ON THE OPTIMALITY OF THE GREEDY POLICY FOR BATTERY LIMITED ENERGY HARVESTING COMMUNICATIONS

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BY

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To my family and my love

Abstract

Wireless network for connecting the devices and sensors to communicate and sense is quite attractive nowadays for a wide range of applications. The scaling of the wireless network to millions of nodes currently is impractical if the process is supplied by battery energy. The batteries need to be periodically replaced or recharged due to the limited battery size. One solution is harvesting ambient energy to power the network. In this thesis, we consider a battery-limited energy harvesting communication system with online power control. Assuming independent and identically distributed (i.i.d.) energy arrivals and the harvest-store-use architecture, it is shown that the greedy policy achieves the maximum throughput if and only if the battery capacity is below a certain positive threshold that admits a precise characterization. Simple lower and upper bounds on this threshold are established. The asymptotic relationship between the threshold and the mean of the energy arrival process is analyzed for several examples. Furthermore, value iteration method is applied for solving the Bellman equation to obtain the optimal power allocation policy. The optimal policy is analyzed for several examples.

Keywords: Bellman equation, energy harvesting, greedy policy, power control, throughput.

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Definitions

Jensen's inequality

$$\varphi\left(E[X]\right) \leq E[\varphi(X)]$$

Stirling's approximation

$$n! \sim \sqrt{2\pi n}(\tfrac{n}{e})^n$$

Notation and Abbreviation

ICT	Information and Communication Technology
IoT	Internet of Things
GHGE	Greenhouse gas emissions
EH	Energy harvesting
HUS	Harvest-Use-Store architecture

HSU	Harvest-Store-Use architecture
MDP	Markov decision process
i.i.d.	Independent and identically distributed

Chapter 1

Introduction

1.1 Background

The main purpose of the fifth generation (5G) cellular network is to provide energy-efficient, low-cost and secure communication system [1]. By the end of 2020, more than 50 billion network devices are expected to use wireless network services [2]. The wide deployment of Information and Communication Technology (ICT) devices, Internet of Things (IoT) applications, and wireless services in the 5G cellular network helps move towards this aim. However, there are still some obstacles in the way to achieve this goal. At first, the lifetime of the wireless network is limited by the lifetime of batteries in the network. When a sufficient number of batteries are exhausted in the network, it will not achieve its designated task and need to replace the batteries every few months [3] which causing the increase of maintenance cost. The network could possibly use the larger size batteries, but in turn, it increases the size, weight, and cost of batteries. Secondly, the emerging applications with high data rate in 5G require careful allocation of the transmission energy to increase the total system capacity and minimize the error rate. The energy consumed in the wireless network is growing rapidly followed by the increase in carbon dioxide emissions. The energy consumption of computers, networking equipment, and other ICT devices (excluding smartphones) amounted to 8% of total world energy consumption in 2007 and is projected to reach 14% by 2020 [4]. The increase in energy consumption of ICT devices associates with the increase in greenhouse gas emissions (GHGE). The total world GHGE from ICT devices grows from roughly 1-1.6% in 2007 to around 3.3% by 2020 [5].

To effectively solve the aforementioned problems, utilizing energy from the natural environment (e.g., solar, wind, and thermal energy) or other energy sources (e.g., body heat and vibration power), namely, *energy harvesting* (EH), has been proposed. Energy harvested from ambient environment supplies wireless networks with green and infinite energy. In this way, the lifetime of the wireless network system will not be constrained by its batteries' lifetime and become self-powered and green. The hardware of energy harvesting technology has developed feasibly for modern wireless networks. For example, the solar-powered (e.g., photo-voltaic), wind-powered base stations have been designed in recent years [6]. There are also some papers considering the system cooperation problems to improve the EH system efficiency because the distribution of ambient energy is random in different places and different times. For example, authors in [7] analyze the power grid energy saving problem and authors in [8] consider how to the plan cellular network. What's more, due to the randomness of the arrival energy process, allocating arrival energy optimally to maximize the throughput is also an important and challenging issue in the energy harvesting communication system. The problem of power control has received significant attention in recent years [9–21].

1.2 Energy harvesting system

Fig. 1.1 illustrates the common EH system classification.

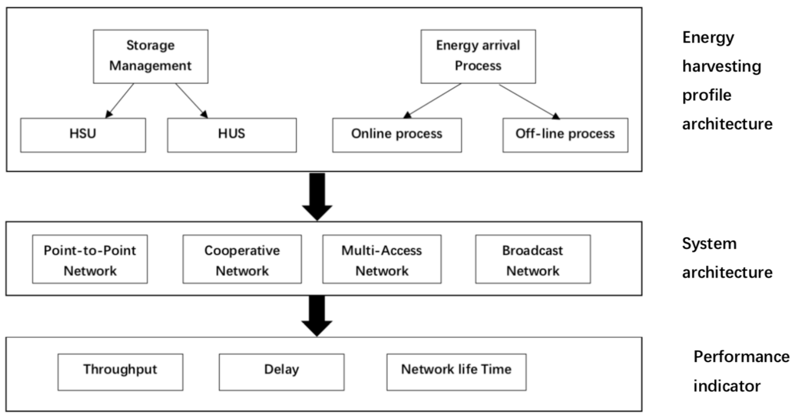


Figure 1.1: Energy harvesting system classification

There are two main storage management ways for energy harvesting system [22]. One is the *Harvest-Use-Store* (HUS) architecture. In HUS, the wireless network directly utilizes the energy from the EH system but when there is no sufficient energy, the network will be disabled. The other is the *Harvest-Store-Use* (HSU) architecture. In HSU, the energy harvesting system is equipped with a battery storing the harvested energy and the system controls stored energy to power the wireless network. In this thesis, we adopt the popular HSU architecture and it will be contrasted with the HUS architecture in our future work. For the energy arrival process, it is commonly studied in two scenarios [10–12]. In the off-line scenario, the amount of harvested energy and the arrival time are known before the communication starts. Authors in [10–12] provide an optimal policy for the off-line case which is consuming the energy as constant as possible over time. This kind of policy ensures nearly zero energy overflow waste for the limited capacity battery. However, the off-line case is hard to apply in the practical communication systems because this realization is under the condition that the future arrival energy can be accurately predicted in the harvesting process. In the online scenario, the harvested energy is unknown and only revealed to the transmitter over time. The allocation policy must adaptively change with the unknown arrival energy at each time slot. In this case, finding the optimal online power control policy is more complicated and practical. Authors in [10] propose the constant policy which requires energy consumption as constant as possible in the communication process. It is easy to show that only when the battery capacity is infinite can guarantee to allocate the same amount of energy in all time slots. In particular, when the given battery capacity is finite, the throughput under the constant policy would be arbitrarily away from the optimal results. We address these above problems and consider online power control for a battery-limited energy harvesting communication system with the goal of maximizing the long-term average throughput.

In this thesis, we consider a single user point-to-point channel with the additive white Gaussian noise (AWGN). The system is slotted, i.e., time is discrete (t=1,2,3,$\cdots$). At time t, the received signal is $y_t = x_t + N_t$, where x_t is transmitted signal and N_t is white Gaussian noise with unit variance and zero mean.

1.3 Thesis Objectives

Though the exact problem formulation varies depending on the system model and the performance metric, the essential challenge remains the same, which is, roughly speaking, to deal with random energy availability. The aforementioned challenge is arguably most pronounced in this setting. Indeed, it is known that the impact of random energy arrivals can be smoothed out if the system is equipped with a battery of unlimited capacity [14], and offline power control can achieve the same effect to a certain extent. The standard approach to the problem under consideration is based on the theory of Markov decision process (MDP). Although in principle the maximum throughput and the associated optimal online power control policy can be found by solving the relevant Bellman equation, it is often very difficult to accomplish this task analytically. To the best of our knowledge, there is no exact characterization of the maximum throughput except for Bernoulli energy arrivals [21]. To circumvent this difficulty, we tackle the problem from a different angle. Specifically, we use the Bellman equation to check whether a given power control policy is optimal. This strategy effectively turns a hard optimization problem into a simple decision problem for which more conclusive results can be obtained (see [23] for the application of a similar strategy in a different context). In particular, it enables us to establish a sufficient and necessary condition for the optimality of the greedy policy, yielding an exact characterization of the maximum throughput in the low-battery-capacity regime. Based on the characterization, we then analyze some properties of the optimal power control policy for some specific i.i.d. energy arrival processes. At last, when the capacity exceeds the regime, we consider the numerical simulations of optimal power control policy and discuss the trade-off relationship between the optimal policy and the available energy in the battery by solving Bellman equation.

The rest of the thesis is organized as follows. Chapter 1 presents the introduction of this thesis. The problem conversion of Markov decision process is presented in

Chapter 2. Chapter 3 provides the problem statement. We state the main results in Chapter 4 and the proofs are provided in Chapter 5. Chapter 6 contains several illustrative examples. The conclusion and our future work are provided in Chapter 7. Throughout this thesis, the base of the logarithm function is e.

Chapter 2

Preliminaries

2.1 Markov Decision Process

Markov Decision Process (MDP) is a stochastic decision-making process in which the decision maker (i.e., the agent) interacts with the environment (i.e., the system) [24]. The process is shown in Fig. 2.1.

A MDP is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$ [25], where

- $\mathcal{S}$ is a finite set of states
- $\mathcal{A}$ is a finite set of actions
- $\mathcal{P}$ is a state transition probability matrix, $P^a_{S,S'} = \mathbb{P}\left[S_{t+1} = s' | S_t = s, A_t = a\right]$
- $\mathcal{R}$ is a reward function, $R^a_s = \mathbb{E}\left[R_{t+1} | S_t = s, A_t = a\right]$

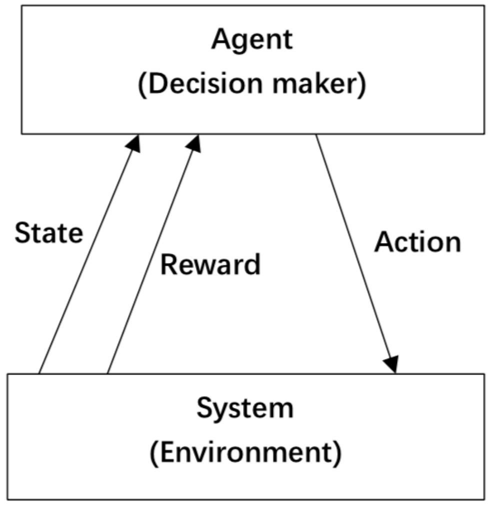


Figure 2.1: MDP model

At MDP, the system stays in a state s and the agent chooses an available action a. After that, the agent gets a reward r and a new state $s^{'}$ according to the transition probability $P^a_{s,s'}$. Policy π is a mapping from a state s to an action a. MDP aims to find an optimal policy π which can maximize a certain objective function. In the infinite time horizon MDP, the goal is to maximize the total reward:

$$\max \mathcal{V}_\pi(s) = \lim_{T\to\infty} \inf \frac{1}{T}\mathbb{E}_{\pi,s}\left[\sum_{t=1}^{T} \mathcal{R}(s_t^{'}|s_t, \pi(a_t))\right],$$

here $\mathbb{E}[\cdot]$ is the expectation of function.

We can find $v^*(s)$ at each state recursively by solving the following Bellman optimal equation [24]:

$$v_t^*(s) = \max_{a\in\mathcal{A}}\left[\mathcal{R}_t(s,a) + \sum_{s^{'}\in\mathcal{S}} \mathcal{P}_t(s^{'}|s,a)v_{t+1}^*(s^{'})\right].$$

There are several methods for solving infinite time horizon MDP [25]: value iteration, policy iteration, linear programming, and online learning. Value iteration (VI) method is based on dynamic programming and it is most efficient and widely used. In this thesis, we adopt VI method and use a stopping criterion (i.e., $||v_t^*(s) - v_{t-1}^*(s)|| < \theta$) to guarantee the convergence instead of running the infinite time horizon [24]. The value iteration algorithm is shown as follows:

Algorithm 1 Value Iteration

Initialize array h arbitrarily (e.g., $h(s) = 0$ for all s $\in S$)
repeat
  $\Delta \leftarrow 0$
  for each $s \in \mathcal{S}$ **do**
    $temp \leftarrow v(s)$
    $v(s) \leftarrow \max_a \left\{ r(a) + \sum_{s'} p\left(s^{'}|s,a\right) v(s') \right\}$
    $\Delta \leftarrow \max(\Delta, |temp - v(s)|)$
  end for
until $\Delta < \theta$ (a small positive number)
Output: optimal policy π:
$\pi(s) = \arg\max_a \left\{ r(a) + \sum_{s'} p\left(s^{'}|s,a\right) v(s') \right\}$

For EH process, it is a Markov decision process and the arriving energy of it is random.

Chapter 3

Problem Statement

3.1 System Model

Consider a discrete-time energy harvesting communication system equipped with a battery of capacity c. Let $X(t)$ denote the amount of energy harvested at time t, $t = 1, 2, \cdots$, where $\{X_t\}_{t=1}^{\infty}$ are assumed to be i.i.d. copies of a non-negative random variable X. An online power control policy is a sequence of mapping $\{f_t\}_{t=1}^{\infty}$ specifying the level of energy consumption G_t in time slot t based on $X^t \triangleq (X_1, \cdots, X_t)$ for all t:

$$G_t = f_t(X^t), \quad t = 1, 2, \cdots .$$

Let B_t denote the amount of energy stored in the battery at the beginning of time slot t. We have

$$B_t = \min\{B_{t-1} - G_{t-1} + X_t, c\}, \quad t = 1, 2, \cdots ,$$

where $B_0 \triangleq 0$ and $G_0 \triangleq 0$. The system model is shown in Fig. 3.1.

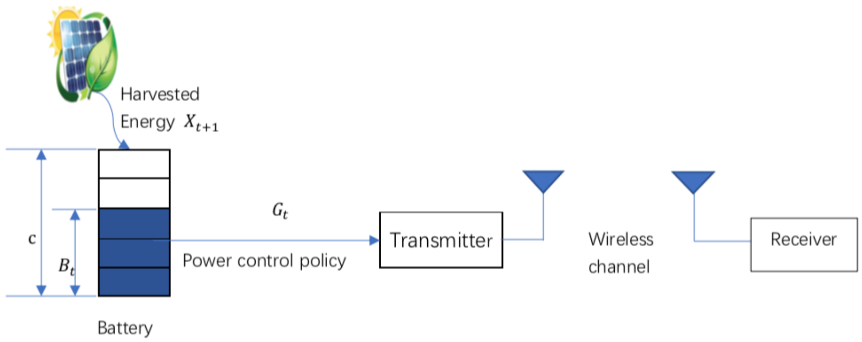


Figure 3.1: The harvest-store-use architecture

An online power control policy is said to be admissible if

$$G_t \leq B_t, \quad t = 1, 2, \cdots,$$

almost surely. The throughput induced by policy $\{f_t\}_{t=1}^{\infty}$ is defined as

$$\gamma(c) = \liminf_{n \to \infty} \frac{1}{n} \mathbb{E} \left[\sum_{t=1}^{n} r(f_t(X^t)) \right],$$

where $r : \mathbb{R}_+ \to \mathbb{R}_+$ is a reward function that specifies the instantaneous rate achievable with the given level of energy consumption. In this paper we assume

$$r(x) = \frac{1}{2} \log(1 + x), \quad x \geq 0.$$

The maximum throughput is defined as

$$\gamma^*(c) \triangleq \sup \gamma(c),$$

where the supremum is taken over all admissible online power control policies.

3.2 Greedy Policy

An online power control policy $\{f_t\}_{t=1}^{\infty}$ is said to be stationary if f_t is time-invariant and the resulting G_t depends on X^t only through B_t. The greedy policy is simple stationary policy of the form

$$G_t = B_t, \quad t = 1, 2, \cdots .$$

Fig. 3.2 illustrates this power control policy.

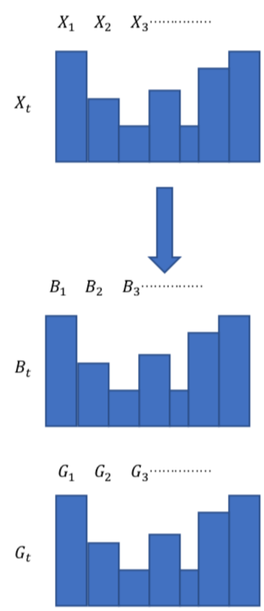


Figure 3.2: Greedy policy model

3.3 Throughput

3.3.1 Throughput of greedy policy

The throughput induced by the greedy policy can serve as a lower bound on $\gamma^*(c)$:

$$\gamma^* \geq \underline{\gamma}(c) \triangleq \mathbb{E}\left[\frac{1}{2}\log(1+\min\{X, c\})\right].$$

On the other hand, the concavity of the reward function implies the following upper bound on $\gamma^*(c)$ [21]:

$$\gamma^* \leq \overline{\gamma}(c) \triangleq \frac{1}{2}\log(1+\mathbb{E}[\min\{X, c\}]). \tag{3.3.1}$$

Proof. See Appendix A □

Let $\rho(x) \triangleq \mathbb{P}(X < x)$, $\underline{x} \triangleq \max\{x \geq 0 : \rho(x) = 0\}$, $\overline{x} \triangleq \inf\{x \geq 0 : \rho(x) = 1)\}$, and $\mu \triangleq \mathbb{E}[X]$. We shall assume $\rho(\mu) > 0$ (i.e., $\mathbb{P}(X = \mu) < 1$, or equivalently, $\underline{x} < \overline{x}$) since otherwise $\underline{\gamma}(c) = \overline{\gamma}(c)$ for all c. It is clear that

$$\lim_{c \downarrow 0} \frac{\underline{\gamma}(c)}{\overline{\gamma}(c)} = 1.$$

In other words, the greedy policy is asymptotically optimal when $c \downarrow 0$. We shall show in this work that the greedy policy is in fact exactly optimal when c is below a certain positive threshold.

3.3.2 Optimal Throughput

The following Bellman equation provides an implicit characterization of the maximum throughput and the associated optimal power control policy.

Proposition 1 ([21]): If there exist a scalar $\gamma \in \mathbb{R}_+$ and a bounded function $h : [0, c] \to \mathbb{R}_+$ that satisfy

$$\gamma + h(b) = \sup_{g \in [0,b]} \{r(g) + \mathbb{E}\left[h(\min\{b - g + X, c\})\right]\} \tag{3.3.2}$$

for all $b \in [0, c]$, then $\gamma^*(c) = \gamma$; moreover, every stationary policy f such that $f(b)$ attains the supremum in (3.3.2) for all $b \in [0, c]$ is throughput-optimal. We apply the value iteration method to solve the Bellman equation and demonstrate the properties of optimal policy. The algorithm is shown as follows, where state s is energy stored in battery b and action a is power allocation policy g.

Algorithm 2 Value Iteration

Initialize array h arbitrarily (e.g., $h(b) = 0$ for all b $\in [0, c]$), then $\gamma = \sum_{b'} p\left(b'|b, g\right) h(b')$)
Input: Battery state $b \in [0, c]$, power allocation policy $g \in [0, b]$
repeat
  $\Delta \leftarrow 0$
  for each $b \in [0, c]$ **do**
    $temp \leftarrow h(b)$
    $\gamma + h(b) \leftarrow \max_g \left\{r(g) + \sum_{b'} p\left(b'|b, g\right) h(b')\right\}$
    $\Delta \leftarrow \max(\Delta, |temp - h(b)|)$
  end for
until $\Delta < \theta$ (a small positive number)
Output: optimal policy π, throughput γ:
$\pi(b) = \arg\max_g \left\{r(g) + \sum_{b'} p\left(b'|b, g\right) h(b')\right\}$
$\gamma = \max_g \left\{r(g) + \sum_{b'} p\left(b'|b, g\right) h(b') - h(b)\right\}$.

Furthermore, we plot the optimal throughput and the throughput of the greedy policy agaist the battery size c by valuation method. We can see that these two kind of throughput the same when c is smaller than one threshold. We define this threshold as c^*. Our goal in this thesis is to find c^*.

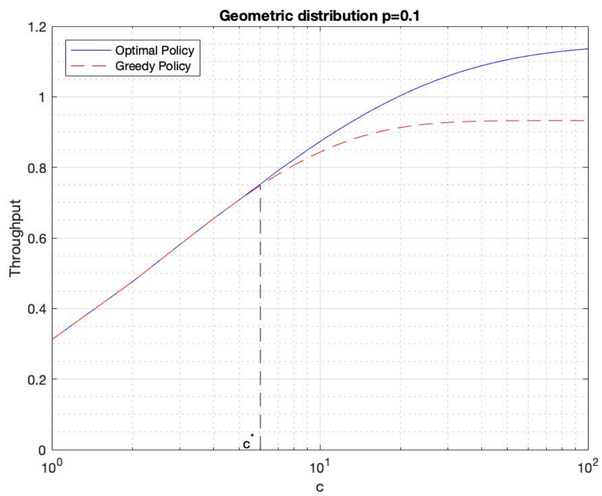


Figure 3.3: Throughput of Geometric distribution

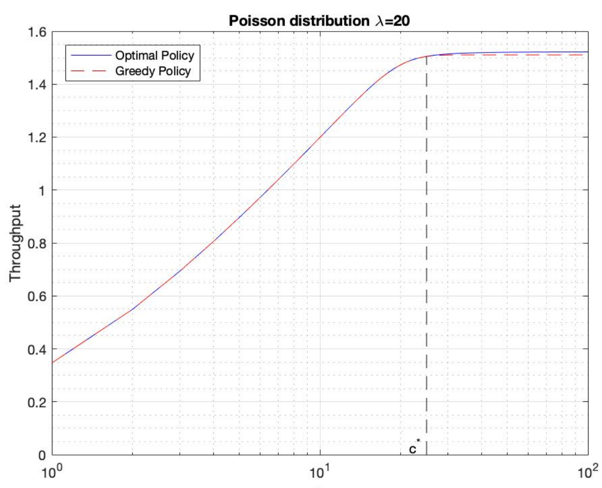


Figure 3.4: Throughput of Poisson distribution

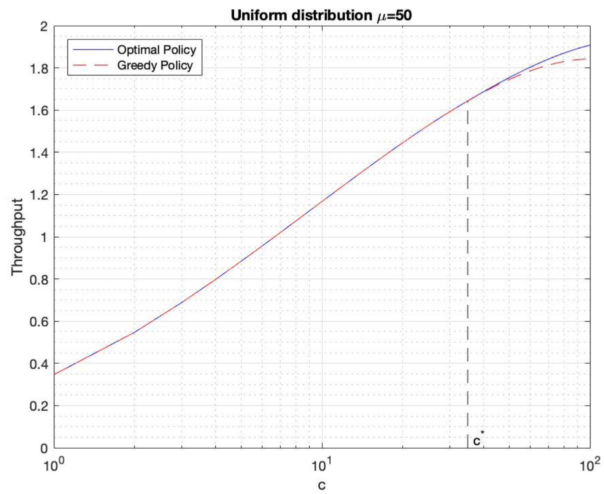


Figure 3.5: Throughput of Uniform distribution

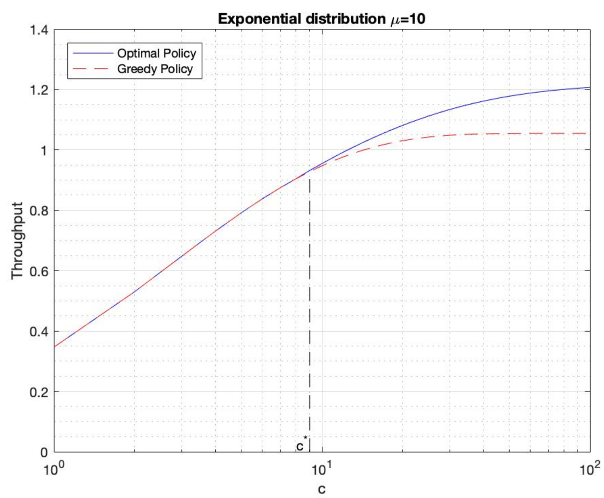


Figure 3.6: Throughput of Exponential distribution

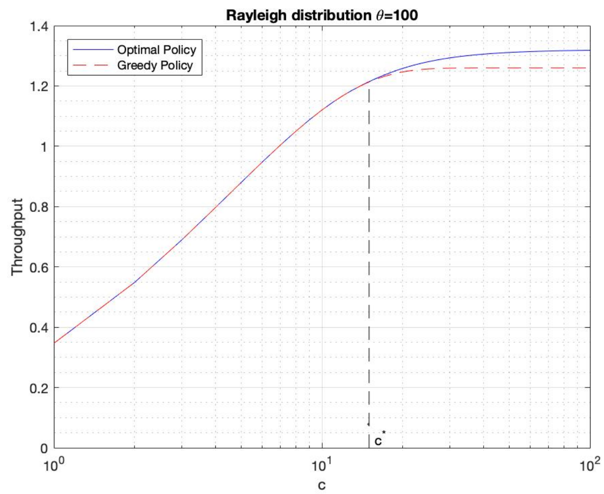


Figure 3.7: Throughput of Rayleigh distribution

Chapter 4

Main Results

4.1 Theorem 1

The greedy policy is optimal, i.e., $\gamma^*(c) = \underline{\gamma}(c)$, if and only if $c \leq c^*$, where

$$c^* \triangleq \max\left\{c \geq 0 : \frac{1}{1+c} \geq \rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle| X < c\right]\right\}.$$

Remark 1. For $c \in [0, \infty)$, let $\phi_1(c) \triangleq \frac{1}{1+c}$ and $\phi_2(c) \triangleq \rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle| X < c\right]$. It is easy to see that $\phi_1(c)$ is a monotonically decreasing continuous function of c, and $\phi_2(c)$ is a monotonically increasing left-continuous function of c; moreover,

$$\phi_1(0) = 1 > 0 = \phi_2(0),$$

$$\lim_{c\to\infty} \phi_1(c) = 0 < \mathbb{E}\left[\frac{1}{1+X}\right] = \lim_{c\to\infty} \phi_2(c),$$

$$\lim_{c\downarrow\underline{x}} \phi_1(c) = \frac{1}{1+\underline{x}} > \frac{\mathbb{P}(X = \underline{x})}{1+\underline{x}} = \lim_{c\downarrow\underline{x}} \phi_2(c),$$

$$\lim_{c \downarrow \overline{x}} \phi_1(c) = \frac{1}{1+\overline{x}} < \mathbb{E}\left[\frac{1}{1+X}\right] = \lim_{c \downarrow \overline{x}} \phi_2(c).$$

These facts imply that c^* is well-defined and more generally

$$\left\{c \geq 0 : \frac{1}{1+c} \geq \rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle| x < c\right]\right\} = [0, c^*]$$

with $\underline{x} < c^* \leq \overline{x}$ (the second inequality is strict if $\overline{x} = \infty$).

Proof. See Chapter 5.1. □

Next we establish bounds on c^* that are in general easier to evaluate than c^* itself.

4.2 Proposition 2: Lower Bound on c^*

$$c^* \geq \underline{c} \triangleq \max\left\{c \geq 0 : \frac{1}{1+c} \geq \frac{\rho(c)}{1+\underline{x}}\right\}$$

Remark 2. A slightly modified version of the argument in Remark 1 can be used to show that

$$\left\{c \geq 0 : \frac{1}{1+c} \geq \frac{\rho(c)}{1+\underline{x}}\right\} = [0, \underline{c}]$$

with $\underline{x} < \underline{c} < \overline{x}$ (the second inequality is strict if $\overline{x} = \infty$).

Proof. See Chapter 5.2. □

4.3 Proposition 3: Upper Bound on c^*

$$c^* \leq \overline{c} \triangleq \frac{1}{3} + \frac{4}{3}\mu.$$

Proof. See Chapter 5.3. □

Consider the case where X is a Bernoulli random variable with $\mathbb{P}(X = \alpha) = 1 - p$ and $\mathbb{P}(X = \beta) = p$, where $0 \leq \alpha < \beta$ and $p \in (0, 1)$. For this special case, a simple calculation shows that

$$c^* = \underline{c} = \begin{cases} \frac{\alpha+p}{1-p}, & \frac{\alpha+p}{1-p} < \beta, \\ \beta & \frac{\alpha+p}{1-p} \geq \beta, \end{cases}$$

$$\overline{c} = \frac{1}{3} + \frac{4}{3}((1-p)\alpha + p\beta).$$

We plot upper bound, lower bound and optimal capacity for Bernoulli distributions with different μ in Fig. 4.1 and compare these characterizations for some discrete and continuous distributions in Appendix B.

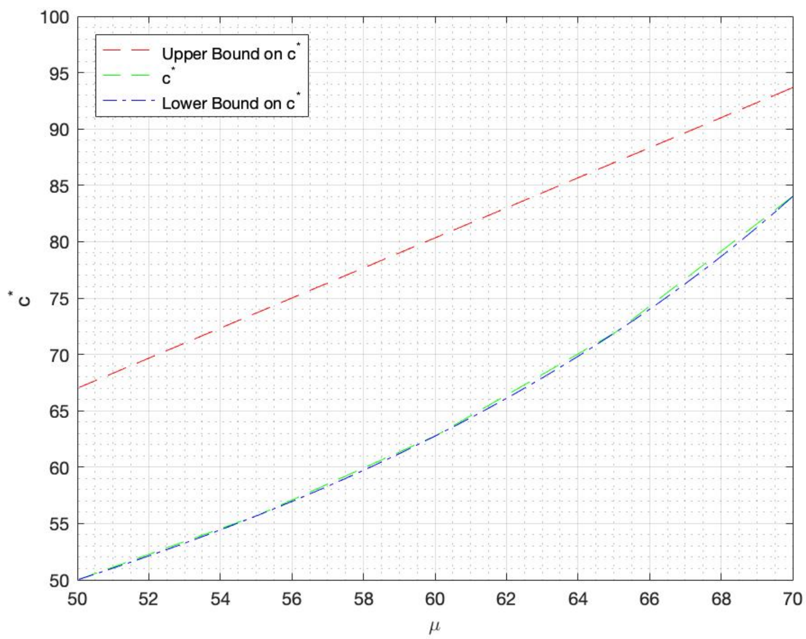


Figure 4.1: Upper and lower bound on c^* for Bernoulli distribution

Moreover, it can be verified that $c^* = \overline{c}$ when $\beta = 2\alpha + 1$ and $p = \frac{1}{2}$. Therefore, the bounds in Proposition 2 and Proposition 3 are tight for non-trivial cases.

Chapter 5

Proofs

5.1 Proof of Theorem 1

Proof.

The main difficulty in solving Bellman equation in Proposition 1 is that the function h associated with the optimal power control policy is in general unknown. However, since we only aim to check the optimality of the greedy policy, it is easy to construct a candidate function h. Specifically, in view of Proposition 1, the greedy policy is optimal if

$$\begin{aligned}&\sup_{g\in[0,b]}\left\{r(g)+\mathbb{E}\left[h(\min\left\{b-g+X,c\right\})\right]\right\}\\&=r(b)+\mathbb{E}\left[h(\min\left\{X,c\right\})\right]\\&=\underline{\gamma}(c)+h(b)\end{aligned}$$

for all $b\in[0,c]$, and the second equality naturally suggests that $h(x)=r(x)$ for

$x \in [0, c]$. Therefore, it suffices to check whether

$$\sup_{g \in [0,b]} \{r(g) + \mathbb{E}[r(\min\{b - g + X, c\})]\}$$
$$= r(b) + \mathbb{E}[r(\min\{X, c\})],$$

i.e., the supremum is attained at $g = b$, for all $b \in [0, c]$. For all $b \in [0, c]$ and $g \in [0, b]$, we have

$$\frac{\mathrm{d}}{\mathrm{d}g} \{r(g) + \mathbb{E}[r(\min\{b - g + X, c\})]\}$$
$$= \frac{1}{2(1 + g)} - \rho(c - b + g)\mathbb{E}\left[\frac{1}{2(1 + b - g + X)}\middle| X < c - b + g\right],$$

which attains its minimum

$$\frac{1}{2(1 + c)} - \rho(c)\mathbb{E}\left[\frac{1}{2(1 + X)}\middle| X < c\right] \tag{5.1.1}$$

at $g = b = c$. Note that the expression in (5.1.1) is non-negative when $c \leq c^*$. This proves the "if" part of Theorem 1.

To prove the "only if" part of Theorem 1, we shall construct an online power control policy that outperforms the greedy policy when $c > c^*$. To this end, we modify the greedy policy as follows: whenever $X_t \geq c - \epsilon$, the modified policy sets $G_t = \min\{X_t, c\} - \epsilon$ and $G_{t+1} = \min\{X_{t+1} + \epsilon, c\}$, where ϵ is a small positive number. As compared to the greedy policy, the modified policy incurs a rate loss approximately $r'(c)\epsilon$ in time slot t, but gains approximately $\rho(c)\mathbb{E}[r'(X)|X < c]\,\epsilon$ in time slot $t + 1$ when $X_t \geq c - \epsilon$ occurs (without loss of generality, we assume $c \leq \overline{x}$ and consequently $\mathbb{P}(X \geq c - \epsilon) > 0$). Since $c > c^*$ is equivalent to $\rho(c)\mathbb{E}[r'(X)|X < c] > r'(c)$, the overall throughput is improved. This proves the "only if" part of Theorem 1. □

5.2 Proof of Proposition 2

Proof.

It is clear that

$$\rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle|X<c\right] = \frac{\rho(c)}{1+\underline{x}} = 0, \quad c \le \underline{x},$$

$$\rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle|X<c\right] \le \frac{\rho(c)}{1+\underline{x}}, \quad c > \underline{x}.$$

Therefore,

$$\left\{c \ge 0 : \frac{1}{1+c} \ge \rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle|X<c\right]\right\} \supseteq \left\{c \ge 0 : \frac{1}{1+c} \ge \frac{\rho(c)}{1+\underline{x}}\right\},$$

from which the desired result follows immediately. □

5.3 Proof of Proposition 3

Proof.

Since $\frac{1}{1+x}$ is convex over $[0,\infty]$, it follows by Jensen's inequality that

$$\rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle|X<c\right] \ge \frac{\rho(c)}{1+\mathbb{E}\left[X|X<c\right]}. \tag{5.3.1}$$

Note that

$$\begin{aligned}\mu &= \rho(c)\mathbb{E}\left[X|X<c\right] + (1-\rho(c))\mathbb{E}\left[X|X\ge c\right] \\ &\ge \rho(c)\mathbb{E}\left[X|X<c\right] + (1-\rho(c))c,\end{aligned}$$

which implies

$$\mathbb{E}\left[X|X<c\right] \leq \frac{\mu-(1-\rho(\mu))c}{\rho(c)}. \tag{5.3.2}$$

Combining (5.3.1) and (5.3.2) gives

$$\rho(c)\mathbb{E}\left[\frac{1}{1+X}\middle|X<c\right] \geq \frac{\rho(c)}{1+\frac{\mu-(1-\rho(c))c}{\rho(c)}}. \tag{5.3.3}$$

In view of Remark 1 and (5.3.3), we have $c^* < c$ for any c satisfying

$$\frac{1}{1+c} < \frac{\rho(c)}{1+\frac{\mu-(1-\rho(c))c}{\rho(c)}},$$

which can be written equivalently as

$$c > \frac{1+\mu}{1-\rho(c)+\rho^2(c)} - 1.$$

Therefore,

$$c^* \leq \max_{\rho(c)\in[0,1]} \frac{1+\mu}{1-\rho(c)+\rho^2(c)} - 1.$$

One can readily complete the proof by noticing that the minimum value of $1-\rho(c)+\rho^2(c)$ is $\frac{3}{4}$ (attained at $\rho(c)=\frac{1}{2}$). □

Chapter 6

Examples

We shall provide a detailed analysis of c^* for a few examples, with a particular interest in understanding how c^* scales with μ as $\mu \to \infty$. In the sequel we adopt the notation $c^* \sim \psi(\mu)$ to denote $\lim_{\mu\to\infty} \frac{c^*}{\psi(\mu)} = 1$.

6.1 Discrete Distribution

Consider the case where X is a discrete random variable with probability mass function p_X. For simplicity, we assume the support of p_X is a countable set $\{\xi_1, \xi_2, \cdots\}$ with $0 \leq \xi_1 < \xi_2 \cdots$. It is easy to show that c^* is the unique positive number satisfying one of the following two conditions.

1) $c^* \in (\xi_j, \xi_{j+1})$ for some j and

$$\frac{1}{1+c^*} = \sum_{i=1}^{j} \frac{1}{1+\xi_i} p_X(\xi_i).$$

2) $c^* = \xi_{j+1}$ for some j and

$$\sum_{i=1}^{j} \frac{1}{1+\xi_i} p_X(\xi_i) \leq \frac{1}{1+c^*} \leq \sum_{i=1}^{j+1} \frac{1}{1+\xi_i} p_X(\xi_i).$$

6.1.1 Geometric Distribution

$$p_X(k) = (1-p)^k p, \quad k = 0, 1, \cdots, \quad p \in (0,1).$$

Note that $\mu = \frac{1-p}{p}$. For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\to\infty} \left(1 + \frac{a\mu}{\log\mu}\right) \sum_{k=0}^{\lfloor \frac{a\mu}{\log\mu} \rfloor} \frac{(1-p)^k p}{1+k} \\
&= \lim_{\mu\to\infty} \left(1 + \frac{a\mu}{\log\mu}\right) \sum_{k=0}^{\lfloor \frac{a\mu}{\log\mu} \rfloor} \frac{(\frac{\mu}{1+\mu})^k}{(1+\mu)(1+k)} \\
&= \lim_{\mu\to\infty} \left(1 + \frac{a\mu}{\log\mu}\right) \sum_{k=0}^{\lfloor \frac{a\mu}{\log\mu} \rfloor} \frac{1}{(1+\mu)(1+k)} \\
&= \lim_{\mu\to\infty} \left(1 + \frac{a\mu}{\log\mu}\right) \frac{1}{1+\mu} \log\left(1 + \left\lfloor \frac{a\mu}{\log\mu} \right\rfloor\right) \\
&= a.
\end{aligned}$$

Therefore, we must have $c^* \sim \frac{\mu}{\log(\mu)}$.

6.1.2 Poisson Distribution

$$p_X(k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, \cdots, \quad \lambda > 0.$$

Note that $\mu = \mathbb{E}\left[(X-\mu)^2\right] = \lambda$. We have

$$1-\mu^{-\frac{1}{3}} \leq \sum_{k=\lceil \mu-\mu^{\frac{2}{3}} \rceil}^{\lfloor \mu+\mu^{\frac{2}{3}} \rfloor} \frac{e^{-\mu}\mu^k}{k!} \leq 1, \tag{6.1.1}$$

where the first " $\leq$ " is due to Chebyshev's inequality (See Appendix A) . For any $a>0$,

$$\begin{aligned} &\lim_{\mu\to\infty}(1+a\mu) \sum_{k=\lceil \mu-\mu^{\frac{2}{3}} \rceil}^{\lfloor \mu+\mu^{\frac{2}{3}} \rfloor} \frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} \\ &= \lim_{\mu\to\infty} a \sum_{k=\lceil \mu-\mu^{\frac{2}{3}} \rceil}^{\lfloor \mu+\mu^{\frac{2}{3}} \rfloor} \frac{e^{-\mu}\mu^k}{k!} \\ &= a, \end{aligned} \tag{6.1.2}$$

$$\begin{aligned} &\lim_{\mu\to\infty}(1+a\mu) \sum_{k=\lfloor \mu+\mu^{\frac{2}{3}} \rfloor}^{\infty} \frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} \\ &\leq \lim_{\mu\to\infty} a \sum_{k=\lfloor \mu+\mu^{\frac{2}{3}} \rfloor}^{\infty} \frac{e^{-\mu}\mu^k}{k!} \\ &= 0, \end{aligned} \tag{6.1.3}$$

where (6.1.2) and (6.1.2) are due to (6.1.1); moreover,

$$\begin{aligned}
&\lim_{\mu\to\infty}(1+a\mu)\sum_{k=0}^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} \\
&\leq \lim_{\mu\to\infty}(1+a\mu)\sum_{k=0}^{\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}\frac{e^{-\mu}\mu^k}{k!} \\
&\leq \lim_{\mu\to\infty}(1+a\mu)\mu(1-\delta)\frac{e^{-\mu}\mu^{\mu(1-\delta)}}{(\mu(1-\delta))!} \qquad (6.1.4)\\
&= \lim_{\mu\to\infty}(1+a\mu)\mu(1-\delta)\frac{e^{-\mu\delta}(1-\delta)^{-\mu(1-\delta)-\frac{1}{2}}}{\sqrt{2\pi\mu}} \\
&= \lim_{\mu\to\infty}(1+a\mu)\mu(1-\delta)\frac{e^{-\frac{\mu\delta^2}{2}+o(\mu^{\frac{1}{3}})}}{\sqrt{2\pi\mu}} \\
&= 0,
\end{aligned}$$

where $\delta \triangleq \frac{\mu-\lfloor\mu-\mu^{\frac{2}{3}}\rfloor}{\mu}$, and (6.1.4) follows by Stirling's approximation. Therefore, we have

$$\lim_{\mu\to\infty}(1+a\mu)\sum_{k=0}^{\lfloor a\mu\rfloor}\frac{e^{-\lambda}\lambda^k}{(1+k)(k!)} = \begin{cases} 0, & a<1, \\ a, & a>1, \end{cases}$$

which implies $c^* \sim \mu$.

We plot c^* against μ in Figure 6.1 for the geometric distribution and the Poisson distribution, which confirms our asymptotic analysis.

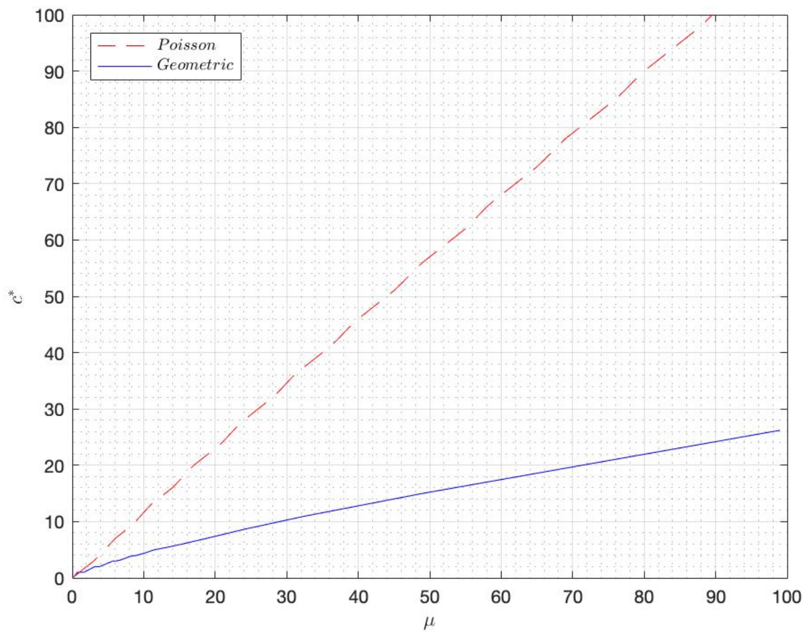


Figure 6.1: The relationship between c^* and μ for some discrete distributions

6.2 Continuous Distribution

Consider the case where X is a continuous random variable with probability density function f_X. It is easy to show that c^* is the unique positive number satisfying

$$\frac{1}{1+c^*} = \int_0^{c^*} \frac{1}{1+x} f_X(x) \mathrm{d}x. \tag{6.2.1}$$

6.2.1 Uniform Distribution

$$f_X(x) = \begin{cases} \frac{1}{\omega}, & x \in [0, \omega], \\ 0, & x \notin [0, \omega], \end{cases} \qquad \omega > 0.$$

We can write (6.2.1) equivalently as

$$\frac{1+c^*}{\omega}\log(1+c^*) = 1.$$

Note that $\mu = \frac{\omega}{2}$. For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\to\infty} \frac{1+\frac{a\mu}{\log\mu}}{\omega}\log\left(1+\frac{a\mu}{\log\mu}\right)\\
&= \lim_{\mu\to\infty} \frac{1+\frac{a\mu}{\log\mu}}{2\mu}\log\left(1+\frac{a\mu}{\log\mu}\right)\\
&= \frac{a}{2}.
\end{aligned}$$

Therefore, we must have $c^* \sim \frac{2\mu}{\log\mu}$.

6.2.2 Exponential Distribution

$$f_X(x) = \begin{cases} \eta e^{-\eta x}, & x \geq 0, \\ 0, & x < 0, \end{cases} \qquad \eta > 0.$$

We can write (6.2.1) equivalently as

$$(1+c^*)\int_0^{c^*} \frac{\eta e^{-\eta x}}{1+x}\mathrm{d}x = 1.$$

Note that $\mu = \frac{1}{\eta}$. For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\to\infty}\left(1+\frac{a\mu}{\log\mu}\right)\int_0^{\frac{a\mu}{\log\mu}}\frac{\eta e^{-\eta x}}{1+x}\mathrm{d}x \\
&=\lim_{\mu\to\infty}\left(1+\frac{a\mu}{\log\mu}\right)\int_0^{\frac{a\mu}{\log\mu}}\frac{e^{-\frac{x}{\mu}}}{\mu(1+x)}\mathrm{d}x \\
&=\lim_{\mu\to\infty}\left(1+\frac{a\mu}{\log\mu}\right)\int_0^{\frac{a\mu}{\log\mu}}\frac{1}{\mu(1+x)}\mathrm{d}x \\
&=\lim_{\mu\to\infty}\left(1+\frac{a\mu}{\log\mu}\right)\frac{1}{\mu}\log\left(1+\frac{a\mu}{\log\mu}\right) \\
&=a.
\end{aligned}$$

Therefore, we must have $c^* \sim \frac{\mu}{\log\mu}$.

6.2.3 Rayleigh Distribution

$$f_X(x) = \begin{cases} \frac{x}{\theta}e^{-\frac{x^2}{2\theta}}, & x \geq 0, \\ 0, & x < 0, \end{cases} \qquad \theta > 0.$$

We can write (6.2.1) equivalently as

$$(1+c^*)\int_0^{c^*}\frac{xe^{-\frac{x^2}{2\theta}}}{\theta(1+x)}\mathrm{d}x = 1.$$

Note that $\mu = \sqrt{\frac{\pi\theta}{2}}$. For any $a > 0$,

$$\begin{aligned}
&\lim_{\mu\to\infty}(1+a\mu)\int_0^{a\mu}\frac{xe^{-\frac{x^2}{2\theta}}}{\theta(1+x)}\mathrm{d}x \\
&= \lim_{\mu\to\infty}(1+a\mu)\int_0^{a\mu}\frac{\pi xe^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2(1+x)}\mathrm{d}x \\
&= \lim_{\mu\to\infty}(1+a\mu)\int_0^{a\mu}\frac{\pi e^{-\frac{\pi x^2}{4\mu^2}}}{2\mu^2}\mathrm{d}x \\
&= \lim_{\mu\to\infty}(1+a\mu)\int_0^{a}\frac{\pi e^{-\frac{\pi y^2}{4}}}{2\mu}\mathrm{d}y \\
&= \frac{\pi a}{2}\int_0^{a} e^{-\frac{\pi y^2}{4}}\mathrm{d}y.
\end{aligned}$$

Therefore, we must have $c^* \sim a^*\mu$, where $a^* \approx 0.875$ is the unique positive number satisfying

$$\frac{\pi a^*}{2}\int_0^{a^*} e^{-\frac{\pi y^2}{4}}\mathrm{d}y = 1.$$

We plot c^* against μ in Figure 6.2 for the uniform distribution, the exponential distribution, and the Rayleigh distribution, which confirms our asymptotic analysis.

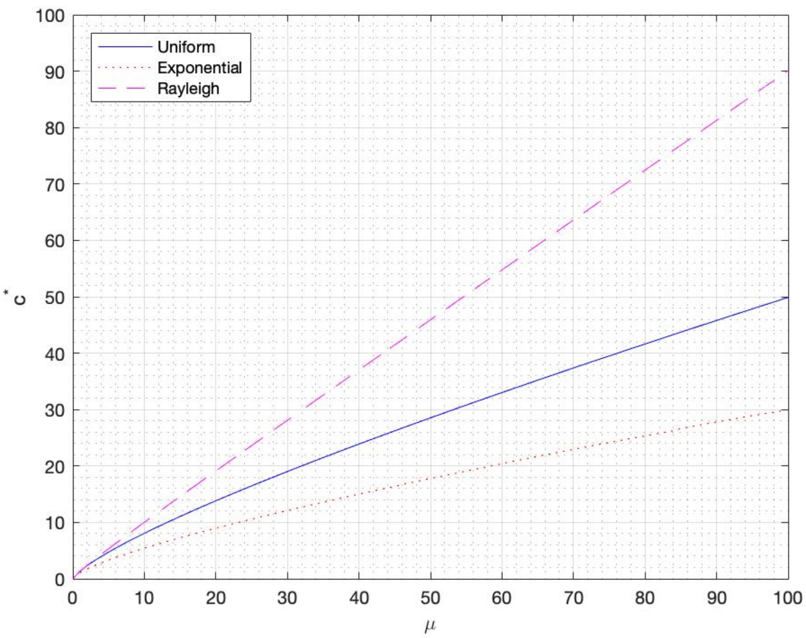


Figure 6.2: The relationship between c^* and μ for some continuous distributions

Chapter 7

Conclusion and Future Work

7.1 Conclusion

We have established a sufficient and necessary condition for the optimality of the greedy policy. Although only a special reward function is considered in this work, this restriction is by no means essential. In particular, it is straightforward to establish an extended version of Theorem 1 that holds for an arbitrary monotonically increasing concave reward function with continuous first-order derivative. Furthermore, our future work is to have a numerical analysis for optimal policy exceeding the greedy range.

7.2 Future Work

Figure. 7.1 below provides the optimal power allocation for Exponential distribution. Figure. C.1, C.2, C.3 and C.4 provide the optimal power allocation for Uniform distribution, Rayleigh distribution, Geometric distribution, and Poisson distribution

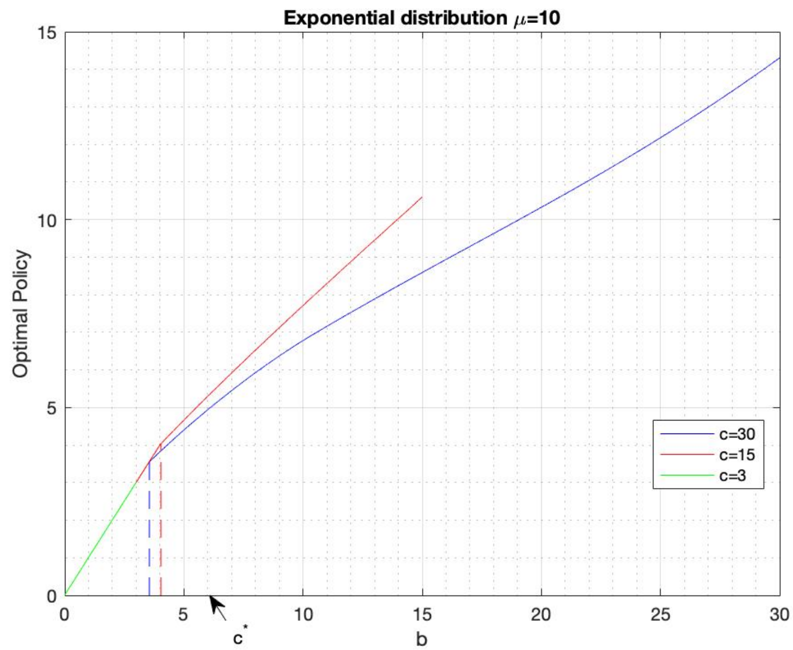


Figure 7.1: Optimal power allocation for Exponential distribution

with different battery capacity.

We illustrate some details for Exponential distribution, Rayleigh distribution, and Poisson distribution at Table. C.1, C.2, and C.3 respectively. It can be seen that when the battery capacity c is smaller than the c^* we calculate at Theorem 1 (i.e. when c is in the low-battery-regime), the greedy policy is always optimal which is consistent with our results. When the battery capacity is larger than c^*, the bigger the capacity is, the smaller the scope of which the greedy policy is optimal is. It is easy to explain. When the battery is nearly full, the following harvested energy may be wasted, so in order to reduce the wastage of energy, more energy needs to be allocated to the transmitter when the energy in the battery is close to its limit. Our future work is to find the threshold c^* when the battery size c exceeds the low-battery-size regime.

Appendix A

Proof of 3.3.1 and 6.1.1

Proof. 3.3.1

$$
\begin{aligned}
\gamma(c) &= \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} r(f_t(X^t))\right] \\
&= \frac{1}{n}\sum_{t=1}^{n}\mathbb{E}\left[\frac{1}{2}\log(1+f_t(X^t))\right] \\
&\leq \frac{1}{2}\log\left(1+\frac{1}{n}\mathbb{E}\left[\sum_{i=1}^{n} f_t(X^t)\right]\right) \\
&\leq \frac{1}{2}\log\left(1+\frac{1}{n}\mathbb{E}\left[B_0+\sum_{t=2}^{n} X_t\right]\right) \\
&= \frac{1}{2}\log\left(1+\frac{1}{n}B_0+\frac{n-1}{n}\mu\right)
\end{aligned}
$$

The first"$\leq$" follows by the concavity of log; the second "$\leq$" follows by the fact that the total of the initial energy in the battery (we define $B_0 \triangleq 0$) and the arrival energy is the maximum allocated energy:

$$\sum_{t=1}^{n} f_t \leq B_0 + \sum_{t=2}^{n} X_t.$$

when $n \to \infty$, the last equation tends to $\frac{1}{2}\log(1+\mu)$. Therefore, we get:

$$\gamma \leq \frac{1}{2}\log(1+\mu),$$

where $\mu = \mathbb{E}[\min\{X, c\}]$. □

Proof. 6.1.1

According to Chebyshev's inequality, we get:

$$\begin{aligned} P(a < X < b) &= P\left(\left|X - \frac{b-a}{2}\right| < \frac{b-a}{2}\right) \\ &\geq 1 - \frac{\lambda + \left(\mu - \frac{a+b}{2}\right)^2}{(\frac{b-a}{2})^2} \\ &= 1 - \frac{\mu}{\mu^{\frac{4}{3}}} \\ &= 1 - \mu^{-\frac{1}{3}} \end{aligned}$$

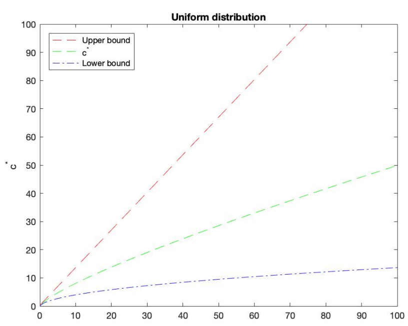
where $a = \mu - \mu^{\frac{2}{3}}, b = \mu + \mu^{\frac{2}{3}}$, $\mu = \mathbb{E}[(X-\mu)^2] = \lambda$. We therefore have

$$1 - \mu^{-\frac{1}{3}} \leq \sum_{k=\lceil \mu - \mu^{\frac{2}{3}} \rceil}^{\lfloor \mu + \mu^{\frac{2}{3}} \rfloor} \frac{e^{-\mu}\mu^k}{k!}$$

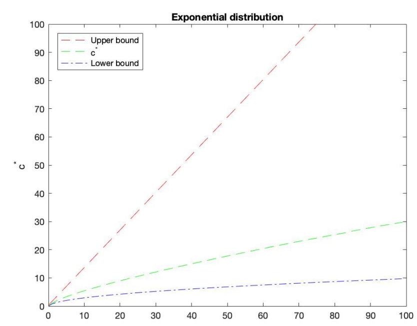
□

Appendix B

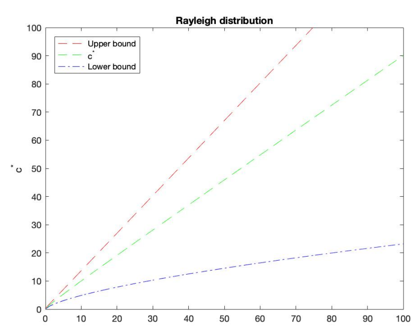
Figures of Bounds on c^*



(a) Uniform

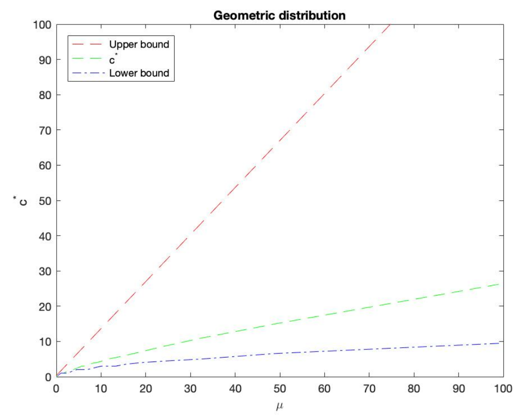


(b) Exponential

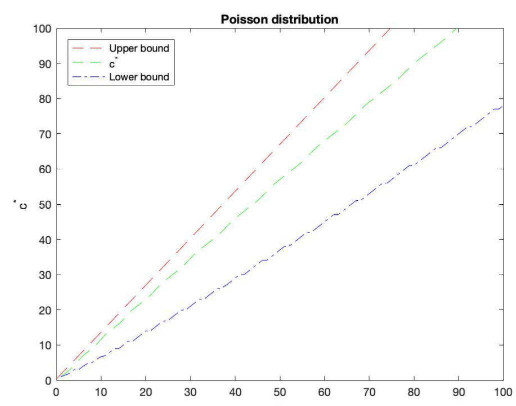


(c) Rayleigh

Figure B.1: Upper and lower bound on c^* for some continuous distributions



(a) Geometric



(b) Poisson

Figure B.2: Upper and lower bound on c^* for some discrete distributions

Appendix C

Optimal Power Control Policy

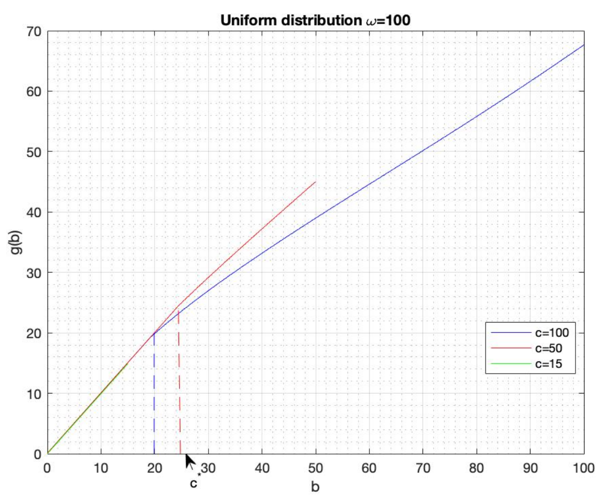


Figure C.1: Optimal power control policy for Uniform distribution

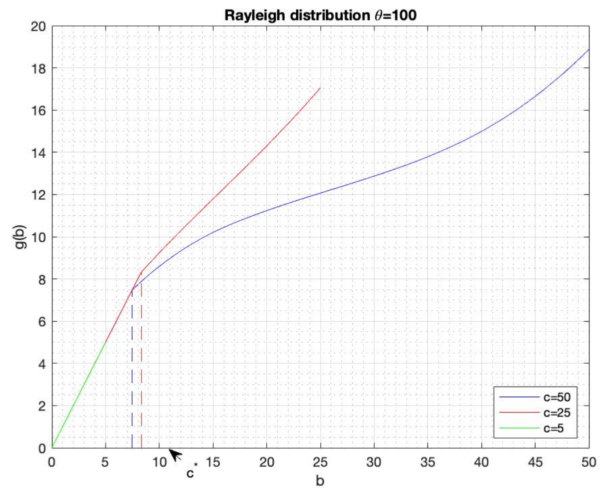


Figure C.2: Optimal power control policy for Rayleigh distribution

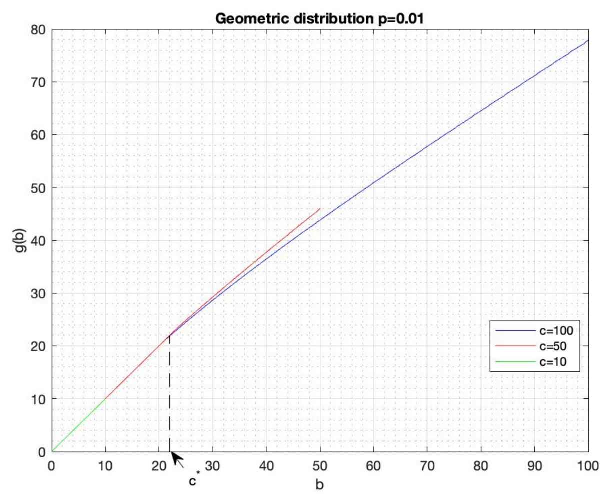


Figure C.3: Optimal power control policy for Geometric distribution

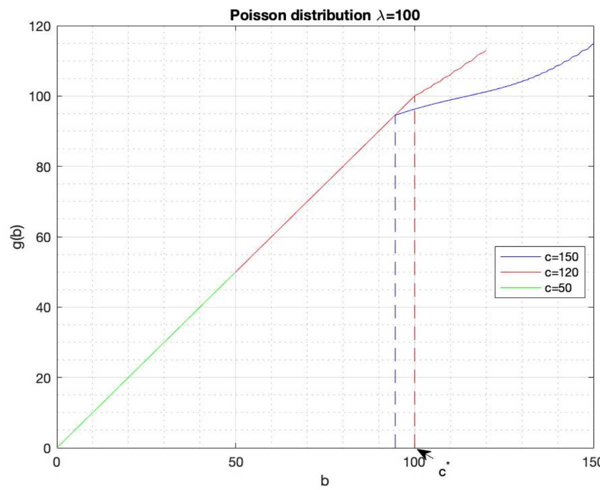


Figure C.4: Optimal power control for policy Poisson distribution

Battery State	c = 3	c = 4	c = 15	c=30
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	×	4	4	3.82
5	×	×	4.66	4.39
6	×	×	5.29	4.93
7	×	×	5.91	5.44
8	×	×	6.52	5.92
9	×	×	7.12	6.37
10	×	×	7.71	6.78

Table C.1: Optimal power allocation for Exponential distribution

Battery State	c = 5	c = 10	c = 25	c=50
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
10	×	10	10	8.6
15	×	×	11.81	10.21
16	×	×	12.29	10.45
17	×	×	12.79	10.67
18	×	×	13.29	10.87

Table C.2: Optimal power allocation for Rayleigh distribution

Battery State	c = 50	c = 90	c = 120	c=150
0	0	0	0	0
20	20	20	20	20
40	40	40	40	40
60	×	60	60	60
80	×	80	80	80
100	×	×	100	96.3
105	×	×	103.0	97.7
110	×	×	106.0	99.0
120	×	×	113.1	101.3
130	×	×	×	104.1
140	×	×	×	108.7

Table C.3: Optimal power allocation for Poisson distribution

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