Combating Climate Change with Green Central Banking: A Continuous-Time, Stock-Flow Consistent Approach

COMBATING CLIMATE CHANGE WITH GREEN CENTRAL BANKING: A CONTINUOUS-TIME, STOCK-FLOW CONSISTENT APPROACH

BY

LUKA MILIĆ, B.Sc.

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AUTHOR:	Luka Milić B.Sc. (Hons), Financial Mathematics Ryerson University, Toronto, Canada
SUPERVISORS:	Dr. Matheus Grasselli and Dr. Ben Bolker
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Život ljudski večno je plakanje.

Sa plačem se radja i umire;

Pri radjanju plače ko dolazi;

Pri umoru plače ko ostaje.

Pevanje je plakanje bez suza.

Dr. Tadija Ž. Pejović

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* In honor of four generations of graduate degrees in the family - my great grandfather's thoughts:

Life is eternal crying.
In tears we are born,
In tears we are dying.
At birth cries who is coming,
At death cries who is staying.
Singing is tearless crying.

Abstract

Climate change is one of the biggest threats humanity has ever confronted as it threatens both environmental and financial stability. Therefore, this thesis aims to add to the current literature surrounding stock-flow consistent continuous-time climate-economy models by exploring the role of innovative green debt instruments, such as green bonds and green loans, in aiding the private sectors' transition towards net-zero emissions. Moreover, the increased financialization of the macroeconomic system opens up the opportunity to explore unique climate-related monetary policies such as green quantitative easing (QE) and a green interest rate and whether they alleviate short-term debt burdens without neglecting the long-term destabilizing effects of climate change. Our key findings include: (i) providing the productive sector with a more efficient alternative to financing mitigation costs via green fixed-income resulted in increased debt levels that the central bank is ill-equipped to reduce; (ii) introducing green loans as a financial avenue towards renewable energies increased the effectiveness of monetary intervention as the green interest rate served its dual purpose of reducing short-term debt burdens, promoting a faster reduction of carbon emissions and minimizing the long-term destabilizing effects of climate change.

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Chapter 1

Introduction

In a world filled with many crises, climate change is increasingly becoming the most pressing to solve. The scientific consensus surrounding the known impacts of rising temperatures is well established, and many of these consequences are already being observed today. For example, more frequent and extreme environmental disasters such as tropical storms and wildfires, rising sea levels, droughts and collapsing ecosystems, to name a few. Moreover, these catastrophic events are not disconnected from the global economy and pose a significant threat to financial stability. This understanding of the destabilizing effects of climate change has led policymakers to take unprecedented action, culminating in the Paris Agreement in 2015, where 197 countries signed a non-binding agreement to reach net-zero emissions by the second half of this century.

In order to meet these ambitious targets, policy actions such as a price on carbon along with large-scale investment into green technologies and infrastructure are required to prevent global temperatures going beyond a 2°C increase from pre-industrial levels. Above this temperature threshold, we would see irreversible damages from climate change. Furthermore, according to the Intergovernmental Panel on Climate Change (IPCC) (2018), an annual investment of $2.4 trillion on a global level, or 2.5% of world GDP, is needed in the energy system alone until 2035 to limit global temperatures from rising beyond 1.5°C (Masson-Delmotte *et al.*, 2018).

Accordingly, Bovari *et al.* (2018a) and (2018b) attempt to capture this complex interplay between the impacts of climate change on the economy and whether the aforementioned climate policies effectively reduce global emissions without over-indebting the economy in the near-term. However, the economic framework postulated in these two works only enables the government sector to enact fiscal policy to prevent a climate-induced economic collapse. Not only does this place an unprecedented burden on governments, but it leaves no consideration of the role green debt instruments, such as green bonds or green loans, can play in providing a dedicated financing method for climate change adaptation and mitigation projects (ICMA, 2021). Likewise, this singular policy view omits the possibility of expanding the central bank's current mandate to include the physical and transitional risks associated with climate change. Ultimately, paving the way for innovative monetary policies such as a green quantitative easing program and a green interest rate.

Thus, the purpose of this thesis is to expand the economic framework provided by Bovari *et al.* (2018a) and (2018b) and explore whether these innovative financial instruments and monetary policies ease the medium-term debt burdens produced by climate policies without neglecting to avoid the long-term consequences of rising temperatures and climate damages.

The structure of this thesis is as follows. Chapter 2 begins by reviewing the relevant literature surrounding our three stock-flow consistent continuous-time climate-economy models and motivating our subsequent work.

In Chapter 3 we review the Bovari *et al.* (2018b) model in great detail. However, we expand the simplified economic framework to include distinct government and central banking sectors. Moreover, we explicitly define the private sector's loan and deposit dynamics along the lines of Grasselli and Huu (2015) and Grasselli and Lipton (2019). This expansion of the stock-flow consistent framework and corresponding behavioural dynamics will allow us to implement our own proposed green policies more readily. Afterwards, we briefly examine a simplified theoretical version of our model to appreciate the asymptotic stability of the dynamical system. Finally, we reproduce and extend the numerical results observed in Bovari *et al.* (2018a) and (2018b) to confirm the robustness of our model and explore the intricate interplay between the destabilizing effects of climate change and the fiscal policy mechanisms that prevent a climate-induced economic collapse.

Next, in Chapter 4, we explore an extension of our baseline model, where we introduce green bonds as an alternative form of financing climate-related abatement efforts. Accordingly, we expand the central bank's role within the economic framework and allow the monetary authorities to engage in an innovative green quantitative easing program to control green bond yields. Much of this work is inspired by the discrete-time model of Dafermos *et al.* (2018). After, we dive directly into the numerical analysis and explore the medium and long-term consequences from increasing the financialization of the economic system and the productive sector's paradoxical behaviour as a result of introducing a more efficient debt instrument to finance mitigation costs.

Finally, Chapter 5 develops a disaggregated version of the Baseline and Green Bond models. More specifically, we separate the private sector's production process into conventional and green methods and allow firms to issue green bonds to finance abatement and acquire a combination of conventional bank loans and green loans to finance new investment. Furthermore, we investigate the central bank's role in implementing a revolutionary green interest rate designed to incentivize the productive sector toward transitioning to renewable energies more rapidly. This work is again motivated by the discrete-time economic framework of Dafermos *et al.* (2018). Subsequently, we investigate the asymptotic stability of the disaggregated dynamical system to show we have not lost any robustness with our expanded framework. Lastly, we numerically explore how incorporating green technologies and loans alters the climate policy landscape and provides a richer set of policy specifications that successfully prevent a climate-driven economic collapse.

Chapter 2

Literature Review

We begin our literature review with the foundational Goodwin (1967) model. In his work, Goodwin developed an elegant two-dimensional differential equation based on the predator-prey dynamics of the Lotka-Volterra equation, which captures the endogenous cycles between the wage share and employment rate. More specifically, he was able to formalize the dynamics where a high level of investment causes the growth rate to increase and, correspondingly, the unemployment rate to fall, leading to higher wages and declining profits for firms. As this behaviour progresses, it ultimately reduces investment and the growth rate, causing a rise in unemployment, diminishing wages, and restoring the profit share, prompting the entire cycle to repeat.

Keen (1995) later extends the Goodwin model to include a third variable, specifically private debt. In doing so, he effectively quantified the famous "Financial Instability Hypothesis" (FIH) theorized by the economist Hyman Minsky. In contrast to general equilibrium economic models, Minsky's FIH does not rely on exogenous shocks to produce economic cycles of differing severity. Instead, Minsky postulates that the financial system is inherently fragile and that during a period of financial prosperity, capitalist economies willingly engage in more debt-financed speculative behaviour causing the economic system to become increasingly unstable. Eventually, the speculation turns to euphoria, and debt levels exceed what borrowers can afford using their current revenues. Consequently, a massive deleveraging ensues and ends with a collapse in asset values (Minsky, 1986).

Moving along, Grasselli and Lima (2012) discovered that the Keen (1995) model contained two locally stable long-run equilibria. What they call a "good" equilibrium is characterized by a finite private debt ratio, non-zero wage share, and employment rate. In contrast, the "bad" equilibrium is defined by an infinite debt ratio with vanishing wage share and employment rates. Naturally, many extensions of the Goodwin-Keen model have followed, which range from considering the effects of inflation and price dynamics (Grasselli and Huu, 2015) to investigating the consequences of narrow banking (Grasselli and Lipton, 2019). We specifically mention these two papers since we adopt their methodology to reduce the dimensionality of our dynamical systems.

In terms of economic and climate change modelling, most of the current literature revolves around Nobel prize-winning economist William Nordhaus's Dynamic Integrated Climate-Economy (DICE) model. In general, the DICE model is a neoclassical integrated assessment model that captures the effects of rising emissions from production, which enter the carbon cycle, increase radiative forcing, and ultimately temperature. Nordhaus connects the effects of climate change to the economy through a damage function, which determines the damages to output resulting from rising temperatures (Nordhaus, 2014). Dietz and Stern (2015) later extend the damage specifications to include both damages to output and the total stock of capital. Thus, asserting that climate change not only impacts current production but any future output, as well.

However, Nordhaus's work has not evaded criticism, especially concerning the convexity of his damage function. Particularly the questionable assumptions made to estimate the percentage loss to GDP for specific temperature thresholds and entirely neglecting climate tipping points, to name a few (Keen, 2020). For this reason, we adopt the more convex damage function of Weitzman (2012).

Now, using the Stock-Flow Consistent (SFC) approach of Godley and Lavoie (2007), Bovari *et al.* (2018a) coupled the ideas of the Goodwin-Keen model with a continuous-time adaption of the DICE model. In doing so, they masterfully combine the effects of endogenous debt accumulation inherent in the financial system while also considering the destabilizing effects of climate change. More specifically, they effectively illustrate how the long-term consequences of climate change may lead to a severe economic collapse without instituting proactive climate policies, such as a carbon tax. However, if an overly aggressive price on carbon is levied, it places additional debt burdens on the private sector, threatening financial stability in the near term. Some of their key findings include that the 2°C target is already out of reach, and therefore, a carbon price should be selected accordingly as not to overburden the private sector in the short-term while aiming for a long-term temperature anomaly target of 2.5°C.

However, as alluded to in the introduction, the Bovari *et al.* (2018a) model makes some simplifying assumptions surrounding the SFC framework. Namely, only assuming a three-sector economy consisting of households, a productive sector and private banks. This economic framework forces the authors to aggregate the household sector with the government sector and presume that households collect a carbon tax from firms. While not entirely atypical for SFC models, it fails to represent the economic system accurately.

In Bovari *et al.* (2018b), the authors seek to correct this simplification by expanding the SFC framework to include a public sector that actively implements a carbon tax and provides subsidies on abatement costs to the private sector. Some of their main results include: absent any climate policy implementation, there is a 0.5% probability of achieving the 2°C target set by the Paris Agreement; a policy intervention of a carbon tax and a carbon tax plus a subsidy increases the likelihood to approximately 6.5% and 25.6%, respectively; and crucially they demonstrate how a public sector subsidy shifts a portion of the debt sharing burden away from the private sector and reduces the possibility of a debt-driven economic collapse.

Given the high dimensionality of the system of differential equations and the uncertainty surrounding both the economic and environmental impacts of climate change, extensions of the long-term behaviour of the Bovari et al. (2018) model have arisen. For example, Bolker *et al.* (2021) study the effects of the Grasselli and Huu (2015) inflation dynamics on the long-term behaviour of the climate-economy model. We give them special recognition as we adopt their economic outcome classification in our long-term analysis.

Notwithstanding its improved accuracy, the Bovari *et al.* (2018b) framework still provides a narrow view of the potential financial instruments and monetary policies that are being put in practice to address the consequences of climate damages. Thus, for inspiration, we turn to the discrete-time stock-flow consistent ecological macroeconomic framework of Dafermos *et al.* (2018). In their work, the authors similarly investigate the financial instability risk posed by climate change. However, they combine the DICE model with an ecosystem block that captures the flows of energy transformation occurring throughout the production process. For example, variables such as the oxygen used for fossil fuel combustion and extracted mattered are considered. This structure enables Dafermos *et al.* (2018) to distinguish between non-renewable and renewable energy processes and the various debt instruments issued to finance each respective production method. In general, they assume conventional loans and bonds are solely dedicated to financing non-renewable production, while green loans and bonds are designed to promote the use of a renewable energy process. Furthermore, the authors consider the effects of a green quantitative easing (QE) program on financial stability by modelling an active bond market where the central bank purchases green bonds, driving bond prices higher and yields lower, creating favourable lending conditions for green investment. The main conclusion of their study reveals that the implementation of a green QE program reduces climate-driven financial instability. However, green QE is not by itself able to facilitate a substantial reduction in global temperatures.

Thus, our research will first expand on the foundational work set by Bovari *et al.* (2018a) and (2018b) to provide a more realistic depiction of the macroeconomic system and the channels in which climate policies are enacted. Afterwards, we take advantage of this expanded economic framework to investigate the impact of incorporating alternative debt instruments dedicated towards financing green mitigation and adaption costs. This increased financialization of the macroeconomy opens up the possibility to explore innovative green monetary policies that promote the adoption of more renewable energies within a continuous-time setting. In essence, we wish to show that the tools at the disposal of policymakers are not a one-size-fits-all solution and that a blend of financial instruments and interventions can and need to be implemented in unison to prevent a climate-induced economic collapse.

Chapter 3

Baseline Model

In this chapter, we review the Bovari *et al.* (2018b) model. However, as argued above, we make various improvements to the model by accounting for a government and central banking sector while explicitly defining the private sector's loan and deposit dynamics, respectively. These preliminary modifications will lay the groundwork for our two subsequent climate-economy models, as the expanded framework will enable us to track the implementation of our proposed green policies more readily. Note, our climate module will remain unchanged from Bovari *et al.* (2018a) and (2018b), but we adjust the damage to capital specification within the damage function to investigate the widespread effects on the economy caused by climate change.

Afterwards, we briefly examine a simplified theoretical version of our model to understand the asymptotic stability of the dynamical system. Subsequently, during the numerical analysis, we recant all assumptions made in the theoretical investigation to solidify the behaviour of the differential system. Through our analysis, we will observe the destabilizing effects climate change has on the economic system. Moreover, we will begin to appreciate the power that green policies such as a carbon tax and subsidy on abatement costs can have on avoiding a catastrophic economic collapse.

3.1 Accounting Framework

We begin by first adopting the Stock-Flow Consistency (SFC) approach from Godley and Lavoie (2007), where the basic idea is that every monetary flow comes from somewhere and goes somewhere. In essence, no "black holes" can exist within the economic framework (Nikiforos and Zezza, 2017). For our purposes, we divide the simplified economy into five sectors, namely households, firms, banks, government and a central bank sector. Accordingly, we create matrices representing the balance sheet, income statement and transactions of all monetary flows between the various sectors of the closed economy.

	Households	Firms	Banks	Government Sector	Central Bank	Sum
Balance Sheet						
Capital Stock		pK				pK
Deposits	M_h	M_f	$-M$			
Loans		$-L$	L			
Treasury Bills (Bonds)				$-B$	B	
Reserves			R		$-R$	
Sum (Net worth)	X_h	X_f	X_b	X_g	$X_{cb} = 0$	X

Table 3.1: Balance Sheet Matrix

The balance sheet matrix (See *Table 3.1*) records all asset and liability stocks within our economic system. More specifically, a positive value (+) indicates an asset to the sector, while a liability is denoted as a negative value (-). Hence, we see that households only hold deposits, M_h, as an asset. Similarly, the productive sector assets are composed of the stock of capital in nominal terms, pK, and deposits, M_f. On the other hand, the firms sector also has liabilities in the form of loans, L_f. Note, we assume that households do not take out bank loans staying in line with

Bovari *et al.* (2018a). Banks have liabilities in the form of total deposits defined as $M := M_h + M_f$, while having loans and reserves, R, as assets. Moreover, reserves are considered a liability to the central bank and are composed entirely of treasury bills, B, which is a liability to the government sector. In essence, we assume that the central bank monetizes any government deficit by simply buying as many government bonds as required. Ultimately, this interplay between the banking, government and central bank sectors ensures that any fiscal or monetary policies implemented do not diminish bank deposits and, by extension, the sector's net worth. Thus, summing each column, the net worth of each sector can be expressed as

$$X_h = M_h \qquad \text{(Household Net Worth)} \tag{3.1}$$

$$X_f = pK + M_f - L_f \qquad \text{(Firms Net Worth)} \tag{3.2}$$

$$X_b = L_f - M + R. \qquad \text{(Banks Net Worth)} \tag{3.3}$$

$$X_g = -B. \qquad \text{(Government Sector Net Worth)} \tag{3.4}$$

$$X_{cb} = B - R. \qquad \text{(Central Bank Net Worth)} \tag{3.5}$$

Moving along, the transaction flow matrix (See *Table 3.2*) accounts for all monetary transactions occurring between each sector. We highlight that a positive sign indicates any transaction involving an incoming monetary flow. In contrast, a negative sign denotes a transaction involving an outgoing flow. In other words, a positive sign symbolizes a source of funds, while a negative sign specifies the use of funds. Note, the current account registers any monetary payments received or made by the productive sector, while the capital account shows how the investment in real and financial assets is funded. Finally, the balance sheet is integrated with the transaction flows through the flow of funds matrix (See *Table 3.3*), which logs the change in stocks resulting from all the monetary transactions occurring at each period of time.

	Households	Firms		Banks	Government Sector	Central Bank	Sum
Transactions		current	capital				
Consumption	$-pC$	pC					
Investment		pI	$-pI$				
Gov. Spending		pG			$-pG$		
Acc. memo [GDP]		$[pY]$					
Wages	W	$-W$					
Capital Depr.		$-\delta_D pK$	$\delta_D pK$				
Carbon Taxes		$-pT^C$			pT^C		
Non-Carbon Taxes	$-pT_h$	$-pT_f$			pT		
Abatement subsidies		pS^C			$-pS^C$		
Int. on loans		$-r_L L$		$r_L L$			
Banks' Dividends	Π_b			$-\Pi_b$			
Firms' Dividends	Π_d	$-\Pi_d$					
Int. on Deposits	$r_M M_h$	$r_M M_f$		$-r_M M$			
Int. on Bonds					$-r_B B$	$r_B B$	
Int. on Reserves				$r_R R$		$-r_R R$	
Central Bank Profits					Π_{cb}	$-\Pi_{cb}$	
Column Sum (Balance)	S_h	Π_r	$-pI + \delta_D pK$	S_b	S_g	$S_{cb} = 0$	

Table 3.2: Transactions Flow Matrix

	Households	Firms	Banks	Government Sector	Central Bank	Sum
Flow of Funds						
Change in Capital Stock		$p\dot{K}$				$p\dot{K}$
Change in Deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$			
Change in Loans		$-\dot{L}$	$\dot{L}$			
Change in Bills (Bonds)				$-\dot{B}$	$\dot{B}$	
Change in Reserves			$\dot{R}$		$-\dot{R}$	
Column Sum (Savings)	S_h	$S_f = \Pi_r$	S_b	S_g	$S_{cb} = 0$	
Change in Net Worth	$\dot{X}_h = S_h$	$\dot{X}_f$	$\dot{X}_b = S_b$	$\dot{X}_g = S_g$	$\dot{X}_{cb} = 0$	$\dot{p}K + p\dot{K}$

Table 3.3: Flow of Funds Matrix

With the closed economic system constructed, we can begin to appreciate the added realism of our updated SFC framework. For instance, even before introducing any behavioural equations, we see that households no longer implement fiscal policy and are now indeed the passive sector in the economic system, as was presumed in

Bovari *et al.* (2018a). Equivalently, the government and central bank sectors have a clear function and can independently implement fiscal and monetary policy instead of operating under an aggregated public sector seen in Bovari *et al.* (2018b). Consequently, this improvement allows us to accurately follow the interactions between the private, banking, government and central banking sectors that would occur in reality.

3.2 Macroeconomic Model

We next explore the set of behavioural assumptions which describe the evolution of the continuous time dynamics for the macro-monetary system.

3.2.1 Production and Sales

We postulate that firms can produce output, Y^0, of a homogeneous good according to

$$Y^0 := \frac{K}{\nu} \tag{3.6}$$

where K denotes the stock of capital and $\nu > 0$ represents the constant capital-to-output ratio. Naturally, the production of goods releases industrial emissions, E_{ind}, into the atmosphere. Therefore, in an effort to reduce CO_2 emissions, the public sector will levy a carbon tax on the productive sector. Hence, to alleviate some of the tax burdens, the private sector may elect to divert a portion of production, A, to various abatement activities aimed at lowering their emission rate. Furthermore, a fraction, $\mathbf{D}^Y$, of the remaining output may be destroyed as a consequence of climate change and rising temperatures, as argued in Nordhaus (2014). Therefore, the portion of production that is available for sales is

$$Y := (1 - \mathbf{D}^Y)(1 - A)Y^0 \tag{3.7}$$

3.2.2 The Labour Market and Inflation

The private sector is assumed to follow a minimum rational behaviour, where firms will minimize their costs by hiring the required amount of labour at full capacity, such that

$$Y^0 = \frac{K}{\nu} = aL \implies L := \frac{Y^0}{a} = \frac{K}{\nu} \tag{3.8}$$

where L refers to the total employed labour and a represents the labour productivity, which grows exogenously at a constant rate of

$$\dot{a} := \alpha a \tag{3.9}$$

since $\alpha \geq 0$. Likewise, we can define the endogenously determined employment rate as

$$\lambda := L/N. \tag{3.10}$$

Note, the dynamics of the global workforce, N, is exogenous and calibrated to the prospective scenarios of the United Nations (2015) medium fertility rate, such that:

$$\beta(N) := \frac{\dot{N}}{N} = \delta_N \left(1 - \frac{N}{\bar{N}}\right) \tag{3.11}$$

where $\bar{N}$ denotes the upper bound of the world's labour force and δ_N represents the rate of convergence towards $\bar{N}$.

Finally, the wage-price dynamics follow Grasselli and Nguyen-Huu (2018) as the wage dynamics follow the short-run Phillips curve expressed by

$$\dot{\mathrm{w}} := \mathrm{w}(\Phi(\lambda) - \gamma i) \tag{3.12}$$

where, w, is the money wage per capita, γ, denotes the money illusion and $\Phi(\cdot)$ is an increasing real-valued function with a calibrated value in $[0, 1)$. Generally, (3.12) describes the phenomenon where wages will rise at an increasing rate as the economy reaches full employment, since workers gain bargaining power due to the lack of competition. Note, the constant parameter, γ, specifies the degree to which workers consider inflation when bargaining for wages. Naturally, these dynamics leads to higher inflation rate, which we can capture through

$$i := \frac{\dot{p}}{p} := \eta(\xi\omega - 1) \tag{3.13}$$

where, $p \geq 0$, is the consumption price, η, is the speed of convergence towards its long-run value, which is set at a constant mark-up, $\xi > 0$, multiplied by the wage share

$$\omega := \frac{\mathrm{w}L}{pY} \tag{3.14}$$

3.2.3 Emissions, Taxation and Abatement Decisions

The production of commodities releases industrial emissions, E_{ind}, according to

$$E_{ind} = Y^0\sigma(1 - n) \tag{3.15}$$

where $\sigma > 0$ defines the exogenous carbon emission intensity, while $n \in [0, 1]$ denotes the endogenously determined emission reduction rate (to be defined). As mentioned above, the public authorities will levy a carbon tax, T^C, on emissions expressed as

$$T^C := p_c E_{ind} \tag{3.16}$$

$$= p_c \left[Y^0 \sigma (1 - n) \right] \tag{3.17}$$

where p_c denotes the inflation-adjusted price per ton of CO_2-e. Naturally, to minimize the burden of a carbon tax, the productive sector might elect to reduce emissions as much as possible, subject to the associated abatement costs. These abatement costs depend on the emission reduction rate chosen by the private sector, the price of back-stop technology, p_{BS} (exogenously decreasing at some rate $\delta_{p_{BS}}$), and the emission intensity of the economy expressed as

$$A := \frac{\sigma p_{BS}}{\theta} n^{\theta} \tag{3.18}$$

where $\theta > 0$ is a constant parameter determining the convexity of the cost of abatement. Furthermore, the government authorities may bankroll a portion of the abatement cost at a rate, s_A, such that firms receive a subsidy in the form of

$$S^C := s_A A Y^0. \tag{3.19}$$

Note, the subsidizing rate will remain constant for all time. Thus, we can define the net transfer from the public to private sector as having a real value of $(S^C - T^C)$.

Hence, firms must choose the emission reduction rate that minimises the abatement cost with a carbon tax and subsidy

$$\underset{n\in[0,1]}{min}\ AY^0 + T^C - S^C \tag{3.20}$$

Consequently, this is achieved through an arbitrage between the carbon price, p_c, the backstop technology price, p_{BS}, and the subsidizing rate by the public authorities, s_A, given by

$$n = \min\Big\{\Big(\frac{p_c}{(1-s_A)p_{BS}}\Big)^{\frac{1}{\theta-1}}; 1\Big\} \tag{3.21}$$

3.2.4 Firm Financing

Using *Table 3.2*, we can define the private sector's nominal profit before dividends as the nominal output minus the cost of production, or

$$\Pi := pY - \mathrm{w}L - \delta_{\mathbf{D}} pK - r_L L_f + r_M M_f + p(S^C - T^C) \tag{3.22}$$

Accordingly, we see that the total cost of production is determined by:

(i) the money wage bill, $\mathrm{w}L$;

(ii) the capital depreciation $\delta_{\mathbf{D}} pK$, where $\delta_{\mathbf{D}} := \delta + \mathbf{D}^K$ is the capital depreciation rate obtained as a constant rate δ plus damages to capital D^K (to be defined);

(iii) the interest paid on bank loans, $r_L L_f$, and the interest received on deposits, $r_M M_f$, where r_L, r_M, represent the interest rate on loans and deposits, respectively; and

(iv) the net transfers, $(S^C - T^C)$, to the productive sector as defined above.

Likewise, we can define the profit ratio, π as:

$$\pi := \frac{\Pi}{pY} \tag{3.23}$$

Now, as long as profits are non-negative, a fraction $\Delta(\pi) \in (0, 1)$, of profits is paid to the households in the form of dividends (See *Figure 3.1*). Thus, the retained earnings of the corporate sector Π_r is described by

$$\Pi_r := \Pi - \Pi_d \tag{3.24}$$

where, $\Pi_d := \Delta(\pi)pY$.

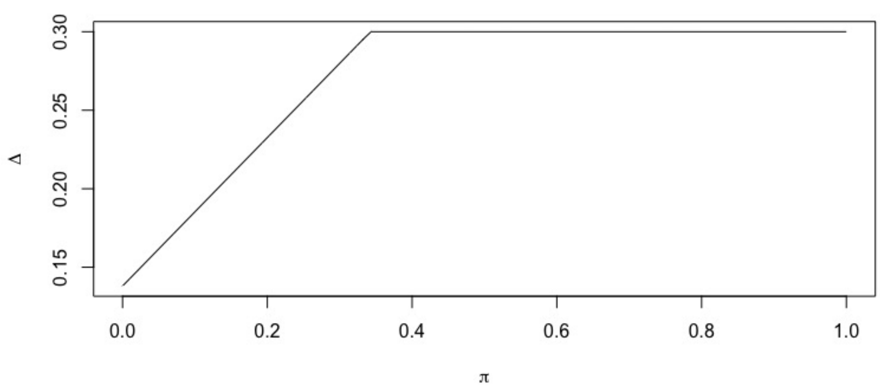


Figure 3.1: Dividend function $\Delta(\pi)$

In addition to paying out dividends, the productive sector will allocate a percentage of output destined for re-investment, I, according to

$$I := \kappa(\pi)Y \tag{3.25}$$

where, $\kappa(\cdot)$ is a increasing function depending on the profit share with a calibrated value in $[0,1]$ (See *Figure 3.2*). Consequently, the change in the stock of capital is calculated by the amount of new investment minus any capital depreciation to the current stock of capital, expressed as

$$\dot{K} := I - \delta_D K \tag{3.26}$$

Now, current profits may not be enough to finance, I, entirely, in which case, firms issue debt through the issuance of bank loans. Note, we slightly depart from Dafermos *et al.* (2018) and assume that the banking sector does not impose in any credit rationing and all financing demands from the private sector are met. Thus, we can express the private sector's loan dynamics as

$$\dot{L_f} := pI - \delta_D pK + r_L L_f - \kappa_L L_f \tag{3.27}$$

where κ_L represents a constant loan principal repayment rate. Moreover, we postulate that private sector deposits evolve following

$$\dot{M_f} = pY - \mathrm{w}L - \delta_D pK + r_M M_f - \kappa_L L_f + p(S^C - T^C) - \Pi_d \tag{3.28}$$

Thus, we can express the change in total debt by the private sector as

$$\dot{D} = \dot{L_f} - \dot{M_f} = pI - \delta_D pK - \Pi_r \tag{3.29}$$

which is identical the total corporate debt dynamics postulated in Bovari *et al.* (2018a) and (2018b).

Lastly, the debt-to-output ratio, loan-to-output and firm's deposits-to-output can be shown as

$$d := \frac{D}{pY}, \quad \ell := \frac{L_f}{pY} \quad m_f := \frac{M_f}{pY} \tag{3.30}$$

respectively.

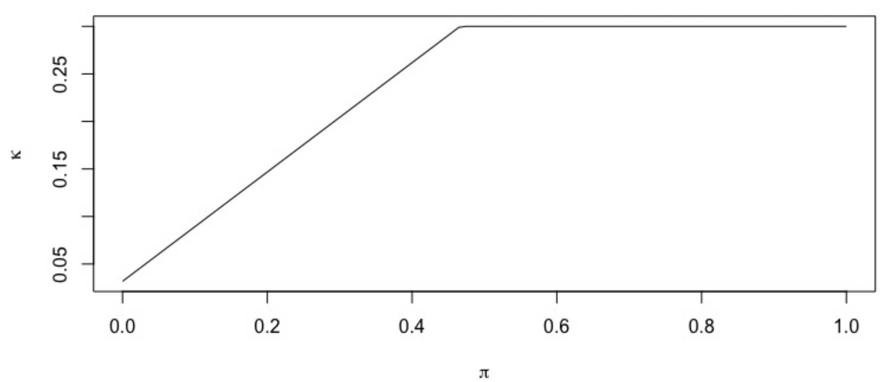


Figure 3.2: Investment function $\kappa(\pi)$

3.2.5 Government Policies, Central Bank and Banking Sector

Given our improved stock-flow consistency framework, it is worth further developing the government sector's role and examining how the central bank is used as a conduit to implement fiscal policy. Firstly, we assume that the government has two roles: collecting a carbon tax and subsidizing climate-related efforts. Thus, apart from those fiscal mechanisms, the government runs a balanced budget in the sense that remaining government spending is always equal to non-carbon taxation, or $pG = pT$. Thus, in order to bankroll climate subsidies, the government will increase its liabilities through the issuance of treasury bills denoted

$$\dot{B} = S_g = p(S^C - T^C) + r_B B - \Pi_{cb} \tag{3.31}$$

Substituting the central bank profits, $\Pi_{cb} = r_B B - r_R R$, expression (3.31) reduces to

$$\dot{B} = p(S^C - T^C) + r_R R \tag{3.32}$$

Consequently, we can define the public debt ratio as

$$b = \frac{B}{pY} \tag{3.33}$$

Naturally, since these subsidies are directed towards the productive sector, the firms' profits and, by extension, their deposit accounts will increase. Note, here we assume that profits first go into the deposit account before firms begin using any retained earnings for investment or dividend payments. Now, in order to maintain SFC, and given that deposits are liabilities to the banking sector, we must also increase the assets held by banks by the amount $p(S^C - T^C)$. Otherwise, any fiscal policy implemented by the government towards the private sector is to the detriment of the banks. Thus, this is achieved by assuming

$$\dot{R} = p(S^C - T^C) + r_R R \tag{3.34}$$

Hence, we see that since reserves are liabilities of the central bank, an increase in reserves is always matched by an increase in assets, $\dot{B}$. Furthermore, the banking sector is now indifferent to any support paid towards the private sector since the increase in reserves offsets the corresponding liability in deposits. Likewise, identical to (3.33), we can express the reserve-to-output ratio as

$$\rho = \frac{R}{pY}. \tag{3.35}$$

Shifting focus, the central bank will adjust the short-term interest rate, r_B, according to the Taylor rule

$$r_B = \max\{0, r^* + i + \phi(i - i^*)\} \tag{3.36}$$

where r^* is the long-term interest rate, i^* is the inflation rate target (2%), and $\phi > 0$ is a parameter that dictates the magnitude of the central bank's response to inflation. Note, we assume that the interest rate on treasury bonds, deposits and reserves are all equal to the short term interest rate. That is, $r_B = r_M = r_R$. Consequently, the interest rate on bank loans is then assumed to be the short-term interest rate plus the prime rate spread, δ_L, or

$$r_L = r_B + \delta_L \tag{3.37}$$

where δ_L is assumed to be constant.

3.3 The Climate Module

Next, we examine the climate component of our integrated assessment model and the damage function that couples the economic and climate modules. The emissions and climate dynamics are a continuous-time adaptation of the famous DICE model from Nordhaus (2018), borrowed directly from Bovari *et al.* (2018a) and (2018b).

3.3.1 Emissions

First, global CO_2-e emissions are the sum of industrial emissions, E_{ind}, as defined above, and land-use emissions denoted, E_{land}. In other words,

$$E = E_{ind} + E_{land}. \tag{3.38}$$

We highlight that the second source of emissions is exogenous and decreases a rate of $\delta_{E_{land}} < 0$ according to

$$\dot{E}_{land} := \delta_{E_{land}} E_{land} \tag{3.39}$$

Similarly, the emission intensity exogenously decreases at a depreciating rate expressed as

$$\dot{\sigma} := g_\sigma \sigma \tag{3.40}$$

$$\dot{g_\sigma} := \delta_{g_\sigma} g_\sigma \tag{3.41}$$

since parameters $g_\sigma < 0$ and $\delta_{g_\sigma} < 0$ (See *Figure 3.3*).

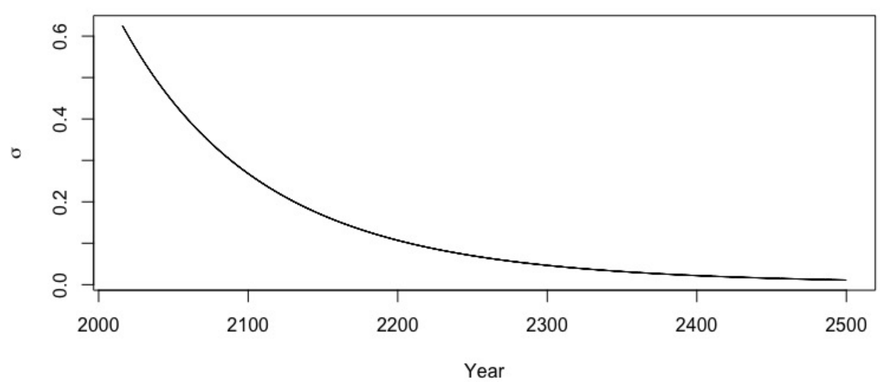


Figure 3.3: Exogenous dynamics for the emission intensity, σ

3.3.2 The Carbon Cycle, Radiative Forcing and Temperature

Moving along, the carbon cycle is captured using a three layer model where emissions can accumulate in the atmosphere (CO_2^{AT}), the upper ocean and the biosphere (CO_2^{UP}) and the deep ocean (CO_2^{LO}). Thus, given global CO_2-e emissions, we can depict the change in CO_2 levels in each layer as

$$\begin{pmatrix} \dot{CO}_2^{AT} \\ \dot{CO}_2^{UP} \\ \dot{CO}_2^{LO} \end{pmatrix} := \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \phi_{12} & \phi_{12}\frac{C^{AT}}{C^{UP}} & 0 \\ \phi_{12} & -\phi_{12}\frac{C^{AT}}{C^{UP}} - \phi_{23} & \phi_{23}\frac{C^{UP}}{C^{LO}} \\ 0 & \phi_{23} & \phi_{23}\frac{C^{UP}}{C^{LO}} \end{pmatrix} \begin{pmatrix} CO_2^{AT} \\ CO_2^{UP} \\ CO_2^{LO} \end{pmatrix} \tag{3.42}$$

where

$$C_i^j := \frac{C_{j_{pind}}}{C_{i_{pind}}}, (i,j) \in \{AT, UP, LO\}^2, \tag{3.43}$$

and ϕ_{12}, ϕ_{23} are parameters.

Moreover, a consequence from the accumulation of greenhouse gases is a rise in radiative forcing, F, which is determined to be the sum of the residual forcing, F_{exo}, and industrial forcing, F_{ind}, expressed as

$$F := F_{ind} + F_{exo} \tag{3.44}$$

where F_{ind} is modeled by

$$F_{ind} := \frac{F_{2\times CO_2}}{log(2)} \left(\frac{CO_2^{AT}}{C_{AT_{pind}}} \right) \tag{3.45}$$

such that the term $F_{2\times CO_2}$ represents the increase in radiative forcing resulting from a doubling of the preindustrial CO_2-e concentration. Note, F_{exo}, grows exogenously and linearly until the year 2100 where it then remains constant (See *Figure 3.4*).

Finally, the rise in radiative forcing results in a corresponding change in the average temperature deviation. This phenomena is captured by the following two equations

$$\dot{T} := \frac{F - \frac{F_{2\times CO_2}}{S} T - \bar{h}(T - T_0)}{C} \tag{3.46}$$

$$\dot{T}_0 := \frac{\bar{h}(T - T_0)}{C_0} \tag{3.47}$$

where, T represents the mean temperature deviations in the atmosphere, biosphere and upper ocean layer, and C denotes the heat capacity of the three layers, respectively. Similarly, T_0, defines the deep ocean average temperature anomaly, while C_0 describes the heat capacity of the layer. In both expressions, $\bar{h}$ signifies the heat exchange between layers, while S is the equilibrium climate sensitivity.

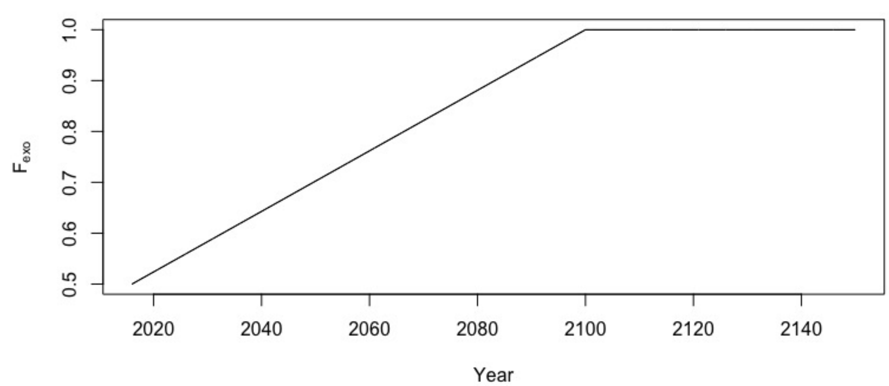


Figure 3.4: Exogenous radiative forcing function

3.3.3 Environmental Damages

Lastly, the environmental damage function quantifies the economic loss due to climate change for a specified temperature anomaly, T, and can be expressed as

$$\mathbf{D} := 1 - \frac{1}{1 + \pi_1 T + \pi_2 T^2 + \pi_3 T^{\zeta_3}} \tag{3.48}$$

where π_1, π_2, π_3 and ζ_3 are parameters.

Furthermore, Dietz and Stern (2015) posit that increasing temperatures will affect not only the real output but capital growth, as well. However, we depart from their damage specifications and postulate that damages from climate change are not a trade-off between output and capital stock. Instead, total output will face the full force of climate damages, with capital stock suffering a fraction of the impacts as well. In other words, we allocate a portion of total damages, $f_k \in [0, 1]$, to the stock of capital, D^K, according to

$$\mathbf{D}^K := f_k \mathbf{D} \tag{3.49}$$

while the damage to output, D^Y, is expressed as

$$\mathbf{D}^Y := \mathbf{D}. \tag{3.50}$$

3.4 Post-Transition Reduced-form Dynamical System

With the climate-economy model constructed, we first explore the asymptotic stability of the differential system.

The model discussed above forms a 21-dimensional differential system, where we see that the climate and macroeconomic models are coupled through the summation of industrial and land emissions (3.38), along with the damage function defined in (3.48) - (3.50). To further our analysis and stay in line with Bovari *et al.* (2018a), we assume that the transition towards net-zero emissions has already been completed. Naturally, since no new emissions are being emitted, the temperature anomaly, T_{eq}, remains constant in the long run. Additionally, since the residual forcing, F_{exo}, grows exogenously and linearly until the year 2100, where it ultimately plateaus, we know that the upper limit is reached at every asymptotic equilibrium. Consequently, both the damage to output and capital converge to a finite limit, $\mathbf{D}^Y_{eq}$ and $\mathbf{D}^K_{eq}$, respectively. Lastly, the portion of abatement undertaken by the productive sector tends towards zero, $A_{eq} = 0$, as well. Note, the methodology to reduce the dimensionality of our model seen below is adopted from Grasselli and Huu (2015) and Grasselli and Lipton (2019).

Thus, we can first show that the growth rate of total output evolves according to

$$g := \frac{\dot{Y}}{Y} := \frac{(1 - \mathbf{D}^{\mathbf{Y}})\kappa(\pi)}{\nu} - \delta_{\mathbf{D}}. \tag{3.51}$$

Likewise, using (3.9) and (3.51), the differential equation describing the labour force can be represented as

$$\frac{\dot{L}}{L} := \frac{\dot{Y}}{Y} - \frac{\dot{a}}{a} = g - \alpha. \tag{3.52}$$

Consequently, combining (3.11) and (3.52), we see that the change in the rate of employment follows

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N} = g - \alpha - \beta(N) \tag{3.53}$$

Furthermore, (3.12), (3.13), (3.51) and (3.52) can be used to obtain the dynamics of the wage share

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{\mathrm{w}}}{\mathrm{w}} + \frac{\dot{L}}{L} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \Phi(\lambda) - i - \alpha. \tag{3.54}$$

For the private sector variables, $\ell = L_f/pY$ and $m_f = M_f/(pY)$ we implement (3.27)-(3.28) along with (3.13) and (3.51) to obtain

$$\frac{\dot{\ell}}{\ell} = \frac{\dot{L_f}}{L_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{pI - \delta_{\mathbf{D}} pK}{L_f} + r_L - \kappa_L - i - g \tag{3.55}$$

$$\frac{\dot{m_f}}{m_f} = \frac{\dot{M_f}}{M_f} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \frac{pY - \mathrm{w}L - \delta_{\mathbf{D}} pK - \kappa_L L_f - \pi_d}{M_f} + r_M - i - g \tag{3.56}$$

Finally, we can depict the evolution of the government debt and reserve ratio using (3.13), (3.32), (3.34) and (3.51)

$$\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = r_R\rho - b(i + g) \tag{3.57}$$

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{R}}{R} - \frac{\dot{p}}{p} - \frac{\dot{Y}}{Y} = \rho\left(r_R - i - g\right) \tag{3.58}$$

Therefore, combining (3.53) - (3.56), we see the main economic model reduces to the following system of differential equations

$$\begin{cases} \dot{\omega} = \omega \left[\Phi(\lambda) - i - \alpha\right] \\ \dot{\lambda} = \lambda \left[g - \alpha - \beta(N)\right] \\ \dot{\ell} = \ell \left(r_L - \kappa_L - i - g\right) + \kappa(\pi) - \frac{\delta_D \nu}{(1-\mathbf{D}^Y)} \\ \dot{m_f} = 1 - \omega - \kappa_L \ell - \Delta(\pi) - m_f \left(r_M - i - g\right) + \frac{\delta_{\mathbf{D}} \nu}{(1-\mathbf{D}^Y)} \end{cases} \tag{3.59}$$

with auxiliary equations defining the profit share, inflation, growth rate of output and population

$$\pi := 1 - \omega - r_L \ell + r_M m_f - \frac{\delta_{\mathbf{D}} \nu}{(1 - \mathbf{D}^Y)} \tag{3.60}$$

$$i := \frac{\dot{p}}{p} := \eta(\xi\omega - 1) \tag{3.61}$$

$$g := \frac{\dot{Y}}{Y} = \frac{(1 - \mathbf{D}^Y)\kappa(\pi)}{\nu} - \delta_{\mathbf{D}} \tag{3.62}$$

$$\beta(N) := \frac{\dot{N}}{N} = \delta_N \left(1 - \frac{N}{\bar{N}}\right) \tag{3.63}$$

We also obtain a secondary differential system containing the state variables (b, ρ) stemming from (3.57) and (3.58)

$$\begin{cases} \dot{b} = r_R \rho - b(i + g) \\ \dot{\rho} = \rho \left(r_R - i - g\right) \end{cases} \tag{3.64}$$

Now, if we assume away inflation, $i = 0$, and that the population has reached its carrying capacity, $\beta = 0$ in (3.63), then we observe the equilibrium growth rate (3.51) simplifies to

$$g_{eq} = \alpha \tag{3.65}$$

which describes the balanced growth path or "Solovian" equilibrium, observed in Bovari *et al.* (2018a). Consequently, we can determine the asymptotic profit rate to be

$$\pi_{eq} = \kappa^{-1}\left[\frac{\nu(\alpha + \delta_{\mathbf{D}_{eq}})}{1 - \mathbf{D}^Y_{eq}}\right] \tag{3.66}$$

Similarly, we can use the the first expression in (3.59) to determine the long-run employment rate

$$\lambda_{eq} = \Phi^{-1}(\alpha) \tag{3.67}$$

Moreover, we utilize (3.65), to derive the equilibrium loans and deposit ratio to be

$$\ell_{eq} = \frac{-\kappa(\pi_{eq}) + \frac{\delta_{\mathbf{D}_{eq}}\nu}{(1-\mathbf{D}^Y_{eq})}}{r_L - \kappa_L - \alpha} \tag{3.68}$$

$$m_{f_{eq}} = \frac{1 - \omega_{eq} - \kappa_L\ell_{eq} - \Delta(\pi_{eq}) - \frac{\delta_{\mathbf{D}_{eq}}\nu}{(1-\mathbf{D}^Y_{eq})}}{r_M - \alpha} \tag{3.69}$$

Finally, for our main dynamical system, we use (3.60) to express the equilibrium wage share as

$$\omega_{eq} = 1 - r_L\ell_{eq} + r_M m_{f_{eq}} - \frac{\delta_{\mathbf{D}_{eq}}\nu}{(1 - \mathbf{D}^Y_{eq})} \tag{3.70}$$

Note, provided α is large enough, the asymptotic dynamics for our public debt and reserve ratios simplifies to

$$b_{eq} = \frac{r_R\rho_{eq}}{\alpha} = 0 \tag{3.71}$$

$$\rho_{eq} = 0 \tag{3.72}$$

Thus, we see that our increased framework observes the same asymptotic behaviour as in Bovari *et al.* (2018a), where if the transition to net-zero is successful, the macroeconomy reaches a balanced growth path characterized by

$$\begin{cases} \omega_{eq} = 1 - r_L \ell_{eq} + r_M m_{f_{eq}} - \frac{\delta_{\mathbf{D}_{eq}}{}^{\nu}}{(1-\mathbf{D}^Y_{eq}} \\ \lambda_{eq} = \Phi^{-1}(\alpha) \\ \ell_{eq} = \frac{\frac{\delta_{\mathbf{D}_{eq}}{}^{\nu}}{(1-\mathbf{D}^Y_{eq})} - \kappa(\pi_{eq})}{r_L - \kappa_L - \alpha} \\ m_{f_{eq}} = \frac{1 - \omega_{eq} - \kappa_L \ell_{eq} - \Delta(\pi_{eq}) - \frac{\delta_{\mathbf{D}_{eq}}{}^{\nu}}{(1-\mathbf{D}^Y_{eq})}}{r_M - \alpha} \end{cases} \tag{3.73}$$

with the corresponding system equalling

$$\begin{cases} b_{eq} = \rho_{eq} = \frac{r_R \rho_{eq}}{\alpha} = 0 \end{cases} \tag{3.74}$$

Furthermore, we can also use (3.73) to begin to comprehend the destabilizing effects of a rising equilibrium temperature and climate damages. More specifically, when looking at (3.66), we see that an increase in either damage to capital or damage to output forces an increase in investment in order to maintain the economy on the balanced growth path. Naturally, this raises the equilibrium profit share and, by extension, the asymptotic loans share. Additionally, climate damages reduce deposits, causing the overall debt share to increase. Ultimately this combined rise in profit share and debt-to-output results in a reduction in the wage share. As expected, this behaviour is magnified with higher temperature anomalies where beyond a particular average temperature, the corresponding debt share tends towards infinity while the wage share and employment rate vanishes to zero. In other words, we observe the bad equilibrium implying an economic collapse.

In the subsequent section, this phenomenon will be demonstrated numerically. We will see that incorporating climate damages causes the economy to converge towards an unstable growth path characterized by an infinite debt ratio and collapsing wage share and employment rate. Hence, to avoid an economic collapse, an aggressive combination of climate policies such as a carbon tax and abatement subsidies must be adopted to reverse the economy back towards a stable growth path. However, as we will see, these climate policies do not come without their short-term costs.

3.4.1 Solovian Steady State, Temperature and Damages

To examine the destabilizing effects that occur from rising temperatures and climate damages, we borrow the methods implemented in Bovari *et al.* (2018a). More precisely, we treat the temperature anomaly as an exogenous parameter instead of letting the climate model endogenously determine the temperature dynamics. In other words, we take (3.73) and manually increase the temperature anomaly and consequently climate damages to numerically study the asymptotic behaviour of the dynamical system. For illustration purposes, we will only demonstrate the *No policy scenario* defined below.

When looking at *Figure 3.5*, we see that as we raise temperatures between 0°C and 3°C, the economy remains on its balanced growth path and converges towards the desired good equilibrium. In this temperature range the asymptotic employment rate and wage share approximately equal 70% and 60%, respectively. However, as global temperatures reach 3°C and 4°C, a bifurcation occurs where the damages from climate change become exceedingly severe, and the economy experiences a collapse. More accurately, the employment rate and wage share vanish to zero, while the debt share converges towards infinity.

Undeniably, our macroeconomic model also observes a tipping point between $3 - 4°C$, as in Bovari *et al.* (2018a). Returning to our coupled climate-economy model, we can then reason that if temperatures and emissions increase too high from industrial production, a climate-induced bifurcation occurs, resulting in the economic system converging away from the Solovian steady state. Therefore, the goal for policymakers is to reduce climate damages by limiting global emissions and pre-industrial temperature levels below the 4°C threshold. The potential avenues to achieve this within our current economic framework are by implementing a carbon tax, a subsidy on abatement costs, or both. Nevertheless, as we will see shortly, the economic bifurcation may be entirely unavoidable depending on the combination of climate policies selected.

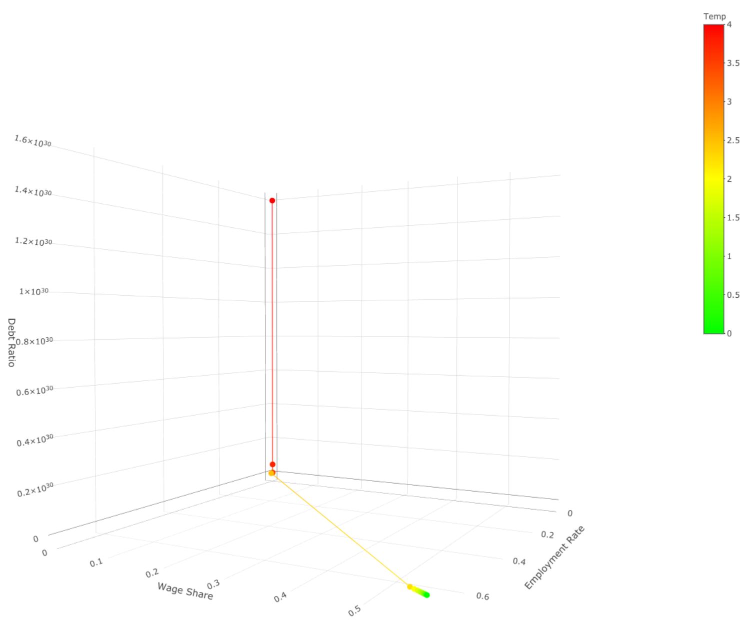


Figure 3.5: Bifurcation from the Solovian steady state as a function of temperature anomaly in the *No Policy Scenario*

3.5 Numerical Analysis

We begin by first introducing our four policy scenarios that will form the basis of all our analyses. Afterwards, we explore the deterministic results of our macroeconomic model to gain insight into the shock of climate change and damages causes on the economy. Finally, we consider the foundational Monte Carlo results for our respective policy scenarios to provide a yardstick for future policy additions in subsequent chapters. Note, all numerical results were obtained by simulating the model in `R` (R Core Team, 2021) using the `deSolve` (Soetaert *et al.*, 2010) package and `lsoda` integration technique.

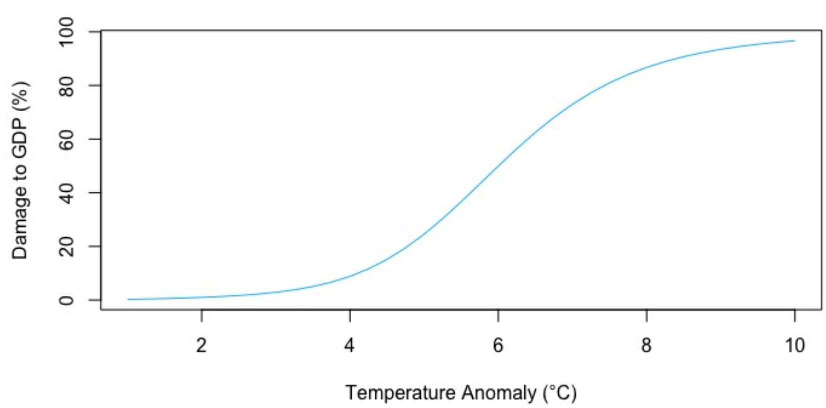


Figure 3.6: Weitzman (2012) Damage Function

3.5.1 Policy Scenarios

For our numerical analysis, we will be comparing four different policy scenarios. Staying in line with Bovari *et al.* (2018a), we will first examine the *No feedback scenario.* In this admittedly unrealistic scenario, we provide a baseline to observe the trajectories of the main economic variables by omitting any climate feedback loop into the economy. We then move into the "business-as-usual" *No policy scenario*, where we introduce the Weitzman (2012) damage function with a damage to capital allocation of $f_k = 1/3$ (See *Figure 3.6*). Here state intervention is minimum as the government implements an inefficient carbon tax with a growth rate of 2% per year. This scenario is in line with Nordhaus (2018) and acts as a benchmark for our remaining scenarios. Moving along, in the *Carbon tax scenario*, we consider the identical damage specifications as before but include an aggressive carbon tax according to the High-Level Commission on Carbon Prices (2017). Under this pricing scheme, the government sector levies a carbon tax of US$80/tCO2 by 2020, with an increase of US$100/tCO2 by 2030. Beyond 2030, we assume the price of carbon continues to grow linearly (See *Figure 3.7*). Finally, we consider the *Strong policy scenario* where in addition to the *Carbon tax scenario*, the government sector implements a 66% subsidy on abatement costs. For quick reference, we summarize the four scenarios in Table *3.4*.

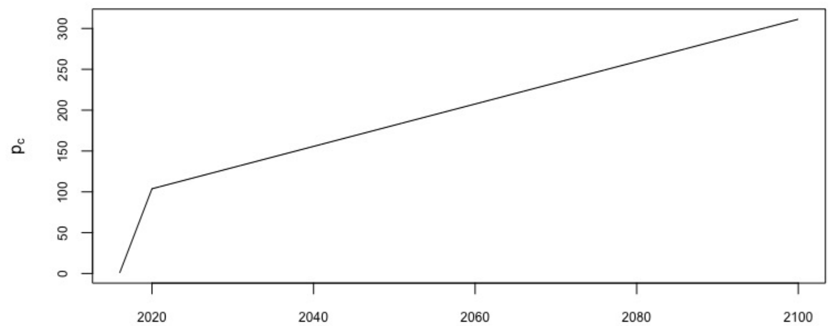


Figure 3.7: Carbon Pricing Scheme

Scenario	*No Feedback*	*No Policy*	*Carbon Tax*	*Strong Policy*
Weak Carbon Tax	-	X	-	-
High Carbon Tax	-	-	X	X
Subsidy (66%)	-	-	-	X
Damage Function (with $f_k = 1/3$)	-	Weitzman	Weitzman	Weitzman

Table 3.4: Summary of Policy Scenarios Considered

3.5.2 Deterministic Analysis

Figure 3.8 depicts some of the key economic and climate-related trajectories for the initial *No Feedback scenario*. If we ignore climate change, we see that by 2050, the GDP growth rate begins to stabilize towards the exogenously set labour productivity growth rate of 2%. Likewise, we see that the employment rate, wage share, and profit share stabilize quickly to approximately 69%, 57% and 32%, respectively. Furthermore, the inflation rate begins at 1.6%, where it ultimately stabilizes between 1.2% and 1.4% by the year 2100. Finally, we see that the debt-to-output ratio remains steady after the year 2100, converging towards 214%.

Thus, by omitting any climate feedback, our preliminary deterministic analysis indicates that the economic dynamics converge towards the proverbial "good" equilibrium, or balanced growth path, defined in the previous section. However, we can already see the disturbing consequences of continuing on this unrealistic scenario as both the temperature anomaly and emissions rise to a frightening 4.2°C and 120 $GtCO_2$ by the year 2100. Note, the dotted red line indicates the target set by the Paris Climate Agreement, where 197 countries agreed to limit the global temperature from reaching 2°C above pre-industrial levels. As we will see in the following section, when the effects of climate damages are included, the long-term shocks to the economic system cause a severe destabilizing effect and take our dynamics outside of the desired Solovian equilibrium.

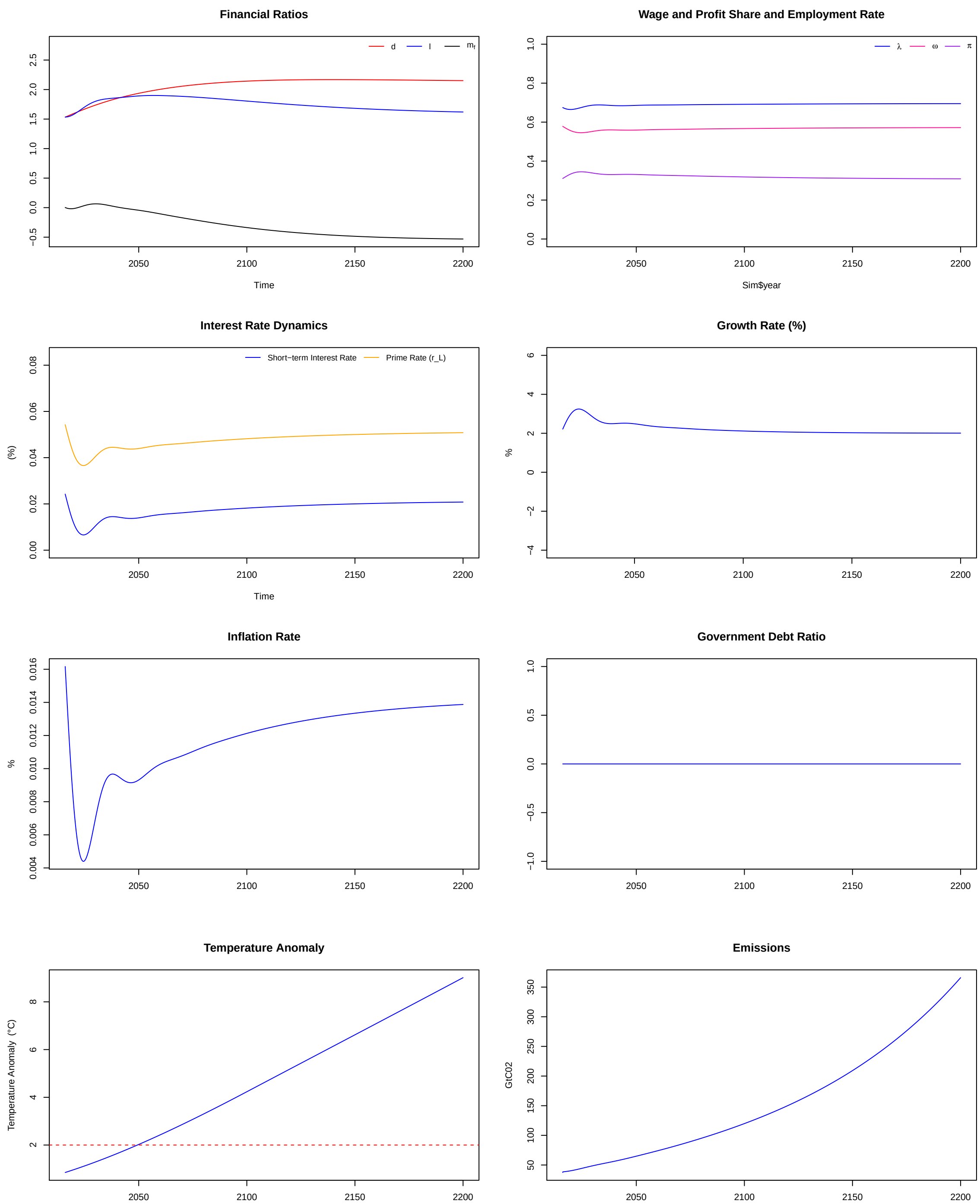


Figure 3.8: Trajectories of the main simulation variables in the *No feedback scenario*. The dotted red line indicates the 2°C target set by the Paris Climate Agreement.

3.5.3 Long-term Analysis

We next turn to the long-term behaviour of the full climate-economy model to study the consequences of global warming and the overall effectiveness of various climate policies. Note, all long-term analyses seen below are simulated from the year 2016-2500 to guarantee the dynamical system has reached equilibrium and to account for the long-term effects of climate change on the macroeconomic system. Afterwards, we will briefly examine the transitional dynamics of our model to capture the trade-off between implementing medium-term environmental policies versus the long-term ramifications of failing to do so.

3.5.4 Sensitivity Analysis

Given the high dimensionality of our model, both in terms of the dynamical system and parameter specifications, we perform a sensitivity analysis to capture a wide variety of economic outcomes based on a range of parameter combinations concerning a carbon tax and subsidies on the cost of abatement.

More specifically, we range the subsidy rate, $s_A \in [0, 0.99]$, and damage to capital, $f_k \in [0, 1]$. Note, both the s_A and f_k parameters are selected from a sequence of 25 equally spaced out points. Furthermore, to illustrate the relationship between the parameters for different damage functions, we compare the subsidy rate and portion of damage to capital against multiple damage specifications categorized by the percent loss to GDP at 4°C. To do this we simply change π_3 in expression (3.48) where $\pi_3 \in [0, 5.07\times10^{-6}, 6.30\times10^{-6}, 1.82\times10^{-5}, 3.35\times10^{-5}, 5.40\times10^{-5}, 8.30\times10^{-5}, 8.19\times10^{-5}]$. *Figure 3.9* summarizes the damage curves for each respective specification of π_3. We select such a wide array of values due to the considerable uncertainty behind climate damages and the necessary amount of government intervention required to reach net-zero emissions as a result. Accordingly, we further consider the potential economic outcomes by comparing the economic model with and without a carbon tax.

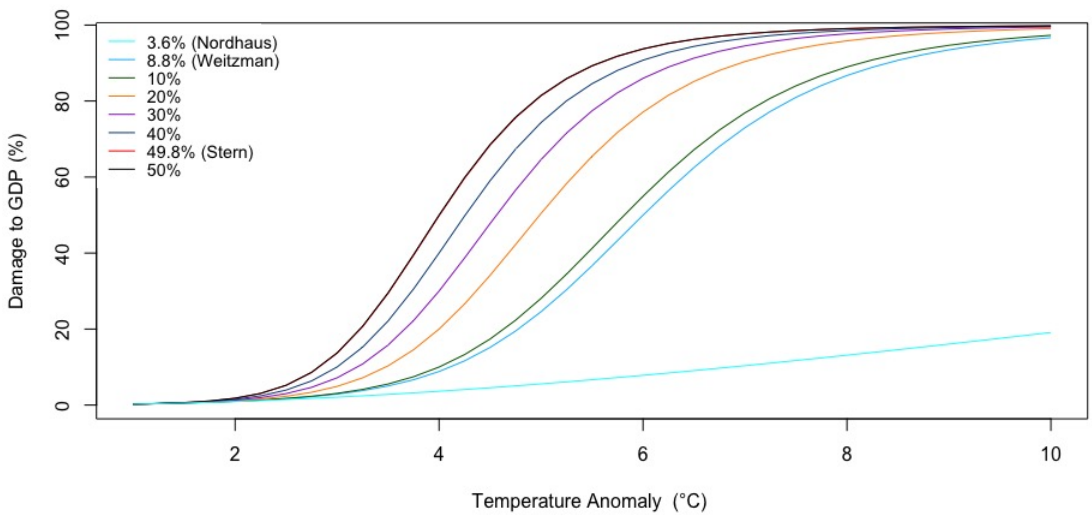


Figure 3.9: Damage curves for different values of $\pi_3 \in [0, 5.07\times10^{-6}, 6.30\times10^{-6}, 1.82\times 10^{-5}, 3.35\times10^{-5}, 5.40\times10^{-5}, 8.30\times10^{-5}, 8.19\times10^{-5}]$. We label each curve by percentage damage at 4°C, from the Nordhaus curve ($\pi_3 = 0$) to the curve with 50% damages at 4°C ($\pi_3 = 8.19\times10^{-5}$).

Following Bolker *et al.* (2021), the economic outcome of each simulation was classified according to three possibilities:

(i) a "good" outcome if $0.4 \leq \omega \leq 1, 0.4 \leq \lambda \leq 1$ and $d \leq 2.7$, which resembles the Solovian equilibrium discussed above, where we observe non-zero wage share, employment rate, and finite debt levels;

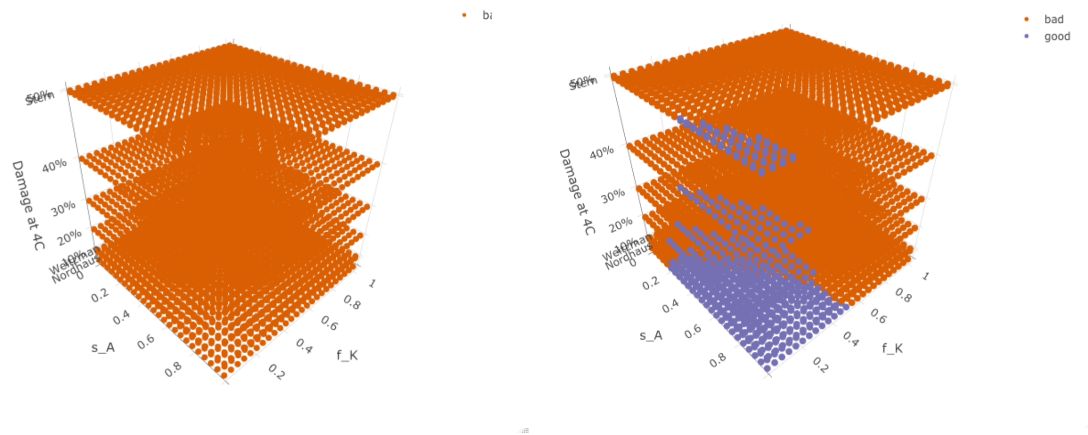
(ii) an "outside bounds" outcome if either $\lambda > 1$ or $\omega > 1$; otherwise

(iii) a "bad" outcome that is synonymous with a vanishing wage share and employment rate, along with infinite debt levels.

We highlight that the 2.7 debt share threshold was borrowed from Bovari *et al.* (2018a) and denotes the point at which the debt levels exceed the total stock of capital, $D = L_f - M_f > pK$. Since capital stock is often used as collateral, defaulting on any debt would make more financial sense whenever the debt-to-output ratio exceeds this level.

The results of the sensitivity analysis can be seen in *Figure 3.10*. Evidently, in the absence of an effective carbon pricing scheme, no amount of subsidies on abatement keeps the economic system on its stable growth path (See *Figure 3.10a)*. On the other hand, if the government sector implements an aggressive carbon tax according to *Figure 3.7*, we observe considerably more good outcomes for a range of subsidy rates (See *Figure 3.10b*). Interestingly, as either the damage at 4°C specification or damage to capital increases, we see a comparable increase in the necessary amount of abatement subsidies required for the economic system to achieve a good equilibrium.

Thus, to maintain the economy on its stable growth path, climate policies such as a carbon tax and abatement subsidies must not be considered mutually exclusive. However, we must stress that the correct policy combination is not dependent only on the long-term economic and climate outcome. Depending on our initial economic conditions, we may observe very different long-term equilibria. Not to mention that imposing a carbon tax raises the productive sector's short-term debt levels and can accelerate the convergence towards an economic collapse. Thus, policymakers must account for the present monetary circumstances when setting carbon prices and subsidy rates. For this, we turn to the basin of attraction analysis.



(a) No Carbon Tax (b) Carbon Tax

Figure 3.10: Outcomes of the climate-economy model with and without a carbon tax for a range of abatement subsidy, s_A, and damage to capital, f_k, parameters. We also vary the damage function parameter π_3 which determines the percent loss to GDP at 4°C.

3.5.5 Basin of Attraction

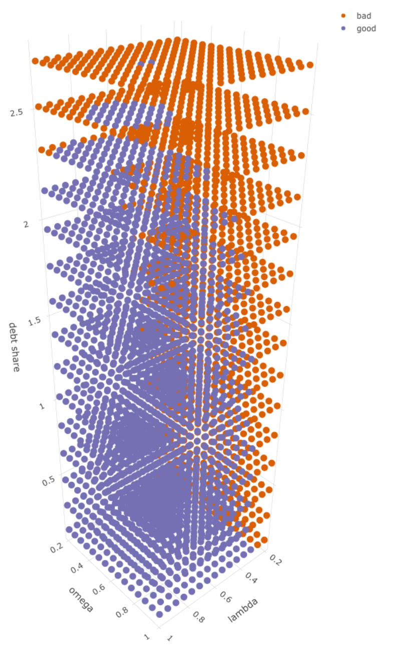
To perform our basin of attraction analysis, we classify economic outcomes for a range of initial conditions concerning the wage share, employment rate and debt share for each of our four scenarios. Note, we make no alterations to the outcome classification from the previous section. The "initial set" for the state variables, $(\omega, \lambda, d) \in [0.2 : 0.99]^2 \times [0.1 : 2.7]$, is borrowed directly from Bovari *et al.* (2018a) and corresponds to a reasonable range of initial conditions where we can observe a resilient economy versus an unviable global economy. By "resilient", we simply mean an economy observing a high initial wage share, employment rate, and low debt levels. In comparison, we imply a heavily indebted productive sector with low employment rates and wage share as an "unviable" economy.

Looking at *Figure 3.11*, we see that in the absence of climate change, almost all sensible initial conditions lead to a good economic outcome for the *No feedback scenario* (See *Figure 3.11a*). This long-term behaviour for the economic model is equivalent to the findings in Bovari *et al.* (2018a). Thus, we do not lose any robustness in the model from our increased economic framework.

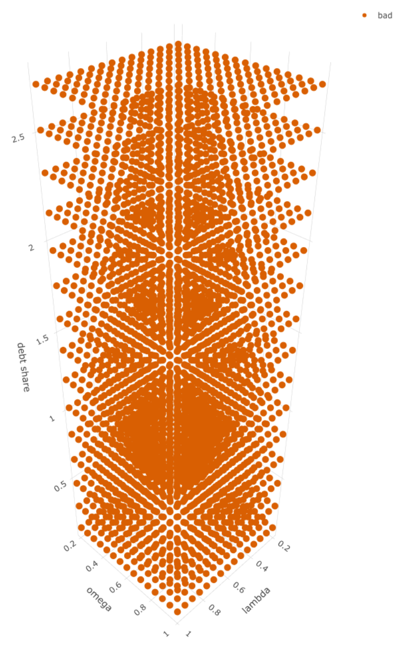
However, if we include the impact of climate change and omit any meaningful policies, we see that the entire set of initial conditions converges towards the bad equilibrium regardless of the economy's resiliency (See *Figure 3.11b*). Similarly, if only a carbon tax is implemented, we see an identical long-term behaviour (See *Figure 3.11c*).

Finally, incorporating an aggressive carbon pricing scheme and a 66% subsidy on abatement returns almost the entire set of initial conditions to the good basin of attraction (See *Figure 3.11d*). However, we must highlight that the set of potential debt share initial values associated with the good equilibrium is reduced in the presence of a carbon pricing scheme. This finding aligns with Bovari *et al.* (2018a), where resiliency against climate change calls for less private debt. That is, an over-indebted economy becomes incapable of carrying the additional burden of a carbon tax. Thus, when setting a carbon price, the government sector must consider the current debt levels to avoid overburdening the productive sector and causing an economic collapse in the near term.

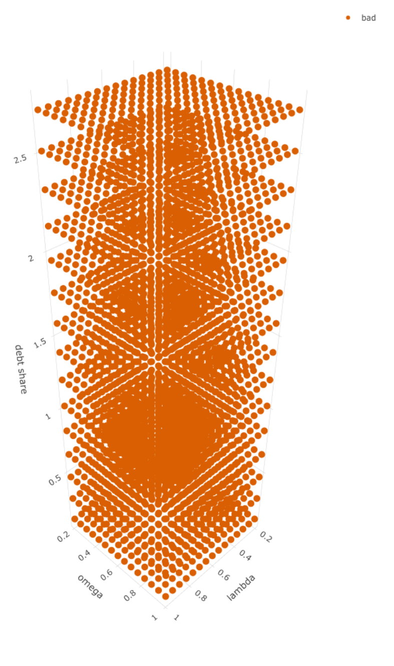
Hence, we see a medium-term and long-term trade-off exists when implementing environmental policies such as a carbon tax and subsidy. More specifically, in order to return the economy to its desired stable growth path, an aggressive combination of climate policies must be imposed. However, this does not come without its cost, as a carbon tax raises near-term debt levels and reduces the good basin of attraction for the global economy. Thus, a carbon tax may accelerate the global economy towards an undesirable equilibrium if our current economic conditions are not resilient. This tight-rope of policy setting is extensively covered in Bovari *et al.* (2018a) and (2018b). Nonetheless, we will illustrate this trade-off in the following section to solidify our interpretations.



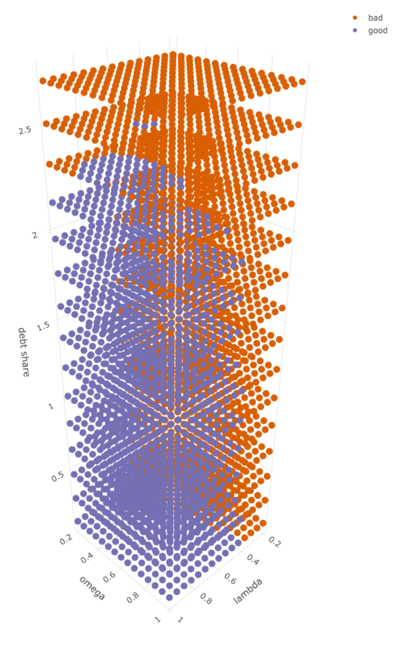
(a) *No feedback scenario*



(b) *No policy scenario*



(c) *Carbon tax scenario*



(d) *Strong policy scenario*

Figure 3.11: Basin of attraction for a range of initial conditions of the wage share, ω, employment rate, λ and debt share, d. We compare the basin of attraction for all four of the policy scenarios.

3.5.6 Transitional Dynamics

Figure 3.12 and *3.13* depict the economic and climate transitional dynamics for the remaining three policy scenarios.

Thus, we see that by coupling the economic and climate modules in the *No policy scenario*, the economy remains on its stable growth path until the year 2080, where increasing emissions and temperatures beyond the 2°C threshold causes damage to output and capital to increase. Absent any climate policies, this behaviour continues, and the damage to GDP significantly reduces the growth rate. Naturally, this raises the debt share and results in the productive sector defaulting on all its obligations by 2086, as it passes the 2.7 limit. As expected, we see the beginning of a collapsing wage share and employment rate, as well.

Shifting the focus to the *Carbon tax scenario*, it becomes clear that imposing a carbon tax on production immediately increases the debt levels of the productive sector, confirming both the sensitivity and basin of attraction results. Naturally, to reduce the burden of a carbon tax, firms increase abatement efforts, reducing total output. As a consequence, the profit share, wage share and employment rate decrease. Given the insufficient government support, represented by a ballooning government surplus, the additional financial strain of a carbon price proves too much as the productive sector exceed the 2.7 threshold on all of its debt obligations by the year 2050.

Finally, in the *Strong policy scenario*, we see the successful implementation of a carbon tax and a subsidy by the government sector. More specifically, we see that despite the carbon tax initially depressing the growth rate and inflating debt levels, the presence of a 66% subsidy on abatement allows firms to reduce emissions quickly and remain on the balanced growth path. In fact, under this policy combination, the economy reaches net-zero emissions by 2043. This is much earlier than the target year of 2050, which many countries that signed the Paris Agreement have set as the deadline to achieve carbon neutrality. Note, the high subsidy rate causes the government debt ratio to reach 1.57 by the year 2100. However, we see that despite successfully returning the economy to its balanced growth path, the climate policies do not prevent global temperatures from increasing beyond the 2°C ceiling.

We must emphasize that our transitional results are deterministic and only show the dynamics for a very particular set of calibrated parameters. Thus, to consider the inherent uncertainty of the economic and climate parameters, we will perform a final sensitivity analysis for our medium-term dynamics in the form of Monte Carlo simulations. Doing so will strengthen our deterministic results and allow us to examine the number of simulations that remain below the 2.7 debt-to-output and 2°C temperature thresholds, respectively.

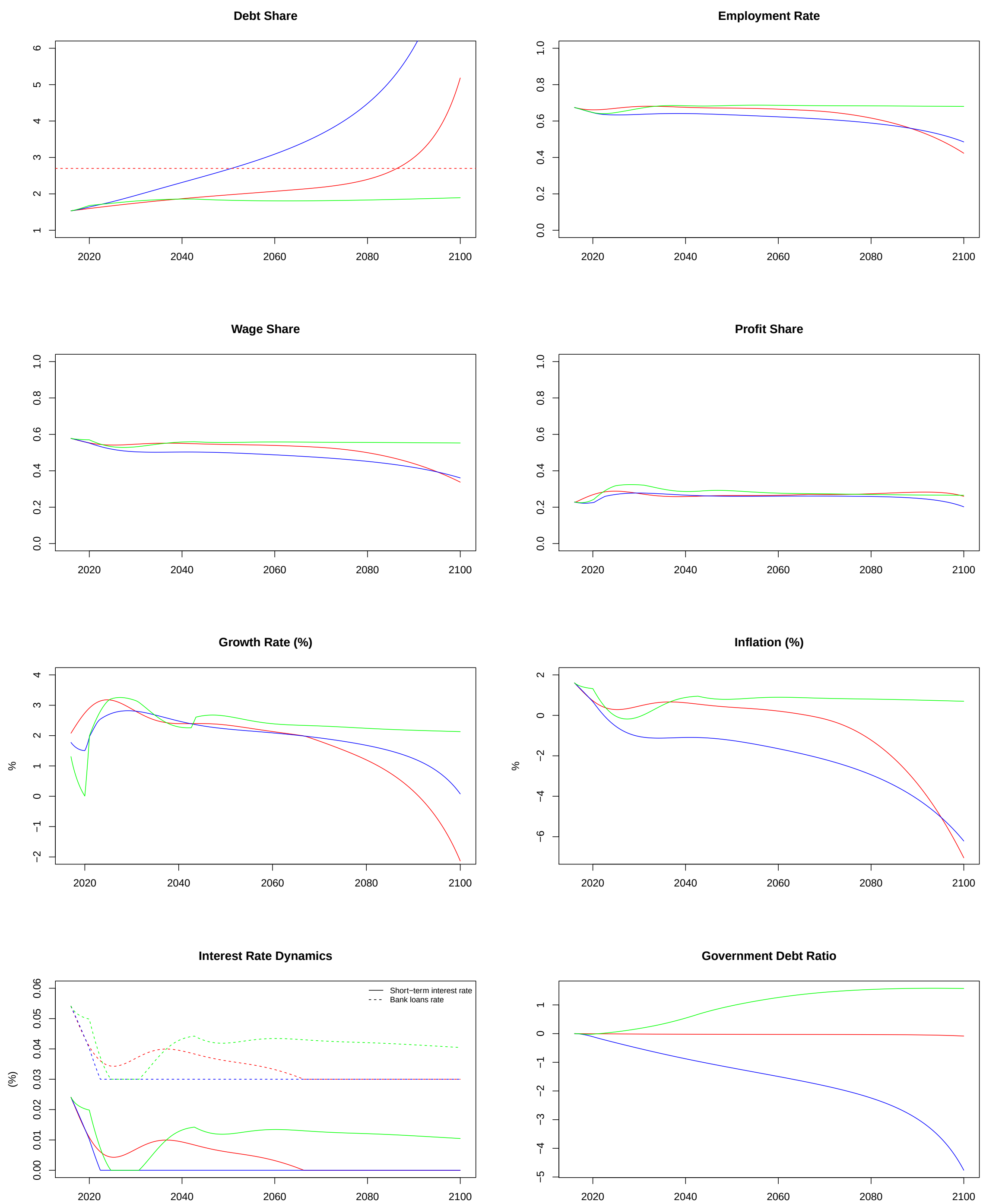


Figure 3.12: Deterministic macroeconomic trajectories for the *No policy scenario* (red line), *Carbon tax scenario* (blue line) and *Strong policy scenario* (green line). The dotted red line in the debt share plot indicates the 2.7 debt-to-output threshold.

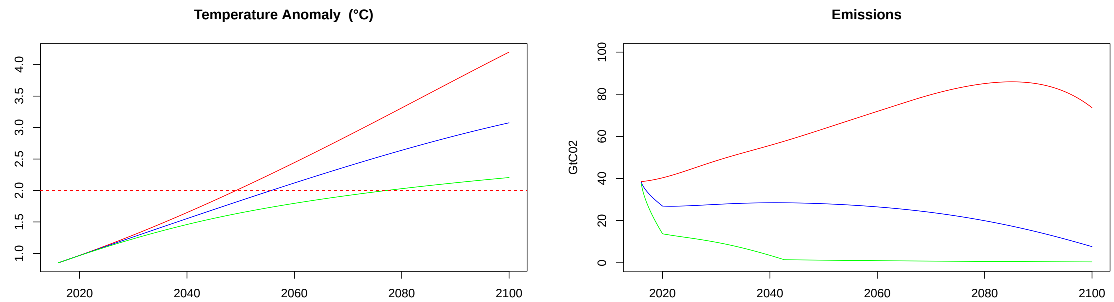
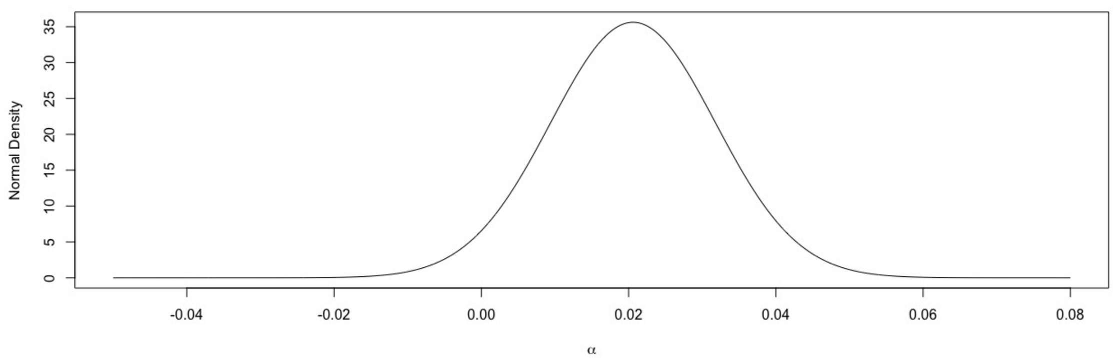


Figure 3.13: Deterministic climate trajectories for the *No policy scenario* (red line), *Carbon tax scenario* (blue line) and *Strong policy scenario* (green line). The dotted red line in the temperature anomaly plot indicates the 2°C target set by the Paris Climate Agreement.

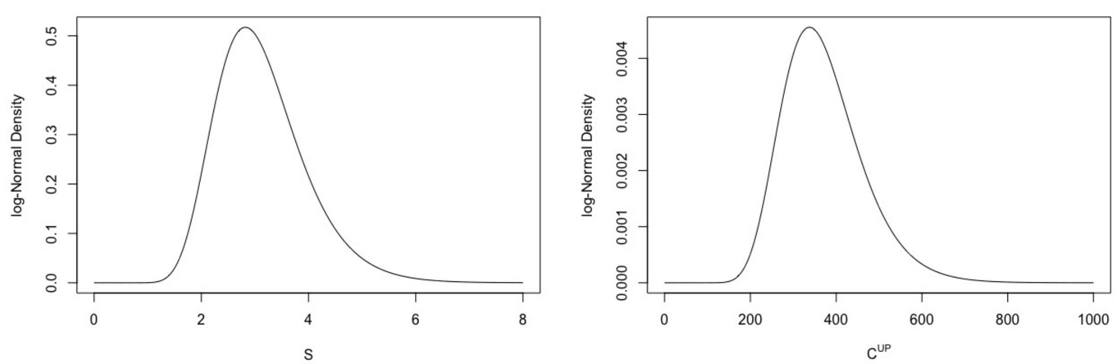
3.5.7 Monte Carlo

For the Monte Carlo simulations, we stay in line with Bovari *et al.* (2018b) and vary the labour productivity, α, climate sensitivity, S, and the concentration of preindustrial CO_2-e in the biosphere and upper ocean, $C_{UP_{pind}}$ for three of our climate policy scenarios. All of these parameters have been widely studied in climate literature, and estimates for each probability density function (PDF) could be found. The PDFs, in our case, are borrowed directly from Bovari *et al.* (2018b) and Nordhaus (2018).

Therefore, we assume that the probability density for the growth rate of labour productivity is a Gaussian distribution with a mean of 0.0206 and a standard deviation of 0.0112. Mathematically, we can denote this as $\alpha \sim \mathcal{N}(0.0206, 0.0112)$. Likewise, we vary the equilibrium climate sensitivity parameter according to a log-Normal distribution with a mean of 1.107 and a standard deviation of 0.264, or $S \sim \log - \mathcal{N}(1.107, 0.264)$. This parameter is vital in dictating how resistant the environment is towards an increase in CO_2 emissions. Finally, we assume that the size of the carbon reservoir follows $C_{UP_{pind}} \sim \log - \mathcal{N}(5.8855763, 0.2512867)$. Note, we sample the PDFs for each parameter 1000 times and run each simulation from 2016-2100.



(a) Probability density function for labour productivity parameters α



(b) Probability density function for climate sensitivity, S, and the CO_2-e preindustrial concentration in the biosphere and upper ocean, $C_{UP_{pind}}$.

Figure 3.14: Probability density functions for the parameters α, S and $C_{UP_{pind}}$

Figure 3.15 depicts the trajectories of our Monte Carlo simulations for each policy scenario. As expected, the transitional dynamics for each scenario do not change from the deterministic results. Thus, we move straight into discussing the probability of staying below the critical temperature and debt thresholds.

In the business-as-usual *No policy scenario*, the temperature anomaly stays below the 2°C target of the Paris Agreement in only 0.4% of the simulated runs with a median value of 4.17°C in the year 2100. Given that no carbon tax is levied by the government, by the end of the century, we only start to observe the impacts of climate damages on debt levels as 30.6% of simulations stay below the 2.7 threshold with a median value of 6.14. Likewise, both the temperature anomaly and debt share stay below their respective thresholds in only 0.4% of the simulations. Note, the median value of the government debt ratio is -0.14.

The outlook does not improve in the *Carbon tax scenario* as the number of simulations remaining under 2°C increases to only 5.2%, while the debt-to-output ceiling is only avoided in 10.7% of runs in the presence of a carbon tax. Combined, the temperature and debt trajectories remain below their targets in 2.5% of simulations. The median values in this instance are 2.97°C and 14.3, respectively. Furthermore, due to the lack of government subsidies, the sector's average debt ratio by the year 2100 is -8.08.

Finally, the climate and economic outcomes drastically improve in the *Strong policy scenario* as the combination of a carbon tax and abatement subsidy results in temperature anomaly staying below the Paris Agreement targets in 44.1% of runs. Meanwhile, the debt-to-output avoids the default ceiling in 69.9% of simulations. Astonishingly, the number of instances where both the temperature anomaly and debt share stay below their limit increases to 33.1% under this policy combination. The median values for the debt-to-output, government debt share and temperature anomaly are 1.91, 2.16 and 2.07°C, respectively. We summarize these results in *Table 3.5* and *3.6*.

Scenario	$T < 2°\text{C}$	$d < 2.7$	$(T < 2°\text{C}) \cup (d < 2.7)$
No policy	0.4%	30.6%	0.4%
Carbon tax	5.2%	10.7%	2.5%
Strong policy	44.1%	69.9%	33.1%

Table 3.5: Percentage of runs where the temperature anomaly and debt share stayed below the 2°C target and 2.7 debt-to-output threshold, respectively. The fourth column details the number of runs that jointly stayed below those limits.

Scenario	Temperature Anomaly (°C)	Financing/Debt Share	Government Debt Share
No policy	4.17	6.14	-0.144
Carbon tax	2.97	14.3	-8.08
Strong policy	2.07	1.91	2.16

Table 3.6: Median values of the temperature anomaly, private and public financing/debt share in the year 2100 for each climate policy scenario.

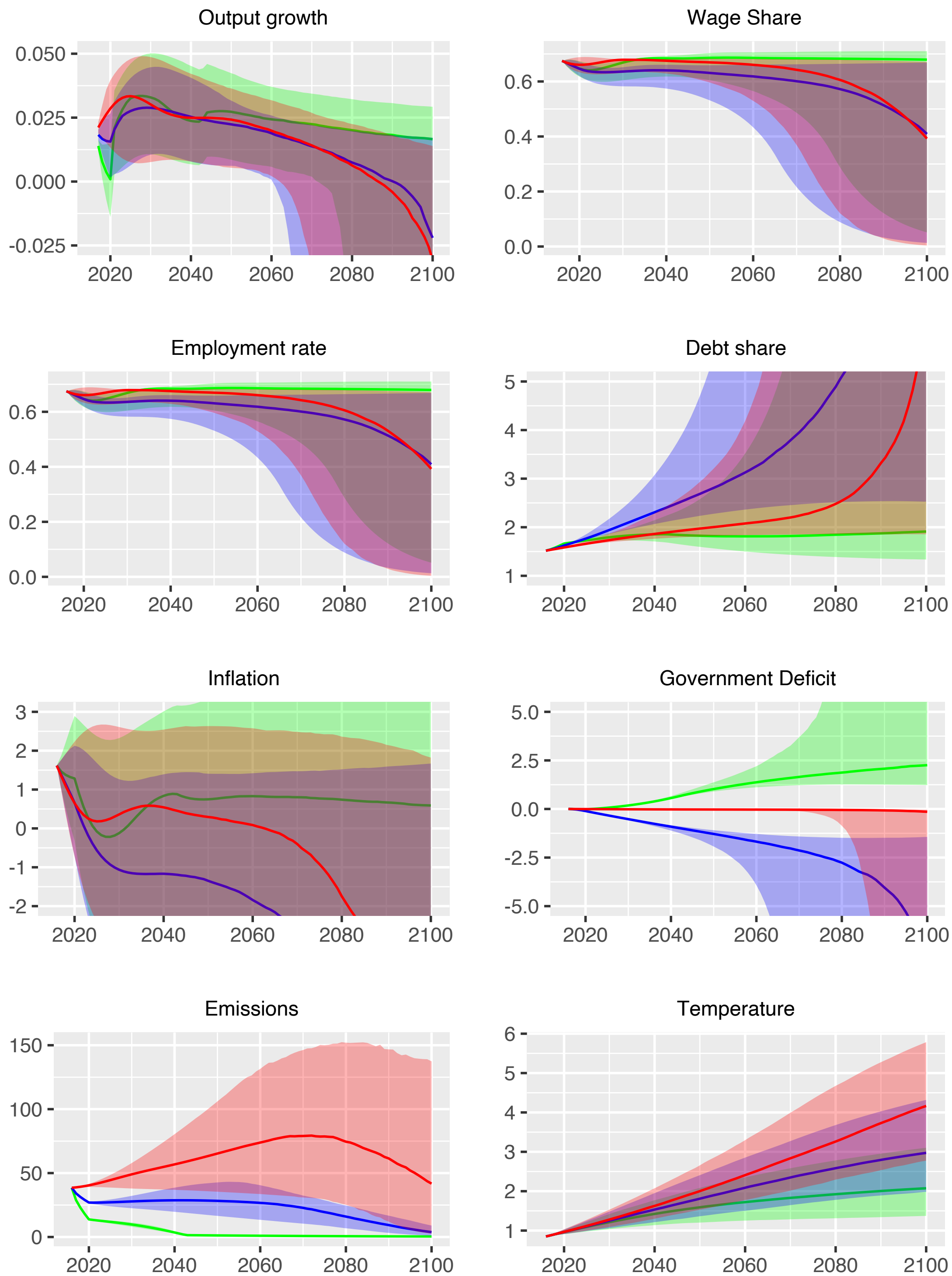


Figure 3.15: [0.05; 0.95] probability interval for the *No policy scenario* (red), *Carbon tax scenario* (blue) and *Strong policy scenario* (green).

3.6 Conclusion

Hence, within this chapter, we have seen that our expanded economic framework resembles and behaves identically to the mathematical models of Bovari *et al.* (2018a) and (2018b). More specifically, without considering the effects of climate change, the macroeconomy equilibrates towards a steady growth path with moderate debt levels, high wage share and employment rates. However, introducing climate change and damages forces the economic system towards an unstable equilibrium, which requires an aggressive combination of a carbon pricing scheme and significant abatement subsidies to reverse the economy towards a good equilibrium. This policy mixture places unprecedented responsibility and expectation on the government sector to solve a global crisis.

Consequently, economists, politicians and policymakers alike have begun to ask, what other sectors and institutions can help in the fight against climate change? More specifically, can innovative green financial instruments and monetary policies such as green bonds and quantitative easing reduce the government sector's burden of maintaining the economy towards a good equilibrium? As we will see in the following chapter, the answer may not be so clear-cut.

Chapter 4

Green Bond Model

The purpose of this chapter is to explore an extension of the baseline model where we allow firms to finance climate-related abatement efforts by first issuing green bonds before turning to more traditional funding options such as bank loans. Furthermore, as a result of introducing green securities into the macroeconomic system, we expand the central bank's mandate and allow the sector to engage in an innovative green quantitative easing program to control green bond yields. This work is motivated by the discrete-time model of Dafermos *et al.* (2018). Note, any model components that are discussed above in the baseline model and not considered here imply that they remain unaltered.

Following the model description, we omit any theoretical asymptotic analysis due to the transitory nature of green bonds and delve straight into the numerical results, exploring the medium-term and long-term consequences from increasing the financialization of the economic system. Our analysis will reveal the paradox of introducing a more efficient financing method and how an asset purchasing program can minimize the counter intuitive effects of green bonds.

4.1 Accounting Framework

Table 4.1, 4.2 and *4.3* depict the balance sheet, transactions and flow of funds matrices for our economic system with the inclusion of green bonds. Focusing on the balance sheet matrix, we see that households remain the passive sector within the economy and continue to hold only deposits, M_h, on their balance sheet. As before, the firms sector assets consist of the nominal stock of capital, pK, and deposits, M_f. However, the sector now has two types of liabilities in the form of bank loans, L_f, and green bonds, Γ. Note that the government and banking sector balance sheet remains unaltered from the baseline model. Moving along, we slightly depart from Dafermos *et al.* (2018), and postulate that the central bank engages in a green quantitative easing (QE) program by purchasing all of the issued green bonds from the private sector. Consequently, green bonds are considered an asset for the central bank, alongside government treasuries, B, while reserves, R, continue to be the sector's only liabilities. The mechanisms through which the central bank participates in green QE will become apparent in the behavioural equations. Thus, we can express the net worth of each sector, as

$$X_h = M_h \qquad \text{(Household Net Worth)} \tag{4.1}$$

$$X_f = pK + M_f - L_f - \Gamma \qquad \text{(Firms Net Worth)} \tag{4.2}$$

$$X_b = L_f - M + R. \qquad \text{(Banks Net Worth)} \tag{4.3}$$

$$X_g = -B. \qquad \text{(Government Sector Net Worth)} \tag{4.4}$$

$$X_{cb} = B + \Gamma - R. \qquad \text{(Central Bank Net Worth)} \tag{4.5}$$

	Households	Firms	Banks	Government Sector	Central Bank	Sum
Balance Sheet						
Capital Stock		pK				pK
Deposits	M_h	M_f	$-M$			
Loans		$-L_f$	L_f			
Treasury Bills (Bonds)				$-B$	B	
Reserves			R		$-R$	
Green Bonds		$-\Gamma$			Γ	
Sum (Net worth)	X_h	X_f	X_b	X_g	X_{cb}	X

Table 4.1: Balance Sheet Matrix with Green Bonds

	Households	Firms		Banks	Government Sector	Central Bank	Sum
Transactions		current	capital				
Consumption	$-pC$	pC					
Investment		pI	$-pI$				
Gov. Spending		pG			$-pG$		
Acc. memo [GDP]		$[pY]$					
Wages	W	$-W$					
Capital Depr.		$-\delta_D pK$	$\delta_D pK$				
Carbon Taxes		$-pT^C$			pT^C		
Non-Carbon Taxes	$-pT_h$	$-pT_f$			pT		
Abatement subsidies		pS^C			$-pS^C$		
Int. on loans		$-r_L L_f$		$r_L L_f$			
Banks' Dividends	Π_b			$-\Pi_b$			
Firms' Dividends	Π_d	$-\Pi_d$					
Int. on Deposits	$r_M M_h$	$r_M M_f$		$-r_M M$			
Int. on Bonds					$-r_B B$	$r_B B$	
Int. on Reserves				$r_R R$		$-r_R R$	
Int. on Green Bonds		$-r_\Gamma \Gamma$				$r_\Gamma \Gamma$	
Central Bank Profits					Π_{cb}	$-\Pi_{cb}$	
Column Sum (Balance)	S_h	Π_r	$-pI + \delta_D pK$	S_b	S_g	S_{cb}	

Table 4.2: Transactions Flow Matrix with Green Bonds

	Households	Firms	Banks	Government Sector	Central Bank	Sum
Flow of Funds						
Change in Capital Stock		$p\dot{K}$				$p\dot{K}$
Change in Deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$			
Change in Loans		$-\dot{L}$	$\dot{L}$			
Change in Bills (Bonds)				$-\dot{B}$	$\dot{B}$	
Change in Reserves			$\dot{R}$		$-\dot{R}$	
Change in Green Bonds		$-\dot{\Gamma}$			$\dot{\Gamma}$	
Column Sum (Savings)	S_h	$S_f = \Pi_r$	S_b	S_g	S_{cb}	
Change in Net Worth	$\dot{X}_h = S_h$	$\dot{X}_f$	$\dot{X}_b = S_b$	$\dot{X}_g = S_g$	$\dot{X}_{cb} = 0$	$\dot{p}K + p\dot{K}$

Table 4.3: Flow of Funds Matrix with Green Bonds

4.2 Macroeconomic Model

We next explore the set of behavioural assumptions concerning private sector financing and central bank policy that are modified due to the introduction of green bonds.

4.2.1 Firm Financing

Since green bonds behave like any other conventional fixed-income instrument, the private sector will be obligated to make interest payments to the debt-holders, in this case, the central bank. Thus, using *Table 4.2*, we redefine the private sector's nominal profit as

$$\Pi := pY - \mathrm{w}L - \delta_{\mathbf{D}} pK - r_L L_f + r_M M_f - r_\Gamma \Gamma + p(S^C - T^C) \tag{4.6}$$

where we see that the total cost of production is now determined by:

(i) the money wage bill, $\mathrm{w}L$;

(ii) the capital depreciation $\delta_{\mathbf{D}} pK$, where $\delta_{\mathbf{D}} := \delta + \mathbf{D}^K$ is the capital depreciation rate obtained as a constant rate δ plus damages to capital D^K;

(iii) the interest paid on bank loans, $r_L L_f$, and $r_M M_f$ denoting the interest received on deposits where r_L, r_M, represent the interest rate on loans and deposits, respectively;

(iv) the interest payment on green bonds, $r_\Gamma \Gamma$, where r_Γ is the yield on green bonds and, Γ, represents the dollar amount of green bonds issued; and

(v) the net transfers, $(S^C - T^C)$, to the productive sector.

Assuming earnings are non-negative, firms will distribute a portion of profits as dividends to households, while the remaining amount will go towards any new investment. Naturally, retained profits may not be enough to finance all desired investments. Hence, the productive sector will turn to two funding sources: green bonds and bank loans. Staying in line with Dafermos *et al.* (2018), we presume that the private sector will first issue green bonds before turning to the banking sector to meet any remaining funding needs. Furthermore, we postulate that green securities are issued exclusively to finance abatement costs. Hence, green bonds evolve according to

$$\dot{\Gamma} := f(1 - s_A)pAY^0 + r_\Gamma \Gamma - \kappa_\Gamma \Gamma \tag{4.7}$$

where f denotes the portion of abatement cost funded by green bonds, κ_Γ represents the repayment rate on green fixed-income, and s_A is the amount of government subsidies. Moreover, we adapt the private sector deposit dynamics to

$$\dot{M}_f = pY - \mathrm{w}L - \delta_D pK + r_M M_f - \kappa_L L_f + \dot{\Gamma} - \kappa_\Gamma \Gamma + p(S^C - T^C) - \Pi_d \tag{4.8}$$

That is to say, after subtracting the amount repaid, the proceeds from the sale of green bonds are added to the firm's deposits. The private sector's loan dynamics are unaffected and evolves following

$$\dot{L_f} := pI - \delta_D pK + r_L L_f - \kappa_L L \tag{4.9}$$

Thus, we can express the change in total financing by the private sector as

$$\dot{F} := (\dot{L_f} + \dot{\Gamma}) - \dot{M_f} \tag{4.10}$$

Likewise, the total debt dynamics are unmodified and can be represented as

$$\dot{D} := \dot{L_f} - \dot{M_f} \tag{4.11}$$

which represents the total amount of loans issued minus any deposits. Note, if we assume that no abatement costs are funded using green bonds, that is $f = 0$ in (4.7), then our total financing dynamics (4.10) reduce to (4.11) as in the baseline model.

Finally, the financing-to-output, debt-to-output, loan-to-output, green bond-to-output and firm's deposits-to-output ratios can be shown as

$$\varphi := \frac{F}{pY}, \quad d := \frac{D}{pY} \quad \ell := \frac{L_f}{pY} \quad \gamma := \frac{\Gamma}{pY} \quad m_f := \frac{M_f}{pY} \tag{4.12}$$

respectively.

4.2.2 Government and Central Bank Policies

Irrespective of green bonds, the government sector continues to implement fiscal policy by imposing a carbon tax and subsidizing any abatement efforts undertaken by the productive sector. Likewise, the central bank controls the short-term interest rate and, by extension, the prime rate on loans. However, the introduction of green financing creates in an innovative role for the central bank within our economic framework. Specifically, controlling the yield on green bonds, r_Γ, according to

$$r_\Gamma := \max\{r_B, r_L - \delta_\Gamma\} \tag{4.13}$$

where δ_Γ represents the spread between the bank loans rate, r_L, and the yield on green bonds which is varied depending on

$$\delta_\Gamma = \phi_2 \left(\frac{(1 - s_A)AY^0}{Y} \right) \tag{4.14}$$

such that ϕ_2 denotes the magnitude of the central bank's response to rising abatement costs. In essence, (4.14) indicates that the central bank considers firms' liquidity when controlling yields. Thus, as abatement costs take up a more significant portion of total output, the central bank will increase the spread between the bank loans rate and the yield on green bonds, and vice versa. However, instead of explicitly establishing a bond market, as in Dafermos *et al.* (2018), we implicitly assume the central bank will purchase the necessary amount of green bonds to ensure the relationship $r_L > r_\Gamma$ holds. Moreover, we postulate that the central bank will engage in QE until bond yields reach the overnight rate, r_B. Beyond this point, we would potentially observe negative yields, which would, in principle, serve as another form of subsidy towards the private sector. If this were the case, it would be more efficient for the government sector to provide additional abatement support by increasing the subsidy rate, s_A. In reality, this practice is commonly observed as the government is always better suited to provide fiscal stimulus to the economy (Hansen, 2020).

Moving along, the central bank reserves now evolve according to

$$\dot{R} := p(S^C - T^C) + r_R R - r_\Gamma \Gamma + \dot{\Gamma} \tag{4.15}$$

Likewise, the government liabilities can be expressed as

$$\dot{B} := p(S^C - T^C) + r_R R - r_\Gamma \Gamma \tag{4.16}$$

Consequently, we see the full effects of the green QE program. More specifically, that the central bank no longer maintains a balanced account, since $\dot{B} \neq \dot{R}$. In other words, the net worth of the central bank decreases whenever the central bank is accumulating green bonds, whereas the net worth rises as the reserve bank tapers its green QE policy.

We conclude by stressing that implementing a green QE program should not be viewed simply as an extension of the current central bank mandate. Since its inception, government and corporate bond purchasing programs were designed to act as an extreme policy action to achieve inflation targets once the central bank's usual monetary toolkit has been exhausted. Thus, QE is designed by nature to be short-term. On the other hand, green QE would require a much longer-term commitment spanning over multiple decades. Therefore, instituting a climate bond purchasing program would entail a redefining, or at the very least, a re-examination of the role the central bank plays and how climate change factors into the tenets of financial stability, price stability and full employment (Dafermos *et al.*, 2018).

Nonetheless, the global COVID-19 crisis has exposed the potential financial stability risks that a widespread crisis such as climate change entails. As a result, leading institutions such as the European Central Bank (ECB) have already started purchasing euro-denominated green debt (Jones and Ranasinghe, 2020). Similarly, the Bank of England (BOE) was assigned a new mandate to purchase green bonds, and "support the government's efforts to make the UK economy greener and achieve zero greenhouse gas emissions by 2050" (Romei, 2021). However, as we will see in the numerical analysis, monetary policymakers may find that a green QE program is ineffective in reducing the financing costs of the private sector in the long run. More specifically, the introduction of green bonds unexpectedly alters the firm sector's behaviour and encourages more borrowing, quickly increasing debt to unsustainable levels that a green QE program cannot curtail perpetually.

4.3 Numerical Analysis

We provide motivation for the numerical analysis by first exploring the deterministic effects of introducing green bonds as an alternative form of financing in the *Strong policy scenario* and set the interest rate spread between bank loans and green bond yields to zero. In doing so, we will reveal the contradictory consequences of providing a more efficient fixed-income instrument for firms to finance abatement efforts. Afterwards, we divided our sensitivity analysis into a medium and long-term comparison to examine the effectiveness of a green QE program over time. Subsequently, we will expand our investigation to combine the range of policy combinations outlined in the previous chapter, including a green asset purchasing program, for the long-term sensitivity and basin of attraction analysis. Finally, we will observe the transitional and Monte Carlo dynamics that incorporate green bonds and an aggressive QE program to verify our interpretations and account for the uncertainty inherent in our parameters.

4.3.1 Deterministic Analysis

Figures 4.1 and *4.2* represent the deterministic economic and climate trajectories for the *Strong policy scenario* with green bonds until the year 2200. For illustration's sake, we have assumed green securities entirely cover abatement costs, $f = 1$, and that the central bank participates in green QE in a manner such that the spread between the bank loans rate and the yield on green bonds is always $\delta_\Gamma = 0$. We achieve this by setting $\phi_2 = 0$. We also include the *Strong policy scenario* from the baseline model to aid in our analysis.

Strangely, we see that the addition of green bonds as an alternative form of financing counterintuitively increases the financing share of the private sector (See *Figure 4.1a*). In essence, green bonds are creating a phenomenon we coin the "Financial Jevon's Paradox". Briefly, Jevon's paradox is the economic theory postulating that improved energy efficiency results in higher energy consumption. For example, as automobiles become increasingly fuel-efficient, consumers will be more willing to get behind the wheel and drive to farther and more obscure places. Naturally, as this cycle progresses, the end-user eventually emits more emissions despite the enhanced efficiency. In our case, green bonds serve as a more practical method to pay for abatement costs. Thus, firms more readily adopt the novel debt instrument but continue to use conventional loans to meet any remaining demand for new capital and production. Thus, despite the increased efficiency of financing abatement costs, firms take on more excessive debt levels than they would originally, using only bank loans. Ultimately, absent any central bank intervention in the form of a reduction in interest paid on green bonds, the introduction of green debt instruments quickly overburdens the firm sector and causes the economy to converge towards an unstable growth path.

Moreover, the employment rate, wage and profit share dynamics for both scenarios remain similar in the short term. However, as the financing-to-output ratio begins to increase in the *Strong policy scenario* with green bonds as a consequence of the Financial Jevon's Paradox, we see the start of the economic ratios collapsing towards the bad equilibrium. Note, green bonds do not improve the climate outcomes as both the temperature anomaly and emissions observe the same trajectories irrespective of the financing methods implemented (See *Figure 4.2*).

Consequently, the financialization of the macroeconomy invites monetary policy-makers to play a pivotal role in reducing the financing costs for the private sector and minimizing the Financial Jevon's Paradox. Thus, the subsequent sections will compare the effectiveness of a green QE program in both the medium and long term. However, as we will see, absent any increased intervention by the government sector, a green asset purchasing program merely delays the effects of the financial paradox and does little to prevent the economy from converging towards an unstable growth path.

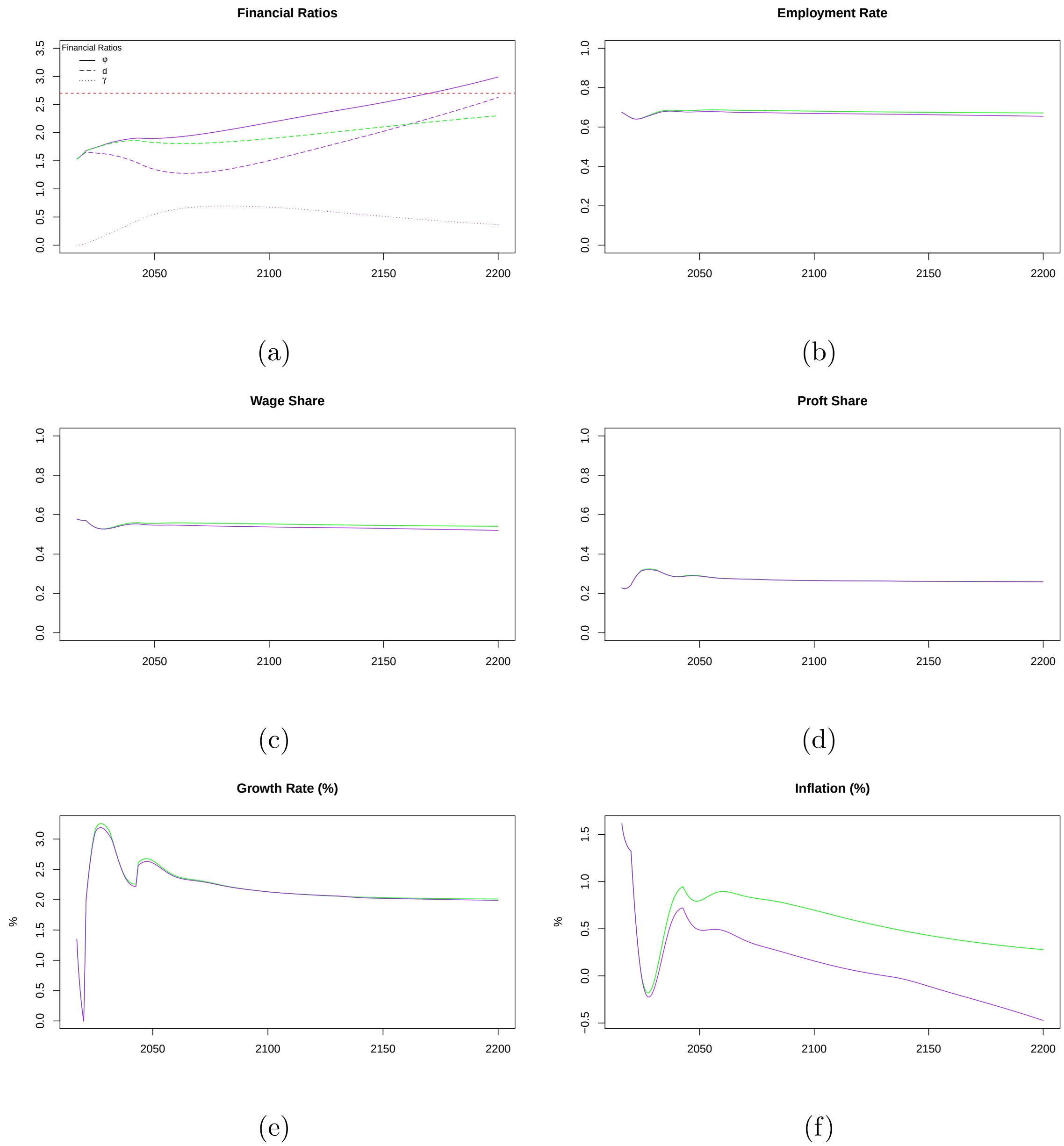


Figure 4.1: Deterministic macroeconomic trajectories for *Strong policy scenario* (green line) and the *Strong policy scenario* with green bonds (purple line). The dotted red line in the debt share plot indicates the 2.7 debt-to-output threshold.

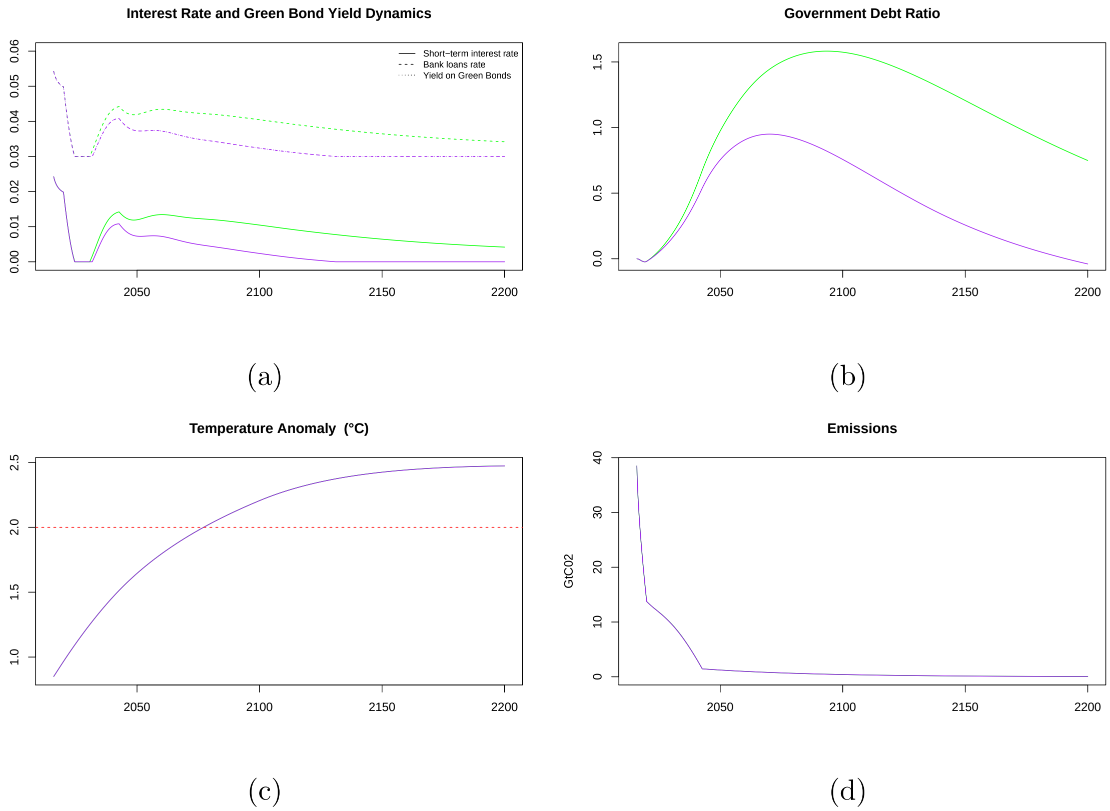


Figure 4.2: Additional deterministic macroeconomic trajectories and deterministic climate trajectories for *Strong policy scenario* (green line) and the *Strong policy scenario* with green bonds (purple line). The dotted red line in the temperature anomaly plot indicates the 2°C target set by the Paris Climate Agreement.

4.3.2 Medium-term Sensitivity Analysis

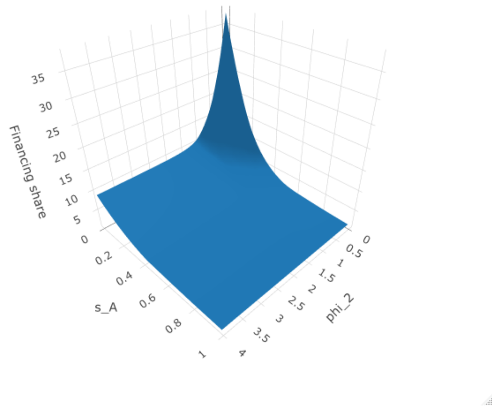
To examine the medium-term success of green QE, we perform a sensitivity analysis, where we vary the subsidy rate, $s_A \in [0, 0.99]$, the magnitude of the central bank's response to rising abatement costs, $\phi_2 \in [0, 4]$, and record the values of a select group of economic and climate variables in the year 2100. Given the shorter simulation period, both the s_A and ϕ_2 parameters are selected from a sequence of 40 equally spaced out points to observe more accurately the policy specifications that cause a shift in dynamics. To aid in our analysis, we incorporate green bonds and QE into the *Strong policy scenario* and solely vary ϕ_2 to examine the financing share and temperature anomaly over time.

When looking at *Figure 4.3*, we see that with no government or central bank support, the combination of a carbon tax and green bonds accelerates the convergence towards the bad equilibrium as the economic variables collapse well before the year 2100. This result is expected, since the Financial Jevon's Paradox accelerates already unsustainable financing levels due to the added burden of a carbon tax. Nevertheless, as the central bank adopts a more aggressive QE policy, the financing-to-output ratio significantly decreases, while the employment rate, wage and profit share improve. However, regardless of how willingly the reserve bank purchases green bonds to maintain low yields, represented by an increase in ϕ_2, the economic system still reaches undesirable outcomes by 2100. It is only when the government sector increases the subsidy rate where we see the economic conditions stabilize in the near-term.

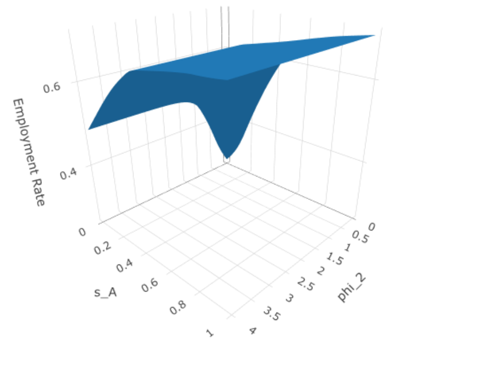
Interestingly, as the subsidy on abatement grows, the effectiveness of the green QE program diminishes. More specifically, there is a maximum ϕ_2 value for each subsidy rate where the financing-to-output ratio no longer decreases. Turning to the *Strong policy scenario* with green QE, we see that beyond $\phi_2 \geq 2.359$ the end values of the financing share remains constant in the year 2100 (See *Figure 4.4a*). This is due to the spread between the rate on bank loans and the yield on green bonds reaching the lower bound, $r_\Gamma = r_B$. As discussed above, beyond this limit, we would observe negative yields that would undoubtedly further decrease the financing share. However, it would be more realistic for the government sector to increase the subsidy rate than allow the central bank to promote negative bond yields. Thus, in the presence of a subsidy, the central bank gains little from adopting an ultra-loose monetary policy beyond a certain maximum spread.

Focusing on the climate side, the size of the central bank's green bond purchasing program has no affect on the temperature anomaly (See *Figure 4.3f*). This is also confirmed by the *Strong policy scenario* with green QE, as the reduction in the temperature anomaly for higher values of ϕ_2, is negligible (See *Figure 4.4b*). Hence, the mixture of a carbon tax and high subsidy rate continue to be the catalyst for reducing CO_2 emissions and limiting the temperature anomaly below the 2°C threshold.

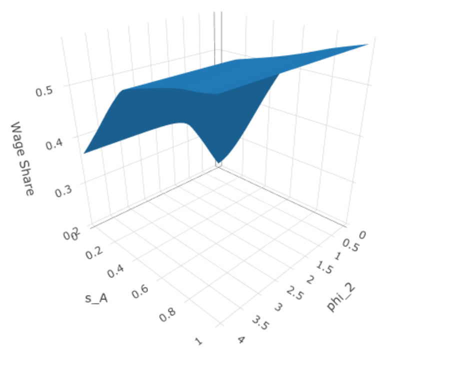
In general, the combination of government subsidies and a green QE program minimizes the Financial Jevon's Paradox and reduces the private sector debt burdens in the medium-term, albeit with diminishing effectiveness. Theoretically, this fiscal and monetary policy mixture should lead the economy toward a balanced growth path post-transition. Although, as we will shortly discover, as abatement costs subside and the central bank begins tapering its green QE program, the large amount of green debt held by firms becomes more costly, pushing the economy towards an undesirable equilibrium.



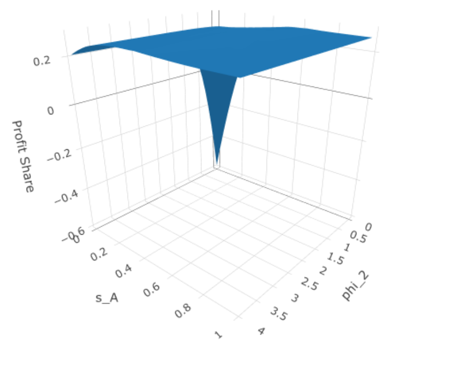
(a) Financing-to-Output Ratio



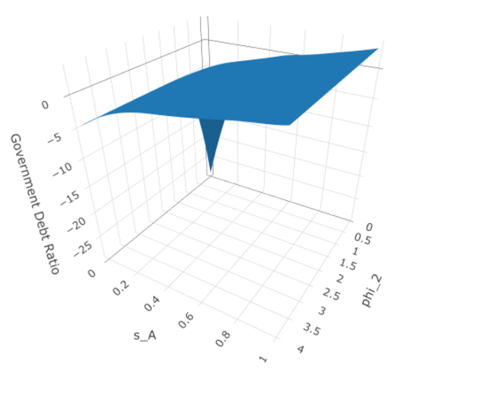
(b) Employment Rate



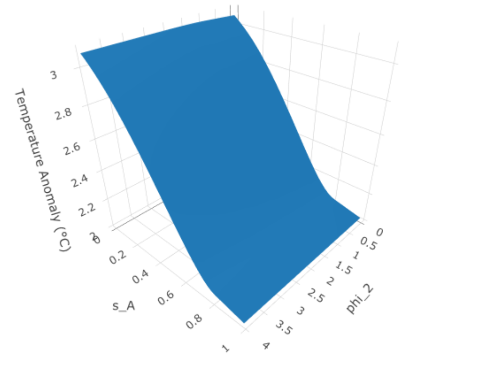
(c) Wage Share



(d) Profit Share

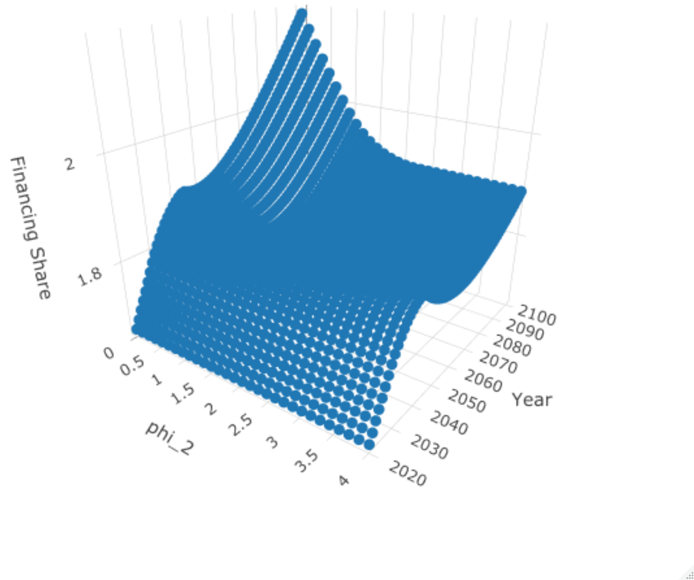


(e) Government Debt Ratio

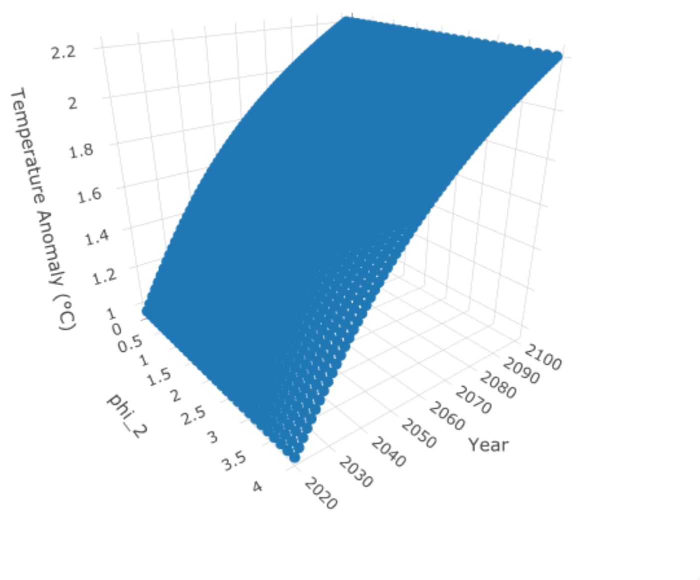


(f) Temperature Anomaly

Figure 4.3: Medium-term sensitivity analysis for a selection of economic and climate dynamics in the year 2100 for a range of abatement subsidy, s_A, and ϕ_2 parameters with a carbon tax. Note, ϕ_2 determines the magnitude of the central banks response to rising abatement costs.



(a) Financing-to-Output Ratio

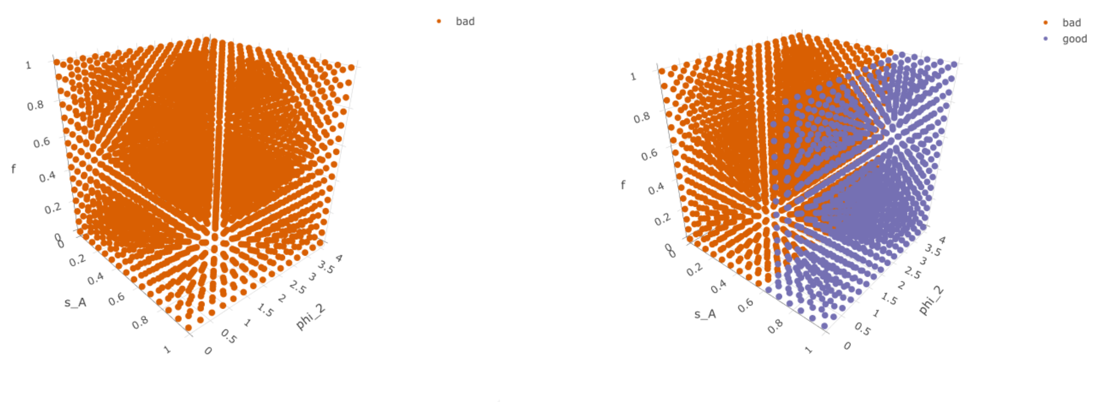


(b) Temperature Anomaly

Figure 4.4: Time plots of the share of private sector financing and temperature anomaly for *Strong policy scenario with green QE* for a range of ϕ_2 values.

4.3.3 Long-term Sensitivity Analysis

To study the post-transition dynamics induced by the central bank tapering its green bond-buying program we perform a sensitivity analysis where we vary the subsidy rate, $s_A \in [0, 0.99]$, the magnitude of the central bank's response to declining liquidity $\phi_2 \in [0, 4]$ and the portion of abatement funded by green fixed-income, $f \in [0, 1]$. Staying consistent with Chapter 3, we consider the potential economic outcomes with and without a carbon tax. Likewise, we keep the outcome classification unaltered, but to account for the addition of green bonds, we use the financing-to-output ratio to measure firm indebtedness levels instead of the debt share. Furthermore, due to the increase in the range of parameters being varied, we modify the number of equally spaced out points from which the parameters were selected to 15.



(a) No Carbon Tax (b) Carbon Tax

Figure 4.5: Economic outcomes for the Green Bond model with and without a carbon tax for a range of abatement subsidies, s_A, portion of abatement funded by green bonds, f, and the magnitude of central banks response to rising abatement costs, ϕ_2.

The results of the sensitivity analysis are summarized in *Figure 4.5*. Unsurprisingly, we see that without a carbon pricing scheme, no amount of fiscal intervention, monetary policy or green bonds maintain the economy on its stable growth path (See *Figure 4.5a*). Hence, we are once again reminded that climate policies should never be viewed as a one-size-fits-all solution.

Nonetheless, we can progress with our long-term analysis by including a carbon tax into the macroeconomic framework. Notably, we see that absent any green bonds; the economy converges towards the Solovian steady state once the subsidy rate reaches 66%, as in the baseline model (See *Figure 4.5b*). However, as the portion of abatement funded by green bonds increases, the paradoxical private sector behaviour becomes apparent. Moreover, since the central bank is restricted from pushing yields into negative territory, the magnitude of the central bank's QE program plays little role in minimizing the long-term effects of the Financial Jevon's Paradox. To make matters worse, once the central bank begins winding down the pace of its climate bond-buying program, the yield on green bonds quickly converges towards the interest rate on bank loans. Consequently, the private sector finds itself holding an excessive amount of debt, of which it can no longer pay the rising interest costs. Hence, only a higher subsidy rate from the government sector will reduce the equilibrium financing share and return the economy towards a balanced growth path.

Thus, a dovish central bank policy only delays the inevitable consequences of the Financial Jevon's Paradox. Nevertheless, as with a carbon tax and subsidy, the initial conditions of our economy may reveal whether or not an aggressive QE program is truly ineffective in sustaining the long-term equilibrium on a stable growth path. To answer this question, we examine the basin of attraction for our economic system.

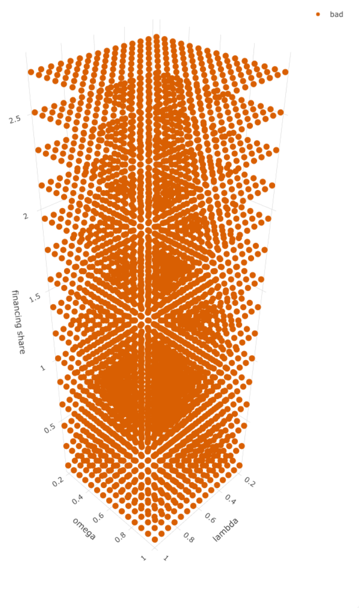
4.3.4 Basin of Attraction

For the basin of attraction analysis, we borrow all three policy scenarios from Chapter 3 and introduce green bonds as an alternative form of financing climate-related abatement efforts. Furthermore, for each scenario, we postulate that the central bank will do, as former ECB president Mario Draghi famously said, "whatever it takes" to reduce the financing costs of green bonds via a green asset purchasing program (Draghi, 2012). We capture this policy stance mathematically by setting $\phi_2 = 2.359$. Note, for the initial set of state variables and classification of economic outcomes, the debt share is replaced with the financing ratio to account for the inclusion of green fixed-income.

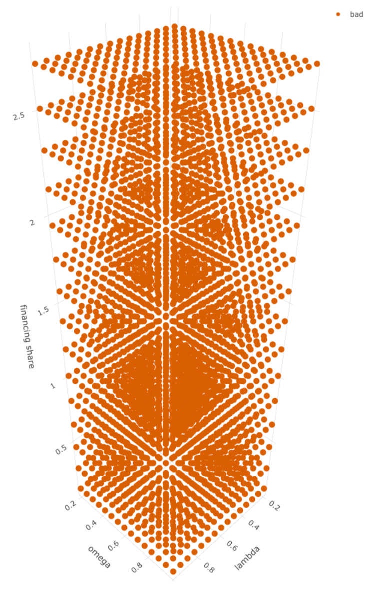
Looking at *Figure 4.6*, we see that regardless of how resilient the economy is, the entire set of initial conditions converges towards the bad equilibrium when green bonds and QE program are implemented as a stand-alone policy measure for the *No policy scenario with green QE* (See *Figure 4.6a*). Equivalently, if a carbon tax and green bond-buying program are enforced, the long-term equilibrium for the entire initial set of state variables again converges to the undesirable equilibrium (See *Figure 4.6b*).

Lastly, the policy mixture of a carbon pricing scheme, 66% subsidy on abatement and a green QE program does produce a region of good economic outcomes (See *Figure 4.6c*). However, when compared to the results for *Strong policy scenario* from the baseline model, it quickly becomes evident that the set of initial values associated with the good basin of attraction is significantly smaller. In other words, only an economy with very high resiliency stands to benefit from an aggressive QE program, which minimizes the effects of green bonds in the short term without accelerating the long-term financing share to unsustainable levels as the reserve bank winds down its corporate bond-buying program.

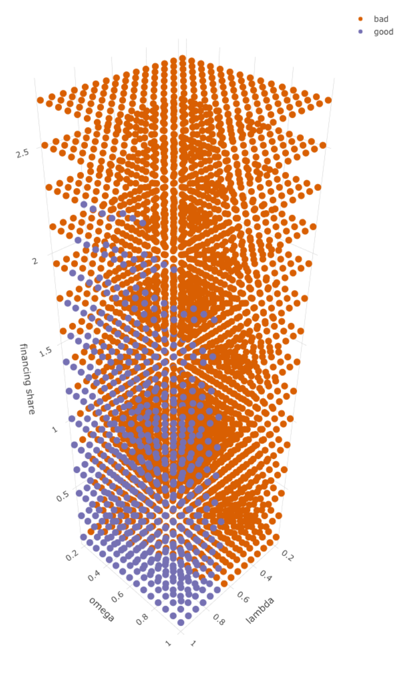
Hence, absent an increase in the subsidy rate or negative yields, we see that under most economic conditions, the central bank green bond-purchasing program is ineffective in minimizing the debt burdens caused by the counterintuitive behaviour of firms. To solidify our long-term analysis, we will demonstrate the deterministic dynamics of the *Strong policy scenario with green QE* in the following section.



(a) *No policy scenario with green QE*



(b) *Carbon tax scenario with green QE*



(c) *Strong policy scenario with green QE*

Figure 4.6: Basin of attraction for a range of initial conditions of the wage share, ω, employment rate, λ and financing share, φ. We include green bonds and quantitative easing (QE) program in all three climate policy scenarios.

4.3.5 Transitional Dynamics

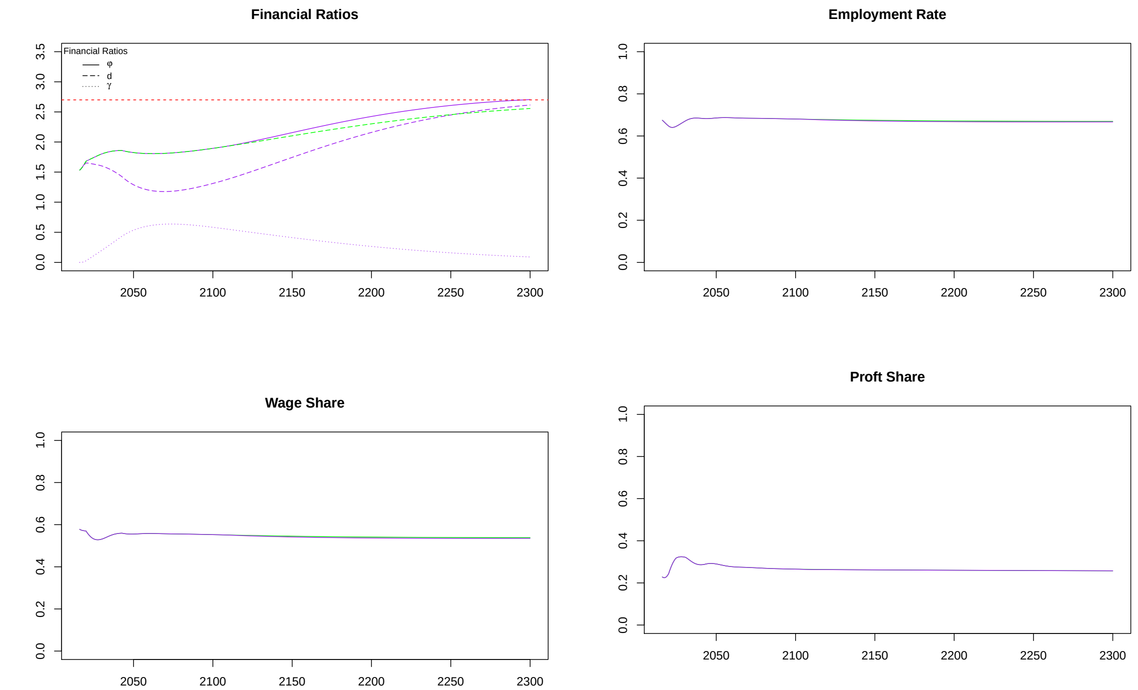
Figure 4.7 and *4.8* depicts the economic and climate dynamics for the *Strong policy scenario with green QE*. We also provide the dynamics of the equivalent scenario from the baseline model for comparison.

Hence, we see that during the transition period, QE keeps the yield on green bonds at the lower bound until the year 2100 and allows the dynamics between the two scenarios to remain essentially identical. In fact, by the end of the century, the dovish monetary policy is not only able to minimize the ramifications of the financial paradox but also manufacture a slightly lower financing share than the baseline *Strong policy scenario*.

As the damages from climate change are mitigated due to the substantial policy measures, the post-transition abatement costs begin to fall. Naturally, the central bank winds down the pace of its climate bond-buying program, reducing the spread between bank loans and green bonds. On the other side, the productive sector finds itself holding an excessive amount of green bonds, which were, up until now, an efficient and artificially cheaper alternative to finance abatement costs. Thus, as the yield on green bonds converges towards the rate on bank loans, the private sector can no longer meet the interest payments on the fixed-income instruments. As the central bank continues to taper green QE, the true level of firms' indebtedness is exposed, and the long-term financing-to-output ratio passes the 2.7 default threshold.

Additionally, we see that green bonds and an aggressive central bank do not affect the climate dynamics for the *Strong Policy Scenario* as net-zero emissions are still achieved by 2043, while the temperature anomaly passes the 2°C ceiling in the year 2077.

Finally, as with the baseline model, we will conclude the numerical analysis by considering the intrinsic uncertainty in our parameters and deterministic results by performing Monte Carlo simulations. Naturally, this will allow us to quantify the percentage of runs for the *Strong Policy Scenario with green QE* that stay below the 2.7 financing-to-output and 2°C thresholds.



(a)

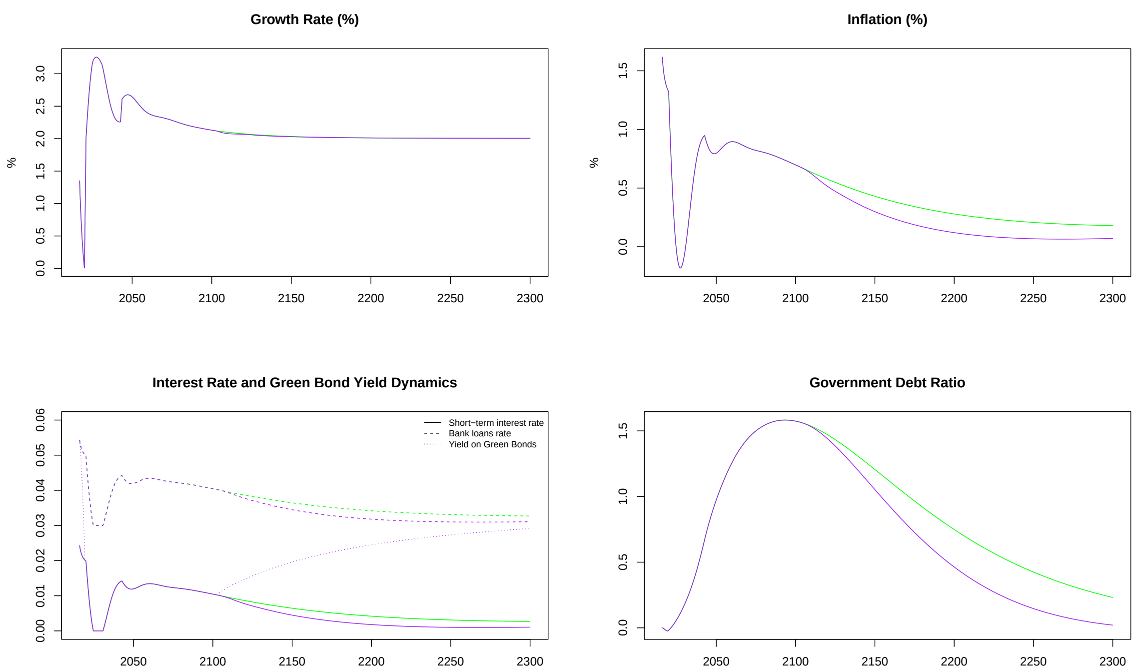


Figure 4.7: Deterministic macroeconomic trajectories for *Strong policy scenario* (green line) and the *Strong policy scenario with green QE* (purple line). The dotted red line in the debt share plot indicates the 2.7 financing-to-output threshold.

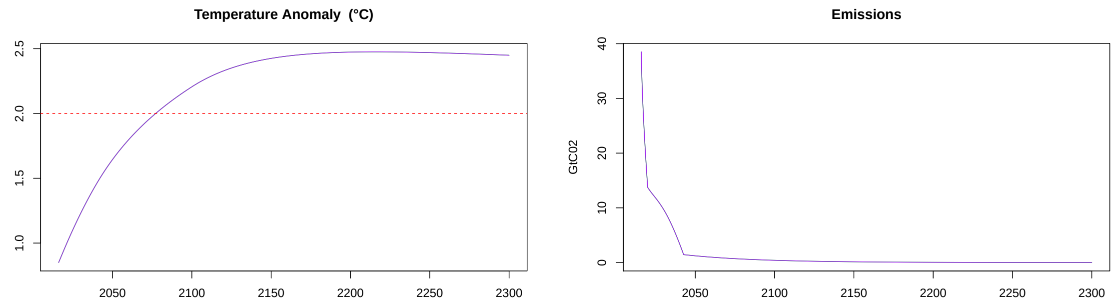


Figure 4.8: Deterministic climate trajectories for *Strong policy scenario* (green line) and the *Strong policy scenario with green QE* (purple line). The dotted red line in the temperature anomaly plot indicates the 2°C target set by the Paris Climate Agreement.

4.3.6 Monte Carlo

In the interest of brevity, we omit paraphrasing the methods and parameters varied for the Monte Carlo simulations as they remain unchanged from Chapter 3 and Bovari *et al.* (2018b).

Figure 4.9 displays the trajectories and probability intervals of the Monte Carlo simulations for both the *Strong policy scenario* and *Strong policy scenario with green QE*. Thus, with the addition of green bonds and a QE program, the total-financing to output ratio stays below the 2.7 default ceiling for 72.5% percent of the simulations. This is a 2.5% improvement when compared to the baseline *Strong policy scenario*. However, we see that the number of simulations that stay below the 2°C Paris Agreement falls to 39.3%. Moreover, the percentage of runs that stay below both limits declines to 31.0%. Note, the mean value for the private financing share, government debt share and the temperature anomaly in the year 2100 is 1.94, 2.23 and 2.16°C, respectively. We update the results from the baseline model Monte Carlo simulations in *Table 4.4* and *4.5*.

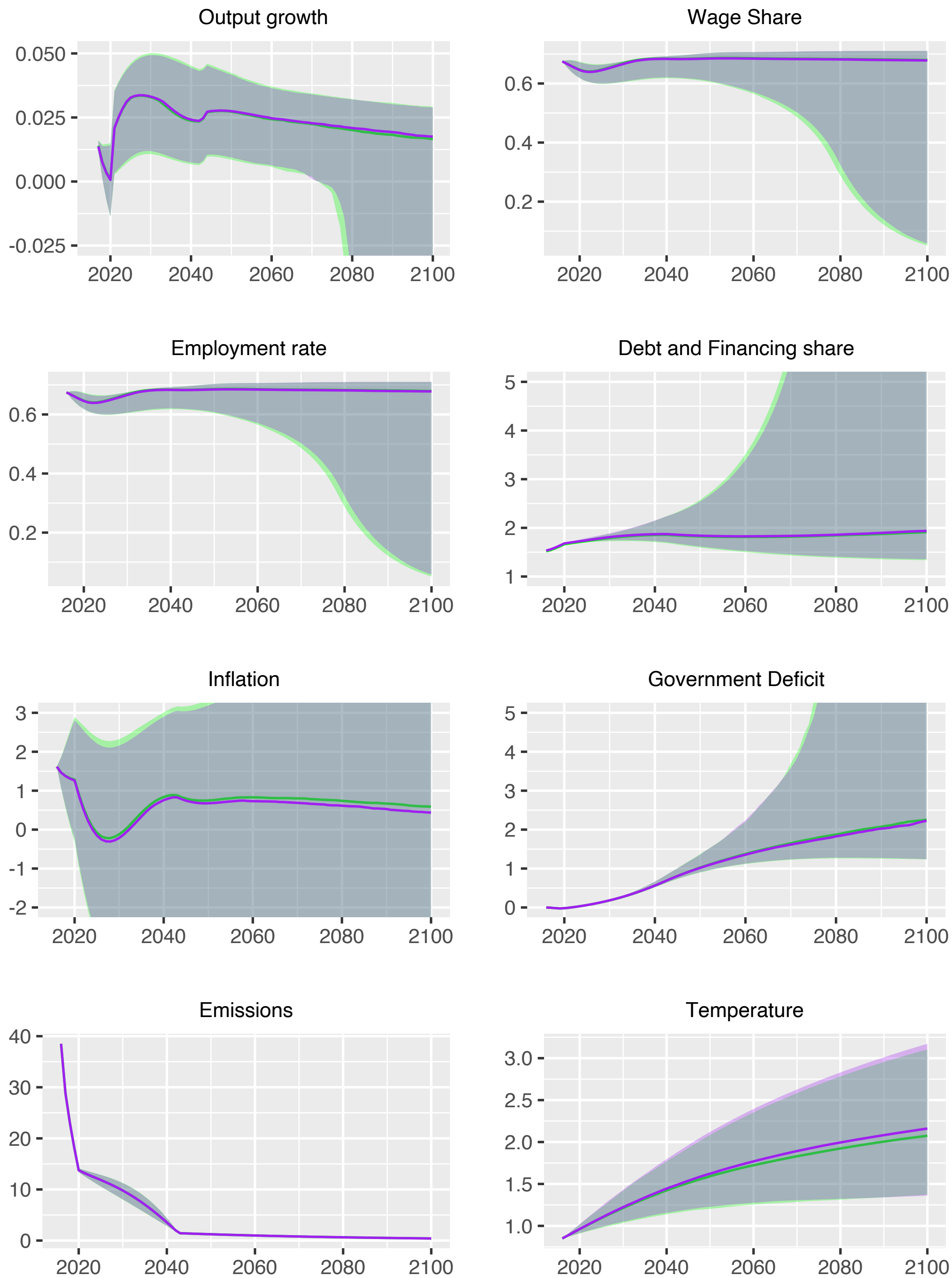


Figure 4.9: [0.05; 0.95] probability interval for the *Strong policy scenario* (green) and the *Strong policy scenario with green QE* (purple) .

Scenario	$T < 2°\text{C}$	$\{\varphi, d\} < 2.7$	$(T < 2°\text{C}) \cup (\{\varphi, d\} < 2.7)$
No policy	0.4%	30.6%	0.4%
Carbon tax	5.2%	10.7%	2.5%
Strong policy	44.1%	69.9%	33.1%
Strong policy with green QE	39.3%	72.4%	31.0%

Table 4.4: Updated percentage of runs where the temperature anomaly and financing/debt share stayed below the 2°C target and 2.7 debt-to-output threshold, respectively.

Scenario	Temperature Anomaly (°C)	Financing/Debt Share	Government Debt Share
No policy	4.17	6.14	-0.144
Carbon tax	2.97	14.3	-8.08
Strong policy	2.07	1.91	2.16
Strong policy with green QE	2.16	1.94	2.23

Table 4.5: Updated median values of the temperature anomaly, private and public financing/debt shares in the year 2100 for each climate policy scenario.

4.4 Conclusion

In this chapter, we have seen that providing the productive sector with a more efficient alternative to financing abatement efforts results in elevated debt levels that are antithetical to the entire purpose of green fixed-income. Moreover, under our current economic framework, the central bank finds itself ill-equipped to minimize the long-term consequences of an increase in the financialization of the macro-economy. This places an additional burden on an already overstretched government sector. To make matters worse, neither green bonds nor a green quantitative easing program accelerates the private sector's transition to net-zero.

However, herein lies the problem. The fundamental objective of green debt and a bond-buying program is to alleviate the firm's financial burden as the sector reduces CO_2 emissions through mitigation efforts such as carbon sequestration. They do nothing to incentivize firms to adopt a cleaner and less emission-intensive production process. In other words, green bonds and, more specifically, a green asset purchasing program are an inefficient economic policy combination. Naturally, this begs the question, are there other financial tools that will effectively promote the use of renewable energies? Moreover, what role can the central bank play in advocating for the issuance of these debt instruments that not only decreases financing costs but equally reduces emissions? As we will see in the following chapter, the solution is twofold: green loans and a green interest rate.

Chapter 5

Disaggregated Model

This chapter develops a disaggregated version of the Baseline and Green Bond models discussed above, where we separate the private sector's production processes between conventional and green methods. Furthermore, we introduce green loans as a third financing alternative to provide firms with a financial avenue towards more renewable energies. Moreover, we expand the central bank's role to incentivize the productive sector toward transitioning to less emission-intensive production methods through an innovative green interest rate. Once again, this work is inspired by the discrete-time model of Dafermos *et al.* (2018). Additionally, any model components discussed above in the previous chapters and not considered here indicate they remain unchanged with our previous work.

Subsequently, we explore the theoretical asymptotic stability of our disaggregated differential system and provide a direct comparison to the analysis covered in Chapter 3. Finally, we strengthen our theoretical interpretations by exploring the behaviour of the dynamical system numerically. More specifically, we will observe how incorporating green loans and technologies alter the climate policy landscape and produce a range of policy measures that successfully avoid a climate-induced economic bifurcation.

5.1 Accounting Framework

When examining the balance sheet matrix (See *Figure 5.1*), we see that the closed macroeconomic system shares many of the same assets and liabilities as to the Green Bond model. However, the productive sector now has a new capital resource in the form of green capital, K_G, while also utilizing green loans, L_f^G, to finance new investment. Moreover, despite explicitly differentiating between conventional or green capital and loans, the financial accounts essentially behave the same way within the accounting framework. More specifically, both conventional and green capital are assets on the firm's balance sheet, while conventional and green loans are denoted as liabilities. Naturally, to maintain stock-flow consistency, both loan types are expressed as assets to the banking sector. We will flesh out these newly introduced variables as we expand on their function through the behavioural assumptions discussed in the following subsection. Nonetheless, we can express the net worth of each sector as:

$$X_h = M_h \qquad \text{(Household Net Worth)} \tag{5.1}$$

$$X_f = pK_C + pK_G + M_f - L_f^C - L_f^G - \Gamma \qquad \text{(Firms Net Worth)} \tag{5.2}$$

$$X_b = L_f^C + L_f^G - M + R. \qquad \text{(Banks Net Worth)} \tag{5.3}$$

$$X_g = -B. \qquad \text{(Government Sector Net Worth)} \tag{5.4}$$

$$X_{cb} = B + \Gamma - R. \qquad \text{(Central Bank Net Worth)} \tag{5.5}$$

Moving along, when looking at the transaction flow matrix of the five sector economy, we see that the incoming and outgoing monetary flows remain identical to our previous SFC framework. However, we have made a further disaggregation between conventional and green investment, capital depreciation and the interest payment on loans (See *Table 5.2*). The flow of fund matrix can be seen in *Table 5.3*.

	Households	Firms	Banks	Government Sector	Central Bank	Sum
Balance Sheet						
Conventional Capital		pK_C				pK_C
Green Capital		pK_G				pK_G
Deposits	M_h	M_f	$-M$			
Conventional Loans		$-L_f^C$	L_f^C			
Green Loans		$-L_f^G$	L_f^G			
Treasury Bills (Bonds)				$-B$	B	
Reserves			R		$-R$	
Green Bonds		$-\Gamma$			Γ	
Sum (Net worth)	X_h	X_f	X_b	X_g	X_{cb}	X

Table 5.1: Balance Sheet Matrix for Disaggregated Model

	Households	Firms		Banks	Government Sector	Central Bank	Sum
Transactions		current	capital				
Consumption	$-pC$	pC					
Conventional Investment		pI_C	$-pI_C$				
Green Investment		pI_G	$-pI_G$				
Gov. Spending		pG			$-pG$		
Acc. memo [GDP]		$[pY]$					
Wages	W	$-W$					
Conventional Capital Depr.		$-\delta_D pK_C$	$\delta_D pK_C$				
Conventional Capital Depr.		$-\delta_D pK_G$	$\delta_D pK_G$				
Carbon Taxes		$-pT^C$			pT^C		
Non-Carbon Taxes	$-pT_h$	$-pT_f$			pT		
Abatement subsidies		pS^C			$-pS^C$		
Int. on Conventional loans		$-r_L L_f^C$		$r_L L_f^C$			
Int. on Green loans		$-r_G L_f^G$		$r_G L_f^G$			
Banks' Dividends	Π_b			$-\Pi_b$			
Firms' Dividends	Π_d	$-\Pi_d$					
Int. on Deposits	$r_M M_h$	$r_M M_f$		$-r_M M$			
Int. on Bonds					$-r_B B$	$r_B B$	
Int. on Reserves				$r_R R$		$-r_R R$	
Int. on Green Bonds		$-r_\Gamma \Gamma$				$r_\Gamma \Gamma$	
Central Bank Profits					Π_{cb}	$-\Pi_{cb}$	
Column Sum (Balance)	S_h	Π_r	$-pI + \delta_D pK$	S_b	S_g	S_{cb}	

Table 5.2: Transactions Flow Matrix for Disaggregated Model

5.2 Macroeconomic Model

In this section, we explore the continuous-time dynamics for the disaggregated economic model. In the interest of brevity, we omit the labour market and inflation dynamics as the differential, and auxiliary equations remain unchanged from the baseline model.

5.2.1 Production and Sales

As before, we continue to presume that firms produce a potential output, Y^0, of a homogeneous good. However, the sector can now choose between two different production methods: conventional capital, K_C, or green capital, K_G. As in Dafermos *et al.* (2018), we make a crucial distinction between conventional and green capital. More specifically, we assume that green capital produces commodities using renewable

	Households	Firms	Banks	Government Sector	Central Bank	Sum
Flow of Funds						
Change in Conventional Capital		$p\dot{K}_C$				$p\dot{K}_C$
Change in Green Capital		$p\dot{K}_G$				$p\dot{K}_G$
Change in Deposits	$\dot{M}_h$	$\dot{M}_f$	$-\dot{M}$			
Change in Conventional Loans		$-\dot{L}_f^C$	$\dot{L}_f^C$			
Change in Green Loans		$-\dot{L}_f^G$	$\dot{L}_f^G$			
Change in Bills (Bonds)				$-\dot{B}$	$\dot{B}$	
Change in Reserves			$\dot{R}$		$-\dot{R}$	
Change in Green Bonds		$-\dot{\Gamma}$			$\dot{\Gamma}$	
Column Sum (Savings)	S_h	$S_f = \Pi_r$	S_b	S_g	S_{cb}	
Change in Net Worth	$\dot{X}_h = S_h$	$\dot{X}_f$	$\dot{X}_b = S_b$	$\dot{X}_g = S_g$	$\dot{X}_{cb} = 0$	$\dot{p}(K_C + K_G) + p(\dot{K}_C + \dot{K}_G)$

Table 5.3: Flow of Funds Matrix for Disaggregated Model

resources, such as wind, solar or hydro. Meanwhile, conventional capital produces goods using non-renewable energies such as fossil fuels. Accordingly, green production has a lower emission intensity than conventional capital and releases less industrial emissions into the atmosphere. In essence, the adoption of green capital leads to a lower-carbon economy. Hence, we can formalize the two distinct production processes as

$$Y_C^0 := \frac{K_C}{\nu} \tag{5.6}$$

$$Y_G^0 := \frac{K_G}{\nu} \tag{5.7}$$

$$Y^0 := Y_C^0 + Y_G^0 = \frac{K_C}{\nu} + \frac{K_G}{\nu} = \frac{K}{\nu} \tag{5.8}$$

where Y_C^0 denotes the potential output produced using conventional capital, Y_G^0 represents the potential output produced using green capital, and $\nu > 0$ is the constant capital-to-output ratio. We note that the summation $K := K_C + K_G$ determines the total stock of capital. Correspondingly, the ratio of green capital to total can be defined as

$$\varkappa := \frac{K_G}{K}. \tag{5.9}$$

Taking into account any private sector abatement activities, A, and the fraction of output destroyed as a consequence of climate change, D^Y, the expression determining the final level of production remains unchanged from before and is represented as

$$Y := (1 - D^Y)(1 - A)Y^0 \tag{5.10}$$

5.2.2 Emissions, Taxation and Abatement Decisions

As expected, the production of commodities releases industrial emissions, E_{ind}, according to

$$E_{ind} := Y^0\sigma(1 - n). \tag{5.11}$$

where, n continues to define the endogenously determined emission reduction rate. However, we redefine the emission intensity to be the convex combination of the carbon intensity released by conventional and green production. More formally,

$$\sigma := \varkappa\sigma_G + (1 - \varkappa)\sigma_C \tag{5.12}$$

where conventional, σ_C, and green emission intensity, σ_G, evolve exogenously according to

$$\dot{\sigma_C} := g_{\sigma_C}\sigma_C \tag{5.13}$$

$$\dot{g_{\sigma_C}} := \delta_{g_{\sigma_C}} g_{\sigma_C} \tag{5.14}$$

$$\dot{\sigma_G} := g_{\sigma_G}\sigma_G \tag{5.15}$$

$$\dot{g_{\sigma_G}} := \delta_{g_{\sigma_G}} g_{\sigma_G} \tag{5.16}$$

Moreover, we posit that green technologies become more efficient over time and reach economies of scale such that conventional capital also emits lower emissions after sufficient time has passed (See *Figure 5.1*). We capture this mathematically by setting the initial conditions for (5.14) and (5.16) to be $g_{\sigma_G} < g_{\sigma_C}$, while also maintaining $\delta_{g_{\sigma_C}} < \delta_{g_{\sigma_G}} < 0$ for the parameters controlling the rate of decrease.

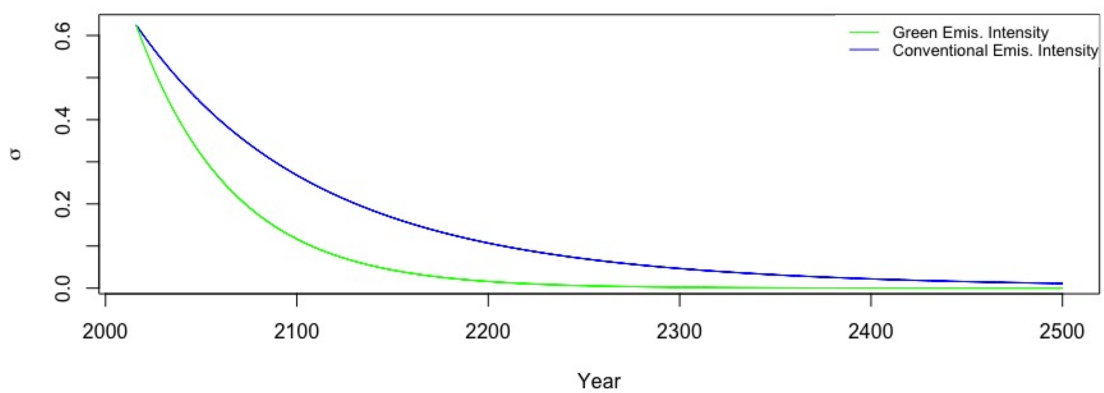


Figure 5.1: Exogenous dynamics for conventional and green emission intensity

Similarly, we postulate that abatement no longer depends on a single production method and that renewable energies can be adopted throughout the abatement process. More specifically, we let

$$\theta := \varkappa\theta_G + (1-\varkappa)\theta_C > 0 \tag{5.17}$$

such that convexity parameter in the abatement cost is now the convex combination of conventional and green parameters. Note, $\theta_G > \theta_C$, implying that green abatement is a cheaper substitute compared to conventional abatement. For completeness, we restate the portion of output dedicated towards mitigation activities and the endogenously determined emission reduction rate as

$$A := \frac{\sigma p_{BS}}{\theta} n^{\theta} \tag{5.18}$$

and

$$n = \min\left\{ \left(\frac{p_c}{(1-s_a)p_{BS}} \right)^{\frac{1}{\theta-1}} ; 1 \right\}, \tag{5.19}$$

respectively.

5.2.3 Firm Financing

Using the transaction flow matrix (*Table 5.2*), we can redefine the private sector's nominal profit before dividends as

$$\Pi := pY - \mathrm{w}L - \delta_{\mathbf{D}} pK - r_L L_f^C - r_G L_f^G + r_M M_f - r_\Gamma \Gamma + p(S^C - T^C). \tag{5.20}$$

Thus, the total cost of production is now contingent on:

(i) the money wage bill, wL;

(ii) the capital depreciation $\delta_{\mathbf{D}} pK$, where $\delta_{\mathbf{D}} := \delta + \mathbf{D}^K$ is the capital depreciation rate obtained as a constant rate δ plus damages to capital D^K;

(iii) the interest paid on conventional bank loans, $r_L L_f^C$, and green loans $r_G L_f^G$ where r_L, r_G, represent the interest rate on conventional and green loans, respectively;

(iv) the interest received on deposits, $r_M M_f$, where r_M represent the interest rate on deposits;

(v) the interest payment on green bonds, $r_\Gamma \Gamma$, where r_Γ is the yield on green bonds and, Γ, represents the dollar amount of green bonds issued; and

(vi) the net transfers, $(S^C - T^C)$, to the productive sector.

Following the distribution of dividends to shareholders, the private sector's investment process is formalized in two stages, as in Dafermos *et al.* (2018). During the first phase, firms decide their aggregate demand for investment, I, which is driven by the profit share and captures the risk appetite of the productive sector

$$I := \kappa(\pi) Y \tag{5.21}$$

where, $\kappa(\cdot)$ is consistent with the investment function adopted in our previous work (See *Figure 3.2)*. At the second stage, firms determine the amount of green investment, I_G, as a proportion, β, of the overall desired investment, according to

$$I_G := \beta(\delta_G) I \tag{5.22}$$

where $\beta(\cdot)$ is an increasing function that depends on the interest rate spread between conventional loans and green loans, δ_G, which is to be defined shortly (See *Figure 5.2)*. Naturally, the demand for conventional investment, I_C, is then determined to be the residual amount

$$I_C := I - I_G. \tag{5.23}$$

Correspondingly, we differentiate between the capital accumulation funded by green investment and conventional investment according to

$$\dot{K}_G := I_G - \delta_D K_G \tag{5.24}$$

$$\dot{K}_C := I_C - \delta_D K_C \tag{5.25}$$

$$\dot{K} := \dot{K}_G + \dot{K}_C \tag{5.26}$$

As before, retained profits may not be enough to fund the private sector's gross demand for investment. Hence, firms can now issue debt through the issuance of either green bonds, conventional or green bank loans. We continue to assume firms will first issue green bonds to cover abatement expenses, before turning to the banking sector to meet any remaining demand for investment. Thus, we can express the private sector's green loan dynamics as

$$\dot{L_f^G} := pI_G - \delta_D p K_G + r_G L_f^G - \kappa_G L_f^G \tag{5.27}$$

where κ_G denotes the repayment rate on green loans. Similarly, we can represent the dynamics of conventional loans as

$$\dot{L_f^C} := pI_C - \delta_D p K_C + r_L L_f^C - \kappa_L L_f^C \tag{5.28}$$

Moreover, we adapt the private sector deposits to evolve according to

$$\dot{M}_f = pY - \mathrm{w}L - \delta_D pK + r_M M_f - \kappa_L L_f^C - \kappa_G L_f^G + \dot{\Gamma} - \kappa_\Gamma \Gamma + p(S^C - T^C) - \Pi_d \quad (5.29)$$

Thus, we can express the change in total financing by the private sector as

$$\dot{F} := (\dot{L_f^C} + \dot{L_f^G} + \dot{\Gamma}) - \dot{M}_f \quad (5.30)$$

Likewise, we redefine the total debt to be the sum of both conventional and green loans minus any deposits, or

$$\dot{D} := (\dot{L_f^C} + \dot{L_f^G}) - \dot{M}_f \quad (5.31)$$

Finally, the financing-to-output, debt-to-output, conventional loan-to-output, green loan-to-output, green bond-to-output and firm's deposits-to-output ratios can be shown as

$$\varphi := \frac{F}{pY}, \quad d := \frac{D}{pY} \quad \ell_C := \frac{L_f^C}{pY} \quad (5.32)$$

$$\ell_G := \frac{L_f^G}{pY} \quad \gamma := \frac{\Gamma}{pY} \quad m_f := \frac{M_f}{pY} \quad (5.33)$$

respectively.

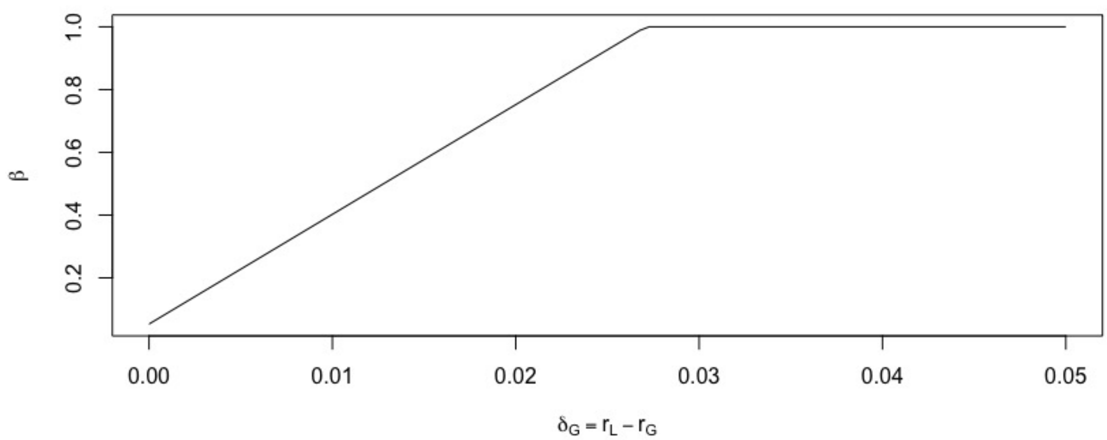


Figure 5.2: $\beta(\cdot)$

5.2.4 Central Bank Policies

With the introduction of green capital and loans, the central bank will act as a catalyst for carbon-intensive firms to adopt more renewable technologies during their production process. With this in mind, we postulate that as emissions prior to any carbon sequestration grows, the central bank will intervene and increase the spread between the green and conventional loan interest rates, causing the green borrowing rate to decline. Naturally, this will encourage the private sector to allocate a more significant portion of funds toward energy-efficient production. On the other hand, if pre-abatement emissions fall, the central bank will reduce the spread between rates resulting in firms becoming indifferent between green and conventional investment. We can capture this policy intervention by defining the spread between the bank loans rate, r_L, and the green interest rate as

$$\delta_G = \phi_3(\frac{\sigma Y^0}{E_{init}}) \tag{5.34}$$

where δ_G is subsequently used to set the green interest rate according to

$$r_G = \max\{r_B, r_L - \delta_G\}. \tag{5.35}$$

Note, E_{init} represents the initial amount of CO_2 emissions released by the productive sector, while ϕ_3 denotes the degree of the central banks' response to a change in carbon emissions.

Now, we must stress that the mechanisms through which our green interest rate is actualized are beyond the scope of our economic framework. Moreover, the exact manner in which a green interest rate should be implemented is still a subject of great debate. For instance, the European Union and Commission are considering introducing a green supporting factor (GSF) and brown penalty (BP) on capital reserve requirements for private banks, where "green" investments would have lower capital requirements than carbon-intensive "brown" assets (Thomä and Gibhardt, 2019). Alternatively, the banking sector has started to adopt sustainability-linked loans (SLLs), which also mirror our green interest rate. Briefly, SSLs behave like any other loan instrument. However, lenders and borrowers agree to financial terms that are predicated on specific sustainability targets. In other words, as firms meet green targets such as increased energy efficiency, lower greenhouse gas emissions, or even savings on water consumption, the interest rate on SLLs decreases. On the other hand, if the borrower fails to meet these green targets, then the interest rate on SLLs increases (Association, 2020). Ultimately, irrespective of the financial mechanism through which the policy measure is realized, the design and purpose remain the same: to penalize the use of non-renewable resources and incentivize a shift towards renewable energy. This concept is successfully symbolized in expressions (5.34) and (5.35), respectively.

5.2.5 Post-Transition Reduced-form Dynamical System

The disaggregated model forms a 28-dimensional differential system, including the climate module from Bovari *et al.* (2018a) and (2018b). Naturally, using the same assumptions from Chapter 3 and methodology from Grasselli and Huu (2015) and Grasselli and Lipton (2019) we can reduce the main macroeconomic model to the following three-dimensional system of differential equations

$$\begin{cases} \dot{\omega} = \omega\left[\Phi(\lambda) - i - \alpha\right] \\ \dot{\lambda} = \lambda\left[g - \alpha - \beta(N)\right] \\ \dot{\varphi} = -\varphi[i+g] + \kappa(\pi) + r_G \ell_G + r_L \ell_C - 1 + \omega - r_M m_f + \Delta(\pi) \end{cases} \tag{5.36}$$

with the auxiliary equations defining the profit share, inflation, growth rate of output and population depicted as

$$\pi := 1 - \omega - r_L \ell_C - r_G \ell_G + r_M m_f - \frac{\delta_{\mathbf{D}} \nu}{(1 - \mathbf{D}^Y)} \tag{5.37}$$

$$i := \frac{\dot{p}}{p} := \eta(\xi\omega - 1) \tag{5.38}$$

$$g := \frac{\dot{Y}}{Y} = \frac{(1 - \mathbf{D}^Y)\kappa(\pi)}{\nu} - \delta_{\mathbf{D}} \tag{5.39}$$

$$\beta(N) := \frac{\dot{N}}{N} = \delta_N \left(1 - \frac{N}{\bar{N}}\right) \tag{5.40}$$

We again obtain the secondary differential system containing the state variables (b, ρ)

$$\begin{cases} \dot{b} = r_R \rho - b(i+g) \\ \dot{\rho} = \rho\left(r_R - i - g\right) \end{cases} \tag{5.41}$$

Thus, assuming no inflation, that the population has reached its carrying capacity, and that pre-abatement emissions have reached zero such that the private sector is indifferent between green and conventional investment, we can represent the Solovian equilibrium for the disaggregated model as

$$\begin{cases} \omega_{eq} = 1 - r_L \ell_{C_{eq}} - r_G \ell_{G_{eq}} + r_M m_{f_{eq}} - \frac{\delta_{\mathbf{D}_{eq}} \nu}{(1 - \mathbf{D}^Y_{eq})} \\ \lambda_{eq} = \Phi^{-1}(\alpha) \\ \varphi_{eq} = \dfrac{\kappa(\pi_{eq}) + r_L \ell_{C_{eq}} - 1 + \omega_{eq} - r_M m_{f_{eq}} + \Delta(\pi_{eq})}{\alpha} \end{cases} \tag{5.42}$$

Additionally, the equilibrium growth rate and profit share can be represented as

$$g_{eq} = \alpha \tag{5.43}$$

$$\pi_{eq} = \kappa^{-1} \left[\frac{\nu(\alpha + \delta_{\mathbf{D}_{eq}})}{1 - \mathbf{D}^Y_{eq}} \right] \tag{5.44}$$

Moreover, provided α is large enough the supplementary system simplifies to

$$\begin{cases} b_{eq} = \rho_{eq} = \frac{r_R \rho_{eq}}{\alpha} = 0 \end{cases} \tag{5.45}$$

Hence, we see that the Disaggregated model is subject to the destabilizing effects of climate change in a similar manner as the Baseline model. More specially, when looking at (5.44) and (5.42), the increase in the investment required to offset a surge in either damage to output or capital results in a corresponding rise in the asymptotic financing share. Naturally, as the asymptotic temperature anomaly increases, this behaviour is magnified, and we observe an infinite financing-to-output ratio with a collapsing wage share and employment rate synonymous with the bad equilibrium.

However, the introduction of green loans creates another channel through which the public authorities can reduce the financing share of the productive sector via the green borrowing rate, r_G. Moreover, by maintaining a wide spread between the conventional and green bank loan rates, the central bank incentivizes the adoption of more green technologies that are conducive to a lower-carbon economy. Naturally, this reduces the equilibrium temperature anomaly and the long-term climate damages absorbed by the financial sector. Furthermore, the burden of the government sector to maintain the economy on a stable growth path is reduced since both fiscal and monetary authorities can now effectively share the responsibility.

This phenomenon will be illustrated numerically in the following section. Likewise, we will see how in combination with a carbon tax, abatement subsidies and quantitative easing, a green interest rate generates a wide range of policy combinations where the economy converges towards the good equilibrium. Naturally, this increased versatility enables a further examination of which policy interventions produce the most desirable outcomes for various initial economic conditions.

5.3 Numerical Analysis

For the numerical analysis, we will begin by examining the deterministic dynamics for the *Strong policy scenario* that incorporates both green loans and a green borrowing rate. We will then turn to the long-term behaviour of the disaggregated model and observe the expanded range of policy specifications that successfully maintain the economy on a balanced growth path. Afterwards, we will briefly deviate from our usual sensitivity analysis and observe how the firm sectors' willingness to adopt green technologies can also play a crucial role in preventing a climate-induced economic bifurcation. Finally, we will solidify our interpretations by exploring the basin of attraction, transitional dynamics, and Monte Carlo results for a range of climate policy scenarios.

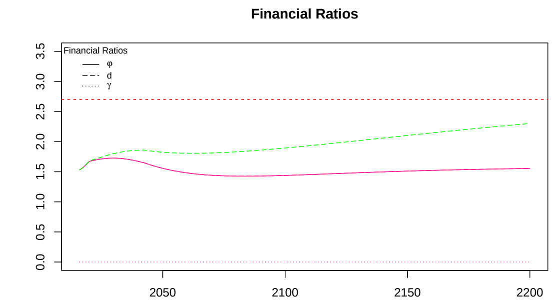
5.3.1 Deterministic Analysis

Figure 5.3 and *5.4* illustrates the economic and climate dynamics for the *Strong policy scenario* with the inclusion of green loans and interest rate, respectively. We prevent firms from issuing green bonds to finance abatement efforts and the central bank implementing a green QE program for illustration purposes. We have also included the *Strong policy scenario* from the baseline chapter to provide a meaningful comparison.

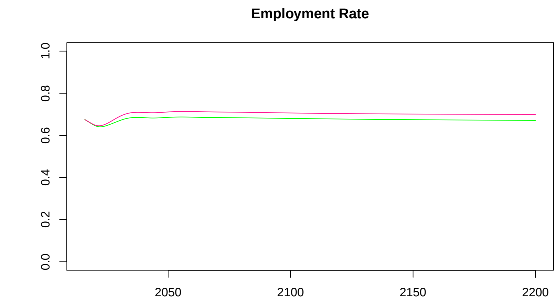
Thus, we see that when combined with a carbon tax and 66% government subsidy on abatement, the addition of green loans and a green interest rate significantly decrease the productive sector's financing share (See *Figure 5.3a*). Moreover, the favourable spread between the conventional and green loan borrowing rates improves the firm's short-term profit-share and entices the sector to seek more investment in green technologies. Naturally, the subsequent spike in production and the growth rate increases the employment rate, wage share, and inflation. Additionally, the rapid adoption of green capital leads to a corresponding decrease in the abatement costs and the government debt ratio, as well (*See Figure 5.4b*). Following the successful transition towards net-zero emissions, we see the growth rate and the economy as a whole beginning to stabilize towards a balanced growth path.

Looking at the climate trajectories, as the productive sector increasingly shifts production towards renewable energies, there is a analogous decrease in the emission intensity (See *Figure 5.4c* and *5.4f*). This reduces CO_2 emissions quicker, which is represented by the area under the curve being smaller compared to the baseline *Strong policy scenario* (See *Figure 5.4e*). Consequently, the temperature anomaly decreases, even if only marginally (See *5.4d*).

Thus, the benefits of including green loans as an alternative financing instrument designed to promote renewable energies in the production process are already visible. Moreover, if the central bank adopts a flexible policy position that effectively entices firms to adopt green technologies, the government's financial burden to bankroll abatement subsidies is significantly reduced. As we will soon discover, a green interest rate can drastically alter the long-term economic outcomes for our policy scenarios.



(a)



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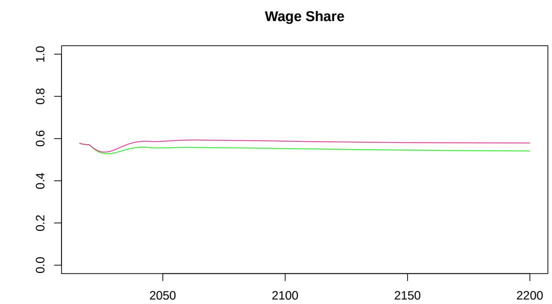
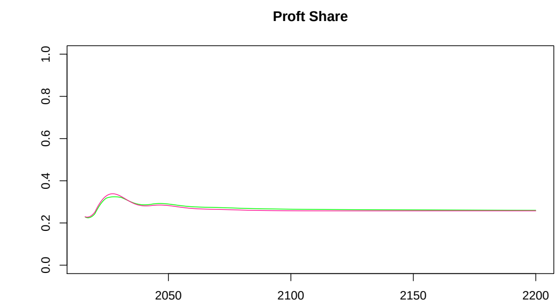


(c)



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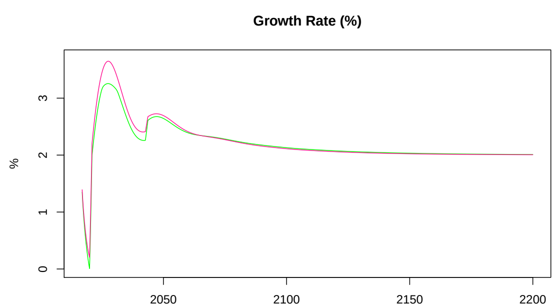
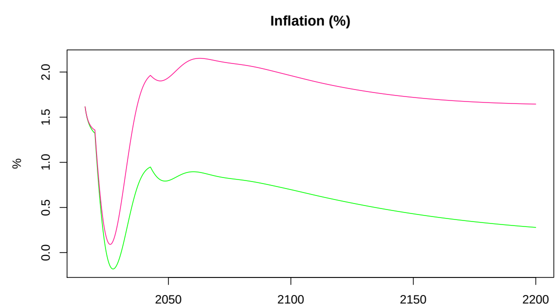


(e)



(f)

Figure 5.3: Deterministic macroeconomic trajectories for *Strong policy scenario* (green line) and the *Strong policy scenario* with green loans and interest rate (pink line). The dotted red line in the debt share plot indicates the 2.7 debt-to-output threshold.

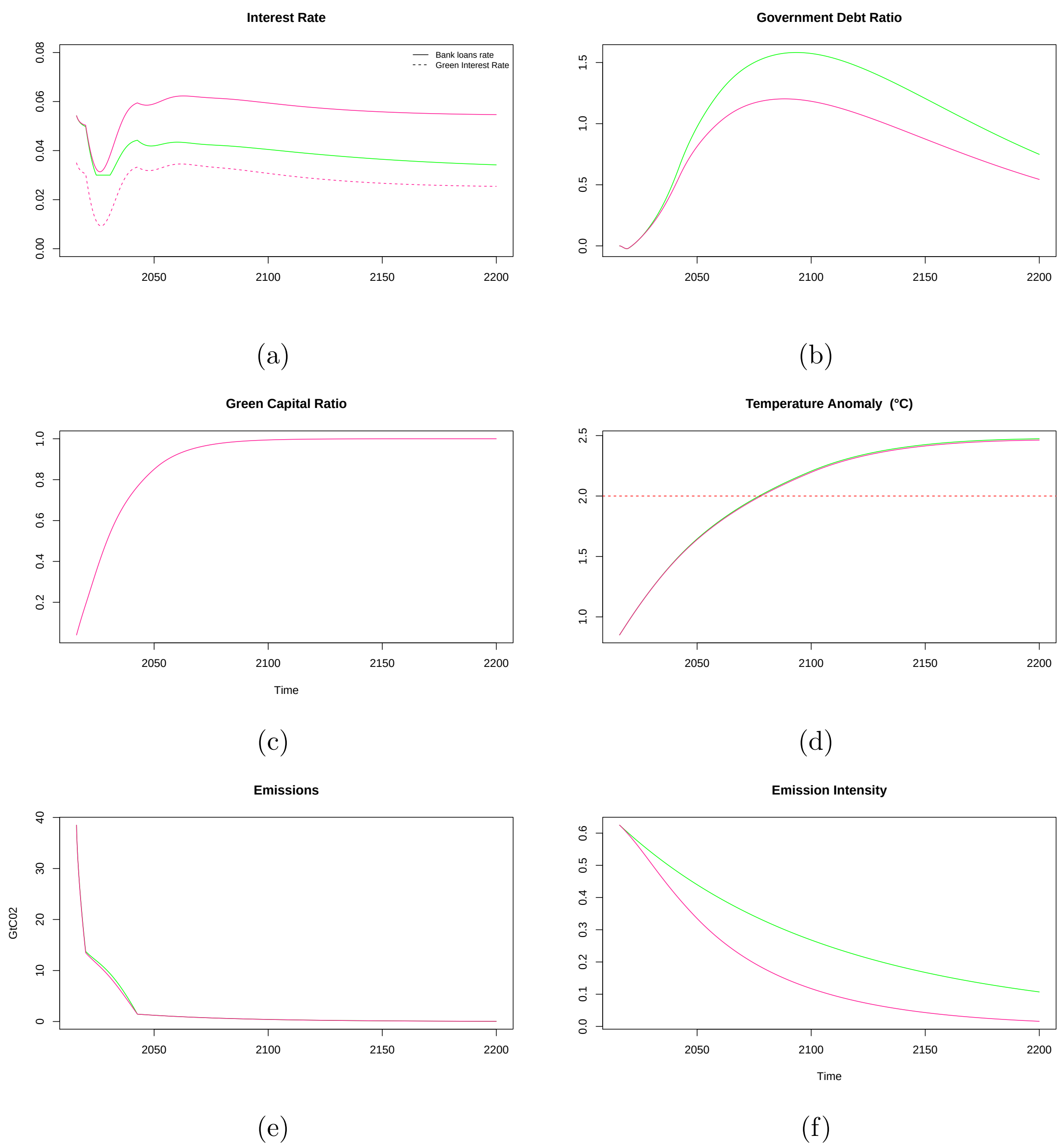
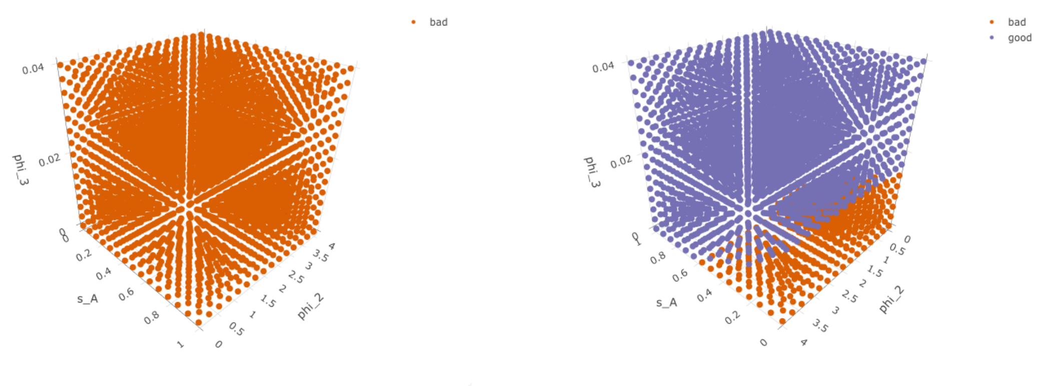


Figure 5.4: Supplementary economic trajectories and deterministic climate trajectories for *Strong policy scenario* (green line) and the *Strong policy scenario* with green loans and interest rate (pink line). The dotted red line in the temperature anomaly plot indicates the 2°C target set by the Paris Climate Agreement.

5.3.2 Long-term Analysis

Within the Disaggregated Model's economic framework, the productive sector can issue green bonds to cover abatement expenses while also using conventional or green loans to change their total stock of capital destined for production. Meanwhile, the government can influence abatement costs through subsidies, while the central bank can ease financial conditions by implementing a green QE program or propel a shift towards renewable energies through a green interest rate.

Thus, to examine the interactions and trade-offs between each of these three policy mechanisms and debt instruments, we vary the subsidy rate, $s_A \in [0, 0.99]$, and the magnitude of the central bank's response to both rising abatement costs, $\phi_2 \in [0, 4]$ and CO_2 emissions, $\phi_3 \in [0, 0.04]$. Staying in line with Chapters 3 and 4, we consider the economic equilibria with and without a carbon tax and maintain the same outcome classification. The results are pictured in *Figure 5.5*.



(a) No Carbon Tax (b) Carbon Tax

Figure 5.5: Economic outcomes for the Disaggregated model with and without a carbon tax for a range of abatement subsidies, s_A, and the central banks responses to both rising abatement costs,ϕ_2, and emissions ϕ_3.

Identical to our previous sensitivity results, in the absence of a carbon pricing scheme, no amount of exotic debt instruments, government or central bank intervention prevents the economy from converging towards the climate-induced bad equilibrium (See *Figure 5.5a*).

However, with a carbon tax in effect, the amount of good economic outcomes drastically improves. Furthermore, we see an inverse relationship emerge between the magnitude of the central bank's green interest rate policy and the government sector's subsidy rate. More specifically, as the central bank becomes increasingly willing to maintain a large spread between the conventional and green interest rates, the subsidy parameter required for the economy to converge to the good equilibrium decreases. In fact, beyond $\phi_3 \geq 0.017$, the government does not need to provide any abatement subsidies as the rapidly falling emission intensity due to the adoption of renewable energies limits abatement costs for the productive sector.

Additionally, although a green QE policy has a more negligible impact on the overall asymptotic economic stability, when merged with green loans and interest rates, the climate bonds and asset-purchasing program slightly reduce the government subsidy rate required to ensure a stable growth path. The rationale here is that firms employ the cheaper green technologies for abatement efforts and not only production. Thus, the dual effect of rapidly decreasing emission intensity and abatement costs minimizes the necessary dollar amount of green bonds. If we include an aggressive QE program, the long-term effects of the Financial Jevon's Paradox are entirely eliminated.

Hence, through a dovish green interest rate policy, green loans ease the debt burdens of the government sector and indirectly improve the efficacy of green securities and quantitative easing programs while also decreasing the long-term climate equilibrium. In addition to the policy implications of incorporating green loans into the macroeconomic system, another aspect worth examining is whether the rate and quantity of green capital the firm sector is willing to adopt plays a role in the economy converging towards the good equilibrium. Thus, we slightly digress from our usual analysis to explore this idea further.

5.3.3 $\beta(\cdot)$ Sensitivity Analysis

To examine how the productive sector's compliance to embrace green technologies impacts the economic equilibrium, we range the calibrated parameters of the linear $\beta(\cdot)$ function, which determines the portion of investment dedicated toward green methods. Accordingly, we vary the slope of the function, denoted $\beta_\pi \in [0, 2]$, the maximum amount of green investment, $\beta_{max} \in [0, 1]$, and the extent of the central banks response to a variation in CO_2 emissions, $\phi_3 \in [0, 0.04]$. Note that we only display the sensitivity grid for the *Carbon tax scenario* with green loans and interest rate since the remaining policy scenarios resulted in either entirely good or bad economic outcomes (*See Figure 5.6*).

Thus, we see there is a well-defined region of good economic outcomes for the range $\phi_3 \in [0.017, 0.04]$, $\beta_\pi \in [0.2857, 2]$ and $\beta_{max} \in [0.85, 1]$. Note, our calibrated $\beta(\cdot)$ values fall well within this region. In economic terms, this implies that the long-term equilibrium is less sensitive to the rate of adoption of renewable energies and more so on the overall amount of green technologies embraced by the productive sector. In fact, if the firm sector's proportion of desired green investment is capped anywhere below $\beta_{max} < 0.85$, then the speed of adoption and the central bank's efforts to incentivize a shift to green production are futile.

Therefore, even though the government and central bank sectors remain the driving force behind ensuring the economy converges towards a good equilibrium, the productive sector must be equally prepared to accept these policy interventions and the structural changes they entail.

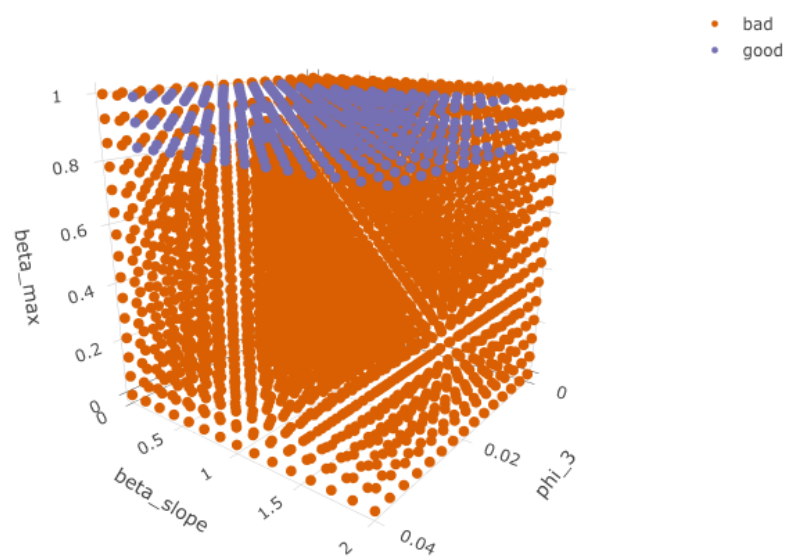


Figure 5.6: $\beta(\cdot)$ sensitivity for the *Carbon Tax Scenario* with green loans and interest rate

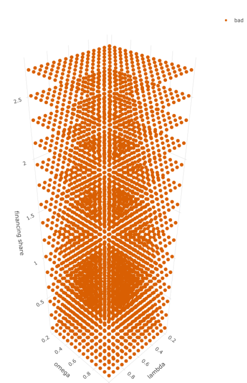
5.3.4 Basin of Attraction

To understand how green loans and borrowing rates affect the stability of our model for a range of initial economic conditions, we again turn to our basin of attraction analysis. Thus, we begin by combining the baseline *No policy scenario* with both green loans and a green interest rate. Out of curiosity, we also consider a 66% subsidy rate and green QE program for this policy scenario. Similarly, we include green loans and a green borrowing rate in the *Carbon tax scenario* and *Strong policy scenario*. Additionally, we extend our *Strong policy scenario* to include a green bond-purchasing program, as well. Staying in line with Chapter 3, we continue to postulate the central banks "whatever it takes" policy stance by setting the magnitude of the sector's response to rising abatement costs and CO_2 emissions as $\phi_2 = 2.359$ and $\phi_3 = 0.02$, respectively. Likewise, the initial set of state variables and classification of economic outcomes remain unaltered. The results of our basin of attraction analysis are displayed in *Figures 5.7* and *5.8*.

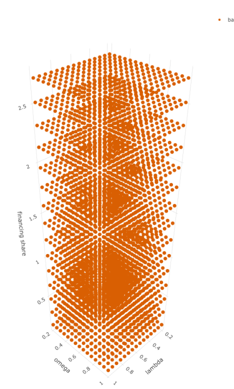
Looking at *Figure 5.7*, we see that without a carbon tax, green loan instruments and a central bank setting ultra-low rates proves ineffective as the entire set of initial values converge towards to undesirable equilibrium (See *Figure 5.7a*). Equivalently, the long-term behaviour does not improve even if firms are allowed to issue green bonds to finance mitigation efforts, the government sector bankrolls all expenses with a 66% subsidy, and the central bank engages in a bond-buying program (See *Figure 5.7b*).

Meanwhile, when incorporating green loans and a green interest rate alongside the *Carbon tax scenario*, we see considerably more good economic outcomes as the convergence set is approximately $(\omega, \lambda, \varphi) \in [0.2 : 0.99] \times [0.53 : 0.99] \times [0.1 : 1.96]$

(See *Figure 5.8a*). Moreover, if the government sector provides a 66% subsidy on abatement costs, the set of initial conditions that converges towards the Solovian equilibrium increases to $(\omega, \lambda, \varphi) \in [0.2 : 0.99] \times [0.25 : 0.99] \times [0.1 : 2.33]$ (See *Figure 5.8b*). This region is a noticeably larger basin of attraction compared to the *No feedback scenario* investigated in Chapter 3 and is a direct consequence of the high subsidy rate. However, we wanted to illustrate that even if the central bank adopts a dovish policy stance concerning the green interest rate, an aggressive government subsidy rate is still necessary to ensure a desirable outcome for economies experiencing meagre employment rates, wage shares and high debt levels. Finally, if we include green bonds and a QE program for the *Strong policy scenario* with green loans and borrowing rate, the convergence set remains comparable to the previous scenario. Once again, confirming that the addition of green loans indirectly minimizes the long-term effects of the financial paradox and improves the function of green bonds within the economic system.

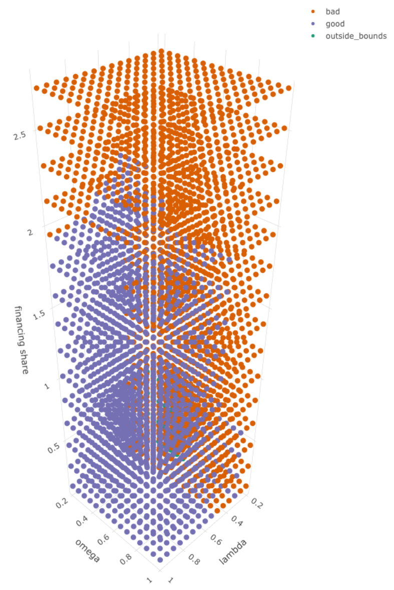


(a) *Green loans and green interest rate*

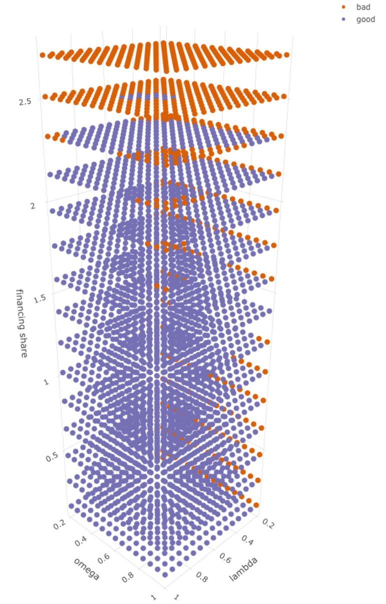


(b) *66% subsidy, green loans, green QE and green interest rate*

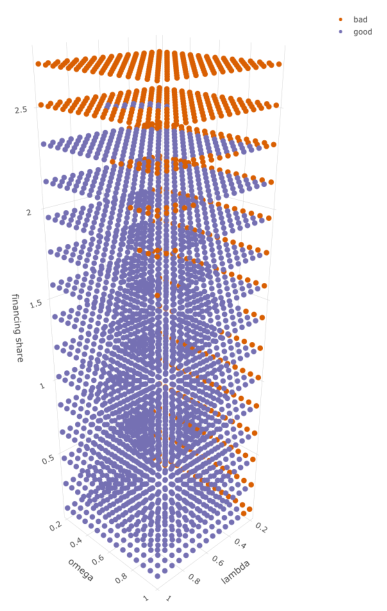
Figure 5.7: Basin of attraction for a range of initial conditions of the wage share, ω, employment rate, λ and financing share, φ. We compare the basin of attraction for the *No policy scenario* that includes green loans, green interest rate and QE program.



(a) *Carbon tax scenario with green loans and green interest rate*



(b) *Strong policy scenario with green loans and green interest rate*



(c) *Strong policy scenario with green loans, green interest rate and QE*

Figure 5.8: Basin of attraction for a range of initial conditions of the wage share, ω, employment rate, λ and debt share, d. Here we compare the baseline *Carbon tax scenario* and *Strong policy scenario* for a combination of green loans, green interest rate and QE program.

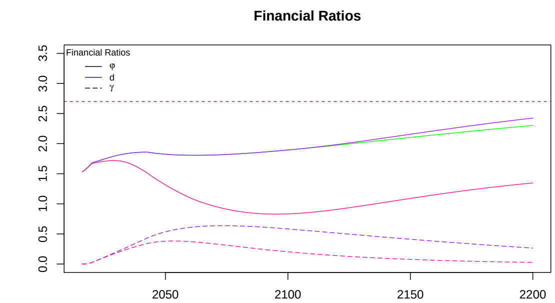
Thus, we see that a carbon price continues to be the kindle that sparks an immediate shift by the productive sector towards reducing CO_2 emissions. Consequently, a carbon tax accompanied by green loans and a green borrowing rate is enough for the more resilient economies to avoid a climate-induced bifurcation. However, as the economic conditions worsen, more drastic policy interventions need to be enacted to avoid the bad equilibrium. Moreover, incentivizing firms to adopt a greener production process indirectly improves the function of green bonds and a green asset purchasing program. Thus, to solidify these interpretations, we will compare the transitional dynamics for the *Strong policy scenario* from each respective chapter.

5.3.5 Transitional Dynamics

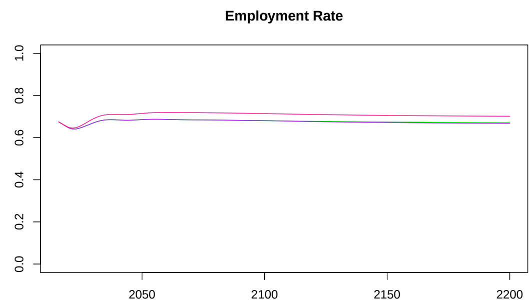
Figures 5.9 and *5.10* illustrate the transitional economic and climate trajectories for the *Strong policy scenario* with green loans, green bonds, and the central bank maintaining a loose monetary policy concerning each debt instrument. For simplicity, we will refer to this policy scenario as the *Strong policy scenario with a green interest rate and QE*. Note, we have included the *Strong policy scenario* from the previous two chapters for comparison.

Thus, we see that when joined with a green QE program, the low green borrowing rate drastically reduces the firm sector's financing-to-output ratio (See *Figure 5.9a*). Furthermore, the favourable green lending rate entices firms to adopt renewable technologies more readily (See *Figure 5.10a*). Naturally, this minimizes the financial paradox inherent in green bonds since the rapidly declining emission intensity and abatement cost subdue the amount of green bonds issued, particularly when compared to the *Strong policy with green QE*. Moving along, the heightened investment in green capital also accelerates the growth rate, which raises the wage share, employment and inflation rate (See *Figures 5.9c, 5.9b* and *5.9f*). Once the transition to net-zero emissions is complete, we see the economy as a whole beginning to stabilize towards a balanced growth path. However, we note that the green interest rate continues to be well below the conventional loan rate since the pre-abatement emissions have not reached zero yet.

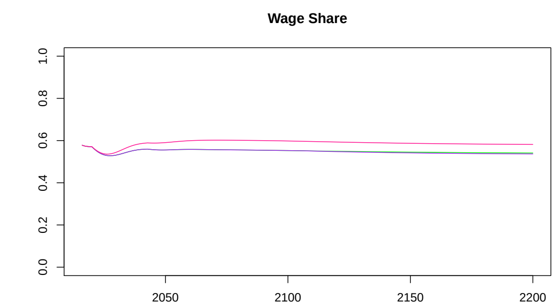
Concerning the climate dynamics, the adoption of renewable energies decreases the emission intensity, and by extension, CO_2 emissions (See *Figures 5.10d* and *5.10f*). Nevertheless, the drastic policy measures prove ineffective as the temperature anomaly only marginally decreases and still passes the 2°C threshold well before the year 2100 (See *Figure 5.10e*).



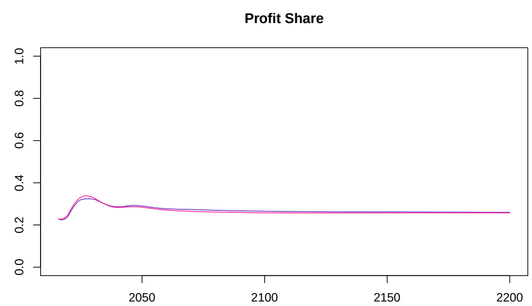
(a)



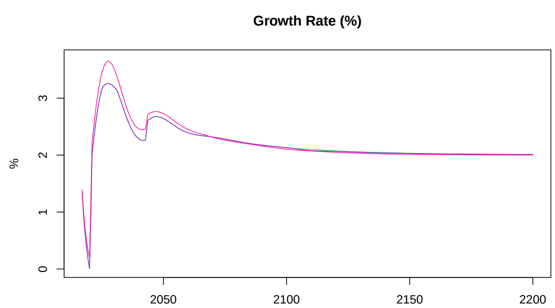
(b)



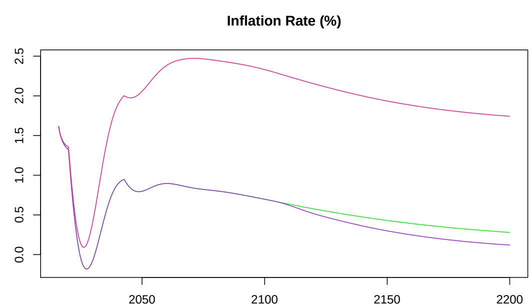
(c)



(d)

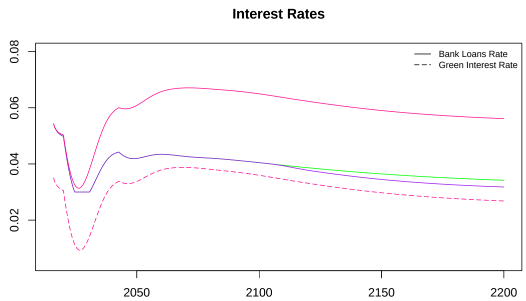


(e)

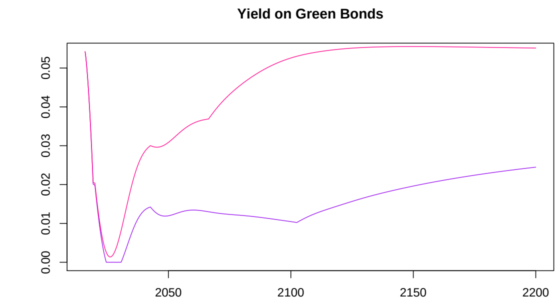


(f)

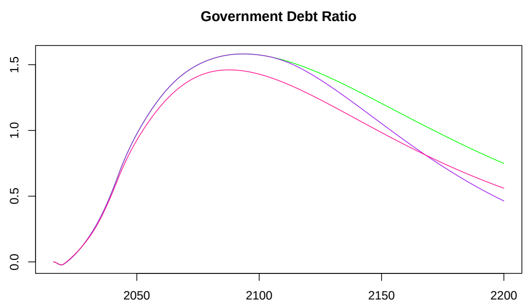
Figure 5.9: Deterministic macroeconomic trajectories for the *Strong policy scenario* (green line), *Strong policy scenario with green bonds and QE* (purple line) and *Strong policy with a green interest rate and QE* (pink line). The dotted red line in the financing ratios plot indicates the 2.7 debt-to-output threshold.



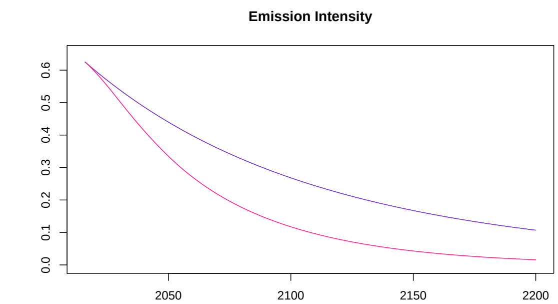
(a)



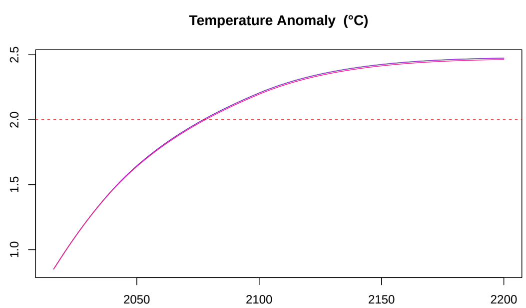
(b)



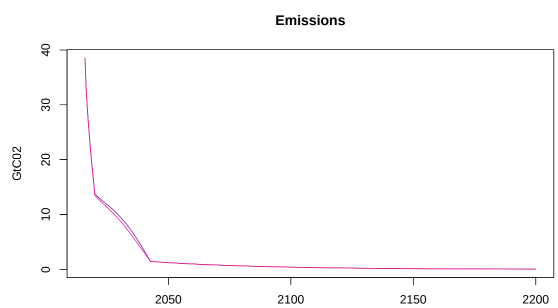
(c)



(d)



(e)



(f)

Figure 5.10: Additional economic trajectories and climate dynamics for the *Strong policy scenario* (green line), *Strong policy scenario* with green bonds and QE (purple line) and *Strong policy with a green interest rate and QE* (pink line). The dotted red line in the temperature anomaly plot indicates the 2°C target set by the Paris Climate Agreement.

5.3.6 Monte Carlo

To close this chapter, we will again account for the uncertainty inherent in our parameters and deterministic results by performing Monte Carlo simulations. Note, we vary the same parameters described in Chapter 3 and Bovari *et al.* (2018b). The near-term trajectories and probability intervals associated with the Monte Carlo simulations for the *Strong policy scenario*, *Strong policy scenario with green QE* and *Strong policy scenario with a green interest rate and QE* are visible in *Figure 5.11.*

Thus, the culmination of a carbon tax, 66% government subsidy rate, green bonds, green loans and a dovish central bank willing to maintain ultra-low green interest rates and bond yields cause the financing share to stay below the 2.7 thresholds for all of the simulations. However, the number of instances where the temperature anomaly stays below the 2°C Paris Agreement surprisingly increases to 41.3%. A sobering reminder of the uncertainties surrounding the environment's response to CO_2 emissions and the necessity to transition to renewable energies immediately. Finally, the total amount of simulations that stayed below both thresholds is 41.3%. The mean value for the private sector financing share, public debt share and temperature anomaly in the year 2100 is for this policy combination is 0.846, 1.37 and 2.14°C, respectively. *Tables 5.4* and *5.5* provide a summary of the Disaggregated model results the Monte Carlo results from the previous two models.

Scenario	$T < 2°\text{C}$	$\{\varphi, d\} < 2.7$	$(T < 2°\text{C}) \cup (\{\varphi, d\} < 2.7)$
No policy	0.4%	30.6%	0.4%
Carbon tax	5.2%	10.7%	2.5%
Strong policy	44.1%	69.9%	33.1%
Strong policy with green QE	39.3%	72.4%	31.0%
Strong policy with a green interest rate and QE	41.3%	100%	41.3%

Table 5.4: Updated percentage of runs where the temperature anomaly and financing/debt share stayed below the 2°C target and 2.7 debt-to-output threshold, respectively.

Scenario	Temperature Anomaly (°C)	Financing/Debt Share	Government Debt Share
No policy	4.17	6.14	-0.144
Carbon tax	2.97	14.3	-8.08
Strong policy	2.07	1.91	2.16
Strong policy with green QE	2.16	1.94	2.23
Strong policy with green interest rate and QE	2.14	0.846	1.37

Table 5.5: Updated median values of the temperature anomaly, private and public financing/debt shares in the year 2100 for each climate policy scenario.

Figure 5.11: [0.05; 0.95] probability interval for the *Strong policy scenario* (green) *Strong policy scenario with green QE* (purple), and the *Strong policy with a green interest rate and QE* (pink).

5.3.7 Conclusion

Thus, by disaggregating our expanded economic framework from Bovari *et al.* (2018a) and (2018b), we can introduce green loans as a financial mechanism for firms to transition towards a less carbon-intensive production process. Furthermore, by design, the green loan instruments allow the central bank to act as a catalyst to incentivize a quicker adoption of renewable energies via a novel green interest rate. Naturally, we see the increased influence of monetary intervention, as the green borrowing rate reduces the debt burden for the private sector and prompts a faster reduction in CO_2 emissions, which decreases the temperature anomaly, and by extension, the climate damages borne by the economy. Likewise, it also eases the government sector's burden to subsidize abatement costs and eliminates the financial paradox inherent in green bonds when implemented as a standalone financial alternative. Thus, in the face of ever-growing uncertainty surrounding the consequences of climate change, introducing a wide range of debt tools dedicated to more than just alleviating private sector indebtedness can increase the versatility of fiscal and monetary policy interventions. Naturally, this will allow economies under various financial conditions to effectively cope with the widespread effects of climate damages and increase the likelihood of avoiding a climate-driven collapse.

Chapter 6

Conclusion

In this thesis, we have explored three stock-flow consistent continuous-time climate-economy models in increasing order of complexity.

Following our literature review in Chapter 2, where we studied the foundations from which our work originates, we set out in Chapter 3 to improve the simplified economic framework assumed in the seminal work of Bovari *et al.* (2018a) and (2018b). In doing so, we provide a much more realistic depiction of the macro-economy by explicitly accounting for the government and central bank sectors. Moreover, these improvements allow us to implement our proposed climate policies more accurately and through the traditional fiscal and monetary channels. Afterwards, we extensively recreated and extended both the theoretical and asymptotic results observed in Bovari *et al.* (2018a) and (2018b) to illustrate that our alterations did not compromise the robustness of our model. Among our main findings, we observed that the climate feedback loop forces the economic system towards an unstable growth path that requires extensive government intervention in the form of a carbon tax and subsidy on abatement efforts to reverse the economy towards the desirable equilibrium.

Chapter 4 strives to alleviate the burden placed on the private sector by introducing green bonds as an alternative form of financing climate-related mitigation costs. Moreover, we expanded the central bank's mandate and investigated how a green quantitative easing program that controls green bond yields can affect the medium and long-term financial stability and climate outcomes. In general, we numerically observed that supplying the private sector with a more efficient alternative to finance climate efforts resulted in a paradoxical increase in debt levels. Likewise, absent negative yields, the central bank sector found itself unsuited to reduce the long-term consequences of an increase in the financialization of the economy. Thus, placing further responsibility on the government sector to support the economy towards a stable growth path.

Our main contribution is achieved in Chapter 5, where we disaggregated the firm sector's production process between conventional and green methods. This enabled us to introduce green loans as a third financing instrument and a dedicated financial pathway toward renewable energies. Accordingly, we enhance the central bank's role to encourage firms to adopt less emission-intensive production methods through a novel green borrowing rate. To our knowledge, this is the first continuous-time stock-flow consistent climate-economy model that explores the concept of a green interest rate. Moving along, we next verified that the asymptotic stability of our disaggregated dynamical system suffered from the same destabilizing effects of climate damages as in the baseline model. Some of our main numerical results included illustrating the increased effectiveness of the monetary intervention as the green interest rate served its dual purpose of reducing the debt burden for the productive sector and abetting a faster reduction of carbon emissions. Naturally, this also reduced the government sector burden and indirectly eliminated the counterintuitive effects of green bonds. Finally, we concluded that policymakers stood the best chance of evading a climate-induced economic collapse by adopting a versatile repertoire of financial instruments and policy interventions that possess the twofold objective of reducing private indebtedness and spurring a faster reduction in CO_2 emissions.

Lastly, this thesis calls for a variety of extensions. The first potential avenue would be to explicitly model an active bond market along the lines of Dafermos *et al.* (2018). This would enable further investigation of a green QE program within our economic framework and allow for the exploration of other potentially destabilizing economic events such as a green taper tantrum. Moreover, a bond market would open the door to include a more active household sector that considers the effects of climate change when determining the asset allocation of their wealth. Similarly, capital requirements on conventional and green loans could be distinctly implemented along the lines of Grasselli and Lipton (2019). More ambitiously, the supply-side driven nature of our macroeconomic framework could be relaxed, paving the route to explore household consumption and inventory dynamics within the larger climate-economy model. Ultimately, we see these as the next crucial steps required to provide policymakers with the most accurate representation of climate change's inherent environmental and financial instability.

Appendix A

Appendix

Symbol	Description	Value
Y	GDP, in trillions USD	59.1394
p	Composite good price level	1
L_f	Corporate loans, in trillions USD	90.7741
M_f	Corporate deposits, in trillions USD	0.124
B	Government bonds issued, in trillions USD	0
R	Central Bank Reserves in trillions USD	0
ω	Wage share	.58
λ	Employment rate	.67
N	Workforce, in billions	4.83
CO_2^{AT}	CO2-e concentration in the atmosphere layer, in Gt C	851
CO_2^{UP}	CO2-e concentration in the biosphere and upper ocean layer, in Gt C	460
CO_2^{LO}	CO2-e concentration in the lower ocean layer, in Gt C	1740
E_{ind}	Industrial CO2-e emissions, in Gt CO2-e	35.85
E_{land}	Exogenous land use CO2-e emissions, in Gt CO2-e	2.6
T	Temperature anomaly, in °C	0.85
T_0	Temperature anomaly in lower ocean layer, in °C	0.0068
F_{exo}	Exogenous radiative forcing, in W/m^2	0.5
σ	Emissions reduction rate	0.03
g_σ	Growth rate of the emissions intensity of the economy	-0.0105
p_{BS}	Price level of backstop technology	547.22
n	Emission reductions rate	0.03

Table A.1: Initial Conditions for the Baseline and Green Bond Model

Symbol	Parameter description	Initial condition
α	Productivity of growth rate	0.02
δ	Depreciation rate of capital	0.04
ν	Capital-to-output ratio	2.7
δ_N	Growth rate of workforce	.0305
$\overline{N}$	Upper limit of workforce dynamics in billions	7.056
div_0	Constant of the dividend function, $\delta(\cdot)$	0.138
div_π	Slope of the dividend function, $\delta(\cdot)$	0.4729
div_{max}	Maximum dividend distributed	0.3
div_{min}	Minimum dividend distributed	0
κ_0	Constant of the investment function, $\kappa(\cdot)$	0.0318
κ_π	Slope of the investment function, $\kappa(\cdot)$	0.575
κ_{max}	Maximum of the investment function	0.3
κ_{min}	Minimum of the investment function	0
ϕ_0	Constant of short-term Phillips curve, $\Phi(\cdot)$	0.0318
ϕ_π	Slope of short-term Phillips curve, $\Phi(\cdot)$	0.575
γ	Money illusion parameter	0
ξ	Markup constant	1.875
r^*	Relaxation parameter of the inflation	0.01
ϕ	Magnitude of reactivity of the monetary policy	0.5
$i*$	Interest rate targeted by the central bank	0.02
δ_L	Prime rate spread on bank loans	0.03
f	Portion of abatement funded using green bonds	1
ϕ_2	Magnitude of central banks response to abatement costs	2.359
C^{AT}_{preind}	Preind. concentration of CO_2 in the atmosphere layer, in Gt C	588
C^{UP}_{preind}	Preind. concentration of CO_2 in the biosphere/upper ocean layer, in Gt C	360
C^{LO}_{preind}	Preind. concentration of CO_2 in the lower ocean layer, in Gt C	1720
ϕ_{12}	Transfer coefficient for carbon from AT to UP	0.0239069
ϕ_{23}	Transfer coefficient for carbon from UP to LO	0.0013409
δ_{g_σ}	Variation rate of the growth of emission intensity	-0.001
$\delta_{E_{land}}$	Growth rate of land use change CO2-e emissions	-0.022
$F_{2\times CO2}$	Change in radiative forcing from a doubling of preind CO_2, in W/m^2	3.6813
F^{start}_{exo}	Initial value of exogenous radiative forcing	0.5
F^{end}_{exo}	Initial value of exogenous radiative forcing	1
T_{preind}	Preindustrial temperature, in degrees Celsius	13.74
C_0	Heat capacity of the lower ocean layer	3.52
$\bar{h}$	Heat exchange coefficient between temperature layers, in SI	0.0176
π_1	Damage function parameter	0
π_2	Damage function parameter	0.00236
π_3	Damage function parameter, Weitzman (2012)	.00000507
θ	Abatement cost function parameter	2.6
$g_{p_{BS}}$	Growth rate of the price of backstop technology	-0.0051

Table A.2: Calibrated Parameters for the Baseline and Green Bond Model

Symbol	Description	Value
Y	GDP, in trillions USD	59.1394
p	Composite good price level	1
L_f^C	Conventional corporate loans, in trillions USD	87.14314
L_f^G	Green corporate loans, in trillions USD	3.63096
M_f	Corporate deposits, in trillions USD	0.124
B	Government bonds issued, in trillions USD	0
R	Central Bank Reserves in trillions USD	0
ω	Wage share	.58
λ	Employment rate	.67
N	Workforce, in billions	4.83
CO_2^{AT}	CO2-e concentration in the atmosphere layer, in Gt C	851
CO_2^{UP}	CO2-e concentration in the biosphere and upper ocean layer, in Gt C	460
CO_2^{LO}	CO2-e concentration in the lower ocean layer, in Gt C	1740
E_{ind}	Industrial CO2-e emissions, in Gt CO2-e	35.85
E_{land}	Exogenous land use CO2-e emissions, in Gt CO2-e	2.6
T	Temperature anomaly, in °C	0.85
T_0	Temperature anomaly in lower ocean layer, in °C	0.0068
F_{exo}	Exogenous radiative forcing, in W/m^2	0.5
σ_C	Conventional emissions reduction rate	0.03
σ_G	Green emissions reduction rate	0.03
g_{σ_C}	Growth rate of the conventional emissions intensity of the economy	-0.0105
g_{σ_G}	Growth rate of the green emissions intensity of the economy	-0.02
p_{BS}	Price level of backstop technology	547.22
n	Emission reductions rate	0.03

Table A.3: Initial Conditions for the Disaggregated Model

Symbol	Parameter description	Initial condition
α	Productivity of growth rate	0.02
δ	Depreciation rate of capital	0.04
ν	Capital-to-output ratio	2.7
$\varkappa_0$	Initial ratio of green capital to total capital	0.04
δ_N	Growth rate of workforce	.0305
$\overline{N}$	Upper limit of workforce dynamics in billions	7.056
div_0	Constant of the dividend function, $\delta(\cdot)$	0.138
div_π	Slope of the dividend function, $\delta(\cdot)$	0.4729
div_{max}	Maximum dividend distributed	0.3
div_{min}	Minimum dividend distributed	0
κ_0	Constant of the investment function, $\kappa(\cdot)$	0.0318
κ_π	Slope of the investment function, $\kappa(\cdot)$	0.575
κ_{max}	Maximum of the investment function	0.3
κ_{min}	Minimum of the investment function	0
ϕ_0	Constant of short-term Phillips curve, $\Phi(\cdot)$	0.0318
ϕ_π	Slope of short-term Phillips curve, $\Phi(\cdot)$	0.575
γ	Money illusion parameter	0
ξ	Markup constant	1.875
r^*	Relaxation parameter of the inflation	0.01
ϕ	Magnitude of reactivity of the monetary policy	0.5
$i*$	Interest rate targeted by the central bank	0.02
δ_L	Prime rate spread on bank loans	0.03
f	Portion of abatement funded using green bonds	1
β_0	Constant of the portion of desired green investment function, $\beta(\cdot)$	0.052825
β_π	Slope of the portion of desired green investment function, $\beta(\cdot)$	0.349616
β_{max}	Maximum of the investment function	1
β_{min}	Minimum of the investment function	0.05
ϕ_2	Magnitude of central banks response to abatement costs	2.359
ϕ_3	Magnitude of of central banks response to change in emissions	0.02
C^{AT}_{preind}	Preind. concentration of CO_2 in the atmosphere layer, in Gt C	588
C^{UP}_{preind}	Preind. concentration of CO_2 in the biosphere/upper ocean layer, in Gt C	360
C^{LO}_{preind}	Preind. concentration of CO_2 in the lower ocean layer, in Gt C	1720
ϕ_{12}	Transfer coefficient for carbon from AT to UP	0.0239069
ϕ_{23}	Transfer coefficient for carbon from UP to LO	0.0013409
$\delta_{g_{\sigma_C}}$	Variation rate of the growth of conventional emission intensity	-0.001
$\delta_{g_{\sigma_G}}$	Variation rate of the growth of green emission intensity	-.00001
$\delta_{E_{land}}$	Growth rate of land use change CO2-e emissions	-0.022
$F_{2\times CO2}$	Change in radiative forcing from a doubling of preind CO_2, in W/m^2	3.6813
F^{start}_{exo}	Initial value of exogenous radiative forcing	0.5
F^{end}_{exo}	Initial value of exogenous radiative forcing	1
T_{preind}	Preindustrial temperature, in degrees Celsius	13.74
C_0	Heat capacity of the lower ocean layer	3.52
$\bar{h}$	Heat exchange coefficient between temperature layers, in SI	0.0176
π_1	Damage function parameter	0
π_2	Damage function parameter	0.00236
π_3	Damage function parameter, Weitzman (2012)	.00000507
θ_C	Conventional abatement cost	2.6
θ_G	Green abatement cost	2.8
$g_{p_{BS}}$	Growth rate of the price of backstop technology	-0.0051

Table A.4: Calibrated Parameters for the Baseline and Green Bond Model

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