

TOTAL REFLECTION OF MICROWAVES BY A PRISM AND SEMICYLINDER

DIFFRACTION, TOTAL REFLECTION, AND REFRACTION
OF
3.2 CM. ELECTROMAGNETIC WAVES BY A DIELECTRIC PRISM
AND
A DIELECTRIC AND METAL SEMICYLINDER

By

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Electromagnetic Waves by a Dielectric Prism and a Dielectric
and Metal Semicylinder

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SCOPE AND CONTENTS:

A description of several experiments carried out to study the existence of evanescent waves behind totally reflecting dielectric surfaces is given in this thesis. Chapter I describes the experimental apparatus used to generate and measure the electromagnetic radiation. A detailed description of the construction of the radiating horns used and the casting of a plastic prism is also given.

Chapters II and III give the results of two experiments in the region behind a totally reflecting face of the plastic prism and similar effects noted behind a lucite semicylinder with its plane face towards the source of radiation. Near field diffraction patterns of this cylinder with its plane face towards the source, away from the source, and parallel to the axis of radiation are also given. The above three cases are compared with results obtained by coating the semicylinder with aluminum foil.

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INTRODUCTION

In recent years considerable interest has been shown in the study of diffraction of microwaves by objects whose size is of the order of a wavelength. Since the frequencies used are such that the wavelengths encountered are a few centimeters it is possible to measure effects very close to the diffracting object. Studies of the diffraction of 3.2 cm. microwaves by long metal and dielectric cylinders have been carried out in this laboratory over the past few years. The indicating apparatus used in these experiments allowed measurements to be taken within one tenth of a wavelength from the surface of the diffracting object. The possibility of such close measurements prompted an investigation of the evanescent wave effects behind a totally reflecting surface.

Electromagnetic wave theory predicts the existence of a wave on the less dense side of a totally reflecting dielectric boundary. This disturbance, termed evanescent, is an inhomogeneous wave travelling parallel to the surface with a very rapid exponential decrease of intensity with increasing distance from the surface. A number of optical experiments have been carried out that give indirect evidence of the existence of evanescent waves, but because of their very nature it would be impossible to detect them directly in the optical region.

D. R. Kneeland of the microwave research group initiated a study of the evanescent wave emerging from a totally reflecting dielectric

prism made of paraffin using 3.2 cm. electromagnetic radiation.⁽¹⁾ At the suggestion of Professor A. B. McLay, the present work continues this study using an improved plastic prism. Effects of a similar nature were noted as the result of studies behind the region of total reflection of a lucite semicylinder with its plane face towards the source. Diffraction patterns in the very near region of this semicylinder with its plane face towards and away from the source are given, and also a study of the near field diffraction when the plane face is parallel to the axis of propagation as a continuation of the work of Subbarao and McLay.⁽²⁾ These patterns are compared with the results obtained by covering the semicylinder with metal foil.

CHAPTER I

EXPERIMENTAL APPARATUS

LOCATION

All the experiments described in this paper were carried out in a room 8.3 x 5.9 x 3.7 meters on the ground floor of the Physical Sciences Building. This room has the advantage of having no windows and no large steel panels to cause scattering of the microwaves, but because of its small size as compared with the room previously used by Kneeland and Subbarao for similar studies, trouble was experienced with interference effects caused by scatter from the walls and the fluorescent lighting fixtures.

To reduce reflection the fluorescent lights in line with the axis of propagation and immediately above it were removed as well as a small sink and its associated fixtures. Reflections from the wall immediately behind the track and probe assembly were minimized by the erection of a large wall of microwave hair absorber 6' high and 8' long. At a later date reflections from this were further reduced by curving the wall outward slightly along a horizontal line on a level with the probe. The placement of absorber found to be the most satisfactory and the one used for all the measurements in this work consisted of two walls of absorber 4' high and 22' long suspended four feet on each side of the axis of propagation with their center line on level with the probe. Even with this arrangement the field was quite sensitive to

changes in the position of the absorbing walls, particularly the sections midway between the horn and the probe.

APPARATUS

The microwave source consisted of a 723-A/B Reflex Klystron powered by a TVN-7BL power supply and square-wave generator as described by Wiles.⁽³⁾ A new micrometer adjusted reaction type wavemeter, Waveline No. 698, replaced the older one previously used and allowed a better determination of the operating frequency of the klystron. A blower was used to direct a constant airstream over the tube as suggested by Kneeland but with the addition of a cardboard diaphragm to control the amount of air. This adjustment allowed compensation to be made for changes in room temperature.

The signal was detected by a 1N23A crystal diode machined to approximate a dipole and the rectified modulated wave was fed through a coaxial cable to a tuned amplifier (described by Kneeland) and then to a Brown Potentiometer Recorder. The amplified controls were modified slightly to allow finer adjustment of the zero line on the chart and to improve the linearity of the diode clipping stage.

The previous method of sighting the position of the probe on the optical bench by means of a telescope was deemed unsatisfactory and a remote indicator was devised using selsyn motors. One of the selsyn motors was mounted on the end of the optical bench opposite the driving motor and was geared to the track lead screw. The second selsyn unit was mounted on the side of the apparatus rack on a level with the recorder. This unit drives a pointer by means of an 80 to 1 worm gear.

The pointer therefore indicates the travel of the probe along the track, making one revolution for each 10 cm. of probe travel. The gear ratio on the indicator is such that it remains in exact synchronization with the probe over the entire 150 cm. length of track, thus the position of the probe may be determined to within ± 0.01 cm. directly from the indicator dial.

RADIATING HORNS

In previous work by members of the microwave research group the incident field could be considered to have a constant amplitude over a range of about 30 cm. Outside this narrow range it tapered off rather rapidly on each side of center. This was due to the very narrow pencil type radiation pattern produced by the horn, a pyramidal type 24λ long from mouth to apex and flared at an angle of 22° in both the E and H planes. It was decided therefore to widen the field of radiation by shortening the horn and decreasing the flare angle. A suitable radiation pattern was chosen by reference to Rhodes⁽⁴⁾ and a horn 10λ long from mouth to apex and flared 10° in the H plane and 14° in the E plane was constructed. This horn produced a field of constant amplitude over a considerable distance, but because of its very wide field it was impossible to eliminate reflections from the walls and ceiling with the limited amount of absorber available. A compromise between the long and short horns was deemed the solution and a horn 12λ long with a flare of 15° in the H plane and 17° in the E plane was made. The length of this horn was limited by the 12" capacity of the metal bending brake in the nuclear building machine shop. The amplitude of the field from this horn was constant over a 70 cm. range. Although there was still consider-

able interference from reflections these were finally minimized by the placement of absorbers as described earlier.

The large horn previously used was made of four separate pieces of $1/16$ " sheet aluminum bolted at the corners to $1/2$ " brass "angle iron". This method of construction resulted in a very large number of discontinuities on the inner surface of the horn. In order to make the new horns as smooth as possible on the inside the first one was folded from a single sheet of $1/32$ " brass, and butt soldered at the one joint. However, due to the pyramidal shape of the horn and the limitations of the bending brake, the last of the three folds had to be finished by hand, leaving a rounded corner rather than a sharp bend and thus causing a slight warping of the horn. The final horn was made of two separate pieces of $1/32$ " brass, each with one fold, and butt soldered together. Some hand finishing of the joints produced an almost perfect finish of the inner surface with no discontinuities.

DIELECTRIC PRISM

The dielectric prism used by Kneeland was made of a special paraffin wax supplied by the Imperial Oil Company. This prism was a right angle one 44.2 cm. x 43.7 cm. x 61.5 cm. and 45.7 cm. high. The prism contained discontinuities consisting of many randomly scattered centers of crystalization as well as uneven faces. It was decided to look into the possibility of casting a plastic prism of about the same size and shape and so a number of plastics manufacturers were contacted for information.

Two products were finally found that would lend themselves to casting in large quantities without cracking. These were Selectron 5026,

a polyester resin manufactured by the Pittsburgh Plate Glass Company, and Epon Resin 828, manufactured by the Shell Oil Company of Canada Ltd. The former plastic was chosen because the latter required oven curing and had too high a dielectric constant. A 90° V shaped trough of 1/16" sheet aluminum was assembled as a mold, 50 cm. long and 32 cm. deep on the inside. The mold was made to fit in the largest sink available in the building for the purpose of providing a water bath that could control the temperature of the casting. Two test castings containing 6.0 kg. of resin, 30 g. of peroxide, and 30 g. of peroxide accelerator were made. The first cracked into many small pieces after about six hours and while still in the mold. One of the larger remaining chunks was placed in an oven and heated very slowly to 70°C at which time it shattered into innumerable fine pieces indicating that the original portion was under very great stress. The second casting, under slightly different temperature conditions, lost all its sharp corners due to cracking while still in the mold but the core remained solid. However after a week at room temperature this core developed transverse fissures again indicating enormous internal stress.

After consulting graduate members of the Chemistry Department it was decided that the polymerization was proceeding at too rapid a rate caused by high temperature and an excess of catalyst. A third test casting was tried, this time containing 6.0 kg. of resin but only 6 g. of peroxide and no peroxide accelerator. The mold was left at room temperature, and after a week the plastic had formed a hard gel. This was further solidified by the application of an intense mercury type spotlight containing a high percentage of long ultra-violet radiation.

After about three more weeks the casting was quite hard and contained no flaws. This casting differed from the other two in that it was translucent rather than transparent, although in the next six months it cleared considerably.

The final large prism was made under the same conditions as the third test casting. 39.0 kg. of resin and 39 g. of peroxide were placed in the mold and allowed to polymerize at room temperature. The plastic formed a hard gel in about two weeks, and after a month and a half exposure to the mercury spotlight it became solid. The three refracting faces of the casting were then machined by the Dominion Pattern Works of Hamilton to form an accurate 45° - 90° - 45° prism. These three faces were further finished by hand grinding with silicon carbide paper and polishing with powdered pumice. The resulting prism is 34.00 cm. x 34.05 cm. x 48.10 cm. and is 50 cm. high. The faces are almost optically smooth and flat. The prism was mounted on top of a 15" square column of styrofoam four feet high. This in turn was placed on a low rubber castered dolly in order to facilitate the placement and orientation of the prism with respect to the incident radiation beam.

CHAPTER II

LUCITE AND ALUMINUM SEMICYLINDERS

ARRANGEMENT

The generating apparatus was placed at one end of the room described in the preceding chapter. The track and probe assembly were placed at the opposite end of the room, 5.5 meters from the apex of the radiating horn on the axis of propagation and oriented so that the probe travelled in a horizontal plane perpendicular to this axis. The horn and probe were on a plane equidistant from the floor and ceiling of the room.

The distance from the horn to the probe was kept constant for all experiments with the dielectric and metal rods. The axis convention is similar to that used in previous work⁽²⁾. The x axis is the axis of propagation (horizontal), the z axis is the longitudinal axis of the semicylinder (vertical), and the probe runs parallel to the y axis (horizontal). The origin is taken as the center of the rod with the positive direction of the x axis on the side away from the transmitter. In order to change the x position of the probe, the rod was moved along the axis of propagation to change the position of the origin, thus the incident field (without the rod) always remained the same for the probe.

All the runs taken in which the probe was not free to pass behind the rod were started with the probe at the surface of the rod. The distance from the center of the crystal to the surface of the rod

and the position of the probe on the track were recorded on the chart. The chart and the probe were then started simultaneously and readings taken out to a distance of 20 cm. from the rod. On runs in which the probe passed behind the rod, the run was started 20 cm. from the rod on one side and continued to a point 20 cm. past the rod on the other side. The distance of closest approach of the probe to the surface of the rod was limited by the radius of the probe's crystal case, 0.3 cm., and in some instances by the probe support. This effect can be seen in all the diagrams as a gap between the outline of the semicylinder and the start of the intensity curves.

The rod was mounted vertically in a wooden stand so that the xy plane passed through a point equidistant from each end in order to eliminate any possible end effects. For Cases I and II where the plane face was towards or away from the source, measurements of the distance between the rod and the probe were used to align the plane face with the y axis. In Case III, with the plane face parallel to the axis of propagation, the rod was aligned by sighting along the plane face using a telescope mounted directly below the radiating horn.

CASE I: PLANE FACE TOWARDS THE SOURCE

Figure 1 gives the diffraction pattern of a lucite semicylinder with its plane face towards the source of radiation. The x axis is plotted as the ordinate and the y axis as the abscissa. The curves are those of the normalized intensity of the diffracted field, that is, the intensity of the diffracted field divided by the intensity of the field at the same point if the rod were removed. Only the field for the positive y axis (to the right of the rod) is shown because a symmetrical object

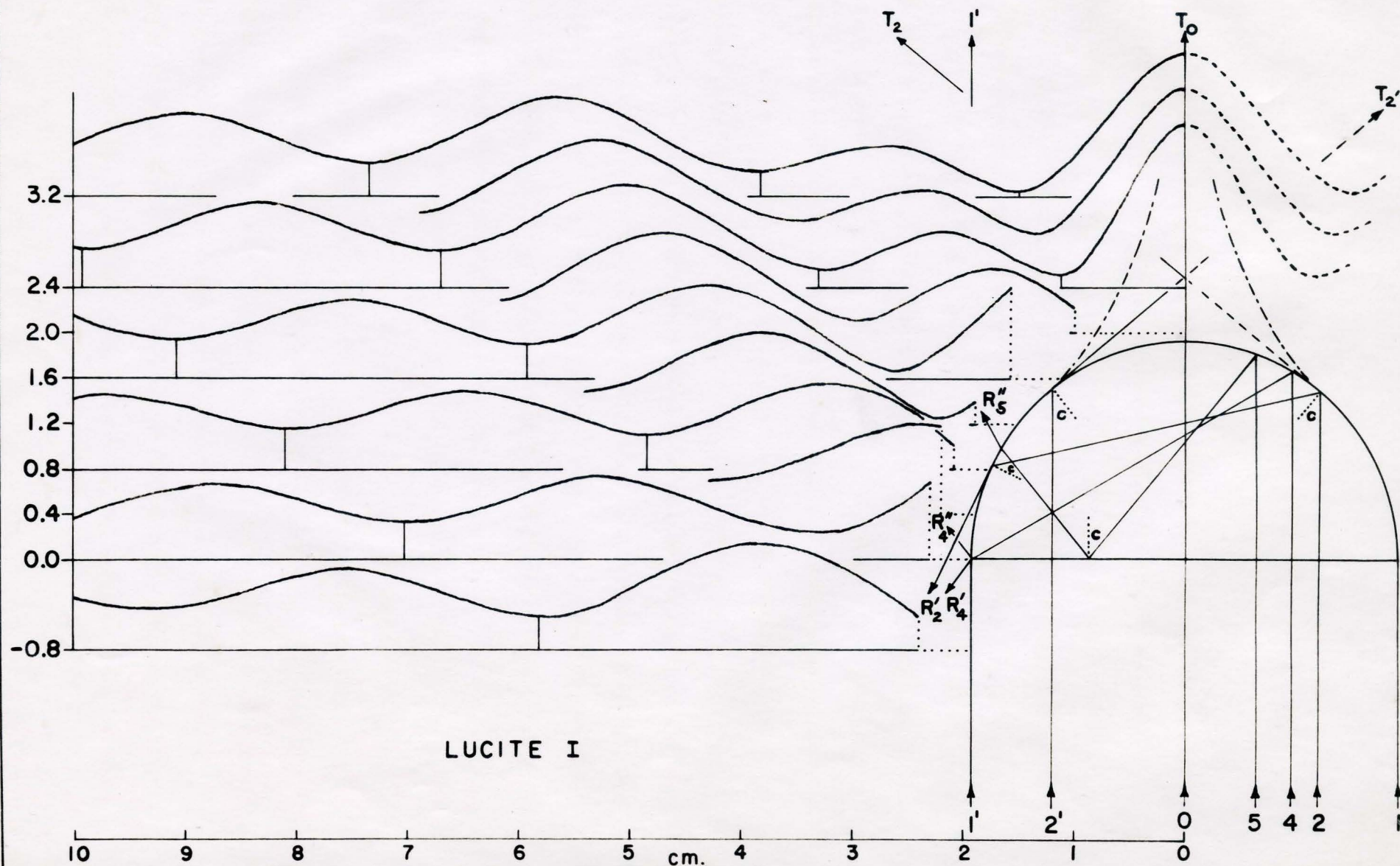


Figure 1

produces a symmetrical field, in fact the symmetry of the field gave a qualitative measure of the uniformity of illumination of the object by the source. The rays shown with the outline of the diffracting object are used to indicate the boundaries of transmitted and surface scattered beams. Only those beams are given which affect the area of the field shown in the drawing. A general explanation of these beams and results of other experiments are given by Subbarao and McLay⁽²⁾.

The purpose of this experiment was to study the region very close to the curved surface of the cylinder. In this region the beam $1'2'$ entering the plane surface is subjected to total internal reflection inside the semicylinder at the curved surface. Ray $2'$ is the limit of this beam since it strikes the curved surface at the critical angle and emerges tangent to the curve as ray T_2' . Field measurements taken at very short intervals behind the curved surface, shown in large scale in Fig. 2, indicate an increase in intensity when approaching the rod in the immediate region of the $2'$ ray. Since no other beam enters this region of total internal reflection the presence of a high field intensity cannot be explained by simple geometrical optics.

This high intensity is not due to diffraction of the field around the edge of the rod as can be seen in Fig. 3. In this experiment the lucite rod was tightly wrapped with a single layer of aluminum foil so as to approximate an infinitely conducting semicylinder with the same dimensions as the lucite rod but opaque to the incident radiation. Ray $1'$ is the limiting ray under these conditions since all the radiation to the left of it passes the rod, and that to the right is reflected back towards the source by the flat metallic surface of the rod. The ray has

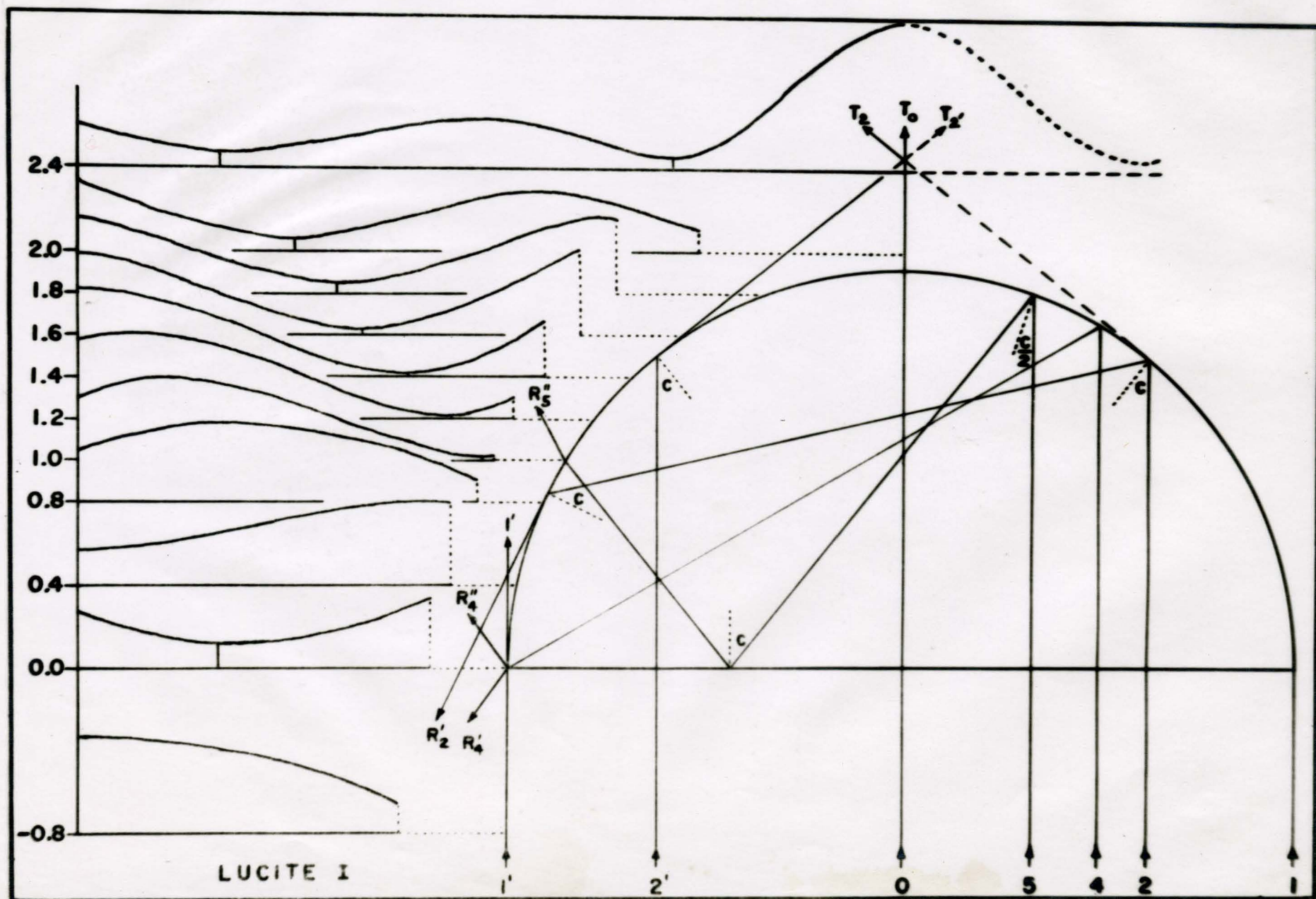
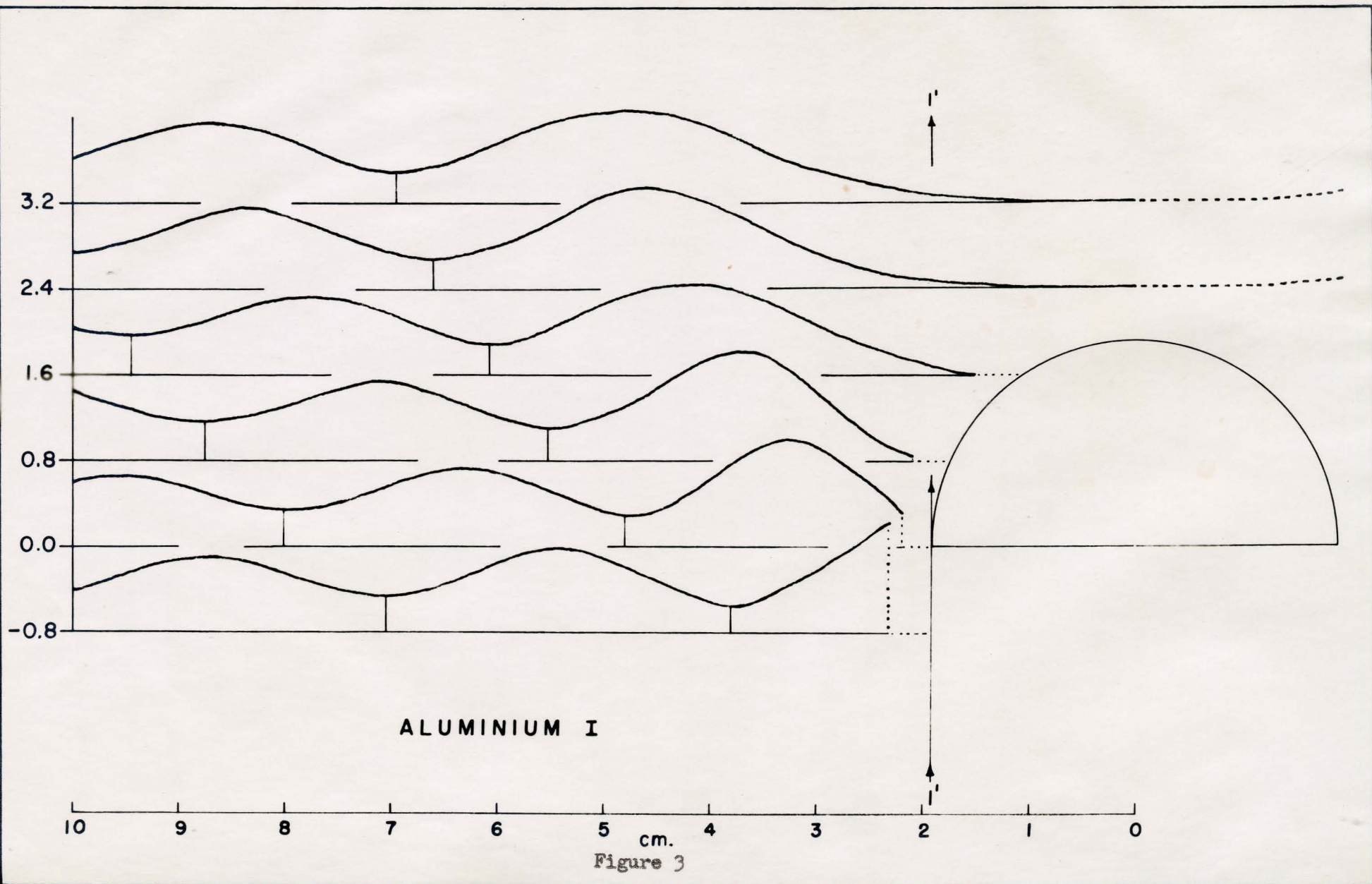


Figure 2



been extended back from the rod to indicate how diffraction effects cause a low but measurable field inside the region of the geometrical optics shadow. This weak field, particularly in the region between the 1.6 and 2.4 cm. runs, bears no resemblance to the high intensity field for the same region behind the lucite rod. The existence of this field in the case of the lucite rod therefore cannot be completely explained by diffraction phenomenon.

This strong field may be explained by electromagnetic wave theory which predicts the existence of a wave on the less dense side of a totally reflecting dielectric boundary. The disturbance, termed evanescent, is an inhomogeneous wave travelling parallel to the boundary of the medium with a very rapid decrease of intensity with increasing distance from the surface of the dielectric. Referring again to Fig. 2, the measured field exhibits this effect near the totally reflecting curved surface of the lucite semicylinder. The maximum field measured is in a region to the left of a radial line extending from the center of curvature through the point where ray 2' strikes the surface at the critical angle. The intensity of this field diminishes quite rapidly, approaching a minimum about one third of a wavelength distant from the surface of the semicylinder.

CASE II: PLANE FACE AWAY FROM THE SOURCE

In the second set of experiments the dielectric semicylinder was rotated through 180° so that the curved surface faced the incident radiation. The results are shown in Fig. 4. Here the measured field consists of the incident radiation, a scattered field formed by the reflection of beam 1'3' from the curved surface, and a third field due to the double

internal reflection of beam 12. This narrow beam enters the right hand curved surface limited on one side by the edge of the rod and on the other side by the pencil of rays that strike the flat surface at the critical angle after refraction by the front curved surface of the semicylinder. The beam is reflected again at the curved surface and then emerges from the rod as the beam R_1R_2 . This beam causes the very pronounced irregularity in the field immediately behind the left corner of the rod and can be seen in the intensity measurements taken at the 0.4 and 0.8 cm. runs.

The next experiment, Fig. 5, in which the rod is again covered with aluminum foil, has only the incident field and the field scattered from the curved surface. The measured field intensity behind the left hand corner of the rod is very nearly zero, in contrast with the irregular field in this region for the previous case. The decrease in intensity is quite obviously due to the shadow cast by the metal rod and the small intensity that does appear is due to diffraction around the edge of the rod. The maximum and minimum variations in the field to the left of the rod are also greater than with the lucite because the reflected beam S_1, S_3 , is stronger from the metal surface, hence causing greater interference with the incident beam.

CASE III: PLANE SURFACE OF SEMICYLINDER PARALLEL TO AXIS OF PROPAGATION

In this experiment the plane surface of the semicylinder was placed parallel to the axis of radiation. This was accomplished by mounting a telescope below the radiating horn and in line with the axis of radiation, and then "sighting in" the plane face of the rod. Since the rod in this experiment does not appear as a symmetrical object to the

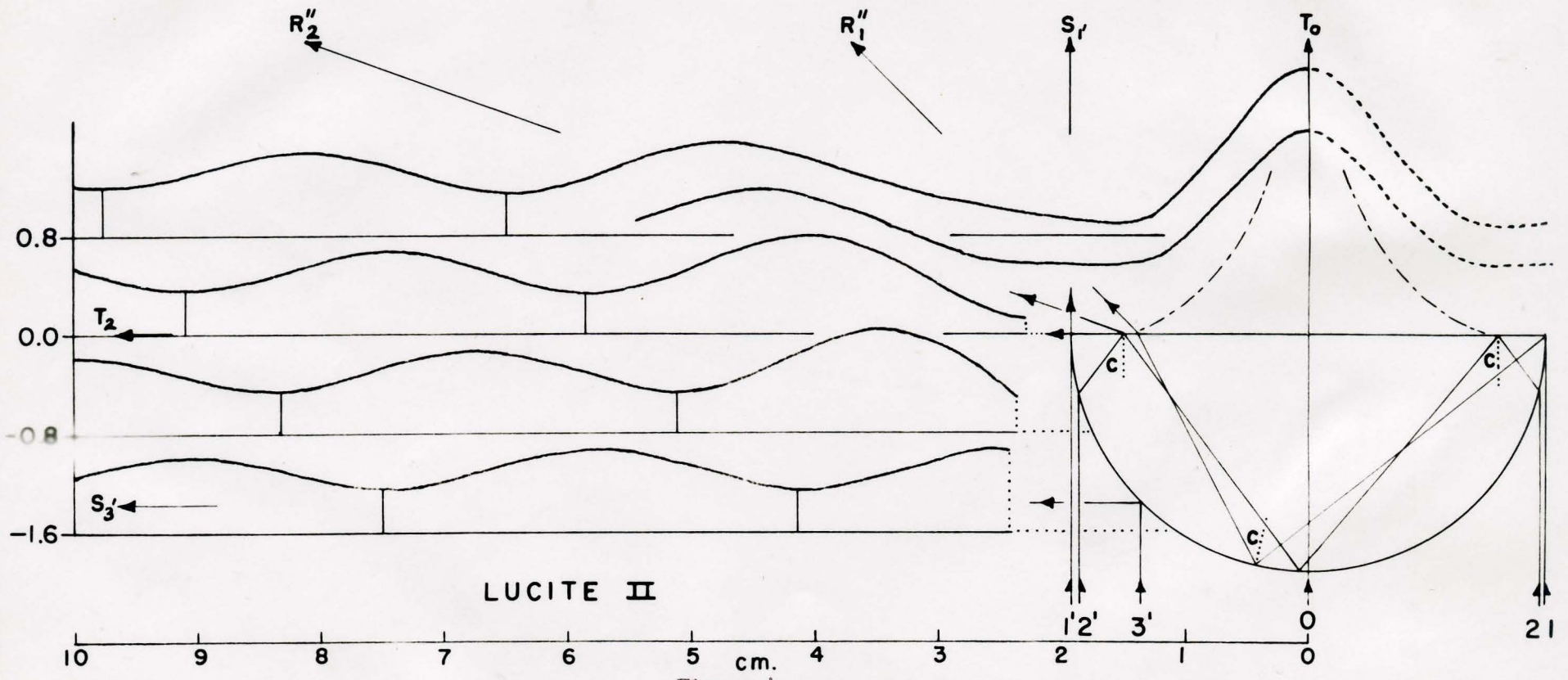
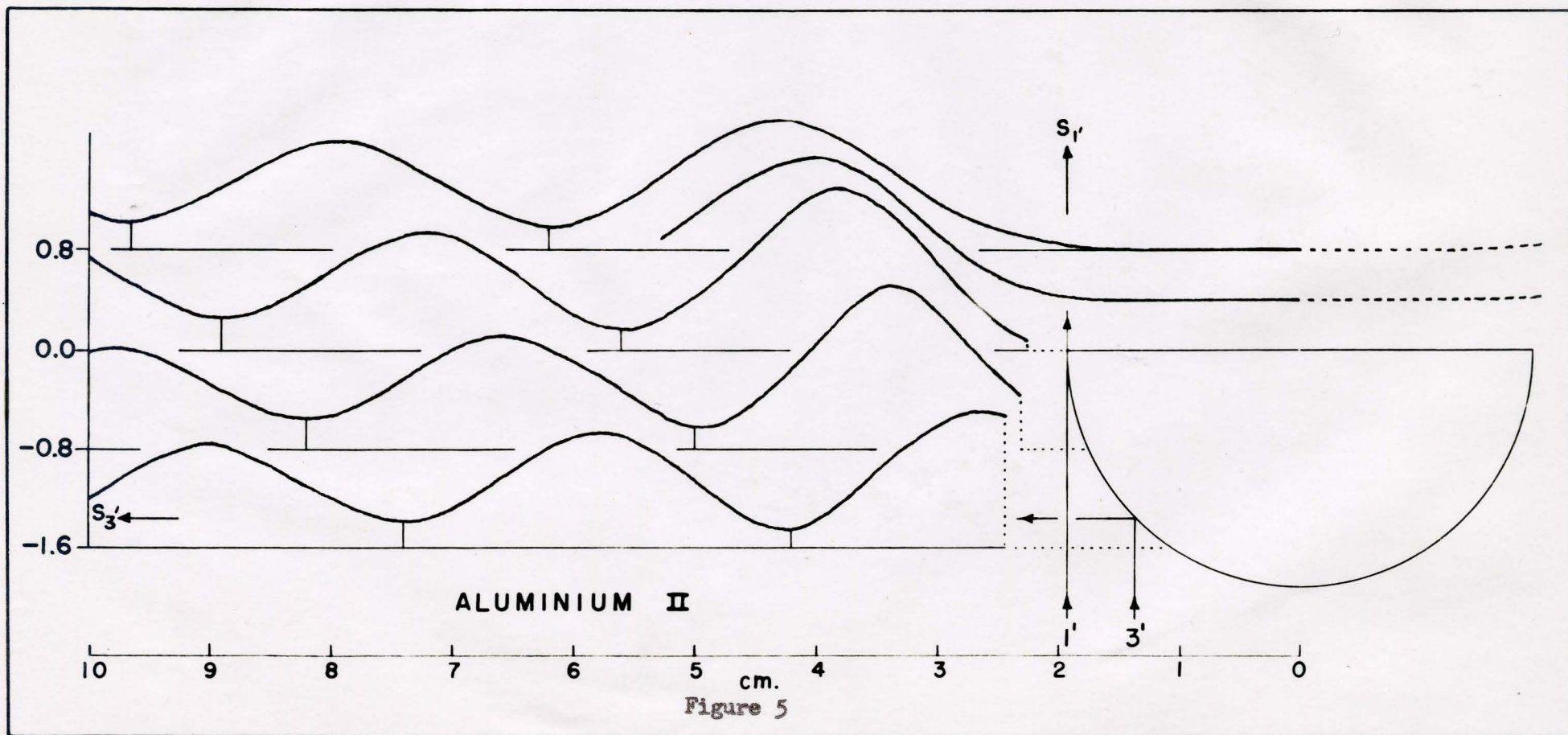


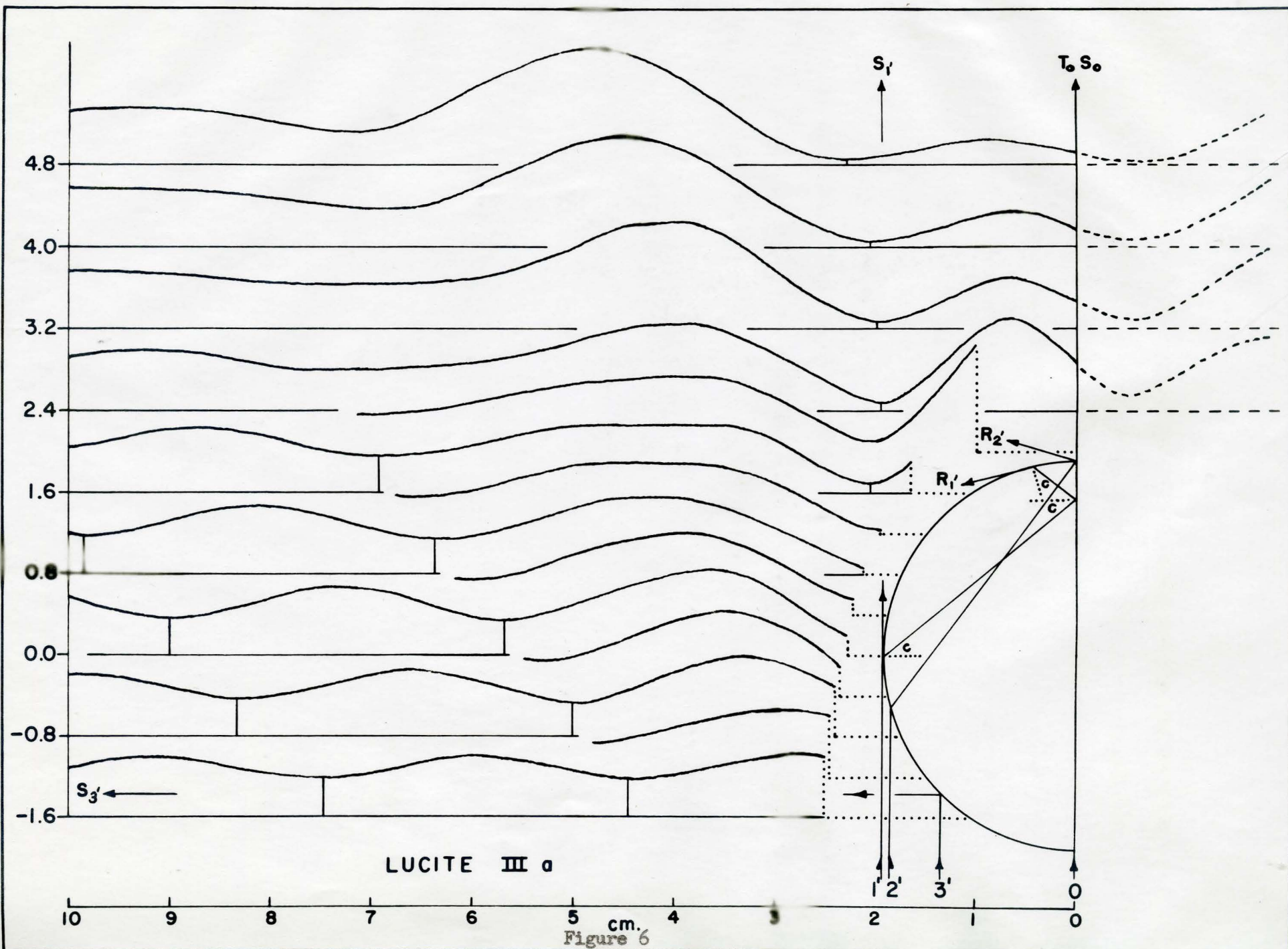
Figure 4



source, the patterns will not possess right-left symmetry. The results therefore are divided into two sections, IIIa and IIIb, the first being the field in the region adjacent to the curved surface and the second in the region adjacent to the plane surface. Because of the asymmetry of the diffracted field for this orientation, the experiment was conducted twice, once with the plane surface facing left and once with it facing right, in order to determine any adverse effects created by asymmetry of the incident field. The results of the two experiments agree very closely and the graphs plotted in cases IIIa and IIIb are averages of the respective orientations for the two experiments.

Figure 6 illustrates lucite case IIIa. The incident rays 1', 2' and 3' indicate regions similar to those in Fig. 4. Rays 1' and 2' however now define a beam which is refracted at the curved surface and reflected internally at the flat face to emerge as R_1, R_2 , behind the rod. This beam gives rise to the high intensity measured 2.0 cm. behind the axis of the rod. The interference of this beam and the normal field set up by the incident wave and the curved surface is evidenced by the flattening of the peaks of the first maximum in the region adjacent to the curved surface. The large peaks immediately behind the rod are from the addition of the incident beam 02' and diffraction around the back corner of rays travelling along the plane surface.

The aluminum counterpart of this case is shown in Fig. 7. Here the field is almost identical to that of a solid rod⁽⁵⁾ with large disturbances created by the scattered beam S_1, S_3 . A small amount of diffraction around the curved face is indicated by the slight field in the shadow region behind the rod.



The field on the plane side of the lucite semicylinder is shown in Fig. 8. The field would have zero intensity along this surface for a right angle dielectric wedge but some modification occurs from beam 1'2' which after double internal reflection emerges as a backward travelling divergent beam R_1 " R_2 " and appears to affect the runs at -0.8 and -1.6 cm.

The plane surface for the aluminum case in Fig. 9 definitely meets the boundary conditions except for the same region (0 to -1.6 cm.), but in this case there is no refracted beam. The field from 2.0 cm. on is similar in both instances except that the lucite shows greater disturbance from the transmitted divergent beam OT_2 .

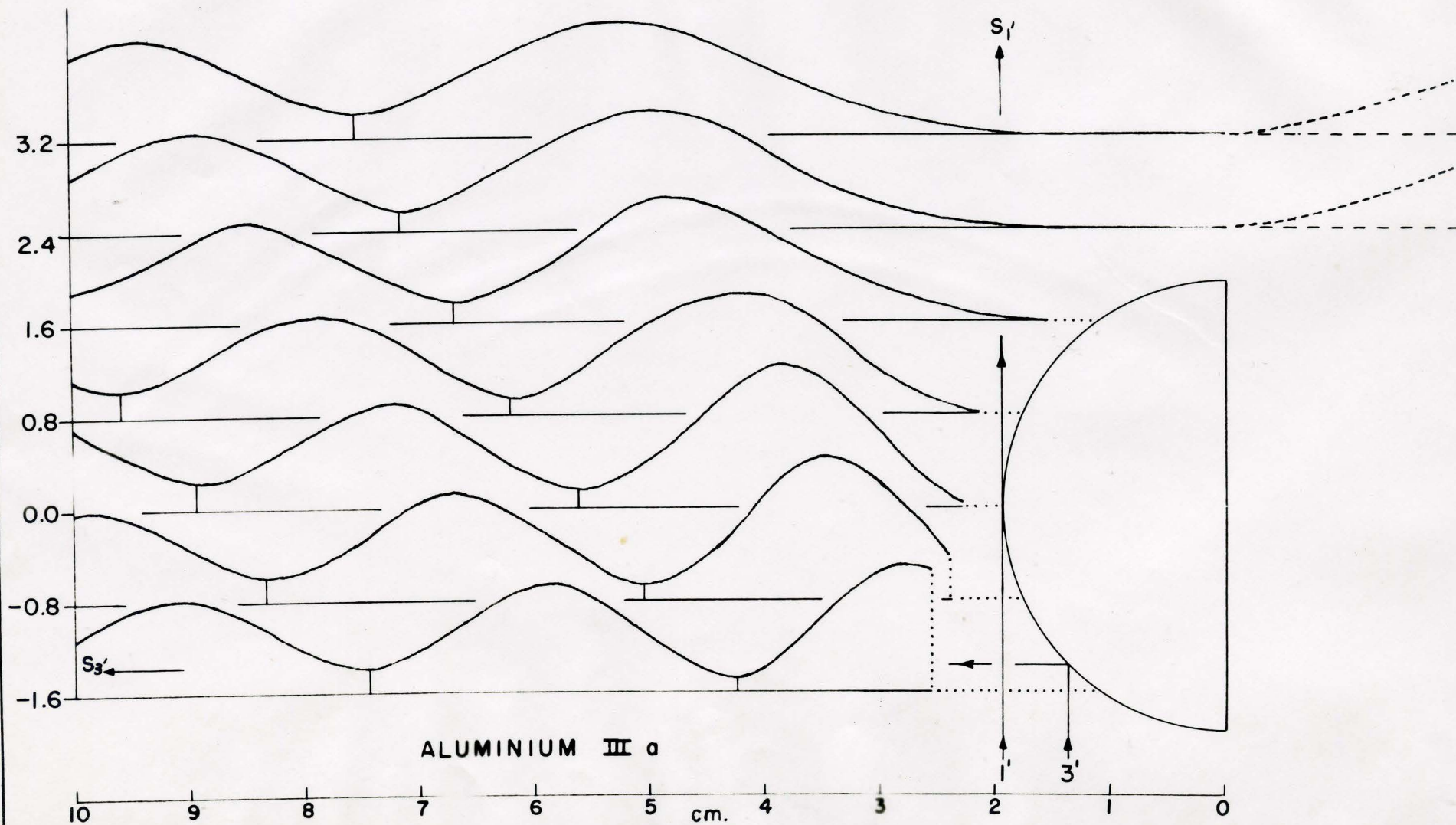
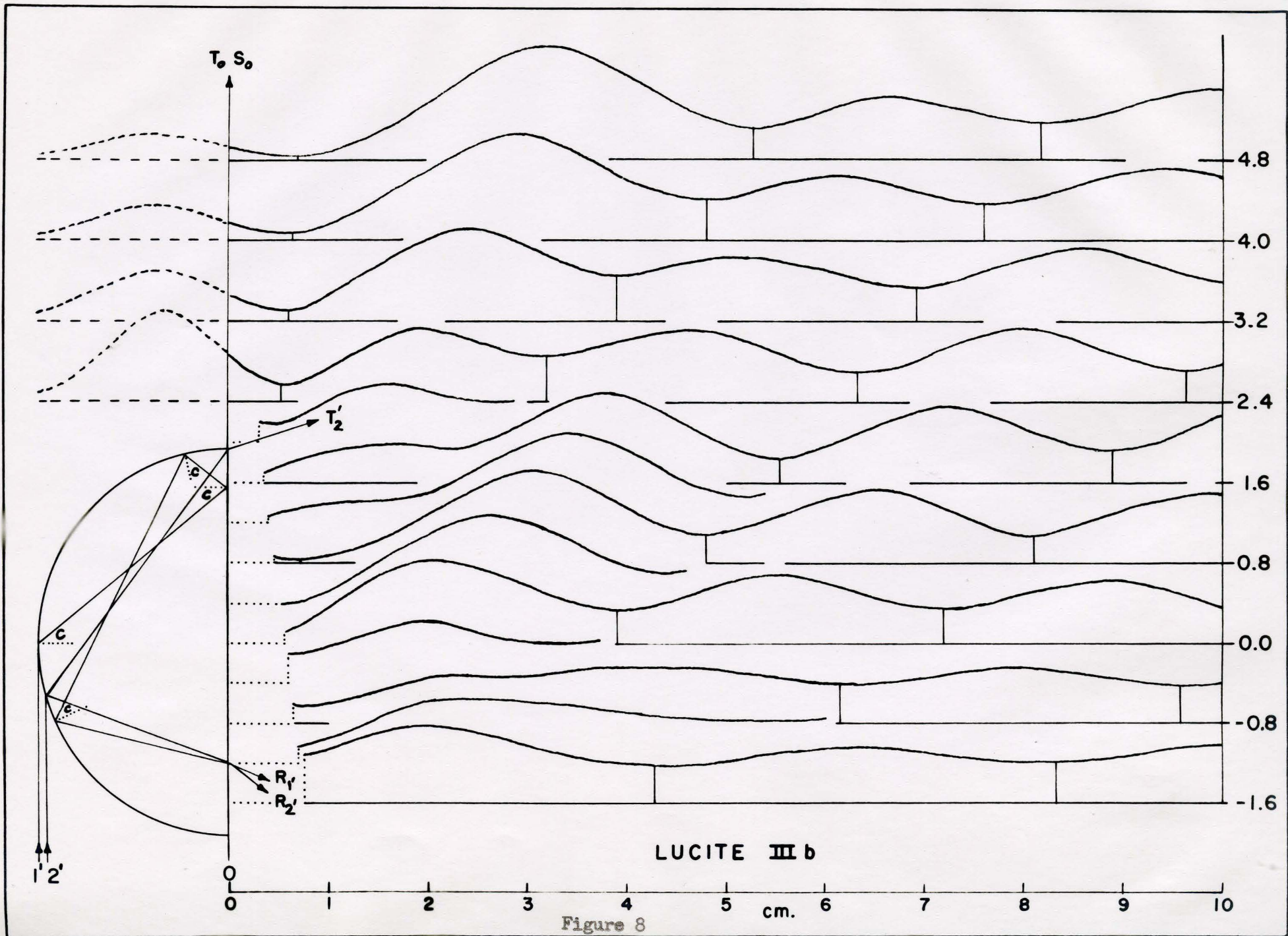
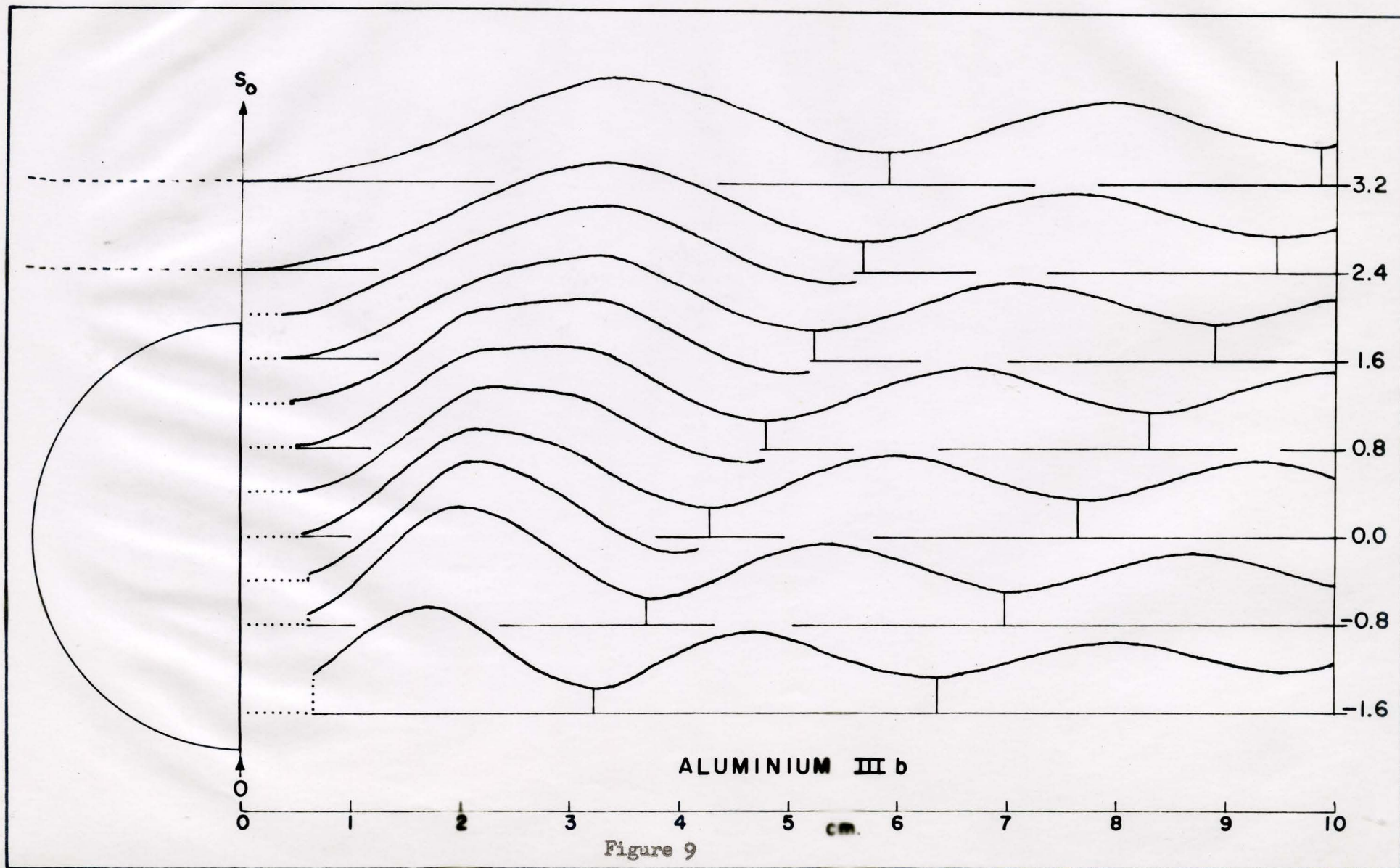


Figure 7





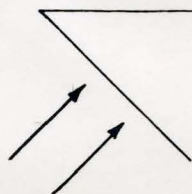
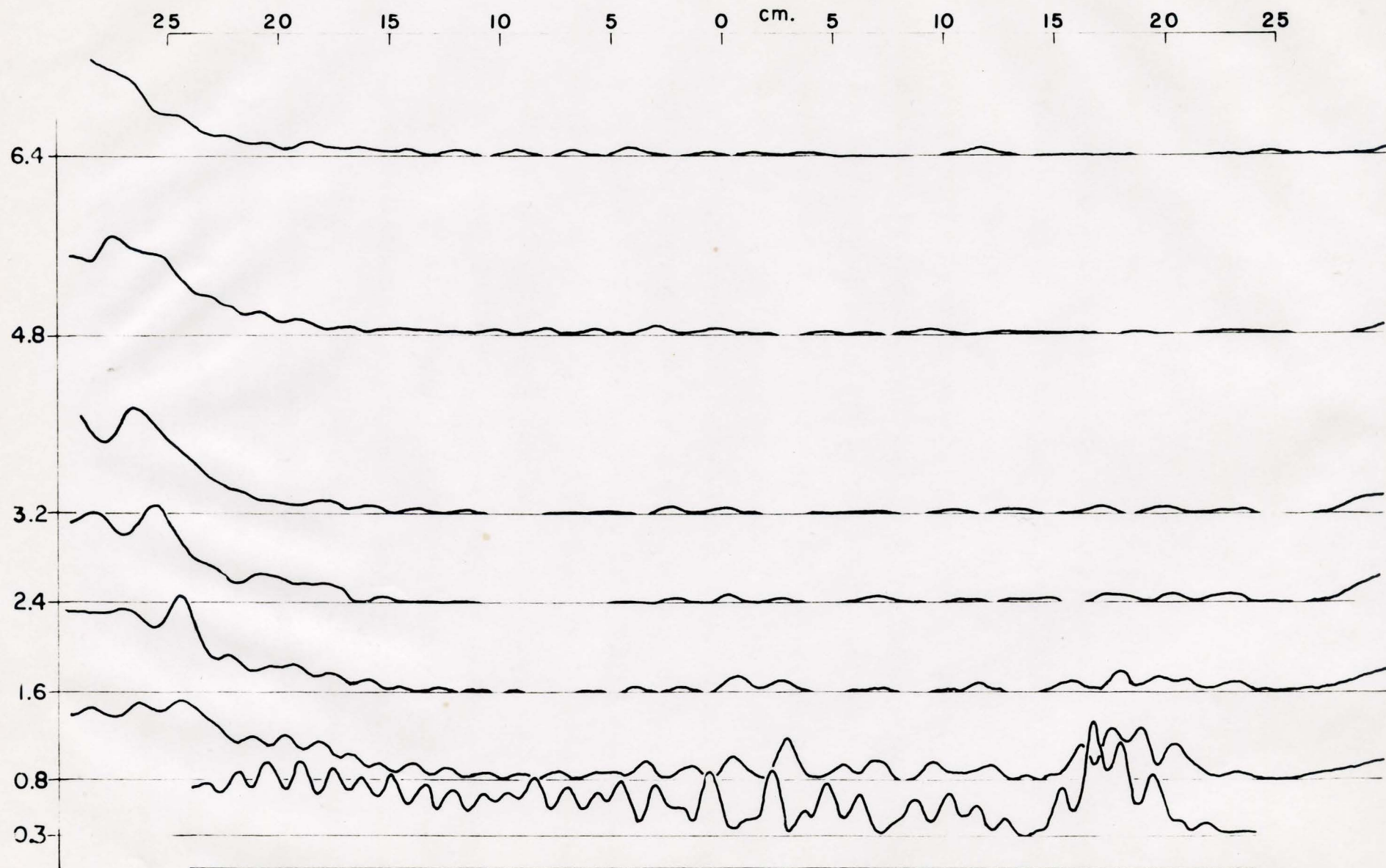
CHAPTER III

DIELECTRIC PRISM

A complete study of the fields produced by the various orientations possible with the dielectric prism had been planned, but the great difficulty encountered in casting such a large mass left time for only the following two measurements. The experimental conditions for the two cases were identical but the field was measured using two different orientations of the track and probe assembly.

In Fig. 10 a partial outline of the prism is shown at the base of the diagram. The arrows indicate the radiation entering the right face from whence it is totally reflected within the prism at the hypotenuse and then emerges normal to the left face. The total internal reflection at the hypotenuse should give rise to an evanescent wave close to the outside surface of this face. The intensity of this wave should diminish rapidly with increasing distance from this surface.

The probe was erected behind the prism so that its direction of travel was parallel with the hypotenuse and its plane of operation half way between the top and bottom of the prism. Measurements of the field intensity were then made at various distances behind this face. The results of some of these runs are given in Fig. 10 where I/I_0 is plotted for each run as it was in the case of the semicylinders. The scale of distance behind the prism (ordinate) is greatly exaggerated in order to bring out the details of the runs very close to the face without too much overlap of the curves. The distance of closest approach was limited



'SELECTRON' PLASTIC PRISM

Figure 10

once again by the radius of the crystal case on the probe and hence the first run is 0.3 cm. behind the face. The decrease in the measured field intensity with increasing distance from the totally reflecting face is very rapid, dropping to zero over almost the entire surface in less than two centimeters. The finite area of the left face in the almost plane incident field creates a slit type diffraction field in the beam as it enters the prism. The non-uniform illumination of the reflecting surface by this diffracted field probably gives rise to the irregularity of the high intensity first run. Diffraction of the incident field around the left hand edge can be seen by the slowly tapering intensity in the shadow region behind the prism. This effect is somewhat masked in the drawing by the expanded vertical scale.

In order to gain a better picture of the intensity decay immediately behind the reflecting face the probe and carriage were arranged in a way to obtain the results shown in Fig. 11. The prism is in the same position as before, but the orientation of its outline of the drawing has been changed so that what was the left face appears at the top of the diagram with the arrows indicating the direction of propagation of the incident radiation. The track was arranged so that the direction of probe travel was perpendicular to the hypotenuse of the prism and the runs were made along lines spaced at one centimeter intervals. Relative intensity is indicated in relief. This form of projection displays the high intensity very close to the totally reflecting surface. The very rapid fall-off in field strength is now quite obvious with only small disturbances in the field from 2 to 20 cm. behind the prism except for the diffraction effects around the right hand corner.

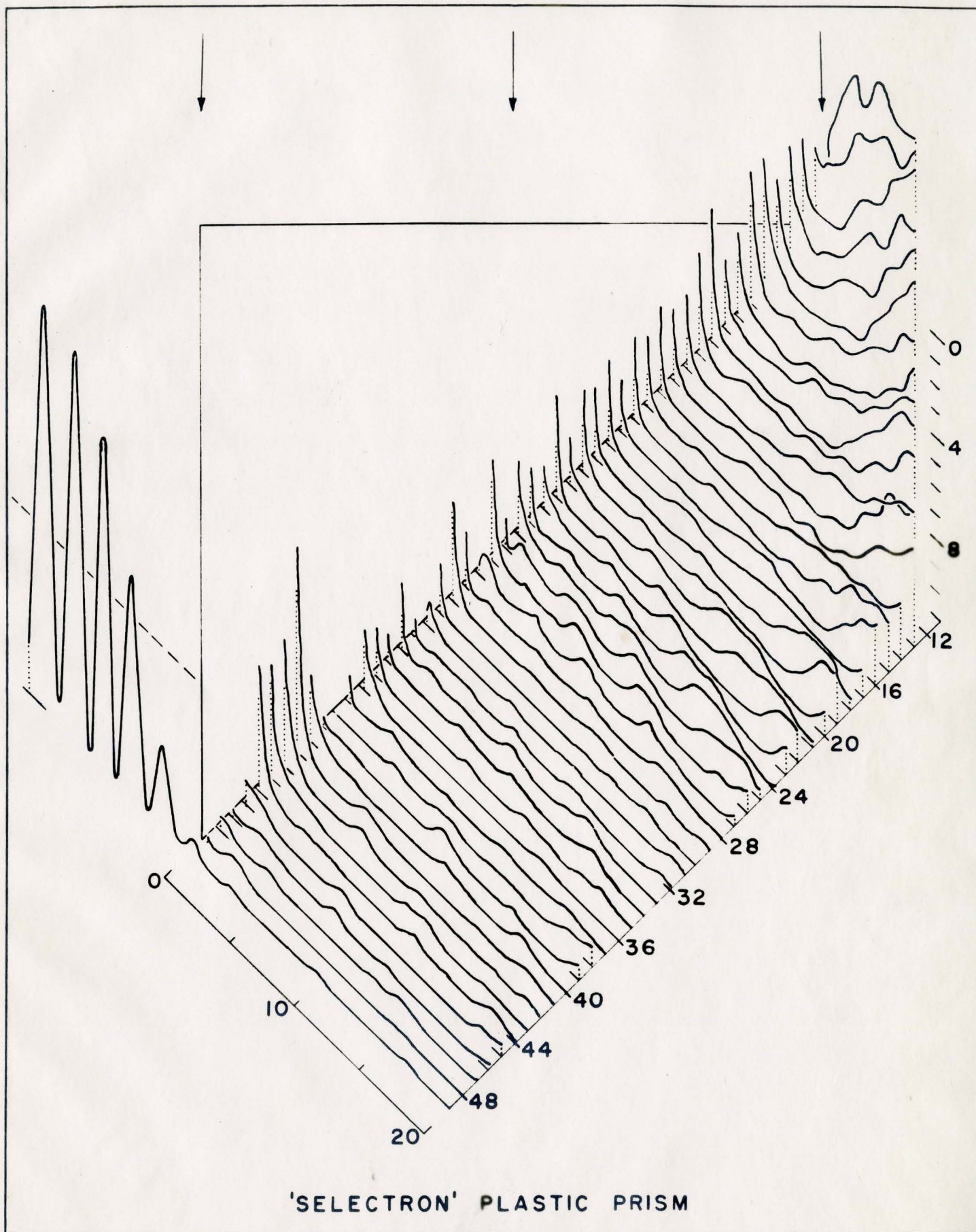


Figure 11

Careful scrutiny of the field at the beginning of each run reveals excellent agreement with the measurements shown for the 0.3 cm. run in Fig. 10 indicating therefore, the reproducibility of the measurements. Agreement also exists between the other runs in Fig. 10 and their respective positions on the runs in Fig. 11 although this is not as easily discernable as that of the 0.3 cm. measurement. Run number 49 in Fig. 11 is beyond the edge and hence the probe enters the region to the left of the prism where the incident field and the internally reflected field meet. The interference of these fields gives rise to very prominent maxima and minima in this area. The heights of the maxima rise to approximately twice the intensity of the normal incident field, whose level is indicated by a dashed line.

CONCLUSIONS:

Experimental measurements of diffracted and reflected 3.2 electromagnetic radiation very near surfaces of dielectric and metallic semi-cylinders in various orientations have been presented. Indications are that geometric optics criteria may be applied to explain the results obtained provided allowance is made for effects due to diffraction. Evidence of the existence of an evanescent wave immediately behind a totally reflecting surface occurring in one of the above orientations has also been presented⁽⁶⁾.

Two other sets of measurements taken behind the totally reflecting surface of a large dielectric prism are shown. Both of these results also show strong evidence of an evanescent wave effect very close to the surface of the prism.

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