PRAGMATICS AND SEMANTICS OF FREE CHOICE DISJUNCTION

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Lay Abstract

A disjunction is a statement using ‘or’, like ‘Anne has a Ford or a Tesla’. From such a statement, we cannot infer either disjunct—e.g. ‘Anne has a Ford’. In choice situations like ‘You may have coffee or tea’ we *can* infer either option. Why this choice inference is legitimate is the problem of free choice disjunction.

I explore the history of solutions to the problem, including semantic solutions that propose a special meaning to choice disjunctions and pragmatic solutions that appeal to the circumstances in which they are uttered. I draw connections between semantics and pragmatics and present a formal account of one major pragmatic approach to the problem.

Where others have sought to explain how 'May(P or Q)' entails 'May P and May Q', I argue instead that the meaning of ‘May (P or Q)’ in choice scenarios translates directly into logical formalism as ‘May P & May Q’.

Abstract

A disjunction is an expression using 'or', such as 'Anne has a Ford or a Tesla'. From such a statement, we cannot usually infer either disjunct, for example, that 'Anne has a Ford'. However, in choice situations like 'You may have coffee or tea' we *can* infer either option. The problem of free choice disjunction is to determine why these choice inferences are legitimate (von Wright 1968, Kamp 1973, Meyer 2016).

Central to this discussion is the observation that a modal possibility operator ranging over a disjunction sometimes implies a conjunction of possibilities. In the case of permission, we express this as the *choice principle* 'May (P or Q)' entails 'May P and May Q' (Zimmerman 2000). Unfortunately, this inference cannot hold in a logical language without significant modification of the systems involved.

I explore the history of proposed solutions to this problem, including semantic solutions that assign a distinctive meaning to free choice disjunctions and pragmatic solutions that use features of their utterance to solve the problem. I draw connections between semantics and pragmatics and, using the tools of dynamic logic (Baltag et al. 1998, van Benthem 2010), I present a formal account of one major (Gricean) approach to the problem (Kratzer & Shimoyama 2002).

Ultimately, I explore the role of logic in this debate and argue that we should formally represent the meaning of these expressions directly as *conjunctions* of possibilities. Thus, rather than trying to account for the choice principle within a logical system, we must instead account for the fact that, in choice situations, the meaning of 'May (P or Q)' translates into logical formalism as (May P & May Q).

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This project comes at the end of a very long road. I'm sure I would not have been able to make the journey without the love and support of my family who have been with me every step of the way. Thanks Mom, Dad, Dez, Trent and Andrea. This thesis is dedicated to you and, more than you know, has been improved by your presence in my life.

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List of Figures and Tables

List of Abbreviations and Symbols

Propositional Logic Symbols:

¬	Negation
&	Conjunction
∨	Disjunction
⊻	Exclusive disjunction
→	Material implication
↔	Material equivalence
(,)	Formula groupers
A, B, C,…	Atomic (unanalyzed) propositions
⊤	Tautology
⊥	Contradiction

Quantifier Logic Symbols:

∀	Universal Quantifier
∃	Existential Quantifier
x, y, z,…	Individual variables
a, b, c,…	Individual constants
A, B, C,…	Predicates

Modal Logic Symbols:

□	Necessity
◇	Possibility
[a], [b],…	Other modal necessity operators (inc. epistemic, deontic)
⟨a⟩, ⟨b⟩,…	Other modal possibility operators (inc. epistemic, deontic)
$\mathcal{P}$, $\mathcal{O}$	Traditional deontic operators (permission, obligation)
$\mathcal{M}$, $\mathcal{K}$	Traditional epistemic operators (possibility, knowledge)
∪	Converse designation (for modal operators)

Set Theory Symbols:

{,}	Set groupers
∈	Element
A, B, C,…	Set symbols

Model Symbols:

$\mathfrak{M}, \mathfrak{N}, \ldots$	Model designators
$\alpha, \beta, \gamma, \ldots$	World (state) labels
$\Gamma, \Delta, \Sigma, \ldots$	World variables
$A, B, C, \ldots$	Accessibility relations
◯	Nodes, worlds
$\longrightarrow$	Arcs, accessibility relations

Metalogical Symbols:

$\varphi, \psi, \ldots$	Proposition/formula variables
φ_a	Atomic, unanalyzed proposition (including $\top$, $\bot$)
$\vdash$	Logical/syntactic consequence
$\vDash$	Semantic consequence
$\therefore$	Therefore
$\stackrel{\text{def}}{=}$	Definition

Other Symbols:

!	Public announcement
/	Given
'	Designation mark (prime symbol)
$_{A, B, C, \ldots}$	Subscript designations
$_{1, 2, 3 \ldots}$	Subscript designations
$>$	Defeasible conditional

Abbreviations:

CP	choice principle
FCD	free choice disjunction
FCP	free choice permission
iff	if and only if
PAL	public announcement logic
RT	Ross' Theorem
wff	well formed formula

Declaration of Academic Achievement

I, Bradley Shubert, declare this thesis to be my own work. I am the sole author of this document. No part of this work has been published or submitted for publication or for a higher degree at another institution.

To the best of my knowledge, the content of this document does not infringe on anyone's copyright.

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Chapter One

Introduction

1.1 Free Choice ‘or’

Imagine you find yourself in a restaurant ordering breakfast. Your favored meal presents you with an apparent choice: your order comes with coffee or tea. The ‘or’ in such a context could be the truth-functional inclusive ‘or’ but convention and experience tells us that it usually isn’t. We believe that we can have one or the other of these options but not both. While greed or gluttony might lead us to be bothered that the ‘or’ in such a situation isn’t of the inclusive variety, this alone should not worry us from any deeper philosophical perspective. We have a perfectly adequate truth-functional exclusive ‘or’ that seems to capture the meaning logically intended.

Table 1.1.1 Exclusive ‘or’ truth-table

Coffee	Tea	Coffee $\veebar$ Tea
T	T	F
T	F	T
F	T	T
F	F	F

So, accepting the ‘or’ as exclusive, imagine that you place your order; for example, you specify the meal that you’d like and that you’d like coffee as the beverage that comes with it. Suppose now the server tells you that this is impossible, and that the meal comes with tea. You might think in this situation that you have been misled but given any strictly truth-functional account of ‘or’ the menu is perfectly accurate. It is of no consequence that the restaurant might not even have coffee or have ever had any coffee. All that matters is that the meal in this case does, in fact, come with tea, and so it is therefore true that it comes with coffee or tea. And, had the menu stated that the meal came with 'coffee or tea or a Ferrari' it would still be accurate.

This is not to suggest that we shouldn't find a situation like this puzzling or even frustrating. One way of getting to the heart of the problem is to imagine that the choice was explicit insofar as the menu actually stated something like '*your choice of* coffee or tea'. Even when not explicit, however, the problem is that we typically understand such situations as suggestive of choice – an option in which we decide which beverage will come with the order. In other words, it will not suffice to evaluate the situation with a simple truth-functional connective (exclusive 'or') as stipulated by the above truth-table.

In any case, we may realize that 'coffee or tea' is not really what the menu is expressing anyway. For ease of exposition I have chosen to introduce the problem in this way but on its own 'Coffee or Tea' does not really contain the content of the menu. Even if we restrict ourselves to a propositional logic, 'Coffee' and 'Tea' are not propositions. Presumably what is meant by the choice on the menu is something like:

> *You may have* coffee or tea

The introduction of 'may' in the above statement is interesting insofar as we have constructed a permission statement which we can understand modally. One common interpretation of modal operators is a deontic reading where we are able to address statements of permissibility and obligation. For the moment, we needn't worry about the fine details of deontic logic, except to note that in such a logic we can qualify propositional formulae φ with the deontic modalities. These have typically been represented as follows:

$\mathcal{P}\varphi$	It is permissible that φ
$\mathcal{O}\varphi$	It is obligatory that φ

As I will be discussing a wide variety of modal operators in this thesis, I will break with convention and, for sake of consistency, adopt an alternative notation. The most basic (alethic) modalities I will denote in the usual manner:

$\Diamond\varphi$	possibly φ
$\Box\varphi$	necessarily φ

but all others will be denoted uniformly by placing characterizing letters inside square or angle braces. Our deontic modalities are therefore:

$\langle p\rangle\varphi$	It is permissible that φ
$[o]\varphi$	It is obligatory that φ[1]

So, by treating 'may' as a modal qualifier we can translate our coffee or tea permission claim as:

$$\langle p\rangle(C \vee T)$$

What was captured by the exclusivity of the 'or' in the propositional case will now require a further clause[2]:

$$\langle p\rangle(C \vee T) \mathbin{\&} \neg\langle p\rangle(C \mathbin{\&} T)$$

And, by ordinary distribution of modal operators[3], this is equivalent to:

$$(\langle p\rangle C \vee \langle p\rangle T) \mathbin{\&} \neg\langle p\rangle(C \mathbin{\&} T)$$

---

[1] Notice, we could have simply used [p] here, or perhaps for sake of uniformity $[d]\varphi$, $\langle d\rangle\varphi$ (deontic). These are merely stylistic concerns and for ease of exposition I will maintain the use of $\langle p\rangle\varphi$ & $[o]\varphi$ for permissibility and obligation respectively.

[2] In the deontic case, $\langle p\rangle(C \veebar T)$ alone does not rule out $\langle p\rangle(C \mathbin{\&} T)$ since there could be a permissible world where $(C \veebar T)$ but another permissible world where $(C \mathbin{\&} T)$.

[3] Unless otherwise indicated, this thesis will assume that modal systems described are *normal* modal logics. These logics are characterized by the following axiom:

K) $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$

This axiom is sometimes known as a *distribution* axiom and the distribution of modal operators given follows from this axiom. Chapter one will contain further discussion of the K axiom, normal modal logics, as well as a proof of the distribution given here.

Translating back to English gives us the disjunction:

You may have coffee or you may have tea (but you can't have both)

Notice, we have not solved our problem. Just as before, we are left with a disjunction and, as such, the claim is made true just in case at least one of the disjuncts is true. Given no other premises, we cannot know more than this, and hence, we are not warranted in inferring a single disjunct.

And yet, when we see the menu statement:

You may have coffee or tea

we feel licensed to make the inferences:

You may have coffee

You may have tea

Moreover, experience seems to support these inferences as we typically find we receive what we want when we make either order in practice. That we feel warranted in making these inferences while the logic does not support them is a problem known as the paradox of free choice permission or alternatively (and perhaps more generally) the paradox of free choice disjunction.

In scenarios like this one, something further than the truth-functional conditions of 'or' satisfaction is required in order to capture the free choice available to us and apparently meant by the menu expression. The choice expression presented is not simply a claim that the above exclusive-or truth table is the right one but also that you (or the agent in question) get to decide which line on the truth table will make the 'You may have coffee or tea' claim true.

In the following thesis, I hope to explore this problem and the sporadic but nevertheless considerable literature that has followed since its initial recognition. The problem, as I will show, is one which has attracted attention from a variety of thinkers at various times in the past half century, but which has never really generated mainstream or sustained interest among logicians and philosophers. As such, there remains valuable work to be done in resolving this issue and, as we will see, the resolution of this paradox brings with it useful insights about the connection between natural language and formal logic.

This project is important for a number of reasons. First, we may draw together work being done in different areas of research where there (as yet) seems to be no clear connection in the literature. Specifically, work being done with relatively new tools of modal logic has a direct and meaningful contribution to make in discussions of free choice permissibility. In addition, I hope to show that the main reason for the insufficiency of past attempts to resolve the problem is the insufficiency of the logical tools employed. Relatively recent advances made in philosophy and logic have not yet been fully brought to bear on the paradox and we will begin to do so here. For example, there have been significant advances made in dynamic modal logics in just the past decade and these will prove illuminating in the following discussion.

Though these tools will prove directly useful in providing some resolution to the paradox, they will also indirectly give us considerable insight into the role logic plays in these kinds of problems. As I hope to argue, the relationship between logic and language is not as straightforward as often supposed and many logical paradoxes (like the general problem of free choice disjunction) are the result of a simplistic insistence on making our logic and language closely match one another.

1.2 Organization

This thesis contains seven additional chapters which are structured in the following way:

Chapter 2

This chapter will begin a survey of past work on the issue, which has typically been confined to research on deontic logic. The problem of free choice permission first appears in Georg Henrik von Wright's *Essay in Deontic Logic and the General Theory of Action* (1968), though even von Wright's concerns have their origin in prior examinations of deontic logic. Alf Ross' 'Imperatives in Logic' (1941) outlines what is essentially a differently formulated version of the same problem that employs obligation rather than permission claims (Ross' Paradox). These two foundational references have shaped most subsequent literature on the problem and will serve as the starting point for further discussion.

Von Wright's essay, in particular, provides much of the foundation for all subsequent work in deontic logic and it is notable that the first theoretical articulation of the paradox of free choice permission appears there. In light of this, we will briefly explore von Wright's system of deontic logic as well as some of the general principles grounding deontic reasoning. Throughout these discussions, I will attempt to provide some new insights into the paradox especially as it relates to logic more generally. Though the deontic formulation is notable it will be my position throughout this thesis that the paradox of free choice permission is not strictly confined to deontic logic. This can be shown using propositional formulations before moving on to more specific examinations in the context of past work on the problem. Finally, I will attempt to characterize the problem as it relates to the inferences involved.

Chapter 3

Here I will provide a framework for subsequent discussion in this thesis concerning modal logic and the most accepted interpretation of modal logic – Kripke semantics. There is a considerable impetus to consider free choice permission in the context of modal logics – a fact which is unsurprising given the deontic flavor of the problem as presented by von Wright. As we will see, however, the approach also has considerable and unexpected fruitfulness in dealing with the issue due to usefulness of modal logic in dealing with dynamic situations (like those involving choice and outcomes). In particular, this chapter will provide an opportunity to reflect on and understand the formulation of free choice permission most accepted in the literature.

This formulation of free choice will follow from model-theoretic considerations of the usual situations and circumstances involved in choice statements and these will be essentially modal. Consider again the specific case presented at the beginning of this thesis:

You may have coffee or tea

When understood as free choice, such a claim concerns an agent's ability to choose between outcomes which can be understood as possible worlds (one in which the agent has coffee and one in which the agent has tea). In this sense, we can model any free choice disjunction using a Kripke semantics in which, from some present world α, there are accessible worlds β, γ and so on in which each of the choice disjuncts is true. As such, the *choice* in any free choice disjunction is represented as a *conjunction* of possibilities where the accessibility relation is understood as a choice relation. Put another way, I will argue that the translation of the above claim as:

$$\langle p\rangle(C \vee T) \ \& \ \neg\langle p\rangle(C \ \& \ T)$$

does not accord with our usual natural language understanding and that we can do better by treating such claims as conjunctions of choice possibilities:

$$(\langle p\rangle C \ \& \ \langle p\rangle T) \ \& \ \neg\langle p\rangle(C \ \& \ T)$$

As I hope to show, this preserves the usual inferences we feel licensed to make and actually accords well with our intuitions about choice situations. Choice claims, after all, cannot merely be disjunctive, but must involve an agent's ability to choose amongst outcomes and hence, each of these possible outcomes must be a live option.

This section will contain a discussion of Kripke semantics for modal logic (Kripke 1963) as well as detailed exposition of the way in which Kripke models can be used to represent choice scenarios. Using Kripke models to represent choice scenarios I will provide precise translations of some common free choice permission claims as well as related problematic claims (e.g.: Ross' Paradox). This approach will be illuminating with respect to the way that natural language free choice claims are strictly ambiguous, and this ambiguity precisely corresponds to differences between modal structures (and hence, between logical expressions). As a result, not only will this approach resolve the paradox on a formal level, but the way in which it does so can help to clear up ambiguities in natural language.

In the course of this exposition it will be useful to explore the manner in which accessibility operators may map onto various kinds of relations. As the most commonly understood modal logics treat accessibility in a very specific way (as *alethic* modalities involving necessity and possibility), it may seem strange to treat accessibility as a *choice* relation. As I will show, this is really no more curious than utilizing choice relations in standard predicate logic (e.g. using the formula aCv to represent a claim like 'Anne chose vanilla'). While such technical results are well-understood (see, e.g., van Benthem 2010), these approaches nevertheless remain somewhat obscure and discussing them here will help both to illustrate my view and to defuse worries about the particular accessibility relation I propose.

As we will see, however, problems remain. A simple translation solution is not a popular option (and indeed not often thought of as a solution at all). More often, thinkers seek to show how the usual free choice permission claim entails that translation. Exploring these problems will be the task of the next three chapters

Chapter 4

In this chapter we will examine a few of the principal purely semantic attempts at providing a resolution to the problem. Many of the initial solutions to the problem of free choice permission were strictly logical insofar as they treated the problem as one in which modifications were required of the formal systems involved. We will explore the main options so far offered in the literature. This discussion will begin with von Wright's system of dyadic deontic logic (1968) though as we will see, what is most interesting about this system is its employment of a notion of *strong* permissibility. This special flavor of permissibility differs from ordinary or *weak* notions of permissibility by stating that all accessible states which make a statement true are permissible states. Compare:

$\langle p \rangle \varphi$	φ is true at some accessible world	(weak permission)
$\langle p \rangle_S \varphi$	All worlds where φ are permissible	(strong permission)
$[o]\varphi$	φ is true at all accessible worlds	(obligation)

In connection with von Wright's attempts to solve the problem by appealing to notions of *weak* and *strong* permissibility I will look at the attempts of Asher and Bonevac (2005) to provide similar semantic solutions. The addition of strong permissibility is not a significant departure from standard modal logic and does resolve the problem. Consider:

$\langle p \rangle_S (C \vee T)$

When understood as a strong permission claim, this means that *all* outcomes making the disjunction true are permissible. Arguably, this will include states where C holds as well as states where T holds. One major problem with this approach is that strong permissions additionally seem to include problematic states we would typically exclude as permissible. After all, for strong permission to work as a solution to the paradox of free choice disjunction, it requires making permissible *all* the possible states where the disjunction is true. This will include not only states that make coffee permissible and states that make tea permissible but also every other state in which one of these holds. So, for example:

It is strongly permissible to have coffee or tea

So, it is permissible to have coffee and pour it on the waiter's head

Asher and Bonevac deal with this problem with additional modifications to the logic involved but in doing so we find ourselves increasingly departing from standard modal logic. This is a theme we will continue to pursue – we are able to modify our logic in a number of ways and this will allow us to express complex ideas but in doing so, we may also increasingly lose much of the simplicity that makes semantic solutions appealing.

Chapter 5

In this chapter I will look at the most common modern strategy for dealing with free choice permission -- to focus on natural language pragmatics (e.g.: Kamp 1973, 1978; Kratzer & Shimayama 2002). Here I will consider some of these approaches as well as the general distinction between semantics and pragmatics and how consideration of each of these may factor into dealing with problems like free choice permission. Typically, while semantics is understood as being about the relationship between language and its objects, pragmatics is understood as being about the relationship between language and its users. So, in this way, free choice permission problems may be understood through subtle conversational contexts that relativize allowable inferences to certain kinds of users given their circumstances.

This discussion will begin with Kamp (1973), whose views serve as the foundational pragmatic approach to dealing with the problem. Kamp's initial account is not widely accepted today, though his turn to pragmatics marks a significant departure from previous attempts to deal with the problem. As we will see, this shift in thinking coincides with other developments in the philosophy of language (e.g. Grice 1975) and this shift will come to dominate discussions of free choice permission.

The view which has ultimately taken hold follows from Gricean ideas of conversational implicature (Grice 1975) in which free choice disjunctions are given and received according to pragmatic maxims of conversation in which we can infer or eliminate logical options. This Gricean approach has been taken up and explicated precisely by Kratzer & Shimayama (2002) and we will examine their position in some detail.

Chapter 6

Here I will come to some original approaches to dealing with free choice permission and disjunction claims. To do this, I will explore the growing literature on modern modal tools and logical dynamics (e.g.: van Benthem 2010, 2014) wherein we see a surprising degree of precedent in the use of modal logic to capture choice and choice outcomes. I will show that the account given by Asher & Bonevac (2005) bears deep similarities to the pragmatic view initially given by Kamp. This similarity is curious insofar as the former is a purportedly semantic account while the latter is pragmatic. I argue that by adding logical tools we essentially move aspects of expressions from the realm of the pragmatic into the realm of semantics.

Making this argument will require some technical examples in order to show that much of what are taken to be pragmatic components of meaning may be expressed semantically with additional modifications to the logical systems involved and that these modifications are already well-established in the broader logical dynamics program.

This argument will culminate with a sketch of how the tools of logical dynamics may be used to give a formal analysis of even Gricean conversational implicature. To do this I will consider the account of Kratzer & Shimoyama (2002) outlined in the previous chapter and provide a parallel inference stream using logical dynamics. I will show how the process of Gricean updating can be understood through the process of dynamic model transformation and will additionally attempt to provide some preliminary logical formulations of the Gricean maxims involved.

It is not my goal here to supply a complete logical theory of Gricean conversational implicature. Rather, this discussion is meant to show that such a theory may be possible and that in a general sense, it may be possible to subsume many so-called *pragmatic* features of language within our semantics if we are willing to complicate the systems involved.

Chapter 7

In this final substantive chapter, I will return to the idea that the paradox of free choice permission may be resolved more simply by translating it into the logic as a conjunction in the first place. It is true that we can tell a complicated story about how the 'or' present in free choice claims leads us to a conjunction of possibilities, but it is less clear that this story needs to be or should be expressible within the logical language. Much of what is valuable about formal logic is its complete precision and utter lack of ambiguity. I argue that natural language free choice claims are ambiguous and that we should expect logical formulations to disambiguate. In doing so, we have no right to expect these logical formulations to 'match-up' with the natural language expressions.

This discussion will involve looking at the role of formal languages in representing the meaning of natural language expressions as well as possible ways of understanding formal logic. Rather than seeing the formal language as an abbreviation of natural language or thought processes, I argue instead that the formal language is best understood as a pure idealized abstraction which needn't mirror natural language to any large degree. In this sense, the job of the formal language is to capture meaning and not particular features of language. While I have given an account of how we can include complex pragmatics within our formal systems, I suggest here that doing so may not be worth the effort. Rather, I will argue that pragmatic accounts provide guidance as to how translations into a formal language take place, as well as the structure of logical models from which our formal expressions obtain their meaning.

As I will show, my approach has some precedent in our translations of ordinary-language expressions to a formal language. After all, many ordinary-language disjunctions are commonly understood to be formal conjunctions. For example:

> Anne or Bill can solve the problem

Similarly, many other English connectives like 'and' or 'therefore' have non-standard translations into formal language. It is an interesting question how exactly this happens with regularity and I think it is plausible that much of the pragmatic free choice analysis involving considerations of conversational implicature may be important in understanding these phenomena. In this regard, the subject of free choice disjunction will serve as an important case for discussions about the role of pragmatics and semantics in broader discourse. As such, I will have opportunity to reflect on the role of semantics and pragmatics more generally and will share what insights I can relevant to the broader philosophical literature.

Chapter 8

This chapter will provide a small summary and final reflection on the issues here discussed as well as offer suggestions for future study.

Chapter Two

Paradoxes of Permission and Obligation

2.1 Preliminaries

Surprisingly, there is no mention of the paradox of free choice disjunction prior to the middle of the twentieth century.[4] The primary reason for this absence seems to be the manner in which this problem was situated in the study of deontic logic as it developed in the past century. Still, one can easily imagine the problem being noticed in purely propositional terms. Ignoring exclusivity for the moment, consider the free choice claim:

Your meal comes with either coffee or tea

As was outlined in the previous chapter, one way of understanding such a claim is to treat the overall expression as a disjunction of options. For example:

Your meal comes with coffee or your meal comes with tea

This could be expressed propositionally as:

$C \vee T$

[4] Conducting a historical search would be a worthwhile project for future study given (in my opinion) the likelihood that this problem or some problem similar to it has been noticed in antiquity. However, to the best of this author's knowledge, there seems to be no such case.

Indeed, my own exposure to this problem began in exactly this way when a friend studying introductory symbolic logic and knowing only the basic truth-tables for propositional logic noticed the exact difficulty described in my introductory ‘coffee or tea’ example while considering a breakfast menu.

If worries remain about how a sentence like 'Your meal comes with coffee or your meal comes with tea' is meant to represent free choice (it does look a lot like an ordinary disjunction), we can even include modal qualification within our atomic propositions. For example:

> You may have coffee, or you may have tea

or even:

> You may choose coffee, or you may choose tea

And, ignoring the complexity of the propositions involved, these can still be represented by:

> $C \vee T$

This alone seems sufficient to generate the paradox. In fact, there is no particular need for even the symbolism of formal propositional logic. The natural-language sentences above (e.g. you may have coffee, or you may have tea) could have inspired the same puzzlement about free choice disjunction with nothing more than a vague sense of the truth-functional character of natural-language disjunction.

One possible explanation as to why the natural-language or strictly propositional cases have not gained attention can be found in the subtle difference in the way the disjunction is worded when compared to the deontic case. As a disjunction of permission statements, the best we can do propositionally is the statement:

> You may have coffee, or you may have tea

That is, each disjunct must individually contain the permission clause ‘you may have’ and this could resolve confusion about the truth-functional disjunctive character of such a claim. Hence, we may be less tempted to see such a claim as adequately expressing choice as opposed to an ordinary disjunction where at least one of the disjuncts is true.

By utilizing a deontic logic with permissibility operators, again ignoring exclusivity, we can express the slightly different English statement:

You may have coffee or tea

by the formalization

$\langle p \rangle (C \vee T)$

And it is possible this formulation is more likely to be understood as a free choice permissibility claim than one with two 'you may' clauses (one present in each disjunct). The fact that the deontic formulation contains a disjunction within the scope of a single permissibility operator may be the impetus for believing we have freedom to choose between these options. In the deontic case, our permission claim appears more like a single permission statement regarding options (coffee or tea).

This reading, with a single modal permission operator (may) ranging over a disjunction is known as a *narrow-scope* disjunction.

Narrow-scope disjunction: May $(\varphi \vee \psi)$

On the other hand, if we take the meaning of this expression as 'you may have coffee or you may have tea', then we call this a *wide-scope* disjunction[5].

Wide-scope disjunction: May $\varphi \vee$ May ψ

Nevertheless, I believe some (if not most) readers will take even a wide scope-disjunction like 'You may have coffee or you may have tea' as expressive of

[5] For a detailed discussion of wide vs narrow scope disjunction see e.g. (Meyer 2016, 4-5, 10-14). As noted in that work, there is varied treatment of the paradox as either fundamentally a wide-scope or narrow-scope disjunction. Most thinkers treat the paradox as a narrow-scope disjunction but there are exceptions (e.g. Zimmerman 2000, Guerts 2005). Though the two are formally equivalent, the difference may matter in a number of ways. A few considerations are how modals might interact with disjunction in these different formulations (see e.g. Simons 2005; Aloni 2003, 2007) as well as the pragmatic effect of these different formulations in natural language – as I mention here.

choice and, in any case, this is supported by the deontic reading. By ordinary distribution of modal operators, the deontic formula ⟨p⟩(C ∨ T) gives us[6]:

⟨p⟩C ∨ ⟨p⟩T

And this translates back into English as our claim:

You may have coffee, or you may have tea

which, as we observed, could be understood in purely propositional terms – so it seems that the two are formally equivalent and there is a case to be made that the paradox is not strictly deontic in character. There is still a deontic flavor to the propositional formulation of the problem given that we are making use of propositions about permissions, but the point remains that past logicians would have needed none of the particular tools of deontic logic in order to recognize the paradox.

In any case, the problem has generated no sustained or noticeable attention prior to its deontic formulation. This attention began not with considerations of permissibility at all, but rather, with a related problem of translation concerning imperative statements and the corresponding obligations they express.

2.2 Ross' Paradox

In his (1941) 'Imperatives and Logic', Alf Ross presents a critique against early developments in deontic logic and, in particular, the way in which a logical operator for obligation fails to capture intuitions about obligation claims. At the time of his writing, deontic logic was not yet developed into its modern formulation but, even so, many of the criticisms presented by Ross can be translated with no loss of philosophical importance into modern systems.

[6] For the statement to be true, there must be some accessible world in which C is true or T is true. If the former is the case, we have ⟨p⟩C and so also, by addition, ⟨p⟩C ∨ ⟨p⟩T. If the latter is the case, we have ⟨p⟩T and so also ⟨p⟩C ∨ ⟨p⟩T.

Indeed, just as the free choice disjunction may be understood without symbolism at all, so too can Ross' objections concerning obligation claims.

In a purely natural language formulation, Ross' paradox proceeds from an ordinary imperative claim such as:

> Slip the letter into the letter-box!

As an imperative claim, we understand that the satisfaction of such a claim comes when one does, in fact, slip the letter into the letter-box. The early deontic logic which Ross considers operates in just this way and evaluates imperatives based on a schema of satisfaction conditions (Ross 40).

This is the Dubislav-Jorgensen transformation method which proceeds by taking an imperative claim and from it, deriving a satisfaction claim. One can then make ordinary logical inferences before reverting back to an imperative claim. Ross presents this in the following way:

Figure 2.2.1 Imperative transformation

$$\begin{tikzcd} I(x) \arrow[r] & S(x) \arrow[d] \\ I(y) & S(y) \arrow[l] \end{tikzcd}$$

We can be more explicit about this reasoning. For example, if $(x \rightarrow y)$ then we can reason as follows:

Figure 2.2.2 Imperative reasoning

$$\begin{tikzcd} I(x) \arrow[r] & S(x) \arrow[d] \\ & S(x \rightarrow y) \arrow[d] \\ I(y) & S(y) \arrow[l] \end{tikzcd}$$

For an unproblematic example, we might take our earlier imperative to slip the letter into the letter-box and imagine that we need to leave the house to do so. In such a case the imperative:

> Slip the letter into the letter-box

is satisfied when:

> The letter is slipped into the letter-box

from which we can reason as follows:

> If the letter is slipped into the letter-box then I have left the house
>
> But (if satisfied) the letter has been slipped into the letter-box
>
> So (if satisfied) I have left the house

And, from this satisfaction conclusion we reason back to the imperative claim:

> Leave the house!

In this case, the transformation method employed by Ross seems to work. But, as we shall see, when we introduce a disjunction we run into difficulties. Consider again the imperative:

> Slip the letter into the letter-box!

And its satisfaction claim:

> The letter has been slipped into the letter box

We can also reason forward by adding disjuncts such as:

> The letter has been slipped into the letter box or the letter has been burned

from which we transform back to the imperative claim:

> Slip the letter into the letter-box or burn it!

In the Dubislav-Jorgensen Schema we can represent this as:

Figure 2.2.3 Imperative disjunction

$$\begin{tikzcd} I(x) \arrow[r] & S(x) \arrow[d] \\ I(x \vee y) & S(x \vee y) \arrow[l] \end{tikzcd}$$

This resulting imperative claim is clearly an absurd logical conclusion from its starting point and for our purposes here serves as an especially interesting claim insofar as logicians and philosophers have been puzzled by the paradoxes of permissibility and obligation. As the conclusion is phrased in natural language:

> Slip the letter into the letter box or burn it!

There is little room for defending such a claim. As a direct imperative it is clearly an absurd conclusion given our starting point and difficult to interpret as anything other than an imperative allowing either option as a satisfaction condition.

Compare this, however, with the claim:

> It is obligatory to slip the letter into the letter-box or to burn it

While troubling to many, the above claim can at least be argued for on the grounds that the obligation does not range over the disjunction. That is, one of the disjuncts will satisfy the obligation, though we may not know which.

We can, for example, view an obligation disjunction in one of two ways:

1) It is obligatory that (A or B)

2) It is obligatory that A *or* It is obligatory that B

In the first case, the disjunction (A or B) is made true just in case at least one of the disjuncts is true, and we can consider the obligation itself as satisfied so long as the disjunction is made true. In the second case, one of the disjuncts being true satisfies the obligation though we may not know which.

As a second attempt to treat the problem, this is exactly what Ross considers, wherein he employs a validity approach to the transformation schema. He proceeds as follows:

Figure 2.2.4 Alternative imperative transformation:

$$\begin{tikzcd} I(x) \arrow[r] & S(x) \arrow[d] \\ I(x) \vee I(y) & S(x \vee y) \arrow[l] \end{tikzcd}$$

In this case the resulting imperative conclusion must now be worded differently. Minimally, we could emphasize the 'or' in order to make the disjunction clear:

Slip the letter into the letter box! *or* Burn it!

This, however, does not seem to be enough to distinguish this conclusion from our earlier troubling result. What is needed is a way of showing a disjunction of imperatives rather than a disjunction within the scope of an imperative. Consider Ross' own formulation (Ross 41):

Either the letter is to be slipped into the letter-box, or it is to be burnt

Even this may be subject to worry (perhaps because we let the subject 'the letter' occur only once at the beginning of the phrase). As Ross treats imperatives and employs the I(x) formulation in the schema above, I suggest we instead simply treat the natural language claims as a disjunction of imperatives exactly as indicated. That is:

It is imperative that the letter be slipped into the letter-box *or*

It is imperative that the letter be burned

I consider the above expression entirely unproblematic. My intuition is that others would find it equally unproblematic. Compare, by contrast, what seems to be an explicitly imperative formulation of our earlier problematic reasoning:

It is imperative that the letter be slipped into the letter-box. Hence,

It is imperative that the letter be slipped into the letter box or burned.

As we shall see, this is a recurring theme in this debate – the natural language formulation of these logical ideas makes a great deal of difference.

I have, of course, done very little justice to either the Dubislav-Jorgensen transformation method or Ross' comments and observations regarding imperative statements generally. In the interests of space I do not intend to do so. Rather than dwell unnecessarily on the early first steps of the logic of imperatives, I will instead turn my attention to more modern formulations and approaches. As we will see, Ross' paradox will find renewed vigour in modern formulations of deontic logic. This begins with von Wright's revival of the paradox as well as its corresponding paradoxes of permissibility.

2.3 Deontic Logic

In the introduction to this topic, I have loosely spoken of deontic logic and its characteristic notions of permissibility and obligation. Recall, as a formal logic of normative or even imperative statements, these notions become logical operators within a formal language. Georg von Wright (1951) was the first to treat these modally and symbolized them as:

$\mathcal{P}\varphi$	It is permissible that φ
$\mathcal{O}\varphi$	It is obligatory that φ

The system of deontic logic outlined by von Wright (1951, 1968) is a milestone in the development of these innovations and is essentially the very same system employed by logicians at the time of this writing[7]. While it has served my purposes to this point to treat this system without much elaboration, we should at this stage make explicit the formal calculus of deontic logic.

In von Wright's initial formulation, the formal calculus *D* is a propositional modal logic consisting of the following elements.

First, we have a formal language which includes:

i) A set of variables {A, B, C...}[8] which represent the basic propositions of the language[9].

ii) A set of truth functional operators $\{\neg, \vee, \&, \rightarrow, \leftrightarrow\}$. Von Wright includes the five listed here, though we do not strictly need this exact list as we could make use of fewer by introducing the rest by definition (e.g. $R \rightarrow Q$ may be replaced by $\neg R \vee Q$).

iii) The deontic operator $\mathcal{P}$, which we commonly read as a permissibility operator.

iv) The deontic operator $\mathcal{O}$, commonly read as an operator for obligation. This is not strictly necessary, as the two deontic operators are interdefinable as follows:

$$\mathcal{P}A \leftrightarrow \neg\mathcal{O}\neg A$$

$$\mathcal{O}A \leftrightarrow \neg\mathcal{P}\neg A$$

---

[7] Some significant changes have been explored within deontic logic -- particularly with the inclusion of preference orderings. Von Wright was himself aware of such possibilities and was a major innovator in this respect. Still, as the most elegant and simple approach to the subject, the system here described remains the framework for our contemporary thinking.

[8] I will throughout this thesis omit the use of quotation marks or other identifying notation when using symbols or strings of symbols to refer to themselves, at least insofar as context makes clear that symbols are being used in this way.

[9] Von Wright initially (1951) treated these variables as representative of *act statements* though he later (1968) adopted the use of propositions or 'proposition-like' entities (1968, 16). Making this shift may be as simple as adding 'it is the case that...' to any ordinary action expression (e.g. 'he goes to the cinema'). Some part of the motivation for this is to allow for nested deontic expressions -- e.g. $\mathcal{P}$ ($\mathcal{P}$ (A)). Von Wright initially represented these propositions with the lowercase alphabet, though more modern treatments use uppercase characters (in keeping with ordinary proposition use in formal languages).

We further supplement this with brackets and a symbol standing for some arbitrary tautology of propositional logic. Von Wright employs the propositional letter T in this capacity, though I will hereafter employ the more modern convention $\top$ (as well as $\bot$ for an arbitrary contradiction). This is consistent with contemporary use and helps to avoid confusion with our ordinary propositional variables.

As earlier mentioned, I will additionally modify even contemporary usage and designate the deontic operators as ⟨p⟩, [o] respectively. I believe there is good reason to take similar steps across all variations of modal systems in order to bring a uniformity of symbolism with the alethic modal operators $\Diamond$, $\Box$. Thus $\Box$, $\Diamond$ function as our most basic modal operators, perhaps undefined or uninterpreted (or at minimum, interpreted as unrestricted possibility & necessity), while all other modal operators simply add specification which we designate by 'breaking' the symbol and inserting whatever additional identifying symbolism we desire (e.g. ⟨p⟩, [o], etc.).

Formulae are formed in accordance with the generative grammar[10]:

$$\varphi ::= \varphi_a \mid \neg\varphi \mid (\varphi \,\&\, \psi) \mid (\varphi \vee \psi) \mid (\varphi \rightarrow \psi) \mid (\varphi \leftrightarrow \psi) \mid \langle p \rangle\varphi \mid [o]\varphi$$

Thus, any atomic proposition is a formula.

The negation of any formula is a formula.

If φ and ψ are formulas, then so are $(\varphi \,\&\, \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$ and $(\varphi \leftrightarrow \psi)$

Placing a formula within the scope of a deontic operator is a formula.

Von Wright adopted a schema of operator strength in order to allow for minimal use of brackets. I will hereafter adopt what I feel is a clearer approach and use brackets in all cases specified by the above formation rules except insofar as I will allow for the convention of dropping pairs of outermost brackets.

Von Wright constructed his deontic calculus axiomatically as follows:

[10] See appendix A if needed for further explanation of these constructions.

A_D1 The ordinary tautologies of propositional logic including those which can be constructed using well-formed formulas of our deontic formal language -- e.g. $\langle p\rangle A \vee \neg\langle p\rangle A$.

A_D2 $\langle p\rangle(\varphi \vee \psi) \leftrightarrow (\langle p\rangle \varphi \vee \langle p\rangle\psi)$

A_D3 $\langle p\rangle\varphi \vee \langle p\rangle\neg\varphi$

Von Wright's rules of inference for his system of deontic logic are:

R1 Substitution of formulae of propositional logic for variables

R2 Detachment (*modus ponens*)

R3 A rule of extensionality which allows us to substitute logically equivalent formulae of propositional logic within well-formed formulae of our deontic calculus. This includes our definition:

$\langle p\rangle\varphi \leftrightarrow \neg[o]\neg\varphi$

Though axiom 2 is used in von Wright's (1968) system, later treatments of deontic logic replace this with a deontic version of the K axiom:

K) $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$

The K axiom is definitive of a large class of modal logics which we designate 'normal modal logics'. This axiom is basic to the normal interpretation of modal logics over modal structures (which we will examine in detail in the following chapter). Converting this ordinary alethic operator to its deontic counterpart we can call the resulting axiom K_O. This axiom is employed in the place of von Wright's Axiom 2 in most standard formulations of deontic logic today.

K_O) $[o](\varphi \rightarrow \psi) \rightarrow ([o]\varphi \rightarrow [o]\psi)$

Additionally, von Wright's axiom 3 can be reformulated using the interdefinability of the modal operators in the following way:

$\neg\langle p\rangle\varphi \rightarrow \langle p\rangle\neg\varphi$

$[o]\neg\varphi \rightarrow \langle p\rangle\neg\varphi$

Removing the negations from φ (since it is uninterpreted) gives us the axiom essential to deontic logic:

D) $[o]\varphi \rightarrow \langle p \rangle \varphi$

This 'D axiom' (for deontic) characterizes deontic logic specifically and modern elementary treatments of general modal or deontic logic treat it in precisely this way. Deontic logics are that class of modal logics which satisfy the (K_o) and (D) axioms. This is, perhaps, unnecessarily suggestive terminology ('deontic' being derived from the Greek *déon* -- "that which ought to be done"). In the strictest sense, the system of logic here described doesn't need to be about permissibility and obligation. All that matters is the way in which the system functions with the axiomatization given. Nevertheless, dealing with permissibility and obligation are standard interpretations of these logics.

2.4 The Paradox of Free Choice Permission

Having made rigorous the formal system of deontic logic, von Wright considers two problematic formulas. He states:

> ...I shall draw attention to one formula which is provable in the calculus but seems to conflict with our intuitions – and to another formula which agrees with intuition but is not provable (von Wright 1968, 20).

The first of these he denotes as Ross' Paradox whereupon he takes up Ross' problematic claim that:

> Slip the letter in the letter-box!

seems to entail

> Slip the letter in the letter box or burn it!

Armed with a now well-developed formal system (deontic logic), von Wright reformulates the problem by considering a theorem he derives showing:

$[o]P \rightarrow [o](P \vee Q)$

This is the first formula von Wright finds troubling. Taking the initial imperative to mail the letter as the antecedent of this conditional, we arrive at the counter-intuitive conclusion:

$[o](P \vee Q)$

While I believe that this formulation does succeed in defusing some of the problems contained in Ross' strictly imperative claim, von Wright nevertheless finds it troubling, believing the proper interpretation of his theorem to be:

> If one ought to mail a letter, one also ought either to mail *or* to burn it (1968, 20).

Von Wright is aware that this interpretation can be understood in such a way that all that is meant is that one or the other is obligatory. He mentions the most common objection to worries about Ross' paradox in the following passage:

> If we see to it that p, and thereby fulfil the obligation expressed by "$\mathcal{O}$p", then we also, by the laws of ordinary logic, see to it that "p or q". But from this it cannot, by the laws of any logic ("ordinary" or "deontic") be concluded that it is obligatory or even permitted that it be the case that q. To say that "$\mathcal{O}$p" entails "$\mathcal{O}(p \vee q)$" is really no more paradoxical than to say that "p" entails "$p \vee q$"... This argument has seemed satisfactory to many people. I have tried myself to be pleased with it, but never quite successfully. I have always felt that there is more to Ross' paradox than can be met by the above piece of reasoning. (von Wright 1968, 21)

A major focus of this thesis will be to argue that von Wright's concerns here are unfounded and that the argument given above is roughly correct. I will make this case in chapters three and seven but for the moment it is easy to see von Wright's intuition and we do well to take it seriously.

In order to explain his deeper worries, he introduces the second of his worrisome formulas, which he later calls a *free-choice permission* (von Wright 1968, 22)[11]:

FCP) $\langle p\rangle(R \vee Q) \rightarrow (\langle p\rangle R \ \& \ \langle p\rangle Q)$

This formula cannot be proven and is not a theorem of the deontic calculus. But, von Wright suggests, the idea seems to accord with the intuition that a statement like

You may work or relax

reasonably implies that both of these are permitted options. That is, from such a statement, one normally feels licensed to perform either of these actions. Despite these intuitions, the deontic calculus does not support such an inference. Moreover, it cannot, since even if we felt inclined to add the above formula as an axiom, absurdities follow. Von Wright (1968, 21) highlights the following derivation and its strange conclusion:

1.	$[o]R \rightarrow \langle p\rangle R$	Axiom D
2.	$\langle p\rangle R \rightarrow \langle p\rangle(R \vee Q)$	Theorem
3.	$\langle p\rangle(R \vee Q) \rightarrow (\langle p\rangle R \ \& \ \langle p\rangle Q)$	FCP
4.	$(\langle p\rangle R \ \& \ \langle p\rangle Q) \rightarrow \langle p\rangle Q$	Simplification Rule
5.	$[o]R \rightarrow \langle p\rangle(R \vee Q)$	Hypothetical Syllogism 1, 2
6.	$[o]R \rightarrow (\langle p\rangle R \ \& \ \langle p\rangle Q)$	Hypothetical Syllogism 3, 5
7.	$[o]R \rightarrow \langle p\rangle Q$	Hypothetical Syllogism 4, 6

Statement Q in the derivation above is introduced by disjunction and can be anything whatsoever. So, if anything is obligatory, then everything is permissible! While devastating to deontic logic just insofar as such a conclusion would undermine all goals of reasoning about permissibility and obligation, conclusion (7) leads quickly to outright contradiction if we think that anything is obligatory. Since Q was chosen arbitrarily in our introduction of the disjunction, we could just as easily have chosen $\neg R$ giving the result:

[11] This expression is identical to the second of von Wright's worrisome formulae (von Wright 1968, 21) though I use slightly different symbolism.

7. $[o]R \rightarrow \langle p\rangle\neg R$	
8. $[o]R \leftrightarrow \neg\langle p\rangle\neg R$	by definition
9. $[o]R \rightarrow \neg\langle p\rangle\neg R$	Simplification 8
10. $[o]R$	Assumption
11. $\langle p\rangle\neg R$	Modus Ponens 7, 10
12. $\neg\langle p\rangle\neg R$	Modus Ponens 9, 10
13. $\perp$	Conjunction 11, 12

Notice, the assumption made here is specific, but the derivation could have been made with any obligation claim. Thus, if *anything* is obligatory then we can derive a contradiction. With these problems in mind, von Wright notes that one may adopt a stance whereby we simply accept that in *some* cases where a disjunction is in the scope of a permission operator:

a) $\langle p\rangle(R \vee Q)$

then it will also be the case that

b) $\langle p\rangle R \,\&\, \langle p\rangle Q$

These are logically compatible expressions (von Wright 1968, 22) and it may be that we simply find the conjunction of permissions somehow normally or usually goes along with a disjunction within the scope of a permissibility operator. But, as we have seen, we cannot regard the conjunction of permissions (b) as a logical necessity given (a). Von Wright states:

> A disjunctive permission for which it holds good that each alternative in the disjunction is permitted too I shall call a *free-choice permission*. And the difficulties connected with the formula "$\mathcal{P}(p \vee q) \rightarrow \mathcal{P}p \,\&\, \mathcal{P}q$" I shall refer to as the Paradox of Free Choice Permission. (von Wright 1968, 22)

2.5 Making Sense of Free Choice Permission

Von Wright's free choice permission definition stipulates that the following two statements are true together:

a) ⟨p⟩(R ∨ Q)

b) ⟨p⟩R & ⟨p⟩Q

Notice, however, that formula (a) is a logical consequence of formula (b) so that strictly speaking von Wright does not need in his definition of free choice permission any explicit mention of disjunctive permission at all. In the simplest form, a free choice permission may just be the expression (b) above.

Then again, there may be more involved than even von Wright has considered. After all, when faced with a free choice permission claim like von Wright's:

You may work or relax

We might actually think the two are mutually exclusive. By choosing to work, I cannot be choosing to relax, and by choosing to relax, I cannot be choosing to work. At a minimum, it seems that when given a free choice permission I should at least have the option to not do both. In the work/relax case, this may be a result of logical incompatibility, but we can imagine other, more revealing cases. For example:

You may accompany Jane or Jill

Perhaps I may also accompany both. Perhaps it is even obligatory that I do so. And yet, supposing I love Jill's company but detest Jane, it seems easy to see that as a free choice I might very much like to accompany Jill while not accompanying Jane. Insofar as I believe I have a choice at all,

⟨p⟩R & ⟨p⟩Q

does not seem to be enough. It must also be the case that:

$\neg[o](R \,\&\, Q)$[12]

Some exclusive permission choice claims seem to be normative while others are a function of logical possibility. Consider, for example, the distinction between the statements:

i) You may have coffee or tea

ii) You may catch the train to Berlin or Madrid

Statement (i) is a permission claim involving the patron of a restaurant and that agent's entitlement to coffee or tea as a meal option. There is nothing logically impossible, however, with that agent's having both coffee and tea. If we assume the situation is a true free choice situation and that the restaurant does, in fact, have both coffee and tea, then the only limitation on having both is a purely permission-based restriction. If we choose to express the disjunctive component of (i) by the formula:

$\langle p\rangle(C \vee T)$

then the exclusivity restriction is adequately expressed by the addition of the clause:

$\neg\langle p\rangle(C \,\&\, T)$ or,

$[o]\neg(C \,\&\, T)$

That is, while we say:

It is permissible to have coffee or tea

We might also say:

It is not permissible to have both

[12] Perhaps even the stronger $\neg[o](R \vee Q)$ which states one needn't make any choice at all. The menu choice seems to be such a choice though I do not consider this feature essential to *choice* generally since we can imagine forced choices which are nevertheless choices.

All well and good. But notice, the train case (ii) may not involve the same kind of restriction. A traveler's inability to catch a train to both Berlin and Madrid may have nothing at all to do with permissibility. Rather the issue may simply concern logical possibility. So, while we can again express the permissibility claim as:

$$\langle p \rangle (B \vee M)$$

The restriction may involve something like an ordinary (*alethic*) possibility modality.

$$\neg \Diamond (B \,\&\, M)$$

Though, of course, introducing this second modal operator will require moving from a strictly deontic logic to a multi-modal logic. As we shall see, this may not be the only additional modal operator we can incorporate meaningfully since free choice scenarios so often involve combinations of deontic, alethic, epistemic and other modal considerations.

These are ideas I will return to in Chapters four and six. For now, von Wright's formulation will suffice, especially insofar as it has set the stage for so much further work on the problem.

We can now make clear some of the connections between Ross' Paradox and the Paradox of Free Choice Permission. As von Wright illustrates, Ross' Paradox concerns a theorem of the deontic calculus that seems counter-intuitive under some interpretations.

RT) $[o]R \rightarrow [o](R \vee Q)$

Conversely, the paradox of free choice permission concerns a formula that seems plausible and yet cannot be a theorem:

FCP) $\langle p \rangle((R \vee Q) \rightarrow (\langle p \rangle R \,\&\, \langle p \rangle Q)$

In each case, the problem seems to be the way in which one formula implies another (or, in the case of FCP, does not):

Paradox of Free Choice Permission:

FCP seems correct, but according to the usual rules of modal/deontic logic and disjunction, is not correct.

Ross' Paradox:

RT seems incorrect but, according to the usual rules of modal/deontic logic and disjunction, is correct.

Recall, axiom D tells us that

$[o]\varphi \rightarrow \langle p \rangle \varphi$

Moreover, the permissibility operator follows the exact same syntactic rules such that, should we want to, we could construct a permissibility version of any obligation claim. This will not always yield the same kinds of conclusions but in some cases comes close. Consider the following permissibility version of Ross' theorem:

$\langle p \rangle R \rightarrow \langle p \rangle (R \vee Q)$

In this variation of Ross' Paradox, we now move from a claim like

You may mail the letter

To the claim

You may mail the letter or burn it

This still seems odd and in many of the same respects as Ross' paradox. Why should the permissibility of mailing a letter have anything to do with a permission to burn it? I can easily imagine, for example, the following stipulation:

Mail the letter if you like, but whatever you do, don't destroy it!

Formally, this could be represented by the following:

$\langle p \rangle R \; \& \; [o]\neg Q$

And, strangely, this is perfectly compatible with either the theorem capturing Ross' Paradox, or its expression in terms of permission statements.

$\langle p \rangle R \rightarrow \langle p \rangle (R \vee Q)$

Now, again, consider our paradox of free choice permission and the problematic formula that *seems* as though it should be correct.

FCP) $\langle p \rangle (R \vee Q) \rightarrow (\langle p \rangle R \,\&\, \langle p \rangle Q)$

The point of this formula is simply that when faced with the free choice disjunction, it seems intuitive that we be warranted in inferring the permissibility of either option. Another way of formulating this intuition is with the more simple formula:

$\langle p \rangle (R \vee Q) \rightarrow \langle p \rangle R$

Thus, we can recognize the overall problem in the following way:

$\langle p \rangle (R \vee Q) \rightarrow \langle p \rangle R$ Seems correct but leads to absurdities

$\langle p \rangle R \rightarrow \langle p \rangle (R \vee Q)$ Seems absurd but is correct

Understood in this way, the general problem is simply that the conditional is intuitively *backwards*.

2.6 Choice Effects and Other Formulations

The general phenomenon whereby we seem licensed to make an inference from a disjunction to a conjunction has been called a *free choice effect* –or just a *choice effect* (see e.g. Nickel 2010, Meyer 2016). This effect is so named for its appearance in situations that involve a choice (explicit or implicit). This choice may be, as in the case of coffee or tea, a concrete choice about actions or outcomes though the 'choices' involved in choice effects may be simply a choice about options concerning available inferences.

I have remarked that we can observe free choice effects in perhaps even a purely propositional disjunction of choice statements but the first point of recognition and analysis for free choice effects appears to have been the deontic possibility operator ('may') and von Wright's early reflections on the problem. As we will see, however, there are many other more complex or varying modal formulations we can consider as well, and there seems to be some correlation between kinds of modal formulations and kinds of choice effects.

Universal modal operators (e.g. obligation, necessity) sometimes suggest a free choice effect, as in the following example from Marie-Christine Meyer (2016, 6, Simons 2005):

> You must write a paper or give a presentation

from which the following conclusions appear legitimate:

> You may write a paper

> You may give a presentation

Presumably, also, the modal necessity indicates that at least one of these *must* be done. As in the case of ordinary free choice permission, the obligation claim could strictly be made true in the case where there is no possibility regarding one of these options (e.g. you must write a paper and have no real option to give a presentation).

Beyond instances of choice effects concerning permission and obligation, we can observe choice effects in a wide variety of other situations as well. One of the most interesting of these concerns the use of disjunctions in *generic* statements. In a generic statement, for example:

> Dogs bark

a claim is made about what is regularly the case, rather than what is universally true. We would not think this statement is invalidated by a single non-barking dog. What matters is that in some regular or typical sense, dogs bark. Now consider the following example involving a disjunction (Nickel 2010):

> Elephants live in Africa or Asia.

In this generic sentence, we can plausibly recognize a similar availability of allowable inferences which Nickel calls a 'free choice like effect' (2010, 479). If we understand this statement as saying:

Elephants (typically) live in Africa or Asia

Then we are arguably warranted in the following inferences:

Elephants (typically) live in Africa

Elephants (typically) live in Asia

Nickel describes the effect as one of *conjunctive strengthening.* Another way of understanding this (which Nickel rejects) treats the generic disjunction as moving from a universal (generic) claim to existential claims (Klinedinst 2007, Fox 2007). Nickel provides the following example (which is also indicative of a free choice effect):

All of the students are boys or girls

So, some of the students are boys

So, some of the students are girls

Thus, on this account, the choice like effect runs as follows (Nickel 2010, 483):

Elephants (typically) live in Africa or Asia

So, there are *some* elephants that live in Africa

So, there are *some* elephants that live in Asia

Nickel rejects this approach on the grounds that existential claims may be too strong in the case of generics which are momentarily uninstantiated. Consider:

Humans have brown, blue, or green eyes

Supposing that there were no green-eyed people at the moment, we might nevertheless consider the claim true, though an inference to the existence of green eyes humans would be incorrect. More importantly, there seem to be structurally identical generics for which this conjunctive strengthening is unwarranted. Nickel provides the example (2010):

Elephants live in Africa or give birth to live young

One way of preserving a universal to existential move in conjunctive strengthening may be to treat these as modal universals and modal existentials. Notice the similarity with:

You must write a paper or give a presentation.

In that case, the free choice effect took us from an obligation to the inference of permission disjuncts. That is, we went from a universal modal operator (obligation) to existential modal operators (permission). In the generic case, *typically* is functioning something like a universal modal operator[13]. Thus, with an understanding of this generic universal operator and the usual interdefinability:

$$\Box\varphi \quad \stackrel{\text{def}}{=} \quad \neg\Diamond\neg\varphi$$

We might better interpret the suggestion that we move from a generic choice effect claim to two existential claims in the following way (which parallels the usual modal treatment):

[typically] Elephants live in Africa or Asia

So, it is not the case that [typically] there are no elephants in Africa

So, it is not the case that [typically] there are no elephants in Asia

We could formally express this with a generic modal operator [g]:

$[g](F \vee S)$

$\therefore \neg[g]\neg F$

$\therefore \neg[g]\neg S$

[13] Or, at least, universal concerning something like regular or normal outcomes. There are interesting connections here with discussion we will see in chapter 4 concerning *normal* worlds in modal models. It is beyond our purposes here to explore these connections though the reader may recognize possibilities for treating generic statements in a similar way. A generic modal operator [g] could be understood as indicating a state of affairs in all accessible *typical* or *normal* worlds.

That choice effects occur in this modal formulation (distinct from permission and obligation) is only the beginning of the story. Choice effects can occur in almost any modal construction under the right circumstances[14].

In accordance with the standard definition of the modal operators, free choice effects can occur under the negation of modal operators as well[15]. For example:

You don't have to submit an electronic copy and a hard copy

This again, we might understand as having a precise formulation as:

$\neg[o](E \,\&\, H)$

which, with some symbol pushing gets us to a version of our familiar free choice permission:

$\langle p\rangle(\neg E \vee \neg H)$

Thus, for example, we might think we can correctly infer

It is permissible to not submit an electronic copy

As we will see in the next chapter, free choice inferences like these seem to frequently involve not merely an acceptable inference to either disjunct, but also to the condition of *not having both*[16].

Unfortunately, the choice effects described here are problematic in exactly the same way as von Wright's paradox of free choice disjunction. In all of these cases, we seem to be able to infer each of the choice options but are not strictly

[14] What these circumstances are exactly is the subject of much speculation in the literature on free choice disjunction and will occupy us for much of what follows in this thesis. One example of a common condition for choice effects is a plural formulation (Kleindinst 2007). Compare, for example 'I have friends in Calgary or Hamilton' with 'I have a friend in Calgary or Hamilton'. Even this plural criterion, however, appears to have exceptions. For a thorough and concise summary of the key linguistic features of choice effects see Meyer (2016, 3-14).

[15] Fox (2007, 87) notices a similar effect concerning the negation of universal quantifier. He gives the example 'We didn't give every student of ours both a stipend and a tuition waiver'. Others have noticed choice effects under different quantifier formulations (see Meyer 2016, 7).

[16] Or, in the case of making a free choice inference from a universal modal operator, *having to choose at least one of the presented options.*

(formally) justified in doing so. The problem at hand is to explain what legitimates these inferences.

With this understanding, we can now begin to consider the ways in which von Wright and many who followed have attempted to solve the problem. As we shall see, there is not only a long and spirited history of discussion on this issue, but this discussion runs parallel to discussions in logic and the philosophy of language whose importance reaches far beyond paradoxes about free choice.

Though von Wright was the first to attempt to solve these paradoxes in deontic logic, we will return to his views after a more general discussion of modal logic and how it can be used to represent choice.

Chapter Three

Modal Logic and Choice

3.1 Free Choice Disjunction

In early discussions of free choice permission, the only motivation to use modal logic to solve the problem was that it was first recognized within a system of modal logic (von Wright's deontic logic). The fact that there was no published instance of a concern prior to Ross' (1941) criticism and von Wright's (1968) consequently focused early research around deontic (permissibility) formulations of the problem and possible resolutions. This was fortunate insofar as many of the tools of modal logic will prove useful in understanding the dynamics of free choice. And yet, the problem can also be grasped and formulated in many ways which do not concern permissibility and are thus not essentially deontic in character[17].

As we will see, there are reasons for using modal logic to deal with the paradox of free choice permission that have nothing to do with the technicalities of deontic logic. Moreover, we should explore beyond just considerations of permissibility as there do seem to be choice situations which have the same modal character as the paradox of free choice permission, but which employ altogether different modal operators.

[17] As we have seen in Chapter 1, the problem can, in fact, be grasped and formulated through considerations of nothing more than truth-functional disjunction in choice contexts. In this way, the problem can be at least conceived of without a modal formulation. As I will argue in this chapter, however, modal tools help in understanding and resolving the problem.

Imagine, for example, a hypothetical thief who has stolen a car and desires to get away with the crime by leaving the immediate area. They reason:

> There is enough gas to make it to New York or Washington

In this case, the thief has a choice very much like the one present in the free choice permission paradox. We might try to symbolize this choice accordingly:

> $\langle p \rangle (N \vee W)$

But this cannot be correct, as we are indicating the choice here as a choice concerning permissible outcomes and surely getting away with a stolen car has nothing to do with what is permissible. Rather, the modal operator involved is a more basic kind of possibility operator. We are speaking here of a choice between two outcomes which come as the deliberate result of action. Hence, a generic 'action' operator could be used to describe this situation though we could be more specific or descriptive about this operator if we wanted (e.g., a possible travel action given the situation). Importantly then, the paradox of free choice permission is just one species of a much broader problem that we can call the *paradox of free choice disjunction*:

> FCD) $\langle x \rangle (\varphi \vee \psi) \rightarrow (\langle x \rangle \varphi \ \& \ \langle x \rangle \psi)$

Notice here that $\langle x \rangle$ is a variable modal operator for which permissibility $\langle p \rangle$ is just one possible substitution instance. How many modal operators might fall into potential free choice disjunction problems is an interesting question. Even in the most basic *alethic* sense concerning bare possibility we may be able to recognize situations of free choice disjunction. Consider the question 'Is the universe finite or infinite?' We might answer:

> It is possible that either is the case

which we might take as implying:

> It is possible that the universe is finite, *and* it is possible that the universe is infinite

Our expression of this, however, seems equivalent to:

It is possible that (the universe is finite, or the universe is infinite)

If we think this translates into the formal language as:

$\Diamond(I \lor \neg I)$

then, just as with free choice permission, we cannot properly conclude the possibility of either disjunct. Thus, perhaps even an *alethic* expression could be considered a substitution instance of FCD (or, at a minimum, this seems to hold if we treat the possibility operator here as the epistemic 'for all we know').

It suffices that we realize there are clear cases where a free choice disjunction paradox is present outside of straightforward permission cases and for our purposes the crucial element is choice options framed as a disjunction. For this reason, it often makes sense to depart from strict deontic systems in treating the problem and indeed, speaking about free choice *disjunction* rather than permissibility is becoming increasingly common (e.g.: Zimmerman 2000, Meyer 2016).

Nevertheless, the problem was recognized within the confines of deontic logic and this has played a large part in subsequent attempts to treat the problem modally. This has been a good development since any formulations capable of accounting for an agent's ability to choose outcomes will almost certainly be modal – among which we can include deontic formulations.

The reason I suggest these formulations will be modal is that in trying to understand this problem it is helpful to consider what exactly lines on a truth table represent and what it would mean to have the ability to choose between them. Lines on a truth table are possible arrangements of the world. They represent states of affairs; ways that the world could be. Each row is a possible world. Indeed, this view is at the heart of modal logic semantics for classical propositional logic and so, in order to account for an agent's ability to choose which disjunct makes a free choice disjunction true, we need to make sense of an agent's ability to choose between possible worlds.

It is unfortunate then that the inclination to formulate the problem (and solutions) within deontic logic was because of deontic logic's treatment of permissibility rather than its capacity to account for choice through a possible worlds semantics. A well formulated possible worlds semantics for modal logic was not available until Kripke's celebrated 1963 'Semantical Considerations on Modal Logic' and so unavailable to Ross (and perhaps unknown or unnoticed by von Wright early in his reflections on the issue). In any case, we do well to consider modern formulations of modal logic as well as Kripke's approach.

3.2 Modal Foundations

The basic modal language is essentially what we saw in the previous chapter on deontic logic, though we now consider the modal operators in more general form:

$$\varphi ::= \varphi_a \mid \neg\varphi \mid (\varphi \,\&\, \psi) \mid (\varphi \vee \psi) \mid (\varphi \rightarrow \psi) \mid (\varphi \leftrightarrow \psi) \mid \Diamond\varphi \mid \Box\varphi$$

Rather than standing for permissibility and obligation, our modal operators will take on a more precise meaning within the semantics of the formal language. This semantics (the meaning of the formal language) is provided by Kripke models.

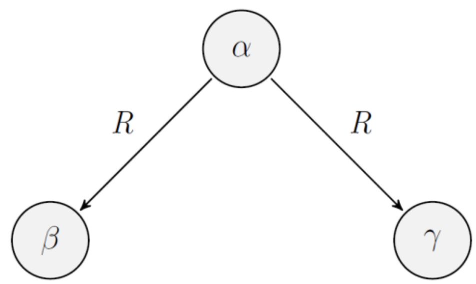
Every Kripke model $\mathfrak{M}$ begins with a *frame* which consists of:

1. A non-empty set S
   (The states, nodes, or 'possible worlds' in the model)

2. A binary relation R on S
   (An accessibility relation between states or 'worlds')

For example, a frame $<$ S, $R >$ consisting of S = $\{\alpha, \beta, \gamma\}$ and $R = \{\alpha R\beta, \alpha R\gamma\}$ could be represented graphically as:

Figure 3.2.1 Example Kripke frame

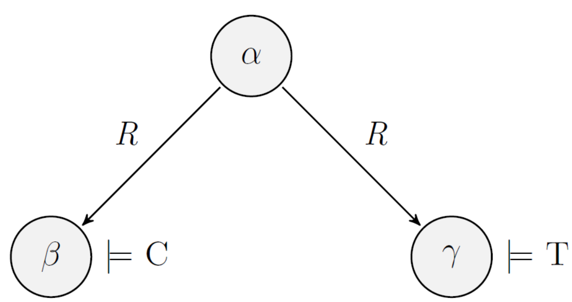


Frames are simple directed graphs, the nodes of which (despite ontologically suggestive terminology like 'possible worlds') might represent anything whatsoever. Similarly the accessibility relation between these nodes (despite suggestive terminology of 'accessing') can be any relation whatsoever. Nevertheless, in our treatment of free choice, it is sometimes useful to think of these nodes as metaphysical possible worlds and the accessibility relation as a choice relation from which some agent may move between worlds.

We add to this frame a valuation V on the members of S which is a specification of propositional letters that are true at each world in S. Adding V to our frame gives us the triple $< S, R, V >$ which is the Kripke model $\mathfrak{M}$. Continuing with our example, if V gives us ($\beta \vDash C$) and ($\gamma \vDash T$), we can continue to represent this graphically as:

Figure 3.2.2 Example Kripke model

$\mathfrak{M}_1$:



Truth in a Kripke model can be defined for each world Γ in S as follows:

1. $\mathfrak{M}, \Gamma \vDash \neg\varphi$ iff $\mathfrak{M}, \Gamma \nvDash \varphi$
2. $\mathfrak{M}, \Gamma \vDash (\varphi \,\&\, \psi)$ iff $\mathfrak{M}, \Gamma \Vdash \varphi$ and $\mathfrak{M}, \Gamma \vDash \psi$
3. $\mathfrak{M}, \Gamma \vDash \Box\varphi$ iff for every $\Delta \in S$, if $\Gamma R \Delta$ then $\mathfrak{M}, \Delta \vDash \varphi$
4. $\mathfrak{M}, \Gamma \vDash \Diamond\varphi$ iff for some $\Delta \in \mathrm{S}$, if $\Gamma R \Delta$ then $\mathfrak{M}, \Delta \vDash \varphi$

Truth definitions 1 and 2 are just the familiar definitions of the Boolean operators (¬, &) and can be used to define all the remaining connectives. Rules 3 and 4 are not strictly *both* required as either can be used to define the other since our modal operators □ and ◇ are themselves interdefinable (ie: □φ iff ¬◇¬φ)[18].

The ‘box’ operator □ and the ‘diamond’ operator ◇ are typically associated with the alethic modalities (necessity & possibility) but again, this is unnecessarily suggestive terminology. All they really mean is the above truth conditions. □φ just says ‘at all accessible worlds φ is true’ and ◇φ just says ‘at some accessible worlds φ is true’.

Notice that even in the case of deontic logic (or other normal modal logics) these same features hold of the respective modal models. In the deontic case, however, the model itself contains additional specification about structure and this is given by the additional axioms of the logical system. As deontic logic is characterized by the D axiom:

$$[o]\varphi \rightarrow \langle p\rangle\varphi$$

this means that in any deontic model, if some statement is made true at *all* accessible worlds, then it must be made true at *some* accessible world. Put another way, there must always be accessible worlds and there can be no worlds which terminate accessibility paths[19].

Though this may seem intuitive, there are often reasons to depart from deontic frameworks – for example, when we consider models of small, concrete,

[18] Similarly, we did not need both □ and ◇ in our language though for clarity we have treated both as basic symbols.

[19] If there were worlds with no further accessibility paths (including reflexive ones), then everything would be obligatory. After all, if there are no accessible worlds, then it is vacuously true that (for example) P is true at *all* accessible worlds.

decision scenarios which have clear end states. Games are well-modelled in this way as are any ordinary decision scenarios which can be treated in a game-theoretic manner. Perhaps unsurprisingly, these frequently include free choice disjunction situations.

3.3 Choice in Kripke Models

Given model 3.2.2 above, we can make all sorts of modal claims. For example:

1. $\mathfrak{M}_1, \alpha \vDash \Diamond C$
2. $\mathfrak{M}_1, \alpha \vDash \Diamond T$
3. $\mathfrak{M}_1, \alpha \vDash \Diamond(C \vee T)$ etc.

While there are an unlimited number of modal formulae true in model 3.2.2, claim (3) is especially interesting in light of the current discussion. For deontic logics, the modal analogues of $\Box\varphi$ and $\Diamond\varphi$ are the familiar $[o]\varphi$ and $\langle p\rangle\varphi$ and so, $\Diamond(C \vee T)$ can be interpreted as a re-expression of our supposed free choice permission claim $\langle p\rangle(C \vee T)$.

Notice however, that in the context of a Kripke semantics for modal logic, $\Diamond(C \vee T)$ may *not* mean free choice at all. We can characterize two ambiguous meanings of 'You may have coffee or tea' as:

Free choice:	You may have your choice of either coffee or tea
Logical options[20]:	You may have one of the options coffee or tea (without knowing which)

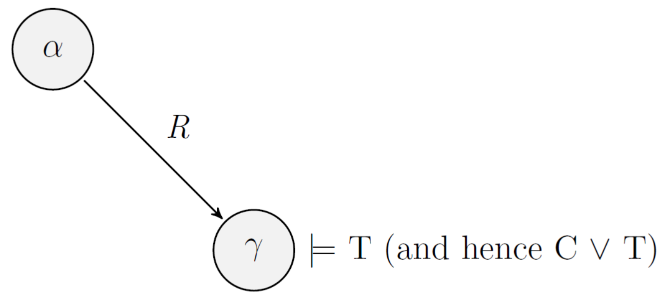
$\Diamond(C \vee T)$ symbolizes that C and T are (at least) logical options. At any world where $\Diamond(C \vee T)$ holds, all we know is that there is some accessible world in which either C is the case or T is the case but the choice in this context (the

[20] Meyer (2016, 9) characterizes this *logical options* reading as an *uncertainty* reading. Typically, those who attempt to account for free choice disjunction within a formal system disambiguate the uncertainty reading by using the wide scope formal disjunction (though there are notable exceptions, e.g. Zimmerman 2000). As presented here (the standard modal meaning), the two *disjunctive* formulations are formally equivalent.

accessibility relation) is not a choice between C and T. Rather, the choice is a choice of worlds. And as the following model counterexample shows, $\Diamond$(C ∨ T) can be true even when C is not a live option:

Figure 3.3.1 Free choice counterexample

$\mathfrak{M}_2$:



In example 3.3.1:

$$\mathfrak{M}_2, \alpha \vDash \Diamond(C \vee T)$$

since: $$\mathfrak{M}_2, \alpha \vDash \Diamond T$$

This, in other words, is a possible model of the restaurant example given at the beginning of this thesis (treating C as having coffee and T as having tea). If we take node α to be the actual world (ordering breakfast), there is no accessible world in which C is true. Coffee in this case is not a live option and we feel that the menu has misled us. In fact, we have just been misled by the ambiguity already mentioned.

Example 3.2.2, on the other hand, is the model that captures free choice. Both C and T are live options here since there are accessible worlds from α (assumed to be the actual world) in which each is true. Rather than expressing a disjunction, free choice between live options is therefore a *conjunction* of possibilities. So, we have a logical representation of our second meaning (free choice) and can express this as:

Free choice: $\Diamond C \;\&\; \Diamond T$

Deontic Free choice: $\langle p\rangle C \;\&\; \langle p\rangle T$

Well… not quite. While the above translations do suffice to capture each as live options, we do not yet have exclusivity. Model 3.2.2 is a model of exclusive options (coffee or tea but not both) but we can easily make counterexamples to exclusivity in which $\Diamond C$ & $\Diamond T$ holds (for example, where $\gamma \Vdash C$ and $\gamma \Vdash T$). As it turns out 'You may have coffee or tea' is even more ambiguous than I have let on when we allow an inclusive 'or' reading.

Fusco (2014, 2) raises a similar concern when she suggests that most natural language free choice expressions like 'You may have the whiskey or the gin' actually seem to entail the falsity of having both. And so, in the coffee or tea case we want it to also be the case that $\neg\Diamond(C \,\&\, T)$. So:

Exclusive Free Choice: $\Diamond C \,\&\, \Diamond T \,\&\, \neg\Diamond(C \,\&\, T)$

Exclusive Free Choice Permission: $\langle p\rangle C \,\&\, \langle p\rangle T \,\&\, \neg\langle p\rangle(C \,\&\, T)$

We can similarly treat Ross' Paradox by recognizing that while

$$[o](R \vee Q)$$

does follow from

$$[o]R$$

the same lack of clarity about live options is present here. This ambiguity is also present in a natural-language expression like 'you must mail the letter or burn it'. Notice, neither the natural language formulation nor the formal expression:

$$[o](R \vee Q)$$

indicate that both are live options. And, the live option version:

$$[o](R \vee Q) \,\&\, \langle p\rangle R \,\&\, \langle p\rangle Q$$

does *not* follow from $[o](P \vee Q)$. It may be that the natural-language expression of Ross' paradox (you must mail the letter or burn it) somehow carries the suggestion that both are live options but the translation into formal language as an obligation ranging over a disjunction should not be mistaken for having any such meaning.

It appears we have a representation and a useful tool (Kripke models) with which to recognize and avoid these kinds of ambiguities. In this way, we can circumvent both Ross' paradox and the paradox of free choice permissibility (as well as free choice disjunction worries in general) by simply being explicit in the formal language about what the permissions are *intended* to mean. In speaking about permissibility, deontic logic seems adequate for the task though free choice disjunction more generally extends well beyond these contexts.

While I have here presented an explicit way of making sense of these paradoxes, the conjunctions used to express free choice are not altogether novel. As far back as von Wright, the consensus has always been that free choice permissions seemed to entail a *conjunction* of permissions. This is immediate from the most basic expressions of the paradox. Rather, the key insight here is that the formalized expressions of these so-called free choice *disjunctions* are not disjunctions at all. The disjunctive character present in the natural language expressions is ambiguous and insofar as we take the meaning to be unambiguous, we should perhaps symbolize the expression as a conjunction of possibilities in the first place.

Notice, this is not as simple as treating the 'or' (disjunction) in these expressions as 'and' (conjunction). In practice, there is something peculiar going on with these statements whereby we are combining some of the properties of disjunction with some of the properties of conjunction. As in the case of straightforward conjunction, free choice disjunction claims allow for an inference to each of the options present. But, unlike conjunction, a free choice disjunction nevertheless disallows an inference to the the conjunction as a whole. For example, the disjunctive claim

> You may have coffee or tea

cannot be merely a misstatement of the conjunctive claim

> You may have coffee *and* tea.

as with the conjunction we want to allow for an inference to either of these options, but we nevertheless want to rule out having *both*. So, when faced with free choice disjunction claims,

> …simply replacing *or, any* with *and, all* in the relevant sentences will not yield two equivalent sentences (Meyer 2016, 3)

We need more. This is a characteristic of free choice disjunction that has been called *quasi-conjunctivity.* I will define this as:

> *Quasi-Conjunctivity* $\stackrel{\text{def}}{=}$
>
> Having the feature of conjunctivity, where in a binary compound expression we can infer either of the available statement components, while also having the feature of disjunctivity, that we cannot infer both.

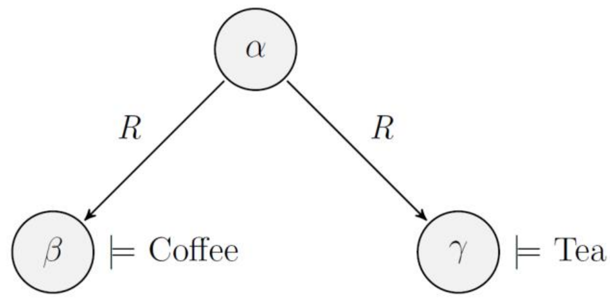
Propositionally (truth-functionally) there is no simple operator that can realize this quasi-conjunctive property consistently. With a model theoretic understanding of modal logic, this *quasi-conjunctivity* is made clear. Rather than simply replacing the disjunction with a conjunction, we take the narrow scope possibility disjunction and replace it with a conjunction of possibilities. In this way, we may say that each is possible (there is an accessible world in which each is true) but we cannot infer the possibility that both are true (we cannot infer that there is an accessible world where both are true).

3.4 Models and Expression

We have so far been treating Kripke models in a completely explicit way. Thus, the valuation of propositions has been fully represented in the examples given and no proposition is made true where not explicitly expressed. Obviously, however, in most useful or accurately descriptive cases there will be many more propositions made true at worlds and there may be no way to practically stipulate these. Thus, we can think of many Kripke models as effectively incomplete. Similarly, we may think that more worlds are present than those explicitly drawn. In this way, whatever models we choose to employ when talking about free choice disjunction, for example:

Figure 3.4.1 Free choice 'coffee or tea' model

$\mathfrak{M}_3$:



should not be taken to be completely illustrative of the situation or as the exact model of the situation unless specified as such[21].

What we may understand a model like this as showing is an extremely simplified case, or possibly a fragment of a much larger model, depending on what exactly we are hoping to represent or express. For example, we might take some other choice options to be:

Run out of the restaurant

Flip the table over

Sing a song etc.

How do we include these options? We know these are, strictly speaking, live options, but the representation we have been discussing is just an expression of what is meant by the semi-explicit statement on the menu, or when told that:

---

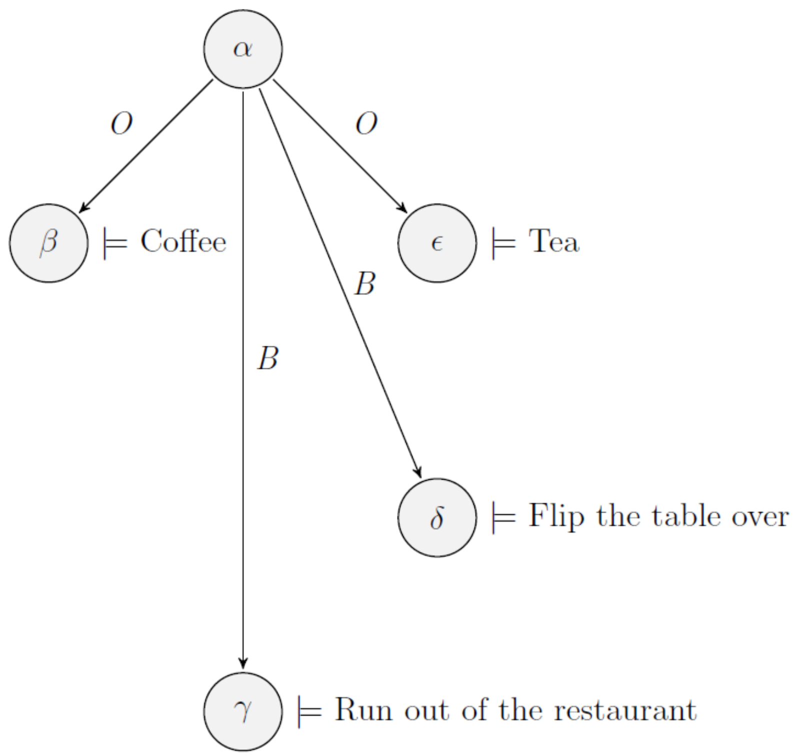
[21] In many cases, additional details of a model will be obvious and can be determined from the operators involved. For example, if we were dealing with permissibility, a model like this could not even be said to be *deontic* since it violates the D axiom. Notice, at worlds β, γ everything is obligatory (true at all *accessible* worlds). This is vacuously the case since there are no accessible worlds. For the model to be deontic we would need to ensure that obligatory outcomes are still permissible ones.

You may go to the beach or to the cinema

If we want to fully capture the complexities of a situation, we can build models appropriately, and may even choose to make them multi-modal with numerous accessibility relations, etc. For example, some choices may be 'ordering' choices, and some may be 'bizarre' choices, and we could represent this easily with a model like:

Figure 3.4.2 Bizarre options model

$\mathfrak{M}_4$:



In such a case, we employ multiple accessibility relations by augmenting our language with different accessibility types. Doing so, we could still say:

$\langle p \rangle C \ \& \ \langle p \rangle T \ \& \ [o](C \veebar T)$

or:

$\langle p \rangle C \ \& \ \langle p \rangle T \ \& \ \neg\langle p \rangle(C \ \& \ T)$

where ordinary permissibility corresponds with ordering choices. Though, now we could also say some other things about other options. For example:

⟨b⟩F & ⟨b⟩R

and:

⟨p⟩(C ∨ T) & ¬⟨p⟩(F ∨ R)

As we will see in the following chapter, the ability to treat bizarre or non-'normal' worlds separately will make sense in order to account for some resolutions of free choice disjunction (e.g. via strong permissibility).

While multi-modal models like these can be interesting in making explicit larger sets of options or complicated dynamics of decision making, even these are usually best understood as fragments of larger models or small models of very specific features of a situation.

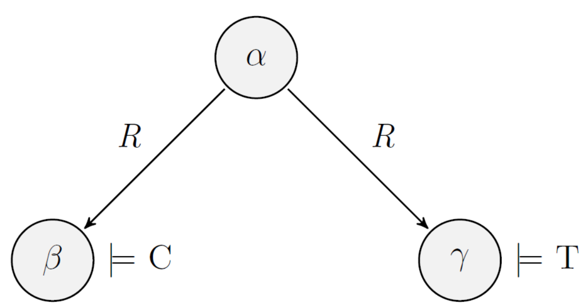
When dealing with well understood models, we face a related problem concerning expressions themselves -- expressions do not tell us much at all about models. Notice that the expression:

◇C & ◇T & ¬◇(C & T)

does not correspond exactly or exclusively with the choice diagram we have been examining:

Figure 3.4.3 Kripke free choice model

$\mathfrak{M}_1$:



While this formula does guarantee that there will be at least two accessible worlds, one where C hold s and one where T holds, it may be that there are multiple such worlds. More troubling still, there may be worlds where some other option(s) hold. So, there is nothing explicitly barring (say) a restaurant patron faced with a choice of coffee or tea from ordering some other item like hot chocolate. While the models given can precisely specify which options are available an expression in the formal language says nothing whatsoever about worlds or options not mentioned.

Given only the formula, it may or may not be the case that hot chocolate is an acceptable substitute, and for most such alternatives it nearly certainly will not be acceptable. We can imagine, for example, looks of confusion that would be given if a patron, faced with a choice of coffee or tea, asked for a Ferrari on the grounds that the menu didn't explicitly state a Ferrari couldn't be chosen.

Notice, this is different from our earlier paradox. Whereas before we might have worried that the menu could have stated Coffee or Tea or Ferrari when there was no Ferrari to be had, we now are presenting coffee and tea as legitimate options. The possibility operator guarantees that there are accessible worlds in which these options hold. The problem is simply that we haven't explicitly enumerated all possible options in such a way as to make clear our tree-structure in the Kripke model.

We may ask then – Is there a formula which can do this? Can we make clear that coffee and tea are the only available options? Using a propositional modal logic, we cannot.

First, our language contains a countably infinite number of atomic propositions, and yet all expressions must contain finite strings of symbols. To outlaw all other options would require a formula like the following:

$$\Diamond C \,\&\, \Diamond T \,\&\, \neg\Diamond(C \,\&\, T) \,\&\, \neg\Diamond P_1 \,\&\, \neg\Diamond P_2 \,\&\, \neg\Diamond P_3 \,\&\, \ldots \,\&\, \neg\Diamond P_n$$

where (P_1, P_2, P_3, …, P_n) are all remaining propositional letters in the language. If we stipulate a finite number of atomic propositions this works but standard accounts of formation rules allow for a countable infinity of atomic propositions and so we cannot exhaust them in a single wff (or even finitely many wffs).[22]

In the interest of keeping things as simple as possible we could have a formula capable of restricting possibilities to only coffee or tea by stating:

$$\Diamond C \,\&\, \Diamond T \,\&\, \neg\Diamond(C \,\&\, T) \,\&\, \Box(C \vee T)$$

And we can avoid some redundancy by reducing this to:

$$\Diamond C \,\&\, \Diamond T \,\&\, \Box(C \veebar T)$$

This gets us closer. With the above restriction, we now know that all accessible worlds (options) are such that they contain C or T. Additionally, this may be preferable as an expression of free choice disjunction compared with the one we have been using thus far:

$$\Diamond C \,\&\, \Diamond T \,\&\, \neg\Diamond(C \,\&\, T)$$

Since

$$\Diamond C \,\&\, \Diamond T \,\&\, \Box(C \veebar T)$$

has the advantage of eliminating other worlds where C or T are not present and it preserves an exclusive disjunction structure, which is nice since we are, after all, talking about free choice *disjunction*. Still, we might imagine there are other substitution allowances (hot chocolate, or even nothing at all) and in such cases the original formulation would still be preferable. And, in either case we still haven't outlawed possibilities where (say) coffee *and* hot chocolate are chosen.

With quantification over propositions added to modal logic, we can perhaps do better. For example:

[22] In his 'Modal logic for Open Minds', van Benthem does mention infinitary formulas when discussing expression of models. He states: "Infinite modal formulas may look daunting, and they go well beyond received ideas of 'syntax' – but infinite logical languages work well in modal logic, model theory, and set theory." (31) This may be an interesting idea to follow up on, though, in any case, I doubt infinitely long formulas will be compelling as a solution to simple free choice expressions like we might see on a menu.

$$\Diamond C \;\&\; \Diamond T \;\&\; \Box(C \veebar T) \;\&\; \forall\varphi\, (\Diamond\varphi \rightarrow (\varphi = C \vee \varphi = T))$$

I have no doubt that many will feel disinclined to such a solution given the difficulties and added complexity associated with quantified propositional modal logic. Those concerns notwithstanding, I think there are more basic reasons to reject quantifier solutions. Though this can show that 'no other options are acceptable', we have still not uniquely identified the structure of the particular model in question.

Just as no single (finite) propositional formula can express an entire 'state' or propositional valuation, neither can any modal formula stipulate a model exactly. What modal formulas can do is express structural features of a model at a world, or, when stated as global truths (axioms), structural features of a whole model.

For example, when the formula

$$\Box\psi \rightarrow \psi$$

is taken as an axiom, we can say of resulting models that they are *reflexive*. But there would, of course, be an infinite number of such models. Logical syntax works this way, and is perhaps even meant to work in this way. Just as the expression:

$$P \vee Q$$

tells us nothing at all about some other propositional letter, say:

$$R$$

so also does the free choice disjunction translation given tell us nothing at all about other possibilities. And, just as in the above purely propositional case, this may not be significant or surprising. What mattered was simply that the options presented were understood to be live options.

This may be a case where natural language pragmatics plays a significant role. To illustrate, suppose I had a tutorial section consisting of the following set of students

{Anne, Bill, Charles, Diane, Elaine, Fred}

Further imagine I am asked “who attended the last tutorial?” And suppose I answer:

A1 “Anne and Bill and Charles and Diane and Elaine were all at the tutorial”

It may seem, from a strictly logical point of view, that I haven’t fully answered the question. I have said nothing about Fred. Was Fred there? If Fred was absent, perhaps a better answer would be:

A2 “Anne and Bill and Charles and Diane and Elaine and not Fred were all at the tutorial”

From a pragmatics point of view, however, this information seems contained in A1, given the question and the context of the questioning and answering. The free choice permissibility translation is, I think, a similar situation with respect to eliminating other options, at least insofar as outrageous options are concerned.

3.5 The Choice Principle

While the aim here has been to provide some rigorous analysis supporting the expression of free choice permission as a conjunction of modal possibilities (permissions), at least the rough ideas behind this approach could not have been unfamiliar to von Wright, or indeed any who have followed by treating choice in terms of permissibility. After all, the basic intuition is that from a permission claim like:

You may go to the beach or to the cinema

We should be able to conclude

You may go to the beach
and You may go to the cinema

As each of these individual permissions is taken to follow, it is a trivial matter to conjoin these as:

You may go to the beach *and* you may go to the cinema

On practically all accounts of the paradox of free choice permission (e.g. Zimmerman 2000), it is exactly this inference that is at the heart of the problem. Zimmerman believes that choice sentences (like all of our free choice examples) suggest a general choice principle of the form:

CP) X may A or B $\vDash$ X may A and X may B

The formal version of this principle would be expressed as (Zimmerman 2000, 257):

Formal CP: $\Delta(A \vee B) \vDash \Delta A \ \& \ \Delta B$

To be clear, Zimmerman does not suggest such a formal principle and understands the difficulties that follow from this formalization. He only presents the formal CP illustratively. In his formalization the Δ is used to express deontic possibility but could presumably acts as a placeholder for any existential modal operator. Once again, such a choice principle echoes von Wright's sentiments regarding free choice disjunctions as those cases where each alternative in a disjunction is permitted.

Even so, as Zimmerman shows, a choice principle of this form quickly leads to problematic conclusions and absurdity. The basic worry has been understood at least as early as von Wright's initial exposition of the paradox of free choice permission (1968, 21 – outlined in the previous chapter) though Zimmerman (2000, 256-257) provides the following example from the rules of the classic boardgame *Scotland Yard*:

a.	Detectives may go by bus	
b.	Anyone who goes by bus goes by bus or boat	
c.	Detectives may go by bus or boat	a, b
d.	Detectives may go by boat	c, CP

In the game, some players (designated 'detectives') have the option of moving from one location to another on the board by bus whereas there is no option for detectives to move by boat in the game! We might formalize this reasoning as:

1.	$\langle p \rangle R$	premise
2.	$\langle p \rangle R \vee \langle p \rangle Q$	1 addition
3.	$\langle p \rangle (R \vee Q)$	2 distribution of $\langle p \rangle$ over $\vee$
4.	$\langle p \rangle (R \vee Q) \rightarrow (\langle p \rangle R \,\&\, \langle p \rangle Q)$	CP
5.	$\langle p \rangle R \,\&\, \langle p \rangle Q$	3, 4 modus ponens
6.	$\langle p \rangle Q$	5 simplification

Hence, with CP as an axiom, $\langle p \rangle R \rightarrow \langle p \rangle Q$ is a theorem where Q can be anything whatsoever. Put another way, if CP holds and some action is permissible then all actions are permissible. So, while:

$\langle p \rangle C \,\&\, \langle p \rangle T$[23]

may function adequately as an expression of free choice, we face serious difficulties in thinking that this expression is entailed by or otherwise directly follows from an initial translation into the formalism as:

$\langle p \rangle (C \vee T)$

This is an idea we will return to in Chapter seven. For the moment we should simply take notice of the similarities between Zimmerman's Choice principle and the formal expression of the paradox of free choice permission (and FCD). Notice that in the above derivation I have treated the formal expression of CP as equivalent to von Wright's formal expression of the paradox[24]. And, this is a substitution instance of:

FCD) $\langle x \rangle (\varphi \vee \psi) \rightarrow (\langle x \rangle \varphi \,\&\, \langle x \rangle \psi)$

[23] Or some variation of this containing additional restrictions. E.g.:

$(\langle p \rangle C \,\&\, \langle p \rangle T) \,\&\, \neg \langle p \rangle (C \,\&\, T)$

[24] This was not strictly necessary. Another way to understand a formal CP inference is to treat formal CP as an inference rule. Hence, from $\Delta(A \vee B)$ to an inference of $\Delta A \,\&\, \Delta B$

There is overlap as well with the general idea of *choice effects* though, perhaps purposefully, that term is less precise.

3.6 Epistemic Models & Disjunction

One final topic of interest concerning expression or resolution of free choice permission is the treatment of modal models with an epistemic accessibility relation. We will make good use of such logics later in this thesis and these are also necessary to understanding the novel approach given by Zimmerman (2000) where he argues that *all* disjunctions are conjunctions of epistemic possibilities.

An epistemic modal language replaces the box/diamond operators with the epistemic modality $[K_i]$ relativized for some agent i.

$$\varphi ::= \varphi_a \mid \neg\varphi \mid (\varphi \mathbin{\&} \psi) \mid (\varphi \vee \psi) \mid (\varphi \rightarrow \psi) \mid (\varphi \leftrightarrow \psi) \mid [K_i]\varphi$$

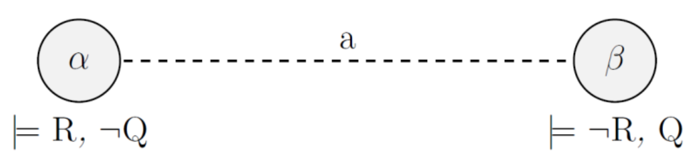
This may be further supplemented by a general knowledge operator $[K_G]$ which indicates knowledge available to all agents i ∈ G. Epistemic models are essentially multi-modal insofar as the individual accessibility relations are relativized to this set of agents i ∈ G. Epistemic models then are defined as tuples:

$$\mathfrak{M} = (W, \{\sim_i \in G\}, V)$$

Thus, we still have a set W of worlds, and a valuation on these worlds V but we now employ multiple accessibility relations $\sim_i$ specific to each agent. In our models these are typically represented by dashed lines connecting worlds.

Figure 3.6.1 Epistemic model

$\mathfrak{M}_5$:



This allows us to make statements about the epistemic state of a given agent relative to some propositions. For example, in the above model agent a is uncertain with respect to the status of propositions R, Q but knows that at least one is true. Thus the following are true:

$\neg[K_a]R \ \& \ \neg[K_a]Q$

$[K_a](R \vee Q)$

Finally, we can further augment this logic with further modal operators like the ordinary alethic modalities or a permission operator.

Though Zimmerman (2000) does not discuss epistemic logics in any detail, he makes use of the ideas behind epistemic logics in an attempt to explain the *choice principle* outlined in the previous section. Recall:

CP) X may A or B $\vDash$ X may A and X may B

The crux of Zimmerman's view is that *all* disjunctions can be seen as conjunctions of epistemic possibilities[25] (Zimmerman 2000, 266-267). In the simplest case (where there is no modal component to the disjunction in question) we can consider an ordinary disjunction. Consider, for example:

$(A \vee B) \vee C$

And, we can further imagine asking the hypothetical question 'what might be the case?' Using with the tools of epistemic logic we can provide an answer:

$(\langle K\rangle A \ \& \ \langle K\rangle B) \ \& \ \langle K\rangle C$

That is, for all we know A is true, and for all we know B is true, and for all we know C is true. They all might be the case and can be conjoined as such. In more complex modal cases, our modalities become nested within these epistemic modal operators. Zimmerman points out the manner in which these become redundant for epistemic situations. Consider Zimmerman's example (2000, 284):

---

[25] Or, equivalently, as *lists* of epistemic possibilities

Mr X might be in Victoria or he might be in Brixton

Which we can symbolize as

⟨K⟩V ∨ ⟨K⟩B

And then apply the same reasoning as in the simple disjunction case. So:

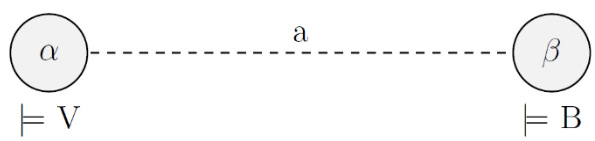
⟨K⟩⟨K⟩V & ⟨K⟩⟨K⟩B

For all I know, Mr X might be in Victoria AND For all I know, he might be in Brixton

Notice the effect in an epistemic model:

Figure 3.6.2 Epistemic 'Victoria or Brixton' model

$\mathfrak{M}_6$:



In this case, the epistemic modalities do collapse in on each other and this is a function of the structure of epistemic models. The accessibility relations of our epistemic modalities are symmetric, reflexive, and transitive (Modal S5)[26] and hence:

⟨K⟩⟨K⟩V → ⟨K⟩V

Thus

⟨K⟩V & ⟨K⟩B

[26] Zimmerman does not explicitly endorse an S5 theory of knowledge and, indeed, there are epistemic logics which do not assume a symmetrical accessibility relation. However, we do not strictly need to assume a modal S5 system for Zimmerman's account to collapse these iterated ⟨K⟩ operators. Only transitivity is required for this to occur and Zimmerman does adopt this through his *self-reflection principle* (Zimmerman 2000, 284)

Of course, the choice principle is no more valid for epistemic logic than it is for deontic logic as counterexamples can easily be constructed. Notice, however, the model in figure 3.6.2 is *not* a counterexample. In this way Zimmerman's attempt to reconcile free choice disjunction is not an entirely semantic solution as he makes great appeal to extra-logical concepts like context, authority and background information. Still, in the relevant cases, Zimmerman believes that similar 'collapsing' of modalities occurs when we combine deontic and epistemic operators.

Given a free choice disjunction:

May R or May Q

We again ask ourselves what might be the case and make the move to the expression of this disjunction as a conjunction of epistemic possibilities:

$\langle K\rangle\langle p\rangle R \,\&\, \langle K\rangle\langle p\rangle Q$

Here, Zimmerman makes appeal to an Authority Principle (2000, 286) such that *knowing* the permission granter is an authority allows us to collapse these modalities:

$\langle p\rangle R \,\&\, \langle p\rangle Q$

This authority principle is arguably a pragmatic feature of Zimmerman's account though, on at least some plausible grounds, many see Zimmerman's view as essentially semantic. The idea is that the *meaning* of permission invokes authority in permission granting in such a way as to collapse the disparate modalities. As Meyer states:

> The authority principle states that an authority on the relevant permissions and obligations… never simply thinks that it's possible that A; rather, either she is certain that A is allowed, or she is certain that A is not allowed (whence her authority). (Meyer 2016, 22)

Following Zimmerman, Guerts (2005) proposes a similar purportedly semantic solution whereby, rather than treating all disjunctions as lists of epistemic possibilities, these lists become modalized in whatever form they appear in the disjunction (with the epistemic operator as a default). In any case, both Zimmerman and Guerts depart from the standard treatment of deontic logic and subscribe to a direct logical formulation of disjunction as modalized conjunction.

While I haven't offered the same analysis as Zimmerman, my proposal earlier in this chapter echoes many elements of Zimmerman's view. While it may be that a natural language entailment like the choice principle makes sense, we do well to realize that the double turnstile operator present at the level of language, e.g.:

CP) X may A or B $\models$ X may A and X may B

is not the same operator present at the level of the formalism. This operator expresses *semantic* consequence. From the perspective of natural language, semantic meaning is about language as it applies to the objects of the world. At the level of the formalism, semantics is derived from the models I have here described. These models are what formal modal logics are about and it is these models that provide the truth conditions for modal formulae. How well these models capture the world is an interesting question (and one I will return to in chapter seven) but for the moment the key point is that the most pressing versions of the problems of free choice disjunction are those expressed in natural language (e.g. the *choice principle, choice effects*).

By contrast, the initial formulation of the paradox was in terms of a formal implication:

FCP) $\langle p\rangle(R \vee Q) \rightarrow (\langle p\rangle R \,\&\, \langle p\rangle Q)$

and these may be importantly different issues. It may be, for example, that the choice principle demands pragmatic explanations while FCP demands semantic explanations. Or, as I have suggested (following Zimmerman), the FCP implication simply does not follow. Rather, the choice principle suggests the meaning of

May R or May Q

is best logically represented by the conjunction of possibilities:

$\langle p \rangle R \;\&\; \langle p \rangle Q$

For the moment I will leave this analysis aside though it is helpful to have introduced these ideas as we will make use of (and expand) epistemic logic later in this thesis. Additionally, we will continue to explore these questions of meaning and logical representation. For now, however, we turn to additional semantic and pragmatic approaches to free choice disjunction.

Chapter Four

Semantics of Free Choice Disjunction

4.1 Strong and Weak Permission

In attempting to capture how permission claims function, an important distinction can be made between *strong* and *weak* permission. These concepts of strong and weak permission originate with Alan R. Anderson's (1957, 1958, 1966) reduction of deontic logic to alethic modal logic. Anderson begins with ordinary alethic modal logic and modifies this with nothing more than the inclusion of a single unanalyzed proposition letter S. In his earliest writings about this account he articulated the meaning of S as merely “sanction”, or, in keeping with a propositional formulation, “a sanction occurs”. He states:

> In addition to the usual primitive notions of alethic modal logic, [this reduction] takes only the notion "sanction" symbolized "S," as primitive. The basic deontic modes are defined with the help of this notion in such a way that von Wright's system (with appropriate qualifications) emerges as a sub- system (...) The only axiom mentioning the sanction states that the sanction is contingent; i.e., it is possible to behave in such a way that the penalty or sanction will occur, and also in such a way that it will not occur. (Anderson 1957, 16)

In later writings, Anderson would adopt more normative characterizations of S as indicative of some "bad" state of affairs (1958, 105) or where some actual sanction has been invoked or is liable to be invoked (1966). In any case, Anderson's interest seems to have had little to do with the meaning of S and more to do with a strictly formal reduction of deontic logic to alethic modal logic. In this sense, for Anderson, S is best understood precisely in terms of the formal notions of permission and obligation such that:

$\Diamond(\varphi \mathbin{\&} \neg S)$ means 'φ is permissible'

$\Box(\neg\varphi \rightarrow S)$ means 'φ is obligatory'

In Anderson's system, obligation can, thus, be understood as indicating that in all possible outcomes, the failure to realize some obligatory outcome φ implies S. Similarly, permissibility is understood as saying that there is some possible outcome where the realization of φ is such that ¬S also holds.

Importantly, Anderson's formulation allows us to differentiate between two different kinds of permissibility. The usual notion of permissibility can now be understood as a *weak* permission. All that is meant by a weak permission is that it is not obligatory to not perform some action. Or, equivalently, that the performance of an action does not necessarily lead to a sanction.

$\neg\Box(\varphi \rightarrow S)$

Notice, however, that we can make sense of a much stronger kind of permission. We can give an account of permission wherein the performance of an action is permitted under all circumstances.

$\Box(\varphi \rightarrow \neg S)$ means 'φ is *strongly* permissible'

The difference between *strong* and *weak* permission is best understood as a distinction between an action's being permitted under all circumstances as opposed to being merely permitted under some particular circumstances. I will consider two simple examples in order to clarify this distinction.

Suppose, in one case, you are permitted to choose amongst several options and may select without sanction in any case. We might imagine this occurring when faced with a selection between dinner options on a menu. If these options include:

> Beef and Baked Potato
> Beef with Rice
> Beef with Vegetables
> Fish with Baked Potato
> Fish with Rice
> Fish with Vegetables

We would correctly infer that

> It is permitted to order fish

> It is permitted to order beef

In this first case a permission statement like:

> You may order fish

is a *strong* permission given the listed options. That is, *all* instances of ordering fish are allowed options. Notice, however, that if for some reason the rice dishes were not permitted (perhaps the restaurant has none in supply) then the permissions above are merely *weak* permissions. Yes, there would still be permitted fish options but not all fish options would be permitted.

In Anderson's system, when rice options are not permitted, we would say that it is obligatory to not order rice.

$$\Box(R \rightarrow S)$$

But fish is nevertheless weakly permitted. Recall, we represent this as:

$$\neg\Box(F \rightarrow S)$$

Were it the case that all fish options were permitted we could make the stronger claim:

$$\Box(F \rightarrow \neg S)$$

Anderson's purpose in making this distinction was merely to differentiate between the kinds of permission one could express. The distinction, however, is one which has been employed repeatedly in trying to make sense of the paradoxes of permission and obligation. As early as von Wright's initial exposition of the problem, the treatment of free choice permission claims as strong permissions has been used to solve the problem. As we shall see, by doing so we are able to license an inference from 'you may do A or B' to 'you may do A and you may do B'. We will thus return to von Wright's initial attempt at a solution in order to better understand how this may help to solve the problem.

4.2 Dyadic Deontic Logic

To deal with the paradoxes he has identified, von Wright introduces an alternative system of deontic logic which can differentiate between kinds of permission and obligation in the same manner as Anderson's alethic modal logic. Von Wright has other motivations besides just the resolution of these paradoxes and, as a result, his system does introduce additional complexity. Even so, the resolution of the paradoxes of permission and obligation is a central concern and it is exactly this capacity to differentiate between strong and weak permissions that von Wright employs to resolve the problem.

The system he introduces is a dyadic modal logic where the modal operator for permission (and, derivatively, obligation) ranges over two separate statements. Von Wright's formulation for these dyadic expressions treats any permission claim as conditional on some other claim:

$$\langle p \rangle(\varphi/\psi)$$

The form of these expressions is at least partly inspired by concepts of conditional probability – though they deal with permission claims, they are interpreted in roughly the same manner. To illustrate, the above formula can generally be interpreted as 'φ is permitted given ψ'. Obligation is defined through the equivalence

$$[o](\varphi/\psi) \overset{\text{def}}{=} \neg\langle p\rangle(\neg\varphi/\psi)$$

and, like conditional probabilities, can be read as 'φ is obligatory given ψ'.

Monadic deontic systems like the one we have established and discussed (basic deontic logic) treat deontic claims as either completely absent any context, or as originating from some already present state of affairs. For example, in the monadic language of deontic logic, a claim like 'It is permissible that Q':

$$\langle p\rangle Q$$

can be understood as either a local truth (true at some world -- e.g. the *actual* world) or simply true under any circumstances (a global consequence of the model). The problem with this approach is that we may be interested in states that are not present at the actual world and may not be deontically accessible from any world. For example, consider the following claim:

You ought to steal as little as possible

Assuming that it is not the case that one ought to steal (at all!), monadic systems are unable to say much about stealing little as opposed to stealing a lot. There is no deontic or normative distinction which can be made between these options. This lack of a distinction among impermissible options underlies a number of interesting paradoxes of obligation. These include the Good Samaritan Paradox (Prior 1958), as well as Forrester's Paradox of gentle murder (Forrester 1984)[27].

While von Wright's dyadic system is not the only way to address these problems, by making obligation and permission conditional, we can treat these cases as instances where a normally forbidden action is permitted or

[27] The good Samaritan paradox arises from ought statements like 'Jones ought to help Smith, who has been hurt' and absurd conclusions which follow like 'Smith ought to have been hurt'. Forrester's paradox similarly treats a claim like 'If Jones murders Smith then he ought to do so gently' from which it follows (again, given some standard formulations of deontic logic) that 'Jones ought to murder Smith'. These are part of a family of related paradoxes and have been much debated with respect to the deontic rules which give rise to the paradoxes. For our purposes we do not need to explore this debate but should note that von Wright's dyadic system does solve many of these problems by conditionalizing obligation.

obligatory *given* some circumstance. For example, given that stealing is an outcome, one should steal as little as possible:

$[o](L/S)$

Importantly, we can express this obligation while maintaining consistency with that circumstance (stealing) being forbidden.

On the surface, a dyadic modal language like this may not seem significantly useful as a means to resolve the free choice permission paradoxes, though as von Wright illustrates, when examined in particular contexts, we can quickly increase our capacity to express a number of different *kinds* of permission and obligation. Some of these are better able to handle intuitions about obligation and permission inferences.

Most importantly, von Wright can differentiate three particular kinds of permission with the following feature:

$\langle p\rangle(R/Q)$ such that *all* worlds where R holds are permitted worlds

The full details of these three distinct permission types[28] are unimportant for our purposes here except insofar as we are able to make sense of the idea that *every* world where some statement is true is a permissible world.

This development is significant for dealing with the paradox of free choice permissions. In von Wright's view, free choice permissions are *strong* permissions. What this means is that in the case of a free choice permission, e.g.:

You may work or relax

it is acceptable to make this true *however* one sees fit. In other words, so long as we are dealing with the strong notions of permissibility described in von Wright's dyadic system, the choice principle holds:

$\langle p\rangle(\varphi \vee \psi) \rightarrow (\langle p\rangle\varphi \mathbin{\&} \langle p\rangle\psi)$

[28] See appendix B for a more detailed account of von Wright's dyadic system.

If, after all, 'φ or ψ' is strongly permitted then all worlds where 'φ or ψ' is true are permitted. Some of these are made true by the truth of φ and some of these are made true by the truth of ψ. So, when 'φ or ψ' is strongly permitted then φ is permitted and ψ is permitted[29].

Moreover, von Wright is able to similarly define varieties of obligation which correspond to his redefined varieties of permission in the usual way[30]. And by doing so, he can similarly establish that in some varieties of obligation Ross' theorem fails. Thus, for some kinds of obligation:

$$\neg([o]\varphi \rightarrow [o](\varphi \vee \psi))$$

Von Wight's Dyadic deontic system is far reaching, not only in its treatment of the paradoxes of permission and obligation but also in its capacity to account for further paradoxes and puzzles of deontic logic. Forrester's paradox of gentle murder, for example, would not be formulated until 1984, and von Wright's conditional account of deontic logic is equally able to meet that challenge[31]. Still, there may be reasons to prefer a monadic deontic operator. Even in that case, strong and weak permissions function similarly to resolve the problem.

As von Wright points out, a monadic system can simply be considered as a limiting case where no particular conditions are supplied. The same strong definitions of permission are able to be specified, and even these are not strictly needed as what is most important with respect to free choice permission is that we are able to separate out strong and weak permissions.

---

[29] Notice, there is an important assumption at work here about the logical comprehensiveness of the models in question. After all, it could be the case in a limited model that this choice principle did not hold even with a strong permission since, for example, the *strong* free choice permission that 'φ or ψ' could still be made true in a model with all worlds where φ being permitted and yet ψ false in all of these. This was not what von Wright had in mind, however, as he saw these models as inclusive of *all logically possible* worlds (1968, 24). Indeed, variations of this assumption are at work in all accounts of strong permission as a solution to free choice permission, since some version of this assumption is required in order to conclude that *all* possible realizations of 'φ or ψ' include some where φ and others where ψ.

[30] i.e. by treating obligation of φ as $\neg\langle p\rangle\neg\varphi$ but according to his dyadic definitions of permission. See appendix B for more details

[31] This can be accomplished by treating a claim like 'If John murders he should do so gently' as a *dyadic* obligation claim (e.g. 'John ought to kill gently given the condition of killing').

This can be done by stipulating distinct modal operators for each type of permission:

$\langle p \rangle \varphi$ iff some possible worlds where φ are permitted

$\langle p \rangle_S \varphi$ iff all possible worlds where φ are permitted

Unfortunately, this is not the end of the story, as weak and strong permissions come with new problems. These problems are best introduced by first considering a related issue in purely propositional formulations of free choice disjunction.

4.3 Conditional Free Choice Disjunction

One way of dealing with free choice permission claims is to treat these claims as conditional statements rather than as simple disjunctions. For example, a claim like:

You may have coffee

could be represented as a conditional where the antecedent expresses an option and the consequent expresses something like the fulfillment of this option. While we are, for the moment, working with purely propositional logic, we can keep with Anderson's notation and express this claim in terms of sanctions:

If coffee is ordered there is no sanction (the order will be fulfilled)

$C \rightarrow \neg S$

The reason why this approach is appealing is that free choice disjunctions now validate von Wright's intuition that a disjunction of permissions licenses an inference to either option. Recall:

You may have coffee or tea

So you may have coffee

Expressed as a conditional we might say

$$(C \vee T) \rightarrow \neg S$$

Notice that, when the permission is formulated in this way, the truth of *either* option in the antecedent (ordering coffee or tea) now warrants the fulfillment of the consequent (you get your order). Put another way, the manner in which free choice disjunctions seem to sometimes behave as conjunctions is captured by the fact that the above conditional can be re-expressed as:

$$(C \rightarrow \neg S) \,\&\, (T \rightarrow \neg S)$$

And so a purely propositional 'choice principle' follows:

$$((C \vee T) \rightarrow \neg S) \rightarrow ((C \rightarrow \neg S) \,\&\, (T \rightarrow \neg S))$$

This idea underlies a number of conditional approaches to free choice disjunction (e.g. Hilpinen 1982, Asher & Bonevac 2005, Barker 2010). Interesting as this solution is, the approach does run into problems which the various thinkers on the subject have attempted to address.

The approach cannot account for some of the *quasi-conjunctive* features of free choice disjunction. If, for example, we know that

$$(C \rightarrow \neg S) \,\&\, (T \rightarrow \neg S)$$

then it must also be the case that

$$(C \,\&\, T) \rightarrow \neg S$$

This difficulty, unfortunately, is just one instance of a much larger problem. If, as in the above example, we believe that the choice of coffee or the choice of tea entails the fulfillment of the order, then so also must either choice conjoined with any other statement whatsoever. For example, if:

$$C \rightarrow \neg S$$

then it must also be true that

$$(C \,\&\, M) \rightarrow \neg S$$

Notice, M can be any statement at all. A particularly illustrative example might be:

> You may have coffee and murder the waiter

Given that we are talking about sanctions (or the lack thereof), the above inference would amount to saying that because ordering coffee implies there will be no sanction, then ordering coffee and committing murder must also imply there will be no sanction!

The conditional approach and its accompanying absurdities are directly analogous to the use of strong permissions in deontic logic. Whenever one has a strong permission then all possible worlds where the permitted action is true are permitted worlds. So, just as in the above example if we think that

$$\langle p \rangle_S\,(C \vee T)$$

then it must also be the case that

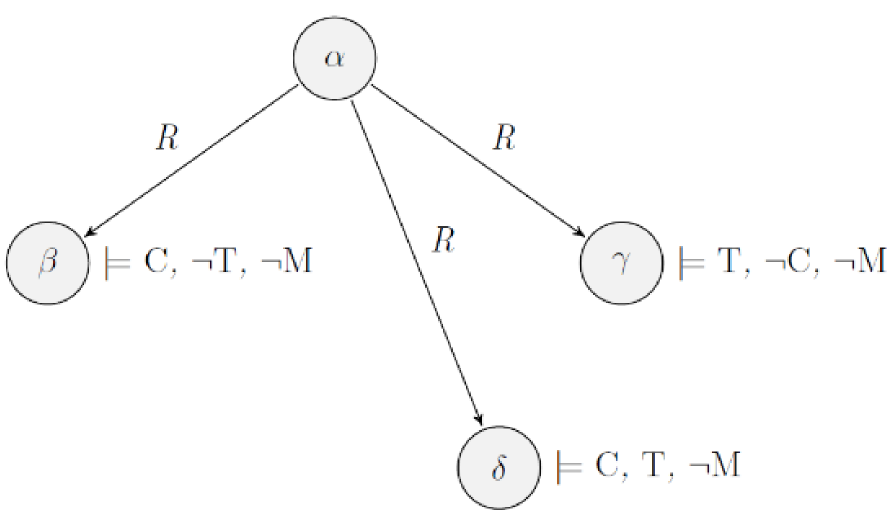
$$\langle p \rangle_S(C \,\&\, M)$$

since any world where C and M are true is a world where C. Thus, you may have coffee and murder the waiter.

In light of our earlier discussion of Kripke models, we could stipulate that the correct model of the situation is one in which C & M never holds. For example:

Figure 4.3.1 Strong permission model

$\mathfrak{M}_7$: $\mathfrak{M}_7, \alpha \models \langle p \rangle_S\,(C \vee T)$



In this limited (finite) model, all that the strong permission amounts to is a claim that in a model all worlds accessible from α where:

(C ∨ T)

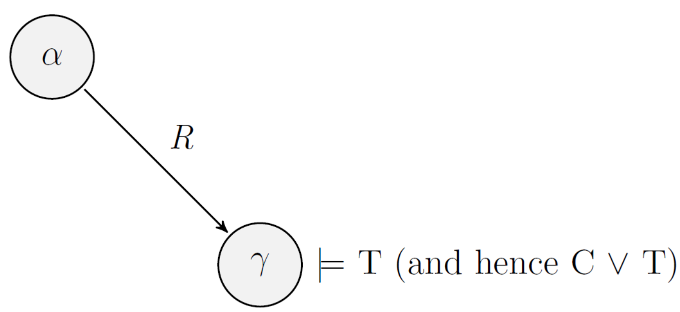
are permitted (deontically accessible) worlds. And, importantly, it may be that there are no α -accessible worlds where

C & M

While this does defuse worries about strong permissions making all possible satisfactions of a permission claim acceptable, it also removes the benefit of strong permissions, in that, depending on a given model, we may not be able to say much at all about what a strong permission actually entails. Recall the model associated with our illusory menu choice:

Figure 4.3.2 Illusory choice model

$\mathfrak{M}_2$:



In this model, not only is it true that

$\langle p\rangle(C \vee T)$

But it is also true that this is strongly permitted!

$\langle p\rangle_S(C \vee T)$

And yet in this case we cannot validly hold the choice principle. Hence:

$\nvDash \langle p\rangle_S(C \vee T) \rightarrow (\langle p\rangle C \mathbin{\&} \langle p\rangle T)$

This apparent contradiction with von Wright's solution is a result of the finite and extremely limited nature of the model provided. For von Wright, deontic models are not stipulated from the outset of a problem. For strong permissions to offer a solution to the problem it must be that the deontically accessible worlds are numerous and varied enough (probably infinite) so as to allow for the choice principle to hold:

$$\langle p \rangle_S (C \vee T) \rightarrow (\langle p \rangle C \ \& \ \langle p \rangle T)$$

Assuming varied and numerous accessible worlds is a natural way of considering the situation since logicians are generally in the business of determining what follows in a language and not what follows from some particular model[32].

In any case, pure strong permissions either get us nowhere towards establishing the choice principle, or they do establish it but simultaneously establish the permissibility of a huge variety of other actions.

One way of making sense of this mess is to concede that there are no strong permissions beyond those of precisely defined models. How could there be, after all, since practically any permissible action might be deemed impermissible under some set of circumstances? If this step is taken, however, strong permissibility ceases to account for the free choice inference (or the choice principle).

A better approach has been to introduce defeasibility into these conditional formulations. This can be done in a number of ways[33] though we will focus on the account given by Asher & Bonevac (2005)

[32] Though importantly, we may often be concerned with formally specifiable features of a model. E.g. Axiom D.

[33] E.g. Barker (2010) employs Linear Logic, Asher & Moreau (1995) propose doing so with generic statements. The topic of inference defeasibility is highly applicable to generic sentences insofar as they are a way to make sense of usual, typical, or otherwise generic circumstances.

4.4 Salvaging Strong Permission

Asher and Bonevac (2005) have proposed an approach through which we may salvage strong permission as a solution to the paradox of free choice disjunction. Their approach makes use of the 'fainthearted' conditional of Asher and Morreau (1991, 1995, Morreau 1997) in which:

$A > B$

is understood as meaning

If A then *normally* B.

This is a defeasible conditional in which A guarantees the truth of B under some standard of *normality*. And, similar to the ordinary conditional as it is used in *modus ponens*, we can make arguments using this defeasible conditional. Asher and Bonevac provide the following example (309):

This a lemon	A
Lemons are *normally* sour	$A > B$
So this is sour	B

The above argument is not strictly valid -- there are counterexamples or cases where the premises are true and yet the conclusion is false. So, we adopt a new standard whereby we say that an argument is *allowed.* That is, the premises necessitate the conclusion in all worlds *regular* with respect to the premises. That this might be an atypical non-sour lemon is a possibility, but we can nevertheless think the above argument reasonable (even if defeasible). So:

$A, A > B \quad \mid\!\sim \quad B$

where the modified turnstile symbol $\mid\!\sim$ represents logical consequence under *normal* circumstances

Consider again the example of ordering coffee. We might stipulate that worlds in which a patron has chosen to have coffee are *normally* never such that they lead to sanction. That is, these are worlds where one does any of the usual things one might do when having coffee. But, importantly, there are worlds in which one might do something abnormal (throwing coffee at another patron, for example) and in these cases a sanction may occur.

As Asher and Bonevac argue (310), this is not simply an *ad hoc* trick in order to make sense of free choice permission. Rather, this sense of regularity seems to be a key component of permission claims. Permissions are given with respect to sometimes large sets of possibilities but rarely, if ever, are they so open as to allow any instance whatever of the permission in question.

Consider again our familiar

> You may have coffee, or you may have tea

Understood as a defeasible permission, this would mean that *normally* the disjunction does not lead to sanction. It is a strong permission under normal circumstances. Further, it would seem to be the case that in ordinary claims where we believe free choice permission is present, there are enough normal worlds permitted to include cases where each of the disjuncts is true.

We can introduce a special 'normal' necessity (obligation) operator to equally well capture these ideas:

$$[n]((C \vee T) \rightarrow \neg S)$$

Even von Wright's earliest reflections on free choice permission seem to admit that this is the nature of free choice permission. Recall his definition:

> A disjunctive permission for which it holds good that each alternative in the disjunction is permitted too I shall call a free choice permission. (von Wright 1968, 22)

Von Wright hasn't invoked any concept of *normality* or *regularity* here but the definition as given nevertheless suggests that in addition to the usual meaning of deontic (weak) permission, free choice permission indicates that there are permitted worlds in which each of the disjuncts is true – and, crucially, this is not so open a permission as to allow *all* possible circumstances which satisfy the permission. Asher and Bonevac's use of a defeasible conditional makes the options permissible under normal circumstances, and so we might modify von Wright's definition as follows:

> A disjunctive permission for which it holds good that there is no sanction for either alternative in the disjunction in any *normal* state of affairs we shall call free choice permission.

As applied to our problem of free choice disjunction, this solution again employs Anderson's sanctioning notation but supplements the idea with a more restricted necessity operator (i.e. one ranging over normal worlds):

$$\langle p \rangle_S \varphi \quad \text{iff} \quad [n](\varphi \rightarrow \neg S)$$

Or, using the notation of the defeasible conditional (where $[n]\rightarrow$ is translated as $>$):

$$\langle p \rangle_S \varphi \quad \text{iff} \quad (\varphi > \neg S)$$

So φ is strongly permissible if it *normally* does not result in sanction.

Asher and Bonevac maintain that 'Free Choice Permission is Strong Permission' (2005) but strictly speaking the defeasible conditional actually amounts to a kind of permission falling somewhere between weak and strong permission. I shall call this *robust* permission:

Weak permission:	$\langle p \rangle \varphi$	iff	φ is permissible in some world
Strong permission:	$\langle p \rangle_S \varphi$	iff	All worlds where φ are permissible
Robust permission	$\langle p \rangle_R \varphi$	iff	All normal worlds where φ are permissible

The idea of normality is an intuitive standard through which we can understand how permissions are typically granted, and in this sense it is easy to see why Asher & Bonevac have chosen *normality* as the constraining feature of the defeasible conditional. Still we may worry that the term is imprecise, vague, or otherwise a kind of sleight-of-hand in making sense of strong permissions (A is strongly permissible… except when it isn't). This loose quality of normality is in some ways desirable and mirrors our everyday ideas about normality. Still, in the interests of rigor, it is at least technically possible to be perfectly precise by stipulating those conditions which are normal and those which are not.

4.5 Normal Worlds

In expressions which employ the defeasible conditional – for example:

$$A > \neg S$$

the defeasible conditional $>$ replaces the ordinary notion of strict implication. But, where strict implication is unrestricted, e.g.:

$$\Box(A \rightarrow \neg S)$$

the defeasible conditional limits this to a strict implication concerning *normal* worlds:

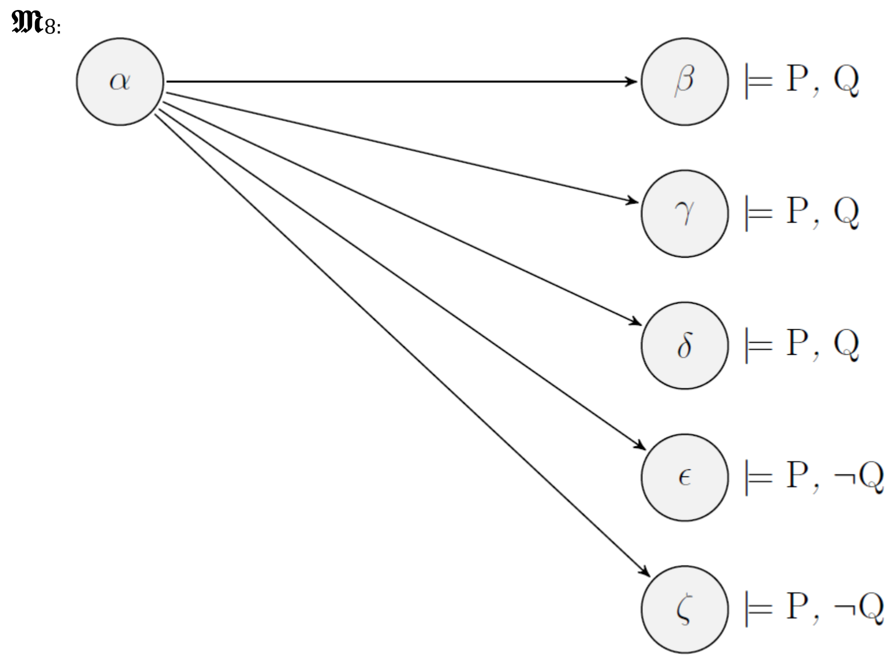
$$[n]\ (A \rightarrow \neg S)$$

Instead of having an unrestricted strong permission concept, where *all* instances of an action are permitted, we now avail ourselves of the notion of *normal* circumstances. That is, A is strongly permitted under all *normal* conditions (A normally implies there will be no sanction). The semantics of this defeasible conditional is based on defining, for any given world ω, and any given proposition φ, some set of worlds N that we stipulate as *normal* or *regular* with respect to ω. Thus:

> $A > B$ is true at ω iff those worlds which are normal with respect to ω, and where A is true, are worlds in which B is true

In this way, we make use of a selection function which relates worlds as *regular* with respect to some given world. Notably, this is no more mysterious than any function relating worlds. That is, just as we might define some set of worlds as accessible given some other world, so too we can define some set of worlds as *regular* given a world and a proposition. For example, if we restrict our model to the following very simple case:

Figure 4.5.1 Regular worlds model



We might imagine that a selection function applied to world α and proposition P generates the set $\{\beta, \gamma, \delta\}$ in which case we could state truthfully of the above model that

$$P > Q$$

Notice, however, that P does not strictly imply Q. That is:

$$\neg\Box(P \rightarrow Q)$$

In this way, the defeasible fainthearted conditional employed by Asher & Bonevac does not necessarily need to be associated with normality at all. For any defeasible conditional $>_i$ there will be some set i of accessible worlds in which it is the case that

$$A \rightarrow \neg S$$

And so, the defeasible conditional merely stipulates a strong permission over these worlds. But, given any model with n worlds, there will be 2^n possible subsets for each of which we could define some defeasible conditional.

For one of these conditionals (applying to all worlds) the defeasible conditional will be equivalent to normal strict implication (and, by extension, strong permission if *necessarily* ($\varphi \rightarrow \neg S$)). That is, when the set of worlds over which a conditional ranges contains *all* worlds, which we could designate as $>_U$ then:

$$\Box(A \rightarrow \neg S) \quad \equiv \quad A >_U \neg S)$$

Hence, normality is just a description for one such defeasible conditional which we think characterizes those worlds in which things are normal (whatever we take that to mean).

Presumably, one feature of worlds where things are normal is that all free choice options are also permitted. But this should not be the standard (or at least the only standard) by which we define normality. If this was the standard by which we characterized a normal world, the solution would be question begging and would take us no further than von Wright's initial suggestion that free choice permission was simply equivalent to a conjunction of permissions. Rather, it is the fact that free choice permissions are *strong* permissions (under normal circumstances) that guarantees that the choice principle holds. More than this, worlds which are normal will exclude those features we'd like to exclude from an unrestricted strong permission (e.g. pouring soup on the waiter's head).

4.6 Making Sense of Normality

Though we can construct this account of *normality* that salvages strong permission, we may still be left with doubts about what exactly we mean by *normal* and how this selection procedure functions in practice. Even if we accept that the logical tools can be made precise, we still seem to be clinging to a practically undefined concept beyond saying that those worlds which are picked out by a particular selector function are normal.

A related worry about creating a selector function that maps permissible outcomes as 'normal' given some world is that we would need to know in advance which worlds are permissible (and normal) just to say of some claim that it is *normally* permissible.

It seems that there may be 'boundary' worlds in which we might not be sure whether to treat them as 'normal' or not and in these cases it may seem arbitrary to classify them as such. Moreover, common-sense ideas about what makes a world normal have nothing to do with mere categorization but seem instead to be about characteristics of the worlds themselves.

One way of getting around these worries is to simply accept that this formal selection of normal worlds is derived from a vague concept. In this way, we should be concerned about boundary cases, but should also realize that there do seem to be cases where we know normality (or abnormality) when we see it. What is important about Asher & Bonevac’s solution is that there do seem to be enough clear-cut cases to establish the choice principle and rule out obviously bizarre exceptions to completely unrestricted strong permission.

On this view, the modal models and the precise selector function may be somewhat mysterious. Still, *we* are not necessarily the ones defining worlds in this way and *we* should not claim omniscience in this regard. Consider:

> Lemons are normally sour
>
> So, this lemon is sour.

Well, how can I be sure this is a *normal* lemon? Similarly, how can I be sure that a free choice scenario:

> Your meal comes with coffee or tea

which I might understand as *normally* implying

> You may have coffee *and* you may have tea

nevertheless holds in the particular case with which I am faced? How can I be sure mine is a *normal* situation?

This, I suggest, is not so troubling, particularly given Asher & Bonevac's weakened concept of *allowance* rather than validity. If we are simply dealing with allowance, we know that the inference is defeasible (even if not precisely defined) since we may not, in fact, be in a normal world. Importantly, however, we will sometimes have very good reason to believe that we are in a normal world (or that a possible outcome is probably not normal). It will sometimes be the case that we find ourselves considering a boundary option where we cannot be quite sure of normality, but perhaps more often, it will be either clear that an option is normal or clear that it is not normal.

Ordering coffee and murdering the waiter seems quite obviously not a normal instance such that its satisfaction would fall under the defeasible *robust* permission:

$$\langle p \rangle_R (C \vee T)$$

On the other hand, going into a well-established restaurant during usual business hours, seeing a credible menu, making a standard order, and otherwise behaving oneself at least suggests that we are dealing with *normal* worlds such that the robust permission:

$$\langle p \rangle_R (C \vee T)$$

would also entail the permission:

$$\langle p \rangle C \ \& \ \langle p \rangle T$$

This may suffice as a justification of strong/robust permissions as a solution to the paradox of free choice permission though I have, admittedly, failed to provide precise criteria by which to classify worlds as *normal*.

For the moment, I will leave this problem aside and move on to discussing pragmatic approaches[34] to dealing with the paradox of free choice disjunction.

[34] There are, of course, additional semantic treatments of free choice disjunction that I have failed to mention here. The most notable of these are approaches which attempt to provide

Still, we should keep this concern in mind as we consider pragmatic approaches to the problem. As we will see, one way to account for a defeasible notion of *normality* involves understanding who is granting permissions, which permissions can be granted by an authority, and the context surrounding the granting and receiving of permissions. This consideration of context is central to language pragmatics and is the direction to which we now turn.

explanation of choice effects through modal interaction with disjunction (e.g. Aloni 2007, Simons 2005). These approaches modify existing semantics for the deontic permission operator (and disjunction) in a number of similar ways. Following Zimmerman's idea that disjunctions generate *lists*, Aloni and Simons believe that disjunctions introduce a set of alternatives. And, given these alternatives, an expression like (may)(A or B) modally ranges over worlds in such a way as to make all of these alternatives permissible. Thus, on a standard modal reading, (may)(A or B) just indicates that the disjunction is true in at least one possible world, but, on the alternative reading, *for each of the alternatives introduced* there is an accessible possible world in which it is true. On such a view, the narrow scope and the wide scope reading are no longer equivalent (the wide scope indicating mere logical options and the narrow scope implying a choice effect).

Chapter Five

Pragmatics of Free Choice Disjunction

5.1 Kamp and the Turn to Pragmatics

Though the paradox of free choice permission originates with Ross and von Wright, a third major paper on the subject is Hans Kamp's 1973 'Free Choice Permission'. Indeed, much modern exposition on the subject of free choice permission begins with Kamp, whose importance stems in part from the novel manner in which he frames both the problem and his own solution.

For Kamp, there is more at work in a permission claim than a purely semantic and logically brute fact from which we can make straightforward inferences. Whereas von Wright's early attempts to solve the problem involved making semantic changes to deontic logic in ways that would allow us to represent free choice intuitions, Kamp does not treat the problem as a simple failure of standard deontic logic. Instead, Kamp explores the nuances of permission statements and the relationships between those who grant permission and those who receive permission. For Kamp, it is this *pragmatic* character of free choice permission that grounds our inferences.

For the sake of simplicity in giving his 'preliminary schematic' of permission contexts (1973, 63), Kamp concentrates his analysis on situations involving only two persons, one of whom has permission-granting authority and the other, to whom the permission is granted. Kamp labels these people *A* and *B*, where *A* is the person issuing the permission and *B* is the person for whom the permission is intended. When this permission-granting authority is legitimate, the function of a permission given by the authority *A* is to remove, for *B*, prohibitions from some standing list of forbidden actions (1973, 63).[35]

The standing prohibitions which bind *B*, the receiver of a permission, stem from various moral laws, codes, rules, or regulations that *A*, the permission granting authority, has the rightful ability to remove in at least some of these cases[36]. So, when a permission is granted, it is only with reference to these prohibitions that we can make sense of how such a permission functions.

Though Kamp seems to consider these prohibitions as ultimately about forbidden *actions*, he indicates that treating them in this way would require an ontology of possible actions that are never actually performed. He states:

> The answers I would *like* to give to these questions are these: Prohibitions are generic actions, or action types … The doctrine I will offer instead has perhaps a less intuitive ring to it. But it avoids the tangles of the general theory of action; and for our present purposes it will do very well indeed. According to this doctrine prohibitions are propositions. (1973, 63-64)

In line with the usual deontic (modal) conception of permissions, prohibitions are thus restrictions on accessible *worlds*. That is, prohibitions indicate propositional states of affairs one is prohibited from realizing. And, when an authority *A* grants a permission, e.g.:

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[35] Though Kamp purposefully considers only these simple two-person scenarios, he does seem to hold the key features of this account as applicable to 'all natural contexts for permission statements' (1973, 63). Insofar as his account could be extended to more complex scenarios, the main point would be that *all* those to whom a permission applies would have *formerly* prohibited states-of-affairs now permitted and that this would come as a result of some permission-granting authority (perhaps a collective of individuals) removing these prohibitions. There are, of course, further potential complications – e.g. the possibility that multiple separate authorities could have related or overlapping prohibitions in place. In any case, Kamp restricts his analysis to only two individuals and assumes there is no other authority besides that of *A* (1973, 65).

[36] How exactly we list or account for these prohibitions is an interesting question though for our purposes it suffices to treat these as explicable in principle, whatever they may be. Likely, as Kamp seems to allow (1973, 63) these will be drawn from not only codified laws and rules but also social norms and moral guidelines that are rooted in practices and convention.

$\langle p \rangle Q$

then a class of worlds where Q is the case is now accessible for *B*. As Kamp indicates, this class will not include *all* worlds where Q since many of these will include worlds with additional prohibitions which are also true. If, for example

$\neg\langle p \rangle R$ (R is prohibited)

Then, despite a granted permission $\langle p \rangle Q$, all worlds where R holds will still be inaccessible (prohibited) even if Q holds there as well.

Kamp considers the free choice disjunction case with this schema in mind (63). Suppose initially that in accordance with one's standing prohibitions one is not permitted to realize worlds where R and one is not permitted to realize worlds where Q. Thus:

$\neg\langle p \rangle R \ \& \ \neg\langle p \rangle Q$

Now, suppose for *B*, some authority *A* grants the following permission:

$\langle p \rangle (R \vee Q)$

What worlds, under these circumstances, are now permitted? Since, in this case, we are now indicating which worlds are now accessible among previously prohibited worlds, *all* worlds where $(R \vee Q)$ hold are now permitted – barring, of course, any worlds where further prohibitions hold.

Notice the difference here from the usual deontic conception of permission. In the usual deontic treatment a statement like the above free choice permission is made true if even one world is accessible which makes it true. And so, from a basic logical fact like:

1. $\langle p \rangle (R \vee Q)$

We cannot straightforwardly infer that any single disjunct is permissible, e.g.

2. $\langle p \rangle (R)$

After all, statement (1) could have been made true by the permissibility of the other disjunct Q and there need not be any permissible world where R holds. Importantly, from a strictly semantic perspective, Kamp agrees that this is the case.

But, if we accept that *pragmatically* statements like this are typically made where standing prohibitions are lifted by some authority, then there is a pragmatic inference involved. In the pragmatic case, where authority *A* lifts prohibitions on *B*, the act of giving permission suggests not only that the permission applies to what were formerly prohibitions, but also that *A* is granting this permission to the greatest degree their authority allows (or at least to a degree large enough to be informative). In this sense, the permission is at least *pragmatically* a kind of strong permission given that no other prohibitions are violated.

For example, if granted the permission

> You may borrow the lawnmower

we would not expect that what was meant was actually only a permissible world in which

> You may borrow the lawnmower and give me a million dollars.

Neither would we expect such a claim to be made if the lawnmower in question already belonged to us and we already knew that access to it was unrestricted. What such a statement seems to mean is something like:

> You may borrow the (previously restricted) lawnmower as long as in doing so you do not violate any other rules or norms.

The pragmatic effect, then, is to treat permissions as moving a class of statements from the realm of the prohibited to the realm of the permissible. So, while semantically (1) is a logically weaker claim than (2), it seems that (1) may be logically strengthened by these pragmatic considerations.

In the case of

> $\langle p \rangle R$

the class of worlds which becomes permissible seems to contain those worlds in which R is true and no other standing prohibitions are violated. Similarly for

⟨p⟩Q.

The class of worlds now permissible contains those formerly prohibited worlds where Q is the case and no other standing prohibitions are violated.

Crucially, if we are moving the class of worlds where

(R ∨ Q)

into the realm of the permissible then all those worlds become accessible where this disjunction is true (and no further norms are violated[37]). As the disjunction is made true by the truth of either of the disjuncts, this means there are now accessible worlds where Q, and there are accessible worlds where R. That is, the class of worlds made permissible is the *union* of those first two classes where R is made true and where Q is made true.

As Kamp illustrates:

> … as logicians have realized with regard to assertions for at least a century, 'or' stands essentially for set theoretic union: The set of possible situations in which a disjunctive assertion is true is the union of the sets of possible situations which realize its disjuncts. (1973, 65)

Consider the case:

---

[37] The authority of the permission-granting agent *A* may have some further effect in removing prohibitions if R is now permitted but this requires the realization of some further (formerly prohibited) proposition which is also in the power of *A* to rescind. For example, suppose a parent (Authority *A*) grants permission to a grounded child (the permission-receiver *B*) to attend a party (R). While attending parties may have been among the list of prohibitions faced by *B*, we can suppose that so also was leaving the house (H). As it may be that, necessarily, going to parties requires leaving the house (☐(R → H)) the permission ⟨p⟩R may require also that ⟨p⟩H. Even where not necessitated, there may be cases where, to be of much use at all, a permission ⟨p⟩R will involve the dropping of further prohibitions which are under the umbrella of *A*'s authority. Kamp does not consider these cases directly, but he circumvents this issue with the assumption (again for sake of simplicity) that all prohibitions under consideration are independent (1973, 64).

You may have Coffee or Tea

Such a permission only makes sense in light of some (fairly ordinary) scenario in which we would not normally expect to be able to just have coffee or tea at our whim. We would not normally expect, for example, to simply walk into any restaurant and take/demand coffee without paying or entering into an expectation of payment. That is, relative to the granting authority (the restaurant), before we order we are in a situation where:

$\neg\langle p\rangle C \,\&\, \neg\langle p\rangle T$

If we order tea, we enter into a payment arrangement, and worlds where T is realized are now permissible as long as they do not violate further prohibitions (we still cannot, for example, order tea and pour it on the waiter's head). The case is the same when we order coffee.

When we order a meal that presents us with the free choice:

$\langle p\rangle(C \vee T)$

those worlds where $(C \vee T)$ are realized (and no further prohibitions are violated) are now permissible. Notice the similarity to *strong* or *robust* permissibility here. While not all worlds where $(C \vee T)$ holds are permissible, all such worlds are permissible where further prohibitions are respected. So, since worlds where C holds are fairly ordinary and in this case do not violate any further prohibitions, we can infer

$\langle p\rangle C$

Similarly for T, we can now infer

$\langle p\rangle T$

And thus,

$\langle p\rangle C \,\&\, \langle p\rangle T$

5.2 The Pragmatics/Semantics Divide

Strong permission approaches to solving the paradox of free choice permission like those of von Wright (1968) and Asher and Bonevac (2005) bear some similarity to Kamp's solution, but a notable difference is that where these approaches make fundamental changes to standard deontic logic (e.g. differing conceptions of permissibility, defeasible inferences) Kamp makes no significant syntactic or semantic modification and instead holds reasonably closely to the basic deontic system presented by von Wright and, indeed, to modern formulations of standard deontic logic.[38]

So, rather than explaining this strengthened permissibility in terms of semantic features of the language, it is pragmatic features of permission-granting scenarios that explain our intuitions. These features include the context of the scenarios as well as the particular relationships and interactions surrounding permission-granting agents and those who receive permissions. As we can see by comparing the approach of Kamp to the approach of Asher and Bonevac, free choice permission is an interesting case resting somewhere on the boundary of the pragmatics/semantics divide. For in some sense we can treat the two approaches as nearly equivalent.

Recall, Asher and Bonevac (2005) explicate free choice using the weakened or defeasible conditional:

$$Q > \neg S$$

That is, R is permissible in such a way that in all *normal* cases, its realization does not result in sanction. As this is not a completely unrestricted strong permission, I suggested we call this *robust* permission:

---

[38] It is true that Kamp makes minor formulation changes and thus differentiates his account of deontic logic from von Wright (58). Nevertheless, the essentials of standard deontic logic remain constant and Kamp's account of the pragmatics of permission can be equally well understood in von Wright's formulation (or entirely contemporary formulations). Kamp does note that the validity of the axioms of deontic logic requires a specific interpretation of the system but this requirement too is consistent with those interpretations required by any standard, Kripkean account of deontic logic.

$\langle p \rangle_R Q \quad \text{iff} \quad (Q > \neg S)$

or

$\langle p \rangle_R Q \quad \text{iff} \quad [n](Q \rightarrow \neg S)$

That is, in all accessible *normal* worlds, if Q is the case then there will be no sanction. Consider what we might mean here by *normal*. The approach of Asher and Bonevac was to simply realize or otherwise identify scenarios which are intuitively *normal* and then stipulate those worlds as such. As outlined in chapter four, this stipulation could be completely arbitrary, yet there is a definite sense that any criteria we use to decide which worlds are normal will be based on what we think intuitively about normality and, perhaps, on what normality actually is in such cases.

It is not implausible to think a reasonable criterion is captured by Kamp's description of the pragmatics. That is, normal worlds may simply be worlds in which none of the other standing prohibitions are violated. This too could be expressed semantically, given such a list of standing prohibitions. For example, if we know and accept that some action is forbidden:

$\neg \langle p \rangle F$

But we are given the permission

$\langle p \rangle R$

We could treat permissibility here as strong permissibility in all worlds not already restricted. That is:

$\langle p \rangle_S R \ \text{ given } \ \neg \langle p \rangle F \qquad \equiv \qquad \Box((R \,\&\, \neg F) \rightarrow \neg S)$

Such an approach actually eliminates the need for Asher and Bonevac's defeasible conditional altogether and we could easily express as much in system *D* with the unproblematic addition of completely strong permissibility:

$\langle p \rangle_S (R \,\&\, \neg F)$

Of course, given usually very extensive lists of prohibitions, these permissions will usually look more like:

$$\langle p \rangle_S(R \,\&\, \neg F_1 \,\&\, \neg F_2 \,\&\, \neg F_3 \,\&\, \neg F_4 \,\&\, \neg F_5 \,\&\, \dots \,\&\, \neg F_{n(n>1)})$$

Unwieldy as this is, Kamp's account too requires a finite standing list of prohibitions and so any objection on grounds that this list is infinite, uncertain, etc. is an objection that may be equally applicable to Kamp's view. Still, where we expect perfect specificity and precision from formal semantics, we may not expect as much from pragmatic accounts.

In any case, for Kamp the paradox of free choice permission has a centrally pragmatic component and it is this focus on natural language pragmatics which has come to dominate discussions of free choice permission. Interestingly, Kamp's particular attempts at solving the problem are less well received than is his general insight to consider the boundary between semantics and pragmatics and the importance of pragmatics in dealing with the problem. This emphasis on pragmatics generally is articulated by Kamp as follows:

> …natural language is of fundamentally greater complexity than is imagined by those who believe that we can describe the semantics of at least the declarative parts of natural language by theories which are based on no other concepts than those of satisfaction and truth (Kamp 1973, 74)

In many respects, this is an idea which must have been circulating around the time of Kamp's paper[39], as shortly after[40], and independently of Kamp's views,

[39] Consider, for example, Kamp's reference within his free choice permission paper to a permission statement as "…the speech act in question" (1973, 70). This draws a clear and timely connection to John Searle's *Speech Acts: An Essay in the Philosophy of Language* (Searle 1970) where Searle draws heavily from (his teacher) J.L. Austin's arguably identical concept of an illocutionary act. In Austin's *How to Do Things with Words* (Austin 1962) – a work which Searle cites repeatedly and acknowledges, a key idea is that not all declarative sentences are statements (needing to be true or false) but may be *performatives*. These utterances may not simply express some truth/falsehood but instead function to change the social reality of which they are part. The similarity to Kamp's position on permission statements and permission-granting is striking.

[40] Though published in 1975, Grice's lectures on logic and conversation were first delivered at Harvard as the William James Lectures in the first part of 1967 (Grice 1989, vi).

Paul Grice would publish ‘Logic and Conversation’ (Grice, 1975). Grice’s views in that paper share Kamp’s intuitions about the subtlety and essentially pragmatic character of natural language and his insights there would ultimately thematize some of the most dominant pragmatic treatments of the paradox of free choice permission.

5.3 Grice and the Maxims of Conversation

Since Kamp’s first suggestion that the problem had an essentially pragmatic component there have been many pragmatic solutions given in response to the paradox of free choice permissibility. None, however, have gained so wide an acceptance as those approaches which appeal to Paul Grice’s maxims of conversation and related ideas about conversational implicature.

Grice’s writings on the pragmatics of ordinary language conversation (1975, 1989) were a turning point in discussions of both pragmatics and the relationship of logic to language. Prior to Grice, common conceptions of logic were rooted in the idea that a fully developed logic should capture our intuitions about inferences, no matter how subtle they seemed to be in our discourse.

In cases where logical connectives or inferences seemed at odds with intuition, there appeared to be room for improvement in our understanding of the semantics of these operators. With Grice, however, a case was made for the adequacy of ordinary logical tools with standard semantics. Where ordinary logic broke with intuition or everyday reasoning, Grice invoked a different explanation.

For Grice, the target of this analysis is at the level of what is implicated in conversational contexts. Frequently, he notes, we are faced with conversational situations in which some expressions seem to correctly imply other expressions where there is no direct logical connection. As it is the conversational context that seems to drive these *implicatures* (1975, 43), we can rightly regard Grice’s focus as similar to Kamp’s insofar as it involves *pragmatic* features of conversation.

Grice calls these cases *conversational implicatures* (1975, 45) and shows that the dynamics of conversational interaction can shape the meaning of expressions and, thus, inferences from these expressions[41]. That is, these inferences are not solely about the meaning of specific expressions and the precise inferences we are able to make from these. Rather, these implicatures are the result of the relationship between language and its users.

Notice the similarity here to Kamp's pragmatic approach. For both Kamp and Grice, the semantics of our logical operators and their use in language are not a subject of concern. These operators do the job they are supposed to do and in exactly the way we would expect given the usual rules of truth-functional logic or satisfaction of the modal operators.

But, crucially, this is not the end of the story. For Kamp, just as for Grice, assumptions are made about the ways in which interlocutors are participating together in conversation. Kamp's solution involved assumptions about authority and information exchange in permission granting. For Grice, the more basic idea is that these users are engaged together in a cooperative enterprise of communication.

It is this critical assumption - that interlocutors in conversation are trying to be helpful in their exchanges – which allows Grice to explain a number of otherwise puzzling communicative inferences. The character of this cooperation is made explicit by Grice in the following general rule which he calls the *cooperative principle*:

> Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. (Grice 1975, 45)

Under the umbrella of his principle of cooperation, Grice includes four categories of conversational cooperation under which fall a number of maxims and sub-maxims. These categories can be roughly summarized as follows:

---

[41] Note that Grice identifies another kind of implicature – *conventional implicature* – in which meaning is shaped by conventional use of terms. Grice provides the example is 'He is an Englishman: he is, therefore, brave' (1975, 44). C*onversational implicatures* are, on Grice's view, distinct from situations like this and, hence, a subclass of nonconventional implicature.

Quantity

This category concerns the quantity of information exchanged in conversation. Here Grice provides two maxims of quantity (1975, 45):

1. Make your contribution as informative as required (for the purposes of the exchange).
2. Do not make your contribution more informative than is required.

Quality

This category concerns the truthfulness of information exchanged in conversation (or, minimally, whether an interlocutor actually believes what they are saying). Grice provides the single supermaxim 'Try to make your contribution one that is true' as well as two sub-maxims of quality (1975, 46):

1. Do not say what you believe to be false.
2. Do not say that for which you lack adequate evidence.

Relation

This category concerns the degree to which information exchanged is relevant or bears directly on the content of the conversation. Grice provides a single maxim of relation (1975, 46):

1. Be relevant.

Manner

This category concerns how things are said in conversation. Grice provides a single supermaxim 'Be perspicuous' under which he suggests various maxims such as (1975, 46):

1. Avoid obscurity of expression.
2. Avoid ambiguity.
3. Be brief (avoid unnecessary prolixity).
4. Be orderly.

These maxims are generally held to be basic to Grice's account, though may be dispensable in favor of a more basic maxim (e.g. Relevance – Sperber & Wilson, 1995). In any event, Grice himself considers the foundation of his view to be the overarching cooperative principle.

To illustrate the manner in which the cooperative principle and the associated maxims of conversation function, consider the following possible exchange:

> Charles: I think I'll start dating again
>
> Diane: My friend Elaine is a nice person

The response given by Diane, strictly speaking, could be completely unrelated to the statement made by Charles. When we assume cooperation, however, the *implicature* is clear. Diane is adhering to the maxim of relation in offering a response that has some relevance to Charles' statement. The meaning of Diane's response seems to be something like 'perhaps my friend Elaine would be a good person for you to date'.

As a matter of semantics, what Diane has said cannot account for this additional information, but assuming that Diane means to be relevant and make a conversational contribution, the path from her explicit claim to the implicature is clear.

Consider another example:

> Anne: Where is the class held?
>
> Bill: Somewhere in the Arts Building.

The response given by Bill is one we can interpret through the cooperative principle and, more specifically, the maxims of quantity. Anne has asked a question and Bill has provided information in his answer pertaining to that question. The correct or appropriate amount of information would ideally be more specific than that Bill has offered and so, assuming that Bill is adhering to the cooperative principle, it follows that Bill has provided as much information as is possible. Bill, it seems, does not know the precise room where the class is held.

Importantly, for Grice, we can purposely flout these maxims in cases like sarcasm or metaphor, but even in such cases, it is the assumption of mutual understanding that cooperative maxims are being flouted which drives implicature.

And, as we will see, this assumption of cooperation can be put to good use in the analysis of free choice disjunction.

5.4 The Standard Account

While there are many Gricean proposals to deal with the free choice permission paradox, the most influential is the solution proposed by Kratzer & Shimoyama in their 'Indeterminate Pronouns: The View from Japanese' (2002). Notably, as Grice's 'Logic in Conversation' (1975) was published just a couple of years after Kamp's 'Free Choice Permission' (1973), Gricean tactics for dealing with the problem have been a natural fit since Kamp's turn toward pragmatics.

For the purpose of analyzing free choice disjunction, the most common proposed relevant maxim in play is the maxim of quantity (e.g. Schultz 2005) and Kratzer & Shimoyama also trade heavily on this aspect of the cooperative principle. On an interpretation of disjunction as truth-functionally inclusive, the maxim of quantity has a significant bearing on the role of disjunction in conversation since, given P, it follows that

$$P \vee Q$$

And yet, it would be uncooperative to assert this where we could truthfully assert P instead. From a cooperative assumption, expressions like the above violate the maxim of quantity by providing less information than is available. This is not to say that statements like 'P or Q' can never be made – after all, there could exist uncertainty about which of these are true, but when interlocutors are cooperating, we would not make disjunction claims where one or the other disjunct is known to be the case. As we will see, however, the matter is complicated somewhat when free choice is introduced.

When a choice is offered the information presented is assumed to be appropriate to the situation at hand. That is, assuming cooperation, we would not expect that more information is being offered than is necessary, and importantly, we do not expect that less information is being offered than could be in order to make the details of our choice comprehensible. As we will see, when we assume that the permission-granting authority (waiter/restaurant) knows what we can have and does their best to convey the right quantity of information, the meaning of the free choice permission becomes clear.

Again, not all disjunctions work this way. In many cases, a disjunction is offered precisely in accordance with the maxim of quantity in order to convey uncertainty. For example:

> Anne: Where is the class held?
>
> Bill: Room 65 or Room 67.

Here, if we assume Bill is cooperating in accordance with the maxim of quantity, the disjunction functions exactly as a disjunction should. If Bill knew that Room 65 was the correct room, he would be in violation of the maxim of quantity when offering the response above. And so, Bill's response indicates that Bill doesn't know for certain which room is the correct one, but he has nevertheless offered as much information as possible to be helpful. Bill's uncertainty follows from the way that adherence to the maxim of quality constrains his response. This constraint has been made explicit by Gamut as:

> Quantity Constraint. If a Speaker asserts "*q*", then for all *p* logically stronger than *q* such that *p* is a relevant alternative to *q*, there must be some reason the speaker refrained from asserting "*p*." (Gamut 1991, 205)

Related to this constraint is the idea that in context we often have some set of salient options, objects, questions or ideas at play in the conversation. How these are spoken about governs directly how the maxim of quantity will operate.

For example, when two options are under discussion – Kratzer & Shimoyama use the example of an algebra book and a biology textbook (2002, 18) - and the claim is made that:

> You may borrow the algebra textbook

we feel licenced to make an exhaustivity inference. That is, it seems we may *not* borrow the other textbook and can *pragmatically* infer as much. By asserting that the one may be borrowed, then in accordance with the maxim of quantity, the possibility of borrowing the other as well is ruled out. Otherwise, the speaker should have asserted the possibility of borrowing the other since both were salient to the discussion. Kratzer & Shimoyama (following Zimmerman 2000) think this kind of exhaustivity inference occurs when we have well-defined options under consideration; the central reasoning behind exhaustivity inferences following from Grice's maxim of quantity.

Were we to know that both were permitted as borrowing options, suggesting the borrowing of only one seems an egregious violation of the maxim. Even in the case that we are unsure of the permissibility of borrowing the other, it seems that (even to avoid erroneous exhaustivity inferences) that information should be offered.

This brings us to the free choice permission case. As a permission is being granted by an authority, we can safely assume that the authority in question *knows* which permissions are legitimate (in at least some if not most free choice cases). Consider for example:

$$\langle p \rangle (R \vee S)$$

From a permission claim like this made in context we generally feel licensed to make a few assumptions. For example, we typically take the disjunction to be *closed* and exhaustive (at least until further permissions are granted). So, given such a permission, were it the case that we discovered that one of the disjuncts was not, in fact, allowed then we could infer that the other is permissible. And, allowing for exhaustivity inferences, were the permission of one stated alone, this would exclude the other as permissible.

Assume there is no chance that the permission-granting agent is unsure of which among {R, S} are truly permissible or impermissible. This parallels the ordinary restaurant scenario where we have good reason to believe that the waiter/restaurant *knows* which options are actually available. We ask ourselves then – why this expression? The speaker either knows that R is permissible or knows that R is not permissible, and so also for S.

If the speaker knows that R is not permissible, then the above free choice disjunction violates the maxim of quantity. For if the speaker knew that R was not a legitimate option and was trying to be as helpful as possible the speaker would have said instead:

$\langle p \rangle S$

But the speaker did not say this, and we assume that the speaker is trying to be helpful. So it must be the case that R *is* permissible. Then why not say this? Kratzer & Shimoyama argue that the explanation must be to avoid the exhaustivity inference. If R is asserted as permissible, we might conclude that S is not permissible, and the intention is to avoid this.

Similarly, if the speaker knew that S was not permissible, they should have said instead:

$\langle p \rangle R$

But again, the speaker did not say this, and so we know that S is permissible. As above, the desire to avoid an exhaustivity inference prevents the assertion of S as permissible.

Hence, from the initial free choice disjunction and the assumption of cooperation, we get the implicature (Kratzer & Shimoyama, 19):

$\langle p \rangle R \ \& \ \langle p \rangle S$

This result falls in line with Zimmerman's *choice principle* (Zimmerman 2000, 256) but does not come as a matter of semantic entailment. Rather, this is a pragmatic inference.

Importantly, the free choice inference is not the result of immediate Gricean implicatures. By making statements with an aim of avoiding or preventing the interpretation of implicatures by an interlocutor (the exhaustivity inference) we are engaged in multiple levels of pragmatic implicature (implicatures *about* implicatures). This multi-levelled implicature determination is a higher-order *recursive pragmatic strengthening*[42]

This solution has had considerable influence and acceptance (Fusco 2014, 276). Where other pragmatic accounts of free choice permission have disagreed with Kratzer & Shimoyama, the disagreement has often been focused only on details or exceptions and has frequently been clarificatory or elaborative in nature (e.g. Alonso-Ovalle 2006, Fox 2007)[43].

5.5 Disjunction vs. Conjunction

In the spirit of Grice and his maxim of quantity, an obvious question remains. If the meaning of a free choice 'or' – for example:

> You may have coffee or tea

is actually a free choice 'and':

> You may have coffee and you may have tea

then why not say this? If one is trying to be helpful and trying to be so in adhering to the maxim of quantity, why not simply express the conjunction of permissions in the first place?

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[42] This higher order pragmatic strengthening can be contrasted with ordinary (immediate) scalar implicature. There have been attempts to derive free choice implicatures directly (e.g. Vainikka 1987) though these have not been the focus of most recent work on free choice pragmatics.

[43] In a survey of pragmatic approaches to the problem of free choice disjunction, Meyer (2016) contrasts Neo-Gricean approaches (with higher-order pragmatic strengthening) like that of Kratzer & Shimoyama with 'grammatical' approaches to the problem (e.g. Chierchia 2004, van Rooij and Schulz 2004, Spector 2006, Fox 2007). The difference is subtle and hinges on implicatures occurring at the level of grammar by including a 'covert' exhaustivity operator at the level of language meaning.

One possible answer rests with the ambiguity of the natural language free choice permission and the manner in which these permissions are frequently exclusive. That is, while we may be able to conclude (via pragmatic implicature) that

$$\langle p \rangle R \,\&\, \langle p \rangle S$$

We nevertheless do not want to be able to conclude that

$$\langle p \rangle (R \,\&\, S)$$

While the free choice 'and' doesn't imply this, it nevertheless doesn't rule it out either. Moreover, we immediately get this faulty implication with any attempt to make the free choice 'and' shorter as in:

You may have coffee and tea

So, the 'or' expression not only is more economical than the 'and' expression, if we assume that the 'or' is exclusive, it additionally helps to rule out the possible ambiguity present in the 'and' case.

All of the reasoning given by Kratzer & Shimoyama in their Gricean analysis of free choice permission can be applied to the exclusive 'or' case and, arguably, the exclusive 'or' is just as natural to ordinary language as the inclusive 'or'. And yet the exclusive 'or' case:

$$\langle p \rangle (R \veebar S)$$

is logically equivalent to the inclusive:

$$\langle p \rangle ((R \,\&\, \neg S) \vee (S \,\&\, \neg R))$$

So, we can follow the Gricean reasoning

If $\neg\langle p \rangle (R \,\&\, \neg S)$ then why not say $\langle p \rangle (S \,\&\, \neg R)$, so $\langle p \rangle (R \,\&\, \neg S)$

If $\neg\langle p \rangle (S \,\&\, \neg R)$ then why not say $\langle p \rangle (R \,\&\, \neg S)$, so $\langle p \rangle (S \,\&\, \neg R)$

to the logically stronger conclusion:

$\langle p \rangle(R \,\&\, \neg S) \,\&\, \langle p \rangle(S \,\&\, \neg R)$

Thus, it is the free choice 'or' which suggests that we cannot have both options together even if each is permissible on its own. After all, when an exclusive disjunction is present outside of a permission, we know that both cannot be true, and it is this exclusivity which the 'or' in a free choice permission intends to convey

Notice, even the above *exclusive* expression still does not completely rule out:

$\langle p \rangle(R \,\&\, S)$

The decision to not express choice with an 'and' is perhaps *pragmatically* suggestive that this is not an allowed permission but there is not a rigorous account of how:

$\langle p \rangle(R \veebar S)$

rules out the permission of both together in a way that

$\langle p \rangle R \,\&\, \langle p \rangle S$

does not. Each of the above expressions is consistent with the possibility that it is permissible to have both options together. The best translation still seems to be:

$\langle p \rangle R \,\&\, \langle p \rangle S \,\&\, \neg\langle p \rangle(R \,\&\, S)$

If the pragmatic account is persuasive, then further work must explain how the 'or' functions to *pragmatically* entail that translation in a way that the 'and' does not. After all, in natural language, we use the 'or' in free choice almost ubiquitously.

One answer is straightforward economy in expression. Compare:

You may have beef or fish

You may have beef and you may have fish

The first is shorter in length, contains fewer syllables and seems to carry the same pragmatic consequences. Most of our natural language assertions are minimally consistent with additional facts not explicitly ruled out by these assertions. For example, the claim:

Alexandria is in Egypt

is minimally consistent with the claim

Alexandria is the name of my friend

As a result, we are faced with some ambiguity when we make most claims. Given context, it might be easy to rule out that my friend Alexandria is in Egypt (e.g. she might have been seen earlier that day). Similarly, in the case of free choice permission, we may rule out the possibility of choosing both options together given past experiences, observations of those around us, or other contextual background information about a choice. Thus, we'd simply use the 'or' expression as a matter of economy in communication.

Another possibility is to follow Kamp in thinking that permission claims remove standing prohibitions (Kamp 1973, 63). If our background assumptions are such that we think before ordering in a restaurant that we cannot just take whatever we want, then ordering a dinner special which comes with beef or fish grants the permission:

$\langle p \rangle (B \vee F)$

Following the standard account, we can make the implicature to

$\langle p \rangle B \;\&\; \langle p \rangle F$

But notice, this still will not get us to the permissibility of both together since the standing prohibition has not been overturned. Thus:

$\neg \langle p \rangle (B \;\&\; F)$

And so, our desired outcome holds:

$\langle p \rangle B \;\&\; \langle p \rangle F \;\&\; \neg \langle p \rangle (B \;\&\; F)$

Following Kamp here needn't be a fallback onto his solution as we may still be uncomfortable with the treatment of the free choice permission as a kind of strong or robust permission. In this way, we can still think that a free choice permission functions as an ordinary permission claim while relying on Gricean implicature to arrive at the conjunction of permissions.

5.6 Uncertainty Implicatures

Perhaps the most plausible explanation as to why we use an 'or' formulation rather than a conjunction of possibilities in our natural language expressions of choice is the manner in which 'or' is so frequently used to express uncertainty. Consider the exchange:

> Anne: Where is the class located?
>
> Bill: Room 65 or Room 67.

In an exchange like this one, where we assume that cooperation is present, we know, in accordance with the maxim of quantity, that Bill is not offering a logically weaker claim here than what he knows to be true. For example, suppose Bill knows that the class is in Room 65. The response given in this case would be accurate (since the class being in room 65 does entail the disjunction given) but in that case it would not be cooperative. Thus, assuming cooperation, it cannot be the case that Bill knows that the class is in Room 65.

The 'or' here is indicative of some ignorance on Bill's part and in cases of cooperative exchange is always indicative of some speaker ignorance. So, let's consider again the familiar coffee or tea choice but framed as a cooperative conversation between a restaurant patron and a server:

> Anne: Does the breakfast come with a beverage?
>
> Bill: Yes, it comes with coffee or tea.

In this case, the situation is somewhat different. There is ample reason to believe that Bill *knows* whether the meal comes with a beverage and, if so, which beverage. So why did Bill use the 'or' in this answer? As mentioned, Bill cannot use an 'and' since the meal does not, in fact come with coffee *and* tea. Though Bill knows precisely what options are available, the 'or' here nevertheless indicates uncertainty on Bill's part – that is, Bill does not know what beverage will actually come with the meal. Bill does not know what the patron will order! I have argued that Bill could correctly say:

Bill: You may have coffee, and you may have tea, but you can't have both.

But, in this case, the 'or' is more economical while also preserving an important pragmatic feature of 'or' usage in conversation – expression of uncertainty. Not uncertainty about the permitted options but, rather, uncertainty about what will be chosen. And, this expression of uncertainty, when we can assume full speaker knowledge regarding the disjuncts presented, indicates that a choice is being offered.

To further illustrate, consider again our classroom example, but assume that Anne is an instructor asking about the location of her class in the coming term and that Bill is a campus scheduling expert. As in the waiter case, we now assume that Bill knows exactly where the class is held (or could be held). The exchange takes place:

Anne: Where is the class located?

Bill: Room 65 or 67.

Now, assuming cooperation and knowledge on Bill's part, the situation takes on exactly the same character as the restaurant example and we could arguably read the exchange as the offering of a choice. That is, Anne is being offered a choice of rooms and in this case, Bill's uncertainty is uncertainty about which room she would prefer.

5.7 The Logic of Implicature

The preceding discussion presents what I consider the two most important pragmatic solutions to the paradox of free choice permission. Kamp's solution is significant because it was the first attempt at analyzing the problem from a pragmatic perspective. The solution of Kratzer & Shimoyama is significant because most in the literature who adopt pragmatic solutions today either accept it or have a sufficiently similar view.

Nevertheless, I do not mean here to suggest that the two solutions offered above capture all possible pragmatic explanations. What's most important for my purposes is to differentiate the motives and considerations behind offering pragmatic solutions rather than semantic ones, though, as we shall see, the boundaries between these approaches may be less clear than is often believed.

A curious feature of most pragmatic analyses of language meaning is that, while we may feel that the implicatures present in language are a result of contextual factors, there is nevertheless an important reasoning process taking place, given whatever features of our situation seem relevant. Put another way, there must be an underlying logic of even the pragmatics of our discourse. The fact that pragmatic analyses are influential at all seems due to this underlying logic; otherwise our pragmatic accounts would be offering little more than doubt regarding the viability of purely semantic accounts of logic in language.

The pragmatic account *does* involve a series of inferences which, for the free choice permission case, have been roughly articulated in the preceding discussion. However well we think that pragmatic accounts generally succeed at bringing us to an informative expression or understanding of language use, we cannot deny that some (perhaps richer or more expressive) logic is at work in these pragmatic inferences. This suggests possibilities for formalization and systematization.

Chapter Six

Formalizing Choice Pragmatics

6.1 From Pragmatics to Semantics

As we have seen in the previous chapter, in dealing with free choice permission the pragmatic approach taken by Kamp (1973) bore some similarity to the semantic approach taken by Asher & Bonevac (2005)[44]. The internal logic of pragmatic accounts seems to partially account for this similarity. For by enriching or otherwise modifying a logical language, we may be able to treat the very rational processes underlying pragmatic accounts as systematic logical processes with a semantics of their own.

In some cases, this may be as simple as treating background assumptions which play a pragmatic role as implicit premises in an argument or as axioms with which we can augment a logical system. Examples of such arguments are commonly found in ordinary discourse and these arguments can be recast in

[44]There are some notable differences – e.g.: Kamp restricts his analysis to a very specific kind of permission granting, whereas Asher & Bonevac consider a much more general solution. This may be especially pertinent in cases like the free choice menu 'coffee or tea' example which may not fall under Kamp's analysis of a single concrete permission granting agent. Still, while Kamp's analysis is carefully restricted, he does make rough suggestions that the general idea can be extended to other if not all free choice permission scenarios.

propositional logic. Consider the following argument (Woods 2004):

> It's raining, so Eveline will not be driving to Calgary.[45]

In propositional logic:

$$R \therefore \neg D$$

As stated, from a formal perspective this argument is truth-functionally invalid. The semantics of a language for propositional logic are given by the concept of an assignment of truth-values to atomic statements and by the truth-functional definitions of the logical operators. In the case above, the truth of R as an atomic proposition has no bearing on the truth (or falsity) of D. If we insist upon making sense of this argument through propositional logic alone, we must stipulate the relationship between these claims. For example

$$R \rightarrow \neg D$$

Even with this relationship established as an implicit premise[46] it may be that all we have done is to formally explicate the pragmatics involved. After all, the claim above is not a part of the argument itself. Nevertheless, treating premises as implicit is quite ordinary in making sense of exactly these kinds of arguments. And, with this additional premise in hand, the argument is now valid. But in any discussion of pragmatic versus semantic solutions to problems like these this is exactly what is at issue – Is this premise an explicit part of the argument? The obvious answer seems to be *no* – at least not as initially presented. We are treating this premise as implicit.

Making implicit premises explicit can be an interesting strategy for making sense of these arguments in a formal way, but it cannot suffice as a fully

[45] Hitchcock (2017, 140) calls these *occasional arguments* following Quine's similar ascription to occasional sentences. Just as for Quine's occasional sentences, where the truth-value of the sentence is determined from the particular context of the occasion of utterance, so too does the conclusion of an *occasional argument* follow only once one has incorporated contextual (pragmatic) information in the interpretation of that argument.

[46] As Hitchcock points out (2017, 140), this may be a flawed premise, as occasional arguments may have such specific contextual factors that a claim like "If it's raining, Eveline doesn't drive to Calgary" may be too general. In any case, it seems plausible that with a sufficiently detailed account given we will be able to construct an appropriate conditional to use as an implicit premise in a valid argument.

semantic treatment of the reasoning or even as a formalization of the pragmatics involved. After all, an implicit premise is likely to be contextual or specific to particular reasoners. For a plausible fully formal analysis, some broader structural aspect of the reasoning must be captured by the system or the meaning of the expressions involved. One way to do this is through an axiomatic treatment which we can understand as non-contextual and fundamental to our understanding of a system of logic.

In the case above, we could trivially try to establish an implicit premise like

$$R \rightarrow \neg D$$

as an axiom of a system, (and thus available to any reasoning) but we are in that case dealing with some system other than ordinary propositional logic (and a highly dubious one at that). No matter what approach we take, in order to make the argument *formally* valid we need the premise stated. At the level of natural language reasoning, the warranted inferences of the argument may plausibly be determined by contextual or pragmatic factors whereas at the level of the formalism this pragmatic component must be included for the argument to be valid.

Examples like this highlight features of analysis present in the free choice disjunction case as well. Those who treat the subject move regularly from natural language claims to formal claims and then conflate or otherwise try to relate the inferences we can make in natural language with those we can make in the formalism. This is, perhaps, the kind of thinking that lies behind the motivations of those seeking semantic solutions to the problem of free choice disjunction.

In the above case, for example, we may realize that a perfectly viable pragmatic explanation for the reasoning exists. But, treated formally, the inference is illegitimate (and the best we can do to deal with it formally is import pragmatic features as implicit premises in order to make it valid). In order to make it legitimate in a fully semantic way, we need a more systematic account of how those features can be accounted for more generally.

Though it is not their intention to provide a semantic version of Kamp's account, Asher & Bonevac (2005) provide a semantic solution to the paradox of free choice permission that is similar to Kamp's original pragmatic solution (1973). But whereas Kamp wants to preserve the ordinary meanings of the logical operators and makes no attempt to augment von Wright's system of deontic logic, Asher & Bonevac augment von Wright's deontic system with their inclusion of a defeasible conditional as well as a concept of strong permission (defined using Anderson's re-translation of deontic logic using sanctions). In this way, it may be that some of the pragmatic features of Kamp's account can find explication through systematic features of a modified deontic logic.

Recall, Kamp claimed that permission givers remove standing prohibitions, so that a permission like

$$\langle p \rangle (R \vee Q)$$

moves all formerly prohibited worlds where it is true that

$$R \vee Q$$

into the realm of permissible worlds (provided that no other standing prohibitions are violated where the permission giver has no authority to lift those prohibitions).

As noted in the previous chapter, Kamp's approach is similar to notions of *strong permission* insofar as both Kamp's approach and strong permissibility treat permission as indicative of more than the existence of some single world where the outcome is permitted. But, where unrestricted strong permission moves *all* worlds where the stated permission is true into the realm of permissible worlds, Kamp's account continues to rule out worlds which have further prohibitions in place. Nevertheless, both the strong permission approach and Kamp's approach include enough worlds in this new class of permitted worlds so that there will be worlds accessible in which each disjunct in the free choice permission is true (and hence, the choice principle follows).

Asher & Bonevac accomplish a similar limitation on pure strong permissibility *within* a logical system by treating this permission as a special defeasible kind of strong permission. I called this *robust permission.* For example:

$\langle p \rangle_R (R \vee Q)$

This permission is defined using the defeasible conditional:

$\langle p \rangle_R (R \vee Q)$ iff $(R \vee Q) > \neg S$

or, $\langle p \rangle_R (R \vee Q)$ iff $[n]((R \vee Q) \rightarrow \neg S)$

That is, the free choice permission makes all *normal* worlds which realize it permissible. At the level of dealing with permission statements already given, this solution moves us towards a semantic version of Kamp's account, especially if we can formally account for *normal* worlds as worlds where the permission giver has authority to lift standing prohibitions. In one example, Kamp characterizes this class of worlds in the following way:

> But how is this class determined? What, for example are the actions rendered permissible by my statement [Michael may go to the beach]? Surely not *every* way of making the statement 'Michael goes to the beach' true. He is not to go by taxi; he is not to go to that section of the beach where there is no swimming guard … and, of course, as always he is to keep out of mischief … All these ways of going to the beach remain proscribed; for they would either violate a standing prohibition which my permission statement has not lifted or else some set of laws, moral rules or codes of conduct … I do not even have the power to modify. (Kamp 1973, 63)

So the granting of a given permission (stated as a proposition) makes all worlds which realize this claim deontically accessible subject to the following two conditions:

i) Worlds which are subject to further sanctions not lifted (or implicitly lifted[47]) by the permission remain forbidden.

ii) Worlds subject to further sanction outside the authority of the

[47] As noted in the previous chapter, the authority of the permission-granting agent *A* may have some further effect in removing prohibitions if an outcome is now permitted but this requires the realization of some further (formerly prohibited) propositions which are also in the power of *A* to rescind.

permission granting agent remain forbidden.

If we understand the *normal* worlds of Asher & Bonevac's approach in exactly this way, the two views converge.

A remaining worry in treating the semantic approach of Asher and Bonevac in this way is that we are merely smuggling pragmatic concerns into these conditions on normal worlds. After all, Asher & Bonevac do not provide an account of permission granting at all – they merely treat permission claims as given and not relativized to any particular authority or as the result of some permission granting action which removes prohibitions. It is certainly true that these pragmatic concerns remain though we may be able to go further in incorporating these ideas formally.

6.2 Kamp's Account Formalized

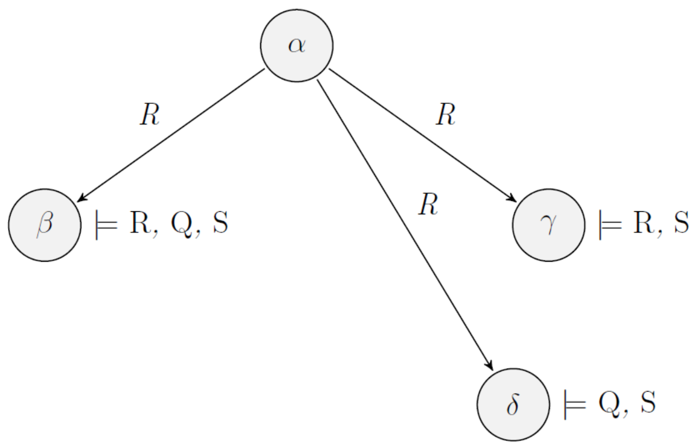
If we want our logical system to fully capture Kamp's intuitions about permission granting as removing standing prohibitions, we can do that as well. Just as choice can be treated as an action which moves agents between worlds, so also can the granting of permissions be treated as an action.

Consider two agents *A, B* such that *A* is an authority capable of lifting standing prohibitions and *B* is the agent from whom *A* will remove those sanctions (and thus grant permissions). For simplicity, let us treat all worlds as normal. That is, we will for the moment exclude all worlds where additional sanctions hold on the actions that *A* will permit or other actions are sanctioned.

In an initial state with standing prohibitions on R, Q for agent *B* we have the following model where the accessibility relation for *B* is the arrow labeled b. This accessibility relation can be understood as corresponding to an action type performed by *B*, the outcome of which is understood as a possible world. This base model is further supplemented by including a propositional letter S standing for 'a sanction occurs' using the (Anderson 1966) reduction of deontic logic. In each of the worlds accessible to *B* we see that the proposition S holds. That is, none of the worlds which agent *B* may access is permissible (not subject to sanction) and thus, all accessible propositional outcomes are prohibited.

Figure 6.2.1 Prohibition model

$\mathfrak{M}_9$:



For simplicity, we will consider this model complete (*B* has no access to possible worlds other than those indicated). Similarly, propositional valuations at worlds will include none other than those indicated. This is, admittedly, a vast oversimplification though it will suffice for the moment to illustrate the formalization proposed.

In model 6.2.1, R and Q are (for *B*) not deontically permissible insofar as any accessible world where they are true is also a world where S is true (a sanction occurs).

By further augmenting this model to include actions corresponding to the permission granting authority *A* (indicated by transition arrows labeled a), we can understand the granting of an ordinary (weak) permission as a kind of action. If *A* is able to grant permission to *B* to realize a world where (R or Q) we could represent this as:

$$[b]((R \vee Q) \rightarrow S) \,\&\, \langle a\rangle\langle b\rangle((R \vee Q) \rightarrow \neg S).$$

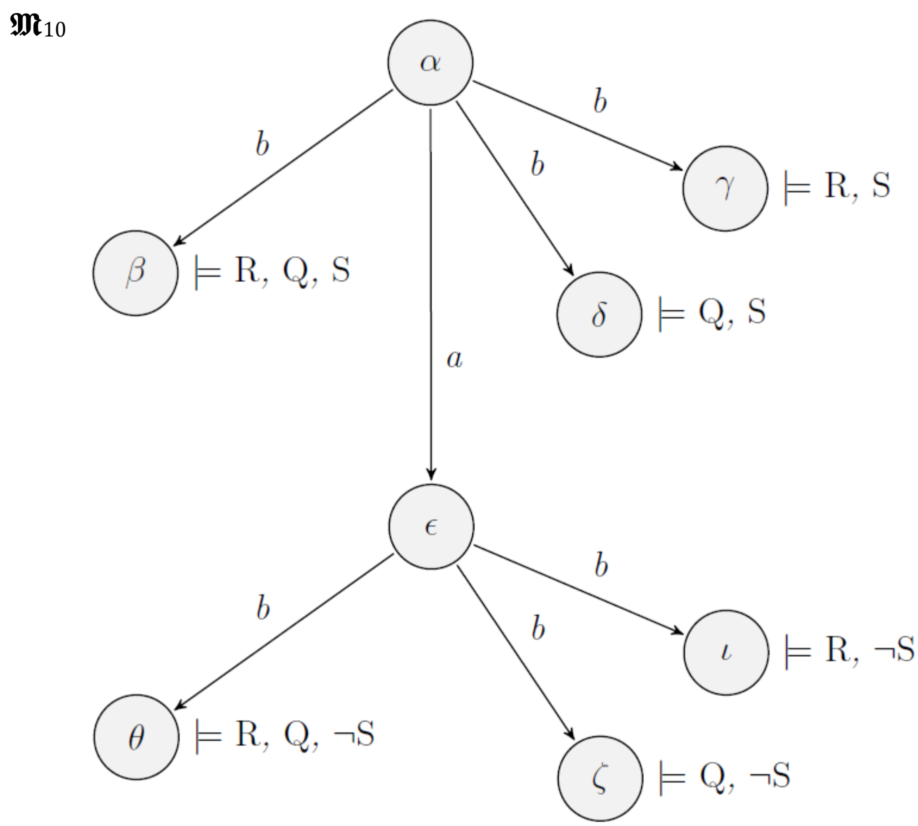
That is, in all currently b-accessible worlds, where (R ∨ Q) is true, a sanction occurs; but there are some a-accessible worlds from which there are further b-accessible worlds in which a sanction does not occur.

Similarly, *A* can be understood to grant the strong permission to *B* as follows:

$$[b]((R \vee Q) \rightarrow S) \;\&\; \langle a\rangle[b]((R \vee Q) \rightarrow \neg S).$$

So there is an a-action where *A* grants a strong permission for *B* to realize (R or Q). This will mean that all further b-accessible worlds where (R ∨ Q) is true will also be worlds where S is false. Returning to our model, we can augment it as follows:

Figure 6.2.2 Strong permission granting model



multimodal model with two separate accessibility relations a, b (one for Agent *A* and one for agent *B*). In world α there are no available b-actions where sanction does not occur. Agent *A*, however, has an available a-action such that those sanctions are removed. This action opens up the possibility for *B* to take any action where R or Q is realized.

Thus, after this permission, the situation is one where (R or Q) is strongly permitted for *B*:

$$\mathfrak{M}_{10}, \varepsilon \vDash [b]((R \vee Q) \rightarrow \neg S).$$

Notice, on this account, even the granting of permissions and the notion of authority itself can be understood as kinds of permissibility. In Model 6.2.2 agent *A* is capable of taking an action without sanction that removes sanctions (for *B*). This is a nice result, since authority can be accounted for in the following general ways.

A can grant a weak permission for *B* to realize φ:

$$[b](\varphi \rightarrow S) \;\&\; \langle a\rangle\langle b\rangle(\varphi \rightarrow \neg S)$$

A can grant a strong permission for *B* to realize φ:

$$[b](\varphi \rightarrow S) \;\&\; \langle a\rangle[b](\varphi \rightarrow \neg S).$$

At this stage, if we want to include additional actions and sanctions that account for the difference between normal and non-normal worlds we can simply introduce Asher & Bonevac's defeasible conditional to account for *normal* worlds and *robust* permissions, though it is possible to do so in other explicit ways as well.

In accordance with Kamp's thinking, we could, for example, treat each action as having sanctions specific to that action. So, in the case of outcomes (expressed propositionally):

$$P, Q, R, T, \ldots$$

we have sanctions[48] (expressed propositionally):

---

[48] This will complicate the logic somewhat, as we will require a function which connects atomic propositions to the appropriate 'atomic' sanctions. One way to do this is by treating particular sanctions as implications from action propositions to separate sanction propositions. For example, $(P \rightarrow S_1)$, $(Q \rightarrow S_2)$, etc. In this way, the removal of a particular sanction amounts to negating those implications.

$S_P, S_Q, S_R, S_T, \ldots$

And ultimately, a permitted world will still be one where no sanction holds. With this in mind we can now consider an example like the restaurant menu where a patron has a choice between two options R, Q.

Recall, part of the problem with treating free choice permission as a strong permission is that it made all worlds which realize the permission true. So, given a strong permission:

$$\langle p \rangle_S (R \vee Q)$$

interpreted as 'any world where (R or Q) does not result in sanction':

$$[p]((R \vee Q) \rightarrow \neg S)$$

we have the unfortunate consequence that some worlds where R will also be worlds where M (murder the waiter). But if we treat sanctions as distinct, we can easily resolve this by instead making the weaker claim:

$$[p]((R \vee Q) \rightarrow (\neg S_R \;\&\; \neg S_Q)).$$

So, as long as no further sanctions are violated, R, Q are strongly permitted. And, clearly it would still be the case that S_M (Murdering the waiter is sanctioned) and so those worlds would still not be permitted.

If we want to include information about the particular permission granting authority in a specific permission claim, then we can do so by making use of converse modalities. These 'backward looking' modal operators simply treat the existing accessibility relations and allow us to make claims about which transitions can have resulted in a given outcome[49]. These will be symbolized:

$$[x^{\cup}]\varphi$$

$$\langle x^{\cup} \rangle \varphi$$

[49] One, perhaps helpful, way of thinking about converse modal operators is that they take existing modal operators and their corresponding accessibility relations and 'flip' the directionality of the modal arrows.

where *x* is any of our usual accessibility relations. These are defined as:

$\mathfrak{M}$, s $\vDash [x^{\cup}]\varphi$ iff for all worlds t *x*-accessing world s, t $\vDash \varphi$

$\mathfrak{M}$, s $\vDash \langle x^{\cup}\rangle\varphi$ iff for some worlds t *x*-accessing world s, t $\vDash \varphi$.

So, for example, given model 6.2.2 above, we could say[50]:

$\mathfrak{M}, \varepsilon \vDash \langle a^{\cup}\rangle\top$

'There is some a-transition which leads to ε'.

Or, perhaps more interestingly:

$\mathfrak{M}, \varepsilon \vDash [b]((R \vee Q) \rightarrow \neg S)\ \&\ \langle a^{\cup}\rangle\ [b]((R \vee Q) \rightarrow S)$.

That is, in this model at world ε, *B* is strongly permitted to realize (R or Q) and there was some a-transition leading to this world before which (R or Q) was impermissible for *B*.[51]

This approach seems to capture much of Kamp's proposal (albeit in the extremely simplified manner presented here), and if we understand the notion of normality associated with the defeasible conditional in exactly this way, captures the *robust* permission of Asher & Bonevac as well[52].

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[50]The symbol ⊤ here is just the usual 0-ary logical operator standing for logical truth, or tautology. Used with a converse modality $\langle x^{\cup}\rangle$ this indicates only that there are possible x-transitions leading to the world where the 'backward' looking *x*-converse operator is true.

[51]Notice, we still have not said that it *was* an a-transition which resulted in the permission state, or that the only possible way of arriving at the permission state was such a transition. Moreover, in an expanded model we would like to differentiate between those prohibitions which are removed (for some agent) as a result of a permission-granting transition and those which continue to hold (perhaps as standing prohibitions unrelated to *A*'s authority). I have no doubt this can all be accomplished, especially if we allow further additions to our logical language (e.g. features of dynamic logics that I will describe later in this chapter). In any case, my purpose is not to provide a fully realized system or to argue strongly for such a system here. The point is merely that, with these tools, we can increasingly incorporate so-called pragmatic ideas into the logical language.

[52]This is not the only way we could accomplish this – while I have tried to keep this analysis as simple as possible, there are a number of altogether different (and sometimes more complex) strategies which may do an even better job. For example, by dealing with permission granting as a *dynamic* modality, we could treat permission granting as model update (see e.g. van der Torre & Tan 1998). In this way, rather than having permission-granting transitions

What are we to make of all this? Does this approach merely formalize the pragmatic considerations given by Kamp or is the problem of free choice permission a semantic problem after all? I will address this question at length in the following chapter but as a first pass I will offer a few thoughts. At the level of natural language[53], the issue is clearly pragmatic. But at the level of formalism it makes no sense to speak of pragmatics at all. At the level of formalism, we simply have symbols and transformation rules along with a semantics given in the form of models, truth tables, etc.

So, if we are attached to a particular formalism, we either accept all of those basic logical rules and consequences (at the level of formalism) and the only way we will be able to make sense of a pragmatic inference formally is to import those pragmatic concerns (for example, as implicit premises) with no expectation that the now fully explicit reasoning will match up with the actual stated argument and no expectation that these pragmatic features will somehow emerge from the formalism itself.

Or, if we wish to treat pragmatic concerns as they occur in a non-explicit way, and we want to do this in a formally, then the underlying structure or mechanism behind the reasoning must become included in the formal system itself. In the case of Kamp's pragmatic account, I think I have (building on Asher & Bonevac) provided a reasonably good sketch of how to treat these ideas formally.

Whether this formal treatment amounts to a fully semantic account is an interesting question. On some level, purportedly semantic approaches like that of Asher and Bonevac seem to subscribe to a kind of semantic minimalism. On this view, given that we have a semantics for the logical system that captures these ideas, and we may consider this logical system a stable semantic base for language, then perhaps the account is semantic. It is, however, a hotly debated topic whether such a stable semantic base can exist at all without 'intrusion' of pragmatic meaning. In any case, the formalization of Kamp's pragmatic solution is at least suggestive of a move towards semantically accounting for

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present in a static model, these permission-granting actions may update or change the existing model.

[53] Understood with a basic logical and semantic structure including only (for example) propositional, quantificational and modal operations.

some features of language beyond just ‘what is said’.

As I will show, we can do the same for the Gricean account, which has considerably more acceptance in contemporary literature on free choice disjunction.

6.3 Logical Dynamics

In order to formalize a Gricean account like that given by Kratzer & Shimoyama (2002) we will need a fairly robust system capable of capturing the kinds of deliberation involved in the Gricean reasoning process. Again, as a first pass at the problem I will do my best to keep the system as simple as possible. This will require some simplification[54] of the problem, though I believe many additional complexities can be dealt with by further supplementing the logic I will here describe. As we will see, even in a simplified form, things become quite complex.

The system employed can be loosely understood as a multi-modal logic containing modal operators encoding:

i) actions (e.g., choice or permissibility) for multiple agents.
ii) epistemic accessibility for multiple agents.
iii) a dynamic modality which shrinks information ranges by modifying the model with updated information.

This logic draws heavily from the tools of dynamic logic, especially the public announcement logic initiated in the work of Alexadru Baltag, Lawrence Moss and Słowomir Solecki (1998) and especially as described by Johan van

[54] In simplifying a problem in order to deal with it, there is always a danger that we fail to address the actual problem at all, but instead resolve the different, more simple problem. The system I here describe is quite complex and my main motivation for simplifying is to keep the resulting *exposition* as simple as possible, rather than to purposefully avoid a larger problem. Still, I am aware of this danger and leave it to the reader to judge whether even some aspects of this system are useful and suggestive of further additions to treat every aspect of the problem.

Benthem (2010, 2014).

The base logic for our system is a standard modal propositional logic with multiple accessibility relations each corresponding to a particular agent i. The accessibility relations may be thought of as ordinary modal accessibility relations between worlds relativized to each agent. With the inclusion of sanction statements, we can treat such as logic as deontic on its own just as we did in the preceding section (using the Anderson reduction). Or, we can treat these relations as explicitly deontic where I will express them as $[p_i]$. We supplement this deontic operator with an epistemic operator $[K_i]$. This language will have the following syntax:

$$\varphi ::= \varphi_a \mid \neg\varphi \mid \varphi \vee \psi \mid [i]\varphi \mid [p_i]\varphi \mid [K_i]\varphi.$$

Models are defined as tuples:

$$\mathfrak{M} = (W, \{\sim_i, i \in G\}, \{R_i, i \in G\}, \{P_i, i \in G\}, V)$$

where W is a set of worlds, $\sim_i$ are epistemic accessibility relations, R_i are ordinary (action) accessibility relations, P_i are deontic accessibility relations, G is a group of agents, and V is a valuation on worlds. Modal operators $\langle i\rangle\varphi$, $\langle K_i\rangle\varphi$, $\langle p_i\rangle\varphi$ are defined in the usual manner as $\neg[i]\neg\varphi$, $\neg[K_i]\neg\varphi$, $\neg[p_i]\neg\varphi$ respectively.

The truth definitions for the basic logical operators ($\neg$, $\vee$, &, $\rightarrow$, $\leftrightarrow$, $\veebar$) are standard. Though they are not explicit in the grammar, we will make use of all the usual binary operators including the exclusive 'or' defined in the usual ways. Tautologies and contradictions can be abbreviated as $\top$, $\bot$ and can function independently as 0-ary logical operators 'true' and 'false'.

Modal claims are made true as follows:

$\mathfrak{M}, s \vDash [i]\varphi$ iff for all worlds t such that s has R_i access to t: $\mathfrak{M}, t \vDash \varphi$

$\mathfrak{M}, s \vDash [p_i]\varphi$ iff for all worlds t such that s has P_i access to t: $\mathfrak{M}, t \vDash \varphi$

$\mathfrak{M}, s \vDash [K_i]\varphi$ iff for all worlds t such that s has $\sim_i$ access to t: $\mathfrak{M}, t \vDash \varphi$

To this stage, such a system, complex as it is, will behave as a standard modal logic with static models showing transitions between worlds as well as epistemic links indicating knowledge or uncertainty. But we will further augment this system with a dynamic modality:

$[!_i\varphi]\psi$.

This dynamic modality allows us to make claims about what follows after a model has been updated by further information. Models will change when the complete set of epistemic agents i represented in a given model ($i \in G$) receive information that will eliminate all counterexamples in the model (van Benthem 2014, 155).

Information φ can be introduced with the notation

$!\varphi$

This notation is not a part of the language on its own but can be thought of as an action encoded in the dynamic modality above. Van Benthem (2014, 156) describes this modality as one of 'public announcement' and indeed, that is how it will at least partially function here. We define it as:

$\mathfrak{M}, s \vDash [!_i\varphi]\psi$ iff if $\mathfrak{M}, s \vDash \varphi$, then $\mathfrak{M}|\varphi, s \vDash \psi$

A natural (if rough) way of understanding this is to interpret dynamic modal claims as saying 'After some update (e.g. R) some other proposition holds (e.g. Q)'. The definition above ensures this by stipulating that such announcement claims are only true when at every world where R is true, the model *given* R makes Q true[55]. For example:

$[!R]Q$

states that 'After an announcement of R, it follows that Q'. In accordance with our definition this means that after such an announcement we can effectively eliminate any worlds where it is not the case that R. As van Benthem states:

> The key idea now is that informational action is model change. The

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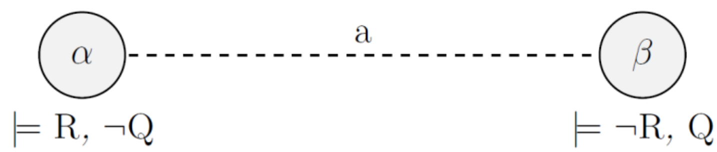
[55] The expression in our definition $\mathfrak{M}|\varphi$, s $\vDash \psi$ is read as 'the model *given* φ, at world s, guarantees ψ.

> simplest case is a *public announcement* $!\varphi$, which conveys all hard information. Learning with total reliability that φ is the case eliminates all worlds with φ false from the current model (2014, 155).

For example, consider the following simple model with a single (epistemic) accessibility relation ‘a’ corresponding to the epistemic uncertainty of an agent *A*:

Figure 6.3.1 Uncertainty model

$\mathfrak{M}_5$:



In this model with a single agent *A*, we see that *A* is uncertain between the truth of R and the truth of Q but does know that exactly one is false. That is:

$[K_a](R \veebar Q)$

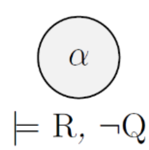
Now consider the reception of the following information:

!R

This ‘announcement’ of R shrinks the model, eliminating all worlds where R is false. Hence, our model becomes:

Figure 6.3.2 Public announcement update model

$\mathfrak{M}_5|R$:



This eliminates the uncertainty link present for A and so, before update, and at all worlds in the model:

$[!R][K_a](R \,\&\, \neg Q)$

This formula conveys that given some public announcement of R as hard information, agent *A* will know that (R and not-Q). Or, *given* R, the model is such that *A* knows (R and not-Q):

$$\mathfrak{M}_5 \,|R \vDash [K_a](R \,\&\, \neg Q)$$

As commonly presented (e.g. van Benthem 2010, 2014), public announcement is not relativized to particular agents as making or conveying the announcement. For the purposes of dealing with Gricean implicature, we will sometimes make use of a subscript i on public announcements as a way of picking out which agent is making the announcement. We can still deal with non-relativized announcements in the usual way and, importantly, relativized announcements do not complicate the system in any way beyond noting which agent made a given announcement[56].

We will make use of a few final modifications.

Each of our modal operators will have a corresponding backward-looking *converse*. These will be symbolized:

$$[i^{\cup}]\varphi \;\;|\;\; [K_i^{\cup}]\varphi \;\;|\;\; [p_i^{\cup}]\varphi \;|\;\; [!\psi^{\cup}]\varphi$$

and are defined as:

$\mathfrak{M}, s \vDash [i^{\cup}]\varphi$ iff for all worlds t such that t has R_i access to s: $\mathfrak{M}, t \vDash \varphi$

$\mathfrak{M}, s \vDash [K_i^{\cup}]\varphi$ iff for all worlds t such that t has $\sim_i$ access to s: $\mathfrak{M}, t \vDash \varphi$

$\mathfrak{M}, s \vDash [p_i^{\cup}]\varphi$ iff for all worlds t such that t has P_i access to s: $\mathfrak{M}, t \vDash \varphi$

$\mathfrak{M}, s \vDash [!\psi^{\cup}]\varphi$ iff for all worlds t such that t has $!\varphi$ access to s: $\mathfrak{M}', t \vDash \varphi$[57]

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[56] The reason for this is that public announcement logic can be considered a limiting case of dynamic epistemic logic including only 'announcement' events with preconditions of the truth of those announcements. As additional 'announcement' events will not change the system given that all agents still receive this information there is no complication beyond that already present in dynamic epistemic logic (see, e.g. Ditmarsch et al 2007).

[57] There is a notable difficulty in adopting a converse dynamic operator. As models update, there is not a functional relationship between worlds in the updated model, and the model 'before' the update announcement. I am here treating a converse dynamic operator loosely as a normal Kripkean accessibility relation between a world in the new updated model and a world in the pre-update model. In this way, we must retain the old model as a part of the new

As we saw in the previous section, these backward-looking modalities will allow us to say that at some world there is a corresponding transition type *leading* to this world. This is important since we want to make sense of Gricean reasoning involving stated claims (which are made in accordance with a cooperative principle). Thus, we will find a converse operator especially useful as applied to the dynamic modality of information update.

For example, if in some world α we want to express that this world is incompatible with some instance of:

Agent *A* having conveyed hard information φ

We can do so in the following way:

$$\mathfrak{M}, \alpha \vDash \neg\langle !_a\varphi^{\cup}\rangle\top$$

That is, there is no possible model update transition to α such that this update arises from a public announcement of φ made by *A*. The function of the ⊤ in the formal expression could be replaced by any tautology – and, since a tautology would be true at any world, there is no such transition.

Finally, we will introduce a special kind of public announcement action

$$!_i\varphi/c$$

As with other public announcements, these are not formulas of the language but are used within dynamic modal operators to convey what follows *after* information update. But, importantly, these will function as *cooperative* public announcements. We can define this as follows:

$!_i\varphi/c$ iff the announcement of φ is made by agent i in a cooperative way.[58]

model (Yap 2007). In what follows I will gloss over this concern for simplicity of exposition. See Appendix C for a more complete analysis of model expansion in the case of free choice disjunction.

[58] A natural worry here is that this *cooperativity* is not defined formally. For the moment we will treat this as a condition on public announcements with no further formal detail but having the usual Gricean meaning. In the next section, I will outline the formal details of cooperativity.

Just as with other kinds of public announcements these will operate in dynamic modalities in such a way that we will make use of expressions like:

$$[!_i\varphi/c]\psi$$

The purpose of such an announcement is to treat a public announcement by a specific agent i as *cooperative* with respect to the conversation or information exchange. As we shall see, it is with such a modal operator that we can make logical claims about Gricean implicature. For example, the maxim of quality is at least roughly captured by:

$$[K_i]\varphi \rightarrow (\neg(\langle !_i\psi/c^{\cup}\rangle\top \;\&\; [K_i](\psi \rightarrow \neg\varphi))$$

That is, if an agent knows some statement φ to be true, then it is not possible that they made a public announcement ψ which is cooperative such that this agent knows that ψ implies the falsity of φ.[59] More simply, you cannot be cooperative if you say what you know to be false (or anything that you know to imply a falsehood).

Admittedly, this formula misses some of the nuances of a fully developed maxim, as it, for example, rules out cases concerning speaker *beliefs*. In at least that case, a modal doxastic addition to this system may do a better job, but for the moment it suffices that we move closer to a workable sketch of Gricean cooperative announcement without imposing further technicalities on an already difficult system.

This resulting logic can be axiomatized by many of the usual axioms of epistemic logic as well as by public announcement logic and (if necessary) deontic logic. These can vary according to language design, but for our purposes here will consist of at least[60]:

[59] Such a principle is guaranteed by common axiomatizations of the public announcement logic (PAL) employed here, though we could construct versions in which misleading public announcements are possible. In any case, we will make use of further principles which are not guaranteed in this way, and this example will serve as a useful first step towards them.

[60] The fullest analysis would require axiomatizations and rules for backward-looking dynamic modalities, as well as precise axiomatization for 'cooperative' public announcement. As this is only meant to be a rough sketch, I will leave this work aside, though the following sections will provide some detail.

i) A classical (multi-modal) K base for standard actions $[i]\varphi$
ii) The classical axiomatization of standard deontic logic (as presented in chapter 1)
iii) Multi agent (multi-modal) S5 for our epistemic operators $[K_i]\varphi$ where each epistemic accessibility relation $\sim_i$ is symmetric, reflexive and transitive (an equivalence relation).
iv) Public announcement logic recursion axioms:

PAL1:	$[!\psi]\varphi$	$\leftrightarrow$	$(\psi \rightarrow \varphi)$
PAL2:	$[!\psi]\neg\varphi$	$\leftrightarrow$	$(\psi \rightarrow \neg[!\psi]\varphi)$
PAL3:	$[!\psi](\varphi \,\&\, \Delta)$	$\leftrightarrow$	$([!\psi]\varphi \,\&\, [!\psi]\Delta)$
PAL4:	$[!\psi][K_i]\varphi$	$\leftrightarrow$	$(\psi \rightarrow [Ki](\psi \rightarrow [!\psi]\varphi)$

For the sake of simplicity, derivation rules will remain open-ended and will include the most common rules of inference. We can, of course, be far more precise than this if we choose.[61]

6.4 Gricean Free Choice Dynamics

Following the outline provided in the previous chapter, I will attempt here to give a formal account of the main Gricean approach to Free Choice permission - the so-called standard account given by Kratzer & Shimoyama (2002). Recall, the crux of this view was an appeal to a Gricean cooperative principle and specifically the Gricean maxims of quantity:

Maxims of Quantity:
1. Make your contribution as informative as required (for the purposes of the exchange).
2. Do not make your contribution more informative than is required. (Grice 1975, 45)

From this maxim Kratzer & Shimoyama make explicit the idea of an exhaustivity inference such that when some set of items are under discussion

[61]For a thorough exposition of axiomatizations for epistemic logic, PAL, as well as derivation rules, see van Benthem (2010).

and a permission claim is made about one of them, in accordance with the maxim of quantity, this must mean that a permission has not been given for the other (otherwise this information would have been provided).

So, when a free choice permission claim is presented and we know that the permission granting authority knows which options are actually permissible, we begin a kind of internal dialogue from which a process of updating our own epistemic state occurs.

When we consider the kinds of problems which are usually solved by dynamic logics – that is, logics which involve model change – we see that these problems are those where information or knowledge is 'updated' in light of inferences or informational events. (e.g. the Muddy children problem, van Benthem 2010, 173) The Gricean deliberation that takes place in the free choice permission case is just this kind of 'updating'.

In the fullest possible model of this reasoning we might begin with all options impermissible and some action by an authority making some options permissible. How we start is worthwhile to think about but for our present purposes, simply consider the situation after some initial permissibility claim has been made which takes the form of a free choice disjunction.

Moreover, it's also worthwhile thinking about wide scope vs narrow scope permissions and exclusive vs inclusive 'or' disjunctions, but again, for the sake of initial simplicity, I'll restrict the current analysis to a single permission statement (announcement) ranging over an exclusive disjunction.

$$!_a\langle p_b\rangle(R \veebar Q)$$

So, an authority A has made a permission statement concerning agent B. In the simplest analysis, we treat this starting from a model in which the permission is true:

$$\langle p_b\rangle(R \veebar Q)$$

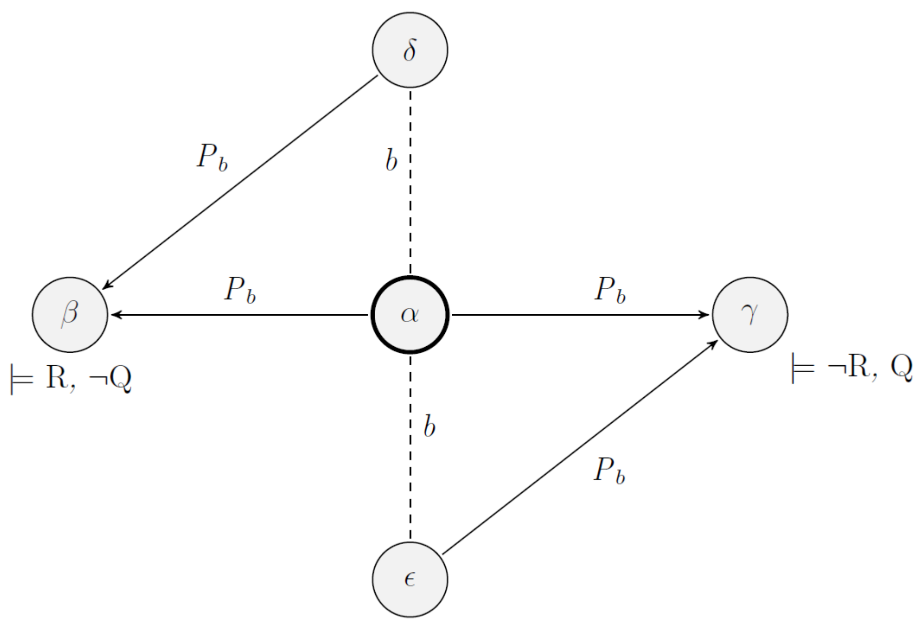
but we do so with the understanding that a model updating has occurred in which, prior to this permission, B had complete uncertainty with regard to all combinations regarding the permissibility of R, Q. Crucially, this is not simply an announcement of a permissibility claim. We take it also to be a *cooperative* announcement made by agent A with respect to what is at issue (the permissibility of R and the permissibility of Q for *B*):

$!_a\langle p_b\rangle(R \veebar Q)/c$

Given the announcement, we arrive at the following epistemic model:

Figure 6.4.1 Free choice announcement model

$\mathfrak{M}_{11}$: $\mathfrak{M}\ |\langle p_b\rangle(R \veebar Q)$:



In this model, we understand the permission granting authority as *A*. Though *A* does not appear in the model, *A* is a member of our group G (agents) and we will be able to make statements about *A*[62]. Crucially, this includes that we have arrived at this model after some update where *A* has (cooperatively) announced the permission state of the freely choosing agent (designated B). We have adopted the convention of bolding the actual world (α).

Given this model, B is uncertain between 3 worlds:

[62] For example, A's epistemic state. Where B is uncertain between worlds α, δ, ε, A is not, and statements about this will follow from the model e.g. $[K_a](\langle p_b\rangle R \,\&\, \langle p_b\rangle Q)$

It could be that B is permitted to realize R and not-Q. (world δ)

It could be that B is permitted to realize Q and not-R. (world ε)

It could be that B is permitted to realize either option. (world α)

Importantly, agent *A* is not uncertain at all (there are no 'a' uncertainty links) and *A* knows which permissions are available. That is, they know the actual world. Further, *B* knows that they know this. So:

$$[K_b][K_a]((\langle p_b\rangle R \,\&\, \neg\langle p_b\rangle Q) \vee (\neg\langle p_b\rangle R \,\&\, \langle p_b\rangle Q) \vee (\langle p_b\rangle R \,\&\, \langle p_b\rangle Q))$$

Whereas normally public announcements take the form of informational communicative events, in this case, further informational events will be a very special kind of deliberation concerning the assumption of cooperation.

Though this is an unusual kind of informational event, such an account of public announcement is not uncommon and useful in treating 'deliberation' updates. As van Benthem states:

> You can think of this [update step] as a typical step of communication, but it is also an act of public *observation*, regardless of language (van Benthem 2010, 172)[63]

The form this takes is exactly like the internal deliberation given in the pragmatic (Gricean) account of free choice permission though I have simplified matters here by assuming an exclusive disjunction.

So *B* realizes that *A* knows which world is the correct one and further, *B* knows that *A* is cooperating in accordance with a Gricean maxim of quantity.

As a first pass, a key component of the maxims of quantity will be that no cooperative announcement made by an agent i about some set of formulae will be logically weaker or logically stronger than what this agent knows to be the case concerning that set of relevant formulae. Here, we take 'logically weaker' and 'logically stronger' to be in line with ideas of *quantity* insofar as a logically

[63] See also (van Benthem 2014, 20). In a fully developed system, we may even go further in treating explicit and varying conceptions of rationality in deliberation mechanics (e.g. van Benthem 2014, 182) though this is somewhat beyond our purposes here.

weaker claim would contain less information while a logically stronger claim would contain more than is necessary for the purposes of conversation[64].

Returning to our analysis, *B* supposes[65]

$$[K_a](\langle p_b\rangle R \,\&\, \neg\langle p_b\rangle Q)$$

If this were the case then A should have made this (more precise) announcement. Or, counting on an exhaustivity inference, they should have made the announcement

$$!_a\langle p_b\rangle R/c$$

But they did not make this announcement, which we can symbolize as:

$$\neg\langle !_a\langle p_b\rangle R\,/c^{\cup}\rangle\top$$

That is, there is not a transition to the current model made by a public announcement (by *A*) that is cooperative with respect to the permissibility (for *B*) of R or Q. Hence,

$$\neg[K_a](\langle p_b\rangle R \,\&\, \neg\langle p_b\rangle Q)$$

This means *B* can rule out world β as a possibility. And it must be the case that:

$$\langle p_b\rangle Q$$

With this in mind we update our information via this 'deliberation' announcement

$$!\langle p_b\rangle Q$$

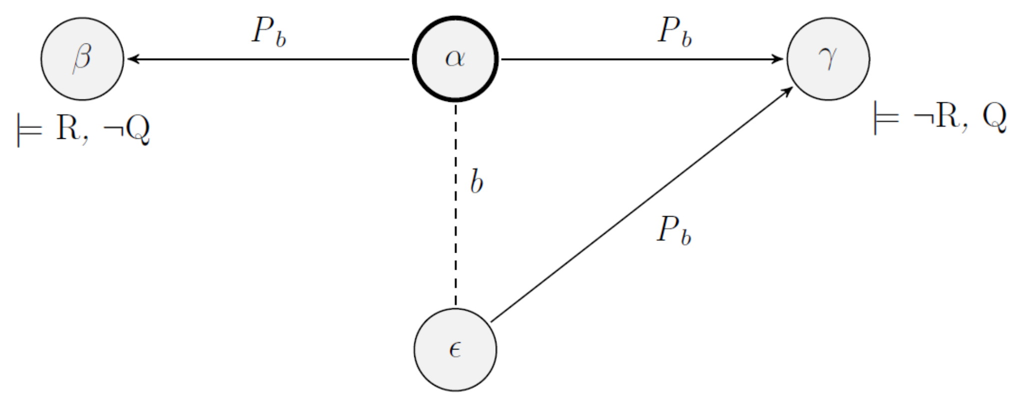
64 It is true that Grice's formulation does allow for a cooperative conversant to say less than what they know, if that information is not required for the purposes of the exchange. This will be accounted for by our stipulation of the set of formulas with respect to which cooperative public announcements are made. In other words, agent *A* could say only φ when they also know that ψ if ψ is not a part of the relevant conversation (e.g.: ψ is not contained in the list of formulae the public announcement is cooperative regarding).

65 This supposition follows Kratzer & Shimoyama's analysis and can be understood as following from what B might suppose given the three-fold uncertainty present in the situation.

And our model changes, eliminating all epistemically accessible worlds where Q is impermissible from the model:

Figure 6.4.2 Free choice update model

$\mathfrak{M}_{12}$: $\mathfrak{M}_{11} | \langle p_b \rangle Q$



Again, this process is repeated. B supposes[66]

$[K_a](\langle p_b \rangle Q \ \& \ \neg\langle p_b \rangle R)$

and concludes that A should have made either this announcement or, counting on an exhaustivity inference:

$!_a\langle p_b \rangle Q/c$

But they did not make this announcement, and so:

$\neg[K_a](\langle p_b \rangle Q \ \& \ \neg\langle p_b \rangle R)$

Which means *B* can rule out world ε as a possibility. And it must be the case that:

$\langle p_b \rangle R$

[66] In keeping with the deliberative steps outlined in the Kratzer & Shimoyama account, we characterize the reasoning of *B* as taking the form of *reductio ad absurdum* in each deliberative step (ruling out these possibilities).

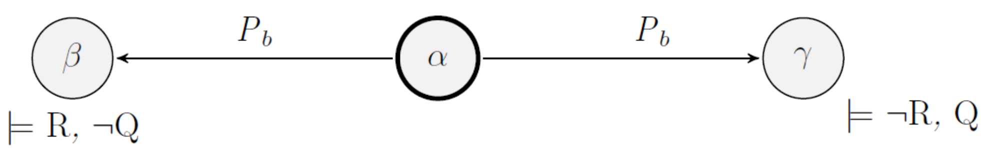
Once more we update our information via this 'deliberation' announcement

$!\langle p_b \rangle R$

and our model changes again:

Figure 6.4.3 Free choice deliberation outcome

$\mathfrak{M}_{13}$: $\mathfrak{M}_{12} | \langle p_b \rangle Q$



We now see that:

$(\langle p_b \rangle R \ \&\ \langle p_b \rangle Q) \ \&\ \neg \langle p_b \rangle (R \ \&\ Q)$

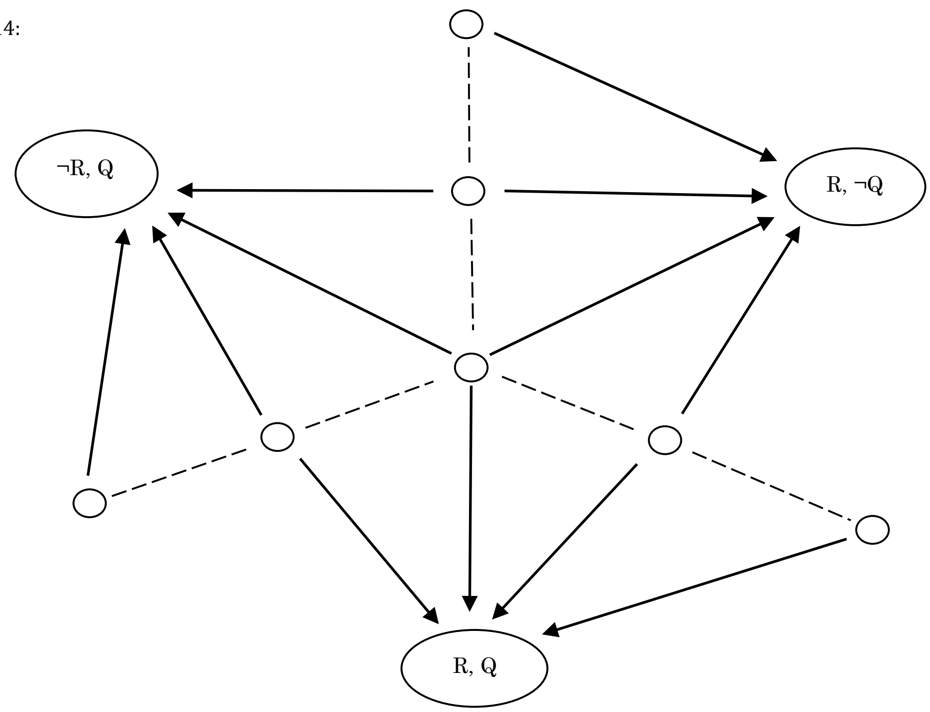
This is a rough sketch of how we might formalize the so-called 'standard' account given by Kratzer & Shimoyama (2002) but (for the sake of simplicity) there are some clear gaps in this analysis. We could, for example, start from the inclusive 'or' permission and this will complicate matters:

$\langle p_b \rangle (R \vee Q)$

In this case, our initial model is:

Figure 6.4.4 Inclusive free choice permission model

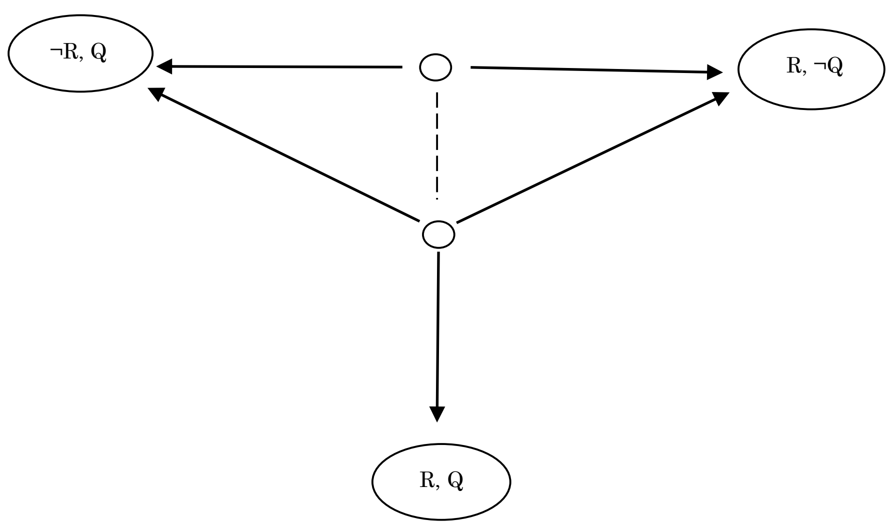
$\mathfrak{M}_{14}$:



As before, we can engage in similar reasoning to that described above and shrink the epistemic ranges. This will eliminate worlds from the model and the result will be the new 'update' model:

Figure 6.4.5 Inclusive free choice deliberation outcome

$\mathfrak{M}_{15}$:



This is an interesting conclusion, given that we typically do not take the possibility of having both as following from a free choice permission (though we sometimes do!). It may be that the way around this is to understand such permissions as exclusive or inclusive as required[67].

Many problems remain and there are clear ways forward in which the sketch provided could be elaborated further (e.g. doxastic additions capturing speaker beliefs). The most obvious concern, however, is that what we mean by 'cooperative' (and associated maxims of quantity) is not explicitly formulated and so, arguably, much of what is pragmatic remains non-formal. In what follows, I will attempt to make these ideas more systematic.

6.5 Formalizing Maxims of Quantity

There are a number of ways we might express the maxims of quantity using the tools of contemporary modal logic and logical dynamics. I do not intend to provide a complete and final analysis here and I have no doubt that there are additional tools which could more closely match many more nuanced intuitions about the precise meaning of such maxims. Nevertheless, I hope to provide at least a glimpse at how formulating these maxims might proceed. Recall, Grice formulates the maxims as:

Maxims of Quantity:

1. Make your contribution as informative as required (for the purposes of the exchange).
2. Do not make your contribution more informative than is required. (Grice 1975, 45)

[67] The issue of whether these permissions are taken to be exclusive disjunctions or inclusive may itself be a pragmatic matter. At minimum, it is not immediately clear why we should prefer inclusive translations of 'or' to exclusive ones, so any impetus to justify the exclusive reading seems no more compelling than an impetus to justify an inclusive reading. Nevertheless, a formal account of why one reading or another is the correct one may rest on deliberative outcomes which make sense under one reading but not under the other.

And, Gamut combines these ideas into a single constraint:

> Quantity Constraint. If a Speaker asserts "*q*", then for all *p* logically stronger than *q* such that *p* is a relevant alternative to *q*, there must be some reason the speaker refrained from asserting "*p*." (Gamut 1991, 205)

Thus, for any cooperative public announcement

$!_x\varphi/c$

It cannot be the case that there is a statement ψ such that:

$(\psi \rightarrow \varphi) \ \& \ \neg(\varphi \rightarrow \psi)$	ψ is logically stronger than φ
$[K_x]\psi$	Agent x knows that ψ
ψ_R	ψ is relevant (thus, a relevant alternative to φ)

Notice, we are here treating Gamut's idea that there be 'some reason' in a very precise way – this reason is based on the epistemic state of x (the reason is that x does not know ψ). There are, of course, other possible reasons, including Gricean 'flouting' of maxims of conversation. For the moment we will leave these other reasons aside.

Defining what it means for ψ to be *relevant* is not a straightforward task. Indeed, much of what is pragmatic about the idea of cooperation is contained in the notion of relevance and worries are justified about a purported Gricean formalization that fails to account for this. One way of thinking about relevance is as a relationship between statements and a *topic* of conversation. Thus, this is cooperation with respect to a *topic*, and in some sense captures the idea that cooperation fundamentally reduces to relevance concerning the topic of conversation (see e.g. Relevance Theory, Sperber & Wilson 1995). There is a sizeable literature on relevance in logic and a number of precedents for formally treating relevance (e.g. Epstein 1979). Many of these approaches will suffice and I do not endorse any one in particular. For present purposes we proceed with an undefined conception of relevance[68].

Given these criteria, we can thus formulate a basic formal maxim of quantity:

Formal MQ) $\langle !_x\varphi/c\rangle\top \rightarrow \neg((\psi_R \,\&\, [K_x]\psi) \,\&\, ((\psi \rightarrow \varphi) \,\&\, \neg(\varphi \rightarrow \psi)))$

We can use this basic formal maxim to treat straightforward implicatures. Consider the following exchange:

Bill: Where is the class located?

Anne: Room D or Room E

If we assume that cooperation is present, we know, in accordance with the maxim of quantity, that Anne is not offering a logically weaker claim here than what she knows to be true. For example, suppose Anne knows that the class is in Room D. The response given in this case would be accurate (since the class being in room D does entail the disjunction given) but in that case it would not be cooperative. Thus, assuming cooperation, it cannot be the case that Anne knows that the class is in Room D. Using the above maxim, we can derive this:

1.	$\langle !_a(D \vee E)/c^{\cup}\rangle\top$	announcement
2.	$\neg((D_R \,\&\, [K_a]D) \,\&\, ((D \rightarrow (D \vee E)) \,\&\, \neg((D \vee E) \rightarrow D)))$	1, MQ
3.	$(D \rightarrow (D \vee E)) \,\&\, \neg((D \vee E) \rightarrow D)$	tautology
4.	D_R	D is relevant
5.	$\neg[K_a]D$	2, 3, 4

Hence, Anne is using the disjunction to indicate uncertainty.

Crucially, this straightforward approach does not have the recursive character we need for an analysis of free choice disjunction (recall, the Kratzer & Shimoyama account is one of *recursive pragmatic strengthening* – implicatures about implicatures).

[68] See appendix D for a rough formal account of relevance that will meet the needs of the present analysis.

If we try to treat free choice disjunction as a single implicature we generate a contradiction. Consider:

1.	$\langle\, !_a\langle p_b\rangle(R \vee Q\,)/c^{\cup}\rangle\top$	announcement
2.	$\neg((\langle p_b\rangle Q_R \;\&\; [K_a]\langle p_b\rangle Q) \;\&\; ((\langle p_b\rangle Q \rightarrow \langle p_b\rangle(R \vee Q)) \;\&\; \neg(\langle p_b\rangle(R \vee Q) \rightarrow \langle p_b\rangle Q))$	1, MQ
3.	$(\langle p_b\rangle Q \rightarrow \langle p_b\rangle(R \vee Q)) \;\&\; \neg(\langle p_b\rangle(R \vee Q) \rightarrow \langle p_b\rangle Q)$	tautology
4.	$\langle p_b\rangle Q_R$	$\langle p_b\rangle Q$ is relevant
5.	$\neg[K_a]\langle p_b\rangle Q$	2, 3, 4

But Anne (the permission granting agent) does know that both are permissible! We need a recursive element to our maxim of quantity so that we can make higher order implicatures.

We modify our definition. For any cooperative public announcement

$$!_x\varphi/c$$

It cannot be the case that there is a statement ψ such that:

$(\psi +\!\!\rightarrow \varphi) \;\&\; \neg(\varphi +\!\!\rightarrow \psi)$	ψ is logically stronger than φ through implication or implicatures
$[K_x]\psi$	Agent x knows that ψ
ψ_R	ψ is relevant

And, crucially, we add a further recursive condition

$\langle !_x\psi/c^{\cup}\rangle\top$	ψ can be announced cooperatively

This is a recursive definition because in order to evaluate the cooperativity of φ we will need to evaluate the cooperativity of alternative announcements, which may in turn require evaluating cooperativity of still further announcements (invoking the definition recursively).

With this condition, the contradictory derivation above breaks down in exactly the manner described by Kratzer & Shimoyama. Though,

$$\langle p_b\rangle Q$$

is relevant, known (by A), and logically stronger than the announcement, it cannot be cooperative. For if:

$$\langle !_a \langle p_b \rangle Q / c^{\cup} \rangle \top$$

Then there is a logically stronger relevant alternative on offer:

$$\langle p_b \rangle R \;\&\; \langle p_b \rangle Q \qquad\qquad \text{(or that A knows this)}$$

Given that A does have complete knowledge, it must be that

$$\neg \langle p_b \rangle R$$

But A knows that this is false. This is the exhaustivity inference and what Agent A blocks by offering the disjunction rather than the stronger individual component claim.

Finally, we may wonder, isn't A still obligated (cooperatively) to make the logically stronger announcement:

$$\langle p_b \rangle R \;\&\; \langle p_b \rangle Q \qquad ?$$

No, A doesn't need to make this statement since we have modified our condition of logical strength as

$$(\psi \mathbin{+\!\!\rightarrow} \varphi) \;\&\; \neg(\varphi \mathbin{+\!\!\rightarrow} \psi) \qquad\qquad \psi \text{ is logically stronger than } \varphi$$

through implication or implicatures

and we can now infer the conjunction of possibilities through implicature. Thus, our refined (recursive) formal maxim of quantity is:

Formal MQ+)

$$\langle !_x \varphi / c \rangle \top \rightarrow \neg((\psi_R \;\&\; [K_x]\psi) \;\&\; ((\psi \mathbin{+\!\!\rightarrow} \varphi) \;\&\; \neg(\varphi \mathbin{+\!\!\rightarrow} \psi)) \;\&\; \langle !_x \psi / c^{\cup} \rangle \top)$$

It may be the case that, in order to determine if an expression satisfies a Gricean maxim, we will need to work backwards from an assumption of cooperation until settling on an equilibrium where no further updates are possible. Though it is beyond my present analysis, it seems likely that further logical tools such as modal fixed-point logics (e.g. the modal μ-calculus) will allow for systematic calculation.

This is a good thing, and in line with common intuitions about Gricean pragmatics, where the dynamics of cooperation can involve successive layers of reasoning and deliberation about what is said relative to what is known before arriving at some final meaning.

6.6 Formal Pragmatics

It may be tempting to view the approach presented here as a springboard for a fully fleshed out logical theory of Gricean conversational implicature employing all the myriad tools of epistemic logic, public announcement logic, propositional dynamic logic, modal μ-calculus, or the many other exciting developments currently emanating from the logical dynamics camp. While I think this would be interesting and almost certainly possible, it is my intention here neither to suggest such a theory nor to hold this work up as a beginning in that regard.

Rather my goal is to merely show that the beginnings of such a formalism in the specific case of free choice permissions can be constructed which captures the pragmatic account involved in one currently popular view. This can be compared with the less complicated example given earlier in this chapter showing the relationship between Kamp's (1973) pragmatic account and the later semantic work of Asher & Bonevac (2005). In each case, where one view sees a basic logical language and invokes pragmatic explanations for felt entailments not warranted in that language, another view can augment the language in order to make the pragmatics formal.

As I have hinted at already and will argue at length in the following chapter, this speaks to a confusion about semantics and pragmatics more generally. Many who make arguments for pragmatic explanations of arguments do so because they recognize that some inference or other is not licensed by the formal logical translation of what is said in a superficial argument or expression and want to explain this inference. In a similar vein, many who argue for semantic solutions to these same problems feel that the information conveyed by an argument or expression actually contains more than is offered by a superficial or surface understanding and want to explain how this deeper rationalizing functions in a formal way.

The distinction between semantics and pragmatics runs deeper than these concerns. For example, even if we modify a logic to capture a pragmatic idea like Gricean conversational implicature, the question stands as to whether this makes an account semantic or merely serves to formalize the pragmatics involved. How lines are drawn concerning how and where semantics and pragmatics interact is a hotly debated subject with recent developments in both theoretical and empirical realms. Initial classifications of pragmatic vs. semantic pivoted around 'what is said' and on that very basic criterion, the account given above is still, very much a pragmatic account.

On the other hand, the dynamic logical picture illustrated above and the formal maxim of quantity I have stipulated are very much more than a simple provision of implicit premises or hidden contextual pragmatic meaning provided in order to analyze (say) validity of choice inferences. Rather, these are general principles, independent of context – and applicable to many (all?!) conversational situations. The above dynamic logic is a *system* with a model theoretic semantics of its own. This suggests that at least a minimal component of language we treat as semantic may potentially be broader than we think. Making sense of this will require getting clear about how semantics and pragmatics interact.

Up to this point we have been discussing the paradox of free choice disjunction and the solutions to this problem as problems and solutions either fundamentally moored in semantic concerns or fundamentally pragmatic in nature. Though we have briefly explored the differences between these views we have not exactly made this distinction clear. To this end, we turn our attention to what precisely such a distinction is about.

Chapter Seven

The Role of Logic

7.1 Semantics & Pragmatics

We have superficially stated the distinction between semantics and pragmatics in a roughly similar manner to that articulated in the first significant and explicit discussion of this issue – C.W. Morris' *Signs, Language & Behavior* (1946)[69]. In that work, Morris breaks the study of signs and sign processes (semiosis) into three 'dimensions' of analysis[70]. The study of these dimensions he refers to as *syntactics*[71], *semantics* and *pragmatics*.

Syntactics concerns signs themselves and the relationships that hold between signs. Morris states:

> …*syntactics* deals with combinations of signs without regard for their specific significations or their relation to the behavior in which they occur (1946, 219)

Importantly, this includes the ways in which signs can be correctly combined (formation rules) – including both single complex units (words, phrases and

[69] Morris' earliest discussion of semantics and pragmatics occurs in his 1938 essay "Foundations of the Theory of Signs" in the *International Encyclopedia of Unified Science* Vol I, Issue 2. This was later modified in his (1946) given further reflection on his own approach and further work on the issue.

[70] Morris describes these three areas of study as dyadic relations abstracted from the triadic relation of semiosis between sign vehicle, designatum and interpreter (1938, 6).

[71] Roughly, syntactics is the study of *syntax.* Modern discussions of the issue (e.g. Birner 2012) simply replace the term *syntactics* with *syntax.* In subsequent discussion, when speaking about the study of syntax I will treat the two terms interchangeably.

sentences) as well as longer groups of units such as proofs. While Morris treats both of these kinds of correct sign construction under a single idea of formation, a common way of understanding these two categories of syntactic construction are in terms of formation rules and transformation rules[72]. Notably, Morris' definition treats syntactics as specifically concerning signs without regard to particular signification or the behavior they produce in interpreters. These two separate concerns are the domain of semantics and pragmatics.

Semantics concerns the ways in which signs can carry meaning with respect to actual things a language is meant to talk about. Morris states:

> …*semantics* deals with the signification of signs in all modes of signifying (1946, 219)

Thus, semantics concerns signs and those things that signs designate. As commonly understood, semantics is a study of meaning but specifically meaning as it arises from designation.

Pragmatics concerns the ways in which signs are employed and interpreted by those in communication. As Morris states:

> …*pragmatics* is that portion of semiotic which deals with the origin, uses, and effects of signs within the behavior in which they occur. (1946, 219)

In this way we could say that syntax is about the relationship between various signs, semantics is understood as capturing the relationship between signs and their corresponding objects and pragmatics is meant to capture the relationship between signs and their users. This view has been influential and formative for most subsequent discussions of natural language pragmatics.

While pragmatics concerns the use of signs by communicators, a key component of this is interpretation and thus, where the communicative effect

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[72] This description of syntax (formation and transformation rules) was included in Morris' own first description of *syntactics* (1938, 14) and is the most common conception of syntax as used in formal systems.

of signs on interpreters differs from semantic meaning, we can differentiate this additional/differing meaning as *pragmatic meaning*.

The most common manner in which semantic meaning is separated from pragmatic meaning is on the basis of two essential properties which distinguish them. Semantic meaning is typically understood to be both:

i) context independent and
ii) truth conditional

The idea of context independence is just that the semantic meaning of expressions does not depend on the particular circumstances of expression/utterance[73]. The idea of truth conditionality is that the meaning of an assertion is equivalent to (or reducible to) the truth conditions of that expression. Thus, a sentence:

Snow is white

is true if, in fact, that thing designated by ‘snow’ has the property designated by ‘white’. The meaning of this expression is just that these truth conditions are satisfied – i.e. ‘snow is white’ means that snow is white[74].

Pragmatic meaning is understood to be context-dependent and/or non-truth conditional (Birner 2012). For example, a claim like

This weather is miserable

Involves the word ‘this’ in such a way as to relativize the meaning to the context of the utterance. If we were to try and establish some objective truth conditions (e.g. miserable weather being defined as involving certain specific weather conditions) then the weather to which the ‘this’ refers matters[75].

[73] Compare, for example, the *occasional* sentences discussed in the previous chapter. The pragmatic meaning of these sentences (or arguments) is dependent upon the occasion (and presumably, context) of utterance/expression.
[74] See, e.g. Davidson (1967).
[75] There are many efforts to treat indexicals semantically, and many semantic minimalists (e.g. Cappelen and Lepore 2005) believe these are easily accommodated in formal semantic models.

Though it is frequently the case that pragmatic meaning results from context sensitivity and a lack of direct (semantic) truth conditions (neither properties hold) there are cases where meaning is context independent and yet truth conditions are still pragmatic. Consider:

> Diane found her keys and opened the door.

The semantic truth conditions of this expression are just the conditions under which this conjunction is true. That is:

> Diane found her keys
>
> Diane opened the door

However, there is a pragmatic component to this expression in that the total meaning seems to be:

> Diane found her keys and Diane opened the door *with the keys*

Notice, this additional information is not a part of the of the truth conditions of the initial sentence.

Perhaps the clearest cases of pragmatic meaning are where the meaning of an expression diverges completely from the semantic meaning of the terms involved (e.g sarcasm or other flouting of maxims of conversation).

The ascription of *pragmatic* to the meaning of a given expression rests heavily on whether we are able to completely determine the meaning of the statement semantically. This phenomenon has led some to adopt the 'pragmatic wastebasket' view (e.g. Bar-Hillel 1971) where all meaning which is not accounted for by semantics falls into the domain of pragmatics. Notice, if all meaning which is not semantic is pragmatic, then how semantic meaning is determined is crucial to our understanding of pragmatic meaning.

Suppose, for example, we think that the semantics of natural language is captured by ordinary first-order quantifier logic. Consider:

> John fell from the ladder. He did not injure himself.

In this expression 'he' is an anaphoric pronoun which refers back to the subject of the first sentence (John). On a strict view of first order semantics, there is no way the expressions alone can provide precise meaning to the pronoun 'he'. The best we are able to do with first order quantification is:

jFl	John fell from the ladder
$\exists$x(Mx & ¬xIx)	Someone is male and that someone did not injure himself

Indeed, problems like this are usually considered boundary problems in the pragmatics/semantics divide. And yet, there is a sense in which the meaning of the above series of expressions should perhaps be viewed as context independent and truth conditional. The meaning of 'he' is, to be sure, strictly ambiguous, but so also is the meaning of 'John' and 'ladder' and perhaps even 'fell from'. To which 'John' are we referring? Which ladder? How did he fall? Are there multiple predicates which might describe this fall? If we go this far, it's difficult to see how *any* expression can retain status as meaningful as a result of pure semantics.

And so, it may make sense to simply recognize some surface ambiguity which is easily resolved and translate accordingly:

jFl & ¬jIj

John fell from the ladder and John did not injure himself

To be clear, this is not to say there are not pragmatic issues involved here. On the contrary, at the level of what is said in natural language there may *always* be pragmatic considerations. But, perhaps, we lose what was valuable in the formalism if we cast it aside in favour of treating everything as better understood through pragmatic lenses. The possibility that pragmatic considerations intrude on the meaning of most/every expression may not be altogether troubling – but, in debates like the one that has occupied us here (free choice disjunction) this seems to push all solutions in a pragmatic direction.

7.2 Semantic Minimalism and Contextualism

More recent debate about language pragmatics elaborates upon this issue and the way semantics and pragmatics interact. Roughly, we can characterize two positions:

Minimalism $\stackrel{\text{def}}{=}$

> The view that within our cognitive language processing system there is a stable base of meaning which is not affected by contextual information. Thus, statements have meaning, though this is frequently enhanced or changed by contextual factors present in the utterance of these statements.

Contextualism $\stackrel{\text{def}}{=}$

> The view that contextual information shapes meaning at every level of analysis in the determination of sentence meaning. In this sense, sentences do not have meaning at all. Rather, utterances have meaning.

It is not my intention to wade into this debate in any deep way[76] but, insofar as this debate concerns free choice disjunction and purported solutions to the problem, we can make a few observations.

Pragmatic and semantic meanings seem to operate in concert as one moves through levels of analysis. This echoes the contextualist sentiment that there is always a pragmatic 'intrusion' (see, e.g. Bezuidenhout 2017). But, where these debates take place, the underlying issue seems to be about which level of analysis is the 'right' level of analysis.

At the limit of pragmatic considerations, virtually all terms require disambiguation and virtually all terms are expressed within a complex bundle of metaphorical, spatio-temporal, linguistic and historical contexts which contributes to extraordinarily complex non-truth conditional ultimate

[76] For a more comprehensive treatment of the minimalism/contextualism debate, see Bezuidenhout (2017). As she illustrates, there is much more to the debate than the rough division I have suggested. Still, I believe this distinction will serve for the moment.

meaning (to say nothing of tone, inflection, expression, or other physical factors which may further enhance the meaning of an actual utterance). And so, at this level of analysis, all expressions have pragmatic meaning to some degree. This is not to say that there is no clear semantic meaning to expressions – just that there will always be considerations involved in the ascription of meaning which go beyond basic semantics. This is the crux of the contextualist view, and on such a view, there can be no purely semantic solution to free choice permission. At best, we can use pragmatics as heuristics to guide formal semantic analyses of choice effects.

But, the minimalist may argue, we can move up a few levels and consider terms as in some sense fixed or standardized. We can treat some pragmatic phenomenon as extraneous or outside our purposes. We might establish in our theorizing some stable fragment of language treating some names as referential despite their ambiguity and predicates as meaningful despite their vagueness. We consider certain operators as functioning logically in precise ways and conduct semantic analysis on this basis. At this level of analysis we have a semantic base which accommodates those 'stable' parts of the language and, where meaning diverges from this semantics, we call it pragmatic.

This, I suggest, is what is happening in current debates (e.g. free choice disjunction as semantic vs. pragmatic), though the decisive factor seems to be whether there is such a semantic base, and if so, what exactly this semantic base accommodates.

7.3 Semantics vs. Formal Pragmatics

For most purposes, the semantic 'base' at work in discussions of semantics vs. pragmatics is not well-defined. Common elements of the semantic base range from ordinary natural-language semantics (definitions of terms), and the usual logical operators as they function *within* natural-language, to formal languages with an associated logic that we take as representative of language. Typically, these formal systems include propositional logic, quantification and sometimes modal constructions (as is typical in discussions of free choice permission).

Where moves are made in natural-language reasoning which go beyond those allowed by this semantic 'base', we have two options; these moves can be explained by factors outside the semantics of the language (pragmatic explanations) or they can be accommodated into the semantic base by expanding the logical tools we take as basic.

As we have seen in the previous chapter, by expanding our logic there may be ways to take some common pragmatic accounts of free choice disjunction and incorporate them into the semantics of the systems involved. Still, there are lingering concerns about expanding or modifying our logic and concluding that the result constitutes a semantic solution.

As mentioned, modifications of logic (or models of a situation within our logic) may not be truly semantic solution but merely formal accounts of the pragmatics. Formal accounts of pragmatics are not uncommon (e.g. Kamp's Discourse Representation Theory, 1981) and those working on formal pragmatics nevertheless see their projects as illustrative of pragmatic processes rather than as an expansion of semantics.

The most common characteristic of formal accounts of pragmatics which seems to keep them in the realm of pragmatics is that these formalisms include much of what we think of as context or treat the formalization as a representation of heuristical generalizations about pragmatics. For example, Kamp's discourse representation theory tracks conversation flow over time in such a way as to treat series of statements as informative regarding future statements (Kamp 1981, 1995). Consider:

John fell from the ladder

He did not injure himself

By tracking statements as they are made, the theory gives tools for linking (for example) pronouns like 'he' with unambiguous referents earlier mentioned. In this case any 'context' of earlier statements is included in the theory and thus many understand this (and other theories like it) as theories of pragmatics.

Still, ordinary first order models and modal models do seem capable of capturing context in a variety of ways. When, for example, we consider first-order languages as representative of natural language, the semantics given by model theory (structures) is similarly meant to be representative of the structure language is operating over (presumably, the world). And yet, in our formal structures we completely disambiguate objects with the same name, and we can treat contexts by considering structures in simplified fashion. For example: the claim

John fell from a ladder

when treated as a first order claim

$\exists x(Lx \,\&\, jFx)$

invokes a first-order structure involving the constants and predicates:

j: John

Lx: x is a ladder

xFy: x fell from y

And, crucially, does so in such a way as to completely disambiguate these terms from, say, other Johns and other ladders. If what is meant is that

'John Smith fell from a ladder in the garage'

We can translate as:

$\exists x((Lx \,\&\, Gx) \,\&\, j_1Fx)$

If a different John falls from a different ladder we must represent this using different constants and predicate extensions. For example:

$\exists x((Lx \,\&\, \neg Gx) \,\&\, j_2Fx)$

The potential ambiguity above is an extremely simple case, and one in which such basic disambiguation is not typically taken as pragmatic or requiring the unpacking of context. But far more complicated cases can be treated similarly. Consider:

Juliet is the Sun

Clearly the literal understanding of this claim is false and any straightforward translation of such a claim into formal logic will similarly be false given some semantics where 'j' refers to an object Juliet and 's' refers to an object which is the Sun.

Were we to attempt to capture the sentiment of 'Juliet is the sun' in a formal system, we must first understand the claim as metaphorical, in which case the 'is' is no longer treated as an identity relation but an altogether different kind of relation.

If we understand this claim as:

Juliet is_M the sun

Where is_M is not an 'is' of identity but rather a metaphorical 'is' in which the two relata share properties, we can make direct sense of this in the formalism. If all that is being said is that

Juliet is like the Sun

Then the statement is at least trivially true. More informatively, however, we might have grounds for treating the statement as suggestive with respect to some likely properties. Presumably such a metaphor expresses qualities like importance and beauty (rather than, say, size or temperature) and, knowing this, we can unpack this as:

Juliet is important and beautiful

Which in the formalism of logic can become:

Ij & Bj

A natural objection here is that there is still a considerable amount of context, interpretation (and pragmatics) involved if we choose to translate in this way. This is certainly true. I do not mean to suggest that these factors are absent.

The suggestion is simply that, if we choose, we can build first order structures with predicates like the 'metaphorical is', along with completely disambiguated names or even temporal modal structures where indexicals like 'today' have precise meaning.

As a matter of practice, the choice of structure we use does not come *prior* to our translation. Rather, the choice of structure (and the formal semantics itself) seems to accompany our formal translation and our purposes.

Crucially, any established semantics of natural language rooted in a formal representative semantics (like first order model theory) requires or at least benefits greatly from simplification in order to keep that analysis rigorous and precise. Much as a map or physical model functions as an idealized and simplified version of what it represents, the formal semantics of logic functions in this way for language itself. But, just as a mapmaker can make choices regarding the use of a map and the salient features to include, so also can we make decisions regarding the design of formal languages and their models.

Linguists initially loosely achieved some semantic base for natural language with the adoption of standard meanings (dictionaries) as well as grammatical and syntactic rules. But as increased rigour was sought, logical systems were developed and incorporated in metalanguages to the natural languages being analyzed. This has, perhaps, placed undue importance on the logical languages as symbolic and representative of natural language.

I will identify this with *Montague's Thesis*, defined as:

> the view that there is 'no important theoretical difference between natural languages and the artificial languages of logicians' and that it is 'possible to comprehend the syntax and semantics of both kinds of language within a single natural and mathematically precise theory' (Montague 1970).

Few today defend the strongest forms of this thesis but in a weaker form, where *some* parts of language are equivalent to formal systems, the view is common. But there are other ways to understand logic that significantly depart from this view.

7.4 Logic as Abbreviation

We can differentiate two views on or about logic. What we mean here by 'logic' will depend in part on these views but for the moment we can just take the word somewhat naively to mean something like principles or rules of right reasoning.

The first view, roughly articulated by a weak form of Montague's thesis, is what I take to be common amongst philosophers, lay persons, philosophers and logicians of antiquity as well as those with only a basic exposure to logic.

The second is more common to very formal practitioners of logic (mathematicians, computer scientists, logicians), though it is also common for these formal practitioners to hold the first view I will describe. Moreover, where formal practitioners adhere to the second view, it is common for these individuals to not really be concerned with this view, or to have not thought about how to express it.

So, what are these two views? Let's start with the first:

View 1) Logical languages are abbreviations of natural languages.

On this view, language, speech, or thought has a logic to it, and structures in language, speech or thought have counterpart structures in any symbolic or formal articulation of this logic. In this way, symbolic logic is a direct representation of reason as it actually occurs in language (or as it occurs in thought or ideas expressed linguistically).

So, for example, any truth-functional operator (e.g. not, and, or) is derived from truth functional qualities of the actual natural language expressions 'and', 'or', etc.

One way of thinking about this view is with the understanding that the symbols of a formal language function as abbreviations of the natural language expressions. Moreover, the symbolism as well as formal inference rules could be dispensed with if we really wanted. We'd lose the succinctness of symbolic logic and some of the precision but any derivation could be carried out in the natural language.

For example, consider the inference rule *modus ponens* as well as a simple application of this rule treated symbolically given some abstract statements – for example:

P

$P \rightarrow Q$

where P='It is raining' and Q='The sidewalk is wet'. *Modus ponens* tells us that given φ, $\varphi \rightarrow \psi$ we can conclude ψ. Moreover, these statements (P, Q) can be used as substitution instances in the rule and hence we can conclude:

Q

But we can just as easily express this directly in the language:

It is raining

If it's raining the sidewalk is wet

So, the sidewalk is wet.

Moreover, we can treat the inference rule itself as a rule which is not particularly concerned with the *abstract* statements of a formal system but is instead concerned with right reasoning even at the level of natural language. When this rule is employed in the formalism, this too functions only in accordance with abbreviations of the reasoning as it appears in natural language.

On this view, it is common to focus on the 'right' symbolization of natural language expressions and to do so in a way that lines up with our actual language use. After all, if the logic is an abbreviation of language, we ought to make sure that expressions like 'and', 'or', 'all', 'some', 'not' (and so on) are doing the logical jobs they are supposed to be doing.

Where our natural language expressions fail to account for these operators, make strange use of these operators, or result in unsanctioned (though apparently warranted) inferences, a common approach is to maintain that the logical operators are doing their job properly – we simply haven't accounted for unstated assumptions, background information, context, or other factors in the language *pragmatics*.

This occurs in the case of free choice permission. We see an expression like

You may have coffee or tea

and we abbreviate, symbolize, or otherwise represent the language structures present as:

$\langle p \rangle (C \vee T)$

only to discover the host of problems which have here been occupying us.

The point is – a problem like the paradox of free choice permission is a direct result of thinking that expressions like 'You may have coffee or tea' need to be symbolized in such a way that the formalism is an accurate abbreviation of the natural language expression. Thus, the appearance and order of logical operators in the natural language expression are preserved in the formal expression. We might do this and see that a felt entailment exists in the natural language. We seem to correctly infer:

'You may have coffee'

and

'You may have tea'

Once again, we can symbolize (abbreviate) this inference in the formalism and we arrive at our familiar:

FCP) $\langle p \rangle (C \vee T) \rightarrow (\langle p \rangle C \,\&\, \langle p \rangle T)$

When this happens, a solution is needed. That entailment cannot be a straightforward logical consequence (or absurdity follows) so we tell a semantic story about strong permissibility, unusual modal operators at work, the relationship between disjunction and conjunctions of possibilities – or we tell a pragmatic story about background assumptions, the relationship between permission claims and permission givers, information and maxims of quantity.

As I hope to show, however, there is another way of thinking about this.

7.5 Logic as Idealized Language

On our second view, logic is not an abbreviation of language. Rather:

View 2) Logical languages are abstract formal systems which model natural language meaning and consequence

This *Formal Systems* view of logic turns questions of symbolization on their head. Rather than asking 'how should we symbolize some natural language statement or other?' – We instead ask, 'what natural language statements are correctly modelled by some formula or other?'

The difference is subtle but powerful. Where natural language is ambiguous, the formal language can differentiate. And, importantly, there needn't be any requirement for structures in the language to map onto structures in the formal system. Rather, the ultimate meaning and consequences of the language (or parts of the language) are preserved as well as possible.

To illustrate the difference between these two views of logic and where we might place ourselves with respect to these accounts, consider the following litmus test. Imagine you are a teacher of elementary logic and ask students to symbolize the following natural language statement using a standard sentential/propositional logic:

If it is raining the sidewalk is wet

We know this is made up of two simple statements

R It is raining

W The sidewalk is wet

And presumably either view of logic will agree this is correctly (truth-functionally) represented by the material conditional:

$R \rightarrow W$

Suppose, however, a student translated this statement into the formal language as:

$\neg R \vee W$

Is this a correct translation? Here we may see a divergence of opinion. On the view of logic as abbreviation we might say that this is not a correct translation (or, perhaps, only partially correct). This is, after all, a disjunction and the natural language expression has a conditional form. A disjunction fails here to translate the right surface grammatical structure in the expression. To consider this a correct translation allows for an infinity of correct translations.

On the *formal systems* view, however, this *is* a correct translation. We are after *meaning* and with that in mind, either translation is perfectly acceptable. While there are infinitely many correct translations, there is only one correct set of truth-conditions. This *meaning* is what matters, and this is accurately captured by either the material conditional, or the corresponding disjunction above.

So, consider again the free choice permission:

You may have coffee or tea.

This could mean a lot of things. The statement is ambiguous. It could mean any of:

You may have your choice of either coffee or tea

You may have (coffee or tea) but I don't know which

It could even mean:

You may have coffee! Or tea is the case

The expression might be a code or cipher or standing for different options or having an altogether different meaning.

But, if we understand the meaning and consequences of what is said, say by recognizing the context and background, we can disambiguate in the formal system. If, for example, we recognize that we are dealing with a typical menu in a typical restaurant and we have background experience with these kinds of situations, we would know to disambiguate by using the following to model the situation:

$$\langle p \rangle C \ \& \ \langle p \rangle T$$

In this case, the conjunction of permissions is not an entailment we need to account for in our logic but rather this expression *is our translation*.

Rather than needing to explain the choice principle

$$\langle p \rangle (C \vee T) \quad \vDash \quad \langle p \rangle C \ \& \ \langle p \rangle T$$

We instead need to explain the translation

You may have coffee or tea *translates as* $\langle p \rangle C \ \& \ \langle p \rangle T$

Notice, to the abbreviation adherent, the choice principle and the translation might be indistinguishable problems and in either case, a paradox needs to be resolved. After all, any disjunctive rules that might govern the narrow scope permission

$$\langle p \rangle \ (C \vee T)$$

will still govern the actual language permission that symbolism represents, especially if we think groupers like brackets can be included in the language somehow (e.g. by pauses, intonation or convention)

You may have (coffee or tea)

For the formalist, this is an altogether different story. Rules about disjunction, modal distribution, well-formedness, etc. are precisely the sorts of things which apply to *formal* languages – and only formal languages. It's true these can be used as a kind of model of how natural language works, but the language and the inferences in the language are not abbreviated by the formal system or the inference rules of that system.

As mentioned in Chapters five and six, there is certainly a rational process at work in much of our derivation of pragmatic meaning. This may lead us to dissatisfaction with the simple translation. After all, by merely translating as a conjunction of possibilities we fail to account formally for the complex (possibly Gricean) procedure involved in deriving that meaning.

But, as we have seen, we can include this in our formal system if we'd prefer. But we cannot keep all of the simplicity or ease of use that comes with a less complex logical system or less complex logical models. We cannot have our cake and eat it too.

7.6 Logical Systems

The choice of semantics (or the choice of logic) involves tradeoffs between richness of expression and simplicity. This tradeoff is well-understood amongst logicians interested in language design, but it is this same tradeoff at work in our discussions of semantics and pragmatics. If we accept a semantic minimalist view, we may also think it possible to expand our semantic base to include tools which capture pragmatics but in doing so the system includes symbolism and concepts with no clear parallels in the natural language. Moreover, the complexity of more robust languages may become unmanageable. Still even with a system as simple and well-received as classical first-order logic, it appears we have already taken this step. When, for example, we consider a statement like:

John only has one daughter

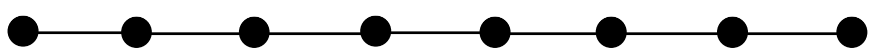
and translate accordingly:

$$\exists x\,(xDj\ \&\ \forall y(yDj \rightarrow y=x))$$

we have arguably gone far afield of how natural language operates. We could tell a complex story about how 'only one' abbreviates the composition present in this formula but even the straightforward use of quantifiers already presents us with serious challenges to any account of logic as preserving natural language structure.

Returning to the analogy of the maps, the design of logical languages and models seems more about how we intend to use them than about detail for detail representational accuracy. For many maps, pure representational accuracy would be a serious deficit. Most subway maps in most major cities, for example contain illustrations of subway track and stops with something like the following layout:

Figure 7.4.1 Idealized path layout



Importantly, these kinds of maps explicitly do *not* preserve many of the features present in the actual systems they represent. Most typically they are not to scale. For the vast majority of subway passengers, the actual distances between stops are irrelevant as are the particular curves present in the path taken. What matters is the clearest possible presentation of the sequence of stops taken along the subway route.

Given different purposes, different features may be emphasized on any map and this may drastically change how a map appears and even the kinds of information that may be reliably derived from a given map. Perhaps notoriously, the standard Mercator projection of the earth creates dramatically skewed representations of relative landmass size while preserving directional bearings for navigation purposes. The Peterson projection preserves landmass size but skews directional bearings.

Logic, I suggest, functions analogously. Both the language and the model we choose can serve particular purposes but in doing so may fundamentally limit or even change other inferences.

For example, classical first order logic, viewed naively, might be seen as quantifying over a domain of objects very much representative of the world. We might think that people, cars, keys, chairs and other objects each have a corresponding object in our model and that each of the properties or relations of these objects have corresponding predicate extensions. Indeed, this is how logic is typically employed. Even in some very sophisticated discussions of logic and philosophy, it is precisely this view of model theory which shapes our understanding of logic and even metaphysics. Quine's suggestion that our metaphysical commitments should be informed by those objects we quantify over in our best scientific theories (to be is to be the value of a bound variable, Quine 1948) seems roughly in line with this perspective even insofar as Quine's models of choice would include abstract objects (e.g. numbers).

But there are altogether different ways that we can construct models and altogether different ways that we can construct languages. A first-order model could, for example, contain as its objects those things we normally understand as modal *worlds* and the predicates applicable to those worlds could be understood as *propositions.* Further, two-place relations between these worlds can be understood as modal accessibility relations. This approach gives us the standard translation of modal logic into first order logic (Van Benthem 2010, 75). Notice, however, this approach treats only *worlds* as objects – a very different picture than the one given above.

Modification of logical languages may be more akin to adding or removing features on a map than to the sorts of perspectival differences obtained by modifying models, though there are cases where the inclusion or removal of logical operators may similarly change what we can say and for what purposes.

Returning to free choice disjunction, we have a few positions we can take. We can accept the contextualist position, in which case there can be no purely semantic solution to the problem, though there may be formal accounts guided by generalizations about pragmatic processes.

Or, if we accept semantic minimalism, we may think that treatment of the problem as essentially pragmatic or semantic is itself a choice. We could design complex languages capable of capturing the Gricean (or other) explanations present in our practical reasoning and thereby move the problem into semantic territory. Or we could maintain the simplicity and typical uses of the systems we currently employ and treat the problem as pragmatic.

If, however, we treat the problem as pragmatic but continue to speak in terms of formal entailment we conflate the roles played by language and logic. In a formal system (e.g. any normal modal logic in which K is an axiom) there are simply no straightforward grounds on which anything like a choice principle will function usefully or consistently. Of course, we could include an inference rule like this if we wanted but in doing so a normal modal logic explodes. Attempts (like those of Asher, Bonevac, etc.) to make the principle work (e.g. strong permissions) can make the inference valid but may not be worth the trouble, given the complexities they add to the logic.

By treating the entailment as pragmatic, we must realize that those concerns operate at the level of language (and usage), whereupon the choice principle functions as a guide to *translation* and not as a formal logical principle at all.

7.7 Dissolution of the Problem

Consider again von Wright's initial consideration that the problem may not be a problem at all but, rather, no more puzzling than ordinary disjunction.

> The argument has seemed satisfactory to many people. I have always tried myself to be pleased with it, but never quite successfully. I have always felt that there was more to Ross' Paradox than can be met by the above piece of reasoning. (von Wright 1968, 21)

I sympathize with von Wright because the problem, I think, *is* a real one. But, as I hope I have shown, I do not think the problem is a necessarily formal one. Moreover, I think there is an unfortunate tendency to conflate ideas about formality with the kinds of inferences we feel entitled to make in natural language. The concern seems to be that some statements like Kamp's (1973) 'You may go to the beach or the cinema' have a *preferred* reading. While it might be true that they are strictly ambiguous, we nevertheless tend to interpret them in a standard way (and in this case as free choice).

Why is this? And if so, shouldn't something like the choice principle be worth fighting for?

CP) X may A or B $\vDash$ X may A and X may B

Maybe so, but we do well to remember that such a principle, if justifiable, cannot be a *formal* principle. In other words, while we might feel that the following is a good piece of reasoning:

a) You may have coffee or tea
b) You may have tea

That does not mean that a formal disjunction in the scope of a permissibility operator licenses a similar formal inference. In this sense I am not suggesting we should be outright predisposed to disagree with something like the above choice principle. Rather, we should be careful what we are doing with it. For example, it cannot be a logical principle operating within a formal proof as:

1. $\langle p \rangle (C \vee T)$
2. $\langle p \rangle (C \vee T) \rightarrow (\langle p \rangle C \,\&\, \langle p \rangle T)$ CP
3. $\langle p \rangle C \,\&\, \langle p \rangle T$ 2,3 modus ponens

Rather, the choice principle seems better suited as a claim about how we should translate an ordinary language sentence like 'You may have coffee or tea'. A revised version might look somewhat like:

Revised CP) Natural language sentences of the form 'X may A or B' *probably* translate formally, in accordance with Gricean or other explanations, to: $\langle p \rangle A \,\&\, \langle p \rangle B) \,\&\, \neg \langle p \rangle (A \,\&\, B)$

This is a move that has a lot of precedent in our translations of ordinary language to formal logic and is essentially the move made by Zimmerman

(2000). After all, many ordinary language disjunctions are commonly understood to be formal conjunctions. For example:

a) Anne or Bill can solve the problem

Similarly, many other English connectives like 'and' or 'therefore' have non-standard translations into formal language. It is an interesting question how exactly this happens with regularity and I think it is plausible that much of the pragmatic free choice analysis involving considerations of conversational implicature may be important in understanding these phenomena. In this respect, I admit I have perhaps minimized these concerns but for my present purposes I hope it is at least clear that at a *formal* level we can make exact translations of exclusive free choice claims and from that perspective there is no formal paradox of free choice permissibility.

Even at the level of natural language we may be doubtful that any ordinary speaker when hearing a free choice permission actually engages in any drawn-out reasoning about Gricean or strong permissibility considerations[77]. The presence of these peculiarities in the language suggests that this kind of reasoning has shaped our discourse and the way our language functions, but we are perhaps long past the point where these deliberations have any place in our understanding.

Rather free choice permissions may function much like dead metaphors. Surely expressions like

John fell in love

must at some point have puzzled hearers, who might then have reasoned about the particular Gricean flouting of the maxim of quality (John didn't actually fall, did he?!). Today, however, this metaphor and others like it seem long dead. We might not, for example, translate

John fell from the ladder

[77] Indeed, much modern empirical research counts against the possibility that actual scalar implicatures are evaluated and derived in cognition (e.g. Bott & Novek 2004). For a discussion of similar empirical findings concerning free choice disjunction, see Meyer (2016).

into formal logic using the same predicate as for falling in love. A reasonable solution might be that falling in love is its own predicate:

$F_L j$

We could, of course, construct (as I have done in chapter six) complex systems to preserve the predicate. But language is flexible, and meanings change. What were once metaphors become so commonplace as to simply carry meaning directly. And other Gricean conversational implicatures become so commonplace as to carry meaning directly too.

Complex systems, like the dynamic logic of conversation presented in this thesis, are theoretically interesting and may take us far in modelling Gricean conversational implicature, but I suspect they do not neatly map on actual cognitive processes at work in deciphering free choice claims. Thus, we have alternative ways to approach the problem. We can treat it theoretically and computationally or, we can simply observe the *meaning* of these claims. In such cases we straightforwardly translate this meaning into our logic. We can do this for free choice disjunction claims as well.

Chapter Eight

Summary and Conclusions

8.1 Overview of the Argument

Introduction to the problem

The introduction to this thesis began with consideration of an ordinary choice scenario – finding oneself in a restaurant with a choice of coffee or tea. As we've seen, a circumstance like this, though simple and ordinary, is characteristic of the problem of free choice permission. We are faced with an 'or' statement which is bounded by a permission claim. For example:

> You may have (coffee or tea)

From this we feel licensed to choose either option and that each option is permitted. For example:

> You may have coffee

Indeed, our felt entitlement to either option seems to be intended by the choice presented. We are, in practice, permitted (in the restaurant case) to make such a choice. And yet, the use of the word 'or' in this expression presents problems. The logic of disjunction does not allow an inference from a disjunction to any single disjunct and so, even when bounded by permission, we similarly cannot infer the permissibility of a single disjunct.

Paradoxes of Permission and Obligation

In chapter two we explored the background to this problem as well as the basics of a formal system to deal with permission claims. This paradox was noticed very early in formal theorizing about obligation and permission as was the related Ross' Paradox. And these paradoxes persist within the formal system developed to treat permission. This system, deontic logic, allows us to translate the above claim into formal language as a disjunction within the scope of a modal permissibility operator:

$\langle p \rangle (C \vee T)$

Again, from such a permission, either of the options seems intuitively permissible. E.g.:

$\langle p \rangle C$, $\langle p \rangle T$

and thus:

$\langle p \rangle C \,\&\langle p \rangle T$

This inference captures von Wright's characterization of free choice permission as well as the formula we can use to express the paradox:

FCP) $\langle p \rangle (C \vee T) \rightarrow (\langle p \rangle C \,\&\, \langle p \rangle T)$

Ross' Paradox captures a related problem in which, from some obligation claim, we add a disjunct to the obligatory action:

It is obligatory to mail the letter.
So it is obligatory to mail the letter or to burn it

RP) $[o]R \rightarrow [o](R \vee Q)$

In the case of free choice permission, the implication feels correct but is not, and quickly leads to absurdity if allowed.

For Ross' paradox, the implication feels absurd but is correct. As we observed, there is a way to make sense of these conflicts with intuition – Ross' statement may only be saying that at least one disjunct is obligatory and the free choice permission may, strictly speaking, only indicate that at least one is permissible. As von Wright considers, free choice permissions may simply be, in practice, such that both disjuncts are permissible with some regularity. Still, this does not conform well with how these expressions are *used* or with the desire that our formal systems represent this use. In each case, the ordinary logic of disjunction does not meet with our intuitions about how deontic expressions function.

Modal Logic and Choice

In chapter three we moved past deontic logic to discuss modal logic more generally. In doing so we recognized that the paradox of free choice permission is merely one species of a larger class of problems we have called free choice disjunction:

FCD) $\langle x \rangle(\varphi \vee \psi) \rightarrow (\langle x \rangle\varphi \ \& \ \langle x \rangle\psi)$

This formulation is more general as many so-called free choice permission problems involve choice between actions that are not properly characterized as permissions at all. Rather, these include scenarios like coming to a fork in the road. E.g.:

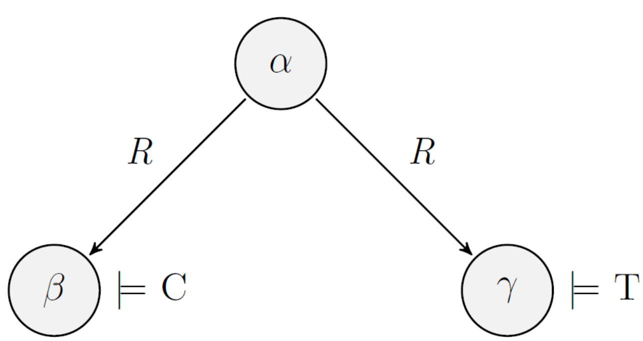
You can travel west, or you can travel east

While a permission is not involved, we nevertheless are faced with a free choice disjunction. So we generalized to include other modal operators (e.g. action operators).

In addition, chapter three made use of Kripke semantics for modal logic in order to make clear the structure of a free choice disjunction. As we saw, we were able to differentiate between two characteristic structures. In the first, the intuition behind true *choice* disjunctions is realized:

Figure 3.2.2
Kripke Model $\mathfrak{M}_1$:
Free Choice

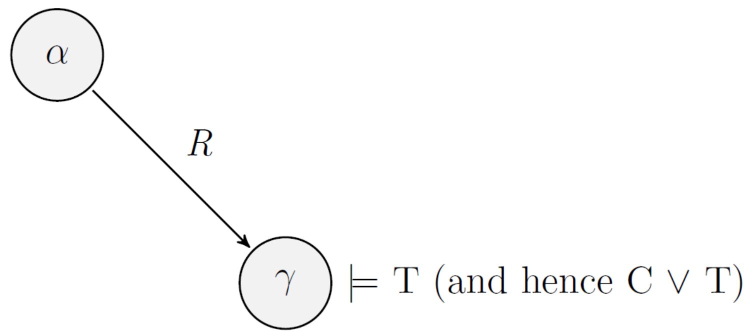
$\alpha \vDash \Diamond(C \vee T), \Diamond C \ \& \ \Diamond T$



In the second, a choice disjunction may be true, even though a real choice is unavailable:

Figure 3.3.1
Free Choice
Counterexample $\mathfrak{M}_2$:

$\alpha \vDash \Diamond(C \vee T), \neg(\Diamond C \,\&\, \Diamond T)$



With this distinction in mind, a formal disambiguation is clear. In the first true free choice scenario (with exclusive options) what is meant is:

$$\Diamond C \,\&\, \Diamond T \,\&\, \neg\Diamond(C \,\&\, T)$$

Whereas, if choice is not necessarily available and we only know that at least one option is real, we can say just:

$$\Diamond(C \vee T)$$

Thus, a formal translation of the free choice is available and has a clear semantics in accordance with our intuitions.

Still, the central worry remains. In practice, we do not use the natural language correlate of the conjunction free choice expression – we use disjunction! And we do engage in a pattern of inference akin to von Wright's FCP claim as well as in what Zimmerman has called the Choice principle:

CP) X may A or B $\vDash$ X may A and X may B

Semantics of Free Choice Disjunction

The earliest attempts to treat the free choice permission paradox were semantic in nature and aimed at modifying the meaning of permission (and, by extension, other modal operators) in such a way that the inference in von Wright's FCP claim holds without generating catastrophic results for the system as a whole. In Chapter four we explored the most common effort of this sort, which was to treat free choice permissions as *strong* permissions.

The ordinary notion of permission in formal deontic logic treats permission *weakly* insofar as it follows the modal notion of possibility – that is, to say that A is permissible only tells us that there is *some* possible outcome where A is the case and that this outcome is permitted. In a *strong* permission for A, *all* outcomes where A is the case are permitted. Treating free choice permissions as strong permissions guarantees the permissibility of each disjunct (since each of these outcomes will make the disjunction true). In the restaurant choice, choosing either option will make the permitted disjunction true, and thus, by treating free choice permission strongly, we get as a consequence:

$$\langle p \rangle_S (C \vee T) \rightarrow (\langle p \rangle C \;\&\; \langle p \rangle T)$$

As we saw, however, this proposed solution is not without difficulties. The problem is that, by treating these permissions strongly, we allow also any other outcome which guarantees the truth of either disjunct. Consider:

$$\langle p \rangle_S (C \vee T) \rightarrow \langle p \rangle (C \;\&\; M)$$

where C & M is the abstract representation of 'have coffee & murder the waiter'. Clearly this should *not* be permissible and is not meant to be implied by a free choice permission.

Asher & Bonevac (2005) have offered an improvement where we adopt a refined version of strong permission in which, rather than unlimited allowance of all possible realizations of the options, we restrict ourselves to allowance of all *normal* outcomes. I have called this *robust* permission and provided some elucidation of the mechanics of this operator. As illustrated, such an operator can be well defined by classifying worlds as normal (or not) relative to a world where the robust permission is available.

Pragmatics of Free Choice Disjunction

While the use of *robust* permission does handle the formal problems associated with free choice permissions, there remains motivation to treat the problem in a non-semantic manner. We do not, for example, specify in natural language what kinds of permission are involved in situations like the restaurant example. And certainly, there are times when we express permissions in very similar ways which are not intended as robust permissions. In practice, there seem to be contextual factors which determine the way in which a permission is intended.

These facts (among other reasons) have led to perhaps the dominant strategy for dealing with the paradox, which is to treat the problem *pragmatically*. In Chapter five we examined two such pragmatic solutions – the pioneering turn to pragmatics made by Kamp (1973) and the now dominant Gricean account of free choice disjunction presented by Kratzer & Shimoyama (2002).

For Kamp, permission has a contextual component in which a permission-granting agent removes prohibition of some outcomes from some permission-receiving agent. In this way, a whole class of formerly prohibited outcomes moves into the status of permissible when a permission is granted. For example:

$$\langle p \rangle (C \vee T)$$

indicates that, insofar as a permission granting agent's authority allows, all states of affairs which realize the permitted claim are now allowed. There is a similarity here with robust permission in that the permissibility of each individual disjunct is practically guaranteed while strange outcomes (such as having coffee and murdering the waiter) remain prohibited. But whereas robust permission achieves this result by opening up as permissible all *normal* worlds in which the disjunction is true, Kamp achieves it in a purely pragmatic way; namely, by stipulating that those (formerly prohibited) worlds in which the permission granting agent has the power to remove prohibitions must, after a permission is granted, be within the realm of the permissible.

In an altogether different pragmatic approach, Kratzer and Shimoyama (2002) adopt a Gricean view in which the permissibility of the individual disjuncts is concluded as a result of conversational implicature. The main thrust of their argument is that, upon seeing a disjunctive permission like the restaurant drink option, we realize that the server does, in fact, know which of these are permissible and so we may ask ourselves why they would have chosen to phrase the permission as a disjunction if only one were actually allowed. It cannot be that either is unavailable as the server would have made a logically stronger claim in accordance with a Gricean maxim of quantity (Grice 1975). And so, it must be that both are available. Hence:

$$\langle p \rangle C \,\&\, \langle p \rangle T$$

In the case of either Kamp or Kratzer & Shimoyama, the crucial step is in treating deontic logic exactly as initially outlined by von Wright. Rather than modifying the system with (for example) an expanded notion of permissibility, the free choice inference is a pragmatic inference.

Formalizing Pragmatics

Though this turn to pragmatics has proven fruitful, there is, as noted, a strong similarity between an approach like Kamp's and the semantic solution outlined by Asher & Bonevac (2005). This similarity attests to characterizations of the paradox of free choice disjunction as falling on the boundary between semantics and pragmatics. In chapter six we turned then to an exploration of this boundary and to ways in which we might move between the pragmatic solutions offered and semantic versions of those solutions.

As illustrated, it is possible to reconceive of the solution offered by Kamp in a completely semantic way by treating the notion of permission granting as a modal action which, once taken, removes prohibitions from some permission-receiving agent. In this way, we can use a multimodal model with transitions corresponding to both the actions of the permission-receiving agent and the actions of a permission-granting agent. Moreover, we can easily stipulate that these permission-granting ‘actions’ have exactly the features described by Kamp (e.g. removing all prohibitions within the authority of the permission granter as stated by the permission itself). We can capture this semantic representation of Kamp’s solution within a multimodal model (e.g. Model 6.2.2) or even make the action of granting permission a kind of dynamic modality in which the model itself transforms in accordance with the permission granted.

In treating the Gricean account given by Kratzer & Shimoyama, I employed a barrage of tools under the umbrella of epistemic and dynamic logics to again give a semantic account which was arguably equivalent to their pragmatic explanation. The idea was that we could express a cooperative principle within a multi-agent dynamic logic in the following way:

Formal MQ+)

$$\langle !_x\varphi/c\rangle\top \rightarrow \neg((\psi_R \,\&\, [K_x]\psi) \,\&\, ((\psi \mathbin{+\!\!\rightarrow} \varphi) \,\&\, \neg(\varphi \mathbin{+\!\!\rightarrow} \psi)) \,\&\, \langle !_x\psi/c^{\cup}\rangle\top)$$

Roughly, this formula states that if some agent x makes a cooperative public announcement of φ, then it cannot be that case that there is a relevant alternative announcement ψ which is also *known by x, logically stronger than* φ and crucially will also be cooperative. This final condition (that alternative announcements must also be cooperative) allows for a formal explication of recursive pragmatic strengthening.

In this way, a free choice permission, while initially allowing for a wide space of possibilities (and hence uncertainty about the permissibility of individual disjuncts), can be deliberated about in the very manner described by Kratzer & Shimoyama and in a way that can be represented *within* the formal system. At each stage in the deliberation, the model is ‘pruned’ until eventually we arrive at the conjunction of possibilities. Chapter six provided an initial sketch of many of these formal details but it was not my intention to advance a formal account of Gricean reasoning. Rather, the key point was that, should we choose to do so, we can. Thus, which parts of this problem (and presumably other problems) are understood semantically or pragmatically may be a decision that we make based upon what exactly we include among our semantic tools.

The Role of Logic and Conclusions

This realization – that we can choose to modify our logical systems in such a way as to move pragmatic concerns into the realm of semantics (and vice versa) – was central to our discussion in chapter seven and the overall conclusions of this thesis. As I argued, whether we treat a problem like the paradox of free choice disjunction as one demanding a semantic solution or as one demanding a pragmatic solution depends heavily on the level of analysis we use in treating the problem.

When we consider the myriad ways in which meaning may be ascribed to even ordinary expressions, there always seem to be pragmatic considerations which may go beyond those stable parts of language we incorporate into our view of semantics. This contextualist view is suggestive that there can be no semantic solution to free choice disjunction.

Alternatively, we may see some parts of language as semantically stable and if so, our purposes matter insofar as we choose some semantic base and frequently discard or ignore those considerations which may complicate semantic analysis. Where we deal with meaning that cannot be accounted for by this semantic base, we move to the realm of pragmatics.

But, as we have seen, we can make decisions about what exactly we do include in this semantic base and, at least in the case of free choice permission if not more generally, many pragmatic concerns may be treatable in a semantic way if we expand or modify the formal systems we take as representative of natural language and natural language inference.

Those who cling strongly to pragmatic accounts of free choice permission, I argued, may be doing so as a result of a naïve understanding of formal logic as functioning only as an abbreviation of natural language. In this way, a desire to translate a natural language expression like:

> You may have coffee or tea

into the formal language as:

$$\langle p \rangle (C \vee T)$$

is rooted in the feeling that each of these formal symbols functions as a kind of shorthand. But this cannot be the case, as so often natural language expressions are themselves ambiguous and the formal system is used to disambiguate by expressing these disparate meanings in different ways.

When we adopt a view of logic as idealized language, we turn questions of translation on their head. Rather than trying to symbolize natural language expressions in as structurally similar a way as possible, we instead preserve meaning as closely as possible. The logic of quantifiers provides ready examples of this, given the fact that few natural language statements which are correctly modelled by first-order quantificational formulas have any significant structural similarities to them.

So too, I argue, with free choice disjunction. So, rather than seeking to explain the free choice disjunction inference:

$$\langle x\rangle(\varphi \vee \psi) \rightarrow (\langle x\rangle\varphi \mathbin{\&} \langle x\rangle\psi)$$

we should instead realize that the correct formal expression of free choice disjunction simply is:

$$\langle x\rangle\varphi \mathbin{\&} \langle x\rangle\psi$$

In this way, rather than needing to account for the free choice disjunction inference or a choice principle, we instead need to account for what I have called the revised choice principle:

Revised CP) Natural language sentences of the form 'X may A or B' *probably* translate formally, in accordance with Gricean or other explanations, to: $\langle p\rangle A \mathbin{\&} \langle p\rangle B) \mathbin{\&} \neg\langle p\rangle(A \mathbin{\&} B)$

Of course, we can alternatively incorporate this principle directly into our semantics, as we illustrated in chapter six, though this comes at the cost of a far more complex logical system. And, should we choose to maintain a simpler formal semantics (say, only basic propositional logic, quantification theory, and basic modal constructions), then there still is a pragmatic story to tell regarding how a natural language 'or' statement has come to convey the meaning of a formal conjunction of possibilities. In this regard, most pragmatic accounts perform admirably. But we should not be confused into thinking either that these solutions are *necessarily* pragmatic, or that they explain a formal inference. Rather, they explain how we come to understand the meaning intended, and thus the proper formal translation.

8.2 Further Directions

This thesis has raised a number of issues that remain open for fruitful exploration. Two avenues of inquiry seem especially promising:

First, it remains to be shown whether a full account of Gricean reasoning can be captured by the tools of the logical dynamics program. I have in chapter six provided a rough sketch of how this might look but I suspect a fuller and more elaborate account would fill in a number of interesting details. In particular, it seems likely that additional tools (e.g. the modal μ-calculus) might allow for recursive analysis of complex and layered Gricean implicatures. Additionally, the remainder of Grice's maxims could be explored, as well as the manner in which cooperation can be incorporated as a kind of public announcement. It has not been my intention in this thesis to argue for such a system but, even so, the possibilities are exciting.

Secondly, I believe there is additional work to do in making explicit the role of logical languages. Though I have here motivated the view that logical languages are best used to represent the *meaning* of natural-language claims, many still tend to use logical languages to represent the structure, appearance, or other peculiarities (e.g. particular operators) that seem involved in natural language claims. This tendency comes from a loose use of logic-as-shorthand in much philosophical argumentation. I suspect that a great number of philosophical problems suffer from similar confusions in translation.

Appendices

Appendix A

Logic and Grammar

The formal languages discussed in this thesis are composed of logical formulae which are constructed in accordance with precise rules. These formation rules are concisely expressed with the use of a generative or transformative grammar. For example:

$$\varphi ::= \varphi_a \mid \neg\varphi \mid (\varphi \vee \psi)$$

where φ_a is a basic unanalyzed proposition drawn from a set of atomic proposition letters {A, B, C, …} as well as the unanalyzed ⊤, ⊥ (always true, always false).

A grammar like this gives a complete set of rules for the construction of logical formulae. The example grammar shown above gives us formation rules for ordinary propositional logic. It states:

i) Any substitution instance of φ is a formula of the language.
ii) The negation symbol '¬' may be placed before any formula φ of the language and the resulting expression '$\neg\varphi$' is also a well-formed formula of the language.
iii) Any two formulae may be enclosed in parentheses and separated by the symbol '∨' and the resulting expression '$(\varphi \vee \psi)$' is also a well-formed formula of the language.[78]

We are here treating {∨, ¬} as the complete set of logical operators. These alone will allow us to construct a propositional logic as additional operators {&, →,

[78] This thesis will also adopt the common convention that outermost groupers may be discarded and will frequently take liberties dropping additional groupers in cases like (A & B & C & D) where there is no possibility of ambiguity.

$\leftrightarrow$, …} can be defined in terms of these (though we could just as easily include them in the grammar of the language).

Thus, a generative grammar like this one allows us to build the complete language of propositional logic in successive steps. Or, alternatively, we can determine recursively, for any string of symbols, whether it is a well-formed formula of the language. For example, the formula:

$$(\neg(P \vee Q) \vee R)$$

can be shown to be well-formed since it can be deconstructed in accordance with the above rules.

Modal Constructions

The most basic modal operators are denoted in the usual manner:

$\Diamond\varphi$	possibly φ
$\Box\varphi$	necessarily φ

These can be understood as the ordinary (alethic) modality or as general (undefined) modal existential/universal operators. I will regularly denote other modal operators uniformly by placing characterizing letters inside square or angle braces. The deontic modalities are therefore:

$\langle p\rangle\varphi$	It is permissible that φ
$[o]\varphi$	It is obligatory that φ

These are added to generative grammar in the manner described above. E.g.:

$$\varphi ::= \varphi_a \mid \neg\varphi \mid (\varphi \vee \psi) \mid \Diamond\varphi \mid \Box\varphi$$

for a propositional modal base logic, or:

$$\varphi ::= \varphi_a \mid \neg\varphi \mid (\varphi \vee \psi) \mid \langle p\rangle\varphi \mid [o]\varphi$$

which we can interpret as a deontic logic (under the correct axiomatization). When providing quotations or referring to historical work (e.g. von Wright's dyadic deontic system), I will occasionally use traditional modal denotations. E.g. $P\varphi$.

Appendix B

Dyadic Deontic Logic

Von Wright's system of dyadic deontic logic (1968) allowed for the expression of *strong permissibility* by identifying three versions of permissibility which expressed strong permission. The system employs a modal operator for permission (and, derivatively, obligation) which is evaluated on the basis of two separate statements. Von Wright's formulation for these dyadic expressions treats any permission claim as conditional on some other claim:

$$\langle p \rangle (\varphi / \psi)$$

The above formula is interpreted as 'φ is permitted given ψ'. Obligation is defined through the equivalence

$$[o](\varphi/\psi) \stackrel{\text{def}}{=} \neg\langle p \rangle\, (\neg\varphi/\psi)$$

and, similarly, is interpreted as 'φ is obligatory given ψ'. Von Wright's dyadic system includes the following six definitions of conditional permission:

> "P_1 (p/q)" shall mean that in *some* possible world in which it is true that q *some* possible world is permitted in which it is true that p.
>
> "P_2 (p/q)" shall mean that in *all* possible worlds in which it is true that q *some* possible world is permitted in which it is true that p.
>
> "P_3 (p/q)" shall mean that *some* possible world in which it is true that p is such that it (this world) is permitted in *every* world in which it is true that q.
>
> "P_4 (p/q)" shall mean that *every* possible world in which it is true that p is such that it is permitted in *some* possible world in which it is true that q.
>
> "P_5 (p/q)" shall mean that in *some* possible world in which it is true that q *all* possible worlds are permitted in which it is true that p.
>
> "P_6 (p/q)" shall mean that in *all* possible worlds in which it is true that q *all* possible worlds are permitted in which it is true that p. (von Wright 1968, 24)

In order to make sense of these six kinds of permission, it is helpful to understand the logical possibilities von Wright is trying to exhaust with these categories. As a start, consider the following four ways in which we can combine *some* and *all* with respect to possible worlds and permission:

> Some worlds where q permit some worlds where p
> All worlds where q permit some worlds where p
> Some worlds where q permit all worlds where p
> All worlds where q permit all worlds where p

These four combinations of ways in which worlds permit worlds are von Wright's P_1, P_2, P_5, P_6. Now consider the following:

> Some worlds where p are permitted by some worlds where q
> All worlds where p are permitted by some worlds where q
> Some worlds where p are permitted by some worlds where q
> All worlds where p are permitted by all worlds where q

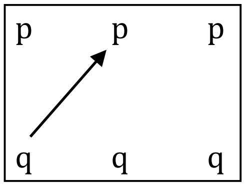
These four combinations of ways in which worlds can be permitted are von Wright's P_1, P_3, P_4, P_6. So, the eight possible combinations of some and all permitting and being permitted by reduce to the six permission types von Wright describes.

In making these definitions, von Wright aims to make explicit a differentiation between weak and strong permissibility. Roughly, q is weakly permitted just in case it is not obligatory for not-q. But we can say something much more powerful about allowable options – with strong permission it is always ok that q.

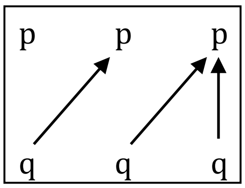
Recall, the standard deontic notion of permissibility is one of weak permission. It is weakly permitted that q iff $\neg[o]\neg q$. But, with the above dyadic expansions to permissibility, we can now separate permissions P_1, P_2, and P_3 as weak permissions while P_4, P_5, and P_6 are strong permissions. The key feature of these strong permissions is that if q is permitted then all worlds where q is true are allowed (at least under some circumstances).

These can be best understood graphically by considering some of the modal models they produce:

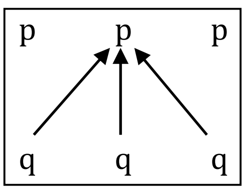
Figure AB.1 Dyadic deontic models



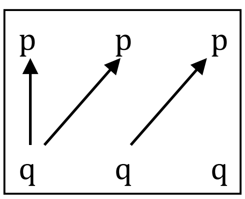
$P_1(p/q)$
Some worlds where q permit *some* worlds where p
Some worlds where p are permitted by *some* worlds where q



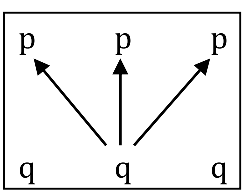
$P_2(p/q)$
All worlds where q permit *some* worlds where p



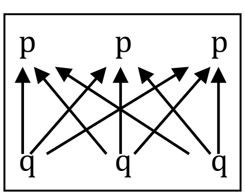
$P_3(p/q)$
Some worlds where p are permitted by *all* worlds where q



$P_4(p/q)$
All worlds where p are permitted by *some* worlds where q (strong permission)



$P_5(p/q)$
Some worlds where q permit *all* worlds where p
(strong permission)



$P_6(p/q)$
All worlds where q permit *all* worlds where p
All worlds where p are permitted by *all* worlds where q
(strong permission)

With the definition of obligation $\neg P_n(\neg\varphi/\psi)$ applied to each of the above permission types, we arrive at six related notions of obligation. Just as the free choice inference holds with some of these permission types, Ross' Theorem $[o]R \rightarrow [o](R \vee Q)$ does not hold for some of these obligation types.

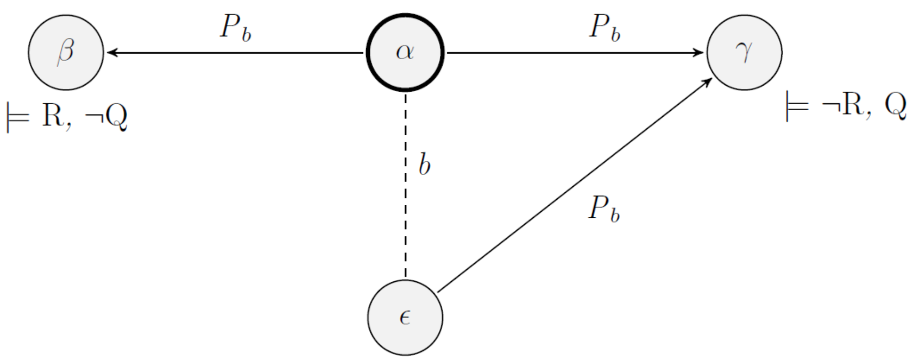
Appendix C

Dynamic Converse Operators

Public announcement logic with converse dynamic operator

Consider the model $\mathfrak{M}$:

Figure AC.1 Free choice mid deliberation



In this model an agent a is permitted (indicated by the P-labeled accessibility relation) to realize (R ∨ Q). They know that Q is permitted but are unsure if R is also permitted.

Public announcement !⟨p⟩R

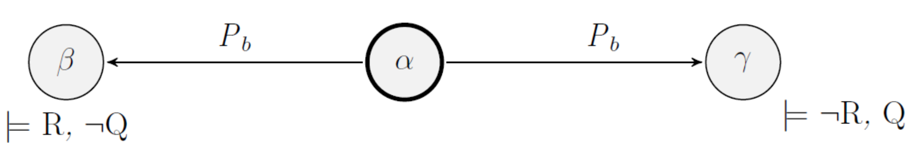
This would normally eliminate the a-uncertainty link between worlds δ, β.

Thus [!⟨p⟩R] [K_a](⟨p⟩R & ⟨p⟩Q) (after announcement a knows that both are permitted)

In an ordinary public announcement logic, the update model would be:

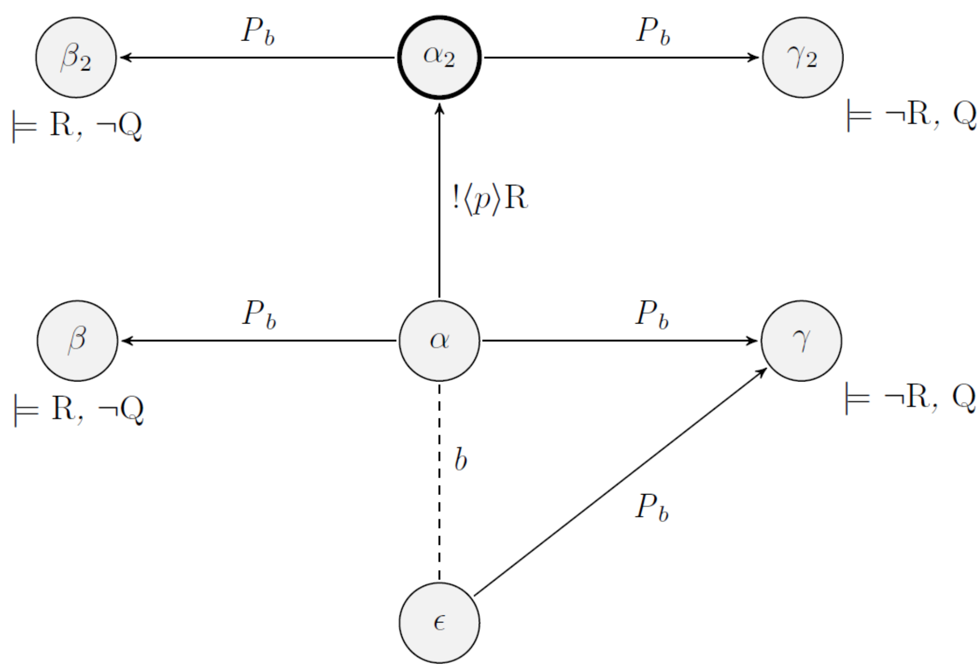
Figure AC.2 Simple choice deliberation outcome

$\mathfrak{M}$/⟨p⟩R:



But, as we need to keep the old model around to use the dynamic converse operator, the update instead expands the model as:

Figure AC.3 Model Updating with Dynamic Converse Operator



In this way, at world β_2 we could say (for example):

$\langle !\langle p\rangle R^{\cup}\rangle \top$

and thereby say something like 'the announcement happened'[79]. This technique has been applied in dynamic epistemic temporal model updating and the transitional arrows are treated as temporal transitions. In this case we have also temporal transitions between all corresponding worlds (e.g. α to α_2).

[79] In the fullest possible analysis, these models would be much larger, having already 'kept around' the models present since the first pre-permission state.

Appendix D

Formal Accounts of Relevance

Commonly, formal accounts of relevance establish a relationship between propositions e.g.:

R(A, B)

This is a relationship which is symmetric, reflexive and non-transitive. That is, all propositions are related to themselves, and if A is related to B, B is also related to A. But it is possible that A is relevant to B and B to C, while A and C are not relevantly related.

This is established against the backdrop of a *topic* of conversation. This can be defined in a number of ways but for our purposes will be a set of propositions:

$T = \{\varphi, \psi, \dots\}$

Relevance is established through the sharing of at least one propositional variable (Epstein 1979, Woods et al., 2000, 141)[80]. For example, given:

$T = \{A, B, C, D\}$

and the statements:

$\neg A$
$C \rightarrow A$
$B \vee C$

We will see that:

$R(\neg A, C \rightarrow A)$
$R(C \rightarrow A, B \vee C)$
$\neg R(\neg A, B \vee C)$

[80] In light of various approaches which use the description ‘relevant’ or ‘relevance’ in arguably different ways that I use here, ‘*relatedness’* is perhaps a better term for these idea – ‘relatedness’ is the term used by Epstein (1979).

In this way, the idea of *relevant alternatives* in our formal Gricean model is established through this relation. In the Kratzer & Shimoyama example (2002, 18) 'Two books are under discussion. An algebra book and a biology book', we can treat this as a set T = {A, B}. As in their example, a statement

$$\langle p \rangle A$$

will generate an exhaustivity inference since

$$\langle p \rangle A \,\&\, \langle p \rangle B$$

is logically stronger and it is the case that

$$R(\langle p \rangle A, \langle p \rangle A \,\&\, \langle p \rangle B)$$

In this way we can evaluate logical strength but in a way that is limited with respect to the valuation of some well-defined set of propositions.

Again, this is only a rough outline of one approach though there are many other more developed approaches we could take (see e.g., Mares 2014 for a survey of other relevance logics and their semantics).

Glossary of Key Concepts

Paradox of Free Choice Permission (1.1)

A paradox of inference that arises when a disjunction is bound by a permission operator, e.g. May(A or B). From such a statement, we cannot infer either disjunct. However, in choice situations like 'You may have coffee or tea' we *can* infer either option. The problem of free choice permission is to determine why these choice inferences are legitimate (von Wright 1968). Formally, the paradox of free choice permission concerns a formula that seems plausible and yet cannot be a theorem:

FCP) $\langle p\rangle((R \vee Q) \rightarrow (\langle p\rangle R \ \& \ \langle p\rangle Q)$

Narrow Scope Disjunction (2.1)

A modalized disjunction where a single modal operator (e.g. may) ranges over a disjunction. E.g.: May $(\varphi \vee \psi)$

Wide Scope Disjunction (2.1)

A modalized disjunction where separate modal statements are disjoined. E.g.: May $\varphi \vee$ May ψ

Formally, narrow and wide scope disjunctions are (standardly) equivalent though their natural language readings are arguably suggestive of differing interpretations.

Ross' Paradox (2.2)

A paradox of obligation where, given an ordinary obligation claim, e.g. 'You must mail the letter', we are able to infer the same obligation disjoined to any other claim, e.g. 'You must mail the letter or burn it'. According to the standard logic of disjunction, the inference is valid. Formally, this paradox is expressible as Ross' Theorem:

RT) $[o]R \rightarrow [o](R \vee Q)$

Choice Effect (2.6)

The effect of 'or' expressions in some formulations to offer a choice and thereby warrant an inference to either disjunct. Choice effects are often noticed in the context of generic expressions like 'Elephants live in Africa or Asia' (Nickel 2010).

Free Choice Disjunction (3.1)

A generalization of the problem of free choice permission under any modal interpretation. E.g.: 'It might be that (A or B)'. This can be formally represented as:

FCD) $\langle x \rangle(\varphi \vee \psi) \rightarrow (\langle x \rangle\varphi \; \& \; \langle x \rangle\psi)$

Kripke Semantics (3.2)

The standard semantics for modal logic (Kripke 1963). Truth conditions for modal statements are given by models consisting of nodes (worlds) and directed arcs between worlds (accessibility relations). Propositions have truth-values at worlds and the modal operators are defined as:

$\Diamond\varphi$ when, at *some* accessible world, φ holds.
$\Box\varphi$ iff $\neg\Diamond\neg\varphi$, thus, $\Box\varphi$ when, at *all* accessible worlds, φ holds.

Quasi-Conjunctivity (3.5)

Having the feature of conjunctivity, where in a binary compound expression we can infer either of the available statement components, while also having the feature of disjunctivity, that we cannot infer both.

Choice Principle (3.5)

The principle of entailment by which to make a free choice inference (Zimmerman 2000). This can be expressed as:

CP) X may A or B $\vDash$ X may A and X may B
Or, formally: $\Delta(A \vee B) \vDash \Delta A \; \& \; \Delta B$

Strong Permission (4.1)

A strengthened version of modal permissibility whereby all possible worlds which realize the permission are accessible. We can contrast this with ordinary (weak) permission:

Weak permission: $\langle p \rangle \varphi$ iff there is some accessible world where φ
Strong permission: $\langle p \rangle_S \varphi$ iff All worlds where φ are accessible

Robust Permission (4.4)

A defeasible version of strong permission that treats an action or proposition as strongly permissible under defeasible (normal) circumstances:

$\langle p \rangle_R \varphi$ iff All normal worlds where φ are permissible

Gricean Conversational Implicature (5.3)

A pragmatic process in which cooperation is assumed among conversational participants and as a result, meaning can be derived (implicated) which goes beyond ordinary inferences or semantic meaning (Grice 1975).

Recursive Pragmatic Strengthening (5.4)

A process by which conversational strategies assume higher-order conversational implicatures. Thus, conversational participants can make (or anticipate) implicatures about implicatures. The 'standard' account of Gricean conversational implicature explanations of free choice inferences is one such higher-order account whereby implicatures are determined *about* implicatures (Kratzer & Shimoyama 2002).

Logical Dynamics (6.3)

A general programme in modal logic which involves a deviation from normal Kripke semantics to include model updating (model change) upon receipt of new information. This includes epistemic additions to modal logic as well as a dynamic modal operator which indicates what follows from a model update (see, e.g. van Benthem 2010).

Semantic Minimalism (7.2)

The view that within our cognitive language processing system there is a stable base of meaning which holds in the absence of contextual information. Thus, statements have meaning, though this is frequently enhanced or changed by contextual factors present in the utterance of these statements.

Contextualism (7.2)

The view that contextual information shapes meaning at every level of analysis in the determination of sentence meaning. In this sense, sentences do not have meaning at all. Rather, utterances have meaning.

Montague's Thesis (7.3)

The view that there is 'no important theoretical difference between natural languages and the artificial languages of logicians' and that it is 'possible to comprehend the syntax and semantics of both kinds of language within a single natural and mathematically precise theory' (Montague 1970).

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