DYNAMIC RESPONSE OF INELASTIC MULTI-STOREY BUILDING FRAMES

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MULTI-STOREY BUILDING FRAMES

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SCOPE AND CONTENTS:

This thesis presents an analytical method based on classical matrix methods for computing the dynamic response of elastic-plastic multi-storey building frames. The method developed is comparatively simple and is of much use for building frames having large number of storeys. By this method, response of multi-storey buildings could be calculated on high-speed digital computers of high storage capacity. The computer program developed saves huge storage locations and thus makes it possible to analyze multi-storey frames which till now were considered as very difficult. Dynamic response of a two-storey and six-storey frame are shown to demonstrate the utility of the method.

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CONTENTS

NOTATION

[A]	Displacement Deformation Matrix
{B}	Column vector as defined in Eq. 3.19
[C]	Damping Matrix
{D}	Joint Displacement Column Vector
$\{D^r\}$	Sub Column Vector of {D}
E	Modulus of Elasticity
F_{oi}	Amplitudes of Applied Dynamic Forces
$F_i(t)$	Dynamic Force acting at ith Mass
[G]	Matrix as defined in Eq. 2.7
[H]	Matrix as defined in Eq. 3.19
I_i	Moment of Inertia of ith Member
J	Matrix as defined in Eq. 2.7
[K]	Frame Deformation-Force Transformation Matrix
$[K^m]$	Member Stiffness Matrix
[L]	Matrix as defined in Eq. 2.7
$\bar{M}$, $\bar{M}^*$	Moment, Plastic Moment
{M}	Column Vector as defined in Eq. 2.8
{N}	Column Vector as defined in Eq. 2.7
{Q}	Joint Load Column Vector
$\{Q^r\}$	Subvector of Q
{R}	Structural Resistance Vector
[S]	Frame Stiffness Matrix

$[T]$	Submatrix of $[S]$
$[W]$	Submatrix of $[S]$
$\{X\}$	Floor Displacement Vector
$[Y]$	Submatrix of $[S]$
$[Z]$	Submatrix of $[S]$
i,j	Indices
ℓ_i	Length of ith Member
m	Number of Members in a Frame
m_i	Lumped Mass at ith Floor
n	Number of degrees of Freedom
$\{p\}$	Frame Force Vector
$\{p^m\}$	Member Force Vector
q_i	ith load at a joint
t	Time
t_1, t_2	Times at Beginning and End of the Small Time Interval Δt
$\{u\}$	Frame Deformation Vector
$\{u^m\}$	Member Deformation Vector
$y^{(i)}$	ith Block of Elements of $[A]$ Matrix as shown in Table 3.1
Δt	Small Time Interval as used in Numerical Integration Procedure
Σ	Indicates Summation
δ_{ij}	Kronecker Delta, $= 1$ if $i=j$, $= 0$ if $i \neq j$
ϕ	Curvature
μ	Exponential Decay Factor of Applied Force
ω	Circular Frequency of Applied Force

ϵ	Strain
σ, σ^*	Stress, Yield Stress
$[\ ]^{-1}$	Inverse Matrix
$[\ ]^T$	Transpose Matrix
$\left.\begin{matrix}\cdot\\ \cdot\cdot\end{matrix}\right\}$	Superscripts Single Dot and Double Dot denote Differentiation w.r.t. Time

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CHAPTER I

INTRODUCTION

1.1 General

Structural systems such as high multi-storey building frames, when subjected to strong dynamic forces, are usually stressed in the inelastic region. Dynamic analysis of such multi-degree of freedom system in the inelastic region is one of the most important and most involved areas in the field of structural dynamics. The importance lies in understanding the dynamic response characteristics in the inelastic region so that suitable design criteria could be formulated. The formulation of design criteria will not only result in the overall economy of the structure but will also enhance the dependability on the behaviour of the structure under strong dynamic forces. The formulation of design criteria of such structures depends entirely on the availability of a simple and reasonably practical method for computing the dynamic response which was hitherto considered as perhaps the most complex and difficult. The methods available so far for carrying out such analysis are a bit cumbersome to use and in addition their use is limited to a small number of storeys due to their requirements of computer having high storage capacity.

In this thesis a method is presented for calculating the dynamic response of inelastic multi-storey frames. The method is particularly developed for analyzing building frames having large number of storeys. This method is much simpler to use and requires minimum storage capacity of the computer. Economy of storage capacity has been achieved by making use of the repetitive geometrical shape of the structural system and elimination of some large matrices through logical programming.

1.2 Nature of the Problem

The complexities involved in the dynamic analysis of multi-degree of freedom structural system are manifold. As the structure vibrates back and forth under strong dynamic forces, there are frequent transformations of the system from one elastic behaviour to another inelastic behaviour and from resulting inelastic behaviour to a different inelastic behaviour and vice-versa. In all these transformations, the properties of one inelastic or elastic behaviour will be entirely different from the previous inelastic or elastic system. Such complex and frequently changing behaviour arises due to the formation of plastic hinges at different sections of the structure where the moments reach the plastic moment. Formation of a single hinge at any section of the structural system completely changes the stiffness of the system. Due to this changed stiffness, response characteristics of the system become altogether different from those existing before the formation of plastic hinge. At subsequent instants, as this new system responds, other sections may plasticize. This may further change the properties of the structure. Subsequently, more sections may either plasticize or some of the plastic hinges may re-elasticize due to reversal of stresses resulting from reversed curvature changes. Under this situation it becomes a formidable task to compute the response of such a structural system possessing multi-degree of freedom and whose properties are changing frequently as it vibrates. The problem becomes still more complicated and challenging when the formation of plastic hinges or re-elasticizing of the formed plastic hinges occur at different instants during a very short time interval. The complications arise due to the fact that at every instant various sections likely to plasticize or sections where plastic hinges exist, should be checked to ascertain whether a plastic hinge is forming or the one already formed is elasticizing respectively or not. In case at any section, a plastic hinge is forming or any plastic hinge already formed is elasticizing, the stiffness of the resulting structural system should be reassessed to determine the future behaviour of the structure.

The process of assessing the changed stiffness of the structure at every transition of its changing from one structural system to another structural system, is itself quite complicated. In addition to this, after each small time interval every elastic section likely to become plastic is required to be checked whether a plastic hinge is occurring there or not. Similarly it is required to ascertain whether a section where a plastic hinge exists, is re-elasticizing or not at the end of each time interval. This whole process elaborated above poses a challenge even now due to limited capacity of digital computers unless some simplifying assumptions are made and special programming techniques are applied.

In the future discussion of inelastic behaviour, the term "phase" refers to a particular state of elastic plastic deformation and the term "transition" refers to a change of phase either by formation or re-elasticizing of one or more hinges.

1.3 Previous Work

To date various approaches pursued in this field could be categorized as (a) Normal Mode Approach and (b) Lumped Mass System. Several authors have proposed methods which fall mainly in either of the above categories.

(a) Normal Mode Approach

A general method using the normal mode approach for dynamic analysis of elastic plastic structures was presented by Bleich and Salvadori.[1*] The method was initially used for dynamic analysis of elastic plastic beams. Its application for dynamic analysis of elasto-plastic structures was extended by DiMaggio.[2] In this method normal modes of vibrations of the elastic structure are computed. As the structure responds, moments at sections likely to develop maximum moments are computed and when these moments become equal to plastic moment, a hinge is inserted at this section with plastic moment constraints applied. A new set of normal modes are now computed for the resultant system. The procedure of computation of normal modes and boundary conditions at every stage of transition limits this approach to relatively simple structures loaded symmetrically, such as a free beam or a simple fixed or two hinged single storey portal frame whose normal modes are usually simple to calculate. This approach is certainly impracticable from the point of view of computational difficulties for a multi-storey building frame in which numerous plastic hinges may occur and re-elasticize during a very short interval of time. At every transition of such a system, the computation of normal modes and boundary conditions for a multi-degree of freedom system

*Numbers refer to the Bibliography listing.

will not only be a formidable task but will also be a sheer waste of time when the transitions occur frequently in a short interval of time. Convergence problems with the series of modes for determining flexural moments further emphasizes the impracticability of the method to multi-storey frames Further difficulties appear in this method when a structure turns into a mechanism. In the mechanism state the consideration of rigid body modes of separate component segments of the structure going through rigid body motion further complicates the whole normal mode approach and makes it unsuitable for analysis of multi-storey frames.

(b) Lumped Mass System

In this approach masses are assumed to be concentrated at floor levels and computation of dynamic response is carried by following some numerical integration procedure.

Berg and DaDeppo[3] presented a method in which masses are assumed to be concentrated at floor levels. Response is calculated by numerical integration of equation of motion for an elastic system. The bending moments are calculated elastically after each time interval. If these moments exceed the plastic moments, linear corrector solutions composed of frames with actual hinges and moment constraints at those points at which a plastic hinge occurs, are superimposed in such a way that none of the moments exceed the yield moment at any point of the frame. At each hinge formed, moment and hinge constraints are introduced so that idealized moment curvature relationship is achieved. At each step plastic hinge rotations are calculated by iteration. For multi-storey frames this method will be too cumbersome and time consuming because of precalculation of the basic corrector solutions for all points and also because of actual computation requiring complex operations during the analysis.

Penzien[4] also uses numerical integration procedure for solution of differential equation of motion. The initial assumptions made are that the masses are concentrated at floor levels, all floor systems are infinitely rigid and all the storey heights are equal. There is only relative horizontal movement between floors. An idealized elastic-plastic force-deformation relationship is assumed. The equations of motions are expressed in terms of inter-floor shear resistance and are integrated by 'mid-acceleration' method. The assumptions made, though simplying the method, make it inapplicable for modern framed buildings with nonrigid floor system.

Heidebrecht[5] developed a method using the single step forward numerical integration procedure. Horizontal resistance to motion at each floor level is expressed in terms of the horizontal deflection at floor levels for any state of elastic plastic behaviour. Yielding of both columns and beams is considered. The horizontal resistance to motion and horizontal floor deflection relationship has been derived using the conjugate frame method developed by Lee[6]. The method is versatile and could be used for large multi-storey frames except that its practical application is limited by the storage capacity of the particular computer being used to perform the computation.

Clough and Benuska[7] developed a method for computing the inelastic earthquake response of tall buildings by assuming a special bilinear moment rotation property prescribed to each member of the structural system. The masses are assumed to be concentrated at floor levels. During a short time interval the acceleration is assumed to vary linearly and displacements are computed using a numerical integration procedure. In assuming a special bilinear moment rotation property associated with each member, the member is assumed to consist of two components in parallel. The first component is a basic elasto-plastic beam which develops a plastic hinge at either end when the respective end moment exceeds the yield moment while the second component remains fully elastic. The elasto-plastic beam component is assumed to possess a rigid plastic moment rotation property. The procedure adopted to calculate the response requires ascertaining the moments at sections at which maximum moments may develop to check whether elasto-plastic component develops a hinge or not. In case any elasto-plastic component develops a hinge the stiffness matrix associated with the structure is modified. In this approach simplifying assumptions prescribing a special bilinear moment curvature relationship makes the computation relatively simple, but renders the method unsuitable for frames consisting of members which do not possess special moment curvature relationship prescribed by the authors. The assumptions made obviously neglect the penetration of plastic zone towards the centre of the member possessing usual bilinear moment curvature relationship.

Saul[8] presented a method of dynamic analysis of structures assuming a piecewise bilinear moment curvature and stress-strain relationship. The masses are assumed to be concentrated at floor levels. The penetration of plastic zone towards the centre of the column has been considered. An iterative method has been adopted to solve the differential equation of motion. Floors are considered as infinitely rigid, thus limiting the analysis only to shear buildings. In this method the effect of a concentrated load on floor system cannot be considered. These limitations renders the method applicable to limited cases.

CHAPTER II

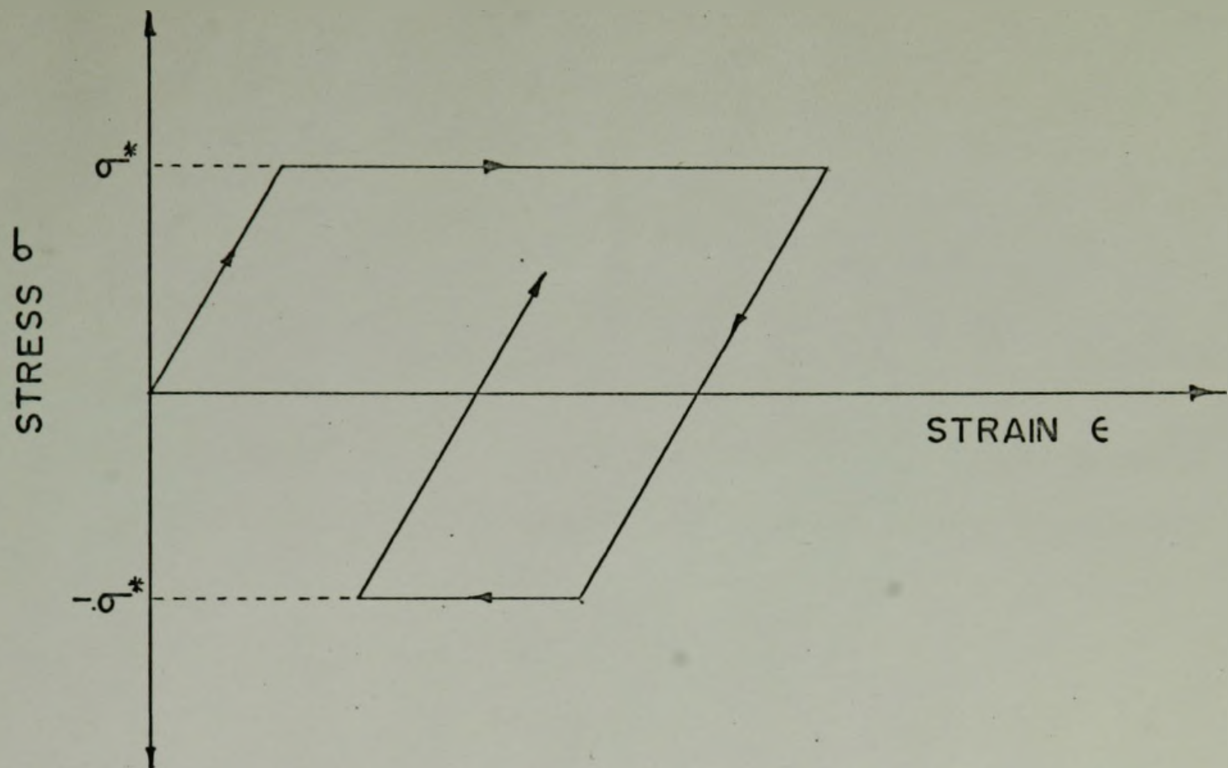
DYNAMIC ANALYSIS

2.1 General

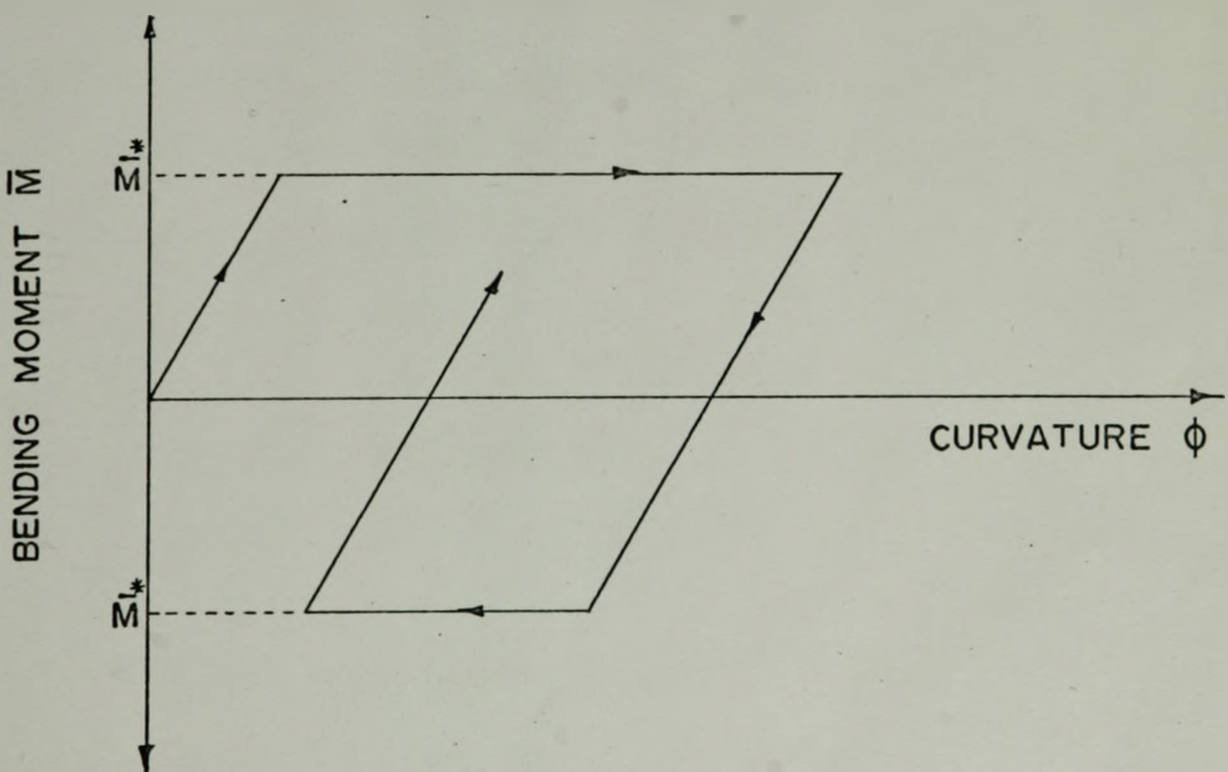
As elaborated earlier, dynamic analysis of a multi-storey building frame stressed in the inelastic region is an extremely complicated matter due to varying characteristics of the structural system resulting from frequent formation of plastic hinges and re-elastification of these hinges at various time instants. The assessment of stiffness at each change of phase could well be done by understanding the stress-strain relationship of the material used and also the moment curvature relationship of components forming the structural system.

2.2 Basic Assumptions

Usually multi-storey building frames are designed using structural steel which is fairly ductile, with ductility factor varying from eight to fifteen for various steels as shown by Beedle[9]. The stress-strain relationship of steel within the strain hardening range is assumed to be of an idealized form as shown in Fig. 2.1a. This type of relationship is usually known as elastic perfectly plastic stress-strain relationship and has been



(a) STRESS-STRAIN RELATIONSHIP



(b) MOMENT-CURVATURE RELATIONSHIP

FIG. 2.1

shown by Beedle[9] to be a very good approximation to the actual stress-strain relationship of mild steel in the normal working range of strains.

The usual shapes used in multi-storey buildings are wide flange and I sections. Using the above mentioned idealized stress-strain relationship, the moment curvature relationship of flexural members, i.e. beams and columns, can reasonably be assumed to be of idealized form as shown in Fig. 2.1b, as shape factor for these shapes is approximately 1.15.

Various authors[9,10,11,12] in this field have confirmed the assumption of idealized moment curvature relationship to be practically the same as that obtained experimentally.

The masses are assumed to be concentrated at floor levels. This assumption is practically justifiable as in multi-storey buildings; the maximum mass is contributed by floor system. The contribution of mass due to columns on either side of the floor is also assumed to be lumped at floor levels. This simplifying assumption has been made by various other authors[3,4,5,6,7,8] in this field.

Any damping is assumed to be of viscous type.

2.3 Differential Equation of Motion

The differential equation of motion for a viscously damped multi-degree of freedom system is given by

$$F_i(t) - R_i - \sum_{j=1}^{n} C_{ij} \dot{X}_j = m_i \ddot{X}_i \quad (i = 1,2....n) \qquad ...(2.1)$$

where $F_i(t)$ is the applied dynamic force, R_i is the structural resistance to deformation, C_{ij} are the damping coefficients, X_i is the horizontal deflection of ith floor, m_i is the ith mass and n is the number of degrees of freedom, i.e. the same as the number of storeys and $\dot{X}_i$ and $\ddot{X}_i$ are the velocity and accelerations of ith mass respectively.

It will be shown later in this thesis that R_i can be expressed in terms of horizontal floor deflections X_i as

$$R_i = \sum_{j=1}^{n} H_{ij} X_j + B_i \qquad ...(2.2)$$

$$(i = 1,2....,n)$$

in which H_{ij} and B_i are constant coefficients and are computed from known external loads and stiffnesses of the members in any phase.

Substituting R_i from Eq. 2.2, Eq. 2.1 yields

$$F_i(t) - \sum_{j=1}^{n} H_{ij} X_j - B_i - \sum_{j=1}^{n} C_{ij} \dot{X}_j = m_i \ddot{X}_i \qquad ...(2.3)$$

$$(i = 1,2, ..., n)$$

2.4 Numerical Integration Procedure

Eq. 2.3 can most conveniently be solved by a single step forward numerical integration procedure developed by Fleming and Romualdi[13]. In the development of this integration procedure, the deflection-velocity and velocity-acceleration relationships are assumed to be linear over a small time interval and are given by

$$\ddot{X}_i(t_2) = \frac{2}{\Delta t}\ [\dot{X}_i(t_2) - \dot{X}_i(t_1)] - \ddot{X}_i(t_1) \qquad \ldots(2.4)$$

and

$$\dot{X}_i(t_2) = \frac{2}{\Delta t}\ [X_i(t_2) - X_i(t_1)] - \dot{X}_i(t_1) \qquad \ldots(2.5)$$

in which $\Delta t = t_2 - t_1$ and t is the time variable. The quantities $X_i(t_1)$, $X_i(t_2)$, etc. are at time t_1, t_2 respectively.

Substituting Eq. 2.5 in Eq. 2.4 yields

$$\ddot{X}_i(t_2) = \frac{4}{(\Delta t)^2}\ [X_i(t_2) - X_i(t_1)] - \frac{4}{\Delta t}\ \dot{X}_i(t_1) - \ddot{X}_i(t_1) \qquad \ldots(2.6)$$

Substituting the values of $\dot{X}_i(t_2)$ and $\ddot{X}_i(t_2)$ from Eq. 2.5 and 2.6 into Eq. 2.3 yields

$$\sum_{j=1}^{n} G_{ij}\ X_j(t_2) = \sum_{j=1}^{n} L_{ij} X_j(t_1) + \sum_{j=1}^{n} J_{ij} \dot{X}_j(t_1) + m_i \ddot{X}_i(t_1) + N_i \qquad \ldots(2.7)$$

in which

$$G_{ij} = \frac{4}{(\Delta t)^2}\ \delta_{ij}\ m_i + \frac{2}{\Delta t}\ C_{ij} + H_{ij}$$

$$L_{ij} = \frac{4}{(\Delta t)^2}\ \delta_{ij}\ m_i + \frac{2}{\Delta t}\ C_{ij}$$

$$J_{ij} = \frac{4}{\Delta t}\ \delta_{ij}\ m_i + C_{ij}$$

$$N_i = F_i(t_2) - B_i$$

and δ_{ij} is Kronecker delta and is defined as

$\delta_{ij} = 1$ if $i=j$ and $\delta_{ij} = 0$ if $i \neq j$.

The numerical solution of differential equation is carried out by using Eq. 2.7 which is expressed in its general form. In matrix form Eq. 2.7 can be written as

$$[G]\ \{X\ (t_2)\} = \{M\} \qquad \ldots(2.8)$$

in which [G] is the matrix of coefficients G_{ij} in Eq. 2.7 and {M} is a column vector consisting of total quantities on the right hand side of Eq. 2.7.

For calculating the deflections $X(t_2)$ vector {M} is evaluated from known velocity, acceleration, deflections $X(t_1)$ at time t_1, the known coefficients B_i and applied force $F_i(t_2)$ at time t_2. Eq. 2.8 is then solved by finding the inversion of [G]. Now using Eq. 2.5 and 2.6 velocities and accelerations at time t_2 are computed. Knowing all quantities at time t_2, the forward integration procedure is repeated over the next time interval. In case a hinge develops at any section or an already existing hinge re-elasticizes, the matrix [G] is modified by taking into account the changed stiffness of the structural system. Similarly, vector {B} is also modified by reassessing the stiffness of the structure.

CHAPTER III

DISPLACEMENT METHOD

3.1 General

Multi-storey building frames are highly indeterminate structures. The degree of indeterminacy of such structures increases with the increase in number of storeys. For carrying out the dynamic response computation of such structures in the inelastic range it is necessary to know the value of moments developed at sections which are known to have extremum value of moments. At sections which have developed plastic hinges, it is necessary to know the hinge rotations in order to ascertain whether a particular hinge is tending to retain its hinge property or if it is reverting back to the elastic state. Apart from the suitability of computation of the above mentioned requisites, the repetitive geometrical shape of multi-storey building frames can best be utilized by adopting the displacement method. This method of analysis is also known as the stiffness method and has been described in detail by McMinn[14], Gennaro[15] and various other authors for the static analysis of elastic structures. It will be shown in later sections that using this method, the structural resistance can be expressed in terms of floor displacements.

3.2 The Displacement Method

The displacement method can be called an organized augmented form of the well known slope deflection method, as in this method, both, the basic assumptions and expressions relating member forces and deformations are the same, except that in the former the set of equations are expressed in the matrix form so that the computation of unknown forces and deformations of highly indeterminate structures can be carried out easily on digital computers.

Using displacement method, member deformations and member forces are expressed in terms of joint displacements which are found by the solution of a set of simultaneous moment equilibrium equations at the joints and shear equilibrium equations for the members. It will be shown further that once the joint displacements are computed, the member deformations and member forces can be obtained easily.

3.3 Member Stiffness Matrix

The relation between end forces and deformations of any ith member of a frame as shown in Fig. 3.1 can be shown to be

$$\begin{Bmatrix} p^{i1} \\ p^{i2} \\ p^{i3} \end{Bmatrix} = \begin{bmatrix} \frac{4EI^i}{\ell^i} & \frac{2EI^i}{\ell^i} & -\frac{6EI^i}{(\ell^i)^2} \\ \frac{2EI^i}{\ell^i} & \frac{4EI^i}{\ell^i} & -\frac{6EI^i}{(\ell^i)^2} \\ \frac{-6EI^i}{(\ell^i)^2} & \frac{-6EI^i}{(\ell^i)^2} & \frac{12EI^i}{(\ell^i)^3} \end{bmatrix} \begin{Bmatrix} u^{i1} \\ u^{i2} \\ u^{i3} \end{Bmatrix} \quad \ldots(3.1)$$

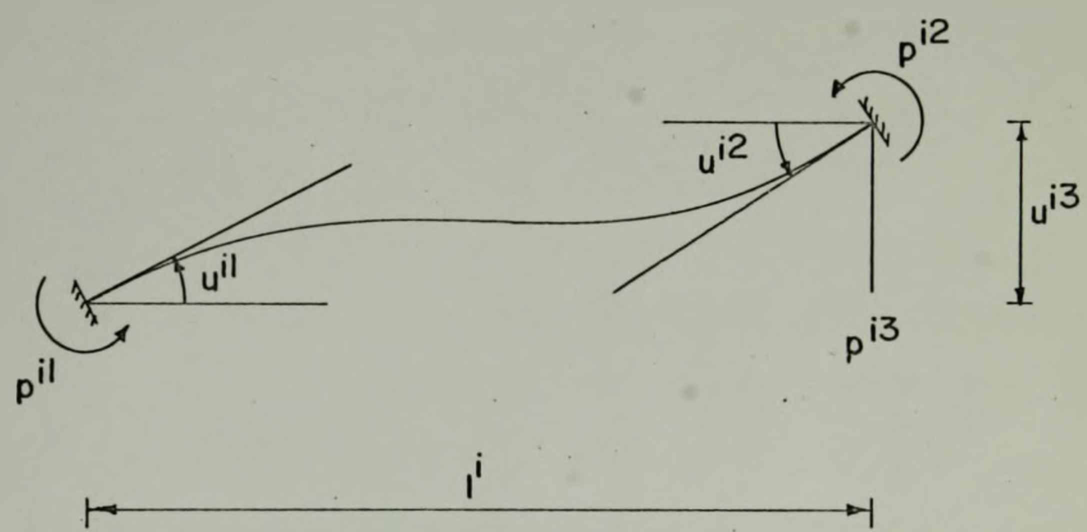


FIG. 3.1 MEMBER FORCES AND DEFORMATIONS

This relation can be expressed as

$$\{p^i\} = [K^{ii}]\ \{u^i\} \qquad \ldots(3.2)$$

The member stiffness matrix $[k^{ii}]$ is also called the deformation-force transformation matrix of ith member as it transforms the deformations into forces.

3.4 Frame Deformation-Force Transformation Matrix

Relation expressed by Eq. 3.2 can be extended to all the members of a frame comprising a number of members as below

$$\{p\} = [K]\ \{u\} \qquad \ldots(3.3)$$

when

$$\{p\} = \begin{Bmatrix} p^1 \\ p^2 \\ \vdots \\ p^i \\ \vdots \\ p^m \end{Bmatrix}$$

$$[K] = \begin{bmatrix} K^{11} & & & \\ & K^{22} & & \\ & & K^{33} & \\ & & & \ddots \; K^{mm} \end{bmatrix}$$

and

$$\{u\} = \begin{Bmatrix} u^1 \\ u^2 \\ \vdots \\ u^i \\ \vdots \\ u^m \end{Bmatrix}$$

where superscript m denotes the total number of members in the frame. K^{11}, K^{22} ....K^{mm} denote the member stiffness matrices of 1st, 2nd, ..., mth member.

$\{p^i\}$ and $\{u^i\}$ are the force and deformation vectors respectively of ith member shown in Fig. 3.1 and are given by

$$\{p^i\} = \begin{Bmatrix} p^{i1} \\ p^{i2} \\ p^{i3} \end{Bmatrix}$$

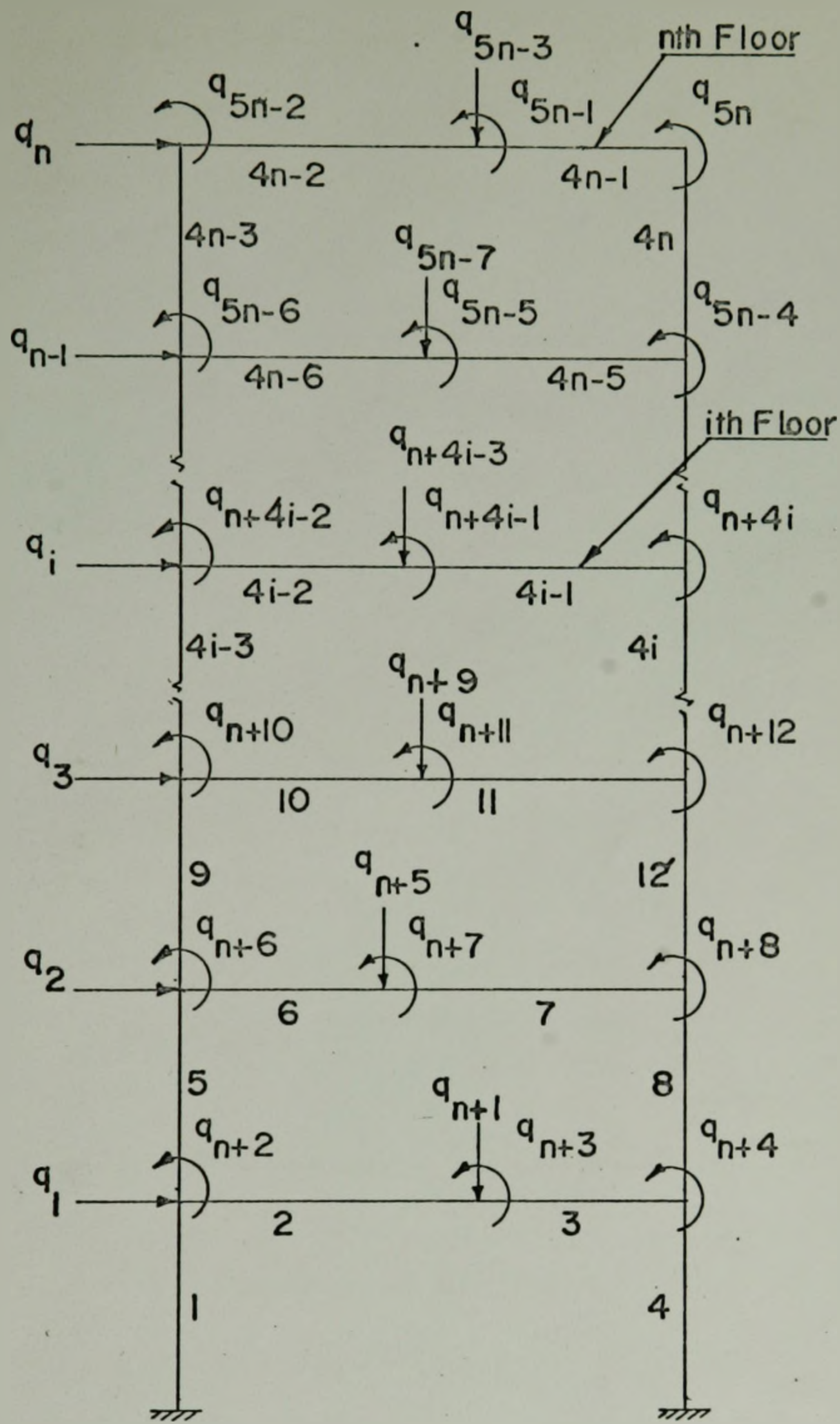
and $$\{u^i\} = \begin{Bmatrix} u^{i1} \\ u^{i2} \\ u^{i3} \end{Bmatrix}$$

and $[K^{ii}]$ is the member stiffness matrix as shown in section 3.3.

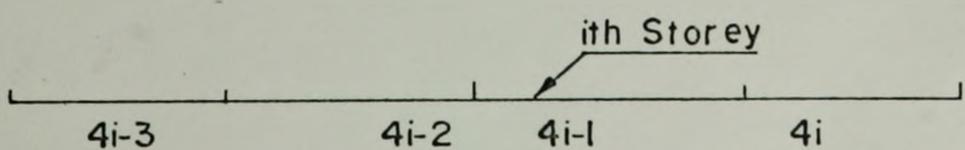
The number of rows and number of columns of [K] matrix will be 3m each.

3.5 Displacement-Deformation Matrix

In order to obtain member deformation produced by the joint displacements, a matrix 'A' called the displacement deformation matrix is obtained from the rigidity of the joints and geometry of the frame. To facilitate the computation of 'A' matrix, the members of the multi-storey frame and the loads acting on each joint are numbered as shown in Fig. 3.2a. The numbering of members starts from the bottom most storey and is carried out upward for successive storeys. Each load point on a floor is considered as a joint hence the beam



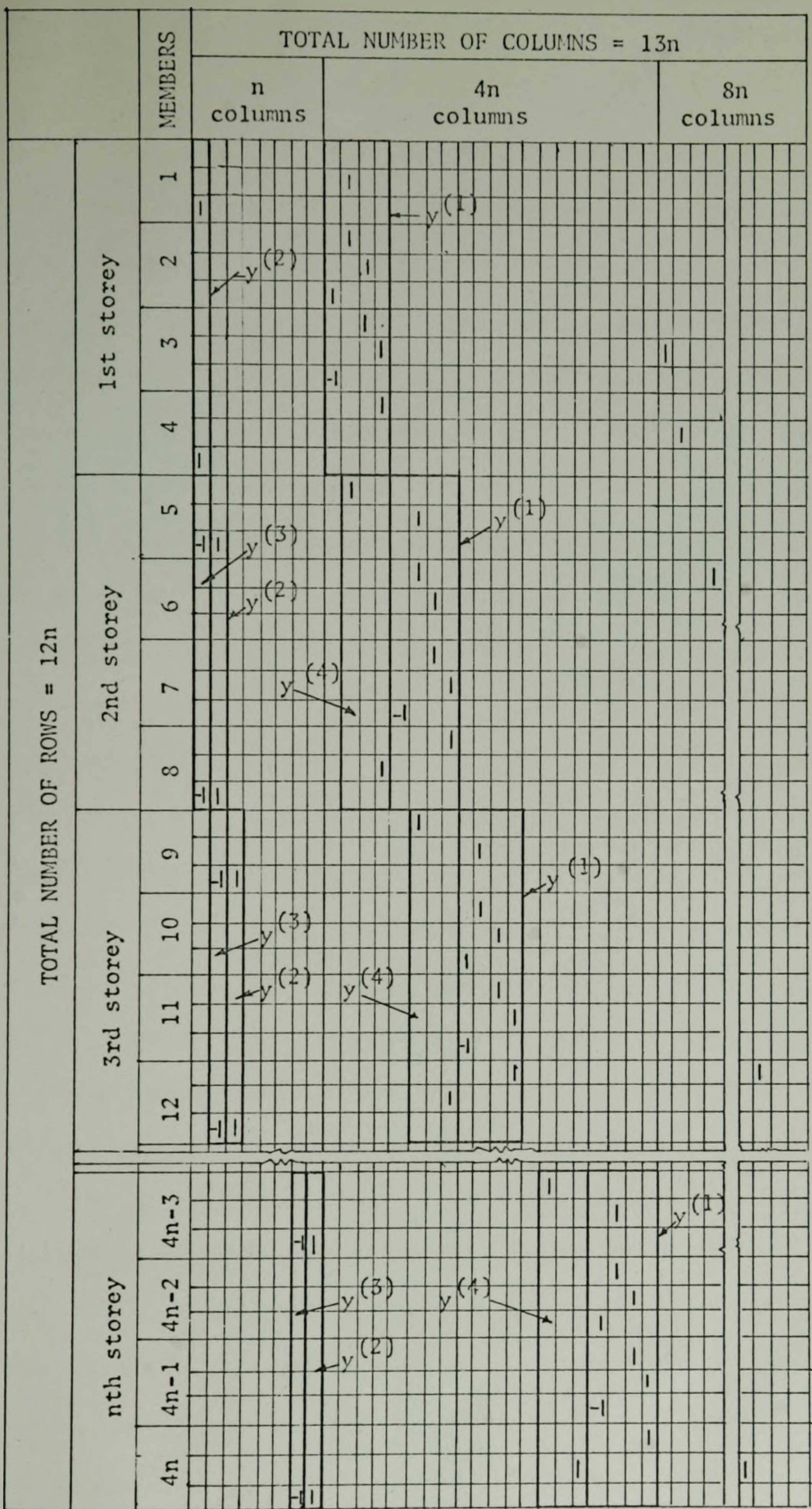
(a)



(b)

FIG. 3.2 NUMBERING OF LOADS AND MEMBERS OF n STOREY FRAME is divided into two components as shown and each of these components is considered as a different member. This is done so that a plastic hinge could be allowed to form at the load point if the moments there become equal to plastic moment of the beam. All the floors are numbered starting from 1st floor and in the increasing order upwards. The horizontal dynamic loads are also numbered starting the 1st load on 1st floor and in increasing order upwards. It will be shown later in this chapter that numbering the loads in such a manner facilitates expressing the structural resistance in terms of horizontal floor deflections. The remainder of the loads on the joints are numbered starting from the concentrated load followed by three joint moments on each floor as shown in Fig. 3.2a. The external moment loads qn+2, qn+3 and qn+4... in Fig. 3.2a are equal to zero.

To obtain 'A' matrix, as shown in Table 3.1, the first three rows of 'A' are assigned to three member deformations u^{11}, u^{12}, u^{13} of the first member in order, the next three rows are assigned to second member deformations and similarly for other members. Thus for a structure comprising m members, which happens to be 4n members for n storey building, the number of rows in 'A' matrix will be 12n Each column of 'A' matrix corresponds to a joint displacement which in turn corresponds to a joint load. First n columns are assigned



Elements of [A] Matrix

<u>TABLE 3.1</u>

to horizontal floor displacements and initially the remainder columns are sequentially assigned to displacements of each storey joint.

Thus, as shown in Table 3.1, columns n+1 to n+4 correspond to vertical displacement of first storey concentrated load, rotation of joints where moments qn+2, qn+3 and qn+4 are acting respectively.

Thus for all n storeys, 'A' initially will have 5n columns. In order to calculate the element A_{ij} of 'A' matrix, a unit displacement at joint j is given. The deformations at i caused by the above displacement gives the value of A_{ij} provided all other joint displacements are kept zero. For example, if a unit horizontal displacement is given to first floor, the first and fourth members are displaced by same amount and fifth and eighth members are displaced by unity in the negative sense. These are entered in the 3rd, 12th, 15th and 24th columns respectively corresponding to lateral deformation of 1st, 4th, 5th and 8th members respectively. Similarly if a unit rotation is applied at 1st storey left joint corresponding to (n+2)nd column of 'A' matrix, 2nd end of first member rotates by unity, and first ends of 2nd and 5th members rotate by unity which are entered in the 2nd, 4th, and 13th rows respectively against (n+2)nd column which corresponds to the above joint rotation. In the same manner all the elements are calculated.

In case a plastic hinge forms at a certain end of a member, the hinge is considered as a separate joint for purposes of rotational displacement. In such a situation, a column is added to 'A' matrix beyond 5n th column and an entry of plus one is made in this column against the row corresponding to rotational deformation of the member where this hinge has formed. The element of 'A' corresponding to rotation of member where hinge has formed is made zero. Thus, as is shown in Table 3.1, if a hinge develops at 6 which is 2nd end of member 3, the element $A_{8,n+4}$ is made zero and column 5n+1 is added and element $A_{8,5n+1}$ becomes unity as a unit rotation at this hinged joint causes unit rotation at the end of this 3rd member. All other elements of this 5n+1 column remain zero as no other member deformations take place. If each column beyond 5n columns of 'A' matrix is reserved for formation of each hinge, another 8n columns would be needed as no. of possible hinges as shown in Fig. 3.3 is 8n. It will be shown in Chapter IV that this huge matrix having 12n rows and 13n columns can be manipulated to reduce storage thereby facilitating the computation of inelastic response of multi-storey frames.

Knowing displacement deformation matrix, the relation between member deformations for whole structure and joint displacements can be expressed as

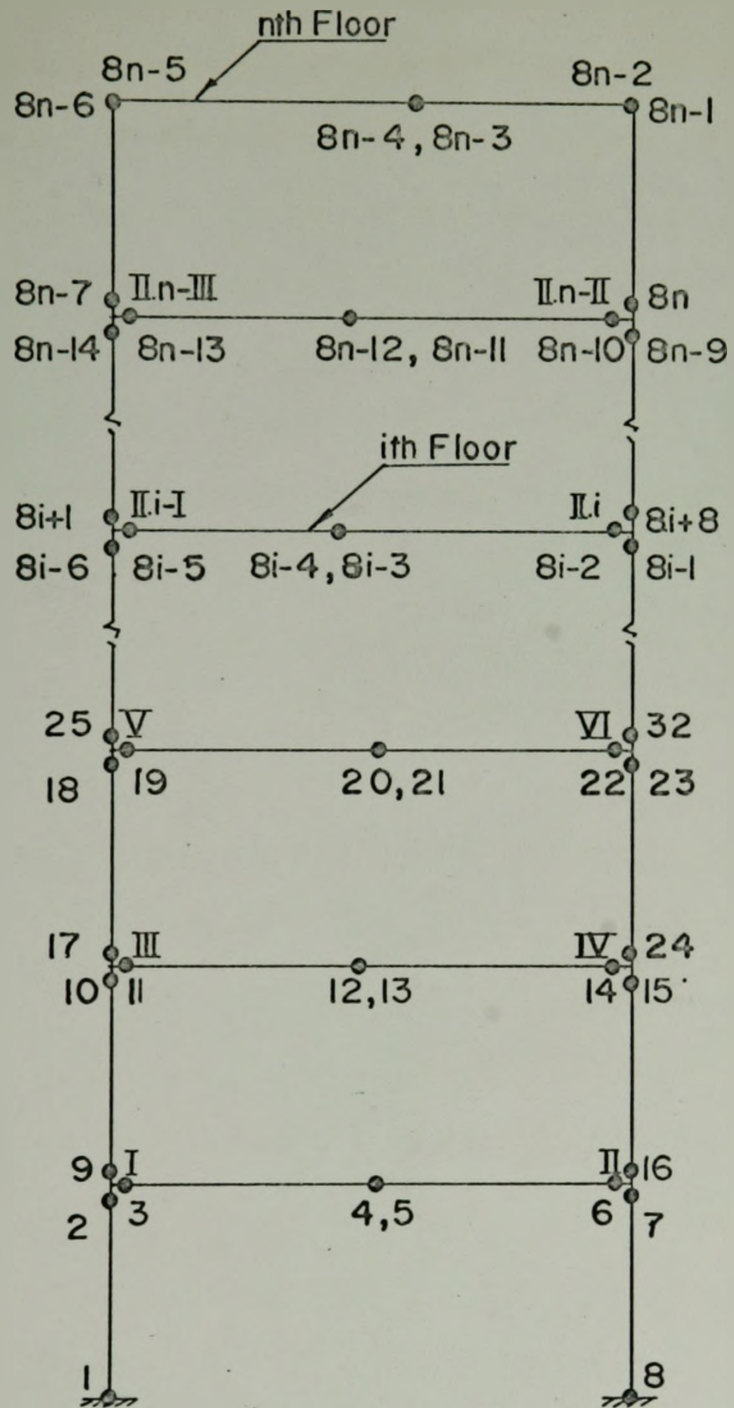


FIG. 3.3 NUMBERING THE ENDS OF MEMBERS AND JOINTS

$$\{U\} = [A]\ \{D\} \qquad \ldots(3.5)$$

where vector {D} represents the joint displacements corresponding to the loads acting on the joints. In case of absence of an external load on the joint, the load is considered to be zero. For instance, all the external moments on the joints are considered zero.

For a multi-storey building frame as shown in Fig. 3.2a, the load vector {Q} will be given by

$$\{Q\} = \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \\ q_{n+1} \\ q_{n+2} \\ \vdots \\ q_{n+4} \\ q_{5n} \\ \vdots \end{Bmatrix}$$

3.6 The Force-Load Matrix

The force load matrix transforms the member forces of a structural system to joint loads. It can be shown by the principle of virtual work that relation between joint loads and member forces is given by

$$\{Q\} = [A]^T\ \{p\} \qquad \ldots(3.6)$$

where $[A]^T$ is the force load matrix, which is the transpose of the previously defined displacement-deformation matrix.

3.7 Displacement-Load Matrix

From the relations expressed in Eqs. 3.5 and 3.6, a relation between joint displacements and loads could be derived. Substituting for {p} from Eq. 3.3, Eq. 3.6 yields

$$\{Q\} = [A]^T [K] \{u\} \qquad ...(3.7)$$

and substituting {u} from Eq. 3.5, Eq. 3.7 yields

$$\{Q\} = [A]^T [K] [A] \{D\} \qquad ...(3.8)$$

$$\text{or} \quad \{Q\} = [S] \{D\} \qquad ...(3.9)$$

where $[S] = [A]^T [K] [A]$. [S] is a square matrix and could be inverted. Thus, joint displacements are obtained from the known load vector {Q} and known [A] as below

$$\{D\} = [S]^{-1} \{Q\} \qquad ...(3.10)$$

3.8 Expression for Moments

Member forces which include moments at the ends of member are given by {p} from Eq. 3.3

$$\{p\} = [K] \{u\}$$

Substituting for {u} from Eq. 3.5 in above equation

$$\{p\} = [K] [A] \{D\} \qquad ...(3.11)$$

again substituting for {D} from Eq. 3.10, Eq. 3.11 yields

$$\{p\} = [K] [A] [S]^{-1} \{Q\} \qquad ...(3.12)$$

As shown in 3.4 vector {p} consists of three rows for each member of the structural system. Thus it will have three times as many elements as the number of structural members. The first two out of these three represent the end moments at the left and right end of the member respectively. The third element represents the shear. Thus, every 1st, 4th, 7th ..... (12n-2)th elements represent the left end moment and 2nd, 5th, 8th .... (12n-1)th elements represent the moment on the right end of the member. These moments are obtained from the corresponding elements of {p} .

In order to designate left and right end of vertical and horizontal members, each storey is considered to be flattened by opening out its lower columns as shown in Fig. 3.2b for ith storey. Thus the left end of left column will be the lower end and right end the upper end. For right column, the left end will be the upper end and right end the lower end. For beam there is no confusion because of its horizontal configuration.

3.9 Hinge Rotations

The sections at which moments may attain extremum values are shown in Fig. 3.3. As soon as moments at these sections become equal to plastic moment, a plastic hinge is inserted at these points. If a hinge develops at sections 1 or 8, the hinge rotations at such points,where only one end of a member exists, are given by angular displacement at the end considered. Similarly, where three members meet, if all the member ends develop hinges, the hinge rotations are the angular displacements of respective members at the end considered. In the situation at such joints when only one or two hinges exist in a particular phase, the hinge rotations are the algebraic difference of the displacements at the end of hinged member and the rotational displacement of the remaining elastic joint.

At sections where beams are loaded by a concentrated vertical load, the beam is divided into two elements as in Fig. 3.2. If a hinge develops at this section, the hinge rotation is given by the algebraic difference of the rotational displacement of the end of member under consideration and that of the other end of the member meeting at the joint. Such sections are 4, 5; 12, 13; 20, 21; ..... 8n-6, 8n-5; 8n-2, 8n-1 th sections of a frame of n storeys.

3.10 Resistance Deflection Relationship

As shown in Fig. 3.2a, the horizontal loads q_1, q_2, ... q_n are the resistances required to hold the frame in its deformed state. For integration of differential equation of motion, Eq. 2.1, it was stated in section 2.3 that the structural resistances R_i could be expressed as a function of horizontal floor displacements X_i.

From Eq. 3.9

$$\{Q\} = [S]\,\{D\}$$

or

$$\left\{\begin{array}{c} R_1 \\ R_2 \\ \vdots \\ R_n \\ \hline q_{n+1} \\ q_{n+2} \\ \vdots \end{array}\right\} = \left[\begin{array}{c|c} T & W \\ \hline Y & Z \end{array}\right] \left\{\begin{array}{c} X_1 \\ X_2 \\ \vdots \\ X_n \\ \hline D_{n+1} \\ D_{n+2} \\ \vdots \end{array}\right\} \qquad \ldots(3.13)$$

where {Q} is partitioned into {R}, the structural resistance vector and $\{Q^r\}$ the remaining external loads vector. Similarly, {D} is partitioned into horizontal floor displacement vector {X} and the remaining displacements vector $\{D^r\}$. Accordingly, [S] is partitioned into [T], [W], [Y] and [Z] matrices in which [T] and [Z] are square matrices and can be inverted. Thus Eq. 3.13 can be written as

$$\begin{array}{c} R \\ Q^r \end{array} = \begin{array}{cc} T & W \\ Y & Z \end{array} \; \begin{array}{c} X \\ D^r \end{array} \qquad \ldots(3.14)$$

or

$$\{R\} = [T]\,\{X\} + [W]\,\{D^r\} \qquad \ldots(3.15)$$

$$\{Q^r\} = [Y]\,\{X\} + [Z]\,\{D^r\} \qquad \ldots(3.16)$$

From Eq. 3.16

$$\{D^r\} = [Z]^{-1}\left\{\{Q^r\} - [Y]\ \{X\}\right\} \qquad ...(3.17)$$

substituting $\{D^r\}$ from Eq. 3.17 in Eq. 3.15 we get

$$\{R\} = \left[[T] - [W]\ [Z]^{-1}\ [Y]\right]\ \{X\} + [W]\ [Z]^{-1}\ \{Q^r\}$$

or $$\{R\} = [H]\ \{X\} + \{B\} \qquad ...(3.18)$$

where $[H] = [T] - [W]\ [Z]^{-1}\ [Y]$

and $\{B\} = [W]\ [Z]^{-1}\ \{Q^r\}$ ...(3.19)

Matrix [H] and vector {B} are constant for a particular phase and are re-calculated after each transition as the structural stiffness matrix is re-calculated after each transition because of addition or subtraction of plastic hinges in the structure.

CHAPTER IV

COMPUTER PROGRAM

4.1 General

The computer program for computing the dynamic response of multi-storey building frames when stressed in the inelastic region is a bit involved due to large matrices such as [K], [S] and [A] which require large storage locations and thus would have limited the analysis to a small number of storeys only. It will be shown in the following paragraphs, as to how the storage necessity of [K] and [A] has been eliminated through logical programming and how the size of [S] is controlled and varied so that minimum storage is required and time is saved in the inversion of [Z] by reducing its size to the minimum possible.

4.2 Computer Program Outline

The first operation in the computer program is to read in the initial data which consists of (a) properties of given structural system, (b) the properties of numerical integration procedure and (c) the properties of the applied dynamic force. The details of these properties are shown in Appendix A. After this the natural frequencies of the system are computed and if desired, the damping matrix can be computed and stored. The computation of damping matrix incorporated in the program is based on percentage of critical damping in the various modes as obtained from modal analysis and discussed by Biggs[16]. Now the initial conditions are calculated which are initial deflections of floors, initial accelerations and velocities of the masses. The matrices [S], [T], [W], [Y], [Z] and {B} are then computed and differential equation of motion Eq. 2.8 is solved for deflections at the beginning of the next time interval. Knowing these deflections, moments at all the elastic sections and hinge rotations at all the plastic sections are computed. All these sections are now tested to ascertain whether any section is passing through a transition from elastic to plastic or plastic to elastic phase. If it is found that elastic-plastic transition is occurring, the computation is reversed back to the beginning of the time interval, and at this time a smaller time interval of $\frac{1}{100}$ th of the previous time interval is adopted and point of transition is approached slowly till it is achieved. If plastic-elastic transition is indicated, it becomes necessary to go two time steps back as shown by Heidebrecht[17] and approach the transition with a smaller time interval. Elastic-plastic transition occurs if any section attains moment equal to the plastic moment for that section. Plastic-elastic transition occurs if the plastic hinge rotation begins reversing direction. This is indicated by the change in sign of the plastic hinge rotation velocity.

The transition procedure adopted is basically the same as described in detail by Heidebrecht[17], except that in the transition loop it is checked to know at what joint how many hinges are being formed and released. If a hinge is formed, a column is added to [A] in the end. If a hinge is released, the corresponding column of [A] is eliminated and all the columns following the one eliminated are shifted one column position to the left so that size of [Z] is kept as small as possible. Matrix [Z] is required to be inverted at each time interval and keeping its size to a minimum possible results in saving of computational time. The procedure of manipulating column numbers and their positions in [A] matrix is explained in details later in this chapter.

After checking the transitions, velocities and accelerations of masses at the beginning of next time interval are computed from Eq. 2.5 and Eq. 2.6 respectively to repeat the procedure. In case transition has taken place, matrices [S], [H] and {B} are re-calculated from new [A] before solving differential equation of motion Eq. 2.8. Thus, knowing all the quantities at the beginning of next time interval, the above procedure is repeated to compute further response.

4.3 Storage of [K] Matrix

As expressed in 3.4, [K] contains three times as many rows and columns as number of members of the structural system. In multi-storey single-bay frames, the number of members in each storey are four. Thus, for a six storey building, number of members will be 24 and number of columns and rows of [K] will be 72 each and hence it will require 72 x 72 = 5184 storage locations. For a multi-storey building of n storeys, the storage required for [K] will be $144n^2$ locations. This will be a heavy drain on the available storage locations.

A careful study of [K] reveals that three rows of [K] are assigned to a particular member. These three rows contain nine elements, three per row which are not equal to zero. For a particular member, say mth member, the locations of these in [K] matrix are given by

(3m -2, 3m - 2)	(3m - 2, 3m - 1)	(3m - 2, 3m)
(3m - 1, 3m - 2)	(3m - 1, 3m - 1)	(3m - 1, 3m)
(3m, 3m - 2)	(3m, 3m - 1)	(3m, 3m)

where first expression within the bracket shows the row number and second the column number in which the element is located.

Out of these nine elements, as shown in Eq. 3.1, for m equal to 1, (3m - 2, 3m - 2)th and (3m -1, 3m - 1)th elements are having the same value. Similarly, (3m - 2, 3m - 1) and (3m - 1, 3m - 2) are identical. The remaining elements,excluding (3m, 3m)th element, are identical. Thus, for each member actually there are four constant values which need real storage. The remaining five are identical to one of these four. It is possible to store only four constant values per member and use these in proper order so that [K] matrix is reproduced. With this technique $144n^2 - 16n$ storage locations are saved. For a ten storey building frame, this figure will be 14240 locations which is a significant economy.

The non-zero elements having different values are four per member and these are stored in an array XA(i,j) where i refers to a particular non-zero element value and j refers to the number of member. Thus, for first member, the four values are XA(1,1), XA(2,1), XA(3,1) and XA(4,1).

4.4 Storage of [A] Matrix

As is evident from Table 3.1, the [A] matrix consists of 12n rows and as many columns as number of external loads, i.e. horizontal loads, vertical loads, and external moments at the joints. (Zero in the elastic phase of the structure). Thus, initially it will have n columns for resistances R, and 4n columns for other static loads. Thus, total number of columns in the elastic phase will be 5n. This will be 30 for six storey building and 50 for 10 storey building. If provision is made for all the possible hinges to develop, the number of columns of [A] matrix will become 5n + 8n = 13n. This will mean 78 columns for a six storey frame and 130 columns for a 10 storey frame. It is quite clear from above figures that [A] matrix will require huge storage capacity of $156n^2$ memory locations unless it is augmented so that these locations could be saved.

A careful examination of [A] shows that except for 2n - 2 rows which are 15th, 27th, 39th ... (12n-9)th and 24th, 36th, 48th ... 12nth, every row contains only one element having a non-zero value and this too is unity and positive except in 9th, 21st, 33rd ... 12n-3 th rows in which it is minus one. The remainder of the elements in each row are zeros. At the hinge points, not only the row contains an element unity but the corresponding column also contains only one element having a value of plus one. All other elements are zero.

These properties are made use of to reproduce the [A] matrix through logical programming and augmentation in such a way that only minimum storage is used. This is achieved by the following technique.

(a) Reproduction of [A] up to 5n columns.

The one dimensional subscripted variable KP(i) is used whose subscript corresponds to the number of the row of [A] and whose numerical value is an integer corresponding to the column number in which the element under consideration has a value of one. Thus, each time an element of [A], say A_{ij}, is used in computation, the value of KP(i) is compared with j. In case it is equal to j, A_{ij} is assigned a value of unity; otherwise, it is taken as zero.

Thus, using only 13n+1 locations for storage, one for each row of [A], $156n^2$ -13n-1 storage locations are saved. For a six storey building this comes to a saving of 5537 locations and a saving of 15469 storage locations for a ten storey building as shown in Table 4.1.

(b) Reproduction of [A] beyond 5n columns.

As already discussed, the size of [A] is increased by one column if a hinge is formed and is decreased by one column if a hinge is released.

At a joint where two members meet, if a hinge is developed, only one column is added as the other hinge which is at the same location is assumed to be formed by giving a value of one to a variable DIC(i) which is multiplied by the element of [A] having the unit value. Similarly, for removing the element when a hinge has developed, the element is multiplied by a variable $(1 - DIA(i)^2)$ where i refers to the location of the particular hinge. The variable DIA(i) is defined as follows:

DIA(i) = +1.0 if i is plastic and moment at i is positive

DIA(i) = -1.0 if i is plastic and moment at i is negative

DIA(i) = 0.0 if i is elastic.

For all such even numbered sections DIC(i) takes a value of 1.0 or 0.0 at elastic-plastic of plastic-elastic transitions respectively.

Matrices	[K]			[A]		
No. of Storeys (1)	Normal Storage Required (2)	Storage Used (3)	% Saving in Storage (4)	Normal Storage Required (5)	Storage Used (6)	% Saving in Storage (7)
n	$144n^2$	16n	$\frac{(2)-(3)}{(2)} \times 100$	$156n^2$	13n + 1	$\frac{(5)-(6)}{(5)} \times 100$
2	576	32	94.5	624	27	95.6
6	5184	96	98.1	5616	79	98.5
10	14400	160	98.8	15600	131	99.2

Storage Requirement and % Saving of Same for [K] & [A] Matrices

TABLE 4.1

At a joint where three members meet, there is possibility of one hinge developing first and a second following or second and third developing at the same time to keep the moment equilibrium. In the worst case all the hinges may develop at the same time.

For the first hinge developing at such a joint, a column is added in the end of [A] matrix. A variable JT(j) is used which assumes a value equal to the number of elastic ends meeting at a joint. Here j corresponds to the number of joint marked in Roman figures as shown in Fig. 3.3. Initially it has a value of three and it becomes less by one if the end of a member meeting at the joint in question develops a hinge. Thus, the value of this variable keeps record of the number of hinges formed at the joint. If a hinge already formed is released, the value of JT(j) increases by one. In case such a joint develops three hinges in a particular phase, no extra column is added to [A] matrix for the last hinge formed. In such a situation, DIC(i) assumes a value of unity for the last hinge formed. The original column for jth joint is used for this last hinge. The record of retaining the column for the last hinge formed in the column corresponding to the joint in question is maintained by another variable LX(j) which assumes a value of i at such an occasion. In a situation where a particular joint has all the hinges formed and if ith hinge is released subsequently, the value of LX(j) is compared with i. If it equals i, DIC(i) assumes a value of zero and the column corresponding to jth joint is restored in its original place. If i does not equal to LX(j), the column corresponding to i beyond 5n columns is eliminated and columns after this removed column are moved to the left by one column to fill this gap. The element corresponding to ith section is restored in the column corresponding to jth joint and another column is added in the end to restore the hinge which was formed in the end and which is indicated by the value of LX(j).

The number of columns added beyond 5n and then reduced for elastic-plastic and plastic-elastic transitions, respectively, are taken care of by the value of a variable KF. Its value initially is zero but is increased by one if a column is added and decreased by one if a column is eliminated. The tracing as to which column corresponds to which hinge is done by another variable KL(j). The value of KL(j) gives the hinge number for which (5n + j)th column was added in [A]. In case of plastic-elastic transition of ith hinge, the value of i is compared with KL(j) by varying j from 1 to KF. At the point where KL(j) becomes equal to i, the particular column, i.e. (5n + j)th column of [A] matrix is eliminated and the rest of the columns beyond (5n + j)th column are shifted by one column space to fill this gap.

In this manner the number of columns of [A] are kept minimum which results in the reduction of the size of [S] as the number of rows and columns of [S] equal the number of columns of [A]. This technique ultimately results in the reduction of the size of [Z] which is to be inverted after each transition.

(c) Repetition of the elements of [A].

Because of the repetitive geometrical shape of the multi-storey frame, a careful examination of the non-zero elements of [A] as shown in Table 3.1 reveals that the elements of block $y^{(1)}$ in first storey repeat in subsequent storeys and the block is shifted by four column positions to the right for every additional storey. Similarly, the elements of block $y^{(2)}$ in first storey repeat in subsequent storeys and this block is shifted by one column position to the right. The elements of block $y^{(3)}$ and $y^{(4)}$ in second storey repeat in subsequent storeys and their positions are shifted by one column space and four column spaces respectively to the right.

The above property is useful in calculating the values of KP(i) variable for a frame of n storeys where i refers to the number of the row of the [A] matrix. The value of KP(i) for a frame of n storeys can be calculated as follows:

KP(12j - 11)	=	4j + n-6
KP(12j - 10)	=	4j + n-2
KP(12j - 9)	=	j
KP(12j - 8)	=	KP(12j - 10)
KP(12j - 7)	=	4j + n-1
KP(12j - 6)	=	4j + n-3
KP(12j - 5)	=	KP(12j - 7)
KP(12j - 4)	=	4j + n
KP(12j - 3)	=	KP(12j - 6)
KP(12j -2)	=	KP(12j - 4)
KP(12j - 1)	=	4j + n-4
KP(12j)	=	KP(12j - 9) when j = 1,2...n

except that KP(1) = KP(11) = 0.

This variable KP(i) is used to reproduce [A] matrix as described in section 4.4b above.

4.5 Computation of [S] Matrix

As per Eq. 3.9

$$[S] = [A^T] [K] [A]$$

It has been described in section 4.3 that all the non-zero elements of [K] are stored in an array XA(i, j). Because of this definition it can be shown that an element $S_{I,j}$ of matrix [S] is given by

$$S_{I,j} = \sum_{i=1}^{4n} \{[(A_{3i-2,\ j}) \cdot XA(1,i) + (A_{3i-1,\ j}) \cdot XA(2,i)$$

$$+ (A_{3i,\ j}) \cdot XA(3,i)] \cdot (A_{3i-2,\ n})$$

$$+ [(A_{3i-2,\ j}) \cdot XA(2,i) + (A_{3i-1,\ j}) \cdot XA(1,i)$$

$$+ (A_{3i,\ j}) \cdot XA(3,i)] \cdot (A_{3i-1,\ n})$$

$$+ [(A_{3i-2,\ j}) \cdot XA(3,i) + (A_{3i-1,\ j}) \cdot XA(3,i)$$

$$+ (A_{3i,\ j}) \cdot XA(4,i)] . (A_{3i,\ n})\}$$

This expression is further simplified by manipulation of [A] matrix as described in details in 4.4. The elements of [A] occurring above are stored in a variable AG(i) where i = 1,2...6. The six elements of [A] corresponding to a particular member are reproduced through logical programming and thus final expression of S_{Ij} becomes

$$S_{Ij} = \sum_{i=1}^{4n} \{[AG(1) \cdot XA(1,i) + AG(2) \cdot XA(2,i)$$

$$+ AG(3) \cdot XA(3,i)] \cdot AG(4)$$

$$+ [(AG(1) \cdot XA(2,i) + AG(2) \cdot XA(1,i)$$

$$+ AG(3) \cdot XA(3,i)] . AG(5)$$

$$+ [(AG(1) \cdot XA(3,i) + AG(2) \cdot XA(3,i) + AG(3) \cdot XA(4,i)]$$

$$\cdot AG(6)\}$$

4.6 Computation of {u} and {P}

Similar techniques as described in section 4.5 are used to reproduce [A] which appears in Eqs. 3.5 and [K] which appears in Eq. 3.3, in order to calculate {u} and {P} vectors. In calculating {u}, [A] is reproduced by a single variable AGX. AGX keeps on attaining values of +1.0 or -1.0 whenever a non-zero element of [A] appears in subroutine for calculating {P}. Logical sequence is developed which reproduces [A] through a single variable AGX. [K] is reproduced through XA(i,j) as already described in section 4.3.

4.7 Saving in Storage Locations

Using the repetitive geometry of the multi-storey frame and developing a logical sequence to reproduce sparse matrices like [K] and [A], which normally require huge storage of $144n^2$ and $156n^2$ memory locations respectively, it has been possible to reduce their storage necessity to only 16n and 13n+1 locations respectively. Table 4.1 shows the details of the saving in storage for frames of varying storeys. The saving in storage of [K] and [A] is 98.1% and 98.5% respectively for a six storey frame which would normally require 10800 memory locations for both these matrices. The corresponding figures for normal storage requirement for [K] and [A] matrices for ten storey frame is 30,000 memory locations but by using the logical sequence this figure has been cut down to only 291 locations which gives 99% saving. Using this technique the program developed could handle a frame of up to ten storeys on a computer having about 32,000 memory locations.

CHAPTER V

ANALYTICAL RESULTS AND CONCLUSIONS

5.1 General

As discussed in Cahpter IV, a computer program has been developed which could handle the computation of response up to ten-storey frame. The program developed as shown in Appendix A is fairly general and could be used for any number of storeys. The IBM 7040 available at McMaster Computing Center has a core memory of 32,000 locations. With this capacity the program developed could handle up to a ten storey frame. Computation of response of two and six storey frames has been carried out and the results obtained are discussed in the following paragraphs.

5.2 Response of Two Storey Frames

The dynamic response of the two storey frame shown in Fig. 5.1 has been computed. The computation has been carried out for various loading conditions, of which two examples are included here. These examples are chosen in particular because the forcing function and damping matrix are such that the frame responds in the inelastic region and has several transitions between the elastic and plastic

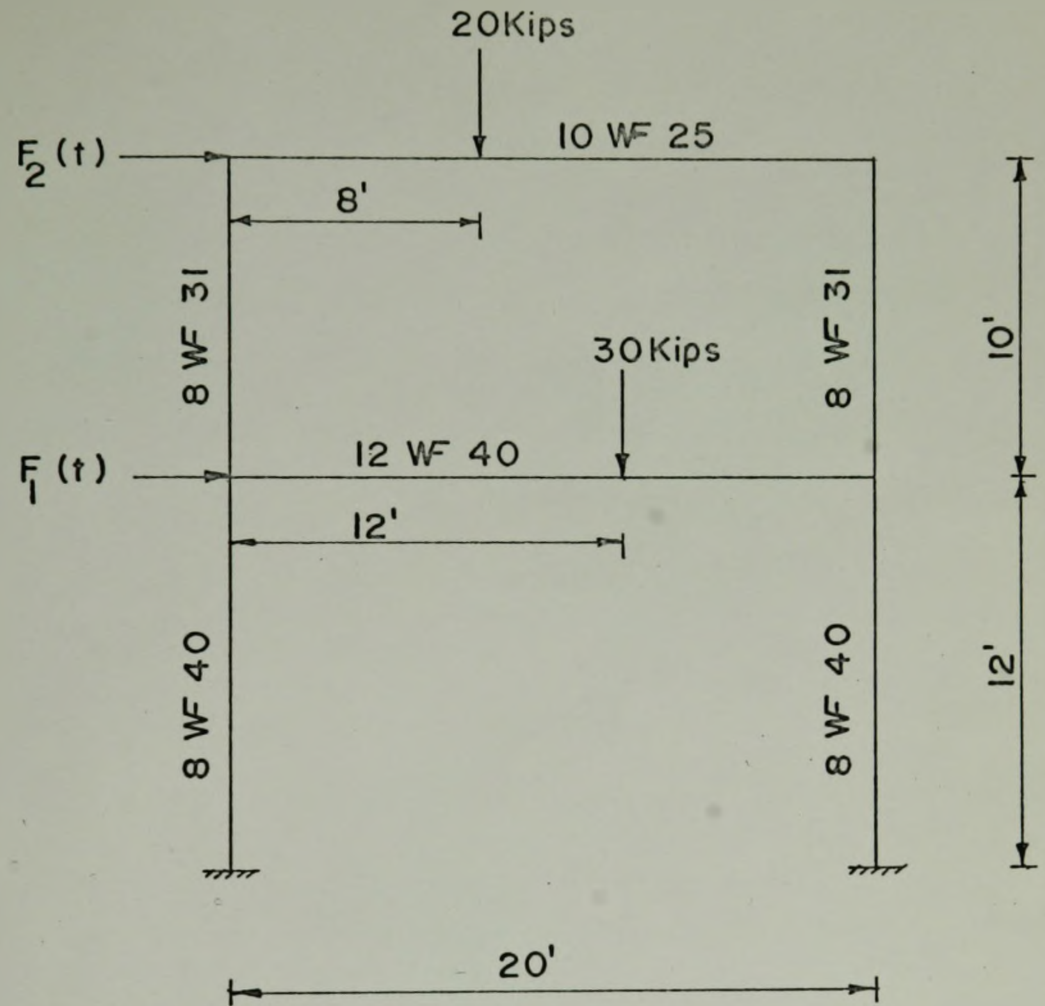


FIG. 5.1 TWO-STOREY FRAME

phases. For both of these examples the forcing function is of the form

$$F_i(t) = F_{oi} e^{-\mu_i t} \text{Cos } \omega_i t \qquad \text{where } i = 1,2$$

The data used in the above expression are shown in Table 5.1.

(a) Example 5.1

The dynamic response curves for the floor deflections X_1 and X_2 are shown in Fig. 5.2.

As the structure responds, hinges appear at sections 6, 14 and 16. These are soon released as the floor deflections move in the opposite direction. Now the hinges appear at 10 and 9 and are soon released. Section 1 and 2 become plastic and then become elastic soon after. In the next cycle of response, hinge forms at 12 and soon released. Beyond this point, i.e. after 0.68 seconds, the forcing function decays so much that the response reamins elastic thereafter.

(b) Example 5.2

The dynamic response curves for this example are shown in Fig. 5.3.

As the structure responds, a plastic hinge appears at section 8 followed by hinges at 6 and 1. Soon after, hinge at 6 is released and section 7 becomes plastic.

Example	Masses $\frac{\text{Kip x sec}^2}{\text{in}}$		Amplitudes Kips		Mus Rad/sec		ω_i Rad/sec		[C] $\frac{\text{Kip x sec}}{\text{in}}$	
	m_1	m_2	F_{o1}	F_{o1}	μ_1	μ_2	ω_1	ω_2	C_{11} C_{21}	C_{12} C_{22}
5.1	0.0817	0.0538	-36.0	-23.0	6.0	6.0	13.0	13.0	0.2816 0.0000	0.0000 0.1855
5.2	0.0041	0.0021	-29.0	-21.75	48.0	48.0	13.0	13.0	0.0456 0.0000	0.0000 0.0234

Data for Examples 1 & 2

TABLE 5.1

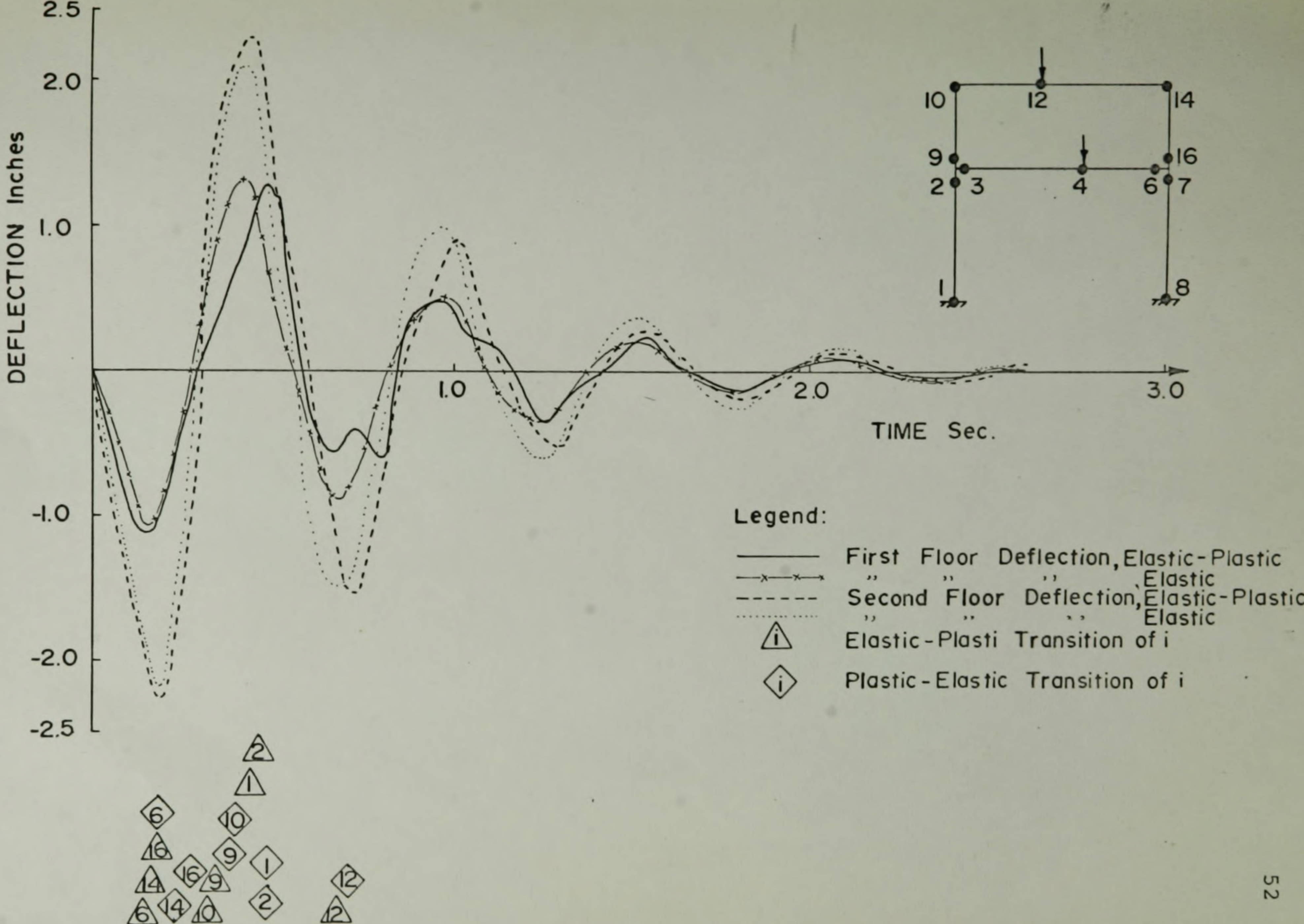


FIG. 5.2 DYNAMIC RESPONSE CURVES, EXAMPLE 5.1

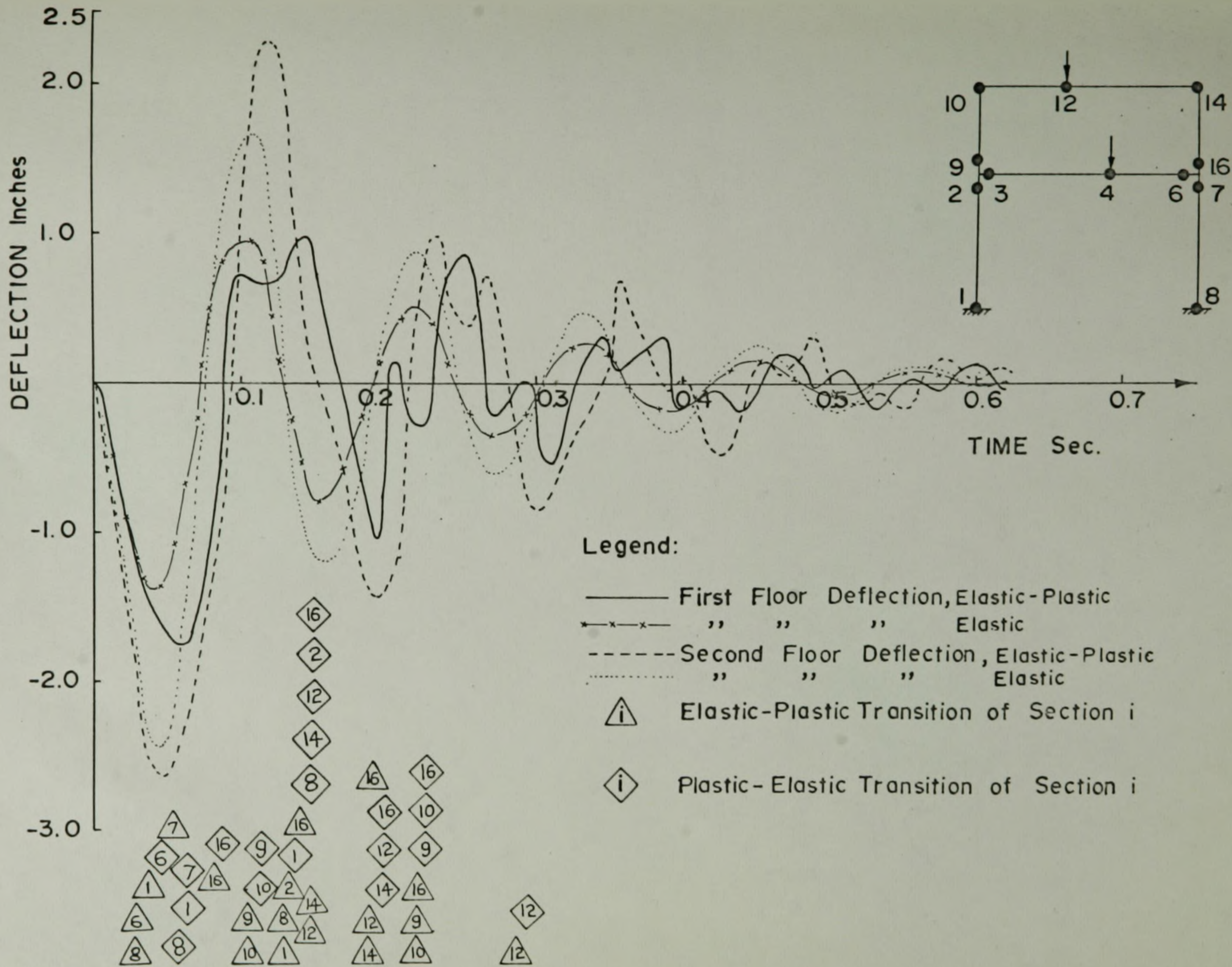


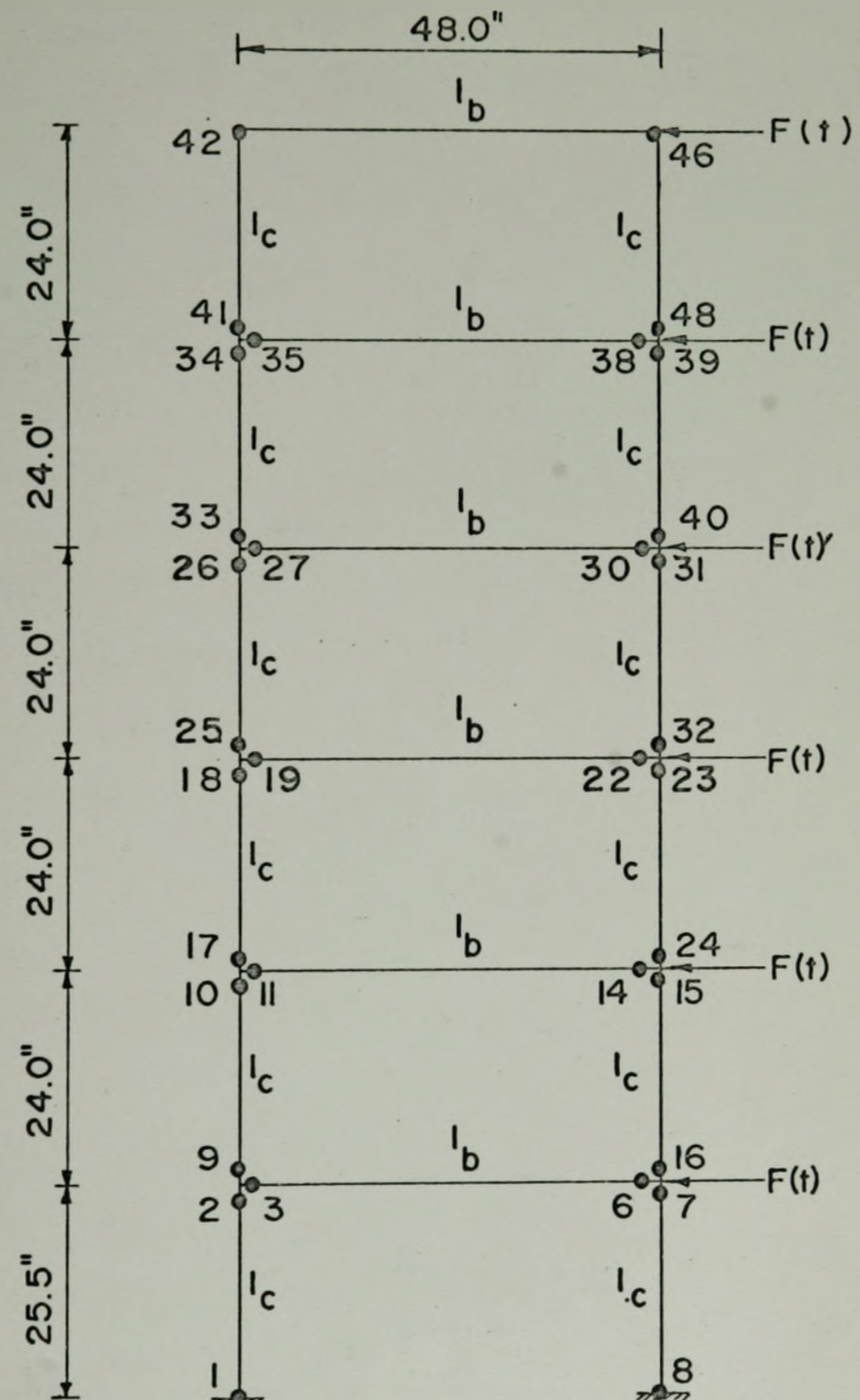
FIG. 5.3 DYNAMIC RESPONSE CURVES, EXAMPLE 5.2

Now the floors start moving in the positive directions and hinges at 8, 1, and 7 are released and structure returns to elastic behaviour. Now after a little lapse of time, hinge forms at 16 and is soon released. Now hinges appear at 10 and 9 and are released immediately after and the structure returns to the elastic phase. The process of formation of hinges and their subsequent release continues till the forcing function decays so much that no hinges form subsequently and the structural response becomes elastic.

5.3 Response of Six Storey Frame

Dynamic response of six storey aluminium frame shown in Fig. 5.4 was computed. The elastic properties of the frame are listed in Fig. 5.4 and the forcing function which is a bilinear pressure wave is shown in Fig. 5.5. As stated in the beginning, the masses of beams and columns were assumed to be lumped at the floor levels. The masses lumped at first through fifth floor are 0.00021 Kip x sec^2/inch each and that at sixth floor level is 0.000205 Kip x sec^2/in.

The response curves of first and sixth floors are shown in Fig. 5.6. The floor deflections are plotted against small time interval which for this particular example has been taken as $\frac{1}{200}$ th of the first natural period. Thus, each time interval represents 6.88×10^{-4} seconds.



Elastic Plastic Properties:

$E = 10 \times 10^6$ psi $\sigma^* = 8 \times 10^3$ psi

$I_c = 0.83$ in.^{4} $I_b = 3.16$ in.^{4}

FIG. 5.4 SIX STOREY FRAME DETAILS & ELASTIC PLASTIC PROPERTIES

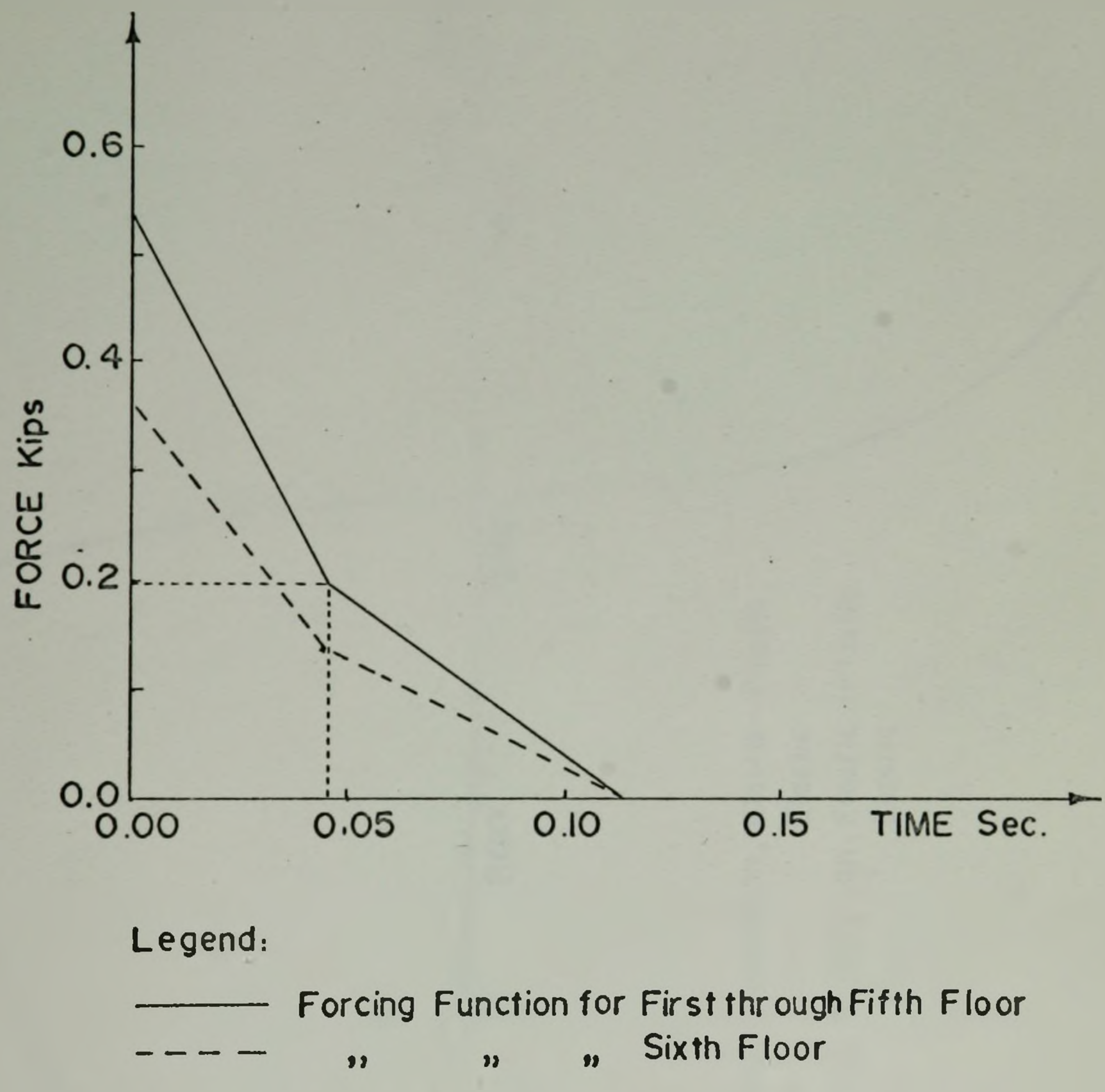


FIG. 5.5 FORCING FUNCTION FOR SIX STOREY FRAME

The response has been computed using a damping factor proportional to masses. The damping matrix is shown in Table 5.2.

As the structure responds, sections 1 and 8 become plastic at the 19th time interval. At the 25th time interval, section 2 and 7 also become plastic. This turns the first storey into sway mechanism. The deflections continue to increase up to 284th interval. At 285th interval, hinges at sections 2 and 7 are released. Immediately after this, hinges at section 1 and 8 are released at 286th interval. As the first storey starts moving backwards, while remaining storeys continue to move forward, hinges form at sections 9, 10, 15 and 16 at 288th interval followed by formation of hinges at 1, 2, 7 and 8 in the negative direction. As the first two storeys become plastic, the deflections of first storey increase rapidly in the negative direction as shown by the dropping curve in Fig. 5.6. The remainder of the storeys continue to vibrate with a small amplitude in the absence of forcing function. This phase continues till at 488th interval hinges are released at 10 and 15 followed by further releasing of hinges at 1, 2, 7, 8, 9 and 16 at 489th interval. Soon after, hinges are formed at 17, 18, 23 and 24, followed by formation of hinges at 1, 2, 7, 8, 9, 10, 11, 14, 15 and 16. This helps in regaining the negative deflection of first storey as shown by the rising

0.009584	0.0	0.0	0.0	0.0	0.0
0.0	0.009584		0.0	0.0	0.0
0.0	0.0	0.009584		0.0	0.0
0.0	0.0	0.0	0.009584	0.0	0.0
0.0	0.0	0.0	0.0	0.009584	0.0
0.0	0.0	0.0	0.0	0.0	0.009356

Damping Matrix for Six Storey Frame

TABLE 5.2

response curve of the first floor. There is little change in the deflections of remaining floors as the amplitude of vibration is very small except the second floor which starts moving in the negative direction with slow rate due to forming of sway mechanism in the first three storeys. The rigid body motion of first and second floor is now very small. The configuration of the frame after 496th interval is shown in Fig. 5.7. At this stage the third storey has again become elastic. There is little change in the position of remaining floors. The residual deflections till this stage are -0.89, 0.83, 2.57, 2.45, 2.43 and 2.43 inches of first through sixth floors respectively. The first floor is still moving in the positive direction. It may be expected that the first mass might reach near about the original position and the remainder of the masses may have permanent delfections of about $2\frac{1}{4}$" or so.

5.4 Conclusions

The object of this investigation has been to develop a simple method which could permit computation of dynamic response of multi-storey frames using high speed digital computer of high storage capacity. The method formulated here is quite simple and is applicable to any number of storeys. Though the computer program developed is meant for a single-bay frame, of n storeys, the same program with slight modification in the procedure for

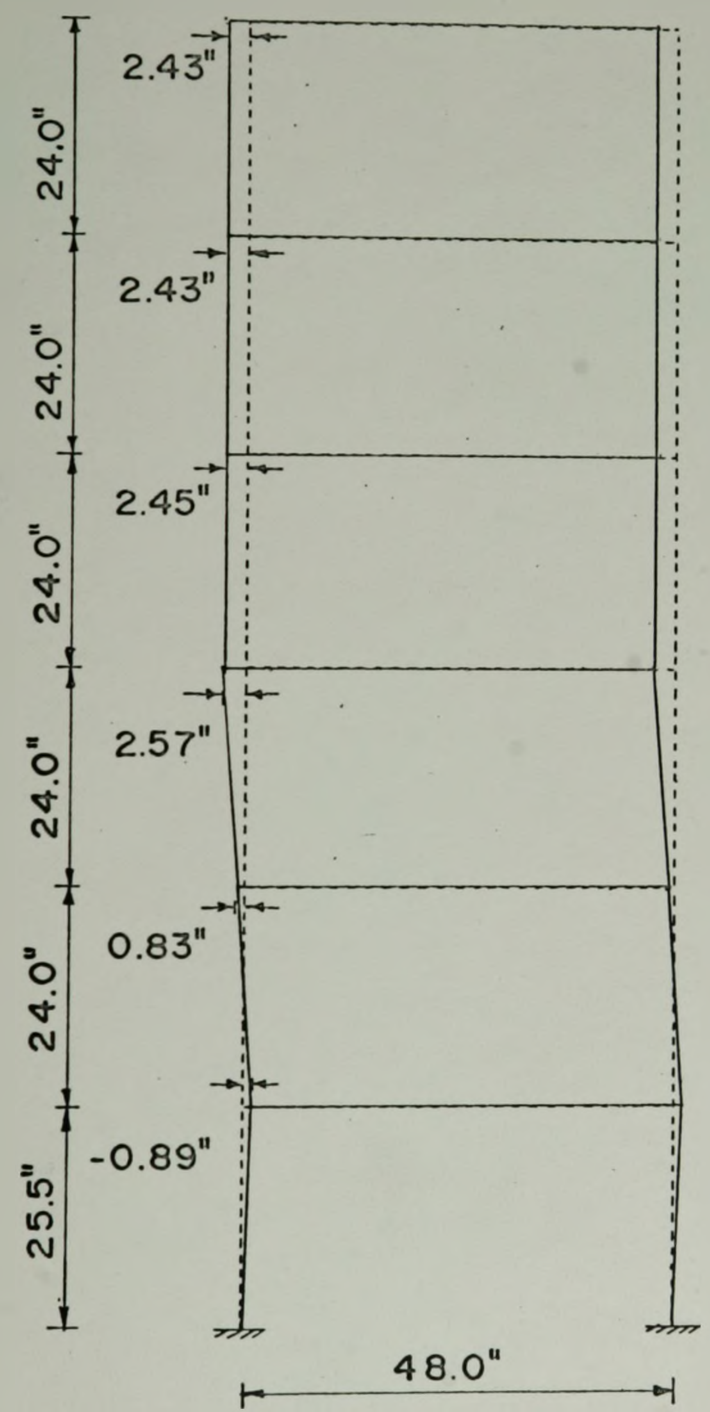


FIG. 5.7 DEFORMED CONFIGURATION OF SIX STOREY FRAME

AT 496TH TIME INTERVAL

reproduction of [A] matrix could be used for a multi-bay multi-storey frame. The generality of the program has been kept such that only the basic data need to be read in along with the value of number of storeys and the program automatically takes care of all the computational work of initial conditions, and response. Any type of forcing function could be used and also the concentrated loads are allowed to act on the beams where hinges may form.

The program developed could compute inelastic dynamic response of up to ten storey frame on a computer having a core memory of 32,000 locations. As the method and program is developed for n number of storeys, the same could be used for computation of response of frames having larger number of storeys depending upon the storage capacity of the particular computer used.

The author feels that the objective of developing a simple and general method for dynamic analysis of inelastic multi-storey frames, which usually have idealized elastic perfectly plastic behaviour, has been attained. However, it is worth mentioning that there is still a vast field lying uncovered in the dynamics of inelastic structures which need to be explored. For example, areas like 'dynamic stability of structures', 'nature of damping in the inelastic region' need special attention due to their paramount importance in the dynamic analysis of structural systems. It still needs further exploration to determine the maximum number of storeys which could be handled for inelastic dynamic analysis of multi-storey frame for a given storage capacity of the computer as the economy in the use of storage locations depends on manipulation of large matrices to eliminate their storage.

APPENDIX A

```
C     PROGRAM FOR COMPUTATION OF DYNAMIC RESPONSE OF INELASTIC
C     MULTI-STOREY FRAMES

C     DIMENSIONS CAN HANDLE A MULTI-STOREY FRAME UP TO TEN STOREYS

C (A) PROPERTIES OF GIVEN STRUCTURAL SYSTEM

C     E           ...   MODULOUS OF ELASTICITY
C     EI(I)       ...   MOMENT OF INERTIA OF ITH MEMBER
C     EL(I)       ...   LENGTH OF ITH MEMBER IN INCHES
C     NF          ...   NO. OF DEGREES OF FREEDOM
C     PM(I)       ...   PLASTIC MOMENT AT ITH SECTION
C     Q(I)        ...   ITH EXTERNAL LOAD

C (B) PROPERTIES OF NUMERICAL INTEGRATION PROCEDURE

C     AMASS(I)    ...   ITH MASS CONCENTRATED AT ITH FLOOR LEVEL
C     DECO(I)     ...   DAMPING COEFFICIENTS
C     DK(I,J)     ...   KRONECKER DELTA
C     TIME(I)     ...   TIME
C     XK(1)       ...   SMALL TIME INTERVAL
C     XK(2)       ...   A FRACTION OF SMALL TIME INTERVAL FOR TRANSITION

C (C) PROPERTIES OF APPLIED DYNAMIC FORCE

C     AMU(I)      ...   EXPONENTIAL DECAY FACTOR
C     APL(I)      ...   AMPLITUDE OF ITH DYNAMIC FORCE
C     OMEG(I)     ...   CIRCULAR FREQUENCY OF APPLIED DYNAMIC FORCE
C     TDF         ...   TIME DURATION OF FORCE, IF TIME EXCEEDS TDF
C                 ...   FORCE =0.
C     TIM         ...   TIME DURATION OF FIRST FORCING FUNCTIONS

C     SUBROUTINES USED ARE THE FOLLOWING

C     FORCE       ...   THIS COMPUTES THE DYNAMIC FORCES
C     STFM        ...   THIS COMPUTES THE STRUCTURAL STIFFNESS MATRIX
C     XFCE        ...   THIS COMPUTES THE MEMBER FORCES

C     OTHER VARIABLES USED

C     ACL(I,J)    ...   ACCELERATIONS
C     D(I,J)      ...   DISPLACEMENT VECTOR
C     DMX(I,J)    ...   DAMPING MATRIX
C     EEP         ...   ELASTIC PLASTIC TRANSITION INDICATOR
C                       IF.EQ.1. TRANSITION OCCURS
C                       IF.EQ.0. EXIT THE LOOP AND START INTEGRATING
C                       AT THE BEGINNING OF PREVIOUS TIME INTERVAL
C                       USING SUB INTERVAL
C     EPE         ...   PLASTIC-ELASTIC TRANSITION INDICATOR
C                       IT FUNCTIONS IN THE SAME WAY AS EEP
C     FCE(I,J)    ...   DYNAMIC FORCE VECTOR
C     KA          ...   IF.EQ.1 MAIN TIME INTERVAL IS USED
C                       IF.EQ.2 SUB  TIME INTERVAL IS USED
```

```
C     KB            ...    IF.EQ.3 IT SETS SLT1=0. AND BECOMES ZERO ITSELF
C     KF            ...    A VARIABLE WHOSE VALUE SHOWS THE TOTAL NUMBER OF
C                          HINGES EXISTING AT A CERTAIN TIME INSTANT
C     KL(J)         ...    AN ARRAY WHOSE VALUE GIVES THE HINGE NO.
C                          CORRESPONDING TO 5*NF+JTH COLUMN OF A MATRIX
C     KLEM          ...    IF INITIALLY .EQ.1 FRESH DATA FOR FORCE IS READ
C                          AFTER TIME EQUALS TIM
C     KP(I)         ...    A VARIABLE WHICH STORES THE NON ZERO ELEMENTS
C                          OF  A MATRIX
C     KREP          ...    IF.EQ.1 COMPUTE DAMPING MATRIX FROM MODAL
C                          ANALYSIS
C                   ...    IF.EQ.2 READ DAMPING MATRIX
C                   ...    IF.EQ.3 COMPUTE ONLY DIAGONAL ELEMENTS OF
C                          DAMPING MATRIX PROPORTIONAL TO MASSES
C     NCTR          ...    NUMBER OF TRANSITIONS OCCURED TILL A CERTAIN
C                          TIME INSTANT
C     NE            ...    TOTAL NO. OF MEMBERS IN THE STRUCTURAL SYSTEM
C     NTL           ...    TOTAL NO. OF EXTERNAL LOADS
C     NTR           ...    MAXIMUM NO. OF TRANSITIONS FOR WHICH RESPONSE
C                          IS COMPUTED
C     S(I,J)        ...    STRUCTURAL STIFFNESS MATRIX
C     SLT1          ...    IF.EQ.1. INTEGRATION IS REVERSED ONLY ONE TIME
C                          STEP BACK
C                          IF.EQ.0. INTEGRATION IS REVERSED TWO TIME
C                          STEPS BACK
C     SLT2          ...    IF .EQ.0. NO TRANSITION HAS OCCURED AND NORMAL
C                          INTEGRATION IS CONTINUED
C                          IF.EQ.1. TRANSITION HAS OCCURED AND STIFFNESS
C                          MATRIX IS MODIFIED
C     U(I)          ...    MEMBER DEFORMATION VECTOR
C     V(I,J)        ...    VELOCITIES
C     TLM           ...    TOTAL TIME LIMIT FOR WHICH COMPUTATION OF
C                          RESPONSE IS TO BE CARRIED
C     XA(I,J)       ...    AN ARRAY WHICH STORES NON ZERO ITH ELEMENT
C                          OF K MATRIX FOR JTH MEMBER
C     XNF(I)        ...    NATURAL FREQUENCIES
      DIMENSION PM(80),WB(90),WA(90),DK(10,10),AA(10,10),W(10),PD(10),
     1WD(10),DMX(10,10),FCE(10,3),V(10,3),ACL(10,3),TAA(10,10),
     2TBB(10,90),TCC(90,10),A(10,10),B(10,10),AMASS(10),XNF(10),AL(10),
     3AM(10),DIB(80),  PA(80),AB(10,10),PHR(80),HR(80,3),THR(80),
     4DECO(10),PPA(80),EL(40),EI(40), IT(18),LX(18),NI(100)
      DIMENSION XK(2),DT(2),TA(2),TB(2),TC(2),TIME(3)
      COMMON KP(120),KL(50),DIA(80),DIC(80),U(120),P(120),APL(10),
     1AMU(10),OMEG(10),Q(100),NTL,NE,NF,IX,KF,KJ,N,E,XA(4,40),S(100,100)
     2,D(100,3)
      READ(5,15)E
      READ(5,19)NF,NTR,KREP,KLEM
      READ(5,15)TIM,TDF,TLM
      READ(5,15)XK(1),XK(2),(TIME(I),I=1,3)
      DO 7001 I=1,NF
      READ(5,15)(DK(I,J),J=1,NF)
 7001 CONTINUE
      READ(5,15)(AMASS(I),I=1,NF)
      READ(5,15)(APL(I),I=1,NF)
      READ(5,15)(AMU(I),I=1,NF)
      READ(5,15)(OMEG(I),I=1,NF)
      READ(5,15)(DECO(I),I=1,NF)
      NH=8*NF
      NE=4*NF
```

```
      N=3*NE
      KJ=5*NF
      KD=2*NF+1
      KK=KJ+1
      NTL=KJ
      IX=NF-1
      MA=2*NF
      LA=NF+1
      IP=8*NF-4
      IK=MA-3
      READ(5,15)(PM(I),I=1,NH)
      READ(5,15)(EL(I),I=1.NE)
      READ(5,15)(EI(I),I=1.NE)
      READ(5,15)(Q(I),I=LA.NTL)
      READ(5,15)(Q(I),I=1.NF)
      DO 7000 J=12,N,12
      K=J/3+NF
      KP(J-11)=K-6
      KP(J-10)=K-2
      KP(J-9)=J/12
      KP(J-8)=KP(J-10)
      KP(J-7)=K-1
      KP(J-6)=K-3
      KP(J-5)=KP(J-7)
      KP(J-4)=K
      KP(J-3)=KP(J-6)
      KP(J-2)=KP(J-4)
      KP(J-1)=K-4
      KP(J)=KP(J-9)
 7000 CONTINUE
      KP(1)=0
      KP(11)=0
      DO 300 M=1,NE
      XA(1,M)=4.*EI(M)/EL(M)
      XA(2,M)=2.*EI(M)/EL(M)
      XA(3,M)=-6.*EI(M)/(EL(M)**2)
  300 XA(4,M)=12.*EI(M)/(EL(M)**3)
      WRITE(6,1019)NF,NTR.KREP.KLEM,(KP(I).I=1.N)
      WRITE(6,20)NH,NE.(PM(I).I=1.NH)
      WRITE(6,1030)E
      WRITE(6,1031)(EL(I),I=1,NE)
      WRITE(6,1032)(EI(I),I=1.NE)
      WRITE(6,1033)XK(1),XK(2).TLM
      WRITE(6,1034)(AMASS(I).I=1.NF)
      WRITE(6,1035)(Q(I),I=LA,NTL)
      WRITE(6,1036)(APL(I).I=1.NF)
      WRITE(6,1037)(AMU(I),I=1.NF)
      WRITE(6,1038)(OMEG(I),I=1,NF)
      WRITE(6,1039)(DECO(I),I=1,NF)
   15 FORMAT(8F10.6)
   19 FORMAT(40I2)
   20 FORMAT(1H-,25H NO OF POSSIBLE HINGES = .I2.4X.18H NO OF ELEMENTS =
     1 ,I2//1X,20H PLASTIC MOMENTS ARE///(1X.8F16.3))
   22 FORMAT(1H-,20H STIFFNESS MATRIX IS///)
  211 FORMAT(1X,8E16.6)
  213 FORMAT(1H-,13H EIGENVALUE =,E16.8)
  215 FORMAT(1H-,26H CORRESPONDING EIGENVECTOR//(1X.8E16.8))
  230 FORMAT(1X,I3)
```

```
 1019 FORMAT(1H-,27HNO OF DEGREES OF FREEDOM = ,4I6/,(40I3))
 1030 FORMAT(1H-,4HE = ,F16.6)
 1031 FORMAT(1H-,11HLENGTHS ARE/(8F16.6))
 1032 FORMAT(1H-,22HMOMENT OF INERTIAS ARE/(8F16.6))
 1033 FORMAT(1H-,18HTIME INTERVALS ARE/(8F16.6))
 1035 FORMAT(1H-,9HLOADS ARE/(8F16.6))
 1034 FORMAT(1H-,10HMASSES ARE/(8F16.6))
 1036 FORMAT(1H-,14HAMPLITUDES ARE/(8F16.6))
 1037 FORMAT(1H-,8HAMUS ARE/(8F16.6))
 1038 FORMAT(1H-,9HOMEGS ARE/(8F16.6))
 1039 FORMAT(1H-,24HDAMPING COEFFICIENTS ARE/(8F16.6))
C     CALCULATION OF INITIAL CONDITIONS
C     CALCULATION OF STIFFNESS MATRIX
      TMS=TIME(3)
      EQUIVALENCE(TIME(3),TMS)
      CALL STFM
      CALL INVMAT(S,100,NTL,0.,IER,NI)
      WRITE(6,230)IER
      DO 314 K=1,NTL
      D(K,3)=0.0
      DO 311 J=1,NTL
  311 D(K,3)=D(K,3)+S(K,J)*Q(J)
      D(K,3)=D(K,3)/E
  314 CONTINUE
      DO 393 I=1,NF
      DO 393 J=1,NF
      A(I,J)=S(I,J)/E
  393 CONTINUE
      WRITE(6,22)
      CALL INVMAT(A,10,NF,0.,IER,NI)
C     A NOW BECOMES STIFFNESS MATRIX
      DO 395 I=1,NF
  395 WRITE(6,211)(A(I,J),J=1,NF)
      DO 322 I=1,NF
      DO 322 J=1,NF
      B(I,J)=0.0
      IF(I.EQ.J)B(I,J)=1.0
  322 AA(I,J)=A(I,J)/AMASS(I)
      CALL EBERVC(AA,NF,1,200,.01,.001,1000.,10,B,-1.0)
C     XNF ARE NATURAL FREQUENCIES
C     B IS EIGENVECTOR
      DO 323 I=1,NF
      XNF(I)=SQRT(AA(I,I))
      WRITE(6,213)XNF(I)
  323 WRITE(6,215)(B(I,J),J=1,NF)
      GO TO 410
  409 CA=AA(J,J)
      AA(J,J)=AA(I,I)
      AA(I,I)=CA
      GO TO 407
  410 K=2
      J=K-1
  407 DO 396 I=K,NF
      IF(AA(J,J).GT.AA(I,I))GO TO 409
  396 CONTINUE
      QP=SQRT(AA(1,1))
      WRITE(6,223)QP
  223 FORMAT(1H+,27H FIRST NATURAL FREQUENCY = ,E20.6)
```

```
      DO 3 I=1,2
      DT(I)=XK(I)*6.28/QP
      TA(I)=2./DT(I)
      TB(I)=TA(I)*2.
    3 TC(I)=TB(I)/DT(I)
      GO TO(933,931,934),KREP
  933 DO 326 K=1,NF
      DO 325 I=1,NF
      DO 324 J=1,NF
  324 AA(I,J)=B(I,J)/B(I,K)
  325 W(I)=2.*AMASS(K)*XNF(I)*DECO(I)
      CALL SOLVE(AA,W,ID,NF,10)
      DO 326 L=1,NF
  326 DMX(K,L)=W(L)
      GO TO 932
  931 READ(5,15)((DMX(I,J),J=1,NF),I=1,NF)
      GO TO 932
  934 DO 935 I=1,NF
      DO 935 J=1,NF
      DMX(I,J)=0.0
      IF(I.EQ.J)DMX(I,J)=2.*AMASS(I)*QP*DECO(I)
  935 CONTINUE
  932 WRITE(6,212)
  212 FORMAT(1H-,30H DAMPING COEFFICIENT MATRIX IS//)
      DO 327 I=1,NF
  327 WRITE(6,211)(DMX(I,J),J=1,NF)
      CALL TICKS
      CALL FORCE(TMS,FCE)
      DO 333 I=1,NF
      D(I,1)=D(I,3)
      ACL(I,1)=FCE(I,3)/AMASS(I)
  333 V(I,1)=0.0
      WRITE(6,214)
  214 FORMAT(1H-,13X,4HTIME,15X,5HDEFLN,16X,3HVEL,16X,5HACCLN,15X,5HFORC
     1E//)
      WRITE(6,219)TIME(1),D(1,1),V(1,1),ACL(1,1),FCE(1,3)
      DO 344 I=2,NF
  344 WRITE(6,218)D(I,1),V(I,1),ACL(I,1),FCE(I,3)
      DO 345 I=1,NF
      D(I,2)=D(I,1)
      V(I,2)=V(I,1)
      PD(I)=0.0
  345 ACL(I,2)=ACL(I,1)
      SLT1=0.0
      KB=0
      K=2*KJ
      DO 899 I=KK,K
  899 Q(I)=0.0
      DO 362 I=1,NH
      DIC(I)=0.0
      PPA(I)=0.0
      DIA(I)=0.0
      PHR(I)=0.0
      HR(I,1)=0.0
      HR(I,2)=0.0
  362 HR(I,3)=0.0
      K=2*IX
      DO 33 I=1,K
```

```
 33 JT(I)=3
    KF=0
402 CALL TICKS
    SLT2=0.0
    EEP=0.0
    EPE=0.0
    KA=1
    CALL STFM
    MX=NTL-NF
    DO 335 I=1,NF
    DO 335 J=1,NF
335 TAA(I,J)=S(I,J)
    DO 336 I=1,NF
    DO 336 J=1,MX
    K=J+NF
336 TBB(I,J)=S(I,K)
    DO 337 I=1,MX
    K=NF+I
    DO 337 J=1,NF
337 TCC(I,J)=S(K,J)
    DO 334 I=1,MX
    K=NF+I
    DO 334 J=1,MX
    L=NF+J
334 S(I,J)=S(K,L)
    CALL INVMAT(S,100,MX,0.,IER,NI)
    WRITE(6,230)IER
    DO 366 I=1,NF
    DO 365 J=1,MX
    C=0.0
    DO 364 L=1,MX
364 C=C+TBB(I,L)*S(L,J)
365 U(J)=C
    DO 366 J=1,MX
366 TBB(I,J)=U(J)
    DO 339 I=1,MX
    K=NF+I
339 WB(I)=Q(K)
    DO 368 I=1,NF
    WA(I)=0.0
    DO 368 J=1,MX
368 WA(I)=WA(I)+TBB(I,J)*WB(J)
    DO 902 I=1,NF
    DO 901 J=1,NF
    C=0.0
    DO 900 L=1,MX
900 C=C+TBB(I,L)*TCC(L,J)
901 U(J)=C
    DO 902 J=1,NF
    TBB(I,J)=U(J)
    TAA(I,J)=TAA(I,J)-TBB(I,J)
902 A(I,J)=TAA(I,J)
    CALL INVMAT(A,10,NF,0.,IER,NI)
419 IF(IER.NE.0)WRITE(6,229)
    CALL TICKS
229 FORMAT(1H-,30H RESISTANCE MATRIX IS SINGULAR)
400 IF(KA.EQ.1)KB=KB+1
    IF(KB.EQ.3)SLT1=0.0
```

```
      IF(KB.EQ.3.OR.KA.EQ.2)KB=0
      TIME(3)=TIME(2)+DT(KA)
      IF(TIME(3).GE.TDF)GO TO 1000
      IF(TIME(3).GE.TIM)KLEM=KLEM+1
      IF(KLEM.EQ.2)GO TO 937
  938 GO TO 88
  937 READ(5,15)(AMU(I),I=1,NF),(OMEG(I),I=1,NF)
      SLT1=1.0
      KLEM=KLEM+1
   88 CALL FORCE(TMS,FCE)
 1003 GO TO 1002
 1000 DO 1001 I=1,NF
 1001 FCE(I,3)=0.
 1002 DO 328 I=1,NF
      DO 328 J=1,NF
  328 AB(I,J)=TC(KA)*DK(I,J)*AMASS(I)+TA(KA)*DMX(I,J)+TAA(I,J)*E
      DO 330 I=1,NF
      AL(I)=0.
      AM(I)=0.
      DO 329 J=1,NF
      AL(I)=AL(I)+(TC(KA)*DK(I,J)*AMASS(I)+TA(KA)*DMX(I,J))*D(J,2)
  329 AM(I)=AM(I)+(TB(KA)*DK(I,J)*AMASS(I)+DMX(I,J))*V(J,2)
  330 WA(I)=AL(I)+AM(I)+AMASS(I)*ACL(I,2)+FCE(I,3)-WA(I)
      CALL SOLVE(AB,WA,ID,NF,10)
      DO 331 I=1,NF
      WD(I)=WA(I)
      WA(I)=WA(I)-PD(I)
  331 D(I,3)=WA(I)
      DO 904 I=1,MX
      X=0.0
      DO 903 J=1,NF
  903 X=X+TCC(I,J)*WA(J)
  904 U(I)=X
      DO 372 I=1,MX
  905 WA(I)=-U(I)
      IF(NCTR.EQ.0)WA(I)=WB(I)/E+WA(I)
  372 CONTINUE
      DO 907 I=1,MX
      X=0.0
      DO 906 J=1,MX
  906 X=X+S(I,J)*WA(J)
  907 U(I)=X
      DO 908 I=1,MX
  908 WA(I)=U(I)
      DO 352 I=LA,NTL
      K=I-NF
  352 D(I,3)=WA(K)
      IF(KF.EQ.0)GO TO 405
      DO 380 I=1,IK,2
      M=2*I+NF
      IL=I+1
      IF(JT(I).NE.0)GO TO 438
      M1=LX(I)
      HR(M1,3)=D(M,3)
  438 IF(JT(IL).NE.0)GO TO 380
      M2=LX(IL)
      HR(M2,3)=D(M+2,3)
  380 CONTINUE
```

```
      DO 360 J=1,KF
      MZ=J+KJ
      L=KL(J)
      MPX=L/8
      MPZ=MOD(L,8)+1
      GO TO (700,701,702,702,360,705,706,706),MPZ
  702 MPX=MPX+1
      IF(MPX.EQ.NF)GO TO 703
  701 IF(L.EQ.1)GO TO 711
      IL=2*MPX-1
      GO TO 707
  706 MPX=MPX+1
      IF(MPX.EQ.NF)GO TO 709
      IL=2*MPX
      GO TO 707
  700 IF(MPX.EQ.1)GO TO 711
      IL=2*MPX-2
  707 IF(JT(IL).EQ.0)GO TO 711
      M=2*IL+NF
      HR(L,3)=D(MZ,3)-D(M,3)
      GO TO 360
  709 M=KJ
      GO TO 708
  703 M=KJ-2
      GO TO 708
  705 M=(L+1)/2+NF
  708 HR(L,3)=D(MZ,3)-D(M,3)
      HR(L-1,3)=-HR(L,3)
      GO TO 360
  711 HR(L,3)=D(MZ,3)
  360 CONTINUE
  405 DO 354 I=1,NH
      DIB(I)=0.0
  354 THR(I)=HR(I,3)+PHR(I)
      CALL XFCE
      DO 390 I=1,NE
      PA(2*I-1)=P(3*I-2)+PPA(2*I-1)
  390 PA(2*I)=P(3*I-1)+PPA(2*I)
C     PLASTIC - ELASTIC TRANSITION
      DO 341 I=1,NH
      IF(DIA(I).EQ.0.0)GO TO 341
      DIB(I)=1.0
  916 IF(DIA(I)*(HR(I,3)-HR(I,2)).LT.0.)GO TO 341
      IF(EPE.EQ.0.0)GO TO 40
      GO TO 42
   40 IF(EEP.EQ.0.0)GO TO 43
   42 DIA(I)=0.0
      DIC(I)=0.
      WRITE(6,220)I
      SLT2=1.0
      SLT1=1.0
      DO 358 M=1,IK,2
      K=4*M-2
      IL=M+1
      IF(I.EQ.K.OR.I.EQ.(K+1).OR.I.EQ.(K+7))GO TO 423
      IF(I.EQ.(K+4).OR.I.EQ.(K+5).OR.I.EQ.(K+14))GO TO 424
  358 CONTINUE
      DO 387 J=4,IP,8
```

```
      IF (I.EQ.J)GO TO 1014
      IF(I.EQ.(J+1))GO TO 425
  387 CONTINUE
      IF(I.EQ.(IP-2))GO TO 1012
      IF(I.EQ.(IP+2))GO TO 1013
      IF(I.EQ.1.OR.I.EQ.8.OR.I.EQ.(IP-1).OR.I.EQ.(IP+2))GO TO 425
      GO TO 341
  424 JT(IL)=JT(IL)+1
      M=IL
      GO TO 802
  423 JT(M)=JT(M)+1
  802 IF(JT(M).GT.1)GO TO 1015
      IF(JT(M).EQ.1.AND.LX(M).NE.I)GO TO 431
      IF(JT(M).EQ.1.AND.LX(M).EQ.I)GO TO 435
      GO TO 341
  431 IT=LX(M)
      LX(M)=0
  434 DIC(IT)=0.0
      LAX=2*M+NF
      Q(LAX)=PA(I)
      KF=KF+1
      KL(KF)=IT
      LBX=KF+KJ
      Q(LBX)=PA(IT)
      GO TO 425
  435 LX(M)=0
      GO TO 432
 1012 Q(KJ-2)=0.0
      GO TO 432
 1013 Q(KJ)=0.0
      GO TO 432
 1014 LCX=J/2+NF+1
 1011 Q(LCX)=0.0
  432 DIC(I)=0.0
  805 GO TO 341
 1015 LDX=2*M+NF
 1010 Q(LDX)=Q(LDX)+PA(I)
  425 DO 301 J=1,KF
      IF(I.EQ.KL(J))GO TO 403
  301 CONTINUE
      GO TO 341
  403 KF=KF-1
      IF(KF.LT.J)GO TO 341
      DO 807 L=J,KF
      KL(L)=KL(L+1)
      LEX=L+KJ
      Q(LEX)=Q(LEX+1)
  807 CONTINUE
  341 CONTINUE
  220 FORMAT(1H-,24H HINGE IS RELEASED AT - ,I2)
      ELASTIC - PLASTIC TRANSITION
      DO 342 I=1,NH
      IF(DIB(I).GT.0.0)GO TO 342
      IF(PA(I).LT.0.0.AND.(PA(I)+PM(I)).GT.0.0)GO TO 342
      IF(PA(I).GT.0.0.AND.(PA(I)-PM(I)).LT.0.0)GO TO 342
      IF(EEP.EQ.0.0)GO TO 45
      GO TO 47
   45 IF(EPE.EQ.0.0)GO TO 48
```

```
 47 DIA(I)=SIGN(1.0,PA(I))
    IF(PA(I).EQ.0.0)GO TO 412
    WRITE(6,221)I,DIA(I),PA(I)
    PM(I)=PA(I)*DIA(I)
    SLT2=1.0
    SLT1=1.0
    DO 303 J=1,IK,2
    K=4*J-2
    IL=J+1
    IF(I.EQ.K.OR.I.EQ.(K+1).OR.I.EQ.(K+7))GO TO 420
    IF(I.EQ.(K+4).OR.I.EQ.(K+5).OR.I.EQ.(K+14))GO TO 421
303 CONTINUE
    DO 386 J=4,IP,8
    IF(I.EQ.J)GO TO 1016
    IF(I.EQ.(J+1))GO TO 1005
386 CONTINUE
    IF(I.EQ.(IP-2))GO TO 1009
    IF(I.EQ.(IP+2))GO TO 1006
    IF(I.EQ.1.OR.I.EQ.8.OR.I.EQ.(IP-1).OR.I.EQ.(IP+3))GO TO 1005
    GO TO 342
421 J=IL
420 JT(J)=JT(J)-1
806 IF(JT(J).GT.0)GO TO 422
    IF(JT(J).EQ.0)LX(J)=I
    GO TO 427
422 KF=KF+1
    KL(KF)=I
    LFX=KF+KJ
    Q(LFX)=PA(I)
    LGX=2*J+NF
    Q(LGX)=Q(LGX)-PA(I)
    GO TO 342
1009 Q(KJ-2)=PA(I)
    GO TO 427
1016 LHX=J/2+NF+1
1004 Q(LHX)=PA(I)
    GO TO 427
1006 Q(KJ)=PA(I)
 427 DIC(I)=1.0
    GO TO 342
1005 KF=KF+1
    KL(KF)=I
    LIX=KF+KJ
    Q(LIX)=PA(I)
 342 CONTINUE
 221 FORMAT(1H+,22H HINGE IS FORMED AT - ,I2,2F20.6)
    DO 391 I=1,NF
    D(I,3)=WD(I)
    C=D(I,3)-D(I,2)
    V(I,3)=TA(KA)*C-V(I,2)
 391 ACL(I,3)=TC(KA)*C-TB(KA)*V(I,2)-ACL(I,2)
    IF(SLT2.EQ.0.0.AND.KA.EQ.2)GO TO 1017
    WRITE(6,219)TIME(3),D(1,3),V(1,3),ACL(1,3),FCE(1,3)
 219 FORMAT(1X,5E20.6)
    DO 343 I=2,NF
 343 WRITE(6,218)D(I,3),V(I,3),ACL(I,3),FCE(I,3)
 218 FORMAT(21X,4E20.6)
1017 DO 359 I=1,NF
```

```
      D(I,1)=D(I,2)
      V(I,1)=V(I,2)
      ACL(I,1)=ACL(I,2)
      D(I,2)=D(I,3)
      V(I,2)=V(I,3)
  359 ACL(I,2)=ACL(I,3)
      DO 302 I=1,NH
      HR(I,1)=HR(I,2)
  302 HR(I,2)=HR(I,3)
      TIME(1)=TIME(2)
      TIME(2)=TIME(3)
  936 IF(TIME(2).GT.TLM)GO TO 406
      IF(SLT2.EQ.0.0)GO TO 400
      NCTR=NCTR+1
  917 IF(NCTR.GT.NTR)GO TO 406
      DO 361 K=1,NH
      PPA(K)=PA(K)
      HR(K,2)=0.0
  361 PHR(K)=THR(K)
      DO 351 I=1,NF
  351 PD(I)=D(I,2)
      NTL=KJ+KF
  918 GO TO 402
C     43 IS EXIT STATEMENT FOR PLASTIC - ELASTIC TRANSITION
   43 EPE=1.0
      KA=2
      IF(SLT1.GT.0.0)GO TO 400
      DO 370 I=1,NF
      V(I,2)=V(I,1)
      D(I,2)=D(I,1)
  370 ACL(I,2)=ACL(I,1)
      DO 930 I=1,NH
  930 HR(I,2)=HR(I,1)
      TIME(2)=TIME(1)
      GO TO 400
C     48 IS EXIT STATEMENT FOR ELASTIC - PLASTIC TRANSITION
   48 EEP=1.0
      KA=2
      GO TO 400
  915 GO TO 406
  412 WRITE(6,222)
  222 FORMAT(1H+,7HERROR=1)
  406 WRITE(6,23)
   23 FORMAT(1H-,17H COMPUTATION ENDS)
      STOP
      END
$IBFTC STFM8
      SUBROUTINE STFM
C     SUBROUTINE TO COMPUTE STIFFNESS MATRIX
      DIMENSION AG(6)
      COMMON KP(120),KL(50),DIA(80),DIC(80),U(120),P(120),APL(10),
     1AMU(10),OMEG(10),Q(100),NTL,NE,NF,IX,KF,KJ,N,F,XA(4,40),T(100,100)
     2,D(100,3)
      DO 38 J=1,NTL
      DO 38 K=1,NTL
      IJ=-1
      IK=0
      T(J,K)=0.0
```

```
      DO 38 M=1,NE
      I=3*M
      IJ=IJ+2
      IK=IK+2
      DO 375 L=1,6
  375 AG(L)=0.0
      IF(J.GT.KJ.AND.K.GT.KJ)GO TO 111
      M1=KP(I-2)
      M2=KP(I-1)
      M3=KP(I)
      IF(J.EQ.M1)AG(1)=1.0-DIA(IJ)**2+DIC(IJ)
      IF(K.EQ.M1)AG(4)=1.0-DIA(IJ)**2+DIC(IJ)
      IF(J.EQ.M2)AG(2)=1.0-DIA(IK)**2+DIC(IK)
      IF(K.EQ.M2)AG(5)=1.0-DIA(IK)**2+DIC(IK)
      IF(J.EQ.M3)AG(3)=1.0
      IF(K.EQ.M3)AG(6)=1.0
      DO 376 IC=1,NF
      MZ=12*IC-3
      IF(J.EQ.M3.AND.I.EQ.MZ)AG(3)=-1.0
      IF(K.EQ.M3.AND.I.EQ.MZ)AG(6)=-1.0
  376 CONTINUE
      IF(J.GT.IX.AND.K.GT.IX)GO TO 200
      DO 378 ID=1,IX
      ML=12*ID
      IF(I.EQ.(ML+3).AND.J.EQ.ID)AG(3)=-1.0
      IF(I.EQ.(ML+3).AND.K.EQ.ID)AG(6)=-1.0
      IF(I.EQ.(ML+12).AND.J.EQ.ID)AG(3)=-1.
      IF(I.EQ.(ML+12).AND.K.EQ.ID)AG(6)=-1.
  378 CONTINUE
  200 IF(KF.EQ.0)GO TO 38
      IF(J.LE.KJ.AND.K.LE.KJ)GO TO 38
  111 DO 384 L=1,KF
      IY=KL(L)
      ID=L+KJ
      IF(IY.EQ.IJ.AND.J.EQ.ID)AG(1)=1.0
      IF(IY.EQ.IJ.AND.K.EQ.ID)AG(4)=1.0
      IF(IY.EQ.IK.AND.J.EQ.ID)AG(2)=1.0
      IF(IY.EQ.IK.AND.K.EQ.ID)AG(5)=1.0
  384 CONTINUE
   38 T(J,K)=(AG(1)*XA(1,M)+AG(2)*XA(2,M)+AG(3)*XA(3,M))*AG(4)+(AG(1)*XA
     1(2,M)+AG(2)*XA(1,M)+AG(3)*XA(3,M))*AG(5)+(AG(1)*XA(3,M)+AG(2)*XA(
     2,M)+AG(3)*XA(4,M))*AG(6)+T(J,K)
      RETURN
      END
$IBFTC XFCE8
      SUBROUTINE XFCE
C     SUBROUTINE TO COMPUTE MEMBER FORCES
      COMMON KP(120),KL(50),DIA(80),DIC(80),U(120),P(120),APL(10),
     1AMU(10),OMEG(10),Q(100),NTL,NE,NF,IX,KF,KJ,N,E,XA(4,40),S(100,100)
     2,D(100,3)
      K=0
      KG=0
      DO 312 I=1,N
      U(I)=0.0
      KG=KG+1
      IF(KG.LT.3)K=K+1
      IF(KG.EQ.3)KG=0
      DO 312 J=1,NTL
```

```
      AGX=0.0
      IF(J.GT.KJ)GO TO 912
      IA=KP(I)
      IF(J.EQ.IA.AND.KG.EQ.0)AGX=1.0
      IF(J.EQ.IA.AND.KG.GT.0)AGX=1.0-DIA(K)**2+DIC(K)
      IF(J.GT.IX)GO TO 7
      M=12*J+3
      IF(I.EQ.M)AGX=-1.
      IF(I.EQ.(M+9))AGX=-1.
    7 CONTINUE
      IF(I.NE.3*(J+2-NF))GO TO 312
      IF(MOD(I,12).NE.9)GO TO 312
      IF(I.LE.(12*NF-3))AGX=-1.0
   70 GO TO 312
  912 DO 71 L=1,KF
      IY=KL(L)
      ID=L+KJ
      IF(IY.EQ.K.AND.J.EQ.ID)GO TO 72
   71 CONTINUE
      GO TO 312
   72 IF(KG.EQ.0)GO TO 312
      AGX=1.0
  312 U(I)=U(I)+AGX*D(J,3)
      DO 313 M=1,NE
      P(3*M-2)=(U(3*M-2)*XA(1,M)+U(3*M-1)*XA(2,M)+U(3*M)*XA(3,M))*E
      P(3*M-1)=(U(3*M-2)*XA(2,M)+U(3*M-1)*XA(1,M)+U(3*M)*XA(3,M))*E
  313 P(3*M)=(U(3*M-2)*XA(3,M)+U(3*M-1)*XA(3,M)+U(3*M)*XA(4,M))*E
      RETURN
      END
$IBFTC FORCE7
      SUBROUTINE FORCE(TME,FCS)
C     SUBROUTINE TO COMPUTE THE DYNAMIC FORCES USED IN THE ANALYSIS
C     OF TWO-STOREY FRAME
      DIMENSION FCS(10,3)
      COMMON KP(120),KL(50),DIA(80),DIC(80),U(120),P(120),APL(10),
     1AMU(10),OMEG(10),Q(100),NTL,NE,NF,IX,KF,KJ,N,E,XA(4,40),S(100,100)
     2,D(100,3)
      DO 332 I=1,NF
  332 FCS(I,3)=APL(I)*EXP(-AMU(I)*TME)*COS(OMEG(I)*TME)
      RETURN
      END
$IBFTC FORCE8
      SUBROUTINE FORCE(TME,FCS)
C     SUBROUTINE TO COMPUTE THE DYNAMIC FORCES USED IN THE ANALYSIS OF
C     SIX-STOREY FRAME
      DIMENSION FCS(10,3)
      COMMON KP(120),KL(50),DIA(80),DIC(80),U(120),P(120),APL(10),
     1AMU(10),OMEG(10),Q(100),NTL,NE,NF,IX,KF,KJ,N,E,XA(4,40),S(100,100)
     2,D(100,3)
      DO 332 I=1,NF
  332 FCS(I,3)=APL(I)*(AMU(I)*TME+OMEG(I))
      RETURN
      END
```

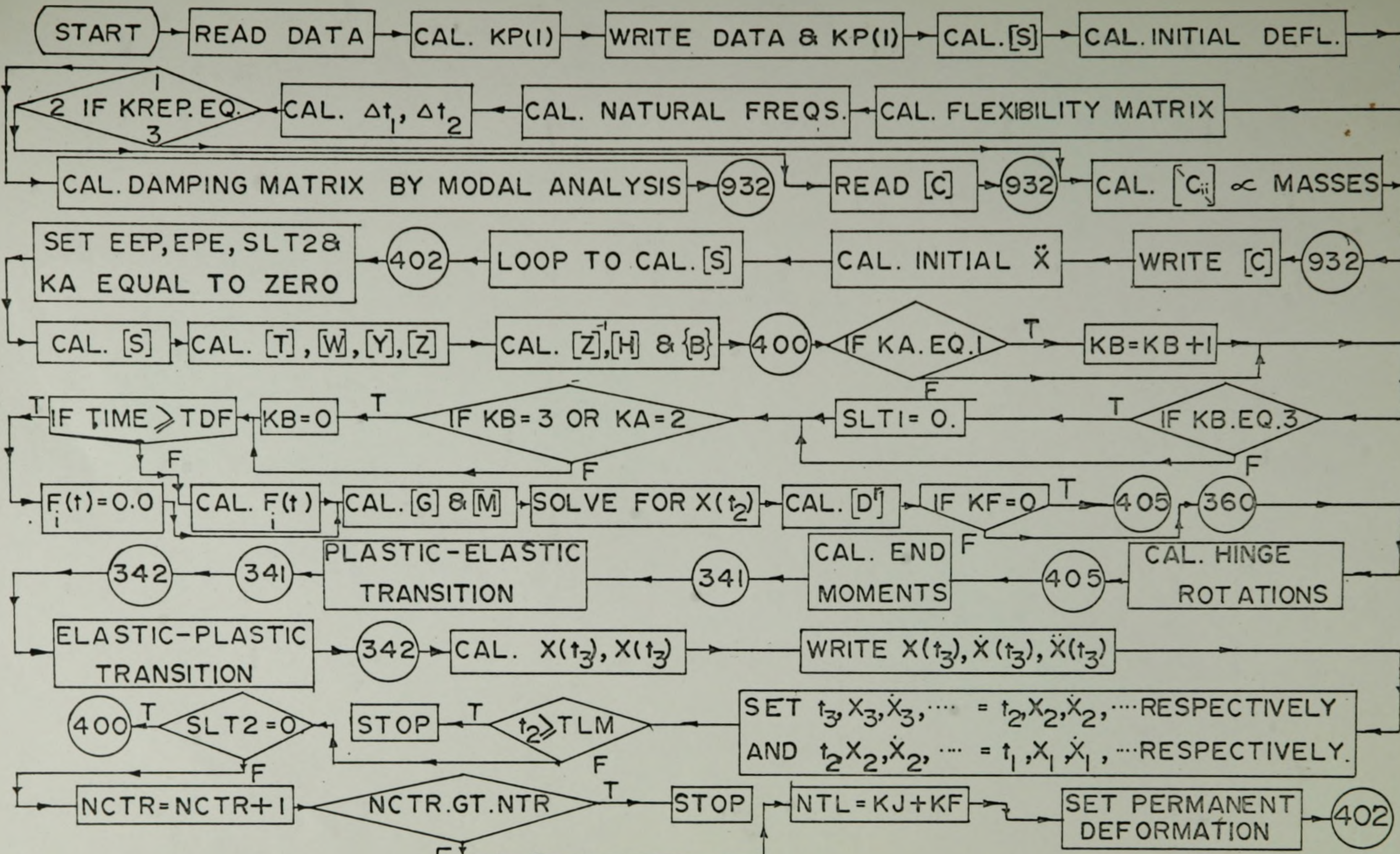


FIG. A.1 FLOW DIAGRAM FOR RESPONSE COMPUTATION

BIBLIOGRAPHY

1. Bleich, Hans H. and Salvadori, Mario G., "Impulsive Motion of Elasto-Plastic Beams", Transactions American Society of Civil Engineers, Vol. 120, 1955, p.499.

2. DiMaggio, Frank L., "Dynamic Elasto-Plastic Response of Rigid Frames", Proceedings American Society of Civil Engineers, Vol. 84, No. EM3, July 1958, Paper 1693.

3. Berg, Glen V. and DaDeppo, Donald A., "Dynamic Analysis of Elasto-Plastic Structures," Proceedings American Society of Civil Engineers, Vol. 86, No. EM2, April 1960, p.35.

4. Penzien, Joseph, "Elasto-Plastic Response of Idealized Multi-Storey Structures Subjected to a Strong Motion Earthquake", Proceedings Second World Conference on Earthquake Engineering, Vol. II, Tokyo, 1960, p.739.

5. Heidebrecht, A. C., Seng-Lip Lee, and Fleming, John F., "Dynamic Analysis of Elasto-Plastic Frames", Journal of the Structural Division, Proceedings of American Society of Civil Engineers, Vol. 90, No. ST2, Proc. Paper 3881, April 1964, pp.315-343.

6. Lee, S. L., "The Conjugate Frame Method and its Application in the Elastic and Plastic Theories of Structures", Journal Franklin Institute, Vol.266, No. 3, September 1958, p.207.

7. Clough, Ray W., Benuska, K. L., and Wilson, E. L., "Inelastic Earthquake Response of Tall Buildings", Proceedings of the Third World Conference on Earthquake Engineering, Vol. II, 1965, New Zealand, p. II-68.

8. Saul, William Edward, "Analysis of Bilinear Inelastic Structural Systems Subjected to Dynamic Loading", Ph.D. Thesis, Northwestern University, 1964.

9. Beedle, Lynn S., Plastic Design of Steel Frames, John Wiley and Sons, Inc., New York, 1958.

10. Baker, J. F., and Roderick, J. W., Proceedings, Inst. of C. E. London, January 1952, p.71.

11. Baker, J. F., and Horne, M. R., "New Methods in the Analysis and Design of Structures in the Plastic Range", British Welding Journal, July 1954, p.307.

12. Symonds, P. S. and Neal, B. G., "Recent Progress in the Plastic Methods of Structural Analysis", Journal Franklin Institute, November 1951, p.383 and December 1951, p.469.

13. Fleming, J. F. and Romualdi, J. P., "A General Procedure for Calculating Dynamic Response Due to Impulsive Loads", Journal Franklin Institute, Vol. 275, No. 2, February 1963, p.107.

14. McMinn, S. J., "Matrices for Structural Analysis", New York: Wiley, 1962.

15. Gennaro, Joseph J., "Computer Methods in Solid Mechanics", New York: The Macmillan Company, 1965.

16. Biggs, John M., "Introduction to Structural Dynamics", McGraw-Hill Book Company, New York, 1964, pp. 140-147.

17. Heidebrecht, A. C., "Analysis of Dynamic Response of Inelastic Structural Systems", Ph.D. Thesis, Northwestern University, Evanston, Illinois, June 1963.