AN ANALYTICAL FRAMEWORK FOR OPTIMAL PLANNING OF LONG-TERM CARE FACILITIES IN ONTARIO

AN ANALYTICAL FRAMEWORK FOR OPTIMAL PLANNING OF LONG-TERM CARE FACILITIES IN ONTARIO

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To my family and late uncle

Abstract

Long-term care facility network in Ontario, and in Canada as a whole, encounters critical issues regarding balancing demand with capacity. Even worse, it is faced with rising demand in the coming years. Moreover, there is an urgent need to provide long-term care for patients in their own language (particularly French). This study proposes a dynamic Mixed-Integer Linear Programming model based on the current standing of the long-term care system in Ontario, which simultaneously optimizes the time and location of constructing new long-term care facilities, adjusting the capacity (namely, human resources and beds) of each facility dynamically, and the assignment of patients to the facilities based on their demand region, gender, language, and age group over a finite time horizon. We apply the diversity-support constraints, based on patients' gender and language, to save patients from loneliness and to comply with the Canadian values of providing care. Finally, we validate the model by performing a case study in Hamilton, Ontario. An extensive set of numerical analyses are explored to provide deeper insights into the whole issue. One set of such analysis is an extensive simulation study to examine the effect of distributional uncertainty in some of the input parameters on the optimal results, hence providing a much more realistic understanding of the optimization model.

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In the end, I declare my deepest gratitude to my parents and siblings for their endless love and support during my life. Without their help, my achievements were simply impossible.

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1 Introduction

Long-term care (LTC) refers to various medical and non-medical services for people, particularly older adults, with disabilities or chronic diseases (Zhang & Puterman, 2013). It helps people with their daily routine, including walking, taking a shower, getting dressed, toileting, and other activities like housework, making meals, and shopping (Li, Zhang, Kong, & Lawley, 2016). Because of the increase in the senior population, chronic illness spread, and injury incidence, the need for LTC is increasing sharply across the world (Li et al., 2016).

According to the white paper on the capacity planning and development by Ontario Association of Non-Profit Homes and Services for Seniors (OANHSS), Ontario has different types of care for the elderly containing seniors' housing, long-term care, home care, and community services (Ontario Association of Non-Profit Homes and Services for Seniors, 2016).

Seniors' housing choices deliver dwelling and assistance to applicants. For example, seniors who are not eligible to enter an LTC home but are at high-risk can receive in-home support by supportive housing. Also, retirement homes are suitable for those who can pay for their care and still are active and independent but need some aid for their daily activities such as housekeeping and making meals.

Long-term care home (sometimes called nursing home or LTC facility), which is the focus of this study, provides housing, nursing care, and support with daily activities to people who need 24-hour care and are not able to live independently in the community.

Home care services, which can be short-term or long-term, propose healthcare-related supports to individuals in their own home. These supports include a different range of assistance, such as personal and home support and nursing. Home care can be a suitable alternative to long-term care home and retirement home, as it is cost-efficient, and patients have the opportunity to stay in a favorable environment with their family and friends. Community services are an addendum to home care services. These services consist of various ranges of supports, such as transportation, meal delivery, social services, and mental care.

In Ontario, the Local Health Integration Networks (LHINs), which are government associations, specify whether applicants are eligible to enter an LTC home. Generally, people can be qualified to enter LTC home when they have serious physical or mental problems, and they cannot live in their own home or retirement home (Ontario Long Term Care Association, 2016). Since 2010, decision-makers have changed the criteria for new patient admission, which require people to be in need for high or very high physical or mental support to be admitted into LTC home (Ontario Long Term Care Association, 2016). This may lead patients to stay in LTC home until the end of their life. Here is an overview of LTC home residents' characteristics in 2016 in Ontario (Ontario Long Term Care Association, 2016):

- Ninety-seven percent need aid with their routines such as leaving the bed,
- Fifty-eight percent employ a wheelchair,
- More than thirty percent are extremely dependant on staff,
- Ninety percent have a cognitive disability (more than 30 percent are in intensive condition),
- Ninety-seven percent have more than one chronic illness such as heart disease,
- Forty percent suffer from a mood disease like depression,
- Thirty-eight percent require supervision for a severe medical status.

LTC facility network in Ontario faces some critical issues which motivate us to conduct this study. There is a high demand for LTC such that in 2017, almost 34,000 people were waiting to receive a bed (Ontario Long Term Care Association, 2018). On the other hand, statistics show that over the next two decades, Ontario will encounter a surge in demand for LTC beds as a result of growth in older adults population (Ontario Association of Non-Profit Homes and Services for Seniors, 2016). This growth results in doubling the population of seniors who are 65 years of old or older, quadrupling those who are 85 years or older, and tripling seniors with the age of 100 or older (Ontario Association of Non-Profit Homes and Services for Seniors, 2016). Improvements in medical care can also be a reason for the increase in the population of older adults. Another study by Munro et al.

(2011) indicates that by 2035, the number of people who require LTC bed in Ontario will be almost 238,000 compared to their number in 2019, which is about 128,000.

Lack of efficient planning in hospitals and LTC network prevents them from optimal efficiencies in the operations. Hospitals, most of the time, are encountered with excessive demand beyond their capacity (Jonathan Patrick, 2011). Around 15 to 20 percent of this excessive demand is because of the so-called alternate level of care (ALC) patients who do not need to remain in acute care services but are waiting to be discharged to a more proper environment such as LTC home (Jonathan Patrick, 2011). According to the president of the Ontario Hospital Association, managing ALC patients is the most pressing problem in hospitals (Jonathan Patrick, 2011). Shortage of capacity in LTC homes is the principal reason in the backlog of ALC patients in hospitals (Zhang & Puterman, 2013). Therefore, an efficient capacity planning in LTC network not only can solve the capacity problem in LTC home but also has a considerable impact on solving the congestion crisis in acute care settings.

Ontario is known as a multicultural land with residents from 200 countries of 130 different languages (Government of Canada, 2019). Although the official language is English, there are various French-speaking communities athwart the province, and the government services in many places are provided in English and French (Government of Ontario, 2019). However, there is a shortage of French-language human resources in LTC facilities, as well as a lack of optimal distribution of patients based on their language across facilities. Therefore, depression is one expected disorder for patients in a healthcare environment such as LTC home where there are no human resource and/or patients whom they can interact with.

Moreover, according to a recent report in The Globe and Mail (2018), there is an urgent need to provide long-term care for patients in their own language (particularly French). This is essential for patients' safety, health, and quality of life. Also, as patients particularly those with chronic diseases, such as asthma and diabetes should have good communication with their health team, language problems may complicate the management of chronic conditions. This report mentions that the lack of language services may lead to some hidden costs to the healthcare system. For example, when human resources cannot communicate efficiently with patients, they perform more diagnostic tests leading to an increased length of stay in health services. Also, when patients are not completely understood to have an accurate diagnosis, they tend to have more medical appointments for the same issue.

Unlike other parts of the healthcare system, in LTC, there are very few quantitative studies that focus on optimizing the capacity planning and its ramifications. More specifically, there is a lack of analytical studies to provide insights on the optimal capacity planning in Ontario's LTC that consider the need of local communities as an urgent matter.

To fill the above gaps, this study presents a dynamic Mixed-Integer Linear Programming (MILP) model based on the current LTC system in Ontario to determine the time and location of constructing new LTC facilities, the capacity (namely, human resources and beds) of each facility at each time, and how to assign patients based on their demand region, gender, language, and age group to facilities over the time horizon. The principal objectives of the optimization model are to minimize expected total costs of planning, while diversity-support criteria (here, gender and language diversity) are explicitly considered as constraints.

The remainder of the thesis is organized as follows. Chapter 2 presents a brief literature review relevant to this research. We define the problem and its formulations as a MILP model in chapter 3. Computational results and numerical analysis are presented in chapter 4. Finally, in chapter 5, we conclude the study and discuss some future works.

2 Literature Review

Over the last decades, Mathematical Programming has been the primary technique for dealing with the delivery of healthcare services (Teresa Cardoso, Oliveira, Barbosa-Póvoa, & Nickel, 2016). In this chapter, we present a brief literature review on the location-allocation and capacity planning problems in healthcare with a focus on the LTC sector. To classify the studies, we examine articles which focus on facility location, allocation, capacity planning, facility location and capacity planning, location-allocation and capacity planning problems, in healthcare, as well as some related works to the LTC sector.

Healthcare facility location problems aim to optimize the location of healthcare facilities when there is a certain number of those facilities, and a set of constraints must be met. Allocation problems, on the other hand, deal with the optimal assignments of patient and resources to various healthcare services. Mitropoulos et al. (2006) present a mathematical programming model with two objectives to determine the location of hospitals and primary healthcare centers. The first objective is to minimize the distance between patients and facilities, while the second objective distributes the facilities among people in an equitable way. In another study, Shroff et al. (1998) use the Data Envelopment Analysis (DEA) tool to determine the optimal potential spots for constructing a long-term care facility in Northern Virginia by considering criteria pertaining the characteristics of their healthcare services.

Optimizing patients and resource assignment to hospitals during the influenza outbreak is examined in Sun et al. (2014). In the objective function, they minimize costs related to patients access to the healthcare facilities. Also, their model can estimate the shortage of medical resources in a specific time and for a particular hospital during the increase in demand. In another study, Kim and Kim (2010) use a branch-and-bound algorithm to specify the location of LTC facilities to optimize the number of patients in each facility. To do so, they consider many assumptions such as no facility at the beginning, and that the patients' need should be met in the nearest facility to the patient groups. In another work related to allocation problem in the healthcare area, Hertz and Lahrichi (2009) present a mixed-integer non-linear model for assigning different types of patients to nurses in home care services while their goal is to have equivalent workloads for nurses and prevent long distance to meet a patient. In their study, they use a Tabu Search algorithm for solving the problem. They solve a simplified version (linear mixed-integer program) of their model using CPLEX.

According to Li et al. (2016), in general, the capacity planning problem deals with balancing supply and demand for an organization, where the capacity planning may be used to determine the number of human resources at a home-health agency, the number of beds, diagnostic devices and operating rooms in a hospital or the number of hospitals in a zone.

Santibáñez et al. (2009) propose a multi-period mathematical programming model to determine the location of hospitals and their bed capacity in British Columbia by using certain weighting methods. As the first study on capacity planning in LTC networks, Li et al. (2016) model transfers among nursing homes and Home and Community-based Services (HCBS) using an open migration network. To specify the optimal capacity of LTC service networks, they use a newsvendor-type model. As a result of their study, they conclude that when there is a financial restriction, decreasing the capacity of nursing homes instead of HCBS has a better outcome in total network profit. Moreover, in limited population size, HCBS need less capacity than nursing homes. Besides, since strong capacity resilience (i.e., the ability to increase capacity in case of sudden demand surge) results in requiring less capacity, decision-makers should increase the capacity resilience in LTC facilities. Finally, they mention that adjusting the quality of care in HCBS can have a substantial effect on reducing costs.

To mention another study relevant to capacity planning in LTC network, Lin et al. (2012) present an optimization problem to determine the infrastructure capacity of HCBS with the goal of minimizing the cost of LTC services through giving services to patients in HCBS instead of LTC facilities which is much more expensive than HCBS. They also use a compartmental model to simulate the transition among different sections of LTC.

There is a list of papers which use simulation techniques for capacity planning in the long-term care network including Patrick (2011), Patrick et al. (2015), Zhang and Puterman (2013), and Zhang et al. (2012). In a seminal work, Patrick (2011) presents a Markov decision process (MDP) to determine the capacity of long-term care that is needed to have a lower limit of the alternative level of care (ALC) patients in hospitals (as mentioned earlier, ALC patients are those who no longer need to stay in an acute care hospital, but remain there because of capacity shortage in an appropriate ALC setting such as a long-term care facility). He also uses simulation to estimate how considering the decisions from the MDP model can affect the community demand wait times for LTC and help decision-makers for planning the LTC capacity in the future. Zhang and Puterman (2013) determine the capacity of LTC in each year over time planning horizon by using simulation optimization techniques. As the demand and Length of Stay (LOS) in long-term care may be different for individuals with various ages and genders, they categorize clients based on these two criteria to have more accurate results.

In order to meet the demand from community and hospitals in a timely manner, Patrick et al. (2015) use simulation technique to determine appropriate capacity for both LTC and supportive housing which is used as decision support to reduce the congestion in LTC for patients who do not need 24-hour supervision. Instead of providing guidelines for managing the demand from all hospitals and community simultaneously, their work provides a policy only for one hospital. Zhang et al. (2012) combine discrete event simulation, optimization, and demographic and survival analysis to satisfy patients' wait time limitations, by guaranteeing the minimum LTC capacity levels over a multi-year planning horizon. One of their significant conclusions is that age, gender, and geographic region noticeably affect the patients LOS.

Researchers have also examined the LTC network for the better estimations of demand. Cardoso et al. (2012) develop a multi-service simulation model based on Markov cycle tree structure to estimate the demand for LTC services at the small-area level. In addition to using Monte Carlo simulation to model demand uncertainty, they consider different types of health and socioeconomic characteristics to approximate demand. An interesting part of their study, which is highly relevant to our research is that they show how LTC demand can be affected by age and gender. Finally, for the validation purposes, they run their model in Excel for the county of Lisbon, Portugal. Hare et al. (2008) present a deterministic multi-state Markov model to predict home cares and accommodation environments (such as residential care) for both publicly and non-publicly funded services in British Columbia, Canada. While some essential features like gender, geographic location, and race can affect the demand, they do not consider such features in their model explicitly.

There are also studies in the literature which investigate facility location, allocation, and capacity planning problems at the same time containing Cardoso et al. (2015a), Cardoso et al. (2015b), Cardoso et al. (2016), Intrevado et al. (2019), Mestre et al. (2015), and Stummer et al. (2004). Mestre et al. (2015) present two stochastic location-allocation models about hospital network planning. Stummer et al. (2004) apply a multi-objective mathematical model to specify the place and size of different medical departments for a given set of hospitals. In the objective function, they minimize travel costs related to patients, location-allocation costs, the number of rejected patients, and the number of units moves to have a new hospital plan. They use a two-phase solution approach to solve their model and perform a numerical case-study based on the data from the hospitals in Germany.

Although there are many studies about facility location problem in healthcare (Kim & Kim, 2010), the number of papers on location-allocation and capacity planning in LTC, especially surveys which use optimization techniques, is not considerable. To the best of our knowledge, Intrevado et al. (2019), Cardoso et al. (2015b), Cardoso et al. (2016), and Cardoso et al. (2015a) are the only studies that determine the optimal location-allocation and capacity of LTC services simultaneously. In none of these articles, important factors such as gender, language, age, patients diversity-support, patients' aging and consequently changing in their LOS or death rate, the need for human resources based on their language, and simulation study are considered explicitly, which, as we will see later, are taken into account in this study.

In this domain, Cardoso et al. (2015b) develop a two-stage stochastic MILP model to schedule delivery of LTC services in a nationalized healthcare system (such as Portugal). They minimize expected costs in the objective function while considering equity criteria (namely geographical equity, socioeconomic equity, equity of access, and equity of utilization) as constraints. Their multi-period and multi-service two-stage model considers demand and LOS as the uncertainty and in the first stage, determines a finite number of facility and capacity decisions while assignment decisions are made in the second stage. In another study, Cardoso et al. (2015a) consider maximizing health gains in the objective function as well as minimizing expected costs. Similar to Cardoso et al. (2015b) and Cardoso et al. (2015a), in the study conducted by Cardoso et al. (2016), the authors develop a multi-objective and multi-period mathematical programming model in order to determine the location and capacity of LTC services for the medium term in a nationalized healthcare system. Unlike their other studies, they treat budget limitation as a constraint, while considering different types of equity-related policies in the objective function. Most recently, Intrevado et al. (2019) develop a new model to design long-term care networks, which considers the patient-centric as a core. They introduce two deterministic dynamic mixed-integer linear programming models to help decision-makers how optimally determine the location of facilities, the capacity of LTC services and patient allocation while considering the so-called patients' quality of life concept which we also use in our optimization model. Treating facility location variables as positive integer variables is another new feature of their model. As one conclusion, they mention that using home cares as possible replacements to long-term care facilities can decrease LTC costs under special status.

Some papers in the literature, instead of location-allocation and capacity planning, focus on other aspects of long-term care. For instance, Xie et al. (2006) analyze the template of LOS in LTC facilities by considering patients features. Also, Robison et al. (2012) discuss the problems that clients have in LTC homes after transferring from home and community-based settings. Finally, the effect of home care on older people from financial perspectives is examined in Greene et al. (1998).

In this thesis, we present an optimization model that varies from the reviewed studies in several ways. As mentioned in the literature review, very few papers have addressed location-allocation and capacity planning in long-term care simultaneously. The significant contributions of this study, which are novel in the literature, are as follows:

- We introduce a mathematical programming model to simultaneously optimize location-allocation and capacity planning in the LTC facility network in the Province of Ontario in Canada, which has its own featuring characteristics.
- We consider diversity-support constraints for each facility. This is to save patients from loneliness (based on patients' gender and language), which is essential for the long-term care system in Ontario, as a Canadian value.
- We specify the need for different types of human resources, particularly based on their language to help decision-makers predict human resources capacity (e.g., English and French language human resources) required in Ontario.
- We characterize patients based on different and new demographic features, such as gender, language, and age.
- We associate the death rate with the patients' age in order to have a more accurate formulation.
- We consider the transition between patients' age groups over time to have more accurate results.
- We apply an extensive set of numerical analyses which provide deeper insights into the whole problem.
- We conduct an extensive simulation study to examine the effect of distributional uncertainty (which is richer than single point uncertainty considered in the Sensitivity Analyses) in the optimal results, hence a much more accurate understanding of the optimization model.

3 Modeling Framework

In this chapter, we define the problem by discussing the planning timeline, location selection for new facilities and capacity planning, patient assignments, and other policies. Then we develop the MILP model to formulate the problem.

3.1 Problem Definition

We consider LTC system that is operating in Ontario to build our model. In Ontario, long-term care facilities prepare nursing and support for individuals unable to live without outside help and need 24-hour care for daily living (Munro et al., 2011). Government agencies, i.e., LHINs, specify who is eligible to receive service in the facilities (Ontario Long Term Care Association, 2018). There are different types of long-term care facilities in Ontario, including for-profit, not-for-profit/charitable, municipal-run homes, and others, which are owned by individuals, companies, not-for-profit organizations, and municipalities and receive funds from the provincial government except for patients' accommodation fees (Ontario Long Term Care Association, 2016, 2018). According to Tanuseputro et al. (2015), the owners of the for-profit facilities can take some parts of funds as profit for the business advantage while not-for-profit facilities should spend all funds and profit only for issues related to the facility. This study focuses on not-for-profit long-term care facilities.

We develop a dynamic mixed-integer linear programming model based on the following characteristics.

Planning Timeline. Our model is dynamic, which means we consider a discrete set of periods in which new events (such as demand, adding facilities and capacity, assigning new patients to facilities, leaving the facility by patients, and transition between age groups that we explain in the coming sections) can happen and change the features of LTC facilities network over time. More specifically, the decisions and events at each period influence the next period.

Location Selection and Capacity Planning. Building LTC facilities, operating beds, and adding the capacity of each facility are all costly. In our optimization model, we are looking for optimal strategies to determine the location and timing of building new facilities among the defined discrete set of periods and sites, where each site can have none or one not-for-profit facility. We also designate the optimal policy for adding the number of beds in each facility over time and investigate how the number/workhours of each type of human resources should change in each facility and period.

Patient Assignments. The model assigns patients to the facilities in each period based on the location, gender, language, and age group of patients. As well as causing the patients' demand to be different in accordance to the demand point, gender, age group, and language, there are other critical factors to consider these patient classifications in the model. We will discuss these factors in the following paragraphs.

First of all, our model aims to allocate patients to the same service points as their demand points; and if it is not possible, to allocate them to the closest service points in which facilities are located. This type of allocation also helps the families and friends of the patients use a shorter journey for visiting patients. Therefore, if we give service to patients in a service point different from their demand point, there will be a penalty called *mis-assignment penalty* (also can be defined as geographic undesirable patient assignment penalty) based on the distance (a function of travel time) between demand and service points.

Secondly, by considering gender, the model can guarantee a minimum proportion of each gender at each facility. Moreover, since the rate of leaving LTC facilities can change by patients' age and in order to have a more accurate formulation, we categorize patients by their age groups.

Finally, considering language for planning the LTC facilities system in Ontario is critical. First, as an important contribution of this thesis, by defining the minimum number of patients, based on their language, at each facility, the model can address the patients' loneliness problem (due to language restrictions), which, based on interviews with several healthcare managers, is a serious issue in Ontario. This problem is mainly caused by the shortage of people speaking the same language. Second, we specify the need for various kinds of human resources who can speak in a specific language to help decision-makers fix the problem of lacking particular types of human resources (especially French language human resources in Ontario).

Other Criteria. In our model, if we do not assign a patient to any not-for-profit LTC facility, there will be a penalty called *un-assignment penalty*. We assume that unassigned patients will remain in acute care hospitals or will be allocated to other types of LTC facilities such as for-profit and municipal facilities. It should be mentioned that to open new facilities and to assign patients to a facility, the cost of un-assignment penalty should be higher than other costs (Li et al., 2016).

As seniors are entering LTC facilities in Ontario when they are older, and in high need of personal care than ever before, it is expected that they stay in LTC homes until the end of their lives. At the end of each period, some patients will leave the facilities because of death. Therefore, there is a death rate based on the patient's age which is used to calculate the number of empty beds (note that one can assume the death rate as departure rate to consider any other types of departure).

Also, since patients who remain in the facilities get older over time, they may transfer to another age group by the end of each period. Figure 3.1 illustrates what happens to residents in a facility over time (assuming three age groups). Accordingly, at the end of each period, some patients leave the facility from every age group (purple arrows), and the remaining patients may stay in their previous age group (blue arrows) or transit to the older age group (brown arrows). Also, at the beginning of each period, there are patient assignments from different classifications to all age groups (black arrows).

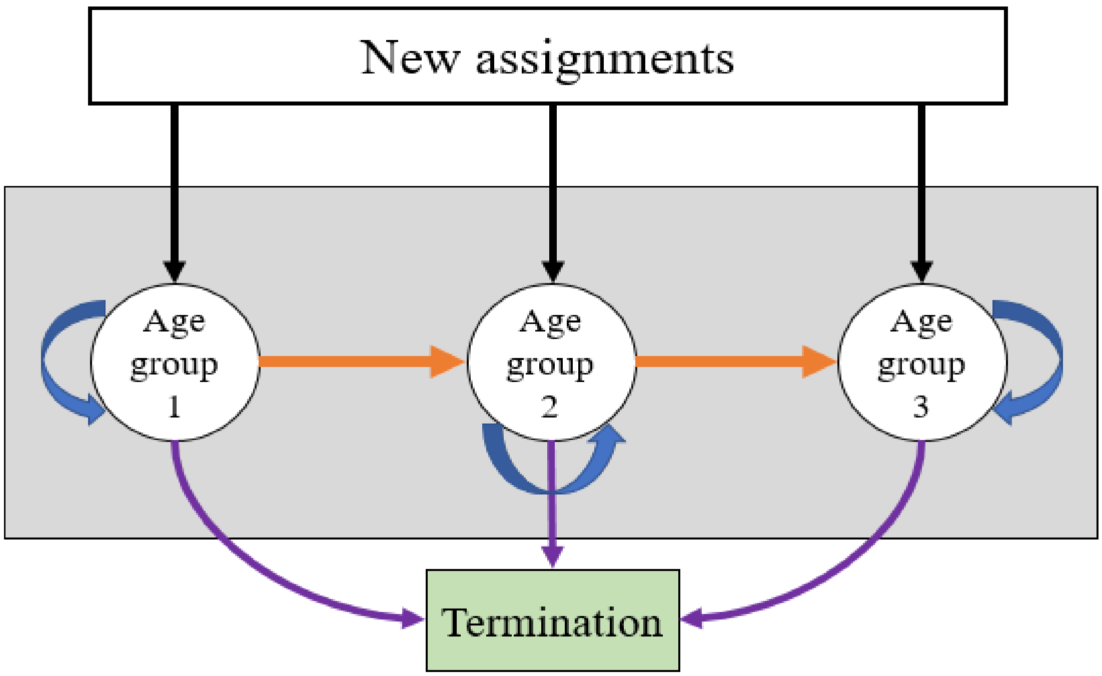

Figure 3.1: Overview of residents transitions in each facility per period

3.2 Problem Formulation

3.2.1 Notations

Our model includes seven indices as follows. We use i to indicate demand region and j to illustrate long-term care service region. h is the index for the human resource type. Indices g, l, and a represent patients' gender, language, and age group, respectively. Finally, we use t to demonstrate each time period. The summary of all notations that we apply in the mathematical model is as follows.

Indices

i index for the demand regions.

j index for the long-term care (LTC) service regions.

h index for the human resource types.

g index for the patients genders.

l index for the patients languages.

a index for the patients age groups.

t index for the time periods (discrete).

We use the following sets for the various elements of our model.

Sets

I set of demand regions.

$J = J^a \cup J^b$ set of LTC service regions; divided into subsets J^a(a subset of regions which already have LTC facility), and J^b(a subset of regions which do not have LTC facility at the beginning of time period 1).

H set of human resource types.

G set of genders.

L set of languages.

A set of age groups.

T set of time periods.

The key parameters in our model are stated as follows.

Parameters

F_{jt} fixed cost of opening an LTC facility in location j at period t.

UP_t single period patient un-assignment penalty at period t.

MP_{ij} mis-assignment penalty from demand region i to location j (as a function of the geometric distance between demand region i and location j, which reflects the undesirability of a patient (who lives in region i but get admitted into a facility in location j)).

NB_j number of beds in LTC facility located in j at $t = 0$.

$MINBC_j$ minimum bed capacity allowed in LTC facility located in j (to control for the fixed cost).

$MAXBC_j$ maximum bed capacity allowed in LTC facility located in j.

N_{iglat} number of individuals from demand region i, gender g, language l, and age group a requiring LTC facility at time t.

NE_t total number of individuals requiring LTC at time t, for all t:

$$NE_t = \sum_{i \in I} \sum_{g \in G} \sum_{l \in L} \sum_{a \in A} N_{iglat} \qquad (1)$$

AC_t cost of adding one bed to any LTC facility at time t.

OC_t cost of operating one bed in LTC facility at time t.

Q_h hours of care that should be provided by human resource type h for each patient in one period.

D_{at} expected rate of death (departure) for patients from age group a at time t.

B_{at} proportion (i.e., expected rate) of patients in the age group a who transit to age group $a+1$ at time t.

MG_{gj} minimum proportion of patients from gender g who should be in the facility located in j.

MLA_{lj} minimum proportion (or number) of patients from language l who should be in the facility located in j.

M upper bound on the number of patient assignments for each language in the LTC facility located in j at time t.

Φ auxiliary coefficient (a large number).

γ correction factor.

The decision and auxiliary variables in our model are listed below.

Variables

X_{jt} binary variable; $X_{jt} = 1$ if and only if LTC facility is located in service region j at time t, 0 otherwise.

Y_{iglajt} number of beds to be made available to the patients from demand region i, gender g, language l, and age group a, requiring LTC, in the facility located in j, at t.

AB_{jt} number of additional beds in the facility located in j, at time t.

P_{iglajt} proportion of new patients from demand region i, gender g, language l, and age group a, receiving LTC in location j, at time t.

PE_t total number of new patients receiving LTC at time t.

HC_{hlajt} number of hours of care that should be provided by human resource h, for patients from language l, and age group a, receiving LTC in the facility located in j, at time t.

TAC_t total cost of adding beds at time t.

TOC_t total cost of operating beds at time t.

$TOFC_t$ total cost of opening facilities at time t.

Z_{ljt} binary variable; $Z_{ljt} = 1$ if and only if there is at least one patient from language l in LTC facility located in j at time t, 0 otherwise.

W_{ljt} minimum patients from language l who can be assigned to LTC facility located in j at time t.

δ_{ljt} binary variable; $\delta_{ljt} = 1$ if and only if $\sum_{i \in I} \sum_{g \in G} \sum_{a \in A} N_{iglat} \leq MLA_{lj} \times Z_{ljt}$, 0 otherwise. This is an indicator variable which identifies the situation where the demand from a specific language is less than the minimum threshold.

3.2.2 Mathematical Formulation

Considering all the criteria, objectives, parameters and decision and auxiliary variables mentioned above, we propose a mixed-integer linear programming model to determine the time and location of building new facilities, patients assignments from each demand region, gender, language, and age group to each LTC facility at any time, number of beds for patients from any demand region, gender, language, and age group in each LTC facility over time (both for new and previous patients), and how to increase the capacity in every facility over time. The objective function and constraints are defined and formulated as follows.

Objective Function. The objective function (2) minimizes the sum of the following cost items over T years:

1) Total costs of adding beds and operational costs (first term). Equation (3) formulates the costs of adding beds at each time by multiplying the number of increasing beds in each facility to the cost of adding every single bed. Equation (4) quantifies the costs of operating beds at each period through multiplying the number of all beds in all facilities to the cost of operating each bed.
2) The total costs of opening LTC facilities (second term) which are computed in equation (5).

3) The un-assignment penalties (third term). Un-assignment penalties are costs related to the patients who do not receive service in not-for-profit LTC facilities which can be calculated via multiplying patient un-assignment penalty by the difference between new patients receiving assistance and the total number of new demands. Using equation (6), we ascertain the total number of new patients assigned to a facility. Also, to minimize the total costs, in some conditions (mostly when un-assignment penalty is almost the same for all periods), the model may prefer to assign a higher percentage of patients in the last periods because of end-of-horizon effect for finite time problems (Puterman, 2005). To prevent this effect, we multiply the un-assignment penalty by a correction factor (γ), which is usually between one and ten percent, such that the un-assignment penalty decreases over time.
4) Mis-assignment penalties (last term), which is calculated by multiplying the total number of mis-assigned patients to the corresponding mis-assignment penalty.

$$Min \left[\left(\sum_{t \in T} (TAC_t + TOC_t) \right) + \left(\sum_{t \in T} TOFC_t \right) + \left(\sum_{t \in T} (NE_t - PE_t) \times UP_t \times (1-\gamma)^{t-1} \right) + \left(\sum_{t \in T} \sum_{i \in I} \sum_{g \in G} \sum_{l \in L} \sum_{a \in A} \sum_{j \in J} (Y_{iglajt}) \times MP_{ij} \right) \right] \tag{2}$$

Where

$$TAC_t = \sum_{j \in J} AB_{jt} \times AC_t \qquad \forall\, t \in T \tag{3}$$

$$TOC_t = \sum_{i \in I} \sum_{g \in G} \sum_{l \in L} \sum_{a \in A} \sum_{j \in J} \left(Y_{iglajt}\right) \times OC_t \qquad \forall\, t \in T \tag{4}$$

$$TOFC_t = \begin{cases} \sum_{j \in J^b} X_{jt} \times F_{jt} & \forall\; t = 1 \\ \sum_{j \in J^b} (X_{jt} - X_{j(t-1)}) \times F_{jt} & \forall\, t \in T, t > 1 \end{cases} \tag{5}$$

$$PE_t = \sum_{i \in I} \sum_{g \in G} \sum_{l \in L} \sum_{a \in A} \sum_{j \in J} P_{iglajt} \times N_{iglat} \qquad \forall\, t \in T \tag{6}$$

Constraints. Next, we can explain and present the set of constraints in our model.

- The constraint in (7) ensures that the number of patients from demand region i, gender g, language l, age group a who are assigned to an LTC facility at t cannot exceed the number of patients from demand region i, gender g, language l, age group a, who need service at t.

$$\sum_{j \in J} P_{iglajt} \leq 1 \qquad \forall\, i \in I, g \in G, l \in L, a \in A, t \in T \tag{7}$$

- The set of constraint (8) states that patients can receive service only in locations where an LTC facility exists.

$$P_{iglajt} \leq X_{jt} \qquad \forall\, i \in I, g \in G, l \in L, a \in A, j \in J, t \in T \tag{8}$$

- Constraints (9) quantify the number of total beds required at t in LTC facility located in j, for individuals from demand region i, gender g, language l, and age group a (which can also be considered as the number of total patient assignments from demand region i, gender g, language l, and age group a, to LTC facility located in j at t). In the first period ($t = 1$, the first inequality) there are only new assignments ($P_{iglajt} \times N_{iglat}$), while in other periods and for age group 1 ($t > 1, a = 1,$ the second inequality) number of total beds for patients from demand region i, gender g, and language l who receive LTC in location j is computed by summing new patient assignments with the remaining patients from the previous period (i.e., $(1 - D_{a(t-1)}) \times Y_{iglaj(t-1)}$) and subtracting patients transitioning to the next age group by the end of the former period ($B_{a(t-1)} \times Y_{iglaj(t-1)}$). Eventually, for calculating the total beds for other age groups ($t > 1, a > 1,$ third inequality), we add the patients who transfer from one age group to the next age group $\left(B_{(a-1)(t-1)} \times Y_{igl(a-1)j(t-1)}\right)$ to the second inequality.

$$Y_{iglajt} = \begin{cases} P_{iglajt} \times N_{iglat} \qquad \forall\, i \in I, g \in G, l \in L, a \in A, j \in J, t = 1 \\ \\ ((1 - D_{a(t-1)} - B_{a(t-1)}) \times Y_{iglaj(t-1)}) + (P_{iglajt} \times N_{iglat}) \\ \qquad \forall\, i \in I, g \in G, l \in L, a = 1, j \in J, t \in T, t > 1 \\ \\ ((1 - D_{a(t-1)} - B_{a(t-1)}) \times Y_{iglaj(t-1)}) + (P_{iglajt} \times N_{iglat}) + \\ \qquad \left(B_{(a-1)(t-1)} \times Y_{igl(a-1)j(t-1)}\right) \\ \qquad \forall\, i \in I, g \in G, l \in L, a > 1, j \in J, t \in T, t > 1 \end{cases} \tag{9}$$

- The number of additional beds per each facility and time (AB_{jt}) is calculated in equation (10), which is the difference between the required capacity in each facility and its previous capacity.

$$\sum_{i\in I}\sum_{g\in G}\sum_{l\in L}\sum_{a\in A} Y_{iglajt} = \begin{cases} NB_j + AB_{jt} & \forall\, j \in J, t = 1 \\ \sum_{i\in I}\sum_{g\in G}\sum_{l\in L}\sum_{a\in A} Y_{iglaj(t-1)} + AB_{jt} & \forall\, j \in J, t \in T, t > 1 \end{cases} \quad (10)$$

- The number of hours of care that should be provided by human resource type h, for patients from language l, and age group a, receiving LTC in the facility located in j, at t is calculated in equation (11). It multiplies the number of existing patients in long-term care by the number of hours of care that should be provided by each human resource for each patient as follows:

$$HC_{hlajt} = \sum_{i\in I}\sum_{g\in G} Y_{iglajt} \times q_h \qquad \forall\, h \in H, l \in L, a \in A, j \in J, t \in T \quad (11)$$

- Constraints (12) and (13) guarantee that, firstly, we consider facilities which already exist in the LTC network (constraint (12)), secondly, when we open a facility, it cannot be closed (constraint (13)).

$$X_{jt} \geq 1 \qquad \forall\, j \in J^a, t \in T \quad (12)$$

$$X_{j(t-1)} \leq X_{jt} \qquad \forall\, j \in J, t \in T, t > 1 \quad (13)$$

- The minimum and maximum bed capacity limitation per location are ensured in constraints (14) and (15).

$$MINBC_j \times X_{jt} \leq \sum_{i\in I}\sum_{g\in G}\sum_{l\in L}\sum_{a\in A} Y_{iglajt} \qquad \forall\, j \in J, t \in T \quad (14)$$

$$MAXBC_j \times X_{jt} \geq \sum_{i\in I}\sum_{g\in G}\sum_{l\in L}\sum_{a\in A} Y_{iglajt} \qquad \forall\, j \in J, t \in T \quad (15)$$

- We formulate the diversity-support constraints as follows.

- The gender diversity constraint in (16) ensures there is a minimum proportion of patients from each gender in every facility over time.

$$\sum_{i \in I} \sum_{l \in L} \sum_{a \in A} Y_{iglajt} \geq MG_{gj} \times \sum_{i \in I} \sum_{g \in G} \sum_{l \in L} \sum_{a \in A} Y_{iglajt} \quad \forall\, g \in G, j \in J, t \in T \quad (16)$$

- The language diversity constraint in (17) guarantees a minimum number of patients from each language in every facility over time. In case that we prefer to use minimum proportion instead of a minimum number of patients, we can use constraint (18). To generalize constraint (17), one can use constraints (19) - (26). Based on these constraints, it is not mandatory to assign patients of all languages to every facility over time. Constraints (19) and (20) consider the binary variable Z_{ljt} to verify whether there is at least one patient allocation of language l to the facility located in j at t. Also, according to the constraint (21), if we assign patients of language l to a facility at time t, the number of those specific patients in that facility at period t onwards should be greater than or equal to a minimum limit (if we have not assigned any patient of language l to the facility located in j at period t and previous periods, the minimum limit equals 0). This minimum restriction is determined via constraint (22). It states that the minimum restriction is equal to the minimum of the threshold that the decision-makers specify as the least number of patients from each language in every facility and the number of demands. Since (22) is non-linear, we replace it by constraints (23) - (26) to maintain the linearity of our model.

$$\sum_{i\in I}\sum_{g\in G}\sum_{a\in A} Y_{iglajt} \geq MLA_{lj} \times X_{jt} \qquad \forall\, l \in L, j \in J, t \in T \tag{17}$$

$$\sum_{i\in I}\sum_{g\in G}\sum_{a\in A} Y_{iglajt} \geq MLA_{lj} \times \sum_{i\in I}\sum_{g\in G}\sum_{l\in L}\sum_{a\in A} Y_{iglajt} \quad \forall\, l \in L, j \in J, t \in T \tag{18}$$

$$\sum_{i\in I}\sum_{g\in G}\sum_{a\in A} Y_{iglajt} \leq M \times Z_{ljt} \qquad \forall\, l \in L, j \in J, t \in T \tag{19}$$

$$\sum_{i\in I}\sum_{g\in G}\sum_{a\in A} Y_{iglajt} \geq Z_{ljt} \qquad \forall\, l \in L, j \in J, t \in T \tag{20}$$

$$\sum_{i\in I}\sum_{g\in G}\sum_{a\in A} Y_{iglajt} \geq W_{ljt} \qquad \forall\, l \in L, j \in J, t \in T \tag{21}$$

$$W_{ljt} = \min\ (MLA_{lj} \times Z_{ljt}\,, \sum_{i\in I}\sum_{g\in G}\sum_{a\in A} N_{iglat}) \qquad \forall\, l \in L, j \in J, t \in T \tag{22}$$

$$W_{ljt} \leq MLA_{lj} \times Z_{ljt} \qquad \forall\, l \in L, j \in J, t \in T \tag{23}$$

$$W_{ljt} \leq \sum_{i\in I}\sum_{g\in G}\sum_{a\in A} N_{iglat} \qquad \forall\, l \in L, j \in J, t \in T \tag{24}$$

$$W_{ljt} \geq\ (MLA_{lj} \times Z_{ljt}) - (\Phi \times\ \delta_{ljt}) \qquad \forall\, l \in L, j \in J, t \in T \tag{25}$$

$$W_{ljt} \geq\ \sum_{i\in I}\sum_{g\in G}\sum_{a\in A} N_{iglat} - (\Phi \times\ (1-\delta_{ljt})) \qquad \forall\, l \in L, j \in J, t \in T \tag{26}$$

- Constraints (27) - (35) determine the variables domain. We use $\mathbb{Z}_+$ and $\mathbb{R}_+$ to represent the set of nonnegative integer and real variables.

$$X_{jt} \in \{0,1\} \qquad \forall\, j \in J, t \in T \tag{27}$$

$$Z_{ljt}, \delta_{ljt} \in \{0,1\} \qquad \forall\, l \in L, j \in J, t \in T \tag{28}$$

$$Y_{iglajt} \in \mathbb{Z}_+ \qquad \forall\, i \in I, g \in G, l \in L, a \in A, j \in J, t \in T \tag{29}$$

$$AB_{jt} \in \mathbb{Z}_+ \qquad \forall\, j \in J, t \in T \tag{30}$$

$$PE_t \in \mathbb{R}_+ \qquad \forall\, t \in T \tag{31}$$

$$W_{ljt} \in \mathbb{Z}_+ \qquad \forall\, l \in L, j \in J, t \in T \tag{32}$$

$$P_{iglajt} \in \mathbb{R}_+ \qquad \forall\, i \in I, g \in G, l \in L, a \in A, j \in J, t \in T \tag{33}$$

$$HC_{hlajt} \in \mathbb{R}_+ \qquad \forall\, h \in H, i \in I, g \in G, l \in L, a \in A, j \in J, t \in T \tag{34}$$

$$TAC_t, TOC_t, TOFC_t \in \mathbb{R}_+ \qquad \forall\, t \in T \tag{35}$$

4 Computational Results

4.1 A Case Study in Hamilton, Ontario

4.1.1 Background

In this section, we apply our MILP model to solve the problem based on the specifications in Hamilton, Ontario. Ontario is the most populous province of Canada, with more than fourteen million residents. In this province, the Local Health Integration Networks (LHINs), run as the government of Ontario agencies, and are responsible for planning, funding, and integrating public healthcare services such as long-term care homes and hospitals at a regional level. There are fourteen LHINs in Ontario, and Hamilton is located in region four, namely the Hamilton Niagara Haldimand Brant (HNHB) LHIN (see Figure 4.1).

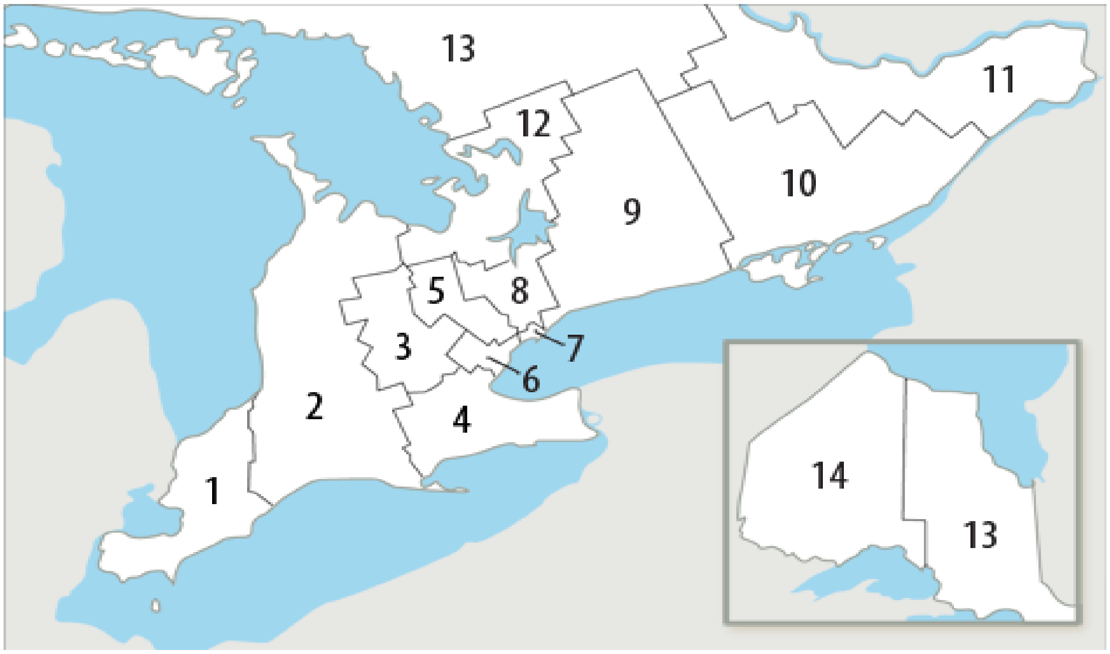

Figure 4.1: LHIN regions in Ontario (Ontario's LHINs, 2019)

Each of these fourteen regions has some smaller geographical areas known as sub-regions. Every sub-region helps its LHIN to have a higher satisfaction of patients' needs at a local level to respond to their distinct requirements in their place and time of need. HNHB LHIN consists of six sub-regions, namely Brant, Burlington, Haldimand Norfolk, Hamilton (the focus of this case study), Niagara, and Niagara North West (see Figure 4.2).

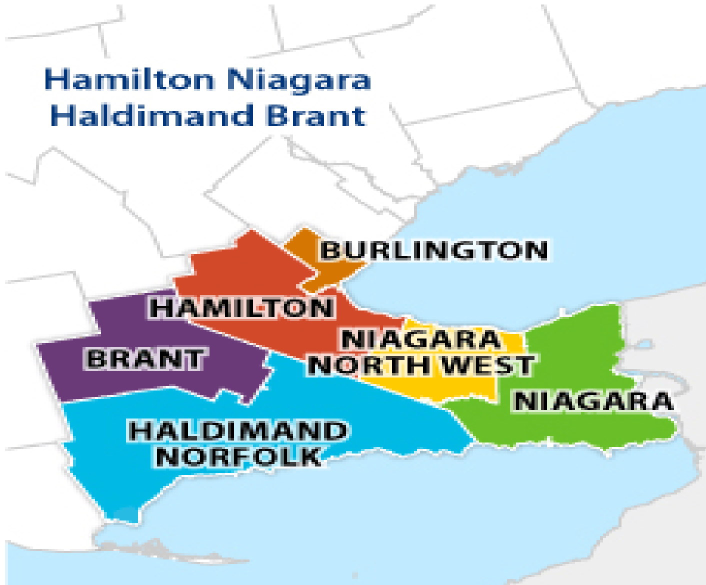

Figure 4.2: HNHB LHIN sub-regions (Health Services for HNHB, 2019a)

4.1.2 Assumptions and Parameters Estimation

Before stating our assumptions and describing our parameter estimations, we specify all the sets in our case study.

Demand and LTC Service Region sets. Hamilton City Council has divided the city into fifteen wards; the first eight wards which are parts of Hamilton city plus seven smaller towns and rural areas (see Figure 4.3). In our case study, we use the map (Figure 4.3) that was used in the 2016 census (City of Hamilton, 2016). We consider these fifteen regions as demand and LTC service regions, namely, in our case-study:

$$i \in I = \{1,2,\dots,15\}\,,\, j \in J = \{1,2,\dots,15\}$$

Based on the existing not-for-profit LTC facilities in Hamilton (Health Services for HNHB, 2019b) we divide service regions into areas which already have a not-for-profit LTC facility and areas which do not have a not-for-profit LTC facility, therefore:

$$J^a = \{1,2,8,9,13\}$$

$$J^b = \{3,4,5,6,7,10,11,12,14,15\}$$

$$(J = \{1,2,8,9,13\} \cup \{3,4,5,6,7,10,11,12,14,15\}).$$

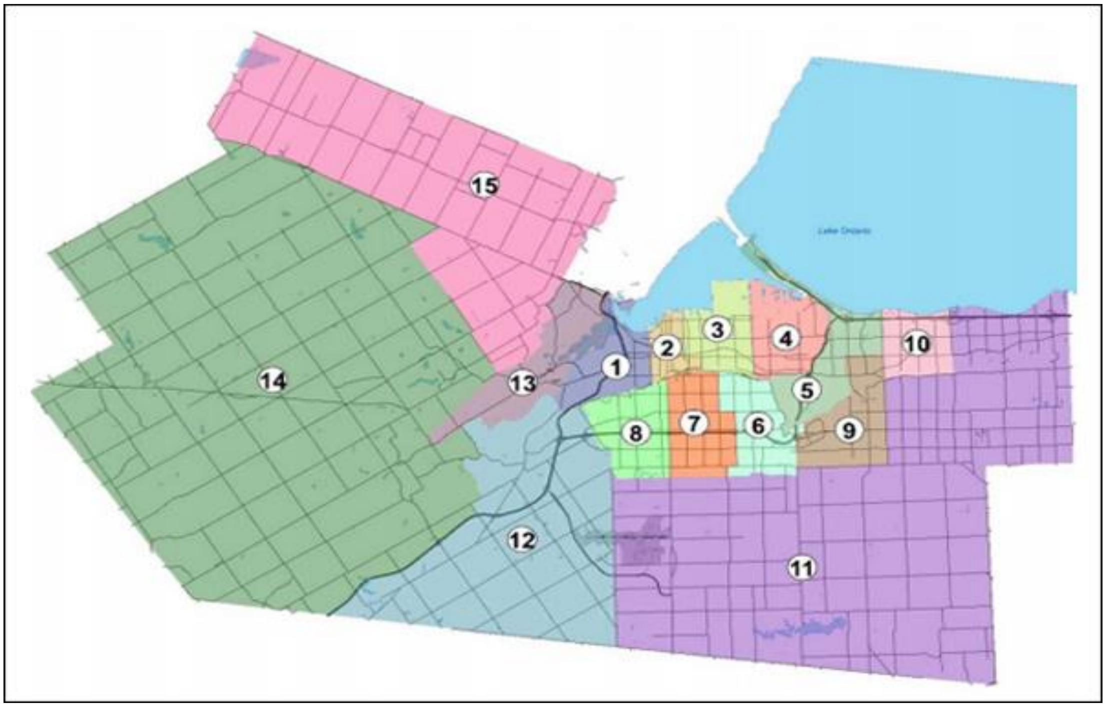

Figure 4.3: Hamilton city wards (City of Hamilton, 2018)

Other sets. Since we could find data just for male and female patients (there could be other gender specifications), we consider these two genders in the case study ($g \in G = \{1,2\}$, where $g = 1$ indicates male patients, and $g = 2$ shows female patients). Also, we divide patients' languages into English and French, as the official languages in Canada ($l \in L = \{1,2\}$, where $l = 1$ denotes English and $l = 2$ denotes French).

We also classify patients into three age groups ($a \in A = \{1,2,3\}$, where $a = 1, 2,$ and 3 represent the age-groups of 60-70 (excluding 70), 70-80 (excluding 80), and greater than or equal to 80, respectively. To determine the need for human resources, among various types of staff, we take into account Health Care Aids (also called Personal Support Workers) and Registered Nurses. Therefore, $h \in H = \{1,2\}$, where 1 refers to Health Care Aids, and 2 refers to Registered Nurses. Health Care Aids help residents in many ways including showering, brushing teeth, and dressing while Registered Nurses have different roles containing nurses leading, delivering direct nursing care to residents and responding patients need through coordinating the delivery of the care plan. Finally, each period accounts for one year, and we analyze the model for five years from 2020 ($t = 1$) to 2024 ($t = 5$), therefore in our case study, $t \in T = \{1,2,3,4,5\}$.

Now, we are ready to state our assumptions and describe the parameter estimations for our case study.

Fixed Cost of Opening an LTC Facility. This cost accounts for expenses regarding constructions, including purchasing land and other infrastructure. To the best of our knowledge, there are no documents that present this value in Hamilton, so we use the equation in Intrevado et al. (2019) which calculates the fixed cost of opening an LTC facility in Montreal, Quebec, assuming no considerable difference between the two cities, as follows:

$$F_{jt} = \$2{,}500{,}000 + \$135{,}000 * physical\ capacity\ of\ facility\ j \qquad \forall j \in J, t \in T$$

Where the first term is the fixed cost of purchasing the land and infrastructure and the second term is the variable cost based on the maximum beds that can be placed in the facility in region j.

Single Period Patient Un-assignment Penalty. There is no data or formulation to specify this value, and it depends on the jurisdictions and their policy regarding the distribution of patients among different kinds of LTC homes such as for-profit, not-for-profit, and municipal facilities and what percentage of clients they can support in LTC homes based on the available budget and other policies. We assume that if a patient cannot receive service from an LTC home, they receive support from acute care hospitals which is almost 6.5 times more expensive for the government to take care of ALC patients in hospitals instead of LTC home. According to Gibbard (2017), in Ontario, it costs hospitals \$949 to operate a bed per ALC patient per day (\$346,000 per ALC patient per year), and we consider that as an un-assignment penalty. Note that in the numerical analysis section, we analyze the sensitivity of our results to this parameter.

Mis-assignment Penalty. Similar to un-assignment penalty, there is no specific method to determine this value, and it depends on how the decision-makers prefer to distribute patients among LTC homes, as well as patients' priority. However, we assume that providing service to patients as close as possible to their home is a very reasonable policy. So, we consider the following equation to estimate the mis-assignment penalty:

$$\frac{TB_{ij} + TC_{ij}}{2} \times DP \qquad \forall\, i \in I, j \in J \tag{36}$$

In equation (36), the first term is the average time distance (in minutes) between the demand region i and service region j by public transportation (TB_{ij}) (we consider bus in the computational results because bus as the public transportation is the primary public vehicle in Hamilton) and private car (TC_{ij}), and second term (i.e., DP) is the penalty per each minute distance. Although DP is a subjective parameter, we define its value such that it just affects the patient distribution among facilities, not on the assignment/un-assignment decision ($DP =$ \$800). Because providing service to a patient far from their home is better than depriving a patient of nursing care in a facility. Note that we presume a higher penalty for demand and service regions in which there is no public transportation service compared to other areas.

The Number of Available Beds in LTC Facility Located in j at $t = 0$. The capacity of not-for-profit LTC facilities in Hamilton at $t = 0$ is listed in Table 4.1, which are derived from (Ontario's LHINs, 2019)

Table 4.1: Number of beds in each LTC facility at the beginning of time horizon

Region	Bed Capacity
1	127
2	128
8	210
9	167
13	378

Minimum and Maximum Bed Capacity Allowed in Each LTC Facility. To determine these numbers, we consider the smallest and biggest LTC facility in Hamilton (Ontario's LHINs, 2019) as the minimum and maximum bed capacity that each facility can have. Therefore, the minimum and maximum bed capacity assumed to be 41 and 378.

Expected New Demand from Every Region, Gender, Language, and Age Group at Each Time. We use the prediction made by Munro et al. (2011) for the demand for LTC homes in Ontario until 2035. We apply their detailed prediction to estimate new demand of various patients groups for five years (2020-2024) in Hamilton based on the following equation:

$$\begin{aligned} N_{iglat} = {} & (Total\ new\ demand\ in\ Ontario\ at\ t) \\ & \times\ (proportion\ of\ Hamilton\ population\ in\ Ontario\ at\ t) \\ & \times\ (ratio\ of\ region\ i\text{'}s\ population\ in\ Hamilton) \\ & \times\ (percentage\ of\ LTC\ patients\ with\ gender\ g\ in\ Hamilton) \\ & \times\ (proportion\ of\ people\ with\ language\ l\ in\ Hamilton) \\ & \times\ (ratio\ of\ LTC\ patients\ in\ Hamilton\ belong\ to\ the\ age\ group\ a) \end{aligned}$$

Where:

$$Total\ new\ demand\ in\ Ontario\ at\ t = total\ demand\ in\ Ontario\ at\ t -$$

$$((total\ demand\ in\ Ontario\ at\ t-1) \times (1 - patients\text{'}\ death\ rate))$$

For the first year (2020), we consider the total demand as the new demand. Note that we take the total LTC demand in Ontario for every year and the percentage of LTC patients with each gender in Hamilton from Munro et al. (2011). The patients' death rate and the ratio of LTC patients in each age group in Hamilton are captured from Tanuseputro et al. (2015). Also, the Ministry of Finance (2018) provides the proportion of the Hamilton population in Ontario over 2017-2041. The ratio of each demand region population in Hamilton is collected from (City of Hamilton, 2016). Finally, we obtain the proportion of languages spoken by people in Hamilton from Statistics Canada (2017). It is noteworthy that we use Ontario data to predict the percentage of LTC patients belong to each gender and age group in the city of Hamilton, as there is no specific data for Hamilton.

The Cost of Adding and Operating Each Bed. Based on the Ontario Long Term Care Association (2019), the cost of adding every single bed in an LTC facility in Ontario in 2019 is $70,000. Gibbard (2017) shows that operating each bed in Ontario costs nearly $52,000 per year, which includes expenses related to medical personnel, support services, food supplies, and so on.

Hours of Care from Each Type of Human Resources to Any Patient in One Period. The average amount of care provided to patients by Registered Nurses equals 0.44 hours per resident per day (160.6 hours per resident per year), while this value is 1.70 hours per resident per day (620.5 hours per resident per year) for Health Care Aids. These data are collected from Hsu et al. (2016).

Other Parameters. The expected rate of death for not-for-profit LTC patients who belong to age group 1, 2, and 3 is 0.108, 0.132, and 0.209, respectively (Tanuseputro et al., 2015). To the best of our knowledge, the transition rates between age groups in Ontario's

LTC facilities are not examined in a study. So, since the division of age group 1 and 2 are based on ten years classification, we presume that ten percent of patients who remain in the LTC home transfer to the next age group at the end of each period (based on uniform distribution). Thus, the patients' transition rate from age group 1 to 2 is 0.089, and from age group 2 to 3 is 0.086 (these rates are not 0.1, because some patients leave the facility at the end of each period). Since there is no data available for the trends in the transition and death rates, we assume that death and transition rates do not change over time.

The decision-makers should define the satisficing levels for the minimum percentage or number of patients in each facility based on the gender (MG_{gj}) and language (MLA_{lj}) specifications. Lacking this data makes us deem that at least twenty percent of patients in each facility should be from each gender. Also, since we are trying to avoid patient loneliness (due to language restrictions), we assume that 2 is the minimum number of patients of the same language in the same LTC home (further, there are usually two beds in a room of an LTC home that makes this value more reasonable), a simple yet practical definition of loneliness.

Finally, for the base scenario, we consider the correction factor equal to one percent as the base case and conduct a separate sensitivity analysis to examine the effect of changing this parameter on the optimal solution in the numerical analysis section.

4.1.3 Numerical Results for the Base Scenario

The model is coded in IBM ILOG CPLEX Optimization Studio 12.8.0, on a PC with 3 GHz Intel Core i5 processor and 8GB of RAM.

For the base scenario, we consider the first eight demand and service regions (i.e., without considering rural areas and towns) and do the numerical analysis based on this scenario. This scenario consists of 8,987 constraints and 8,260 variables (3,920 integer variables and 4,340 continuous variables), and all results are obtained while the optimality gap and computation time are less than 0.09% and 20 minutes, respectively. Also, in the Appendix (A.1), we show the summary of results for all 15 demand and service regions.

Table 4.2 shows the total bed capacity for LTC facilities in each service region and period and indicates how this capacity should increase over time. As the demand in the first period is more than other periods (because this demand consists of demand from its previous years as well as new demand) and at the end of each period, there are some empty beds that we can use for upcoming years, the number of additional beds in period 1 is much more than other periods.

Table 4.2: Number of total and additional beds in each service region and time (base scenario)

Service Region	Time (year)	Additional Beds	Total Beds
1	1	135	262
1	2	5	267
1	3	7	274
1	4	12	286
1	5	92	378
2	1	196	324
2	2	5	329
2	3	6	335
2	4	17	352
2	5	26	378
3	1	328	328
3	2	4	332
3	3	11	343
3	4	16	359
3	5	19	378
4	1	301	301
4	2	7	308
4	3	13	321
4	4	16	337
4	5	41	378
5	1	328	328
5	2	5	333
5	3	11	344
5	4	16	360
5	5	18	378
6	1	332	332
6	2	8	340
6	3	8	348
6	4	11	359
6	5	19	378
7	1	378	378
7	2	0	378
7	3	0	378
7	4	0	378

7	5	0	378
8	1	155	365
8	2	3	368
8	3	4	372
8	4	6	378
8	5	0	378

Figure 4.4 demonstrates the percentage of satisfied demand in each period. It shows that in each year, we give service to more than 80% of patients who need an LTC facility. By changing the un-assignment penalty, we can observe changes in the results (we show this in the numerical analysis section). Also, as a result of considering the correction factor a small value (equal to 0.01), it is evident that the proportion of people who receive service in the last period is more than other periods. In the numerical analysis section, we discuss how this proportion can change to provide more reasonable results.

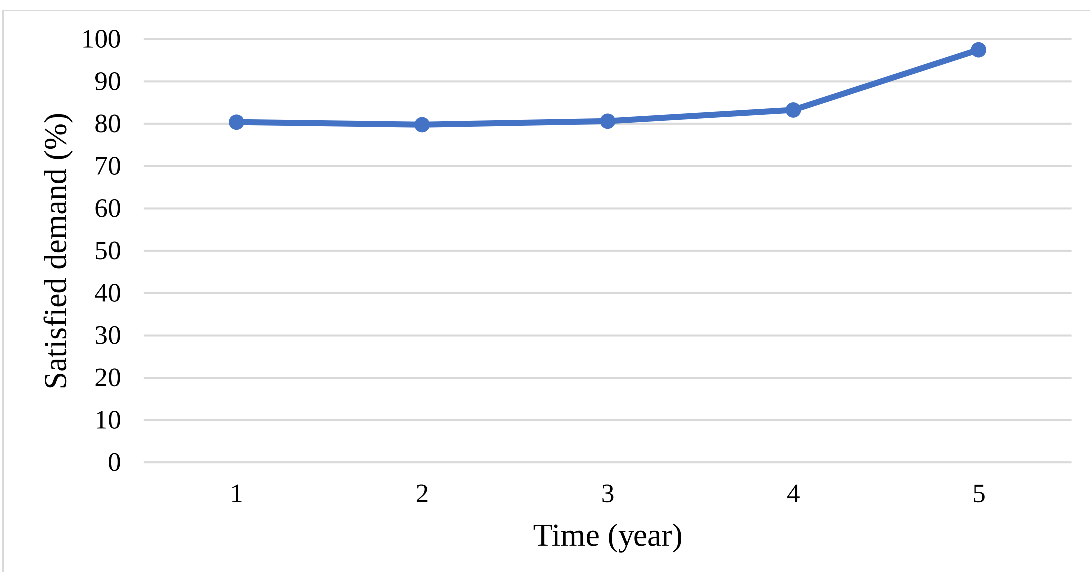

Figure 4.4: Satisfied demand in each year (base scenario)

Table 4.3 provides information about the human resources capacity required based on their language in each year. It is clear that as a majority of the population of patients in Hamilton speaks English, we need more human resources for this type of patients. Figure 4.5 illustrates the value of each portion of the objective function. We observe that, in this scenario, the most costly part of our LTC network is related to operating beds (47% of total cost), while the mis-assignment expenses represent the lowest portion (0.4% of total cost), mostly because we have a small number of mis-assigned patients. Eventually, Figure 4.6 shows the percentage of each source of the total cost (values are rounded to their nearest integer).

Table 4.3: Human resources capacity (base scenario)

Human Resource Type	Language	Time (year)	Hours of Care
1	1	1	1,500,369.0
1	1	2	1,526,430.0
1	1	3	1,564,280.5
1	1	4	1,623,228.0
1	1	5	1,747,328.0
1	2	1	124,100.0
1	2	2	120,997.5
1	2	3	120,377.0
1	2	4	119,756.5
1	2	5	129,064.0
2	1	1	388,330.8
2	1	2	395,076.0
2	1	3	404,872.6
2	1	4	420,129.6
2	1	5	452,249.6
2	2	1	32,120.0
2	2	2	31,317.0
2	2	3	31,156.4
2	2	4	30,995.8
2	2	5	33,404.8

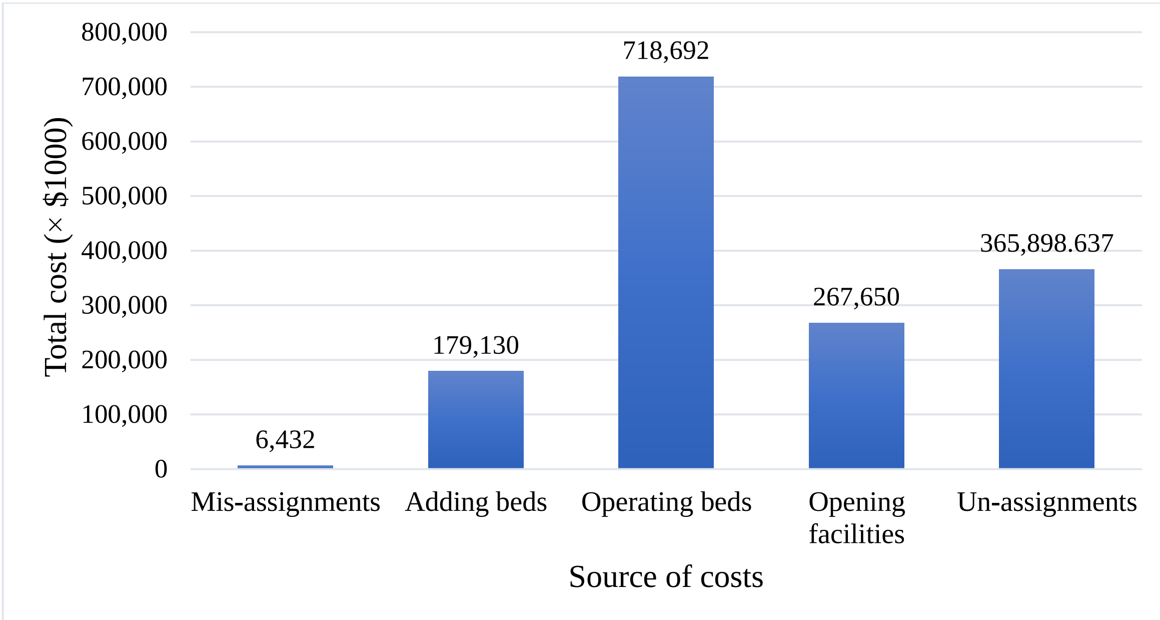

Figure 4.5: The share ($) of each source of cost in the objective function (base scenario)

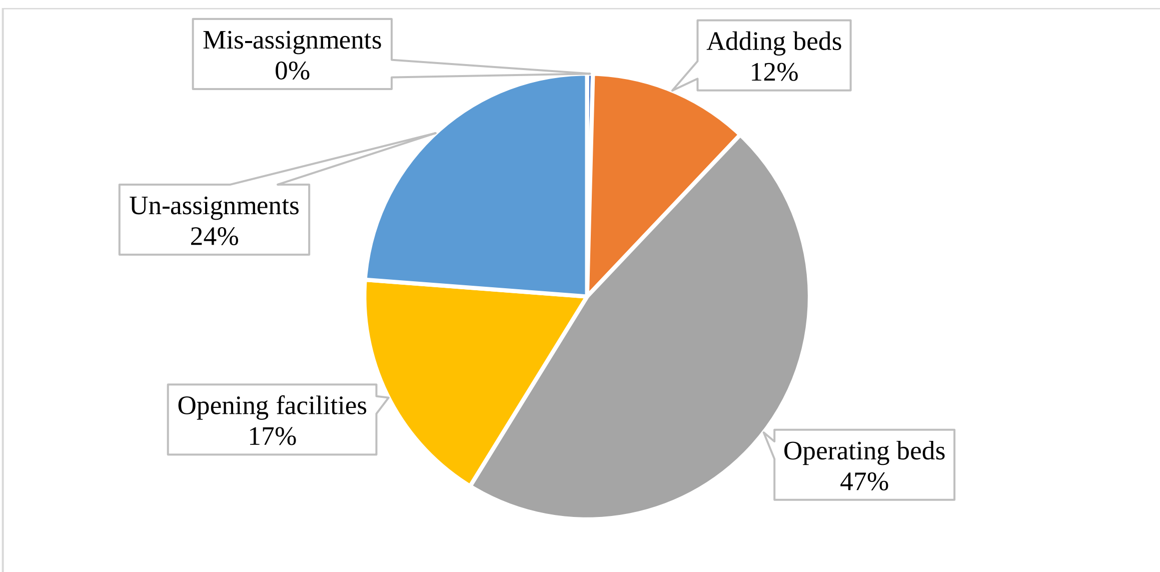

Figure 4.6: The share (in %) of each source of cost in the objective function (base scenario)

4.2 Numerical Analysis

In this section, we conduct an extensive set of numerical analysis to investigate the sensitivity of our results to:

a) un-assignment penalty

b) correction factor

c) mis-assignment penalty

4.2.1 Sensitivity to the Un-Assignment Penalty

Figure 4.7 shows how changing the un-assignment penalty can affect the percentage of patients receiving service in LTC facilities at each time. As expected, by increasing the un-assignment penalty, more patients receive service in LTC homes. When $UP = \$100{,}000$, we assign patients only to beds and facilities which already exist in the LTC network. Also, by increasing the un-assignment penalty from $300,000 to $500,000, the model prefers to take more proportion of patients in the first periods, which in total results in assigning more patients, as the demand in the first period is more than other periods. One noticeable trend that can be seen, especially when $UP = \$200{,}000$, is that model tends to assign more patients in the last periods (because of considering a small correction factor in the base scenario).

The effect of changing the un-assignment penalty on different types of expenses is provided in Figure 4.8. As one can see, only when $UP = \$100{,}000$, the mis-assignments cost equals zero. The model, also, does not allow for opening any new facility when the un-assignment penalty is equal to $100,000 and $200,000. There is no cost associated with adding beds when considering the un-assignment penalty of \$100,000, while this cost is similar and higher when the un-assignment penalty is \$300,000, \$400,000, and \$500,000 compared to the \$200,000. Eventually, by raising this penalty and as a result of assigning more patients to facilities, the operating beds' cost grows. Note that we have already discussed $UP = \$346{,}000$ (i.e., the base scenario) in Figure 8 and 9 in the previous section.

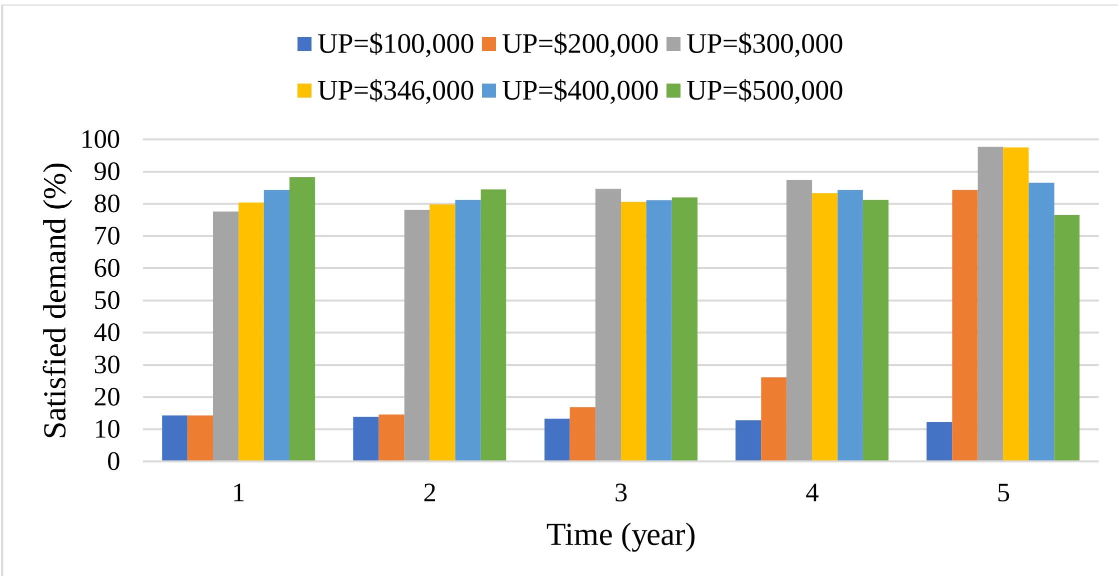

Figure 4.7: The effect of UP value on satisfied demand

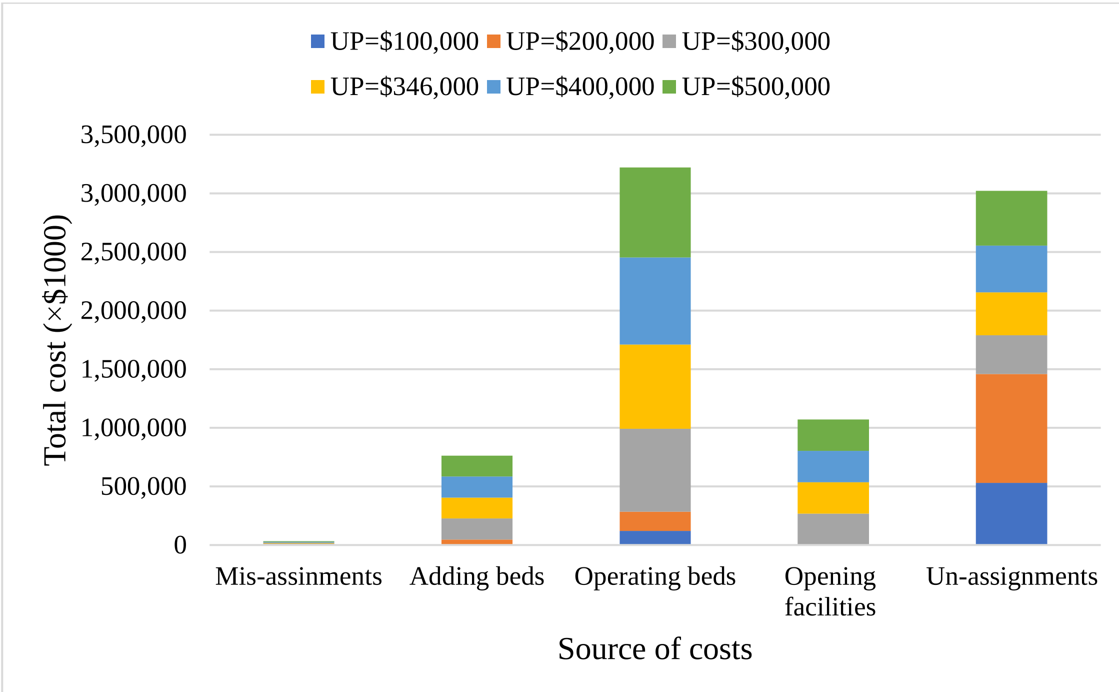

Figure 4.8: The effect of UP value on Costs

4.2.2 Sensitivity to the Correction Factor (γ)

Figure 4.9, clearly, indicates that correction factor (γ) can reduce the end-of-horizon effect for finite time problems. In a case that correction factor is equal to 1% (base scenario), the proportion of satisfied demand in the last period is 17% more than the first period. While a more moderate value for correction factor (e.g., 5%) makes the model assigns almost the same percentage of patients in first and last period, putting 10% as correction factor leads in allocating a higher proportion of patients to $t = 1$.

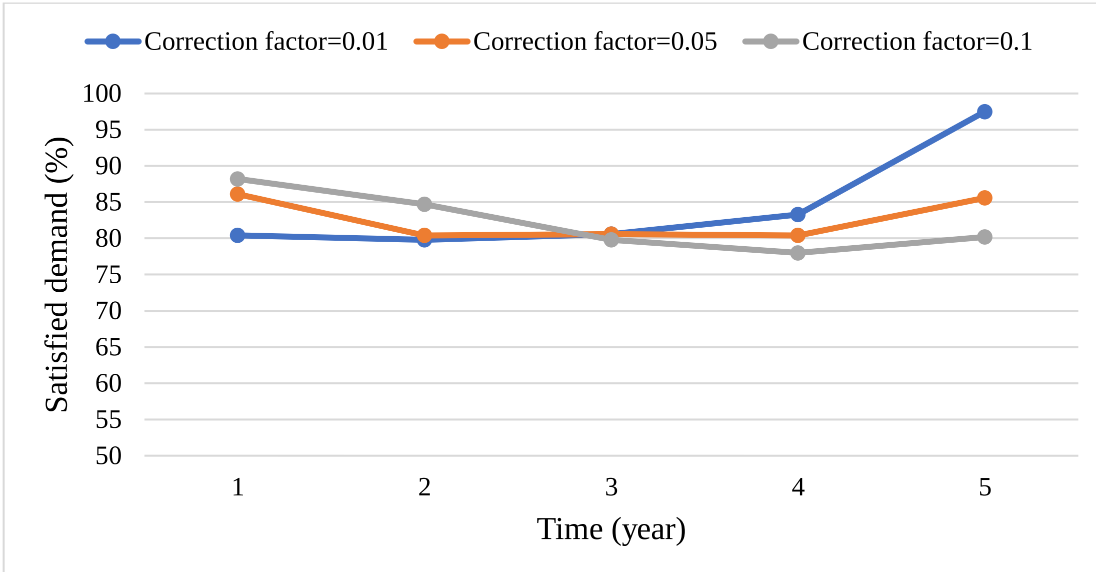

Figure 4.9: The effect of the correction factor (γ) value on the satisfied demand

4.2.3 Sensitivity to the Mis-Assignment Penalty

Figure 4.10 compares the number of mis-assigned patients in each time for different mis-assignment penalties. It implies that higher penalties result in fewer mis-assignments until there are no mis-assignments. Although it is not remarkable, we can observe the impact of the end-of-horizon effect for finite time problems even on the distribution of patients among facilities (as there are larger mis-assigned patients in the last period compared to other periods). Therefore, similar to the un-assignment equation in the objective function, we can multiply the mis-assignment penalty by a correction factor to avoid or reduce the impact of this problem. Unlike the un-assignment penalty, we should multiply the new correction factor (β) by the mis-assignment penalty in the objective function in a way that there will be more penalty for mis-assigned patients in last periods as follows:

$$\sum_{t\in T}\sum_{i\in I}\sum_{g\in G}\sum_{l\in L}\sum_{a\in A}\sum_{j\in J}(Y_{iglajt}) \times MP_{ij} \times (1+\beta)^{t-1} \quad (37)$$

The correction factors for un-assignment (γ) and mis-assignment (β) penalties are different because γ and β can take different values at the same time. Also, we use distinct formulation compared to one that we use for un-assignment penalty ($(1-\gamma)^{t-1}$) because we should decrease the un-assignment penalty and increase the mis-assignment penalty to avoid or reduce the end-of-horizon effect.

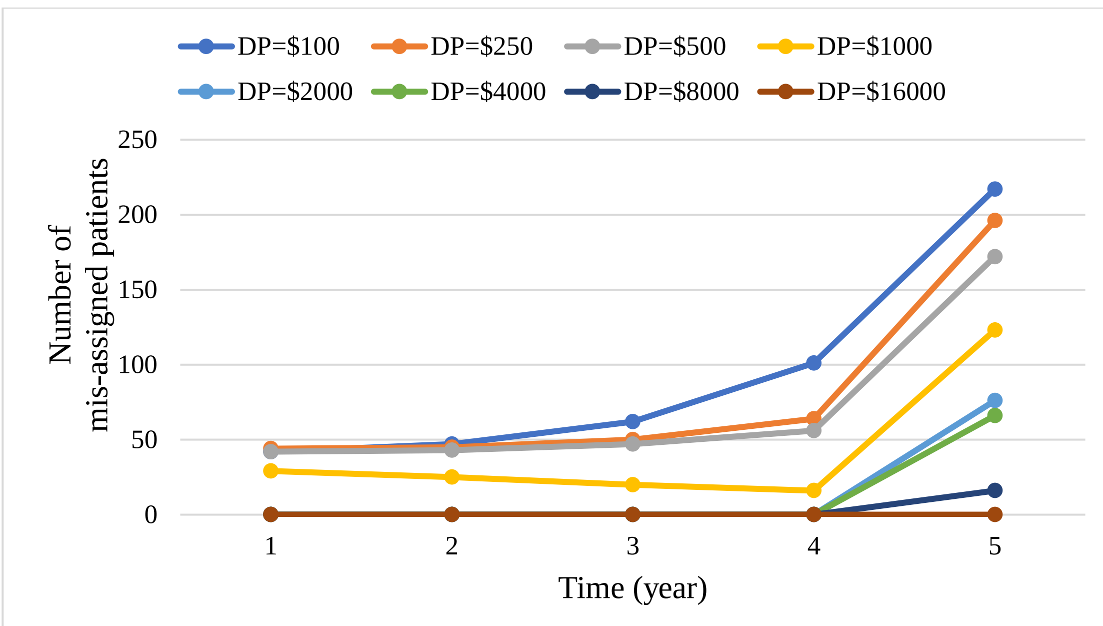

Figure 4.10: the effect of MP value on the number of mis-assignments

4.3 Simulation

As a key contribution in this study, we have also conducted an extensive simulation study, particularly to provide deeper insight regarding the influence of the various sources of uncertainties in our model. Note that this analysis is beyond and richer than the traditional sensitivity analysis where the impact of the change in a parameter is investigated through changing the parameter over a limited range of values of that parameter without explicit consideration of the probability distributions of the uncertainty of the parameter.

Turning a deterministic model into a stochastic one makes the problem more complex. Also, solving a stochastic problem can be more difficult and may need to develop new solution methods for stochastic problems. One important purpose of this simulation study is to inform how sensitive is our whole model to the various sources of parameter uncertainties. This analysis highly informs the choice between deterministic and stochastic optimization models.

To mimic the real-world LTC system and figure out how changes in uncertain parameters affect the system and optimal results, we simulate our LTC network by applying ARENA software version 15.00.00001.

The Normal distribution is one of the most important and common probability distributions, as it illustrates many uncertain events. Every Normal distribution has the following three features. First, it is most likely that the uncertain variable takes some special value called mean. Second, the probability that uncertain variable be above or below than mean is equal. Third, it is more likely that the uncertain variable takes value around the mean (Mun, 2008). There are some uncertain events, such as patients' demand, death rate, and transition rate between age groups, that we considered as constant numbers in the mathematical model. In the simulation model, we assign a Normal distribution to each of these parameters to capture the influence of their uncertainties in the model. All of these parameters posses the three mentioned features of the Normal distribution. To specify Normal distribution parameters, the value of uncertain parameters in the optimization model are considered as mean, and we assume that the standard deviation equals 10% of the mean.

Figure 4.11 illustrates the screenshot of the simulation model in ARENA for the first two years of the planning horizon (note that we simulate the model for all five years). In part 1 of the simulation model, we generate the total demand. Parts 2 and 3 divide the demand based on patients' region, and age group, respectively. In part 4, a patient is assigned to an LTC facility or leave the system (indicating un-assignment). Part 5 occurs at the end of each period when an assigned patient may stay in the same age group, transfer to the next age group, or die (i.e., leaving the system). This process repeats for all periods.

The patients' assignments and the time and location of opening facilities are obtained from the optimization model. Also, as we consider a probability distribution for uncertain parameters (instead of constant values), each time we run a simulation model, we get different results. Therefore, we run the simulation model with 200 replications (a sufficiently large number for simulating the process which can also be implemented in the computer used for conducting the simulation experiment) and consider the average results of these replications.

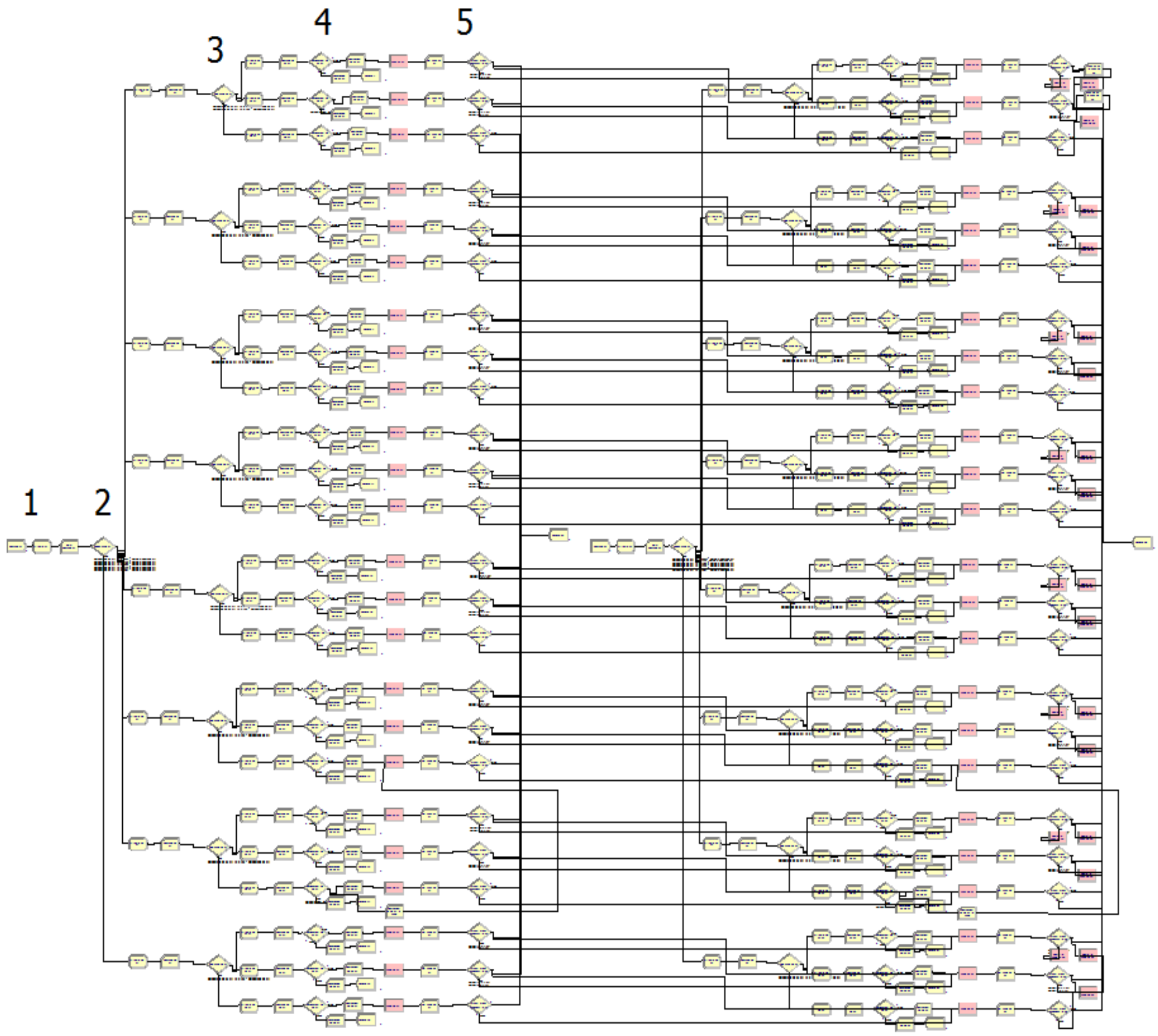

Figure 4.11: Simulation for the first two years

Figure 4.12 shows how the percentage of satisfied demand in each period can change in a real-world LTC system (simulation results), and Figure 4.13 compares the total cost of adding and operating beds and un-assignment penalty in simulation results with the deterministic optimization results. The decision-maker can decide whether these changes are significant or not.

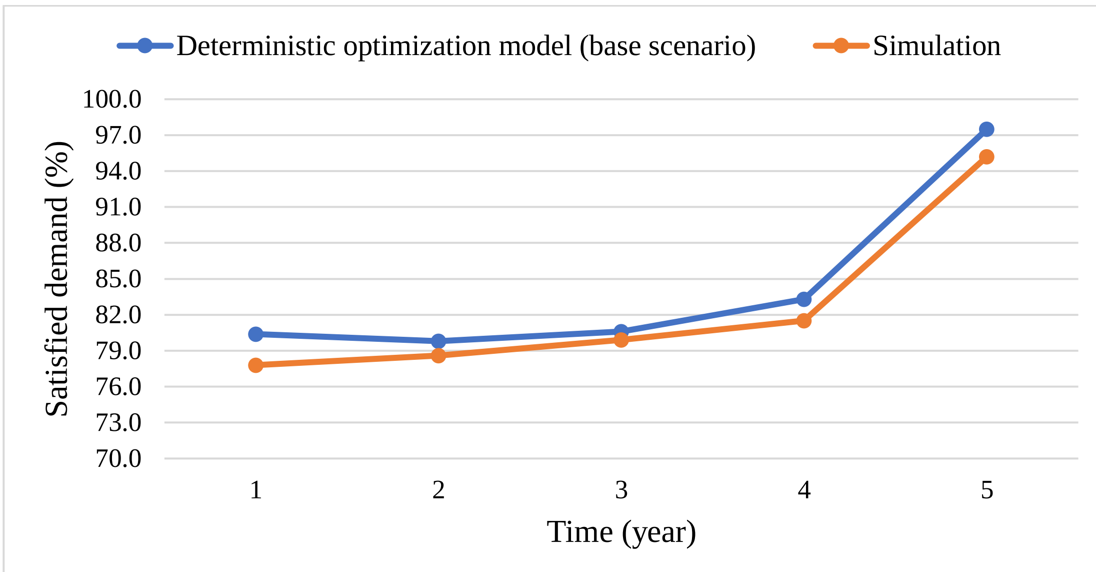

Figure 4.12: Comparison between satisfied demand in the deterministic optimization model (base scenario) and simulation results

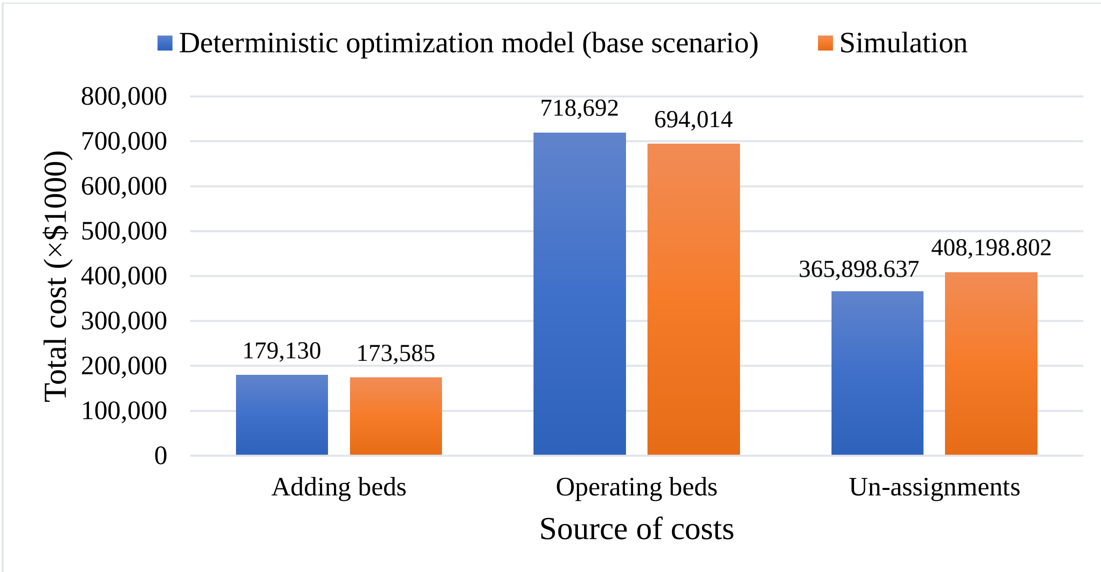

Figure 4.13: Comparison between costs in the deterministic optimization model (base scenario) and simulation results

5 Conclusion and Future Work

This study presents a mathematical programming model to simultaneously optimize location-allocation and capacity planning for the LTC facility network in the province of Ontario in Canada, which has its own featuring characteristics. Accordingly, the proposed dynamic mixed-integer linear programming model optimizes the timing and locations of building new LTC facilities, in addition to the capacity (human resources and beds) of each facility at each time, and the assignments of patients, based on their demand region, gender, language, and age group to the LTC facilities over a finite time horizon.

Our model helps authorities in Canada, in general, and in Ontario, in particular, tackle the crucial issues in the LTC network which are characterized by the excessive demand, the surge in demand in the coming years, language considerations, and the lack of efficient capacity planning. It also provides guidance to help patients have a better life in LTC facilities, by assigning them to the nearest facilities and rescue them from gender/ language loneliness by ensuring the minimum number of patients from each gender and language in every facility and period.

Our computational results show that increasing the un-assignment and mis-assignment penalties result in fewer patient un-assignments and mis-assignments, respectively. Plus, depending on the values of these penalties and other parameters, the proportion of each source of costs (mis-assignments, adding beds, operating beds, opening facilities, un-assignments) can change. Accordingly, in the base scenario, the cost of operating beds has the highest proportion, while the mis-assignment cost has the least proportion of the total cost. Moreover, we demonstrate that correction factor (γ) can avoid model from end-of-horizon effect for finite time problems. Finally, we illustrate the impact of uncertainty in the optimal results through simulating the LTC facility network. It is up to the policymakers (here healthcare authorities in Ontario) to decide whether the changes in satisfied demand and expenses are significant.

Although the model is developed based on the LTC network specifications in the province of Ontario, it can be generalized and adapted to the LTC network in other provinces in Canada and even other countries with similar characteristics and criteria.

Several ways can be taken to extend the current work. First, the model can incorporate all types of care for the elderly containing seniors' housing, long-term care, home care, and community services. Second, the uncertain feature of patients demands, mortality, and transition between age groups make the problem suitable to extend into the stochastic model. Lastly, as a result of a stochastic model and/or bigger problem (in terms of variables and constraints), it may be desirable to develop exact or even approximate solution algorithms.

A Appendix

A.1 Summary of computational results for all Hamilton city regions (including the first eight regions as well as rural areas and towns) with 15 demand and service regions

This scenario includes 29,430 constraints and 28,070 variables (13,650 integer variables and 14,420 continuous variables), and results are obtained while the optimality gap is 2.95%, and the processing time is 85 minutes.

Table A.1 presents the total bed capacity for LTC facilities in each service region and time and shows how this capacity changes over time. The percentage of satisfied demand in each period is provided in Figure A.1. In Table A.2, we show the human resource capacity required in each year, based on their language. Finally, Figure A.2 indicates the cost for each part of the objective function, and Figure A.3 shows the percentage of each source of the total cost.

Table A.1: Number of total and additional beds in each service region and time (by considering rural areas and towns)

Service Region	Time (year)	Additional Beds	Total Beds
1	1	59	186
1	2	10	196
1	3	17	213
1	4	22	235
1	5	143	378
2	1	152	280
2	2	14	294
2	3	5	299
2	4	34	333
2	5	45	378
3	1	347	347
3	2	9	356
3	3	4	360
3	4	11	371
3	5	7	378
4	1	322	322
4	2	0	322
4	3	17	339
4	4	27	366
4	5	12	378
5	1	0	0
5	2	0	0
5	3	0	0
5	4	0	0
5	5	0	0
6	1	348	348
6	2	6	354
6	3	8	362
6	4	16	378
6	5	0	378
7	1	363	363
7	2	0	363
7	3	0	363

7	4	15	378
7	5	0	378
8	1	154	364
8	2	1	365
8	3	8	373
8	4	5	378
8	5	0	378
9	1	96	263
9	2	5	268
9	3	19	287
9	4	31	318
9	5	60	378
10	1	378	378
10	2	0	378
10	3	0	378
10	4	0	378
10	5	0	378
11	1	338	338
11	2	0	338
11	3	2	340
11	4	3	343
11	5	35	378
12	1	293	293
12	2	2	295
12	3	23	318
12	4	25	343
12	5	34	377
13	1	0	378
13	2	0	378
13	3	0	378
13	4	0	378
13	5	0	378
14	1	0	0
14	2	0	0
14	3	0	0
14	4	0	0
14	5	0	0

15	1	0	0
15	2	0	0
15	3	0	0
15	4	0	0
15	5	0	0

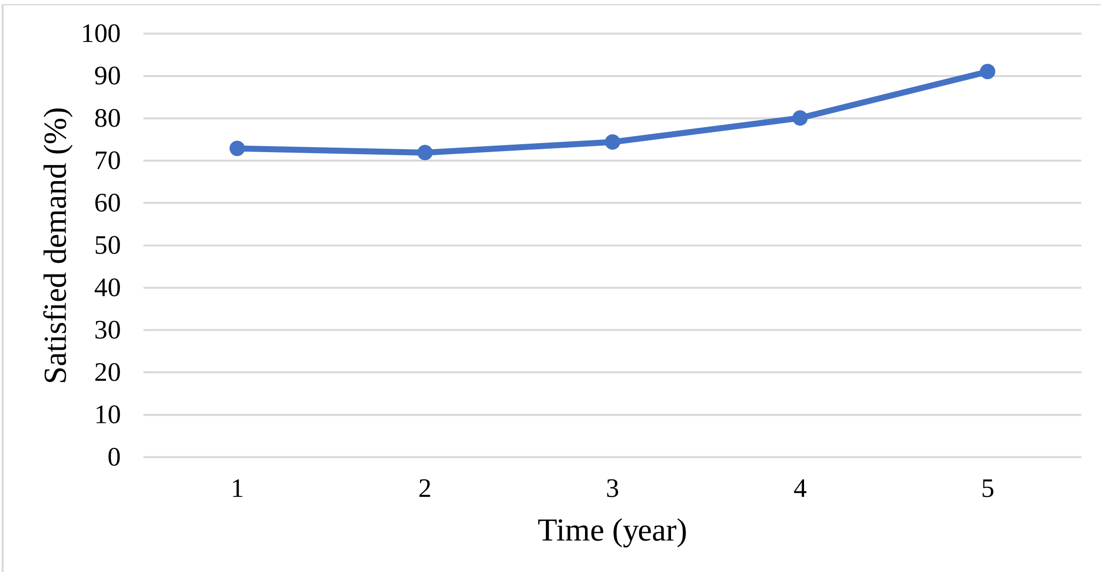

Figure A.1: Satisfied demand in each year (by considering rural areas and towns)

Table A.2: Human resources capacity (by considering rural areas and towns)

Human Resource Type	Language	Time (year)	Hours of Care
1	1	1	2,211,462.0
1	1	2	2,244,348.5
1	1	3	2,308,260.0
1	1	4	2,424,914.0
1	1	5	2,619,130.5
1	2	1	183,668.0
1	2	2	179,945.0
1	2	3	179,945.0
1	2	4	180,565.5
1	2	5	194,837.0
2	1	1	572,378.4
2	1	2	580,890.2
2	1	3	597,432.0
2	1	4	627,624.8
2	1	5	677,892.6
2	2	1	47,537.6
2	2	2	46,574.0
2	2	3	46,574.0
2	2	4	46,734.6
2	2	5	50,428.4

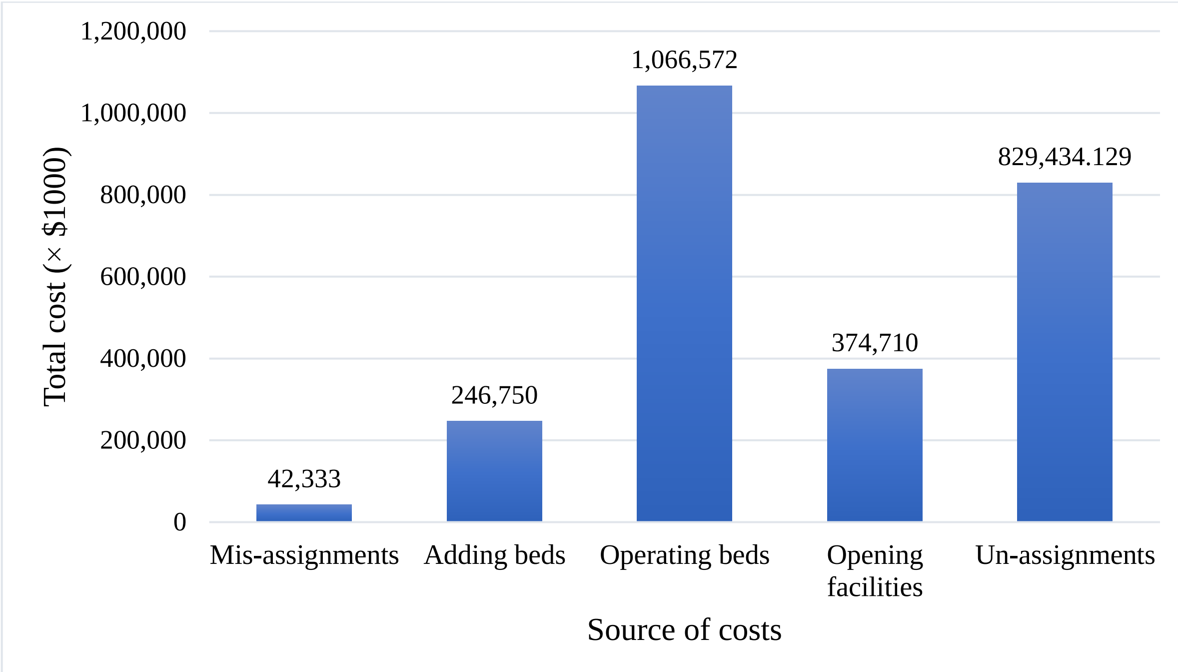

Figure A.2: The share ($) of each source of cost in the objective function (by considering rural areas and towns)

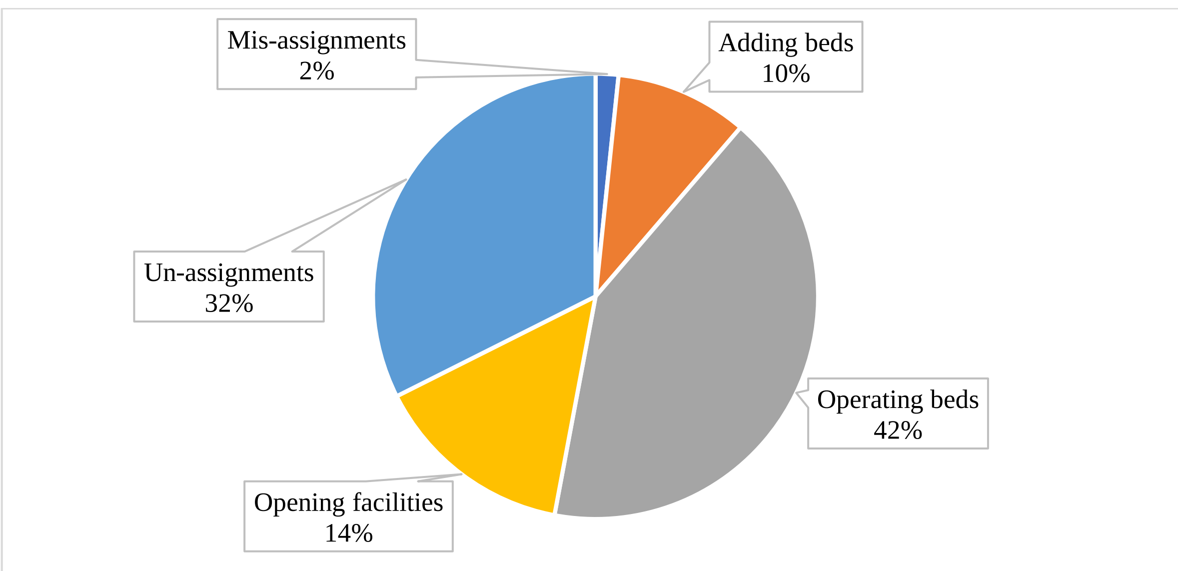

Figure A.3: The share (in %) of each source of cost in the objective function (by considering rural areas and towns)

Bibliography

Cardoso, T., Oliveira, M. D., Barbosa-Póvoa, A., & Nickel, S. (2015). Introducing health gains in location-allocation models: A stochastic model for planning the delivery of long-term care. Journal of Physics: Conference Series, 616.

Cardoso, Teresa, Oliveira, M. D., Barbosa-Póvoa, A., & Nickel, S. (2012). Modeling the demand for long-term care services under uncertain information. Health Care Management Science, 15(4), 385–412.

Cardoso, Teresa, Oliveira, M. D., Barbosa-Póvoa, A., & Nickel, S. (2015). An integrated approach for planning a long-term care network with uncertainty, strategic policy and equity considerations. European Journal of Operational Research, 247(1), 321–334.

Cardoso, Teresa, Oliveira, M. D., Barbosa-Póvoa, A., & Nickel, S. (2016). Moving towards an equitable long-term care network: A multi-objective and multi-period planning approach. Omega, 58, 69–85.

City of Hamilton. (2016). Census Data for Hamilton. Retrieved July 2, 2019, from https://www.hamilton.ca/moving-hamilton/community-profile/census-data-hamilton

City of Hamilton. (2018). Ward Boundary Changes. Retrieved June 26, 2019, from https://www.hamilton.ca/municipal-election/election-information/ward-boundary-changes

Gibbard, R. (2017). Sizing up the challenge: Meeting the demand for long-term care in Canada. Retrieved from https://www.conferenceboard.ca/e-library/abstract.aspx?did=9228

Government of Canada. (2019). Learn about Ontario. Retrieved June 23, 2019, from https://www.canada.ca/en/immigration-refugees-citizenship/services/new-immigrants/prepare-life-canada/provinces-territories/ontario.html

Government of Ontario. (2019). About Ontario. Retrieved June 23, 2019, from Ontario.ca website: https://www.ontario.ca/page/about-ontario

Greene, V. L., Ondrich, J., & Laditka, S. (1998). Can home care services achieve cost savings in long-term care for older people? The Journals of Gerontology: Series B, 53B(4), S228–S238.

Hare, W. L., Alimadad, A., Dodd, H., Ferguson, R., & Rutherford, A. (2008). A deterministic model of home and community care client counts in British Columbia. Health Care Management Science, 12(1), 80.

Health Services for HNHB. (2019a). Health services for Hamilton Niagara Haldimand Brant—Hnhbhealthline.ca. Retrieved June 25, 2019, from https://www.hnhbhealthline.ca/

Health Services for HNHB. (2019b). Long-Term Care Homes-Hamilton-Hnhbhealthline.ca. Retrieved June 26, 2019, from https://www.hnhbhealthline.ca/listServices.aspx?id=10665

Hertz, A., & Lahrichi, N. (2009). A patient assignment algorithm for home care services. Journal of the Operational Research Society, 60(4), 481–495.

Hsu, A. T., Berta, W., Coyte, P. C., & Laporte, A. (2016). Staffing in Ontario’s long-term care homes: Differences by profit status and chain ownership. Canadian Journal on Aging; Toronto, 35(2), 175–189.

Intrevado, P., Verter, V., & Tremblay, L. (2019). Patient-centric design of long-term care networks. Health Care Management Science, 22(2), 376–390.

Kim, D.-G., & Kim, Y.-D. (2010). A branch and bound algorithm for determining locations of long-term care facilities. European Journal of Operational Research, 206(1), 168–177.

Li, Y., Zhang, Y., Kong, N., & Lawley, M. (2016). Capacity planning for long-term care networks. IIE Transactions, 48(12), 1098–1111.

Lin, F., Kong, N., & Lawley, M. (2012). Capacity planning for publicly funded community based long-term care services. In M. P. Johnson (Ed.), Community-Based Operations Research: Decision Modeling for Local Impact and Diverse Populations (pp. 297–315).

Mestre, A. M., Oliveira, M. D., & Barbosa-Póvoa, A. P. (2015). Location–allocation approaches for hospital network planning under uncertainty. European Journal of Operational Research, 240(3), 791–806.

Ministry of Finance. (2018). Ontario Population Projections Update. Retrieved July 2, 2019, from https://www.fin.gov.on.ca/en/economy/demographics/projections/

Mitropoulos, P., Mitropoulos, I., Giannikos, I., & Sissouras, A. (2006). A biobjective model for the locational planning of hospitals and health centers. Health Care Management Science, 9(2), 171–179.

Mun, J. (2008). Advanced analytical models: Over 800 models and 300 applications from the Basel ii accord to wall street and beyond. John Wiley & Sons.

Munro, D., Downie, M., & Stonebridge, C. (2011). Elements of an effective innovation strategy for long term care in Ontario. Retrieved from The Conference Board of Canada website: www.conferenceboard.ca

Ontario Association of Non-Profit Homes and Services for Seniors. (2016). Improving seniors' services in Ontario: OANHSS position paper on capacity planning and development.

Ontario Long Term Care Association. (2016). This is Long-Term Care 2016. Retrieved from Ontario Long Term Care Association website: https://www.oltca.com/OLTCA/Documents/Reports/TILTC2016.pdf

Ontario Long Term Care Association. (2018). This is Long-Term Care 2018. Retrieved from Ontario Long Term Care Association website:

https://www.oltca.com/OLTCA/OLTCA/Public/MediaEvents/News_Releases/TILTC2018Report.aspx

Ontario Long Term Care Association. (2019). Long-term care that works. For seniors. For Ontario. 2019 Budget Submission. Retrieved June 29, 2019, from https://www.oltca.com/oltca/OLTCA/ReportsPublications/AssociationPublications/OLTCA/Public/ReportsPublications/Association_Publications_Main.aspx

Ontario's LHINs. (2019). Local Health Integration Network (LHIN). Retrieved June 25, 2019, from http://www.lhins.on.ca/

Patrick, J., Nelson, K., & Lane, D. (2015). A simulation model for capacity planning in community care. Journal of Simulation, 9(2), 111–120.

Patrick, Jonathan. (2011). Access to long-term care: The true cause of hospital congestion? Production and Operations Management, 20(3), 347–358.

Puterman, M. L. (2005). Markov decision processes: Discrete stochastic dynamic programming. Hoboken, NJ: Wiley-Interscience.

Robison, J., Shugrue, N., Porter, M., Fortinsky, R. H., & Curry, L. A. (2012). Transition from home care to nursing home: Unmet needs in a home- and community-based program for older adults. Journal of Aging & Social Policy, 24(3), 251–270.

Santibáñez, P., Bekiou, G., & Yip, K. (2009). Fraser health uses mathematical programming to plan its inpatient hospital network. INFORMS Journal on Applied Analytics, 39(3), 196–208.

Shroff, HomeéF. E., Gulledge, Th. R., Haynes, K. E., & O'Neill, M. K. (1998). Siting efficiency of long-term health care facilities. Socio-Economic Planning Sciences, 32(1), 25–43.

Statistics Canada. (2017). Census Profile, 2016 Census—Hamilton, Census division, Ontario and Ontario. Retrieved July 2, 2019, from https://www12.statcan.gc.ca/census-recensement/2016/dp-pd/prof/details/page.cfm?Lang=E&Geo1=CD&Code1=3525&Geo2=PR&Code2=35&Data=Count&SearchText=Hamilton&SearchType=Begins&SearchPR=01&B1=Language&TABID=1

Stummer, C., Doerner, K., Focke, A., & Heidenberger, K. (2004). Determining location and size of medical departments in a hospital network: A multiobjective decision support approach. Health Care Management Science, 7(1), 63–71.

Sun, L., DePuy, G. W., & Evans, G. W. (2014). Multi-objective optimization models for patient allocation during a pandemic influenza outbreak. Computers & Operations Research, 51, 350–359.

Tanuseputro, P., Chalifoux, M., Bennett, C., Gruneir, A., Bronskill, S. E., Walker, P., & Manuel, D. (2015). Hospitalization and mortality rates in long-term care facilities: Does for-profit status matter? Journal of the American Medical Directors Association, 16(10), 874–883.

The Globe and Mail. (2018). Long-term and dementia care urgently needed in mother tongue for multilingual patients, experts say. Retrieved from https://www.theglobeandmail.com/life/health-and-fitness/article-long-term-and-dementia-care-urgently-needed-in-mother-tongue-for/

Xie, H., Chaussalet, T. J., & Millard, P. H. (2006). A model-based approach to the analysis of patterns of length of stay in institutional long-term care. IEEE Transactions on Information Technology in Biomedicine, 10(3), 512–518.

Zhang, Y., & Puterman, M. L. (2013). Developing an adaptive policy for long-term care capacity planning. Health Care Management Science, 16(3), 271–279.

Zhang, Y., Puterman, M. L., Nelson, M., & Atkins, D. (2012). A simulation optimization approach to long-term care capacity planning. Operations Research, 60(2), 249–261.