Sensitivity and Stability: An Investigation of Stock-Flow Consistent Climate-Economic Models

SENSITIVITY AND STABILITY: AN INVESTIGATION OF STOCK-FLOW CONSISTENT CLIMATE-ECONOMIC MODELS

BY

DANIEL PRESTA, B.Sc.

A THESIS

SUBMITTED TO THE DEPARTMENT OF MATHEMATICS & STATISTICS

AND THE SCHOOL OF GRADUATE STUDIES

OF MCMASTER UNIVERSITY

IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

Master of Science (2021)	McMaster University
(Mathematics & Statistics)	Hamilton, Ontario, Canada

TITLE:	Sensitivity and Stability: An Investigation of Stock-Flow Consistent Climate-Economic Models
AUTHOR:	Daniel Presta B.Sc., (Mathematics and Statistics) McMaster University, Hamilton, Canada
SUPERVISOR:	Dr. Matheus R. Grasselli and Dr. Benjamin M. Bolker
NUMBER OF PAGES:	xvii, 144

Abstract

We investigate the stability of various stock-flow consistent economic models, and the potential causes for economic collapse therein. Through parameter sensitivity analysis, we study models that feature a public sector, an active central bank, and a household sector with independent consumption. Our final, most comprehensive economic system combines all of the intricacies of each model, prominently featuring a demand-driven economy that is stabilized by an expansionary monetary policy. In addition, we incorporate a climate module for each economic system, and analyze public sector intervention through carbon taxes and abatement subsidies. We find that the most common feature of economic instability is a lack of demand, driven by decreases in capital investment from firms, as well as a decline in household consumption. In order to maintain a stable growth path and prevent a permanent economic contraction in the face of climate-driven economic stresses, we propose the implementations of an expansionary monetary policy, increased public sector subsidies of abatement costs, and stricter carbon taxes.

Acknowledgements

I would first like to thank my supervisors, Dr. Matheus Grasselli and Dr. Ben Bolker, for their insights and instruction throughout my degree. Their guidance, patience, and support have helped me grow and mature as a researcher, and I owe much of my passion for my research to them.

I would also like to acknowledge Luka Milic and Emma Holmes for their ideas, advice, and collaboration throughout my thesis.

Last but certainly not least, I would like to thank my parents, my brother, and my girlfriend for their constant support and encouragement. They continue to inspire me to this day; this thesis is dedicated to them.

Contents

List of Figures

Chapter 1

Introduction

Economics affects every aspect of our lives. On an elementary level, economic structures decide where and how we live, what and when we will eat, and the livelihoods that we pursue. On a macroscale, economics determines the health and well-being of entire nations, the relationships between countries, and the future stability of our world. Understanding how these life-shaping systems can be affected and altered is crucial in our eternal quest to understand and realize the vast possibility of the future.

Our research is rooted in the work of economist Hyman Minsky, who uncovered a pivotal feature of modern economic systems. Traditional general equilibrium or optimal behaviour economic models assume that macroeconomic systems are inherently stable, exhibiting sustained growth paths so long as there are no external shocks to the system. These models are ineffective at predicting, understanding, and mitigating the disastrous effects and damages caused by financial crises and economic collapses, and thus we look to Minsky's work for motivation. In Minsky (1986), he postulates that our financial systems are intrinsically unstable in his now-famous "Financial Instability Hypothesis". He argues that due to this inherent instability, we should expect that financial crises are an expected outcome of our macroeconomic system. His hypothesis was later modelled by economist Steve Keen (Keen, 1995). The Keen model builds on the Goodwin Lotka-Volterra growth model (Goodwin, 1967), and attempts to quantify Minsky's idea of inherent instability. This three-dimensional model analyzes the time-evolution of important economic ratios such as the wage and private debt shares of a closed economy, as well as the employment rate.

The Keen model was later found to contain two locally stable long-run equilibria (Grasselli and Costa Lima, 2012). The "good" equilibrium is characterized by a finite private debt ratio, and positive wages and employment, whereas the "bad" equilibrium is characterized by vanishing wages and employment, and a private debt ratio that explodes to infinity. The Keen model was then further analyzed and explored in a stock-flow consistent (SFC) setting in Grasselli and Nguyen-Huu (2015), with later sections in that paper adding inflation and speculation to the original model. They uncovered an important deflationary regime, in which employment falls and the economy enters into a recession, all while the wage share remains stable, and the debt share potentially remains at safe levels.

The value of analyzing Keen-based models through an SFC lens is two-fold: first, it ensures that the economic system under study remains closed, with all nominal amounts of money flowing from one sector to another. There can be no "black holes"; all money must flow from one agent to another. Second, it provides a simple accounting framework from which we can analyze the balance sheets and savings equations for each sector, therefore gaining further insight into the relationships between different sectors in an economic system. In addition to the many continuous-time SFC models that are inspired by the work of Keen (such as the models featured in Grasselli and

Costa Lima (2012) or Grasselli and Nguyen-Huu (2015), for example), there are also numerous discrete-time SFC models that investigate financial instability as well, such as Godley and Lavoie (2006), and in the field of ecological macroeconomics, the work of Dafermos *et al.* (2017), (2018).

Returning to our discussion of continuous-time SFC models, we turn to the work of Grasselli and Nguyen-Huu (2018), who add inventory dynamics and explicit consumption modelling to the framework presented in Grasselli and Nguyen-Huu (2015). Their research is inspired by the growth cycles model of Franke (1996), and departs from previous Keen-based models by allowing a disequilibrium in the goods market. The disequilibrium between expected sales and aggregate demand is accounted for through the use of inventories, specifically unplanned investment in inventory, and follows many of the accounting assumptions presented in Godley and Lavoie (2006). Furthermore, the independent modelling of consumption enables the behaviour of households to explicitly enter the main dynamical system, with a higher wage share corresponding to greater levels of consumption, and thus increased demand.

Economic models that aim to mitigate and alleviate the effects of economic recessions are important to consider, as well. For example, Costa Lima *et al.* (2014) attempt to curb a fall in private profits through a corresponding increase in government expenditures, whenever employment rates are low. By doing so, their fiscal policy regime can prevent economic trajectories from going to the bad equilibrium, avoiding a permanently depressed economic system. Grasselli and Lipton (2019a) take a different approach, extending the Goodwin-Keen model to include time deposits, government bills, cash, and central bank reserves. Their model describes a fractional reserve banking system, and develops a monetary policy that explores narrow banking, in which a full reserve requirement is imposed on time deposits. Their work reveals that narrow banking does not suppress economic growth when the fractional reserve and full reserve systems converge to a finite ("good") equilibrium. Furthermore, they demonstrate that the narrow banking model can prevent financial breakdowns when the private debt share explodes.

Our work also explores the potentially catastrophic effects of climate change on economic systems. The modelling of economic effects caused by climate change originates from William Nordhaus' Dynamic Integrated model of Climate and the Economy (DICE) (Nordhaus, 1994). This was the first climate model to introduce a closed feedback loop, taking emissions, the carbon cycle, radiative forcing, temperature, and damages into account. Furthermore, the DICE model made major strides in the field of climate control, by testing different methods and policies for mitigating the effects of climate change, along with their respective costs and benefits. It also introduces uncertainties regarding key parameters such as labour productivity growth, the equilibrium climate sensitivity, and the inertia of the climate system were also introduced.

In recent years, there has been much research performed on the convexity and impact of the damage function in climate models. The DICE model employs a quadratic damage curve in Nordhaus (1994), and therefore produces damages that are often deemed to be unrealistically low, as argued by Weitzman (2012) and Dietz and Stern (2015). The *Weitzman* and *Stern* specifications increase the convexity of the damage curve, and are thus more suitable for examining the impact of possible climate catastrophes on our economic and financial systems.

Our climate research is grounded in the work of Bovari *et al.* (2018a), (2018b). They first develop a continuous-time, SFC macroeconomic model that considers the economic impacts of climate change, and the equally calamitous prospect of over-indebtedness in the private sector. They observe that climate change will not only cause damages by reducing output and capital, but also by forcing the private sector to leverage in order to compensate for their losses. This leads to a potentially explosive private debt, which then produces a major economic collapse (Bovari *et al.*, 2018a). Later, Bovari *et al.* (2018b) builds upon their earlier research by performing a sensitivity analysis on four of their key economic and climate variables, similar to the work done in Nordhaus (2018). They find that as the level of public intervention increases, so does the probability of achieving the +2°C target outlined in the Paris Agreement. Furthermore, they remark that even though private debt increases with public intervention, the implementation of public policy limits the risk of surpassing the problematic threshold of a private debt ratio equal to approximately 2.7. This threshold is important; once the private debt ratio attains values of approximately 2.5 to 3, the nominal debt levels in the economy would exceed the market value of capital, which would likely cause a systematic cascade of defaults in the private sector.

Public policy plays a critical role in climate-economic models. In Bovari *et al.* (2018b), the public sector can subsidize abatement technology costs, and can also levy carbon taxes on the emissions of firms, in line with the suggested growth path of the Stern-Stiglitz Commission (Stern and Stiglitz, 2017). Bovari *et al.* (2018b) observe that taxing carbon emissions and subsidizing a transition to green technology will decrease the amount of emissions by the private sector, thus mitigating the temperature anomaly as well as the subsequent damages. We follow their climate module, and their modelling approach regarding emissions, carbon pricing, and abatement.

The thesis is organized as follows. Chapter 2 analyzes the Keen model with inflation (originally developed in Grasselli and Nguyen-Huu (2015)), but initially disaggregates private loans and deposits, through the assumption that the lending and deposit rates are not equal. Chapter 3 adds a public sector to the existing framework of Chapter 2. Chapter 4 retains the framework of Chapter 3, while adding a monetary policy regime, conducted by an active central bank. Chapter 5 studies a modified version of the inventory model presented in Grasselli and Nguyen-Huu (2018). Chapter 6 combines the work of the two preceding chapters, analyzing a larger model complete with independent household consumption, inventories, a public sector, and an active central bank. Chapter 7 concludes.

In each of Chapters 2 through 6, the first section presents the accounting framework and model equations. Chapters 3, 4, and 6 build upon the work of previous chapters, and thus contain a reduced opening section, that discusses changes or additions to the preceding models. Section 2 of each chapter derives a reduced-form dynamical system for the respective models, and performs a basic numerical local stability analysis (for a calibrated run), if applicable. Section 3 of each chapter presents and discusses numerical results (basic plots, sensitivity analyses, and Monte Carlo simulations), while Section 4 provides a brief description of the climate-economic model, along with an analysis of climate results.

Chapter 2

Keen Model with Inflation

2.1 Economic Module

2.1.1 Accounting Framework

We begin by investigating the original Keen model (Keen, 1995) with inflation, a system first explored in Grasselli and Nguyen-Huu (2015). The economy contains three different sectors: households, firms, and banks. Observing Tables 2.1, 2.2, and 2.3, we can investigate the balance sheets, income statements, and transaction flow matrices of the economy. Note that all entries in the economy's balance sheet are measured in nominal monetary amounts, whereas transaction entries and items on the flow of funds sheet are measured in monetary units per unit of time.

In this model, households are only permitted to hold deposits, denoted by M_h. They hold no other assets, and furthermore, have no liabilities. Firms hold deposits, M_f, and the nominal stock of capital, denoted by pK. The only liabilities of firms are bank loans, L_f, which serve as assets for the banking sector. Accordingly, the

	Households	Firms	Banks	Sum
Balance Sheet				
Capital Stock		pK		pK
Deposits	M_h	M_f	$-M$	0
Loans		$-L_f$	L_f	0
Sum (net worth)	X_h	X_f	X_b	X

Table 2.1: Balance sheet for the economy.

deposits of households and firms, denoted by $M = M_h + M_f$, are held by banks as liabilities. Lastly, we assume that constant interest is earned on each of the monetary items on the balance sheets.

	Households	Firms		Banks	Sum
Transactions		Current	Capital		
Consumption	$-pC_h$	pC		$-pC_b$	0
Investment		pI_k	$-pI_k$		0
[GDP]		$[pY]$			$[pY]$
Wages	W	$-W$			0
Depreciation		$-p\delta K$	$p\delta K$		0
Int. on loans		$-r_L L_f$		$r_L L_f$	0
Int. on deposits	$r_M M_h$	$r_M M_f$		$-r_M M$	0
Sum (balance)	S_h	S_f	$-pI_k$	S_b	0

Table 2.2: Transactions in the given economy

2.1.2 Output, Profits, and Firm Financing

As in Grasselli and Nguyen-Huu (2015), we assume a Leontieff-type production function for a given country's economy, and then suppose that a good is produced to a real level of output Y using the real stock of capital K, such that

$$Y = \frac{K}{\nu}. \tag{2.1}$$

	Households	Firms	Banks	Sum
Balance Sheet				
Change in capital stock		pI_k		pI_k
Change in deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$	0
Change in loans		$-\dot{L}_c$	$\dot{L}_c$	0
Sum (savings)	S_h	S_f	S_b	pI_k
Change in net worth	$\dot{X}_h = S_h$	$\dot{X}_f = \Pi + (\dot{p} - \delta p)K$	$\dot{X}_b = S_b$	$\dot{X}$

Table 2.3: Flow of funds in economy

The ratio $1/\nu$ represents capital productivity; conversely, ν represents the capital-to-output ratio in the model. Next, we assume that capital evolves according to

$$\dot{K} = I_k - \delta K, \tag{2.2}$$

where I_k represents the level of real capital investment by firms, and δ denotes a constant depreciation rate. The investment decisions of firms are funded both internally and externally, with internal funds obtained through pre-depreciation corporate profits, denoted by

$$\Pi_f = pY - W - r_L L_f + r_M M_f. \tag{2.3}$$

Actual nominal profit is defined as nominal output pY, minus wages W paid to workers and interest paid on existing loans $r_L L_f$, plus interest received on corporate deposits $r_M M_f$. If we further assume that the short-term interest rate r is equal for both loans and deposits ($r = r_L = r_M$), then we can focus on the net borrowing of firms from banks. This can be expressed as $D = L_f - M_f$; i.e. the difference between the loans and deposits of firms. The expression for corporate profits would then follow the prior work of Grasselli and Nguyen-Huu (2015):

$$\Pi = pY - W - rD. \tag{2.4}$$

We will make this simplifying assumption later, when performing sensitivity analysis (to be explained in Section 2.3.1) on key parameters in the model. For now, however, we will assume distinct lending and deposit rates, and thus express the profit share of the economy as

$$\pi_f = \frac{\Pi_f}{pY} = 1 - \omega - r_L \ell_f + r_M m_f, \tag{2.5}$$

where ω and d denote the wage share and private debt share of the economy, respectively. The wage share is given by the ratio of the wage bill $W = \mathrm{w}L$ to nominal output pY, while the private debt share is given by the ratio of corporate debt D to nominal output:

$$\omega = \frac{\mathrm{w}L}{pY} \tag{2.6}$$

$$d = \frac{D}{pY}. \tag{2.7}$$

Similarly, the private loan-to-output ratio and deposit-to-output ratios are given by $\ell_f = L_f/(pY)$ and $m_f = M_f/(pY)$, respectively.

Turning our attention to the investment decisions and financing needs of firms, we begin with the equation for real capital investment:

$$I_k = \kappa(\pi_f)Y, \tag{2.8}$$

where $\kappa(\cdot)$ is a linear increasing function of the profit share, bounded from below by

0, and above by 1. We can then model the aggregate credit demand of firms as the difference between corporate nominal investment and the nominal profits of firms. The credit demand of firms is not rationed, and can be equivalently expressed as the total financing needs for firms:

$$\dot{D} = \dot{L}_f - \dot{M}_f = pI_k - \Pi_f. \tag{2.9}$$

From equation (5.20), we can determine the evolution of corporate loans and deposits, while also taking into account any repayment of existing debt. The loans and deposits of firms evolve according to

$$\dot{L}_f = pI_k + r_L L_f - k_f L_f \tag{2.10}$$

$$\dot{M}_f = pY - W + r_M M_f - k_f L_f. \tag{2.11}$$

Following the simplifying assumption made in Grasselli and Nguyen-Huu (2015), we set the repayment rate on loans k_f equal to the lending rate r_L, thus resulting in the simpler differential equations

$$\dot{L}_f = pI_k \tag{2.12}$$

$$\dot{M}_f = \Pi_f. \tag{2.13}$$

2.1.3 Prices, Labour Markets, and Households

The price inflation dynamics follow Grasselli and Nguyen-Huu (2015) and are given by

$$i = \frac{\dot{p}}{p} := \eta_p \left(\xi \left(\frac{c}{p} \right) - 1 \right). \tag{2.14}$$

The ratio of the unit cost of labour to the price of a commodity is equivalent to the wage share ω, and follows the work of Keen (2013):

$$c = p\omega = \frac{\mathrm{w}}{a} \tag{2.15}$$

We take the markup $\xi \geq 1$ and the adjustment speed of prices η_p to be fixed.

A country's workforce N and the labour productivity of workers a follow exogenous exponential dynamics:

$$\dot{N} = \beta N \tag{2.16}$$

$$\dot{a} = \alpha a. \tag{2.17}$$

Labour is assumed to be the abundant factor of production. As a result, we assume that labour is hired at full capacity, such that

$$L = \frac{Y}{a}, \tag{2.18}$$

and so the employment rate is endogenously given by $\lambda = \frac{L}{N}$. For workers, nominal wages are modeled by

$$\dot{\mathrm{w}} = \mathrm{w}(\Phi(\lambda) + \gamma i). \tag{2.19}$$

The wage dynamics follow Desai (1973) and Grasselli and Costa Lima (2012), where workers bargain for wages by taking into account both the current labour market, and observed changes in the pricing dynamics (given by the inflation rate). The parameter γ thus corresponds to the degree of money illusion; a value of $\gamma = 1$ signifies a complete consideration and implementation of inflation into the workers' bargaining.

Lastly, the consumption of households and banks is not modelled explicitly (explicit household consumption is modelled beginning in Chapter 5) but is instead expressed as an accommodating variable:

$$C = Y - I_k = (1 - \kappa(\pi_f))Y. \tag{2.20}$$

Total output is assumed to be sold either through consumption (by households and banks) or investment (by firms), and is determined solely through the investment behaviour of firms.

2.1.4 Functional Forms

Following the work of Grasselli and Nguyen-Huu (2015), a rational function is employed to help model the growth of wages

$$\Phi(\lambda) = \frac{\phi_1}{(1-\lambda)^2} - \phi_0. \tag{2.21}$$

The use of a rational curve (instead of a linear form) enables wage stickiness, since for lower employment rates, nominal wages are slower to decrease. Therefore, temporary price deflation or a relatively small contraction in the economy will not crash the entire system, as nominal wages will not quickly decrease to zero. In addition, this form prevents the employment rate from surpassing the economically possible value of 1. As the economy reaches 100 percent employment, wages increase dramatically, causing a decrease in profits for firms, and a corresponding decrease in investment (see equation (2.8)). Consequently, we observe that capital decreases in equation (2.2), leading to a contraction in output (equation (2.1)). This change in turn leads to a decrease in employed labour (equation (2.18)), therefore forcing the employment rate down away from 1.

As for capital investment, the $\kappa(\cdot)$ investment function is bounded above by 1, and below by 0, and follows a linear form given by

$$\kappa(\pi_f) = \max\{0, \min\{1, \mathrm{inv}_0 + \mathrm{inv}_1 \pi_f\}\}. \tag{2.22}$$

We proceed with this form, since it produces fewer outcomes that are sensitive to changes in the respective parameters, when compared with the original exponential function, adopted in Grasselli and Nguyen-Huu (2015). This functional form is easier to understand than the rational Phillips curve: as corporate profits increase, the firms' desire for investment increases as well (equation (2.8)).

2.2 Main Dynamical System

As shown in Grasselli and Nguyen-Huu (2015), the equations and expressions from Section 2.1 can be reduced to a three-dimensional system of differential equations.

From the expression for the employment rate $\lambda = L/N = Y/(aN)$, and equations (2.16) and (2.17), we denote the dynamics for the employment rate below:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{Y}}{Y} - \frac{\dot{a}}{a} - \frac{\dot{N}}{N} = g(\pi_f) - \alpha - \beta, \tag{2.23}$$

where $g(\pi_f)$ represents the long-run growth rate of the economy, and is given by

$$g(\pi_f) := \frac{\dot{Y}}{Y} = \frac{\kappa(\pi_f)}{\nu} - \delta. \tag{2.24}$$

Then, from the expression for the wage share $\omega = \mathrm{w}L/(pY) = \mathrm{w}/(ap)$ and equations (2.17) and (2.14), we similarly obtain the closed-form dynamics for the ω:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} - \frac{\dot{a}}{a} - \frac{\dot{p}}{p} = \Phi(\lambda) - \alpha - (1-\gamma)i(\omega). \tag{2.25}$$

Then, using the expression $\ell_f = L_f/(pY)$ and the expression for loan dynamics (equation (2.12)), we can obtain the dynamics for the private loan-to-output ratio:

$$\begin{aligned}\frac{\dot{\ell}_f}{\ell_f} = \frac{\dot{L}_f}{L_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \left[\frac{pI_k}{L_f}\right] - i(\omega) - g(\pi_f) \\ &= \left[\frac{p\kappa(\pi_f)Y}{\ell_f pY}\right] - i(\omega) - g(\pi_f) \\ &= \left[\frac{\kappa(\pi_f)}{\ell_f}\right] - i(\omega) - g(\pi_f)\end{aligned} \tag{2.26}$$

Similarly, we can use the expression $m_f = M_f/(pY)$ and the expression for corporate deposit dynamics (equation (2.13)) to obtain the differential equation for the private deposit-to-output ratio:

$$\begin{aligned}\frac{\dot{m}_f}{m_f} = \frac{\dot{M}_f}{M_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \left[\frac{\Pi_f}{M_f}\right] - i(\omega) - g(\pi_f) \\ &= \left[\frac{pY\pi_f}{m_f pY}\right] - i(\omega) - g(\pi_f) \\ &= \left[\frac{\pi_f}{m_f}\right] - i(\omega) - g(\pi_f). \end{aligned} \tag{2.27}$$

Thus, we can reduce the model presented in Section 2.1 to a differential rate model with four state variables (ω, λ, ℓ_f, m_f):

$$\begin{cases} \dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega)\right] \\ \dot{\lambda} &= \lambda\left[g(\pi_f) - \alpha - \beta\right] \\ \dot{\ell}_f &= \kappa(\pi_f) - \ell_f(i(\omega) + g(\pi_f)) \\ \dot{m}_f &= \pi_f - m_f(i(\omega) + g(\pi_f)). \end{cases} \tag{2.28}$$

As explained by Grasselli and Nguyen-Huu (2015), the model succinctly summarizes the interplay between prices and private debt. Rising prices will stabilize the private loan-to-output ratio (and thus the private debt-to-output ratio) at lower values; conversely, deflation will cause the value of the real aggregate debt D to increase, leading to a higher private debt to output ratio, and a lower profit share π_f. This antecedent in turn reduces investment, and causes the economy to contract, as real output Y decreases.

We follow the work of Grasselli and Nguyen-Huu (2015) and obtain an interior equilibrium $(\omega^*, \lambda^*, \ell_f^*, m_f^*)$ for the reduced four-dimensional system. From Grasselli and Costa Lima (2012), we know that the growth rate of the economy at equilibrium is equal to

$$g(\pi_f^*) = \frac{\kappa(\pi_f^*)}{\nu} - \delta = \alpha + \beta. \tag{2.29}$$

Consequently, we can obtain the equilibrium profit share

$$\pi_f^* = g^{-1}(\alpha + \beta) = \kappa^{-1}(\nu(\alpha + \beta + \delta)), \tag{2.30}$$

and from equation (2.5), we derive the equilibrium wage share:

$$\omega^* = 1 - \pi_f^* - r_L \ell_f^* + r_M m_f^*. \tag{2.31}$$

Then, from the first differential equation in System 2.28, we obtain the equilibrium employment rate:

$$\lambda^* = \Phi^{-1}(\alpha + (1 - \gamma) i(\omega^*)). \tag{2.32}$$

Lastly, we use equation (2.29) and obtain the expressions for the private loan-to-output and deposit-to-output ratios at equilibrium:

$$\ell_f^* = \frac{\kappa(\pi_f^*)}{\alpha + \beta + i(\omega^*)} \tag{2.33}$$

$$m_f^* = \frac{\pi_f^*}{\alpha + \beta + i(\omega^*)}. \tag{2.34}$$

If we substitute equations (2.33) and (2.34) into equation (2.31) (comparable to the approach of Grasselli and Nguyen-Huu (2015)), we will find that the equilibrium wage share ω^* is a solution of the quadratic equation $a_0(\omega^*)^2 + a_1\omega + a_2 = 0$, where

$$
\begin{aligned}
a_0 &= \xi\eta_p > 0 \\
a_1 &= (\alpha + \beta) - \eta_p(1 + \xi(1 - \pi_f^*)) \\
a_2 &= (\eta_p - \alpha - \beta)(1 - \pi_f^*) + r_L\kappa(\pi_f^*) - r_M\pi_f^*.
\end{aligned}
$$

Thus, turning our attention to the conditions for existence and stability, we note that our adjustment speed parameter η_p is always greater than the equilibrium growth rate of the economy $\alpha+\beta$. Therefore, from Grasselli and Nguyen-Huu (2015), we can infer that if the condition

$$((\alpha + \beta - \eta_p) + \eta_p\xi(1 - \pi_f^*))^2 > 4\eta_p\xi(r_L\kappa(\pi_f^*) - r_M\pi_f^*) \tag{2.35}$$

is satisfied, then the equilibrium profit share $\pi_f^* < 1$ must also hold, and that there must be at least one non-negative equilibrium solution for ω^*. This non-negative equilibrium wage share is not often unique, as pointed out by Grasselli and Nguyen-Huu (2015).

2.2.1 Local Stability Analysis

We now present a brief local stability analysis of the aforementioned four dimensional model. Assuming there exists a non-negative equilibrium solution $(\omega^*, \lambda^*, \ell_f^*, m_f^*)$ as previously defined in Section 2.2 ($\omega^* > 0,\ \lambda^* \in (0, 1)$), we follow Grasselli and Nguyen-Huu (2015) by examining the following Jacobian matrix $J^*(\omega^*, \lambda^*, \ell_f^*, m_f^*) = J^*$ in order to investigate the local stability of the system:

$$J^* = \begin{bmatrix} J_{11}^* & \omega^* \Phi'(\lambda^*) & 0 & 0 \\ -\lambda^* g'(\pi_f^*) & g(\pi_f^*) - \alpha - \beta & -r_L \lambda^* g'(\pi_f^*) & r_M \lambda^* g'(\pi_f^*) \\ J_{31}^* & 0 & J_{33}^* & J_{34}^* \\ m_f^*(g'(\pi_f^*) - \eta_p \xi) - 1 & 0 & r_L(m_f^* g'(\pi_f^*) - 1) & J_{44}^* \end{bmatrix}. \tag{2.36}$$

In the matrix, we have that:

$$J_{11}^* = \Phi(\lambda^*) - \alpha + (1-\gamma)\eta_p(1 - 2\xi\omega^*) \tag{2.37}$$

$$J_{31}^* = -\kappa'(\pi_f^*) + \ell_f^*(g'(\pi_f^*) - \eta_p \xi) \tag{2.38}$$

$$J_{33}^* = r_L \ell_f^* g'(\pi_f^*) - r_L \kappa'(\pi_f^*) - g(\pi_f^*) - i(\omega^*) \tag{2.39}$$

$$J_{34}^* = r_M(\kappa'(\pi_f^*) - \ell_f^* g'(\pi_f^*)) \tag{2.40}$$

$$J_{44}^* = r_M - r_M m_f^* g'(\pi_f^*) - i(\omega^*) - g(\pi_f^*) \tag{2.41}$$

In the interest of simplicity and brevity, we will verify a numerical example, checking that the base parameters we have chosen for this system produce a locally stable equilibrium. It can be verified that our base numerical simulation equilibrates at the values:

$$(\omega^*,\, \lambda^*,\, \ell_f^*,\, m_f^*) = (0.91013,\, 0.97851,\, 0.3989,\, 0.18777). \tag{2.42}$$

Other useful equilibrium values are

$$
\begin{aligned}
i(\omega^*) &= 0.36864 \\
\kappa(\pi_f^*) &= 0.165 \\
g(\pi_f^*) &= \frac{\kappa(\pi_f^*)}{\nu} - \delta = \frac{0.165}{3} - 0.01.
\end{aligned}
$$

Important derivatives of $\kappa(\pi_f)$ and $g(\pi_f)$ include $\kappa'(\pi_f^*) = \text{inv}_1 = 15$ and $g'(\pi_f^*) = \frac{\kappa'(\pi_f^*)}{\nu} = \frac{15}{3} = 5$. Lastly, we calculate the derivative of the Phillips curve $\Phi(\lambda)$:

$$\Phi'(\lambda^*) = \frac{2\phi_1}{(1-\lambda^*)^3} = \frac{2(0.0000641)}{(1-0.97851)^3}. \tag{2.43}$$

We substitute the above equilibrium values into the Jacobian J^*, along with all necessary parameter values (see Appendix A.2), and obtain:

$$J^* = \begin{bmatrix} -0.87375 & 11.757 & 0 & 0 \\ -4.8926 & 0 & -0.19570 & 0.097851 \\ -14.992 & 0 & -1.0934 & 0.26011 \\ -0.96245 & 0 & -0.002446 & -0.41242 \end{bmatrix}. \tag{2.44}$$

The eigenvalues of this matrix are approximately

$$(\epsilon_1,\, \epsilon_2,\, \epsilon_3,\, \epsilon_4) = (-0.462 \pm 0.024i,\, -0.728 \pm 7.563i). \tag{2.45}$$

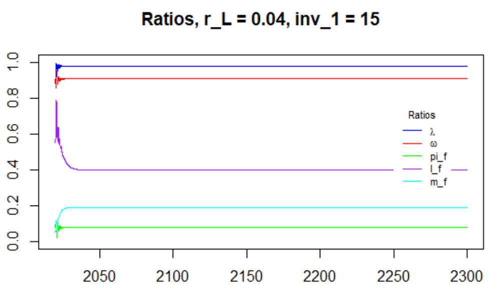
Since all four eigenvalues have negative real parts, we can say that our calibrated model converges to a locally stable equilibrium. In addition, our four complex eigenvalues ensure that our system will illustrate damped oscillatory trajectories en route to stabilization, as will be shown in Section 2.3. This type of behaviour is common to Keen models (see Keen (1995)), and gives a quasi-analytical explanation for the similar results of Grasselli and Nguyen-Huu (2015).

2.3 Results

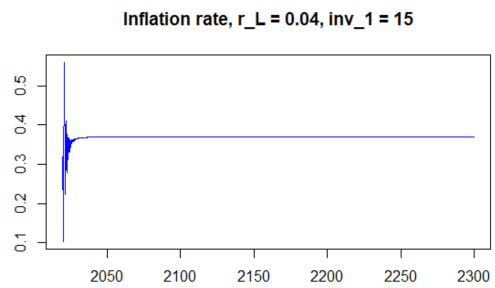
We now present basic simulated trajectories of variables from the model discussed in Section 2.1, followed by sensitivity grids, Monte Carlo plots, and an analysis of the basin of attraction. All computational work is performed in R (R Core Team (2020)), using the `lsoda` integrator from the `deSolve` package (Soetaert *et al.*, 2010). Parameter values and initial conditions for all of our numerical simulations can be found in Appendix A.

Figure 2.1 illustrates the importance of two key parameters in the model: the lending rate r_L, and the slope of the investment function inv_1. Each of the four state variables from the reduced model are plotted (ω, λ, ℓ_f, m_f), as well as the profit share π_f, all from the years 2020 to 2300. In Subfigures 2.1 (a) and (b), we observe a simple calibrated model. As we derived earlier in our local stability analysis, the system converges to a stable equilibrium. There is not much of interest otherwise, as the employment rate and wage share equilibrate to stable values, each greater than 0.9, while the private debt ratio $d = \ell_f = m_f$ equilibrates to a rather low value of just over 0.2. The inflation rate equilibrates at a high value of 0.36864, but this is to be expected, as we have not yet introduced any policy mechanisms to cool down our economy.

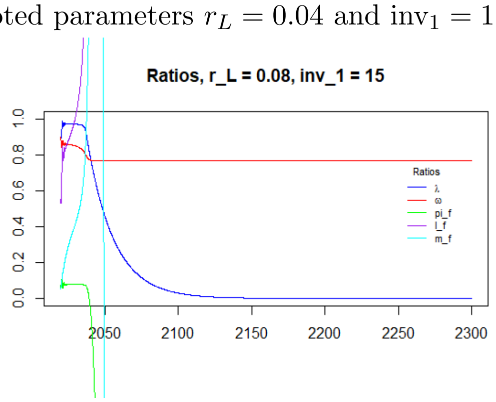
Subfigures 2.1 (c) and (d) tell a completely different story. We raise the lending rate r_L from 0.04 to 0.08, and observe a private sector that becomes extremely over-indebted. The private loan-to-output ratio explodes immediately, while the private deposit-to-output ratio crashes by 2050. This crash in turn causes the profit share π_f to plummet, which thus reduces investment. Consequently, the economy contracts and enters a permanently deflationary state, resulting in a vanishing employment rate. This result is achieved in a more direct fashion in subfigures 2.1 (e) and (f), where the lending rate returns to its original value of $r_L = 0.04$, but the slope of the investment function inv_1 is reduced to 5, from its original value of 15. Therefore, we conclude that the base Keen model with inflation is strongly influenced by interest rate specifications, as well as the shape of the investment function $\kappa(\cdot)$.



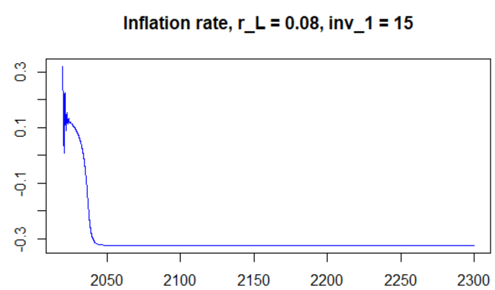
(a) Basic plots of key economic ratios, with noted parameters $r_L = 0.04$ and $\text{inv}_1 = 15$.



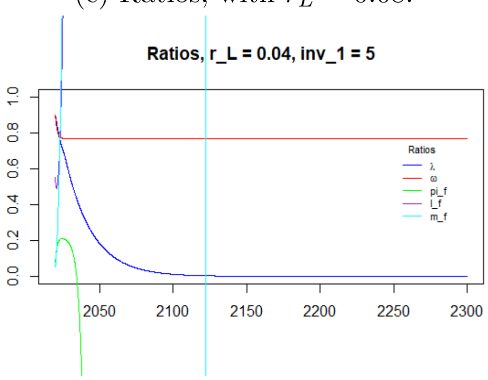
(b) Inflation rate plot, with noted parameters $r = 0.04$ and $\text{inv}_1 = 15$.



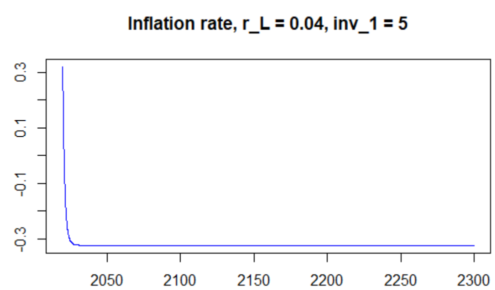
(c) Ratios, with $r_L = 0.08$.



(d) Inflation rate, with $r_L = 0.08$.



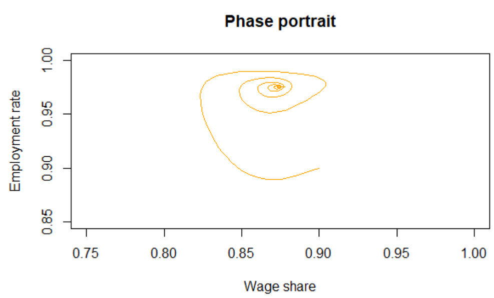
(e) Ratios, with $\text{inv}_1 = 5$.



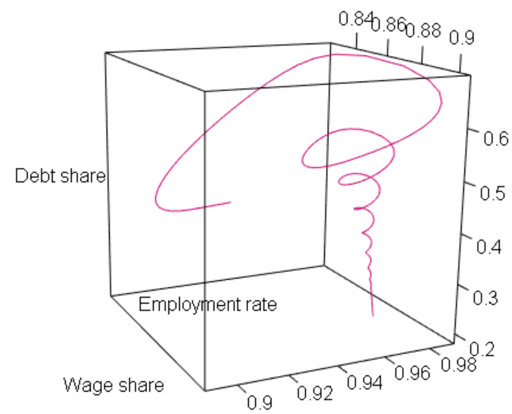
(f) Inflation rate, with $\text{inv}_1 = 5$.

Figure 2.1: Subfigures (a), (c), and (e), we observe simulations of the employment rate λ (blue), wage share ω (red), profit share π_f (green), private loan-to-output ratio ℓ_f (purple), and private deposit-to-output ratio m_f (cyan), for the model presented in Section 2.1, for lending rates $r_L = \{0.04, 0.08\}$ and investment function slopes $\text{inv}_1 = \{15, 5\}$. Plots (b), (d), and (f) depict simulations of the inflation rate $i(\omega)$ for the same parameter values. All simulations are run from 2020 to 2300.

Figure 2.2 (a) shows the relationship between the wage share and employment rate in a two-dimensional phase portrait, while Figure 2.2 (b) portrays the relationship between the wage share, employment rate, and private debt share in a three-dimensional phase portrait. The base calibration featured in Figures 1 2.2 (a) and (b) is plotted in these phase portraits, which capture the oscillatory convergence to a locally stable equilibrium.



(a) Two-dimensional phase portrait.



(b) Three-dimensional phase portrait.

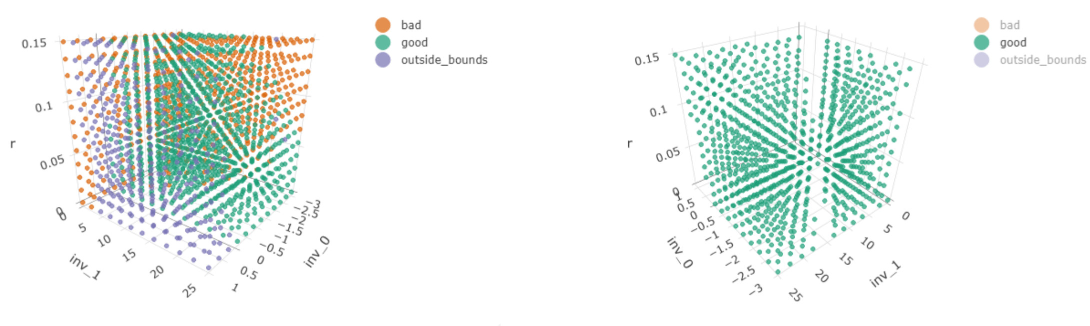
Figure 2.2: Phase portraits for the four dimensional Keen model with pricing. The two-dimensional phase portrait in (a) plots employment rate λ with respect to the wage share ω, whereas the three-dimensional phase portrait in (b) plots the employment rate λ, the wage share ω, and the private debt-to-output ratio d.

2.3.1 Parameter Sensitivity

One of the most important methods of computational analysis in our research involves investigating the sensitivity of certain parameters, to understand which parameters are most responsible for convergence to either the good or bad equilibrium. This level of analysis involves varying different economic parameters, each over some designated interval, and running simulations. We will use this parameter sensitivity method to determine an interval of values for which the model *converges* to an equilibrium, and will thus compute simulations on a relatively longer time scale, from 2020 to 2300.

Our work is similar to Holmes (2020) and Bolker *et al.* (2021). For each of the simulations, if we encounter an error, then the outcome is designated as an "error". All simulations that do not reach the specified final year (2300, in each of these runs), or that have final values of `NaN` for the employment rate λ or the wage share ω are classified as "bad". Simulations that cause the integrator to crash immediately and produce a final value of `NA` for either λ or ω are classified as "error" runs.

Furthermore, simulations that produce tail values of $\lambda < 0.4$ or $\omega < 0.4$ are classified as "bad". Simulated runs that are classified as "outside_bounds" occur if the final values for employment rate or wage share are not sensible, i.e. $\lambda > 1$ or $\omega > 1$. All other simulations are classified as "good". Final plots shown in this paper for all parameter sensitivity grids and basins of attraction are created using the `plotly` package (Sievert (2020)) in R.



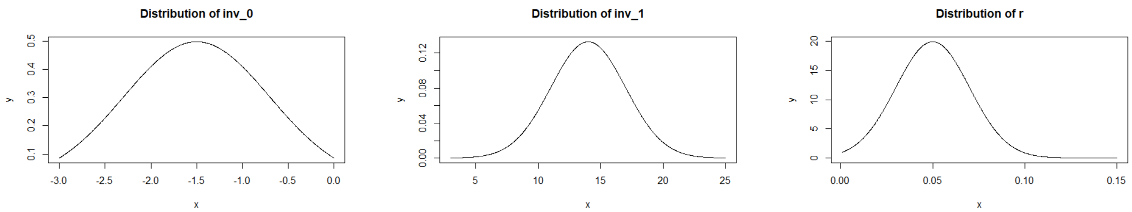
(a) Sensitivity of investment function parameters, alongside the interest rate.

(b) Sensitivity plot (a), with good simulations highlighted.

Figure 2.3: Parameter sensitivity for investment function parameters (along with the interest rate) (a), and for solely the good simulations of (a), illustrated in (b). The sensitivity plot in (b) helps depict the interesting vertical plane of simulations classified as "bad", which is found sandwiched between two "good" regions.

For our sensitivity analysis, we make the simplifying assumption that $r_L = r_M = r$, i.e. the lending and deposit rates are equal. In Figure 2.3, we observe the sensitivity grid for the general short-term interest rate r, varied from 0.001 to 0.15, the investment function slope inv_1, varied from 0.05 to 25, and the investment function intercept inv_0, varied from -3 to -1. Twelve equally spaced points are chosen within each of these specified intervals, creating a three-dimensional grid of $12 \times 12 \times 12$ simulations.

Figure 2.3 (b) displays the region of simulations to which the model converges to a good equilibrium. The region illustrates two general trends. First, the interest rate r and the intercept inv_1 are positively related, which is understandable, since lower investment values caused by a lower intercept in the investment function must be countered with a lower interest rate. A higher interest rate would produce dynamics similar to Figure 2.1 (c), in which an excessive interest rate will lead to a collapse in the profit share (and thus an overall economic contraction), in the absence of sufficiently strong capital investment. The second observation is a narrow region of bad classifications, sandwiched between the two good regions. We have not yet identified a potential explanation for this phenomenon, and leave the potential link between this "bad" region and any theoretical conditions of stability for future research.



(a) $\mu = -1.5$, $\sigma = 0.8$ (b) $\mu = 14$, $\sigma = 3$ (c) $\mu = 0.05$, $\sigma = 0.04$

Figure 2.4: Probability density functions for parameters inv_0, inv_1, and r. Each of the three parameters are sampled from a normal distribution; consequently, normal density is plotted on the y-axis.

Turning our attention to Monte Carlo simulations, we attempt to sample our three parameters from their "good" regions (in the sensitivity grid) alone. Correlations between parameters often make this a difficult task, and so it is certainly possible that some parameter combinations which result in economic collapse will be chosen. The inclusion of some poor economic outcomes, however, will give a wider understanding of potential trajectories for the model.

Figure 2.4 plots the probability density functions for inv_0, inv_1, and r. Each of the three parameters is sampled from a normal distribution; their means and standard deviations are stated in the sub-captions of Figure 2.4. In Figure 2.5, we observe 200 Monte Carlo replications of the wage share ω, employment rate λ, private debt ratio d, and inflation rate $i(\omega)$, all from 2020 to 2100. The shaded blue region represents the [0.025;0.975] probability interval, while the solid blue line represents the median trajectory for each variable. As mentioned earlier, the less restrictive sampling of parameters (combined with the large prediction interval) really shows a large range of economic trajectories for the base Keen model with inflation.

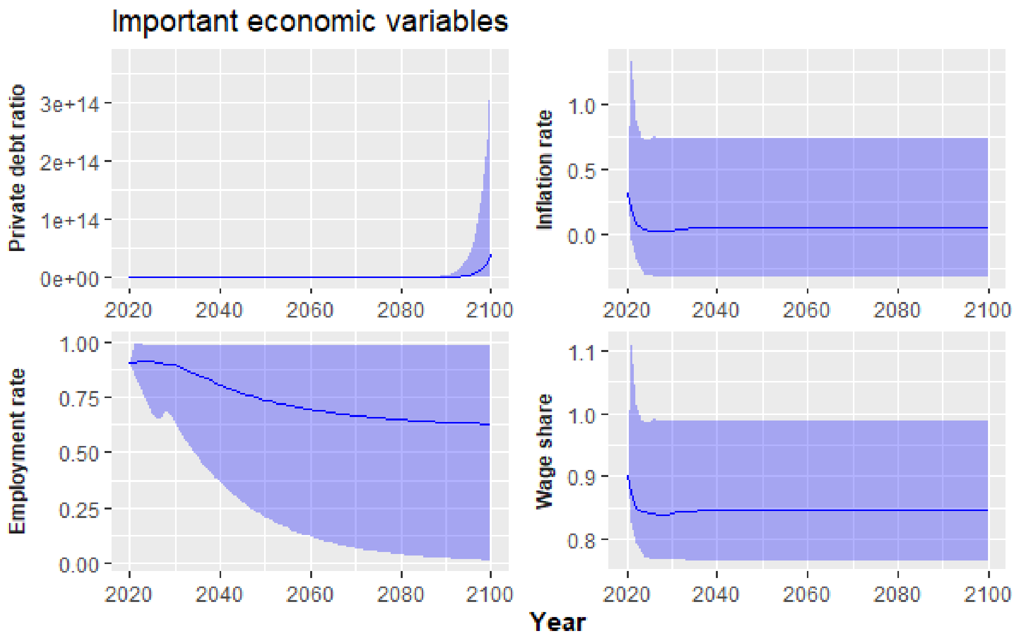


Figure 2.5: Monte Carlo simulations for the Keen model with pricing. Two hundred replications of the private debt share d, inflation rate $i(\omega)$, employment rate λ, and wage share ω are plotted, from 2020 to 2100. The shaded regions represent a [0.025; 0.975] (95%) probability interval for each variable, while the darker lines represent the median trajectories.

In order to better understand the range of possible trajectories, we can identify the fraction of simulations that stay within certain key thresholds. In Figure 2.5, 134/200 replications produce a private debt share of $d < 2.7$, while 126/200 replications produce an employment rate of $\lambda \in [0.4, 1]$. These fractions should be similar, as overindebtedness in the private sector will lead to economic contraction via diminished profits and capital investment, thus resulting in worker layoffs and a vanishing employment rate over time. For reference, 196/200 simulations produce a wage share of $\omega \in [0.4, 1]$.

2.3.2 Basin of Attraction

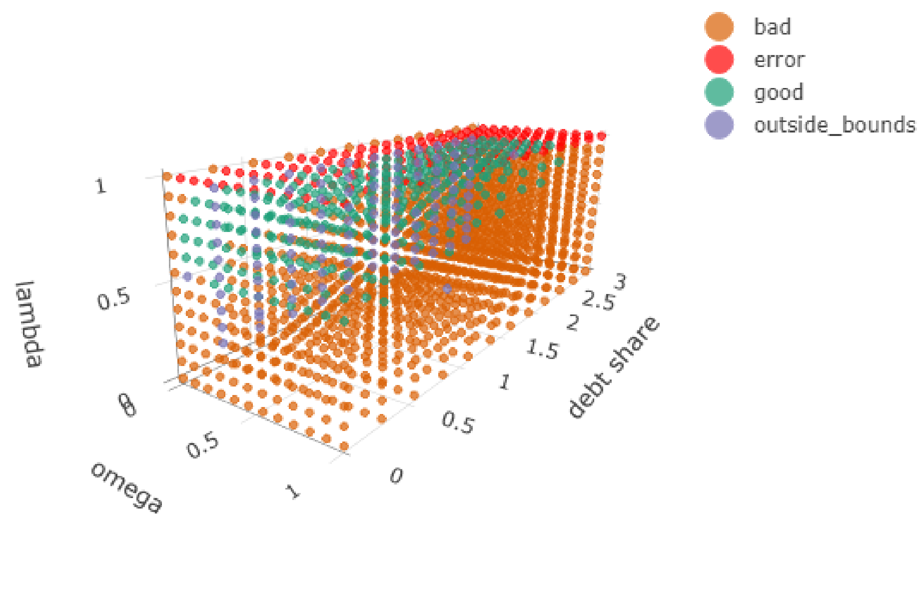


Figure 2.6: Basin of attraction, varying the initial conditions for wage share ω, employment rate λ, and private debt ratio d.

Using the same classification method as that for parameter sensitivity, we now vary three key initial conditions in the model. The initial wage share and employment rate are both varied according to ω, $\lambda \in [0, 1]$, while the initial private debt ratio is varied according to $d \in [0, 3]$. Twelve equally spaced points are chosen within each of the three specified intervals, creating a $12 \times 12 \times 12$ grid. Figure 2.6 displays the basin of attraction for the Keen model with inflation, most notably depicting the aforementioned relationship between the private debt ratio and the employment rate. Higher initial values for the private debt share must correspond with higher initial values for the employment rate, lest the model collapse (output, capital, investment all fall to zero) due to vanishing employment and overindebtedness in the corporate sector.

2.4 Climate Model

2.4.1 Emissions, Carbon Taxes, and Abatement

We now attempt to understand the basic implications of climate change in the model, following many of the same assumptions from Bovari *et al.* (2018a), (2018b). Given a carbon pricing function p_C, firms will determine a desired emission reduction rate $n \in (0,1)$. Industrial emissions are modelled by

$$E_{ind} = \sigma(1-n)Y_{\max} \tag{2.46}$$

where $\sigma_i > 0$ is the carbon intensity of the economy, with initial growth rate function g_{σ_i}. (Maximum potential output $Y_{\max}$ will be defined shortly.) These functions are coupled and are given by

$$\dot{\sigma} = g_\sigma \sigma_i \tag{2.47}$$

$$\dot{g_\sigma} = \delta_{g_\sigma} g_\sigma, \tag{2.48}$$

where $\delta_{g_\sigma} < 0$ is a parameter denoting the growth rate of the growth of emission intensity. Then, as in Nordhaus (2018), the abatement cost function A is defined as a function of the emissions reduction rate n. It is then normalized by the emission intensity of the economy σ and the price of a backstop technology p_{BS}:

$$A = \frac{\sigma p_{BS}}{\theta} n_\theta. \tag{2.49}$$

The parameter $\theta > 0$ controls the convexity of the cost. The price of the backstop technology then grows exogenously according to

$$\dot{p_{BS}} = \delta_{p_{BS}} p_{BS}. \tag{2.50}$$

Examining the public sector and its role in abatement activities, we introduce carbon taxes paid to the government, and abatement subsidies paid back to the firms from the public sector. Following the work of Bovari *et al.* (2018b), a carbon tax will be levied on the emissions of firms, such that

$$T_f = p_C E_{ind}, \tag{2.51}$$

and a fraction s_A of abatement costs paid by firms may be subsidised by the government, resulting in a national transfer of

$$S_f = s_A A Y_{\max}. \tag{2.52}$$

Therefore, net transfers from the public sector to the private sector are denoted by

$$NT = S_f - T_f. \tag{2.53}$$

The research of Bovari *et al.* (2018b) assumes two different carbon pricing strategies. The first is a low carbon tax with growth rate 2% per year (denoted $p_{C_{low}}$, in line with the *Baseline* scenario of Nordhaus (2014). The second is a high carbon tax (denoted $p_{C_{high}}$), following the recommendation of the Stern-Stiglitz report (Stern and Stiglitz (2017)). For our medium and high carbon taxes, we follow a modified version of the upper barrier of the corridors presented in their report: from US\$50-80/tCO_2 by 2020 to US\$80-100/tCO_2 by 2035. Our lower carbon tax is far less severe, growing from US\$40/tCO_2 to US\$50/tCO_2 by 2035. The growth rate of p_C is then interpolated linearly until 2150, for each of the three specifications.

Firms then choose their abatement emissions rate depending on the carbon price p_C, the cost of backstop technology p_{BS}, and the public subsidization of green technology s_A, according to

$$n = \min\left\{\left(\frac{p_C}{(1-s_A)p_{BS}}\right)^{\frac{1}{\theta-1}}, 1\right\} \tag{2.54}$$

2.4.2 Climate Module

The climate module presented below follows the same framework laid out by Nordhaus in his DICE model (Nordhaus, 1994), and later reintroduced by Bovari *et al.* (2018a), (2018b). Global CO_2 emissions are assumed to be the sum of industrial and land-use emissions, where land-use emissions are assumed to be exogenous and decrease at the rate $\delta_{E_{land}}$.

$$E = E_{ind} + E_{land} \tag{2.55}$$

$$\dot{E}_{land} = \delta_{E_{land}} E_{land} \tag{2.56}$$

The carbon cycle is then modeled in three layers. CO_2 emissions can accumulate in the atmosphere (CO_2^{AT}), the upper ocean and biosphere (CO_2^{UP}), or the lower ocean

(CO_2^{LO}). The dynamics of CO_2 accumulation progress as such:

$$\begin{pmatrix} \dot{CO_2}^{AT} \\ \dot{CO_2}^{UP} \\ \dot{CO_2}^{LO} \end{pmatrix} = \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\phi_{12} & \phi_{12} C_{UP}^{AT} & 0 \\ \phi_{12} & -\phi_{12} C_{UP}^{AT} - \phi_{23} & \phi_{23} C_{LO}^{UP} \\ 0 & \phi_{23} & -\phi_{23} C_{LO}^{UP} \end{pmatrix} \begin{pmatrix} CO_2^{AT} \\ CO_2^{UP} \\ CO_2^{LO} \end{pmatrix} \tag{2.57}$$

where $C_i^j = \frac{C_{j_{preind}}}{C_{i_{preind}}}$ for $i, j \in \{AT, UP, LO\}$ and ϕ_{12}, ϕ_{23} are parameters, as in Bovari *et al.* (2018b). The accumulation of CO_2 then increases radiative forcing, F, of CO_2. Radiative forcing F is modeled as follows, where F_{dbl} is an exogenous parameter that represents the effect on forcing of a doubling of pre-industrial CO_2 levels, and F_{exo} increases at an exogenous rate over time.

$$F_{ind} = \frac{F_{dbl}}{\log(2)} \log\left(\frac{CO_2^{AT}}{C_{AT_{preind}}}\right) \tag{2.58}$$

$$F = F_{ind} + F_{exo} \tag{2.59}$$

Then, we model the effects of radiative forcing on temperature, given by T. Mean temperature is divided into two layers: the atmosphere, land, and upper ocean layer T, and the lower ocean layer T_0. The heat capacities of each layer are given by C and C_0, respectively, while γ^* represents the heat transfer between layers. The equilibrium climate sensitivity is given by S, and thus temperature in each layer evolves according to

$$\dot{T} = \frac{F - \frac{F_{dbl}}{S} T - \gamma^*(T - T_0)}{C} \tag{2.60}$$

$$\dot{T_0} = \frac{\gamma^*(T - T_0)}{C_0}. \tag{2.61}$$

2.4.3 Damages

Following the research of Dietz and Stern (2015), we assume that rising temperatures can reduce both output and capital, thereby further reducing output. Temperature factors into the initial damage function originally presented in Dietz and Stern (2015), which is given by

$$D = 1 - \frac{1}{1 + \pi_1 T + \pi_2 T^2 + \pi_3 T^\zeta} \tag{2.62}$$

Some fraction of initial damages further diminishes capital, according to a proportion $f_k \in [0, 1]$. Then, the total damages to output are given as the sum of initial damages D and damages to capital D_K:

$$D_K = f_k D \tag{2.63}$$

$$D_Y = (1 + f_k)D \tag{2.64}$$

This formulation departs from Bovari *et al.* (2018b), in which damages to capital and output were zero-sum; that is to say, $D_Y = (1 - f_k)D$.

In previous research, three different specifications of the damage curve are employed. As illustrated in Bovari *et al.* (2018a), the *Nordhaus* specification (from Nordhaus (1994)) is the least convex, while the *Weitzman* specification (from Weitzman (2012)) and the *Stern* specification (from Dietz and Stern (2015)) are progressively more convex. Our simulations explore damages incurred under the Nordhaus and Stern specifications.

2.4.4 Changes to Economic Module

The introduction of the climate module requires certain revisions to the previously defined economic module. We begin by redefining output Y as potential output, prior to damages and abatement costs:

$$Y_{\max} = \frac{K}{\nu} \tag{2.65}$$

Firms will base employment decisions on potential output; consequently, labour will be hired at full capacity, such that

$$L = \frac{Y_{\max}}{a}. \tag{2.66}$$

After accounting for abatement costs and observing damages to output, we have that actual production Y is given by

$$Y = (1 - D_Y)(1 - A)Y_{\max}. \tag{2.67}$$

Considering damages to capital, we now observe the evolution of capital over time:

$$\dot{K} = I_k - (\delta + D_K)K. \tag{2.68}$$

Finally, corporate profits now account for the net public transfer (abatement subsidies minus carbon taxes):

$$\Pi_f = pY - W - r_L L_f + r_M M_f + pNT. \tag{2.69}$$

2.4.5 Climate Scenario Results

We test two damage function specifications, the milder damage function provided by Nordhaus (2018), and the more convex specification introduced in Dietz and Stern (2015). The three scenarios below will be considered for a brief deterministic scenario analysis. We briefly summarize the three scenarios:

1. **Low Damages, Low Policy.** This scenario uses the *Nordhaus* damage function. Public intervention is minimized; the least severe carbon tax is implemented, with no public subsidies of abatement costs.

2. **High Damages, Medium Policy** This scenario assumes the *Stern* damage function. The medium carbon tax follows a similar (but less severe) path to that outlined by the Stern-Stiglitz commission (from Stern and Stiglitz (2017)). The public sector also subsidizes 40% of abatement technology costs.

3. **High Damages, Strong Policy**. This scenario assumes the *Stern* damage function. The high carbon tax follows the recommended path outlined by the Stern-Stiglitz commission (from Stern and Stiglitz (2017)). The public sector now subsidizes 80% of abatement technology costs.

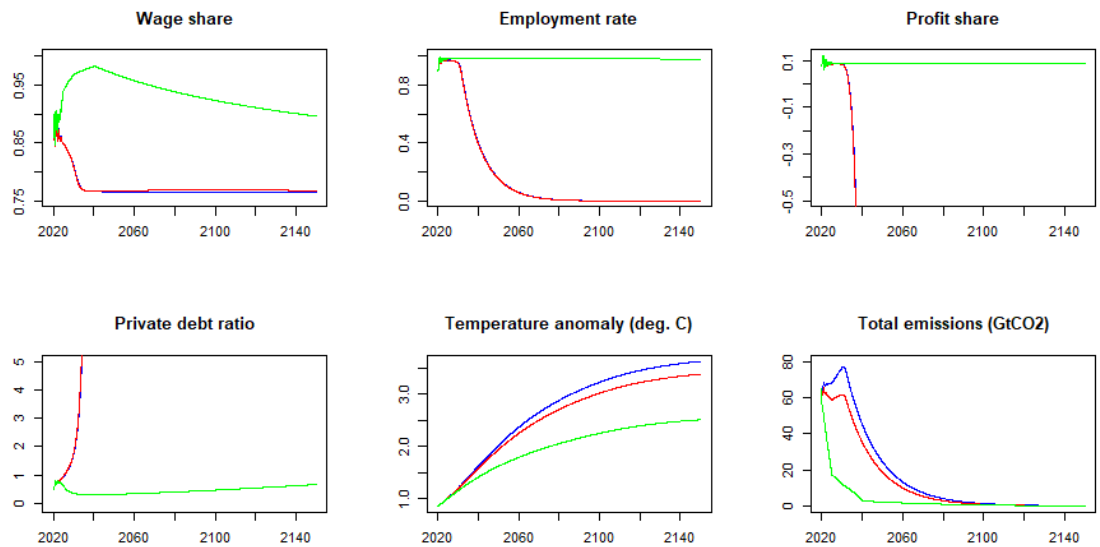


Figure 2.7: Climate scenario analysis for the basic Keen model with prices, from 2020 to 2150. The blue curve represents the *Low Damages, Low Policy* scenario, the red curve represents the *High Damages, Medium Policy* scenario, and the green curve represents the *High Damages, Strong Policy* scenario. The damages to capital is set at $f_k = 0.1$.

Figure 2.7 plots the results for the basic scenario analysis outlined above. Holding all parameters from the economic module equal, and setting the damages to capital f_K equal to 0.1, we observe rather unsurprising results. In the first two scenarios, the economy suffers a major long-term contraction, with the employment rate falling to zero and the private sector becoming brutally indebted, all while the temperature anomaly surpasses the desired threshold of +2°C. The strongest policy scenario is able to combat under-investment in the private sector due to the generous subsidy from the public sector, which in turn increases the profit share of firms. This subsidy has the desired result of preventing deflation in the long-run, thus preventing a long-term economic contraction. The economy attains a stable employment rate and profit share, with a private debt share that remains below 1. Furthermore, while the

Strong Policy scenario reaches carbon-neutrality ($E_{ind} = 0$) by 2040, the temperature anomaly still exceeds $+2^{\circ}$ C in this scenario, with a value of approximately $T = 2.25$ in the year 2100.

Chapter 3

Introducing a Public Sector

3.1 Economic Module

3.1.1 Accounting Framework

Our first extension to the original Keen model with inflation (from Grasselli and Nguyen-Huu (2015)) involves adding an active public sector. The economy now contains four sectors: households, firms, banks, and the public sector. Observing Tables 3.1, 3.2, and 3.3, we can investigate the balance sheets, income statements, and transaction flow matrices of the new economic model. Once again, all entries in the country's balance sheet are measured in nominal monetary amounts, whereas transaction entries and items on the flow of funds sheet are measured in monetary units per year.

Observing Table 3.1, we see that in terms of assets and liabilities, this new model is very similar to the previous model introduced in Chapter 2. The only difference involves the introduction of the public sector, which does not necessarily run a balanced

	Households	Firms	Banks	Public Sector	Sum
Balance Sheet					
Capital Stock		pK			pK
Deposits	M_h	M_f	$-M$		0
Loans		$-L_c$	L_c		0
Bonds			B	$-B$	0
Sum (net worth)	X_h	X_f	X_b	X_g	X

Table 3.1: Balance sheet for country's economy

budget, and finances government debt through the issuance of bonds B. All bonds are purchased by banks and held as assets by the financial sector. The government enters the transactions flow matrix (Table 3.2) through government expenditures G and the taxation of firms and households according to $T = \tau(\mathrm{w}L + \Pi_f)$, while also earning interest on government debt $r_B B$. Otherwise, the model remains the same, both in terms of the monetary stocks held by households, firms, and banks, and the financial flows between the original three sectors.

	Households	Firms		Banks	Public Sector	Sum
Transactions		Current	Capital			
Consumption	$-pC_h$	pC		$-pC_b$		0
Investment		pI_k	$-pI_k$			0
Govt. Spending		G			$-G$	0
[GDP]		$[pY]$				$[pY]$
Wages	W	$-W$				0
Taxes		$-T_f$			T_f	0
Int. on loans		$-rL_c$		rL_c		0
Int. on deposits	rM_h	rM_f		$-rM$		0
Int. on bonds				$r_B B$	$-r_B B$	0
Sum (balance)	S_h	$\Pi_r = (1-\tau)\Pi_f$	$-pI_k$	S_b	S_g	0

Table 3.2: Transactions in the given economy

	Households	Firms	Banks	Public Sector	Sum
Balance Sheet					
Change in capital stock		pI_k			pI_k
Change in deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$		0
Change in loans		$-\dot{L}_c$	$\dot{L}_c$		0
Change in bonds			$\dot{B}$	$-\dot{B}$	0
Sum (savings)	S_h	Π_r	S_b	S_g	pI_k
Change in net worth	$\dot{X}_h = \dot{M}_h$	$\dot{X}_f = \Pi_r + (\dot{p} - \delta p)K$	$\dot{X}_b = S_b$	$\dot{X}_g = -\dot{B}$	$\dot{X}$

Table 3.3: Flow of funds in economy

3.1.2 Changes to Original Model

We build upon the work of Grasselli and Nguyen-Huu (2015) by assuming the existence of a public sector. First, we characterize government expenditures as a proportion ψ_g of total nominal output:

$$G = \psi_g pY. \tag{3.1}$$

Then, we suppose that the government taxes household earnings and the profits of the corporate sector; tax revenues are thus given by

$$T = \tau(\mathrm{w}L + \Pi_f). \tag{3.2}$$

The public sector issues bonds in order to finance its debt. Banks purchase all bonds B, and pay interest according to $-r_B B$, where r_B denotes the policy rate. Public debt therefore evolves according to:

$$\dot{B} = G - T + r_B B. \tag{3.3}$$

Turning our attention to the private sector, we now consider the *retained* (i.e.

after-tax) profits of firms Π_r, expressed as

$$\Pi_r = (1-\tau)\Pi_f. \tag{3.4}$$

While firms will still use actual nominal profits to inform their investment decisions, firms will now consider their after-tax retained profits when accessing loans from the banks. Consequently, the corporate deposit dynamics are now expressed as

$$\dot{M}_f = \Pi_r, \tag{3.5}$$

with the aggregate debt dynamics following

$$\dot{D} = pI_k - \Pi_r. \tag{3.6}$$

The rest of the model proceeds as before, with no other explicit expressions changing. Finally, consumption by households and banks is expressed as an accommodating variable, and is simply the difference between output and the sum of investment and government spending. Consumption is not solely determined by the investment behaviour of firms; government expenditures also have some role in determining consumption:

$$C = Y - I_k - \frac{G}{p} = (1-\kappa(\pi_f) - \psi_g)Y \tag{3.7}$$

From the right side of equation (3.7), we obtain the only change to any of the behavioural functions. The investment function is still linear, capped from below by 0. The upper bound, however, is now capped from above by $1-\psi_g$, resulting in the expression:

$$\kappa(\pi_f) = \max\{0,\, \min\{(1-\psi_g),\ \mathrm{inv}_0 + \mathrm{inv}_1\pi_f\}\}, \tag{3.8}$$

3.2 Main Dynamical System

The equations for the employment rate, wage share, and private loan-to-output ratio remain the same as in Chapter 2. The inflation dynamics, the equilibrium growth rate, and the lending dynamics remain unchanged after the addition of a public sector to the model from Chapter 2, and thus the first three differential equations of our reduced three-dimensional system are exactly the same as before.

Our expression for the private deposit-to-output ratio changes, however, due to the current use of retained profits in the debt dynamics, instead of pre-taxed profits. Using the expression $m_f = M_f/(pY)$ and the revised equation (3.5) for the corporate deposit dynamics, we can obtain the dynamics for the private deposit-to-output ratio:

$$\begin{aligned}
\frac{\dot{m}_f}{m_f} = \frac{\dot{M}_f}{M_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \left[\frac{\Pi_r}{M_f}\right] - i(\omega) - g(\pi_f) \\
&= \left[\frac{(1-\tau)\Pi_f}{M_f}\right] - i(\omega) - g(\pi_f) \\
&= \left[\frac{(1-\tau)\pi_f pY}{m_f pY}\right] - i(\omega) - g(\pi_f) \\
&= \left[\frac{(1-\tau)\pi_f}{m_f}\right] - i(\omega) - g(\pi_f)
\end{aligned} \tag{3.9}$$

When there is no taxation in the model ($\tau = 0$), the gradient for the private deposit ratio (and thus the overall system) is the same as in Chapter 2. We now observe a revised four-dimensional rate model with state variables $(\omega, \lambda, \ell_f, m_f)$:

$$\begin{cases} \dot{\omega} &= \omega\left[\phi(\lambda) - \alpha - (1-\gamma)i(\omega)\right] \\ \dot{\lambda} &= \lambda\left[g(\pi_f) - \alpha - \beta\right] \\ \dot{\ell_f} &= \kappa(\pi_f) - \ell_f(i(\omega) + g(\pi_f)) \\ \dot{m_f} &= (1-\tau)\pi_f - m_f(i(\omega) + g(\pi_f)). \end{cases} \tag{3.10}$$

The revised model contains the same observations from Chapter 2, and now features additional interplay between the public and private sectors. Higher tax rates reduce the amount of after-tax retained profits, and thus increase the private debt ratio $d = \ell_f - m_f$. This is reflected in System 3.10, as increases in τ correspond to a decrease in the private deposit-to-output ratio. The decrease in the private deposit ratio is responsible for the increase in the private debt ratio. In addition, modest increases in government spending (brought upon by raising ψ_g) will reduce the upper bound of the $\kappa(\pi_f)$ investment share of the economy, thus slightly reducing the private loan-to-output ratio and the private debt share. The danger comes from larger increases in ψ_g, which reduce the upper bound of $\kappa(\pi_f)$ to the point that capital investment vanishes to zero, thus causing a permanent economic contraction, with zero employment and skyrocketing private debt in the long run.

We can further understand the relationship between the public and private sectors by observing the public debt-to-output ratio b, obtained through equation (3.3) and the expression $b = B/(pY)$:

$$\begin{aligned}\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \frac{(G-T) + r_B B}{B} - i(\omega) - g(\pi_f)\\ &= \frac{(\psi_g pY - \tau(\mathrm{w}L + \Pi_f)) + r_B(bpY)}{bpY)} - i(\omega) - g(\pi_f)\\ &= \frac{pY(\psi_g - \tau\pi_f + r_B b)}{bpY} - \frac{\tau\omega}{b} - i(\omega) - g(\pi_f)\\ &= \frac{(\psi_g - \tau(\omega + \pi_f))}{b} + r_B - i(\omega) - g(\pi_f) \end{aligned} \tag{3.11}$$

Then, using the household deposit dynamics $\dot{M}_h = S_h = \mathrm{w}L + r_M M_h - pC$ and the ratio $m_h = M_h/(pY)$, we obtain the expression for the evolution of household deposits:

$$\begin{aligned}\frac{\dot{m}_h}{m_h} = \frac{\dot{M}_h}{M_h} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \frac{\mathrm{w}L + r_M M_h - pC}{M_h} - i(\omega) - g(\pi_f)\\ &= \frac{(\mathrm{w}L + r_M(m_h pY) - pY(1 - \kappa(\pi_f) - \psi_g)}{m_h pY} - i(\omega) - g(\pi_f)\\ &= \frac{(\omega + \kappa(\pi_f) + \psi_g - 1)}{m_h} + r_M - i(\omega) - g(\pi_f)\end{aligned} \tag{3.12}$$

We thus present a supplementary system, consisting of the public debt to output ratio b and the household deposit-to-output ratio m_h. These equations do not factor into the main System 3.10, and thus can be solved separately.

$$\begin{cases} \dot{b} &= (\psi_g - \tau(\omega + \pi_f)) + b(r_B - i(\omega) - g(\pi))\\ \dot{m}_h &= \omega + \kappa(\pi_f) + \psi_g - 1 + m_h(r_M - i(\omega) - g(\pi_f))\end{cases} \tag{3.13}$$

From the supplementary System 3.13, we observe that increases in the level of government spending, as well as decreases in the tax rate, correspond to increases in the public debt ratio. Increases in the policy rate do not affect the main system (and thus the private sector) in any way; they do, however, increase the public debt-to-output ratio. The gradient expression for household deposits is also affected by the introduction of the public sector, as increases in government spending reduce household consumption, thus enabling households to retain more of their earnings as deposits.

The interior, non-negative equilibrium (ω^*, λ^*, ℓ_f^*, m_f^*) is almost identical to its counterpart in Chapter 2. The introduction of the public sector affects the equilibrium deposit share:

$$\omega^* = 1 - \pi_f^* - r_L \ell_f^* + r_M m_f^* \tag{3.14}$$

$$\lambda^* = \varphi^{-1}(\alpha + (1-\gamma) i(\omega^*)) \tag{3.15}$$

$$\ell_f^* = \frac{\kappa(\pi_f^*)}{\alpha + \beta + i(\omega^*)} \tag{3.16}$$

$$m_f^* = \frac{(1-\tau)\pi_f^*}{\alpha + \beta + i(\omega^*)} \tag{3.17}$$

Following our work from Chapter 2 (and the original work from Grasselli and Nguyen-Huu (2015)), we substitute the expressions for the private loan-to-output and deposit-to-output ratios (ℓ_f^* and m_f^*, respectively) into the expression for the equilibrium wage share ω^*. As in Chapter 2, we find that the equilibrium wage share ω^* is a solution of the quadratic equation $a_0(\omega^*)^2 + a_1\omega + a_2 = 0$, where

$$a_0 = \xi\eta_p > 0$$

$$a_1 = (\alpha + \beta) - \eta_p(1 + \xi(1 - \pi_f^*))$$

$$a_2 = (\eta_p - \alpha - \beta)(1 - \pi_f^*) + r_L\kappa(\pi_f^*) - r_M(1-\tau)\pi_f^*$$

Since our adjustment speed parameter η_p is always greater than the equilibrium growth rate of the economy $\alpha + \beta$, we follow the work of Chapter 2 and Grasselli and Nguyen-Huu (2015), and infer that if the condition

$$((\alpha + \beta - \eta_p) + \eta_p \xi(1 - \pi_f^*))^2 > 4\eta_p \xi(r_L(\kappa(\pi_f^*) - r_M(1 - \tau)\pi_f^*) \tag{3.18}$$

is satisfied, then the equilibrium profit share $\pi_f^* < 1$ must also hold. Consequently, there exists at least one non-negative equilibrium solution for ω^*.

3.2.1 Local Stability Analysis

We now delve into a brief local stability analysis of the four dimensional model with government, presented in Section 3.2. Assuming there exists a non-negative equilibrium solution (ω^*, λ^*, ℓ_f^*, m_f^*) as previously defined in Section 3.2 ($\omega^* > 0$, $\lambda^* \in (0, 1)$), we follow our work from Chapter 2 in examining the following Jacobian matrix $J^*_{\text{govt}}(\omega^*, \lambda^*, \ell_f^*, m_f^*) = J^*_{\text{govt}}$ in order to investigate the local stability of the system:

$$J^*_{\text{govt}} = \begin{bmatrix} J^*_{\text{govt},11} & \omega^*\Phi'(\lambda^*) & 0 & 0 \\ -\lambda^* g'(\pi_f^*) & g(\pi_f^*) - \alpha - \beta & -r_L\lambda^* g'(\pi_f^*) & r_M\lambda^* g'(\pi_f^*) \\ J^*_{\text{govt},31} & 0 & J^*_{\text{govt},33} & J^*_{\text{govt},34} \\ J^*_{\text{govt},41} & 0 & r_L(m_f^* g'(\pi_f^*) - (1-\tau)) & J^*_{\text{govt},44} \end{bmatrix} \tag{3.19}$$

In this matrix, we have that:

$$J^*_{\text{govt},11} = \Phi(\lambda^*) - \alpha + (1-\gamma)\eta_p(1 - 2\xi\omega^*) \tag{3.20}$$

$$J^*_{\text{govt},31} = -\kappa'(\pi_f^*) + \ell_f^*(g'(\pi_f^*) - \eta_p\xi) \tag{3.21}$$

$$J^*_{\text{govt},33} = r_L\ell_f^* g'(\pi_f^*) - r_L\kappa'(\pi_f^*) - g(\pi_f^*) - i(\omega^*) \tag{3.22}$$

$$J^*_{\text{govt},34} = r_M(\kappa'(\pi_f^*) - \ell_f^* g'(\pi_f^*)) \tag{3.23}$$

$$J^*_{\text{govt},41} = m_f^*(g'(\pi_f^*) - \eta_p\xi) - (1-\tau) \tag{3.24}$$

$$J^*_{\text{govt},44} = (1-\tau)r_M - r_M m_f^* g'(\pi_f^*) - i(\omega^*) - g(\pi_f^*) \tag{3.25}$$

We now verify a numerical example, checking that the base parameters we have chosen for this system produce a locally stable equilibrium. Our base numerical simulation for the four dimensional Keen model with government equilibrates at the values:

$$(\omega^*, \lambda^*, \ell_f^*, m_f^*) = (0.91111, 0.97858, 0.68129, 0.13055). \tag{3.26}$$

Other equilibrium values of note are

$$i(\omega^*) = 0.37333$$

$$\kappa(\pi_f^*) = 0.285$$

$$g(\pi_f^*) = \frac{\kappa(\pi_f^*)}{\nu} - \delta = \frac{0.285}{3} - 0.01$$

As before, our important derivative values include $\kappa'(\pi_f^*) = \text{inv}_1 = 20$, $g'(\pi_f^*) = 20/3$, and $\Phi'(\lambda^*) = \frac{2(0.0000641)}{(1-0.97858)^3}$. Substituting all necessary values into the Jacobian, we obtain

$$J^* = \begin{bmatrix} -0.87472 & 11.885 & 0 & 0 \\ -6.5239 & 0 & -0.26100 & 0.13048 \\ -18.728 & 0 & -1.0367 & 0.30916 \\ -0.60631 & 0 & 0.00081333 & -0.41874 \end{bmatrix}. \tag{3.27}$$

The eigenvalues of this matrix are approximately

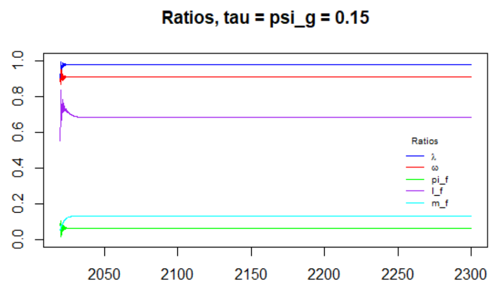
$$(\epsilon_1,\, \epsilon_2,\, \epsilon_3,\, \epsilon_4) = (-0.298,\, -0.418,\, -0.807 + 8.793i,\, -0.807 - 8.793i). \tag{3.28}$$

Since all four eigenvalues have negative real parts, our calibrated parameters produce a locally stable equilibrium. Furthermore, the fact that only two eigenvalues are complex (instead of four) will likely result in the new system illustrating less oscillatory behaviour en route to stabilization, when compared with the analogous results of Chapter 2. The quicker stabilization (when all parameters and initial conditions are held equal from Chapter 2) emerges from the introduction of a public sector, and will be explored in greater depth in Section 3.3.

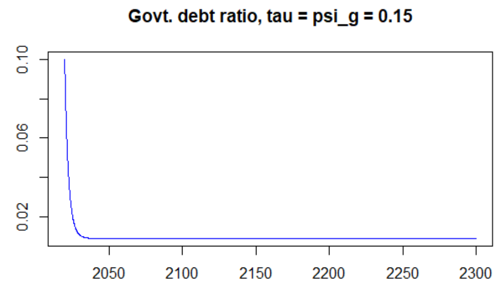
3.3 Results

Figure 3.1 illustrates the basic effects of the public sector on the new model. We vary the tax rate τ, and the share of nominal output allocated as government expenditures ψ_g. Once again, each of the four state variables from the reduced model is plotted (ω, λ, ℓ_f, m_f, as well as the profit share π_f, and the public debt-to-output ratio b. In Figures 3.1 (a) and (b), we observe a simple balanced-budget model, in which $\tau = \psi_g = 0.15$. As the eigenvalues of the Jacobian demonstrated earlier, the system converges to a stable equilibrium, via damped oscillations. The implementation of the tax rate reduces corporate deposits, as was pointed out in 3.2, thus slightly increasing the private debt ratio. Meanwhile, a slight increase in the slope of the investment function to $\text{inv}_1 = 20$ (from 15) further increases the private debt ratio, as firms must demand more in loans. Since the tax rate and government spending parameter are set to be equal, the public debt ratio equilibrates to just slightly above zero, with the main discrepancy arising from the interest that is earned on government bonds.

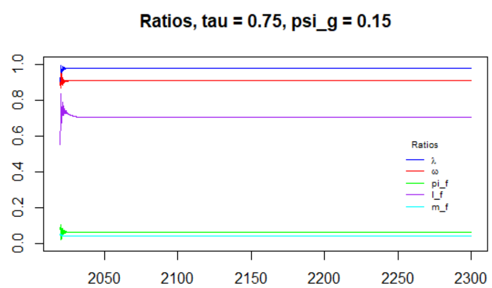
Figures 3.1 (c) and (d) illustrate an extreme case for the tax rate. We raise τ from 0.15 to 0.75, while holding all other parameters equal, and observe that the increase forces the corporate deposit-to-output ratio even lower, thus slightly increasing d, while creating a sizeable surplus for the public sector. A more extreme result is achieved in Figures 3.1 (e) and (f), where the tax rate returns to its original value of $\tau = 0.15$, but ψ_g is increased to 0.75, from its original value of 0.15. The sharp increase drives investment to zero in the long run, thus reducing capital K, and ultimately causing an economic collapse. Employment vanishes to zero, and the private debt ratio explodes before 2060.



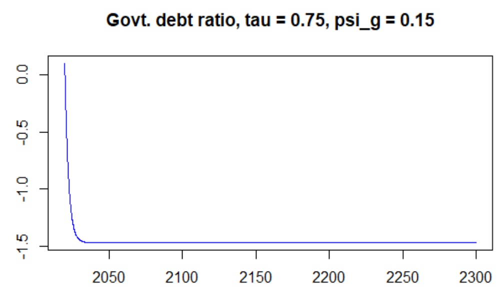
(a) Basic plots of key economic ratios, with $\tau = \psi_g = 0.15$.



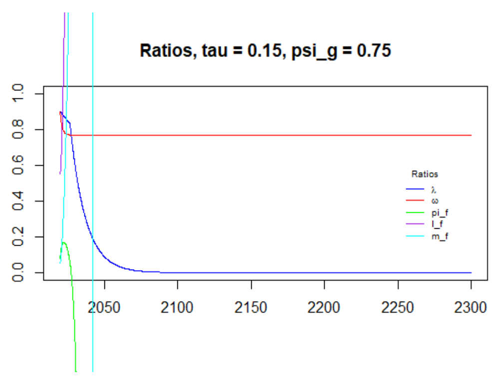
(b) Public debt ratio, with $\tau = \psi_g = 0.15$.



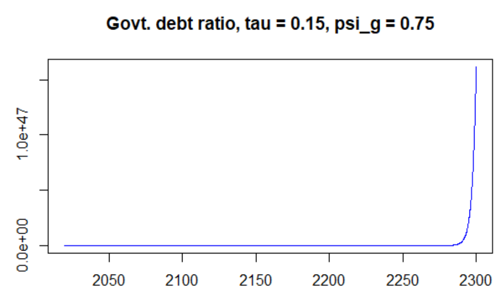
(c) Ratios, with a tax rate of $\tau = 0.75$.



(d) Public debt ratio, with a tax rate of $\tau = 0.75$.



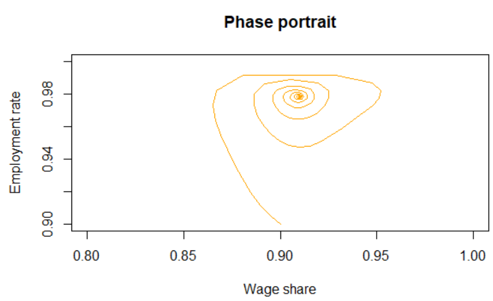
(e) Ratios, with government expenditures parameter $\psi_g = 0.75$.



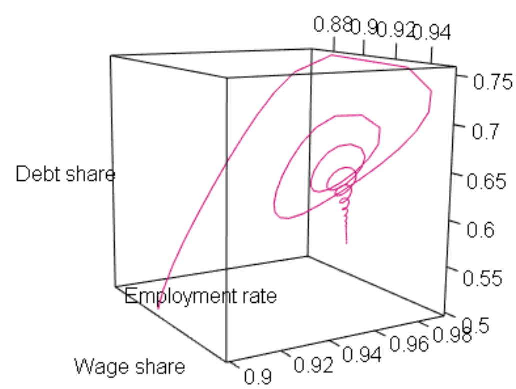
(f) Public debt ratio, with government expenditures parameter $\psi_g = 0.75$.

Figure 3.1: Subfigures (a), (c), and (e), we observe simulations of the employment rate λ (blue), wage share ω (red), profit share π_f (green), private loan-to-output ratio ℓ_f (purple), and private deposit-to-output ratio m_f (cyan), for the model presented in Section 3.1, for tax rates $\tau = \{0.15, 0.75\}$ and government spending parameters $\psi_g = \{0.15, 0.75\}$. Plots (b), (d), and (f) depict simulations of the public debt ratio b for the same parameter values. All simulations are run from 2020 to 2300.

Figure 3.2 is of mild interest, displaying once again the relationship between the employment rate and the wage share, and the relationship between the employment rate, wage share, and private debt ratio, for the calibrated result found in Figures 3.1 (a) and (b). The oscillatory behaviour of the trajectories en route to a locally stable equilibrium is captured again by both the two-dimensional and three-dimensional phase portraits.



(a) Two-dimensional phase portrait.



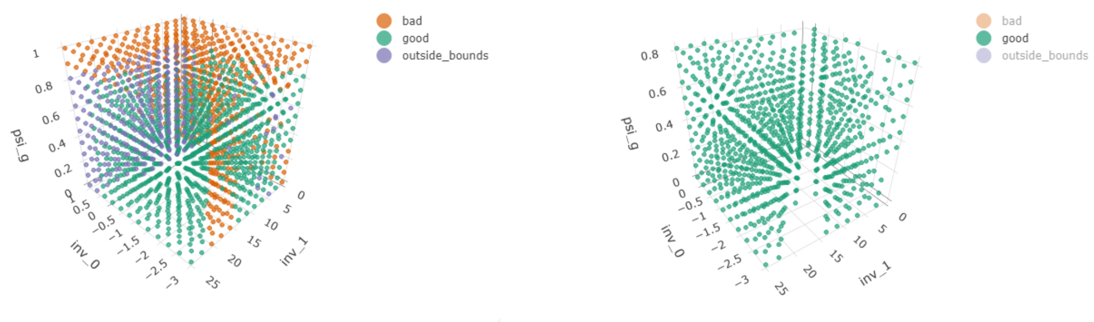
(b) Three-dimensional phase portrait.

Figure 3.2: Phase portraits for the four-dimensional model with government. The two-dimensional phase portrait in (a) plots employment rate λ with respect to the wage share ω, whereas the three-dimensional phase portrait in (b) plots the employment rate λ, the wage share ω, and the private debt-to-output ratio d.

3.3.1 Parameter Sensitivity

Following the same classification methods outlined in Chapter 2 and assuming again that $r_L = r_M = r$, we first vary the intercept and the slope of the investment function, according to $\text{inv}_0 \in [-3, 1]$ and $\text{inv}_1 \in [0.05, 25]$, respectively, Furthermore, we vary the fraction of nominal output dedicated to public expenditures according to $\psi \in [0, 1]$. Twelve equally spaced points are chosen within each of these intervals, and in

Figure 3.3, we observe the resulting sensitivity grid, when the tax rate is set to $\tau = 0$. The general features are very similar to the sensitivity plot from Chapter 2; intercept values of $\text{inv}_0 > 0$ and slope values of $\text{inv}_1 < 3$ produce bad runs more often than not, as insufficient capital investment is likely to cause collapses in capital and output, thus driving the employment rate to zero. The public sector parameter ψ_g must be less than approximately 0.8 for similar reasons, as an initial spike in government spending will reduce capital investment, resulting in a permanently contracted, deflationary economy.

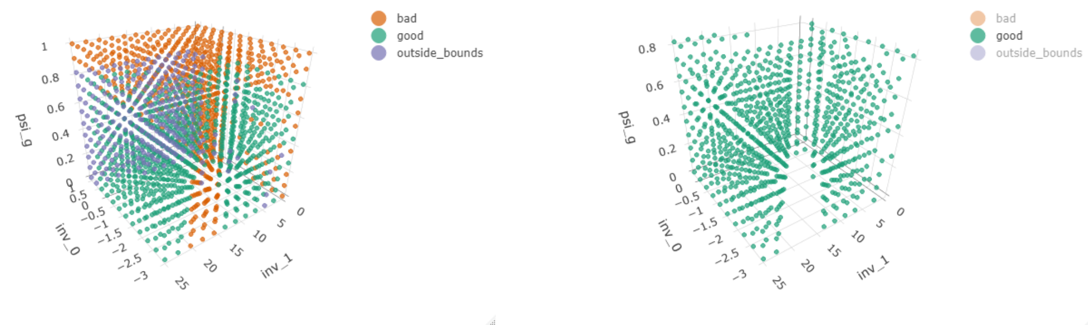


(a) Sensitivity of investment function parameters, alongside ψ_g.

(b) Sensitivity plot (a), with good simulations highlighted.

Figure 3.3: Parameter sensitivity for investment function parameters, along with the share of nominal output dedicated to government spending, ψ_g. The sensitivity plot in (b) helps depict the interesting vertical plane of simulations classified as "bad", which is found sandwiched between two "good" regions. For all simulated runs in the grid, the tax rate is set to zero ($\tau = 0$).

Figure 3.4 is very similar to Figure 3.3, with the parameter sensitivity analysis proceeding exactly as before, but with the government now attempting to run a balanced budget. The tax rate is set at $\tau = 0.15$ (up from zero), thus increasing private indebtedness, and resulting in 61 more runs classified as "bad". In general, the trends from the previous sensitivity analysis hold as before, but with slightly fewer "good" classifications. The phenomenon of the "bad" region sandwiched between the two separate "good" regions occurs once again for both analyses. We now hypothesize that its existence is likely tied to the structure or behaviour of the investment function, as those two parameters are varied in Chapter 2 as well.

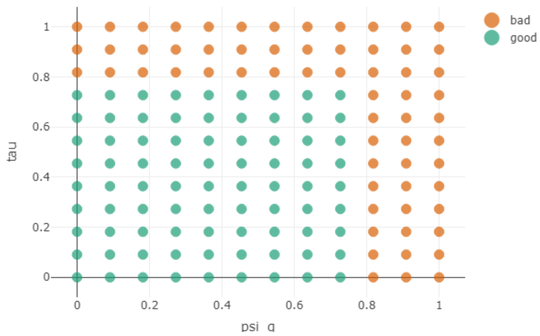


(a) Sensitivity of investment function parameters, alongside ψ_g.

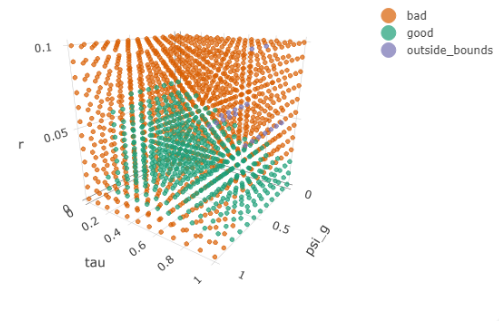
(b) Sensitivity plot (a), with good simulations highlighted.

Figure 3.4: Parameter sensitivity for investment function parameters, along with the share of nominal output dedicated to government spending, ψ_g. For all simulated runs in the grid, $\tau = 0.15$.

Turning to sensitivity for public sector parameters, we observe a two-dimensional sensitivity grid in Figure 3.5 (a). Both the tax rate τ and the government spending parameter ψ_g are varied from 0 to 1, with 12 equally spaced points chosen within both of those intervals. An easily determinable "good" region appears; simulations that converge to a good, stable equilibrium must have a tax rate and government spending parameter less than 0.7 or so. If either of these two parameters are any higher, investment will be minimized as the private debt ratio increases, and as corporate profits fall. This decrease in private profits would to a long-term economic contraction, with the resulting simulations classified as "bad".



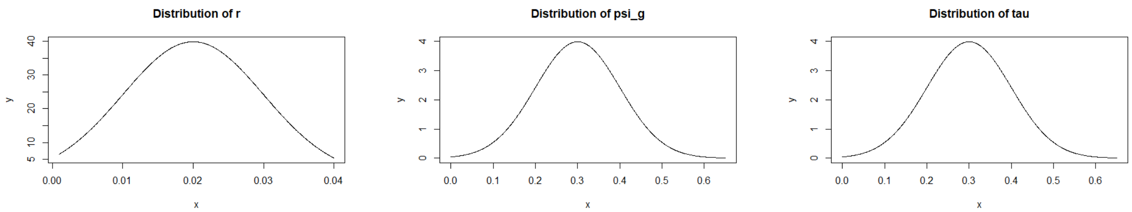
(a) Two-dimensional sensitivity grid of τ and ψ_g.



(b) Sensitivity plot of τ, ψ_g, and r.

Figure 3.5: Plot (a) depicts a two-dimensional parameter sensitivity grid for tax rate τ, along with the share of nominal output dedicated to government spending, ψ_g. Plot (b) illustrates a sensitivity of grid of τ, ψ_g, and the short-term interest rate r.

Figure 3.5 (b) then varies the short-term interest rate r from 0.001 to 0.1, with twelve equally spaced points chosen from that region. The intervals and points remain the same for τ and ψ_g as in Figure 3.5 (a), and we observe the same region of "good" simulations for ψ_g, as well. Furthermore, we note that higher tax rates can now result in convergence to the good equilibrium, provided that the interest rate r is sufficiently low. In fact, we uncover an inverse relationship between r and τ; higher tax rates lead to a greater private debt ratio, but if firms pay less interest on their nominal debt, they will retain a sufficiently high profit share, such that investment does not decrease to zero. As a result, capital and output do not collapse, and the employment rate stays above the 40% threshold required for convergence to the good equilibrium.



(a) $\mu = 0.02$, $\sigma = 0.01$ (b) $\mu = 0.3$, $\sigma = 0.1$ (c) $\mu = 0.3$, $\sigma = 0.1$

Figure 3.6: Probability density functions for parameters r, ψ_g, and τ. Each of the three parameters are sampled from a normal distribution; consequently, normal density is plotted on the y-axis.

From the good regions of our sensitivity grids, we determine suitable probability density functions for r, ψ_g, and τ, each of which are plotted in Figure 3.6. The three parameters are all sampled from a normal distribution, with their means and values for standard deviation stated in the sub-captions of Figure 3.6. In addition, we sample inv_0 and inv_1 from the same distributions as in Chapter 2 (not shown). Our 200 Monte Carlo replications of the Keen model with government are then examined, with all runs occurring from 2020 to 2100. In Figure 3.7, the shaded blue region represents the [0.025;0.975] probability interval for the wage share, employment rate, and public and private debt-to-output ratios. The solid blue line represents the median trajectory for each variable, as before.

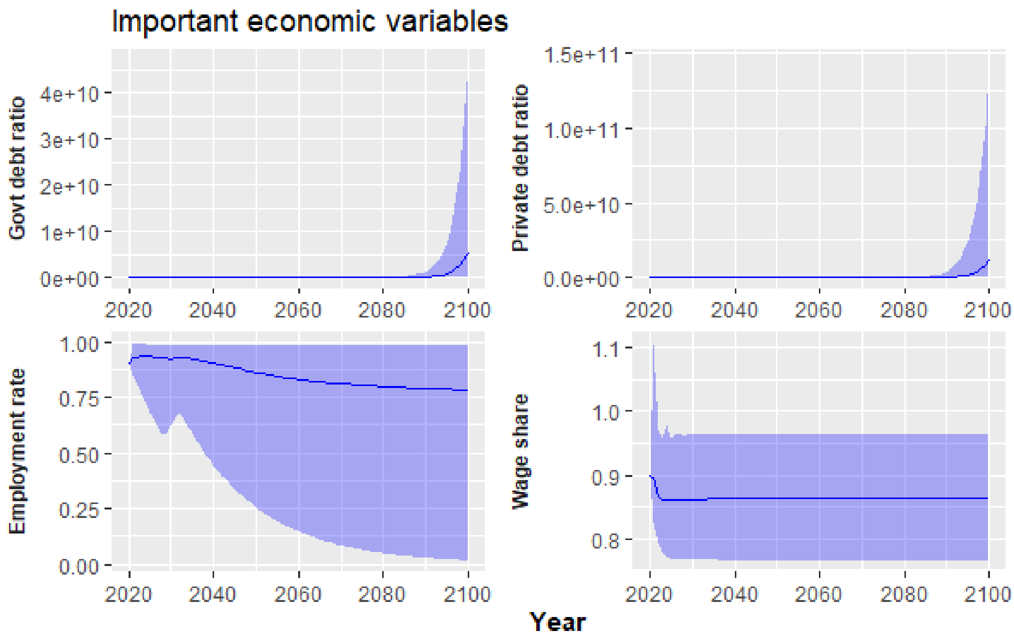


Figure 3.7: Monte Carlo simulations for the Keen model with government. Two hundred replications of the private debt-to-output ratio d, public debt-to-output ratio b, employment rate λ, and wage share ω are plotted, from 2020 to 2100. The shaded regions represent a [0.025; 0.975] (95%) probability interval for each variable, while the darker lines represent the median trajectories.

In Figure 3.7, 159/200 replications produce a private debt share of $d < 2.7$, and 159/200 replications produce an employment rate of $\lambda \in [0.4, 1]$. The addition of the public sector has created easily identifiable regions of good simulations in our sensitivity grids, thus enabling probability density functions that are more consistent with those good regions. Furthermore, the public sector seems to have stabilized the system overall, with fewer runs resulting in explosions of the private debt share, or collapses of the employment rate. For reference, 199/200 simulations produce a wage share of $\omega \in [0.4, 1]$.

3.3.2 Basin of Attraction

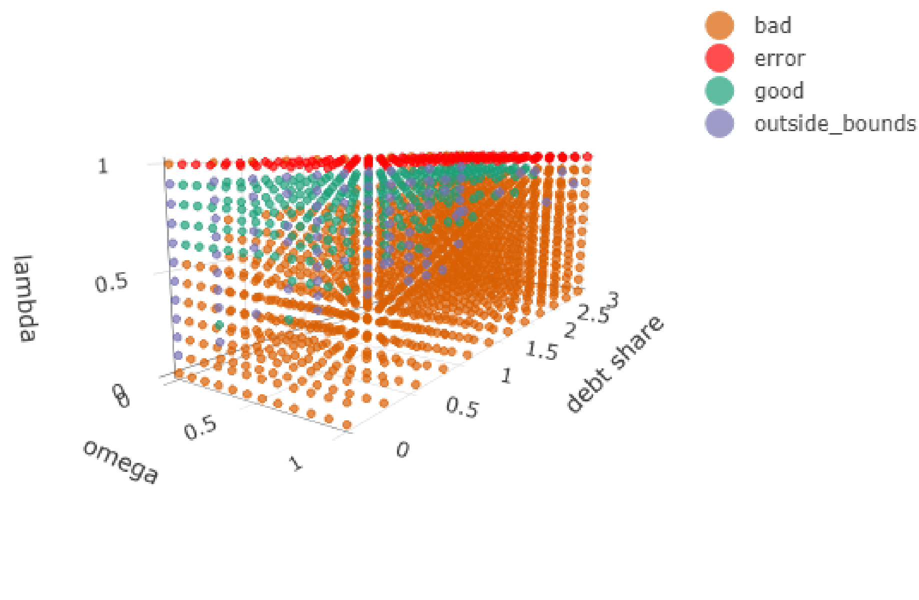


Figure 3.8: Basin of attraction, varying the initial conditions for wage share ω, employment rate λ, and private debt ratio d.

Once again, the initial wage share and employment rate are both varied according to ω, $\lambda \in [0, 1]$, while the initial private debt ratio is varied according to $d \in [0, 3]$. Twelve equally spaced points are chosen within each of the three specified intervals, creating a $12 \times 12 \times 12$ grid. Figure 3.8 displays the basin of attraction for the Keen model with government, once again depicting the aforementioned relationship between the private debt ratio and the employment rate.

3.4 Climate Results

The climate module proceeds exactly as before, with the same changes to the economic module as well. Due to carbon taxes T_f collected by the government and the abatement subsidy S_f given to firms, public debt now evolves according to

$$\dot{B} = (G + S_f) - (T + T_f) + r_B B. \tag{3.29}$$

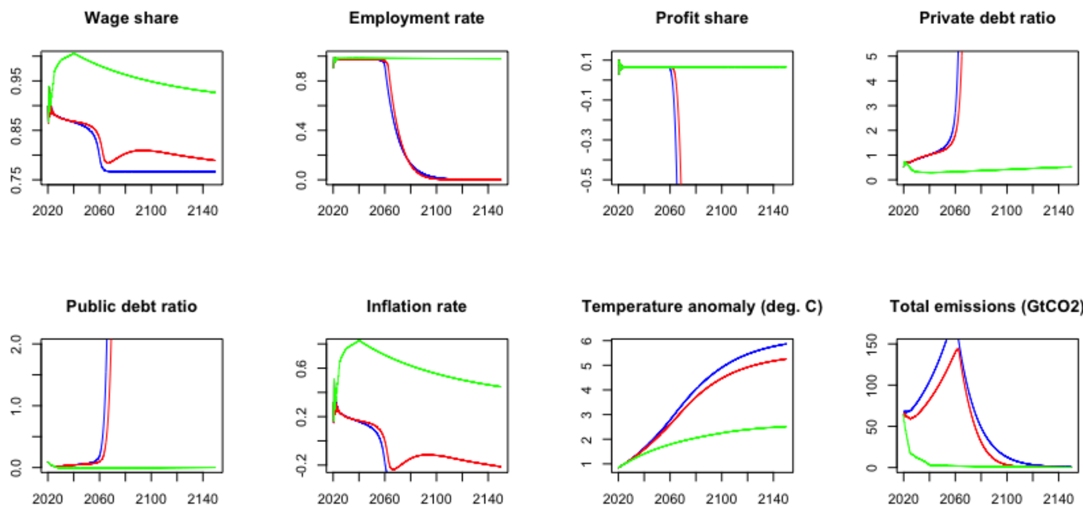


Figure 3.9: Climate scenario analysis for the Keen model with government, from 2020 to 2150. The blue curve represents the *Low Damages, Low Policy* scenario, the red curve represents the *High Damages, Medium Policy* scenario, and the green curve represents the *High Damages, Strong Policy* scenario. The damages to capital is set at $f_k = 0.1$.

Then, considering the three climate scenarios outlined in Chapter 2, we observe Figure 3.9, which plots the basic scenario analysis results for the Keen model with government. The strongest policy scenario performs the best, with very similar results as those presented in Chapter 2. The inclusion of the public sector and the increase in inv_1 from 15 to 20 appear to have staved off economic collapse for a few decades, with the first two scenarios exhibiting very similar trajectories to its counterparts from the previous chapter, with a delay of about 30 years. Collapses in the employment rate λ

or the profit share π_f, or an explosion in the private debt share d, do not occur until after 2060 in this simulation (instead of after 2030 in Chapter 2).

The delay in economic collapse is also accompanied by an increase in the temperature anomaly and level of total emissions, relative to Chapter 2. An increase in capital investment leads to subsequent increases in capital and output, which raise the level of industrial emissions, and ultimately the temperature anomaly, as well. In the strongest policy scenario, the employment rate, wage share, profit share, and private debt ratio all exhibit very similar dynamics to those presented in Chapter 1. As a result, we conclude that the inclusion of a tax-and-spend government does not destabilize the economy, and instead produces slightly better economic outcomes for the *Strong Policy* scenario. The first two scenarios also exhibit modest economic improvement, at the cost of worse climate outcomes, however.

Chapter 4

Monetary Policy

4.1 Economic Module

4.1.1 Accounting Framework

We extend the Keen model with inflation (from Grasselli and Nguyen-Huu (2015)) further in this chapter, building upon the work of Chapter 3. In addition to the existing public sector, we introduce an active central bank, through which simple monetary policy is conducted. The economy now contains five sectors: households, firms, banks, the public sector, and a central bank. Observing Tables 4.1, 4.2, and 4.3, we can investigate the balance sheets, income statements, and transaction flow matrices of the new economic model. Once again, all entries in the country's balance sheet are measured in nominal monetary amounts, whereas transaction entries and items on the flow of funds sheet are measured in monetary units per unit of time.

The balance sheets for households and firms remain identical to those from Chapters 1 and 2. The changes in this model are concentrated solely to the banking sector,

	Households	Firms	Banks	Public Sector	Central Bank	Sum
Balance Sheet						
Capital Stock		pK				pK
Deposits	M_h	M_f	$-M$			0
Loans		$-L_f$	L_f			0
Bonds			B_b	$-B$	B_{cb}	0
Reserves			R		$-R$	0
Sum (net worth)	X_h	X_f	X_b	X_g	0	X

Table 4.1: Balance sheet for economy with public sector and monetary policy.

as well as the newly included central bank. We observe that banks hold firms' loans L_f and domestically distributed government bonds B_b as assets. The deposits of households and firms, denoted by $M = M_h + M_f$, are held by banks as liabilities, but a constant proportion of these deposits are then held as reserves (assets), R, by the central bank. Banks distribute all profits to households as dividends, such that $\Pi_b = \Delta_b$.

The central bank holds government bonds B_{cb} as assets, and reserves R as liabilities. We assume that the central bank has net worth equal to zero, and that as a result, $B_{cb} = R$. Turning our attention to the government, we see that the aggregate public sector completes the economy, and since it has no assets, and only distributes bonds to the domestic banks and central bank as short-term liabilities, its simple balance sheet consists solely of government debt, financed through bonds, denoted as B. Lastly, we assume again that interest is earned on each of the monetary items on the balance sheet, except for reserves.

	Households	Firms		Banks	Public Sector	Central Bank	Sum
Transactions		Current	Capital				
Consumption	$-pC$	pC		pC_b			0
Investment		pI_k	$-pI_k$				0
Govt. Spend.		pG			$-pG$		0
[GDP]		$[pY]$					$[pY]$
Wages	W	$-W$					0
Capital depr.		$-\delta pK$	δpK				0
Non-carbon taxes	$-T_h$	$-T_f$		$-T_b$	T		0
CB profits					Π_{cb}	$-\Pi_{cb}$	0
Bank dividends	Δ_b			$-\Delta_b$			0
Int. on loans		$-r_L L_f$		$r_L L_f$			0
Int. on deposits	$r_M M_h$	$r_M M_f$		$-r_M M$			0
Int. on bonds				$r_B B_b$	$-r_B B$	$r_B B_{cb}$	0
Sum (balance)	S_h	Π_r	$-pI_k + \delta pK$	S_b	S_g	0	0

Table 4.2: Transactions in economy with public sector and monetary policy

4.1.2 Bank Financing

As mentioned earlier, we build upon the work of Grasselli and Nguyen-Huu (2015) and Chapter 3 by assuming the existence of an active central bank, through which monetary policy is conducted. This subsection introduces a simple bank financing module, while the following subsection will briefly describe the role of the central bank in this model.

In line with Grasselli and Lipton (2019a), we assume that there is a constant required reserves ratio for the banking sector, denoted by $f_M \in [0, 1)$, such that

$$R = f_M M \quad \text{and} \quad \dot{R} = f_M \dot{M}. \tag{4.1}$$

We follow Grasselli and Lipton (2019a), and propose that the amount of government bonds held by banks is obtained through open-market operations with the central

	Households	Firms	Banks	Public Sector	Central Bank	Sum
Balance Sheet						
Change in capital stock		$p\dot{K}$				$p\dot{K}$
Change in deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$			0
Change in loans		$-\dot{L}_f$	$\dot{L}_f$			0
Change in bills			$\dot{B}_b$	$-\dot{B}$	$\dot{B}_{cb}$	0
Change in reserves			$\dot{R}$		$-\dot{R}$	0
Sum (savings)	S_h	Π_r	S_b	S_g	0	0
Change in net worth	$\dot{X}_h = S_h$	$\dot{X}_f = \Pi_r$	$\dot{X}_b = S_b$	$\dot{X}_g = -\dot{B}$	$\dot{X}_{cb} = 0$	$\dot{X}$

Table 4.3: Flow of funds in economy with public sector and monetary policy.

bank, thus enabling the banks to attain the "correct" amount of reserves. The evolution of the financial sector's holding of bonds $\dot{B}_b$ is thus given by:

$$\dot{B}_b = \dot{M} - \dot{L}_f - \dot{R} = (1 - f_M)\dot{M} - \dot{L}_f. \tag{4.2}$$

From the balance sheet, we can then write the pre-tax profits of banks as

$$\Pi_b = r_L L_f + r_B B_b - r_M M. \tag{4.3}$$

4.1.3 Central Bank and Government

Assuming that the central bank sets the interest rate on bonds and advances through open market operations, we propose that the policy rate r_B evolves according to

$$\dot{r}_B = \beta_{rB}(g(\pi_f) - \alpha - \beta) = \beta_{rB}\left(\left(\frac{\kappa(\pi_f)}{\nu} - \delta\right) - (\alpha + \beta)\right). \tag{4.4}$$

This dynamic policy rate allows the central bank to adjust the interest rate on bonds in accordance with any deviations in the short-term, actual growth rate of output, from that of its long-term asymptote. Then, we allow for small positive perturbations to the policy rate, thus determining the short-run nominal interest rate on both loans and deposits:

$$r_L = r_B + \delta_L \tag{4.5}$$

$$r_M = r_B + \delta_M. \tag{4.6}$$

Returning to the bond market, we assume that the central bank purchases a supply of government bonds equal to the reserves it issues to the banks as liabilities:

$$\dot{B}^{cb} = \dot{R}. \tag{4.7}$$

The only remaining change involves taxation; bank profits taxed in addition to household earnings and corporate profits:

$$T = \tau(wL + \Pi_b + \Pi_b). \tag{4.8}$$

4.2 Main Dynamical System

The equations for the employment rate, wage share, loan dynamics, and corporate deposit dynamics remain the same as in Chapter 3, and can be derived using the exact same steps. However, due to the introduction of a policy rate r_B that evolves dynamically (instead of following a Taylor rule, for example) and influences the short-term rates on loans and deposits, the differential equation $\dot{r}_B$ must also be included in the reduced system. As a result, we obtain a reduced, five-dimensional model, given by

$$\begin{cases} \dot{\omega} &= \omega\left[\phi(\lambda) - \alpha - (1-\gamma)i(\omega)\right] \\ \dot{\lambda} &= \lambda\left[g(\pi_f) - \alpha - \beta\right] \\ \dot{\ell}_f &= \kappa(\pi_f) - \ell_f(i(\omega) + g(\pi_f)) \\ \dot{m}_f &= (1-\tau)\pi_f - m_f(i(\omega) + g(\pi_f)) \\ \dot{r}_B &= \beta_{rB}(g(\pi_f) - \alpha - \beta) \end{cases} \tag{4.9}$$

The revised model replicates many of the findings that were explained in Chapter 3. Increases in taxation depress corporate deposits, thus leading to a higher private debt ratio, but a lower public debt ratio, whereas increases in government spending only increase the public debt ratio (there is no effect on firms). The introduction of monetary policy enters most prominently through the β_{rB} parameter, a speed adjustment parameter that the central bank can set. The parameter allows the central bank to heat up or cool down the economy, depending on how far the actual, short-term growth rate of output is deviating from its long-term counterpart.

The effects of an increase in β_{rB} appear at first glance to be rather ambiguous. Smaller increases should shift the policy rate upwards, leading to increases in both the lending and deposit rates. As a result, the public debt-to-output ratio could decrease, and the private debt-to-output ratio should slightly increase, due to a slight increase in the profit share, and a corresponding increase in nominal investment. We hypothesize that larger spikes in β_{rB} can cause an economic contraction; if interest rates are too high, then investment will decrease, causing the short-term growth rate to underperform its long term counterpart. A decrease in the actual growth rate of output would then result in a sharp decrease in the policy rate (as well as the lending and deposit rates), with potential stabilization at negative values, and an explosion in the public and private debt ratios.

The effect of monetary policy on banks is observed through the ratio of bonds held by banks to nominal output, denoted by b_b, and obtained through equation (4.2) and the expressions m_f, m_h, ℓ_f, and $b_b = B_b/(pY)$:

$$\begin{aligned}
\frac{\dot{b}_b}{b_b} = \frac{\dot{B}_b}{B_b} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \frac{(1-f_M)\dot{M} - \dot{L}_f}{B_b} - i(\omega) - g(\pi_f) \\
&= \frac{(1-f_M)(m_f pY + m_h pY) - \ell_f pY}{b_b pY)} - i(\omega) - g(\pi_f) \\
&= \frac{pY((1-f_M)(m_f + m_h) - \ell_f)}{b_b pY} - i(\omega) - g(\pi_f) \\
&= \frac{(1-f_M)(m_f + m_h) - \ell_f}{b_b} - i(\omega) - g(\pi_f). \qquad (4.10)
\end{aligned}$$

We now present a supplementary system, consisting of the public debt to output ratio b, the ratio of bill holdings of banks to nominal output b_b and the household deposit-to-output ratio m_h. These equations do not factor into the main System 4.9, and thus can be solved separately.

$$\begin{cases}
\dot{b} &= (\psi_g - \tau(\omega + \pi_f + \pi_b)) + b(r_B - i(\omega) - g(\pi)) \\
\dot{b}_b &= (1-f_M)(m_f + m_h) - \ell_f - b_b(i(\omega) - g(\pi_f) \\
\dot{m}_h &= \omega + \kappa(\pi_h) + \psi_g - 1 + m_h(r_M - i(\omega) - g(\pi_f))
\end{cases} \qquad (4.11)$$

Note that π_b in the first equation represents the ratio of bank profits to nominal output, expressed by $\pi_b = \Pi_b/(pY)$. Smaller increases in the policy rate r_B should lead to a higher public debt ratio, as we observed from System 4.9, as well as a higher household deposit ratio, as can be directly observed from System 4.11. Increases in the policy rate will also lead to more bill holdings by banks, as the amount of public bonds issued rises in order for the government to finance its larger public debt. In addition, the amount of bonds held by banks can be influenced by the central bank through open-market operations. If the central bank issues more reserves to banks through an increase in the reserve ratio f_M, then the amount of government bonds held by the central bank will increase. This in turn will lead to a corresponding decrease in the amount of government bonds held by banks.

The interior, non-negative equilibrium values for the wage share, employment rate, private loan-to-output ratio, and private deposit-to-output ratio are identical to those presented in Chapter 3. The equilibrium policy rate can by found by first considering the expression for the equilibrium profit share for firms:

$$\pi_f^* = 1 - \omega^* - r_L \ell_f^* + r_M m_f^*. \tag{4.12}$$

Then, using equations (4.5) and (4.6) to ultimately solve for r_B^*:

$$r_B^* = \frac{1 - \omega^* - \pi_f^* - \delta_L \ell_f^* + \delta_M m_f^*}{\ell_f^* + m_f^*}. \tag{4.13}$$

Thus our equilibrium (ω^*, λ^*, ℓ_f^*, m_f^*, r_B^*) is given by

$$\omega^* = 1 - \pi_f^* - r_L\ell_f^* + r_M m_f^* \tag{4.14}$$

$$\lambda^* = \varphi^{-1}(\alpha + (1-\gamma)i(\omega^*)) \tag{4.15}$$

$$\ell_f^* = \frac{\kappa(\pi_f^*)}{\alpha + \beta + i(\omega^*)} \tag{4.16}$$

$$m_f^* = \frac{(1-\tau)\pi_f^*}{\alpha + \beta + i(\omega^*)} \tag{4.17}$$

$$r_B^* = \frac{1 - \omega^* - \pi_f^* - \delta_L\ell_f^* + \delta_M m_f^*}{\ell_f^* + m_f^*}. \tag{4.18}$$

As in Chapters 1 and 2, our adjustment speed parameter η_p will always be greater than the equilibrium growth rate of the economy $\alpha + \beta$. Thus we can infer that if the condition

$$((\alpha + \beta - \eta_p) + \eta_p\xi(1 - \pi_f^*))^2 > 4\eta_p\xi(r_L(\kappa(\pi_f^*) - r_M(1-\tau)\pi_f^*) \tag{4.19}$$

is satisfied, then the equilibrium profit share $\pi_f^* < 1$ must also hold, and as a result, there must be at least one non-negative equilibrium solution for ω^*.

4.2.1 Local Stability Analysis

Following earlier chapters, we investigate the local stability analysis of the five dimensional model with a public sector and monetary policy, presented in Section 4.2. Assuming there exists a non-negative equilibrium solution (ω^*, λ^*, ℓ_f^*, m_f^*, r_B^*) as previously defined in Section 4.2 ($\omega^* > 0$, $\lambda^* \in (0, 1)$), we follow our work from Chapter 3 and examine the following Jacobian matrix $J_{5\times5}^*(\omega^*, \lambda^*, \ell_f^*, m_f^*, r_B^*) = J_{5\times5}^*$ in order to investigate the local stability of the system:

$$J^*_{5\times5} = \begin{bmatrix} J^*_{5\times5,\,11} & \omega^*\Phi'(\lambda^*) & 0 & 0 & 0 \\ -\lambda^* g'(\pi^*_f) & 0 & -r^*_L\lambda^* g'(\pi^*_f) & r^*_M\lambda^* g'(\pi^*_f) & \lambda^* g'(\pi^*_f)(m^*_f - \ell^*_f) \\ J^*_{5\times5,\,31} & 0 & J^*_{5\times5,\,33} & J^*_{5\times5,\,34} & J^*_{5\times5,\,35} \\ J^*_{5\times5,\,41} & 0 & J^*_{5\times5,\,43} & J^*_{5\times5,\,44} & J^*_{5\times5,\,45} \\ -\beta_{rB} g'(\pi^*_f) & 0 & J^*_{5\times5,\,53} & J^*_{5\times5,\,54} & J^*_{5\times5,\,55} \end{bmatrix} \tag{4.20}$$

In the matrix, we have that:

$$J^*_{5\times5,\,11} = \Phi(\lambda^*) - \alpha + (1-\gamma)\eta_p(1 - 2\xi\omega^*) \tag{4.21}$$

$$J^*_{5\times5,\,31} = -\kappa'(\pi^*_f) + \ell^*_f(g'(\pi^*_f) - \eta_p\xi) \tag{4.22}$$

$$J^*_{5\times5,\,33} = r^*_L\ell^*_f g'(\pi^*_f) - r^*_L\kappa'(\pi^*_f) - g(\pi^*_f) - i(\omega^*) \tag{4.23}$$

$$J^*_{5\times5,\,34} = r^*_M(\kappa'(\pi^*_f) - \ell^*_f g'(\pi^*_f)) \tag{4.24}$$

$$J^*_{5\times5,\,35} = (m^*_f - \ell^*_f)(\kappa'(\pi^*_f) - \ell^*_f g'(\pi^*_f)) \tag{4.25}$$

$$J^*_{5\times5,\,41} = m^*_f(g'(\pi^*_f) - \eta_p\xi) - (1-\tau) \tag{4.26}$$

$$J^*_{5\times5,\,43} = r^*_L(m^*_f g'(\pi^*_f) - (1-\tau)) \tag{4.27}$$

$$J^*_{5\times5,\,44} = (1-\tau)r^*_M - r^*_M m^*_f g'(\pi^*_f) - i(\omega^*) - g(\pi^*_f) \tag{4.28}$$

$$J^*_{5\times5,\,45} = (m^*_f - \ell^*_f)(1 - \tau - m^*_f g'(\pi^*_f)) \tag{4.29}$$

$$J^*_{5\times5,\,53} = -\beta_{rB} g'(\pi^*_f)(r^*_B + \delta_L) \tag{4.30}$$

$$J^*_{5\times5,\,54} = \beta_{rB} g'(\pi^*_f)(r^*_B + \delta_M) \tag{4.31}$$

$$J^*_{5\times5,\,55} = \beta_{rB} g'(\pi^*_f)(m^*_f - \ell^*_f) \tag{4.32}$$

Once again, we will verify a numerical example, checking that the base parameters we have chosen for this system produce a locally stable equilibrium. Our base numerical simulation for the five dimensional Keen model with government and monetary policy equilibrates at the values:

$$(\omega^*,\, \lambda^*,\, \ell_f^*,\, m_f^*,\, r_B^*) = (0.88797,\, 0.97665,\, 0.53702,\, 0.26858,\, 0.0081732). \tag{4.33}$$

Other equilibrium values of note are

$$\begin{aligned} i(\omega^*) &= 0.26225 \\ \kappa(\pi_f^*) &= 0.165 \\ g(\pi_f^*) &= \frac{\kappa(\pi_f^*)}{\nu} - \delta = \frac{0.165}{3} - 0.01 \\ r_L^* &= 0.033173 \\ r_M^* &= 0.010673 \end{aligned}$$

As before, our important derivative values include $\kappa'(\pi_f^*) = \mathrm{inv}_1 = 15$, $g'(\pi_f^*) = 5$, and $\Phi'(\lambda^*) = \frac{2(0.0000641)}{(1-0.97665)^3}$. Substituting all necessary values into the Jacobian, we obtain

$$J^* = \begin{bmatrix} -0.85243 & 8.9418 & 0 & 0 & 0 \\ -4.8833 & 0 & -0.16199 & 0.052119 & -1.3109 \\ -14.893 & 0 & -0.71577 & 0.13144 & -3.3058 \\ -0.79628 & 0 & 0.016351 & -0.31251 & 0.13231 \\ -0.5 & 0 & -0.016587 & 0.0053366 & -0.13422 \end{bmatrix}. \tag{4.34}$$

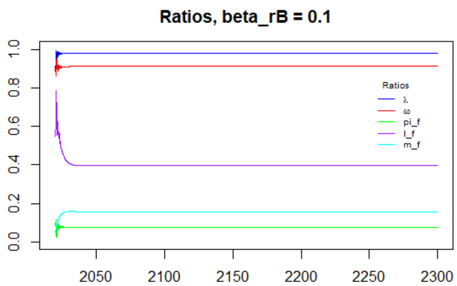
The eigenvalues of this matrix are approximately

$$(\epsilon_1,\, \epsilon_2,\, \epsilon_3,\, \epsilon_4,\, \epsilon_5) = (-0.318,\, -0.223,\, 0,\, -0.737 \pm 6.599i). \tag{4.35}$$

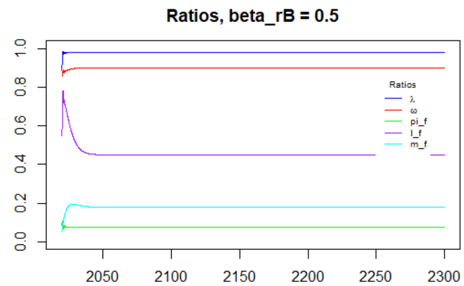
Four out of five eigenvalues have negative real parts, with the remaining eigenvalue equal to zero. This indicates that our attempt to determine the local stability of our system through linearization is inconclusive. Asymptotic stability would need to be proven through the usage of a suitable Lyapunov function. While this theoretical approach can be a potential avenue for future research, we can observe the stability of our system through basic numerical simulations, which will be explored in Section 4.3 of this chapter.

4.3 Results

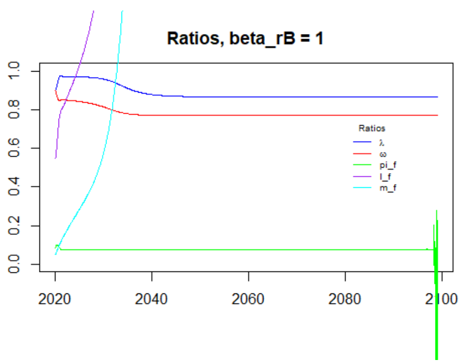
Turning our attention to some numerical results, we first observe Figure 4.1, which illustrates the basic effects of monetary policy on the model. We vary the policy rate adjustment parameter β_{rB}, as well as the lending rate scaling parameter δ_L. We plot four of the five state variables from the reduced model: ω, λ, ℓ_f, and m_f, as well as the profit share π_f. In Figures 4.1 (a), (b), and (c) we observe a simple balanced-budget model, in which $\tau = \psi_g = 0.15$, and vary β_{rB}. Observing Figure 4.1, we note that the base five-dimensional system converges to a stable long-term equilibrium, via damped oscillations (recall that our quasi-theoretical stability analysis in Section 4.2 was inconclusive).



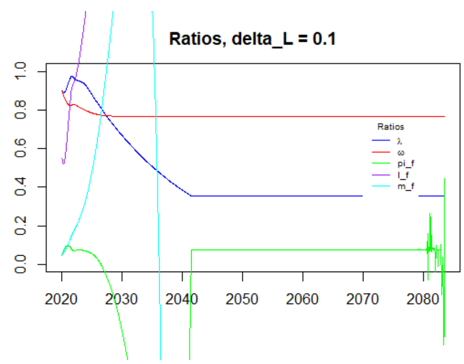
(a) Plots of key economic ratios, with policy rate adjustment parameter $\beta_{rB} = 0.1$.



(b) Key economic ratios, with $\beta_{rB} = 0.5$.



(c) Key economic ratios, with $\beta_{rB} = 1$.



(d) Key economic ratios, with lending parameter $\delta_L = 0.1$.

Figure 4.1: Simulations of the employment rate λ, wage share ω, profit share π_f, private loan-to-output ratio ℓ_f, and private deposit-to-output ratio m_f for the model with government and monetary policy presented in Section 4.1. Plots (a), (b), and (c) vary the policy rate adjustment parameter β_{rB} according to $\beta_{rB} = \{0.1, 0.5, 1\}$ while (d) holds all parameters equal, but raises the lending rate scaling parameter δ_L to 0.1 (from 0.025). All simulations are run from 2020 to 2300.

Slightly increasing β_{rB} in Figure 4.1 (b) does not greatly affect the model. The system does not equilibrate as quickly, but exhibits very similar dynamics to the base plot. The second row of Figure 4.2 shows that raising β_{rB} to a value of 0.5 (from 0.1) slightly decreases the inflation rate and the public debt ratio. Consequently, we can think of a slight increase in β_{rB} as an attempt by the central bank to cool down the economy with a higher policy rate. The increase in the policy rate slightly increases the private debt ratio, due to the relatively larger increase in the lending rate. A higher private debt share slightly diminishes the profit share, which slightly reduces investment and output, causing inflation to slightly decrease, and the wage share to equilibrate at a lower value. This attempt to cool down the economy will be successful as long as the short-term, actual growth rate of output does not fall below its long-run growth part, and cause the policy rate to decrease.

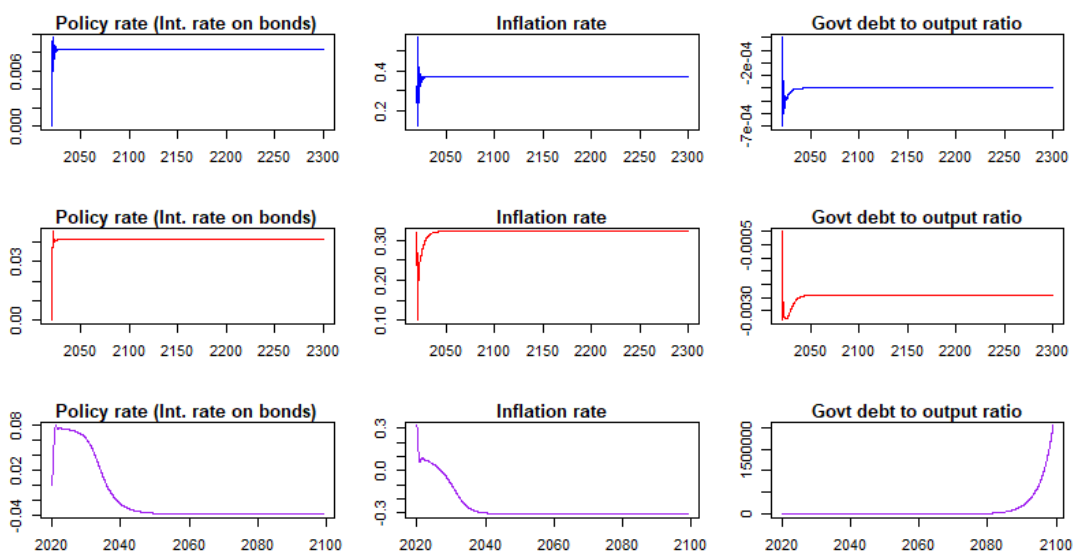
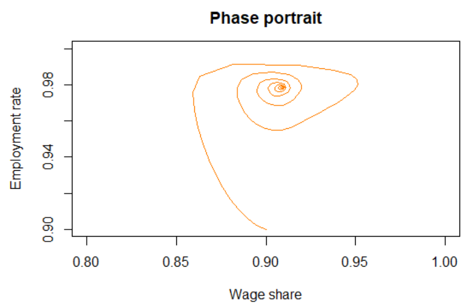


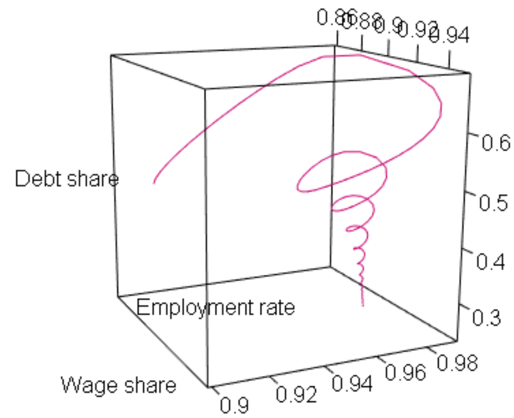
Figure 4.2: Plots of the policy rate r_B, inflation rate $i(\omega)$, and public debt-to-output ratio b, from 2020 to 2300. The top row of plots (blue) sets $\beta_{rB} = 0.1$, the middle row (red) sets $\beta_{rB} = 0.5$, while the bottom row (purple) sets $\beta_{rB} = 1$. All other parameters are held equal throughout the adjustments in β_{rB}.

Figure 4.1 (c) confirms a key hypothesis made in Section 4.2. Raising the adjustment parameter β_{rB} to a much higher value of 1 will cause an initial spike in the policy rate, but as the private loan-to-output ratio begins to drastically outpace the private deposit-to-output ratio, the economy will begin to contract rapidly, due to overindebtedness. The extreme cooling down of the economy occurs when the high policy rate (and correspondingly high lending rate) pushes us into an economic regime in which the short-run growth rate falls below the long-term equilibrium growth rate, thus causing a rapid decrease in the policy rate, and a shift to a deflationary state. The policy rate and the lending rate eventually settle at negative values, but the shift to a permanently deflationary regime results in a decrease in the wage bill, which ultimately forces government spending down to a low, non-zero value, and taxes down to zero. As the private debt ratio continues to increase the public debt ratio explodes, the profit share eventually collapses, causing the entire model to crash before 2100. For a more detailed visual of the crashes in r_B, $i(\omega)$, and b, see the bottom row of Figure 4.2.

Figure 4.1 (d) is less interesting than the first three subplots, but gives valuable information nonetheless. As we determined earlier (in Chapter 2), an increase in the lending rate causes the corporate profit share to collapse due to overindebtedness, also causing investment to plummet. This decrease in π_f further decreases the deposit-to-output ratio m_f, causing further indebtedness that ultimately reduces output to the point of economic collapse. This type of economic contraction is observed in our five-dimensional model in Figure 4.1, where the lending rate scaling parameter is raised to a relatively high $\delta_L = 0.1$, from the original calibrated value of $\delta_L = 0.025$.



(a) Two-dimensional phase portrait.



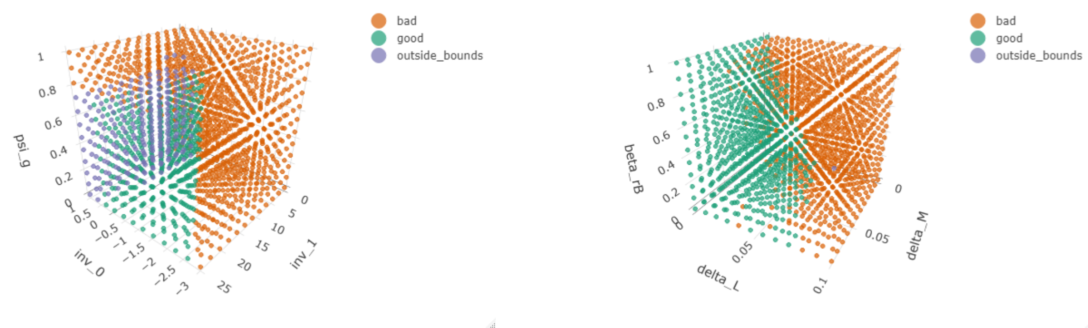
(b) Three-dimensional phase portrait.

Figure 4.3: Phase portraits for the five-dimensional model with government and monetary policy. The two-dimensional phase portrait in (a) plots employment rate λ with respect to the wage share ω, whereas the three-dimensional phase portrait in (b) plots the employment rate λ, the wage share ω, and the private debt-to-output ratio d.

Once again, the oscillatory convergence to a locally stable, long-term equilibrium is captured in the phase portraits displayed in Figure 4.3. These plots portray the relationship between the employment rate and the wage share, and the relationship between the wage share, employment rate, and private debt ratio, for the base calibrated model first observed in Figure 4.1 (a).

4.3.1 Parameter Sensitivity

Following the classification criteria outlined in Chapter 2, we perform another round of parameter sensitivity analysis. In Figure 4.4 (a), we vary the intercept of our linear investment function according to $\mathrm{inv}_0 \in [-3, 1]$, the slope according to $\mathrm{inv}_1 \in [0.05, 25]$ and the share of nominal output dedicated to government expenditures according to $\psi_g \in [0, 1]$. Twelve equally spaced points are chosen across each of the three intervals. The resulting sensitivity grid has a considerably smaller region of runs that converge to the "good", stable equilibrium, when compared to its counterpart from Chapter 3. This occurs for the reasons outlined earlier: under-investment (which occurs if a relatively lower inv_0 value is not countered with a relatively higher inv_1 value, or vice versa) pushes the short-term growth rate of output below the long-term equilibrium rate, thus causing a sharp cooling down of the economy, and an overindebted, deflationary regime that ultimately results in economic collapse.

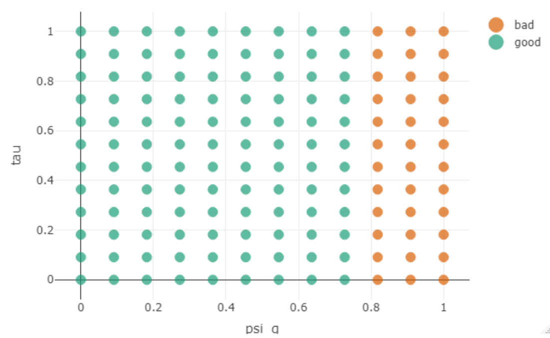


(a) Sensitivity of investment function parameters, alongside ψ_g.

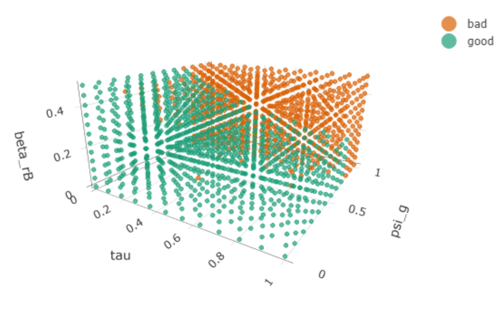
(b) Sensitivity of interest rate parameters β_{rB}, δ_L, and δ_M.

Figure 4.4: Plot (a) depicts the parameter sensitivity for investment function parameters inv_0 and inv_1, along with the share of nominal output dedicated to government spending, ψ_g. The sensitivity plot in (b) portrays the parameter sensitivity of the interest rate parameters β_{rB}, δ_L, and δ_M. For all simulated runs in (a) and (b), we assume that $\tau = 0.15$, while for all simulated runs in (b), we assume that $\psi_g = 0.15$.

Figure 4.4 (b) displays a similar trend, this time for the interest rate parameters. Lower values for β_{rB} are generally more stable, corresponding with a larger region of values for δ_L. If the equilibrium policy rate is lower due to a lower adjustment value, then the lending rate can be higher, without running into the problem of overindebtedness in the private sector. As β_{rB} increases, and we risk entering a deflationary regime, the size of the "good" region shrinks. Higher β_{rB} values must be accompanied by lower δ_L values and higher δ_M values. This counterbalance keeps the profit rate from decreasing too drastically, which in turn prevents from investment, capital, and ultimately output from collapsing. In general, higher lending rate parameter values δ_L must be countered with higher deposit rate parameter values δ_M, in order to keep a healthy profit share and growth rate, and in order to prevent a rapidly increasing private debt ratio.



(a) Two-dimensional sensitivity grid of τ and ψ_g.

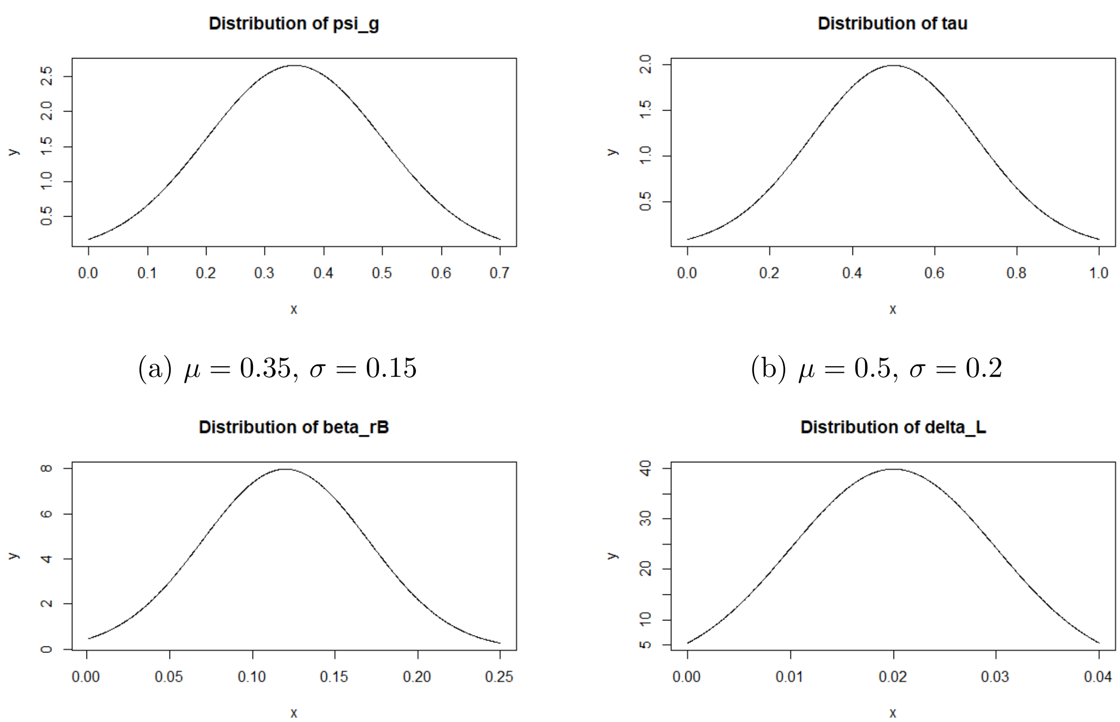


(b) Sensitivity plot of τ, ψ_g, and β_{rB}.

Figure 4.5: Plot (a) depicts a two-dimensional parameter sensitivity grid for tax rate τ, along with the share of nominal output dedicated to government spending, ψ_g. Plot (b) illustrates a sensitivity of grid of τ, ψ_g, and the policy rate parameter β_{rB}.

In Figure 4.5 (a), we observe a similar two-dimensional grid as the one presented in the previous chapter (with the same specifications). The base calibrated monetary policy results in lower lending and deposit rates, thus ensuring that as the tax rate rises (all the way to 1), the decrease in m_f (and corresponding increase in d) does not reduce the profit share to the point of an economic contraction. The behaviour of ψ_g remains the same as before; its value cannot exceed approximately 0.73, as too much government spending will reduce investment to zero in the long run, thus resulting in economic collapse.

Figure 4.5 (b) varies the policy rate adjustment parameter β_{rB} from 0 to 0.5, along with the parameters tau and ψ_g from 0 to 1. Lower policy rates produce lower lending and deposit rates, thus enabling a higher level of taxation. The central bank in the model can exert monetary policy through increasing and decreasing β_{rB}, which correspond to heating up or cooling down the economy. A heated up economy will produce lower policy rates with lower private debt ratios, which will thus allow for higher taxation, since even very high increases in the tax rate will not dramatically increase the level of private indebtedness.



(a) $\mu = 0.35$, $\sigma = 0.15$

(b) $\mu = 0.5$, $\sigma = 0.2$

(c) $\mu = 0.12$, $\sigma = 0.05$

(d) $\mu = 0.02$, $\sigma = 0.01$

Figure 4.6: Probability density functions for parameters ψ_g, τ, β_{rB}, and δ_L. All four parameters are sampled from a normal distribution; consequently, normal density is plotted on the y-axis.

From the good regions of our sensitivity grids, we determine suitable probability density functions for ψ_g, τ, β_{rB}, and δ_L, each of which are plotted in Figure 4.6. The four parameters are all sampled from a normal distribution, with their means and values for standard deviation stated in the sub-captions of Figure 4.6. Our 200 Monte Carlo replications of the Keen model with a public sector and monetary policy are then examined, with all runs occurring from 2020 to 2100. In Figure 4.7, we note that the shaded blue region represents the [0.025;0.975] probability interval for the wage share, employment rate, and public and private debt-to-output ratios. The solid blue line represents the median trajectory for each variable, as before.

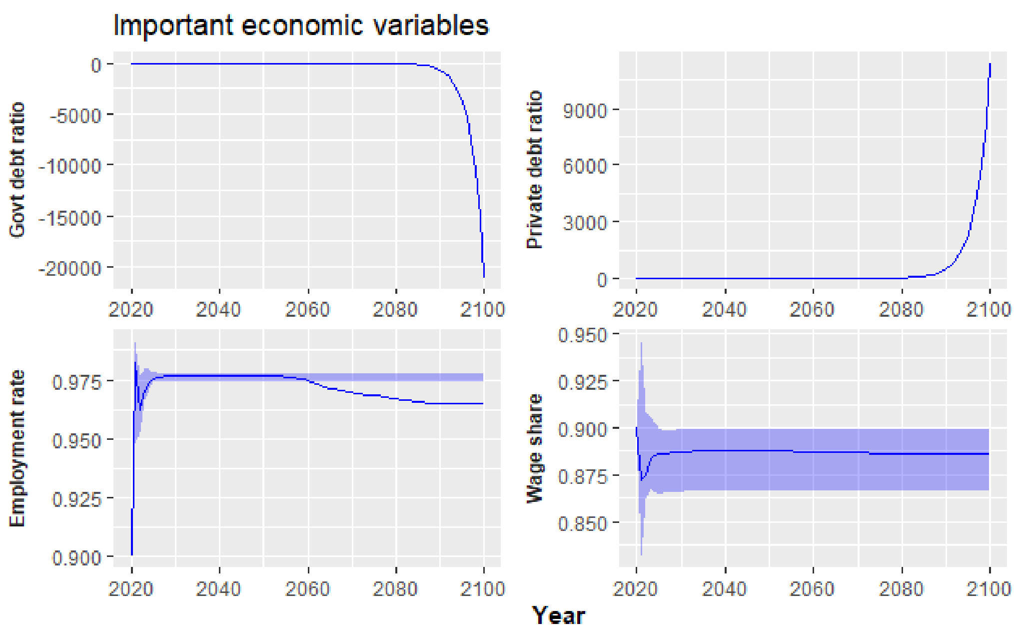


Figure 4.7: Monte Carlo simulations for the Keen model with government and monetary policy. Two hundred replications of the private debt-to-output ratio d, public debt-to-output ratio b, employment rate λ, and wage share ω are plotted, from 2020 to 2100. The shaded regions represent a [0.025; 0.975] (95%) probability interval for each variable, while the darker lines represent the median trajectories.

In Figure 4.7, 193/200 replications produce a private debt share of $d < 2.7$, and 195/200 replications produce an employment rate of $\lambda \in [0.4, 1]$. The introduction of monetary policy has further created easily identifiable regions of good simulations in our sensitivity grids, thus enabling probability density functions that are more consistent with those good regions. Furthermore, the addition of the public sector and monetary policy to the base Keen model with inflation has stabilized the system, with far fewer runs resulting in explosions of the private debt share, or collapses of the employment rate, and far less variance in our replicated results, as well. For reference, 197/200 simulations produce a wage share of $\omega \in [0.4, 1]$.

4.3.2 Basin of Attraction

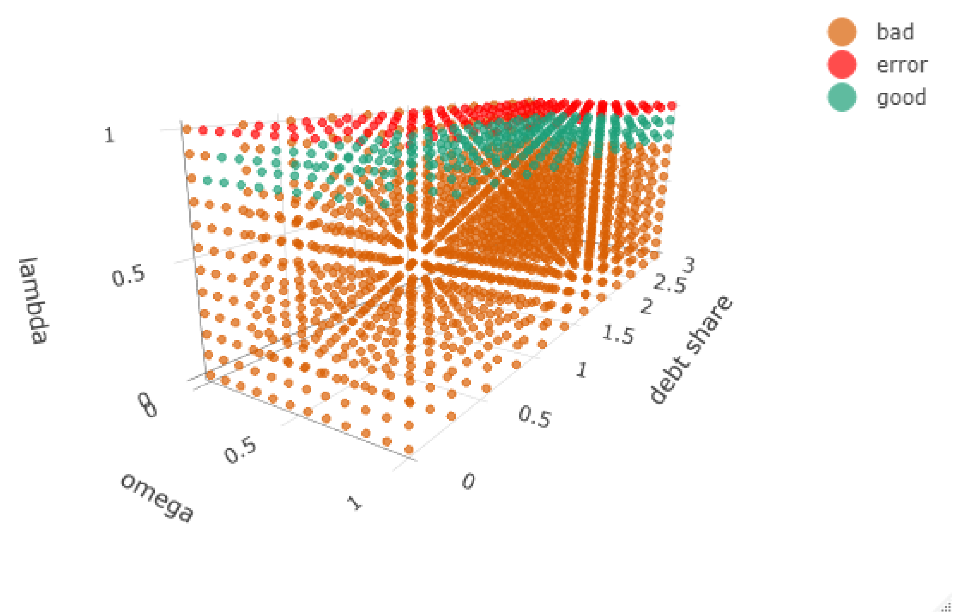


Figure 4.8: Basin of attraction, varying the initial conditions for wage share ω, employment rate λ, and private debt ratio d.

The initial wage share, employment rate, and private debt share are varied according to the specifications discussed in earlier chapters. The resulting basin of attraction for the Keen model with government and monetary policy is plotted in Figure 4.8. We note that the introduction of monetary policy stabilizes the existing model slightly, with five more runs (out of 1728) classified as having converged to the "good" region. Most notably, the introduction of an active policy rate necessitates higher initial employment rates (for convergence to the good equilibrium), as lower employment rates will result in depressed short-term growth, therefore spurring a deflationary-driven economic collapse. However, the introduction of a stabilizing, lower policy rate (and thus lower lending rates) allows for higher initial levels of d, since firms are now paying relatively less interest on higher amounts of debt.

4.4 Climate Results

The climate module proceeds exactly as explained in Chapter 2, with the same changes to the economic module as well. Similar to the model presented in Chapter 3, due to carbon taxes T_f collected by the government and the abatement subsidy S_f given to firms, public debt now evolves according to

$$\dot{B} = (G + S_f) - (T + T_f) + r_B B_b. \tag{4.36}$$

Figure 4.9 plots the results for the basic scenario analysis explained in Chapter 2. Holding all other parameters from the economic module equal, we raise inv_1 from 15 to 20, in order to remain in line with the results from Chapter 3. We observe that the strongest policy scenario performs the best, with very similar results to those from previous chapters, *up until 2100*. The collapses for the weaker two scenarios are more gradual, with an eventual recovery in the profit share for both, even as the private debt ratio explodes past the $d = 2.7$ threshold. The gradual collapse stems from the adoption of the monetary policy, which allows for firms to pay less interest on their debt. Therefore, firms can retain a greater level of profit and refrain from immediate mass layoffs, even as their private debt shares rapidly increase.

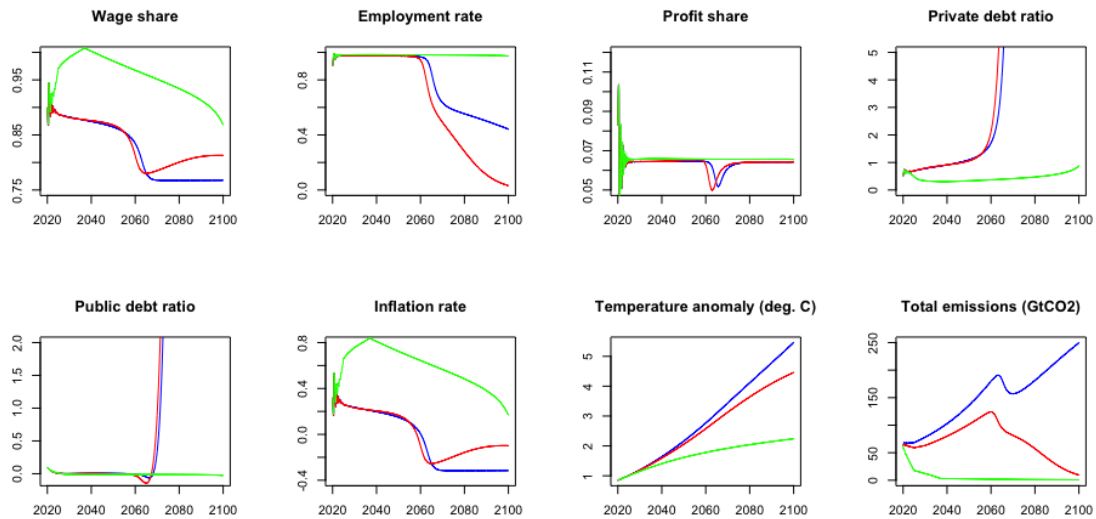


Figure 4.9: Climate scenario analysis for the Keen model with government and monetary policy, from 2020 to 2100. The blue curve represents the *Low Damages, Low Policy* scenario, the red curve represents the *High Damages, Medium Policy* scenario, and the green curve represents the *High Damages, Strong Policy* scenario. The damages to capital parameter is set at $f_k = 0.1$.

The beginning of the following century brings economic collapse to all three scenarios (not plotted; the collapse caused the integrator to fail). Major long-term contractions are observed, due to the over-accumulation of private debt. The monetary policy stabilizes the economy such that the private sector pays little interest on its growing private debt, even as output is decreasing. This generates a regime in which the short-term growth rate of output begins to outpace the long-term, equilibrium rate, with the caveat of a rapidly increasing private debt ratio. This drives interest rates up, with the subsequent effect of an economic cool-down. Firms pay more interest on their debt, which causes output to decrease further, which thus leads to the short-term growth rate stabilizing below its long-term counterpart. This produces a deflationary regime characterized by a falling growth rate, which thus leads to a massive economic contraction. Therefore, we conclude that while an expansionary monetary policy can certainly stabilize an economy in the short-run, continual investment in carbon-intensive goods can still lead to a climate-induced economic collapse in the long-run.

Chapter 5

Consumption and Inventories

5.1 Economic Module

5.1.1 Accounting Framework

In this chapter, we will introduce a Keen model with explicit consumption and inventories. We return to a three-sector economy: households, firms, and banks. Tables 5.1, 5.2, and 5.3, display the balance sheets, income statements, and transaction flow matrices of the economy. Once again, all entries in the country's balance sheet are measured in nominal monetary amounts, whereas transaction entries and items on the flow of funds sheet are measured in monetary units per unit of time. We follow the exact same assumptions of the original Keen model with inflation, but now follow Grasselli and Nguyen-Huu (2018) and propose that firms hold both the nominal stock of capital pK *and* nominal amount of inventories cV as assets. The transaction flows proceed as before, with the biggest change originating from our assumption that Say's

Law (aggregate demand must equal aggregate supply) no longer holds. This disequilibrium in the goods market will become more prominent through the introduction of an explicit, behavioural consumption function.

	Households	Firms	Banks	Sum
Balance Sheet				
Capital Stock		pK		pK
Inventories		cV		cV
Deposits	M_h	M_f	$-M$	0
Loans		$-L_f$	L_f	0
Sum (net worth)	X_h	X_f	X_b	X

Table 5.1: Balance sheet for country's economy

	Households	Firms		Banks	Sum
Transactions		Current	Capital		
Consumption	$-pC$	pC			0
Investment		pI_k	$-pI_k$		0
Change in inventories		$c\dot{V}$	$-c\dot{V}$		0
[GDP]		$[Y_n]$			$[Y_n]$
Wages	W	$-W$			0
Capital depreciation		$-p\delta K$	$p\delta K$		0
Int. on loans		$-rL_f$		rL_f	0
Int. on deposits	rM_h	rM_f		$-rM$	0
Profits		$-\Pi_f$	Π_f		0
Sum (balance)	S_h	0	$S_f - p(I_k - \delta K) - c\dot{V}$	S_b	

Table 5.2: Transactions in the given economy

5.1.2 Output and Inventories

We again assume a Leontieff-type production function for a given country's economy, and then suppose that a good is produced to a potential level $Y_{\max}$ of output using

	Households	Firms	Banks	Sum
Balance Sheet				
Change in capital stock		$p\dot{K}$		$p\dot{K}$
Change in inventories		$c\dot{V}$		$c\dot{V}$
Change in deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$	0
Change in loans		$-\dot{L}_f$	$\dot{L}_f$	0
Sum (savings)	S_h	S_f	S_b	$pI_k + c\dot{V}$
Change in net worth	$\dot{X}_h = S_h$	$\dot{X}_f = S_f + \dot{p}K + c\dot{V}$	$\dot{X}_b = S_b$	$\dot{X}$

Table 5.3: Flow of funds in economy

available capital K, such that

$$Y_{\max} = \frac{K}{\nu}. \tag{5.1}$$

The ratio $1/\nu$ represents capital productivity. In this model, the explicit modelling of consumption is accompanied by an assumption that Say's law does not hold. This causes a disequilibrium in the goods market, in which aggregate production no longer equals the demand for goods. In order to explore the economic effects of this disequilibrium, we examine the concept of unplanned changes in inventory, while following many assumptions of Grasselli and Nguyen-Huu (2018). Given the level of expected sales/output of firms, denoted by Y_e (whose dynamics will be defined shortly), we define the actual output of firms by

$$Y = Y_e. \tag{5.2}$$

We depart from the work of Grasselli and Nguyen-Huu (2018), and assume that there is no planned investment in inventories that is added to expected sales. From the expressions for potential and total production, we obtain an expression for capital utilization, given by

$$u = \frac{Y}{Y_{\max}} = \frac{\nu Y}{K}. \tag{5.3}$$

We then assume that capital evolves according to

$$\dot{K} = I_k - \delta K. \tag{5.4}$$

Given the real consumption of goods C and real capital investment I_k, total sales demand is given by

$$Y_d = C + I_k. \tag{5.5}$$

Then, changes in the level of inventory — which can alternatively be thought of as investment in inventory — can be determined by the difference in total aggregate output and total domestic demand, such that

$$\dot{V} = Y - Y_d. \tag{5.6}$$

By assuming that there is no planned investment in inventory, we find that the inventory stock evolves solely according to *unplanned* changes in inventory, which act as an accommodating variable, accounting for any discrepancies between actual and expected sales.

We then introduce nominal output, which is first defined as the sum of nominal consumption of goods (by households) pC and nominal capital investment by firms pI_k. Following Godley and Lavoie (2006), we value changes in inventory at the unit cost c, and account for firms' purchases of their own unsold goods through the investment term $c\dot{V}$:

$$Y_n = pC + pI_k + c\dot{V} = pY_d + c\dot{V}. \tag{5.7}$$

Finally, total investment is given by the sum of real capital investment and any changes in the investment in inventory. After assuming that there is no planned investment in inventory stock, we conclude that

$$I_T = I_k + \dot{V}. \tag{5.8}$$

5.1.3 Prices, Profits, and Behavioural Rules

The price inflation dynamics are given by

$$i = \frac{\dot{p}}{p} := \eta_p \left(\mu \left(\frac{c}{p} \right) - 1 \right) - \eta_q \left(\frac{Y_e - Y_d}{Y} \right) \tag{5.9}$$

It is through this second term that inventory dynamics are considered in the pricing mechanism. This second term adds to the original construction featured in Grasselli and Nguyen-Huu (2015), in which pricing dynamics were defined solely by the first term in equation (5.9). The ratio of the unit cost of changes in inventory to the price of a commodity is equivalent to the wage share ω:

$$c = p\omega \tag{5.10}$$

We take the markup μ and the relaxation parameters η_p and η_q to be fixed. Turning our attention to profits earned by firms, we first observe that the actual and expected profits (pre-depreciation) are given by

$$\Pi_f = Y_n - \mathrm{w}L + rM_f - rL_f \tag{5.11}$$

$$\Pi_e = pY_e - \mathrm{w}L + rM_f - rL_f. \tag{5.12}$$

Actual nominal profit (resp. expected nominal profit) is defined as nominal output (resp. expected nominal output) minus production costs and interest from loans, plus interest earned from deposits. The profit share and expected profit share of nominal production π_f are given by

$$\pi_f = \frac{\Pi_f}{pY} \tag{5.13}$$

$$\pi_e = \frac{\Pi_e}{pY}. \tag{5.14}$$

Originally, the long-run growth rate $g_e(u, \pi_e)$ presented in Grasselli and Nguyen-Huu (2018) was assumed to be equal to the equilibrium growth rate of the economy $(\alpha+\beta)$, as assumed in Franke (1996):

$$\dot{Y}_e = (\alpha + \beta)Y_e + \eta_e(Y_d - Y_e). \tag{5.15}$$

We revise this expression by replacing the equilibrium growth rate of the economy $(\alpha + \beta)$ with the expected growth rate of capital, denoted by $(\kappa(\pi_e,\, u)/\nu - \delta)$. These two expressions are equivalent in the long run, as both should converge to $(\alpha + \beta)$ at equilibrium. In the short run, however, the use of the expected growth rate of capital causes the expected sales dynamics to be more sensitive to the expected profit share, thus reducing output explosions where aggregate demand vastly outpaces the level of expected sales. After making this modification, we obtain the following dynamics for expected sales:

$$\dot{Y}_e = \left(\frac{(\kappa(\pi_e,\, u)}{\nu} - \delta \right) Y_e + \eta_e (Y_d - Y_e). \tag{5.16}$$

For both specifications, firms are assumed to adjust their short-run expectations based on the observed level of demand, and thus the parameter η_e acts as the speed of convergence to the observed level of output demand.

5.1.4 Firm Financing

The total financing needs of firms are represented by the difference in capital investment and corporate profits. These needs are satisfied by loans, and we assume that firms face no rationing by banks. With this information, we present the loan dynamics in a given economy as

$$\dot{L}_f = pI_k + c\dot{V} \tag{5.17}$$

where real capital investment is given by

$$I_k = \kappa(\pi_e,\, u) \left(\frac{K}{\nu} \right). \tag{5.18}$$

The investment function, $\kappa(\cdot)$, depends on the expected profit rate, and follows a linear specification that will be discussed shortly. Then, we assume that deposits evolve according to

$$\dot{M}_f = \Pi_f, \tag{5.19}$$

as in the previous chapters. In order to reduce dimensionality and focus solely on the new model with consumption and inventories, we forego explicitly modelling private loans and deposits again until the next chapter. We thus model the aggregate credit demand dynamics of firms by

$$\dot{D} = \dot{L}_f - \dot{M}_f = pI_k + c\dot{V} - \Pi_f, \tag{5.20}$$

and we use the interest rate r on net debt, defined by the relation $rD = r_L L_f - r_M M_f$. The corporate subsection concludes with the debt-to-output ratio, once again defined as

$$d = \frac{D}{pY}. \tag{5.21}$$

5.1.5 Labour Markets and Households

This subsection remains mostly the same as in previous chapters, with the sole change occurring in the expression for household consumption. We model consumption C as a function of the wage share, *not* as a function of the total wealth of households (as will be explored in the following section):

$$C = \theta(\omega)Y. \tag{5.22}$$

Households do not hold financial assets such as bonds or loans, and can only hold deposits. Therefore, we say that household savings are equivalent to the change in deposits:

$$\dot{M}_h = S_h. \tag{5.23}$$

5.1.6 Functional Forms

A rational curve is employed to model the growth of wages, as in the three preceding chapters. The $\kappa(\cdot)$ investment function presented in Grasselli and Nguyen-Huu (2018) followed

$$\kappa(\pi_e,\, u) = \kappa_0 + \exp \kappa_1 + \kappa_2 \pi_e + \eta_u (u - \bar{u}), \tag{5.24}$$

taking the form of an uncapped exponential function. The dependence on the profit share π_f that was used in previous chapters is now replaced by a dependence on the *expected* profit share (π_e), as well as on capital utilization u. The parameter $\bar{u}$ represents the baseline level of utilization, estimated by firms, while the parameter η_u denotes the speed at which capital utilization u adjusts to this baseline value. We choose to employ the linear investment function given by

$$\kappa(\pi_e,\, u) = \mathrm{inv}_0 + \mathrm{inv}_1 \pi_e + + \eta_u (u - \bar{u}), \tag{5.25}$$

as it produces fewer outcomes that are sensitive to changes in the respective parameters.

Turning our attention to the consumption function, we first observe the generalized logistic form featured in Grasselli and Nguyen Huu (2015).

$$\theta(\omega) = \max \left\{ C_0, A_c + \frac{K_c - A_c}{(C_c + Q_c e^{-B_c \omega})^{\frac{1}{\nu_c}}} \right\}. \tag{5.26}$$

The share of consumption of the overall economy is modelled by $\theta(\omega)$, and depends on the wage share. The hard lower bound of the consumption share represents the subsistence level of consumption in the economy, denoted by C_0. The asymptotic lower bound is given by A_c, the upper bound for the consumption share is denoted by K_c, and the growth rate of the household consumption share of the economy is given by B_c.

A simple linear form replaces this consumption function, since its parameters are less sensitive to perturbations than the parameters of the original logistic form. The linear consumption function is given by

$$\theta(\omega) = \max\{c_{\min}, \min\{c_0 + c_1\omega,\ c_{\max}\}\}. \tag{5.27}$$

Here, the lower bound/subsistence level of consumption in the economy is represented by $c_{\min}$, while the upper bound of the consumption share is given by $c_{\max}$.

5.2 Main Dynamical System

Before deriving our main dynamical system, we first make note of a new auxiliary function $g(\pi_e, u, y_d)$, which represents the growth rate of output. The form of this function is given by

$$g(\pi_e, u, y_d) := \frac{\dot{Y}}{Y} = \left(\frac{\kappa(\pi_e, u)}{\nu} - \delta\right) + \eta_e(y_d - 1), \tag{5.28}$$

where the expected profit share π_e is denoted by

$$\pi_e = (1 - \omega) - rd. \tag{5.29}$$

Lastly, the normalized level of sales demand y_d is simply given by

$$y_d = \frac{Y_d}{Y} := \theta(\omega) + \frac{\kappa(\pi_e, u)}{u}. \tag{5.30}$$

The reduced-form equations for the wage share ω and employment rate λ follow the steps outlined in Chapter 2. The reduced private debt ratio dynamics can be derived using the expression $d = \frac{D}{pY}$ and the dynamics for corporate debt (equation (5.20)):

$$\begin{aligned}
\frac{\dot{d}}{d} = \frac{\dot{D}}{D} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \left[\frac{pI_k + c\dot{V} - \Pi_f}{D}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{pI_k + c\dot{V} - (pY_d - c\dot{V} - \mathrm{w}L - rD)}{dpY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{pI_k - p(C + I_k) + \mathrm{w}L + rdpY}{dpY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{\omega pY - pY\theta(\omega) + rdpY}{dpY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{\omega - \theta(\omega)}{d}\right] + r - i(\omega, y_d) - g(\pi_e, u, y_d)
\end{aligned} \tag{5.31}$$

The utilization dynamics can be obtained from equation (5.3):

$$\begin{aligned}
\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} &= g(\pi_e, u, y_d) - \left(\frac{I_k - \delta K}{K}\right) \\
&= \left(\frac{\kappa(\pi_e, u)}{\nu} - \delta\right) + \eta_e(y_d - 1) - \left(\frac{\kappa(\pi_e, u)}{\nu} - \delta\right) \\
&= \eta_e(y_d - 1)
\end{aligned} \tag{5.32}$$

This results in the following four dimensional system, given by

$$\begin{cases} \dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega, y_d)\right] \\ \dot{\lambda} &= \lambda\left[g(\pi_e, u, y_d) - \alpha - \beta\right] \\ \dot{d} &= d\left[r - g(\pi_e, u, y_d) - i(\omega, y_d)\right] + \omega - \theta(\omega) \\ \dot{u} &= u\left[\eta_e(y_d - 1)\right]. \end{cases} \tag{5.33}$$

Increases in the consumption share of the economy $\theta(\omega)$ will reduce the private debt ratio. Households increasing their demand for goods will result in greater profits for the corporate sector, thus increasing deposits and lowering the debt ratio for firms. Furthermore, increases in y_d (caused by increases in household consumption or by increases in corporate investment) that result in $y_d > 1$ will cause the economy to perform *above capacity*. An economy performing above capacity will increase employment and decrease the private debt ratio in the short-term, as both inflation and the growth rate of output increase. As the employment rate approaches 1, however, wages increase dramatically, leading to a decrease in the profit share, which ultimately causes the economy to contract and cool down.

Turning our attention once again to the four-dimensional system of equations introduced in (5.33), we proceed similarly to the work of Grasselli and Nguyen-Huu (2018), and obtain an interior equilibrium (ω^*, λ^*, d^*, u^*). First, we assume that the equilibrium growth rate of output is equal to the sum of the labour productivity growth rate and the population growth rate:

$$g(\pi_e^*, u^*, y_d^*) = \alpha + \beta, \tag{5.34}$$

as is required at equilibrium from the second equation in System (5.33). Next, we can observe that the last equation implies that $y_d = 1$. This, in turn, implies that the equilibrium expression for inflation is independent of inventory effects, and is simply given by

$$i(\omega^*, y_d^*) = i(\omega^*) = \eta_p(\mu\omega^* - 1). \tag{5.35}$$

As a result, the equilibrium expression for the private debt-to-output ratio is represented by

$$d^* = \frac{\omega^* - \theta(\omega^*)}{\alpha + \beta + i(\omega^*) - r}. \tag{5.36}$$

From the expression for $g(\pi_e, u, y_d)$ and the fact that $y_d^* = 1$, we observe that at equilibrium, the investment share of the economy satisfies

$$\kappa(\pi_e^*, u^*) = \nu(\alpha + \beta + \delta). \tag{5.37}$$

This result can be substituted into equation (5.30) (for normalized sales demand), to obtain the expression for capacity utilization at equilibrium:

$$u^* = \frac{\nu(\alpha + \beta + \delta)}{1 - \theta(\omega^*)} \tag{5.38}$$

As in earlier chapters, we say that the equilibrium employment rate is expressed by

$$\lambda^* = \Phi^{-1}(\alpha + (1 - \gamma)i(\omega^*)). \tag{5.39}$$

Lastly, we can use the equilibrium profit share to derive the expression for the equilibrium wage share:

$$\omega^* = 1 - \pi_e^* - rd^*. \tag{5.40}$$

As is explained in Grasselli and Nguyen-Huu (2018) (and in previous chapters) the general conditions for existence and uniqueness of the interior equilibrium determined above depend strongly on the behavioural functions for investment and consumption. Calculations similar to those performed in Chapters 2 through 4 can be completed for this chapter as well. In the interest of brevity, however, we choose to forego this analytical work and proceed to our computational analysis. Through parameter sensitivity analysis, we can explore the functional forms for $\kappa(\cdot)$ and $\theta(\cdot)$, and gain greater information as to the numerical sensitivity of the parameters in these functions.

5.3 Results

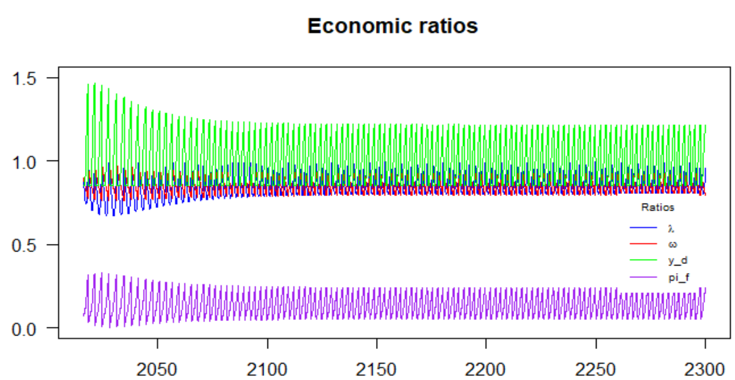


Figure 5.1: Simulations of employment rate λ (blue), wage share ω (red), demand-to-output ratio $y_d = \frac{Y_d}{Y}$ (green), and profit share π_e (purple), for the Keen model with consumption and inventories, described in Section 5.1. All simulations are run from 2020 to 2300.

The most notable change to the trajectories of the key economic variables and ratios is undamped oscillatory behaviour. We observe limit cycles traversing the "good" equilibrium of the wage share, employment rate, and profit share, displayed in a sample run in Figure 5.1. In this simulation, the wage share ultimately oscillates between values just under 0.8, and just over 0.9, while the employment rate ultimately cycles through values just over 0.8 and just under 1. Perhaps most alarmingly, the demand to output ratio y_d oscillates between 0.85 and just above 1.2. The average of the final 500 values of y_d is equal to 1.0052, thus representing a very reasonable potential equilibrium of just slightly over 1. A potential issue stems from these regular mini-explosions of demand, which can be interpreted as causing a backlog of "orders", causing firms' inventories to regularly dip below 0, as pictured in Figure 5.2.

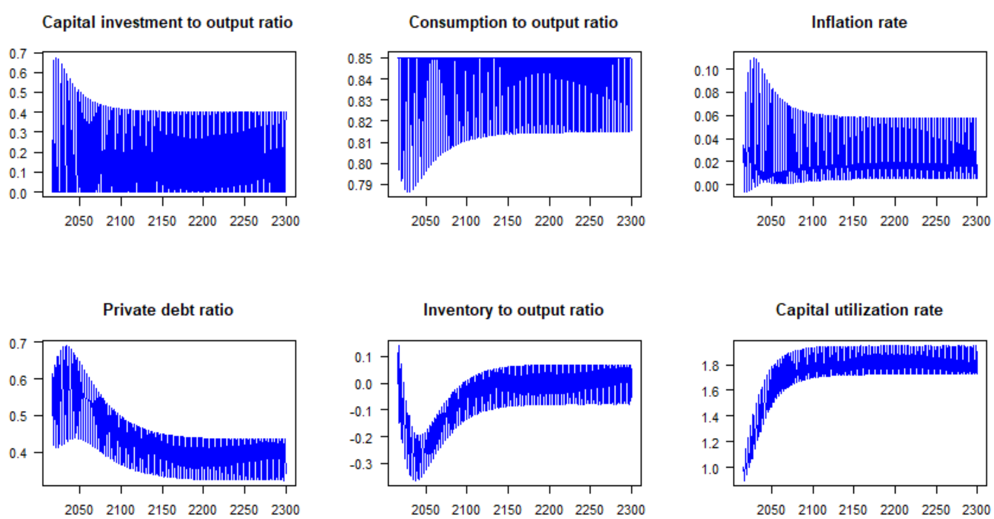
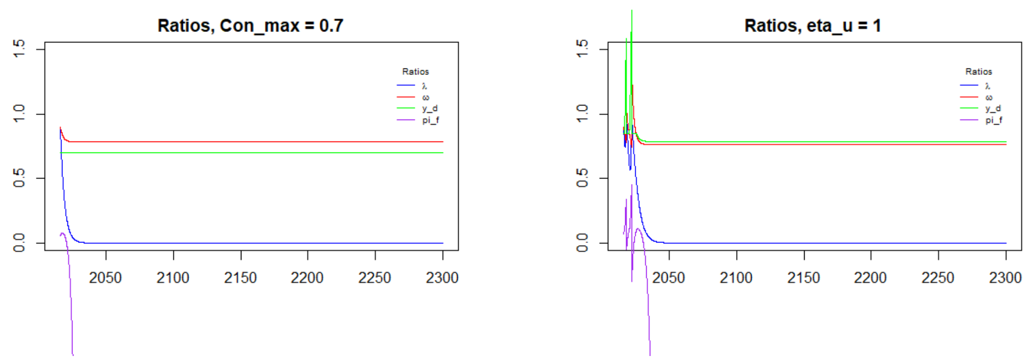


Figure 5.2: Simulations of other important economic variables, such as the capital investment to output ratio $\frac{I_k}{Y}$, consumption to output ratio $\theta(\omega)$, inflation rate i, private debt-to-output ratio d, inventory to output ratio $\frac{V}{Y}$, and capital utilization rate u. All simulations are run from 2020 to 2300.

In fact, turning our attention to Figure 5.2, we can determine that explosions in demand are caused by explosions in capital investment. While the consumption to output ratio is capped at a value of 0.85, the investment to output ratio is free to increase without bound, as in theory, unplanned investments in inventory should prevent over-investment. The short-term, actual growth rate of output $\left(\frac{\kappa(\pi_e,u)}{\nu} - \delta\right)$ signifies an increased reactivity of firms to the expected profit share. Consequently, the capital investment share of the economy ultimately oscillates between 0 and just over 0.4. Short-term bursts in profit will boost investment and heat up the economy, until the economy is performing well above capacity, at which point it will begin to cool down. The private debt ratio and inflation rate follow reasonable paths as well, with d finally cycling through values between roughly 0.32 and 0.44 (after 2200), and the inflation rate ultimately oscillating between positive values just above 0 and just below 0.06. As was the case in the original model presented in Grasselli and Grasselli and Nguyen-Huu (2018), the economy is performing well above capacity, with a limit cycle for u occurring way above 1.

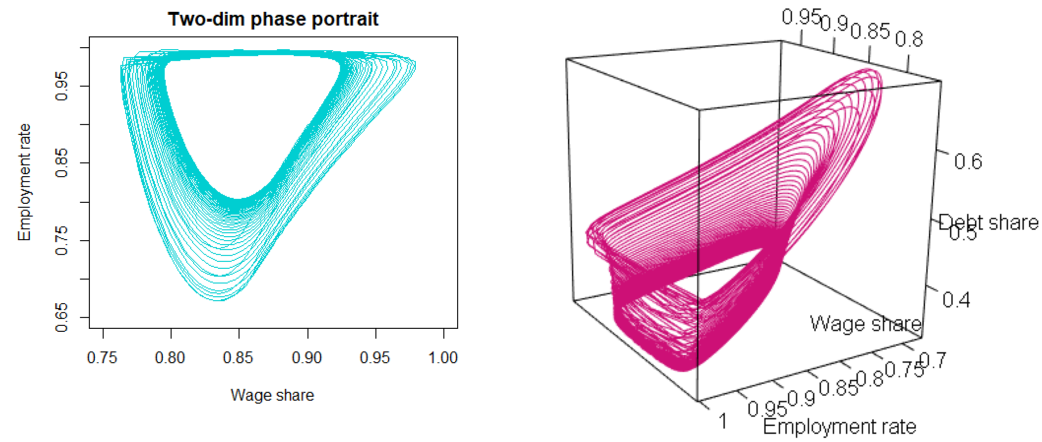
At this point, we understand the effects that the investment parameters or the interest rate have on the model, so it is instructive to learn about some of the new parameters. After some exploration, we can determine that the maximum share of consumption in the economy $c_{\max}$ and the utilization adjustment parameter η_u can potentially destabilize the model due to under-consumption or over-investment, respectively. For example, in Figure 5.3 (a), we observe that decreasing the maximum share of consumption to 0.7 (from 0.85 originally) results in a decrease in demand y_d. Since aggregate demand now lags well behind production, firms experience a collapse in their profit share. The economy contracts further due to under-investment (and the continuing under-consumption), which results in mass layoffs, and a vanishing employment rate. In Figure 5.3 (b) the increase in η_u from 0.2 to 1 causes initial over-investment, with a corresponding spike in demand. Production cannot keep up with demand, which eventually causes y_d to drop below 1 and stabilize. This results in a collapsing profit share, vanishing employment, and permanent economic recession.



(a) Economic ratios, with $c_{\max} = 0.7$. (b) Economic ratios, with $\eta_u = 1$.

Figure 5.3: Simulations of employment rate λ (blue), wage share ω (red), demand-to-output ratio $y_d = \frac{Y_d}{Y}$ (green), and profit share π_e (purple), for the Keen model with consumption and inventories. Bad runs due to initial under-consumption (a) or over-investment (b) are depicted. All simulations are run from 2020 to 2300.

Figure 5.4 displays the relationships between key economic variables in the model. The phase portraits presented in Figures 5.4 (a) and (b) correspond to the sample run presented above in Figure 5.1. The two-dimensional phase portrait plots the employment rate λ with respect to the wage share ω, whereas the three-dimensional phase portrait plots the employment rate λ, the wage share ω, and the private debt-to-output ratio d.



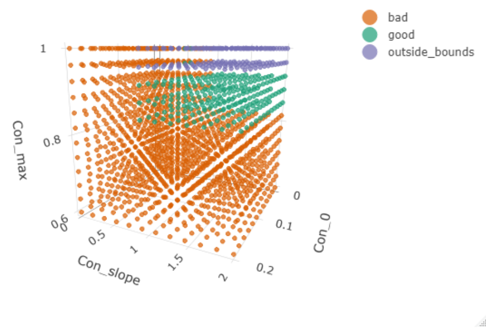
(a) Two-dimensional phase portrait.

(b) Three-dimensional phase portrait.

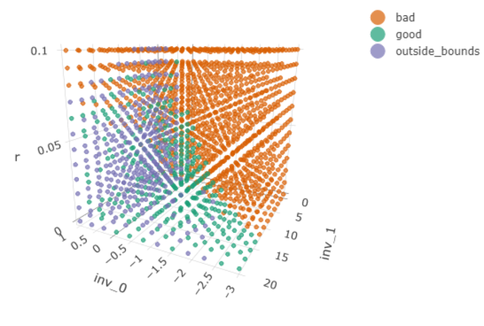
Figure 5.4: Phase portraits for the four-dimensional model with consumption and inventories.

5.3.1 Parameter Sensitivity

Once again, a deeper understanding of the model can be gained through an analysis of the sensitivity of key parameters and functions in the model. Following the same classification conditions as before, we vary the parameters for the linear consumption and investment functions. For the household consumption share of the economy, we vary the intercept of the linear function, denoted by c_0, according to $c_0 \in [0, 0.2]$, and the slope of the function, denoted by c_1, according to $c_1 \in [0, 2]$. The maximum share of consumption is varied according to $c_{\max} \in [0, 1]$. The sensitivity of two of the investment function parameters are analyzed as well, along with the constant interest rate r. The intercept parameter is varied according to $\text{inv}_0 \in [-3, 1]$, the slope parameter according to $\text{inv}_1 \in [0.05, 20]$, and the interest rate according to $r \in [0, 0.1]$. For each of these six parameters, 12 equally-spaced values are chosen within the specified intervals.



(a) Sensitivity of linear consumption function parameters.

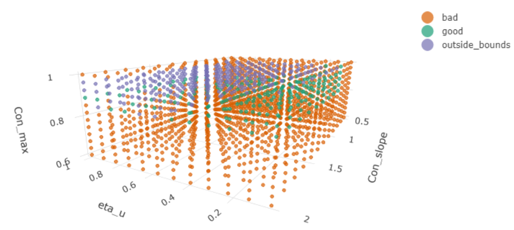


(b) Sensitivity of investment function parameters, with the interest rate.

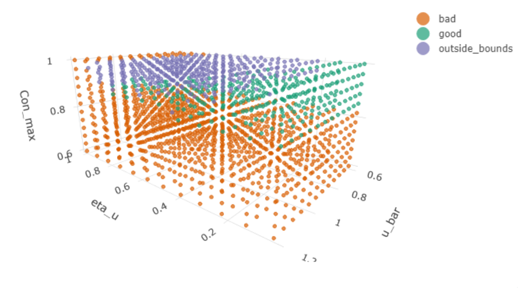
Figure 5.5: Parameter sensitivity for consumption function parameters (a) and investment function parameters (along with the interest rate) (b).

The corresponding sensitivity plots are presented in Figure 5.5. For the consumption sensitivity grid, the parameters `Con_0`, `Con_slope`, and `Con_max` in the figure correspond to c_0, c_1, and $c_{\max}$, respectively. The revised consumption model (and overall model) is considerably less sensitive to changes in parameter values, with a well-defined region of good simulations occurring for all tested values of $c_0 \in [0, 0.2]$, so long as c_1 is approximately greater than 0.9 (and less than the tested upper bound of 2), and more importantly so long as the upper bound of the share of household consumption is roughly between 0.85 and 0.93. This upper bound for $c_{\max}$ is necessary to prevent limit cycles for the demand to output ratio y_d from greatly surpassing 1, thus preventing the wage share from oscillating above 1 (resulting in the simulation obtaining an "outside_bounds" classification). The lower bound of the $c_{\max}$ interval ensures that demand is sufficiently high, thus ultimately preventing the economy from contracting, and wages from vanishing (i.e. a "bad" equilibrium).

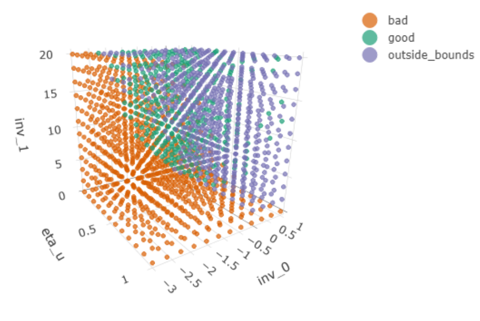
The investment function parameters can be interpreted more generally, as for each parameter that was considered, any of the values could potentially result in a good simulation. The associated caveat is that the investment function parameters and interest rate are highly dependent *on each other*. As the intercept inv_0 decreases/becomes more negative, the slope of the investment function inv_1 must increase, in order to maintain a sufficient level of investment necessary to prevent a long-term economic contraction. The opposite also holds true; an increasing intercept must be counteracted with a decreasing slope, in order to prevent excessive demand in the economy and a wage share that will ultimately exceed 1. As for the interest rate, greater r values generally correspond with lower values for inv_1 and higher values for inv_0.



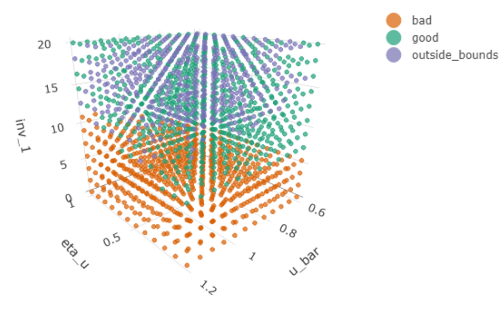
(a) Sensitivity of consumption function parameters c_1 (`Con_slope`) and $c_{\max}$ (`Con_max`), along with utilization parameter η_u.



(b) Sensitivity of consumption function parameter $c_{\max}$ (`Con_max`), along with utilization parameters η_u and $\bar{u}$.



(c) Sensitivity of investment parameters inv_0 and inv_1, along with utilization parameter η_u.



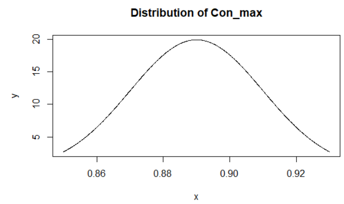
(d) Sensitivity of investment parameter inv_1, along with utilization parameters η_u and $\bar{u}$.

Figure 5.6: Parameter sensitivity for various consumption, investment, and utilization parameters, for the Keen model with consumption and inventories.

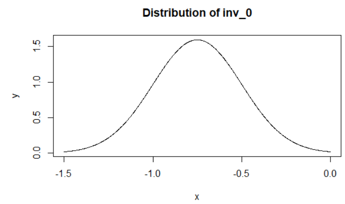
Moving on, we investigate the effects of capacity utilization in the model, most notably altering two parameters: the adjustment speed for investment as a function of utilization, denoted by η_u, and the baseline level of utilization, denoted by $\bar{u}$. In Figure 5.6, different parameters from the consumption and investment functions are varied along with the utilization parameters. The parameters from these functional forms are varied according to the specifications described earlier; in addition, we now vary η_u and $\bar{u}$ according to $\eta_u \in [0, 1]$ and $\bar{u} \in [0.6, 1.2]$, respectively, with 12 equally spaced points chosen within each of these intervals.

Figures 5.6 (a) and (b) illustrate the long-term relationship between consumption and utilization in the model. When we are permitted to alter the investment behaviour of firms through utilization, we observe that a larger range of $c_{\max} \in [0.85, 1]$ values produce good simulations than previously ($c_{\max} \in [0.85, 0.93]$). While the baseline utilization $\bar{u}$ does not seem to be very sensitive to changes in the designated interval, the adjustment speed parameter η_u exhibits much greater sensitivity, along with a corresponding inverse relationship with the maximum consumption share of the economy $c_{\max}$. Larger $c_{\max}$ values associated with "good" runs must be countered with smaller η_u values, in order to curb investment and prevent excess-demand, and keep the equilibrium wage share under 1. Conversely, smaller $c_{\max}$ require larger η_u values, in order to stimulate investment and demand, and to prevent the economy from contracting, and the wage share from ultimately collapsing.

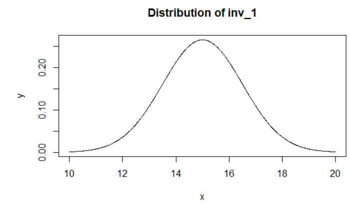
The relationship between the original investment function parameters and the new utilization parameters is also worth exploring. From Figures 5.6 (c) and (d), we see that the inverse relationship between the parameters inv_0 and inv_1 still holds. As for the relationship between the utilization and investment parameters, we notice that almost any value in the designated intervals of η_u and $\bar{u}$ produces a "good" simulation. In general, we note that larger values of η_u and $\bar{u}$ are associated with convergence to the good equilibrium only if they correspond to greater values of inv_1. This occurs in order to prevent the economy from producing too far above capacity, and to ultimately keep the equilibrium wage share under 1.



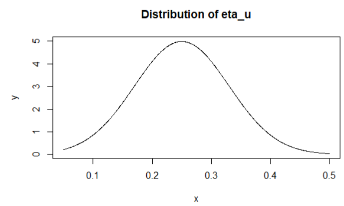
(a) $\mu = 0.89$, $\sigma = 0.02$



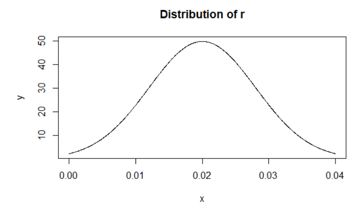
(b) $\mu = -0.75$, $\sigma = 0.25$



(c) $\mu = 15$, $\sigma = 1.5$



(d) $\mu = 0.25$, $\sigma = 0.08$



(e) $\mu = 0.02$, $\sigma = 0.008$

Figure 5.7: Probability density functions for parameters $c_{\max}$, inv_0, inv_1, η_u, and r. All five parameters are sampled from a normal distribution; consequently, normal density is plotted on the y-axis.

From the good regions of our sensitivity grids, we determine suitable probability density functions for $c_{\max}$, inv_0, inv_1, η_u, and r, each of which are plotted in Figure 5.7. The four parameters are all sampled from a normal distribution, with their means and values for standard deviation stated in the sub-captions of Figure 5.7. Our 200 Monte Carlo replications of the Keen model with explicit consumption and inventories are then examined, with all runs occurring from 2020 to 2100.

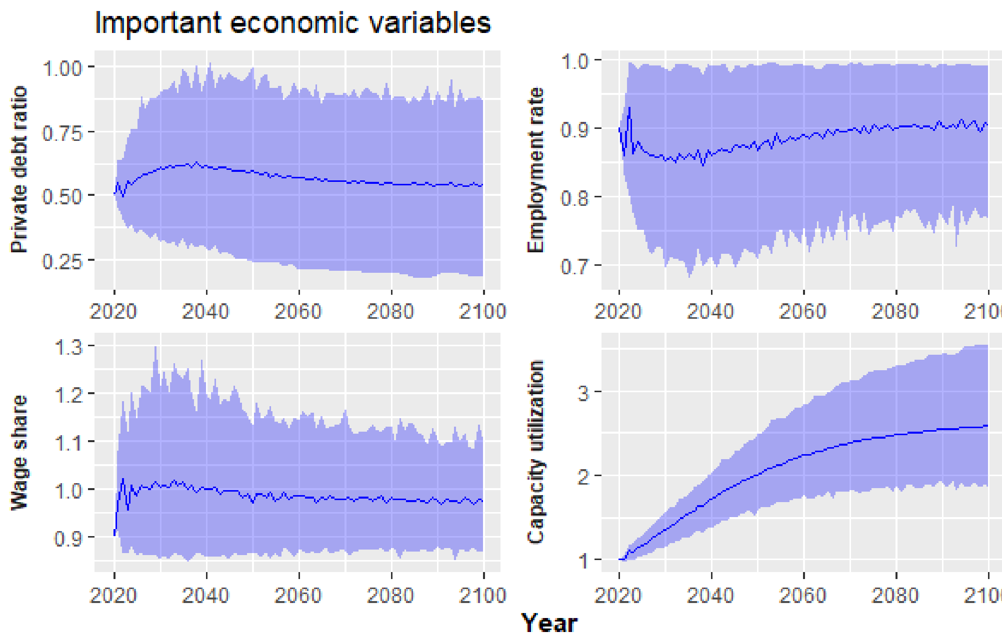


Figure 5.8: Monte Carlo simulations for the Keen model with consumption and inventories. Two hundred replications of the private debt-to-output ratio d, employment rate λ, wage share ω, and capacity utilization u are plotted, from 2020 to 2100. The shaded regions represent a [0.025; 0.975] (95%) probability interval for each variable, while the darker lines represent the median trajectories.

In Figure 5.8, we observe that the more restrictive values for the maximum consumption share of the economy $c_{\max}$ and the utilization parameter η_u prevent the previously discussed issues of over-demand due to over-consumption or over-investment (which would thus lead to a wage share that oscillates well above 1) or under-demand relative to supply, due to depressed capital investment (which would ultimately cause a massive economic contraction). Furthermore, the decision to restrict the short-term interest rate r to lower values staves off overindebtedness in the private sector. Firms will pay less interest on their debt, thus ensuring that even if private debt does slightly increase, the lower values for r will prevent corporate profits from further decreasing and destabilizing the economy. On the whole, we observe that all 200 replications produce final private debt ratios of $d < 2.7$ and final employment rates of $\lambda \in [0.4, 1]$. The issue of over-demand still persists, however, as the final value for the wage share could potentially be at or near the peak of its limit cycle, and above the maximum desired value of 1. Consequently, we have that only 133/200 final values of ω are in $[0.4, 1]$, with all of the problematic runs producing final wage share values above 1.

5.3.2 Basin of Attraction

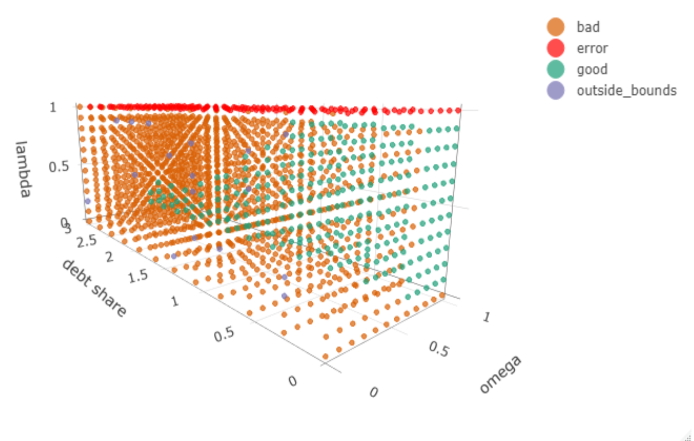


Figure 5.9: Basin of attraction for the Keen model with consumption and inventories, using the linear consumption function. We vary the initial conditions for employment rate λ, wage share ω, and private debt ratio d.

As in previous chapters, it is instructive to analyze the basin of attraction to the various equilibria, given perturbations in the initial conditions for the employment rate, wage share, and private debt-to-output ratio. Once again, we vary the initial wage share and employment rate according to ω, $\lambda \in [0, 1]$, and the initial private debt-to-output ratio according to $d \in [0, 3]$. The classification of simulations remains the same as before. Most notably, we observe that greater initial private debt-to-output ratios must be accompanied by lower initial wage shares, so as to maintain a stable profit share that does not collapse and become very negative.

5.4 Climate Model and Results

The climate module proceeds exactly as explained in Chapter 2. We summarize the changes to the model below, beginning with the newly created production variable Y_0, expressed by

$$Y_0 = \frac{Y_e}{(1 - D_Y)(1 - A)}. \tag{5.41}$$

This variable represents actual production, taking into account expected sales, abatement costs (which are planned by firms), and damages to output, which can be expected, to some degree. This determines labour according to

$$L = \frac{Y_0}{a} \tag{5.42}$$

and utilization according to

$$u = \frac{Y_0}{Y_{\max}}. \tag{5.43}$$

After accounting for abatement costs and damages, the output that is available to sell is given by

$$Y = (1 - D_Y)(1 - A)Y_0. \tag{5.44}$$

Profits and expected profits now consider the net transfer from the public to private sector:

$$\Pi_f = Y_n - \mathrm{w}L - rD + pNT \tag{5.45}$$

$$\Pi_e = pY_e - \mathrm{w}L - rD + pNT. \tag{5.46}$$

Lastly, damages to capital now factor into the capital dynamics:

$$\dot{K} = I_k - (\delta + D_K)K. \tag{5.47}$$

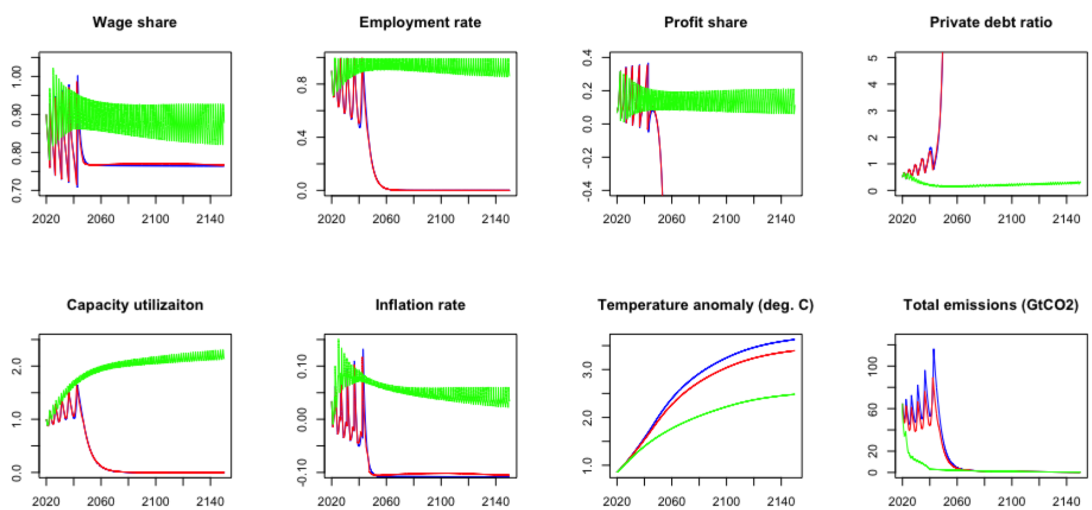


Figure 5.10: Climate scenario analysis for the Keen model with consumption and inventories, from 2020 to 2150. The blue curve represents the *Low Damages, Low Policy* scenario, the red curve represents the *High Damages, Medium Policy* scenario, and the green curve represents the *High Damages, Strong Policy* scenario. The damages to capital parameter is set at $f_k = 0.1$.

Figure 5.10 plots the results for the basic scenario analysis explained in Chapter 2. Holding all parameters from the economic module equal, and setting the damages to capital f_K equal to 0.1, we observe that the strongest policy scenario performs considerably better than the *Low Damages* scenario, and appreciably better than the *Medium Policy* scenario. The weaker policy scenarios are characterized by a deflationary economy that is performing well below capacity, featuring a collapsing employment rate, and an exploding private debt-to-output ratio. The elimination of the government sector and the monetary policy regime produce quicker economic collapses, as there are no mechanisms to counter drastic decreases in capital investment.

Alternatively, the introduction of a disequilibrium in the goods market enables demand to either outpace or lag behind production. When wages are higher, and households thus have more money to consume goods, the level of consumption is higher, with the corresponding decrease in corporate profits leading to a decrease in investment. This results in a decrease in production and thus a corresponding decrease in emissions. This is temporary, of course, as lower profits will ultimately result in fewer workers, which will drive the wage share and consumption down, and result in increased profits and production for firms. Thus, the rejection of Say's law (aggregate demand must equal aggregate supply) and the subsequent explicit modelling of consumption will result in oscillatory behaviour that could conceivably delay an economic contraction for a couple of decades (provided interest rates are sufficiently low).

Once again, the *Strong Policy* scenario generates higher employment rates, a much lower private debt ratio, and a stable profit share. To conclude, we observe that while the model does not appear to be as stable as the previous model with government and a monetary policy, we manage to avoid a long-term economic contraction in the strongest policy scenario. This likely occurs due to the adoption of independent consumption, which allows production to decrease at times, thus slowing down the accumulation of private debt, and decreasing the short-term growth rate (relative to the long-term equilibrium rate).

Chapter 6

Bringing it all together

6.1 Economic Module

6.1.1 Accounting Framework

Our work culminates in this final modelling chapter, where we aim to combine our previous work into a larger, cohesive model. Most notably, we blend the work of Chapter 4, which analyzed the Keen model with a public sector and monetary policy, with Chapter 5, which discussed the Keen model with consumption and inventories. The economy thus contains all five sectors presented in Chapter 4 (households, firms, banks, the government, and a central bank), but assumes an additional disequilibrium in the goods market, which creates unplanned investment in inventory stock. This disequilibrium also leads to the explicit modelling of consumption, as was explained in Chapter 5.

The balance sheet for the economy (see Table 6.1) thus remains very similar to the one presented in Chapter 4, with firms now holding inventories as assets (as in

	Households	Firms	Banks	Public Sector	Central Bank	Sum
Balance Sheet						
Capital Stock		pK				pK
Inventories		cV				cV
Deposits	M_h	M_f	$-M$			0
Loans		$-L_f$	L_f			0
Bonds			B_b	$-B$	B_{cb}	0
Reserves			R		$-R$	0
Sum (net worth)	X_h	X_f	X_b	X_g	0	X

Table 6.1: Balance sheet for complete economy with inventories, a public sector, and a monetary policy conducted by the central bank.

Chapter 5). The only other accounting changes are shown in the transaction flow matrix (Table 6.2), where consumption is no longer treated as an accommodating variable, and is instead modelled explicitly, with only households consuming (but not banks). We make the minor assumption that some fixed fraction of profits Π_f will be paid to households as dividends Π_d, thus slightly increasing household wealth, while reducing the retained earnings of firms. We also eliminate the concept of bank dividends, as a means of simplifying the household sector.

The remaining sectors are untouched from Chapter 4. Once again, we propose that the central bank holds government bonds B_{cb} as assets, and reserves R as liabilities. We assume that the central bank has net worth equal to zero, and that as a result, $B_{cb} = R$. The public sector completes the economy, and since it has no assets, and only sells bonds to banks and the central bank as short-term liabilities, its balance sheet consists solely of government debt, financed through bonds, denoted as B. Interest is earned on each of the monetary items on the balance sheet, except for reserves.

	Households	Firms		Banks	Public Sector	Central Bank	Sum
Transactions		Current	Capital				
Consumption	$-pC$	pC					0
Investment		pI_k	$-pI_k$				0
Govt. Spend.		pG			$-pG$		0
[GDP]		$[pY]$					$[pY]$
Wages	W	$-W$					0
Capital depr.		$-\delta pK$	δpK				0
Non-carbon taxes	$-T_h$	$-T_f$		$-T_b$	T		0
CB profits					Π_{cb}	$-\Pi_{cb}$	0
Firm dividends	Π_d	$-\Pi_d$					0
Bank dividends	Δ_b			$-\Delta_b$			0
Int. on loans		$-r_L L_f$		$r_L L_f$			0
Int. on deposits	$r_M M_h$	$r_M M_f$		$-r_M M$			0
Int. on bonds				$r_B B_b$	$-r_B B$	$r_B B_{cb}$	0
Sum (balance)	S_h	Π_r	$-pI_k + \delta pK$	S_b	S_g	0	0

Table 6.2: Transactions in complete economy with consumption, inventories, and a public sector, with a monetary policy conducted by the central bank.

6.1.2 Changes to Model

There are very few changes from the relevant previous chapters. Equations for output and inventories are identical to the ones presented in Chapter 5, with the exception that demand Y_d and nominal output Y_n are now given by

$$Y_d = C + I_k + \frac{G}{p} \tag{6.1}$$

$$Y_n = pC + pI_k + G + c\dot{V} = pY_d + c\dot{V} \tag{6.2}$$

Turning our attention to prices and profits, all existing equations remain the same. We introduce dividend (resp. expected dividend) payments to households (from firms), denoted by Π_d and expressed as a fixed fraction of total profits Π_f (resp. expected

	Households	Firms	Banks	Public Sector	Central Bank	Sum
Balance Sheet						
Change in capital stock		$p\dot{K}$				$p\dot{K}$
Change in deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$			0
Change in loans		$-\dot{L}_f$	$\dot{L}_f$			0
Change in bills			$\dot{B}_b$	$-\dot{B}$	$\dot{B}_{cb}$	0
Change in reserves			$\dot{R}$		$-\dot{R}$	0
Sum (savings)	S_h	Π_r	S_b	S_g	0	0
Change in net worth	$\dot{X}_h = S_h$	$\dot{X}_f = \Pi_r$	$\dot{X}_b = S_b$	$\dot{X}_g = -\dot{B}$	$\dot{X}_{cb} = 0$	$\dot{X}$

Table 6.3: Flow of funds in economy with inventories, a public sector, and a monetary policy regime conducted by the central bank.

profits Π_e):

$$\Pi_d = f_d \Pi_f \tag{6.3}$$

$$\Pi_{de} = f_d \Pi_e \tag{6.4}$$

Consequently, the retained earnings (resp. expected retained earnings) of firms are expressed as

$$\Pi_r = (1 - f_d)(1 - \tau)\Pi_f \tag{6.5}$$

$$\Pi_{re} = (1 - f_d)(1 - \tau)\Pi_e \tag{6.6}$$

The financing of firms follows from Chapter 5, with corporate deposits now dependent on the level of retained earnings, thus evolving according to

$$\dot{M}_f = \Pi_r, \tag{6.7}$$

with the evolution of private debt now following

$$\dot{D} = pI_k + c\dot{V} - \Pi_r. \tag{6.8}$$

The public sector and central bank are identical in structure to Chapter 4. The banking sector is very similar, with the sole exception of bank dividends, which are now set as equal to the after-tax profits of banks:

$$\Delta_b = (1-\tau)\Pi_b \tag{6.9}$$

This leads to a small change in the dynamics for government bonds held by banks:

$$\dot{B}_b = \Pi_b - \Delta_b + \dot{M} - \dot{R} - \dot{L}_f = \tau\Pi_b + (1-f_M)\dot{M} - \dot{L}_f \tag{6.10}$$

Turning our attention to households, we now introduce total household income Π_h, represented as the sum of the wage bill, interest earned on household deposits, bank dividend payments, and expected dividends earned from firms:

$$\Pi_h = \mathrm{w}L + r_M M_h + \Pi_{de} + \Delta_b \tag{6.11}$$

The household profit share excludes dividends for simplicity, since its inclusion results in complicated reduced dynamics that depend on firms and banks. It is thus expressed by

$$\pi_h = \frac{\Pi_h - \Pi_{de} - \Delta_b}{pY}. \tag{6.12}$$

Households then use their total income (minus expected corporate dividends) to determine consumption decisions, with the consumption share of the economy now dependent on the household share π_h, instead of the wage share ω:

$$C = \theta(\pi_h)Y. \tag{6.13}$$

Finally, household savings (equal to the evolution of deposits $\dot{M}_h$, since households hold no other assets) is expressed as the difference between after-tax household income and nominal consumption:

$$S_h = (1-\tau)\Pi_h - pC. \tag{6.14}$$

All behavioural equations remain the same as before, with the rational function $\Phi(\lambda)$ employed to determine wages, and the linear function $\kappa(\pi_e, u)$ used to determine investment. The consumption function $\theta(\pi_h)$ now depends on the household income share, but it still follows the linear form introduced in the previous chapter.

6.2 Main Dynamical System

The reduced form equations for the wage share ω, employment rate λ, and capacity utilization u follow the same expressions as in Chapter 5, while the evolution of the policy rate r_B is identical to the equation presented in Chapter 4. The explicit modelling of corporate loans and deposits require two new differential equations, one for the private loan-to-output ratio ℓ_f, and a second for the private deposit-to-output ratio m_f. We can obtain the former using the expression $\ell_f = L_f/(pY)$, and the dynamics for corporate loans presented in Chapter 5 (i.e. $\dot{L}_f = pI_k + c\dot{V}$):

$$\begin{aligned}
\frac{\dot{\ell}_f}{\ell_f} &= \frac{\dot{L}_f}{L_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} \\
&= \left[\frac{pI_k + c\dot{V}}{L_f}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{p\kappa(\pi_e, u)Y + p\omega(Y - Y_d)}{\ell_f pY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{p\kappa(\pi_e, u)Y - p\omega Y - p\omega\left(C + I_k + \frac{G}{p}\right)}{\ell_f pY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{p\kappa(\pi_e, u)Y - p\omega Y - p\omega C - p\omega I_k - \omega G}{\ell_f pY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{pY(\kappa(\pi_e, u) - \omega - \omega\theta(\pi_h) - \omega\kappa(\pi_e, u) - \omega\psi_g)}{\ell_f pY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{\kappa(\pi_e, u) - \omega(1 + \theta(\pi_h) + \kappa(\pi_e, u) + \psi_g)}{\ell_f}\right] - i(\omega, y_d) - g(\pi_e, u, y_d)
\end{aligned} \tag{6.15}$$

Similarly, we can use equation (6.7) and the expression $m_f = M_f/(pY)$ to derive the expression for the evolution of the private deposit-to-output ratio.

$$
\begin{aligned}
\frac{\dot{m}_f}{m_f} = \frac{\dot{M}_f}{M_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} &= \left[\frac{\Pi_r}{M_f}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{(1-f_d)(1-\tau)\Pi_f}{m_f pY}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \\
&= \left[\frac{(1-f_d)(1-\tau)\pi_f}{m_f}\right] - i(\omega, y_d) - g(\pi_e, u, y_d) \qquad (6.16)
\end{aligned}
$$

Therefore, we obtain the following six-dimensional, reduced-form dynamical system for the Keen model with explicit consumption, inventories, a public sector, and monetary policy (conducted by the central bank):

$$
\begin{cases}
\dot{\omega} &= \omega\left[\Phi(\lambda) - \alpha - (1-\gamma)i(\omega, y_d)\right] \\
\dot{\lambda} &= \lambda\left[g(\pi_e, u, y_d) - \alpha - \beta\right] \\
\dot{\ell}_f &= \kappa(\pi_e, u) - \omega(1 + \theta(\pi_h) + \kappa(\pi_e, u) + \psi_g) - \ell_f[i(\omega, y_d) + g(\pi_e, u, y_d)] \\
\dot{m}_f &= (1-f_d)(1-\tau)\pi_f - m_f[i(\omega, y_d) + g(\pi_e, u, y_d)] \\
\dot{u} &= u\left[\eta_e(y_d - 1)\right] \\
\dot{r}_B &= \beta_{rB}[g(\pi_e, u, y_d) - (\alpha + \beta)]
\end{cases} \qquad (6.17)
$$

The main findings from earlier chapters still hold, even as the dimensionality of the reduced-form system increases. An increase in taxes reduces the private deposit-to-output ratio m_f, which in turn causes the private debt share d to rise. Drastic increases in government spending, brought about when ψ_g is raised dramatically, reduce the maximum economic share of capital investment $\kappa(\pi_e, u)$, thus causing a long-term reduction in capital, and an ensuing economic collapse. Increases in the policy rate adjustment parameter β_{rB} have the effect of cooling down the economy, but increases past a certain threshold can shift the economy into a deflationary regime, in which the short-term, actual growth rate of output remains considerably below the long-term equilibrium growth rate. Increases in household consumption will result in a greater demand-to-output ratio y_d, thus reducing the inventory stock of firms, and the private loan-to-output ratio, as well. The one new parameter in the reduced-form system is f_d, which dictates the level of private profits paid out to households in dividends. It simply raises the private debt ratio d, by decreasing the private deposit-to-output ratio m_f.

Turning our attention to the secondary System 6.18, we observe very similar equations to those presented in Chapter 4. The variable π_b corresponds to the profit share for banks, expressed by $\pi_b = \Pi_b/(pY)$. We once again observe that increases in the policy rate lead to a greater amount of bill holdings by banks, as the amount of public bonds issued rises in order for the government to finance its larger public debt. The bond market is also influenced by the central bank and its ability to increase the reserve ratio f_M. An increase in f_M will cause the amount of government bonds held by the central bank to rise, which in turn leads to a corresponding decrease in the amount of government bonds held by banks. The household deposit-to-output ratio m_h can surge due to increases in the wage share, expected dividend to output ratio (denoted by $f_d\pi_e$), profit share for banks, or deposit rate r_M. It decreases as a result of a larger consumption share $\theta(\pi_h)$ in the economy.

$$\begin{cases} \dot{b} &= (\psi_g - \tau(\omega + \pi_f + \pi_b) + r_B b_b) - b[i(\omega) - g(\pi)] \\ \dot{b_b} &= (1 - f_M)(m_f + m_h) + \pi_b - \ell_f - b_b[i(\omega) - g(\pi_f)] \\ \dot{m_h} &= \omega + f_d \pi_e + (1 - \tau)\pi_b - \theta(\pi_h) + m_h(r_M - i(\omega) - g(\pi_f)) \end{cases} \tag{6.18}$$

The theoretical work for a model of this size is difficult to perform, often yielding results that are not easy to comprehend. The work done in previous chapters can be reproduced to solve for the equilibrium points and the Jacobian, but we will forego this in the interest of brevity. Since our final model represents the culmination of the previous models we have studied, and since we have shown in this current section that very similar trends and relationships can be observed, we will now turn to the numerical results for this chapter, in order to examine the effects of these findings in a larger, more comprehensive model.

6.3 Results

Observing a basic calibrated run in Figure 6.1, we first notice that the limit cycles that were discovered in Chapter 5 persist. The undamped cycles appear to originate from the disequilibrium in the goods market, which allows for explicit modelling of consumption. An increase in demand (through consumption) will lead to a subsequent increase in the profit share π_f, which causes the economy to heat up as back-orders are filled, leading to an increase in capital investment, and a decrease in inventories. The short-term growth rate rises as capital increases, leading to an increase in employment. As employment gets closer to 1, wages increase dramatically, leading to a reduction in profits, capital investment, and demand. This reduces production and sales (aggregate supply), and results in lower employment. Consequently, the wage share decreases, consumption decreases, and demand begins to fall. As wages fall, the profit share increases, leading once again to an increase in capital investment. This causes demand to rise once again, perpetuating the described cycle.

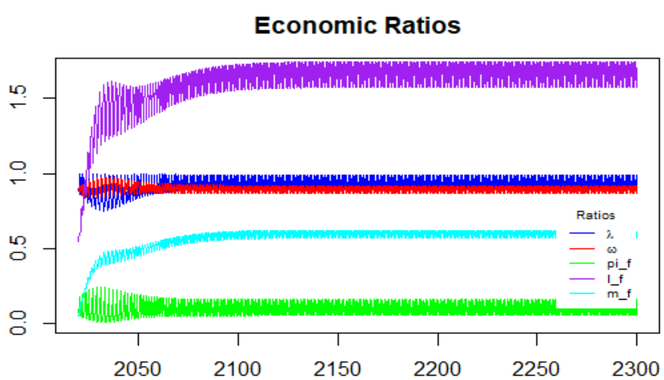


Figure 6.1: Simulations of employment rate λ (blue), wage share ω (red), profit share π_f (green), private loan-to-output ratio ℓ_f (purple), and private deposit-to-output ratio m_f (cyan), for the model presented in Section 6.1. All simulations are run from 2020 to 2300.

In Figure 6.1, the wage share ultimately oscillates between values of 0.87 and 0.92 or so, while the employment rate ultimately cycles through values just over 0.9 and just under 1. The private debt ratio cycles between approximately 1 and 1.12, after 2100. We note that the introduction of the public sector and monetary policy stabilize the consumption and inventories model presented in Chapter 5. The amplitude of the oscillations has decreased, with smaller economic recessions in the recession-expansion cycle.

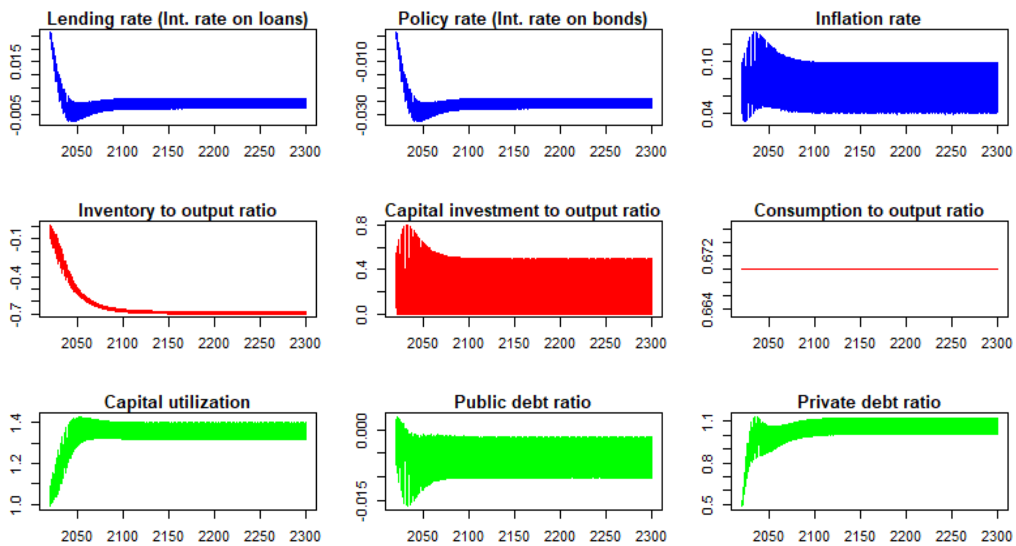
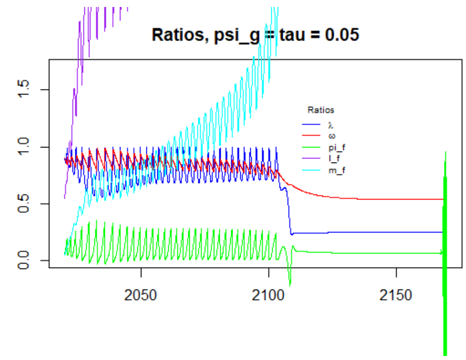
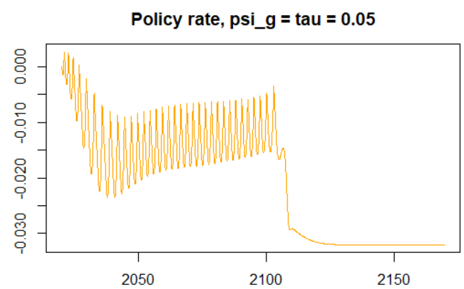


Figure 6.2: Simulations of the lending rate r_L, policy rate r_B, inflation rate i, inventory-to-output ratio v, capital investment to output ratio $\kappa(\pi_e, u)$, consumption to output ratio $\theta(\omega)$, capacity utilization u, public debt ratio b, and private debt ratio d. All simulations are run from 2020 to 2300.

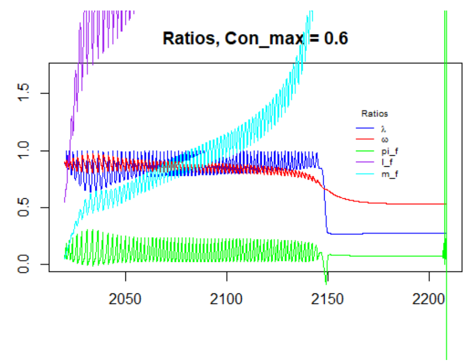
Observing Figure 6.2, we observe an economy in which demand always outpaces supply, as there is always a supply of back-orders waiting to be filled (observe the inventory to output ratio). As a result, the economy is always performing above capacity, as evidenced by $u > 1$ for the entirety of the simulation. The ratio of real capital investment to output can get alarmingly high or low, due to the increased reactivity of firms to the expected profit share, but perhaps the most interesting feature of this model is the policy rate r_B. It oscillates below zero almost immediately, cycling through values around -3 percent.



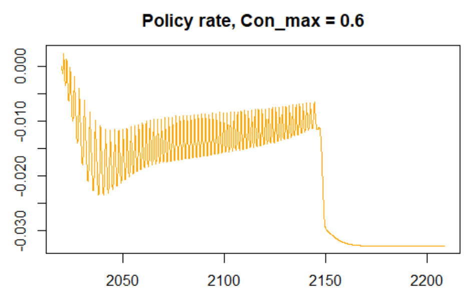
(a) Plots of key economic ratios, with $\tau = \psi_g = 0.05$.



(b) Policy rate, with $\tau = \psi_g = 0.05$.



(c) Ratios, for $c_{\max} = 0.6$.



(d) Policy rate, for $c_{\max} = 0.6$.

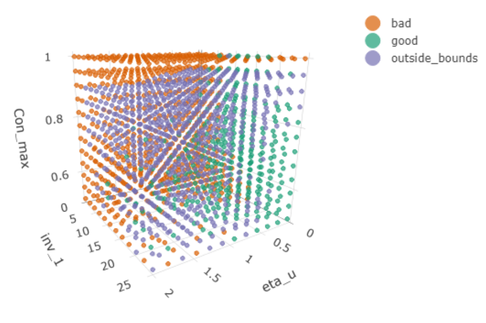
Figure 6.3: Simulations of the employment rate λ (blue), wage share ω (red), profit share π_f (green), private loan-to-output ratio ℓ_f (purple), and private deposit-to-output ratio m_f (cyan), for the model presented in Section 6.1. Plots (a) and (b) decrease the tax rate and government spending parameter to 0.05, from 0.15, while (c) and (d) set $\tau = \psi_g = 0.15$ again, but decrease the maximum share of consumption in the economy $c_{\max}$ to 0.6, from 0.67.

Figure 6.2 shows that the cyclical negative policy rate in our model occurs after a sharp decline in the first 30 years. The steady decrease in r_B implies under-investment, leading to a short-term growth rate of output that lags behind its long-term counterpart. Attempts to spur investment through decreasing government spending and consumption appear to be unsuccessful, as observed in Figure 6.3. By decreasing ψ_g or $c_{\max}$, we allow for greater capital investment, and as soon as the actual growth rate of output begins to exceed the long-term equilibrium growth rate, we observe an increase in the policy rate (see Figures 6.3 (b) and (d)). An increase in r_B, however, is accompanied by an increase in the short-term lending and deposit rates, as well. The combination of an increase in investment and an increase in the lending rate leads to a declining profit share, and a skyrocketing private debt-to-output ratio. As corporate profits decline, the growth rate of output begins to fall, culminating in the economic crash observed in Figures 6.3 (a) and (c).

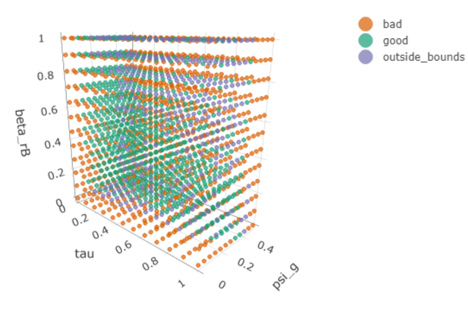
As Grasselli and Lipton (2019b) pointed out, a negative policy rate is not absurd by any means. They argue that a negative policy rate can be a stabilizing economic force, provided that r_B is permitted to become sufficiently negative whenever the short-term growth rate of output declines below the long-term equilibrium rate. Lower interest rates prevent firms from becoming over-indebted, thus enabling an increase in investment, and theoretically, an increase in economic growth. Contrary to Grasselli and Lipton (2019b), we observe a model that does not rebound after a negative policy rate regime, but instead remains in a permanent state of reduced economic growth, with any further attempts to increase investment leading to extreme overindebtedness. More research might be required to further examine such an economic situation, and the feasibility of a negative interest rate policy in the context of our model.

6.3.1 Parameter Sensitivity

Many of the relationships between parameters that were explored in the sensitivity grids of previous chapters still hold, but it is nonetheless useful to observe the interplay between key parameters in a more comprehensive economic model. In Figure 6.4 (a), we observe a sensitivity grid in which the maximum consumption share of the economy is varied according to $c_{\max} \in [0.5, 1]$, the investment function slope is varied according to $\text{inv}_1 \in [0.05, 25]$, and the utilization adjustment parameter (in the investment function) is varied according to $\eta_u \in [0.05, 2]$. Twelve points are chosen within each interval. We observe that higher values of $c_{\max}$ can only result in convergence to the good equilibrium when accompanied by lower values for inv_1 and η_u. This is unsurprising, as greater levels of consumption must be accompanied by lower levels of investment, lest the amount of aggregate demand greatly exceed the level of aggregate supply, forcing the wage share to equilibrate above 1.



(a) Sensitivity plot of $c_{\max}$, inv_1, and η_u.

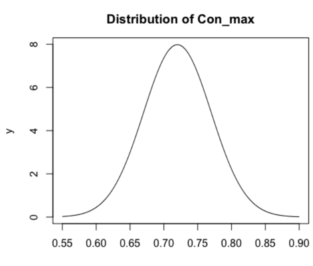


(b) Sensitivity plot of ψ_g, τ, and β_{rB}.

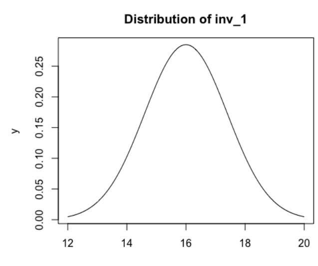
Figure 6.4: Plot (a) depicts a sensitivity grid for the maximum share of output allocated for consumption $c_{\max}$, the investment parameter inv_1, and the utilization parameter η_u. Plot (b) illustrates a sensitivity of grid of the government spending parameter ψ_g, the tax rate τ, and the policy rate adjustment parameter β_{rB}.

Figure 6.4 (b) varies the policy rate adjustment parameter β_{rB} and the tax rate

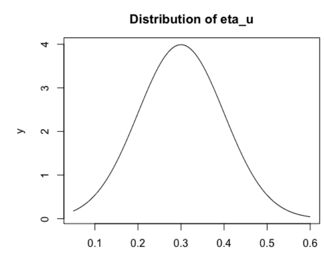
τ from 0 to 1, and the government expenditures parameter ψ_g from 0 to 0.5. Due to the permanently suppressed growth path in this particular economic model, both β_{rB} and τ can potentially take on any value in the grid, as a reduction in the policy rate enables further investment (thus preventing a collapse from under-investment), and the increase in private indebtedness that arises from an increase in τ is not sufficient to cause an economic collapse. The most sensitive parameter of the three is ψ_g; increasing government expenditures past a certain level (approximately $\psi_g = 0.4$) greatly reduces investment, causing an extreme reduction in capital investment, and an eventual economic collapse.



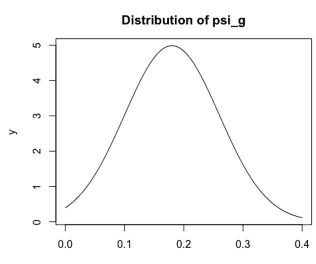
(a) $\mu = 0.72$, $\sigma = 0.05$



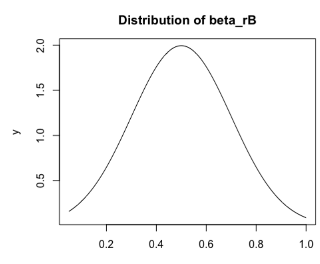
(b) $\mu = 16$, $\sigma = 1.4$



(c) $\mu = 0.3$, $\sigma = 0.1$



(d) $\mu = 0.18$, $\sigma = 0.08$



(e) $\mu = 0.2$, $\sigma = 0.1$

Figure 6.5: Probability density functions for parameters $c_{\max}$, inv_1, η_u, ψ_g, and β_{rB}. All five parameters are sampled from a normal distribution; consequently, normal density is plotted on the y-axis.

From the good regions of our sensitivity grids, we then determine suitable probability density functions for $c_{\max}$, inv_0, inv_1, η_u, ψ_g, τ and β_{rB}, each of which are plotted in Figure 6.5, with the exceptions of inv_0 and τ, which follow the distributions outlined in Chapter 5 and Chapter 4, respectively. The four parameters are all sampled from a normal distribution, with their means and values for standard deviation stated in the sub-captions of Figure 6.5. For reference, the mean and standard deviation for the inv_0 probability density function are given by $\mu = -0.75$, $\sigma = 0.25$, while the mean and standard deviation for the τ probability density function are denoted by $\mu = 0.5$, $\sigma = 0.2$. Our 200 Monte Carlo replications of the final model are then examined, with all runs occurring from 2020 to 2100.

In Figure 6.6, we observe similar results to the Monte Carlo plot of Chapter 5, in that the more restrictive values for $c_{\max}$ and η_u prevent the model from collapsing due to under-investment, while also preventing the wage share ω from exhibiting limit cycles above 1 due to over-demand. We observe very similar results to the previous chapter, with 189/200 replications producing final private debt ratios of $d < 2.7$ and 197/200 replications producing final employment rates of $\lambda \in [0.4, 1]$. The issue of over-demand is still present, however, as only 123/200 final values of ω are in $[0.4, 1]$.

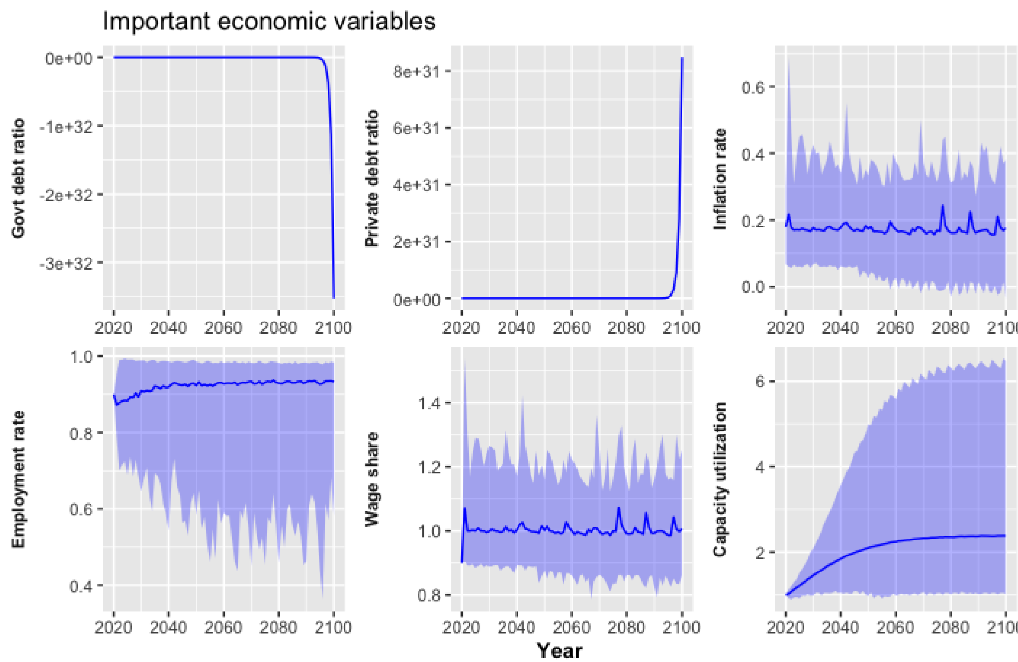


Figure 6.6: Monte Carlo simulations for the Keen model with consumption, a public sector, and a central bank. Two hundred replications of the public debt-to-output ratio b, private debt-to-output ratio d, inflation rate i, employment rate λ, wage share ω, and capacity utilization u are plotted, from 2020 to 2100. The shaded regions represent a [0.025; 0.975] (95%) probability interval for each variable, while the darker lines represent the median trajectories.

6.3.2 Basin of Attraction

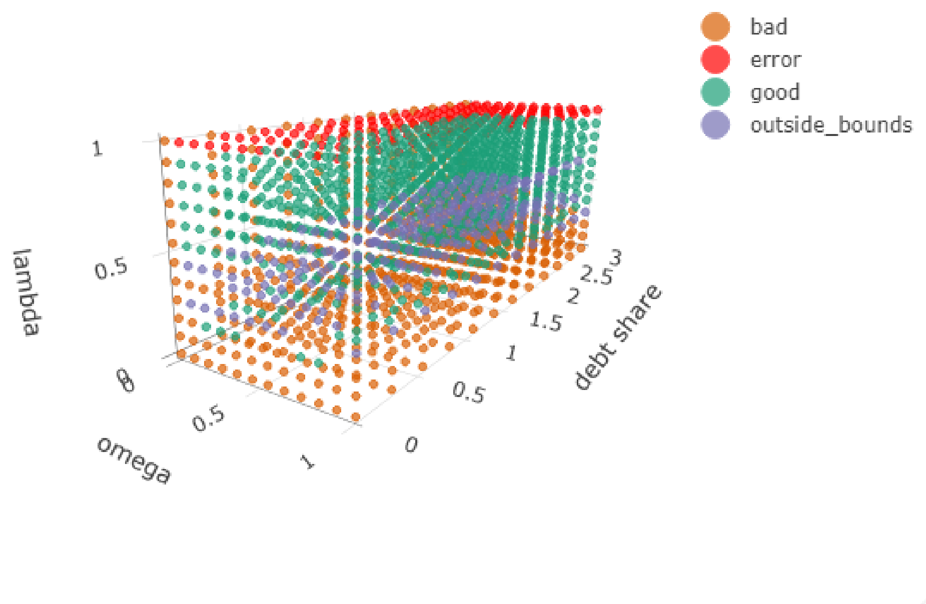


Figure 6.7: Basin of attraction, varying the initial conditions for wage share ω, employment rate λ, and private debt ratio d.

The basin of attraction for the final model is displayed in Figure 6.7, where the initial conditions for the employment rate, wage share, and private debt-to-output ratio are varied once again according to $\omega \in [0, 1]$, $\lambda \in [0, 1]$, and $d \in [0, 3]$, respectively. The negative policy rate regime allows simulations with higher initial private debt ratios or lower initial employment rates to converge to a limit cycle in the "good" region. Since firms do not need to pay higher interest rates on their debt, they can afford to retain more profit and thus increase capital investment, which in turn prevents mass layoffs, a decrease in demand, and an overall economic contraction.

6.4 Climate Results

The incorporation of the climate module causes the household and private sector to change in line with the modifications made in Chapter 5, Section 4, and the public sector to change in line with the modifications made in Chapter 4, Section 4. Figure 6.8 plots the results for the basic scenario analysis explained in Chapter 2. Holding all parameters from the economic module equal, and setting the damages to capital f_K equal to 0.1, we again observe that the strongest policy scenario performs considerably better than *Medium Policy* scenario, and about as well as the Low Damages scenario. All three scenarios, however, exhibit declining employment rates, rising private debt shares, and a temperature anomaly that surpass the threshold of +2°C.

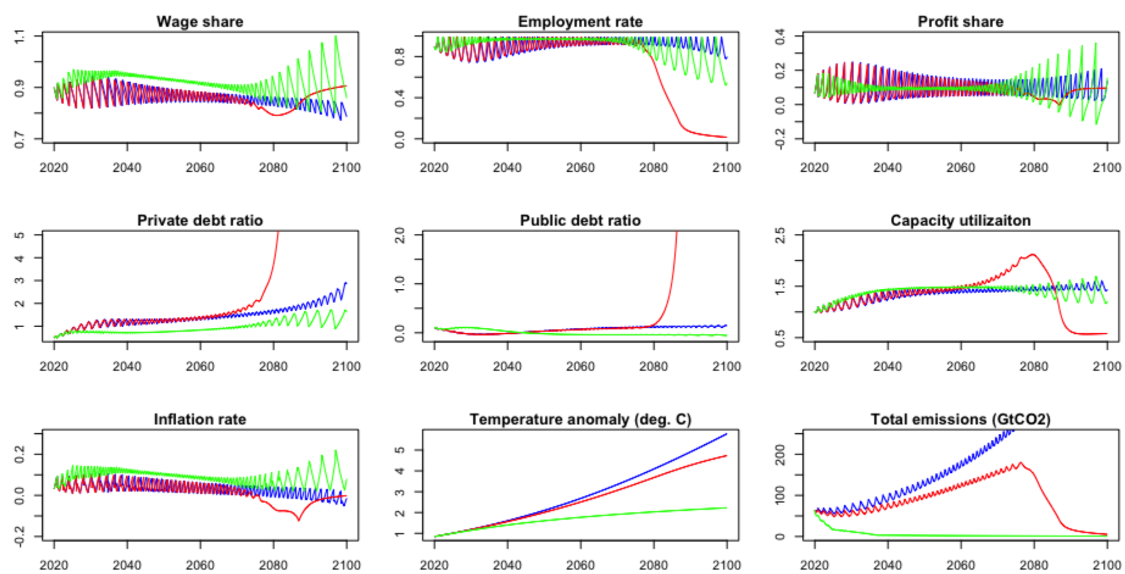


Figure 6.8: Climate scenario analysis for the model presented in Section 6.1.2, from 2020 to 2100. The blue curve represents the *Low Damages, Low Policy* scenario, the red curve represents the *High Damages, Medium Policy* scenario, and the green curve represents the *High Damages, Strong Policy* scenario. The damages to capital parameter is set at $f_k = 0.1$.

We observe once again the related issues of short-term stability and long-term collapse. The monetary policy produces better economic trajectories for the weaker policy scenarios, relative to Chapter 5. However, it also leads to an increase in investment and a decrease in interest rates, all while the private debt share is increasing, and the economy is contracting. In the case of the *Strong Policy* scenario, demand is driven up via an increasing wage share, which decreases the profit share, and further increases the private debt ratio. This further decreases production, which results in a declining employment rate, culminating in an economic collapse after 2100 (not pictured, as the collapse caused the integrator to fail). In order to alleviate the issue of an expansionary monetary policy leading to over-investment and an eventual climate-induced economic depression, the disaggregation of conventional and green capital should be explored, as in Dafermos *et al.* (2017). Adopting their framework to our continuous-time model (which already includes explicit consumption, inventories, a public sector, and a central bank) is left as an intriguing avenue for future research.

Chapter 7

Concluding Remarks and Next Steps

This thesis aims to create a more complete, realistic framework for a macroeconomic system, to determine potential drivers and causes of economic instability, and to identify policy regimes that are capable of reducing overindebtedness in the private sector.

In Chapter 2, we observe the base model presented in Grasselli and Nguyen-Huu (2015): the macro-economy has no government or central bank, and is thus susceptible to economic collapse due to under-investment, or due to excessively high lending rates. In both cases, the private sector suffers from a rapid explosion in private debt, which in turn reduces the profit share of firms and further lowers capital investment. The economy ultimately suffers a permanent contraction, with mass worker layoffs and suppressed economic growth. The addition of the public sector in Chapter 3 does not fundamentally alter the base model, and produces very similar results. Therefore, its inclusion is determined to not have destabilizing effects on the macro-economy. From our sensitivity analysis and Monte Carlo plots, we ascertain that modest increases in taxes or government spending cool down the economy by limiting investment. Consequently, we observe that considerably lower interest rates must be adapted in order to prevent the private sector from becoming overindebted.

The principal finding from our thesis occurs in Chapter 4, in which an active central bank is introduced to the existing framework from Chapter 3. The central bank pursues a monetary policy by lending reserves to banks, which in turn determines the amount of government bonds purchased by banks through open market operations. More importantly, the central bank sets a policy rate, from which the lending and deposit rate are developed. By observing the short-term growth rate of output and comparing it to its long-term counterpart, the central bank can lower the policy rate in order to increase the profit share of firms and spur economic growth. The necessity of an expansionary monetary policy for economic stability is observed in our sensitivity analysis, where we also observe the additional benefit of higher taxation capabilities that can be used to fund social programs and government projects.

We remove the government and central bank in Chapter 5, instead choosing to focus on the household and firm sectors. We modify the work of Grasselli and Nguyen-Huu (2018), but retain the central assumption that aggregate demand need not be equal to aggregate supply. Our sensitivity analysis establishes new stability conditions regarding consumption and nominal demand. The maximum share of consumption in the economy is required to be sufficiently high, such that production does not greatly outpace demand. If this happens, and the economy performs well below capacity, then the profit share of firms will decrease, resulting in a vanishing employment rate and a permanently contracted economy. The stable region expands in Chapter 6, when we re-introduce our monetary policy regime to an economic arrangement that features suppressed short-term growth. The negative interest rate policy featured in the previous chapter improves the financial position of firms by decreasing the amount of interest paid on liabilities, thus preventing a long-term economic contraction.

Our climate results produce the same general findings of Bovari *et al.* (2018b). A government that imposes stricter carbon taxes and commits to subsidizing larger proportions of the total abatement costs in the economy performs better than its less active counterparts. Economies that combine strong climate policy with an expansionary monetary policy appear best suited to mitigate climate damages *in the short run*. In the long-run, however, an expansion in capital investment will increase the short-term economic growth rate to the point that firms will pay too much interest on their debt, thus leading to a collapse in the profit share, and an eventual economic contraction.

The essential findings from our sensitivity analyses can be integrated with the climate model as a potential future avenue of research. A disaggregation of conventional and green capital (as in Dafermos *et al.* (2017)) can be pursued, with a lower green policy rate encouraging investment in carbon-neutral capital goods. Furthermore, governments could also consider financing their debt through the issuance of green bonds, which could be used to further encourage abatement activities or other environmentally-conscious projects. Lastly, a green fiscal policy regime could be designed in similar fashion to Costa Lima *et al.* (2014). The objective of the fiscal policy regime would be to stimulate green employment through a federal jobs program, thus ensuring that the process of transitioning from a conventional economy to a green economy does not result in a permanent economic contraction characterized by increased, long-term unemployment.

These policy prescriptions may seem ambitious, but could very well be essential to the long-term survival of our planet! This thesis articulates various potential causes of financial and economic instability, and proposes a policy regime that, if adapted to a climate-focused framework, could be successful at mitigating the long-term damages of climate change, thus creating a sustainable future for all.

Appendix A

Appendix

A.1 Initial conditions

Symbol	Variable description	Initial condition	Sources and remarks
$Y_{\max}$	Potential output	100	Climate only, Chapters 5, 6
Y	Output	100	All chapters, Grasselli and Costa Lima (2012)
p	Composite good price level	1	Normalization constant, all chapters
V	Inventory stock	0	Chapters 5, 6, Grasselli and Nguyen-Huu (2018)
L_c	Corporate loans	55	Chapters 2-4, 6
M_f	Corporate deposits	5	Chapters 2-4, 6
B	Public debt	10	Chapters 3, 4, 6
r_B	Policy rate	0	Chapters 4, 6
M_h	Household deposits	50	Chapters 4, 6
N	Workforce	0.9	All chapters
ω	Wage share	0.9	All chapters, Grasselli and Nguyen-Huu (2018)
λ	Employment rate	0.9	All chapters, Grasselli and Nguyen-Huu (2018)
d	Private debt ratio	0.5	All chapters, Grasselli and Nguyen-Huu (2018)
T	Temperature anomaly, in °C	0.85	All chapters, Nordhaus (2018)
T_0	Temperature anomaly in lower ocean layer, in °C	0.0068	All chapters, Nordhaus (2018)
CO_2^{AT}	CO2-e concentration in the atmosphere layer, in Gt C	851	All chapters, Nordhaus (2018)
CO_2^{UP}	CO2-e concentration in the biosphere and upper ocean layer, in Gt C	460	All chapters, Nordhaus (2018)
CO_2^{LO}	CO2-e concentration in the lower ocean layer, in Gt C	1740	All chapters, Nordhaus (2018)
E_{ind}	Industrial CO2-e emissions, in Gt CO2-e	60	All chapters, calibrated
E_{land}	Exogenous land use CO2-e emissions, in Gt CO2-e	4.7	All chapters, calibrated
F_{exo}	Exogenous radiative forcing, in W/m^2	0.5	All chapters, Nordhaus (2018)
g_σ	Growth rate of the emissions intensity of the economy	-0.0105	All chapters, Nordhaus (2018)
p_{BS}	Price level of backstop technology	547.22	All chapters, Nordhaus (2018)
n	Emission reductions rate	0.03	All chapters, Nordhaus (2018)

A.1.1 Calibration parameters

Symbol	Parameter description	Initial condition	Sources and remarks
α	Productivity growth rate	0.025	Grasselli and Nguyen-Huu (2018)
f_d	Dividend constant	0.2	Calibrated, Chapter 6
η_e	Convergence parameter for Y_e	0.01	Chapters 5, 6 Grasselli and Nguyen-Huu (2018)
δ	Depreciation rate of capital	0.05 [1]	All chapters, Grasselli and Nguyen-Huu (2018)
ν	Capital-to-output ratio	3	All chapters, Grasselli and Nguyen-Huu (2018)
β	Growth rate of workforce	0.02	All chapters, Grasselli and Nguyen-Huu (2018)
ϕ_0	Phillips curve parameter	0.0401	All chapters, Grasselli and Nguyen-Huu (2018)
ϕ_1	Phillips curve parameter	-0.0000641	All chapters, Grasselli and Nguyen-Huu (2018)
inv_0	Investment function intercept	-1	All chapters, calibrated
inv_1	Investment function slope	{15, 20, 15, 8, 12}	Calibrated [2]
η_u	Adjustment speed for investment as function of utilization	0.2	Chapters 5, 6, Grasselli and Nguyen-Huu (2018)
$\bar{u}$	Baseline utilization	0.9	Chapters 5, 6, Grasselli and Nguyen-Huu (2018)
f_M	Reserves ratio requirement	0.1	Chapters 4, 6, calibrated
r_L	Fixed short-term interest rate	0.03	Chapters 2, 3, 5 [3], Grasselli and Nguyen-Huu (2018)
δ_L	Lending rate adjustment parameter	0.025	Chapters 4, 6, calibrated
r_L	Fixed lending rate	0.04	Chapters 2, 3, calibrated
δ_M	Deposit rate adjustment parameter	0.0025	Chapters 4, 6, calibrated
r_M	Fixed deposit rate	0.03	Chapters 2, 3, calibrated
β_{rB}	Policy rate adjustment speed parameter	0.1	Chapters 4, 6, calibrated
r_B	Fixed policy rate	0.01	Chapters 3, calibrated
γ	(1 - γ) is degree of monetary illusion	0.8 [4]	Chapters 2-4, 6 Grasselli and Nguyen-Huu (2015)
ξ	Markup value	1.2	All chapters, calibrated
η_p	Adjustment speed for prices with respect to cost	{4, 0.8}	Grasselli and Nguyen-Huu (2015), Grasselli and Nguyen-Huu (2018) [5]
η_q	adjustment speed for prices with respect to demand	0.2	Chapters 5 and 6, Grasselli and Nguyen-Huu (2018)
τ	Base tax rate	0.15	Chapters 3, 4, 6, calibrated
ψ_g	Base government spending parameter	0.15	Chapters 3, 4, 6, calibrated
c_0	Consumption function intercept	{0.1, 0}	Chapters 5 (= 0.1), 6 (= 0), calibrated
c_1	Consumption function slope	0.9	Chapters 5, 6, calibrated
$c_{\min}$	Minimum consumption share	0	Chapters 5, 6, calibrated
$c_{\max}$	Maximum consumption share	{0.85, 0.67}	Chapters 5 (= 0.85), 6 (= 0.67), calibrated
C^{AT}_{preind}	Preind. concentration of CO_2 in the atmosphere layer, in Gt C	588	All chapters, Nordhaus (2018)
C^{UP}_{preind}	Preind. concentration of CO_2 in the biosphere/upper ocean layer, in Gt C	360	All chapters, Nordhaus (2018)
C^{LO}_{preind}	Preind. concentration of CO_2 in the lower ocean layer, in Gt C	1720	All chapters, Nordhaus (2018)
ϕ_{12}	Transfer coefficient for carbon from AT to UP	0.0239069	Nordhaus (2018), Bovari *et al.* (2018b)
ϕ_{23}	Transfer coefficient for carbon from UP to LO	0.0013409	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
δ_{g_σ}	Variation rate of the growth of emission intensity	-0.001	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
$\delta_{E_{land}}$	Growth rate of land use change CO2-e emissions	-0.022	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
F_{dbl}	Change in radiative forcing from a doubling of preind CO_2, in W/m^2	3.6813	All chapters, Nordhaus (2018)
F^{start}_{exo}	Initial value of exogenous radiative forcing	0.5	All chapters, Nordhaus (2018)
F^{end}_{exo}	Initial value of exogenous radiative forcing	1	All chapters, Nordhaus (2018)
T_{preind}	Preindustrial temperature, in degrees Celsius	13.74	All chapters, Nordhaus (2018)
C_0	Heat capacity of the lower ocean layer	3.52	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
γ^*	Heat exchange coefficient between temperature layers, in SI	0.0176	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
π_1	Damage function parameter	0	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
π_2	Damage function parameter	0.00236	All chapters, Nordhaus (2018)
π_3	Damage function parameter, Dietz and Stern	0.0000819	All chapters, Dietz and Stern (2015)
θ	Abatement cost function parameter	2.6	All chapters, Nordhaus (2018)
$g_{p_{BS}}$	Growth rate of the price of backstop technology	-0.0051	All chapters, Nordhaus (2018), Bovari *et al.* (2018b)
f_K	Damages to capital	0.1	All chapters, calibrated

[1] The value of δ in Chapter 1 is 0.01 Grasselli and Nguyen-Huu (2015)

[2] Note that $\text{inv}_1 = 15$ in Chapter 2, and 20 in Chapter 3. For sensitivity plots, $\text{inv}_1 = 10$ (for Chapters 2 and 3). Additionally, $\text{inv}_1 = 15$ in Chapter 4, $\text{inv}_1 = 8$ in Chapter 5, and $\text{inv}_1 = 12$ in Chapter 6 for all plots.

[3] Sensitivity for Chapters 2 and 3, everything for Chapter 5.

[4] The value of γ in Chapter 4 is 0.4 (Grasselli and Nguyen-Huu, 2018)

[5] Note that $\eta_p = 4$ for Chapters 2-4, and $\eta_p = 0.8$ for Chapters 5, 6

Bibliography

Bolker, B. M., Grasselli, M. R., and Holmes, E. (2021). Sensitivity analysis of an integrated climate-economic model. *SIAM Journal on Financial Mathematics*, **12**(2), SC–44.

Bovari, E., Giraud, G., and Mc Isaac, F. (2018a). Coping with collapse: a stock-flow consistent monetary macrodynamics of global warming. *Ecological Economics*, **147**, 383–398.

Bovari, E., Lecuyer, O., and Mc Isaac, F. (2018b). Debt and damages: What are the chances of staying under the 2°C warming threshold? *International Economics*, **155**, 92–108.

Costa Lima, B., Grasselli, M., Wang, X.-S., and Wu, J. (2014). Destabilizing a stable crisis: Employment persistence and government intervention in macroeconomics. *Structural Change and Economic Dynamics*, **30**, 30–51.

Dafermos, Y., Nikolaidi, M., and Galanis, G. (2017). A stock-flow-fund ecological macroeconomic model. *Ecological Economics*, **131**, 191–207.

Desai, M. (1973). Growth cycles and inflation in a model of the class struggle. *Journal of economic theory*, **6**(6), 527–545.

Dietz, S. and Stern, N. (2015). Endogenous growth, convexity of damage and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. *The Economic Journal*, **125**(583), 574–620.

Franke, R. (1996). A Metzlerian model of inventory growth cycles. *Structural Change and Economic Dynamics*, **7**(2), 243–262.

Godley, W. and Lavoie, M. (2006). *Monetary economics: an integrated approach to credit, money, income, production and wealth*. Springer.

Goodwin, R. M. (1967). A growth cycle. In C. H. Feinstein, editor, *Socialism, Capitalism and Economic Growth*, pages 54–58. Cambridge University Press, Cambridge.

Grasselli, M. R. and Costa Lima, B. (2012). An analysis of the Keen model for credit expansion, asset price bubbles and financial fragility. *Mathematics and Financial Economics*, **6**(3), 191–210.

Grasselli, M. R. and Lipton, A. (2019a). The broad consequences of narrow banking. *International Journal of Theoretical and Applied Finance*, **22**(01), 1950007.

Grasselli, M. R. and Lipton, A. (2019b). On the normality of negative interest rates. *Review of Keynesian Economics*, **7**(2), 201–219.

Grasselli, M. R. and Nguyen-Huu, A. (2015). Inflation and speculation in a dynamic macroeconomic model. *Journal of Risk and Financial Management*, **8**(3), 285–310.

Grasselli, M. R. and Nguyen-Huu, A. (2018). Inventory growth cycles with debt-financed investment. *Structural Change and Economic Dynamics*, **44**, 1–13.

Holmes, E. (2020). Sensitivity analysis of a Keen-based economic model with climate.

Keen, S. (1995). Finance and economic breakdown: Modeling Minsky's 'financial instability hypothesis'. *Journal of Post Keynesian Economics*, **17**(4), 607–35.

Keen, S. (2013). A monetary Minsky model of the Great Moderation and the Great Recession. *Journal of Economic Behavior & Organization*, **86**, 221–235.

Minsky, H. P. (1986). Stabilizing an unstable economy. *Hyman P. Minsky Archive*, **144**.

Nordhaus, W. (2014). Estimates of the social cost of carbon: concepts and results from the DICE-2013R model and alternative approaches. *Journal of the Association of Environmental and Resource Economists*, **1**(1/2), 273–312.

Nordhaus, W. (2018). Projections and uncertainties about climate change in an era of minimal climate policies. *American Economic Journal: Economic Policy*, **10**(3), 333–60.

Nordhaus, W. D. (1994). *Managing the global commons: the economics of climate change*. MIT Press, Cambridge, MA, USA.

Sievert, C. (2020). *Interactive Web-Based Data Visualization with R, plotly, and shiny*. Chapman and Hall/CRC.

Soetaert, K., Petzoldt, T., and Setzer, R. W. (2010). Solving differential equations in R: Package deSolve. *Journal of Statistical Software*, **33**(9), 1–25.

Stern, N. and Stiglitz, J. E. (2017). Report of the high-level commission on carbon prices.

R Core Team (2020). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria.

Weitzman, M. L. (2012). GHG targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, **14**(2), 221–244.