Development of a Cloud-Based Dual-Objective Nonlinear Programming Model for Irrigation Water Allocation

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ABSTRACT

Irrigation water allocation is essential to the management of agricultural water use in irrigation districts. Many irrigation optimization models were proposed from previous studies to provide decision support for water managers. In order to capture the complex nonlinear relationships and meet different water demands, more advanced multi-objective nonlinear programming models were developed in the past decade. However, it is still a challenging task to address varies uncertainties associated with irrigation optimization. Fuzzy programming, interval programming, and chance-constrained programming can be used to quantify uncertainties in simplified formats, but none of them can represent complex uncertainty in a composite format. In this thesis, a cloud-based dual objective nonlinear programming (CDONP) model is developed by implementing a cloud modeling method in an irrigation model to address the uncertainties of reference evapotranspiration (ET0) and surface water availability (SWA). The cloud modeling method is used to generate 2,000 data samples from historical data. The results show that the generated samples are consistent with historical data. Optimized allocation schemes are provided, and the performance of the CDONP model are discussed. This is the first Canadian study that used the cloud modeling method in irrigation water allocation. This method provides a solution to quantify composite uncertainties based on limited data, which represents a unique contribution to irrigation water allocation modeling. This study provides valuable decision support for agriculture management to improve water use efficiency.

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PUBLISCATION LIST

This M.A.Sc thesis consists of the following submitted paper:

- **Zehao Yan**, Brian Baetz, Zhong Li, "A cloud-based dual-objective nonlinear programming model for irrigation water allocation in Northwest China" submitted in November. Zehao Yan processed the data, formulated the model, wrote the code, performed results analysis, prepared the figures and tables, and drafted the manuscript. He made a significant original contribution to this paper and is the first author.

CO-AUTHORSHIP

This thesis was prepared in a "sandwich" format in accordance with the guidelines provided by the School of Graduate Studies, McMaster University and was co-authored.

For the paper presented in Chapter 3, the model development and data analysis were conducted by Z. Yan in consultation with Drs. Z. Li and B. Baetz. The paper was written by Z. Yan and edited by Drs. Z. Li and B. Baetz.

CONTENTS

LIST OF FIGURES AND TABLES

ABBREVIATIONS

CCP	Chance-Constrained Programming
CDONP	Cloud-Based Dual Objective Nonlinear Programming
DONP	Dual-Objective Nonlinear Programming
En	Entropy
ENP	Economic Net Profit
ET0	Reference Evapotranspiration
Ex	Expect Value
FMP	Fuzzy Mathematical Programming
GA	Genetic Algorithm
He	Hyper Entropy
ILMP	Interval Linear Multi-Objective Programming
ILP	Interval Linear Programming
LMOFP	Linear Multi-Objective Fuzzy Programming
ISE	Irrigation System Efficiency
SMP	Stochastics Mathematical Programming
SWA	Surface Water Availability
YID	Yingke Irrigation District

CHAPTER 1 – INTRODUCTION

1.1 Background

Water demand never stops due to continuous and rapid population growth (Lalehzari et al., 2016). Industrial and agricultural activities, as well as the increasing population result in a considerable global water consumption. Water is an irreplaceable resource to support daily life, and it also is the foundation for technology development. Among different water demands, agriculture water usage accounts for approximately 60% of the global water usage (Pimentel et al., 2004). Irrigation water management has become a critical problem for decision makers.

Optimization of irrigation water allocation is a complex and challenging problem. A major challenge in irrigation water allocation is system complexity. The water allocation process is governed by complex interactions among different factors. For example, the relationship between system economic net profit (ENP) and water allocation is often complex and nonlinear, which cannot be described by linear models. Meanwhile, maximizing ENP may not be the only objective in irrigation water allocation. When water availability is limited, decision makers need to minimize the use of water resources or maximize irrigation system efficiency (ISE), which makes the optimization of irrigation water allocation a complex multi-objective programming problem. Another major challenge is model uncertainty. There are many factors in irrigation water allocation systems that are associated with various uncertainties. For example, meteorological data, crop market price, and resource availability. The uncertainties of such parameters can lead to huge differences in the optimal water allocation plan. These uncertainties must be addressed when developing an irrigation water allocation model.

1.2 Irrigation Optimization Models

To address those challenges in irrigation water allocation, a number of optimization models have been proposed. Irrigation optimization models can be used to generate optimal water allocation plans. The decision variables in irrigation water allocation are mostly the amount of water allocated for each crop (Li et al., 2018). Crop cultivation area can also be used with water allocations as decision variables (Kumar et al., 1998). The objective functions can be maximizing ENP, total crop yield, or the ratio of crop yield over water usage, or minimizing water usage, crops' water shortage, irrigated area, water loss level, agricultural pollution, or groundwater exploitation (Li et al., 2018; Ren et al., 2017; Kilic et al., 2010). Examples of commonly used constraints include availability of water and land resources, food demand.

A major challenge for the development of irrigation optimization models is how to adequately analyze and quantify the parameter uncertainty. Rainfall, evapotranspiration, land use, and river inflow can change the result of irrigation models, and the uncertainties associated with those critical parameters can lead to different optimal decisions. In the past ten years, several uncertainty modelling approaches were developed to address this issue such as fuzzy sets, random variables, and intervals. However, there are more advanced uncertainty quantification approaches that are yet to be investigated for irrigation water allocation.

1.3 Objective

The goal of this study is to introduce a new uncertainty quantification method, the cloud model, to address parameter uncertainties in a dual-objective nonlinear programming (DONP) model for the optimization of irrigation water allocation. The DONP model is used to generate the optimal irrigation water allocation scheme that maximizes ENP and ISE. The cloud model is used to address the complex uncertainties associated with two model parameters, reference evapotranspiration (ET0) and surface water availability (SWA). The new cloud-based dual-objective nonlinear programming (CDONP) model is applied to a real-world case study in the Yingke Irrigation District (YID), Northwest China. A risk evaluation approach is also developed to analyze the optimal irrigation water allocation schemes under different scenarios of extreme conditions.

1.4 Thesis Outline

Chapter 2 presents a literature review of irrigation optimization models and methodologies used in irrigation models to address uncertainty. The advantages and limitations of existing models and methods are reviewed and discussed.

Chapter 3 introduces the CDONP model and its application in YID. The results and performance of CDONP model are presented. A drought risk analysis is conducted based on four scenarios of extreme conditions. This chapter includes a journal article that has been submitted for publication.

Chapter 4 summarizes the conclusions and recommendations for future work.

CHAPTER 2 – LITERATURE REVIEW

2.1 Multi-Objective Optimization Models

The most commonly used objective function in irrigation optimization models is to maximize economic profit. Ganji et al. (2006) developed a nonlinear programming model to obtain maximum net return from the irrigation system under deficit irrigation conditions. The same objective function was also used by Kumar et al. (1998), Maqsood et al. (2005), Georgiou et al. (2008), Kilic et al. (2010), Lalehzari et al. (2016) and Li et al (2017),

The increase of economic profit often requires and leads to an increase in irrigation water usage. As water resources become scarce, water reduction becomes another goal that irrigation decision makers seek for. To address this issue of multiple objectives, a number of multi-objective programming models were developed. Lalehzari et al. (2016) proposed a DONP model with objective functions of maximizing total benefits and minimizing water usage. Li et al. (2017) developed a multi-objective programming model to maximize crop yield and water saving. This model has three objective functions, including the maximization of crop yield increase and water saving, and the minimization of water supply cost. Regulwar et al. (2011) used the maximization of employment in addition to the maximization of crop production a, considering that employment growth can enhance social development and it also can facilitate economic growth. Kilic et al. (2010) develop an irrigation optimization model to minimize both the production area and the water loss at the irrigation network. Minimizing the production area is an alternative way to save water.

The DONP framework can effectively address the complexity of multiple objectives in irrigation optimization. However, there are various uncertainties that exist in irrigation water allocation processes. These uncertainties in DONP models should be tackled to provide reliable and robust decision support for irrigation water allocation.

2.2 Uncertainty Analysis for Optimization Models

There are many existing uncertainty analysis methods for optimization models. Different techniques, such as chance-constrained programming, fuzzy programming, and interval programming, have been developed and applied to quantify uncertainties in the formats of randomness, fuzziness, or intervals.

Fuzzy programming (Sakawa et al., 1998) represents uncertainty using the likelihood or satisfaction degree of an estimated value. The likelihood is determined by its fuzzy membership function, and the commonly used fuzzy membership functions are triangle and trapezoid functions. The triangular fuzzy membership function can be defined using the most possible value and the range of all possible values. The satisfaction degree describes the possibility that a constraint with fuzzy parameter(s) is satisfied. For example, Ren et al. (2017) developed a linear multi-objective fuzzy programming (LMOFP) irrigation optimization model. The LMOFP model integrates rainfall, evapotranspiration and soil water as fuzzy sets and it generates optimization schemes under different satisfaction degrees. Guo et al. (2014) used fuzzy chance-constrained programming to address the uncertainty associated with cultivate area and crop price.

Xu et al. (2017) used two stage fuzzy chance-constrained programming to address the uncertainty associated with water quantity, water quality, and economical targets. Zhang et al. (2018) used interval fuzzy constrained programming to address the uncertainty associated with available water, crop area, evapotranspiration and crop price.

The essence of interval programming (Crarnes et al., 1976) is to use intervals with an upper and a lower bound values to describe parameter uncertainty. Interval programming models can be solved by formulating two deterministic submodules corresponding to the upper and lower bounds of the objective function (Huang et al., 1995). For example, Li et al. (2018) proposed an interval linear multi-objective programming (ILMP) model for irrigation water allocation. Interval social-economic and hydrological parameters were generated through statistical simulation, and then introduced into a linear multi-objective programming framework for the optimization of irrigation water allocation.

Although the abovementioned techniques are useful in quantifying simple format uncertainties, uncertainty often exists in a hybrid format in real world problems, for example, in the prediction of meteorological data. Those types of predictions lack reliability because it is difficult to define one probability distribution or one fuzzy membership that can reflect a real world scenario from existing data. There is a lack of techniques for solving uncertainties with high complexity.

2.3 The Cloud Model

The cloud model was proposed by Li et al (2009) as a new method to describe uncertainty. It can represent randomness and fuzziness at the same time. The cloud model uses historic data to define a fuzzy membership function and a probability distribution for an uncertain parameter, and then generate cloud drops to describe the composite uncertainty of the parameter. The cloud drops form a cloud membership function, and the characteristics of a cloud membership function can be described by three parameters: expect value (Ex), entropy (En), and hyper entropy (He). With this property, the cloud generator is developed, and it can use a historical data set to form a new data set that has the same characteristic in cloud membership.

Liu et al. (2017) used the cloud model in a failure mode analysis. Cheng et al. (2018) used the cloud model in water resource carrying capacity evaluation analysis. Those studies indicate that the cloud model has been successfully applied in some studies, and it has a potential to address the complex uncertainties in irrigation optimization models. Although the cloud model has been successfully applied to a number of real world case studies, it has yet to be investigated for irrigation water allocation.

CHAPTER 3 – CLOUD-BASED DUAL-OBJECTIVE NONLINEAR PROGRAMMING MODEL

A cloud-based dual-objective nonlinear programming model for irrigation water allocation in Northwest China

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Abstract

Agricultural water management has become an essential problem in recent years due to the increasing water demands. Irrigation water resources allocation is a dynamic decision making process associated with various uncertainties, which often exist in a complex and composite format. In this study, a new uncertainty quantification technique, the cloud model, is introduced to a dual-objective nonlinear programming (DONP) framework, and a cloud-based dual-objective nonlinear programming (CDONP) model is developed to support irrigation water allocation and agricultural water planning under composite uncertainties. The cloud model is applied to address the complex composite uncertainties associated with reference evapotranspiration (ET0) and surface water availability (SWA). A case study of the Yingke irrigation district (YID) in Northwest China is conducted to demonstrate the applicability of the developed model. The results show that the net economic profit (ENP) and irrigation system efficiency (ISE) are influenced by ET0 more than SWA. The obtained results are also compared to those of a traditional dual-objective nonlinear programming model to illustrate the advantages of the proposed CDONP model. In addition, four water shortage scenarios are built and discussed for risk analysis.

1.0 Introduction

Water resources play an irreplaceable role in the agriculture field, which has resulted in abundant research in irrigation. As the population grows, the increasing food demand leads to increasing water demand. Both groundwater and surface water resources have their limits. Overuse of water resources will bring devastating consequences that may require considerable time and money to fix. Therefore, how to allocate limited water resources to maximize agricultural benefits has become an important topic in agriculture management.

Optimization models have been widely used to optimize water allocation for agricultural use. For example, Maji and Heady (1981) built a linear programming model with deterministic and stochastic inflows to determine the optimal storage for reservoir. Bozorg et al. (2008) developed a linear programming model to optimize multi-crop irrigation areas. Satari et al (2006) developed a nonlinear deterministic model for the capacity planning of a small irrigation reservoir. Zhang et al. (2007) applied a nonlinear optimization model in corn cultivation and used the genetic algorithm (GA) to solve the nonlinear model. Most coefficients in these models are deterministic. These deterministic models provided theoretical interpretation to understand agricultural water allocation processes; however, they were unable to tackle the uncertainties associated with the water management systems and thus could not provide robust decision support for agricultural water management and planning.

In the past two decades, more advanced optimization techniques have been developed to address various uncertainties in agricultural water management systems. There are four types of techniques to address uncertainty in the agriculture field: interval linear programming (ILP), fuzzy mathematical programming (FMP), chance-constrained programming (CCP) and stochastic mathematical programming (SMP). Paudyal et al. (1990) integrated SMP in an irrigation water allocation model. Li et al. (2014) developed a CCP model for irrigation water allocation. Zhang et al. (2018) proposes an enhanced CCP model, i.e., the double-sided stochastic chance-constrained programming model, to manage irrigation water uncertainty. Li et al. (2018) applied ILP in a multi-objective framework for irrigation water allocation. Ren et al. (2017) implement FMP in a multi-objective model for optimal use of irrigation water and land resources. In these previous irrigation water allocation models, only uncertainties in a simple format, described as interval, fuzzy or random, were tackled. The fuzziness and randomness are described using possibility and probability distributions, respectively. In practice, as water resources allocation is a dynamic decision making process influenced by subjective factors, uncertainties often exist in a more complex, composite format. The lack of appropriate interpretation of such uncertainties could lead to misrepresentation of the system and underestimation of the system-failure risks.

Recently, a new approach, called the cloud model, was proposed to address uncertainties in a complex format. The cloud model can synthetically describe the randomness and fuzziness of a system element and efficiently obtain quantitative information of the composite uncertainty (Li et al., 2009). It provides a straightforward way to decode complex uncertainties; it can also be used to generate reliable data for facilitating risk assessment when there is a data scarcity issue. Cheng et al. (2018) applied the cloud model in a factor analysis of water resource capacity, to address the varied index value in a risk evaluation framework. Liu (2017) integrated the cloud model in a failure mode and effect analysis approach to assess possible failure modes of process, products, services, and systems. The cloud model has shown a great potential to address complex uncertainties that are both fuzzy and random; however, it has not been investigated for irrigation water allocation.

Therefore, the objective of this study is to introduce the cloud model and develop an advanced optimization model to support irrigation water allocation and agricultural water planning under composite uncertainties. Based on a dual-objective nonlinear programming (DONP) framework, a cloud-based dual-objective nonlinear programming (CDONP) model will be developed. The cloud model will be applied to address the complex composite uncertainties associated with reference evapotranspiration (ET_0) and surface water availability (SWA). A case study of the Yingke irrigation district (YID) in Northwest China will be conducted to demonstrate the applicability of the developed model. The risk of water shortage will be analyzed based on the cloud model results, and optimal water allocation schemes will be generated for different risk scenarios.

2.0 Methodology

2.1 The cloud model

2.1.1 Cloud membership

The cloud model synthetically describes fuzziness and randomness, which are the two most common uncertainties inherent in decision-making processes. Given a concept T, and let $x \in T$ be a random realization of the concept T and $\mu(x)$ is the degree of x belonging to T. Each x is called a cloud drop, and all the possible x realizations form a cloud C_T.

The cloud model integrates possibility and probability distributions and uses a unique cloud membership to describe uncertainty. Figure 1 shows an example cloud model of daily average inflow. Traditionally, when there is not enough historical data, a triangular fuzzy membership function can be used to describe the uncertainty of daily average inflow. To define a traditional triangular fuzzy membership, it requires a minimum of three estimates (the maximum, minimum, and the most likely value) from experts. In the cloud model, each inflow value has a corresponding membership degree, and each membership degree has a probability. The blue dots represent 500 samples generated by the example cloud model. For each value, the model has multiple values for the membership degree. For example, when the inflow is 45 m^3/s, the membership degree varies from 0.5 to 0.7. The probability of the membership degree being 0.65 is higher than the two bounds, as there are more dots clustered near 0.65.

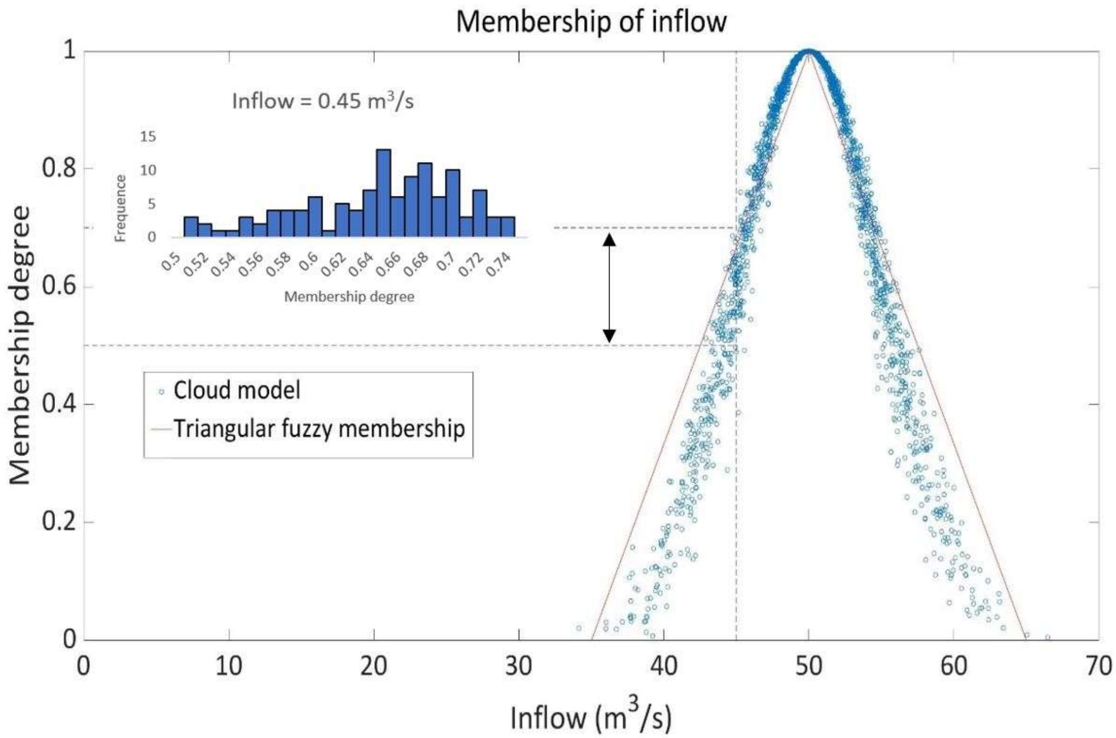


Figure 1 Cloud membership

The cloud model can describe the composite uncertainty of randomness and fuzziness using three characteristics: the expect value (Ex), the entropy (En), and the hyper entropy (He). For an uncertain parameter u, let U be a cloud that includes n cloud drops $C_1, C_2, \ldots\ C_n$. Each cloud drop is a possible value for parameter u. Let $\mu(C_i)$ be the degree of the i^{th} drop C_i belonging to cloud U (membership degree). Ex is the most representative drop in cloud U, defined as the expected value of cloud U. En is the uncertainty magnitude of cloud U. In the fuzzy concept, this represents the range of all drops that belong to cloud U. In the randomness concept, this represents the dispersing extent of C_i. He is the uncertainty magnitude of En, representing the range of En and the dispersing extent of En.

There are different types of cloud models depending on the probability distribution and fuzzy membership types. The types of probability distribution and fuzzy membership can be changed for different datasets. In this study, a Gaussian cloud model based on the integration of a normal distribution and a Gaussian fuzzy membership function is used.

2.1.2 Forward cloud generator

A forward cloud generator takes Ex, En, He, number of clusters (n) as input and returns n cloud drops $C_1, C_2, \ldots\ C_n$ with corresponding membership $\mu(\ C_1), \mu(\ C_2)\ ,\ldots.\ \mu(\ C_n)$. This can be achieved using the following procedure:

Step 1: Generate n number of normally distributed random numbers with a mean of En and a variance of He^2. Record as $Enn\ (Enn_1, Enn_2, \ldots\ Enn_n)$.

Step 2: Generate n number of normally distributed random numbers with a mean of Ex and a variance of Enn^2, record as $C\ (C_1, C_2, \ldots\ C_n)$.

Step 3: Calculate the cloud membership degree for each cloud drop of $C\ (C_1, C_2, \ldots\ C_n)$. The equation for membership degrees is as follows (Li, D et.al, 2009):

$$\mu(\ C_i) = \exp\left(-\frac{(C_i - Ex)^2}{2(Enn_i)^2}\right) \tag{1}$$

where $\mu(\ C_i)$ is the membership degree of C_i.

2.1.3 Backward cloud generator

A backward cloud generator takes a set of data as a group of cloud drops, then returns Ex, En, and He values for this data set (i.e., cloud). The equations to calculate Ex, En, He are as follows (Li, D et al., 2009):

$$Ex = \frac{1}{I}\sum_{i=1}^{I} C_i \tag{2}$$

$$En = \sqrt{\frac{\pi}{2}} * \frac{1}{I}\sum_{i=1}^{I} |C_i - Ex| \tag{3}$$

$$He = \sqrt{\frac{1}{I-1}\sum_{i=1}^{I}(C_i - Ex)^2 - En^2} \tag{4}$$

where I is the total number of cloud drops.

2.2 CDONP model for irrigation water allocation

In this study, reference evapotranspiration (ET0) and surface water availability (SWA) are identified as two major sources of uncertainties. These two parameters are common uncertainty sources in irrigation models (Li et al., 2014; Zhang et al., 2018; Ren et al., 2017). By introducing the cloud model to address the uncertainties associated with ET0 and SWA, a cloud-based dual-objective nonlinear programming (CDONP) model was proposed for irrigation water allocation. The objective functions and model constraints are described as below.

2.2.1 Objective functions of CDONP model

The two objective functions of the CDONP model are economical net profit (ENP) and irrigation system efficiency (ISE). The ENP objective is defined as follows:

$$Max\ ENP = \sum_{k=1}^{K} P_k[a_k(x_k^s + x_k^g)^2 + b_k(x_k^s + x_k^g) + c_k] * A_k + \sum_{m=1}^{M} P_m(x_m^s + x_m^g) * A_m \\ - \sum_{k=1}^{K} C_k\left[a_k(x_k^s + x_k^g)^2 + b_k(x_k^s + x_k^g) + c_k\right] * A_k + \sum_{m=1}^{M} C_m * A_m \\ - \left[\frac{W_c^s(\sum_{k=1}^{K} x_k^s * A_k + \sum_{m=1}^{M} x_m^s * A_m)}{\eta^s} + \frac{W_c^g(\sum_{k=1}^{K} x_k^g * A_k + \sum_{m=1}^{M} x_m^g * A_m)}{\eta^g}\right] \tag{5a}$$

where ENP is in the unit of Chinese Yuan (CNY); *k* is the crop type of grain crops, and the total number of grain crops is *K*; *m* is the crop type of economic crops, and the total number of economic crops is *M;* P_k is the price of grain crop *k* (Yuan/kg); P_m is the economic benefit per unit water of economic crop *m* (Yuan/m^3); a_k, b_k, c_k are coefficients of the water production function for grain crop *k*; x_k^s is the amount of surface water allocated to grain crop *k* (m^3/ha); x_k^g is the amount of groundwater allocated to grain crop *k* (m^3/ha); x_m^s is the amount of surface water allocated to economic crop *m* (m^3/ha); x_m^g is the amount of groundwater allocated to economic crop *m* (m^3/ha); A_k is the area of grain crop *k* (ha); A_m is the area of the economic crop *m* (ha); C_k is the cost of grain crop *k* per unit yield (Yuan/kg); C_m is the cost of economic crop *m* per unit area (Yuan/ha); W_c^s is the cost of surface water (Yuan/m^3); W_c^g is the cost of groundwater (Yuan/m^3); η^s and η^g are the surface water and groundwater irrigation efficiencies, respectively.

The change in ET0 has significant effects on the ISE. The ISE objective is defined as:

$$Max\ ISE = \frac{\sum_{k=1}^{K} K_{c,k} * ET0 * A_k + \sum_{m=1}^{M} K_{c,m} * ET0 * A_m}{\sum_{k=1}^{k}\left(\frac{x_k^s}{\eta^s} + \frac{x_k^g}{\eta^g}\right)A_k + \sum_{m=1}^{M}\left(\frac{x_m^s}{\eta^s} + \frac{x_m^g}{\eta^g}\right)A_m + (EP + \Delta_w) * (\sum_{k=1}^{K} A_k + \sum_{m=1}^{M} A_m)} \tag{5b}$$

where ISE is the efficiency of irrigation water utilization; $K_{c,k}$ is the crop coefficient of grain crop *k*; $K_{c,m}$ is the crop coefficient of economic crop *m*; ET0 is the reference evapotranspiration (m^3/ha); *EP* is the effective precipitation (m^3/ha); Δ_w is the variation of soil water (m^3/ha); $EP + \Delta_w$ represents the rainfall water and the water in soil absorbed by crops.

2.2.2 Constraints

The constraints of the CDONP model are listed as below:

SWA:

$$\widetilde{Q_s} = \{Q_s{}^1, Q_s{}^2, \dots, Q_s{}^n\} \tag{5c}$$

$$\sum_{k=1}^{K} (x_k^s/\eta^s) * A_k + \sum_{m=1}^{M} (x_m^s/\eta^s) * A_m \leq \widetilde{Q_s} \tag{5d}$$

Groundwater availability:

$$\sum_{k=1}^{K} (x_k^g/\eta^g) * A_k + \sum_{m=1}^{M} (x_m^g/\eta^g) * A_m \leq Q_g \tag{5e}$$

Irrigation water requirement:

$$\widetilde{ET}0 = \{ET0^1, ET0^2, \dots, ET0^n\} \tag{5f}$$

$$x_k^s + x_k^g + EP \geq K_{c,k} * \widetilde{ET}0 \tag{5g}$$

Minimum food demand:

$$\sum_{k=1}^{K} (x_k^g/\eta^g) * A_k + \sum_{m=1}^{M} (x_m^g/\eta^g) * A_m \leq Q_g \tag{5h}$$

Maximum irrigation water for economic crops:

$$x_m^s + x_m^g \leq IQ_m \tag{5I}$$

Non-negativity constraints:

$$x_k^s, x_k^g, x_m^s, x_m^g \geq 0 \tag{5J}$$

where Q_g is the maximum allowable groundwater (m^3); Q_s is the SWA (m^3); $\widetilde{Q_s}$ is the cloud set of SWA and its uncertainty is quantified using the cloud model; ${Q_s}^1, {Q_s}^2, \dots, {Q_s}^n$ are the cloud drops of SWA; $\widetilde{ET_0}$ is the cloud set of ET0 and is the second cloud parameter in the CDONP model; $ET0^1, ET0^2, \dots, ET0^n$ are the cloud drops of ET0; P_p is the population (person); G_p is the minimum food demand (kg/person); IQ_m is the maximum irrigation amount for economic crop m (m^3/ha).

There are two functional constraints relating to ET0 and SWA. ET0 affects the minimum crop irrigation amount (Equation 5g) and SWA affects the maximum allowable surface water usage (Equation 5d).

2.3 Solution method

To transform the composite uncertainties associated with ET0 and SWA to deterministic values and thus solve the proposed CDONP model, the following solution procedure is developed and applied:

Step 1: Collect historical data of ET0 and SWA;

Step 2: Use the backward cloud generator to obtain the Ex, En and He values, and characterize the clouds for ET0 and SWA;

Step 3: Use the forward cloud generator and the Ex, En and He values obtained from Step 2 to populate the cloud drops of ET0 and SWA, and record the synthetic cloud drops as $ET0^j$ and SWA^j, respectively.

Step 4: Pair the generated ET0 to SWA cloud drops, record them as $P^j(ET0^j, SWA^j)$, and convert the CDONP model to a number of deterministic DONP models.

These deterministic models are then solved using the genetic algorithm (GA). GA is a common tool to solve optimization models, and it is widely used for irrigation planning (Kuo, 2000). GA randomly generates a candidate solution (i.e., an individual) to the decision variables, and repeats the process to obtain a set of solutions (i.e., a population). In each generation, the non-dominant individuals are removed, and the dominant individuals are kept to create new individuals for the next generation through crossover with other dominant individuals. After a given number of generations, the final populations are recorded as Pareto front solutions. In this study, Pareto front solutions are evaluated using Equation 6, and the individual with the highest score is recorded as the final solution. The optimal irrigation water allocation scheme is then obtained based on the final solution.

$$F_i = \sum_{j=1}^{J} \omega_j * d_{ij} \tag{6}$$

where F_i is the evaluation score of solution i; ω_j is the weight coefficient for objective function j; d_{ij} is the standardized value of the j^{th} objective function solution i.

3.0 Application of developed methodology

A case study of the Yingke Irrigation District (YID) in Northwest China is presented to demonstrate the applicability of the proposed model. Figure 2 shows the location of YID (38°50'-38°58'N, 100°17'-100°34'E). It is the third largest irrigation area in the Heihe River basin. This area has an elevation of 1450-1600 m, with a total area of 19,200 ha, within which 13,147 ha is irrigated. YID is a major irrigation district in Zhangye City, which has a population of 1.2 million. There are four main crops grown in YID: grain corn, forage corn, wheat, and vegetables. The cultivated areas are 6,025 ha for grain corn, 4,449 ha for forage corn, 832 ha for wheat, and 2119 ha for vegetables.

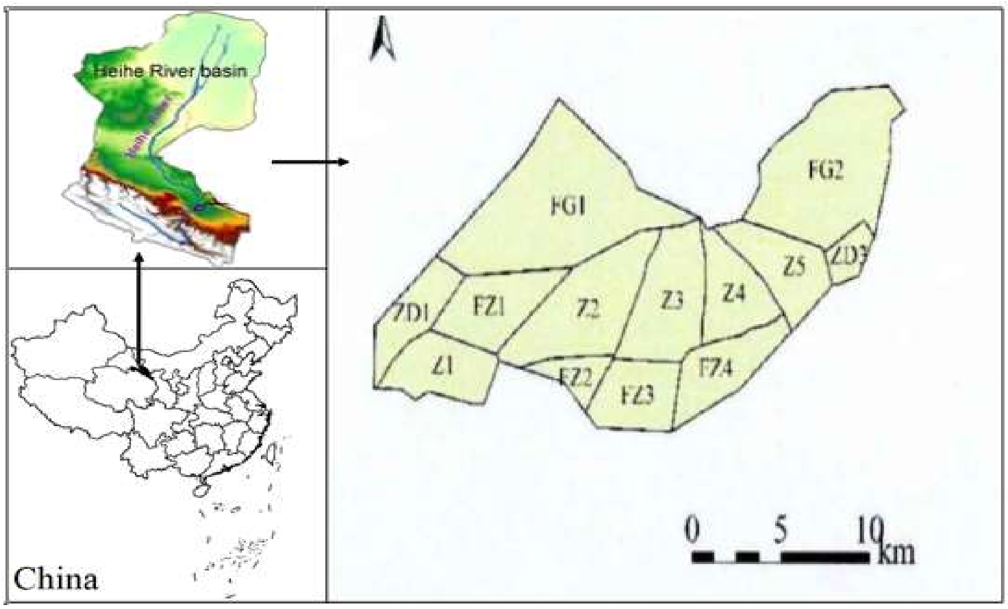


Figure 2 Study area

YID has a cold, dry continental climate. The highest, lowest, and average daily temperatures during April to September in 1996-2012 are 33.5 ◦C, -28 ◦C, and 6.8 ◦C, respectively. The average ET0 during the same period, computed by the PM-ET equation described in Allen et al. (1998), is 862mm. The estimated average SWA is 1.48 × 108 m3. Due to the arid climate, YID does not receive much rainfall. The annual mean precipitation is only 125 mm, approximately 80% of which occurs during July to September. The main water sources for irrigation are surface water and groundwater. Irrigation water is transferred to the field through a multi-level canal system that consists of one main canal and 11 sub canals. The transportation efficiency for surface water and groundwater are 0.68 and 0.8, respectively.

Water scarcity has always been a critical problem in YID. Due to the limited access to surface water and the inefficient water allocation, local people took large amounts of groundwater to cope with water shortages during the 1990s (Qi and Luo, 2005; Jiang et al., 2015). As a result, the water table dropped significantly. To protect groundwater resources, groundwater usage in YID is now limited to 0.6 × 108 m3.per year. The historical data of SWA and ET0 for April to September during 1996-2012 are shown in Figure 3. It is shown that there are significant uncertainties associated with SWA and ET0. For example, the SWA in 1998 is twice as much as that in 2001.

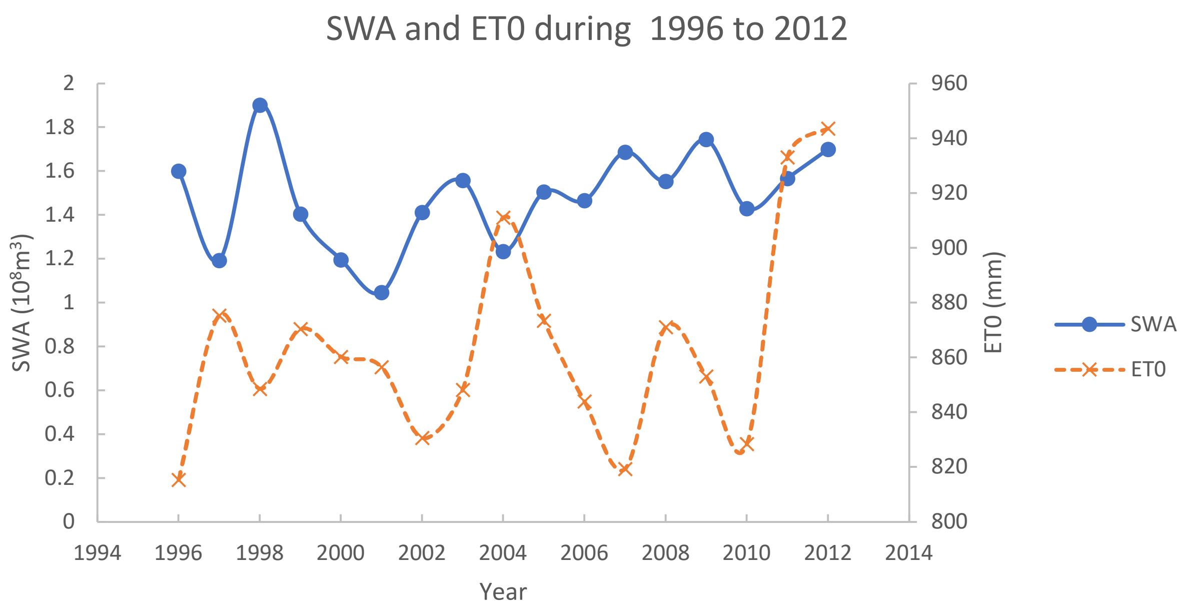


Figure 3 SWA and ET0 during 1996 to 2012

In this study, meteorological data such as rainfall and temperature are collected from the Zhangye weather station (100º25'E, 38º51'N, 1425 m). ET0 is calculated using the PM-ET equation described in Allen et al. (1988). The streamflow data are obtained from the simulation results of an agro-hydrological model (SWAP-EPIC) proposed by Jiang et al. (2015). Water price, crop price, cultivated area, crop type, and cultivation cost data are obtained from the local government's website and a previous study by Li et al. (2014).

4.0 Results and discussion

In this study, two Gaussian cloud models are built to analyze the uncertainties associated with SWA and ET0, respectively. Each cloud model has a total of 2,000 drops created from historical data using the forward and backward cloud generators as described in Section 2.1. The SWA and ET0 drops are paired and fed into the CDONP model to generate optimal irrigation water allocation schemes. The decision variables include monthly surface water usage and monthly groundwater usage for the four main crops in YID. The developed CDONP model is solved using a GA with a population size of 2,500 and a maximum generation of 100. With the population size of 2,500, both objective functions can converge to a stable maximum value. If the population is too small, the solutions will lack diversity. On the other hand, if the population is too large, it will take a long time for the program to generate solutions that can meet the constraints in the initial population. A population of 100 can ensure the diversity of solutions and limit the computation time for each run within 3 minutes.

4.1 Solution feasibility

The 2,000 paired cloud drops of SWA and ET0 are presented in Figure 4. Feasible solutions are obtained for 1,625 pairs, while the models fed with the other 375 pairs are infeasible. Although no exact threshold of SWA or ET0 is found, it is clear that the model constraints are violated under extreme conditions of ET0 and SWA. The highest ET0 value with a feasible solution is 949 mm, and the lowest feasible SWA value is 0.93x10^8 m^3.

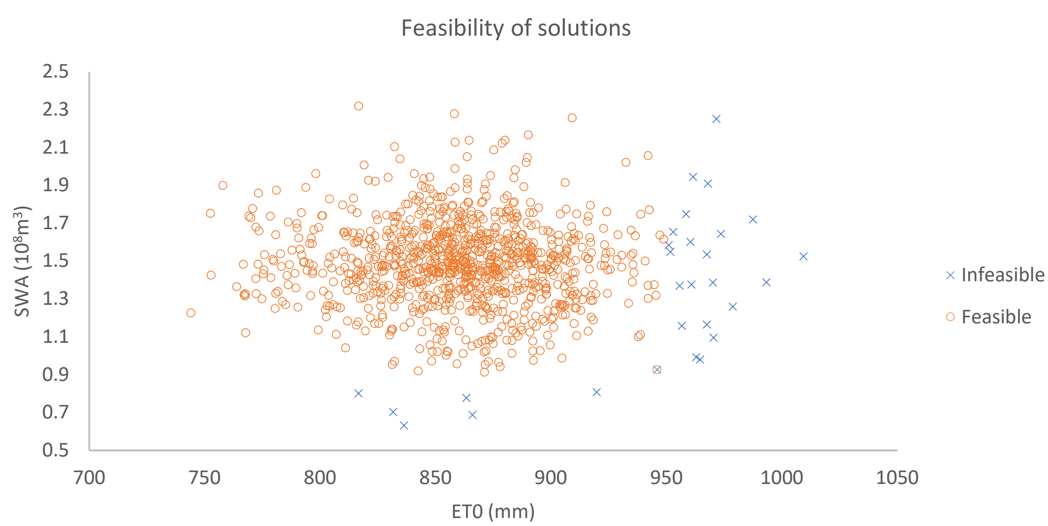


Figure 4 Solution feasibility

The solution robustness is tested by substituting a random solution back into the CDONP model for 2,000 runs, where the 2,000 paired cloud drops of SWA and ET0 are used as inputs. As an example, the solution of the CDONP model with an ET0 value of 862.97mm and an SWA value of 1.48×10^8 m^3 is used, and its robust test result is presented in Figure 5. No constraint violation is detected for approximately 51% of the paired ET0 and SWA drops. It is also found that there is a clear threshold of ET0: when the ET0 values exceed 860 mm, one or more constraints are violated. There is no clear threshold of SWA that leads to constraint violation, which implies that the uncertainty of SWA has more complex implications on irrigation water allocation and that the water allocation system might be more resilient to changes in SWA.

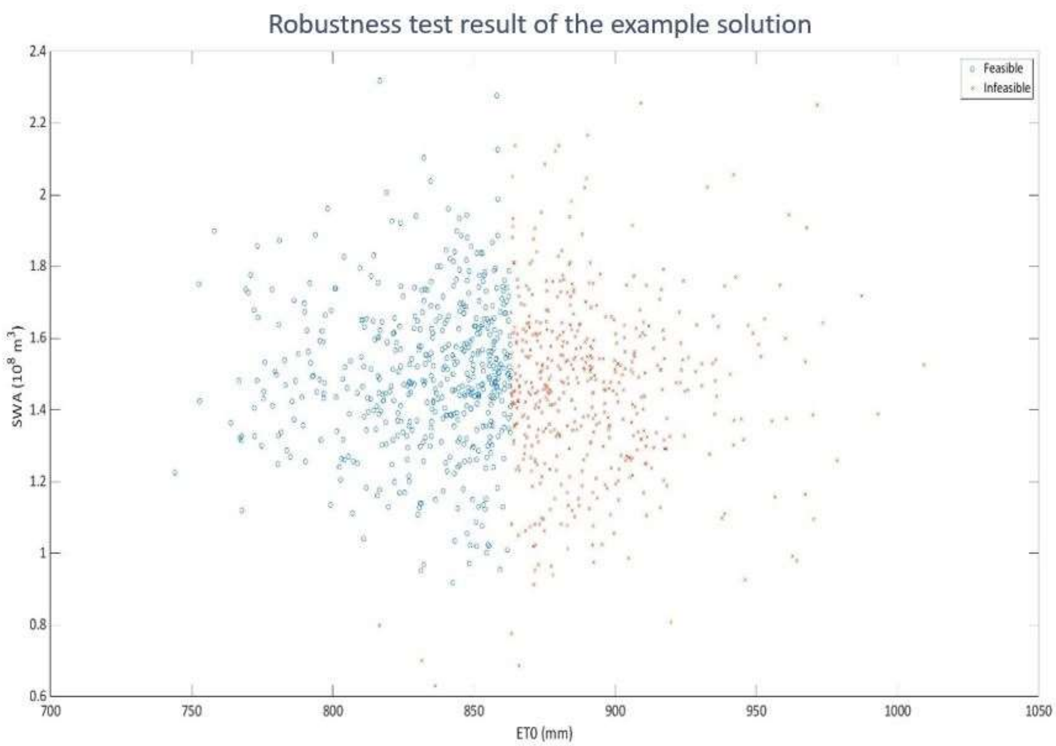
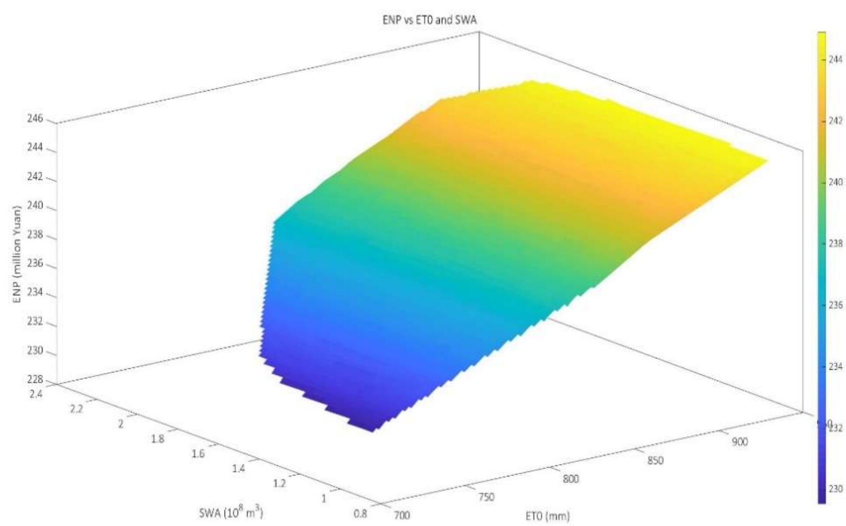


Figure 5 Robustness test result of the example solution

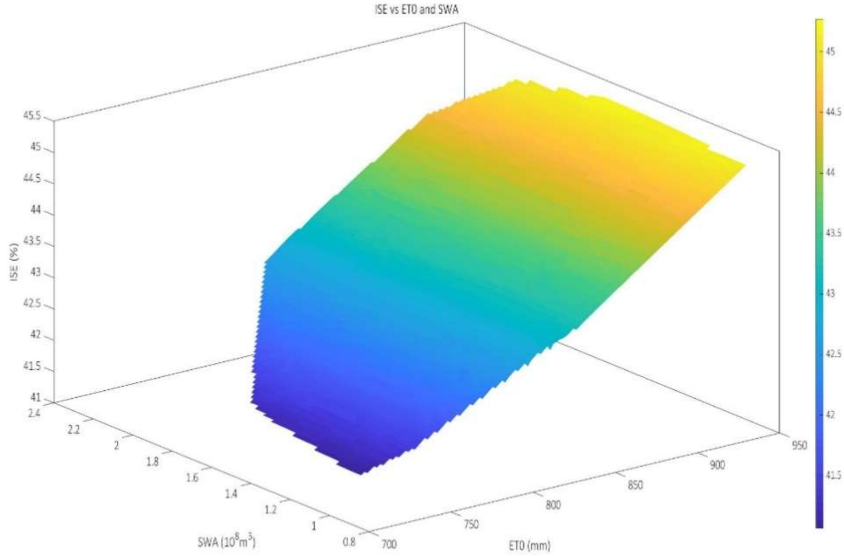
4.2 Sensitivity to changes in ET0 and SWA

4.2.1 Objective functions and total water usage

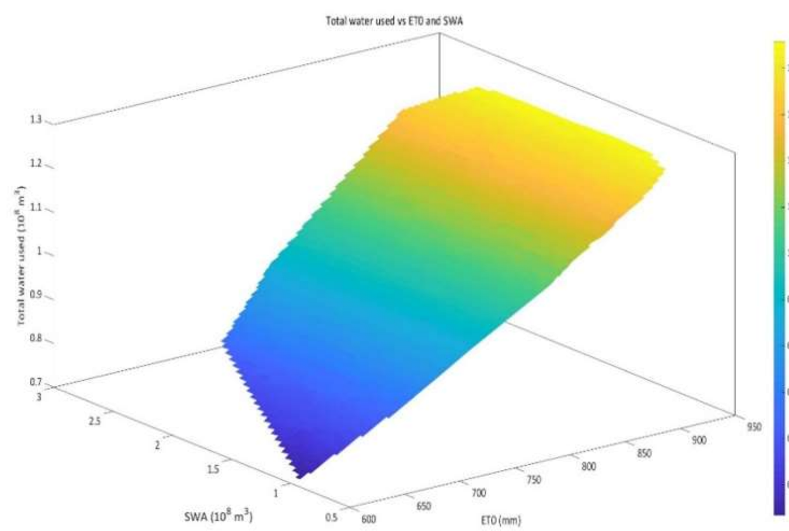
Values of the two objective functions (i.e., ENP and ISE) in the 1,625 feasible runs are calculated. Figure 6 shows the variations of ENP, ISE, and total water usage as ET0 and SWA change. As ET0 changes from 740 mm to 1,009 mm and SWA changes from 0.63×10^8 m^3 to 2.31×10^8 m^3, ENP changes from 229.27 million to 245.00 million; ISE changes from 40% to 45%; and the total water usage changes from 0.94×10^8 m^3 to 1.22×10^8 m^3. It is worth mentioning that when ET0 is fixed, ENP, ISE, and total water usage are not sensitive to changes in SWA. Compared with SWA, ET0 has more significant effects on ENP, ISE, and total water usage. Both objective function values and the total water usage increase as ET0 increases. According to Equation 5g, a high ET0 value results in an increase of the minimum irrigation requirement. An increase of the minimum irrigation requirement implies a higher water usage for crops, which often leads to increases in production yields and thus higher ENP. When the minimum irrigation requirement increases, according to Equation 5b, ISE also increases. The results show that the optimal allocation scheme is sensitive to ET0.



(a)



(b)

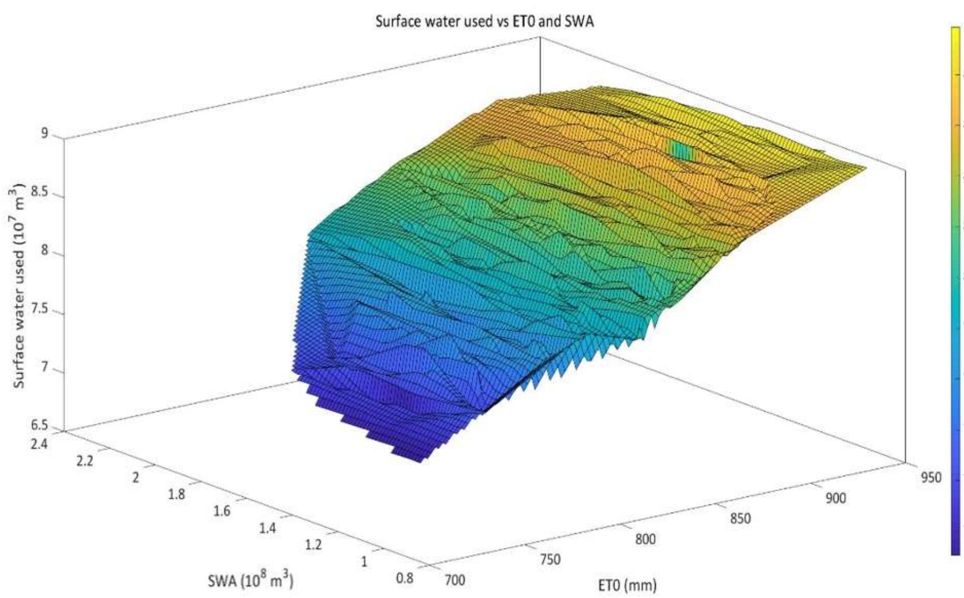


(c)

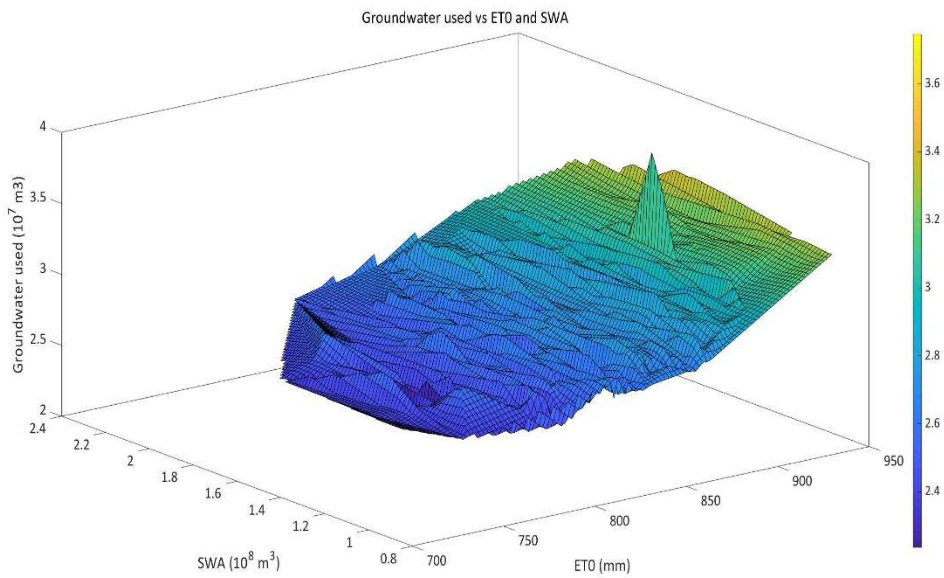
Figure 6 (a) ENP vs ET0 and SWA, (b) ISE vs ET0 and SWA, and (c) Total water usage vs ET0 and SWA

4.2.2 Surface water and groundwater usage

To analyze the sources of allocation water, the amounts of surface water and groundwater in the 1,625 feasible runs are further calculated. Figure 7 shows the surface water usage's and groundwater usage's response to ET0 and SWA. The response surfaces in Figure 7 are rougher than that of Figure 6c, where total water usage is computed as the sum of surface water and groundwater usage. Figure 7b has a clear peak near ET0 of 897 mm and SWA of 1.35 x 10^7 m^3, corresponding to the valley in Figure 7a. The surface water usage has an average value of 8.24 x 10^7 m^3 and a range of [6.52, 8.99] x 10^7 m^3; while the average and range of groundwater usage are 2.77 x 10^7 m^3 and [2.11, 3.72]x10^7 m^3, respectively. This indicates that although surface water is the major water source, groundwater still plays an important role. Groundwater can offset the changes in SWA. This also explains why the objective functions and total water usage are not sensitive to SWA as shown in Figure 6.



(a)



(b)

Figure 7 Surface water (a) and groundwater (b) usage vs ET0 and SWA

4.2.3 Crop water usage

Further analysis is performed to investigate how water allocation among the four types of crops responds to changes in ET0 and SWA. As shown in Figure 8, the water usages, ranking from highest to lowest, are: grain corn, forage corn, vegetable, and wheat. Figures 8a-c show similar patterns with different magnitudes, while the response of water allocated to wheat (Figure 8d) shows a significantly different pattern. Compared with the other crops, wheat has two remarkable features. Firstly, wheat has the smallest amount of water use. It has a growth period of 4 months, while the growth period of the other three crops is 6 months. Wheat also has the smallest cultivation area (831.57 ha) among the four crops. Secondly, wheat has the lowest unit profit. Wheat's unit price (2.28 Yuan/kg) is lower than those of forage corn (2.31 Yuan/kg), grain corn (3.00 Yuan/kg), and the average vegetable profit (4.16 Yuan/kg). Due to the small cultivation area and the low profit, wheat contributes the least to ENP. High priority is given to the water demand and production of the other three crops. Water is allocated to the three crops to maintain a stable production rate, while the water allocated to wheat needs to be frequently adjusted to cope with the complex uncertainties of ET0 and SWA.

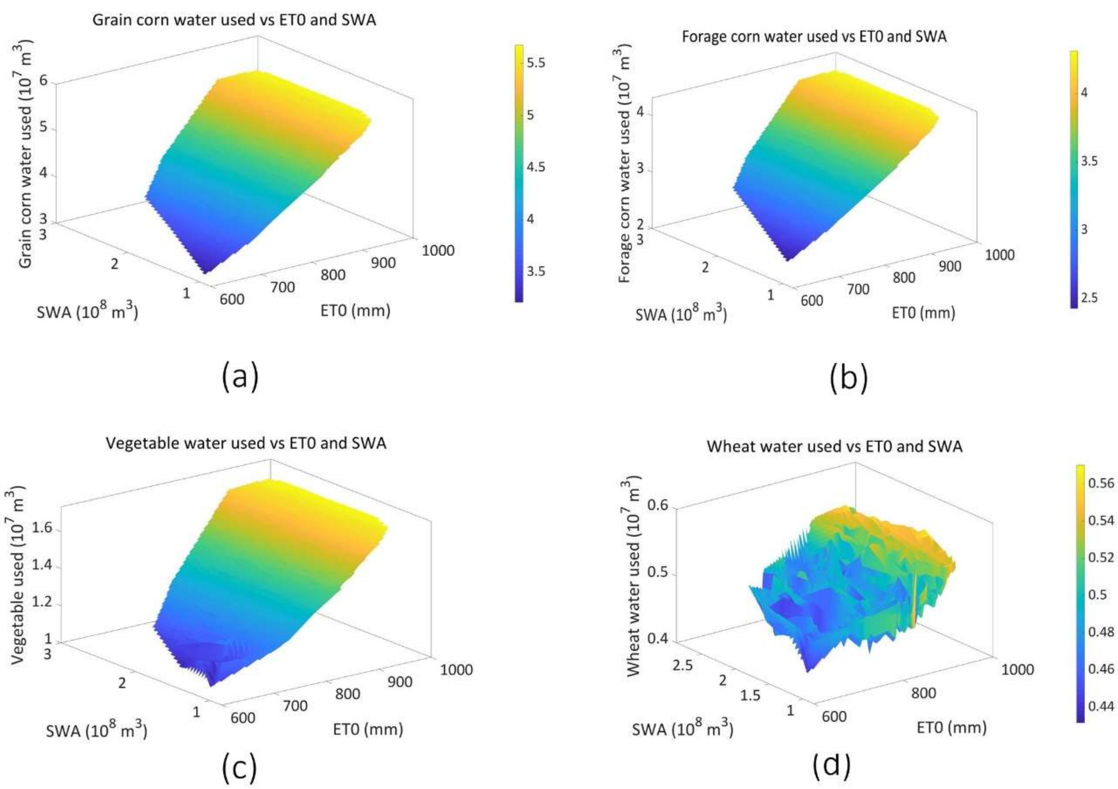


Figure 8 Forage corn (a), grain corn (b), vegetable (c), and wheat (d) water allocation vs ET0 and SWA

4.3 Optimized irrigation water allocation strategy

One optimal irrigation water allocation scheme is obtained for each of the 1,625 paired cloud drops using the CDONP model. The generated monthly water allocation schemes are analyzed in detail and are compared to historical irrigation water usage.

4.3.1 Monthly water usage

Figure 9 shows the maximum, 25th percentile, median, 75th percentile, and minimum values of the monthly usage of surface water and groundwater, as well as the monthly total water usage. The lowest variation of surface water use across the 1,625 runs is found in April. The peak of surface usage is in July with the highest variation across the 1,625 runs. It is worth mentioning that the peak of total water usage is also in July (Figure 3c). The temporal pattern of groundwater usage is different from that of surface water. The variation across the 1,625 cloud drop pairs is consistent throughout the growing season, and the peak occurs in July. All of the interpercentile ranges in Figure 9 are small, which indicates that most solutions are clustered near the median of the cloud model. The results can provide decision makers with an important basis to evaluate their current allocation plan, as well as a set of optimal ranges for monthly water usage under complex uncertainties.

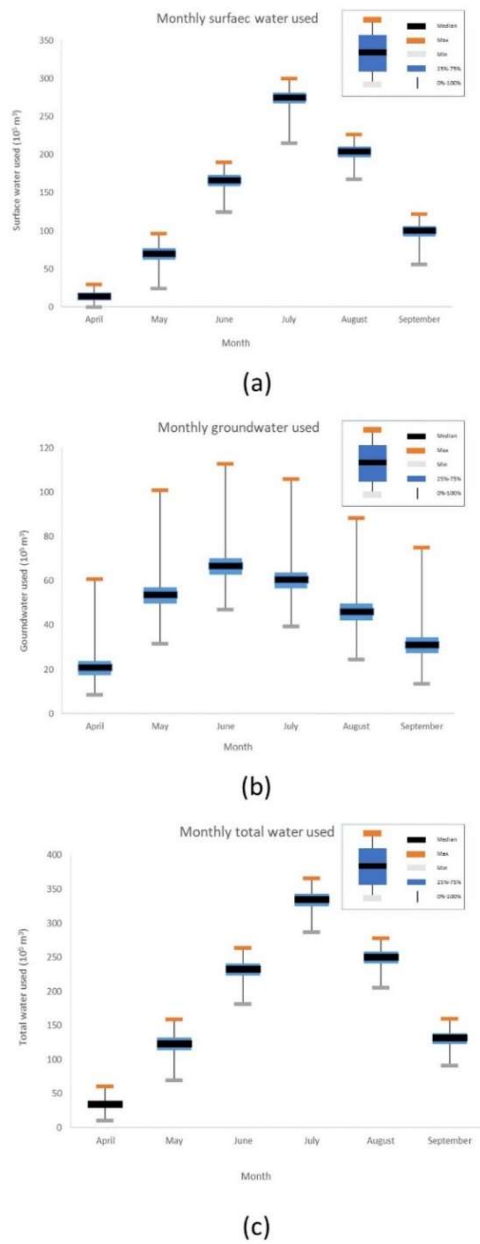


Figure 9 Box plots of monthly surface water usage (a), groundwater usage (b), and total water usage (c)

4.3.2 Sources of irrigation water

The surface water and groundwater uses are calculated for the 1,625 feasible *CDONP* runs, and the resulting histograms are shown in Figure 10 The use of surface water is significantly higher than that of groundwater. The 95% confidence interval of groundwater usage is [0.23, 0.33]x10^8 m^3, and that of surface water usage is [0.65, 0.9]x10^8 m^3.

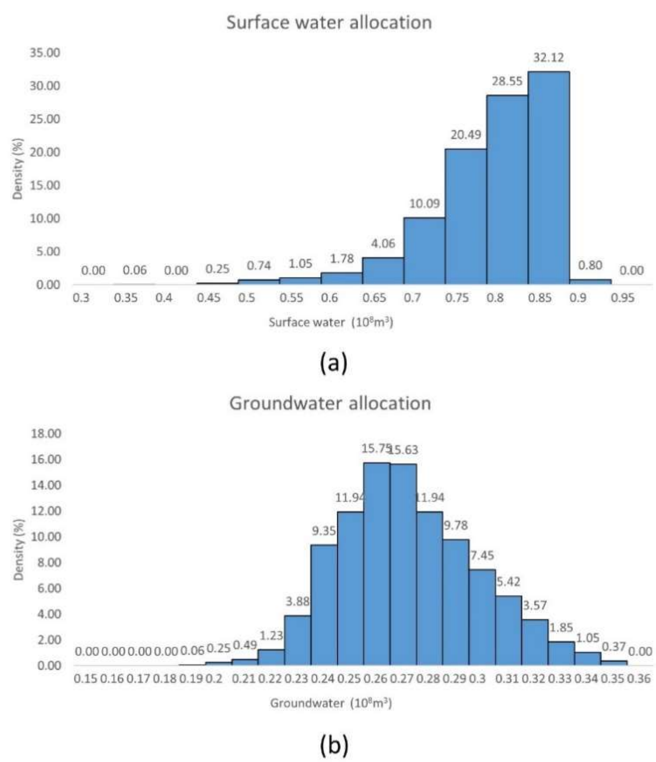


Figure 10 Histograms of surface water (a) and groundwater (b) use obtained from the CDONP model

4.3.3 Comparison between CDONP model and traditional DONP model

To demonstrate the advantages of the proposed CDONP model, a traditional dual-objective nonlinear programming (DONP) model is built, and optimal solutions are obtained by feeding the DONP model with historic ET0 and SWA data. The historical dataset contains annual average ET0 and SWA during April to September from 1996 to 2012. It is a very small dataset with only 17 pairs of ET0 and SWA, which is not sufficient for representing the complex uncertainties of ET0 and SWA. On the other hand, the cloud model integrated in the proposed CDONP model can extract the statistical characteristics of ET0 and SWA from the historic data and generate a much larger sample set to illustrative and represent their uncertainties. The histograms of water allocation across the four crop types obtain from CDONP and DONP are compared and presented in Figure 10.

Figures 10 shows that the distributions of water allocation from the CDONP model and the traditional DONP model are similar, which validates the cloud model's ability to provide meaningful and statistically reliable uncertainty analysis results. The density (%) is calculated by the frequency of solutions in a specific bin over the total number of feasible solutions. It is noteworthy that more extreme allocation values are found from the CDONP model. These extreme values represent possible combinations of ET0 and SWA conditions under the complex uncertainty, where irrigation contingency analysis is desired. The results demonstrate the need for robust uncertainty analysis for irrigation water allocation planning, and the CDONP model can provide the reliable and robust uncertainty analysis results to meet this need.

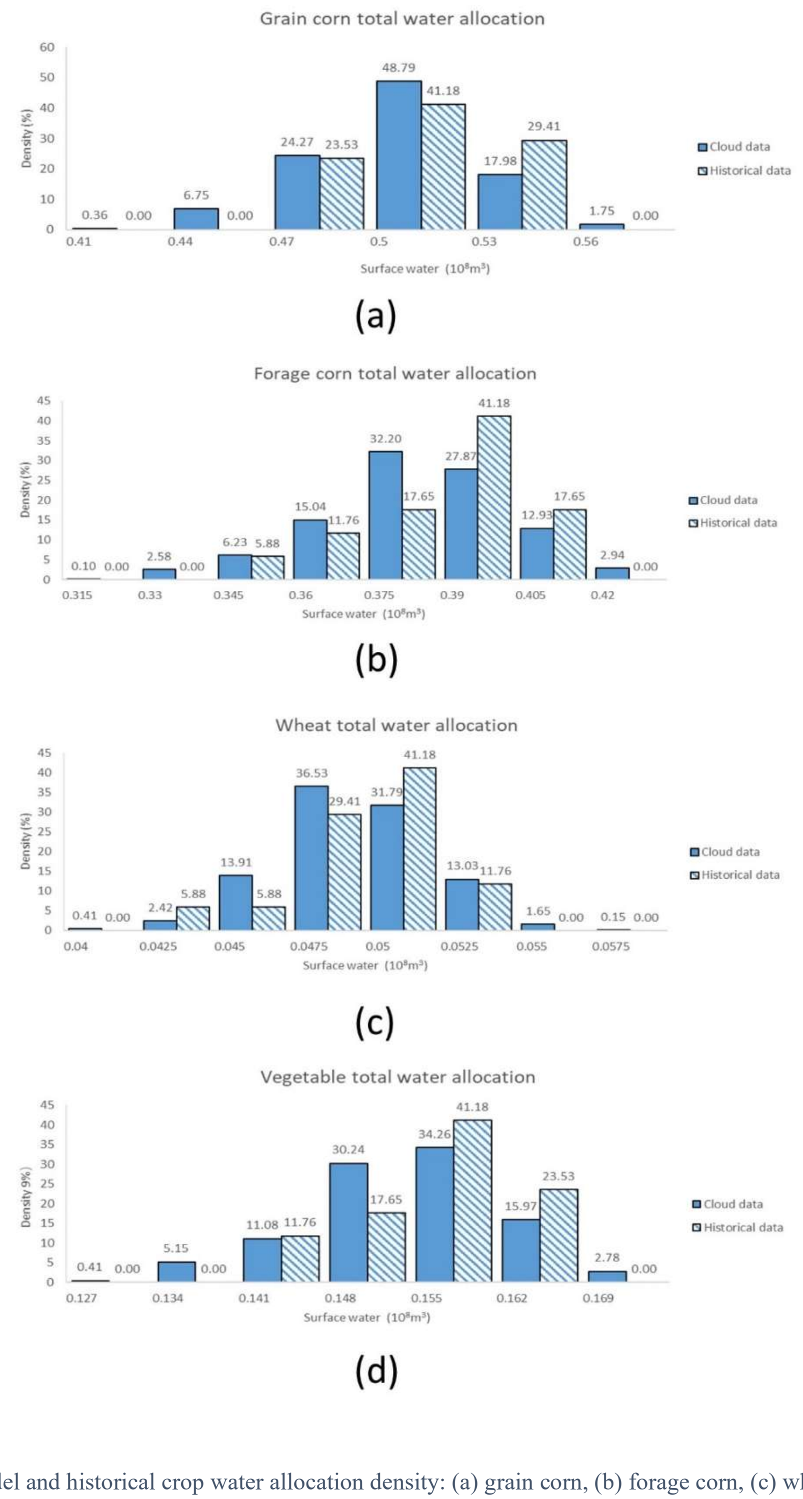


Figure 11 Cloud model and historical crop water allocation density: (a) grain corn, (b) forage corn, (c) wheat, and (d) vegetable

4.4 Analysis of extreme conditions

Irrigation planning under extreme water availability conditions is critical for risk management. As shown in Figure 4, the solution is very likely to become infeasible when ET0 is higher than 863 mm or SWA is lower than 1.48×10^8 m^3. To further investigate the impacts of extreme conditions, four scenarios with different levels of drought risk are created. The highest and lowest 500 values from both the ET0 and SWA clouds are paired to establish the four scenarios. The four scenarios include: 1) high risk (high ET0 and low SWA), 2) medium risk – A (low ET0 and low SWA), 3) medium risk - B (high ET0 and high SWA), and 4) low risk (low ET0 and high SWA). Ten risk levels are defined under each drought risk scenario, with each level corresponding to 50 cloud drop pairs. The drought risk increases from Level 1 to Level 10. For ET0, the drought risk increases as ET0 increases; for SWA, the drought risk increases as SWA decreases. When pairing the ET0 and SWA cloud drops, the highest 500 values are sorted from high to low, while the lowest 500 are sorted from low to high. For example, the Level 10 risk under a high risk scenario is defined using the 451st to 500th highest ET0 values paired with the 1st to 50th lowest SWA values. The ranges of ET0 and SWA for the 10 risk levels under the four drought risk scenarios are shown in Table 1.

Table 1 *Ranges of ET0 and SWA values in scenario analysis*

Risk level	High risk		Medium risk – A		Medium risk – B risk		Low risk	
	ET0 (mm)	SWA (10^8 m^3)	ET0 (mm)	SWA (10^8 m^3)	ET0 (mm)	SWA (10^8 m^3)	ET0 (mm)	SWA (10^8 m^3)
1	[888, 890]	[1.4, 1.3]	[888, 891]	[2.3, 2.0]	[664, 780]	[1.4, 1.3]	[664, 837]	[2.0, 2.3]
2	[891, 893]	[1.3, 1.3]	[891, 894]	[2.0, 1.9]	[780, 793]	[1.3, 1.3]	[780, 834]	[1.9, 2.0]
3	[894, 898]	[1.3, 1.3]	[893, 898]	[1.8, 1.8]	[794, 804]	[1.3, 1.3]	[794, 831]	[1.8, 1.8]
4	[898, 901]	[1.3, 1.3]	[898, 901]	[1.8, 1.8]	[804, 812]	[1.3, 1.3]	[804, 827]	[1.8, 1.8]
5	[901, 906]	[1.3, 1.3]	[901, 906]	[1.8, 1.7]	[812, 818]	[1.3, 1.3]	[812, 823]	[1.7, 1.8]
6	[906, 911]	[1.3, 1.2]	[906, 911]	[1.7, 1.7]	[818, 823]	[1.3, 1.2]	[819, 818]	[1.7, 1.7]
7	[911, 918]	[1.2, 1.2]	[911, 918]	[1.7, 1.7]	[823, 827]	[1.2, 1.2]	[823, 812]	[1.7, 1.7]
8	[919, 931]	[1.2, 1.1]	[919, 931]	[1.7, 1.7]	[827, 831]	[1.2, 1.1]	[827, 804]	[1.7, 1.7]
9	[931, 946]	[1.1, 1.0]	[931, 946]	[1.7, 1.6]	[831, 834]	[1.1, 1.0]	[831, 793]	[1.6, 1.7]
10	[946, 1017]	[1.0, 0.6]	[946, 1016]	[1.6, 1.6]	[834, 837]	[1.0, 0.6]	[834, 780]	[1.6, 1.6]

Figure 12 shows the scenario analysis results. The left column shows the changes in ENP and the right column shows the resulting changes in ISE. It is found that the medium risk B scenario has a significantly lower value for both objective functions compared to the other three scenarios, and that the other three scenarios show similar patterns of changes in ENP and ISE. This indicates that the highest drought risk does not necessarily lead to the highest ENP and ISE. This is because under high ET0 conditions, more water will be drawn to meet the increased irrigation requirement, as discussed in Section 4.2.1. According to Equation 5b, 5f, and 5g, ET0 determines the minimum irrigation requirement, and ET0 is part of numerator in the ISE objective function. Under the medium risk – B scenario, the low SWA value forces the model to search for solutions with low water allocations, and the low ET0 value decreases the minimum water required for irrigation. As a result, the smallest amount of water is used for irrigation, leading to the lowest system benefit and water efficiency. Moreover, the ranges for ENP and ISE under the medium risk B scenario are the smallest across the four scenarios, and only the ranges of risk level 10 are narrow under the other three scenarios. This implies that as the uncertainties of ET0 and SWA propagates through the optimization process, their impacts on the uncertainties of ENP and ISE reduce as the risk level increases.

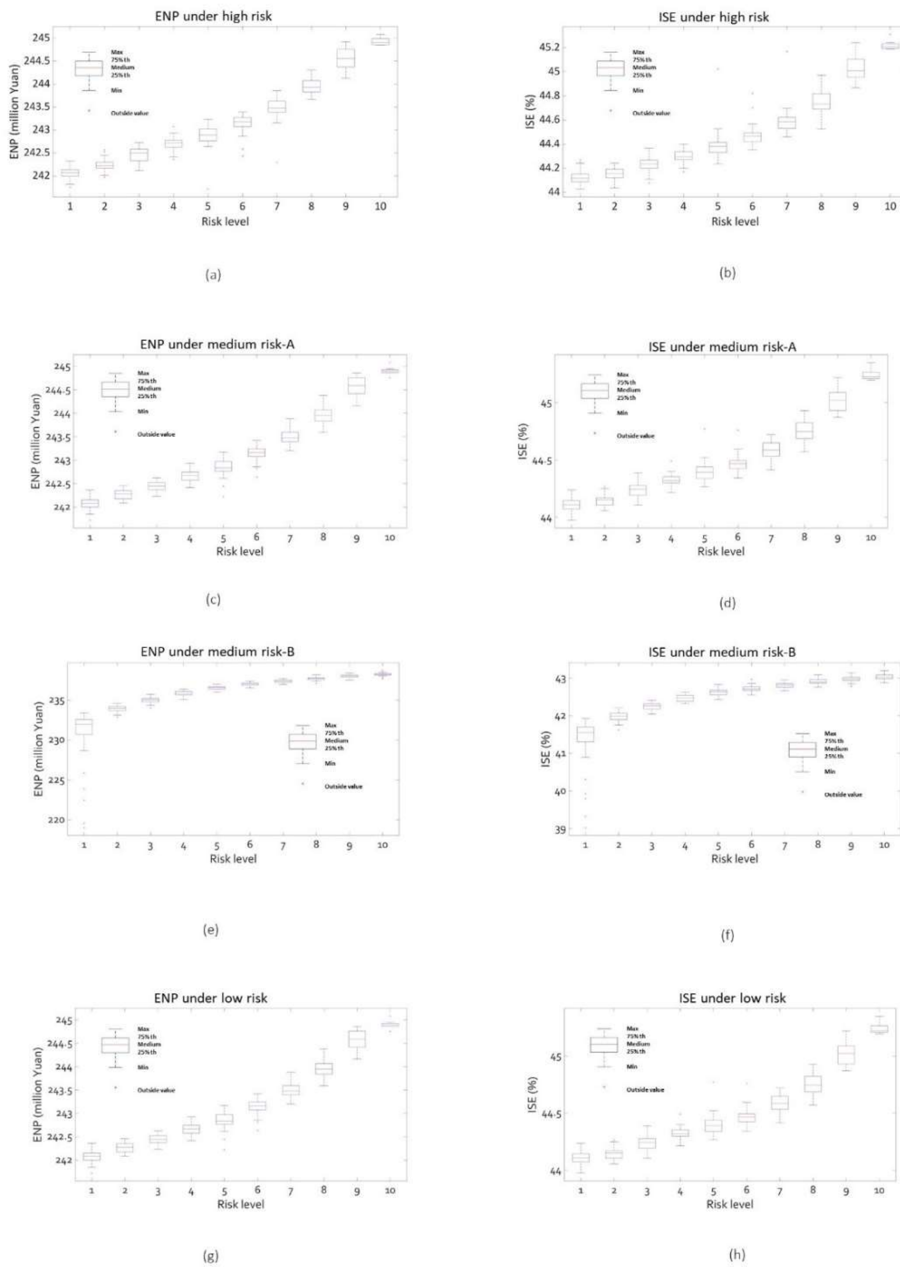


Figure 12 Changes of ENP and ISE under different drought risk scenarios

5.0 Conclusions

In this study, a cloud-based dual-objective nonlinear programming (CDONP) model for irrigation water allocation was developed by introducing the cloud model to a dual-objective nonlinear programming framework. The integrated cloud model can quantify the complex and composite uncertainty associated with evapotranspiration (ET0) and surface water availability (SWA) using possibility and probability distributions. The proposed model can be used to provide reliable and robust decision support for optimizing irrigation water allocation under complex uncertainties.

To demonstrate the applicability of the proposed model, a case study of Yingke Irrigation District (YID) was conducted. Based on historic data, 2,000 pairs of ET0 and SWA cloud drops (samples) were obtained through the cloud generator. Each pair of ET0 and SWA was passed to the CDONP model to generate its corresponding optimal solutions with the goals to maximize both economic net profit (ENP) and irrigation system efficiency (ISE). The feasibility of the 2,000 solutions was tested and analyzed. The effects of ET0 and SWA on ENP, ISE, and crop water usage were analyzed. The obtained results were also compared to those of a traditional dual-objective nonlinear programming model to illustrate the advantages of the proposed CDONP model. Moreover, four drought risk scenarios based on different combinations of high/low ET0 and SWA values were created and discussed.

It was found that the impact of ET0 on ENP and ISE is greater than that of SWA. This is because groundwater can serve as a backup source of irrigation water when SWA changes, while there is no compensation mechanism for ET0 loss. The peak usage of surface water is most likely to occur in July, and the peak usage of groundwater is most likely to happen in June. The results also demonstrated the CDONP model's advantage in providing reliable and robust uncertainty analysis results when compared to the traditional optimization model. The risk level analysis also showed that there is a significant drop in the values of both objective functions under the medium risk B scenario, and that the highest drought risk does not necessarily lead to the lowest ENP and ISE.

In this study, a simple weighted sum method was used to select only one optimal solution from the Pareto frontier. More solutions on the Parato frontier should be investigated in further studies. Meanwhile, only two uncertain parameters, ET0 and SWA, were and analyzed in this study. In future studies, more uncertain parameters, such as rainfall related parameters could be considered.

ACKNOWLEDGEMENTS

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CHAPTER 4 – CONCLUSIONS

The demand for effective irrigation water allocation is urgent since water demands keep increasing across the world. Previous studies proved that optimization models have a potential to provide decision support for irrigation water allocation in water shortage areas. Irrigation water allocation models could support decision makers to obtain higher profits and to save more water.

In this study, a cloud-based dual objective nonlinear model programming (CDONP) model was developed for monthly irrigation water allocation. The objective functions included net economic profit (ENP) and irrigation system efficiency (ISE). The composite uncertainties of reference evapotranspiration (ET0) and surface water availability (SWA) were addressed by implementing cloud modeling. The cloud model generated a large number of data samples from a relatively small historical dataset for uncertainty quantification. Furthermore, a risk analysis was performed based on four scenarios of extreme conditions, which allowed the decision makers to better understand the water shortage risks and the corresponding risk mitigation schemes. The CDONP model can provide technical support for irrigation water allocation under complex uncertainties and with limited data.

For future studies, the uncertainties of other parameters could be investigated using the proposed CDONP model. Different types of distributions could be considered for building the cloud model. Other solution algorithms for dual-objective nonlinear optimization could be investigated and incorporated. Additionally, ET0 and SWA were assumed to be independent when using the cloud generator. The correlation between ET0 and SWA should be further investigated. More work could also be carried out to integrate the develop model with existing irrigation simulation models.

LIST OF NOTATIONS

Notation	Definition
ENP	Economical net profit in the unit of Chinese Yuan (CNY)
k	The crop type of grain crops (1… K)
m	The crop type of economic crops (1… M)
P_k	The price of grain crop k (Yuan/kg)
P_m	The economic benefit per unit water of economic crop m (Yuan/m^3)
a_k,	Coefficient of the water production function for grain crop k
b_k,	Coefficient of the water production function for grain crop k
c_k	Coefficient of the water production function for grain crop k
x_k^s	The amount of surface water allocated to grain crop k (m^3/ha)
x_k^g	The amount of groundwater allocated to grain crop k (m^3/ha)
x_m^s	The amount of surface water allocated to economic crop m (m^3/ha)
x_m^g	The amount of groundwater allocated to economic crop m (m^3/ha)
A_k	The area of grain crop k (ha)
A_m	The area of the economic crop m (ha)
C_k	The cost of grain crop k per unit yield (Yuan/kg)
C_m	The cost of economic crop m per unit area (Yuan/ha)
W_c^s	The cost of surface water (Yuan/m^3)
W_c^g	The cost of groundwater (Yuan/m^3)
η^s	The surface water irrigation efficiencies
η^g	The groundwater irrigation efficiencies
ISE	The efficiency of irrigation water utilization
$K_{c,k}$	The crop coefficient of grain crop k
$K_{c,m}$	The crop coefficient of economic crop m
ET0	The reference evapotranspiration (m^3/ha)
EP	The effective precipitation (m^3/ha)
Δ_w	The variation of soil water (m^3/ha)
Q_g	The maximum allowable groundwater (m^3)
Q_s	The maximum allowable surface water (m^3)
$\widetilde{Q_s}$	The cloud set of maximum allowable surface water (${Q_s}^1, {Q_s}^2, \dots, {Q_s}^n$)
$\widetilde{ET}_0$	The cloud set of reference evapotranspiration ($ET0^1, ET0^2, \dots, ET0^n$)
P_p	The population (person)
G_p	The minimum food demand (kg/person)
IQ_m	The maximum irrigation amount for economic crop m (m^3/ha)
F_i	The evaluation score of solution i
ω_j	The weight coefficient for objective function j
d_{ij}	The standardized value of the j^{th} objective function solution i

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