A Thesis Submitted to the School of Graduate Studies in the Partial Fulfillment of the Requirements for the Degree

The Analysis of Asymptotic Rate-Distortion of Symmetric Remote Gaussian Source Coding: A Comparison of Centralized Encoding and Distributed Encoding

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TITLE: The Analysis of Asymptotic Rate-Distortion of Symmetric Remote Gaussian Source Coding: A Comparison of Centralized Encoding and Distributed Encoding

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Lay Abstract

Many applications involve potentially noise-corrupted when collecting and transmitting data. It is generally needed to compress the collected data in order to reduce the transmission cost. In this work, we study a new and innovative way to compress noise corrupted data.

Abstract

Take into account a symmetric multivariate Gaussian source with ℓ components, which are corrupted by independent and identically distributed Gaussian noises; these noisy components are compressed at a certain rate, and the compressed version is leveraged to reconstruct the source subject to a mean squared error distortion constraint. We analyze rate-distortion performance for both centralized encoding (where the noisy source components are jointly compressed) and distributed encoding (where the noisy source components are separately compressed). Among other things, it is indicated that the gap between the rate-distortion functions associated with these two scenarios admits a simple characterization in the large ℓ limit.[1]

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Declaration of Authorship

I, Mengzhen WANG, declare that this thesis titled, " The Analysis of Asymptotic Rate-Distortion of Symmetric Remote Gaussian Source Coding: A Comparison of Centralized Encoding and Distributed Encoding" and the work presented in it are my own. I confirm that:

- validation
- formal analysis
- investigation

Chapter 1

Introduction

1.1 Motivation

Data compression is one of the oldest and most important signal processing questions. A famous historical example is the Morse code, created in 18,38, which gives shorter codes to letters that appear more frequently in English (such as letter 'e' and 't').

The original source information is represented by a sequence of bits. Classically, all the information to be compressed was available in one place, leading to centralized encoding problems. However, with the advent of multimedia, sensor and ad-hoc networks, there is a new problem called distributed encoding. The source information appears at serval separate encoding terminals.

In many of the most source coding scenarios, the encoders do not get to observe directly the information that is of interest to the decoder. Rather, they may observe a noisy function thereof. This occurs, for example, in camera and sensor networks. This such source coding problem called remote source coding. While in the remote compression problem, the encoders only access that information indirectly through a noisy observation process. There is a famous example called CEO problem. A chief executive officer is interested in estimating a random process. M agents observe noisy versions of the random process and have noiseless bit pipes with finite rate to the CEO. Under the assumption that the agents cannot communicate with one another, one wants to analyze the fidelity of the CEO's estimate of the random process subject to these rate constraints.

An interesting insight discussed in this thesis concerns rate distortion performance difference of symmetric remote Gaussian source coding between centralized encoding and distributed encoding.

1.2 Introduction

In this paper, we learn a quadratic Gaussian version of the remote source coding problem, where compression is performed on the noise-corrupted components of a symmetric multivariate Gaussian source.

A prescribed mean squared error distortion constraint is imposed on the reconstruction of the noise-free source components; furthermore, it is pretended that the noises across different source components are independent and obey the same Gaussian distribution. We need to consider both scenarios: centralized encoding (Figure 1.1) versus distributed encoding (Figure 1.2). It is worth noting that the distributed encoding scenario is firmly related to the CEO problem, which has been studied extensively. [1-19].

The present paper is primarily devoted to the analysis of the rate-distortion functions associated with the aforementioned two scenarios in the asymptotic regime where the number of source components, denoted by ℓ, is sufficiently large. Indeed, it will be seen that the gap between the two rate-distortion functions admits a relatively simple characterization in the large ℓ limit, yielding useful insights into the fundamental difference between centralized encoding and distributed coding, which are hard to obtain otherwise.

The rest of this paper is organized as follows. We state the problem definitions and the main results in Chapter 2. The proofs are provided in Chapter 3. We conclude the paper in Chapter 4.

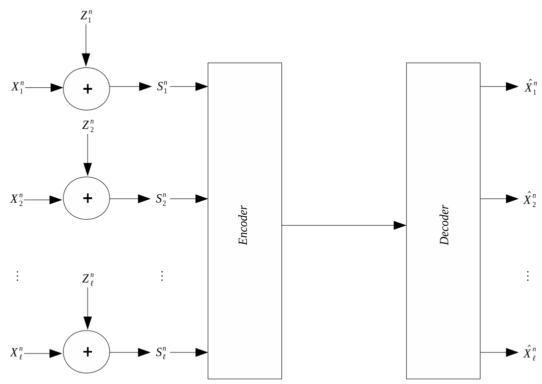

FIGURE 1.1: Symmetric remote Gaussian source coding with centralized encoding

Notation: The expectation operator and the transpose operator are denoted by $\mathbb{E}[\cdot]$ and $(\cdot)^T$, respectively. An ℓ-dimensional all-one row vector is written as 1_ℓ. We use W^n as an abbreviation of $(W(1), \cdots, W(n))$. The cardinality of a set $\mathcal{C}$ is

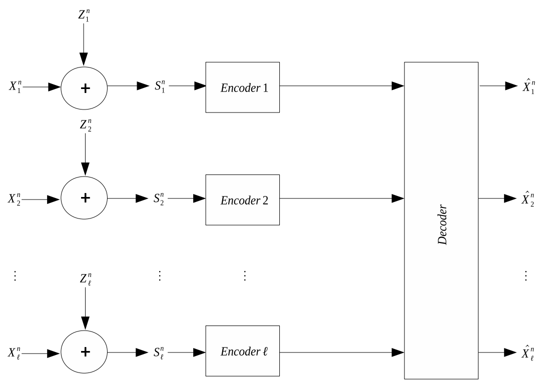

FIGURE 1.2: Symmetric remote Gaussian source coding with distributed encoding

denoted by $|\mathcal{C}|$. We write $g(\ell) = O(f(\ell))$ if the absolute value of $\frac{g(\ell)}{f(\ell)}$ is bounded for all sufficiently large ℓ. Throughout this paper, the base of the logarithm function is e, and $\log^+ x \triangleq \max\{\log x, 0\}$.

Chapter 2

Problem Definitions and Main Results

Let $S \triangleq (S_1, \cdots, S_\ell)^T$ be the sum of two mutually independent ℓ-dimensional ($\ell \geq 2$) zero-mean Gaussian random vectors, source $X \triangleq (X_1, \cdots, X_\ell)^T$ and noise $Z \triangleq (Z_1, \cdots, Z_\ell)^T$, with

$$\mathbb{E}[X_i X_j] = \begin{cases} \gamma_X, & i = j, \\ \rho_X \gamma_X, & i \neq j, \end{cases}$$

$$\mathbb{E}[Z_i Z_j] = \begin{cases} \gamma_Z, & i = j, \\ 0, & i \neq j, \end{cases}$$

where $\gamma_X > 0$, $\rho_X \in [\frac{1}{\ell-1}, 1]$, and $\gamma_Z \geq 0$. Moreover, let $\{(X(t), Z(t), S(t))\}_{t=1}^{\infty}$ be i.i.d. copies of (X, Z, S).

2.1 Definition 1(centralized encoding)

A rate-distortion pair (r, d) is said to be achievable with centralized encoding if, for any $\epsilon > 0$, there exists an encoding function $\phi^{(n)} : \mathbb{R}^{\ell \times n} \to \mathcal{C}^{(n)}$ such that

$$\frac{1}{n} \log |\mathcal{C}^{(n)}| \leq r + \epsilon,$$
$$\frac{1}{\ell n} \sum_{i=1}^{\ell} \sum_{t=1}^{n} \mathbb{E}[(X_i(t) - \hat{X}_i(t))^2] \leq d + \epsilon,$$

where $\hat{X}_i(t) \triangleq \mathbb{E}[X_i(t) | (\phi^{(n)}(S^n)]$. For a given d, we denote by $\underline{r}(d)$ the minimum r such that (r, d) is achievable with centralized encoding.

2.2 Definition 2(Distributed encoding)

A rate-distortion pair (r, d) is said to be achievable with distributed encoding if, for any $\epsilon > 0$, there exist encoding functions $\phi_i^{(n)} : \mathbb{R}^n \to \mathcal{C}_i^{(n)}$, $i = 1, \cdots, \ell$, such that

$$\frac{1}{n} \sum_{i=1}^{\ell} \log |\mathcal{C}_i^{(n)}| \leq r + \epsilon,$$
$$\frac{1}{\ell n} \sum_{i=1}^{\ell} \sum_{t=1}^{n} \mathbb{E}[(X_i(t) - \hat{X}_i(t))^2] \leq d + \epsilon,$$

where $\hat{X}_i(t) \triangleq \mathbb{E}[X_i(t) | (\phi_1^{(n)}(S_1^n), \cdots, \phi_\ell^{(n)}(S_\ell^n))]$. For a given d, we denote by $\overline{r}(d)$ the minimum r such that (r, d) is achievable with distributed encoding.

Suppose we want to transmit information about a source to the user with a distortion not exceeding d. Rate–distortion theory tells us that at least r(d) bits/symbol of information from the source must reach the user. We will refer to $\underline{r}(d)$ as the rate-distortion function of symmetric remote Gaussian source coding with centralized encoding, and $\overline{r}(d)$ as the rate-distortion function of symmetric remote Gaussian source coding with distributed encoding. It is clear that $\underline{r}(d) \leq \overline{r}(d)$ for any d since distributed encoding can be simulated by centralized encoding. Moreover, it is easy to show that $\underline{r}(d) = \overline{r}(d) = 0$ for $d \geq \gamma_X$ (since the distortion constraint is trivially satisfied with the reconstruction set to be zero) and $\underline{r}(d) = \overline{r}(d) = \infty$ for $d \leq d_{\min}$ (since $d_{\min}$ is the minimum achievable distortion when $\{S(t)\}_{t=1}^{\infty}$ is directly available at the decoder), where (see Chapter 3.1 for a detailed derivation)

$$d_{\min} \triangleq \frac{1}{\ell}\mathbb{E}[(X-\mathbb{E}[X|S])^T(X-\mathbb{E}[X|S])] = \begin{cases} \frac{(\ell-1)\gamma_X\gamma_Z}{\ell\gamma_X+(\ell-1)\gamma_Z}, & \rho_X = -\frac{1}{\ell-1}, \\ \frac{(\ell\rho_X\gamma_X+\lambda_X)\gamma_Z}{\ell(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)} + \frac{(\ell-1)\lambda_X\gamma_Z}{\ell(\lambda_X+\gamma_Z)}, & \rho_X \in (-\frac{1}{\ell-1}, 1), \\ \frac{\gamma_X\gamma_Z}{\ell\gamma_X+\gamma_Z}, & \rho_X = 1, \end{cases}$$

with $\lambda_X \triangleq (1-\rho_X)\gamma_X$. Henceforth we shall focus on the case $d \in (d_{\min}, \gamma_X)$.

2.3 Lemma 1

For $d \in (d_{\min}, \gamma_X)$,

$$\underline{r}(d) = \begin{cases} \frac{\ell-1}{2}\log\frac{\ell(\ell-1)\gamma_X^2}{(\ell\gamma_X+(\ell-1)\gamma_Z)((\ell-1)d-\gamma_X)}, & \rho_X = -\frac{1}{\ell-1}, \\ \frac{1}{2}\log^+\frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)\xi} + \frac{\ell-1}{2}\log^+\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)\xi}, & \rho_X \in (-\frac{1}{\ell-1}, 1), \\ \frac{1}{2}\log\frac{\ell\gamma_X^2}{(\ell\gamma_X+\gamma_Z)d-\gamma_X\gamma_Z}, & \rho_X = 1, \end{cases}$$

where

$$\xi \triangleq \begin{cases} d - d_{\min}, & d \le \min\{\frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{\ell\rho_X\gamma_X+\lambda_X+\gamma_Z}, \frac{\lambda_X^2}{\lambda_X+\gamma_Z}\} + d_{\min}, \\ \frac{\ell(d-d_{\min})}{\ell-1} - \frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{(\ell-1)(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)}, & d > \frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{\ell\rho_X\gamma_X+\lambda_X+\gamma_Z} + d_{\min}, \\ \ell(d-d_{\min}) - \frac{(\ell-1)\lambda_X^2}{\lambda_X+\gamma_Z}, & d > \frac{\lambda_X^2}{\lambda_X+\gamma_Z} + d_{\min}. \end{cases}$$

See Chapter 3.1.

The following result can be deduced from ([20] Theorem 1) (see also [12,16]).

2.4 Lemma 2

For $d \in (d_{\min}, \gamma_X)$,

$$\overline{r}(d) = \frac{1}{2}\log\frac{\ell\rho_X\gamma_X + \lambda_X + \gamma_Z + \lambda_Q}{\lambda_Q} + \frac{\ell-1}{2}\log\frac{\lambda_X + \gamma_Z + \lambda_Q}{\lambda_Q},$$

where

$$\lambda_Q \triangleq \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

with

$$a \triangleq \ell(\gamma_X - d),$$

$$b \triangleq (\ell\rho_X\gamma_X + \lambda_X)(\lambda_X + 2\gamma_Z) + (\ell - 1)\lambda_X(\ell\rho_X\gamma_X + \lambda_X + 2\gamma_Z) - \ell(\ell\rho_X\gamma_X + 2\lambda_X + 2\gamma_Z)d,$$

$$c \triangleq \ell(\ell\rho_X\gamma_X + \lambda_X + \gamma_Z)(\lambda_X + \gamma_Z)(d_{\min} - d).$$

The expressions of $\underline{r}(d)$ and $\overline{r}(d)$ as shown in Lemma 1 and Lemma 2 are quite complicated, rendering it difficult to make analytical comparisons. Fortunately, they become significantly simplified in the asymptotic regime where $\ell \to \infty$ (with d fixed). To perform this asymptotic analysis, it is necessary to restrict attention to the case $\rho_X \in [0, 1]$; moreover, without loss of generality, we assume $d \in (d_{\min}^{(\infty)}, \gamma_X)$, where

$$d_{\min}^{(\infty)} \triangleq \lim_{\ell\to\infty} d_{\min} = \begin{cases} \frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z}, & \rho_X \in [0, 1), \\ 0, & \rho_X = 1. \end{cases}$$

2.5 Theorem 1 (Centralized encoding)

1. $\rho_X = 0$: For $d \in (d_{\min}^{(\infty)}, \gamma_X)$,

$$\underline{r}(d) = \frac{\ell}{2}\log\frac{\gamma_X^2}{(\gamma_X + \gamma_Z)d - \gamma_X\gamma_Z}.$$

2. $\rho_X \in (0,1]$: For $d \in (d_{\min}^{(\infty)}, \gamma_X)$,

$$\underline{r}(d) = \begin{cases} \frac{\ell}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z} + \frac{1}{2}\log\ell + \underline{\alpha} + O(\frac{1}{\ell}), & d < \lambda_X, \\ \frac{1}{2}\log\ell + \frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X+\gamma_Z)}{\lambda_X^2} + \frac{\gamma_Z^2}{2\lambda_X^2} + O(\frac{1}{\ell}), & d = \lambda_X, \\ \frac{1}{2}\log\frac{\rho_X\gamma_X}{d-\lambda_X} + O(\frac{1}{\ell}), & d > \lambda_X, \end{cases}$$

where

$$\underline{\alpha} \triangleq \frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X+\gamma_Z)}{\lambda_X^2} + \frac{\gamma_Z^2}{2((\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z)}.$$

See Chapter 3.2.

2.6 Theorem 2 (Distributed encoding)

1. $\rho_X = 0$: For $d \in (d_{\min}^{(\infty)}, \gamma_X)$,

$$\overline{r}(d) = \frac{\ell}{2}\log\frac{\gamma_X^2}{(\gamma_X+\gamma_Z)d-\gamma_X\gamma_Z}.$$

2. $\rho_X \in (0,1]$: For $d \in (d_{\min}^{(\infty)}, \gamma_X)$,

$$\overline{r}(d) = \begin{cases} \frac{\ell}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z} + \frac{1}{2}\log\ell + \overline{\alpha} + O(\frac{1}{\ell}), & d < \lambda_X, \\ \frac{(\lambda_X+\gamma_Z)\sqrt{\ell}}{2\lambda_X} + \frac{1}{4}\log\ell + \frac{1}{2}\log\frac{\rho_X}{1-\rho_X} - \frac{(\lambda_X+\gamma_Z)(\lambda_X-\rho_X\gamma_Z)}{4\rho_X\lambda_X^2} + O(\frac{1}{\sqrt{\ell}}), & d = \lambda_X, \\ \frac{1}{2}\log\frac{\rho_X\gamma_X}{d-\lambda_X} + \frac{(\lambda_X+\gamma_Z)(\gamma_X-d)}{2\rho_X\gamma_X(d-\lambda_X)} + O(\frac{1}{\ell}), & d > \lambda_X, \end{cases}$$

where

$$\overline{\alpha} \triangleq \frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X-d)}{\lambda_X^2}+\frac{(\lambda_X+\gamma_Z)d^2}{2(\lambda_X-d)((\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z)}.$$

See Chapter 3.3.

2.7 Remark 1

One can readily recover ([21] Theorem 3) for the case $m=1$ (see **[21]** for the definition of parameter m) and Oohama's celebrated result for the quadratic Gaussian CEO problem ([4] Corollary 1) by setting $\gamma_Z=0$ and $\rho_X=1$, respectively, in Theorem 2.

The following result is a simple corollary of Theorem 1 and Theorem 2.

Corollary 1(Asymptotic gap)

1. $\rho_X=0$: For $d\in(d_{\min}^{(\infty)},\gamma_X)$,

$$\overline{r}(d)-\underline{r}(d)=0.$$

2. $\rho_X\in(0,1]$: For $d\in(d_{\min}^{(\infty)},\gamma_X)$,

$$\lim_{\ell\to\infty}\overline{r}(d)-\underline{r}(d)=\psi(d)\triangleq\begin{cases}\frac{1}{2}\log\frac{\lambda_X-d}{\lambda_X+\gamma_Z}+\frac{\gamma_Z+d}{2(\lambda_X-d)}, & d<\lambda_X,\\ \infty, & d=\lambda_X,\\ \frac{(\lambda_X+\gamma_Z)(\gamma_X-d)}{2\rho_X\gamma_X(d-\lambda_X)}, & d>\lambda_X.\end{cases}$$

2.8 Remark 2

When $\rho_X = 0$, we have $\psi(d) = \frac{\gamma_Z(\gamma_X - d)}{2\gamma_X d}$, which is a monotonically decreasing function over $(0, \gamma_X)$, converging to ∞ (here we assume $\gamma_Z > 0$) and 0 as $d \to 0$ and γ_X, respectively. When $\rho_X \in (0,1)$, it is clear that the function $\psi(d)$ is monotonically decreasing over (λ_X, γ_X), converging to ∞ and 0 as $d \to \lambda_X$ and γ_X, respectively; moreover, since $\psi'(d) = \frac{\gamma_Z + d}{2(\lambda_X - d)^2} > 0$ for $d \in (d_{\min}^{(\infty)}, \lambda_X)$, the function $\psi(d)$ is monotonically increasing over $(d_{\min}^{(\infty)}, \lambda_X)$, converging to $\tau(\gamma_Z) \triangleq \frac{1}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)^2} + \frac{2\lambda_X\gamma_Z+\gamma_Z^2}{2\lambda_X^2}$ and ∞ as $d \to d_{\min}^{(\infty)}$ and λ_X, respectively. Note that $\tau'(\gamma_Z) = \frac{2\lambda_X\gamma_Z+\gamma_Z^2}{\lambda_X^2(\lambda_X+\gamma_Z)} \geq 0$ for $\gamma_Z \in [0,\infty)$; therefore, the minimum value of $\tau(\gamma_Z)$ over $[0,\infty)$ is 0, which is attained at $\gamma_Z = 0$. See Figures 2.1 and 2.2 for some graphical illustrations of $\psi(d)$.

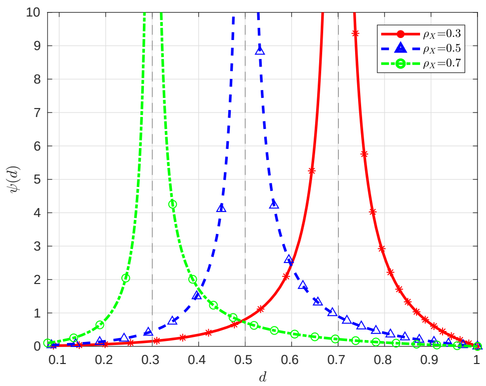

FIGURE 2.1: Illustration of $\psi(d)$ with $\gamma_X = 1$ and $\gamma_Z = 0.1$ for different ρ_X.

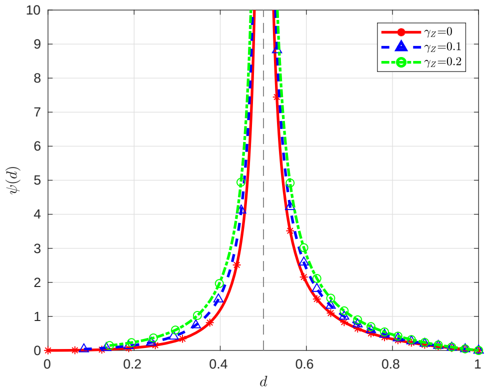

FIGURE 2.2: Illustration of $\psi(d)$ with $\gamma_X = 1$ and $\rho_X = 0.5$ for different γ_Z.

Chapter 3

Proofs

3.1 Proof of Lemma 1

It is known [22] that $\underline{r}(d)$ is given by the solution to the following optimization problem:

$$(\mathbf{P}_1) \quad \min_{p_{\hat{X}|S}} I(S;\hat{X}) \tag{3.1}$$

$$\text{subject to} \quad \mathbb{E}[(X-\hat{X})^T(X-\hat{X})] \leq \ell d, \tag{3.2}$$

$$X \leftrightarrow S \leftrightarrow \hat{X} \text{ form a Markov chain.} \tag{3.3}$$

Let $\tilde{X} \triangleq \Theta X$, $\tilde{Z} \triangleq \Theta Z$, and $\tilde{S} \triangleq \Theta S$, where Θ is an arbitrary (real) unitary matrix with the first row being $\frac{1}{\sqrt{\ell}} 1_\ell$. Since unitary transformations are invert-able and preserve the Euclidean norm, we can write $(\mathbf{P}_1)$ equivalently as

$$
\begin{aligned}
(\mathbf{P}_2) \quad & \min_{p_{\hat{X}|\tilde{S}}} I(\tilde{S};\hat{X}) \\
\text{subject to} \quad & \mathbb{E}[(\tilde{X}-\hat{X})^T(\tilde{X}-\hat{X})] \leq \ell d, \\
& \tilde{X} \leftrightarrow \tilde{S} \leftrightarrow \hat{X} \text{ form a Markov chain.}
\end{aligned}
$$

For the same reason, we have

$$\ell d_{\min} = \mathbb{E}[(\tilde{X}-\mathbb{E}[\tilde{X}|\tilde{S}])^T(\tilde{X}-\mathbb{E}[\tilde{X}|\tilde{S}])]. \tag{3.4}$$

Denote the i-th components of $\tilde{X}$, $\tilde{Z}$, and $\tilde{S}$ by $\tilde{X}_i$, $\tilde{Z}_i$, and $\tilde{S}_i$, respectively, $i = 1, \cdots, \ell$. Clearly, $\tilde{S}_i = \tilde{X}_i + \tilde{Z}_i$, $i = 1, \cdots, \ell$. Moreover, it can be verified that $\tilde{X}_1, \cdots, \tilde{X}_\ell, \tilde{Z}_1, \cdots, \tilde{Z}_\ell$ are independent zero-mean Gaussian random variables with

$$\mathbb{E}[(\tilde{X}_1)^2] = \ell\rho_X\gamma_X + \lambda_X, \tag{3.5}$$

$$\mathbb{E}[(\tilde{X}_i)^2] = \lambda_X, \quad i = 2, \cdots, \ell, \tag{3.6}$$

$$\mathbb{E}[(\tilde{Z}_1)^2] = \gamma_Z, \quad i = 1, \cdots, \ell.$$

Now denote the i-th component of $\hat{S} \triangleq \mathbb{E}[\tilde{X}|\tilde{S}]$ by $\hat{S}_i$, $i = 1, \cdots, \ell$. We have

$$\hat{S}_i = \mathbb{E}[\tilde{X}_i|\tilde{S}_i], \quad i = 1, \cdots, \ell,$$

and

$$\mathbb{E}[(\hat{S}_1)^2] = \begin{cases} 0, & \rho_X = -\frac{1}{\ell-1}, \\ \frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{\ell\rho_X\gamma_X+\lambda_X+\gamma_Z}, & \rho_X \in (-\frac{1}{\ell-1}, 1], \end{cases} \tag{3.7}$$

$$\mathbb{E}[(\hat{S}_i)^2] = \begin{cases} \frac{\lambda_X^2}{\lambda_X+\gamma_Z}, & \rho \in [-\frac{1}{\ell-1}, 1), \\ 0, & \rho_X = 1, \end{cases} \quad i = 2, \cdots, \ell. \tag{3.8}$$

Note that

$$\mathbb{E}[(\tilde{X}-\hat{S})^T(\tilde{X}-\hat{S})] = \sum_{i=1}^{\ell}\mathbb{E}[(\tilde{X}_i)^2] - \sum_{i=1}^{\ell}\mathbb{E}[(\hat{S}_i)^2],$$

which, together with (3.4)–(3.8), proves

$$d_{\min} = \frac{1}{\ell}\mathbb{E}[(\tilde{X}-\hat{S})^T(\tilde{X}-\hat{S})] = \begin{cases} \frac{(\ell-1)\gamma_X\gamma_Z}{\ell\gamma_X+(\ell-1)\gamma_Z}, & \rho_X = -\frac{1}{\ell-1}, \\ \frac{(\ell\rho_X\gamma_X+\lambda_X)\gamma_Z}{\ell(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)} + \frac{(\ell-1)\lambda_X\gamma_Z}{\ell(\lambda_X+\gamma_Z)}, & \rho_X \in (-\frac{1}{\ell-1}, 1), \\ \frac{\gamma_X\gamma_Z}{\ell\gamma_X+\gamma_Z}, & \rho_X = 1. \end{cases}$$

Clearly, $\hat{S}$ is determined by $\tilde{S}$; moreover, for any ℓ-dimensional random vector $\hat{X}$ jointly distributed with $(\tilde{X}, \tilde{S})$ such that $\tilde{X} \leftrightarrow \tilde{S} \leftrightarrow \hat{X}$ form a Markov chain, we have

$$\begin{aligned} \mathbb{E}[(\tilde{X}-\hat{X})^T(\tilde{X}-\hat{X})] &= \mathbb{E}[(\hat{S}-\hat{X})^T(\hat{S}-\hat{X})^2] + \mathbb{E}[(\tilde{X}-\hat{S})^T(\tilde{X}-\hat{S})^2] \\ &= \mathbb{E}[(\hat{S}-\hat{X})^T(\hat{S}-\hat{X})^2] + \ell d_{\min}. \end{aligned}$$

Therefore, $(\mathbf{P}_2)$ is equivalent to

$$(\mathbf{P}_3) \quad \min_{p_{\hat{X}|\hat{S}}} I(\hat{S};\hat{X})$$
$$\text{subject to} \quad \mathbb{E}[(\hat{S}-\hat{X})^T(\hat{S}-\hat{X})] \leq \ell(d-d_{\min}).$$

One can readily complete the proof of Lemma 1 by recognizing that the solution to $(\mathbf{P}_3)$ is given by the well-known reverse water-filling formula ([23] Theorem 13.3.3).

3.2 Proof of Theorem 1

Setting $\rho_X = 0$ in Lemma 1 gives

$$\underline{r}(d) = \frac{\ell}{2}\log\frac{\gamma_X^2}{(\gamma_X+\gamma_Z)d-\gamma_X\gamma_Z}$$

for $d \in (\frac{\gamma_X\gamma_Z}{\gamma_X+\gamma_Z}, \gamma_X)$. Setting $\rho_X = 1$ in Lemma 1 gives

$$\underline{r}(d) = \frac{1}{2}\log\frac{\ell^2\gamma_X^2}{\ell(\ell\gamma_X+\gamma_Z)d-\gamma_X\gamma_Z}$$

for $d \in (\frac{\gamma_X\gamma_Z}{\ell\gamma_X+\gamma_Z}, \gamma_X)$; moreover, we have

$$\frac{1}{2}\log\frac{\ell^2\gamma_X^2}{\ell(\ell\gamma_X+\gamma_Z)d-\gamma_X\gamma_Z} = \frac{1}{2}\log\frac{\gamma_X}{d} + O(\frac{1}{\ell}),$$

and $\frac{\gamma_X\gamma_Z}{\ell\gamma_X+\gamma_Z} \to 0$ as $\ell \to \infty$.

It remains to treat the case $\rho_X \in (0,1)$. In this case, it can be deduced from Lemma 1 that

$$\underline{r}(d) = \begin{cases} \frac{1}{2}\log\frac{(\ell\rho_X\gamma_X+\lambda_X)^2(\lambda_X+\gamma_Z)}{\lambda_X^2(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)} + \frac{\ell}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)(d-d_{\min})}, & d \in (d_{\min}, \frac{\lambda_X^2}{\lambda_X+\gamma_Z}+d_{\min}], \\ \frac{1}{2}\log\frac{(\ell\rho_X\gamma_X+\lambda_X)^2(\lambda_X+\gamma_Z)}{(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)(\ell(\lambda_X+\gamma_Z)(d-d_{\min})-(\ell-1)\lambda_X^2)}, & d \in (\frac{\lambda_X^2}{\lambda_X+\gamma_Z}+d_{\min}, \gamma_X), \end{cases}$$

and we have

$$\begin{aligned} d_{\min} &= \frac{(\ell\rho_X\gamma_X+\lambda_X)\gamma_Z}{\ell(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)} + \frac{(\ell-1)\lambda_X\gamma_Z}{\ell(\lambda_X+\gamma_Z)} \\ &= \frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z} + \frac{\rho_X\gamma_X\gamma_Z^2}{(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)(\lambda_X+\gamma_Z)} \qquad (3.9) \\ &= \frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z} + \frac{\gamma_Z^2}{(\lambda_X+\gamma_Z)\ell} + O(\frac{1}{\ell^2}). \qquad (3.10) \end{aligned}$$

Consider the following two subcases separately.

- $d \in (\frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z}, \lambda_X]$

  It can be seen from (3.9) that $d_{\min}$ is a monotonically decreasing function of ℓ and converges to $\frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z}$ as $\ell \to \infty$. Therefore, we have $d \in (d_{\min}, \frac{\lambda_X^2}{\lambda_X+\gamma_Z} + d_{\min}]$ and consequently

  $$\underline{r}(d) = \frac{1}{2}\log\frac{(\ell\rho_X\gamma_X+\lambda_X)^2(\lambda_X+\gamma_Z)}{\lambda_X^2(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)} + \frac{\ell}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)(d-d_{\min})}, \qquad (3.11)$$

  when ℓ is sufficiently large. Note that

  $$\frac{1}{2}\log\frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{\ell\rho_X\gamma_X+\lambda_X+\gamma_Z} = \frac{1}{2}\log\ell + \frac{1}{2}\log(\rho_X\gamma_X) + O(\frac{1}{\ell}) \qquad (3.12)$$

and

$$\frac{1}{2}\log(d-d_{\min}) = \frac{1}{2}\log\left(d - \frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z} - \frac{\gamma_Z^2}{(\lambda_X+\gamma_Z)\ell} - O(\frac{1}{\ell^2})\right) \tag{3.13}$$

$$= \frac{1}{2}\log\frac{(\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z}{\lambda_X+\gamma_Z} - \frac{\gamma_Z^2}{2((\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z)\ell} + O(\frac{1}{\ell^2}), \tag{3.14}$$

where (3.13) is due to (3.10). Substituting (3.12) and (3.14) into (3.11) gives

$$\underline{r}(d) = \frac{\ell}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z} + \frac{1}{2}\log\ell + \frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X+\gamma_Z)}{\lambda_X^2} + \frac{\gamma_Z^2}{2((\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z)} + O(\frac{1}{\ell}).$$

In particular, we have

$$\underline{r}(\lambda_X) = \frac{1}{2}\log\ell + \frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X+\gamma_Z)}{\lambda_X^2} + \frac{\gamma_Z^2}{2\lambda_X^2} + O(\frac{1}{\ell}).$$

- $d \in (\lambda_X, \gamma_X)$

  Since $d_{\min}$ converges to $\frac{\lambda_X\gamma_Z}{\lambda_X+\gamma_Z}$ as $\ell \to \infty$, it follows that $d \in (\frac{\lambda_X^2}{\lambda_X+\gamma_Z}+d_{\min}, \gamma_X)$ and consequently

$$\underline{r}(d) = \frac{1}{2}\log\frac{(\ell\rho_X\gamma_X+\lambda_X)^2(\lambda_X+\gamma_Z)}{(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)(\ell(\lambda_X+\gamma_Z)(d-d_{\min})-(\ell-1)\lambda_X^2)} \tag{3.15}$$

when ℓ is sufficiently large. One can readily verify that

$$\begin{aligned}&\frac{1}{2}\log\frac{(\ell\rho_X\gamma_X+\lambda_X)^2}{(\ell\rho_X\gamma_X+\lambda_X+\gamma_Z)(\ell(\lambda_X+\gamma_Z)(d-d_{\min})-(\ell-1)\lambda_X^2)}\\&=\frac{1}{2}\log\frac{\rho_X\gamma_X}{(\lambda_X+\gamma_Z)(d-\lambda_X)}+O(\frac{1}{\ell}).\end{aligned} \tag{3.16}$$

Substituting (3.16) into (3.15) gives

$$\underline{r}(d)=\frac{1}{2}\log\frac{\rho_X\gamma_X}{d-\lambda_X}+O(\frac{1}{\ell}).$$

This completes the proof of Theorem 1.

3.3 Proof of Theorem 2

One can readily prove part one of Theorem 2 by setting $\rho_X=0$ in Lemma 2. So only part two of Theorem 2 remains to be proved. Note that

$$b=g_1\ell^2+g_2\ell,$$
$$c=h_1\ell^2+h_2\ell,$$

where

$$g_1 \triangleq \rho_X \gamma_X (\lambda_X - d),$$
$$g_2 \triangleq \lambda_X^2 + 2\gamma_X\gamma_Z - 2(\lambda_X + \gamma_Z)d,$$
$$h_1 \triangleq \rho_X \gamma_X (\lambda_X + \gamma_Z)(d_{\min}^{(\infty)} - d),$$
$$h_2 \triangleq \rho_X \gamma_X \gamma_Z^2 + \lambda_X \gamma_Z (\lambda_X + \gamma_Z) - (\lambda_X + \gamma_Z)^2 d.$$

We shall consider the following three cases separately.

- $d < \lambda_X$

  In this case $g_1 > 0$ and consequently

$$\lambda_Q = \frac{-b + b\sqrt{1 - \frac{4ac}{b^2}}}{2a} \tag{3.17}$$

  when ℓ is sufficiently large. Note that

$$\sqrt{1 - \frac{4ac}{b^2}} = 1 - \frac{2ac}{b^2} - \frac{2a^2c^2}{b^4} + O(\frac{1}{\ell^3}). \tag{3.18}$$

  Substituting (3.18) into (3.17) gives

$$\lambda_Q = -\frac{c}{b} - \frac{ac^2}{b^3} + O(\frac{1}{\ell^2}). \tag{3.19}$$

It is easy to show that

$$-\frac{c}{b} = -\frac{h_1}{g_1} - \frac{g_1 h_2 - g_2 h_1}{g_1^2 \ell} + O(\frac{1}{\ell^2}), \tag{3.20}$$

$$-\frac{ac^2}{b^3} = -\frac{(\gamma_X - d)h_1^2}{g_1^3 \ell} + O(\frac{1}{\ell^2}). \tag{3.21}$$

Combining (3.19), (3.20) and (3.21) yields

$$\lambda_Q = \eta_1 + \frac{\eta_2}{\ell} + O(\frac{1}{\ell^2}),$$

where

$$\eta_1 \triangleq -\frac{h_1}{g_1},$$

$$\eta_2 \triangleq -\frac{g_1^2 h_2 - g_1 g_2 h_1 + (\gamma_X - d)h_1^2}{g_1^3}.$$

Moreover, it can be verified via algebraic manipulations that

$$\eta_1 = \frac{(\lambda_X + \gamma_Z)d - \lambda_X \gamma_Z}{\lambda_X - d},$$

$$\eta_2 = -\frac{\lambda_X^2 d^2}{(\lambda_X - d)^3}.$$

Now we write $\overline{r}(d)$ equivalently as

$$\overline{r}(d) = \frac{1}{2}\log\frac{\ell\rho_X\gamma_X + \lambda_X + \gamma_Z + \lambda_Q}{\lambda_X + \gamma_Z + \lambda_Q} + \frac{\ell}{2}\log\frac{\lambda_X + \gamma_Z + \lambda_Q}{\lambda_Q}. \tag{3.22}$$

Note that

$$\frac{1}{2}\log\frac{\ell\rho_X\gamma_X+\lambda_X+\gamma_Z+\lambda_Q}{\lambda_X+\gamma_Z+\lambda_Q} = \frac{1}{2}\log\ell+\frac{1}{2}\log\frac{\rho_X\gamma_X}{\lambda_X+\gamma_Z+\eta_1}+O(\frac{1}{\ell})$$
$$= \frac{1}{2}\log\ell+\frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X-d)}{\lambda_X^2}+O(\frac{1}{\ell}) \tag{3.23}$$

and

$$\frac{1}{2}\log\frac{\lambda_X+\gamma_Z+\lambda_Q}{\lambda_Q}$$
$$=\frac{1}{2}\log\frac{\lambda_X+\gamma_Z+\eta_1}{\eta_1}-\frac{(\lambda_X+\gamma_Z)\eta_2}{2(\lambda_X+\gamma_Z+\eta_1)\eta_1\ell}+O(\frac{1}{\ell^2})$$
$$=\frac{1}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z}+\frac{(\lambda_X+\gamma_Z)d^2}{2(\lambda_X-d)((\lambda_X+\gamma_X)d-\lambda_X\gamma_Z)\ell}+O(\frac{1}{\ell^2}). \tag{3.24}$$

Substituting (3.23) and (3.24) into (3.22) gives

$$\overline{r}(d)=\frac{\ell}{2}\log\frac{\lambda_X^2}{(\lambda_X+\gamma_Z)d-\lambda_X\gamma_Z}+\frac{1}{2}\log\ell+\frac{1}{2}\log\frac{\rho_X\gamma_X(\lambda_X-d)}{\lambda_X^2}$$
$$+\frac{(\lambda_X+\gamma_Z)d^2}{2(\lambda_X-d)((\lambda_X+\gamma_X)d-\lambda_X\gamma_Z)}+O(\frac{1}{\ell}).$$

- $d=\lambda_X$

In this case $g_1=0$ and consequently

$$\lambda_Q=\frac{-g_2+\sqrt{g_2^2-4(\gamma_X-\lambda_X)(h_1\ell+h_2)}}{2(\gamma_X-\lambda_X)}. \tag{3.25}$$

Note that

$$\sqrt{g_2^2 - 4(\gamma_X - \lambda_X)(h_1\ell + h_2)} = \sqrt{-4(\gamma_X - \lambda_X)h_1\ell} + O(\frac{1}{\sqrt{\ell}}). \tag{3.26}$$

Substituting (3.26) into (3.25) gives

$$\lambda_Q = \mu_1\sqrt{\ell} + \mu_2 + O(\frac{1}{\sqrt{\ell}}),$$

where

$$\begin{aligned} \mu_1 &\triangleq \sqrt{-\frac{h_1}{\gamma_X - \lambda_X}}, \\ \mu_2 &\triangleq -\frac{g_2}{2(\gamma_X - \lambda_X)}. \end{aligned}$$

Moreover, it can be verified via algebraic manipulations that

$$\begin{aligned} \mu_1 &= \lambda_X, \\ \mu_2 &= \frac{(1-\rho_X)^2\gamma_X - 2\rho_X\gamma_Z}{2\rho_X}. \end{aligned}$$

Now we proceed to derive an asymptotic expression of $\overline{r}(d)$. Note that

$$\begin{aligned} \frac{1}{2}\log\frac{\ell\rho_X\gamma_X + \lambda_X + \gamma_Z + \lambda_Q}{\lambda_X + \gamma_Z + \lambda_Q} &= \frac{1}{4}\log\ell + \frac{1}{2}\log\frac{\rho_X\gamma_X}{\mu_1} + O(\frac{1}{\sqrt{\ell}}) \\ &= \frac{1}{4}\log\ell + \frac{1}{2}\log\frac{\rho_X}{1-\rho_X} + O(\frac{1}{\sqrt{\ell}}) \end{aligned} \tag{3.27}$$

and

$$
\begin{aligned}
\frac{1}{2}\log\frac{\lambda_X+\gamma_Z+\lambda_Q}{\lambda_Q} &= \frac{\lambda_X+\gamma_Z}{2\lambda_Q} - \frac{(\lambda_X+\gamma_Z)^2}{4\lambda_Q^2} + O(\frac{1}{\ell^{\frac{3}{2}}}) \\
&= \frac{\lambda_X+\gamma_Z}{2\mu_1\sqrt{\ell}} - \frac{(\lambda_X+\gamma_Z)(\lambda_X+\gamma_Z+2\mu_2)}{4\mu_1^2\ell} + O(\frac{1}{\ell^{\frac{3}{2}}}) \\
&= \frac{\lambda_X+\gamma_Z}{2\lambda_X\sqrt{\ell}} - \frac{(\lambda_X+\gamma_Z)(\lambda_X-\rho_X\gamma_Z)}{4\rho_X\lambda_X^2\ell} + O(\frac{1}{\ell^{\frac{3}{2}}}). \quad (3.28)
\end{aligned}
$$

Substituting (3.27) and (3.28) into (3.22) gives

$$
\bar{r}(\lambda_X) = \frac{(\lambda_X+\gamma_Z)\sqrt{\ell}}{2\lambda_X} + \frac{1}{4}\log\ell + \frac{1}{2}\log\frac{\rho_X}{1-\rho_X} - \frac{(\lambda_X+\gamma_Z)(\lambda_X-\rho_X\gamma_Z)}{4\rho_X\lambda_X^2} + O(\frac{1}{\sqrt{\ell}}).
$$

- $d > \lambda_X$

In this case $g_1 < 0$ and consequently

$$
\lambda_Q = \frac{-b-b\sqrt{1-\frac{4ac}{b^2}}}{2a} \quad (3.29)
$$

when ℓ is sufficiently large. Note that

$$
\sqrt{1-\frac{4ac}{b^2}} = 1 + O(\frac{1}{\ell}). \quad (3.30)
$$

Substituting (3.30) into (3.29) gives

$$
\lambda_Q = -\frac{b}{a} + O(1). \quad (3.31)
$$

It is easy to show that

$$-\frac{b}{a} = \frac{\rho_X \gamma_X (d - \lambda_X)\ell}{\gamma_X - d} + O(1). \tag{3.32}$$

Combining (3.31) and (3.32) yields

$$\lambda_Q = \frac{\rho_X \gamma_X (d - \lambda_X)\ell}{\gamma_X - d} + O(1).$$

Now we proceed to derive an asymptotic expression of $\overline{r}(d)$. Note that

$$\frac{1}{2}\log\frac{\ell\rho_X\gamma_X + \lambda_X + \gamma_Z + \lambda_Q}{\lambda_X + \gamma_Z + \lambda_Q} = \frac{1}{2}\log\frac{\rho_X\gamma_X}{d - \lambda_X} + O(\frac{1}{\ell}) \tag{3.33}$$

and

$$\begin{aligned}\frac{1}{2}\log\frac{\lambda_X + \gamma_Z + \lambda_Q}{\lambda_Q} &= \frac{\lambda_X + \gamma_Z}{2\lambda_Q} + O(\frac{1}{\ell^2}) \\ &= \frac{(\lambda_X + \gamma_Z)(\gamma_X - d)}{2\rho_X\gamma_X(d - \lambda_X)\ell} + O(\frac{1}{\ell^2}).\end{aligned} \tag{3.34}$$

Substituting (3.33) and (3.34) into (3.22) gives

$$\overline{r}(d) = \frac{1}{2}\log\frac{\rho_X\gamma_X}{d - \lambda_X} + \frac{(\lambda_X + \gamma_Z)(\gamma_X - d)}{2\rho_X\gamma_X(d - \lambda_X)} + O(\frac{1}{\ell}).$$

This completes the proof of Theorem 2.

Chapter 4

Conclusion

We have learned the problem of symmetric remote Gaussian source coding and made a systematic analysis, with asymptotic rate-distortion, for centralized encoding and distributed encoding. We can extend our work by considering the more general source and noise models in future.

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