as they consider the shelf-life as a constant parameter. In another deterministic work, Arya and Mittendorf (2015) studied the subtle effects of government incentives on a supplier-retailer supply chain and total welfare. We also presume that the government offers subsidies to accelerate CSR activities, but develop a stochastic demand framework with a continuous quality monitoring scheme to address the issue of organic waste as well.

In a recent study, Chu *et al.* (2018) develop a two-period newsvendor-pricing model with tax planning and charitable donations when the second period demand is assumed to be deterministic. They consider donation as a means of salvaging left-over inventory at the end of the selling season and use dynamic programming to model the problem and solve it numerically. There are significant structural differences between their work and the present study. We assume random demand in both periods and consider donation at the end of the first period, before the realization of the randomness in the second period. Also, we incorporate the condition of the items into the demand function and solve the problem analytically.

3.3 Quality-dependent Newsvendor Problem (QDNP)

Problem definition, model assumptions and development of QDNP are described in this section. We also mention how the underlying quality loss model is integrated into the demand function.

3.3.1 Assumptions

We study a food-retailer's pricing and inventory management problem for a continuously deteriorating product. The selling season is divided into two periods where the firm decides on stocking quantity and regular selling price at the beginning of period 1, and donation amount and adjusted price at the beginning of period 2. The retailer faces demand uncertainty during both periods. Table 3.1 summarizes the main notation used in this chapter.

We use a two-period single-product newsvendor model to study our problem dynamics. Unlike Ferguson and Koenigsberg (2007); Chu *et al.* (2018), which assume demand certainty in the second period, we consider demand uncertainty in both periods. The random component of demand, ε_i, in any period $i = 1, 2$ follows a distribution with non-decreasing hazard rate, finite mean, μ_i, and standard deviation, $\sigma_i > 0$, and has a twice differentiable and invertible c.d.f., $F_i(.)$, defined on the interval $[A_i, B_i]$. We also note that we do not restrict B_i to be a finite number so that we can use common distributions, such as the exponential distribution. Without loss of generality, we assume that the random components are i.i.d. so that we can drop the subscripts (i.e., $\mu_1 = \mu_2 = \mu$). The newsvendor model is appropriate since fresh

	Notation	Definition
Parameters	a	Demand parameter depending on market size
	b	Price sensitivity
	C_0	Purchasing cost per unit
	C_d	Disposal cost per unit
	C_s	Shortage cost per unit
	ϕ	Quality sensitivity
	q	Initial quality at supplier warehouse
	R	Benefit of donation per unit
	d	Estimated quality drop during transportation
	I	Inventory on hand at the end of period 1
	λ	Quality deterioration rate at retailer's site
	T	Product shelf-life
	T_1	The length of period 1
	T_2	The length of period 2
	ν	Maximum discernible quality
	η	Minimum acceptable quality
	$\delta(t)$	Perceived quality at time t
	$\varepsilon \in [A, B]$	Random component of demand
	$f(.)$, $F(.)$	p.d.f. and c.d.f. of ε, respectively
	e_t	Effective quality of the product at time t
	μ, σ	Mean and standard deviation of ε
Decision Variables	p_1	Price in period 1
	z_1	Stocking factor in period 1
	p_2	Price in period 2
	z_2	Stocking factor in period 2
Consequence variables	Q	Stocking quantity
	γ	Portion of I kept for selling in the second period

Table 3.1: Main notation.

foods are mostly seasonal and the quality varies from one harvest to another. Thus, it is plausible to perceive each batch of items being unique due to its quality. Therefore, another purchase in the next period means another batch of items of different quality as either the supplier's inventory ages or is replaced by a new one. However, we also acknowledge there are limitations to this model as it does not account for the substitution effect or repurchasing during the selling season.

It is assumed that consumers are sensitive not only to price, but also to quality. As expected, fresher goods attract more customer demand for the same price level. To reflect this, demand is represented as a function of price and quality. We provide more detailed discussions regarding demand in the next section. It is also assumed that the quality level and deterioration rate are known at all times. This can be achieved by using IoT-enabled sensors such as time-temperature indicators (TTIs). We input time-temperature data to characterize quality deterioration following the prevalent models in the food science literature. In addition, the length of the selling season is taken as the product shelf-life which the retailer is able to determine internally by using TTI data. The reliability of TTI technology and current quality loss models have been tested in several studies (see, for example, Giannakourou and Taoukis, 2002, 2003; Labuza, 1982, 1984).

Most supermarkets update the price (often in the form of a markdown) of perishable food products during their life-cycle to enhance earnings. The time and depth of the price adjustment may vary depending on inventory on-hand and remaining shelf-life of the foods. Price discounts are useful particularly when the retailers have higher than necessary inventory to satisfy the forecasted demand and/or when goods are about to spoil/expire (moderate or low quality). On the other hand, when the inventory is exhausted too early, the price may be adjusted upwards; although it is not a common practice. In this price-adjustment scenario, goods are either stored until they are unfit for consumption and disposed of afterwards (Alexander and Smaje, 2008), or they are disposed of while they are still edible to make room for fresher items and attract more customers. There are several operations management studies suggesting that disposal of usable inventory may increase profits (see, for example, Ferguson and Koenigsberg, 2007; Li *et al.*, 2012; Sainathan, 2013).

This does not mean that supermarkets do not suffer from waste. On the contrary, many municipalities in North America and Europe implement a pay-as-you-throw (PAYT) policy. However, food-retailers are profit-driven organizations and they may be willing to bear any expense associated with waste as long as it is offset by a revenue. Therefore, some governments, including those of the U.S. and some European countries, offer tax deductions for charitable donations to incentivize CSR activities and reduce waste (Alexander and Smaje, 2008; Giuseppe *et al.*, 2014; Chu *et al.*, 2018). The retailer may also derive some intangible benefits, such as improved public image and consumer recognition, due to her CSR acts, referred as "warm glow"(Arya and Mittendorf, 2015). Thus, in this study, we assume that the retailer incurs a disposal cost, $C_d > 0$, for the leftover inventory, if there is any, at the end of the selling season, and that she collects a revenue, $R > 0$, for donated goods.

Figure 3.1 illustrates the sequence of events. To incorporate continuous quality deterioration, the lifetime of the product is characterized by equal time units (e.g., hours) with T denoting the shelf-life. The selling season is composed of two periods, where period 1 refers to time-span $[0, T_1)$ and period 2 refers to the rest, $[T_1, T]$. We assume that T_1 is exogenous and can be any time-point within the interval $(0, T)$, as long as the effective quality of the goods at T_1 is high enough to alleviate the food-bank's safety concerns. In practice the value of T_1 would be signaled by the food bank with which the retailer has partnered to donate the food. At the beginning of the selling season (at time 0), the retailer acquires an inventory of size Q and sets a selling price, p_1. At the end of the first period (at time T_1), the retailer observes the remaining inventory, I, determines how many units to donate, $(1-\gamma)I$, where $0 \leq \gamma \leq 1$, and sets selling price p_2 for the goods carried forward, γI. Finally, any leftover inventory is disposed of at the end of the selling season (at time T).

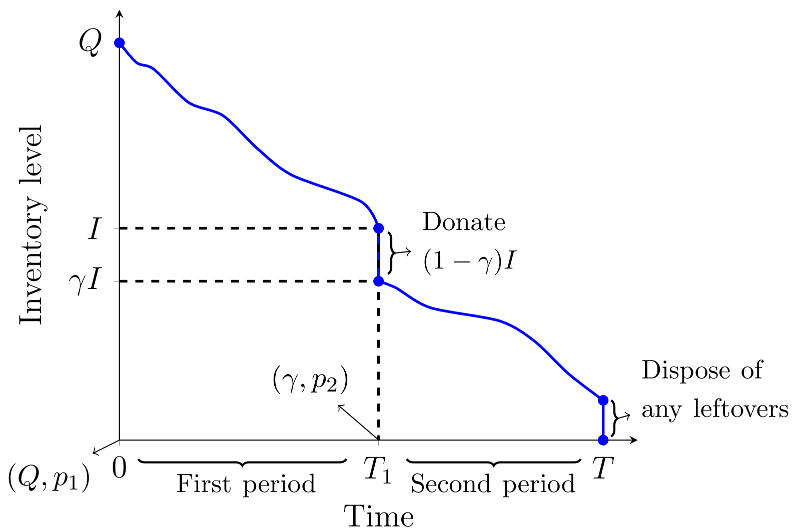


Figure 3.1: Sequence of events for QDNP.

3.3.2 Demand Function

Because demand is sensitive not only to price but also to quality, we characterize demand as a function of price and time.

$$D_i(p_i, t) = y_i(p_i, t) + \varepsilon, \quad i = 1, 2, \tag{3.1}$$

where $y_i(p_i, t)$ is a linear function that is decreasing in price and increasing in quality: $y_i(p_i, t) = \hat{a}(t) - bp_i$. Here we abuse the conventional notation by setting $D_i(p_i, t) \equiv D_i(p_i, t, \varepsilon)$ for convenience. In our model, the deterministic part of demand is not only price-dependent, but also time-dependent to represent quality deterioration. Note that $D_1(p_1, t)$ and $D_2(p_2, t)$ are defined for $t \in [0, T_1)$ and $t \in [T_1, T]$, respectively. The market size $\hat{a}$ is comprised of a constant, a, and a time-variant component, $\delta(t)$, which is called the perceived quality at time t. Studies using utility functions to derive demand often use a constant quality value normalized between 0 and 1 (Ferguson and Koenigsberg, 2007; Huang *et al.*, 2013b). To account for *continuous* quality deterioration, we present $\delta(t) \geq 0$ as a decreasing function of time and define the market size as $\hat{a} = a + \phi\,\delta(t)$, where $\phi > 0$ is the quality sensitivity parameter of demand. We note that, without a loss of generality, all our results can be extended to a multiplicative form by defining $\hat{a} = a\,\phi\,\delta(t)$. The form of $\delta(t)$ is specified based on the chemical characteristics of the goods being studied and more details about it shall be provided in the upcoming sections. Let T_1 and T_2 be the length of period 1 and period 2, respectively. Denoting the shelf-life of the goods as $T = T_1 + T_2$, which is determined internally according to perceived quality, we define the random demand over the selling period as,

$$\int_0^{T_1} D_1(p_1, t)\, dt + \int_{T_1}^{T} D_2(p_2, t)\, dt = [\bar{y}_1(p_1, T_1) + \varepsilon]T_1 + [\bar{y}_2(p_2, T_2) + \varepsilon]T_2,$$

where $\bar{y}_1(p_1, T_1) = (1/T_1) \int_0^{T_1} y_1(p_1, t)\, dt = a - bp_1 + \phi\, \bar{\delta}_1(T_1)$ and $\bar{y}_2(p_2, T_2) = (1/T_2)$ $\int_{T_1}^{T} y_2(p_2, t)\, dt = a - bp_2 + \phi\, \bar{\delta}_2(T_2)$. One can consider $\bar{y}_i(p_i, T_i)$ and $\bar{\delta}_i(T_i)$ as average (deterministic part of) demand and average (perceived) quality of the goods per unit time, respectively. We define the upper bound of price in period $i = 1, 2$ as $\bar{p}_i$, where $\bar{y}_i(\bar{p}_i, T_i) + A = 0$. Here, the integration of a random variable may appear unconventional, but it is, in fact, equivalent to considering a random variable with mean μT_1 and variance $\sigma^2 T_1^2$ in the first period, and a random variable with mean μT_2 and variance $\sigma^2 T_2^2$ in the second period.

3.3.3 Quality Loss Function

The derivation of perceived quality function $\delta(t)$ is articulated in this section. To model the quality deterioration of fresh groceries, we extend the quality loss models of Bowman *et al.* (2009) and Osvald and Stirn (2008). Food starts losing its nutritional value just after harvesting/production, but changes become apparent to customers when ripeness of the food exceeds a threshold of discernibility level, ν, which is determined based on chemical characteristics of the food item. However, unlike the earlier published characterizations of quality degradation, we account for changing environmental conditions, such as temperature, throughout the supply chain as they affect the deterioration rate and thereby remaining lifetime of the products. For instance, improved storage facilities at a producer's warehouse is able to keep the units under optimal conditions, whereas trucks that are equipped for cold storage may still increase the ripening rate during transportation. Retailers can provide better storage conditions than shippers, but may not maintain an environment as favorable as that of the producers since the products must be moved and displayed.

We assume that the retailer observes the product's initial quality as q at the supplier's site. Effective quality, e_t, may not be apparent to the naked eye, but can be estimated from TTI data. During the transportation, depending on the travel time and conditioning technology, the quality of the product decays d units. This leads to two possible product types: (1) a *fresh product* that is at or above the maximum discernible quality level ν or (2) an *aged product* that is below that threshold by time t_0 when it arrives to the shelves. Figure 3.2 illustrates the quality changes during a period for fresh products.

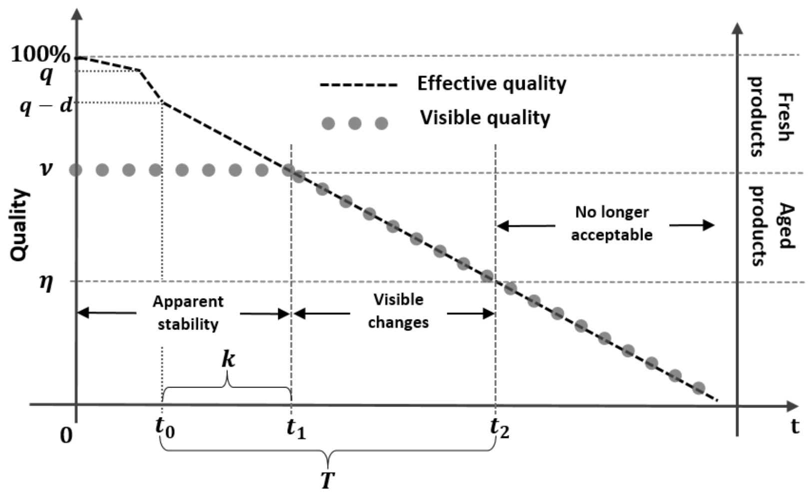


Figure 3.2: Linear quality deterioration of fresh products.

The retailer can monitor the quality drop during transportation and act accordingly. Items that are below the minimum acceptable quality level η are assumed to be unsellable either due to their appearance or inedibility. Impact of the quality is calibrated by demand sensitivity parameter, ϕ, and the maximum quality is taken as 100%. For the sake of simplicity, the clock is reset to zero at the beginning of the selling season t_0 and we use the shelf-life, T, in our calculations. The product quality changes begin to be visible to customers at time t_1. When $t_1 \geq t_0$ we denote the time during which the quality is perceived to be stable by $k = t_1 - t_0$. The products become unacceptable to the customers at time $t_2 \geq t_1$, so we have $T = t_2 - t_0$. The products' shelf-life depends on the quality upon arrival, $q - d$, minimum acceptable quality, η, and deterioration rate at the retailer's site, $\lambda > 0$.

For meat products we follow Wang and Li (2012) to model quality loss. Unlike vegetables, the quality of meat products drops in an exponential manner. The shelf-life and deterioration rate are both identified based on the environmental conditions during transportation and storage by using the reaction kinetics of the items (Labuza, 1982). Once again we assume a maximum quality of 100%. The concepts maximum discernibility and minimum acceptability do not apply in the degradation process of meat products.

3.3.3.1 Linear Quality Loss Function

For the linear quality decay pattern, we let $\delta(t) = \min\{\nu, e_t\}$, where $e_t = q - d - \lambda t$ when $t \leq T$ and $e_t = 0$ otherwise. This framework relies on the fact that customers have a limited capacity to judge the quality and always perceive it as ν for fresh products (when $q - d \geq \nu$). The value of the perceived quality changes only when the effective quality, e_t, drops below the maximum discernible quality level ν. Therefore, the demand function is contingent on the product type. For fresh products, $\delta(t)$ can be decomposed as follows.

$$\delta(t) = \begin{cases} \nu, & t \leq (q - d - \nu)/\lambda, \\ (q - d - \lambda t), & t > (q - d - \nu)/\lambda. \end{cases} \tag{3.2}$$

By setting $k \equiv (q - d - \nu)/\lambda$, average perceived quality for the fresh products over the first selling period can be calculated as

$$\begin{aligned}\bar{\delta}_1(T_1) = (1/T_1)\int_0^{T_1} \delta(t)\,dt &= (1/T_1)\left[\int_0^k \nu\,dt + \int_k^{T_1} (q - d - \lambda t)\,dt\right],\\ &= (1/T_1)\{k\nu + [q - d - \lambda(T_1 + k)/2](T_1 - k)\},\end{aligned}$$

where, without loss of generality, it is assumed that quality decay becomes visible during the first period. The rationale behind this assumption is that the retailer would consider repricing typically after the quality becomes visibly low. Average perceived quality for aged products over period two (when $t > (q - d - \nu)/\lambda$) takes a relatively straightforward form:

$$\bar{\delta}_2(T_2) = (1/T_2)\int_{T_1}^{T} (q - d - \lambda t)\,dt = q - d - \lambda(2T - T_2)/2.$$

As a result, the product shelf life is estimated by $T = (q - d - \eta)/\lambda$.

3.3.3.2 Exponential Quality Loss Function

Some perishable products' quality degrades exponentially over time. To represent this case we use $\delta(t) = qe^{-\lambda t}$. This gives rise to the following average quality formulations

$$\begin{aligned}\bar{\delta}_1(T_1) &= (1/T_1)\int_0^{T_1} qe^{-\lambda t}\,dt = (q/\lambda)(1 - e^{-\lambda T_1})/T_1,\\ \bar{\delta}_2(T_2) &= (1/T_2)\int_{T_1}^{T} qe^{-\lambda t}\,dt = (q/\lambda)e^{-\lambda T}(e^{\lambda T_2} - 1)/T_2.\end{aligned}$$

3.3.4 Model Analysis

We solve the retailer's two-period perishable inventory management problem by using backward induction starting with the second period.

3.3.4.1 Second Period

In this period, we propose that a food retailer considers both price adjustment and inventory donation as a means to increase the expected profit at the end of the selling season as shown in Figure 3.1.

First, the retailer observes the remaining inventory I whose effective quality is known. Next, she adjusts the price from p_1 to p_2 and donates $1-\gamma$ $(0 \leq \gamma \leq 1)$ fraction of I at a reward R per unit. One can argue that R is quite often less than the unit purchase cost, C_0, meaning that it can cover only a part of the retailer's expenses. The reward can be considered as a per unit return derived from a blend of (1) government incentives, such as tax deductions (e.g., see Alexander and Smaje, 2008; Giuseppe *et al.*, 2014), and (2) intangible fringe benefits, such as improved public image due to corporate social responsibility acts, also referred as "warm-glow"(Arya and Mittendorf, 2015).The random profit function of the newsvendor is given as below.

$$\Pi_2(\gamma, p_2) = \begin{cases} p_2 \int_{T_1}^{T} D_2(p_2,t)\,dt - C_d\big[\gamma I - \int_{T_1}^{T} D_2(p_2,t)\,dt\big] & \\ \qquad +R(1-\gamma)I, & \int_{T_1}^{T} D_2(p_2,t)\,dt \leq \gamma I, \\ p_2\gamma I - C_s[\int_{T_1}^{T} D_2(p_2,t)\,dt - \gamma I] + R(1-\gamma)I, & \int_{T_1}^{T} D_2(p_2,t)\,dt > \gamma I, \end{cases}$$

where, $D_2(p_2,t) = a + \phi(q - d - \lambda t) - bp_2 + \varepsilon$. In this setting, we define $z_2 = \gamma I/T_2 - \bar{y}_2(p_2, T_2)$ so that $\max\{A, -\bar{y}_2(p_2,T_2)\} \leq z \leq I/T_2 - \bar{y}_2(p_2,T_2)\}$ since $0 \leq \gamma \leq 1$.

Recall that $\bar{y}_2(p_2, T_2)$ is defined as $a + \phi\,\bar{\delta}_2(T_2) - bp_2$. The model can be extended to include all types of quality loss models by defining the appropriate function $\delta(t)$. Rewriting the profit as a function of z_2 and p_2 we have,

$$\Pi_2(z_2, p_2) = \begin{cases} \Big\{p_2[\bar{y}_2(p_2, T_2) + \varepsilon] - C_d(z_2 - \varepsilon) - R[\bar{y}(p_2, T) + z_2]\Big\}T_2 + RI, & \varepsilon \leq z_2, \\ \Big\{p_2[\bar{y}_2(p_2, T_2) + z_2] - C_s(\varepsilon - z_2) - R[\bar{y}_2(p_2, T_2) + z_2]\Big\}T_2 + RI, & \varepsilon > z_2. \end{cases} \tag{3.3}$$

In fact, (3.3) is essentially a newsvendor formulation with shortage and surplus costs, where R takes the place of the unit purchasing cost. The retailer's expected profit in the second period can be given as,

$$\begin{aligned} \mathbb{E}[\Pi(z_2, p_2)] &= \big\{(p_2 - R)[\bar{y}_2(p_2, T_2) + \mu] - (R + C_d)\Lambda(z_2) - (p_2 + C_s - R)\Theta(z_2)\big\}T_2 \\ &\quad + RI, \\ &= \Gamma_2(z_2, p_2) + RI, \end{aligned} \tag{3.4}$$

where $\Lambda(z) = \int_A^z (z - u)\, dF(u)$ and $\Theta(z) = \int_z^B (u - z)\, dF(u)$ are the expected surplus and shortage, respectively. Before solving the retailer's second period problem, we analyze the expected profit function. Let us lay out the first and second order conditions of $\Gamma_2(z_2, p_2)$:

$$\frac{\partial \Gamma_2(z_2,p_2)}{\partial z_2} = -(R+C_d) + (p_2 + C_s + C_d)[1 - F(z_2)], \tag{3.5}$$

$$\frac{\partial^2 \Gamma_2(z_2,p_2)}{\partial z_2^2} = -(p_2 + C_s + C_d) f(z_2), \tag{3.6}$$

$$\frac{\partial \Gamma_2(z_2,p_2)}{\partial p_2} = a + \phi\,\bar{\delta}_2(T_2) - b(2p_2 - R) + \mu - \Theta(z_2), \tag{3.7}$$

$$\frac{\partial^2 \Gamma_2(z_2,p_2)}{\partial p_2^2} = -2b, \tag{3.8}$$

where we omitted T_2, as it is a scalar coefficient and RI is a constant by Equation (3.4). Note that the second period profit function of the retailer is concave in p_2 for a given z_2 and vice versa by conditions (3.6) and (3.8). This means that one can maximize $\Gamma_2(z_2, p_2)$ over a single variable by using either of the first order conditions. Similar to Petruzzi and Dada (1999), we determine the unique price in terms of stocking factor z_2 as,

$$p_2(z_2) = \frac{a + \phi\,\bar{\delta}_2(T_2) + bR + \mu}{2b} - \frac{\Theta(z_2)}{2b}, \tag{3.9}$$

which is increasing in z_2. Let us denote the deterministic part of the price as $\tilde{p}_2 = p_2(B)$. To assure the positivity of demand for any $z_2 \in [A, B]$, we assume $\tilde{p}_2 \leq \bar{p}_2$. The next theorem directly follows from the above conditions.

Theorem 1. *The second period expected profit function of the retailer, $\Gamma_2(z_2, p_2(z_2))$, is concave for $z_2 \in [A, B]$, if $F(.)$ has a non-decreasing hazard rate and $f(A)[\bar{y}_2(-R - 2(C_s + C_d), T_2) + A] > 1$.*

Proof. All appendices are in the Appendices. See Appendix B.1. □

Petruzzi and Dada (1999) showed that $\Gamma_2(z_2, p_2(z_2))$ is unimodal for similar conditions on $F(.)$, whereas, Theorem 1 specifies the conditions under which $\Gamma_2(z_2, p_2(z_2))$ is concave and we assume these conditions hold throughout this chapter. Now, we can present the retailer's profit maximization problem in the second period as,

$$\max \quad \Gamma_2(z_2, p_2) + RI \tag{3.10}$$

$$\text{s.t.} \quad \underline{z} \leq z_2 \leq \bar{z}, \tag{3.11}$$

$$p_2 \leq \bar{p}_2, \tag{3.12}$$

where $\underline{z} = \max\{A, -\bar{y}_2(p_2, T_2)\}$ and $\bar{z} = I/T_2 - \bar{y}_2(p_2, T_2)$. The lower-bound, $\underline{z}$, characterizes the optimal solution for a trivial case, where $\gamma = 0$ meaning that the optimal policy is to donate the whole inventory I. Therefore, we drop it and focus on the upper-bound constraint, which may be binding when $B > \bar{z}$ as we assume repurchase is not available. The following proposition characterizes the solution for the retailer's second period problem.

Proposition 1. *The KKT conditions are necessary and sufficient for the constrained problem* (3.10) *when the condition in Theorem 1 is satisfied. Let $\mathcal{L}$ denote the Lagrangian function, and let ω_z and ω_p denote the Lagrangian multipliers, respectively, corresponding to constraints* (3.11) *and* (3.12)*. Then, the Lagrangian and the KKT* *conditions can be listed as below:*

$$
\begin{aligned}
&\mathcal{L} = \Gamma_2(z_2, p_2) + RI - \omega_z(z_2 - \bar{z}) - \omega_p(p_2 - \bar{p}_2),\\
&\frac{\partial \mathcal{L}}{\partial z_2} = \frac{\partial \Gamma_2}{\partial z_2} - \omega_z = 0,\\
&\frac{\partial \mathcal{L}}{\partial p_2} = \frac{\partial \Gamma_2}{\partial p_2} + \omega_z \frac{d\bar{z}}{dp_2} - \omega_p = 0,\\
&\omega_z(z_2 - \bar{z}) = 0, \quad \omega_p(p_2 - \bar{p}_2) = 0,\\
&z_2 \leq \bar{z}, \quad p_2 \leq \bar{p}_2, \quad \omega_z, \omega_p \geq 0.
\end{aligned}
$$

Proof. Because $\Gamma_2(z_2, p_2)$ is concave by Theorem 1 and both upper-bound constraints are linear, the KKT conditions are necessary and sufficient. □

Using the KKT conditions, the following theorem builds up and analyzes the final-period value function.

Theorem 2. *The second period value function is a concave function of on-hand inventory, I, and defined as such,*

$$
\Phi(I) = \begin{cases} \Gamma_2(z_2^0, p_2(z_2^0)) + RI, & I > I^0, \\ H(I, \min\{p_H(I), \bar{p}_2\}), & I \leq I^0, \end{cases} \tag{3.13}
$$

where $I^0 \equiv [\bar{y}_2(p_2(z_2^0), T) + z_2^0]T_2$ with $(z_2^0, p_2(z_2^0))$ being the unique maximizer of $\Gamma_2(z_2, p_2)$ without the inventory constraint (3.11)*, $H(I, p_2) = \Gamma_2(\bar{z}(I, p_2), p_2) + RI$ is the expected profit function when $\gamma^* = 1$, and $p_H(I)$ is derived from the first order condition (FOC) of $H(I, p_2)$ with respect to price. As a result, we obtain the following* *optimal inventory carrying and pricing policies:*

$$\gamma^* = \begin{cases} I^0/I, & I > I^0, \\ 1, & I \leq I^0. \end{cases}, \qquad p_2^* = \begin{cases} p_2(z_2^0), & I > I^0, \\ \min\{p_H(I), \bar{p}_2\}, & I \leq I^0, \end{cases} \tag{3.14}$$

Proof. See Appendix B.2. □

A simple example derived from a case study by Wang and Li (2012) is provided to illustrate the optimal policy in period 2:

Example 1. *Let* $a = 7.92$, $b = \phi = 4.86$, $C_d = C_s = 0.05$, $R = 0.5$, $T = 2T_2$ *with linear quality deterioration parameters,* $q = 0.75$, $\lambda = 0.009$, $\nu = 0.80$, $\eta = 0.20$, *leading to a shelf-life of* $T_2 = 61$ *hours. The random component of demand follows an exponential distribution with mean,* $\mu = 1$.

Figure 3.3 depicts the solution of the above example for different on-hand inventory values. Following the optimal policy given in Theorem 2, when $I \leq I^0 = 278.71$ units, $\gamma^* = 1$ and $p_2^* = p_H(I)$ (as $\bar{p}_2 = 2.11 > 1.61$); whereas, when $I > I^0$, we obtain $\gamma^* = I^0/I$ and $p_2^* = p_2(z_2^0) = 1.37$. The shaded area shows the donated part of the on-hand inventory, which reaches up to 30% for $I = 400$ units.

3.3.4.2 First Period

At the beginning of the first period, the retailer jointly determines the purchasing quantity, Q, to which she will commit until the end of the selling season, and first period selling price, p_1. Assuming that the purchasing expenses are incurred during

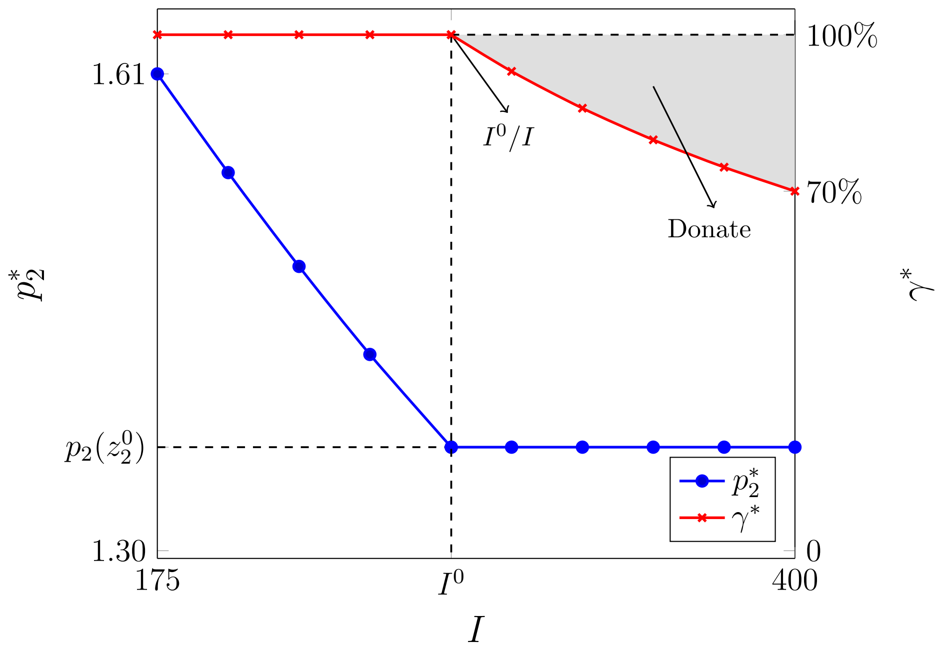


Figure 3.3: Plot of the optimal second period price and donation policy for Example 1.

the first period, the firm's random profit function can be given as ,

$$\Pi_1(Q,p_1)=\begin{cases} p_1\int_0^{T_1} D_1(p_1,t)\,dt - C_0Q, & \int_0^{T_1} D_1(p_1,t)\,dt \le Q, \\ (p_1-C_0)Q - C_s[\int_0^{T1} D_1(p_1,t)\,dt - Q], & \int_0^{T_1} D_1(p_1,t)\,dt > Q. \end{cases}$$

Once again, we apply the stocking factor substitution by defining $z_1 = Q/T_1 - \bar{y}_1(p_1,T_1)$ and obtain first period expected profit as,

$$\Gamma_1(z_1,p_1) = (p_1-C_0)[\bar{y}_1(p_1,T_1)+\mu]T_1 - C_0\Lambda(z_1)T_1 - (p_1+C_s-C_0)\Theta(z_1)T_1, \quad (3.15)$$

which is a newsvendor problem without disposal cost. Therefore, it exhibits the good properties of $\Gamma_2(z_2,p_2)$, in other words, the conditions given in equations (3.5)–(3.8), and thereby Theorem 1 apply to $\Gamma_1(z_1,p_1)$ with mild adjustments.

The retailer aims to maximize her expected profit over two periods which can be formulated as,

$$V = \Gamma_1(z_1, p_1) + \alpha \mathbb{E}[\Phi(\max(0, (z_1 - \varepsilon)T_1))], \tag{3.16}$$

where α is the discount rate. Note that both terms on the right are concave, and so is V. The following theorem characterizes the optimal solution to the retailer's operational planning problem over two periods:

Theorem 3. *The optimal stocking quantity and first period pricing policies can be determined according to the range and possible realizations of the first period demand.*

Proof. See Appendix B.3 □

3.4 Managerial Insights

In this section, we examine the decisions advised by QDNP, their relation with some of the key input parameters, and discuss possible managerial implications of the model.

3.4.1 Donation Behavior of the Retailer

Here, we analyze the donation behavior of the retailer and identify the factors that influence it.

Proposition 2 (Impact of quality/remaining shelf-life)**.** *When $z_2^* < \bar{z}$, optimal decisions, z_2^*, p_2^*, and γ^* are all monotone increasing in T_2.*

Proof. See Appendix B.4. □

Proposition 2 shows that the higher the quality (or equivalently the further the due date) of on-hand inventory, the more units the retailer is willing to carry forward and the fewer units she is willing to donate. Moreover, longer shelf-life drives the second period price up.

Proposition 3 (Impact of donation reward)**.** *When* $z_2^* < \bar{z}$*, optimal decisions,* z_2^* *and* γ^* *are both decreasing in* R*.*

Proof. See Appendix B.5 □

There is a monotonic relationship between the retailer's donation behavior and its associated reward. Expectedly, a higher reward motives more donations. The following theorem directly follows from Propositions 1–3.

Theorem 4 (Donation threshold)**.** *A donation threshold can be found in the linear demand case by fixing any two of the factors* T_2*,* R*, and* I *and solving the following equations for* z_2 *and the unfixed factor:*

$$\bar{z}(p_2(z_2)) - z_2 = 0, \tag{3.17}$$

$$\frac{d\Gamma_2(z_2, p_2(z_2))}{dz_2} = 0. \tag{3.18}$$

Proof. Because γ^* is monotone in quality, donation reward, and on-hand inventory at the end of the first period, such a threshold exists. □

Theorem 4 extends the optimal policy given by (3.14) to consider shelf-life and donation reward as active actors, instead of constant parameters. Practically speaking, having a threshold in terms of R or I could be more useful. Thus, the retailer can either find the minimum reward that incentivizes her to donate some of her stock for a given shelf-life and inventory on hand, or find the minimum inventory level that allows the retailer to benefit from donation for a given shelf-life and donation reward.

Figure 3.4 illustrates the threshold for Example 1 in terms of I for various R and T_2 values. The threshold decreases as the benefit of donation increases and as the expiration date approaches. Therefore, the structure is somewhat similar to that of an optimal stopping problem. It is also worth recalling that T_2 represents the quality as the shelf-life of the items are determined internally based on the environmental conditions. Thus, the benefit of donation offsets opportunity cost of depleting the inventory early as the items deteriorate. For example, let us consider a case where the food bank with which the retailer has partnered requires the donated foods to arrive at least 2 days before the due date (assume T_2 is given in hours). Recall that the retailer estimates per unit donation would bring in $R = \$0.5$. Hence, she would donate only if she has more than 210 units of inventory.

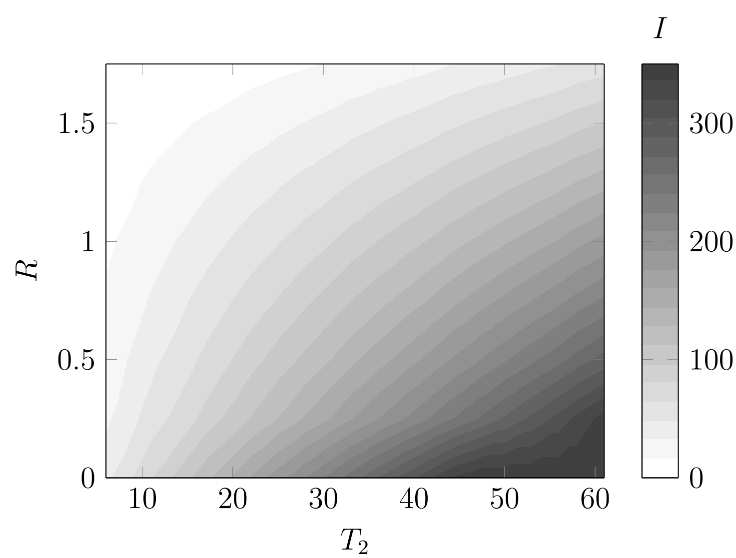


Figure 3.4: A contour map of donation thresholds in terms of inventory for various donation reward and shelf-life values when ε follows an exponential distribution with mean 1.

3.4.2 Impact of Donation on the First Period Decisions

Having donation as a viable option at the end of the first period impacts the firm's operational planning over two periods. Let ξ_1 be the realization of randomness in the first period. The following theorem outlines the impact of the retailer's donation behavior on her first period decisions.

Theorem 5. *The purchasing quantity and the first period selling price under QDNP are both greater than or equal to those under the no-donation policy. Also, the optimal donation amount is decreasing in ξ_1, when $\xi_1 < z_1^* - I^0/T_1$, and it is independent of ξ_1 otherwise.*

Proof. See Appendix B.6. □

Donations provide the retailer with another tool to deal with uncertainty and Theorem 5 shows that this, in return, leads to a larger purchasing quantity and a higher first period price. However, the retailer does not need that extra tool unless the realized demand at the end of the first period is low. In other words, if the ending inventory of period 1 is less than the threshold value, that is $I \leq I^0$ (or $\xi_1 \geq z_1^* - I^0/T_1$), we obtain $\gamma^* = 1$ (no donation) by the optimal policy in (3.14). On the other hand, if $I > I^0$ (or $\xi_1 > z_1^* - I^0/T_1$), the optimal inventory carrying amount becomes $\gamma^* = I^0/I$, which is increasing in ξ_1 as the larger the first period demand, the fewer the donated inventory.

3.4.3 Expected Waste

As a final note, we analyze the changes in expected waste with respect to the donation behavior of the retailer. Let $\hat{p}_2$ be the optimal price under the no-donation policy. Theorem 6 shows that donations not only can be profitable but also can be environmentally conscious.

Theorem 6. *The expected waste in the absence of donation is at least as large as the expected waste under the optimal donation policy. Also, the gap is increasing in on-hand inventory, I, when $I > I^0$. The reduction in expected waste can be quantified as*

$$\Lambda(\bar{z}(I,\hat{p}_2)) - \Lambda(z_2^*) = \begin{cases} \Lambda(\bar{z}(I,\hat{p}_2)) - \Lambda(z_2^0), & I > I^0, \\ 0, & I \leq I^0, \end{cases}$$

where the first expression can be further simplified as $\Lambda(\bar{z}(I,\hat{p}_2)) - \Lambda(z_2^0) = \bar{z}(I,\hat{p}_2)\, F(\bar{z}(I,\hat{p}_2)) + z_2^0 F(z_2^0) - \int_{z_2^0}^{\bar{z}(I,\hat{p}_2)} u\, dF(u)$.

Proof. See Appendix B.7. □

Donating inventory does not necessarily imply zero waste as the future demand is still assumed to be uncertain. However, it does bring down expected waste and thereby promises both social and economic improvements. Figure 3.5 depicts the expected waste at the end of the selling season under both models for Example 1. Accordingly, expected waste ascends rapidly as I increases under the no-donation policy, whereas, it is fixed to $\Lambda(z_2^0) = 22.71$ for all $I > I^0 = 278.71$ under QDNP, as specified by Theorem 6. An interesting observation is that the expected waste under the no-donation policy grows not only in magnitude, but also in proportion as inventory on hand rises. For instance, the expected waste proportion is only around 7.8% at $I = I^0 = 271.78$, whereas, it is more than 19% at $I = 400$. This implies that the larger the I, the greater the improvement offered by the donation-enabled policy.

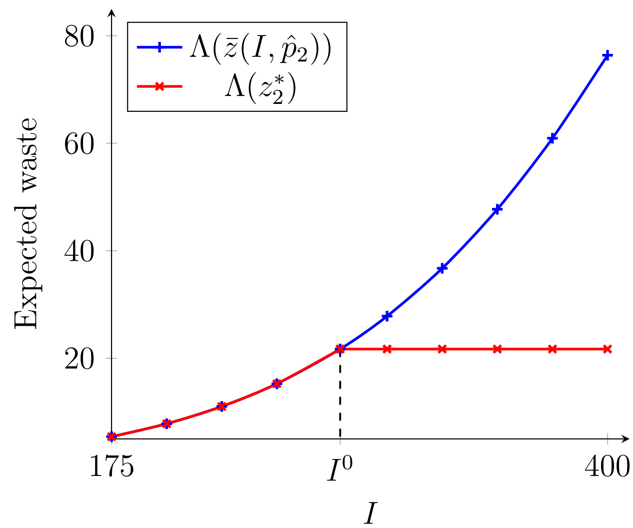


Figure 3.5: Plot of the expected waste (in units) under QDNP and the no-donation policy.

3.5 Numerical Study

In this section, we test the proposed model with the empirical data provided by Wang and Li (2012) and offer insights on the role of donations. We perform a numerical sensitivity analysis to explore the impact of uncertainty and the length of the first selling period on the optimal decisions and expected profit.

3.5.1 A case study from Wang and Li (2012)

Wang and Li (2012) provide data on four branches of a supermarket with the assumption of exponential quality degradation. We use the data as it is for meat products where it makes sense for quality to decay exponentially (Labuza, 1982). We note that the term 'fresh' is used to refer to the condition (quality) of the items. Therefore, we consider the product group called 'fresh vegetables' by Wang and Li (2012) as 'vegetables' in our study. To model for vegetables, however, we modify the deterioration rate λ to use the data for a linear quality loss model. In all examples, $C_s = 0.05$, $\nu = 0.80$, $\eta = 0.20$, and $\alpha = 1$. The random component of demand is assumed to follow a uniform distribution with zero mean. We set $[A, B] = [-2, 2]$ for vegetables and $[A, B] = [-1, 1]$ for meat products. The per unit reward of donated inventory is $R = 0.8$ for vegetables and $R = 3$ for meat products.

As shown in Table 3.2, all stores have fresh (i.e., $q - d \geq \nu$) vegetables at the beginning of the selling season. Because there is no maximum discernibility level in exponential quality loss models, we do not make such distinction for meat products, the data for which is included in Table 3.3. It is worth noting that there are some structural differences between the present work and the work of Wang and Li (2012). In particular, they seek offering a markdown policy by using a deterministic demand model which takes the quality of the perishable items into account. On the other hand, this chapter develops a two-period pricing-newsvendor model with donation. However, comparing the supermarket's price and inventory positions given in the last two columns with the outputs of the proposed model may help us find out if she has excess inventory and/or bad pricing policy.

Vegetables	a	b	ϕ	$q-d$	λ/hr	T_1 (hrs)	T (hrs)	C_0	C_d	Initial Price	Inventory
Store 1	7.92	4.86	4.86	0.90	0.009	48	78	1	0.05	1.49	280
Store 2	6.73	4.13	4.13	0.92	″	″	80	″	″	″	240
Store 3	10.30	6.32	6.32	0.83	″	″	70	″	″	″	360
Store 4	6.34	3.89	3.89	0.85	″	″	72	″	″	″	220

Table 3.2: Parameters with pricing and stocking quantity policies of the supermarket for vegetables as provided by Wang and Li (2012). We modified the parameters λ and T, which were originally 0.0216 and 72, respectively for all stores.

Meat products	a	b	ϕ	$q-d$	λ/hr	T_1 (hrs)	T (hrs)	C_0	C_d	Initial Price	Inventory
Store 1	9.79	1.83	1.83	0.95	0.0067	168	240	3.5	0.05	4.5	740
Store 2	8.32	1.56	1.56	0.97	″	″	″	″	″	″	630
Store 3	12.73	2.38	2.38	0.88	″	″	″	″	″	″	950
Store 4	7.83	1.46	1.46	0.90	″	″	″	″	″	″	590

Table 3.3: Parameters with pricing and stocking quantity policies of the supermarket for meat products as provided by Wang and Li (2012). Quality is assumed to decay exponentially.

To gain more insights, we also represent the optimal decisions and corresponding expected profits suggested by the two-period model without donation, denoted by $\hat{Q}$, $\hat{p}_1$, $\hat{V}$, and the single-period model with the period length of T, denoted by $\check{Q}$, $\check{p}_1$, $\check{V}$. Tables 3.4 and 3.5 list the results for vegetables and meat products, respectively, as well as the the performance of QDNP with respect to the alternative approaches.

First of all, we note that incorporating quality into the decision making process may alter the retailer's operational decisions significantly. For all stores and both product types, both QDNP and the no-donation model set higher prices and lower quantities than those set by the supermarket. These observations indicate a faster inventory turnover which is likely to reduce waste.

	QDNP			No-donation			Single-period			Performance	
	V^*	Q^*	p_1^*	$\hat{V}$	$\hat{Q}$	$\hat{p}_1$	$\check{V}$	$\check{Q}$	$\check{p}$	$\frac{V^*-\hat{V}}{\hat{V}}(\%)$	$\frac{V^*-\check{V}}{\check{V}}(\%)$
Store 1	103.82	236.79	1.65	99.73	214.44	1.64	70.97	206.61	1.50	4.10	46.28
Store 2	87.17	210.06	1.66	82.80	187.12	1.65	54.59	177.83	1.48	5.28	59.68
Store 3	123.09	273.88	1.62	117.84	246.64	1.61	91.94	239.78	1.50	4.45	33.89
Store 4	69.29	186.75	1.62	62.18	154.58	1.60	39.97	144.41	1.46	11.45	73.37

Table 3.4: Comparison of the optimal ordering-pricing policies and expected profits for vegetables.

Secondly, for both product groups, a monotone relation emerges, where QDNP offers the largest decisions and expected profit, whereas, the single-period model offers the lowest ones. It is already shown in Theorem 5 that $\hat{Q} \leq Q^*$ and $\hat{p}_1 \leq p_1^*$, and it is

	QDNP			No-donation			Single-period			Performance	
	V^*	Q^*	p_1^*	$\hat{V}$	$\hat{Q}$	$\hat{p}_1$	$\check{V}$	$\check{Q}$	$\check{p}$	$\frac{V^*-\hat{V}}{\hat{V}}(\%)$	$\frac{V^*-\check{V}}{\check{V}}(\%)$
Store 1	498.05	533.67	4.71	480.79	485.81	4.70	386.23	450.90	4.50	3.59	28.95
Store 2	406.72	459.48	4.71	386.19	406.88	4.69	298.58	372.75	4.46	5.32	36.22
Store 3	656.19	670.23	4.69	644.34	633.55	4.69	540.51	594.97	4.52	1.84	21.40
Store 4	371.40	431.86	4.70	349.54	377.20	4.68	265.67	343.19	4.45	6.25	39.80

Table 3.5: Comparison of the optimal ordering-pricing policies and expected profits for meat products.

expected to have $\hat{V} \leq V^*$ due to the option of donation at the end of the first period. The contribution of donation compared to the no-donation setting can be as large as 11.45%.

Finally, the results reveal that introducing another decision making opportunity changes the decisions drastically, in particular price, and boosts the expected profit up to 73%. Besides the value of additional decisions, the two-period models enjoy the reduced variation: $Var(T_1\varepsilon_1 + T_2\varepsilon_2) = (T_1^2 + T_2^2)\sigma^2 < (T_1 + T_2)^2\sigma^2 = Var(T\varepsilon)$. Therefore, the enhancements in expected profits, in part, can be attributed to diminishing uncertainty. We should note here that our comparison is biased by the fact that the single period model does not have the opportunity to update prices. Another alternative benchmark model could be to use a donate-all policy at the end of the first period.

Overall, for all instances, QDNP offers significant improvements in terms of expected profit. In the next section we analyze the impact of two drivers of the retailer's problem: variation of the randomness in demand and the length of the first period.

3.5.2 Sensitivity Analysis

We investigate the impact of the variation in demand uncertainty (by varying the support of ε) and the length of the first period, on operational decisions as well as the expected profit of the firm.

Vegetables data in Store 1 is taken as the base case. The optimal decisions and expected profit are determined under $B = -A \in \{1, 2, 3\}$ and $T_1 \in \{40, 50, 60\}$ for QDNP, No-Donation model, and Single-Period model. Varying T_1 has two consequences: it impacts (1) the overall variation in demand, $(T_1^2 + T_2^2)\sigma^2$, and (2) the operational planning of the firm as the length of the second period is also tied to T_1, that is $T_2 = T - T_1$, where $T = 78$. We also note that the results of the single-period model are not sensitive to the changes in T_1. The findings are shown in Table 3.6.

		QDNP			No-Donation			Single-Period			Performance	
$[A, B]$	T_1	V^*	Q^*	p_1^*	$\hat{V}$	$\hat{Q}$	$\hat{p}_1$	$\check{V}$	$\check{Q}$	$\check{p}$	$\frac{V^*-\hat{V}}{\hat{V}}(\%)$	$\frac{V^*-\check{V}}{\check{V}}(\%)$
$[-1, 1]$	40	117.22	219.93	1.67	117.12	217.75	1.67	99.81	216.31	1.54	0.09	17.44
$[-2, 2]$	40	100.82	224.36	1.67	99.49	212.40	1.67	70.97	206.61	1.50	1.33	42.04
$[-3, 3]$	40	85.60	226.90	1.66	82.24	206.54	1.66	43.84	191.73	1.44	4.09	95.28
$[-1, 1]$	50	119.12	227.31	1.65	118.17	219.08	1.65				0.80	19.34
$[-2, 2]$	50	104.04	241.59	1.65	99.17	214.98	1.64	″	″	″	5.27	47.09
$[-3, 3]$	50	90.13	260.36	1.64	79.10	211.11	1.61				13.95	105.60
$[-1, 1]$	60	119.82	238.49	1.63	115.81	218.85	1.62				3.46	20.05
$[-2, 2]$	60	106.23	265.49	1.62	92.73	213.13	1.59	″	″	″	14.55	49.67
$[-3, 3]$	60	92.82	292.92	1.61	69.51	206.08	1.55				33.53	111.73

Table 3.6: Comparison of the optimal stocking quantity, first period price, and expected profit.

We observe that the performance of QDNP compared to the no-donation model increases in T_1. For instance, when $[A, B] = [-3, 3]$, QDNP outperforms the no-donation framework by around 4% for $T_1 = 40$ and by around 34% for $T_1 = 60$. This implies that the postponement of the donation decision has a positive impact on profits. In other words, as suggested by Theorem 4, the shorter the second period, the more willing the retailer is to donate. As a result, the firm stocks more inventory and changes the price only slightly. Therefore, the discrepancy between the optimal quantities and prices grow along with the discrepancy in expected profits as the first period stretches longer. We also note that an increase in T_1 drives up the variation when $T_1 > T_2$. However, V^* is monotone increasing in T_1 for all values of $B = -A$, meaning that the negative impact of larger variation is offset by the value derived from the postponement of the donation decision.

We also note that as the degree of uncertainty in demand grows, the relative performance of QDNP to the no-donation model increases. For example, the relative difference in expected profits hits 0.80% for $B = -A = 1$ and around 14% for $B = -A = 3$ when $T_1 = 50$. Considering changes in both parameters, T_1 and B, the difference can be as large as 34%. More interestingly, the two models react to this change oppositely: QDNP enlarges the order size while the no-donation policy shrinks it. Thus, the flexibility imposed by donation prevails under a high degree of uncertainty. This finding also aligns with that of Chu *et al.* (2018), despite the structural differences between their study and the present one.

Lastly, using a two-period model, with the opportunity to update decisions in the second period, dampens the impact of uncertainty. As $B = -A$ increases from 1 to 3, the expected profit of the single-period model is about halved, whereas neither of the two-period models shows such a sharp change in the expected profit.

3.6 Conclusion

This chapter introduces the quality-dependent newsvendor problem (QDNP), which is a combined inventory and price adjustment model with a special focus on quality. In particular, we consider a two-period problem, where a socially responsible food-retailer jointly determines the stocking quantity and first period price at the start of the selling season, and modifies the price and decides on her donation policy at the end of the first period. The problem utilizes the data provided by TTI technologies to make accurate judgments about the condition of the perishable items and thereby aims to help food retail chains mitigate food waste and raise the profit at the same time. We find that incorporating donations in the model can improve a firm's profit while at the same time increase its social responsibility impact.

It is assumed that consumers are both price- and quality-sensitive and therefore, we propose a demand framework that takes into account the quality deterioration process of perishables. We consider two different quality loss models, linear and exponential, for two product types, vegetables and meat, respectively. We extend the work of Osvald and Stirn (2008) to design a linear quality loss model and use the approach suggested by Wang and Li (2012) for exponential quality loss. In both periods, demand is assumed to be a linear function of price with an additive randomness.

Our findings revealed that the donation behavior of the retailer at the end of the first period depends on the quality (time to expiration), on-hand inventory, and the per unit reward of donation derived from the tax deductions and/or the firm's corporate social responsibility act. Furthermore, for a given inventory level, expected waste under a donation enabled-policy is always less than or equal to the expected waste in the absence of donation.

In our numerical study, we compared QDNP with a two-period model without donation and a single-period model. In all instances, the proposed framework outperformed both of the benchmark models. We also found that the relative performance of QDNP is particularly higher when the uncertainty is high and/or the length of the second period is short.

The present study has a few limitations. Firstly, we assume that the realized demand in the first period has no effect on the second period uncertainty. This assumption can be relaxed by considering demand update models, though it would impose additional technical difficulty to the problem. Next, we only consider a linear demand function, which may fall short of explaining demand for some products. Therefore, integrating the continuous quality deterioration concept into other demand function schemes could be a promising extension. Finally, we consider a single-product scenario. It would be interesting to see how the donation behavior of the retailer is affected by a competition between two products of different quality. We leave these issues as future research directions.

Chapter 4

Can Tax Incentives Induce Donation of Fresh Goods?

4.1 Introduction

Demand for food items has shown a steady growth at 3.6% per year on average since 2012, generating US $7 trillion in revenues globally in 2019 (Frimpong, 2020). Due to the COVID-19 pandemic environment, an extra 11% growth is estimated in 2020, increasing the worldwide food revenues up to US $8 trillion (Frimpong, 2021). According to Nielsen (2013), fresh food constitutes approximately 30–60% of total grocery sales.

Despite this constantly growing trillions of dollars worth economy, many people still rely on food-banks. According to the United Nations, 690 million, 8.9 percent of the global population, were undernourished (FAO, 2020) while over 135 million have suffered from acute food insecurity (FAO, 2021; WFP, 2020a). Moreover, it is estimated that this number has been doubled during the COVID-19 pandemic (WFP, 2020b). To address these challenges, some governments offer tax relief to retailers who donate food (Alexander and Smaje, 2008; Giuseppe *et al.*, 2014; Chu *et al.*, 2018).

However, the retailers often tend to see donation as an opportunity to salvage low-quality inventory, whereas, the food-banks prefer goods that will stay fresh during the distribution, since they do not have the resources and capabilities to manage inventories. Fresh groceries usually have tentative due dates and visual inspection alone might be insufficient to estimate the true (effective) quality. Fortunately, the internet of things (IoT)-enabled condition tracking technologies, such as time temperature indicators (TTIs), have been miniaturized and become cost-effective (Dada and Thiesse, 2008). Such technology can ease the food-safety concerns of charities and help firms better plan for the selling season.

This chapter studies a food retailer's operational planning problem for a continuously deteriorating inventory over two periods, where the retailer faces uncertainty only in the first period. The length of the selling season is determined dynamically by the shelf-life of the products. We assume a price- and quality-sensitive demand. Before the start of the selling season, the retailer jointly determines the stocking quantity and regular price. At the end of the first period, the uncertainty in demand resolves and the retailer decides on the amount of inventory to be donated with the second period price. We model the enhanced tax deduction of the firm by incorporating the U.S. government's tax subsidy for food donations into the retailer's after-tax profit function and analyze the impact of the tax subsidy parameters on the retailer's optimal decisions and profit.

Our analysis revealed that as opposed to the conventional wisdom, the retailer's optimal donation volume may decline with respect to the amount of leftovers at the end of period 1, their effective quality and the tax incentive coefficients. Such unorthodox findings arise as a result of the government's tax deduction being tied to the retailer's second period price. Moreover, we observe that the enhanced tax deduction benefits the retailer most when the degree of uncertainty is high. Finally, donations trigger only a slight increase in price while significantly increasing the stocking quantity.

In the next section, we review the literature. In Section 3, we lay out the assumptions, introduce the demand functions, and develop the retailer's problem. We present the managerial insights about the firm's donation behavior in Section 4. In Section 5, we conduct numerical experiments to find out the impact of several problem factors, such as, degree of uncertainty and the length of period 1 on the performance of the retailer's optimal decisions with and without tax deductions. Finally, we outline our findings and point out venues for future research in the last section.

4.2 Literature Review

Our study is related to three research streams in the literature: freshness-dependent demand models of perishable goods in the era of IoT, two-period Newsvendor-pricing models, and government subsidies for charitable donations.

Fresh grocery products most often have tentative *best before* dates as opposed to consumer packaged goods which show definite due dates. The rationale behind this practice is that the aging process of fresh goods after harvest/production primarily depends on the environmental conditions during transportation and storage. Labuza (1984) developed chemical reaction kinetics of fresh foods with respect to changing environmental temperatures to quantify their deterioration process trough time. With the advent and miniaturization of the IoT-enabled condition tracking technologies, e.g., TTIs, it is now more cost-effective and convenient for firms to access timely temperature data to estimate accurate (effective) quality of the fresh goods they sell. The reliability of TTI data was examined by the extant literature (see, for example, Dada and Thiesse, 2008; Bowman *et al.*, 2009). Osvald and Stirn (2008); Rong *et al.* (2011) use a linear quality loss scheme in their problem while optimizing the routing problem of perishable foods. Wang and Li (2012) incorporate an exponentially decaying quality loss model into a deterministic demand function to find the optimal markdown policy for a supermarket. Following Chapter 3, we develop a quality deterioration function, which can be either linear or exponential depending on the product type. However, unlike their paper, this study derives demand from a quadratic consumer valuation model that depends on price and quality.

This chapter develops a two-period newsvendor-pricing model for a continuously deteriorating product, where the uncertainty in demand is revealed at the end of the first period. Ferguson and Koenigsberg (2007) used a similar methodology to study a situation where a firm faces the competition between new and old products during the second period. However, they do not take donation as a viable option and consider the quality in a binary-fashion (new and old). Cachon and Kök (2007) presented a work on salvage value estimation of a newsvendor and, recently, Chu *et al.* (2018) developed a model built on their theoretical framework. However, neither of these studies the quality as a factor in their models.

The issue of charitable donations in the presence of government subsidies have been only scarcely studied. Giuseppe *et al.* (2014) presented a deterministic model where a food retailer can salvage some of her inventory by donating to a food-bank and/or sending to a livestock market. Arya and Mittendorf (2015) developed a deterministic framework and analyzed the subtle effects of government subsidies on a supply chain that consists of a retailer and a supplier. In a similar vein, Wang *et al.* (2019c) followed a deterministic approach to model a manufacturer-retailer supply-chain, where either of the parties may donate some inventory depending on the government's tax deduction. We have found only two stochastic frameworks that incorporate donation into a firm's operational planning. Among them, Chu *et al.* (2018) developed a two-period newsvendor model with a general demand function without quality considerations, and solved the problem numerically. On the other hand, in Chapter 3 we developed a two-period newsvendor problem with random demand in both periods and considered overage and shortage costs. However, we assumed a constant reward for donated inventory and used a different demand function than the one used in this chapter.

In line with Chu *et al.* (2018), we incorporate the tax subsidy offered by the U.S. government into the retailer's after-tax profit function. However, using a quality-dependent demand function, we solve our problem analytically, and present some insights on the tax deduction parameters. Our numerical analysis indeed yields some contrasting results. For example, we observe the enhanced tax deduction raises the first period price while Chu *et al.* (2018) reported the otherwise. Moreover, assuming the actual tax deduction policy offered by the U.S. government, the reward derived from donation is tied to the retailer's second period price, which results in a substantially different optimal policy than the one suggested in Chapter 3.

4.3 Model Framework

In this section, we lay out the problem assumptions, introduce quality deterioration schemes and develop demand functions.

4.3.1 Assumptions

We study the operational planning problem of a food-retailer for a continuously deteriorating inventory. The retailer stocks up her inventory at the beginning of the selling season and depletes it over two periods.

The market demand is assumed to be both price- and quality-dependent, and to have an uncertain component that is to be revealed at the end of the first period. The length of the selling season is characterized by the shelf-life of the items at the beginning of period 1, T. Note that fresh produce and frozen meat products often have tentative due dates, but we assume that the retailer can estimate the true (effective) quality of the goods by inputting the timely temperature data into the appropriate reaction kinetics model as proposed by Labuza (1984). Such data can be easily provided via IoT-enabled sensors, such as time temperature indicators (TTIs), or via smart packages that are equipped with freshness sensors. Thus, she can internally determine the shelf-life of the products and plan for the selling season accordingly. The accuracy of TTI technology and quality degradation models using TTI data have been validated by the past literature (see, for example, Giannakourou and Taoukis, 2002, 2003).

The retailer's operational planning comprises of inventory and pricing decisions, where she plans not only for the purchase of the goods but also for donating them.

She considers donation as a part of her corporate social responsibility (CSR) act and as a means to maximize her after-tax profit. The sequence of events is depicted by Figure 4.1 and can be summarized as follows:

1) Prior to the selling season, the retailer observes the effective quality of the goods at the supplier's site and jointly determines the ordering quantity Q, that she will commit to until the end of the selling season, and regular price, P_1.

2) At time $t = 0$, the retailer receives the goods and the selling season kicks off.

3) At time $t = T_1$, the uncertain component of demand becomes known and the retailer observes the remaining inventory, I. Then, she carries s_2 units forward to be sold in the second period and donates $I - s_2$ units. Because the demand is deterministic in period 2, the clearance price, $P_2(s_2)$, is defined as a function of s_2.

We also assume that the length of the first period, $T_1(< T)$, is exogenous and can be determined based on experience when the due date is known. This assumption is reasonable for most perishables as supermarkets sell similar goods for consecutive selling seasons and, therefore, are able to develop know-how on estimating when the uncertainty in the demand of a particular product resolves. Besides, such assumptions are common in the literature (see, for example, Ferguson and Koenigsberg, 2007; Cachon and Kök, 2007).

In the first period, the retailer incurs a cost c per unit of inventory, where c is composed of purchasing expenses and inventory carrying cost over $[0, T_1)$. On the other hand, the company incurs an additional holding cost, h, per unit of inventory sold in the second period. To quantify the tax deduction, we consult the guidelines

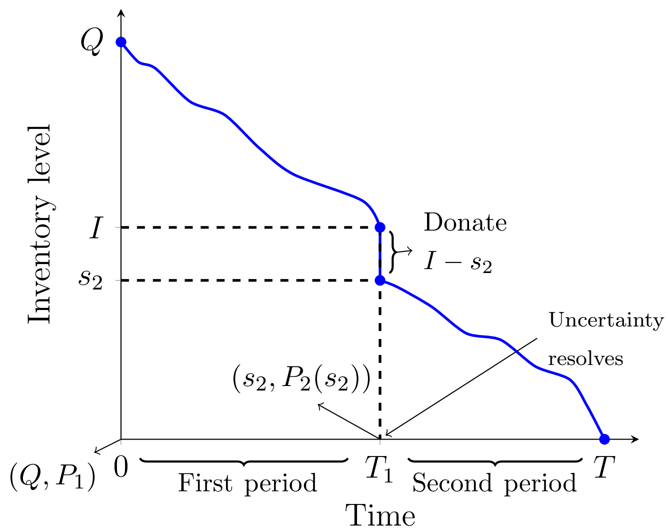


Figure 4.1: Sequence of events.

provided by the U.S. Department of the Treasury, Internal Revenue Service (IRS). Accordingly, there are two dynamics involved while calculating charitable contribution deductions for food: fair market value and cost basis. As Publication 526 (Charitable Contributions for use in preparing 2020 returns) states "When determining the fair market value [...]take into account the price at which the same or substantially the same food items (as to both type and quality) were sold by you at the time of the contribution." Thus, we assume the second period price, $P_2(s_2)$, as the fair market value of the product. Publication 526 defines the basis as "The basis of contributed inventory is any cost incurred for the inventory in an earlier year that you would otherwise include in your opening inventory for the year of the contribution." Hence, we take c as the basis cost of the donated goods.

Such assumptions lead to a scheme where per unit donation of an inventory of cost c reduces the taxable income by $\min\{2c, c+(P_2(s_2)-c)/2\} = c+\min\{c, (P_2(s_2)-c)/2\}$. In line with Chu *et al.* (2018), for a given federal tax rate τ, a general formula can be adopted by defining per unit tax subsidy as $c + \min\{\alpha c, \beta(P_2(s_2) - c)\}$. We also follow their assumptions on the tax deduction parameters, $\alpha, \beta \in [0, \frac{1-\tau}{\tau})$. Under the current tax law, we have $\alpha = 1$, $\beta = 0.5$, and $\tau < 40\%$. Therefore, we assume that the retailer receives an extra deduction (besides the unit inventory cost) of $r(P_2(s_2)) := \min\{\alpha c, \beta(P_2(s_2) - c)\}$ in her taxable per unit donated.

4.3.2 Quality Loss Function

The customers are quality-sensitive and the retailer's inventory degrades continuously during the selling season. However, the retailer can estimate the quality deterioration pattern of the goods over their lifetime by leveraging TTI data. As first described by Labuza (1984), the reaction kinetics of food can be estimated by using historical temperature data. With the advances of the technology, TTI labels are now miniaturized, affordable, and able to collect and communicate timely temperature data. We adopted the framework suggested in Chapter 3, where we consider two quality loss models, linear and exponential, corresponding to fresh groceries and frozen meat/fish products, respectively.

The goods are at their best condition just after harvesting/production and have the maximum quality of %100. The retailer observes the quality of the items at the supplier's site, $0 < q \leq 1$, and can predict their effective quality upon their arrival to the shelves, $q - d$, where $0 < d < q$ is considered as the quality drop during the transportation of goods. At the retailer's store, the condition of the goods deteriorate at a constant rate of $\lambda > 0$.

We note that the effective quality of the goods may not be the same as their perceived quality as the customers have a limited ability to determine the condition of the products, whereas, the retailer can make use of the TTI output to estimate it accurately. Thus, we need to know the perceived quality of the goods, denoted by $\delta(t)$ for $t \in [0, T]$, to understand the impact of quality on customer demand. Because the selling season is divided into two periods, we also define the average quality over period 1 as $\bar{\delta}_1(T_1) = (1/T_1)\int_0^{T_1} \delta(t)\, dt$ and over period 2 as $\bar{\delta}_2(T_2) = (1/T_2)\int_{T_1}^{T} \delta(t)\, dt$, where $T = T_1 + T_2$.

4.3.2.1 Linear Quality Loss Function

We use a linear quality decay scheme to model deterioration kinetics of fresh grocery items (Labuza, 1982). Following Chapter 3, which extends the works of Bowman *et al.* (2009) and Osvald and Stirn (2008), changes in quality is not apparent to customers until the ripeness of the goods hit a threshold called maximum discernibility level, denoted by $\nu > 0$. Therefore, when the effective quality is greater than or equal to the maximum discernibility threshold, customers perceive it as ν. We also specify the minimum acceptable quality level, $\eta > 0$, below which the goods are unfit for human consumption.

Now, we define the perceived quality as $\delta(t) = \min\{\nu, q - d - \lambda t\}$ for $t \in [0, T]$, where $q - d - \lambda t$ is the effective quality. At $t = (q - d - \nu)/\lambda$, we realize $\nu = q - d - \lambda t$ and, therefore, we can rewrite the perceived quality as,

$$\delta(t) = \begin{cases} \nu, & t \leq (q - d - \nu)/\lambda, \\ q - d - \lambda t, & t > (q - d - \nu)/\lambda. \end{cases} \tag{4.1}$$

We see that $q - d$ may or may not be greater than ν, resulting in two possible scenarios while calculating the average perceived quality. For $q - d > \nu$, let $k \equiv$

$(q-d-\nu)/\lambda$, this gives rise to the following average perceived quality over the first period:

$$\bar{\delta}_1(T_1) = (1/T_1)\int_0^{T_1} \delta(t)\,dt = (1/T_1)\left[\int_0^k \nu\,dt + \int_k^{T_1}(q-d-\lambda t)\,dt\right],$$
$$= (1/T_1)\{k\nu + [q-d-\lambda(T_1+k)/2](T_1-k)\},$$

where, without loss of generality, we assume that physical changes become visible during period 1. This assumption is reasonable as the retailer would consider repricing the goods after their quality becomes apparently low. The average perceived quality over period 2 (when $t > (q-d-\nu)/\lambda$) takes the form:

$$\bar{\delta}_2(T_2) = (1/T_2)\int_{T_1}^{T}(q-d-\lambda t)\,dt = q-d-\lambda(2T-T_2)/2.$$

As a result, the product shelf-life can be described as $T = (q-d-\eta)/\lambda$.

4.3.2.2 Exponential Quality Loss Function

Some perishables experience an exponential quality deterioration over time. To model this behavior, we adopt a quality deterioration scheme used by Wang and Li (2012), $\delta(t) = qe^{-\lambda t}$. Thus, the average perceived quality can be given as,

$$\bar{\delta}_1(T_1) = (1/T_1)\int_0^{T_1} qe^{-\lambda t}\,dt = (q/\lambda)(1-e^{-\lambda T_1})/T_1,$$
$$\bar{\delta}_2(T_2) = (1/T_2)\int_{T_1}^{T} qe^{-\lambda t}\,dt = (q/\lambda)e^{-\lambda T}(e^{\lambda T_2}-1)/T_2.$$

4.3.3 Inverse Demand Functions

We adopt an inverse demand function framework that reflects the customers' sensitivity to price and quality. Accordingly, the price in period $i = 1, 2$, P_i, can be described as,

$$P_2(s_2) = \bar{\delta}_2(T_2)(a + x - s_2), \tag{4.2}$$

$$P_1(s_1) = \bar{\delta}_1(T_1)(a + X - s_1), \tag{4.3}$$

where s_i denotes the sales amount in period $i = 1, 2$, a is the market potential parameter, and X is the stochastic part of the market potential with realization x. This framework can be derived from a quadratic utility. Let U_i denote the utility of a representative customer at period $i \in \{1, 2\}$ and $q_i = \bar{\delta}_i(T_i)$ is defined for the sake of brevity, then we obtain:

$$U_i = q_i(As_i - Bs_i^2/2) - P_i s_i,$$

where maximizing the above equation with respect to s_i for $B = 1$ gives rise to $P_i(s_i) = q_i(A - s_i)$, $i \in \{1, 2\}$. The market potential, A, has a deterministic part, a, and a stochastic part, X, which is to be revealed at the end of the first period. We assume X has a finite mean, μ, standard deviation, $\sigma > 0$, and it is drawn from cumulative distribution $F(.)$, which is defined over $[0, \bar{X}]$, twice-differentiable in its domain and has a non-decreasing hazard rate. We also assume $aq_i > c + h$, $i = 1, 2$, to assure that the sale of goods generates a positive profit margin to the firm.

4.4 Model Analysis

We develop and solve the retailer's operational planning problem over two periods by using backward induction starting with the second period.

4.4.1 Second Period (No Uncertainty)

We first define the before-tax profit function of the firm without donations as,

$$\pi_2(s_2) = (P_2(s_2) - h)s_2 = [q_2(a + x - s_2) - h]s_2, \tag{4.4}$$

where h is the additional holding cost per unit of inventory carried forward to period 2. Note that the firm's optimal sales amount under the no-donation policy is given as,

$$\hat{s}_2 = \min\left\{I, s_2^0\right\}, \quad s_2^0 = \frac{q_2 A - h}{2q_2}.$$

Next, we incorporate the government subsidy for donated items into the firm's profit function:

$$J_2 = (1 - \tau)\pi_2(s_2) + \tau r(P_2(s_2))(I - s_2),$$

where $r(P_2(s_2)) = \min\{\alpha c, \beta(P_2(s_2) - c)\}$ denotes the per unit donation subsidy offered by the government and $(I - s_2)$ is the amount of inventory to be donated. Obviously, $s_2 \leq I$ as the retailer cannot carry forward more units than she has.

Let us define $\tilde{s}_2 = A - (1 + \alpha/\beta)(c/q_2)$ to facilitate the discussions. To maximize her after-tax profit, the firm needs to consider two optimization problems depending on the realization of $r(P_2(s_2))$: 1) (**P1**) when $s_2 \leq \tilde{s}_2$ leading to $r(P_2(s_2)) = \alpha c$ and 2) (**P2**) when $s_2 \geq \tilde{s}_2$ leading to $r(P_2(s_2)) = \beta(P_2(s_2) - c)$. Thus, we obtain

$J_2^* = \max\{J_2^{1^*}, J_2^{2^*}\}$, where $J_2^{1^*}$ and $J_2^{2^*}$ are the optimal objective values of $(\mathbf{P1})$ and $(\mathbf{P2})$, respectively.

$$\begin{aligned} (\mathbf{P1}):\quad & \max_{s_2} J_2^1 \\ \text{s.t.}\quad & 0 \le s_2 \le \min\{I, \tilde{s}_2\} \end{aligned}, \qquad \begin{aligned} (\mathbf{P2}):\quad & \max_{s_2} J_2^2 \\ \text{s.t.}\quad & \max\{0, \tilde{s}_2\} \le s_2 \le \min\{A, I\} \end{aligned}$$

The first-order-conditions (FOCs) of J_2^1 and J_2^2 are as follows:

$$J_2^{1'} = (1-\tau)(q_2 A - 2q_2 s_2 - h) - \tau\alpha c, \tag{4.5}$$

$$J_2^{2'} = (1-\tau)(q_2 A - 2q_2 s_2 - h) - \tau\beta[q_2(A+I) - 2q_2 s_2 - c]. \tag{4.6}$$

Henceforth, we denote the sales amount derived from the FOCs as s_2^i, $i = 1, 2$:

$$s_2^1 = \frac{1}{2q_2}\left(q_2 A - h - \frac{\tau}{1-\tau}\alpha c\right), \tag{4.7}$$

$$s_2^2 = s_2^2(I) = \frac{1}{2q_2\left[1 - \frac{\tau}{1-\tau}\beta\right]}\left(q_2 A - h - \frac{\tau}{1-\tau}\beta[q_2(A+I) - c]\right). \tag{4.8}$$

Theorem 7. *Both J_2^1 and J_2^2 are concave functions.*

The proof follows from the FOCs given in equations (4.5) and (4.6). The following theorem characterizes the optimal second period decisions of the firm.

Theorem 8. *Three main scenarios summarize the firm's second period optimal solution.*

(1) $q_2 \le (1 + \alpha/\beta)(c/A)$ *(Low quality):*

$s_2^* = \max\{0, \min\{s_2^2(I), I\}\}$.

(2) $(1 + \alpha/\beta)(c/A) < q_2 \leq (1/A)\left[2(1 + \alpha/\beta)c - h - \alpha c\tau/(1 - \tau)\right]$ *(Medium quality):*

$s_2^* = \min\{I, \max\{s_2^2(I), \tilde{s}_2\}\}.$

(3) $q_2 > (1/A)\left[2(1 + \alpha/\beta)c - h - \alpha c\tau/(1 - \tau)\right]$ *(High quality):*

$s_2^* = \min\{I, s_2^1\}.$

Proof. See Appendix C.1.

□

The second-period problem has a closed-form solution, but we also need the second period value function before moving forward to the first period analysis. Let us define $H_i(s_2, I) = J_2^i(s_2)$, $i = 1, 2$. Following Theorem 8 we can introduce the second period value function as,

(1) Low quality:

$$\Phi(I) = \begin{cases} H_2(I, I), & 0 < I \leq \bar{I}, \\ H_2(s_2^2(I), I), & \bar{I} < I \leq \check{I}, \\ H_2(0, I), & I > \check{I}, \end{cases} \tag{4.9}$$

(2) Medium quality:

$$\Phi(I) = \begin{cases} H_1(I, I), & 0 < I \leq \bar{I}, \\ H_2(s_2^2(I), I), & \bar{I} < I \leq \tilde{I}, \\ H_1(\tilde{s}_2, I), & I > \tilde{I}, \end{cases} \tag{4.10}$$

(3) High quality:

$$\Phi(I) = \begin{cases} H_1(I, I), & 0 < I \leq s_2^1, \\ H_1(s_2^1, I), & I > s_2^1, \end{cases} \tag{4.11}$$

where $\check{I}$, $\tilde{I}$, and $\bar{I}$ are defined such that $s_2^2(\check{I}) = 0$, $s_2^2(\tilde{I}) = \tilde{s}_2$, and $s_2^2(\bar{I}) = I$. Because $s_2^2(I)$ is monotone (decreasing) in I, such threshold values can be established. Moreover, we note that $H_1(I, I) = H_2(I, I) = (1 - \tau)\pi_2(I)$, $H_1(\tilde{s}_2, I) = H_2(\tilde{s}_2, I)$.

When I is below an inventory-threshold, which is determined according to the quality of the items ($\bar{I}$ for low and medium quality, s_2^1 for high quality), the firm carries forward all leftover inventory from the first period to the clearance period, and thereby realizes profit $H_1(I, I) = H_2(I, I)$. However, when I is greater than the threshold value, the optimal donation amount gradually grows as it increases. A counter-intuitive finding is that $s_2^2(I)$ is decreasing in I, meaning that the company sells fewer units (and charges a higher price) as the amount of leftover inventory increases, when the products' quality is low or medium. For instance, when the firm has more than $\check{I}$ units of low quality products, she donates her whole inventory and sells nothing during the second period. The motive behind this unorthodox behavior of the firm is that when $I \leq \bar{I}$, the sales generate more after-tax profit than the government's tax subsidy does, whereas, when $I > \bar{I}$, instead of dropping the price to sell all leftovers, the firm is better of by increasing the volume of donations and clearance period price. On the other hand, when the leftover inventory is of high quality, the firm's optimal decision is to sell up to s_2^1 units and donate any spillovers. This result contrasts with the findings of Chapter 3, where we have reported a monotone relation between ending inventory of period 1 and the optimal donation policy. The following example illustrates the optimal inventory carrying/donation policy of the firm for medium quality.

Example 2 (Medium quality)**.** *Let* $A = 12.9$*,* $c = 1$*,* $h = 0.05$*,* $q_2 = 0.3515$ *with tax rate* $\tau = 0.35$ *and government subsidy parameters* $\alpha = 1$ *and* $\beta = 0.5$*. This set-up leads to* $\tilde{s}_2 = 4.37$*,* $s_2^1 = 5.61$*,* $\tilde{I} = 13.63$*, and* $\bar{I} = 5.81$*.*

Figure 4.2 shows the optimal sales amount and clearance price with respect to different leftover inventory volumes.

As described by policy (4.10), the optimal sales amount increases in on-hand inventory until $I = \bar{I}$ and then decreases when $I \in [\bar{I}, \tilde{I}]$, and settles at $s_2^* = \tilde{s}_2$ afterwards. As we assume one-to-one correspondence between demand and price, the clearance price follows a similar pattern, but in the reverse direction until it settles at $P_2(\tilde{s}_2)$. We also compare our findings with the optimal policy under no-donation

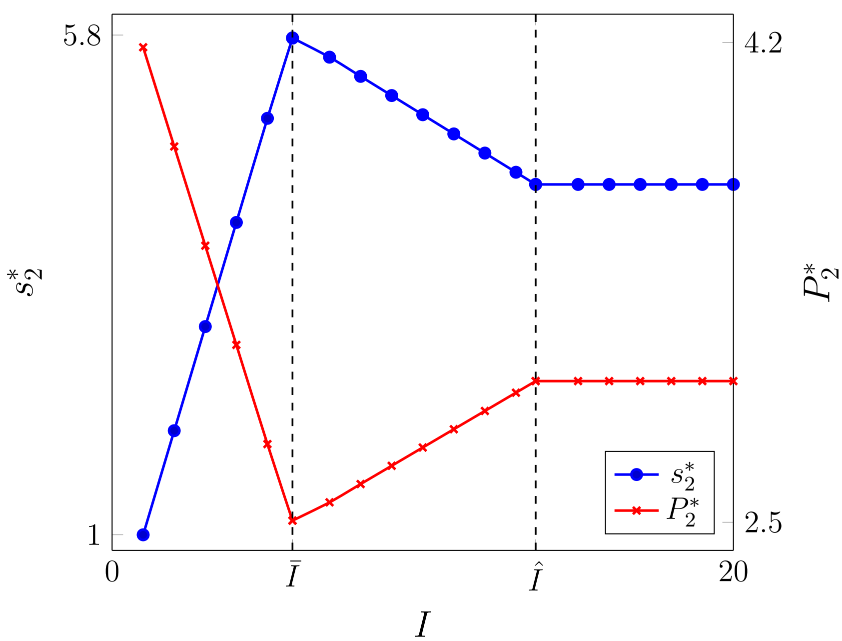


Figure 4.2: The optimal sales amount and clearance price with respect to different leftover inventory volumes.

subsidy. Figures 4.3 and 4.4 depict the comparisons of after-tax profits, and the optimal clearance period sales and price, respectively. A couple of observations are in order. First of all, the donation enabled policy of the firm outperforms the no-donation policy when $I > \bar{I}$. The profit gap reaches up to 50%. Also, as we assume the products have no salvage value in the absence of the tax subsidy, the marginal contribution of additional leftover inventory hits zero when $I > \bar{I}$. However, the firm's after-tax profit under the tax subsidy grows linearly with I as $H_1(\tilde{s}_2, I)$ is a linearly increasing function of I. Finally, as can be seen from Figure 4.4, the firm achieves a higher after-tax profit by selling fewer units and charging a higher price.

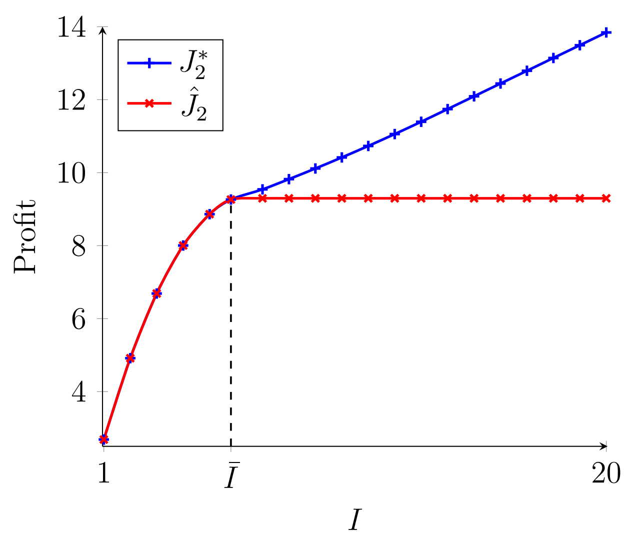


Figure 4.3: The profits with respect to different leftover inventory volumes with and without the government's tax subsidy.

The following proposition analyzes the value function.

Proposition 4. *The second period value function, $\Phi(I)$, is strictly increasing, but may not be concave.*

Proof. We know that $H_1(I, I) = H_2(I, I) = (1 - \tau)\pi_2(I)$ is concave as $\pi_2(s_2)$ is a

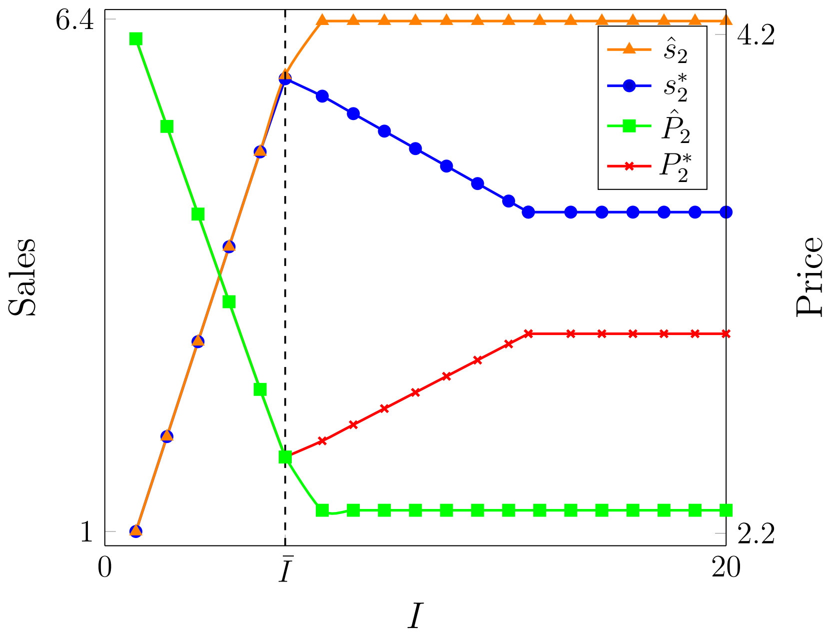


Figure 4.4: The optimal sales amount and clearance price with respect to different leftover inventory volumes.

concave function. However, one can show that $H_2(s_2^2(I), I)$ is convex and $H(\tilde{s}_2, I)$ is a linear function of I. Because the purchasing costs are charged in the first period, any additional inventory generates a positive profit margin to the firm. $\square$

Now that we have developed the second period value function and set forth its properties, we can analyze the firm's operational planning problem in period 1.

4.4.2 First Period (with Uncertainty)

At the start of period 1, the retailer jointly determines the purchasing quantity, Q, to be sold over two periods, and the first period price, P_1. The market potential, $a + X$, has an uncertain component, X, defined on the interval $[0, \bar{X}]$ with twice differentiable cumulative distribution function $F(x)$, density function $f(x)$, mean μ, and standard deviation σ.

The first period demand can be written as $s_1(P_1) = a + X - P_1/q_1 = y(P_1) + X$, where $y(P_1)$ is defined as the deterministic part of the demand, $a - P_1/q_1$. To assure the positivity of demand, P_1 must be less than its upper-bound, $\bar{p} = q_1 a$. Following Petruzzi and Dada (1999), we define the stocking factor variable, $z = Q - y(P_1)$, and optimize the problem over (z, P_1). We start by developing the firm's random before-tax profit function in the first period:

$$\pi_1(z, P_1) = \begin{cases} P_1[y(P_1) + X] - c[y(P_1) + z], & X \le z, \\ (P_1 - c)[y(P_1) + z], & X > z, \end{cases}$$

which is essentially a newsvendor problem with demand having an additive randomness. The retailer's expected before-tax profit can be represented as,

$$\Gamma(z, P_1) = P_1[y(P_1) + \mu] - c[y(P_1) + z] - P_1\Theta(z), \tag{4.12}$$

where $\Theta(z) = \int_z^{\bar{X}} (x - z)\, dF(x)$ is the expected shortage. Now, we derive some properties of the retailer's first period before-tax expected profit function. Following the past literature (Petruzzi and Dada, 1999; Zabel, 1970), in search of the maximum value of (4.12), we first define the price for a fixed z and search over the resulting optimal trajectory.

Lemma 1. *For a given z, $\Gamma(z, P_1)$ is maximized by a unique price given by*

$$P_1^* \equiv p(z) = \begin{cases} \tilde{p} - q_1 \frac{\Theta(z)}{2}, & z \in [0, \bar{X}], \\ \tilde{p}, & z > \bar{X}, \end{cases}$$

where $\tilde{p} = (q_1/2)(a + \mu + c/q_1) = p(\bar{X})$ *is the riskless price that maximizes the profit in the absence of uncertainty.*

Proof. The proof follows from the FOC and SOC of $\Gamma(z, P_1)$ with respect to P_1. See Appendix C.2 for details. □

We note that $p(z)$ is an increasing function for $z \in [0, \bar{X}]$ and that we assume $\tilde{p} < \bar{p}$ to assure the positivity of demand for $z \geq \bar{X}$. We consider z values outside the range $[0, \bar{X}]$ because when $\bar{X} < \infty$, the retailer may realize $z^* \geq \bar{X}$ as she will commit to the initial stocking quantity, Q^*, for two periods. In such a case, the retailer never faces shortages during the first period as she has sufficient inventory to satisfy demand even if the uncertainty in demand is realized at its upper-bound, $x = \bar{X}$. Therefore, the firm's optimal first period price becomes the riskless price, $p(z) = \tilde{p}$ for $z \geq \bar{X}$. Theorem 9 demonstrates that $\Gamma(z, p(z))$ has a unique maximizer.

Theorem 9. *The retailer's first period expected before-tax profit function,* $\Gamma(z, p(z))$, *is concave for* $z \in [0, \bar{X}]$ *if* $f(0)y(-c) > 1$, *and it is unimodal with a local maximum, otherwise. Also,* $\Gamma(z, p(z))$ *is a linearly decreasing function for* $z \geq \bar{X}$ *with* $\Gamma'(z, \tilde{p})|_{z \geq \bar{X}} = -c < 0$.

Proof. See Appendix C.3. □

Petruzzi and Dada (1999) already showed $\Gamma(z, p(z))$ is unimodal for $z \in [0, \bar{X}]$, whereas, Theorem 9 extends their results by (1) introducing the conditions under which $\Gamma(z, p(z))$ is also concave, and (2) evaluating the behavior of $\Gamma(z, \tilde{p})$ for $z \geq \bar{X}$.

The retailer aims to maximize her expected after-tax profit over two periods which can be formulated as,

$$J_1 = (1 - \tau)\Gamma(z, p(z)) + \rho\mathbb{E}[\Phi(\max\{0, z - X\})], \tag{4.13}$$

where ρ is the discount rate. Notice that the conditions of Lemma 1 still hold as the second term in (4.13) is independent of the first period price and we can optimize the problem over the stocking factor. At the start of the selling season, the retailer can accurately predict the effective quality of the goods at the end of period 1, as we assume a stable deterioration rate at the retailer's site, and can determine her purchasing quantity and first period price accordingly. If the deterioration rate changes for some reason during the first period, she shall still follow the optimal policy given in Theorem 8. The following theorem characterizes the optimal solution to the retailer's operational planning problem over two periods:

Theorem 10. *The optimal stocking factor z^* that maximizes J_1 can be found according to the quality of the goods at the end of the first period.*

- ***(a)*** *Low or medium quality: $J_1' = 0$ can have at most three roots. If it has only one root, it corresponds to the optimal solution; if it has two roots, the larger one corresponds to the optimal solution; and if it has three roots, one of them corresponds to a local minimum and, therefore, the optimal solution can be found by searching the maximum of J_1 over the other two roots.*

- ***(b)*** *High quality: J^1 is concave and there is a unique optimal solution.*

Proof. See Appendix C.4 □

It is worth mentioning that, in our numerical analysis, we have always found a unique z^* for low and medium quality scenarios implying that a large variety of parameter combinations lead to a unimodal objective.

4.5 Managerial Insights

In this section, we analyze the drivers of the firm's donation decision in the second period and reveal their impact on the optimal clearance price.

4.5.1 Impact of the Tax Subsidy Parameters on Donations

In this part, we analyze the role of the government's tax subsidy parameters on the firm's optimal decisions. Let us start with the clearance period:

Lemma 2. *The impact of the government's tax subsidy on s_2^1, $s_2^2(I)$, and $\tilde{s}_2$ is summarized as,*

1) *s_2^1 is decreasing in α and the tax rate, τ.*

2) *$s_2^2(I)$ is decreasing in β and the tax rate, τ, if $I \geq (c-h)/q_2$, and increasing in both if $I < (c-h)/q_2$.*

3) *$\tilde{s}_2$ is decreasing in α/β.*

Notice that q_2 is determined by using the effective quality of the items, whereas, the boundaries set forth to categorize the quality as high, medium, low are subjective and depend on the other problem parameters. Lemma 2 suggests that the tax parameters not only influence the sales amounts derived from the FOCs, given in (4.7)–(4.8), but also influence the perceived quality of the goods via shifting $\tilde{s}_2$. Thus, their impact on the firm's optimal decision may be subtle. Theorem 11 demonstrates the relation between the government's tax incentive parameters and the firm's donation behavior. Note that we use increasing/decreasing in the weak sense, unless otherwise stated.

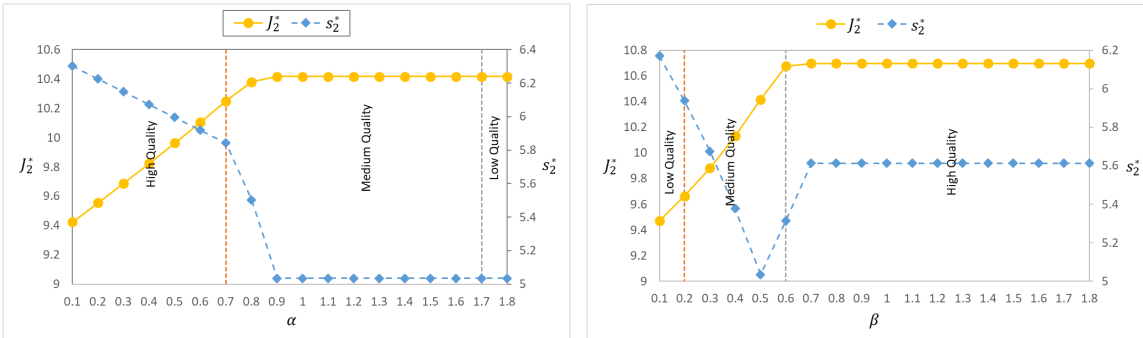
Theorem 11. *For given q_2 and I values, the firm's optimal donation amount is increasing in α for $\tilde{s}_2 > \max\{0, s_2^2(I)\}$ and stable, otherwise, whereas it is increasing in β for $\tilde{s}_2 < \min\{s_2^2(I), s_2^1\}$, decreasing for $s_2^2(I) < \tilde{s}_2 < s_2^1$, and stable for $\tilde{s}_2 > s_2^1$.*

Proof. The proof directly follows from the findings of Theorem 8 and Lemma 2. For details, see Appendix C.5. □

The above theorem suggests a peculiar relation between the firm's optimal donation amount and the government's tax incentive. In particular, a larger tax deduction does not always bring in more charitable donations and, in fact, it may cause an adverse effect, where the retailer prefers to carry more units forward to the clearance period and donate less. This unexpected finding can partially be attributed to the fact that varying tax incentive parameters can drag down s_2^1 and s_2^2 while, at the same time, promoting (demoting) the goods to a higher (lower) quality category. Example 3 illustrates the results of Theorem 11.

Example 3. *Consider Example 2 with $I = 10$ units. We vary α for $\beta = 0.5$ and β for $\alpha = 1$. Also, the retailer's optimal solution without the government's tax incentive is $\hat{s}_2 = 6.38$ leading to $\hat{J}_2 = 9.30$.*

Figures 4.5a and 4.5b depict the optimal sales and profits for different values of α and β. Recall that we focus on the case $\alpha, \beta \in [0, \frac{1-\tau}{\tau})$ with $\tau = 0.35$, as given in Example 2. When one of the parameters is equal to zero, the resulting scenario is equivalent to the no-donation policy, therefore, we analyze $\alpha, \beta \in [0.1, 1.8]$.



(a) The optimal clearance period sales, s_2^*, and profits, J_2^*, for different α values when $\beta = 0.5$.

(b) The optimal clearance period sales, s_2^*, and profits, J_2^*, for different β values when $\alpha = 1$.

Figure 4.5: The optimal clearance period sales, s_2^*, and profits, J_2^*, for different tax incentive parameters.

Supporting the findings of Theorem 11, as α (β) grows, the goods demote (promote) to a lower (an upper) quality category. As can be seen from Figure 4.5a, the optimal sales in period 2 is strictly decreasing in α until $\tilde{I} > I$ and settles at $s_2^2(I)$ thereafter. On the other hand, as shown in Figure 4.5b, the firm faces a trade-off: ascending quality category of the goods motivates the retailer to sell more while increasing per unit revenue of the donation motivates her to donate more. In particular, the retailer is better off by raising the donation amount for $\tilde{I} > I$, whereas, she is better of carrying more inventory forward for $\tilde{I} < I$. The optimal sales settles at s_2^1 (high quality) for large values of β. Interestingly, the maximum donation amount is the same in both cases and it happens around $\alpha = 1, \beta = 0.5$ which are the original values of the parameters set by the government. Moreover, when $\beta > 0.5$, an increase in β may result in the firm receiving a larger tax deduction by donating less, suggesting that keeping β over 0.5 is inefficient for the government in this example.

4.5.2 Effective Quality of the Leftover Inventory

Another dynamic affecting the retailer's optimal decision in the second period is the effective quality (or due date) of the goods. The following example aims to shed light on the role of the quality on the retailer's donation behavior and to compare the performance of the optimal and no-donation policies for different quality scenarios.

Example 4. *Consider Example 2 with* $I = 10$ *units, but for various* q_2 (T_2) *values.*

Figure 4.6–4.8 demonstrates the findings. First of all, for given T_2 values the products fall into two categories, medium quality for $T_2 \in [24, 49)$ and high quality for $T_2 \in [49, 59]$. It looks like the changes in the effective quality does not trigger a significant change in sales, but rather drives the clearance price up under the no-donation policy. On the other hand, although the optimal clearance price is non-decreasing in the remaining shelf-life of the goods, the optimal donation amount may be increasing or decreasing in it. In particular, for $T_2 \in [24, 39)$, the retailer is better off by reflecting the increase in demand (due to longer shelf-life) on the price, which also boosts the per unit reward of donation, and thereby raise her profit by donating more. For $T_2 \geq 39$, the firm sells more as the effective quality elevates, but the rate of increase in sales changes (drops) at the point where the goods are of high quality. As expected, the longer the shelf-life of the products, the more the firm's second period profit. However, the rate of increase appears to be larger under the optimal donation policy.

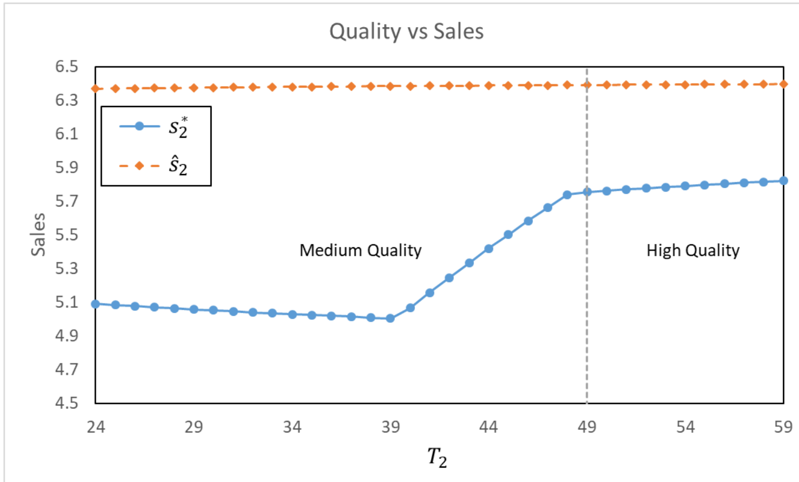


Figure 4.6: The optimal clearance period sales with and without tax incentives, s_2^* and $\hat{s}_2$, respectively, for different remaining shelf-life, T_2, values.

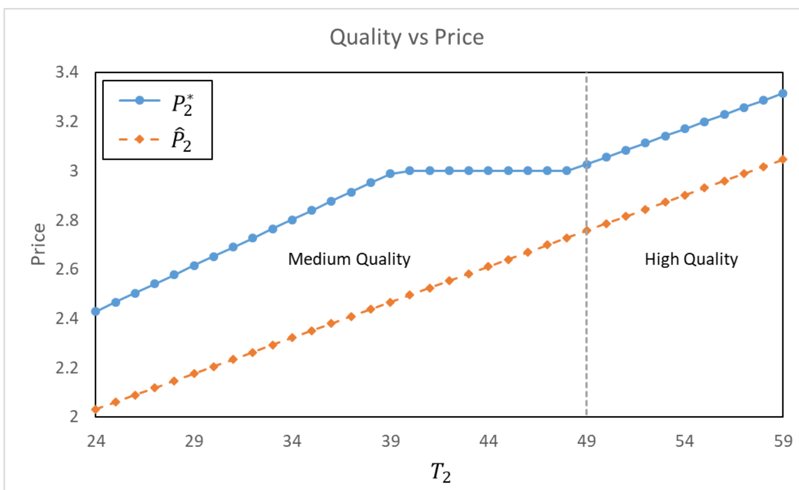


Figure 4.7: The optimal clearance period prices with and without tax incentives, P_2^* and $\hat{P}_2$, respectively, for different remaining shelf-life, T_2, values.

4.6 Numerical Analysis

In this section, we analyze the impacts of randomness, length of the first period, and tax deduction parameters on the retailer's expected after-tax profit over two periods

Figure 4.8: The optimal clearance period profits with and without tax incentives, J_2^* and $\hat{J}_2$, respectively, for different remaining shelf-life, T_2, values.

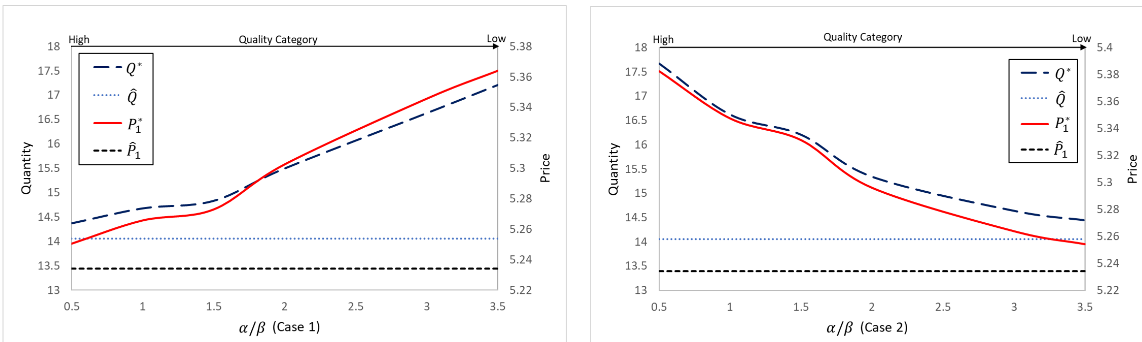
and on her first period decisions. We also compare our results with those found under the no-donation policy to provide some insights on the role of the tax incentives.

We construct scenarios by combining the data presented in 4.1. We consider a linear quality degradation pattern with $q-d = 0.9$, $\nu = 0.8$, $\eta = 0.2$, and $\lambda = 0.009/\text{hr}$ leading to $T = 78$ hours of shelf-life, but we find the optimal solution under different T_1 values. The purchasing and holding costs are given as $c = 1.25$ and $h = 0.05$, respectively. Given the fast deteriorating nature of the goods, ρ is set to 1. The random component of demand follows an exponential distribution with parameter $1/\mu$. We normalize the expected market potential, $a + \mu$, to 15 and vary the (a, μ) pair to alter the prominence of the uncertainty. According to the current tax law, the U.S. government imposes the corporate tax rate of $\tau = 0.21$. This gives rise to a feasibility region of $\alpha, \beta \in [0, 3.7619)$ for the tax incentive parameters. We consider 5 different values of α/β, where $\alpha/\beta = 0.5$ indicates the goods will be of high quality in the second period, whereas $\alpha/\beta = 3.5$ indicates they will be of low quality. Thus, we can examine all three quality categories in our numerical tests. This setting generates 54 different scenarios. However, as shown by Theorem 11, α and β affect the firm's optimal decisions in the second period in different ways. Therefore, we consider two cases: in Case 1, both parameters ascend and in Case 2, both descend as α/β moves from 0.5 to 3.5. As a result, we solve 108 scenarios and present the highlights of our analysis in figures 4.9–4.11.

$T = 78$,	$T_1 \in \{40, 50, 60\}$
$X \sim \text{Exp}(1/\mu)$,	$c = 1.25, h = 0.05, \rho = 1$
$a + \mu = 15$,	$(a, \mu) \in \{(5, 10), (7.5, 7.5), (10, 5)\}$
$\tau = 0.21$,	$\alpha/\beta \in \{0.5, 1, 1.5, 2, 3, 3.5\}$
Case 1:	$(\alpha, \beta) \in \{(0.25, 0.5), (0.5, 0.5), (0.75, 0.5), (1.5, 0.75), (3, 1), (3.76, 1.075)\}$
Case 2:	$(\alpha, \beta) \in \{(1.875, 3.75), (1.5, 1.5), (1.5, 1), (1.4, 0.7), (1.2, 0.4), (1, 0.286)\}$

Table 4.1: Numerical analysis parameters. We consider all parameter combinations for two cases: Case 1 and 2 for different α and β values.

We start by comparing the optimal stocking quantity and first period price under the donation-enabled policy, (Q^*, P_1^*), with the those under the no donation policy, $(\hat{Q}, \hat{P}_1)$, for varying α/β values. Figures 4.9a and 4.9b depict the optimal decisions (averaged) for Case 1 and Case 2, respectively. Accordingly, predicting a superior quality level in the second period does not always imply larger stocking quantity and regular price. On the contrary, the retailer may be better of stocking up more units and sell them at a higher price when the goods are of low quality in period 2 due to enhanced tax deductions. However, the tax deduction raises the quantity drastically (up to 25%) while it raises the price only slightly (less than 3%). This result challenges the findings of Chu *et al.* (2018), who (in their numerical analysis) reported that the regular price can be diminished under the enhanced tax deduction policy. It is worth noting that Chu *et al.* (2018) used a multiplicative demand formulation in their analysis whereas, we adopt an additive one in this study. Thus, the discrepancy between our findings and theirs may partially be attributed to the difference in the two demand modeling approaches.

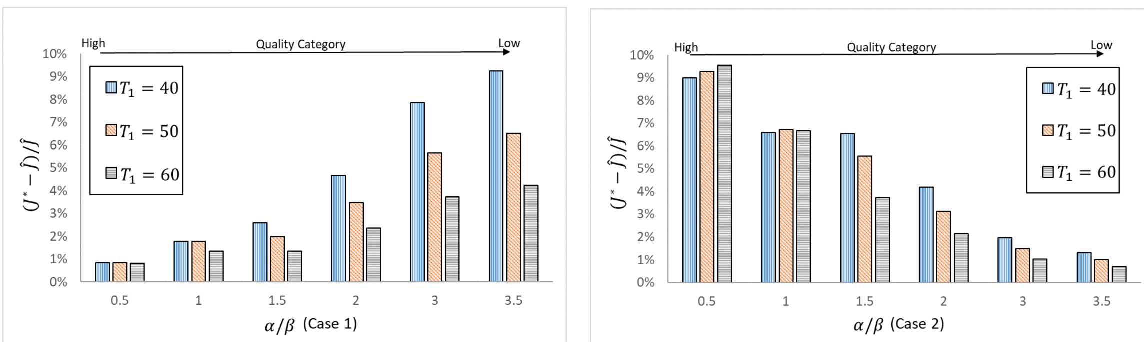


(a) The optimal stocking quantity and regular price values with (without) tax deductions, Q^* ($\hat{Q}$) and P_1^* ($\hat{P}_1$), respectively, for Case 1.

(b) The optimal stocking quantity and regular price values with (without) tax deductions, Q^* ($\hat{Q}$) and P_1^* ($\hat{P}_1$), respectively, for Case 2.

Figure 4.9: The optimal stocking quantity and regular price values with respect to different tax incentive parameters given in Table 4.1.

Next, we analyze changes in the expected after-tax profit, given as $(J^* - \hat{J})/\hat{J}$, with respect to various α/β and T_1 scenarios. Note that T_1 specifies the time at which the uncertain part of the demand is revealed. It is expected that the shorter the first period, the simpler the operational planning of the firm and thereby the more the expected after-tax profits J^* and $\hat{J}$. Thus, we focus on the performance of the donation-enabled policy compared to the no donation policy. Once again, we run the problem instances for both Case 1 and Case 2, and present (averaged) results in figures 4.10a–4.10b. Even though for a given α/β, the gap between the expected profits grows as T_1 shrinks in most instances, Figure 4.10b shows that the opposite may happen for high quality scenarios. When T_1 is small, the company sells less inventory during the first period and donates more. Therefore, the donation-enabled policy often performs better for a shorter first period. On the other hand, when the goods are of high quality and the tax subsidy α is sufficiently large, the possibility of donation serves as an extra pillar against the uncertainty and the profit gap (in percentage) enlarges as the length of period 1 stretches longer.



(a) The difference (in %) between the optimal after-tax profits under the enhanced tax deduction and no-donation policies for different first period lengths and tax incentive parameters given in Case 1.

(b) The difference (in %) between the optimal after-tax profits under the enhanced tax deduction and no-donation policies for different first period lengths and tax incentive parameters given in Case 2.

Figure 4.10: The difference (in %) between he optimal after-tax profits under for different first period lengths and tax incentive parameters given in Table 4.1.

Finally, we examine the impact of uncertainty on the profits as illustrated by Figure 4.11. Because both Case 1 and Case 2 display similar results, we only present the graph for Case 2. We see that the retailer suffers from higher degree of uncertainty with or without the tax deductions. However, the donation-enabled policy seems to be more resilient to uncertainty than the no donation policy as the profit gap (in percentage) ascends with the prominence of the randomness in demand.

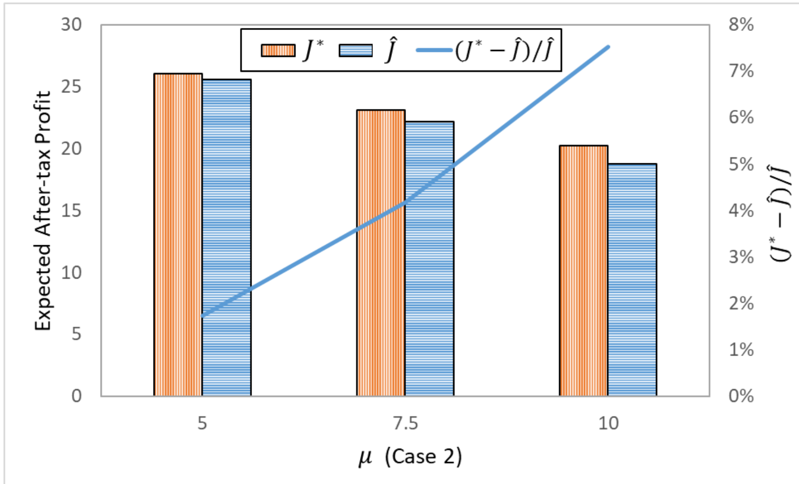


Figure 4.11: The optimal after-tax profits, J_2^* and $\hat{J}_2$, and the improvement (in %) offered by the enhanced tax deduction with respect to different μ values for Case 2.

4.7 Conclusion

This chapter studies a food retailer's integrated operational and CSR planning act for a continuously deteriorating inventory over two periods. Before the start of the selling season, the retailer jointly determines the stocking quantity and regular price. With the end of period 1, uncertainty in demand resolves and the retailer decides on how much inventory to donate and the second period price for the units (if any) that are carried forward to the next period. Any donated item yields an enhanced tax deduction to the firm. We incorporate the U.S. government tax subsidy policy for donated foods into the retailer's expected after-tax profit function, and analyze the impact of tax incentive parameters on the optimal decisions as well as the profit.

The retailer utilizes the data from IoT-enabled condition tracking technologies to estimate the true (effective) quality of the goods. She also categorizes the first period's ending inventory as high, medium, or low depending on the problem parameters. Our findings revealed that as opposed to the conventional wisdom, the retailer may donate less when she has more leftover inventory at the end of the first period, and that her donation amount is not always monotone in her categorization ranks. In particular, the donation amount is tied to tax incentive parameters, α and β, and the retailer may prefer to donate more inventory of higher-ranked quality. Interestingly, raising the tax incentive parameters may result in the retailer donating less inventory, though her earnings from donation is always non-decreasing in those parameters. This adverse effect is caused by the fact that the government's subsidy is linked to the retailer's second period price.

Our numerical analysis revealed that the donations benefit the retailer particularly when the uncertainty is more prominent. The performance of the donation-enabled policy compared to the no-donation policy is not monotone in the length of the first period, though donations prove more efficient for shorter first-period scenarios in many cases. Moreover, the retailer may be better off stocking up more units when the quality of the leftover inventory is predicted as low than when it is predicted as high, depending on the tax incentive coefficients. Finally, planning for donations at the end of the first period boosts both the first period price and the ordering quantity up. However, we observe that the enhanced tax deductions barely affect the first period price and rather impact the retailer's quantity decision. This implies that only a slight portion of the economic burden of the donations is o transferred to the customers.

We recognize that the present study has limitations. First of all, we assume that the uncertainty in demand is revealed at the end of the first period. It would be interesting to investigate the impact of demand uncertainty in the second period and analyze the influence of the enhanced tax deduction on the expected waste. However, when demand is uncertain, the second period value function is not guaranteed to be concave, which imposes more technical difficulty. Another venue for future research is to incorporate a competition between two products of different quality into the problem to see how the firm's product assortment decision is affected by donations. Finally, developing a game theory model between a retailer and a local government to draw some policy insights on tax incentive parameters is a promising research direction. In this context, it is also important to consider the government's trade-off between income tax returns and the donation tax incentives.

Chapter 5

Perils and Merits of Cross-Channel Returns

5.1 Introduction

The retail market landscape has been evolving ever since the introduction of e-commerce. As more customers around the globe are attracted to the web-based stores (Statista, 2020), the competition gets tougher. To stay competitive, most bricks-and-mortar (B&M) firms had to augment their operations by an online channel and have become bricks-and-clicks (also called, dual-channel) retailers.

The COVID-19 pandemic has only escalated the digital transformation. In the United States, in the first quarter of 2021, the e-commerce sales hit $1,581 billion, 39.1% growth since the first quarter of 2020, while the total retail sales only grew by 16.8% during the same period (U.S. Department of Commerce, 2021). The online market constituted 13.6% of the first quarter sales in 2021, which was only 6.4% in 2015 U.S. Department of Commerce (2016).

However, the proliferation of the e-commerce has also brought up new challenges and management of returns can be enlisted among the most persistent ones. According to the pre-COVID data, on average, the in-store returns rates are estimated to be 8–10%, whereas, the online return rates are estimated to be 30%, but may be as large as 40% (for apparel merchandise), depending on the product type (Reagan, 2016). During the COVID-19 outbreak, many companies stretched their return window to attract more online shoppers while many pandemic-weary customers changed their shopping habits which together put additional pressure on the online returns (Reuters, 2021). For example, the rate of consumers bracketing (deliberately purchasing multiple versions of a merchandise to try at home, and then, returning those that do not work) was 40% in 2017, but climbed up to 62% in 2020 (Narvar, 2021).

The online returns may undermine the retailers' profits significantly. Return processing costs range between 20% and 65% of the cost of goods sold Ellis (2017). Thus, many dual-channel retailing giants are looking for alternative ways to handle returns while keeping the customers satisfied. One way many dual-channel retailers, such as, Hudson's Bay (2021) and SportChek (2021), follow is to utilize their physical facilities to handle the return of online purchases, also referred as cross-channel returns. In this model, they charge a shipment fee to the customers for the mailed returns and offer in-store services for free.

The retailer's cross-channel service offering boosts not only the demand but also the returns, and requires a partial channel integration. Therefore, it is not guaranteed cross-channel returns enhance the profits. In this paper, we study this cross-channel returns phenomenon by developing an analytical model for a retailer, who sells her products through both a B&M facility and an online store. We draw insights on if and when welcoming online returns to the physical facility would benefit the retailer. Our model has two versions: (1) exogenous returns, and (2) refund-dependent returns. To the best of our knowledge, we present the first study that incorporates refund-dependent returns to analyze idiosyncrasies of cross-channel returns in a dual-channel retailing environment.

The rest of the paper is organized as follows: We present the relevant literature in Section 5.2. Then, we develop our models and draw managerial insights for the retailer in Section 5.3. Finally, Section 5.4 summarizes the findings and suggests avenues for future research

5.2 Relevant Literature

While e-commerce returns have been a popular topic in the literature (see, for example, Vlachos and Dekker, 2003; Choi *et al.*, 2004; Altug and Aydinliyim, 2016; Fan and Chen, 2020), it was only recently that cross-channel returns have started to receive attention.

While modeling the refund-return relationship we extend the framework first proposed by Mukhopadhyay and Setaputra (2006). They studied the problem of returns in a pure e-tailer. Liu *et al.* (2011); Choi (2013) adopted a similar approach under mass customization. We use their formulation and extend it to study a dual-channel retailer's problem.

The impact of refund policy in dual-channels has been studied in a game-theoretical framework, where a manufacturer sells his products through a direct online channel as well as a traditional retailer. Among the salient works, Li *et al.* (2018c) investigated the impact of the money-back-guarantees, and Li *et al.* (2019c) examined the full-, partial-, and no-refund policies in a dual-channel supply chain. In a similar vein, Batarfi *et al.* (2017) studied a situation where the manufacturer sells the standard products in the conventional channel and the customized as well as refurbished products in the online channel. Zhang *et al.* (2020) considered a green supply-chain and examined the impact of cross-channel returns on the greening strategy of the manufacturer. Our work has a different context as we consider a bricks-and-clicks retailer.

There is a growing literature on the cross-channel returns in a dual-channel retailing environment. Besides few exceptions (Mahar and Wright, 2017; Dijkstra *et al.*, 2019), most studies assume the firm has exactly one conventional store and one online store. Yan *et al.* (2020b) and Nageswaran *et al.* (2020) used consumer valuation models to derive the customer demand and online returns, but ignore the returns of in-store customers and assume uniform pricing. Under similar assumptions, Jin *et al.* (2020) studied the competition between two dual-channel retailers. There are also two newsvendor-type models to analyze effect of cross-channel returns on the firm's optimal procurement decision (Radhi and Zhang, 2019; He *et al.*, 2020). They also ignored the returns of in-store purchases and assumed uniform pricing across both channels to assure mathematical tractability. However, we explicitly model the returns coming from both channels, and consider differentiated pricing across the channels to make our work more relevant to the current practices.

Overall, the extant dual-channel literature lacks studies on the impact of cross-channel returns on individual channel-demand, -prices, and -profits. Therefore, we develop a stylized model where a retailer selling a single product through an online channel as well as a physical store wants to determine the optimal price in each channel as well as the potential benefit and hindrance of cross-channel returns.

5.3 Model Frameworks

This paper investigates the optimal pricing and return policies of a dual-channel retailer, who sells her products through an online channel as well as a bricks-and-mortar (B&M) store.

The retailer chooses between (1) a dedicated returns policy corresponding to a multi-channel setting (M), where the retailer manages two channels separately meaning that only the same channel returns are allowed, and (2) a hybrid returns policy corresponding to a cross-channel setting (C), where the retailer partially integrates the channels by allowing the return of online purchases to her physical facility. Furthermore, the retailer implements a differentiated pricing policy, therefore, we determine the optimal price for each channel under both settings. This assumption allows the retailer to match the prices across both channels only when it is optimal to do so. Multiple-channel firms match prices of clothing products more than 80% of the time while they match the price of office supplies and perishables only less than 40% of the time (Cavallo, 2017). That said, many dual-channel retailers often offer promotions/rebates/coupons in their web-based store, therefore, an online shopper seldom pays the same price with a B&M customer for the same product.

Throughout the paper, we use subscripts r and e to denote the traditional retail channel and the online channel, respectively. The retailer incurs a cost of $c_i > 0$ per unit of product sold in channel $i \in \{r, e\}$, where c_i reflects all operational costs associated with acquiring and selling the product in channel i. In line with the shipping and return policies many dual-channel retailers implement, e.g., Hudson's

Bay, Sportchek, Sephora, etc., we assume,

- the customers incur a shipping fee, $s > 0$, for the last-mile delivery,
- when they return a product to the B&M store, they receive full-refund regardless of which channel they bought the product from,
- when they return an online purchase via shipment, they receive the product value minus return shipping fee, s,
- the returned items are salvaged at the channel they were returned to.

In particular, each returned product generates a return handling cost, $\omega_i > 0$, and has salvage value ν_i in channel $i \in \{r, e\}$. We assume $\nu_i > \omega_i$ to avoid a situation where refunding the unsatisfied customers without collecting the unsatisfactory items is optimal. Despite such a policy is being implemented by some firms, such as Amazon, for low-value items only, it may pose problems when implemented by a fashion retailer or a department store. Some customers may exploit the retailer's policy by claiming refunds for their satisfactory purchases as well.

We analyze two situations: (1) exogenous returns and (2) refund-dependent returns. In the former case, we assume the firm can estimate the return amounts in both channels and they are constants, whereas, in the latter case, we assume the return amount in a channel is tied to the refund amount, i.e., the selling price at the same channel, as the retailer implements full-refund policy. We also draw managerial insights throughout this section whenever necessary.

5.3.1 Multi-channel vs Cross-channel with Exogenous Returns

In this section, we develop the retailer's optimal solution under both channel settings and compare the optimal prices, demands, and profits for given return amounts.

5.3.1.1 Multi-channel Setting

Demand in each channel under no cross-returns is given as,

$$D_r^M(p_r, p_e) = a_r - bp_r + \beta p_e + \beta s \tag{5.1}$$

$$D_e^M(p_e, p_r) = a_e - bp_e + \beta p_r - bs, \tag{5.2}$$

where $a_i > 0$ is the market size parameter and p_i is the selling price in channel $i \in \{r, e\}$. Also, $b > 0$ and $\beta > 0$ are the self- and cross-price sensitivity parameters, respectively. We assume $b > \beta$, i.e., self-price sensitivity is prominent. Because the customers incur the shipping cost, s, it is considered to influence the customer demand in the same way the e-channel price does.

Return amounts are exogenous constants and denoted by $R_r^M = \alpha_r$ and $R_e^M = \alpha_e$. Here, we also define the net demand in channel i as $N_i^M(p_i, p_j) = D_i^M(p_i, p_j) - R_i^M$, $i, j \in \{r, e\}$ with $i \neq j$. To ensure that the net demand in both channels will be positive for some range of the prices, we assume $N_i^M(c_i, c_j) > 0$. Now we can write down the retailer's profit over the two channels as,

$$\Pi^M = \sum_{i,j\in\{r,e\},i\neq j} (p_i^M - c_i)D_i^M(p_i^M, p_j^M) - \alpha_i(p_i^M - v_i + w_i)$$
$$= \sum_{i,j\in\{r,e\},i\neq j} (p_i^M - c_i)N_i^M(p_i^M, p_j^M) - \alpha_i(c_i - v_i + w_i), \tag{5.3}$$

where the first term corresponds to the firm's profit from sales and the second term corresponds to the expenses associated with the returns. In Theorem 12, we establish the optimal prices.

Theorem 12. Π^M *is concave and the optimal prices are as follows:*

$$p_r^M = \frac{1}{2}\left[c_r + A - K^M\right], \quad p_e^M = \frac{1}{2}\left[c_e - s + B - L^M\right], \tag{5.4}$$

where $A = (ba_r+\beta a_e)/(b^2-\beta^2)$, $B = (ba_e+\beta a_r)/(b^2-\beta^2)$, $K^M = (b\alpha_r+\beta\alpha_e)/(b^2-\beta^2)$, *and* $L^M = (b\alpha_e + \beta\alpha_r)/(b^2 - \beta^2)$.

Proof. Let us write down the first order conditions (FOCs) of Π^M:

$$\frac{\partial \Pi^M}{\partial p_r^M} = a_r - \alpha_r - 2bp_r^M + 2\beta p_e^M + bc_r - \beta c_e + \beta s,$$
$$\frac{\partial \Pi^M}{\partial p_e^M} = a_e - \alpha_e - 2bp_e^M + 2\beta p_r^M + bc_e - \beta c_r - bs.$$

This gives rise to the following Hessian:

$$H^M = \begin{bmatrix} -2b & 2\beta \\ 2\beta & -2b \end{bmatrix},$$

which is negative definite since $b > \beta > 0$ by assumption. □

The optimal prices have a linear relationship with almost all problem parameters. Only the self- and cross-price sensitivity of demand have a more sophisticated relationship with the selling prices, whereas, salvage values, $\nu_i - \omega_i$, $i \in \{r, e\}$, have no effect on the firm's pricing decision. This finding is somewhat expected as the return rates are independent of the prices. In Section 5.3.2, we relax the constant return rate assumption obtain a different result. In Proposition 5, we show that the optimal price in any channel is larger than the unit operational cost in the same channel.

Proposition 5. *A loss leader strategy is not optimal for the dual-channel retailer, i.e.,* $p_i^M \geq c_i, i \in \{r, e\}$.

Proof. By plugging the optimal prices into the (net) demand functions, one can obtain:

$$N_r^M(p_r^M, p_e^M) = \frac{1}{2}(a_r - \alpha_r - bc_r + \beta c_e + \beta s) = \frac{1}{2}N_r^M(c_r, c_e)$$
$$N_e^M(p_e^M, p_r^M) = \frac{1}{2}(a_e - \alpha_e - bc_e + \beta c_r - bs) = \frac{1}{2}N_e^M(c_e, c_r),$$

where both $N_r^M(c_r, c_e)$ and $N_e^M(c_e, c_r)$ are non-negative by assumption. Hence, we obtain $a_r \geq \alpha_r + bc_r - \beta c_e - \beta s$ and $a_e \geq \alpha_e + bc_e - \beta c_r - bs$. By substituting a_r and a_e in equations (5.4) with the right-hand-side of the inequalities, we obtain,

$$p_r^M \geq \frac{1}{2}\left[c_r + \frac{(b^2 - \beta^2)c_r + b\alpha_r + \beta\alpha_e}{b^2 - \beta^2} - \frac{b\alpha_r + \beta\alpha_e}{b^2 - \beta^2}\right] = c_r,$$
$$p_e^M \geq \frac{1}{2}\left[c_e - s + \frac{(b^2 - \beta^2)(c_e + s) + b\alpha_e + \beta\alpha_r}{b^2 - \beta^2} - \frac{b\alpha_e + \beta\alpha_r}{b^2 - \beta^2}\right] = c_e.$$

□

Finally, we can write the retailer's profit in a closed-form as,

$$\Pi^M = \frac{1}{2}\left[N_r^M(c_r, c_e)(-c_r + A - K^M) + N_e^M(c_e, c_r)(-c_e - s + B - L^M)\right] \\ - \alpha_r(c_r - \nu_r + \omega_r) - \alpha_e(c_e - \nu_e + \omega_e).$$

In the next section, we carry out a similar analysis for the cross-channel setting.

5.3.1.2 Cross-channel Setting

Demand for each channel under the cross-channel setting is described as,

$$D_r^C(p_r, p_e) = a_r - bp_r + \beta p_e + \beta s - \phi \tag{5.5}$$

$$D_e^C(p_e, p_r) = a_e - bp_e + \beta p_r - bs + h, \tag{5.6}$$

where $h > \phi > 0$. This framework captures the fact that the firm's cross-return service offering boosts up the demand in the online channel by h units, where ϕ units of it is cannibalized from the B&M channel.

Return amounts are given as $R_r^C = R_r^M = \alpha_r$ and $R_e^C = R_e^M + \alpha_{er} = \alpha_e + \alpha_{er}$, where α_{er} denotes the cross-channel returns. As in the multi-channel setting, we define the net demand in channel i as $N_i^C(p_i, p_j) = D_i^C(p_i, p_j) - R_i^C$, and assume $N_i^C(p_i, p_j) > 0$ $i, j \in \{r, e\}$, $i \neq j$ to assure that demand will be positive for some range of the prices. The retailer's profit under the cross-channel setting can be given as,

$$\Pi^C = \sum_{i,j\in\{r,e\}, i\neq j} \left[(p_i^C - c_i)D_i^C(p_i^C, p_j^C) - \alpha_i(p_i^C - v_i + w_i)\right] - \alpha_{er}(p_e^C - \nu_r + \omega_r) \\ = \sum_{i,j\in\{r,e\}, i\neq j} \left[(p_i^C - c_i)N_i^C(p_i^C, p_j^C) - \alpha_i(c_i - v_i + w_i)\right] - \alpha_{er}(c_e - \nu_r + \omega_r),$$

where the terms inside the brackets stand for the profit from the sales and the costs generated by the same channel returns, whereas, the last term stands for the costs generated by the cross-channel returns. The optimal prices are as stated in following theorem.

Theorem 13. Π^C *is concave and the optimal prices are*

$$p_r^C = \frac{1}{2}\left[c_r + A + E - K^C\right], \quad p_e^C = \frac{1}{2}\left[c_e - s + B + F - L^C\right], \tag{5.7}$$

where $E = (\beta h - b\phi)/(b^2 - \beta^2)$, $F = (bh - \beta\phi)/(b^2 - \beta^2)$, $K^C = (bR_r^C + \beta R_e^C)/(b^2 - \beta^2)$, *and* $L^C = (bR_e^C + \beta R_r^C)/(b^2 - \beta^2)$.

Proof. We first lay out the FOCs of Π^C:

$$\frac{\partial \Pi^C}{\partial p_r^C} = a_r - \alpha_r - 2bp_r^C + 2\beta p_e^C + bc_r - \beta c_e + \beta s - \phi,$$
$$\frac{\partial \Pi^C}{\partial p_e^C} = a_e - \alpha_e - \alpha_{er} - 2bp_e^C + 2\beta p_r^C + bc_e - \beta c_r - bs + f.$$

Then, we develop the Hessian:

$$H^C = \begin{bmatrix} -2b & 2\beta \\ 2\beta & -2b \end{bmatrix},$$

which is negative definite since $b > \beta > 0$ by assumption. □

Similar to the multi-channel case, the optimal prices under the cross-channel setup are independent of the net salvage value of the goods in both channels and have a linear relationship with the rest of the problem parameters, but the price sensitivity parameters. Proposition 6 shows that the optimal prices are greater than the respective unit cost.

Proposition 6. *A loss leader strategy is not optimal for the dual-channel retailer under the cross-channel set-up, i.e.,* $p_i^C \geq c_i, i \in \{r, e\}$.

Proof. The proof is very similar to the proof of Proposition 5 and follows from the assumption $N_i^C(p_i, p_j) > 0$. □

We can write the retailer's profit under cross-channel in a closed-form as,

$$\Pi^C = \frac{1}{2}\left[N_r^C(c_r, c_e)(-c_r + A + E - K^C) + N_e^C(c_e, c_r)(-c_e - s + B + F - L^C)\right]$$
$$- \alpha_r(c_r - \nu_r + \omega_r) - \alpha_e(c_e - \nu_e + \omega_e) - \alpha_{er}(c_e - \nu_r + \omega_r).$$

Now that we have developed the optimal prices, demands, profits under both channel settings, we can compare them to derive some insights.

5.3.1.3 Impact of Cross-Channel Returns on the Optimal Prices and Channel Profits

We analyze the impact of cross-channel returns by comparing the prices, channel profits, and total profits under M and C. We assume $h - \phi - \alpha_{er} > 0$ through our analysis. Recall that h denotes the additional demand in the online channel due to the allowance of cross-channel returns and $\phi < h$ denotes the amount of additional demand that is cannibalized from the B&M channel. Thus, $h - \phi - \alpha_{er} > 0$ implies that the firm's cross-channel initiative boosts up both total demand and total returns with the former being larger than the latter. In other words, the demand grows larger than the returns so that the net demand increases under this assumption. We use π_i^S to denote the optimal profit in channel $i \in \{r, e\}$ under channel setting $S \in \{M, C\}$

Proposition 7 summarizes the impact of cross-channel returns on the firm's pricing decisions and customer demand.

Proposition 7. *The availability of cross-channel returns,*

1. *raises the optimal price in the traditional channel only if* $\frac{b}{\beta} < \frac{h-\alpha_{er}}{\phi}$*,*
2. *decreases (net) demand in the B&M channel by* $\phi/2$ *units, regardless of the price change,*
3. *raises both (net) demand and price in the online channel. The demand increases by* $(h - \alpha_{er})/2$ *and the price increases by*

$$\frac{1}{2(b^2-\beta^2)}\left[b(h-\alpha_{er})-\beta\phi\right] > \frac{1}{2(b^2-\beta^2)}b(h-\alpha_{er}-\phi) > 0.$$

4. *raises the total (net) demand by* $(h - \alpha_{er} - \phi)/2$*.*

Proof. The proof follows from Theorems 12–13. □

A couple of remarks are worth mentioning. First of all, notice that the condition given in the first part of Proposition 7 may not be satisfied even when $h-\phi-\alpha_{er} > 0$, i.e., the traditional channel price may still decline when the cross-channel set-up generates more demand than return. Next, recall that the cannibalization effect of the retailer's cross-channeling initiative is taken as ϕ units. However, the optimal demand drops only by $\phi/2$. This finding suggests that the firm is able to cushion the undesirable impact of channel integration on the B&M demand. A more interesting finding, that contrasts with the conventional wisdom, is that allowing cross-channel returns may move the optimal price in the physical channel up, despite a certain drop in the channel demand.

Expectedly, adopting the cross-channel set-up boosts up both the optimal price and demand in the online channel. The increase in the demand over both channels is sufficiently large to compensate the loss in the B&M demand, therefore, the overall demand grows by a half of the total impact, $h - \alpha_{er} - \phi$. However, it is still not clear if the retailer is better off allowing the return of the online purchases to her physical facilities. She enjoys the enhanced customer traffic, on the one hand, but suffers from the additional return costs, on the other. To gain more insights, we compare the channel profits under M and C in the following proposition. Note that all revenues and costs associated with the cross-channel returns are considered under the e-channel's profit function.

Proposition 8. *The relationship between the retailer's profits under M and C can be described as follows:*

1. $\pi_r^C \geq \pi_r^M$ *if* $\Delta\pi_r \geq 0$,
2. $\pi_e^C \geq \pi_e^M$ *if* $\Delta\pi_e \geq 0$,
3. $\Pi^C \geq \Pi^M$ *if* $\Delta\pi_r + \Delta\pi_e \geq 0$,

where $\Delta\pi_r = \frac{1}{2}[N_r^M(c_r, c_e)(p_r^C - p_r^M) - \phi(p_r^C - c_r)]$ *and* $\Delta\pi_e = \frac{1}{2}[N_e^M(c_e, c_r)(p_e^C - p_e^M) + (h - \alpha_{er})(p_e^C - c_e)] - \alpha_{er}(c_e - \nu_r + \omega_r)$.

Proof. The proof follows from Propositions 5–6. □

A number of observations are in order. Firstly, $p_r^C > p_r^M$ does not assure $\pi_r^C \geq \pi_r^M$, but $p_r^C \leq p_r^M$ is sufficient to imply $\pi_r^C < \pi_r^M$, i.e., unless the B&M price is sufficiently large to compensate the losses due to the cannibalization effect, the channel profit declines. As such, the retailer's effort of maximizing the systemwide profit disrupts the conventional channel. Another finding is that when $\nu_r \geq \omega_r + c_e$, $\pi_e^C \geq \pi_e^M$ always holds, that is when the salvaged cross-channel returns do not generate loss, the internet-enabled channel's profit always improves. However, even this condition alone is not sufficient to determine if the retailer's overall profit grows or not. The results in this part are summarized in Table 5.3.1.3.

Channel	Price	Demand	Profit
R	$\uparrow\downarrow$	$\downarrow$	$\uparrow\downarrow$
E	$\uparrow$	$\uparrow$	$\uparrow\downarrow$
Total	-	$\uparrow$	$\uparrow\downarrow$

Table 5.1: Impact of the cross-channel returns on channel prices, demands, and profits as well as total demand and profit when $h > \phi + \alpha_{er}$. Symbols $\uparrow$ and $\downarrow$ indicate that the respective component increases and decreases, respectively, whereas, $\uparrow\downarrow$ indicates that the respective component may increase or decrease.

In the next section, we examine the retailer's problem under refund-dependent return rates.

5.3.2 Multi-channel vs Cross-channel with Refund-dependent Returns

As in the constant return scenario, in this section, we develop the retailer's optimal solution under both channel settings but derive our insights via numerical analysis as the problem becomes analytically intractable when cross-channel returns are included.

In contrast with the previous sections, here return rates depend on the refund amount. As such, the price becomes an active influencer of returns as the retailer follows a full-refund policy. There is an additional trade-off that the firm has to deal with: raising the prices not only reduces the demand but also increases the return amount.

5.3.2.1 Multi-channel Setting

The retailer's demand formulation here is the same with the one established for the exogenous returns case under no cross-returns. However, for the modeling of returns, we extend a framework first proposed by Mukhopadhyay and Setaputra (2006), and later studied by Liu *et al.* (2011); Choi (2013). The extant literature develops the framework for an e-tailer and, in this study, we extend it to a dual-channel environment. Accordingly, the returns are given as,

$$R_r^M(p_r) = \alpha_r + yp_r, \tag{5.8}$$

$$R_e^M(p_e) = \alpha_e + y(p_e - s), \tag{5.9}$$

where α_i is the intercept point of the returns in channel $i \in \{r, e\}$ and $y > 0$ is the refund sensitivity parameters of returns, i.e., a one dollar change in the refund will change the returns by y units. One can argue that the online returns may be more refund-sensitive for some products, but we assume the difference is negligible for the sake of analytical tractability. Recall that the company adopts a full-refund policy meaning that the total refund in the B&M store it is p_r and in the online store is p_e, but the online shoppers also pay for the return shipping fee, s. We develop the retailer's profit as,

$$\begin{aligned}\Pi^M &= \sum_{i,j\in\{r,e\},i\neq j} (p_i^M - c_i)D_i^M(p_i^M, p_j^M) - R_i^M(p_i^M)(p_i^M - v_i + w_i),\\ &= \sum_{i,j\in\{r,e\},i\neq j} (p_i^M - c_i)N_i^M(p_i^M, p_j^M) - R_i^M(p_i^M)(c_i - v_i + w_i),\end{aligned} \tag{5.10}$$

where we assume $N_i^M(c_i, c_j) = D_i^M(c_i, c_j) - R_i^M(\omega_i - \nu_i) > 0$ to assure that demand is non-negative for some region of the prices.

The following theorem characterizes the optimal prices for this case.

Theorem 14. Π^M *is concave and the optimal prices are as follows:*

$$p_r^M = \frac{1}{2}(A^M + E^M - K^M), \quad p_e^M = \frac{1}{2}(B^M + F^M - L^M). \tag{5.11}$$

where

$$\begin{aligned}A^M &= \frac{(b+y)a_r + \beta a_e}{(b+y)^2 - \beta^2}, \quad B^M = \frac{(b+y)a_e + \beta a_r}{(b+y)^2 - \beta^2},\\ E^M &= \frac{(b+y)(bc_r - \beta c_e + \beta s) + \beta(bc_e - \beta c_r - bs)}{(b+y)^2 - \beta^2},\\ F^M &= \frac{(b^2 - \beta^2)(c_e - s) + y(bc_e - \beta c_r - bs)}{(b+y)^2 - \beta^2},\\ K^M &= \frac{(b+y)R_r^M(\omega_r - \nu_r) + \beta R_e^M(\omega_e - \nu_e)}{(b+y)^2 - \beta^2},\\ L^M &= \frac{(b+y)R_e^M(\omega_e - \nu_e) + \beta R_r^M(\omega_r - \nu_r)}{(b+y)^2 - \beta^2}.\end{aligned}$$

Proof. Let us write down the FOCs of Π^M:

$$\frac{\partial \Pi^M}{\partial p_r^M} = a_r - \alpha_r - 2(b+y)p_r^M + 2\beta p_e^M + bc_r - \beta c_e + \beta s + y(\nu_r - \omega_r),$$
$$\frac{\partial \Pi^M}{\partial p_e^M} = a_e - \alpha_e - 2(b+y)p_e^M + 2\beta p_r^M + bc_e - \beta c_r - bs + y(\nu_e - \omega_e).$$

This gives rise to the following Hessian:

$$H^M = \begin{bmatrix} -2(b+y) & 2\beta \\ 2\beta & -2b(b+y) \end{bmatrix},$$

which is negative definite since $b > \beta > 0$ by assumption. □

Incorporating a refund-dependent return model has elevated the level of complexity of the price expressions drastically. Although the objective function is still concave, the optimal prices have taken complex forms. We can still make an important observation nevertheless: net salvage value in a channel, $\nu_i - \omega_i$, $i \in \{r, e\}$, now have a linear relationship with both of the optimal prices, which were independent of the salvaging parameters under the exogenous returns scenario.

Proposition 9. *A loss leader strategy is not optimal for the dual-channel retailer under the multi-channel set-up, i.e.,* $p_i^M \geq c_i, i \in \{r, e\}$.

Proof. The proof is very similar to the proof of Proposition 5 and follows from the assumption $N_i^M(c_i, c_j) > 0$. □

The refund-dependent model under the multi-channel setting still preserves the good properties of the exogenous refund model. However, the closed-form profit function becomes too complicated to analyze, therefore, we do not present it here and leave any further analysis to Section 5.3.2.3.

5.3.2.2 Cross-channel Setting

Demand for each channel with cross-channel returns is given as,

$$D_r^C(p_r, p_e) = a_r - bp_r + \beta p_e + \beta s - \mu p_e \tag{5.12}$$

$$D_e^C(p_e, p_r) = a_e - bp_e + \beta p_r - bs + mp_e, \tag{5.13}$$

where $0 < m < \beta$ and $0 < \mu < m$ are the self- and cross-refund sensitivity of demand for cross-channel returns, respectively. As such, a dollar increase in the refund for online purchases that are returned to B&M store raises the online demand by m units, but reduces the retail demand by μ units. Since the retailer implements a full-refund policy, the customers receive p_e for their cross-channel returns. We have similar modification in return set-up as well:

$$R_r^C(p_r) = R_r^M(p_r) = \alpha_r + yp_r, \tag{5.14}$$

$$R_e^C(p_e) = \alpha_e + y(p_e - s) - \eta p_e = \alpha_e + (y - \eta)p_e - ys, \tag{5.15}$$

$$R_{er}^C(p_e) = \alpha_{er} + yp_e - \eta(p_e - s) = \alpha_{er} + (y - \eta)p_e + \eta s, \tag{5.16}$$

where $R_{er}^C(p_e)$ is the amount of cross-channel returns for refund p_e and $0 < \eta < y$ is the cross-refund sensitivity of online-purchased returns. Here, η captures the fact that allowing cross-channel returns decreases the return shipments and the degree of change depends on the refund amount, p_e. In this way, instead of assuming a constant change in channel demands and returns, we modify the model of Mukhopadhyay and

Setaputra (2006) to capture the entwined nature of demand-return relationship in a dual-channel retailing environment.

The retailer's profit under this setting can be described as,

$$\begin{aligned}\Pi^C &= \sum_{i,j\in\{r,e\},i\neq j} \big[(p_i^C - c_i)D_i^C(p_i^C, p_j^C) - R_i^C(p_i^C)(p_i^C - v_i + w_i)\big] - R_{er}(p_e)(p_e - \nu_r + \omega_r),\\ &= \sum_{i,j\in\{r,e\},i\neq j} \big[(p_i^C - c_i)N_i^C(p_i^C, p_j^C) - R_i^C(p_i^C)(c_i - v_i + w_i)\big] - R_{er}(p_e)(c_e - \nu_r + \omega_r),\end{aligned} \tag{5.17}$$

where $N_e^C(p_e^C, p_r^C) = D_e^C(p_e^C, p_r^C) - R_e^C(p_e^C) - R_{er}(p_e)$ is updated. The objective function given above may not be always concave. The following proposition frames the condition for concavity and lays out the optimal prices.

Proposition 10. *The retailer's profit function,* Π^C*, is concave if* $4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2 > 0$ *and the resulting optimal prices are given as,*

$$p_r^C = (A^C + E^C - K^C), \quad p_e^C = (B^C + F^C - L^C), \tag{5.18}$$

where

$$\begin{aligned}A^C &= \frac{2[b-m+2(y-\eta)]a_r + (2\beta-\mu)a_e}{4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2},\\ B^C &= \frac{2(b+y)a_e + (2\beta-\mu)a_r}{4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2},\\ E^C &= \frac{2[b-m+2(y-\eta)](bc_r - \beta c_e + \beta s) + (2\beta-\mu)[(b-m)c_e - (\beta-\mu)c_r - bs]}{4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2},\end{aligned}$$

$$F^C = \frac{2(b+y)[(b-m)c_e - (\beta-\mu)c_r - bs] + (2\beta-\mu)(bc_r - \beta c_e + \beta s)}{4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2},$$
$$K^C = \frac{2[b-m+2(y-\eta)]R_r^C(\omega_r-\nu_r) + (2\beta-\mu)[R_e^C(\omega_e-\nu_e) + R_{er}^C(\omega_r-\nu_r)]}{4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2},$$
$$L^C = \frac{2(b+y)[R_e^C(\omega_e-\nu_e) + R_{er}^C(\omega_r-\nu_r)] + (2\beta-\mu)R_r^C(\omega_r-\nu_r)}{4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2}.$$

Proof. We first lay out the FOCs of Π^C:

$$\frac{\partial \Pi^C}{\partial p_r^C} = a_r - \alpha_r - 2(b+y)p_r^C + (2\beta-\mu)p_e^C + bc_r - \beta c_e + \beta s + y(\nu_r - \omega_r),$$
$$\frac{\partial \Pi^C}{\partial p_e^C} = a_e - \alpha_e - \alpha_{er} - 2(b-m+2y-2\eta)p_e^C + (2\beta-\mu)p_r^C + (b-m)c_e - (\beta-\mu)c_r$$
$$- (b+\eta)s + (y-\eta)(\nu_r - \omega_r + \nu_e - \omega_e).$$

Then, we develop the Hessian:

$$H^C = \begin{bmatrix} -2(b+y) & 2\beta-\mu \\ 2\beta-\mu & -2(b-m)-4(y-\eta) \end{bmatrix},$$

which is negative definite only when $4(b+y)[b-m+2(y-\eta)] - (2\beta-\mu)^2 > 0$. □

We have run a Monte Carlo simulation by assigning a uniform random variable with appropriate support to each parameter value to generate possible scenarios that satisfy our assumptions. Overall, we report that more than 99% of the time, the condition given in Theorem 10 is satisfied, i.e., the Hessian of the objective is negative-definite. Due to the analytical difficulty, we analyze the optimal demand and profit numerically in the next section.

5.3.2.3 Numerical Analysis

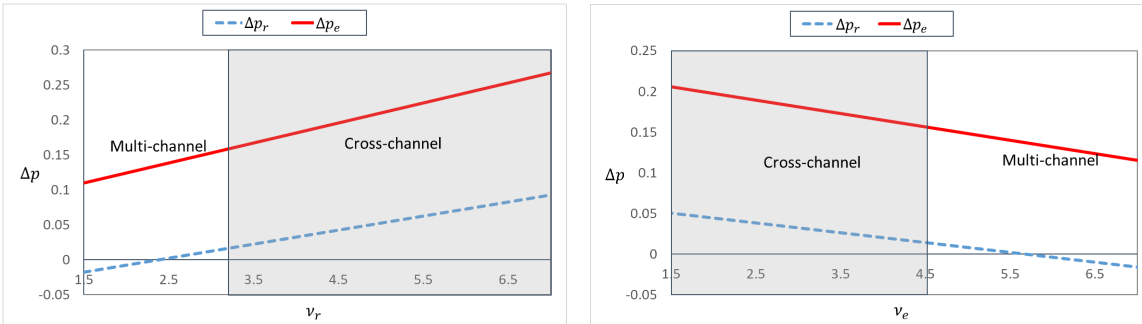
To derive some managerial insights and compare our findings with the those of the exogenous returns case, we conduct a numerical study for the refund-dependent returns case.

In particular, we look at the impact of salvage values and return-related parameters on the optimal profit, demand, and price in each channel. We denote the optimal profit in channel $i \in \{r, e\}$ under setting $S \in \{M, C\}$ as π_i^S, and use notations $\Delta\pi_i = \pi_i^C - \pi_i^M$ and $\Delta p_i = p_i^C - p_i^M$ to facilitate discussions. We first construct a base case for our analysis: $a_r = 35.75, a_e = 19.25, b = 4, \beta = 3, c_r = 5, c_e = 3, s = 1.5, \nu_r = 4, \nu_e = 3, \omega_r = 1, \omega_e = 0.5, m = 0.15, \mu = 0.07, \alpha_r = 0.03a_r = 1.0725, \alpha_e = 0.04a_e = 0.77, \alpha_{er} = 0.01a_e = 0.1925, y = 0.18, \eta = 0.07$. Notice that the retailer's B&M store has 65% of the total market share and the online store has the rest, 35%. The return base point of a channel is tied to the respective market potential. However, the return rate is smaller in the conventional channel as the customers are able to examine the products at the B&M store. We also observe a high cross-price sensitivity, as $b - \beta$ is relatively small, meaning that the customers tend to switch channels when the price discrepancy across the channels is large. A dollar change in the refund amount of a channel shifts the return volume in the same channel by $y = 0.18$ units, whereas a dollar change in the refund amount of cross-channel returns shifts the online demand by $m = 0.15$ units and the in-store demand by $\mu = 0.07$ units in the opposite directions. Finally, any returned item is salvaged at $\nu_r - \omega_r = 3$ in the physical store and at $\nu_e - \omega_e = 2.5$ in the online channel.

We first vary the salvage values. Referring to the closed-form solutions in equations (5.11) and (5.18), both prices grow with $\nu_i - \omega_i$, $i \in \{e, r\}$ under both channel settings.

As one may expect a larger salvage value brings in a larger profit and the retailer achieves it by simply marking up the selling price. However, by doing so, she also endures reduced demand and increased returns. On the other hand, the rate of change in the optimal channel prices and profits may be different for M and C. Furthermore, because the cross-channel returns are salvaged at the B&M store, varying v_r and ν_e may result in different outcomes. Figures 5.1a and 5.1b show that an increase in ν_r enlarges the price gap in both channels while an increase in ν_e closes it. The rationale behind this result is that the retailer's e-channel price grows with ν_e and the online returns become less detrimental, but the introduction of cross-channel returns shifts some online returns to the B&M store, where the products have a smaller salvage value. We also observe that when setting C is optimal, the retailer raises the prices in both channels. As shown in Figures 5.2a and 5.2b, the channel profit follow a similar pattern to the prices. An interesting finding can be cited as the retailer's in-store profit and sales both decline under the cross-channel setting. Recall that although the optimal prices are independent of the salvage values in the case of the exogenous return, we reported a similar finding for different parameter combinations.

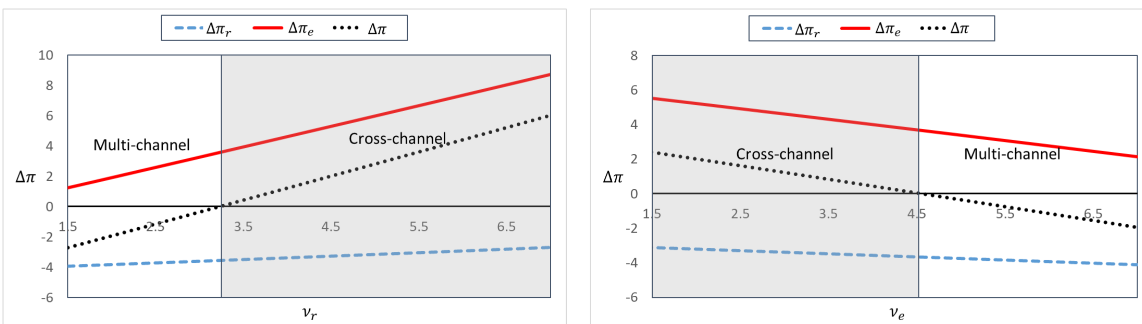
We finally analyze the impact of refund sensitivity parameter on the firm's optimal channel setting, prices and profits in Figures 5.3a and 5.3b. The retailer's optimal channel setting is very responsive to the changes in y. Since the retailer adopts a full-refund policy, raising the price in a channel may result in an immense increase in the returns when they are highly sensitive to the refund, i.e., when y is large. We know the retailer tends to mark up the online price when she accepts cross-channel returns, but due to the additional returns induced by this policy, it only drags down the overall profit.



(a) The difference between optimal prices under M and C with respect to different salvage values in the B&M channel, ν_r.

(b) The difference between optimal prices under M and C with respect to different salvage values in the online channel, ν_e.

Figure 5.1: The difference between optimal prices under M and C with respect to salvage values ν_r and ν_e.



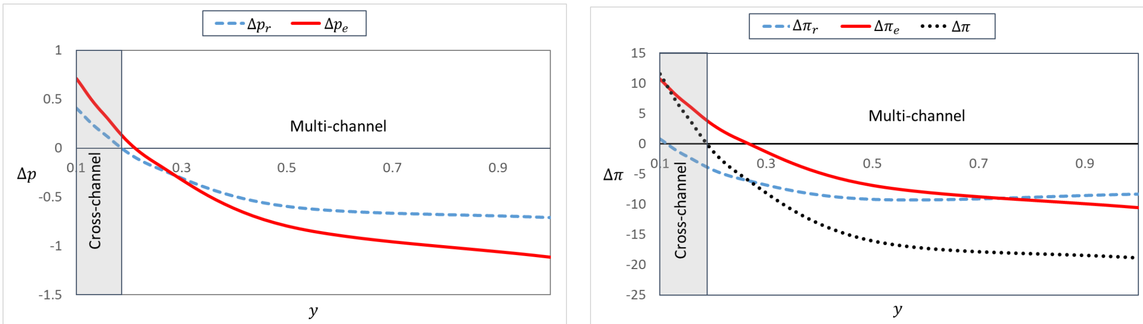
(a) The difference between optimal profits under M and C with respect to different salvage values in the B&M channel, ν_r.

(b) The difference between optimal profits under M and C with respect to different salvage values in the online channel, ν_e.

Figure 5.2: The difference between optimal profits under M and C with respect to salvage values ν_r and ν_e.

5.4 Conclusion

In this paper, we study the impacts of cross-channel returns on a bricks and clicks dual-channel retailer's operational planning. Inspired by the multiple-channel retailing giants, such as Hudson's Bay and Sportchek, the retailer differentiates the selling price across the channels, adopts a full-refund policy, but reflects all shipping expenses



(a) The difference between optimal prices under M and C with respect to different refund sensitivity parameter of returns, y.

(b) The difference between optimal profits under M and C with respect to different refund sensitivity parameter of returns, y.

Figure 5.3: The difference between optimal prices and the optimal profits under M and C with respect to y.

to the customers.

We assume the retailer sells her products through a physical store as well as an online website. By accepting the returns of online purchases to the B&M store, the retailer engages in a partial channel integration. As such, we call the business model without cross-channel returns *multi-channel* (M) an in the presence of them *cross-channel* (C). We explore the conditions that favor C under two scenarios: (1) exogenous returns, and (2) refund-dependent returns. For the former we were able to develop analytical insights, but the latter one the problem was analytically intractable, therefore, we conduct numerical experiments to derive insights.

In both scenarios, cross-channel returns raise not only the demand but also the returns in the online channel and result in some of the in-store demand cannibalized by the online channel. Interestingly, the retailer may still mark up the B&M price, despite a drop in the in-store demand due to the cannibalization effect. In the exogenous returns case, even though the net additional demand is larger than the sum of the cross-channel returns and the cannibalized demand from the conventional channel, it is not assured that the firm is better off offering this service. Unless the growth in online customers compensates the retailer for the enhanced returns and abused B&M sales, she should not accept cross-channel returns. However, we still observe an increase in both demand and price in the online channel even when the overall profit drops. One drawback of the exogenous returns is that the optimal price and demand in both channels are independent of the salvaging values.

In the refund-dependent returns case, we extended the work of Mukhopadhyay and Setaputra (2006); Liu *et al.* (2011); Choi (2013) to a dual-channel framework. Since the objective function may not be concave under C for some extreme cases, we developed sufficient conditions to assure it. Our numerical analysis revealed that most of the findings under the exogenous returns case still persist, but the salvage values and refund-sensitivity of returns play an important role in the determination of whether or not the retailer should allow cross-channel returns under this case. In particular, because the retailer adopts a full-refund policy, if the returns are highly refund-sensitive, she may suffer from a sizable jump in the online returns (both to online and offline stores) when allowing the cross-channel returns. Also, the salvage values in different channels may have contrasting effects: a large ν_r favors a cross-channel setting, whereas, a large ν_e favors a multi-channel setting.

Overall, we observe the retailer's conventional channel is disturbed and her in-store demand is likely to drop, yet the growth in online profit may be sufficiently large to offset any losses in the conventional channel. This work can be extended to incorporate uncertainty in demand and/or in returns. It would be also interesting to see what happens when the retailer has multiple physical facilities and online returns can be delivered to any of them. A third possible extension is to incorporate competition from other retailers that may have single or multiple channels.

Chapter 6

Conclusion and Future Research

In this dissertation, we examined the operational planning problem of a retailer under single- and dual-channel systems with product returns and donations considerations. This chapter highlights our major contributions and proposes directions for future research.

Since we have found no comprehensive reviews of the bricks-and-clicks dual channels literature, we proposed one in Chapter 2. We reviewed 263 published contributions and found two main research themes in the literature: (1) a novel channel competition between a manufacturer/supplier and a traditional retailer where either of the players augments their operations with an e-channel, (2) inventory management and demand fulfillment policies in a dual-channel environment. The former research stream, which constitutes around 82% of the whole literature, lacks studies that consider demand uncertainty, information asymmetry and multi-period models, whereas, the latter seeks more research on cross-channel delivery/return and refund policy making. We also discovered a paradigm shift in the OM literature as the e-commerce adoption of B&M retailers have thrived during the last few years. The challenges faced by dual-channel companies have become increasingly popular and, therefore, industry-specific channel management issues (high-tech, grocery, fashion) and the disruptions caused by the COVID-19 pandemic present promising research opportunities. This chapter also illustrated how existing research can be leveraged to address this new trend.

The issue of charitable donations has been largely ignored by the existing operational planning models under the single-channel framework. In addition, the food safety concern of the food banks necessitates an objective metric for the quality of donated fresh groceries. Thus, in Chapter 3, we introduced the quality-dependent newsvendor problem (QDNP), which incorporates the philanthropic act of a food retailer in the form of charitable donations into her operational planning problem over two periods. In particular, the retailer jointly determines the stocking quantity and first period price at the start of the selling season, and modifies the price and decides on her donation policy at the end of the first period. In each period, the customer demand is uncertain, and both price- and quality-sensitive. The company uses the data provided by IoT-enabled labels to estimate the effective (true) quality of the goods. Our analysis showed that the charitable donations can improve a firm's profit while at the same time reduce inventory waste. This work can be extended by considering multiple (substitutable) products or using demand update models at the end of period 1. Also, considering T_1 as an endogenous variable to be determined by the retailer may yield interesting results. In such a model, the food-bank can state the minimum shelf-life condition for its food-safety concerns and the retailer may evaluate if it is optimal to donate or not.

In Chapter 4 we considered a similar problem to the one in Chapter 3 with the addition of a tax subsidy policy. We also used a different demand model with the assumption that the demand uncertainty is revealed at the end of the first period. Our goal was to analyze the impact of tax subsidy parameters on the retailer's optimal decisions as well as on the charitable donations. Our assumptions on demand enabled us to isolate this effect and improve the analytical tractability. The model yielded some unexpected results: the retailer's optimal donation volume may decline with respect to the amount of leftover inventory at the end of period 1, their effective quality and the tax subsidy coefficients. These results are contrasting with the findings of the previous chapter. This work can be improved by relaxing the assumption of having deterministic demand in the second period. It would also be interesting to analyze the government's tax incentive mechanism in a game theoretical framework, where the government faces a trade-off between income tax returns and the donation tax incentives.

Finally, in Chapter 5, we addressed one of the open questions pointed out in Chapter 2 by proposing an analytical model to investigate the impacts of cross-channel returns on the operational planning of a dual-channel retailer, who sells her products through a web-based channel as well as a B&M store. Welcoming the returned online purchases to her B&M store, the retailer engages in a partial channel integration. Thus, with the decision of whether to accept cross-channel returns or not, she also chooses between operating under a multi-channel (separately managed channels) or a cross-channel (partially integrated channels) setting. The customer demand is sensitive to the prices as well as the refund amount in both channels. The allowance of cross-channel returns drives the online sales up, but a part of that growth is cannibalized from the conventional channel. We found that despite the cannibalization effect, the B&M price may still increase with the introduction of the cross-channel returns. We also reported that when the online returns are highly refund sensitive, channel integration may hurt the retailer. Possible future research directions for this chapter are the consideration of demand uncertainty and a retailer having multiple physical facilities. Also, one can argue that the model is having too many exogenous variables resulting in many assumptions. Therefore, this model can be reconsidered by deriving some parameters using a consumer valuation model and evaluate a customer's decision at the returning point (in-store or via shipment) by comparing her inconvenience cost for both options.

Appendix A

Appendices of Chapter 2

A.1 Derivation of Vertical Differentiation Model

There are a couple of ways to develop demand functions given in equations (2.2) and (2.3). We use a probabilistic approach. Recall that the reservation price, V, is a uniformly distributed random variable over $[0, 1]$. A marginal consumer would have $U_r = U_e$ or $V - p_r = \theta V - p_e$, which brings the indifference threshold $\hat{V} = (p_r - p_e)/(1 - \theta)$.

A consumer prefers the traditional channel over the online channel if and only if $V \geq p_r$ and $V \geq \hat{V}$. This can be expressed as,

$$D_r = \mathbb{P}\{V \geq \max(p_r, \hat{V})\} = \begin{cases} 1 - \hat{V} & \text{if } \hat{V} \geq p_r, \\ 1 - p_r & \text{otherwise,} \end{cases}$$

where $\hat{V} \geq p_r$ is equivalent to $p_e/\theta \leq p_r$.

On the other hand, a consumer prefers the online channel over the traditional channel if and only if $V \geq p_e/\theta$ and $V \leq \hat{V}$. This can be expressed as,

$$D_e = \mathbb{P}\{p_e/\theta \leq V \leq \hat{V}\} = \begin{cases} \hat{V} - p_e/\theta & \text{if } \hat{V} \geq p_e/\theta, \\ 0 & \text{otherwise,} \end{cases}$$

where, once again, $\hat{V} \geq p_e/\theta$ is equivalent to $p_e/\theta \leq p_r$. Note that when $p_e/\theta \geq p_r$, the online channel is dominated by the retail channel.

A.2 Derivation of Horizontal Differentiation Model

As in the vertically differentiated valuation model, we start with finding the indifference threshold from $U_r = U_e$ or $v - p_r - tX = v - p_e - s$, where X is a uniform random variable with support $[0, 1]$. In what follows, we obtain the threshold $\hat{X} = (p_e - p_r + s)/t$. Note that $\hat{X}$ is nonnegative as $p_r \leq p_e + s$ holds by assumption. Then, demand functions (2.4) and (2.5) can be found as,

$$D_r = \mathbb{P}\{v - p_r - tX \geq \max(0, V - p_e - s)\} = \mathbb{P}\{X \leq \hat{X}\} = \hat{X},$$
$$D_e = \mathbb{P}\{v - p_e - s \geq \max(0, V - p_r - tX)\} = \mathbb{P}\{X \geq \hat{X}\} = 1 - \hat{X}.$$

A.3 Derivation of Demand Functions in (2.7)-(2.8)

Because the net utility given in (2.6) is to be maximized over the quantities D_r and D_e, we simply differentiate the function with respect to those quantities. This gives rise to the following first-order condition,

$$\frac{\partial U}{\partial D_i} = A_i - B_i D_i - \gamma D_j - p_i = 0, \quad i, j \in \{r, e\}, \quad i \neq j, \tag{A.1}$$

where this expression can be considered as the net marginal utility a representative consumer receives by purchasing from channel $i \in \{r, e\}$. Note that the Hessian satisfies the sufficiency conditions since it is assumed that $B_r B_e - \gamma^2 > 0$. Thus, one can obtain the demand functions (2.7) or (2.8) by solving the first-order conditions given in (A.1) for D_r and D_e.

Appendix B

Appendices of Chapter 3

B.1 Proof of Theorem 1

One can write the second order condition of $\Gamma(z_2, p_2(z_2))$ as,

$$\frac{d^2\Gamma_2(z_2, p_2(z_2))}{dz_2^2} = \frac{[1 - F(z_2)]}{2b} \left\{[1 - F(z_2)] - 2b(p_2(z_2) + C_s + C_d)n(z_2)\right\},$$

where $n(z) = f(z)/[1-F(z)]$ denotes the hazard rate function. Let us define $N(z_2) = [1-F(z_2)]-2b(p_2(z_2)+C_s+C_d)n(z_2)$. The objective is concave if and only if $N(z_2) \leq 0$. When $f(A)[\bar{y}_2(-R - 2(C_s + C_d), T_2) + A] > 1$, $N(z_2)|_{z_2=A} \leq 0$ and $N(z_2) \leq 0$ holds if $dN(z_2)/dz_2 \leq 0$, which can be shown as,

$$\frac{dN(z_2)}{dz_2} = -f(z_2) - 2b(p_2(z_2) + C_s + C_d)(dn(z_2)/dz_2) - [1 - F(z_2)]n(z_2) \leq 0.$$

Finally, we note that, the optimal z_2^* is always an interior point as $d\Gamma_2(z_2, p_2(z_2))/dz_2$ is positive for $z_2 = A$ and negative for $z_2 = B$.

B.2 Proof of Theorem 2

Using Proposition 1, we analyze two cases:

1) $z_2 < \bar{z}$ ($\gamma < 1$):

   In this case, the inventory constraint is redundant and we realize $\omega_z = \omega_p = 0$ (as $p_2(z_2) < \bar{p}_2$ for $z_2 \in [A, B]$) resulting in $\partial\mathcal{L}/\partial z_2 = \partial\Gamma_2/\partial z_2$ and $\partial\mathcal{L}/\partial p_2 = \partial\Gamma_2/\partial p_2$. Following Theorem 1, and the conditions given in (3.7) and (3.5), we obtain the optimal solution $(z_2^0, p_2(z_2^0))$. This gives rise to $[z_2^0 + \bar{y}(p_2(z_2^0), T_2)]T_2 = I^0 = \gamma^* I$, which means $\gamma^* = I^0/I$ and $I > I^0$ as $\gamma^* < 1$.

   Thus, the value function becomes $\Phi(I) = \Gamma_2(z_2^0, p_2(z_2^0)) + RI$, which leads to $\Phi'(I) = R$ and $\Phi''(I) = 0$.

2) $z_2 = \bar{z}$ ($\gamma = 1$):

   In this case, the inventory constraint is binding, which makes R irrelevant. Let $H(I, p_2) = \Gamma_2(\bar{z}(I, p_2), p_2) + RI$. We first derive some properties of $H(I, p_2)$:

$$\begin{aligned}
\frac{\partial H(I,p_2)}{\partial p_2} &= \left.\frac{\partial \Gamma_2}{\partial p_2}\right|_{z_2=\bar{z}(I,p_2)} + \left.\frac{\partial \Gamma_2}{\partial z_2}\right|_{z_2=\bar{z}(I,p_2)} \cdot \bar{z}'(p_2), \\
&= a + \mu + \phi\bar{\delta}_2(T_2) - bC_d - 2bp_2 - \Theta(\bar{z}(I,p_2)) \\
&\quad + b(p_2 + C_s + C_d)[1 - F(\bar{z}(I,p_2))], \qquad \text{(B.2)}\\
\frac{\partial^2 H(I,p_2)}{\partial p_2^2} &= -2b + 2b\left[1 - F\big(\bar{z}(I,p_2)\big)\right] - b^2 f\big(\bar{z}(I,p_2)\big)(p_2 + C_s + C_d) < 0. \qquad \text{(B.3)}
\end{aligned}$$

   Note that, equation (B.3) ensures that p_2 can be uniquely determined for a given I, call it $p_H(I)$, by using equation (B.2). Now, we consider the behavior of $H(I, p_H(I))$:

$$\begin{aligned}
\frac{dH(I,p_H(I))}{dI} &= -C_d + \big[1 - F\big(\bar{z}(I, p_H(I))\big)\big]\,(p_H(I) + C_d + C_s),\\
\frac{d^2H(I,p_H(I))}{dI^2} &= \big[1 - F\big(\bar{z}(I, p_H(I))\big)\big]\\
&\qquad \cdot \{-n\big(\bar{z}(I, p_H(I))\big)[1/T_2 + bp'_H(I)][p_H(I) + C_s + C_d]\\
&\qquad\quad + p'_H(I)\} < 0,
\end{aligned}$$

as one can show that $p'_H(I) < 0$ and $\bar{z}'(I, p_H(I)) = 1/T_2 + bp'_H(I) > 0$.

When $I = I^0$, we obtain $\bar{z}(I^0, p_H(I^0)) = z_2^0$ (two solutions align at $\gamma^* = 1$), meaning that $I \leq I^0$ when $\gamma^* = 1$. Now, let us analyze the boundary values of the price. We define an inventory threshold, $\check{I}$, such that $p_H(\check{I}) = \bar{p}_2$. If there is no such threshold, let $\check{I} = 0$. As $p_H(I)$ is monotone decreasing in I, we have $p_2^* = p_H(I)$ when $I^0 \geq I \geq \check{I}$ and $p_2^* = \bar{p}_2$ when $0 \leq I \leq \check{I}$. Next, we check the lower-bound of price, which happens at another threshold value, call it $\bar{I}$, where $\bar{z}(\bar{I}, p_H(\bar{I})) = B$. It is plausible to assume that $I^0 < \bar{I}$, which ensures that the optimal price will always be larger than its lower-bound as $p_2(z_2^0) = p_H(I^0) > p_H(\bar{I})$. Therefore, the optimal pricing policy is realized as $p_2^* = \min\{p_H(I), \bar{p}_2\}$ for $I \leq I^0$.

Hence, the value function can be defined as $\Phi(I) = H(I, \min\{p_H(I), \bar{p}_2\})$ meaning that $\Phi''(I) < 0$, which concludes the proof.

B.3 Proof of Theorem 3

Let ξ_1 denote the realization of the random component in the first period. We outline the value function and resulting optimal policies for each case as follows:

1) $B - A \geq I^0/T_1$:

   (a) $A \leq z_1^* < \check{I}/T_1 + A$ (no donation):

$$V = \Gamma_1(z_1, p_1(z_1)) + \int_A^{z_1} H((z_1 - u)T_1, \bar{p}_2)\, dF(u),$$

   $p_1^* = p_1(z_1^*)$, $Q^* = [\bar{y}_1(p_1(z_1^*)) + z_1^*]T_1$, $Q^* - [\bar{y}_1(p_1^*) + \xi_1]T_1 < I^0$, and $p_2^* = \bar{p}_2$.

   (b) $\check{I}/T_1 + A \leq z_1^* < I^0/T_1 + A$ (no donation):

$$V = \Gamma_1(z_1, p_1(z_1)) + \int_A^{z_1 - \check{I}/T_1} H((z_1 - u)T_1, p_H((z_1 - u)T_1))\, dF(u) + \int_{z_1 - \check{I}/T_1}^{z_1} H((z_1 - u)T_1, \bar{p}_2)\, dF(u),$$

   $p_1^* = p_1(z_1^*)$, $Q^* = [\bar{y}_1(p_1(z_1^*)) + z_1^*]T_1$, $Q^* - [\bar{y}_1(p_1^*) + \xi_1]T_1 < I^0$, and $p_2^* = \min\{p_H(I), \bar{p}_2\}$.

   (c) $I^0/T_1 + A \leq z_1^* < B$:

$$V = \Gamma_1(z_1, p_1(z_1)) + \int_A^{z_1 - I^0/T_1} [\Gamma_2(z_2^0, p_2(z_2^0)) + R(z_1 - u)T_1]\, dF(u) + \int_{z_1 - I^0/T_1}^{z_1 - \check{I}/T_1} H((z_1 - u)T_1, p_H((z_1 - u)T_1))\, dF(u) + \int_{z_1 - \check{I}/T_1}^{z_1} H((z_1 - u)T_1, \bar{p}_2)\, dF(u),$$

$p_1^* = p_1(z_1^*)$, $Q^* = [\bar{y}_1(p_1(z_1^*)) + z_1^*]T_1$, and $p_2^* = \min\{p_H(I), \bar{p}_2\}$.

(d) $B \le z_1^* < \check{I}/T_1 + B$:

$$\begin{aligned} V = \Gamma_1(z_1, \tilde{p}_1) &+ \int_A^{z_1 - I^0/T_1} [\Gamma_2(z_2^0, p_2(z_2^0)) + R(z_1 - u)T_1] \, dF(u) \\ &+ \int_{z_1 - I^0/T_1}^{z_1 - \check{I}/T_1} H((z_1 - u)T_1, p_H((z_1 - u)T_1)) \, dF(u) \\ &+ \int_{z_1 - \check{I}/T_1}^{B} H((z_1 - u)T_1, \bar{p}_2) \, dF(u), \end{aligned}$$

$p_1^* = \tilde{p}_1$, $Q^* = [\bar{y}_1(\tilde{p}_1) + z_1^*]T_1$, and $p_2^* = \min\{p_H(I), \bar{p}_2\}$.

(e) $\check{I}/T_1 + B \le z_1^* < I^0/T_1 + B$:

$$\begin{aligned} V = \Gamma_1(z_1, \tilde{p}_1) &+ \int_A^{z_1 - I^0/T_1} [\Gamma_2(z_2^0, p_2(z_2^0)) + R(z_1 - u)T_1] \, dF(u) \\ &+ \int_{z_1 - I^0/T_1}^{B} H((z_1 - u)T_1, p_H((z_1 - u)T_1)) \, dF(u), \end{aligned}$$

$p_1^* = \tilde{p}_1$, $Q^* = [\bar{y}_1(\tilde{p}_1) + z_1^*]T_1$, and $p_2^* = p_H(I)$.

(f) $z_1^* = I^0/T_1 + B$:

$$\begin{aligned} V &= \Gamma_1(z_1, \tilde{p}_1) + \int_A^{B} [\Gamma_2(z_2^0, p_2(z_2^0)) + R(z_1 - u)T_1] \, dF(u), \\ &= \Gamma_1(z_1, \tilde{p}_1) + \Gamma_2(z_2^0, p_2(z_2^0)) + RT_1(z_1 - \mu), \end{aligned}$$

$p_1^* = \tilde{p}_1$, $Q^* = [\bar{y}_1(p_1^o) + z_1^*]T_1$, $Q^* - [\bar{y}_1(p_1^*) + \xi_1]T_1 > I^0$, and $p_2^* = p_H(I)$. Note that z_1^* cannot exceed $I^0/T_1 + B$ due to $V' = -(C_0 - R)T_1 < 0$ as $R \le C_0$ by assumption.

2) $B - A \le I^0/T_1$ and $\check{I}/T_1 + B > I^0/T_1 + A$:

(a) $A \leq z_1^* < \check{I}/T_1 + A$ (no donation): same as 1)-(a).

(b) $\check{I}/T_1 + A \leq z_1^* < B$ (no donation): same as 1)-(b).

(c) $B \leq z_1^* \leq I^0/T_1 + A$ (no donation):

$$V = \Gamma_1(z_1, \tilde{p}_1) + \int_A^{z_1 - \check{I}/T_1} H((z_1 - u)T_1, p_H((z_1 - u)T_1))\, dF(u) + \int_{z_1 - \check{I}/T_1}^{B} H((z_1 - u)T_1, \bar{p}_2)\, dF(u),$$

$p_1^* = \tilde{p}_1$, $Q^* = [\bar{y}_1(\tilde{p}_1) + z_1^*]T_1$, and $p_2^* = \min\{p_H(I), \bar{p}_2\}$.

(d) $I^0/T_1 + A \leq z_1^* < \check{I}/T_1 + B$: same as 1)-(d).

(e) $\check{I}/T_1 + B \leq z_1^* < I^0/T_1 + B$: same as 1)-(e).

(f) $z_1^* = I^0/T_1 + B$: same as 1)-(f).

3) $B - A \leq I^0/T_1$ and $\check{I}/T_1 + B < I^0/T_1 + A$:

(a) $A \leq z_1^* < \check{I}/T_1 + A$ (no donation): same as 1)-(a).

(b) $\check{I}/T_1 + A \leq z_1^* < B$ (no donation): same as 1)-(b).

(c) $B \leq z_1^* < \check{I}/T_1 + B$ (no donation): same as 2)-(c).

(d) $\check{I}/T_1 + B \leq z_1^* < I^0/T_1 + A$ (no donation):

$$V = \Gamma_1(z_1, \tilde{p}_1) + \int_A^B H((z_1 - u)T_1, p_H((z_1 - u)T_1))\, dF(u),$$

$p_1^* = \tilde{p}_1$, $Q^* = [\bar{y}_1(\tilde{p}_1) + z_1^*]T_1$, and $p_2^* = p_H(I)$.

(e) $I^0/T_1 + A \leq z_1^* < I^0/T_1 + B$: same as 1)-(e).

(f) $z_1^* = I^0/T_1 + B$: same as 1)-(f).

4) $B - A \leq \check{I}/T_1$: We skip this scenario as it is a very unlikely situation. A set of cases could have been developed similar to 1) – 3).

B.4 Proof of Proposition 2

We first need to show some properties of the perceived quality function in the second period, $\bar{\delta}_2(T_2)$. We have the following observation:

Observation 1. *$\bar{\delta}_2(T_2)$ is increasing in T_2 for both linear and exponential quality degradation schemes.*

Proof. For the linear deterioration scheme, we have $\bar{\delta}_2'(T_2) = \lambda(1/2) > 0$. For the exponential deterioration scheme, calculations result in a more complex expression:

$$\bar{\delta}_2'(T_2) = \frac{1}{T_2^2}\left[qe^{-\lambda(T-T_2)}(T_2 - 1/\lambda) + (q/\lambda)e^{-\lambda T}\right],$$

where the first expression inside the brackets, $qe^{-\lambda(T-T_2)}(T_2 - 1/\lambda)$, is monotone increasing in T_2, and when $T_2 = 0$, we have $\bar{\delta}_2'(T_2) = 0$. This completes the proof. □

Now, to prove the proposition, we take implicit differentiation of $d\Gamma_2(z_2, p_2(z_2))/dz = 0$ with respect to T_2 as we consider the case $z_2^* < \bar{z}$ (and $\omega = 0$) in Proposition 1.

$$\frac{d^2\Gamma_2(z_2, p_2(z_2))}{dz_2^2} \cdot \frac{\partial z_2}{\partial T_2} + [1 - F(z_2)]\frac{\phi\bar{\delta}_2'(T_2)}{2b} = 0,$$

where the second expression is positive. The first expression must be negative to maintain the right hand side of the equation, therefore, $\partial z_2/\partial T_2 > 0$ is realized.

Next, we show the same property for the price:

$$\frac{\partial p_2(z_2)}{\partial T_2} = \frac{\phi \bar{\delta}_2'(T_2)}{2b} + \frac{[1 - F(z_2)]}{2b}\frac{\partial z_2}{\partial T_2} > 0.$$

Finally, for $\gamma = (T_2/I)[\bar{y}_2(p_2, T_2) + z_2]$, we obtain:

$$\frac{\partial \gamma}{\partial T_2} = (T_2/I)\frac{\partial z_2}{\partial T_2}\left[1 - \frac{[1 - F(z_2)]}{2}\right] + (1/I)[\bar{y}_2(p_2, T_2) + z_2] - \phi(T_2/I)\frac{1}{2}\bar{\delta}_2'(T_2),$$

where one can show that $(1/I)\bar{y}_2(p_2, T_2) > \phi(T_2/I)\frac{1}{2}\bar{\delta}_2'(T_2)$ for both quality degradation schemes, which completes the proof.

B.5 Proof of Proposition 3

Once again, we use implicit differentiation to prove our claim. By taking the implicit derivative of $d\Gamma_2(z_2, p_2(z_2))/dz_2 = 0$ with respect to R, we obtain,

$$1 - \frac{1 - F(z_2)}{2} = \frac{d^2\Gamma_2(z_2, p_2(z_2))}{dz_2^2}\frac{\partial z_2}{\partial R},$$

where $\partial z_2/\partial R \leq 0$ as the left-hand-side is positive while the second derivative of $\Gamma_2(z_2, p_2(z_2))$ is negative by Theorem 1. Now we can take the implicit derivative of $\gamma = [\bar{y}_2(p_2(z_2), T_2) + z_2](T_2/I)$ (we ignore T_2/I as it is a positive constant):

$$\frac{\partial \gamma}{\partial R} = \frac{\partial z_2}{\partial R}\left[1 - \frac{1 - F(z_2)}{2}\right] - \frac{b}{2} \leq 0,$$

which completes the proof.

B.6 Proof of Theorem 5

Let $\hat{V}$ denote the firm's value function over two periods under the no-donation policy. Clearly, no donation cases in Theorem 3 yield $\hat{V} = V$. We show the proof for scenario 1)-(c) as it is one of the most common instances that the retailer may face. Findings can be extended to other donation-enabled scenarios with mild adjustments.

One can write the FOC of V as,

$$V'(z_1) = \hat{V}'(z_1) + \alpha T_1 \Big[RF(z_1 - I^0/T_1) - \int_A^{z_1 - I^0/T_1} H'((z_1-u)T_1, p_H((z_1-u)T_1))\, dF(u) \Big].$$

We need to analyze the sign of the expression inside the brackets. Note that $H'(I, p_H(I))$ is decreasing in I. Substituting $u = z_1 - I^0/T_1$ into $H'((z_1 - u)T_1, p_H((z_1 - u)T)$, we obtain $H'(I^0, p_H(I^0)) = \Phi'(I^0) = R$ as two solutions collide at I^0 by Theorem 2. This gives rise to $V'(z_1) > \hat{V}'(z_1)$ meaning that $z_1 > \hat{z}_1$. Also, note that V and $\hat{V}$ have the same FOC for price resulting in $p_1(z_1) > p_1(\hat{z}_1)$ and $Q > \hat{Q}$, where $\hat{Q}$ is the stocking quantity in the absence of donation.

As for the second part of the proposition, note that $I = (z_1^* - \xi_1)T_1$. Therefore, if $\xi_1 \leq z_1^* - I^0/T_1$, we have $I \geq I^0$ and $\gamma^* = I^0/I$, which increases as ξ_1 approaches to $z_1^* - I^0/T_1$. On the other hand, if $\xi_1 \leq z_1^* - I^0/T_1$, we have $I < I^0$ and $\gamma^* = 1$ meaning that all unsold units should be carried forward to the second period.

B.7 Proof of Theorem 6

Let $\hat{p}_2$ and $\hat{\Gamma}_2(\bar{z}(I, \hat{p}_2), \hat{p}_2)$ denote the pricing decision and expected profit, respectively, in the absence of donation. We note that $\hat{\Gamma}_2(\bar{z}(I, \hat{p}_2), \hat{p}_2) = H(I, p_2)$ for an extended interval of $I \in [0, \infty)$. We have two cases:

1) $I \leq I^0$:

   In this case, $\gamma^* = 1$ by the optimal policy (3.14) and the solutions align, meaning that $\Lambda(\bar{z}(I, p_2^*)) = \Lambda(\bar{z}(I, \hat{p}_2))$.

2) $I > I^0$:

   In this case, we have $\gamma^* = I^0/I$ and $\Gamma_2(z_2^*, p_2^*) = \Gamma_2(z_2^0, p_2^0)$ by the optimal policy (3.14). Therefore, the expected waste under the optimal donation policy is constant and equals to $\Lambda(z_2^0)$.

   On the other hand, $\hat{\Gamma}_2(\bar{z}(I, \hat{p}_2), \hat{p}_2)$ leads to $\Lambda(\bar{z}(I, \hat{p}_2))$, where $\bar{z}(I, \hat{p}_2) > z_2^0$ as $\bar{z}'(I, \hat{p}_2(I)) = 1/T_2 + b\hat{p}_2'(I) > 0$. Since $\Lambda(z)$ is an increasing function of z, we obtain $\Lambda(z_2^0) < \Lambda(\bar{z}(I, \hat{p}_2))$.

Appendix C

Appendices of Chapter 4

C.1 Proof of Theorem 8

The proof relies on the fact that both J_2^1 and J_2^2 are strictly concave. We check the FOCs at point $s_2 = 0$ and $s_2 = A$:

$$J_2^{1'}|_{s_2=0} = (1-\tau)\left[Aq_2 - \frac{\tau}{1-\tau}\alpha c + h\right] > (1-\tau)(Aq_2 - c - h) > 0,$$

$$J_2^{1'}|_{s_2=A} = -(1-\tau)(q_2 A + h) - \tau\alpha c < 0,$$

$$J_2^{2'}|_{s_2=0} = (1-\tau)(q_2 A - h) - \tau\beta[q_2(A+I) - c] > 0, \quad \text{when} \quad I < \check{I}$$

$$J_2^{2'}|_{s_2=A} = -(1-\tau)\left[1 - \frac{\tau}{1-\tau}\beta\right] q_2 A - (1-\tau)h - \tau\beta(q_2 I - c) < 0 \quad \text{when} \quad A \leq I,$$

where $\alpha, \beta < (1-\tau)/\tau$ by the tax law, $Aq_2 > c + h$ by assumption, and $\check{I}$ is defined such that $s_2^2(\check{I}) = 0$. It is straightforward that $s_2^1 \in (0, A)$. We also obtain $s_2^2(I) < A$ as there will be no tax subsidy for $s_2^2 = A$ when $A > I$ and we realize $J_2^{2'}|_{s_2=A} = -(1-\tau)(q_2 A + h) < 0$. However, $J_2^{2'}$ is monotone decreasing in I with $J_2^{2'}|_{s_2=0,I=0} > 0$, therefore, a threshold value, $\check{I}$, satisfying $s_2^2(\check{I}) = 0$ exists.

There are three possible scenarios depending on the value of $\tilde{s}_2 = A-(1+\alpha/\beta)c/q_2$:

(1) Suppose $\tilde{s}_2 \leq 0 \Leftrightarrow q_2 \leq (1+\alpha/\beta)(c/A)$, then **(P1)** is infeasible and the solution is $s_2^* = \max\{0, \min\{s_2^2(I), I\}\}$.

(2) Suppose $0 < \tilde{s}_2 \leq s_2^1 \Leftrightarrow (1+\alpha/\beta)(c/A) < q_2 \leq (1/A)[2(1+\alpha/\beta)c-h-\alpha c\tau/(1-\tau)]$

As $J_2^{1'}|_{s_2=\tilde{s}_2} > 0$, there are only two possible outcomes: **(a)** $J_2^{2'}|_{s_2=\tilde{s}_2} < 0$ and **(b)** $J_2^{2'}|_{s_2=\tilde{s}_2} > 0$. Outcome **(a)** leads to $s_2^* = \min\{I, \tilde{s}_2\}$, whereas, outcome **(b)** leads to $s_2^* = \min\{I, s_2^2(I)\}$. Notice that $\tilde{s}_2 > s_2^2(I)$ under **(a)** and $\tilde{s}_2 < s_2^2(I)$ under **(b)**. Thus, the conditions boil down to $s_2^* = \min\{I, \max\{s_2^2(I), \tilde{s}_2\}\}$.

(3) Suppose $\tilde{s}_2 > s_2^1 \Leftrightarrow q_2 > (1/A)\,[2(1+\alpha/\beta)c - h - \alpha c\tau/(1-\tau)]$, then **(P2)** is infeasible and the solution is $s_2^* = \min\{I, s_2^1\}$.

C.2 Proof of Lemma 1

Let us outline the FOCs and SOCs of $\Gamma(z, P_1)$:

$$\frac{\partial\Gamma(z,P_1)}{\partial z} = -c + P_1[1-F(z)], \tag{C.4}$$

$$\frac{\partial^2\Gamma(z,P_1)}{\partial z^2} = -P_1 f(z), \tag{C.5}$$

$$\frac{\partial\Gamma(z,P_1)}{\partial P_1} = a - (2P_1 - c)/q_1 + \mu - \Theta(z), \tag{C.6}$$

$$\frac{\partial^2\Gamma(z,P_1)}{\partial P_1^2} = -2/q_1 \tag{C.7}$$

where the expected profit is concave in P_1 for a fixed z so that we can use condition (C.6) to derive the price uniquely as a function of z, $p(z) = (q_1/2)(a+\mu+c/q_1-\Theta(z))$. Notice that when $\bar{X} < \infty$ we may realize $z^* \geq \bar{X}$ as the firm will commit her stocking quantity for two-periods. In such a case, there will be no shortage during the first period and the optimal first-period price will be the riskless price, $\tilde{p} = (q_1/2)(a+\mu+c/q_1)$.

C.3 Proof of Theorem 9

One can write the second order condition of $\Gamma(z, p(z))$ as,

$$\frac{d^2\Gamma(z,p(z))}{dz^2} = \frac{q_1[1-F(z)]}{2}\left[1-F(z)-(2/q_1)p(z)n(z)\right],$$

where $n(z) = f(z)/[1-F(z)]$ denotes the hazard rate function. Let us define $N(z) = 1-F(z)-(2/q_1)p(z)n(z)$. The expected profit is concave if and only if $N(z) \leq 0$. When $f(0)y(-c) > 1$, $N(z)|_{z=0} \leq 0$ and $N(z) \leq 0$ holds if $dN(z)/dz \leq 0$, which can be shown as,

$$\frac{dN(z_2)}{dz_2} = -f(z)-(2/q_1)p(z)(dn(z)/dz)-[1-F(z)]n(z) \leq 0.$$

We note that the maximizer of $\Gamma(z, p(z))$ is always an interior point as $d\Gamma(z, p(z))/dz$ is positive for $z = 0$ and negative for $z = \bar{X}$. Therefore, if $f(0)y(-c) \leq 1$, the expected profit is unimodal with a unique local maximum.

When $z^* \geq \bar{X}$, the firm never faces shortages in the first period and, therefore, $\Gamma(z, \tilde{p})'|_{z \geq \bar{X}} = -c < 0$.

C.4 The Proof of Theorem 10

The first expression in J_1, given in (4.13), is unimodal, but the second expression may be concave for some values of $z - X$ and convex for the others. However, we know that $\Phi(I)$ is a strictly increasing function for $I > 0$, therefore, the second expression is a non-decreasing function of z, regardless of the quality category for the leftover products. This implies that J_1 is non-decreasing for $z \in [0, z^0]$, where z^0 is the unique solution of $\Gamma'(z^0, p(z^0)) = 0$, and that $z^* > z^0$. Let us examine the behavior of J_1 under each quality category for $z > z^0$:

(a) *Low and Medium quality*: Following the optimal policies given in (4.9) and (4.10), $H_2(I, I) = H_1(I, I)$ is concave, $H_2(s_2^2(I), I)$ is convex, and $H_i(., I)$, $i = 1, 2$ is linear in I. Moreover, we have $J_1'|_{z=\min\{\infty, \bar{X}+\check{I}\}} = -(1-\tau)c + \tau\beta(q_2A - c) < -(1-\tau)c[1 - \alpha\tau/(1-\tau)] < 0$ for the low quality case and $J_1'|_{z=\min\{\infty, \bar{X}+\tilde{I}\}} = -(1-\tau)c[1 - \alpha\tau/(1-\tau)] < 0$ for the medium quality case as $\alpha \in [0, \frac{1-\tau}{\tau})$ by assumption and $q_2A < (1 + \alpha/\beta)c$ under the low quality scenario. Thus, $J_2' = 0$ has at least one root.

Because the rate of increase in $H_2(s_2^2(I), I)$ first declines and then climbs up, when $z^0 < \check{I}$ ($z^0 < \tilde{I}$) for the low (medium) quality case, J_1 may have two critical points with the smaller one corresponding to a stationary point and the larger one corresponding to a local maximum, or it may follow a decrease, increase, decrease pattern leading to three critical points, call them $z^1 < z^2 < z^3$. Notice that when there are three critical points, z^2 corresponds to a local minimum meaning that $z^* = z^i$ such that $J_1(z^i) > J_1(z^j)$, $i, j = 1, 3$ and $i \neq j$. On the other hand, when $z^0 > \check{I}$ ($z^0 > \tilde{I}$) for the low (medium) quality case, J_1 is unimodal and has a unique maximizer.

(b) *High quality*: In this case, both expressions in equation (4.13) are concave, and so is J_1. Moreover, $J_1'|_{z=\min\{\infty,\bar{X}+s_2^1\}} = -(1-\tau)c[1-\alpha\tau/(1-\tau)] < 0$ meaning that z^* is always an interior point of $[0, \min\{\infty, \bar{X}+s_2^1\})$.

C.5 The Proof of Theorem 11

We show our claims over the firm's optimal inventory carrying policy as there is a one-to-one relation between the firms selling amount in the clearance period, s_2^*, and donation amount, $I - s_2^*$.

Let us start with analyzing the firm's optimal solution for varying α values in $[0, \frac{1-\tau}{\tau})$. Notice that $\tilde{s}_2$ and s_2^1 are both decreasing in α with $\tilde{s}_2|_{\alpha\to 0} = A - c/q_2 > (1/2)(A-h/q_2) = s_2^1|_{\alpha\to 0}$ as $qA > 2c-h$ by assumption. However, the rate of decrease is larger for $\tilde{s}_2$. Thus, for fairly small values of α, we obtain $\tilde{s}_2 > s_2^1$ leading to a high quality scenario, where the optimal solution suggests $s_2^* = \min\{I, s_2^1\}$, which is decreasing in α. For medium values of α, we realize $0 < \tilde{s}_2 < s_2^1$ corresponding to a medium quality case, where the optimal solution is $s_2^* = \min\{I, \max\{s_2^2(I), \tilde{s}_2\}\}$. We know $s_2^2(I)$ is independent of α, therefore, s_2^* is decreasing in α. Also, notice that $s_2^2(I) < \tilde{s}_2$ for $I > \tilde{I}$, where $\tilde{I}$ is increasing in α. Thus, for large values of α, we realize either $I \leq \tilde{I}$ or $\tilde{s}_2 \leq 0$, whichever comes first, and the solution becomes independent of α.

The threshold value, $\tilde{s}_2$, is increasing in $\beta \in [0, \frac{1-\tau}{\tau})$. For fairly small values of β, we obtain $\tilde{s}_2 \leq 0$ corresponding to a low quality scenario, where the optimal solution is $s_2^* = \max\{0, \min\{s_2^2(I), I\}\}$. Note that $s_2^2(I) < I$ for $I > \bar{I}$, we also have

$\bar{I} > (c-h)/q_2$, as $q_2 A > 2c-h$ by assumption, meaning that $s_2^2(I)$ is decreasing in β when $s_2^* = s_2^2(I)$. One can also show that $\bar{I}$ and $\tilde{I}$ are always decreasing in β, while $\check{I}$ is decreasing in β for $s_2^2(I) > 0$ (equivalent to $I < \check{I}$). Therefore, as β grows, we observe either $\check{I} > I$ (leading to $s_2^2(I) \leq 0$ and $s_2^* = 0$), or $\tilde{s}_2 > 0$ (leading to medium quality scenario). If the former happens first, the firm donates all leftover units for $\tilde{s}_2 \leq 0$, and we realize $s_2^* = \tilde{s}_2$, which is increasing in β, for $0 < \tilde{s}_2 < s_2^1$. On the other hand, if the latter happens first, $s_2^* = s_2^2(I)$ keeps shrinking as β increases until the firm realizes $\tilde{I} < I$, leading to $s_2^* = \tilde{s}_2$. For large values of β, we observe $\tilde{s}_2 > s_2^1$ (high quality), and the optimal solution settles at $s_2^* = \min\{I, s_2^1\}$, which is independent of β.

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