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**3D multi-scale multi-physics modelling of hot cracking in welding**

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## Abstract

A 3D multi-scale and multi-physics numerical model has been developed and validated to predict the occurrence of hot cracking during fusion welding of Al-Mg-Si alloys. The new model consists of four modules: (I) a welding solidification module that creates the desired weld microstructure consisting of both columnar and equiaxed grains and varies as a function of welding conditions; (II) a thermo-mechanical analysis module that predicts the deformation of the weld mushy zone due to solidification contraction and the response of the base metal; (III) a fluid flow module that calculates the variation in fluid velocity and pressure within the micro liquid channels of the semisolid; and (IV) a crack initiation module that applies Kou's hot cracking criterion [1] to identify cracked liquid channels based on inputs from the solidification, thermo-mechanical and fluid-flow modules. The results identify the underlying mechanisms by which welding process parameters (current and travel speed) and external restraining conditions influence hot cracking susceptibility during Gas Tungsten Arc welding. Interestingly, micro hot cracks seem to initiate near the fusion zone but then localize and form a macroscopic hot crack at the core of the weld

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## 1. Highlights

2 A 3D multi-scale and multi-physics mathematical model has been developed  
and validated to simulate the fusion welding of Al-Mg-Si alloys. The metal-  
4 lurgical mechanisms linking welding process parameters, external restraining  
conditions and hot cracking are discussed. The localization of hot cracks at  
6 individual grain boundaries are predicted; interestingly micro hot cracks seem  
to initiate near the fusion zone.

## 8 2. Introduction

Of the different fusion welding processes, Gas Tunsten Arc Welding (GTAW)  
10 and Gas Metal Arc Welding (GMAW) are most commonly used to join metal  
alloys because of their affordability, mobility, and reliability [2]. However, fusion  
12 welding is not defect-free. One of the main defects that forms within the weld  
pool is hot cracking [3]. This defect, also known as hot tearing in metal casting  
14 processes, occurs during the final stages of solidification when the solid fraction  
( $f_s$ ) is close to unity. Several factors are reported to affect the formation of  
16 hot cracks in both welding and casting [4]. These include: a network of thin  
liquid channels surrounding the solid grains; deformation of the mushy zone  
18 that induces cracks at the solid-liquid interface; and low permeability inhibiting  
liquid flow through the channels to prevent refilling (also called healing) of the  
20 initiated cracks [5, 6].

Experimentally, many tests have been conducted to assess alloy and process  
22 susceptibility to hot cracking (e.g. [4, 7, 8]). However, the high temperatures  
and the very short lifetime of the semisolid restrict the use of experimental  
24 methods when investigating the transient phenomena involved in crack initia-  
tion. Instead, numerical methods must be used as they allow for concurrent

26 study of solidification [6, 9, 10], semisolid deformation [11–13], and defect for-  
mation [14, 15].

28 In the past few years, there has been significant interest in using a granular  
modelling approach to study this defect [6, 12, 13, 16–25] as this method allows  
30 for simulation of the most-relevant physics within a large <sup>1</sup> and sometimes non-  
isothermal mushy zone. Unlike macro-scale simulations that model transport  
32 phenomena during welding (e.g. [26]) and casting (e.g. [27]) based on averaged  
equations, granular methods are able to simulate the interactions between the  
34 solid and liquid phases at the scale of the microstructure while at the same  
time taking into account long-range stresses and strains. Granular methods are  
36 also able to provide all of the inputs for hot cracking criteria (e.g. [1]) at the  
scale of the microstructure, along with spatial variations in these parameters.  
38 Combined, this allows for investigation of localization of hot cracking at grain  
boundaries taking into account both stochastic variations in grain morphology,  
40 and process parameters.

In granular models of semi-solids, the microstructure is approximated by a  
42 set of polyhedrons based on the Voronoi diagram of a random set of nuclei,  
resulting in irregular grain arrangements. Solidification is then simulated by  
44 advancing the grain edges towards the final grain boundaries along a linear seg-  
ment connecting the nuclei with a Voronoi vertex. At a specified  $f_s$ , flow and  
46 deformation are simulated through the liquid channels and within the grains,  
respectively. Vernede et al. [20, 21] used this modelling technique to simulate,  
48 in 2D, the solidification sequence, grain percolation, and fluid flow of an Al-Cu  
binary alloy. Phillion et al. [17] extended Vernede’s approach to 3D, showing  
50 that the third dimension allows for concurrent continuity of both the liquid  
and solid phase. Sistaninia et al. [6, 12, 18, 19] then developed a sophisticated  
52 3D hydro-mechanical model to simulate the semisolid constitutive behaviour of  
metallic alloys. Following reconstruction of the semisolid microstructure com-  
54 posed of a network of micro liquid channels and solid grains, the separation of

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<sup>1</sup>On the order of 1000s of grains

the liquid channels as a result of semisolid deformation and fluid flow was pre-  
dicted. The calculated pressure and fluid velocity inside each channel were then  
used in conjunction with a cracking criteria to directly model hot crack initia-  
tion and growth. Phillion et al. [16] and Zaragoci et al. [22] have also developed  
granular-type models to investigate additional aspects of semisolid deformation  
and crystal rearrangement.

Despite many similarities between casting and fusion welding processes, fun-  
damental differences do not allow for direct use of prior granular models to  
investigate hot cracking in welding. First, the prior models are intended to  
develop equiaxed-globular microstructure and cannot reconstruct the columnar  
grains that form an essential part of welding microstructure. Second, semisolid  
deformation occurs during welding as a result of a complex and evolving thermo-  
mechanical stress field. Third, the weld mushy zone contains large thermal gra-  
dients and consequently large spatial variations in  $f_s$ . Ploshikhin et al. [28] intro-  
duced a preliminary integrated mechanical-metallurgical model for hot cracking  
during welding, examining strain accumulation in the liquid at the centerline of  
a laser weld using the finite element method. In this previous work, the liquid  
weld was modelled as a weak solid, which then interacted with the surrounding  
base metal. Recently, Zareie Rajani and Phillion have presented a set of numer-  
ical modules as initial steps towards developing a comprehensive 3D multi-scale  
and multi-physics model of hot cracking in welding. In the first contribution,  
the evolving 3D semisolid weld microstructure consisting of both equiaxed and  
columnar grains was simulated within a granular context [29] as a function  
of GTAW current and travel speed. In the second contribution, the transient  
thermo-mechanical stresses acting on the weld mushy zone were simulated to  
directly quantify the separation rate of the micro liquid channels [30].

Although the previous multi-scale and multi-physics models of welding so-  
lidification and semisolid deformation have provided fundamental insight into  
hot crack formation, they do not directly predict damage initiation that eventu-  
ally leads to a hot crack. In this study, a comprehensive 3D model is proposed  
that couples the deformation within the weld mushy zone, fluid flow through

the micro liquid channels existing at high  $f_s$ , and hot crack initiation. This has been achieved through development of a sequential method that links the interactions between the solidification shrinkage, thermo-mechanical forces, restraining forces, and the transient pressure within the melt pool. The simulation results are applied to GTAW of AA6061 and then compared with experimental studies to assess the performance of the model.

### 3. Model Development

The comprehensive multi-physics and multi-scale model of hot cracking during welding is described below. It consists of four modules: a welding solidification module (WSM), a thermo-mechanical analysis module (TMM), a fluid-flow module (FFM) and a crack initiation module (CIM). The WSM creates the desired weld mushy zone microstructure for a given average  $f_s$ . This is then used by both the TMM and the FFM as the input semisolid geometry for simulating deformation and fluid flow. Physical quantities from all three modules are combined within the CIM to predict crack initiation in welding micro liquid channels.

#### 3.1. Welding Solidification Module

The solid-liquid geometry is created using a 3-D granular model for welding. The model simulates the evolution of the weld mushy zone composed of both continuous liquid films and solidifying grains. This is accomplished by coupling a modified Voronoi tessellation to provide grain structure, the variation of  $f_s$  with temperature based on Scheil-type solidification, and imposed thermal fields calculated using the Rosenthal equation. The model is sensitive to welding conditions, creating different microstructures based on the input welding parameters by varying the number and position of the grain nuclei that define the Voronoi diagram.

The specifics are as follows (more details can be found in [29]). First, an unstructured mesh representing the as-solidified weld microstructure is generated using individual Voronoi diagrams to create the base metal, columnar zone,

and equiaxed zone, with each Voronoi cell/polyhedron denoting one grain. The  
 116 nodes and elements of these diagrams are combined together to create a single  
 continuous mesh. Second, each grain is further divided into small tetrahedral  
 118 elements having one vertex at the grain nuclei and the other three on the grain  
 surface. Third, 1D solidification is simulated within an element given a specified  
 120  $f_s$  versus temperature curve and imposed temperature evolution. The temper-  
 ature at any position in any element is calculated via linear interpolation of the  
 122 four nodal temperatures. At the end of solidification, the solid-liquid interfaces  
 from opposing grains come into contact with each other and coalesce following  
 124 the thermodynamic criterion proposed by Mathier [23].

The temperature field is calculated from the 3D Rosenthal equation,

$$\frac{2\pi(T - T_0)KR}{Q} = \exp\left[\frac{-V(R - X)}{2\alpha}\right], \quad (1)$$

where  $R$  is the radial distance from the weld centre,  $X$  is the distance from the  
 126 weld centre along the weld line,  $T_o$  is the initial temperature of the workpiece,  
 $V$  is the travel speed of the torch,  $Q$  is the heat transferred from the torch  
 128 to the metal, and  $K$  and  $\alpha$  are the thermal conductivity and diffusivity of  
 the base metal. The value of  $Q$  is used as a fitting parameter to match the  
 130 predictions made by the Rosenthal equation to thermocouple data from welding  
 experiments, with each experiment having a different  $Q$  value.

132 The output of the WSM is a 3D representative volume element (RVE)  
 consisting of solid grains and micro liquid channels that characterizes the mi-  
 134 crostructure within a region of the mushy zone formed during GTAW at a given  
 average  $f_s$ . An example microstructure near the end of solidification and show-  
 136 ing the grains and liquid channels is given in Fig. 1. Only one side of the weld  
 is modelled due to symmetry. Due to the strong thermal gradients inherent in  
 138 the process, the local  $f_s$  will vary considerably from the average value. The  
 size of the RVE is given by the welding experimental data, which specify the  
 140 weld half-width and penetration depth for each set of welding parameters. The  
 dimension along the weld line is set to 1000  $\mu\text{m}$ . This value was chosen as a  
 142 compromise between capturing the entire mushy zone and computational cost.

### 3.2. Thermo-Mechanical Analysis Module

Semisolid deformation during welding occurs mainly as a result of solidification shrinkage and externally-applied mechanical deformation. Assuming that the walls of the micro liquid channels within the RVE remain parallel during welding [6], the deformation rate  $\dot{\delta}_z^i$  of each channel can be decomposed into an internal component  $\dot{\delta}_{z,int}^i$  and an external component  $\dot{\delta}_{z,ext}^i$ ,

$$\dot{\delta}_z^i = \dot{\delta}_{z,int}^i + \dot{\delta}_{z,ext}^i. \quad (2)$$

where  $i$  is a counter representing the channel number (or more generally the element number) and  $z$  is a local coordinate having a direction perpendicular to the solid-liquid interface. The internal component represents solidification shrinkage. The external component can be further decomposed into contributions from thermo-mechanical deformation between the base metal and the weld, and the restraining forces acting along the edge of the workpiece. The output of the TMM are the  $\dot{\delta}_z^i$  values for each channel. As can be seen through Eq. (2), each channel will be subject to a different deformation rate.

The internal deformation rate resulting from solidification shrinkage is given by  $\beta v^*$  where  $v^*$  is the velocity,  $\beta = (\rho_s/\rho_l - 1)$  is the shrinkage factor, and  $\rho_l$  and  $\rho_s$  are the densities of the liquid and solid. Due to the strong thermal gradients, the liquid channel walls can have different velocities especially at the columnar/equiaxed interface. Thus the general form must be decomposed into

$$\dot{\delta}_{z,int}^i = \beta(v_1^* + v_2^*), \quad (3)$$

where  $v_1^*$  and  $v_2^*$  represent the velocities of the two channel walls resulting from solidification shrinkage and given by the WSM for each micro liquid channel.

The external deformation rate is a result of solidification shrinkage in combination with mechanical deformation of the base metal. Recently, Zareie Rajani and Phillion [30] modelled the evolution of thermo-mechanical stresses that act laterally on the fusion surface to deform the mushy zone during GTAW for a given set of welding parameters at high  $f_s$ . As described in [30], these stresses can be coupled with an appropriate visco-plastic constitutive equation in order

to calculate the thermal strain rate perpendicular to the weld line,  $\dot{\epsilon}_{th}$ . Similarly, the restraining forces will result in an imposed strain rate applied to the edge of the work-piece also in a direction perpendicular to the weld line,  $\dot{\epsilon}_{re}$ . In the case of self-restraint, such as when the work-piece is clamped,  $\dot{\epsilon}_{re} = 0$ . The external strain rate  $\dot{\epsilon} = \dot{\epsilon}_{th} + \dot{\epsilon}_{re}$  is then translated into an external deformation rate as

$$\dot{\chi}_{ext} = \dot{\epsilon} \cdot l, \quad (4)$$

where  $l$  is the initial lateral length of the mushy zone. The value of  $l$  varies with position due to the shape of the weld pool and thus the mushy zone must be discretized into a series of lateral bar elements as shown in Fig. 2 with each having a unique length  $l^j$  resulting in a unique  $\dot{\chi}^j$ , where  $j$  is a counter representing the bar number. Following a partitioning technique proposed by Coniglio and Cross [31],  $\dot{\chi}^j$  is distributed equally among every micro liquid channel within a bar element that has a local  $f_s$  below the solid fraction for grain coalescence,  $f_s^{coh}$ . In elements having  $f_s > f_s^{coh}$ , it is assumed that the dendrite arms on the walls of the micro liquid channels have merged to form a solid bridge that is much stronger than unbridged channels and thus do not preferentially deform. Thus,

$$\dot{\delta}_{ext}^k = \eta \frac{\dot{\chi}^j}{N_u^j}, \quad (5)$$

where  $N_u^j$  is the number of unbridged channels within the bar element  $j$ ,  $k$  is a subset of  $i$  that identifies all the micro liquid channels within the bar element  $j$ , and  $\eta$  has a value of 0 if  $f_s$  is greater than  $f_s^{coh}$  and 1 otherwise. Finally, as the micro liquid channels have random orientations within the mushy zone whereas the term  $\dot{\chi}_{ext}$  acts lateral to the fusion surface, Eq. (5) is further modified to yield the component of  $\dot{\delta}_{ext}^k$  along the local  $z$  direction of the micro liquid channel  $k$ ,

$$\dot{\delta}_{z,ext}^k = \eta \frac{\dot{\chi}^j (\vec{e}_Y \cdot \vec{e}_z^k)}{N_u^j}, \quad (6)$$

154 where  $\vec{e}_Y$  and  $\vec{e}_z$  represent unit vectors in the global  $Y$  direction and the local  $z$  direction for channel  $k$ .

### 156 3.3. Fluid Flow Module

The solidification shrinkage as well as the externally-applied deformations, both calculated within the TMM on individual channels, act as a driving force to induce fluid flow in the weld mushy zone. Flow occurs from the weld pool into the network of micro liquid channels. The FFM then calculates the variation in fluid velocity and pressure on all the micro liquid channels within the domain. The FFM is based on Sistaninia et al. [6] granular fluid flow model, uses the unstructured mesh from the WSM to describe the geometry of the micro liquid channels, assumes that flow occurs only within the interconnected intergranular regions (i.e. not through interdendritic regions within grains) and parallel to the walls of each micro liquid channel, and is laminar, incompressible and irrotational.

The Navier-Stokes equation with the above assumptions leads to the Poiseuille flow equation between two parallel plates [6],

$$\vec{v}_\ell = \frac{1}{2\mu_\ell} \vec{\nabla} p_\ell [z^2 - h^2], \quad (7)$$

where  $\vec{v}_\ell$  is the fluid velocity parallel to the walls of the micro liquid channel having  $x$  and  $y$  components,  $p_\ell$  is the pressure in the liquid,  $\mu_\ell$  is the dynamic viscosity, and  $h$  is the half-width of the micro-liquid channel. By combining Eq. (7) with a local mass balance taking into account channel deformation resulting from Eq. (2), the pressure in a micro liquid channel is given by

$$\frac{2h^3}{3\mu_\ell} \nabla^2 p_\ell = \dot{\delta}_z^k, \quad (8)$$

$$i.e. \quad \frac{2h^3}{3\mu_\ell} \nabla^2 p_\ell = \dot{\delta}_{z,int}^k + \dot{\delta}_{z,ext}^k. \quad (9)$$

The left hand side of Eq. (9) provides the pressure gradient required to compensate the solidification shrinkage, thermo-mechanical deformations and restraining forces can be seen on the right hand side. Further, Eq. (9) implies that the thickness of the micro liquid channel, itself a function of the local  $f_s$ , will significantly affect the local pressure and fluid velocity. The term relating to

the restraining forces,  $\delta_{z,ext}^k$  was added in this work to Sistaninia's FFM model  
 174 due to its importance on fluid flow during welding.

The FFM boundary conditions require careful consideration to ensure their  
 176 applicability to GTAW. As Fig. 3(a) illustrates, solving Eq. (9) over the RVE  
 given by the WSM requires knowledge of boundary conditions on the following  
 178 surfaces: S1 inlet surface through which the molten material enters the RVE,  
 S2 outlet surface through which the liquid leaves the RVE, S3 a top surface  
 180 representing the weld/atmosphere boundary, S4 a fusion surface separating the  
 weld mushy zone from the base metal, and S5 a symmetry plane at the centre  
 182 of the weld. The specifics are as follows. S3 and S4 are set to  $\vec{v} = 0$  since the  
 molten metal cannot flow into the solid, nor escape into the atmosphere. S5 is  
 184 homogeneous with  $\frac{\partial p}{\partial n} = 0$ . The inlet and outlet boundary conditions are more  
 complex. Since the RVE can be located anywhere within the mushy zone, the  
 186 boundary fluid velocity and boundary pressure values on the inlet and outlet  
 are, in general, also unknown. However, if the RVE is located right behind the  
 188 weld pool, the boundary pressure on the inlet surface equals the static pressure  
 inside the weld pool, and this static pressure can be assumed to be the same  
 190 as the arc pressure that acts on top of the weld pool. Similarly, if the RVE is  
 located at the end of the mushy zone where the average  $f_s$  approaches unity, it  
 192 can be assumed that  $\vec{v} = 0$  since liquid cannot flow into the fully solidified weld.

The above discussion implies that the fluid flow module can only be applied  
 194 to a model containing the entire weld mushy zone, from the weld pool to the  
 fully solidified weld. This would significantly increase computation time. To  
 196 overcome this limitation, the entire mushy zone is sub-divided into five smaller  
 RVEs having continuity in average  $f_s$  at the interface between each RVE. The  
 198 fluid flow and pressure in each RVE are then determined in a sequential manner  
 as first proposed in [6] and shown schematically in Fig. 3(b). In the first step,  
 200 the analysis of fluid flow begins by solving Eq. (9) in a small RVE that is in  
 contact with the solid weld. Since there is no flux possible at the interface with  
 202 fully solid material, the outlet boundary condition becomes  $\vec{v} = 0$ . Since fluid  
 flow is a function of pressure gradient and not the actual pressure, the inlet

204 boundary condition is an imposed pressure set equal to zero. The inlet flow  
 into the small RVE is then calculated. This value is then imposed on the outlet  
 206 side of the next RVE (i.e.  $q_{1,2}$  in Fig. 3(b)) and the inlet pressure set equal to  
 zero, to again calculate the inlet flow. The process is continued for each small  
 208 RVE. In the last RVE, in contact with the weld pool, the imposed pressure on  
 the inlet side is set to the arc pressure instead of being set equal to zero. In the  
 210 second step, the analysis is reversed, beginning with the RVE in contact with  
 the weld pool. Here, the fluid velocity obtained in the last small RVE of the  
 212 first step is applied as the boundary condition at the outlet, while the imposed  
 pressure on the inlet side is set to the weld arc pressure [32]. The pressure drop  
 214 across the RVE is then calculated. This process is continued until the pressure  
 drop across the entire weld mushy zone is determined. For these simulations,  
 216 the arc pressure was assumed to be 3 atm. This value was chosen based on  
 measurements of the average stagnation pressure on the free surface of the weld  
 218 pool made during the welding experiments reported in [29].

### 3.4. Crack Initiation Module

The crack initiation module applies Kou's micro-scale hot cracking crite-  
 rion [1] to simulate the occurrence of hot cracks during welding. According to  
 Kou's criterion, a micro liquid channel located between two solidifying grains  
 will crack (i.e. form a void) if the separation rate of the channel walls exceeds  
 the sum of the wall growth rate and the local feeding rate. The criterion is  
 expressed as,

$$KCC = \frac{d\epsilon_\ell}{dt} - (1 - \beta)^m \frac{df_s^m}{dt} + ((1 - (1 - \beta)^m) f_s^m) \frac{dv_\eta}{d\eta}, \quad (10)$$

220 where  $KCC$  is a numeric value representing the Kou cracking index,  $\frac{d\epsilon_\ell}{dt}$  is the  
 local strain rate perpendicular to the flow direction caused by the solidification  
 222 shrinkage and external strains,  $m$  is a microstructure parameter with value of  
 0.5 and 0.333 for columnar and equiaxed grains respectively, and the term  $\frac{dv_\eta}{d\eta}$   
 224 represents the first derivative of the intergranular liquid flow velocity along the

dominant flow direction,  $\eta$ . If  $KCC > 0$ , micro liquid channels are susceptible to cracking.

The CIM uses inputs from the WSM, TMM and FFM to evaluate Kou's criterion and thus to identify micro liquid channels that have cracked as a function of average  $f_s$ . Specifically,  $f_s^i$ , and  $\frac{df_s^i}{dt}$ , are given by the WSM,  $\frac{d\epsilon_\ell^i}{dt} = \frac{\dot{\delta}_z^i}{h}$  is given by the TMM, and the local fluid velocity vector is given by the FFM to obtain  $\frac{dv_\eta^i}{d\eta}$ . The method to determine this last term requires further explanation. As Fig. 4 shows, the FFM does not yield a single flow direction within a micro liquid channel but rather a fluid flux  $\vec{q}$  and thus a fluid velocity vector,  $\vec{v}_\ell$  at each edge. To determine the dominant flow direction, the arithmetic mean is taken from the components of each edge as

$$v_\eta^{i,x} = \frac{\sum_{ii=1}^3 v_x^{ii}}{3} \text{ and } v_\eta^{i,y} = \frac{\sum_{ii=1}^3 v_y^{ii}}{3}, \quad (11)$$

where  $v_\eta^{i,x}$  and  $v_\eta^{i,y}$  are the velocity components in  $x$  and  $y$  along the dominant flow direction in each channel  $i$ . The summation range is 1...3 because each micro liquid channel is prismatic/triangular with fluid entering along the three "sides" of the prism. This flow direction is then used to identify the dominant flow line within the element. The edges that intersect the flow line represent the inlet and outlet edges, respectively. Since the triangular elements in this model are very small, the derivative of the fluid velocity along the dominant flow direction can be obtained by,

$$\frac{dv_\eta}{d\eta} = \frac{|\vec{v}^{outlet}| - |\vec{v}^{inlet}|}{L_{flowline}}. \quad (12)$$

### 3.5. Model Implementation

Hot cracking during aluminum alloy welding is assessed as follows. First, the WSM, a purpose-written C++ code that uses the Voro++ open-source library to create a modified Voronoi tessellation, is executed to create welding microstructure / numerical mesh at a specified average  $f_s$  for a given set of welding parameters. Second, the TMM, a purpose-written C++ code that interfaces with the Abaqus FEA suite, is used to determine the mechanical response of

234 the weld mushy zone for a given welding parameters (i.e. the  $\dot{\delta}_{z,int}^i$  and  $\dot{\delta}_{z,ext}^i$   
 values for each micro liquid channel). Specifically, this is determined by (i) ap-  
 236 plying Eq. 3 using the  $v_1^*$  and  $v_2^*$  values from the WSM in order to calculate  
 the internal deformation rates for each micro liquid channel; (ii) performing a  
 238 mechanical simulation [30] on the base metal to determine the lateral stress  
 acting on the fusion surface; and (iii) applying Eqs. 4-6 to determine the cor-  
 240 responding external deformation rates resulting from the lateral stresses and  
 partition to each micro liquid liquid channel. Then, with the  $\dot{\delta}_{z,int}^i$  and  $\dot{\delta}_{z,ext}^i$   
 242 values, the pressure  $p_\ell$  and the fluid velocity  $\vec{v}_\ell$  are calculated using a modified  
 version of Sistaninia's C++ FFM code [6] using the numerical mesh from the  
 244 WSM. Combined, the WSM, TMM, and FFM provide all the variables/inputs  
 required to apply Kou's criterion. Finally, the CIM is then applied to identify  
 246 all the micro liquid channels that have cracked. Although not simulated in this  
 research, the newly-formed micro cracks would join together with additional  
 248 deformation to form a visible hot crack. Note that the coupling is one way only,  
 i.e. the failed channels identified within the CIM do not influence the FFM, nor  
 250 the TMM. Note also that the model is not transient but instead assesses the  
 state of hot crack formation for a specified average  $f_s$ . Individual simulations  
 252 are then performed for different values of  $f_s^{avg}$  to quantify the variation in hot  
 crack susceptibility.

#### 254 **4. Results and Discussion**

The simulations presented below investigate hot cracking tendency during  
 256 GTAW of the Al-Mg-Si alloy AA6061 for different welding currents and travel  
 speeds. The domain size is  $\approx 2$  mm (penetration depth) by  $\approx 3$  mm (weld half-  
 258 width), and  $\approx 1000 \mu\text{m}$  (along the weld line), and consists of  $\approx 10,000$  grains.  
 These values are approximate, since each set of welding parameters will result  
 260 in a different weld size and grain density. First, the internal and external defor-  
 mations are presented to investigate their relative contributions to weld mushy  
 262 zone dynamics. Second, the fluid flow through the mushy zone under differ-

ent welding conditions is examined to gain insight into flow behaviour and the  
link between flow and microstructure. Then, crack formation in micro liquid  
channels is predicted and explored.

#### 4.1. Mushy Zone Deformation

A comparison of the average semisolid deformation rates during welding re-  
sulting from solidification shrinkage (internal deformation rate) and externally-  
applied mechanical deformations as calculated by the TMM is shown in Fig. 5.  
As can be seen, the internal deformation rate is quite high at low solid frac-  
tion but decreases as the solidification process advances towards the fully solid  
state. This is a result of the solidification rate  $\frac{df_s}{dT}$ , which continually decreases  
with increasing fraction solid for alloys having mostly a primary solid phase  
such as AA6061. For the problem of hot cracking, fortunately, the low solid  
fraction at high internal deformation rates enables the healing or liquid refilling  
of any initiated micro crack. In contrast, the external deformation rate induced  
by thermomechanical stresses and lateral restraining deformations is seen to  
be initially small but then becomes quite influential, and will strongly affect  
hot cracking susceptibility. Further, this term is quite sensitive to welding pa-  
rameters, whereas the internal deformation rate is not; modifying the welding  
conditions while still applying a tensile restraining rate of  $0.1 \text{ s}^{-1}$  (curve 3 to  
curve 4) is seen to double the average external deformation rate at high solid  
fraction. Thus, it would appear from these simulations that both welding pa-  
rameters and restraining strains mainly affect hot cracking susceptibility due to  
their effect on the semisolid deformations resulting from the thermomechanical  
response of the base metal.

#### 4.2. Fluid Flow

Fig. 6 compares the FFM results for two different welding conditions at  $f_s$   
 $= 0.85$ . The images on the left show the cross-sectional distribution of the  
magnitude of the fluid velocity vectors, i.e.  $\|\vec{v}_\ell\|$ , while the images on the right  
show the corresponding semisolid microstructure. The semisolid weld shown in

292 Fig. 6(a) corresponds to a weld fabricated on a clamped plate using a welding  
 current of 140 A and a welding speed of  $5 \text{ mm s}^{-1}$ . As shown in prior exper-  
 294 iments [29], GTAW of a 3 mm sheet of AA6061 under these conditions create  
 in microstructure having an average columnar zone length of  $140 \text{ }\mu\text{m}$ , and an  
 296 equiaxed grain size of  $68 \text{ }\mu\text{m}$ . The figure reveals that fluid flow is not uniform  
 across the cross-section of the weld, but rather experiences severe localization  
 298 near the centre. This is a result of the localization of deformation within the  
 weld mushy zone. The TMM, Figure 6 of [30], has shown that the micro liquid  
 300 channels located near the centre of the weld experience much higher deforma-  
 tion rates. This will induce higher fluid velocities in this region, to compensate  
 302 for the faster separation rates of the channel walls. The semisolid geometry  
 shown in Fig. 6(b) represents the same welding parameters at Fig. 6(a) but  
 304 because weld microstructure manipulation [29] is assumed, the columnar grains  
 are much longer, at  $800 \text{ }\mu\text{m}$  but the equiaxed grain size remains the same.. Now,  
 306 the fluid flow has localized to within the columnar region of the weld. This is  
 because the WSM predicts that the liquid films separating the columnar and  
 308 equiaxed regions scale with the length of the columnar zone; wider liquid films  
 will transfer higher volumes of molten metal towards the columnar grains thus  
 310 increasing the volumetric feeding rate.

Fig. 7 plots the average fluid velocity as a function of average  $f_s$  across the  
 312 cross-section of the weld mushy zone for three different externally-applied tensile  
 strain rates. Examining first the data points<sup>2</sup> without external deformation, it  
 314 can be seen that the fluid velocity decreases significantly with increasing  $f_s$ . At  
 very high  $f_s$ , the micro liquid channels are quite small, making the permeability  
 316 of the mush too low to allow for significant fluid flow. Further, the no-flow  
 condition at  $f_s = 1$  applies a force acting in the opposite direction, inducing  
 318 fluid deceleration. The application of external lateral tensile strain raises the  
 fluid velocity throughout the weld mushy zone. These external deformations

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<sup>2</sup>Please note that the curves on Fig. 7, as well as Fig. 9, and 11 represent trend lines; the  
 data points come from the simulations

320 increase the micro liquid channel deformation rates  $\dot{\delta}_{z,ext}^i$ . Consequently, faster  
 flows are needed to compensate for the semisolid deformation. Fig. 8 illustrates  
 322 the corresponding cross-sectional distribution of the absolute pressure within the  
 semisolid weld at four locations between the weld pool and the fully solidified  
 324 weld. The solid grains are shaded with the corresponding pressure value of  
 their neighbour channels. As can be seen in the figure, the pressure field is  
 326 not uniform throughout the micro liquid channel network; the channels near  
 the centre have lowest pressure values. It should be noted that since the arc  
 328 pressure (boundary condition) was assumed to be 3 atm, the entire set of of  
 micro liquid channels shows a significant pressure drop, approaching nearly 2.8  
 330 atm of pressure loss in some regions of the mushy zone. According to Darcy's  
 law, two major factors affect the pressure drop within a porous medium: 1) the  
 332 velocity of the fluid flowing through the medium, and 2) the permeability of the  
 porous medium, with pressure drop being directly proportional to velocity but  
 334 inversely proportional to permeability. Therefore, one can conclude that that  
 the higher fluid velocities seen at the centre of the weld in Fig. 6 lead to the  
 336 localization of pressure drop in this region.

Fig. 9(a) plots the average pressure as a function of average  $f_s$  across the  
 338 cross-section of the weld mushy zone for six different sets of welding parameters,  
 each clamped to a base plate. The figure's legend indicates the welding condi-  
 340 tions; the numbers following the V represent the welding speed ( $\text{mm s}^{-1}$ ), while  
 the numbers following I show the welding current (A). In all cases, the average  
 342 pressure is seen to initially decrease to a minimum value near an average  $f_s$  of  
 0.8 and then rise again close to the final solidification. The initial pressure drop  
 344 along the weld mushy zone can be linked to the variation in permeability. For a  
 semisolid structure, the permeability is strongly influenced by the micro liquid  
 346 channel widths; wider channels at lower  $f_s$  have higher permeability [6, 20].  
 Therefore, as the average  $f_s$  behind the weld pool increases from 0 to 0.8, the  
 348 permeability of the weld mushy zone decreases and thus the average pressure  
 drop must increase in order to maintain a relatively fast fluid flow in that re-  
 350 gion. Interestingly, the increase in the pressure drop does not extend into the

regions with high  $f_s$ . This observation is quite different than results for granular  
solidification models applied to casting, where the pressure drop continues to  
increase until final solidification. One possible explanation is that at high  $f_s$  the  
external deformation rates applied to the micro liquid channels are smaller since  
most of the solid network has percolated. At  $f_s$  nearing 0.8, there are many  
micro liquid channels in each lateral bar, each deforming and thus requiring a  
pressure drop to achieve sufficient fluid flow. By comparing the different sets of  
data points in Fig. 9(a), it can be seen that welding parameters also affect the  
average pressure variation with increasing  $f_s$ . The model indicates that a weld  
fabricated by a welding speed of 2 mm s<sup>-1</sup> and a welding current of 95 A has  
the smallest minimum average pressure value inside the weld mushy zone, corre-  
sponding to the largest pressure drop. This prediction can be explained through  
the microstructure of the weld where V2I95 has a long columnar zone [29]. As  
demonstrated through Fig. 6, larger columnar grains intensify the fluid velocity  
through the columnar zone, correspondingly requiring larger pressure drops.

Fig. 9(b) shows the effect of welding constraints on the average pressure  
along the weld mushy zone for four different external lateral tensile strain rates.  
The weld was fabricated at a welding current of 120A and a welding speed  
of 4 mm/s. Based on the obtained results, increasing the external strain rate  
from 0 to 0.1 s<sup>-1</sup> slightly amplifies the pressure drop inside the weld mushy  
zone. The pressure drop decreases significantly at larger external strains, as  
shown by the data points for 1 s<sup>-1</sup>. This phenomenon can be linked to the  
effects of welding constraints on the fluid velocity field. As depicted in Fig. 7,  
increasing the external lateral tensile strain rate to even 0.1 s<sup>-1</sup> significantly  
raises the average fluid velocity through the weld mushy zone, inducing larger  
pressure drops. For a clamped weld, the results of the model show that the  
pressure drop between the weld pool and high average  $f_s$  regions at the end of  
the weld mushy zone is nearly 2 atm (Fig. 9(a)). This result is fairly similar to  
the pressure drop calculated by Coniglio and Cross during GTAW of clamped  
aluminum plates [31]. Using the RDG model, they have reported that the  
pressure drop between the aluminum weld pool and the end of the mushy zone

382 with  $f_s = 0.98$  varies in the range of 0.11 and 0.55 atm. Multi-physics models of  
hot cracking provide much more detailed information as compared to the RDG  
384 criterion, however, such as the occurrence and location of defects as explored  
below.

### 386 4.3. Defect Formation

The CIM detects defected micro liquid channels within a semisolid RVE.  
388 Fig. 10 shows qualitatively the location of defected micro liquid channels as a  
function of average  $f_s$ . These RVEs can be interpreted as either the various  
390 states of a specific location within a weld at different times during the welding  
process or at different sections along the weld mushy zone as shown in Fig. 3.  
392 Note that the defected channels formed at one average  $f_s$  are not transferred  
to the next in order to obtain a better understanding regarding the role of  
394 the mushy zone structure on defect localization. As can be seen, the defected  
channels formed at low average  $f_s$  are mainly located inside the columnar zone  
396 where the fluid experiences high deformation rates [30]. Upon approaching the  
fully solidified end of the weld mushy zone, however, the fluid velocity within  
398 each channel decreases significantly (Fig. 7 while the external deformation rate  
increases significantly. This causes a noticeable jump in the number of defected  
400 channels, which are mostly located within the equiaxed region and near the weld  
centreline. Such a shift from the columnar zone to the centre of the weld can be  
402 linked to the fact that with increasing  $f_s$ , the mushy zone deformation localizes  
to the centre of the weld since it is the last region to solidify.

404 To quantify the occurrence of channel defects that would initiate a hot crack,  
a *Kou Crack Index* is defined and given by the percentage of total micro liquid  
406 channels having  $KCC > 0$  at a given  $f_s$ . Hence, the defect index only accounts  
for the newly defected channels and not the defects formed at a previous  $f_s$ .  
408 Note also that RVEs with  $f_s$  smaller than 0.7 are not considered since it was  
assumed that there is sufficient fluid flow to heal any newly-formed hot crack  
410 in these areas of the mushy zone. Fig. 11(a) shows the Kou crack index for  
six welds fabricated under an external lateral tensile strain rate of  $0.1 \text{ s}^{-0.1}$  with

412 different welding parameters. A number of salient observations can be made.  
 First, the areas with higher average  $f_s$  are, relatively, very susceptible to hot  
 414 cracking as zones closer to the weld pool show negligible defected micro liquid  
 channels and thus have negligible contribution to the formation of hot cracks.  
 416 As discussed in Section 2.4, the Kou cracking criterion indicates that a con-  
 junction of fast deformation rates (the first term on the r.h.s. of Eq. 10 and  
 418 slow slow fluid flow (the third term on the r.h.s. of Eq. 10 induce hot tearing.  
 As seen in Fig. 10, the relatively fast fluid flow within the RVEs occurring at  
 420 low average  $f_s$  can accommodate for the deformation of the semisolid structure  
 and hence limits the occurrence of hot cracking to a few channels. Second, the  
 422 role of welding parameters on hot cracking, and specifically the travel speed,  
 is revealed by comparing the six provided sets of data points. As can be seen,  
 424 the model results reproduce the well-known behavior whereby increased travel  
 speed lowers the susceptibility of the weld mushy zone to hot tearing. Anal-  
 426 ysis of the model results reveals the mechanism resulting in this phenomenon.  
 Specifically, a higher travel speed increases the cooling rate, which lowers the  
 428 number of unbridged channels capable of developing defects at high average  $f_s$   
 where the accumulated deformation and slow fluid velocity can cause hot crack-  
 430 ing. Fig. 11(b) examines the effect of external lateral tensile strain rates on the  
*Kou Crack Index*. As can be seen, increasing the external strain rate, espe-  
 432 cially above a value of  $0.1 \text{ s}^{-1}$  significantly increases this index to a point where  
 more than one percent of all the micro liquid channels have cracked. Thus, this  
 434 value could be used as the critical strain rate for the formation of hot cracks  
 during GTAW of AA6061. The observed correlation between the externally-  
 436 applied strain rate and the formation of hot cracks is a result of the fact that as  
 shown by the TMM the external strain rate will increase  $\dot{\delta}_{z,ext}^i$  for each micro  
 438 liquid channel, making the weld mushy zone more susceptible to hot cracking.

#### 4.4. Model Verification

440 The results of the proposed multi-physics welding model are quite close to  
 the outcomes of several experimental studies. In terms of hot cracking, Arata

et al. [33] reported a critical strain rate value of  $0.25 \text{ s}^{-1}$  in arc welding of Al-Mg-Si alloys, while Coniglio and Cross reported a critical external strain rate value during GTAW of Al alloys of between  $0.05$  and  $0.3 \text{ s}^{-1}$  [31]. Both values are quite similar to the value of  $0.1 \text{ s}^{-1}$  obtained in this study. In terms of welding travel speed, Matsuda et al. have reported that, in Al alloys, increasing the travel speed from  $2.5 \text{ mm s}^{-1}$  to  $13 \text{ mm s}^{-1}$  during GTAW can improve the hot cracking resistance [34]. Chihoski [35] showed that at high welding travel speeds, a compressive stress field forms near the mushy zone, preventing hot cracking. Niel et al. [10] showed that for a constant current in GTAW, increasing the welding travel speed can prevent hot cracking in AA 6061 aluminum alloys. These findings are consistent with the finding of the present model. Prior welding experiments have also shown that reducing the size of the columnar zone through grain size manipulation techniques will reduce hot cracking, qualitatively matching the results shown in Fig 6. In separate studies, Mousavi et al. [36], Warrington et al. [37], and Dvornak et al. [38] each demonstrated that a reduction in Al alloy hot crack susceptibility could be achieved through the addition of grain refiners that shorten the columnar region inside the weld. Also, a recent synchrotron radiography experimental study on initiation and growth kinetics of solidification cracking during welding by Aucott et al. [39] confirms several hot cracking features predicted by the present model. Specifically, hot cracking is mainly driven by the strain acting on the solidifying weld, and further, micro cracks first initiate inside the columnar region near the fusion boundary and at later stages of solidification they begin to form at the core of the weld. This fracture growth pattern is quite similar to the crack advancement predicted by the model in Fig. 10 where hot cracks first initiate near the fusion boundary but the migrate towards the center of the weld as the average  $f_s$  increases.

## 5. Conclusions

470 A 3D multi-scale multi-physics model of GTAW has been developed to pre-  
dict hot crack formation in aluminum alloys. The model is made up of four  
472 modules: (I) a welding solidification module, (II) a thermo-mechanical analysis  
module, (III) a fluid flow module, and (IV) a crack initiation module. The model  
474 considers the effects of solidification contraction, the response of the base metal  
during welding, the application of external deformation, and fluid flow through-  
476 out the mushy zone on hot cracking susceptibility. With this information, the  
localization of hot cracks at individual grain boundaries as well as the overall  
478 hot cracking susceptibility are predicted. The model was verified against prior  
numerical and experimental data available in the literature, and qualitatively  
480 agree with these prior results.

Analysis of the simulations results help to explain the metallurgical mecha-  
482 nisms underlying hot crack occurrence during GTAW. First, the welding current  
and travel speed affect hot tearing susceptibility by strongly influencing the de-  
484 formation rates of the micro liquid channels, and the number of unbridged liquid  
channels capable of developing defects. Second, microstructure manipulation  
486 methods that lengthen the columnar zone increases the volumetric feeding rate.  
Third, the non-uniform liquid flow within the weld mushy zone that results from  
488 complex microstructure creates regions of very low pressure that act as crack  
nucleation sites. When the mushy zone cannot be fed, a hot crack will occur.  
490 Interestingly, micro cracks were predicted to initiate near the fusion zone but  
then localize at the core of the weld.

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494 Alcan, and the Natural Sciences and Engineering Research Council of Canada  
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## 496 6. Data Availabilitydd

The raw/processed data required to reproduce these findings cannot be  
498 shared at this time due to its size, but representative sample data is included  
in the Figures.

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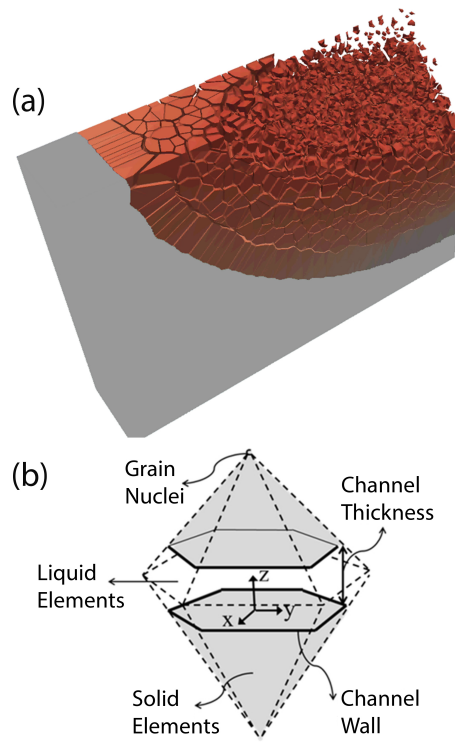


Figure 1: (a): Welding semisolid microstructure predicted by the WSM for GTAW of a 3 mm AA60601 sheet using a welding current of 140 A and a travel speed of 5 mm/s. (b) Schematic showing the elementary solid and liquid elements.

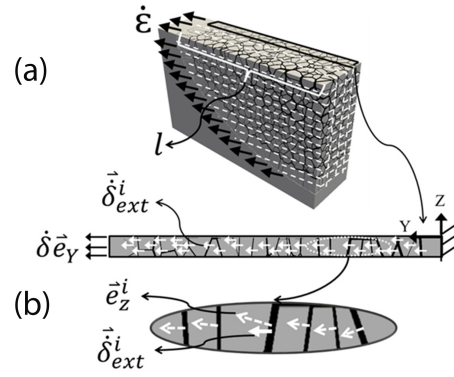


Figure 2: (a): Discretization of the welding mushy zone into lateral bar elements. Each bar element is indicated by dotted white lines, while the black arrows indicate the lateral deformation applied by the base material. (b) Schematic depicting the decomposition of the global deformation rate into external normal deformation rate vectors. After [30].

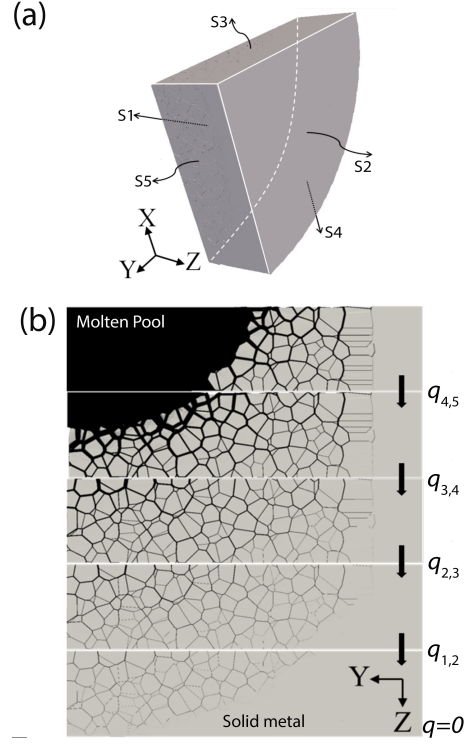


Figure 3: (a) Boundary conditions applied to the FFM RVE. (b) Schematic illustrating the mushy zone segments used to sequentially compute fluid flow within the weld mushy zone; the term  $q$  denotes the flux transfer from one segment to another.

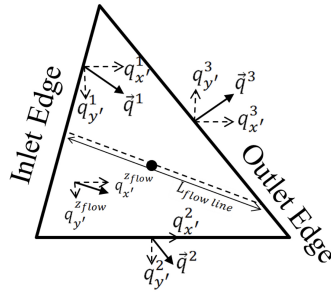


Figure 4: Fluid flux components within a micro liquid channel showing the dominant flow line.

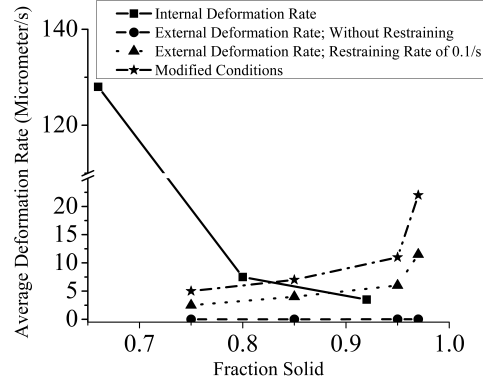


Figure 5: Comparison of the internal deformation rate resulting from the solidification shrinkage, and the external deformation rates for a clamped weld and one where a tensile restraining rate of  $0.1 \text{ s}^{-1}$  is applied, as calculated by the TMM, for a weld fabricated using a welding current of 140 A and a welding speed of  $5 \text{ mm s}^{-1}$ . The curve *Modified Conditions* was produced using welding conditions of 95 A, a welding speed of  $2 \text{ mm s}^{-1}$ , and a tensile restraining rate of  $0.1 \text{ s}^{-1}$ .

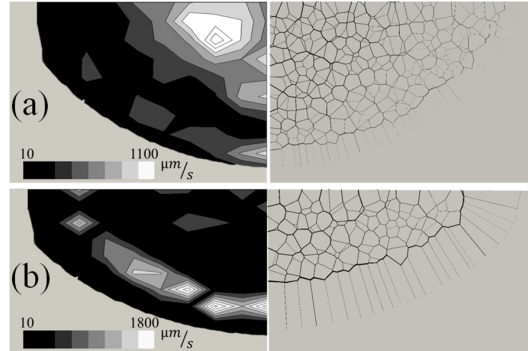


Figure 6: Cross-sectional distribution of the magnitude of the fluid velocity vector at an average  $f_s$  of 0.85 for a weld fabricated on a clamped plate using a welding current of 140 A and a welding speed of  $5 \text{ mm s}^{-1}$ , and having (a) an average columnar zone length of  $140 \text{ } \mu\text{m}$ ; (b) an average columnar zone length of  $800 \text{ } \mu\text{m}$ . The contours on the left show the distribution of the magnitude of the fluid velocity, while the images on the right show their corresponding microstructure.

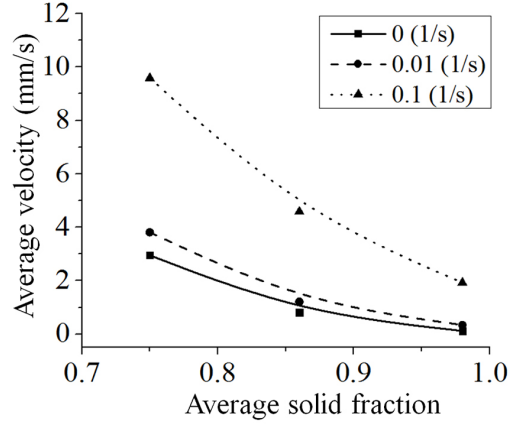


Figure 7: The variation in the average fluid velocity as a function of the average  $f_s$  for a weld fabricated for three different external lateral tensile strain rates. The weld was fabricated at a welding current of 140 A and a welding speed of 5 mm/s.

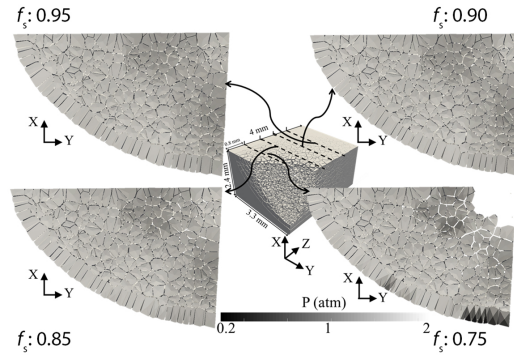
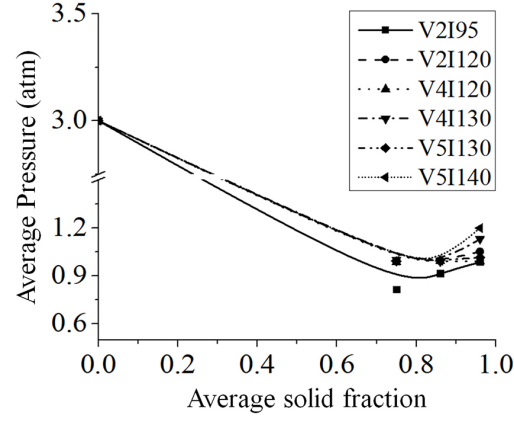
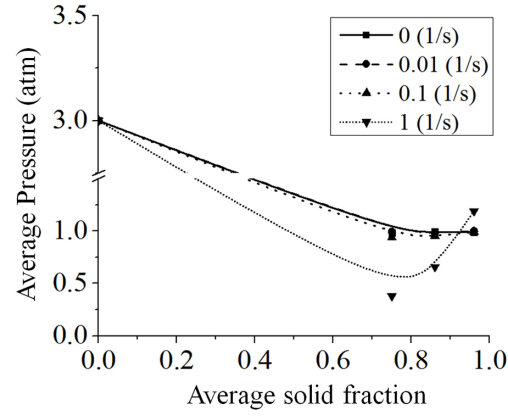


Figure 8: Cross-sectional distribution of the absolute pressure at different average  $f_s$  for a semisolid weld fabricated using a welding current of 140 A and a welding speed of 5 mm s<sup>-1</sup>. All pressures are below the applied arc pressure of 3 atm.



(a)



(b)

Figure 9: (a) The variation in average pressure as a function of average  $f_s$  for a clamped weld with various welding parameters. (b) The variation in the average pressure along the weld mushy zone for four different external lateral tensile strain rates. The weld was fabricated at a welding current of 120 A and a welding speed of 4 mm s<sup>-1</sup>.

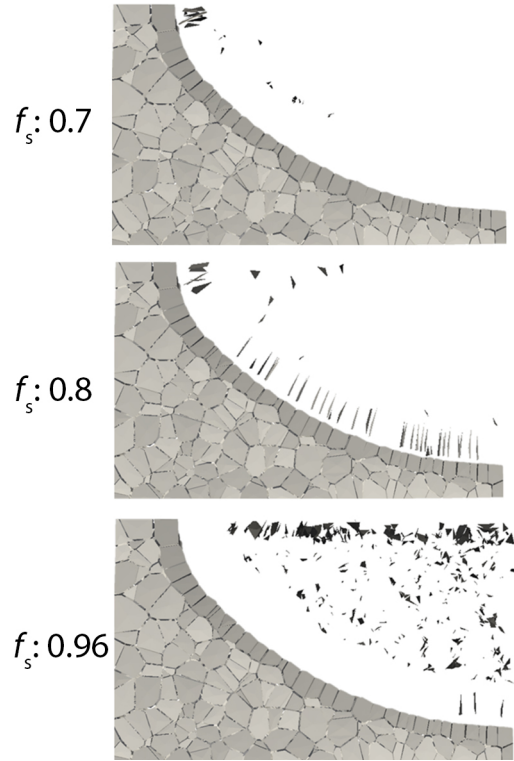
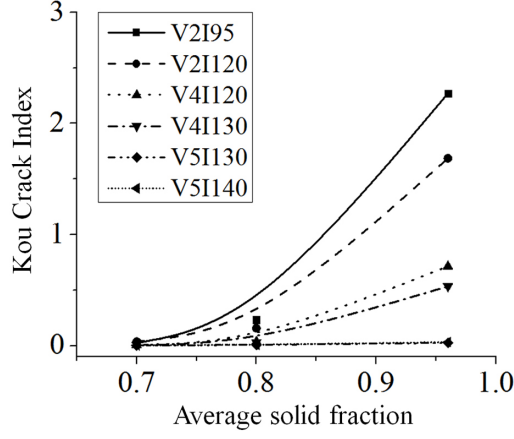
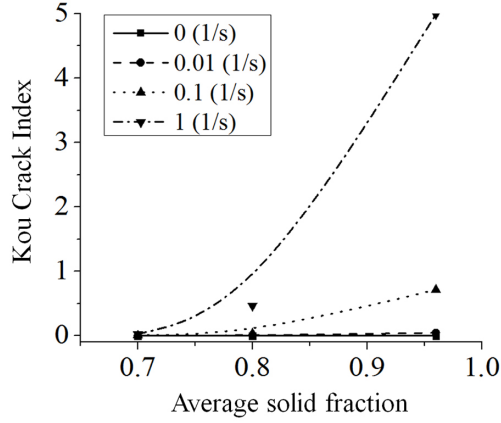


Figure 10: The distribution of newly defected channels at three different average  $f_s$  of 0.7, 0.8, and 0.96 based on the Kou cracking criterion. The weld was fabricated at a welding current of 120 A and a welding speed of 4 mm s<sup>-1</sup>.



(a)



(b)

Figure 11: (a) The Kou crack index for different RVE's with different average  $f_s$  along the weld mushy zone. The welds are fabricated on a plate under an external lateral tensile strain rate of  $0.1 \text{ s}^{-1}$  with six different sets of welding parameters. (b) The variation of the Kou crack index along the mushy zone of a weld fabricated at a welding speed of  $4 \text{ mm s}^{-1}$  and a welding current of  $120 \text{ A}$  under various external lateral tensile strain rates.