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RATIO OF COUPLING CONSTANTS IN THE BETA-DECAY OF Rb^{87}

THE RATIO OF VECTOR AND AXIAL VECTOR COUPLING CONSTANTS IN THE BETA-DECAY OF Rb^{87}

By

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SCOPE AND CONTENTS: The purpose of this work is to evaluate the ratio of the vector and axial vector coupling constants in the beta-decay of Rb^{87}. Particular attention is given to determining the sign of the ratio. All this is accomplished mainly by a conic analysis of the beta-spectrum and the calculation of a matrix element ratio. The results obtained indicate that the ratio of the coupling constants is negative. Due to uncertainties in nuclear matrix elements, only an approximate absolute value for the ratio can be secured. These results are consistent with other present evidence.

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CONTENTS

INTRODUCTION

1. ORIGIN OF PROBLEM

In the theory of beta-decay the main problem has been to determine the form of the interaction between the heavy and light particle fields. The theory is based upon Fermi's (1934) neutrino hypothesis and Dirac theory for leptons and nucleons. According to the Dirac theory, five independent Lorentz invariant expressions can be chosen for the interaction Hamiltonian (e.g. Bethe and Bacher (1936)). Due to their method of formation these are called scalar, vector, tensor, axial vector, and pseudo-scalar interactions; and are denoted by S, V, T, A, P respectively. Excluding derivatives of the wave functions, a linear combination of these five invariants is the most general interaction possible (e.g. Fierz (1937)). Since there is no good theoretical reason for preferring any particular term or linear combination, the interaction must be determined by experiment. The constants used to form the linear combination are the coupling constants g_i.

At present it is believed that the beta-decay interaction contains only the vector and axial vector terms (e.g. Gatlinburg Conference, 1958). The absolute value of the ratio of the coupling constants can be determined experimentally with considerable accuracy. A recent value for $|g_A/g_V|$ is 1.19 ± 0.04 reported at the Rochester conference (1958).

The coupling constants are believed to be real since it appears that time-reversal invariance is perserved in beta-decay (e.g. Burgy et al. (1958)). There remains some question as to the relative sign of the coupling constants g_A and g_V. It is the main purpose of this present work to determine this sign. Further, a check will be made to see if the value quoted above will fit our analysis. There is a theoretical question as to whether the coupling constants are independent of the nucleus decaying. Feynman and Gell-Mann (1958) show the vector coupling constant to be so (except for small electromagnetic corrections) but nothing definite is said about the axial vector coupling constant. Indeed, the sign itself may depend upon the nucleus decaying.

The coupling constants g_V and g_A are those originally introduced for the parity-conserving interaction. Allowing for the nonconservation of parity, Lee and Yang (1956) introduced another five terms into the interaction Hamiltonian. They showed that measured quantities out of which pseudoscalars cannot be formed, such as spectrum shapes and lifetimes, could be adjusted correctly by replacing g_V^2 by $g_V^2 + g_V'^2$, $g_V g_A$ by $g_V g_A + g_V' g_A'$, etc. (here their results have been particularized). g_V' and g_A' are the new coupling constants for the parity-nonconserving interactions. Evidence indicates, however, that $g_A = g_A'$ and $g_V = g_V'$ (e.g. see Wu (1958)). Thus, the measured quantity which is really $|(g_A^2 + g_A'^2)/(g_V^2 + g_V'^2)|^{\frac{1}{2}}$ reduces to $|g_A/g_V|$ as it was before the nonconservation of parity.

The analysis of forbidden beta-decay spectra is useful in studying the beta-decay interaction. For the present work the third forbidden decay of Rb^{87} has been chosen ($\Delta J = 3$, parity change). There are three reasons for this choice.

First, the method of conics as developed in chapter II depends strongly upon the Fermi plot of the observed spectrum not being a straight line. In the case of allowed spectra the method is not applicable. It seems that the chances for a successful analysis would be best with a high degree of forbiddeness. Second, the nucleon structure of $_{37}Rb^{87}$ allows for a shell model assumption. Reasons for such an assumption are found in chapter I. Finally, in the theoretical group here at McMaster there has been past interest in this decay. Calculations employing scalar-tensor theory have been carried out by Zernik (1956) and Pearson (1956). I am indebted to the latter for tables of certain functions which he calculated to five significant figures and for the normalized shape correction ρ-factors (see chapter II).

2. THE SHAPE CORRECTION FACTOR

If the interaction is vector and axial vector the Hamiltonian may be written as

$$H_{\beta} = g_V V + g_A A + g_V' V' + g_A' A'$$

V and A are respectively the vector and axial vector parity-conserving invariants, and g_V and g_A are the respective coupling constants; the primed quantities refer to the corresponding parity-nonconserving parts. Then the formula for the shape of the beta-spectrum is

$$P(E)dE = (1/2\pi)^3 E (E^2 - 1)^{\frac{1}{2}} (E_o - E)^2 F(Z,E) C_n(E,Z) dE$$

where P(E) is the decay probability

E is the total energy of the electron

E_o is the end-point energy of the spectrum

F(Z,E) is the Fermi function

C_n is the shape correction factor for the n-th forbidden transition considered. C also depends on certain nuclear matrix elements.

Z is the charge of the daughter nucleus.

The units $\hbar = c = m_e = 1$ are used here and elsewhere, unless otherwise stated.

If the beta-decay interaction consists only of vector and axial vector terms (as above), the shape correction factor for a third forbidden transition where $\Delta J = 3$ can be written as

$$C = (g_V{}^2 + g_V'^2)\, C_{3V} + (g_A{}^2 + g_A'^2)\, C_{3A} + (g_A g_V + g_A' g_V')\, C_{3VA}$$

Using Pursey's (1951) notation for the nuclear matrix elements Q_n,

$$C_{3V} = |Q_3(\underline{r})|^2 A(q) + |Q_3(\underline{\alpha})|^2 B(q) + i(Q_3(\underline{\alpha}).Q_3^*(\underline{r}) - c.c.)D(q)$$

$$C_{3A} = |Q_3(\underline{\sigma}x\underline{r})|^2 F(q) + |Q_4(\underline{\sigma})|^2 k(q)$$

$$C_{3VA} = (Q_3(\underline{\alpha}).Q_3^*(\underline{\sigma}x\underline{r}) + c.c.)G(q) + i(Q_3(\underline{r}).Q_3^*(\underline{\sigma}x\underline{r}) - c.c.)H(q)$$

q is the momentum of the neutrino, and $\underline{\sigma}$ and $\underline{\alpha}$ are the usual Dirac operators. The expressions for C_{3V} and C_{3A} are taken from Greuling (1942); C_{3VA} from Pursey (1951). Notation unexplained here will be found in Greuling's article. In the above

$B(q) = (L_0/30)q^4 + L_1q^2 + (15/2)L_2$

$k(q) = (L_0/630)q^6 + (L_1/10)q^4 + (5/2)L_2q^2 + (35/2)L_3$

$A(q) = Q_1 + Q_2 - 2n$

$D(q) = Q_3 - m$

$F(q) = Q_1 + Q_2 + 2n - k/4$

$G(q) = Q_3 + m$

$H(q) = Q_1 + Q_2 - k$, and

$Q1 = (M_0/30)q^4 + M_1q^2 + (15/2)M_2$

$Q_2 = (L_0/630)q^6 + (L_1/15)q^4 + (3/2)L_2q^2 + 10L_3$

$Q_3 = (L_0/210)q^5 + (L_1/5)q^3 + (5/2)L_2q$

$n = (N_0/210)q^5 + (N_1/5)q^3 + (5/2)N_2q$

$m = (N_0/30)q^4 + N_1q^2 + (15/2)N_2$

The L_ν, M_ν, N_ν functions are defined precisely by Greuling (1942). They are combinations of radial lepton wave functions evaluated at the position of the transforming neutron. These have been tabulated by Rose et al. (1953) to four significant figures for the effective radial position $R = 1.41 A^{1/3} \times 10^{-13}$ cm. For the conic analysis of chapter II greater accuracy was found to be needed.

Pearson (1956) has made expansions for L_ν, M_ν, and N_ν (see Zernik (1956)) and tabulated values to five significant figures for the Rb^{87} decay. The latter has been done for three effective radial positions to investigate the dependence of this uncertain quantity.

CHAPTER I: MATRIX ELEMENT RATIO $Q_3(\underline{\alpha})/Q_3(\underline{r})$

1. SHELL MODEL ASSUMPTION

In this chapter the matrix ratio $Q_3(\underline{\alpha})/Q_3(\underline{r})$ is calculated. This calculation was suggested by the results found in chapter II and it turns out to be useful in completing the analysis.

To evaluate this matrix ratio we need to assume some nuclear model. From the energy level scheme of the extreme single particle model (eg. Lane and Elliot (1957)) it is seen that this shell model of the nucleus predicts our beta-decay process to be the transition of a $1g_{9/2}$ neutron into a $2p_{3/2}$ proton. This event leaves a hole in the $1g_{9/2}$ neutron level and fills up the $2p_{3/2}$ proton level of Sr^{87}. The measured ground state spins of Rb^{87} and Sr^{87} are 3/2 and 9/2 respectively (so $\Delta J = 3$). The measured magnetic moments of Rb^{87} and Sr^{87} lie fairly close to the Schmidt limits. Both are near their respective "$j = L + \frac{1}{2}$ limits" indicating good agreement with the parity assignments predicted by the extreme single particle model (eg. Blin-Stoyle (1957)).

It is known from the change of spin and the measurements of half-life, $\sim 5 \times 10^{10}$ years, and end-point energy, 280 kev (Goodman (1956)), that the Rb^{87} beta-decay is 3rd forbidden. The above argument is consistent with this result.

Now, we wish to write wave functions for the initial and final states. Let each total wave function be written as a linear combination of properly antisymmetrized product wave functions; each term being formed from a possible set of eighty-seven single particle shell states. This allows for possible configuration mixing. The resulting total matrix elements will each be a sum of terms, each such term being a matrix element between two product wave functions. Next, it is important to remember the nature of our operators--they are single particle operators. This means that matrix elements between two product wave functions can be non-zero only if these product wave functions differ in only one of their single particle states.

Let us look at the two nuclei and their shell structure. Since fifty is a magic number it is expected that the fifty neutrons in Rb^{87} will have only one possible configuration--filling all levels up to and including the $1g_{9/2}$. Also, the thirty-eight protons in Sr^{87} are expected to have only the configuration which fills all levels up to and including the $1f_{5/2}$. We can write

$$\Psi(Sr^{87}) = \Omega^{p}(\text{38 protons filling levels to } 1f_{5/2}) \times \sum_i \alpha_i \varphi_i(39)\varphi_i(40)\ldots\varphi_i(87)$$

$$= \Omega^{p} \times \sum_i \alpha_i \Psi_i^{n}$$

$$\Psi(Rb^{87}) = \Omega^{n}(\text{50 neutrons filling levels to } 1g_{9/2}) \times \sum_j \beta_j \varphi_j(1)\varphi_j(2)\ldots\varphi_j(37)$$

$$= \Omega^{n} \times \sum_j \beta_j \Psi_j^{p}$$

Ω^p and Ω^n are the fixed core wave functions for Sr^{87} and Rb^{87} respectively. $\varphi(k)$ are the single particle nucleon wave functions; $k \leq 37$ for protons, $k \geq 39$ for neutrons. Particle number 38 undergoes beta-decay.

The $Rb^{87} \xrightarrow{\beta^-} Sr^{87}$ decay is the transition of a $1g_{9/2}$ neutron into a $2p_{3/2}$ proton. Contributions to the matrix elements may be obtained only when

a. Ω^n and ψ^n are identical for particles 39 to 87 which means that ψ^n represents the neutron configuration

$\ldots (1g_{9/2})^9 \quad : \quad \psi^n_s$

b. Ω^p and ψ^p are identical for particles 1 to 37 which means that ψ^p represents the proton configuration

$\ldots (2p_{3/2})^3 \; (1f_{5/2})^6 \quad : \quad \psi^p_r$

In taking the ratio of the matrix elements the product of the configuration mixing constants $\alpha_s \beta_r$ will cancel out. Therefore, any configuration mixing present will not influence our result.

2. OUTLINE OF CALCULATION

The calculation is simplified by using the theorem of Longmire and Messiah (1951). This states that the ratio of matrix elements is independent of the values of the indicies and of the magnetic quantum numbers of the initial and final states. We choose to calculate

$$\frac{Q_{zzz}(\underline{\alpha})}{Q_{zzz}(\underline{r})} = \frac{\int d\tau \; \psi^*_f(2p_{3/2}) \; O_{zzz}(\underline{\alpha}) \; \psi_i(1g_{9/2})}{\int d\tau \; \psi^*_f(2p_{3/2}) \; O_{zzz}(\underline{r}) \; \psi_i(1g_{9/2})}$$

The sums which appear in the general expressions for the numerator and denominator (e.g. Greuling (1942)) reduce to single terms. The integrations over the eighty-six particles not affected by the operators cancel out.

The operator $O_{zzz}(\underline{r})$ is easily written in terms of spherical harmonics Y_ℓ^m. The spherical harmonics used here are those defined by Condon and Shortley (1935). Note that $(Y_\ell^m)^* = (-1)^m Y_\ell^{-m}$. We have

$$O_{zzz}(\underline{r}) = z^3 - (1/5)z\, r^2 - (2/5)z\, r^2$$
$$= (4\pi/7)^{\frac{1}{2}}(2/5)\, r^3\, Y_3^0$$

The procedure for the operator $O_{zzz}(\underline{\alpha})$ is more involved. The odd Dirac operator $\underline{\alpha}$ is replaced by the even operator $-\underline{p}/M$ (e.g. Rose and Osborn (1954)). $\underline{p}$ and M are the nucleon momentum and mass, respectively. One finds

$$O_{zzz}(\underline{\alpha}) = (-1/M)\, (z^2\, p_z - (1/5)r^2\, p_z - (2/5)z\, (\underline{r}.\underline{p}))$$
$$= (4i/5M)\, (\pi/21)^{\frac{1}{2}}\, (3^{\frac{1}{2}} Y_3^0\, r^2 \frac{\partial}{\partial r} - Y_3^{-1}\, r\, L_+ - Y_3^1\, r\, L_-)$$

Here L_+ and L_- are the ladder operators. They satisfy

$$L_+\, Y^m = (\ell - m)^{\frac{1}{2}}\, (\ell + m + 1)^{\frac{1}{2}}\, Y_\ell^{m+1}$$
$$L_-\, Y^m = (\ell + m)^{\frac{1}{2}}\, (\ell - m + 1)^{\frac{1}{2}}\, Y_\ell^{m-1}$$

It is noted that $O_{zzz}(\underline{r})$ will have matrix elements only between states of equal magnetic quantum numbers, J_z. Choose $J_z = 3/2$. We have

$$\Psi_{final} = R_{2p}(r)\, Y_1^1(\theta,\varphi)\, \alpha$$
$$\Psi_{inital} = R_{1g}(r)\, (\, (2/3)^{\frac{1}{2}}\, Y_4^1(\theta,\varphi)\, \alpha + (1/3)^{\frac{1}{2}}\, Y_4^2(\theta,\varphi)\, \beta\,)$$

α and β are the usual spin wave functions; α for spin up, β for spin down.

Using the relationship

$$\langle Y_L^M, Y_{\ell_1}^{m_1} Y_{\ell_2}^{m_2} \rangle = \int (Y_L^M)^* Y_{\ell_1}^{m_1} Y_{\ell_2}^{m_2} d\Omega$$

$$= \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)}{4\pi(2L + 1)}} \quad (\ell_1 \ell_2 m_1 m_2 / LM) \; (\ell_1 \ell_2 00/L0)$$

where the latter two factors are the usual Clebsch Gordan coefficients (e.g. Condon and Shortley), the ratio reduces to

$$\frac{Q_{zzz}(\underline{\alpha})}{Q_{zzz}(\underline{r})} = \frac{i}{M} \frac{\int_0^\infty R_{2p} r^4 (dR_{1g}/dr) dr + 5 \int_0^\infty R_{2p} r^3 R_{1g} dr}{\int_0^\infty R_{2p} R_{1g} \, r^5 \, dr}$$

Now an assumption needs to be made about the radial wave functions R_{2p} and R_{1g}. It is felt that most of the error in the final ratio will enter here. For this type of a calculation, however, the spherical harmonic oscillator radial wave functions should give a fair approximation. From Mayer and Jensen (1955),

$$R_{1g} = N_{1g} \, e^{-\gamma r^2} \, r^4$$

$$R_{2p} = N_{2p} \, e^{-\gamma r^2} \, r(1 - (4\gamma/5) \, r^2)$$

The same γ is used for both wave functions. As shown below, this is a safe assumption since the measured nuclear radius which is used to fix γ is the same for both nuclei.

Using these radial functions,

$$\frac{Q_{zzz}(\underline{\alpha})}{Q_{zzz}(\underline{r})} = i \, \frac{2}{3} \, \frac{\gamma}{M}$$

The constant γ is fixed by computing the root mean square radius. We take

$\sqrt{\langle r^2 \rangle_{av}} = (3/5)^{\frac{1}{2}} \, 1.20 \, (87)^{1/3} \; 10^{-13}$ cm. (e.g. Hofstadter (1956))

$$= 1.06 \times 10^{-2} \text{ relativistic units.}$$

From Mayer and Jensen (1955),

$$\langle r^2 \rangle_{n\ell} = (1/2\gamma)\,(\,2(n-1) + \ell + 3/2\,)$$

Averaging over the protons of Rb^{87},

$$\langle r^2 \rangle_{av} = 1.84/\gamma \quad ,$$

or averaging over all the nucleons

$$\langle r^2 \rangle_{av} = 1.96/\gamma$$

For Sr^{87} the respective values are $1.86/\gamma$ and $1.96/\gamma$.

Using the first of these values, $1.84/\gamma$, since the measured radius is from a "charge-dependent" experiment and letting $M = 1837$ we have

$$\frac{Q_{zzz}(\underline{\alpha})}{Q_{zzz}(\underline{r})} \cong 6.0\ i$$

The value $1.96/\gamma$ would give a value for the ratio of 6.4 i.

Yamada (1953) has calculated this matrix ratio using the model-independent method developed by Ahrens and Feenberg (1952). Adjusting his value to correspond to our choice of nuclear radius, one obtains

$$\frac{Q_{zzz}(\underline{\alpha})}{Q_{zzz}(\underline{r})} \cong 2.8\ i$$

This result does agree with ours for its sign which is important in the final analysis. The disagreement of the values is not too unreasonable.

CHAPTER II: CONIC ANALYSIS

1. METHOD OF CONICS

The following method is based upon that of Lee-Whiting (1955). We let

$$x = i \frac{g_V}{g_A} \frac{Q_3(\underline{r})}{Q_3(\underline{\sigma} x \underline{r})}, \quad y = \frac{g_V}{g_A} \frac{Q_3(\underline{\alpha})}{Q_3(\underline{\sigma} x \underline{r})}, \quad u^2 = \frac{|Q_4(\underline{\sigma})|^2}{|Q_3(\underline{\sigma} x \underline{r})|^2}$$

The notation $|Q_n(\underline{a})|^2 = \sum_{ijk\ldots m} |Q_{ij\ldots m}(\underline{a})|^2$

By a time-reversal argument one can show that x and y are real. This means that their squares will be positive. Using the theorem of Longmire and Messiah (1951), we have

$$x^2 = \left(\frac{g_V}{g_A}\right)^2 \frac{|Q_3(\underline{r})|^2}{|Q_3(\underline{\sigma} x \underline{r})|^2}, \quad y^2 = \left(\frac{g_V}{g_A}\right)^2 \frac{|Q_3(\underline{\alpha})|^2}{|Q_3(\underline{\sigma} x \underline{r})|^2}$$

The expression for the shape factor can then be written as

$$\frac{C(q)}{(g_A^2 + g_A'^2)|Q_3(\underline{\sigma} x \underline{r})|^2} = A(q)\, x^2 + B(q)\, y^2 - 2D(q)\, xy + 2G(q)\, y + 2H(q)\, x + F(q) + k(q)\, u^2$$

Let $\rho = C(q)/C(q_o)$. Thus, we have

$$A'\, x^2 + B'\, y^2 - 2D'\, xy + 2G'\, y + 2H'\, x + F' + k'\, u^2 = 0$$

where $A'(q) = A(q) - \rho(q)\, A(q_o)$, etc.

Although the order of magnitude of u^2 is unity, the smallness of its coefficient k' allows the term $k'\, u^2$ to be considered as a correction to the constant F'.

Then, our equation represents a conic in x and y. In principle, an exact solution for x and y could be determined by obtaining the common point of intersection of a few conics for different values of momentum. As will be seen in the next section, uncertainties in the L_ν, M_ν, N_ν functions and the ρ factors require the replacement of the above conics with bands of finite width. The intersection of these bands is employed in our analysis.

Finally, we may again use our shell model assumption here to evaluate the matrix ratio in x. Morita, Fujita, and Yamada (1953) give

$$\frac{Q_3(\underline{r})}{Q_3(\underline{\sigma} x \underline{r})} = i \; .$$

This result is dependent only on the angular part of the wave functions since the radial integrals cancel out. Thus, we get

$$x = - \frac{g_V}{g_A}$$

2. DATA AND ANALYSIS

Using the spectrum obtained by Goodman (1955), Pearson (1956) calculated the normalized shape ρ-factors, normalizing at p = 0.8 (p is the electron's momentum in relativistic units). For a discrete set of values of the electron's momentum, he gives two extreme ρ-values pertaining to his 65% statistical calculations.

The special functions A', B', etc. (coefficients in conic-bands) were calculated employing Pearson's five-place L_ν, M_ν, N_ν. Due to the slight uncertainty in end-point energy the three energies 275, 280 and 285 Kev were considered. It turns out that this effect is not important. The dependence of the L_ν, M_ν, N_ν functions upon the effective radial position R is investigated by considering the three radii 1.13, 1.41, and 1.55 $A^{1/3} \times 10^{-13}$ cm.

The approach to the numerical analysis is apparent from the method of conics already described. For each value of end-point energy and of radial position R, a set of conic-bands are drawn and their common area is found. Each conic-band is the allowed x-y area for a certain value of the electron's momentum. Our allowed values for x and y will lie within the common area of the intersection of these bands.

There remains one point regarding u^2. An attempt was made in a preliminary analysis of the spectrum to include this as a variable and to determine its maximum value, if any. An upper limit was found (as a function of y) but this was larger than the expected order of magnitude. It was decided to test the effect of u^2 by doing the conic analysis for $u^2 = 0$ and 2, and for further values if any large scale effects seemed to be present.

Most of the numerical work for the conic analysis was done on the Bendix G-15 computer here at McMaster.

3. RESULTS OF CONIC ANALYSIS

In the cases studied, the conics which serve as boundary curves for the bands of allowed x and y are all hyperbola of a similar character. For each case, the bands--one for each value of the electron's momentum chosen--differ only by small amounts. Their common area of intersection, when this exists, occurs all along one arm of the hyperbolas defining almost a linear relationship for x and y (e.g. Fig. 1). This will be used in connection with the relationship found in chapter I by the matrix ratio calculation to complete our analysis.

It is found that there is very little dependence of this common area of intersection upon the end-point energy chosen (cf. Fig. 2). The effect of the term containing u^2 is smaller yet, and the term can be neglected.

The influence of the effective radial position R upon the system of conic-bands is appreciable. This is illustrated by the systems for $R = 1.13$ and $1.41\ A^{1/3} \times 10^{-13}$ cm. (cf. Fig. 2 and 3). The structures of the common areas of intersection are similar but their positions are shifted somewhat. The case $R = 1.55\ A^{1/3} \times 10^{-13}$ cm. does not give any allowed areas for either $u^2 = 0$ or 2. This is not unexpected for such a radial position is well out into the mass distribution tail of the nucleus. The beta-decay process is thought to take place more within the nuclear matter. Although the variation of the common area of intersection with the radial position R as indicated by Fig. 2 and 3 produces small changes in the values of x and y obtained, the important elements remain unchanged. This study justifies the statement that our final conclusions (as to sign and approximate values) are independent of reasonable values of the effective radial position R.

Since our conic analysis gives both positive and negative values of x of a reasonable magnitude, it alone is not sufficient to determine the sign. All these values of x also satisfy the necessary requirement of giving a positive shape factor. From the position of the common area of intersection in the x-y plane one sees that a value of y/x should be helpful. Essentially, this has been calculated in chapter I.

4. EFFECT OF FINITE SIZE OF NUCLEUS

Rose and Holmes (1951) have shown that taking into account the finite size of the nucleus the order of magnitude of the corrections to the L_ν, M_ν, N_ν functions can be rather large. The largest correction occurs in the M_ν functions where, for example, the corrected value of M_0 is about $\frac{1}{2}$ the value for a point nucleus for Z = 83 (the value is roughly energy independent).

To determine whether the finite size correction would be important in our result a sample calculation was carried out. Fig. 4 shows two conic-bands for p (electron's momentum) = 0.5, E_0 = 275 Kev. and R = 1.41 $A^{1/3} \times 10^{-13}$ cm., one of which is corrected for the finite size of the nucleus. The correction factors for Z = 38 are approximated from the figures in the Oak Ridge report of Rose and Holmes (1951). Assuming the corrections to be energy-independent for our range of energy, we made the approximations $\Delta L_0 \simeq -0.01$, $\Delta L_1 \simeq -0.0012$,

$\Delta L_2 \simeq -0.0003$, $\Delta L_3 \simeq -0.0001$, $\Delta M_0 \simeq -0.30$, $\Delta M_1 \simeq -0.12$, $\Delta M_2 \simeq -0.08$, $\Delta N_0 \simeq -0.15$, $\Delta N_1 \simeq -0.07$, $\Delta N_2 \simeq -0.04$. The corrected $L_0 = (1 + \Delta L_0)$ x uncorrected L_0, etc. Although the conic-bands were modified a little, the correction is not important here.

CONCLUSION

1. DISCUSSION

From the definitions of x and y and the matrix ratio calculated in chapter I we have $y = 6x$. This line is plotted on Fig. 1 and 3 to show its intersection with the allowed areas of x and y values found in the conic analysis.

In Fig. 1 with $R = 1.41\ A^{1/3}\ 10^{-13}$ cm. the intersection of $y = 6x$ with the allowed area gives for x ($\simeq -g_V/g_A$) values between 0.53 and 0.58. In Fig. 3 with $R = 1.13\ A^{1/3} \times 10^{-13}$cm. the corresponding values are 0.83 and 0.93. From our study it is believed also that intermediate values of R will give intermediate ranges of x values.

Since our matrix calculation assumed a single particle model with harmonic oscillator radial wave functions, we do not wish to insist too strongly on y/x being equal to 6, although it should be recalled that the simplest forms of configuration mixing do not change this value. The x values obtained become smaller but are definitely positive when this y/x value is increased. On decreasing this y/x value, the x values increase and are positive until $y/x \simeq 2$, when large negative values of x are obtained (cf. Fig. 1). Allowing for an error of a factor of 2 in the result $y/x \simeq 6$, we may definitely say that the sign of x ($\simeq -g_V/g_A$) is positive. That is, the sign of the ratio of the coupling constants is negative.

This conclusion is strengthened if we accept $|x| < 1$.

The latter has been found in other beta-decays which are discussed in the next section. To get a negative x for $R = 1.41\ A^{1/3}\ 10^{-13}$ cm., a large negative y/x value is needed. For $R = 1.13\ A^{1/3}\ 10^{-13}$ cm., the conic analysis alone is sufficient for the above conclusion.

2. OTHER EVIDENCE FOR g_V/g_A

Burgy et al. (1958)II have studied the asymmetrical distribution of electrons in the decay of polarized neutrons. This Argonne experiment gives $g_V/g_A = -0.80 \pm 0.03$.

The comparison of the lifetimes of the neutron (Sosnovskij et al. (1958)) and of O^{14} (Gerhart (1958)) gives the absolute value of the ratio, $|g_V/g_A| = 0.84 \pm 0.03$. This assumes that g_V is the same for both beta-decays.

Vlasov and Rudakov (1959) have found that the ratio g_V/g_A is negative in the first forbidden decay of Ba^{139}. Their method of analysis was $\beta-\gamma$ angular correlation.

3. SUMMARY OF RESULTS

The assumption of only vector and axial vector terms in the interaction Hamiltonian allows for a consistent analysis of the observed spectrum.

The approximate absolute value of the ratio found in our analysis for Rb^{87} is in the region of the measured values for other decays. All the values given in the previous section will fit our analysis.

The sign of the ratio of the coupling constants for

Rb^{87} is negative. This agrees with that found for the other decays mentioned above.

Our conic analysis was only slightly sensitive to end-point energy, to neglecting the term containing $Q_4(\underline{\sigma},\underline{r})$, and to the finite size of the nucleus correction. Although the conic-bands were more sensitive to the effective radial position R, our conclusions are valid for reasonable values of R--unreasonable ones give no agreement for any values of x and y (see chapter II, section 3).

Because of the uncertainties in the nuclear matrix elements, one cannot use these results to argue that the value of g_V/g_A is independent of the nucleus. It is however interesting to remark that if we take the average experimental value $g_V/g_A = -0.82$, we find that, for $R = 1.41\ A^{1/3}\ 10^{-13}$ cm., the point on the conic is $y \simeq 3.8$ and for $R = 1.13\ A^{1/3}\ 10^{-13}$ cm., $y \simeq 5.2$. These values correspond to $y/x \simeq 4.7$ and 6.3, in good agreement with the "theoretical" value 6.

CAPTIONS FOR FIGURES

FIG. 1: Intersection of Conic-Bands for $E_o = 275$ Kev, $R = 1.41\ A^{1/3}\ 10^{-13}$ cm., and $u^2 = 0$. The five conic-bands are shown by the line segment pairs offset to the right of the integral values of y to which they refer. Each conic-band corresponds to a value of the electron's momentum p; from left to right p = 0.5, 1.0, 0.6, 0.9 and 0.7.

FIG. 2: Intersection of Conic-Bands for $E_o = 280$ Kev and 275 Kev, $R = 1.41\ A^{1/3}\ 10^{-13}$ cm., and $u^2 = 0$.

FIG. 3: Intersection of Conic-Bands for $E_o = 280$ Kev, $R = 1.13\ A^{1/3}\ 10^{-13}$ cm., and $u^2 = 0$.

FIG. 4: Effect of Finite Size of Nucleus. Two conic-bands are drawn for $E_o = 275$ Kev, $R = 1.41\ A^{1/3}\ 10^{-13}$ cm., $u^2 = 0$, and p = 0.5; one corrected for the finite size of nucleus effect.

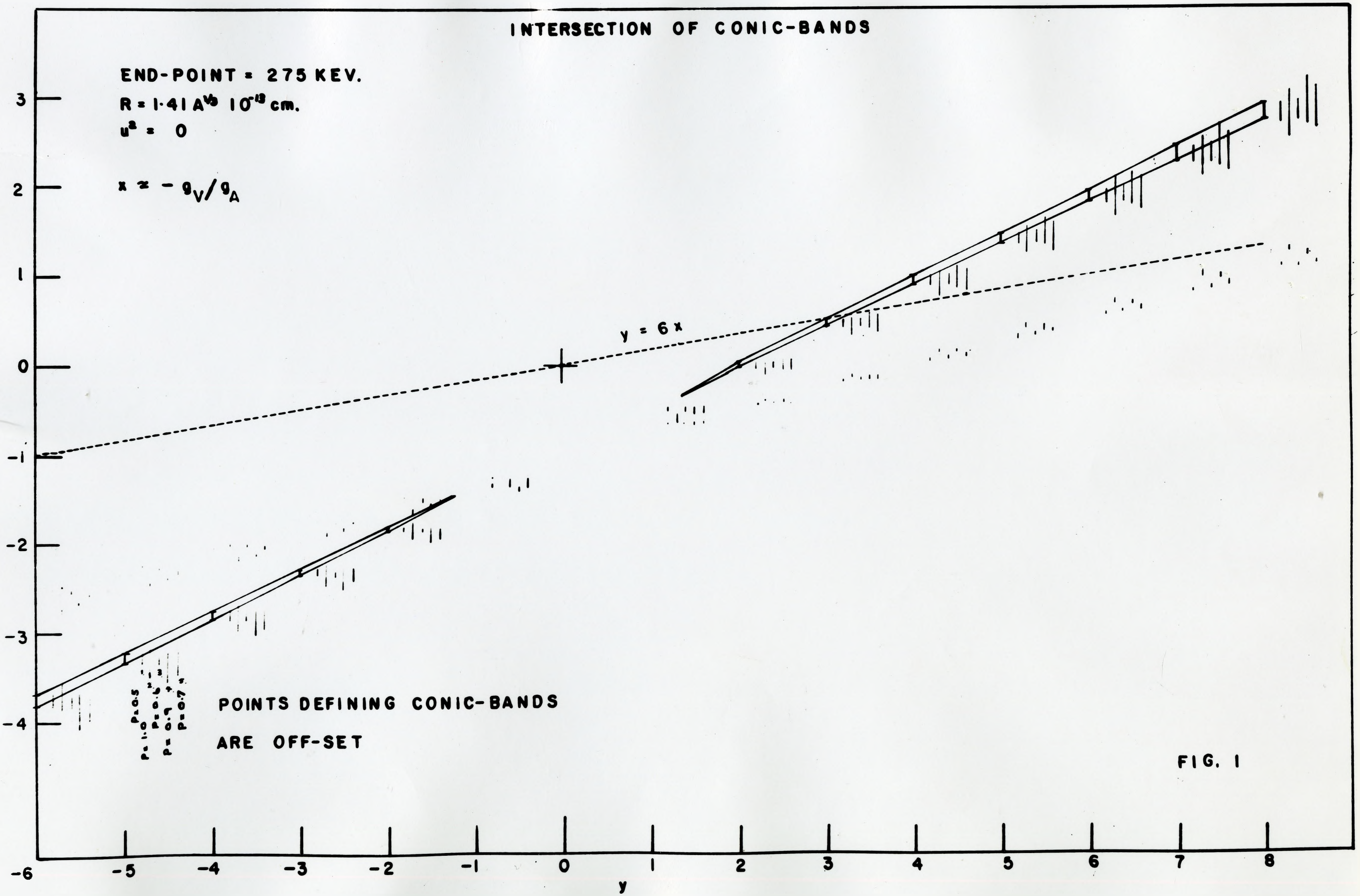

INTERSECTION OF CONIC-BANDS
END-POINT = 275 KEV.
R = 1.41 A^1/3 10^-13 cm.
u² = 0
x = - g_V/g_A
y = 6x
POINTS DEFINING CONIC-BANDS
ARE OFF-SET
x
y
3
2
1
0
-1
-2
-3
-4
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8

FIG. 1

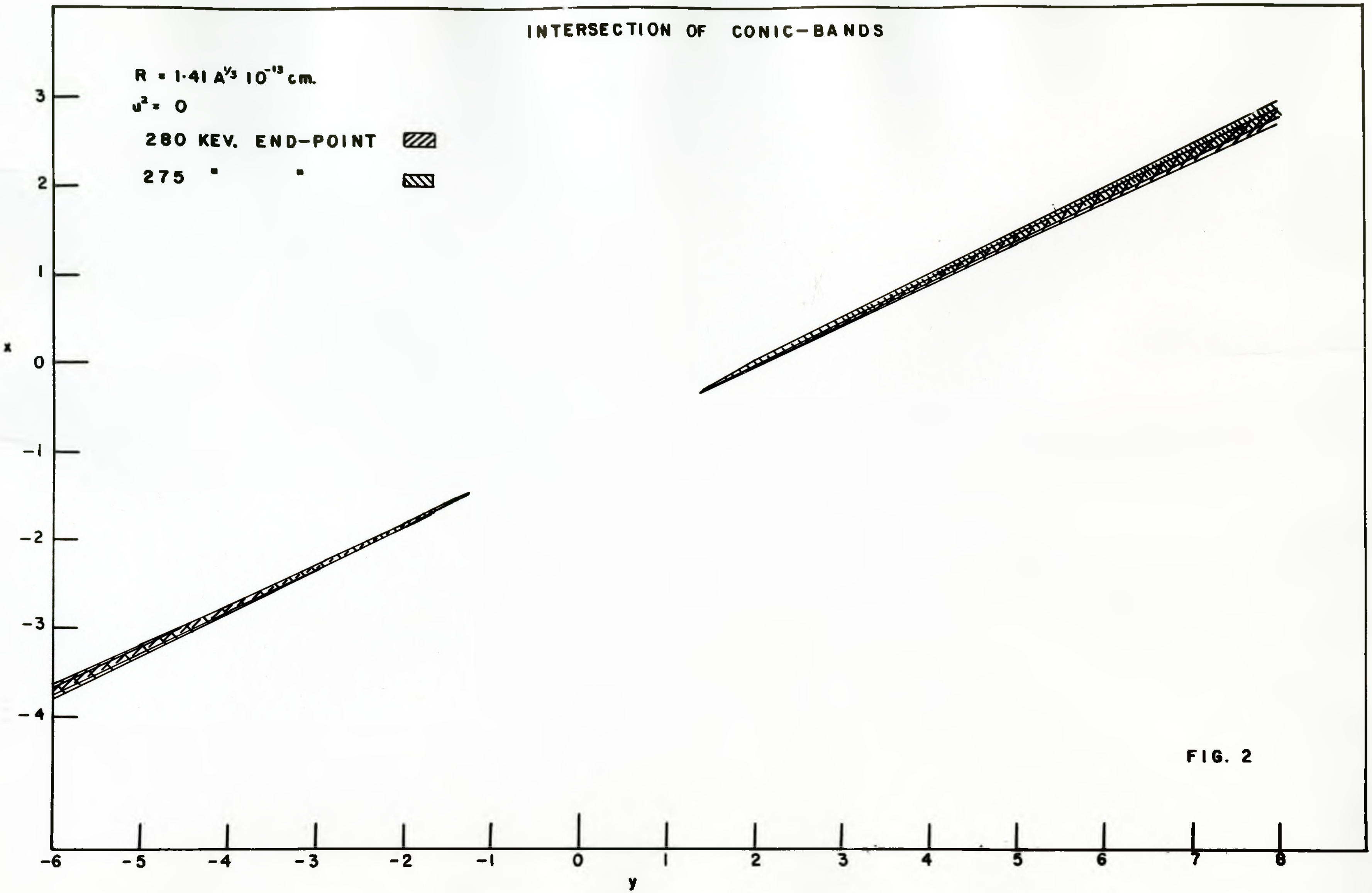

INTERSECTION OF CONIC-BANDS
R = 1·41 A^1/3 10^-13 cm.
u² = 0
280 KEV. END-POINT
275 " "
3
2
1
x
0
-1
-2
-3
-4
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
y

FIG. 2

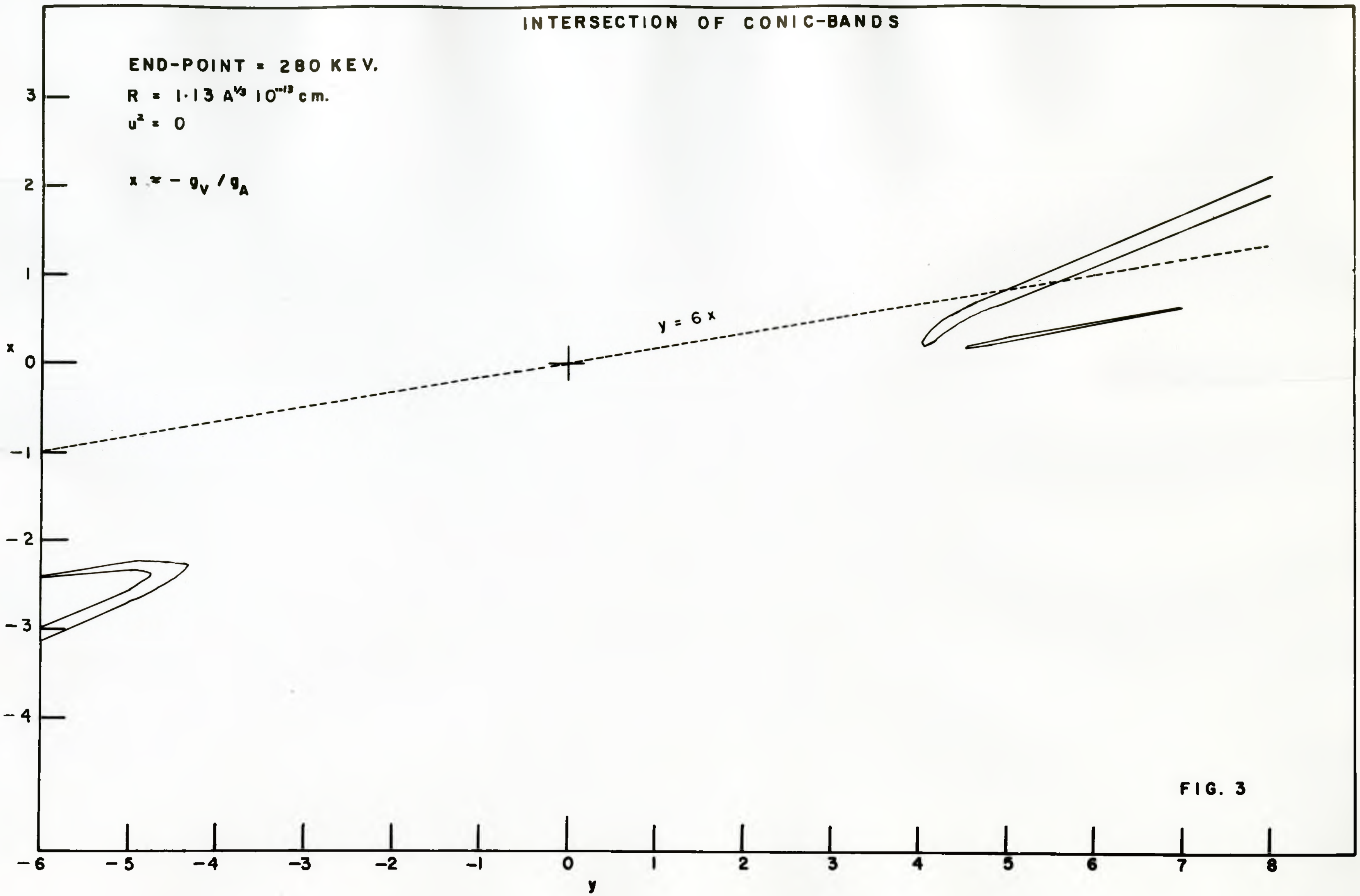
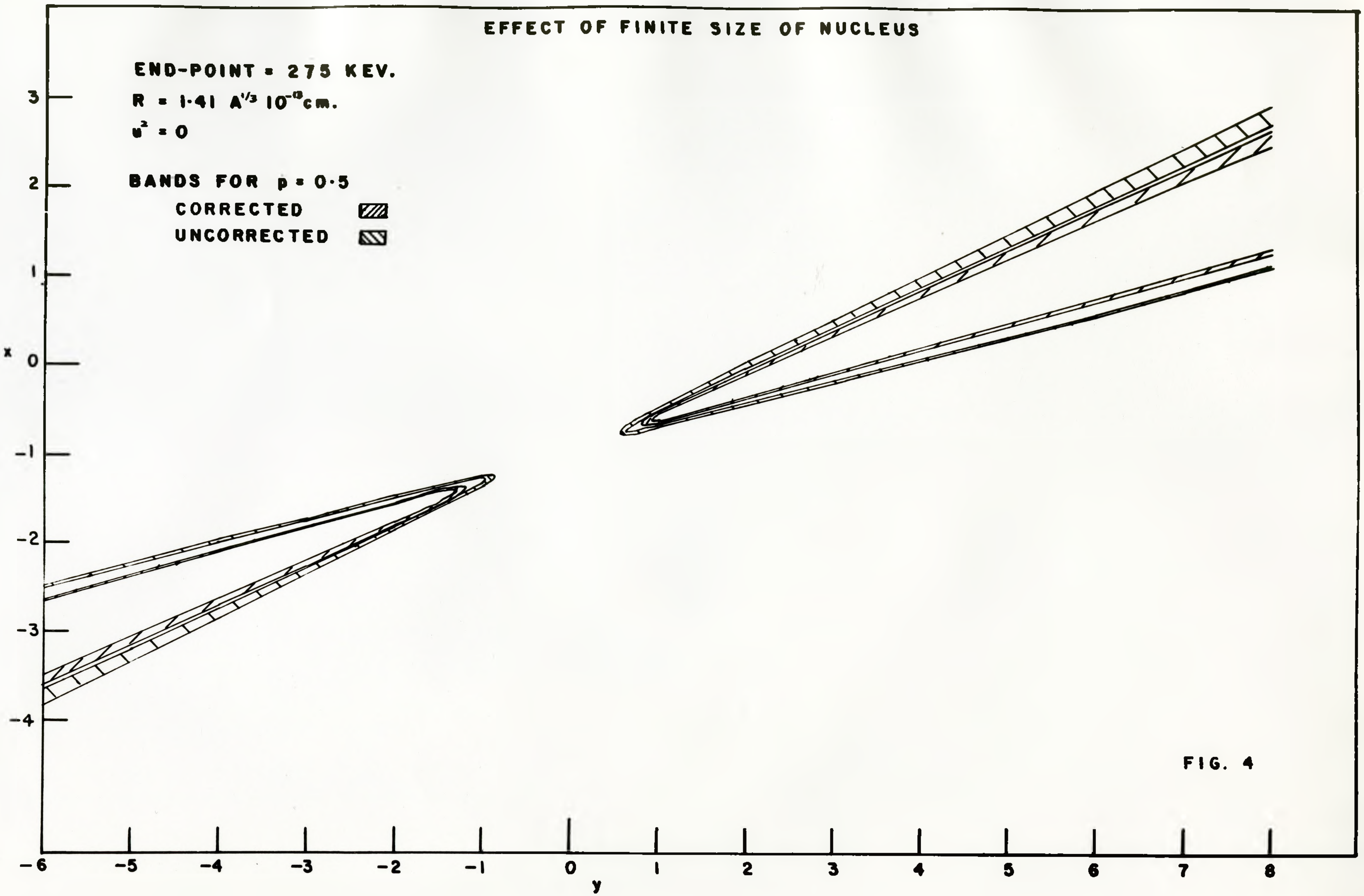

INTERSECTION OF CONIC-BANDS
END-POINT = 280 KEV.
R = 1·13 A^1/3 10^-13 cm.
u² = 0
x = − g_V / g_A
y = 6 x
x
y
3
2
1
0
−1
− 2
−3
− 4
− 6
− 5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8

FIG. 3



EFFECT OF FINITE SIZE OF NUCLEUS
END-POINT = 275 KEV.
R = 1·41 A^1/3 10^-13 cm.
u² = 0
BANDS FOR p = 0·5
CORRECTED
UNCORRECTED
x
y
3
2
1
0
-1
-2
-3
-4
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8

FIG. 4

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