RISK MANAGEMENT FOR RAILROAD TRANSPORTATION OF HAZMAT

ANALYTICAL APPROACHES TO RISK ASSESSMENT AND MANAGEMENT FOR RAILROAD TRANSPORTATION OF HAZARDOUS MATERIALS

By

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تقدیم به پدر و مادر عزیزم

به خاطر فداکاری، صبر، حمایت و مهربانی بی دریغ شان

و همچنین به خاطر رنج دوری که متحمل شدند

To my beloved parents, **Fatemeh Zahra** and **Mirbaba**

For their personal sacrifice, patience, wholehearted support and deep affection

And also, for the pain they experienced because of me being far away from home

Abstract

Hazardous materials (hazmat), such as crude oil and gasoline, are harmful to humans and the environment because of their toxic ingredients, but their transportation is integral to sustain our industrial lifestyle. In North America, a significant portion of hazmat shipments is moved via the railroad network. Rail hazmat incidents are rare though the consequences could be catastrophic. The low probability–high consequence nature of such events mandate that a risk-averse plan be implemented for routing hazmat shipments.

We propose a value-at-risk (VaR) and conditional value-at-risk (CVaR) methodology to route rail hazmat shipments, using the best train configuration, over a given railroad network with limited number of train services, such that the transport risk is minimized. Freight train derailment records of the Federal Railroad Administration (FRA) were analyzed to model the behavior of railroad accidents, and to estimate their conditional probabilities. The proposed methodologies were used to study several problem instances generated using the realistic network of a railroad operator in Midwest United States, and to demonstrate that they are superior to the existing risk measures in the literature in regard to providing risk-averse routing of hazmat shipments and being versatile enough to yield various routes based on the risk preferences of decision makers.

Next, we propose a CVaR model, as a risk-averse routing plan for multiple rail hazmat shipments and multiple origin-destinations pairs, such that the total transport risk in the railroad network as measured by CVaR is minimized. However, it may happen that certain links and yards of the railroad network tend to be overloaded with hazmat traffic and risk. To overcome this issue, we also promote equity in the spatial distribution of risk. Therefore, the main problem is to find minimum risk routes, while limiting and equitably spreading the risk in any zone where the railroad network is embedded. The problem is mathematically formulated, and a heuristic algorithm is proposed for its solution, which takes into consideration the risk load limits on arcs and transferring yards and spreads the risk equitably throughout the network. Moreover, a lower bound based on a Lagrangian relaxation of the mathematical formulation is also provided. Finally, several computational experiments are developed using the above realistic railroad network.

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Declaration of Academic Achievement

This thesis has already resulted in two publications listed below. Currently, we are working on the outcomes of Chapter 5 to be submitted to another top-tier journal.

- SD Hosseini, M Verma. *Conditional Value-at-Risk (CVaR) Methodology to Optimal Train Configuration and Routing of Rail Hazmat Shipments*. **Transportation Research Part B: Methodological 2018**; 110: 79-103.

- SD Hosseini, M Verma. *A Value-at-Risk (VaR) Approach to Routing Rail Hazmat Shipments*. **Transportation Research Part D: Transport and Environment 2017**; 54: 191-211.

Chapter 1. Introduction

Hazardous materials (hazmat), as defined by the U.S. Department of Transportation Pipeline and Hazardous Materials Agency (2007), are substances or materials capable of posing an unreasonable risk to health, safety, or property when transported in commerce. Hazmat, such as crude oil and gasoline, are harmful to humans and the environment because of their toxic ingredients, but their transportation is integral to sustain our industrial lifestyle. In North America, a significant portion of the hazmat shipments is moved via the railroad network. Based on the latest commodity flow survey for 2012, railroad carried around 111 million tons of hazmat in the United States (U.S. Department of Transportation, 2015), whereas the number for Canada was 26 million tons (Searag et al., 2015). It may appear that railroads are not the predominant mode for surface transportation of hazmat, but they are almost always preferred to move shipments over long distances. In fact, in the United States, railroads account for around 29% of hazmat movement in ton-miles compared to 32.2% for trucks, which in turn translated into a 27.9% increase from 2002 (Bagheri et al., 2014). The quantity of hazmat traffic on railroad networks has been increasing steadily since 2009, in large part due to the need to move crude oil shipments from the Bakken shale formation region in the United States and Canada to the refineries along the southern and eastern coast of the continent, and the increased utilization of intermodal transportation to move chemicals (AAR, 2014; CAPP, 2014).

Railroad is one of the safest modes for transporting hazmat, but the possibility of spectacular events resulting from multicar incidents, however small, does exist (Verma and Verter, 2013). Most recently, as an example of the possible catastrophe associated with rail hazmat shipments, the Lac-Mégantic (Quebec) freight train derailment tragedy, in July 2013, resulted in 47 deaths and mass evacuation of 2,000 persons, equivalent to one-third of the town's population. 63 tank cars derailed, many ruptured and escaping crude oil ignited, leading to a succession of powerful explosions which destroyed much of the downtown core. The perforated tank cars disgorged an estimated 6 million liters of oil, much of which caught fire and burned, or seeped into the soil, but about 100,000 liters spilled into the waters. In fact, around 125 train accidents involving release from multiple tank cars have been reported over the past decade in the United States, which translates into an average of seven accidents every year (FRA, 2014). Verma (2011) collected information on freight derailment from 1995 to July 2009 (FRA, 2010) and analyzed around 25,000 derailment instances to conclude that the five main causes of train accidents are as follows: Human factors (39%), track, roadbed, and structures (30%), mechanical and electrical failures (10%), signal and communication (2%), and finally miscellaneous causes (19%). It is important to note that, over the past four decades, the railroad industry has spent considerable effort in reducing the frequency of tank car accidents as well as the likelihood of releases in the event of an accident. The more recent academic and industry initiatives have focused on analyzing past accident data in an effort to increase railroad safety by improving rail-tracks or railcar tank designs; and, on the development of risk assessment methodologies that incorporate the specific nature of railroad shipments.

Though the last decade has witnessed the development of risk assessment methodologies that incorporate specific nature of railroad shipments, most of the risk assessment methodologies for hazmat shipments were developed in the highway domain, and given the *low probability –high consequence* nature of rail hazmat incidents, their efficacy is rather limited. For example, the most popular measure of hazmat risk *viz.*, expected consequence is risk neutral, and hence would be unable to prevent high consequence events. In addition, all the existing risk assessment measures developed within the railroad domain yield a single route between a given origin-destination pair, which is not suitable from the perspective of a regulator interested in not overloading any segment or part of the network. Thus, there is a need for a risk assessment methodology that cannot only ensure risk-averse routing of rail hazmat shipments, but also incorporate the risk tolerance of the decision maker to generate multiple routes between a given origin-destination pair.

We make the first attempt to fill that gap by developing a *Value-at-Risk* (VaR) and *Conditional Value-at-Risk* (CVaR) risk assessment methodology to facilitate simultaneously risk-averse and flexible routing of rail hazmat shipments. VaR and CVaR as measures of risk have their origin in the portfolio management area of finance, but have found applications outside finance and economics such as agriculture (Prruzo et al., 2003), and for highway hazmat shipments (Kang et al., 2014a; Kwon 2011). Unlike the other existing approaches for rail hazmat routing, VaR and CVaR are able to generate alternative route choices given different confidence levels, instead of a single optimal route output. In other words, depending on the decision makers' attitude to risk, multiple planning decisions can be made according to each individual risk preferences. In addition, while most existing hazmat routing methods study the entire risk distribution, these new models, especially CVaR, focus more on the long tail to avoid extreme events (catastrophic rail hazmat accidents), which is more reasonable for rail hazmat transportation.

The rest of the proposal is organized as follows. *Chapter 2* reviews the relevant literature, while *Chapter 3* comprises an outline of the railroad transportation system and a detailed discussion of the proposed VaR methodology. It also describes the case study and the parameter estimation technique used in both chapters 3 and 4. Then it presents the solution and analyses gained from solving several problem instances. Finally the chapter throws light on the performance of the proposed methodology vis-à-vis the three most popular measures of hazmat risk in railroad transportation. *Chapter 4* contains a detailed discussion about the proposed CVaR methodology. Several problem instances are then solved and analyzed to gain insights and the results are compared with those of *Chapter 3*. Finally, in *Chapter 5*, we extend the work of *Chapter 4* and adapt the developed CVaR methodology to be applied to the more realistic case where there are multiple origin-destination and multiple hazmat shipments in the railroad network. Additionally, we address the risk equity constraints in the proposed optimization model in order to avoid risk congestion in specific rail arcs and transferring yards of the network.

Chapter 2. Literature Review

Although railroads move a significant quantity of hazmat both in the United States and in Canada, which has translated into increased research over the past decade, an overwhelming majority of academic initiatives in the preceding periods focused on road shipments (Erkut et al. 2007). The tremendous strides made in the highway domain, unfortunately, could not be extended to railroads because of differences between the two modes. For example, a train usually carries both regular and hazmat cargo together, whereas these two are almost never mixed in a truck shipment. Secondly, a rail tank car has roughly three times the capacity of a truck-tanker, and the number of hazmat railcars varies significantly among different trains. The resulting variability in the total amount of hazmat needs to be taken into account in assessing the rail transport risk, wherein railroads typically have much less routing flexibility compared to trucks. Finally, hazmat incidents involving freight trains could entail content-release from multiple railcars (Verma and Verter, 2013). For example, in the United States, between 1995 and 2009, around 120 train accidents resulted in release from multiple tank cars, which translates into an average of eight accidents every year (FRA, 2010). In December 1999, Canadian National Ultratrain released 2.7 million liters of petroleum products due to the derailment of 35 tank cars just outside Montreal. Thirty cars were seriously punctured and had to be demolished at the accident site (Railway Investigation, 2002). Thus, a train accident can have more severe consequences than those involving trucks, mainly due to the higher volumes of hazmat being shipped and the interaction between railcars.

Though the last few years have witnessed the development of risk assessment methodologies that incorporate the specific nature of railroad shipments, they were preceded mostly by works focusing on accident rate analysis. Glickman and Rosenfield (1984) used past train derailment data to derive three forms of risk: the probability distribution of the number of fatalities in a single accident; the probability distribution of the total number of fatalities from all the accidents in a year; and, the frequency of accidents that result in any given number of fatalities. On the other hand, Barkan et al. (2003) conducted a statistical analysis of the railroad accident data to conclude that the speed of derailment and the number of derailed cars are highly correlated with hazmat release, and then proposed estimating direct and conditional probabilities in conducting risk analysis (Anderson and Barkan, 2004). While these engagements made use of empirical data for insights and conclusions, the recent efforts geared towards developing assessment methodologies (Verma and Verter, 2013).

There are two groups of decision makers in hazmat transportation *viz.*, hazmat carriers and regulatory agencies. The former group considers each hazmat shipment independently, and thus their objective is to seek the least risky path through the network in question between a given origin and destination. Erkut et al. (2007) and Bianco et al. (2013) review the pertinent papers for highway hazmat shipments. The second group on the other hand consider shipments between multiple origins and destinations, and thus strive to regulate hazmat risk for the entire network. To that end, several policies can be employed by the authorities such as restricting hazmat carriers from using certain road segments (Bianco et al., 2009; Dadkar et al., 2010, Erkut and Alp, 2007; Erkut and Gzara, 2008; Kara and Verter, 2004) and/or diverting hazmat carriers to less risky areas by assigning tolls to road segments (Bianco et al., 2016; Esfandeh et al., 2016; Marcotte et al., 2009; Wang et al., 2012).

In an effort to appropriately position the proposed works in the context of the existing literature, we organize the pertinent published works under two themes: *first*, risk assessment efforts specific to rail hazmat shipments; and *second*, peer reviewed engagements dealing with catastrophe avoidance (i.e., high consequence).

Risk assessment for rail hazmat shipments: Risk is most commonly defined as the product of the *probability* and the *consequence* of an undesirable event, and is referred to as *traditional risk* (TR). Bubbico et al. (2004a, b) made use of the traditional risk approach to study rail hazmat shipments in Italy, but ignored the characteristics of railroad accidents, i.e., sequence of events resulting in release from a railcar, and the possibility of multiple sources of release. Subsequently, Verma (2011) outlined an approach that incorporated hazmat release from multiple sources and where conditional derailment probabilities for different positions along the train length were approximated based on the ten deciles. However, the most recent efforts have strived to incorporate position-specific derailment probabilities in the determination of hazmat risk (Bagheri et al., 2014; Cheng et al., 2016). It is important that there are two challenges with using the traditional risk approach: *first*, dearth of reliable data often limits its usage; and *second*, it cannot be used to prepare risk-averse routes.

Researchers attempted to overcome the first challenge by focusing either on the probability or on the consequence of hazmat incidents. The first group proposed *incident probability* (IP) as the measure of risk, in the highway context, which is appropriate for hazmat with relatively small danger zones (Saccomanno and Chan, 1985; Abkowitz et al. 1992). Consequently the assessment methodology adapted for railroads incorporated the pertinent operational characteristics, and endeavored to determine the best position to place hazmat railcars such that in-transit risk is minimized (Bagheri et al. 2011). To that end, the proposed framework recommends assigning hazmat railcars to those positions along the train that have the lowest probability of derailing along the different route segments, which could be done by making use of service engines at the rail marshaling yards. Clearly this measure ignores consequence and is not suitable for capture the *low probability –high consequence* feature of rail shipments. The second group focused on the total number of people exposed to the possibility of an undesirable consequence, and proposed *population exposure* (PE) as the measure of risk. For example, according to the Emergency Response Guidebook (ERG, 2012), 800 m around a fire that involves a chlorine tank, railcar or tank-truck must be isolated and evacuated.

Therefore, the people within the predefined threshold distance from the railroad are exposed to the risk of evacuation. This fixed bandwidth approach was originally suggested by Batta and Chiu (1988), and ReVelle et al. (1991) for highway shipments, and was subsequently adapted for rail hazmat shipments in Verma and Verter (2007). It is important that, in contrast with the traditional "average" risk measure, population exposure constitutes a "worst-case" approach to transport risk. Though it yields conservative routes, it fails to provide multiple routes to cater to the varying levels of risk-tolerance of the decision makers.

Catastrophic avoidance: The second challenge associated with traditional risk approach is the failure to capture the public posture against hazmat shipments, i.e., it is risk-neutral. Abkowitz et al. (1992) made the first attempt to overcome the challenge by proposing a perceived risk (PR) model for highway hazmat shipments, where risk-averseness was incorporated via a consequence perception factor. However, the value of the perception factor is difficult to both understand and quantify. Subsequently, Erkut and Ingolfsson (2000) analyzed three catastrophic avoidance models for highway hazmat shipments–i.e., maximum risk (MM); mean-variance (MV); and, disutility (DU). The objective of the MM model is to minimize the maximum consequence of the path in order to avoid significant damages and casualties, while the MV approach is primarily used to perform trade-off between risk and return of an investment portfolio. Finally, by placing more importance on each incremental life lost, the DU approach makes use of utility theory to conduct risk-averse routing of highway hazmat shipments. It is important to note that these models depend on the historical data about one or more input parameters, and often result in a unique optimal highway route irrespective of the risk preference of the decision maker. Most recently, Kang et al. (2014a, b) made the first attempt to incorporate risk preference of the decision maker, and proposed a value-at-risk model for highway hazmat shipments.

To sum, to the best of our knowledge, there is an absence of risk-averse assessment methodology for rail hazmat shipments though a number of efforts are noticeable in the highway domain. In light of the demonstrated advantage of the value-at-risk model over other catastrophic avoidance techniques (Kang et al. 2014a), we make the first attempt to develop a VaR-based risk assessment framework for rail hazmat shipments. It is important that the proposed framework is distinct from that in Kang et al. (2014a) in the following ways: *first*, characteristics of railroad accidents –i.e., sequence of events resulting in release from railcar, derailment and conditional probabilities of release; and the possibility of multiple sources of release, need to be incorporated; *second*, unlike highway shipments, one has to work with only the pre-defined train services to move shipments, which may involve transfer operations at rail yards; *third*, optimal train configuration to move shipments are determined. Thus, the proposed methodology is more complex as it captures the dynamics of railroad transportation as outlined in Chapter 3, in contrast to the routing of a one hazmat truck between a given origin-destination pair.

Chapter 3. A Value-at-Risk (VaR) Approach to Routing Rail Hazmat Shipments

3.1 Introduction

Given the *low probability –high consequence* nature of rail hazmat shipments, the efficacy of the existing risk assessment measures developed within the railroad domain is rather limited, because they either are risk neutral, and hence would be unable to prevent high consequence events, or yield a single route between a given origin-destination pair, which is not suitable from the perspective of a regulator interested in not overloading any segment or part of the network. Thus, there is a need for a risk assessment methodology that cannot only ensure risk-averse routing of rail hazmat shipments, but also incorporate the risk tolerance of the decision maker to generate multiple routes between a given origin-destination pair.

We make a first attempt to develop a *Value-at-Risk* (VaR) assessment methodology to facilitate risk-averse routing of rail hazmat shipments. This chapter has a three-fold contribution: *first*, this is the first work that incorporates the characteristics of railroad accidents, and then outlines a VaR-based assessment methodology to measure risk from rail hazmat shipments; *second*, this is the only effort that proposes a optimization program to route hazmat shipments over a given railroad network using the optimal train configuration; and *third*, this is the only work that demonstrates that the proposed methodology not only facilitates risk-averse routing of rail hazmat shipments, but can also yield distinct routes based on the risk preference (or tolerance) of the decision makers.

The rest of the chapter is organized as follows. To facilitate exposition, *Section 3.2* briefly outlines a railroad transportation system and introduces the relevant notations, which are then used to develop the Value-at-Risk (VaR) methodology in detail in the subsequent section, *Section 3.3*. *Section 3.4* outlines the parameter estimation technique. The method to find and use the best train configuration setting is explained in *Section 3.5*, followed by the description of the case study in *Section 3.6*. *Section 3.7* presents the solution and analyses gained from solving several problem instances, and then throws light on the performance of the proposed methodology vis-à-vis the three most popular measures of hazmat risk. Finally, conclusion and directions of future research are outlined in *Section 3.8*. Also a detailed explanation for calculating the TR model is provided in *Section 3.9* as the appendix.

3.2 Railroad Transportation System

A rail transportation system can be represented via a network, whose nodes represent yards (or stations) and arcs represent tracks (or service legs) on which trains carry freight (or passengers). A sequence of service legs and intermediate yards constitute a route available to a railcar for its journey (Verma et al., 2011), which is completed using the finite number of train services available in the network. The objective is to transport a specific number of railcars including some with hazmat (N) between a given origin-destination yards. Clearly, hazmat railcars pose an inherent risk of releasing the dangerous contents following an accident on the tracks or in the yards. Let $G = (\mathcal{Y}, \mathcal{A}, S)$ show the rail network, where $\mathcal{Y}$ is the set of yards, $\mathcal{A}$ is the set of service legs, and S is the set of available train services. We assume that each service leg and each yard has two attributes: probability that a hazmat railcar meets with an accident; and, the resulting consequence. We define the following notations:

$\mathcal{Y}$: Set of yards, indexed by i, j, k

$\mathcal{A}$: Set of service legs, indexed by (i, j)

S: Set of train services, indexed by s (and/or $\acute{s}$)

$\mathcal{Y}_s$: Set of yards for train service s, indexed by i_s, j_s, k_s

$\mathcal{A}_s$: Set of service legs for train service s, indexed by (i_s, j_s)

N: Number of hazmat railcars to be shipped

p_k: Probability of hazmat incident in yard k

p_{ij}: Probability of hazmat incident on arc (i, j)

c_k: Consequence from N hazmat railcars accident in yard k

c_{ij}: Consequence from N hazmat railcars accident on arc (i, j)

It is pertinent that although it is possible to compute position-specific derailment probability, such numbers do not have much use since freight train-lengths vary. Hence, we compute derailment probabilities based on the ten deciles of the train (i.e., the length of the train is divided into 10 equal parts). Conceivably, a decile-based approach should result in better analysis, but only if train-lengths are similar (or constant). We make use of the information presented in Bagheri (2009) that freight-trains with up to 40 railcars be called *short*; between 41 and 120, *medium*; and, the rest *long*. Thus, for similar train lengths, we define the positive integer y_r as the number of hazmat railcars in decile *r* of the train. Thus, $\sum_{r=1}^{10} y_r = N$, where $y_r \leq \frac{\text{train length}}{10}$. Hence, p_{ij} and p_k would be determined as follows:

$$p_{ij} = P(A_{ij}) \times \sum_{r=1}^{10} y_r \times \left(P\left(D^r \middle| A_{ij}\right) \times P\left(H \middle| D^r, A_{ij}\right) \times P\left(R \middle| H, D^r, A_{ij}\right)\right) \quad (3.1)$$

$$p_k = P(A_k) \times \sum_{r=1}^{10} y_r \times \left(P(D^r|A_k) \times P(H|D^r, A_k) \times P(R|H, D^r, A_k)\right) \quad (3.2)$$

where $P(A_{ij})$ (or $P(A_k)$) is the probability that a train meets with an accident on service leg (i, j) (or at yard k); $P\left(D^r \middle| A_{ij}\right)$ (or $P(D^r|A_k)$) is the probability of derailment of a railcar in the r^{th} decile of the train given the accident on service leg (i, j) (or at yard k); $P\left(H \middle| D^r, A_{ij}\right)$ (or $P(H|D^r, A_k)$) is the probability that a hazmat railcar derailed in the r^{th} decile of the train given the accident on service leg (i, j) (or at yard k); $P\left(R \middle| H, D^r, A_{ij}\right)$ (or $P(R|H, D^r, A_k)$) is probability of release from a hazmat railcar derailed in the r^{th} decile of the train given the accident on service leg (i, j) (or at yard k).

Finally, the consequence, i.e., c_{ij} and c_k, would be estimated as the population exposure due to the release from N hazmat railcars given the accident on service leg (i, j) and at yard k, respectively. It should be evident that the both the probability of a hazmat railcar meeting with an accident in a yard and the resulting consequence are pertinent only if a yard operation is being performed on the hazmat railcars. We elaborate on the parameter estimation technique in Section 3.4.

3.3 Value-at-risk (VaR) Measure

As alluded we propose a VaR based assessment methodology to measure hazmat risk from rail shipments, and a optimization model to route them over the given network. VaR is a sophisticated risk measurement tool widely used in portfolio management (Sarykalin et al., 2008), and has found applications outside finance and economics including routing highway hazmat shipments (Kang et al., 2014a, b). However, there are two important differences between standard VaR models in finance, and the hazmat application. *First*, the standard VaR models in finance express both the investment and the loss in the same units, say dollars. This is in contrast to the hazmat routing application, where the input (or investment) is the route and consequence (i.e., number of people exposed) is the loss. *Second*, losses in portfolio management domain are additive, whereas in hazmat transportation the risk of each road segment in a path is non-additive (Toumazis et al., 2013). We next outline the VaR-based risk assessment methodology for rail hazmat shipments.

For expositional reasons, we first compute VaR for a pre-specified route between a pair of railyards representing an origin and a destination for rail shipments, and then present the optimization program that could be used to determine the best route through the given railroad network.

<u>VaR for a pre-specified route *l*:</u> Let R^l denote the discrete random variable for the risk along route l. The probability of R^l not exceeding a threshold β is then given by:

$$F_{R^l}(\beta) = \Pr(R^l \le \beta).$$

where, $F_{R^l}(\beta)$ is the cumulative distribution function (CDF) for hazmat risk along route l, which completely determines the behavior of the random variable R^l and is fundamental in defining VaR (Rockafellar and Uryasev 2000). For a specific confidence level α in $(0,1)$, the α-VaR value for the risk associated with the random variable for route l will be denoted by VaR_α^l, and given by:

$$\mathrm{VaR}_\alpha^l = \min\left\{\beta \mid F_{R^l}(\beta) \ge \alpha\right\} \tag{3.3}$$

or

$$\mathrm{VaR}_\alpha^l = \min\{\beta \mid \Pr(R^l \le \beta) \ge \alpha\} \tag{3.4}$$

That is, VaR is the minimal threshold level β such that the hazmat risk R^l does not exceed β with the least probability of α. In fact, VaR shows the population exposure that a route is expected to cause with a given probability (α). For example, a route that is expected to cause no more than 1,000 casualties 95% of the time, has a VaR of 1,000 with confidence level 0.95. On the downside, 5% of the time the route is expected to cause at least 1,000 casualties. Therefore, in order to calculate VaR_α^l, we first need to specify the values that R^l can take and then calculate the cumulative distribution function $F_{R^l}(\beta)$.

Consider a specific route *l* that uses *T* (of the *S*) train services for transporting N hazmat railcars between a pre-determined pair origin-destination yards in the railroad network, $G = (\mathcal{Y}, \mathcal{A}, S)$, as defined in Section 3.2. This route consists of a set of yards $\mathcal{Y}^l = \bigcup_{s \in T} \mathcal{Y}_s^l$ and a set of service legs $\mathcal{A}^l = \bigcup_{s \in T} \mathcal{A}_s^l$ (i.e., $n_l = |\mathcal{Y}^l \cup \mathcal{A}^l|$ items), which have been engaged to move the railcars from the origin yard to the destination yard. Let $C_{(t)}^l$ denote the t th smallest value in the set $\{c_k \cup c_{ij} : k \in \mathcal{Y}^l \;\&\; (i,j) \in \mathcal{A}^l\}$, and $P_{(t)}^l$ be the corresponding accident probability. As indicated earlier, both the accident probabilities and resulting consequences of only the yards involved in performing a transfer operation are pertinent, and thus considered. More specifically, if at yard k; the hazmat shipment unloads from an inbound train and is then loaded to an outbound train, p_k and c_k of this yard will be added to the above set. If a yard k' performs just a transshipment role (i.e., no actual handling of railcars, but merely a transit point), $p_{k'}$ and $c_{k'}$ will not be considered. In light of the above, hazmat risk associated with the discrete random variable R^l can take the following values in ascending order (Kang et al., 2014a):

$$R^l = \begin{cases} C^l_{(0)} = 0, & \text{with probability } P^l_{(0)} = 1 - \sum_{i=1}^{n_l} P^l_{(i)} \\ C^l_{(1)}, & \text{with probability } P^l_{(1)} \\ \vdots & \vdots \\ C^l_{(t)}, & \text{with probability } P^l_{(t)} \\ \vdots & \vdots \\ C^l_{(n_l)}, & \text{with probability } P^l_{(n_l)} \end{cases}$$

and, given R^l as above, $F_{R^l}(\beta)$ is as follows:

$$F_{R^l}(\beta) = \Pr(R^l \le \beta)$$

$$= \begin{cases} P^l_{(0)}, & \text{if } 0 \le \beta < C^l_{(1)} \\ P^l_{(0)} + P^l_{(1)}, & \text{if } C^l_{(1)} \le \beta < C^l_{(2)} \\ \vdots & \vdots \\ P^l_{(0)} + P^l_{(1)} + \cdots + P^l_{(t-1)}, & \text{if } C^l_{(t-1)} \le \beta < C^l_{(t)} \\ \vdots & \vdots \\ 1, & \text{if } C^l_{(n_l)} \le \beta \end{cases} \tag{3.5}$$

For expositional reasons, and also to facilitate flow, we graph the cumulative distribution function in Figure 3.1. Note that α is placed between $\sum_{i=0}^{t-1} P^l_{(i)}$ and $\sum_{i=0}^{t} P^l_{(i)}$. It is evident that for that specific α, the minimum β that can satisfy the condition $F_{R^l}(\beta) \ge \alpha$ is $C^l_{(t)}$. In fact, if α is chosen anywhere in the interval $\left(\sum_{i=0}^{t-1} P^l_{(i)}\ ,\ \sum_{i=0}^{t} P^l_{(i)}\right]$, then $C^l_{(t)}$ will still satisfy the above condition.

Therefore, for any α in the above interval, we have $\mathrm{VaR}_{\alpha}^{l} = C_{(t)}^{l} = \min\{\beta \mid F_{R^l}(\beta) \geq \alpha\}$. Equation (3.5) and Figure 3.1 enable us to develop Equation (3.6), which could be used to obtain $\mathrm{VaR}_{\alpha}^{l}$ for all other possible values of α.

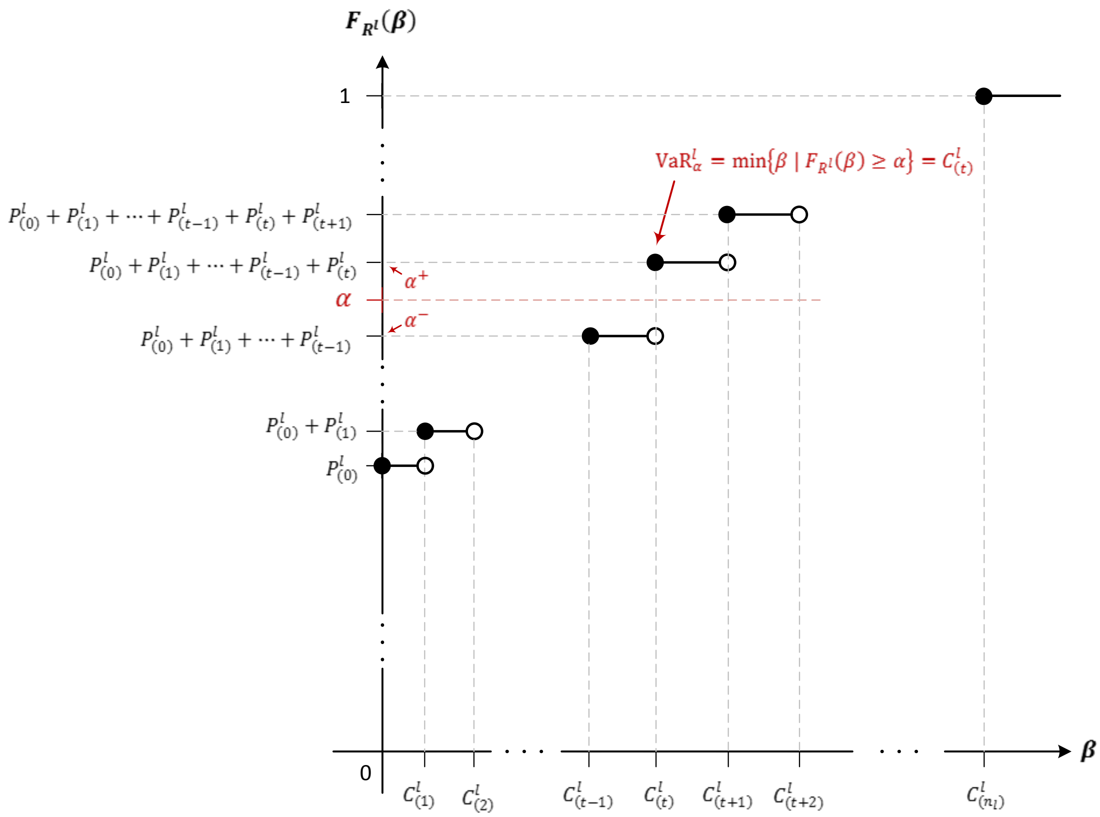


Figure 3.1. Cumulative distribution function $F_{R^l}(\beta)$, and $\mathrm{VaR}_{l}^{\alpha}$

$$\mathrm{VaR}_{\alpha}^{l} = \begin{cases} C_{(0)}^{l} = 0, & \text{if } 0 < \alpha \le P_{(0)}^{l} \\ C_{(1)}^{l}, & \text{if } P_{(0)}^{l} < \alpha \le P_{(0)}^{l} + P_{(1)}^{l} \\ & \vdots \\ C_{(t)}^{l}, & \text{if } P_{(0)}^{l} + P_{(1)}^{l} + \cdots + P_{(t-1)}^{l} < \alpha \le P_{(0)}^{l} + P_{(1)}^{l} + \cdots + P_{(t-1)}^{l} + P_{(t)}^{l} \\ & \vdots \\ C_{(n_l)}^{l}, & \text{if } P_{(0)}^{l} + P_{(1)}^{l} + \cdots + P_{(n_l-1)}^{l} < \alpha < 1 \end{cases} \tag{3.6}$$

Therefore from (3.6) we obtain

$$\mathrm{VaR}_{\alpha}^{l} = \begin{cases} 0, & \text{if and only if} \quad 0 < \alpha \le P_{(0)}^{l} \\ C_{(t)}^{l}, & \text{if and only if} \quad \sum_{i=0}^{t-1} P_{(i)}^{l} < \alpha \le \sum_{i=0}^{t} P_{(i)}^{l}, \quad t \in \{1,2,\dots,n_l\} \end{cases} \tag{3.7}$$

We observe that there is always a VaR for the route l with the confidence level $\alpha \in (0,1)$. However, since the distribution function $F_{R^l}(\beta)$ has a vertical discontinuity gap (or jump) at VaR, that VaR remains the same for an interval of α. The (open) lower and (closed) upper endpoints of that interval, respectively, are:

$$\alpha^{-} = F_{R^l}\left(\mathrm{VaR}^{-l}_{\ \alpha}\right) = \Pr(R^l < \mathrm{VaR}_{\alpha}^{l}) \tag{3.8}$$

and

$$\alpha^{+} = F_{R^l}(\mathrm{VaR}_{\alpha}^{l}) = \Pr(R^l \le \mathrm{VaR}_{\alpha}^{l}). \tag{3.9}$$

Because the difference $\alpha^{+} - \alpha^{-} = \Pr(R^l = \mathrm{VaR}_\alpha^l)$ is always positive ($F_{R^l}(\beta)$ has a jump at VaR_α^l), a probability "atom" is said to be present at VaR_α^l (Rockafellar and Uryasev 2002). We continue our calculation for VaR_α^l by considering

$$\sum_{i=0}^{t} P_{(i)}^l = P_{(0)}^l + P_{(1)}^l + \cdots + P_{(t-1)}^l + P_{(t)}^l = 1 - \left[P_{(t+1)}^l + \cdots + P_{(n_l)}^l\right], \quad t \in \{0,1,2,\dots,n_l\}$$

And so according to the definition of $P_{(t)}^l$, we obtain

$$\sum_{i=0}^{t} P_{(i)}^l = 1 - \left[\sum_{k \in \mathcal{Y}^l,\ c_k > C_{(t)}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij} > C_{(t)}^l} p_{ij}\right], \quad t \in \{0,1,2,\dots,n_l\}$$

Hence from (3.7) we obtain

$$\mathrm{VaR}_\alpha^l = \begin{cases} 0, & \text{if and only if} \quad \left[\sum_{k \in \mathcal{Y}^l,\ c_k > 0} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij} > 0} p_{ij}\right] \le 1 - \alpha < 1 \\ C_{(t)}^l, & \text{if and only if} \\ \left[\sum_{k \in \mathcal{Y}^l,\ c_k > C_{(t)}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij} > C_{(t)}^l} p_{ij}\right] \le 1 - \alpha < \left[\sum_{k \in \mathcal{Y}^l,\ c_k \ge C_{(t)}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij}} \right. \\ t \in \{1,2,\dots,n_l\}. \end{cases} \tag{3.10}$$

Let

$$\mathbb{P}_{(t)}^l = \sum_{k \in \mathcal{Y}^l,\ c_k > C_{(t)}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij} > C_{(t)}^l} p_{ij}, \quad t \in \{0,1,2,\dots,n_l\} \tag{3.11}$$

($\mathbb{p}_{(t)}^{l}$: the summation of the accident probabilities of the yards and tracks in route l, whose accident consequences are greater than $C_{(t)}^{l}$). So, we have

$$\sum_{k \in \mathcal{Y}^l,\ c_k \geq C_{(t)}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij} \geq C_{(t)}^l} p_{ij}$$

$$= \sum_{k \in \mathcal{Y}^l,\ c_k > C_{(t)}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l,\ c_{ij} > C_{(t)}^l} p_{ij} + P_{(t)}^l = \mathbb{p}_{(t)}^l + P_{(t)}^l$$

Therefore (3.10) can be written as:

$$\mathrm{VaR}_\alpha^l = C_{(t)}^l \quad \text{if and only if} \quad \mathbb{p}_{(t)}^l \leq 1 - \alpha < \mathbb{p}_{(t)}^l + P_{(t)}^l, \quad t \in \{0,1,2,\dots,n_l\}$$

Consequently, finding VaR_α^l is equivalent to solving the following problem

$$\min_t C_{(t)}^l$$

$$\text{subject to:}$$

$$\mathbb{p}_{(t)}^l \leq 1 - \alpha < \mathbb{p}_{(t)}^l + P_{(t)}^l$$

$$t \in \{0,1,2,\dots,n_l\}.$$

VaR for finding the best route in the given railroad network: So far, we had assumed that the route was pre-specified for a given shipment. However, if the route between a pair of origin-destination yards is not given, then the objective is to find the route with minimum VaR. To that end, consider that a set of routes for a given shipment is denoted by L, and indexed by l. Hence, $\mathrm{VaR}_\alpha^* = \min\{\mathrm{VaR}_\alpha^l : l \in L\}$, subject to a set of constraints (depicted compactly as:

$(X \in \psi)$). More specifically, we need to define two binary decision variables and conservation of flow constraints, which collectively would capture routing of rail shipments. The two decision variables keep track of the service legs and yards utilized in moving shipments are:

$$x_{i_s j_s} = \begin{cases} 1, & \text{if service leg } (i_s, j_s) \text{ from train service } s \text{ is used in the route} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{k_{s\acute{s}}} = \begin{cases} 1, & \text{if yard } k \text{ is used in the route as a transferring yard between train services } s \text{ and } \acute{s} \\ 0, & \text{otherwise} \end{cases}$$

and, the conservation of flow constraints are:

$$\sum_{j_s} x_{i_s j_s} = 1 \quad \text{for origin yard } i \text{ and train service } s$$

$$\sum_{j_s} x_{j_s i_s} - \sum_{j_s} x_{i_s j_s} = 0 \quad \text{for any non} - \text{transferring yard } i \text{ and train service } s$$

$$\sum_{j_s} x_{j_s k_s} - \sum_{j_{\acute{s}}} x_{k_{\acute{s}} j_{\acute{s}}} = 0 \quad \text{for any transferring yard } k \text{ and train service } s \text{ and } \acute{s}$$

$$\sum_{j_s} x_{j_s i_s} = 1 \quad \text{for destination yard } i \text{ and train service } s$$

We next sort the accident consequences of all (M) yards and service legs in the network:

$$C_{(r)} = \{C_{(0)} = 0, C_{(1)}, C_{(2)}, \dots, C_{(M)} \ : \ 0 < C_{(1)} < C_{(2)} < \cdots < C_{(M)}\}, \quad r \in \{0,1,2,\dots,M\} \tag{3.12}$$

and, label the corresponding accident probabilities as: $P_{(r)}$, $r \in \{0,1,2,\dots,M\}$: $\sum_{r=0}^{M} P_{(r)} = 1$. Hence, according to the definition in (3.11), we can define $\mathbb{p}_{(r)}$ as:

$$\mathbb{p}_{(r)} = \sum_{s,\acute{s}} \left(\sum_{k_{s\acute{s}} \in \mathcal{Y}_s \& \mathcal{Y}_{\acute{s}},\ c_{k_{s\acute{s}}} > C_{(r)}} p_{k_{s\acute{s}}} + \sum_{(i_s,j_s) \in \mathcal{A}_s,\ c_{i_s j_s} > C_{(r)}} p_{i_s j_s} \right), \quad r \in \{0,1,2,\dots,M\}$$

and,

$$\mathbb{p}_{(r)} X = \sum_{s,\acute{s}} \left(\sum_{k_{s\acute{s}} \in \mathcal{Y}_s \& \mathcal{Y}_{\acute{s}},\ c_{k_{s\acute{s}}} > C_{(r)}} p_{k_{s\acute{s}}} x_{k_{s\acute{s}}} + \sum_{(i_s,j_s) \in \mathcal{A}_s,\ c_{i_s j_s} > C_{(r)}} p_{i_s j_s} x_{i_s j_s} \right), \quad r \in \{0,1,2,\dots,M\} \tag{3.13}$$

Hence, the optimization program required to generate the minimal VaR route (i.e., $\mathrm{VaR}_{\alpha}^{*}$) between a given pair of origin-destination yards to route rail shipments is:

$$\min_{r} C_{(r)}$$

$$f^{r} = \min_{X} \mathbb{p}_{(r)} X \tag{3.14}$$

subject to:

$$f^{r} \leq 1 - \alpha < f^{r} + P_{(r)}$$

$$X \in \psi$$

$$r \in \{0,1,2,\dots,M\}$$

Note that problem (3.14) has two levels, and that solving the lower level problem, i.e., $f^r = \min_X \mathbb{p}_{(r)} X$ necessitates using (3.13) to make the following modification:

$$\bar{p}_{k_{s\acute{s}}} = \begin{cases} p_{k_{s\acute{s}}}, & \text{if} \quad c_{k_{s\acute{s}}} > C_{(r)} \qquad \forall\, k_{s\acute{s}}, \forall s, \forall \acute{s} \\ 0, & \text{otherwise} \end{cases}$$

$$\bar{p}_{i_s j_s} = \begin{cases} p_{i_s j_s}, & \text{if} \quad c_{i_s j_s} > C_{(r)} \qquad \forall\, (i_s, j_s), \forall s \\ 0, & \text{otherwise} \end{cases} \tag{3.15}$$

This way $\mathbb{p}_{(r)} X$ transforms to

$$\mathbb{p}_{(r)} X = \sum_{s,\acute{s}} \left(\sum_{k_{s\acute{s}} \in \mathcal{Y}_s \& \mathcal{Y}_{\acute{s}}} \bar{p}_{k_{s\acute{s}}} x_{k_{s\acute{s}}} + \sum_{(i_s, j_s) \in \mathcal{A}_s} \bar{p}_{i_s j_s} x_{i_s j_s} \right), \qquad r \in \{0,1,2,\dots,M\} \tag{3.16}$$

and $f^r = \min_{X \in \psi} \mathbb{p}_{(r)} X$ can be solved using an efficient shortest path algorithm, such as Dijkstra's. We conclude this section with a summary of the proposed *VaR Algorithm* in Figure 3.2.

Step 1. Generate $C_{(r)}$ and their corresponding $P_{(r)} : r \in \{0,1,2,\dots,M\}$ using (3.12).

Step 2. Set $r \leftarrow 0$

Step 3. Consider (3.15) and (3.16) and solve $f^r = \min_X \mathbb{p}_{(r)} X$ using an efficient shortest path algorithm like Dijkstra's Algorithm.

Step 4. If condition $f^r \leq 1 - \alpha < f^r + P_{(r)}$ holds, stop; $\mathrm{VaR}_\alpha^* = C_{(r)}$ and the route found in Step 3 is the optimal VaR route for confidence level α.

Step 5. If condition $f^r \leq 1 - \alpha < f^r + P_{(r)}$ does not hold, set $r \leftarrow r + 1$ and go to Step 3.

Figure 3.2. Summary of *VaR Algorithm*

3.4 Parameter Estimation

In preparation for using the proposed methodology, we briefly outline the parameter estimation technique in this section, and note that the values are borrowed from published works.

<u>Train accident rate:</u> Although the United States Federal Railroad Administration (FRA) provides comprehensive data of railroad accidents, it is not inconceivable that accident rates for every rail-segment and rail-yard may not be available given that hazmat episodes are rare. Given the above limitations, it would not be unreasonable to use network-wide accident rates. Hence, we have assumed an average train accident rate of 1.48×10^{-6} per mile for service legs, and 17.13×10^{-6} for transferring yards, as presented in Verma (2011). It means $P(A_{ij}) = \text{service leg } (i,j)'\text{s distance (mile)} \times 1.48 \times 10^{-6}$ and $P(A_k) = 17.13 \times 10^{-6}$ in equations (3.1) and (3.2), respectively.

Conditional probabilities: Given our intent of making use of the FRA data set, and be able to assess risk at a higher resolution as discussed in Section 3.2, we work with decile-specific conditional probabilities of derailment of a railcar, presence of hazmat railcar, and of release. This approach was proposed in Verma (2011) for three train lengths (i.e., *short*, *medium*, and *long*), and we use the resulting estimates as input in (3.1) and (3.2). For expositional reasons, we reproduce the estimates from Verma (2011) in Table 3.1 and Table 3.2.

Decile	$P(D^r \mid A_{ij})$			$P(H \mid D^r, A_{ij})$			$P(R \mid H, D^r, A_{ij})$		
	short	***medium***	***long***	***short***	***medium***	***long***	***short***	***medium***	***long***
1st	0.1666	0.1884	0.2012	0.0668	0.0417	0.0216	0.0093	0.0156	0.0060
2nd	0.0957	0.1001	0.1088	0.0609	0.0838	0.0413	0.0047	0.0141	0.0163
3rd	0.0952	0.0897	0.0983	0.0398	0.0746	0.0685	0.0038	0.0108	0.0037
4th	0.0908	0.0947	0.1029	0.0760	0.0803	0.0614	0.0112	0.0127	0.0259
5th	0.0938	0.0895	0.0831	0.0905	0.0792	0.0525	0.0016	0.0120	0.0150
6th	0.0877	0.0834	0.0818	0.0611	0.0761	0.0588	0.0075	0.0121	0.0201
7th	0.0783	0.0816	0.0831	0.0853	0.0718	0.0318	0.0009	0.0061	0.0040
8th	0.0839	0.0870	0.0765	0.0726	0.0776	0.0432	0.0073	0.0087	0.0021
9th	0.0841	0.0826	0.0831	0.0720	0.0579	0.0410	0.0076	0.0070	0.0055
10th	0.1239	0.1032	0.0811	0.0625	0.0883	0.0531	0.0045	0.0054	0.0024

Table 3.1. Conditional probabilities for service legs

Decile	$P(D^r \mid A_k)$			$P(H \mid D^r, A_k)$			$P(R \mid H, D^r, A_k)$		
	short	***medium***	***long***	***short***	***medium***	***long***	***short***	***medium***	***long***
1st	0.1336	0.1937	0.2081	0.0663	0.0693	0.0347	0.0036	0.0052	0.0072
2nd	0.1187	0.1117	0.0905	0.0756	0.0738	0.0922	0.0050	0.0063	0.0023
3rd	0.0914	0.0915	0.1086	0.0780	0.0787	0.1146	0.0046	0.0027	0.0083
4th	0.1000	0.0982	0.1086	0.0917	0.0770	0.0243	0.0041	0.0012	0.0021
5th	0.1072	0.0896	0.0950	0.0734	0.0757	0.0224	0.0030	0.0171	0.0020
6th	0.0687	0.0838	0.0724	0.0831	0.0631	0.0547	0.0043	0.0039	0.0020
7th	0.0759	0.0672	0.1041	0.0744	0.0638	0.0662	0.0038	0.0011	0.0071
8th	0.0784	0.0804	0.0814	0.0772	0.0791	0.0374	0.0025	0.0028	0.0019
9th	0.0702	0.0709	0.0543	0.0963	0.0758	0.1207	0.0041	0.0087	0.0179
10th	0.1559	0.1130	0.0769	0.0989	0.1109	0.0611	0.0038	0.0090	0.0023

Table 3.2. Conditional probabilities for yards

Consequence: We make use of an aggregate measure, i.e., *population exposure*, to estimate consequence. As alluded in the literature review chapter, this approach was proposed by Batta and Chiu (1988) and ReVelle et al. (1991) in the highway domain, and was subsequently extended to incorporate the possibility and volume of hazmat released from multiple sources, i.e., for rail hazmat shipments in Verma and Verter (2007). This dependence relationship can be represented by:

$$PE_{(i,j)} = f\left(V_{(i,j)}, \rho\left(V_{(i,j)}\right)\right)$$

where $V_{(i,j)}$ is the volume of hazmat released due to the accident on rail-link (i,j) and $\rho\left(V_{(i,j)}\right)$ is the population density of centers exposed due to $V_{(i,j)}$. If $v_{(i,j)}^{n}$ is the quantity of hazmat released from railcar n due to the accident on link (i,j), the total volume of hazmat released from all the sources (N hazmat railcars) due to the accident on link (i,j), can be determined by:

$$V_{(i,j)} = \sum_{n=1}^{N} v_{(i,j)}^{n}$$

In an effort to simulate most conservative scenario, Verma and Verter (2007) assume loss of entire lading, and focus on airborne shipments whose behavior is emulated via a Gaussian dispersion plume model. Finally, they can make suitable adaptations to the (GPM) to capture hazmat release from multiple sources, which was subsequently augmented in Verma (2011) to incorporate the characteristics of railroad accidents. For expositional reasons, and for brevity, we are not repeating the complete methodological details, and invite the reader to consult Verma and Verter (2007) and Verma (2011).

3.5 Train Configuration Setting

Before applying the proposed methodology to realistic size problem instances, we delineate the steps involved in determining train configuration. Recall that equations (3.1) and (3.2) estimate hazmat incident probabilities in the service legs and at the yards, which in turn depend on the number of hazmat railcars in each train-decile (i.e., *train configuration*). It should be evident that different train configurations can result in different values of risk, including VaR. Hence, we endeavor to determine the train configuration that would result in minimum total risk, and the corresponding route. Recall that the positive integer y_r shows the number of hazmat railcars in decile r ($r = \{1,2,\dots,10\}$) of the train; $\sum_{r=1}^{10} y_r = N$, where $y_r \leq \frac{\text{train length}}{10}$. Let us define constants TCP^r and YCP^r as the multiplication of conditional probabilities in decile r of the train for service legs and yards, respectively, i.e. $TCP^r = P\left(D^r \middle| A_{ij}\right) \times P\left(H \middle| D^r, A_{ij}\right) \times P\left(R \middle| H, D^r, A_{ij}\right)$ and $YCP^r = P(D^r|A_k) \times P(H|D^r, A_k) \times P(R|H, D^r, A_k)$. Thus, equation (3.1) and (3.2) could be expressed as:

$$p_{ij} = P\left(A_{ij}\right) \times \sum_{r=1}^{10} y_r \times TCP^r, \tag{3.1'}$$

$$p_k = P(A_k) \times \sum_{r=1}^{10} y_r \times YCP^r. \tag{3.2'}$$

Given the above explanation, we can define the following minimization model to find the best train configuration:

$$\min_{\mathcal{W},y_r} \mathcal{W}\left(\sum_{r=1}^{10} y_r \times TCP^r\right) + (1-\mathcal{W})\left(\sum_{r=1}^{10} y_r \times YCP^r\right) \tag{3.17-1}$$

subject to:

$$\min_{l\in L} \mathrm{VaR}_\alpha^l \tag{3.17-2}$$

$$\sum_{r=1}^{10} y_r = N \tag{3.17-3}$$

$$0 \le y_r \le \frac{\text{train length}}{10} \tag{3.17-4}$$

$$y_r\text{: integer,} \quad r = \{1,2,\dots,10\} \tag{3.17-5}$$

$$0 \le \mathcal{W} \le 1 \tag{3.17-6}$$

To solve the above model, we assign different values to $\mathcal{W}$ (weight) in the objective function (3.17-1) subject to the constraints (3.17-3) to (3.17-6), which results in various train configurations, but we finally select the one that minimizes the VaR (constraint 3.17-2) and consequently find its corresponding optimal route. $\mathcal{W} = 0$ (or 1) means that we are going to use a train configuration that minimizes accident probabilities in the network's transferring yards (or service legs), i.e., determining y_r for p_k $(or\ p_{ij})$ minimization. Such a configuration is gained by placing as many hazmat railcars as possible in the deciles with the least multiplication of the conditional probabilities for transferring yards (or service legs), i.e. YCP^r (or TCP^r), using Table 3.2 (or Table 3.1).

3.6 Case Study

In this section, we describe the case study that would be solved and analyzed in Section 3.7. Figure 3.3 depicts the railroad infrastructure, introduced in Verma et al. (2011), which will be used to perform computational experiments. The indicated network has 25 yards, where each can be an origin and destination for the others, i.e., 600 origin-destination pairs. A total of 31 different train services –identified by origin and destination yards, intermediate stops, and service legs, connect the yards. Finally, ArcGIS (ESRI, 2007) was used to estimate population exposure, which serves a measure for consequence. The objective is to *determine the best way to move a given number of hazmat railcars, on the available train services, between various origin-destination pairs such that hazmat transport risk as measured by VaR is minimized*. It is important that given the nature of railroad accidents, and the preceding discussions, one needs to determine both the route and the placement of hazmat railcars in a train.

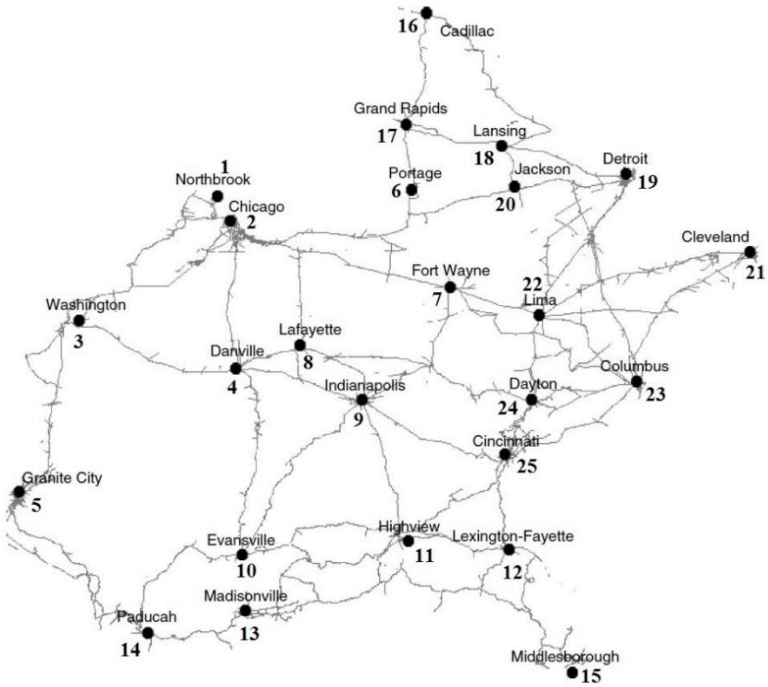


Figure 3.3. Railroad network in the Midwest United States [Source: Verma et al., (2011)]

3.7 Computational Experiments

In this section, we use the risk assessment methodology developed in Section 3.3 to study a number of problem instances generated using the real infrastructure of a Class I railroad operator in the United States (i.e., the case study outlined in Section 3.6), which are further analyzed to develop managerial insights.

3.7.1. An Illustrative Example

In an effort to facilitate the discussions and analyses to follow, we make use of a small illustrative example to demonstrate how each step of the methodology is operationalized. We focus on a shipment going from Middlesborough to Chicago, i.e., yards 15 and 2, respectively, in Figure 3.3. Furthermore, we assume that only 5 train services are available, and their routes are indicated in Table 3.3. For example, train service number 5 originates in yard 2, stops at yards 4, 9 and 25 before terminating at yard 12.

Train service number	**Route provided by the train service**
{5}	2 – 4 – 9 – 25 – 12
{12}	9 – 10 – 14 – 13 – 15
{16}	9 – 8 – 4 – 3 – 2
{24}	12 – 11 – 10 – 4 – 2
{27}	15 – 11 – 9 – 8 – 7

Table 3.3. Train services for the illustrative example

Assume that a total of 70 hazmat railcars have to be shipped from yard 15 to yard 2 using a *medium* length train with 120 railcars, while the confidence level α is 0.999999. We next outline the details of the six steps involved in preparing the routing and train configuration plan.

1. <u>Ascertaining train configuration</u>

The very first decision involves the placement of the 70 hazmat railcars in the train, i.e., determining the positive integer y_r as the number of hazmat railcars in decile r of the train. This way, $\sum_{r=1}^{10} y_r = 70$, where $y_r \leq 12 \quad \forall r$. Although this would be ascertained by solving (3.17), for illustration purposes, we assume that minimum VaR is achieved when $\mathcal{W} = 0$, which implies placing as many hazmat railcars as possible in the deciles with the least YCP^r (using Table 3.2 for *medium* trains), yielding the configuration given in Table 3.4:

Decile No. r	1	2	3	4	5	6	7	8	9	10
y_r	0	0	12	12	0	12	12	12	10	0

Table 3.4. Train configuration for the illustrative example

2. *Calculating consequence*

Accident consequences of the service legs and transferring yards in the network (i.e., c_{ij} and c_k, respectively) are determined as the population exposure (PE) due to the release from 70 hazmat railcars, given the accident on service leg (i, j) and at yard k, respectively. These values are depicted in column 4 of Table 3.6.

3. *Calculating probabilities of train accident*

To calculate accident probabilities on the service legs and transferring yards in the network (i.e. p_{ij} and p_k, respectively) using equations (3.1) and (3.2), we first need to calculate $\sum_{r=1}^{10} y_r \times \left(P\left(D^r \middle| A_{ij}\right) \times P\left(H \middle| D^r, A_{ij}\right) \times P\left(R \middle| H, D^r, A_{ij}\right)\right)$ and $\sum_{r=1}^{10} y_r \times \left(P(D^r|A_k) \times P(H|D^r, A_k) \times P(R|H, D^r, A_k)\right)$. This is done using the train configuration in Table 3.4 and the conditional probabilities provided in Table 3.1 and Table 3.2 as follows (see Table 3.5):

Decile r	1	2	3	4	5	6	7	8	9	10
y_r	0	0	12	12	0	12	12	12	10	0
$y_r \times \left(P\left(D^r \middle| A_{ij}\right) \times P\left(H \middle| D^r, A_{ij}\right) \times P\left(R \middle| H, D^r, A_{ij}\right)\right)$	0	0	0.000867	0.001158	0	0.000921	0.000429	0.000705	0.000335	0
$y_r \times \left(P(D^r\|A_k) \times P(H\|D^r, A_k) \times P(R\|H, D^r, A_k)\right)$	0	0	0.000233	0.000109	0	0.000247	0.000057	0.000214	0.000468	0

Table 3.5. Conditional probabilities for calculating p_{ij} and p_k for the illustrative example

$\sum_{r=1}^{10} y_r \times \left(P\left(D^r \middle| A_{ij}\right) \times P\left(H \middle| D^r, A_{ij}\right) \times P\left(R \middle| H, D^r, A_{ij}\right)\right) = 0.004415$, and

$\sum_{r=1}^{10} y_r \times \left(P(D^r|A_k) \times P(H|D^r, A_k) \times P(R|H, D^r, A_k)\right) = 0.001328$.

Now, we can calculate p_{ij} and p_k using equations (3.1) and (3.2), respectively. For instance, $P(A_{ij}) =$ service leg $(i,j)'$s distance (mile) $\times 1.48 \times 10^{-6}$, and appropriate accident probabilities are depicted in the last column of Table 3.6. On the other hand, $P(A_k) = 17.13 \times 10^{-6}$, which implies that the accident probability at all potential transfer yards (p_k) is equal to2.274×10^{-8} (i.e., $17.13 \times 10^{-6} \times 0.001328$).

4. *Sorting consequence and accident probabilities for the given network*

In this step, we sort all accident consequences of the service legs and potential transferring yards in the network in ascending order. We call them $C_{(r)}$ and their corresponding accident probabilities $P_{(r)}$, where $0 \le r \le 26$ and $C_{(0)} < C_{(1)} < C_{(2)} < \cdots < C_{(26)}$. Recall that $C_{(0)} = 0$ with $P_{(0)} = 1 - \sum_{r=1}^{26} P_{(r)}$. All the relevant values are shown in Table 3.7.

5. *Calculating optimal VaR value and the Route*

We make use of the VaR Algorithm as outlined in Figure 3.2 to calculate the optimum value (i.e., VaR*). Note that there are 26 elements in the sorted list in Table 3.7, and hence the algorithm could have a maximum of 27 iterations. However, the algorithm find VaR* in the 19th iteration. For brevity, we report the results of iterations 1, 3, 10 and 19.

	Train Service	$(\boldsymbol{i}, \boldsymbol{j})$ or $\boldsymbol{k}$	c_{ij} *or* c_k	Length (miles)	p_{ij} *or* p_k
Service legs	**{5}**	2 – 4	12,015	126	8.2×10^{-7}
		4 – 9	5,658	91	5.9×10^{-7}
		9 – 25	3,937	103	6.7×10^{-7}
		25 – 12	4,528	81	5.3×10^{-7}
	{12}	9 – 10	3,115	158	1.0×10^{-6}
		10 – 14	7,835	128	8.3×10^{-7}
		14 – 13	5,027	74	4.8×10^{-7}
		13 – 15	2,944	267	1.7×10^{-6}
	{16}	9 – 8	5,676	68	4.4×10^{-7}
		8 – 4	1,173	50	3.3×10^{-7}
		4 – 3	10,539	118	7.7×10^{-7}
		3 – 2	12,752	146	9.5×10^{-7}
	{24}	12 – 11	2,885	80	5.2×10^{-7}
		11 – 10	3,258	119	7.7×10^{-7}
		10 – 4	1,735	156	1.0×10^{-6}
		4 – 2	12,015	126	8.2×10^{-7}
	{27}	15 – 11	15,834	185	1.2×10^{-6}
		11 – 9	5,138	115	7.5×10^{-7}
		9 – 8	5,676	68	4.4×10^{-7}
		8 – 7	1,832	110	7.2×10^{-7}
Potential transfer yards	**{5}, {16}, {24}**	[2]	217,630		2.274×10^{-8}
		[4]	21,840		
	{16}, {27}	[8]	22,190		
	{5}, {12}, {16}, {27}	[9]	26,740		
	{12}, {24}	[10]	31,010		
	{24},{27}	[11]	20,160		
	{5}, {24}	[12]	59,290		
	{12}, {27}	[15]	54,460		

Table 3.6. Consequence and accident probability of 70 hazmat railcars

r	(i,j) or k	Train Service	$C_{(r)}$	$P_{(r)}$
0	---	---	0	0.999986
1	8 – 4	**{16}**	1,173	3.3×10^{-7}
2	10 – 4	**{24}**	1,735	1.0×10^{-6}
3	8 – 7	**{27}**	1,832	7.2×10^{-7}
4	12 – 11	**{24}**	2,885	5.2×10^{-7}
5	13 – 15	**{12}**	2,944	1.7×10^{-6}
6	9 – 10	**{12}**	3,115	1.0×10^{-6}
7	11 – 10	**{24}**	3,258	7.8×10^{-7}
8	9 – 25	**{5}**	3,937	6.7×10^{-7}
9	25 – 12	**{5}**	4,528	5.3×10^{-7}
10	14 – 13	**{12}**	5,027	4.8×10^{-7}
11	11 – 9	**{27}**	5,138	7.5×10^{-7}
12	4 – 9	**{5}**	5,658	5.9×10^{-7}
13	9 – 8	**{16}, {27}**	5,676	4.4×10^{-7}
14	10 – 14	**{12}**	7,835	8.4×10^{-7}
15	4 – 3	**{16}**	10,539	7.7×10^{-7}
16	2 – 4 & 4 – 2	**{5} & {24}**	12,015	8.2×10^{-7}
17	3 – 2	**{16}**	12,752	9.5×10^{-7}
18	15 – 11	**{27}**	15,834	1.2×10^{-6}
19	[11]	**{24}, {27}**	20,160	2.3×10^{-8}
20	[4]	**{5}, {16}, {24}**	21,840	
21	[8]	**{16, {27**	22,190	
22	[9]	**{5}, {12}, {16}, {27}**	26,740	
23	[10]	**{12}, {24}**	31,010	
24	[15]	**{12}, {27}**	54,460	
25	[12]	**{5}, {24}**	59,290	
26	[2]	**{5}, {16}, {24}**	217,630	

Table 3.7. Sorted consequence and accident probability

Iter.	r	$C_{(r)}$	$P_{(r)}$	VaR Route	f^r	$f^r \leq 1-\alpha < f^r + P_{(r)}$
1	0	0	0.999986	15 - 11 - [9] - 8 - [4] - 2 **{27}** **{16}** **{24}**	3.6×10^{-6}	Does not hold.
3	2	1,735	1.0×10^{-6}	15 - [11] - 10 - 4 - 2 **{27}** **{24}**	2.8×10^{-6}	
10	9	4,528	5.3×10^{-7}	15 - [11] - 10 - 4 - 2 **{27}** **{24}**	2.1×10^{-6}	
19	18	15,834	1.2×10^{-6}	15 - 11 - 9 - [8] - 4 - 3 - 2 **{27}** **{16}**	2.3×10^{-8}	Holds!

Table 3.8. Iteration and VaR*

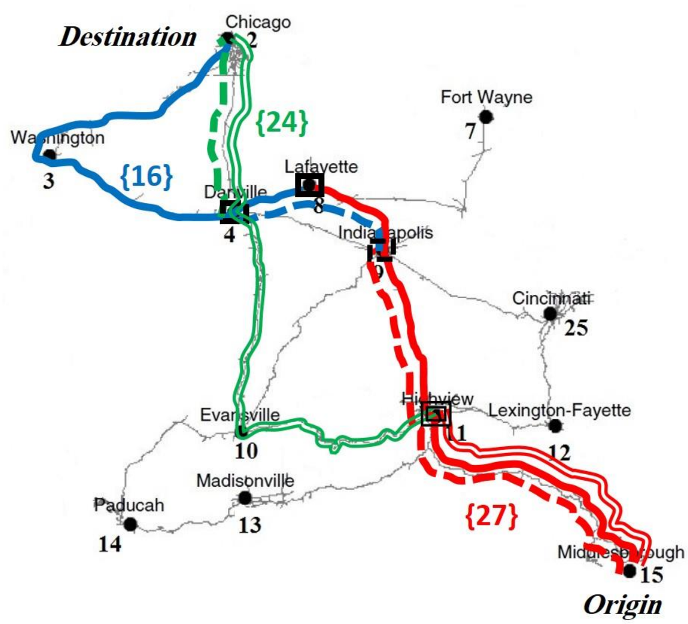


Figure 3.4. Optimal VaR route for the illustrative example

The result of iteration 19 shows that the VaR route generated in this iteration holds the condition $f^r \leq 1 - \alpha < f^r + P_{(r)}$ and so $\text{VaR}_{\alpha}^{*} = C_{(r)} = 15{,}834$. The optimal VaR route shows that the hazmat shipment should be placed on train service number 27, and travel non-stop crossing service-legs 15-11, 11-9, and 9-8 before being transferred to train service number 16 at yard 8. Subsequently, the second train service brings the shipment to the destination node via yards 4 and 3. The optimal route is shown in Figure 3.4 in bold line, while the other two routes are given in dot and double lines. Note that the train services 27, 16, and 24 are shown in red, blue, and green color, respectively.

Finally, we conclude the discussion by providing some additional details needed to calculate VaR for the optimal route (i.e., last row in Table 3.8). The optimal route is composed of the service legs and transfer yards as indicated in the second column of Table 3.9.

t	(i,j) or k	Train Service	$C^{l}_{(t)}$	$P^{l}_{(t)}$	$\sum_{i=0}^{t} P^{l}_{(i)}$
0	---	---	0	0.99999552	0.99999552
1	8 – 4	**{16}**	1,173	3.3×10^{-7}	0.99999585
2	11 – 9	**{27}**	5,138	7.5×10^{-7}	0.99999660
3	9 – 8	**{27}**	5,676	4.4×10^{-7}	0.99999704
4	4 – 3	**{16}**	10,539	7.7×10^{-7}	0.99999781
5	3 – 2	**{16}**	12,752	9.5×10^{-7}	0.99999877
6	15 – 11	**{27}**	15,834	1.2×10^{-6}	0.99999998
7	[8]	**{16}, {27}**	22,190	2.3×10^{-8}	1.00000000

Table 3.9. Components of the optimal VaR route for the illustrative example

According to Equation (3.7), $\mathrm{VaR}^{l}_{\alpha} = 15{,}834$, since

$$\sum_{i=0}^{5} P^{l}_{(i)} < \alpha \leq \sum_{i=0}^{6} P^{l}_{(i)}$$

or

$$0.99999877 < 0.999999 \leq 0.99999998\,.$$

3.7.2. Solution to Problem Instances

In an effort to conduct focused analyses, we will consider shipments from Chicago to Highview, i.e., nodes 2 and 11, respectively. For expositional reasons and for brevity, we consider seven distinct hazmat volumes, i.e., $N = \{5, 20, 40, 60, 80, 100, 120\}$ and for four different confidence levels: $\alpha = \{\alpha_1 = 0.9, \alpha_2 = 0.99999, \alpha_3 = 0.999997, \alpha_4 = 0.999999\}$, and note that other values could be similarly evaluated. In an effort to find the best train configuration for the above problem instances using (3.17), we apply eleven different weights to the model: $\mathcal{W} = \{1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0\}$. Finally, we consider only *medium* trains of 120 railcars in length, and thus the output would give information about hazmat railcars in each decile. For expositional reasons, and for brevity, we only show the results of the best train configurations for different number of hazmat railcars and for various weights, in Table 3.10. For example, for problem instances involving moving five hazmat railcars from Chicago to Highview, only two train configurations indicated by C# emerge: *1st* depicts the scenario when the entire weight is placed on hazmat risk on the service-leg, and all the five hazmat railcars are placed in the 9th decile; and, *2nd* configuration results for other weight setting, and places all the hazmat railcars in the 7th decile. Other settings, and configurations could be interpreted similarly. Thus, a total of 24 distinct configurations were observed for medium length trains, which resulted in 24*4=96 problem instances.

It is important that similar configurations were generated for long and short trains, but for brevity are not reported here. For *long* trains, 26 distinct configurations were observed when assuming a length of 200 railcars, and 26*4=104 problem instances were solved. On the other hand, for *short* trains 8 distinct configurations were determined with 40 railcars train length, and 8*4=32 problem instances were solved. Hence, a total of 232 problem instances were solved to gain managerial insights. The VaR algorithm was coded in Matlab R 2015b, and (3.17) was solved in GAMS 24.1.3 using Cplex 12.5.1.0 as the solver.

We ran them on a 2.90 GHz Intel Core i7 computer system. The computation times are less than 3 seconds.

N	C#	$\mathcal{W}$	Train Configuration									
			y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
5	*1st*	1	0	0	0	0	0	0	0	0	5	0
	2nd	0.9 - 0	0	0	0	0	0	0	5	0	0	0
20	*1st*	1	0	0	0	0	0	0	8	0	12	0
	2nd	0.9 - 0.6	0	0	0	0	0	0	12	0	8	0
	3rd	0.5 - 0.2	0	0	0	0	0	0	12	8	0	0
	4th	0.1 - 0	0	0	0	8	0	0	12	0	0	0
40	*1st*	1	0	0	0	0	0	0	12	4	12	12
	2nd	0.9	0	0	0	0	0	0	12	12	12	4
	3rd	0.8 - 0.5	0	0	4	0	0	0	12	12	12	0
	4th	0.4	0	0	12	0	0	0	12	12	4	0
	5th	0.3	0	0	12	4	0	0	12	12	0	0
	6th	0.2 - 0	0	0	4	12	0	0	12	12	0	0
60	*1st*	1 - 0.8	0	0	12	0	0	0	12	12	12	12
	2nd	0.7 - 0.4	0	0	12	0	0	12	12	12	12	0
	3rd	0.3 - 0	0	0	12	12	0	12	12	12	0	0
80	*1st*	1	0	0	12	0	8	12	12	12	12	12
	2nd	0.9 - 0.7	0	0	12	8	0	12	12	12	12	12
	3rd	0.6 - 0.5	0	0	12	12	0	12	12	12	12	8
	4th	0.4 - 0	0	8	12	12	0	12	12	12	12	0
100	*1st*	1 - 0.7	0	4	12	12	12	12	12	12	12	12
	2nd	0.6	0	12	12	12	4	12	12	12	12	12
	3rd	0.5 - 0.4	4	12	12	12	0	12	12	12	12	12
	4th	0.3 - 0	12	12	12	12	0	12	12	12	12	4
120	*Only*	All	12	12	12	12	12	12	12	12	12	12

Table 3.10. Best medium train configurations for different N and $\mathcal{W}$

It should be evident from equations (3.1) and (3.2) that different train configurations would yield different accident probabilities on the service legs and the transfer yards, and consequently may result in different optimal VaR values and the resulting route associate with each confidence level α. For instance, for 80 hazmat railcar setting depicted in Table 3.10, there are four possible train configurations resulting from attaching eleven different weights on the objective in (3.17). The resulting optimal VaR and the associated route at $\alpha = 0.999999$ are indicated in Table 3.11. Note that either of the first two configurations leads to the optimal VaR value of 5996, and result in the same route.

<table>
<tr><th>C#</th><th>VaR*</th><th>VaR-Route*</th></tr>
<tr><td>1st</td><td rowspan="2">5,996</td><td rowspan="2">$\underset{\{4\}}{\underline{2 - 6 - 8 - [}}\ 9\ \underset{\{5\}}{\underline{] - [}}\ 25\ \underset{\{18\}}{\underline{] - 11}}$</td></tr>
<tr><td>2nd</td></tr>
<tr><td>3rd</td><td>6,221</td><td>$\underset{\{5\}}{\underline{2 - 4 - 9 - [}}\ 25\ \underset{\{18\}}{\underline{] - 11}}$</td></tr>
<tr><td>4th</td><td>8,914</td><td>$\underset{\{3\}}{\underline{2 - 6 - 7 - 22 - [}}\ 23\ \underset{\{18\}}{\underline{] - 25 - 11}}$</td></tr>
</table>

Table 3.11. Optimal VaR and associated route for N=80 and α=0.999999

3.7.3. Analysis and Insights

In this subsection, the 232 problem instances are further analyzed to gain managerial insights, and the discussion is organized by insights.

3.7.3.1. <u>Determinants of optimal VaR</u>

The proposed VaR algorithm is applied to each configuration indicated in Table 3.10, and for the four confidence levels being considered. Table 3.12 depicts the configuration resulting in minimal VaR, and the resulting route for each confidence level. For brevity, unlike in Table 3.11, we are only reporting the configuration resulting in minimum VaR*. Note that at both α_1 and α_2, VaR algorithm finds the same route for the seven specified hazmat volumes, and the optimal VaR is zero.

This implies that with the least probability equivalent to the given confidence level, the hazmat risk associated with the route does not exceed zero. However at α_3 three routes are returned: a single route for the first four hazmat volumes; another distinct route when 80 hazmat railcars have to be shipped; and, yet another distinct route for higher hazmat volumes. Finally, for the highest confidence level, one notices four different optimal VaR routes for the seven specified hazmat volumes.

Two other things are evident from Table 3.12 and Figure 3.5: *first*, at a given confidence level, the optimal VaR values increase with the increase in the number of hazmat railcars; and *second*, for a given number of hazmat railcars, the optimal VaR values increase with an increase in confidence level. For instance, for 80 hazmat railcar instance under both train configurations, the optimal VaR increases from 1877 to 5996 when α increases from 0.999997 to 0.999999.

Confidence Level		Number of hazmat railcars (*N*)						
		5	**20**	**40**	**60**	**80**	**100**	**120**
$\alpha_1 = 0.9$ $\alpha_2 = 0.99999$	*C#*	*All*						*Unique*
	*VaR**	0						
	*VaR-Route**	*(a)*						
$\alpha_3 = 0.999997$	*C#*	*All*				*1st 2nd*	*1st 2nd 3rd*	*Unique*
	*VaR**	0				1,877	4,845	6,078
	*VaR-Route**	*(a)*				*(b)*	*(c)*	
$\alpha_4 = 0.999999$	*C#*	*All*		*1st 2nd*	*1st*	*1st 2nd*	*All*	*Unique*
	*VaR**	0		1,980	4,038	5,996	10,115	11,426
	*VaR-Route**	*(a)*		*(d)*	*(e)*		*(f)*	
(a) =	$\underbrace{2 - 4 - 9 -}_{\{5\}} [\ 25\ ] \underbrace{- 11}_{\{18\}}$							
(b) =	$\underbrace{2 - 6 -}_{\{1\}} [\ 20\ ] \underbrace{- 22 -}_{\{10\}} [\ 24\ ] \underbrace{- 25 - 11}_{\{22\}}$							
(c) =	$\underbrace{2 - 6 - 8 -}_{\{4\}} [\ 9\ ] \underbrace{- 25 -}_{\{5\}} [\ 12\ ] \underbrace{- 11}_{\{24\}}$							
(d) =	$\underbrace{2 - 6 -}_{\{1\}} [\ 20\ ] \underbrace{- 7 -}_{\{9\}} [\ 23\ ] \underbrace{- 25 - 11}_{\{18\}}$							
(e) =	$\underbrace{2 - 6 - 8 -}_{\{4\}} [\ 9\ ] \underbrace{-}_{\{5\}} [\ 25\ ] \underbrace{- 11}_{\{18\}}$							
(f) =	$\underbrace{2 - 6 - 7 - 22 -}_{\{2\}} [\ 21\ ] \underbrace{- 23 - 24 - 25 - 11}_{\{22\}}$							

Table 3.12. Optimal configuration, VaR and associated route at various confidence levels

3.7.3.2. ***Longer but less risky routes***

Figure 3.6 depicts the best optimal routes for the top two hazmat volumes (i.e., *N=100* and *N=120*) at the four confidence levels. More specifically, Figure 3.6-1 depicts the combination of train services 5 and 18 to move shipments at both α_1 and α_2 (route *(a)*); Figure 3.6-2 shows that train services 4, 5 and 24 are needed at α_3

(route *(c)*), and that the optimal VaR value becomes non-zero; and, Figure 3.6-3 indicates usage of two train services, i.e., 2 and 22, to move shipments at α_4 (route *(f)*). It should be evident that the maximum VaR is registered at the highest confidence level and hazmat volume, and that the VaR algorithm seeks to find safer but longer routes which generally include service legs and transfer yards at the periphery of the given network.

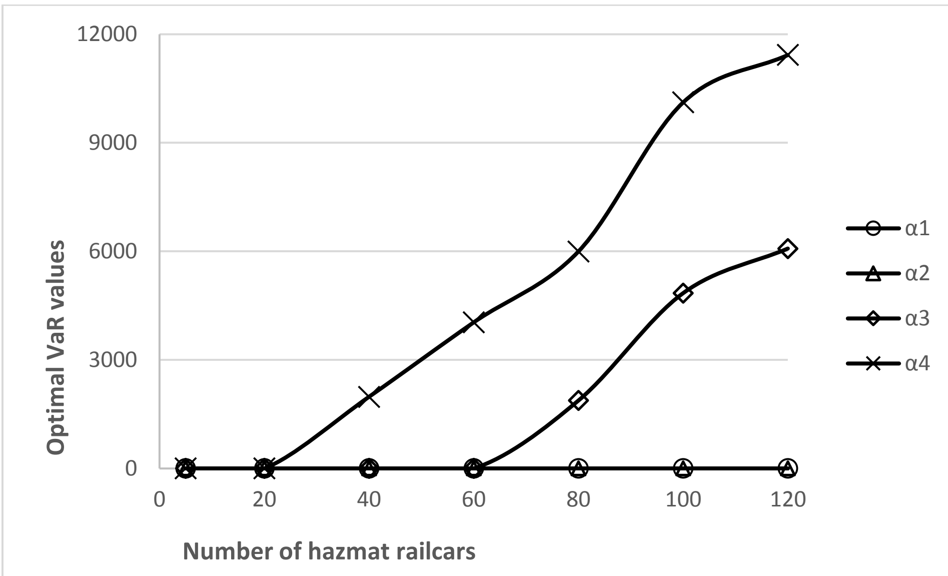
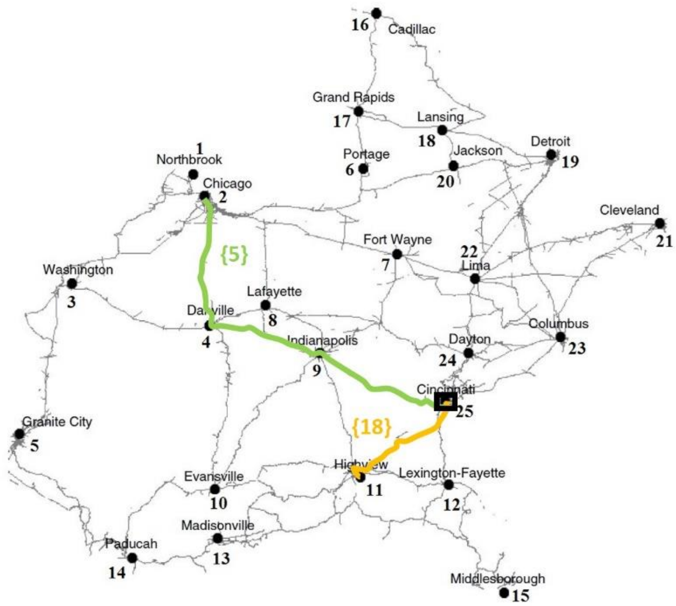
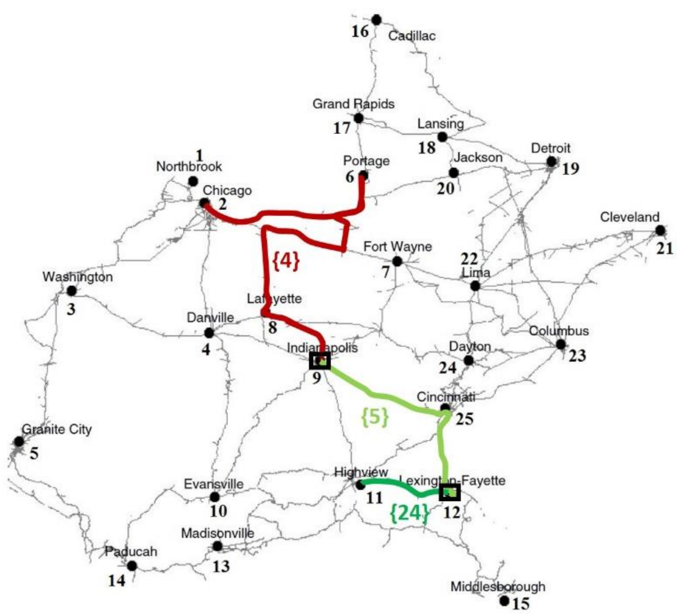


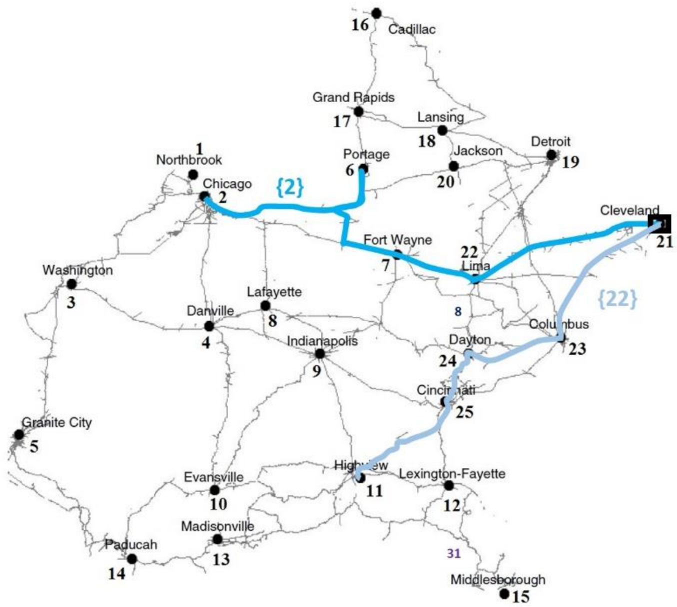
Figure 3.5. Optimal VaR values for different number of hazmat railcars


16 Cadillac
Grand Rapids
17
Lansing
18
Jackson
Detroit
19
1
Northbrook
Portage
6
20
Chicago
2
Cleveland
21
{5}
Fort Wayne
7
22
Lima
Washington
3
Lafayette
8
Danville
4
Indianapolis
9
Dayton
24
Columbus
23
Cincinnati
25
{18}
Granite City
5
Highview
11
Lexington-Fayette
12
Evansville
10
Madisonville
13
Paducah
14
Middlesborough
15

(3.6-1)


16 Cadillac
Grand Rapids
17
Lansing
18
Jackson
Detroit
19
1
Northbrook
Portage
6
20
Chicago
2
Cleveland
21
{4}
Fort Wayne
7
22
Lima
Washington
3
Lafayette
8
Danville
4
Indianapolis
9
Dayton
24
Columbus
23
Cincinnati
25
{5}
Granite City
5
Highview
11
Lexington-Fayette
12
{24}
Evansville
10
Madisonville
13
Paducah
14
Middlesborough
15

(3.6-2)



(3.6-3)

Figure 3.6. Optimal routes for 100 and 120 hazmat railcars

3.7.3.3. Impact of train length

We also investigated the impact of train length on optimal VaR value. To that end, the VaR algorithm was applied to *short* and *long* trains with 40 and 200 railcar lengths, respectively. It was noticed that the optimal VaR for all hazmat volumes for both these train lengths was zero for the first three confidence levels. Recall that the aforementioned is exactly alike medium length trains, except at α_3 for 80, 100, and 120 hazmat railcars (Table 3.13). Figure 3.7 depicts the best VaR values, for the three train lengths, at α_4. It should be evident from Figure 3.7 that long train length would yield lower VaR values, in part because of the potential to exploit the decile-based configuration more appropriately.

Train Length	Confidence Level	Number of hazmat railcars (N)						
		5	20	40	60	80	100	120
short	α_1	0	0	0	-	-	-	-
	α_2	0	0	0	-	-	-	-
	α_3	0	0	0	-	-	-	-
	α_4	0	0	1,980	-	-	-	-
medium	α_1	0	0	0	0	0	0	0
	α_2	0	0	0	0	0	0	0
	α_3	0	0	0	0	1,877	4,845	6,078
	α_4	0	0	1,980	4,038	5,996	10,115	11,426
long	α_1	0	0	0	0	0	0	0
	α_2	0	0	0	0	0	0	0
	α_3	0	0	0	0	0	0	0
	α_4	0	0	0	0	0	3,482	5,628

Table 3.13. Optimal VaR values for three train lengths at various confidence levels

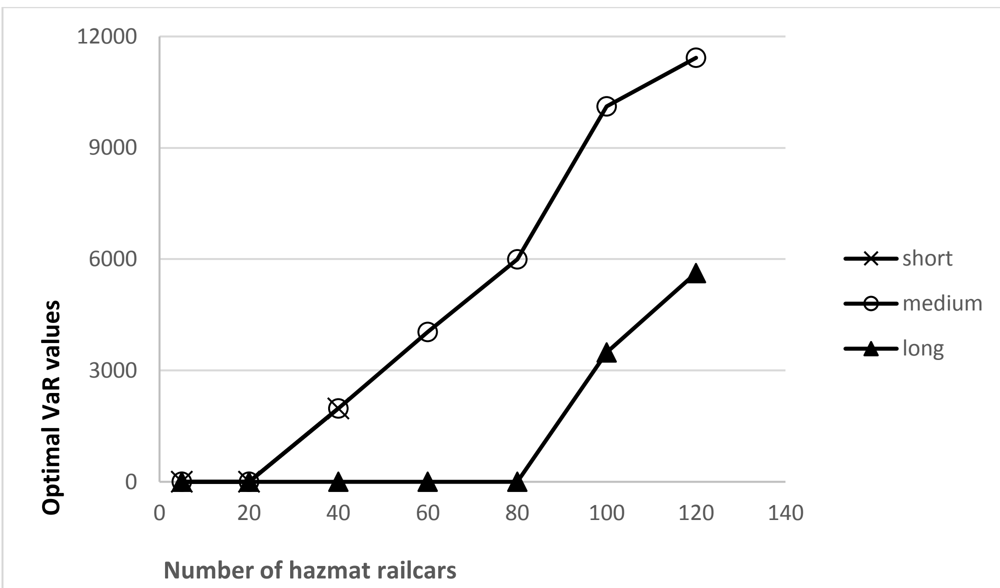


Figure 3.7. Optimal VaR values for different *N* for three train lengths at α_4

3.7.3.4. *VaR=0. What does it mean?*

It is worth emphasizing here why VaR value is zero for many of the instances and they have the same optimal route. Consider route l and recall that $P^l_{(0)} = 1 -$

$\sum_{i=1}^{n_l} P_{(i)}^{l}$, where $P_{(i)}^{l}$ shows the accident probability of service legs and yards (p_{ij} and p_k, respectively) in route l. As it is seen in the illustrative example, these accident probabilities are very small, which in turn make $P_{(0)}^{l}$ big. On the other hand, we know that as long as the confidence level is less than or equal to $P_{(0)}^{l}$ ($0 < \alpha \leq P_{(0)}^{l}$), the VaR value of route l will be equal to zero: $\text{VaR}_{\alpha}^{l} = 0$. Therefore if a VaR value greater than zero is desired for a route, big confidence levels should be considered. That is why for many of the problem instances, the optimal VaR value is equal to zero and the optimal VaR route is the same, because the confidence level is not big enough in comparison with the total accident probability of the route. Note that for all of the problem instances (short, medium, and long trains) with optimal VaR value equal to zero, the optimal VaR route is as follows (route *(a)*):

$$\underbrace{2 \quad - \quad 4 \quad - \quad 9 \quad - \quad [}_{\{5\}} \quad 25 \quad \underbrace{] \quad - \quad 11}_{\{18\}}$$

This means that with the least confidence of the given confidence levels, we can claim that the above route does not expose any threat to the population in those problem instances.

3.7.3.5. <u>VaR superiority over the existing measures</u>

Finally, in an effort to demonstrate both the distinctness and superiority of VaR over the three most popular risk assessment techniques – i.e., traditional risk (*TR*), incident probability (*IP*), and population exposure (*PE*), we solved the case study using each of the three risk assessment techniques. Table 3.14 depicts the appropriate expressions for the three measures after appropriate adaptations for rail shipments on the network indicated for the case study in Section 3.6. For precise comparison, we again consider the problem of sending N hazmat railcars from Chicago (i.e., node 2) to Highview (i.e., node 11) using a medium train (with 120 railcars), and the resulting solutions are depicted in Table 3.15.

Model	**Risk Measure**	**Objective**
TR	Expected consequence	$\min_{l \in L} \left(\sum_{k \in \mathcal{Y}^l} p_k c_k + \sum_{(i,j) \in \mathcal{A}^l} p_{ij} c_{ij} \right)^*$
IP	Incident probability	$\min_{l \in L} \left(\sum_{k \in \mathcal{Y}^l} p_k + \sum_{(i,j) \in \mathcal{A}^l} p_{ij} \right)$
PE	Population exposure	$\min_{l \in L} \left(\sum_{k \in \mathcal{Y}^l} c_k + \sum_{(i,j) \in \mathcal{A}^l} c_{ij} \right)$

Table 3.14. The existing risk assessment models in the literature for rail hazmat shipments

** A detailed explanation for calculating the objective of TR model is provided in the Appendix (Section 3.9)*

Risk Model		Number of hazmat railcars (*N*)						
		5	**20**	**40**	**60**	**80**	**100**	**120**
TR	*C#*	*2nd*	*2nd*	*3rd*	*2nd*	*2nd*	*1st*	*Unique*
	*TR**	0.00015	0.00139	0.00549	0.01292	0.02322	0.03867	0.05967
	*TR-Route**	*(g)*	*(c)*					
IP	*C#*	*1st*						*Unique*
	*IP**	1.14E-07	4.64E-07	1.13E-06	2.02E-06	3.09E-06	4.41E-06	6.03E-06
	*IP-Route**	*(a)*						
PE	*C#*	*All*						*Unique*
	*PE**	9,338	25,608	44,736	62,306	78,000	93,029	108,949
	*PE-Route**	*(h)*						
(a) =	$\underbrace{2 - 4 - 9 - [}_{\{5\}}\ 25\ \underbrace{] - 11}_{\{18\}}$							
(c) =	$\underbrace{2 - 6 - 8 - [}_{\{4\}}\ 9\ \underbrace{] - 25 - [}_{\{5\}}\ 12\ \underbrace{] - 11}_{\{24\}}$							
(g) =	$\underbrace{2 - 6 - [}_{\{4\}}\ 8\ \underbrace{] - [}_{\{11\}}\ 4\ \underbrace{] - 9 - 25 - [}_{\{5\}}\ 12\ \underbrace{] - 11}_{\{24\}}$							
(h) =	$\underbrace{2 - 6 - 8 - [}_{\{4\}}\ 9\ \underbrace{] - 10 - 14 - [}_{\{12\}}\ 13\ \underbrace{] - 11}_{\{29\}}$							

Table 3.15. Optimal configuration, objective and associated route for the three risk measures

A total of four routes were generated for the three risk measures across the different hazmat volumes (Table 3.15). In contrast, the VaR approach took into consideration the risk preference (or tolerance) of the decision maker, and yielded six routes (Table 3.12). It is pertinent that of the two routes generated by the *TR* approach, the first one holds only for 5 hazmat railcars (and thus could be an anomaly). More importantly, the second route being used for the remaining seven hazmat volumes is exactly the same as the one with the VaR model when N=100 and 120 hazmat railcars at a confidence level of $\alpha_3 = 0.999997$, i.e., route *(c)*.

In addition, the route generated by the *IP* approach is the route generated by VaR at all four confidence levels, and whose optimal value is equal to zero. Note that this route is indicated by index *(a)* in both Table 3.12 and Table 3.15, and the interpretation of VaR=0 was provided in Section 3.7.3.4. More specifically, for the appropriate hazmat volume and train configuration, none of the four confidence levels is large enough compared to the total accident probability of the route –i.e., the route that minimizes total incident probability when using the *IP* approach.

Finally, the objective of the *PE* approach is to minimize the total consequence of the hazmat route, which neither depends on the accident probabilities on the service legs or in the yards, and nor on the train configuration. Therefore, for all hazmat volumes, all train configurations result in the same optimal route. On the other hand, for all hazmat volumes using the *IP* approach, only the 1st train configuration results in the optimal route. This is because of the objective to minimize total accident probability of a route, which in turn is mostly composed of service legs –and thus the model selects train configuration likely to minimize accident probabilities on the service legs, i.e., determining y_r for p_{ij} minimization, which happens when $\mathcal{W}$ is close to 1. Note that unlike these models, VaR approach utilizes various train configurations for different hazmat volumes to find the optimal routes.

3.8 Conclusion

Railroad is one of the safest modes for transporting hazmat, however, the possibility of spectacular events resulting from multicar incidents, however small, does exist. Though the last two decades has witnessed the development of numerous risk assessment methodologies, most of them were developed in the highway domain and thus have limited effectiveness in capturing the *low probability –high consequence* nature of rail hazmat incidents, i.e., prepare risk-averse routes. We make a first attempt to develop a *Value-at-Risk* (VaR) assessment methodology to facilitate risk-averse routing of hazmat shipments. It is important that developing such routing plans in a railroad setting is more complex than in highway transportation because of three reasons: *first*, the characteristics of railroad operations need to be taken into consideration; *second*, one could only work with the given set of pre-defined train services, which would entail transfer operations at yards and thus the corresponding risk; and *third*, decision about optimal train configuration should also be taken into consideration.

The proposed risk assessment methodology was used to study 232 problem instances generated using the realistic infrastructure of a railroad operator in Midwest United States, which were further analyzed to arrive at the following insights. *First*, unlike the three most popular risk assessment methodologies, VaR incorporates the risk-preference of the decision maker to generate (multiple) routes between a given origin-destination pair. More specifically, as confidence level goes from the theoretical minimum value of zero to the theoretical maximum value of one, the decision maker moves away from being risk-neutral and towards becoming risk-averse. Having such flexibility could be important from the perspective of the regulators, who is interested in a balanced distribution of hazmat risk in the network. *Second*, for a given origin-destination pair, optimal VaR value has a positive relationship with both the hazmat volume, and the confidence level (or risk-preference) of the decision maker. *Third*, at high confidence levels (i.e., risk-averse), safer but longer routes that make use of the service legs and yards at the periphery of the network are utilized. *Fourth*, for a given route, longer trains would result in lower VaR values because of the potential to exploit the decile-based configurations more appropriately.

Though the proposed approach was a useful first attempt at understanding the nuances of risk-averse routing of rail hazmat shipments, it has some shortcomings. Some instances with optimal VaR value of zero were noticed, which implies that complete risk measure for the corresponding route could not be generated. In some other with positive VaR values, it was still not possible to capture information about the extra population at risk due to the accident.

To sum, though by focusing on the adverse tail of the distribution, VaR seems a more suitable measure of hazmat risk than the three most popular measures (i.e., traditional risk, incident probability, and population exposure), the aforementioned shortcomings does create an incentive to search for alternative measures that could quantify the affected populace beyond the threshold, i.e., VaR. One such alternate measure is conditional value-at-risk (CVaR) (Toumazis et al., 2013; Toumazis and Kwon, 2013; Faghih-Roohi et al., 2015; Toumazis and Kwon, 2016), which is the immediate area of interest for this dissertation.

3.9 Appendix

As explained in Chapter 2, various models that use different risk measures have been developed so far to quantify the risk generated by a route. The traditional risk (TR) model, however, is the most popular one in railroad hazmat transportation. It calculates the expected value of the consequence along the route (Sherali et al., 1997; Erkut and Verter, 1998). Suppose a route l which uses $T(\text{out of } S)$ train services and consists of an ordered set (from the route's origin to its destination) of service legs $\mathcal{A}^l = \bigcup_{s\in T} \mathcal{A}^l_s$ and yards $\mathcal{Y}^l = \bigcup_{s\in T} \mathcal{Y}^l_s$, such that $\mathcal{A}^l = \{(i^r, j^r): 1 \leq r \leq n_l\}$ and $\mathcal{Y}^l = \{k^r: 1 \leq r \leq n_l\}$, where $n_l = |\mathcal{Y}^l \cup \mathcal{A}^l|$ (the cardinality of $\mathcal{Y}^l \cup \mathcal{A}^l$), (i^r, j^r) is the rth item in the route showing a service leg, and k^r is the rth item in the route showing a yard. Recall that R^l denotes the discrete random variable for the risk along route l. Using TR, we have

$$\begin{aligned} \mathrm{E}\,[R^l] = & \sum_{(i^r,j^r)\in\mathcal{A}^l} \prod_{(i^h,j^h)\in\mathcal{A}^l,\ k^h\in\mathcal{Y}^l,\ h<r} \left(1 - p_{i^h j^h}\right)\left(1 - p_{k^h}\right) p_{i^r j^r}\ c_{i^r j^r} \\ & + \sum_{k^r\in\mathcal{Y}^l} \prod_{(i^h,j^h)\in\mathcal{A}^l,\ k^h\in\mathcal{Y}^l,\ h<r} \left(1 - p_{i^h j^h}\right)\left(1 - p_{k^h}\right) p_{k^r}\ c_{k^r} \end{aligned} \tag{A3.1}$$

which states that immediately after an accident occurs on a service leg $\left((i^r, j^r)\right)$ or at a yard (k^r), the hazmat shipment will be terminated.

Harwood et al. (1993), using North American data, estimate hazmat accident probabilities to be on the order of 10^{-6} per trip per kilometer, which are extremely small. This means that the probability $(1-p_{i^h j^h})(1-p_{k^h})$ that no accident occurs on service leg (i^h, j^h) nor at yard (k^h) is close to 1. Therefore following (Jin and Batta, 1997), (A3.1) can be approximated by

$$\mathrm{E}\,[R^l] \approx \sum_{(i^r,j^r)\in\mathcal{A}^l} p_{i^r j^r}\, c_{i^r j^r} + \sum_{k^r\in\mathcal{Y}^l} p_{k^r}\, c_{k^r} \tag{A3.2}$$

The TR model (A3.2) can be optimized much easier than (A3.1), since it is a shortest-path problem in which the traversing costs of service leg (i, j) and yard k are $p_{ij}c_{ij}$ and $p_k c_k$, respectively.

Chapter 4. A Conditional Value-at-Risk (CVaR) Methodology to Optimal Train Configuration and Routing of Rail Hazmat Shipments

4.1 Introduction

Motivated by the risk-neutral behavior of traditional risk (TR) model and consequent failure to capture the public posture against hazmat transportation, recently, there have been efforts to generate different routes based on the risk preferences of the decision maker. To that end, Kang et al. (2014a, b) proposed a value-at-risk (VaR) model for highway shipments. Subsequently, Hosseini and Verma (2017) proposed a VaR-based assessment framework for routing rail hazmat shipments, as explained in the previous chapter. Within hazmat transportation setting, VaR has a simple interpretation, *viz.*, how many people are exposed to hazmat risk given a certain confidence level? Though the straightforward interpretation has its appeal, VaR in general has not found widespread acceptance as a measure of risk since it is not a coherent risk measure (Artzner et al., 1999; Dowd and Blake, 2006), and might lead to inaccurate perception of risk (Einhorn, 2008; Nocera, 2009) because it cuts off and ignores what happens in the tail of the distribution –and thus overlook catastrophic events (see Figure 4.1). The indicated shortcomings of VaR motivated the development of a more sophisticated measure, i.e. conditional value-at-risk (CVaR) which is capable of quantifying population exposure that may be encountered in the unfavorable tail of the distribution to avoid extreme events. Toumazis et al. (2013) were the first to adapt the CVaR notion to simultaneously generate flexible and risk averse route for highway hazmat shipments, and the notion was further developed in Toumazis and Kwon (2013), Faghih-Roohi et al. (2015), and Toumazis and Kwon (2016).

It is important that, given the possibility of hazmat release from multiple railcars, consequences are much more catastrophic within a railroad setting than for highway shipments. Hence, the proposed work makes a first attempt to develop a CVaR-based risk assessment methodology for rail hazmat shipments that incorporates the characteristics of railroad accidents, and then utilizes it to prepare rail hazmat routing plans for different confidence levels wherein risk-averse phenomenon is sustained. Note that, given the physical infrastructure of railroad transportation system, the proposed methodology should work given the limited number of pre-defined train services, which in turn might require transfer operations at intermediate yards and determining optimal train configuration. Thus, the proposed methodology is more complex than routing a single hazmat truck between a given origin-destination pair in an unconstrained highway network. In addition, we have also attempted to provide a succinct and clear definition of CVaR for hazmat shipments, which we felt was done un-satisfactorily by the published peer-reviewed works. Finally, we use the proposed CVaR methodology to study and analyze several problem instances generated using the realistic infrastructure of a railroad operator, and to gain insights. Through numerical experiments we demonstrate that the proposed methodology is superior to other measures for risk-averse routing of hazmat shipments and versatile enough to yield routes based on risk preferences of the decision makers.

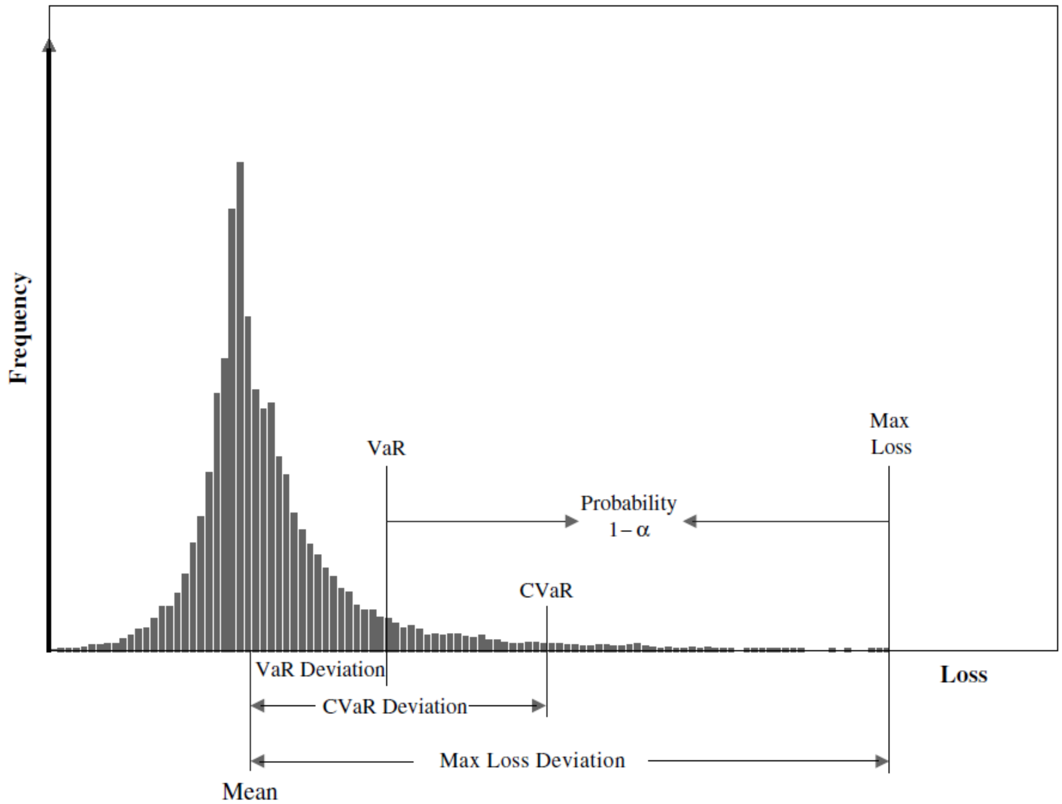


Figure 4.1. VaR and CVaR deviations [Source: Sarykalin et al. (2008)]

The rest of the chapter is organized as follows. *Section 4.2* provides a detailed discussion about the proposed conditional value-at-risk methodology. *Section 4.3* contains the discussion about the railroad network and the parameters used to generate problem instances, which are then solved and analyzed to gain insights. Finally, conclusions and directions of future research are presented in *Section 4.4*.

In order to facilitate the discussion of the proposed CVaR methodology in *Section 4.2*, we first refer the reader to the two building blocks presented in Chapter 3; appropriate sets and notations pertinent to railroad transportation system introduced in Section 3.2, followed by an exposition of the concept of value-at-risk in Section 3.3.

4.2 Conditional value-at-risk (CVaR) Methodology

CVaR and its minimization formula were first developed in Rockafellar and Uryasev (2000) to optimize a portfolio so as to reduce the risk of high losses, and focused on continuous distributions with smooth density functions. However, in a subsequent work (Rockafellar and Uryasev, 2002), they developed a general definition of CVaR for random variables with possibly a discontinuous distribution function. In this section, we will provide complete details of how this measure was adapted to assess risk from rail hazmat shipments.

4.2.1. Definition

For a random variable X that represents loss and has a continuous distribution function $F_X(x)$, the CVaR at a given confidence level α is defined as the expected loss given that the loss is greater than or equal to the VaR, i.e. $\mathrm{CVaR}_\alpha(X) = E[X \mid X \geq \mathrm{VaR}_\alpha(X)]$ (see Figure 4.2). However, in the general case when there are discontinuities in the distribution function, CVaR is replaced by two terms: CVaR^- and CVaR^+, called lower and upper CVaR, respectively. Hence, for our problem, we have:

$$\mathrm{CVaR}^{-l}_{\alpha} = E(R^l \mid R^l \geq \mathrm{VaR}^l_{\alpha}) \tag{4.1}$$

$$\mathrm{CVaR}^{+l}_{\alpha} = E(R^l \mid R^l > \mathrm{VaR}^l_{\alpha}) \tag{4.2}$$

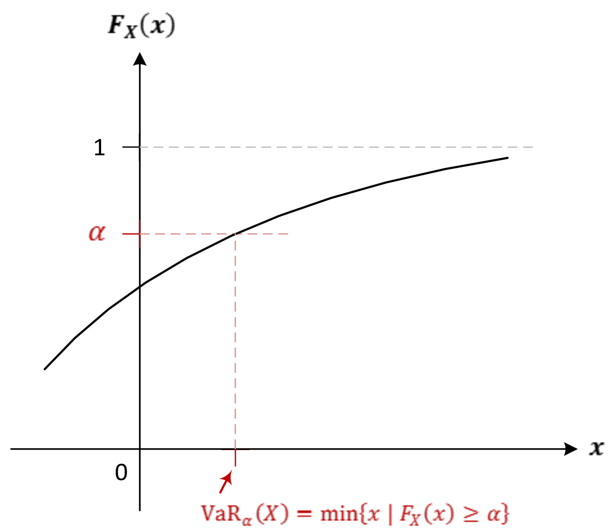


Figure 4.2. For continuous $F_X(x)$: $\mathrm{CVaR}_{\alpha}(X) = E[X \mid X \geq \mathrm{VaR}_{\alpha}(X)]$

It is important that contrary to popular opinion, in the general case, CVaR is not always equal to the average of the consequences greater than (or equal to) VaR. For problems, similar to what we are studying, where the distribution function has possible discontinuities (see Figure 3.1), CVaR has a more elusive definition. More specifically, CVaR of the risk associated with the route l given confidence level $\alpha \in (0,1)$ is defined as:

$$\mathrm{CVaR}^l_{\alpha} = \text{mean of the } \alpha\text{-tail distribution of } R^l, \tag{4.3}$$

where by the α-tail distribution of R^l we mean the one with distribution function $F^{\alpha}_{R^l}(\beta)$ defined by

$$F^{\alpha}_{R^l}(\beta) = \begin{cases} 0, & \text{for } \beta < \mathrm{VaR}^l_\alpha \\ \dfrac{F_{R^l}(\beta) - \alpha}{1-\alpha}, & \text{for } \beta \geq \mathrm{VaR}^l_\alpha \, . \end{cases} \tag{4.4}$$

Note that $F^{\alpha}_{R^l}(\beta)$ is truly another distribution function, but like $F_{R^l}(\beta)$; it is non-decreasing and right-continuous, with $F^{\alpha}_{R^l}(\beta) \to 1$ as $\beta \to \infty$. This way CVaR^l_α can be obtained as follows:

$$\mathrm{CVaR}^l_\alpha = \int_0^\infty \beta \, dF^{\alpha}_{R^l}(\beta). \tag{4.5}$$

The elusiveness of the above definition is because of the probability atom that exists at VaR^l_α in the original distribution function of risk R^l ($F_{R^l}(\beta)$), as explained in Figure 3.1. Recall that α^- and α^+ (defined in (3.8) and (3.9), respectively) mark the bottom and top of the vertical gap in $F_{R^l}(\beta)$, which makes an interval of confidence level α having the same VaR^l_α. The issue that arises here is that what should really be meant by the α-tail distribution given in the definition of CVaR in (4.3), since that term presumably refer to the ‘‘upper $1-\alpha$ part’’ of the full distribution, but neither of the intervals $[\mathrm{VaR}^l_\alpha, \infty)$ and $(\mathrm{VaR}^l_\alpha, \infty)$ has probability $1-\alpha$:

$$[\mathrm{VaR}^l_\alpha, \infty): \qquad \Pr(R^l \geq \mathrm{VaR}^l_\alpha) = 1 - \Pr(R^l < \mathrm{VaR}^l_\alpha) = 1 - \alpha^- \, ,$$

$$(\mathrm{VaR}_\alpha^l, \infty): \qquad \Pr(R^l > \mathrm{VaR}_\alpha^l) = 1 - \Pr(R^l \leq \mathrm{VaR}_\alpha^l) = 1 - \alpha^+ \, .$$

In fact, the interval $[\mathrm{VaR}_\alpha^l, \infty)$ has probability $1 - \alpha^-$ which is greater than $1 - \alpha$ and the interval $(\mathrm{VaR}_\alpha^l, \infty)$ has probability $1 - \alpha^+$ which is less than $1 - \alpha$, when $\alpha^- < \alpha < \alpha^+ < 1$. To resolve this issue, one can "split" the atom, which has a total probability of $\alpha^+ - \alpha^-$, into two pieces with probabilities $\alpha - \alpha^-$ and $\alpha^+ - \alpha$. Then by considering only the upper piece which is adjoining the interval $(\mathrm{VaR}_\alpha^l, \infty)$, one will obtain the probability $1 - \alpha$ through $(\alpha^+ - \alpha) + (1 - \alpha^+)$. This conceptually means one is rescaling that portion of the graph of the original distribution $F_{R^l}(\beta)$ which is between the horizontal lines at levels α and 1, and spans it instead between 0 and 1, so that $F_{R^l}^\alpha(\beta)$ in (4.4) is gained. The α-tail distribution in (4.3) can be then stated using this new distribution. Figure 4.3 shows the result of such a conversion for Figure 3.1.

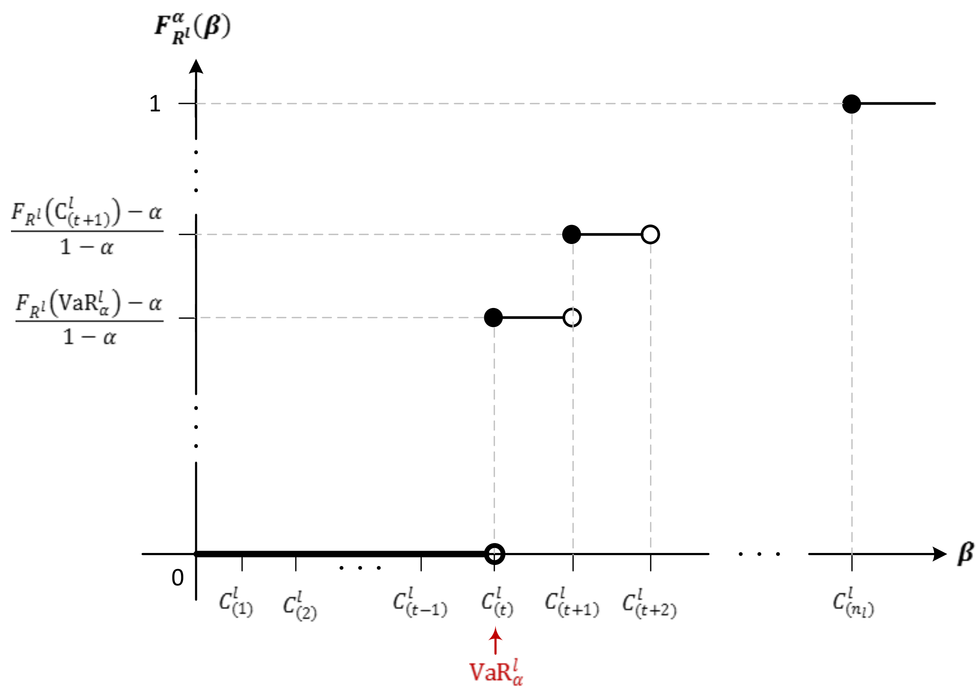


Figure 4.3. $F^{\alpha}_{R^l}(\beta)$ is obtained by rescaling $F_{R^l}(\beta)$ in the interval $[\alpha, 1]$

4.2.2. Proposed approach

The splitting technique outlined in the previous section implies that CVaR can be presented as a weighted average of VaR and the risk greater than that (see (4.5)) as follows:

$$\mathrm{CVaR}^l_{\alpha} = \lambda^l_{\alpha}\ \mathrm{VaR}^l_{\alpha} + (1-\lambda^l_{\alpha})\ \mathrm{CVaR^+}^l_{\alpha} \tag{4.6}$$

where λ^l_{α} signifies the probability that is allotted to the risk amount VaR^l_{α} by the α-tail distribution defined in (4.3) and (4.4), i.e.,

$$\lambda_\alpha^l = \frac{F_{R^l}(\mathrm{VaR}_\alpha^l) - \alpha}{1 - \alpha} \tag{4.7}$$

where $F_{R^l}(\mathrm{VaR}_\alpha^l) \geq \alpha$ always by definition in (3.3). Note that $F_{R^l}(\mathrm{VaR}_\alpha^l) = 1$ and so $\lambda_\alpha^l = 1$ means VaR_α^l has gained the highest possible risk (there is no chance of a risk greater than VaR_α^l), therefore $\mathrm{CVaR}_\alpha^l = \mathrm{VaR}_\alpha^l$, although ${\mathrm{CVaR}^+}_\alpha^l$ is ill defined. Whereas as long as $F_{R^l}(\mathrm{VaR}_\alpha^l) < 1$, meaning there is a chance of a risk greater that VaR_α^l, then $\mathrm{CVaR}_\alpha^l > \mathrm{VaR}_\alpha^l$. Therefore generally $\mathrm{CVaR}_\alpha^l \geq \mathrm{VaR}_\alpha^l$, which is intuitively evident from the definition of CVaR.

However, our problem consists of scenarios, where the distribution function $F_{R^l}(\beta)$ is a step function with jumps at discrete points (Figure 3.1). Hence, CVaR can be computed in an explicit manner as explained next. Consider the route l and recall that the distribution of risk R^l is concentrated in finitely many points; $C_{(i)}^l, i \in \{0, 1, 2, \dots, n_l\}$, where $C_{(0)}^l = 0 < C_{(1)}^l < C_{(2)}^l < \cdots < C_{(n_l)}^l$ and that the probability of $C_{(i)}^l$ is $P_{(i)}^l$. Let K be equal to 0 if $0 < \alpha \leq P_{(0)}^l$, otherwise let it be the unique index such that

$$\sum_{i=0}^{K-1} P_{(i)}^l < \alpha \leq \sum_{i=0}^{K} P_{(i)}^l \, .$$

Using (3.7), the α-VaR of the risk is then given by:

$$\mathrm{VaR}_\alpha^l = C_{(K)}^l \, .$$

According to (4.7)

$$\lambda_\alpha^l = \frac{\Pr\left(R^l \leq C_{(K)}^l\right) - \alpha}{1-\alpha} = \frac{1}{1-\alpha}\left(\sum_{i=0}^{K} P_{(i)}^l - \alpha\right),$$

and, according to (4.2)

$$\mathrm{CVaR^+}_\alpha^l = E(R^l \mid R^l > \mathrm{VaR}_\alpha^l) = \sum_\beta \beta\, f(\beta \mid R^l > \mathrm{VaR}_\alpha^l)\,,$$

where

$$f(\beta \mid R^l > \mathrm{VaR}_\alpha^l) = \Pr(R^l = \beta \mid R^l > \mathrm{VaR}_\alpha^l) = \frac{\Pr(R^l = \beta,\ R^l > \mathrm{VaR}_\alpha^l)}{\Pr\left(R^l > \mathrm{VaR}_\alpha^l\right)}\,,$$

so

$$\mathrm{CVaR^+}_\alpha^l = \sum_\beta \beta\ \frac{\Pr(R^l = \beta,\ R^l > \mathrm{VaR}_\alpha^l)}{\Pr\left(R^l > \mathrm{VaR}_\alpha^l\right)} = \sum_{i=K+1}^{n_l} C_{(i)}^l\ \frac{P_{(i)}^l}{\sum_{i=K+1}^{n_l} P_{(i)}^l}$$

$$= \frac{\sum_{i=K+1}^{n_l} C_{(i)}^l\, P_{(i)}^l}{\sum_{i=K+1}^{n_l} P_{(i)}^l}\,,$$

therefore,

$$(1-\lambda_\alpha^l)\ \mathrm{CVaR^+}_\alpha^l = \frac{1}{1-\alpha}\left(\sum_{i=K+1}^{n_l} C_{(i)}^l\, P_{(i)}^l\right).$$

Consequently, according to (4.6), the α-CVaR is given by

$$\mathrm{CVaR}_\alpha^l = \frac{1}{1-\alpha}\left[\left(\sum_{i=0}^{K} P_{(i)}^l - \alpha\right)\mathrm{VaR}_\alpha^l + \sum_{i=K+1}^{n_l} P_{(i)}^l\, C_{(i)}^l\right]. \tag{4.8}$$

Similarly, we can compute $\mathrm{CVaR^-}_\alpha^l$ using (4.1)

$$\mathrm{CVaR}^{-}{}_{\alpha}^{l} = E(R^l \mid R^l \geq \mathrm{VaR}_{\alpha}^{l}) = \sum_{\beta} \beta \, \frac{\Pr(R^l = \beta,\ R^l \geq \mathrm{VaR}_{\alpha}^{l})}{\Pr\left(R^l \geq \mathrm{VaR}_{\alpha}^{l}\right)}$$

$$= \frac{\sum_{i=K}^{n_l} C_{(i)}^{l} \, P_{(i)}^{l}}{\sum_{i=K}^{n_l} P_{(i)}^{l}}.$$

4.2.3. Relations

As explained in the definition of CVaR, for distributions with possible discontinuities, CVaR can differ from either of CVaR^{+} and CVaR^{-} quantities. The terms ‘‘mean shortfall’’ (Mausser and Rosen, 1999) and ‘‘expected shortfall’’ (Acerbi et al., 2001) have been used in the literature to call CVaR^{+}, while the term ‘‘tail VaR’’ has been suggested for CVaR^{-} (Artzner et al., 1999). Compared to the definition of $\mathrm{CVaR}_{\alpha}^{l}$ in (4.3) and (4.4), the $\mathrm{CVaR}^{-}{}_{\alpha}^{l}$ value in (4.1) is the mean of the risk distribution associated with

$$F_{R^l}^{-\,\alpha}(\beta) = \begin{cases} 0, & \text{for } \beta < \mathrm{VaR}_{\alpha}^{l} \\ \dfrac{F_{R^l}(\beta) - \alpha^{-}}{1 - \alpha^{-}}, & \text{for } \beta \geq \mathrm{VaR}_{\alpha}^{l} \end{cases}$$

similarly the $\mathrm{CVaR}^{+}{}_{\alpha}^{l}$ value in (4.2) is the mean of the risk distribution associated with

$$F_{R^l}^{+\,\alpha}(\beta) = \begin{cases} 0, & \text{for } \beta < \mathrm{VaR}_{\alpha}^{l} \\ \dfrac{F_{R^l}(\beta) - \alpha^{+}}{1 - \alpha^{+}}, & \text{for } \beta \geq \mathrm{VaR}_{\alpha}^{l} \end{cases}$$

This way, the following CVaR relations may happen:

- If there was no probability atom at $\mathrm{VaR}_{\alpha}^{l}$ $\left(\alpha^{-}=\alpha=\alpha^{+}\in(0,1)\right)$, we would simply have ${\mathrm{CVaR}^{-}}_{\alpha}^{l}=\mathrm{CVaR}_{\alpha}^{l}={\mathrm{CVaR}^{+}}_{\alpha}^{l}$.

But with a probability atom at $\mathrm{VaR}_{\alpha}^{l}$:

- If $\alpha=\alpha^{+}$, we get $\alpha^{-}<\alpha^{+}<1, \Rightarrow {\mathrm{CVaR}^{-}}_{\alpha}^{l}<\mathrm{CVaR}_{\alpha}^{l}={\mathrm{CVaR}^{+}}_{\alpha}^{l}$,
- If $\alpha^{+}=1, \Rightarrow {\mathrm{CVaR}^{-}}_{\alpha}^{l}=\mathrm{CVaR}_{\alpha}^{l}$, with ${\mathrm{CVaR}^{+}}_{\alpha}^{l}$ then being ill defined ,
- If $\alpha^{-}<\alpha<\alpha^{+}<1, \Rightarrow {\mathrm{CVaR}^{-}}_{\alpha}^{l}<\mathrm{CVaR}_{\alpha}^{l}<{\mathrm{CVaR}^{+}}_{\alpha}^{l}$.

In general, $\mathrm{CVaR}^{-}\leq\mathrm{CVaR}\leq\mathrm{CVaR}^{+}$. If there is no jump (probability atoms induced by discreteness) at the VaR threshold in the distribution function, the equalities hold, but when a jump occurs, both inequalities can be strict.

4.2.4. Examples

CVaR definitions and relations for our scenario-based model are illustrated further with the following examples inspired by Sarykalin et al. (2008). Suppose that route l is composed of totally 5 service legs and transferring yards ($n_l=5$) with ordered accident consequences $C_{(1)}^{l},\ldots,C_{(5)}^{l}$ and equal corresponding accident probabilities of $P_{(1)}^{l}=\cdots=P_{(5)}^{l}=0.01$, hence $P_{(0)}^{l}=0.95$. For the first case, let $\alpha=0.98$, hence $\alpha^{-}=0.97$ and $\alpha^{+}=\alpha$ (see Figure 4.4). In this case, α does not split any probability atom. Then $\lambda_{\alpha}^{l}=0$, $\mathrm{VaR}_{\alpha}^{l}<{\mathrm{CVaR}^{-}}_{\alpha}^{l}<\mathrm{CVaR}_{\alpha}^{l}={\mathrm{CVaR}^{+}}_{\alpha}^{l}$, where

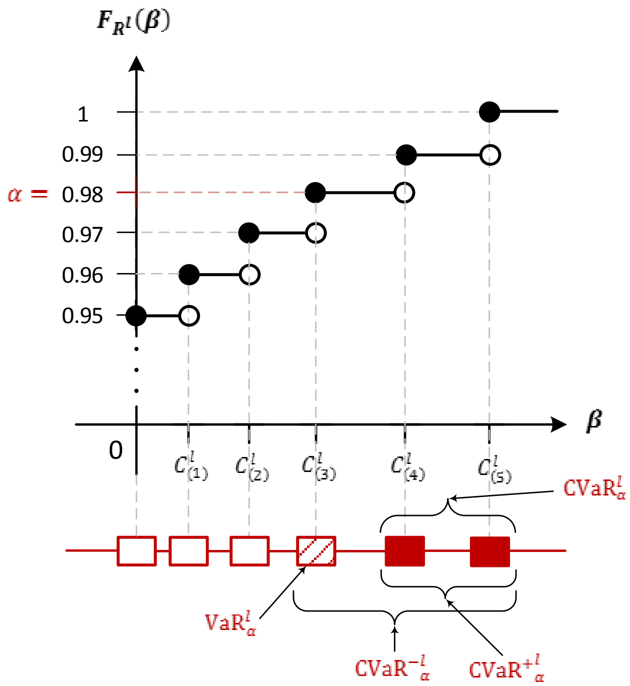


Figure 4.4. CVaR Example 1: computation of CVaR when α does not split the atom

$$\mathrm{VaR}^l_\alpha = C^l_{(3)},$$

$$\mathrm{CVaR}^l_\alpha = \mathrm{CVaR^+}^l_\alpha = \frac{1}{2}C^l_{(4)} + \frac{1}{2}C^l_{(5)},$$

$$\mathrm{CVaR^-}^l_\alpha = \frac{1}{3}C^l_{(3)} + \frac{1}{3}C^l_{(4)} + \frac{1}{3}C^l_{(5)}\,.$$

Now, let $\alpha = 0.975$, hence $\alpha^- = 0.97$ and $\alpha^+ = 0.98$ (see Figure 4.5). In this case, α does split the VaR^l_α atom, $\lambda^l_\alpha > 0$, $\mathrm{VaR}^l_\alpha < \mathrm{CVaR^-}^l_\alpha < \mathrm{CVaR}^l_\alpha < \mathrm{CVaR^+}^l_\alpha$, and the following equations hold (VaR^l_α, $\mathrm{CVaR^+}^l_\alpha$, and $\mathrm{CVaR^-}^l_\alpha$ have not changed):

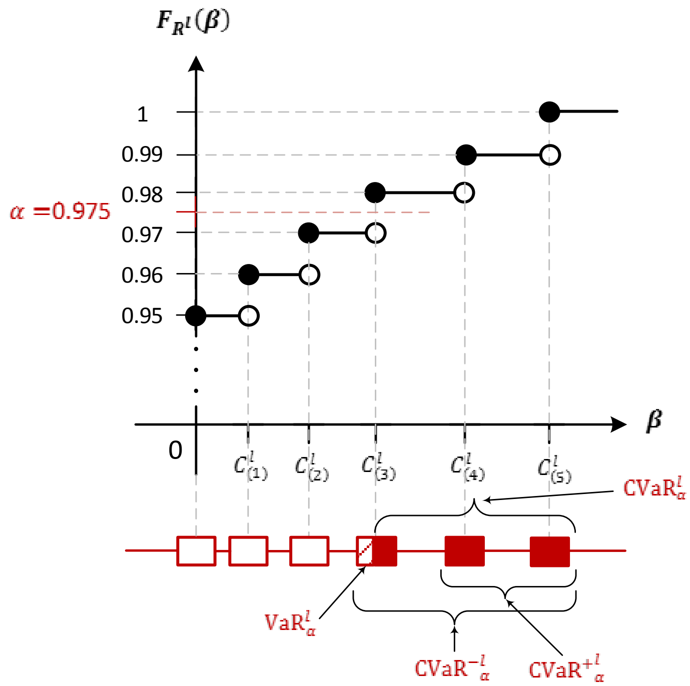


Figure 4.5. CVaR Example 2: computation of CVaR when α splits the atom

$$\mathrm{VaR}^l_\alpha = C^l_{(3)},$$

$$\lambda^l_\alpha = 0.2,$$

$$\mathrm{CVaR^+}^l_\alpha = \frac{1}{2}C^l_{(4)} + \frac{1}{2}C^l_{(5)},$$

$$\mathrm{CVaR}^l_\alpha = 0.2\mathrm{VaR}^l_\alpha + 0.8\mathrm{CVaR^+}^l_\alpha = 0.2C^l_{(3)} + 0.4C^l_{(4)} + 0.4C^l_{(5)},$$

$$\mathrm{CVaR^-}^l_\alpha = \frac{1}{3}C^l_{(3)} + \frac{1}{3}C^l_{(4)} + \frac{1}{3}C^l_{(5)}\,.$$

In the last case, we consider $\alpha = 0.995$, hence $\alpha^- = 0.99$ and $\alpha^+ = 1$, which splits the last atom (see Figure 4.6). Now $\lambda_\alpha^l = 1, \mathrm{VaR}_\alpha^l = \mathrm{CVaR}^{-}{}_\alpha^l = \mathrm{CVaR}_\alpha^l = C_{(5)}^l$, and $\mathrm{CVaR}^{+}{}_\alpha^l$ is not defined.

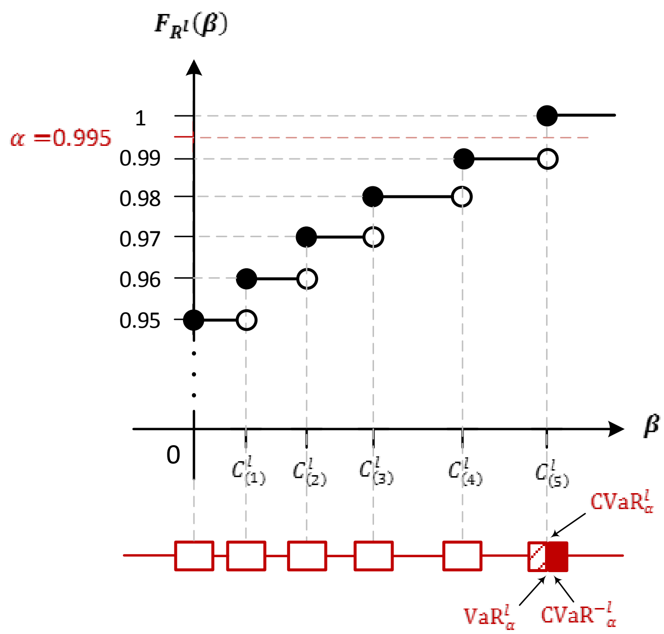


Figure 4.6. CVaR Example 3: computation of CVaR when α splits the last atom

4.2.5. Properties

We now provide some important properties of CVaR for our scenario-based problem.

1) Consider route l and suppose that $0 < \alpha \leq P_{(0)}^l$, i.e. $K = 0$, and so $\mathrm{VaR}_\alpha^l = 0$. According to (4.8), CVaR_α^l then will be $\frac{\sum_{i=1}^{n_l} P_{(i)}^l C_{(i)}^l}{1-\alpha}$ which is (almost) equal to

$\frac{\mathrm{E}\,[R^l]}{1-\alpha}$ (see (A3.2) in the Appendix of Chapter 3). Similarly, let us define $P_{(0)}^{min} = \min_{l\in L} P_{(0)}^l$, then for any $0 < \bar{\alpha} \le P_{(0)}^{min}$ and $l \in L$ we will have: $\mathrm{VaR}_{\bar{\alpha}}^l = 0$ and $\mathrm{CVaR}_{\bar{\alpha}}^l = \frac{\sum_{i=1}^{n_l} P_{(i)}^l\, C_{(i)}^l}{1-\bar{\alpha}} = \frac{\mathrm{E}\,[R^l]}{1-\bar{\alpha}}$. This means that by having $\bar{\alpha}$ as the confidence level, the VaR model is not effective for any route and minimizing the CVaR measure is equivalent to minimizing the TR risk measure for all routes. Therefore the route generated by the CVaR model is the same as the route that the TR model generates.

2) Consider route l and suppose that $0 < \alpha_1 < \alpha_2 \le P_{(0)}^l$, so $\mathrm{VaR}_{\alpha_1}^l = \mathrm{VaR}_{\alpha_2}^l = 0$. Therefore, according to (4.8), $\mathrm{CVaR}_{\alpha_2}^l = \frac{1-\alpha_1}{1-\alpha_2}\,\mathrm{CVaR}_{\alpha_1}^l$, i.e. $\mathrm{CVaR}_{\alpha_2}^l$ is a multiple of (and greater than) $\mathrm{CVaR}_{\alpha_1}^l$.

3) Suppose that $\sum_{i=0}^{n_l-1} P_{(i)}^l < \alpha < 1$, i.e. $K = n_l$, and so $\mathrm{VaR}_{\alpha}^l = C_{(n_l)}^l$, which is the highest possible risk. According to (4.8), CVaR_{α}^l then will be equal to $C_{(n_l)}^l$ as well. Similarly, let us define $P_{(n)}^{max} = \max_{l\in L} \sum_{i=0}^{n_l-1} P_{(i)}^l$, then for any $P_{(n)}^{max} < \bar{\bar{\alpha}} < 1$ and $l \in L$ we will have: $\mathrm{VaR}_{\bar{\bar{\alpha}}}^l = \mathrm{CVaR}_{\bar{\bar{\alpha}}}^l = C_{(n_l)}^l$. This means that by choosing $\bar{\bar{\alpha}}$ as the confidence level, both VaR and CVaR models result in the maximum risk that is possible for the routes available for the hazmat shipment and there is no chance of a risk greater than that.

For the next two properties, suppose that the attributes of route l, $C_{(i)}^l$ and $P_{(i)}^l$, $i \in \{0, 1, 2, \dots, n_l\}$, are in the following form:

$C_{(0)}^l$	$C_{(1)}^l$	$C_{(2)}^l$	…	$C_{(m)}^l$	…	$C_{(K)}^l$	…	$C_{(n_l)}^l$
$P_{(0)}^l$	$P_{(1)}^l$	$P_{(2)}^l$	…	$P_{(m)}^l$	…	$P_{(K)}^l$	…	$P_{(n_l)}^l$

where

$$\begin{gathered} C_{(0)}^l = 0 < C_{(1)}^l < C_{(2)}^l < \cdots < C_{(m)}^l < \cdots < C_{(K)}^l < \cdots < C_{(n_l)}^l, \\ m \in \{1, 2, \dots, K-1\}, \qquad K \in \{2, \dots, n_l\}, \end{gathered} \tag{4.9}$$

$$P_{(0)}^l = 1 - \sum_{i=1}^{n_l} P_{(i)}^l \,,$$

$$\sum_{i=0}^{K-1} P_{(i)}^l < \alpha \leq \sum_{i=0}^{K} P_{(i)}^l \,,$$

therefore

$$\mathrm{VaR}_\alpha^l = C_{(K)}^l, \text{and}$$

$$\mathrm{CVaR}_\alpha^l = \frac{1}{1-\alpha}\left[\left(\sum_{i=0}^{K} P_{(i)}^l - \alpha\right)\mathrm{VaR}_\alpha^l + \sum_{i=K+1}^{n_l} P_{(i)}^l \, C_{(i)}^l\right].$$

4) Suppose that route $\acute{l}$ is the same as route l except that it does not include the mth component (service leg or transferring yard), $C_{(m)}^l$ and $P_{(m)}^l$, of route l. Let define set $J = I - \{m\}$, where $I = \{i | i = 0, 1, 2, \dots, m, \dots, K, \dots, n_l\}$. Therefore we have:

$$n_{\acute{l}} = n_l - 1$$

$$P_{(0)}^{\acute{l}} = 1 - \sum_{j=1}^{n_{\acute{l}}} P_{(j)}^{\acute{l}} = 1 - \left(\sum_{i=1}^{n_l} P_{(i)}^l - P_{(m)}^l\right) = P_{(0)}^l + P_{(m)}^l$$

therefore

$$\sum_{i=0}^{K} P_{(i)}^{l} = P_{(0)}^{l} + P_{(m)}^{l} + \sum_{i=1,\ i\neq m}^{K} P_{(i)}^{l} = P_{(0)}^{\acute{l}} + \sum_{i=1,\ i\neq m}^{K} P_{(i)}^{l}$$
$$= P_{(0)}^{\acute{l}} + \sum_{j=1}^{\acute{K}} P_{(j)}^{\acute{l}} = \sum_{j=0}^{\acute{K}} P_{(j)}^{\acute{l}}, \tag{4.10}$$

where $\acute{K} = K - 1$. Similarly we can show that

$$\sum_{i=0}^{K-1} P_{(i)}^{l} = \sum_{j=0}^{\acute{K}-1} P_{(j)}^{\acute{l}}\,. \tag{4.11}$$

In addition, because $C_{(j)}^{\acute{l}} = C_{(i)}^{l}$ and $P_{(j)}^{\acute{l}} = P_{(i)}^{l},\ \ \forall j = i - 1,\ i \geq K,\ j \geq \acute{K}$, we will have

$$\sum_{i=K+1}^{n_l} P_{(i)}^{l}\, C_{(i)}^{l} = \sum_{j=\acute{K}+1}^{n_{\acute{l}}} P_{(j)}^{\acute{l}}\, C_{(j)}^{\acute{l}}\,. \tag{4.12}$$

This way, according to $\sum_{i=0}^{K-1} P_{(i)}^{l} < \alpha \leq \sum_{i=0}^{K} P_{(i)}^{l}$ and (4.10) and (4.11), we will have:

$$\sum_{j=0}^{\acute{K}-1} P_{(j)}^{\acute{l}} < \alpha \leq \sum_{j=0}^{\acute{K}} P_{(j)}^{\acute{l}}$$

therefore

$$\mathrm{VaR}_{\alpha}^{\acute{l}} = C_{(\acute{K})}^{\acute{l}} = C_{(K)}^{l} = \mathrm{VaR}_{\alpha}^{l}\,.$$

Also taking into consideration (4.12), we will have

$$\mathrm{CVaR}_{\alpha}^{\acute{l}} = \frac{1}{1-\alpha}\left[\left(\sum_{j=0}^{\acute{K}} P_{(j)}^{\acute{l}} - \alpha\right)\mathrm{VaR}_{\alpha}^{\acute{l}} + \sum_{j=\acute{K}+1}^{n_{\acute{l}}} P_{(j)}^{\acute{l}}\, C_{(j)}^{\acute{l}}\right]$$

$$= \frac{1}{1-\alpha}\left[\left(\sum_{i=0}^{K} P_{(i)}^{l} - \alpha\right)\mathrm{VaR}_{\alpha}^{l} + \sum_{i=K+1}^{n_l} P_{(i)}^{l}\, C_{(i)}^{l}\right] = \mathrm{CVaR}_{\alpha}^{l}\,.$$

We can also utilize the reverse of the above equations to prove that if we add component m, with the attributes $C_{(m)}^{l}$ and $P_{(m)}^{l}$, to the route $\acute{l}$ (given the condition (4.9)), then the new route l will have the same VaR and CVaR.

These equations simply mean that if we eliminate (or add) a component (service leg or transferring yard) from (or to) a route, given that component's consequence is less than VaR value of the route, the new route will hold the same VaR and CVaR values.

5) A similar result can be derived for a route that has only one component different from route l. Suppose that route $l^{"}$ is the result of replacing the mth component of route l with a component (q) that has the same probability but different accident consequence $C_{(q)}^{l^{"}}$, given that q has the same properties of m in (4.9). Note that m and q do not need to be equal. Therefore we have:

$$n_{l^{"}} = n_l$$

$$P_{(0)}^{l^{"}} = 1 - \sum_{i=1}^{n_{l^{"}}} P_{(i)}^{l^{"}} = 1 - \sum_{i=1}^{n_l} P_{(i)}^{l} = P_{(0)}^{l}$$

therefore

$$\sum_{i=0}^{K} P_{(i)}^{l} = \sum_{i=0}^{K} P_{(i)}^{l''}, \tag{4.13}$$

$$\sum_{i=0}^{K-1} P_{(i)}^{l} = \sum_{i=0}^{K-1} P_{(i)}^{l''}, \tag{4.14}$$

and also

$$\sum_{i=K+1}^{n_l} P_{(i)}^{l}\, C_{(i)}^{l} = \sum_{i=\acute{K}+1}^{n_{l''}} P_{(i)}^{l''}\, C_{(i)}^{l''} \tag{4.15}$$

because $C_{(i)}^{l''} = C_{(i)}^{l}$ and $P_{(i)}^{l''} = P_{(i)}^{l}$, $\forall\ i \geq K$. Therefore $\sum_{i=0}^{K-1} P_{(i)}^{l} < \alpha \leq \sum_{i=0}^{K} P_{(i)}^{l}$ with (4.13) and (4.14) lead to $\sum_{i=0}^{K-1} P_{(i)}^{l''} < \alpha \leq \sum_{i=0}^{K} P_{(i)}^{l''}$, which means $\mathrm{VaR}_{\alpha}^{l''} = C_{(K)}^{l''} = C_{(K)}^{l} = \mathrm{VaR}_{\alpha}^{l}$. With considering (4.15) as well, we will have $\mathrm{CVaR}_{\alpha}^{l''} = \mathrm{CVaR}_{\alpha}^{l}$.

This means that if we replace a component (service leg or transferring yard) of a route with a new component, given that the both components' consequences are less than VaR value of the route and they both have the same accident probability, the new route will hold the same VaR and CVaR values.

Note that when we remove/add a service leg or transferring yard from/to a route or we replace that with a new one, the subsequent route should still make sense and be feasible. The results and examples of the above properties will be provided in Section 4.3.3.

4.2.6. Optimization Program

In this sub-section, we provide a method to reach an algorithm that can find the optimal CVaR value and its corresponding route for the rail hazmat shipment in the network. Consider the route l and its attributes $C_{(i)}^{l}$ and $P_{(i)}^{l}$, $i \in \{0, 1, 2, \dots, n_l\}$, where $C_{(0)}^{l} = 0 < C_{(1)}^{l} < C_{(2)}^{l} < \cdots < C_{(n_l)}^{l}$. Recall that we defined K be equal to 0 if $0 < \alpha \le P_{(0)}^{l}$, otherwise it is the unique index such that $\sum_{i=0}^{K-1} P_{(i)}^{l} < \alpha \le \sum_{i=0}^{K} P_{(i)}^{l}$. The α-VaR of the risk is then $\mathrm{VaR}_{\alpha}^{l} = C_{(K)}^{l}$. We now rewrite equation (4.8) as follows:

$$\mathrm{CVaR}_{\alpha}^{l} = \frac{1}{1-\alpha}\left[\left(\sum_{i=0}^{K} P_{(i)}^{l} - \alpha\right) C_{(K)}^{l} + \sum_{i=K+1}^{n_l} P_{(i)}^{l}\, C_{(i)}^{l}\right]$$

$$= \frac{1}{1-\alpha}\left[\left(\left(1 - \sum_{i=K+1}^{n_l} P_{(i)}^{l}\right) - \alpha\right) C_{(K)}^{l} + \sum_{i=K+1}^{n_l} P_{(i)}^{l}\, C_{(i)}^{l}\right]$$

$$= \frac{1}{1-\alpha}\left[(1-\alpha) C_{(K)}^{l} - \left(\sum_{i=K+1}^{n_l} P_{(i)}^{l}\right) C_{(K)}^{l} + \sum_{i=K+1}^{n_l} P_{(i)}^{l}\, C_{(i)}^{l}\right]$$

Therefore the α-CVaR of the risk associated with the route l can be written as

$$\mathrm{CVaR}_{\alpha}^{l} = C_{(K)}^{l} + \frac{1}{1-\alpha}\left[\sum_{i=K+1}^{n_l} P_{(i)}^{l}\left(C_{(i)}^{l} - C_{(K)}^{l}\right)\right]. \qquad (4.16)$$

Considering different alternative routes in the network for the shipment (set L), our objective is to determine the route $l \in L$ which has the minimum CVaR. That is,

$$\mathrm{CVaR}_{\alpha}^{*} = \min_{l \in L} \mathrm{CVaR}_{\alpha}^{l} \, .$$

To find this minimum and according to the expression in the brackets of (4.16), we define $\acute{\mathbb{p}}_{(r)}$ as follows:

$$\acute{\mathbb{p}}_{(r)} = \sum_{s,\acute{s}} \left(\sum_{k_{s\acute{s}} \in \mathcal{Y}_s \& \mathcal{Y}_{\acute{s}},\ c_{k_{s\acute{s}}} > C_{(r)}} p_{k_{s\acute{s}}} \left(c_{k_{s\acute{s}}} - C_{(r)} \right) + \sum_{(i_s, j_s) \in \mathcal{A}_s,\ c_{i_s j_s} > C_{(r)}} p_{i_s j_s} \left(c_{i_s j_s} - C_{(r)} \right) \right)$$

where $C_{(r)}; r \in \{0, 1, 2, \dots, M\}$ $\left(C_{(0)} = 0\right)$ is the rth smallest value in the set $\left\{c_k \cup c_{ij} \ : \ k \in \mathcal{Y} \ \& \ (i,j) \in \mathcal{A}\right\}$. Then we have

$$\acute{\mathbb{p}}_{(r)} X = \sum_{s,\acute{s}} \left(\sum_{k_{s\acute{s}} \in \mathcal{Y}_s \& \mathcal{Y}_{\acute{s}},\ c_{k_{s\acute{s}}} > C_{(r)}} p_{k_{s\acute{s}}} \left(c_{k_{s\acute{s}}} - C_{(r)} \right) x_{k_{s\acute{s}}} + \sum_{(i_s, j_s) \in \mathcal{A}_s,\ c_{i_s j_s} > C_{(r)}} p_{i_s j_s} \left(c_{i_s j_s} - C_{(r)} \right) x_{i_s j_s} \right)$$

where $X \in \psi$, which is defined in Section 3.3 for routing of rail shipments (it contains two binary decision variables and conservation of flow constraints). This way, the $\mathrm{CVaR}_{\alpha}^{*}$ minimization problem will be

$$\begin{aligned}
&\mathrm{CVaR}_{\alpha}^{*} = \min_{r} \mathrm{CVaR}_{\alpha}^{r} \\
&\text{subject to} \\
&\mathrm{CVaR}_{\alpha}^{r} = C_{(r)} + \frac{1}{1-\alpha} f^{r} \\
&f^{r} = \min_{X} \acute{\mathbb{p}}_{(r)} X \\
&X \in \psi \\
&r = 0, 1, 2, \dots, M
\end{aligned} \tag{4.17}$$

f^r minimization could be done using an efficient shortest path algorithm like Dijkstra's Algorithm if we make the following modifications:

$$\begin{aligned}
&\bar{\bar{p}}_{k_{s\acute{s}}} = \begin{cases} p_{k_{s\acute{s}}}\left(c_{k_{s\acute{s}}} - C_{(r)}\right), & \text{if} \quad c_{k_{s\acute{s}}} > C_{(r)} \qquad \forall\, k_{s\acute{s}}, \forall s, \forall \acute{s} \\ 0, & \text{otherwise} \end{cases} \\
&\bar{\bar{p}}_{i_s j_s} = \begin{cases} p_{i_s j_s}\left(c_{i_s j_s} - C_{(r)}\right), & \text{if} \quad c_{i_s j_s} > C_{(r)} \qquad \forall\, (i_s, j_s), \forall s \\ 0, & \text{otherwise} \end{cases}
\end{aligned} \tag{4.18}$$

therefore

$$f^{r} = \min_{X \in \psi} \sum_{s,\acute{s}} \left(\sum_{k_{s\acute{s}} \in \mathcal{Y}_s \& \mathcal{Y}_{\acute{s}}} \bar{\bar{p}}_{k_{s\acute{s}}} x_{k_{s\acute{s}}} + \sum_{(i_s, j_s) \in \mathcal{A}_s} \bar{\bar{p}}_{i_s j_s} x_{i_s j_s} \right) \tag{4.19}$$

Consequently we reach the following algorithm, named *CVaR Algorithm*, for solving the problem (4.17):

1) Generate $C_{(r)}$ and their corresponding $P_{(r)}\,:\; r = 0, 1, 2, \dots, M$.

2) For $r = 0$ to M do:

   2.1) Consider (4.18) and (4.19) and solve $f^r = \min_{X \in \psi} \acute{\mathbb{p}}_{(r)} X$ using an efficient shortest path algorithm like Dijkstra's Algorithm.

   2.2) Calculate $\text{CVaR}_{\alpha}^{r} = C_{(r)} + \frac{1}{1-\alpha}\, f^r$.

3) Let $r^* = \arg \min_{r=0,1,\dots,M} \text{CVaR}_{\alpha}^{r}$, consequently $\text{CVaR}_{\alpha}^{*} = \text{CVaR}_{\alpha}^{r^*}$.

4.2.7. Parameter Estimation

The technique to estimate the parameters used in the proposed CVaR methodology is exactly the same as the one explained in Section 3.4 for the VaR approach.

4.2.8. Train Configuration Setting

To find the best train configuration for applying the proposed methodology, we follow the strategy given in Section 3.5 and similarly define the following minimization model:

$$\min_{\mathcal{W}, y_r} \mathcal{W} \left(\sum_{r=1}^{10} y_r \times TCP^r \right) + (1 - \mathcal{W}) \left(\sum_{r=1}^{10} y_r \times YCP^r \right) \tag{4.20-1}$$

subject to:

$$\min_{l \in L} \mathrm{VaR}_{\alpha}^{l} \quad \text{or} \quad \min_{l \in L} \mathrm{CVaR}_{\alpha}^{l} \tag{4.20-2}$$

$$\sum_{r=1}^{10} y_r = N \tag{4.20-3}$$

$$0 \leq y_r \leq \frac{\text{train length}}{10} \tag{4.20-4}$$

$$\begin{gathered} y_r\text{: integer}, \quad r = \{1,2,\dots,10\} \\ 0 \leq \mathcal{W} \leq 1 \end{gathered} \tag{4.20-5}$$

4.3 Computational Experiments

In this section, we use the proposed CVaR risk assessment methodology to study a number of problem instances generated using the realistic infrastructure of a Class 1 railroad operator in the United States, and then develop some relevant managerial insights.

4.3.1 Problem setting

To perform computational experiments, we make use of the case study described in Section 3.6, which is a railroad infrastructure originally introduced in Verma et al. (2011), and used in Azad et al. (2016). The resulting network has 25 yards (Figure 4.7), where each is an origin and destination for the others, i.e., 600 origin-destination pairs. A total of 31 different train services –identified by origin and destination yards, intermediate stops, and service legs, connect the yards. Finally, ArcGIS (ESRI, 2007) was used to estimate population exposure which serves a measure for consequence. The objective is to *determine the best way to move a* *given number of hazmat railcars, on the available train services, between various origin-destination pairs such that hazmat transport risk as measured by CVaR is minimized*. It is important that given the nature of railroad accidents, and in light of the preceding discussions, both the route and the placement of the hazmat railcars in a train needs to be determined.

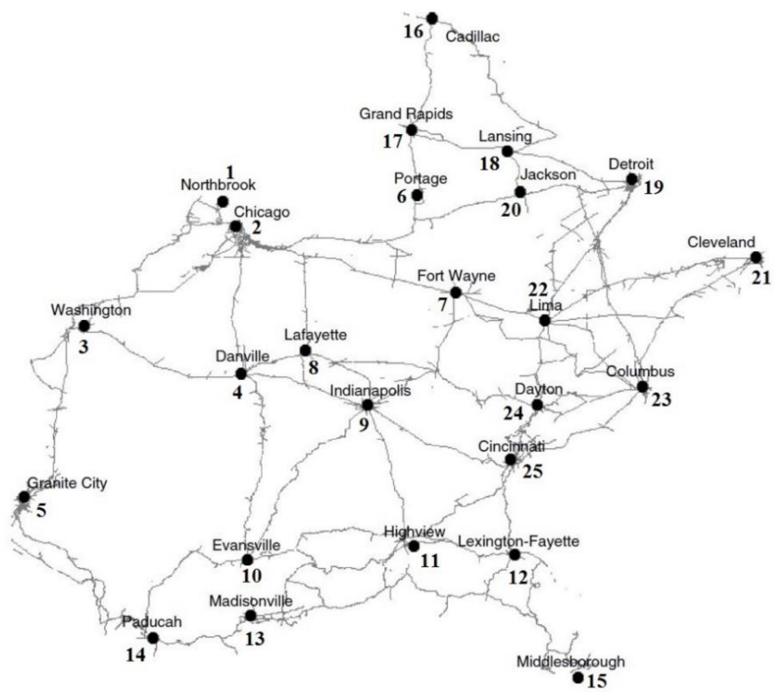


Figure 4.7. Railroad network in the Midwest United States [Source: Verma et al., (2011)]

4.3.2 Solution

In an effort to conduct focused analyses, we consider shipments from Chicago to Highview, i.e., nodes 2 and 11, respectively. We consider seven distinct hazmat volumes, i.e., $N = \{5, 20, 40, 60, 80, 100, 120\}$, and solve the problem for four different confidence levels: $\alpha = \{\alpha_1 = 0.9, \alpha_2 = 0.99999, \alpha_3 = 0.999997, \alpha_4 = 0.999999\}$. In addition, with the objective of finding the best train configuration for the above problem instances using (4.20), we apply eleven different weights to the model: $\mathcal{W} = \{1, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, 0\}$. Finally, we consider only *medium* trains of 120 railcars in length, and thus the output would give information about hazmat railcars in each decile. For expositional reasons, and for brevity, we only show the results of the *best* train configurations (indicated by C#) for different number of hazmat railcars and for various weights, in Table 4.1. Thus, a total of 24 distinct configurations were observed for *medium* length trains, which resulted in 24*4=96 problem instances.

It is important that similar configurations were generated for *long* and *short* trains, but for brevity are not reported here. For *long* trains, 26 distinct configurations were observed when assuming a length of 200 railcars, and 26*4=104 problem instances were solved. On the other hand, for *short* trains 8 distinct configurations were determined with 40 railcars train length, and 8*4=32 problem instances were solved. Hence, a total of 232 problem instances were solved to gain managerial insights. The VaR and CVaR algorithms were coded in Matlab R 2015b, and (4.20) was solved in GAMS 24.1.3 using Cplex 12.5.1.0 as the solver. We ran them on a 2.90 GHz Intel Core i7 computer system. The computation times are less than 5 seconds.

N	C#	$\mathcal{W}$	Train Configuration									
			y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
5	*1^st^*	1	0	0	0	0	0	0	0	0	5	0
	*2^nd^*	0.9 - 0	0	0	0	0	0	0	5	0	0	0
20	*1^st^*	1	0	0	0	0	0	0	8	0	12	0
	*2^nd^*	0.9 - 0.6	0	0	0	0	0	0	12	0	8	0
	*3^rd^*	0.5 - 0.2	0	0	0	0	0	0	12	8	0	0
	*4^th^*	0.1 - 0	0	0	0	8	0	0	12	0	0	0
40	*1^st^*	1	0	0	0	0	0	0	12	4	12	12
	*2^nd^*	0.9	0	0	0	0	0	0	12	12	12	4
	*3^rd^*	0.8 - 0.5	0	0	4	0	0	0	12	12	12	0
	*4^th^*	0.4	0	0	12	0	0	0	12	12	4	0
	*5^th^*	0.3	0	0	12	4	0	0	12	12	0	0
	*6^th^*	0.2 - 0	0	0	4	12	0	0	12	12	0	0
60	*1^st^*	1 - 0.8	0	0	12	0	0	0	12	12	12	12
	*2^nd^*	0.7 - 0.4	0	0	12	0	0	12	12	12	12	0
	*3^rd^*	0.3 - 0	0	0	12	12	0	12	12	12	0	0
80	*1^st^*	1	0	0	12	0	8	12	12	12	12	12
	*2^nd^*	0.9 - 0.7	0	0	12	8	0	12	12	12	12	12
	*3^rd^*	0.6 - 0.5	0	0	12	12	0	12	12	12	12	8
	*4^th^*	0.4 - 0	0	8	12	12	0	12	12	12	12	0
100	*1^st^*	1 - 0.7	0	4	12	12	12	12	12	12	12	12
	*2^nd^*	0.6	0	12	12	12	4	12	12	12	12	12
	*3^rd^*	0.5 - 0.4	4	12	12	12	0	12	12	12	12	12
	*4^th^*	0.3 - 0	12	12	12	12	0	12	12	12	12	4
120	*Only*	All	12	12	12	12	12	12	12	12	12	12

Table 4.1. Best *medium* train configurations for different N and $\mathcal{W}$

As indicated earlier, different train configurations would yield different accident probabilities on the service legs and the transfer yards, and consequently may result in different optimal VaR and CVaR values and the resulting routes associated with each confidence level α. For instance, for 80 hazmat railcar setting depicted in Table 4.1, there are four possible train configurations resulting from attaching eleven different weights to the objective in (4.20-1). The resulting optimal VaR and CVaR and the associated routes at $\alpha = 0.999999$ are indicated in Table 4.2. The optimal VaR route generated using the third train configuration (3rd C#) shows that the hazmat shipment should be placed on train service number 5, and travel non-stop crossing service-legs 2-4, 4-9, and 9-25 before being transferred to train service number 18 at yard 25. Subsequently, the second train service brings the shipment to the destination node via service leg 25-11. Note that either of the first two configurations leads to the best optimal VaR value of 5,996 and results in the same route, while the best optimal CVaR value (10,913) is achieved using the 4th train configuration, whose route is different from the best VaR route. Similarly, we can apply the VaR and CVaR algorithms to each configuration indicated in Table 4.1, and for the four confidence levels being considered. For each hazmat volume, Table 4.3 to Table 4.5 depict the configuration resulting in minimal VaR and CVaR, and the resulting routes for each confidence level. Note that, unlike in Table 4.2, we are only reporting the configurations resulting in minimum (or best) VaR* and CVaR*. For expositional reasons, and also to demonstrate the distinctness of the proposed methodology, we also report the best train configurations using the traditional risk (TR) model in Table 4.6, and note that confidence level has no impact on the resulting routes.

For a given confidence level, the optimal values of both VaR and CVaR increase with the increase in hazmat volume (Figure 4.8). Also, for a given hazmat volume, the optimal values of both VaR and CVaR increase with the increase in confidence level (Figure 4.9). Although in most cases the optimal VaR value is equal to zero.

<table>
<tr><th>C#</th><th>VaR*</th><th>VaR-Route*</th><th>CVaR*</th><th>CVaR-Route*</th></tr>
<tr><td>1st</td><td rowspan="2">5,996</td><td rowspan="2">2 - 6 - 8 - [9] - [25] - 11
{4} {5} {18}</td><td>11,840</td><td rowspan="2">2 - 6 - [8] - 4 - [3] - 5 - 14 - [13] - 11
{4} {11} {6} {29}</td></tr>
<tr><td>2nd</td><td>11,629</td></tr>
<tr><td>3rd</td><td>6,221</td><td>2 - 4 - 9 - [25] - 11
{5} {18}</td><td>11,442</td><td rowspan="2">2 - 6 - 7 - [22] - 24 - 25 - 12 - [15] - 12 - 11
{2} {8} {31}</td></tr>
<tr><td>4th</td><td>8,914</td><td>2 - 6 - 7 - 22 - [23] - 25 - 11
{3} {18}</td><td>10,913</td></tr>
</table>

Table 14.2. Optimal VaR, CVaR and associated routes for *N=80* and α=0.999999

<table>
<tr><th rowspan="2">N</th><th rowspan="2">C#*</th><th rowspan="2">VaR*</th><th rowspan="2">VaR-Route*</th><th rowspan="2">C#*</th><th>CVaR*
α1</th><th rowspan="2">CVaR-Route*</th></tr>
<tr><th>α2</th></tr>
<tr><td rowspan="2">5</td><td rowspan="2">All</td><td rowspan="2">0</td><td rowspan="14">2 - 4 - 9 - [25] - 11
{5} {18}</td><td rowspan="2">2nd</td><td>0.0015</td><td rowspan="2">2 - 6 - [8] - [4] - 9 - 25 - [12] - 11
{4} {11} {5} {24}</td></tr>
<tr><td>15</td></tr>
<tr><td rowspan="2">20</td><td rowspan="2">All</td><td rowspan="2">0</td><td rowspan="2">2nd</td><td>0.0139</td><td rowspan="12">2 - 6 - 8 - [9] - 25 - [12] - 11
{4} {5} {24}</td></tr>
<tr><td>139</td></tr>
<tr><td rowspan="2">40</td><td rowspan="2">All</td><td rowspan="2">0</td><td rowspan="2">3rd</td><td>0.0549</td></tr>
<tr><td>549</td></tr>
<tr><td rowspan="2">60</td><td rowspan="2">All</td><td rowspan="2">0</td><td rowspan="2">2nd</td><td>0.1292</td></tr>
<tr><td>1,292</td></tr>
<tr><td rowspan="2">80</td><td rowspan="2">All</td><td rowspan="2">0</td><td rowspan="2">2nd</td><td>0.2322</td></tr>
<tr><td>2,322</td></tr>
<tr><td rowspan="2">100</td><td rowspan="2">All</td><td rowspan="2">0</td><td rowspan="2">1st</td><td>0.3867</td></tr>
<tr><td>3,867</td></tr>
<tr><td rowspan="2">120</td><td rowspan="2">Unique</td><td rowspan="2">0</td><td rowspan="2">Unique</td><td>0.5967</td></tr>
<tr><td>5,967</td></tr>
</table>

Table 4.3. Optimal configuration, VaR, CVaR and associated routes for α_1=0.9 and α_2=0.99999

<table>
<tr><th>N</th><th>C#*</th><th>VaR*</th><th>VaR-Route*</th><th>C#*</th><th>VaR
CVaR*</th><th>CVaR-Route*</th></tr>
<tr><td>5</td><td>All</td><td>0</td><td rowspan="4">2 - 4 - 9 - [25] - 11
{5} {18}</td><td>2nd</td><td>0
49</td><td>2 - 6 - [8] - [4] - 9 - 25 - [12] - 11
{4} {11} {5} {24}</td></tr>
<tr><td>20</td><td>All</td><td>0</td><td>2nd</td><td>0
462</td><td rowspan="6">2 - 6 - 8 - [9] - 25 - [12] - 11
{4} {5} {24}</td></tr>
<tr><td>40</td><td>All</td><td>0</td><td>3rd</td><td>0
1,831</td></tr>
<tr><td>60</td><td>All</td><td>0</td><td>2nd</td><td>0
4,307</td></tr>
<tr><td>80</td><td>1st, 2nd</td><td>1,877</td><td>2 6 [20] 22 [24] 25 11
{1} {10} {22}</td><td>3rd</td><td>3,032
7,219</td></tr>
<tr><td>100</td><td>1st, 2nd, 3rd</td><td>4,845</td><td rowspan="2">2 - 6 - 8 - [9] - 25 - [12] - 11
{4} {5} {24}</td><td>4th</td><td>5,481
10,125</td></tr>
<tr><td>120</td><td>Unique</td><td>6,078</td><td>Unique</td><td>6,078
13,822</td></tr>
</table>

Table 4.4. Optimal configuration, VaR, CVaR and associated routes for α_3=0.999997

N	C#*	VaR*	VaR-Route*	C#*	VaR / CVaR*	CVaR-Route*
5	All	0	2 - 4 - 9 - [25] - 11 {5} {18}	2nd	0 146	2 - 6 - [8] - [4] - 9 - 25 - [12] - 11 {4} {11} {5} {24}
20	All	0		2nd	0 1,385	2 - 6 - 8 - [9] - 25 - [12] - 11 {4} {5} {24}
40	1st, 2nd	1,980	2 - 6 - [20] - 7 - [23] - 25 - 11 {1} {9} {18}	3rd	2,206 4,729	2 - 6 - [8] - [4] - 9 - 25 - [12] - 11 {4} {11} {5} {24}
60	1st	4,038	2 - 6 - 8 - [9] - [25] - 11 {4} {5} {18}	3rd	4,987 7,979	2 - 6 - 8 - [9] - 25 - [12] - 11 {4} {5} {24}
80	1st, 2nd	5,996		4th	8,914 10,913	2 - 6 - 7 - [22] - 24 - 25 - 12 - [15] - 12 - 11 {2} {8} {31}
100	All	10,115	2 - 6 - 7 - 22 - [21] - 23 - 24 - 25 - 11 {2} {22}	4th	10,115 14,288	2 - 6 - [8] - 4 - [3] - 5 - 14 - [13] - 11 {4} {11} {6} {29}
120	Unique	11,426		Unique	11,426 18,020	

Table 4.5. Optimal configuration, VaR, CVaR and associated routes for α_4=0.999999

N	C#*	$E[R]^*$	TR-Route*
5	2nd	0.00015	2 - 6 - [8] - [4] - 9 - 25 - [12] - 11 {4} {11} {5} {24}
20	2nd	0.00139	2 - 6 - 8 - [9] - 25 - [12] - 11 {4} {5} {24}
40	3rd	0.00549	
60	2nd	0.01292	
80	2nd	0.02322	
100	1st	0.03867	
120	Unique	0.05967	

Table 4.6. Optimal configuration, $E[R]$ and associated routes

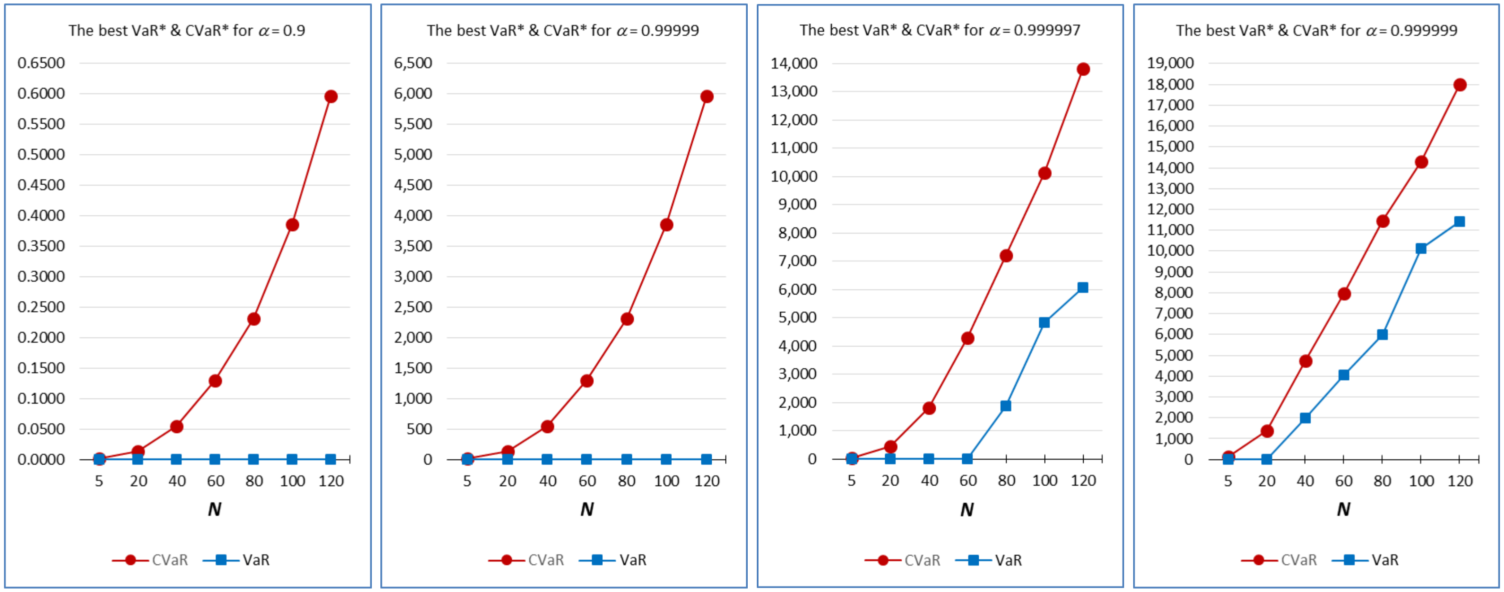


Figure 4.8. Optimal VaR and CVaR values for different hazmat volumes

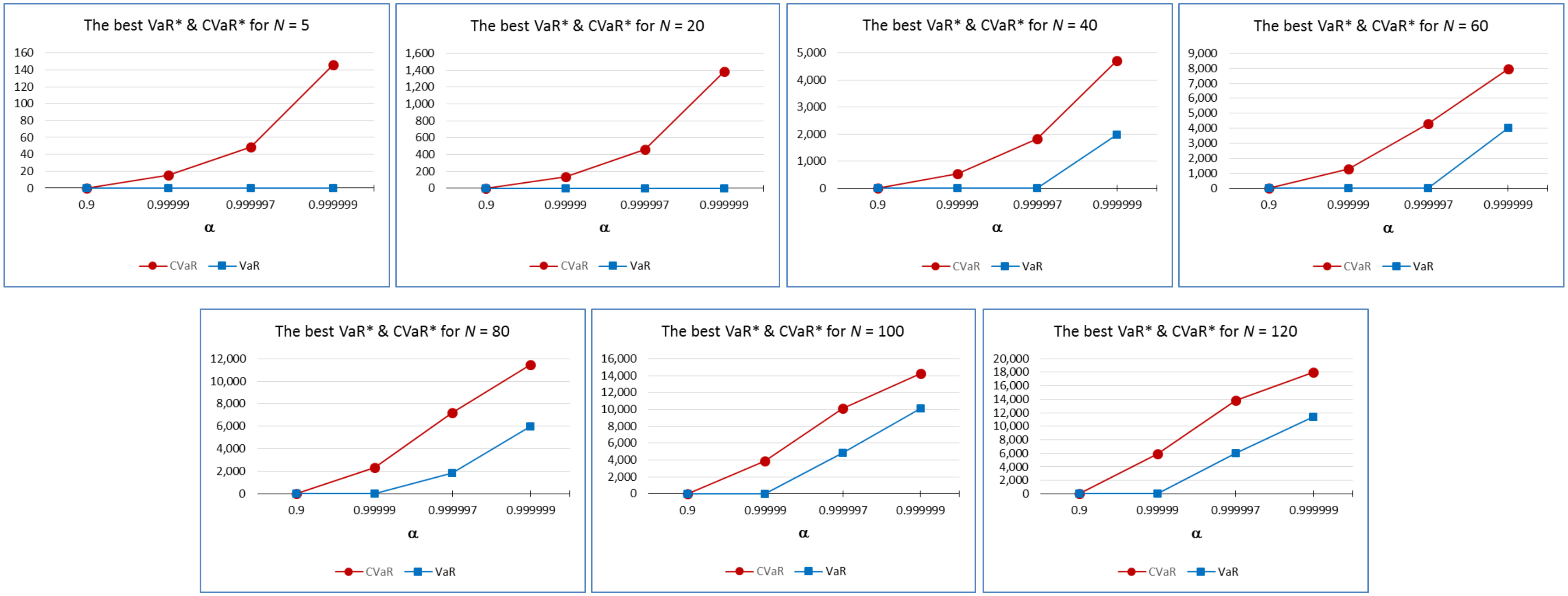


Figure 4.9. Optimal VaR and CVaR values for different confidence levels

4.3.3 Analyses and insights

In an effort to concretely demonstrate the effectiveness of the proposed CVaR measure, we will organize the analyses under five themes: CVaR relation with and its superiority over TR measure; CVaR superiority over VaR measure; risk averse routing of shipments by CVaR; CVaR value stability; and, impact of train length.

4.3.3.1. <u>CVaR relation with and its superiority over TR measure</u>

TR model can find only two distinct routes for all seven specified hazmat volumes (one for $N = 5$, and one for the rest; see Table 4.6), which are exactly as the optimal routes found by CVaR for confidence levels of 0.9 and 0.99999 (see Table 4.3) while VaR optimal values are zero. This result is consistent with the first property of CVaR as outlined in Section 4.2.5. To analyze the second property as well, consider the following route as l, which is the best optimal CVaR route for $N = \{20, 40, 60, 80, 100, 120\}$ and $\alpha = \{\alpha_1 = 0.9, \alpha_2 = 0.99999\}$. As alluded, this is also the only best optimal route that the TR model can find for all of these hazmat volumes. Also, the VaR of this route for the both αs and all these hazmat volumes is equal to zero.

$$\underset{\{4\}}{\underline{2\ -\ 6\ -\ 8\ -\ [}}\ \ 9\ \ \underset{\{5\}}{\underline{]\ -\ 25\ -\ [}}\ \ 12\ \ \underset{\{24\}}{\underline{]\ -\ 11}}$$

Therefore, we have

$$\mathrm{CVaR}^{l}_{\alpha_1} = \frac{1-\alpha_2}{1-\alpha_1}\ \mathrm{CVaR}^{l}_{\alpha_2} = \frac{\mathrm{E}\ [R^l]}{1-\alpha_1}$$

$$\mathrm{CVaR}^{l}_{\alpha_2} = \frac{1-\alpha_1}{1-\alpha_2}\ \mathrm{CVaR}^{l}_{\alpha_1} = \frac{\mathrm{E}\,[R^l]}{1-\alpha_2}$$

For instance, consider *N=80, C#2* in Table 4.3, and Table 4.6, then

$$\mathrm{CVaR}^{l}_{0.9} = \frac{1-0.99999}{1-0.9}\ \mathrm{CVaR}^{l}_{0.99999} = \frac{\mathrm{E}\,[R^l]}{1-0.9} \quad \rightarrow \quad 0.2322 = \frac{1}{1000}2{,}322 = \frac{0.02322}{0.1}$$

$$\mathrm{CVaR}^{l}_{0.99999} = \frac{1-0.9}{1-0.99999}\ \mathrm{CVaR}^{l}_{0.9} = \frac{\mathrm{E}\,[R^l]}{1-0.99999} \quad \rightarrow \quad 2{,}322 = 10{,}000 \times 0.2322 = \frac{0.02322}{0.00001}$$

If we still consider *N=80, C#2* but for $\alpha_3 = 0.999997$, the CVaR of the above route will be equal to 7,324 but the above equations do not hold for $\mathrm{CVaR}^{l}_{\alpha_3}$ anymore:

$$\mathrm{CVaR}^{l}_{\alpha_3} \neq \frac{1-\alpha_1}{1-\alpha_3}\ \mathrm{CVaR}^{l}_{\alpha_1}, \qquad \neq \frac{1-\alpha_2}{1-\alpha_3}\ \mathrm{CVaR}^{l}_{\alpha_2}, \qquad \neq \frac{\mathrm{E}\,[R^l]}{1-\alpha_3}$$

The reason is that the VaR of the above route for *N=80, C#2* and α_3 is not zero: $\mathrm{VaR}^{l}_{\alpha_3} = 3{,}094$. This reasoning similarly works for *N=80, C#2* and $\alpha_4 = 0.999999$, where $\mathrm{VaR}^{l}_{\alpha_4} = 5{,}996$ and $\mathrm{CVaR}^{l}_{\alpha_4} = 12{,}636$.

4.3.3.2. CVaR superiority over VaR measure

For confidence levels of 0.9 and 0.99999 (i.e., Table 4.3), VaR finds only one route for all seven specified hazmat volumes, while the optimal value is zero. On the other hand, all the CVaR values are greater than zero, and that two distinct routes are generated. As evident from the solutions, in most cases the optimal VaR value is equal to zero, which essentially implies that with the least probability equivalent to the given confidence level, the hazmat risk associated with the route does not exceed zero. Such mathematical interpretation may not be acceptable to a decision maker, and in turn is likely to undermine the efficacy of VaR as a measure of risk. On the other hand, even when VaR generates route with values greater than zero, it fails to capture information about the additional populace exposed to the consequence of the accident even if the probability is extremely low. Note that the latter is critical in avoiding catastrophic events associated with hazmat routing. All of the above limitations are resolved when using CVaR.

4.3.3.3. *Risk averse routing of shipments by CVaR*

In an effort to highlight the risk averse routing aspect of CVaR, we focus the discussion on the results from Table 4.5 (i.e., $\alpha_4 = 0.999999$). The four optimal CVaR routes associated with the seven hazmat volumes are depicted in Figure 4.10, which shows that the proposed algorithm seeks to find safer routes which in general involves transferring yards and service legs on the periphery of the network. Note that such a tendency is not reflected in the TR model, where only the first two CVaR optimal routes for α_4 are generated (Figure 4.10a and 4.10b).

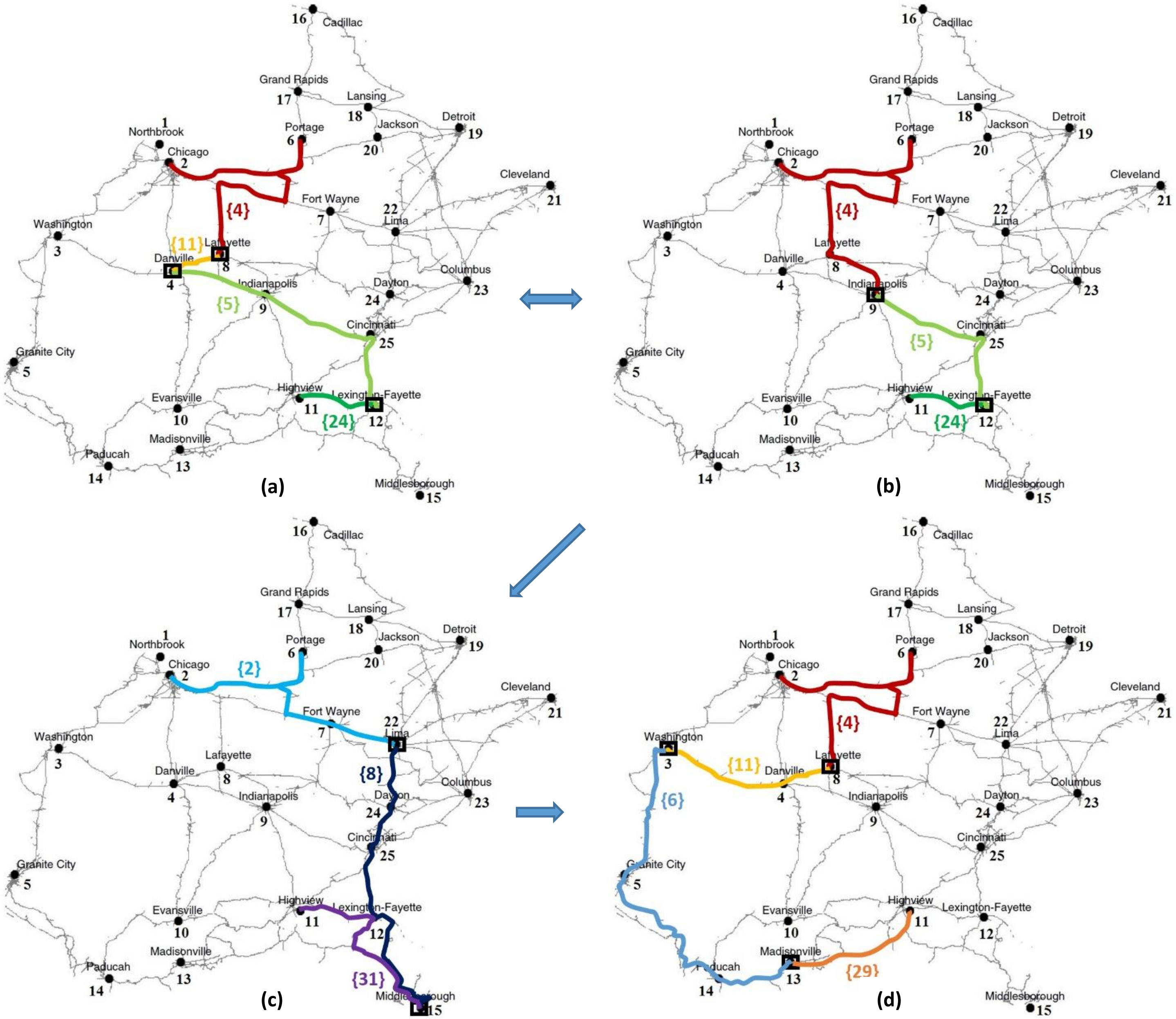


Figure 4.10. Different optimal CVaR routes for $\alpha_4 = 0.999999$

To further underline the ability of CVaR measure to generate risk averse routing of hazmat shipments, we explicitly discuss two problem instances from Table 4.5 (i.e., $\alpha_4 = 0.999999$).

For the *first* instance, consider Figure 4.10d where the CVaR measure generates optimal route for *N=120* using the service legs and transfer yards at the periphery of the network. Why does the proposed route go from the origin yard (i.e., #2) to yard # 6, and then undergoes transfer operations at yard #8? Though not evident, but two other routes could have been proposed using the service legs and transfer yards at the periphery, and they are:

$$\underbrace{2 - 1 - 3 - 5 - 14 - [}_{\{6\}}\ 13\ \underbrace{] - 11}_{\{29\}}$$

$$\underbrace{2 - [}_{\{5\}}\ 4\ \underbrace{] - [}_{\{11\}}\ 3\ \underbrace{] - 5 - 14 - [}_{\{6\}}\ 13\ \underbrace{] - 11}_{\{29\}}$$

Let these two routes be l_1 and l_2, respectively. Figure 4.11 depicts these two routes along with the shaded part of the original (optimal) route. We use equation (4.8) to calculate the CVaR value of the above routes (for $N = 120$ and $\alpha = \alpha_4$): $\mathrm{CVaR}_{\alpha_4}^{l_1} = 28{,}600$ and $\mathrm{CVaR}_{\alpha_4}^{l_2} = 19{,}433$, which are both greater than the CVaR value of the optimal route: 18,020 as reported in Table 4.5. Therefore, we answer the above question claiming that what the proposed CVaR measure has done is to generate a less risky (optimal) route compared to the other alternatives even if it apparently requires sending hazmat shipments to the service legs and transferring yards not on the periphery of the network. In fact, the reason that the algorithm has not reported routes l_1 and l_2 as the optimal route is that they include service legs with very high population exposure (PE) compared to that of the service legs in the optimal route. More specifically, in route l_1 service leg 2-1 has the PE of 55,737 and in route l_2 service leg 2-4 has the PE of 17,512 which are much higher than the PE of the different service legs used in the optimal route, where service leg 4-3 has the largest PE of 12,007.

For the *second* instance, consider Figure 4.10c, where CVaR measure generates the optimal route when *N=80*. Note that the shipment arrives at yard #12, goes to transfer yard #15, and comes back to yard #12. Why did it not go directly to the destination node (i.e., #11) following a transfer operation at yard #12? More specifically:

$$\underbrace{2 - 6 - 7 - [}_{\{2\}}\; 22 \;\underbrace{] - 24 - 25 - [}_{\{8\}}\; 12 \;\underbrace{] - 11}_{\{31\}}$$

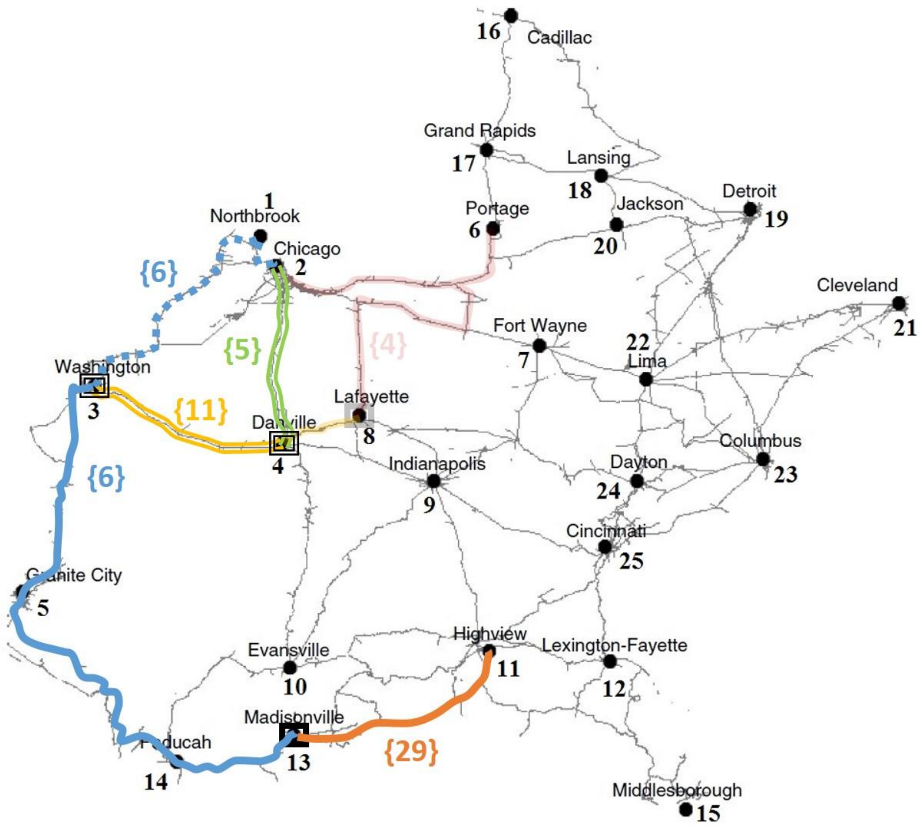


Figure 4.11. Using side service legs and transferring yards for the largest N=120 and $\alpha = \alpha_4 = 0.999999$

The answer to this question is that, the CVaR value of the above route is 11,086 (while still considering the configuration *N=80*, *C#4* and for α_4), which is greater than the CVaR value of the optimal route in Figure 4.10c (10,913). This confirms that the algorithm has correctly reported the best optimal CVaR route and the reason is that it avoids doing the transferring operation at yard #12 and postpones it to be done at yard #15 even it requires passing the service leg 12-15 twice, which has the population exposure of 3,934 (for carrying 80 hazmat railcars).

That is because the population exposure (for 80 hazmat railcars) at yard #15 is 62,240 while at yard #12 is 67,760.

4.3.3.4. CVaR value stability

In the fourth and fifth property given in section 4.2.5, we have shown how rail routes can slightly change while their CVaR values remain stable. To provide an example for them, suppose that for the transportation of 60 hazmat railcars from the origin (node #2) to the destination (node #11), we use the configuration *N=60, C# 1* and confidence level $\alpha = \alpha_4$. The optimal CVaR route will be:

$$\underbrace{2 \; - \; 6 \; - \; 8 \; - \; [}_{\{4\}} \; 9 \; \underbrace{] \; - \; 10 \; - \; [}_{\{12\}} \; 14 \; \underbrace{] \; - \; [}_{\{6\}} \; 13 \; \underbrace{] \; - \; 11}_{\{29\}}$$

Let name this route l, then the $C^l_{(t)}$ and $P^l_{(t)}$, where $t \in \{0,1,2,\ldots,10\}$ will be:

t	Service Leg / Transferring Yard	Train Service	$C^l_{(t)}$	$P^l_{(t)}$	$\sum_{i=0}^{t} P^l_{(i)}$
0	---	---	0	0.99999628	0.99999628
1	6-8	{4}	596	3.23E-07	0.99999660
2	**[13]**	{6}, {29}	**2,760**	4.14E-08	0.99999664
3	9-10	{12}	2,796	7.00E-07	0.99999734
4	14-13	{6}	4,563	3.28E-07	0.99999767
5	8-9	{4}	5,312	3.01E-07	0.99999797
6	**[14]**	{12}, {6}	**5,940**	4.14E-08	0.99999801
7	10-14	{12}	7,364	5.67E-07	0.99999858
8	2-6	{4}	7,560	5.67E-07	0.99999914
9	13-11	{29}	8,435	8.15E-07	0.99999996
10	[9]	{4}, {12}	22,920	4.14E-08	1.00000000

Therefore $\text{VaR}^l_\alpha = 7{,}560$ and $\text{CVaR}^l_\alpha = 8{,}909$. In addition to what is shown in route l, train service $\{12\}$ continues from node #14 to node #13 as well. Therefore we can remove transferring yard [14] (and consequently train service {6}) from route l and gain route $\grave{l}$ as follows (the new route makes sense):

$$\underbrace{2 \ - \ 6 \ - \ 8 \ - \ [}_{\{4\}}\ \ 9\ \ \underbrace{] \ - \ 10 \ - \ 14 \ - \ [}_{\{12\}}\ \ 13\ \ \underbrace{] \ - \ 11}_{\{29\}}$$

Because the consequence of transferring yard [14] is less than $\mathrm{VaR}_{\alpha}^{l}$ ($5{,}940 < 7{,}560$), the new route $\grave{l}$ has the same VaR and CVaR values: $\mathrm{VaR}_{\alpha}^{\grave{l}} = 7{,}560$ and $\mathrm{CVaR}_{\alpha}^{\grave{l}} = 8{,}909$ as shown below:

t	Service Leg / Transferring Yard	Train Service	$C_{(t)}^{l}$	$P_{(t)}^{l}$	$\sum_{i=0}^{t} P_{(i)}^{l}$
0	---	---	0	0.99999632	0.99999632
1	6-8	{4}	596	3.23E-07	0.99999664
2	**[13]**	{12}, {29}	**2,760**	**4.14E-08**	0.99999668
3	9-10	{12}	2,796	7.00E-07	0.99999738
4	14-13	{12}	4,563	3.28E-07	0.99999771
5	8-9	{4}	5,312	3.01E-07	0.99999801
6	10-14	{12}	7,364	5.67E-07	0.99999858
7	2-6	{4}	7,560	5.67E-07	0.99999914
8	13-11	{29}	8,435	8.15E-07	0.99999996
9	[9]	{4}, {12}	22,920	4.14E-08	1.00000000

Now suppose we replace transferring yard [13] of route $\grave{l}$ with transferring yard [14] and route l'' is generated as follows:

$$\underbrace{2 \ - \ 6 \ - \ 8 \ - \ [}_{\{4\}}\ \ 9\ \ \underbrace{] \ - \ 10 \ - \ [}_{\{12\}}\ \ 14\ \ \underbrace{] \ - \ 13 \ - \ 11}_{\{29\}}$$

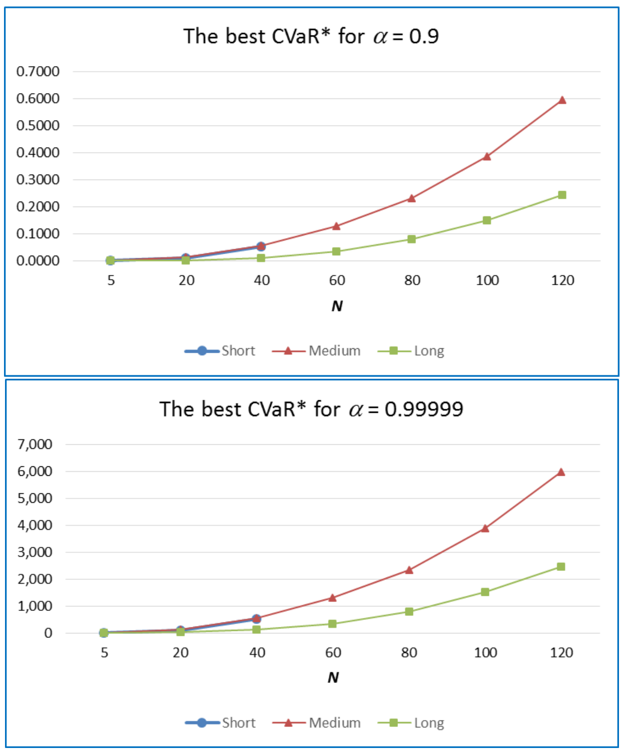
This replacement makes sense because train service {29} includes service leg 14-13 as well. Since the consequences of both transferring yards [13] and [14] are less than $\mathrm{VaR}_{\alpha}^{l}$ ($2{,}760$ & $5{,}940 < 7{,}560$) plus they have the same accident probability

(4.14E-08), the new route l'' has the same VaR and CVaR values: $\mathrm{VaR}_{\alpha}^{l''} = 7{,}560$ and $\mathrm{CVaR}_{\alpha}^{l''} = 8{,}909$ as shown below:

t	Service Leg / Transferring Yard	Train Service	$C_{(t)}^{l}$	$P_{(t)}^{l}$	$\sum_{i=0}^{t} P_{(i)}^{l}$
0	---	---	0	0.99999632	0.99999632
1	6-8	{4}	596	3.23E-07	0.99999664
2	9-10	{12}	2,796	7.00E-07	0.99999734
3	14-13	{29}	4,563	3.28E-07	0.99999767
4	8-9	{4}	5,312	3.01E-07	0.99999797
5	**[14]**	{12}, {29}	**5,940**	**4.14E-08**	0.99999801
6	10-14	{12}	7,364	5.67E-07	0.99999858
7	2-6	{4}	7,560	5.67E-07	0.99999914
8	13-11	{29}	8,435	8.15E-07	0.99999996
9	[9]	{4}, {12}	22,920	4.14E-08	1.00000000

4.3.3.5. ***Impact of train length***

Finally, we investigate the impact of train length on optimal CVaR values. To that end, the CVaR Algorithm was applied to *short* and *long* trains with 40 and 200 railcar lengths, respectively. In Figure 4.12, the best optimal CVaR for different number of hazmat railcars and confidence levels, which are found using short and long trains are compared with the ones found by the medium train. This shows that the long train has a better performance as it yields lower optimal values for CVaR, in part because of the potential to exploit the decile-based configuration more appropriately. More specifically, each decile in a long train can carry up to 20 hazmat railcars compared to a maximum of 4 and 12 hazmat railcars in short and medium trains, respectively. Therefore, given a fixed number of hazmat railcars to be shipped, when using a long train, decision makers are more able to put as many hazmat railcars as possible in the deciles with lower conditional derailment probabilities (see Tables 3.1 and 3.2). Consequently, according to (3.1) and (3.2), smaller accident probabilities in the service legs and transferring yards of the network will be expected when long trains are used, which in turn will result in lower CVaR values.



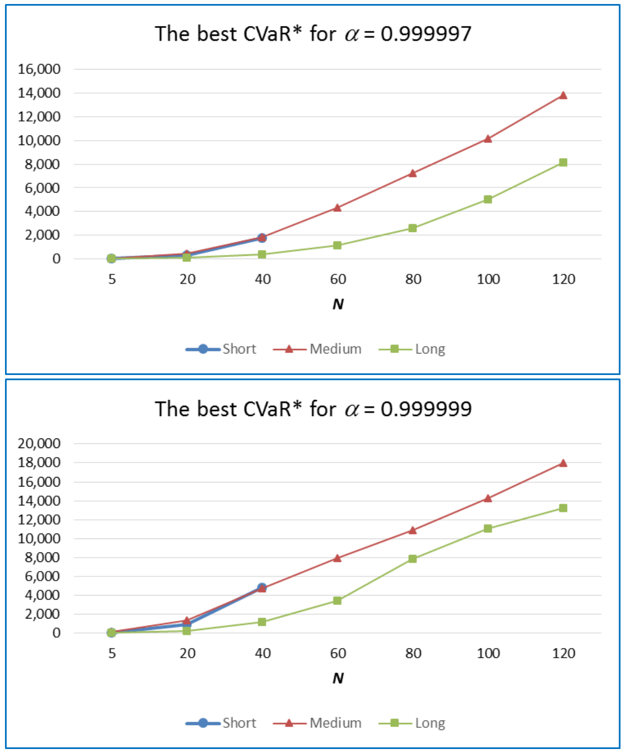


Figure 4.12. The best optimal CVaR found using short, medium, and long trains

4.4 Conclusion

Rail hazmat incidents, conceivably much lower in probability, could be significantly more catastrophic because of the involvement of multiple railcars than those from highway hazmat episodes. This phenomenon necessitated the development of a risk-averse routing methodology for routing rail hazmat shipments, and the emergence of value-at-risk (VaR) measure. Though VaR overcomes the risk-neutral behavior of the existing traditional risk approach and also has a relatively easy interpretation, it has a crucial limitation in that it cuts off and thus ignores the tail of the distribution, i.e., the region where catastrophic outcomes reside.

We have made the first attempt to propose a CVaR-based risk assessment methodology for rail hazmat shipments, which considers the confidence level of the decision maker to generate risk-averse routes. It is important that the proposed methodology is more complex than that for highway hazmat shipments since it incorporates information about pre-defined train services, transfer operations at intermediate yards, and the optimal configuration of trains. The methodology development entailed: providing a clear definition of CVaR for hazmat shipments, which was missing in the existing literature; delineating several relevant properties; and, an optimization program. The validation of the proposed methodology was accomplished by applying it to study several realistic size problem instances, which were further analyzed to conclude: *first*, CVaR is both distinct and superior to both TR and VaR measures; *second*, CVaR provides risk-averse routing of hazmat shipments; and *third*, CVaR generates different optimal routes based on the confidence-level of the decision makers.

There are a number of future research directions, including applying the proposed risk measure in the railroad blocking, railroad routing, train make-up decisions, train scheduling, etc. Two immediate research areas of interest to the authors including the railroad blocking problem taking into consideration both transport cost and hazmat risk, and the tactical planning problem of routing hazmat shipments.

Chapter 5. Routing Plan for Multiple Rail Hazmat Shipments with Optimal Conditional Value-at-Risk (CVaR) and Risk Equity

5.1 Introduction

Railroad is one of the safest modes for hazardous materials (hazmat) transportation. There is always, however, a small risk of catastrophic repercussion associated with the train carrying a shipment composed of multiple hazmat railcars. The tragedy occurred in Lac-Megantic (Quebec, Canada), in July 2013, provides a dramatic example of damage and loss of human life that rail hazmat shipments can cause. It is, therefore, crucial to implement a *risk-averse* routing plan for railroad transportation of hazmat, by reason of *low probability–high consequence* nature of rail hazmat incidents.

The risk-neutral behavior of the existing traditional risk models in the literature on routing rail hazmat shipments called for the development of value-at-risk (VaR) measure in Hosseini and Verma (2017), as a risk-averse methodology. VaR, however, has been criticized because of its deficiency in capturing the whole long tail of the risk distribution which is the residence of catastrophic outcomes. To overcome such a defect that brings about ignorance of extreme rail hazmat events, Hosseini and Verma (2018) developed conditional value-at-risk (CVaR) measure as an extension of VaR in a more sophisticated manner. The proposed CVaR-based risk assessment methodology not only incorporates the characteristics of railroad accidents, which is then utilized to provide risk-averse routing of rail hazmat shipments with the capability to generate various optimal routes based upon the confidence-level of decision makers, but also establishes that it is superior to both traditional risk (TR) and VaR measures. Readers are referred to Kang et al. (2014a) and Hosseini and Verma (2018) for a review of risk models for hazmat transportation in road and rail domain, respectively. CVaR measure was first adapted by Toumazis et al. (2013) to provide a risk-averse and flexible routing plan for highway hazmat shipments. It was then extended in the works of Toumazis and Kwon (2013), Faghih-Roohi et al. (2015), and Toumazis and Kwon (2016).

In this research we extend the work of Hosseini and Verma (2018) and adapt the CVaR methodology they developed for a single rail hazmat shipment, single origin-destination (O-D) pair, to *multiple* rail hazmat shipments. This aspect leads to a harder class of problems that involve *multi-commodity* and *multiple O-D* hazmat routing decisions. On the other hand, it may happen that certain links and yards of the railroad network tend to be overloaded with hazmat traffic and risk. This becomes crucial when certain populated zones are exposed to an intolerable level of risk resulting from the routing decisions. To overcome this issue, we also promote *equity* in the spatial distribution of risk throughout the railroad network. Therefore, the main problem is to find minimum risk routes, as measured by CVaR methodology, while limiting and equitably spreading the risk in any zone where the railroad network is embedded.

To the best of our knowledge, no work in the literature of rail hazmat transportation has incorporated risk equity. There are, however, several models that have addressed risk equity in the context of road hazmat transportation. Risk equity has been defined as the largest difference in the level of risk among a set of individuals in Keeney (1980). Gopalan et al. (1990) developed an integer programming model to generate an equitable set of routes for road hazmat shipments, while the total risk of travel is minimized, and risk is equitably spread among the population zones of the transportation network by ensuring that the difference in risk entailed in the zones is less than a threshold. They showed a high degree of equity can be achieved by modest increase in the total risk. Current and Ratick (1995) proposed a multi-objective model considering both the location of facilities that handle hazmat, and the routing of hazmat to these facilities. Risk equity is imposed by minimizing the maximum allowable risk exposure for each zone. Carotenuto et al. (2007)'s model generates minimal risk paths for the road transportation of hazmat, which minimize the total risk of hazmat shipments. The risk imposed on population is spread in an equitable way by limiting the risk on each link to be less than an upper threshold. Lagrangian relaxation method is then applied to provide a lower bound on the optimal solution. Kang et al. (2014b) used the same method of modeling equity proposed in Gopalan et al. (1990), but unlike Current and Ratick (1995) and Carotenuto et al. (2007) which studied a single hazmat trip, they developed a more general model that allows multiple O–D pairs and multiple trips for each O–D pair. Additionally, instead of traditional risk measure used in the majority of the literature, they utilized VaR as the risk measure to be minimized in the objective. A Lagrangian relaxation heuristic is then developed to obtain an efficient solution method.

Unlike road domain, transportation in railroad network is performed using a set of pre-defined *train services*, each of which starts from its origin yard, traverses a set of intermediate rail arcs and yards, and finally stops at its destination yard. Being constrained to these itineraries requires us to commonly exploit a combination of different train services for moving shipments between given O-D pairs. Thus, each hazmat shipment, before reaching its destination yard, typically undergoes several *transfer* operations in the intermediate yards where it is first unloaded from one train service then gets loaded into the next one. To assure risk equity in the proposed model, we not only restrict the expected risk on rail arcs to be less that an upper limit, but also enforce the expected risk at the transferring yard to not exceed a set threshold. Of course, such assurance is achieved simultaneously with minimizing the total risk of transporting hazmat shipments as measured by CVaR.

The remainder of the chapter is organized as follows. The next section defines the problem under consideration in detail and presents the mathematical formulation. In Section 5.3, we develop the corresponding solution framework which consists of a Lagrangian, subgradient, and heuristic algorithm. Section 5.4

presents the computational results based on data in Midwest United States. Finally, in Section 5.5, conclusion remarks and suggestions for future research are provided.

5.2 Problem Description

In an effort to formulate the problem, we first introduce appropriate concepts and notations pertinent to railroad transportation system in Section 5.2.1 It is then followed by Sections 5.2.2 and 5.2.3 in which the application of CVaR measure to multiple rail hazmat shipments with risk equity consideration is discussed.

5.2.1. Railroad Transportation System

A railroad transportation network $G = (\mathcal{Y}, \mathcal{A}, S)$ consists of a set of yards $\mathcal{Y}$, a set of (undirected) arcs $\mathcal{A}$, and also a set of available train services S. Each train service is a pre-determined itinerary composed of a set of (directed) service legs and a set of yards. There is a set of hazmat shipments V, each of which contains $N(v)$ hazmat railcars that is to be transported throughout the network from the origin $O(v)$ to the destination $D(v)$ using the on-hand train services. The notations to represent the sets and parameters used in the chapter are as follows:

Network $G = (\mathcal{Y}, \mathcal{A}, \mathcal{S})$

$\mathcal{Y}$: set of yards in the networks, indexed by i, j, k

$\mathcal{A}$: set of $(undirected)$ arcs in the networks, indexed by (i, j) and (k, j)

$\mathcal{S}$: set of train services in the network, indexed by s (and/or $\acute{s}$)

$\mathcal{Y}_s \in \mathcal{Y}$: set of yards in train service $\{s\}$, indexed by i_s, j_s, k_s

$\mathcal{A}_s \in \mathcal{A}$: set of $(directed)$ service legs in train service $\{s\}$, indexed by (i_s, j_s) and (k_s, j_s)

V: set of shipments in the network, indexed by v

$N(v)$: number of hazmat railcars in shipment v

$O(v)$: origin of shipment v

$D(v)$: destination of shipment v

The above notations are then used to define the following binary decision variables for routing the shipments through the railroad network using the available train services:

$$x_{i_s j_s}^{v} = \begin{cases} 1, & \text{if shipment } v \text{ is carried using arc } (i,j) \text{ of train service } \{s\} \text{ (service leg } (i_s, j_s)) \\ 0, & \text{otherwise} \end{cases}$$

$$x_{k_{\acute{s}} k_s}^{v} = \begin{cases} 1, & \text{if } k \text{ is a transferring yard between train services } \{\acute{s}\} \text{ and } \{s\} \ (k_{\acute{s}s}) \text{ for shipment } v \\ 0, & \text{otherwise} \end{cases}$$

Next, we can build the routing constraints $(X \in \psi)$ based on the decision variables as follows:

$$\sum_{j_s} x_{i_s j_s}^v - \sum_{j_s} x_{j_s i_s}^v$$
$$= \begin{cases} 1, & \text{if } i_s = O(v) \\ 0, & \text{if } i_s \neq O(v) \text{ or } D(v) \\ & \text{(for any non–transferring yard } i_s \text{ for shipment } v) \\ -1, & \text{if } i_s = D(v) \end{cases} \qquad \forall v, \forall i_s$$

$$\sum_{j_s} x_{k_s j_s}^v - \sum_{j_{\acute{s}}} x_{j_{\acute{s}} k_{\acute{s}}}^v$$
$$= 0, \qquad \text{for any transferring yard } k_{\acute{s}s} \text{ for shipment } v \qquad \forall v, \forall k_{\acute{s}s}$$

Shipment v traverses through route $O(v) - D(v)$ using $\mathbb{S}$ out of $\mathcal{S}$ available train services in the network, which consists of a set of transferring yards $\mathcal{Y}^v = \bigcup_{s \in \mathbb{S}} \mathcal{Y}_s$ and a set of service legs $\mathcal{A}^v = \bigcup_{s \in \mathbb{S}} \mathcal{A}_s$, i.e. totally $n^v = |\mathcal{Y}^v \cup \mathcal{A}^v|$ items. We define the following parameters for the route $O(v) - D(v)$ of shipment v:

p_k^v: Accident probability at transferring yard k resulting from transporting $N(v)$ hazmat railcars

c_k^v: Accident consequence at transferring yard k resulting from transporting $N(v)$ hazmat railcars

p_{ij}^v: Accident probability in arc (i, j) resulting from transporting $N(v)$ hazmat railcars

c_{ij}^v: Accident consequence in service leg (i, j) resulting from transporting $N(v)$ hazmat railcars

The techniques used to estimate the above parameters are borrowed from other peer-reviewed works, whose details are provided in Hosseini and Verma (2017). They categorized trains into three groups: *short* can carry up to 40 railcars; *medium*, between 41 and 120 railcars; and, *long*, the rest. To calculate the accident probability in the yards and arcs, the derailment probability of train and then multiplication of *conditional probabilities* which finally lead to release from a hazmat railcar derailed are computed based on the ten deciles of the train (i.e., the length of the train is divided into 10 equal parts). This means, p_k^v and p_{ij}^v depend on the numbers of hazmat railcars in each train-decile (i.e., *train configuration*), which in total equal $N(v)$. Hosseini and Verma (2018) followed the same strategy for the accident probabilities, although showed that long trains yield a lower optimal value for CVaR and so have a better performance compared to short and medium trains. Regarding this fact and also by reason of remaining close to the main theme of this research, we assume that all trains used in the train services are long, and we use the average values of the conditional probabilities. Hence, p_k^v and p_{ij}^v do not depend on the train configuration any more, although they are still dependent upon the number of hazmat railcars $N(v)$. On the other side, the consequences, i.e., c_k^v and c_{ij}^v, are estimated as the population exposure due to the release from $N(v)$ hazmat railcars traversing transferring yard k and arc (i, j), respectively, given the accident in them. In brief, the accident probability and consequence in the transferring yards and arcs of the network enlarge with increase in the total number of hazmat railcars (hazmat volume) traversing them. More numerical details will be provided in Section 5.4.

5.2.2. CVaR for Rail Hazmat Shipment

Let $C_{(t)}^{v}$ denote the t^{th} smallest value in the set $\{c_k^v \cup c_{ij}^v \ : \ k \in \mathcal{Y}^v \ \& \ (i,j) \in \mathcal{A}^v\}$, $P_{(t)}^{v}$ be the corresponding accident probability, and R^v be the discrete random variable denoting the risk associated with route $O(v) - D(v)$. Then R^v can take the following values:

$$R^v = \begin{cases} C_{(0)}^v = 0, & \text{with probability } P_{(0)}^v = 1 - \sum_{i=1}^{n^v} P_{(i)}^v \\ C_{(1)}^v, & \text{with probability } P_{(1)}^v \\ \vdots & \vdots \\ C_{(t)}^v, & \text{with probability } P_{(t)}^v \\ \vdots & \vdots \\ C_{(n^v)}^v, & \text{with probability } P_{(n^v)}^v \end{cases}$$

where $C_{(0)}^v < C_{(1)}^v < C_{(2)}^v < \cdots < C_{(n^v)}^v$ and $t \in \{0, 1, 2, \ldots, n^v\}$. According to Hosseini and Verma (2017), for a specific confidence level $\alpha \in (0,1)$, VaR is the minimal threshold level β such that the hazmat risk R^v does not exceed β with the least probability of α : $\mathrm{VaR}_\alpha(v) = \min\{\beta \mid \Pr(R^v \leq \beta) \geq \alpha\} = \min\{\beta \mid F_{R^l}(\beta) \geq \alpha\}$. Hosseini and Verma (2018) demonstrated that since there are discontinuities in the risk distribution function, CVaR cannot be interpreted simply as the average of the consequences greater than or equal to VaR. Instead, it has a more subtle delineation: CVaR is represented as a weighted average of VaR and the risk greater than that:

$$\mathrm{CVaR}_\alpha(v) = \lambda_\alpha^v\, \mathrm{VaR}_\alpha(v) + (1-\lambda_\alpha^v)\, E\big(R^v \mid R^v > \mathrm{VaR}_\alpha(v)\big)$$

where λ_α^v indicates the probability allocated to the risk amount $\mathrm{VaR}_\alpha(v)$ by the α-tail distribution of R^v as follows:

$$\lambda_\alpha^v = \frac{F_{R^v}\big(\mathrm{VaR}_\alpha(v)\big) - \alpha}{1-\alpha}$$

where always $F_{R^v}\big(\mathrm{VaR}_\alpha(v)\big) \geq \alpha$ by definition of VaR.

Considering confidence level $\alpha \in (0,1)$, we define T be equal to 0 if $0 < \alpha \leq P_{(0)}^v$, otherwise it is the unique index such that $\sum_{t=0}^{T-1} P_{(t)}^v < \alpha \leq \sum_{t=0}^{T} P_{(t)}^v$. The α-VaR of the risk along route $O(v) - D(v)$ is then $\mathrm{VaR}_\alpha(v) = C_{(T)}^v$, and the α-CVaR of the risk associated with train service $\{s\}$ is

$$\mathrm{CVaR}_\alpha(v) = C_{(T)}^v + \frac{1}{1-\alpha}\left[\sum_{t=T+1}^{n^v} P_{(t)}^v \left(C_{(t)}^v - C_{(T)}^v\right)\right] \tag{5.3}$$

The detailed methods to calculate the above VaR and CVaR are described in Hosseini and Verma (2017) and Hosseini and Verma (2018), respectively. In our problem, the objective at the given confidence level α is to

$$\min \sum_{v} \mathrm{CVaR}_{\alpha}(v)$$

which means to do the routing of all rail hazmat shipments using the available train services in the network in such a way that the summation of the risks generated by them and measured by CVaR is minimized.

5.2.3. Risk Equity for Rail Hazmat Shipment

Each yard in the network may be used as a transfer operation node between two train services for several shipments. Likewise, it is very likely that (undirected) arcs of the network be utilized by a (directed) service leg of several train services for carrying hazmat shipments. We enforce threshold restrictions on the both two types of the risks which might be imposed on the transferring yards and arcs of the network, so that equitable distribution of risk over the network is guaranteed. In order to define the equity constraints, we first need to calculate the total hazmat volume transferred at each transferring yard k and also the total hazmat volume carried via arc (i,j) of the network, which are gained as follows

$$N(k) = \sum_{v,\acute{s},s} N(v)\, x^{v}_{k_{\acute{s}} k_s} \tag{5.4}$$

$$N(i,j) = \sum_{v,s} N(v)\, x^{v}_{i_s j_s} \tag{5.5}$$

Then we define the following parameters for the yards and arcs in the network:

$\mathcal{P}_k$: Accident probability at transferring yard k resulting from hazmat volume $N(k)$

$\mathcal{C}_k$: Accident consequence at transferring yard k resulting from hazmat volume $N(k)$

$\mathcal{P}_{ij}$: Accident probability in arc (i,j) resulting from hazmat volume $N(i,j)$

$\mathcal{C}_{ij}$: Accident consequence in arc (i,j) resulting from hazmat volume $N(i,j)$

Consequently, we define the equity constrains as follows:

$$\mathcal{P}_k\, \mathcal{C}_k \le \delta_k \qquad \forall k$$

$$\mathcal{P}_{ij}\, \mathcal{C}_{ij} \le \delta_{ij} \qquad \forall (i,j)$$

They ensure that the expected consequence in each transferring yard and arc of the network resulting from transporting multiple rail hazmat shipments will not exceed the threshold values δ_k and δ_{ij}, respectively. Therefore, the proposed problem P is

$$\min \sum_v \mathrm{CVaR}_\alpha(v) \tag{5.6}$$

Subject to

$$\mathcal{P}_k\, \mathcal{C}_k \le \delta_k \qquad \forall k \tag{5.7}$$

$$\mathcal{P}_{ij}\, \mathcal{C}_{ij} \le \delta_{ij} \qquad \forall (i,j) \tag{5.8}$$

$$X \in \psi \tag{5.9}$$

5.3 Optimization Program

Considering different alternative routes in the network available for each shipment, our objective is to determine the route $O(v) - D(v)$ which has the minimum CVaR. To do that, we extend (5.1) as follows

$$\begin{aligned}\mathrm{CVaR}_{\alpha}^{r}(v) = {} & C_{(r)}^{v} \\ & + \frac{1}{1-\alpha}\left(\sum_{s,\acute{s}} \sum_{k, c_k^v > C_{(r)}^{v}} p_k^v \left(c_k^v - C_{(r)}^{v}\right) x_{k_{\acute{s}} k_s}^{v}\right. \\ & \left. + \sum_{s} \sum_{(i,j), c_{ij}^v > C_{(r)}^{v}} p_{ij}^v \left(c_{ij}^v - C_{(r)}^{v}\right) x_{i_s j_s}^{v}\right)\end{aligned}$$

where $C_{(r)}^{v}; r \in \{0, 1, 2, \dots, M\}$ $\left(C_{(0)}^{v} = 0\right)$ is the rth smallest value in the set $\left\{c_k^v \cup c_{ij}^v \;:\; k \in \mathcal{Y} \;\&\; (i,j) \in \mathcal{A}\right\}$, and M is the total number of yards and arcs of all train services available in the rail network. Therefore, for each shipment the objective is to $\mathrm{CVaR}_{\alpha}^{*}(v) = \min_{r} \mathrm{CVaR}_{\alpha}^{r}(v)$, and so the objective of problem P, (5.4), can be rewritten as

$$\min_{r} \sum_{v} \left(C_{(r)}^{v} + \frac{1}{1-\alpha}\, g^{r}(v)\right) \tag{5.10}$$

where

$$g^r(v) = \min_{X\in\psi}\left(\sum_{s,\acute{s}}\sum_{k,c_k^v>C_{(r)}^v} p_k^v\left(c_k^v - C_{(r)}^v\right)x_{k_{\acute{s}}k_s}^v \right.$$

$$\left. + \sum_{s}\sum_{(i,j),c_{ij}^v>C_{(r)}^v} p_{ij}^v\left(c_{ij}^v - C_{(r)}^v\right)x_{i_s j_s}^v\right)$$

5.3.1. Lagrangian Relaxation Method

To solve our problem, we apply Lagrangian relaxation method and dualize the constraint sets (5.5) and (5.6). The exact left-hand-side values of these constraints are calculated using $N(k)$ and $N(i,j)$ in (5.2) and (5.3), respectively, as follows

$$p_k^{\sum_{v,\acute{s},s} N(v)\, x_{k_{\acute{s}}k_s}^v}\, c_k^{\sum_{v,\acute{s},s} N(v)\, x_{k_{\acute{s}}k_s}^v} \le \delta_k \qquad \forall k$$

$$p_{ij}^{\sum_{v,s} N(v)\, x_{i_s j_s}^v}\, c_{ij}^{\sum_{v,s} N(v)\, x_{i_s j_s}^v} \le \delta_{ij} \qquad \forall (i,j)$$

Although, to use the Lagrangian method, we have to approximate them as follows

$$\left(\sum_{v}\sum_{s,\acute{s}} N(v)\, x_{k_{\acute{s}}k_s}^v\right) p_k\, c_k \le \delta_k \qquad \forall k \tag{5.11}$$

$$\left(\sum_{v}\sum_{s} N(v)\, x_{i_s j_s}^v\right) p_{ij}\, c_{ij} \le \delta_{ij} \qquad \forall (i,j) \tag{5.12}$$

where p_k (or p_{ij}) and c_k (or c_{ij}) are the accident probability and consequence, respectively resulting from transporting *one* hazmat railcar at transferring yard k (or in arc (i,j)). Hence, the Lagrangian function $L(u)$ will be

$$
\begin{aligned}
L(u) = \min_r \sum_v \Bigg(& C^v_{(r)} \\
& + \frac{1}{1-\alpha}\Bigg(\sum_{s,\acute{s}} \sum_{k, c^v_k > C^v_{(r)}} p^v_k \left(c^v_k - C^v_{(r)}\right) x^v_{k_{\acute{s}} k_s} \\
& + \sum_s \sum_{(i,j), c^v_{ij} > C^v_{(r)}} p^v_{ij} \left(c^v_{ij} - C^v_{(r)}\right) x^v_{i_s j_s} \Bigg)\Bigg) \\
& + \sum_k \left(\sum_v \sum_{s,\acute{s}} N(v) x^v_{k_{\acute{s}} k_s}\right) u_k \, p_k \, c_k \\
& + \sum_{(i,j)} \left(\sum_v \sum_s N(v) \, x^v_{i_s j_s}\right) u_{ij} \, p_{ij} \, c_{ij} \\
& - \sum_k u_k \delta_k - \sum_{(i,j)} u_{ij} \delta_{ij}
\end{aligned}
$$

Subject to

$$X \in \psi$$

where $u_k \geq 0, \forall k$ and $u_{ij} \geq 0, \forall (i,j)$ are the vectors of dual variables (Lagrangian multipliers) for constraint sets (5.9) and (5.10), respectively. After some computation, we can rewrite the Lagrangian function as follows

$$L(u) = \min_r \sum_v \left(C_{(r)}^v + \sum_{s,\acute{s}} \sum_{k, c_k^v > C_{(r)}^v} \left(\frac{p_k^v \left(c_k^v - C_{(r)}^v \right)}{1-\alpha} + N(v)\, u_k\, p_k\, c_k \right) x_{k_{\acute{s}} k_s}^v \right.$$

$$+ \sum_s \sum_{(i,j), c_{ij}^v > C_{(r)}^v} \left(\frac{p_{ij}^v \left(c_{ij}^v - C_{(r)}^v \right)}{1-\alpha} \right.$$

$$\left. \left. + N(v)\, u_{ij}\, p_{ij}\, c_{ij} \right) x_{i_s j_s}^v \right)$$

$$- \sum_k u_k \delta_k - \sum_{(i,j)} u_{ij} \delta_{ij}$$

Subject to

$$X \in \psi$$

This Lagrangian function separates into $|V|$ distinct shortest path problems; it will become more evident if we make the following modifications

$$w_k^v = \begin{cases} \dfrac{p_k^v \left(c_k^v - C_{(r)}^v \right)}{1-\alpha} + N(v)\, u_k\, p_k\, c_k\ , & \text{if} \quad c_k^v > C_{(r)}^v \qquad \forall k\ , \forall v \\ N(v)\, u_k\, p_k\, c_k, & \text{otherwise} \end{cases} \tag{5.13}$$

$$w_{ij}^{v} = \begin{cases} \dfrac{p_{ij}^{v}\left(c_{ij}^{v} - C_{(r)}^{v}\right)}{1-\alpha} + N(v)\, u_{ij}\, p_{ij}\, c_{ij}\ , & \text{if} \quad c_{ij}^{v} > C_{(r)}^{v} \\ N(v)\, u_{ij}\, p_{ij}\, c_{ij}, & \text{otherwise} \end{cases} \qquad \forall (i,j)\ , \forall v \tag{5.14}$$

This way the Lagrangian function will be

$$L(u) = \min_{r} \sum_{v} \left(C_{(r)}^{v} + \sum_{s,\acute{s}} \sum_{k} w_{k}^{v}\, x_{k_{\acute{s}}k_{s}}^{v} + \sum_{s} \sum_{(i,j)} w_{ij}^{v}\, x_{i_{s}j_{s}}^{v} \right) - \sum_{k} u_{k}\delta_{k} - \sum_{(i,j)} u_{ij}\delta_{ij}$$

Subject to

$$X \in \psi$$

which can be restated as follows

$$L(u) = \min_{r} \sum_{v} \left(C_{(r)}^{v} + f^{r}(v) \right) - \sum_{k} u_{k}\delta_{k} - \sum_{(i,j)} u_{ij}\delta_{ij}$$

where

$$f^{r}(v) = \min_{X\in\psi} \left(\sum_{s,\acute{s}} \sum_{k} w_{k}^{v}\, x_{k_{\acute{s}}k_{s}}^{v} + \sum_{s} \sum_{(i,j)} w_{ij}^{v}\, x_{i_{s}j_{s}}^{v} \right) \tag{5.15}$$

which is a shortest path problem. We then develop an algorithm to solve $L(u)$ as follows

Lagrangian Function Algorithm

1. For $v = 1$ to $|V|$ (all shipments) do:

    1.1 Generate $C_{(r)}^{v}$ and their corresponding $P_{(r)}^{v}$: $r = 0, 1, 2, \ldots, M$.

    1.2 For $r = 0$ to M do:

        1.2.1 Do the modifications (5.11) and (5.12), then solve problem (5.13), $f^{r}(v)$, using Dijkstra's Shortest Path Algorithm. It also gives the corresponding route $X^{r}(v)$.

        1.2.2 Calculate $C_{(r)}^{v} + f^{r}(v)$.

    1.3 Let $r^{*} = \underset{r=0,1,\ldots,M}{\arg\min} \left(C_{(r)}^{v} + f^{r}(v) \right)$. Save the best route $X^{r^{*}}(v)$.

2. Calculate $L(u) = \sum_{v} \left(C_{(r^{*})}^{v} + f^{r^{*}}(v) \right) - \sum_{k} u_k \delta_k - \sum_{(i,j)} u_{ij} \delta_{ij}$.

5.3.2. Subgradient Search Algorithm

The Lagrangian dual problem, $\max\{L(u) \mid u \geq 0\}$, is in general difficult to solve exactly, and may be approximately solved by means of the well-known and widely-used subgradient technique (Held et al., 1974). Let $\Gamma(\bar{u})$ denote the set of optimal solutions for the Lagrangian function at $\bar{u}$, if $\bar{x} \in \Gamma(\bar{u})$, then the following vectors $\bar{\gamma}$ are *subgradients* of $L(u)$ at $\bar{u}$

$$\bar{\gamma}_k = -\delta_k + \left(\sum_{v} \sum_{s,\acute{s}} N(v)\, \bar{x}_{k_{\acute{s}} k_s}^{v} \right) p_k\, c_k \qquad \forall k$$

$$\bar{\gamma}_{ij} = -\delta_{ij} + \left(\sum_v \sum_s N(v)\, \bar{x}^v_{i_s j_s} \right) p_{ij}\, c_{ij} \qquad \forall (i,j)$$

Next, we utilize them to develop the following algorithm:

Subgradient Optimization Algorithm

Step 1: (Initialization) Let $q \leftarrow 0, u^q \geq 0, \epsilon > 0$.

Step 2: Do the Lagrangian Function Algorithm using u^q and find

$$X^q \in \Gamma(u^q\,), \text{and}$$

$$L(u^q) = \sum_v \left(C^v_{(r^*)} + f^{r^*}(v) \right) - \sum_k u^q_k \delta_k - \sum_{(i,j)} u^q_{ij} \delta_{ij}$$

Step 3: Calculate the subgradients using the result of the Lagrangian Function Algorithm (X^q)

$$\gamma^q_k = -\delta_k + \left(\sum_v \sum_{s,\acute{s}} N(v)\, {x^v_{k_{\acute{s}} k_s}}^q \right) p_k\, c_k \qquad \forall k$$

$$\gamma^q_{ij} = -\delta_{ij} + \left(\sum_v \sum_s N(v)\, {x^v_{i_s j_s}}^q \right) p_{ij}\, c_{ij} \qquad \forall (i,j)$$

Step 4: Let $u^{q+1} \leftarrow \max\{0, u^q + t_q \gamma^q\}$ where t_q is a positive scalar called the *step size*.

Step 5: If $\|u^{q+1} - u^q\| < \epsilon$ stop, else let $q \leftarrow q + 1$ and go to Step 2.

By means of the above algorithm, the sequence of dual solutions $\{u^q\}$ approaches an optimal solution $\{u^*\}$, however the value of $L(u^{q+1})$ is not necessarily greater than $L(u^q)$. In fact, according to the fundamental theoretical result (Held et al., 1974), the sequence $\{L(u^q)\}$ converges to $L(u^*)$ if the sequence $\{t_q\}$ converges to zero and $\sum_{q=0}^{\infty} t_q = \infty$. It is common to determine t_q by a formula such as

$$t_q = \frac{\theta_q \left(L^{ub} - L(u^q)\right)}{\|\gamma^q\|^2}$$

where θ_q is a positive scalar between 0 and 2 (Martin, 1999). We determine θ_q by setting $\theta_0 = 2$ and halving θ_q whenever $L(u)$ fails to increase for some fixed number of iterations. L^{ub} is an upper bound on the optimal value $L(u)$, which can be the value of any primal feasible solution.

Note that the Lagrangian relaxation method has been introduced here to supply us with a lower bound on the optimal solution. This enables us to evaluate the maximum distance from the optimality of the solutions generated by the heuristic to be described in the following section.

5.3.3. A Heuristic Algorithm for k-minimal CVaR Paths Determination

In this section, we propose a greedy heuristic algorithm to determine k-minimal CVaR paths while the equity constraints are satisfied. This algorithm is developed based upon Yen's k-shortest path algorithm (Yen, 1971). It not only provides a good initial primal feasible solution to be used in the Subgradient algorithm, but also can be regarded as a stand-alone solution algorithm, as will be shown in the next section.

The main idea in this algorithm is that we start with the shipment which has the highest number of hazmat railcars and do its routing in such a way that the CVaR risk associated with the generated route is minimized. We repeat this process for the next highest number of hazmat railcar shipment, but then check if the created minimum CVaR route violates any of the risk equity constraints for the arcs and/or transferring yards or not. If so, we remove the corresponding arcs and/or transferring yards from the network and do a *rerouting* to find the next minimal CVaR route for this shipment. This process is repeated as many times as required until a minimal CVaR route is found for the shipment. Likewise, this procedure is iterated until all hazmat shipments in the network are routed. It is worthwhile mentioning that when a yard is removed from the network as a transferring yard, it may still be used in the routing of the next shipments as a transshipment yard.

To implement the proposed heuristic algorithm, we first rewrite the objective function of problem P, (5.4), by using (5.8) as follows:

$$\min_{r} \sum_{\bar{v}} \mathrm{CVaR}_{\alpha}^{r}(\bar{v}) = \min_{r} \sum_{\bar{v}} \left(C_{(r)}^{\bar{v}} + \bar{f}^{r}(\bar{v}) \right)$$

where

$$\bar{f}^{r}(\bar{v}) = \min_{X \in \psi} \left(\sum_{s,\acute{s}} \sum_{k} \bar{w}_{k}^{\bar{v}} \, x_{k_{\acute{s}} k_{s}}^{\bar{v}} + \sum_{s} \sum_{(i,j)} \bar{w}_{ij}^{\bar{v}} \, x_{i_{s} j_{s}}^{\bar{v}} \right) \tag{5.16}$$

where

$$\overline{w}_k^{\bar{v}} = \begin{cases} \dfrac{p_k^{\bar{v}}\left(c_k^{\bar{v}} - C_{(r)}^{\bar{v}}\right)}{1-\alpha} \;, & \text{if} \quad c_k^{\bar{v}} > C_{(r)}^{\bar{v}} \qquad \forall k \,, \forall \bar{v} \\ 0, & \text{otherwise} \end{cases} \tag{5.17}$$

$$\overline{w}_{ij}^{\bar{v}} = \begin{cases} \dfrac{p_{ij}^{\bar{v}}\left(c_{ij}^{\bar{v}} - C_{(r)}^{\bar{v}}\right)}{1-\alpha} \;, & \text{if} \quad c_{ij}^{\bar{v}} > C_{(r)}^{\bar{v}} \qquad \forall (i,j) \,, \forall \bar{v} \\ 0, & \text{otherwise} \end{cases} \tag{5.18}$$

Next, we develop the following steps to obtain the k-minimal CVaR paths as the equity considerations are taken into account.

Step 1. Sort the shipments in descending order in terms of $N(v)$. It gives a new set of index $\bar{v} = \{1, 2, \dots, |V|\}$ such that $N(1) \geq N(2) \geq \cdots \geq N(|V|)$

Step 2. Define the cumulative risk on yard k and arc (i,j) of the network after routing the first $\mathbb{V}$ voluminous shipments as follows

$$\mathrm{CR}_k^{\mathbb{V}} = \left(\sum_{\bar{v}=1}^{\mathbb{V}} \sum_{s,\acute{s}} N(\bar{v})\, x_{k_{\acute{s}} k_s}^{\bar{v}}\right) p_k\, c_k$$

$$= \left(\sum_{\bar{v}=1}^{\mathbb{V}-1} \sum_{s,\acute{s}} N(\bar{v})\, x_{k_{\acute{s}} k_s}^{\bar{v}}\right) p_k\, c_k + \left(\sum_{s,\acute{s}} N(\mathbb{V})\, x_{k_{\acute{s}} k_s}^{\mathbb{V}}\right) p_k\, c_k$$

$$\mathrm{CR}_{ij}^{\mathbb{V}} = \left(\sum_{\bar{v}=1}^{\mathbb{V}} \sum_{s} N(\bar{v})\, x_{i_s j_s}^{\bar{v}}\right) p_{ij}\, c_{ij}$$

$$= \left(\sum_{\bar{v}=1}^{\mathbb{V}-1} \sum_{s} N(\bar{v})\, x_{i_s j_s}^{\bar{v}}\right) p_{ij}\, c_{ij} + \left(\sum_{s} N(\mathbb{V})\, x_{i_s j_s}^{\mathbb{V}}\right) p_{ij}\, c_{ij}$$

Step 3. Set initial $\mathrm{CR}_k^0 = 0\ \forall k$, and $\mathrm{CR}_{ij}^0 = 0\ \forall (i,j)$

Step 4. Let $\bar{v} \leftarrow 1$

Step 4.1. $\forall k \in \mathcal{Y}$ (for all yards in the network) do: If $\mathrm{CR}_k^{\bar{v}-1} = \delta_k$, the risk capacity of yard k is already reached, so remove the yard from the network for routing the rest of shipments.

Step 4.2. $\forall (i,j) \in \mathcal{A}$ (for all arcs in the network) do: If $\mathrm{CR}_{ij}^{\bar{v}-1} = \delta_{ij}$, the risk capacity of arc (i,j) is already reached, so remove the arc from the network for routing the rest of shipments.

Step 4.3. Generate $C_{(r)}^{\bar{v}}$ and their corresponding $P_{(r)}^{\bar{v}} : r = 0, 1, 2, \ldots, M$.

Step 4.4. For $r = 0$ to M do:

Step 4.4.1. Do the modifications (5.15) and (5.16), then solve problem (5.14), $\bar{f}^r(\bar{v})$, using Dijkstra's Shortest Path Algorithm. It gives the corresponding route $X^r(\bar{v})$.

Step 4.4.2. Calculate $C_{(r)}^{\bar{v}} + \bar{f}^r(\bar{v})$.

Step 4.5. Let $r^* = \underset{r=0,1,\ldots,M}{\arg\min} \left(C_{(r)}^{\bar{v}} + \bar{f}^r(\bar{v}) \right)$. $\mathrm{CVaR}_\alpha^{r^*}(\bar{v}) = C_{(r^*)}^{\bar{v}} + \bar{f}^{r^*}(\bar{v})$. Hold the best route $X^{r^*}(\bar{v})$ for shipment $\bar{v}$.

Step 4.6. $\forall k \in X^{r^*}(\bar{v})$ (for each yard in the current shipment's route) do:

Step 4.6.1. Update $\mathrm{CR}_k^{\mathbb{V}}$ by setting $\mathbb{V}$ up to shipment $\bar{v}$, as the summation of $\mathrm{CR}_k^{\mathbb{V}-1}$ and the risk added to yard k by the current shipment $\bar{v}$.

Step 4.6.2. If $\mathrm{CR}_k^{\mathbb{V}} > \delta_k$, remove yard k from all train services of the network temporarily (only for the routing of this shipment).

Step 4.7. $\forall(i,j) \in X^{r^*}(\bar{v})$ (for each arc in the current shipment's route) do:

Step 4.7.1. Update $\mathrm{CR}_{ij}^{\mathbb{V}}$ while $\mathbb{V}$ up to shipment $\bar{v}$, as the summation of $\mathrm{CR}_{ij}^{\mathbb{V}-1}$ and the risk added to arc (i,j) by the current shipment $\bar{v}$.

Step 4.7.2. If $\mathrm{CR}_{ij}^{\mathbb{V}} > \delta_{ij}$, remove arc (i,j) from all train services of the network temporarily (only for the routing of this shipment).

Step 4.8. If at least one yard or arc is temporarily removed from the network in steps 4.6.2 and/or 4.7.2, go to Step 4.3, else go to Step 5.

Step 5. Add the temporarily removed yards and arcs to the network.

Step 6. If $\bar{v} = |V|$ stop, else let $\bar{v} \leftarrow \bar{v} + 1$ and go to Step 4.1.

Step 7. Calculate $\sum_{\bar{v}} \mathrm{CVaR}_\alpha^{r^*}(\bar{v}) = \sum_{\bar{v}} \left(C_{(r^*)}^{\bar{v}} + \bar{f}^{r^*}(\bar{v}) \right)$.

5.4 Computational Analysis

The proposed routing methodology was tested on a realistic infrastructure of a Class 1 railroad operator in the Midwest United States (Figure 5.1). This network has 25 yards each of which is an origin and destination for the others, i.e., at most 600 origin-destination pairs. However, we considered 560 different hazmat shipments

(O-D pairs) in the computational study, whose number of hazmat railcars to be shipped from origin to destination, $N(v)$, ranges in [1,15]. We also assume there is a total of 31 different train services available in the network, each of which is identified by origin and destination yards, intermediate stops, and service legs that connect them. All algorithms have been implemented in Java 1.9 and run on a 2.90 GHz Intel Core i7 PC with 8 GB of RAM.

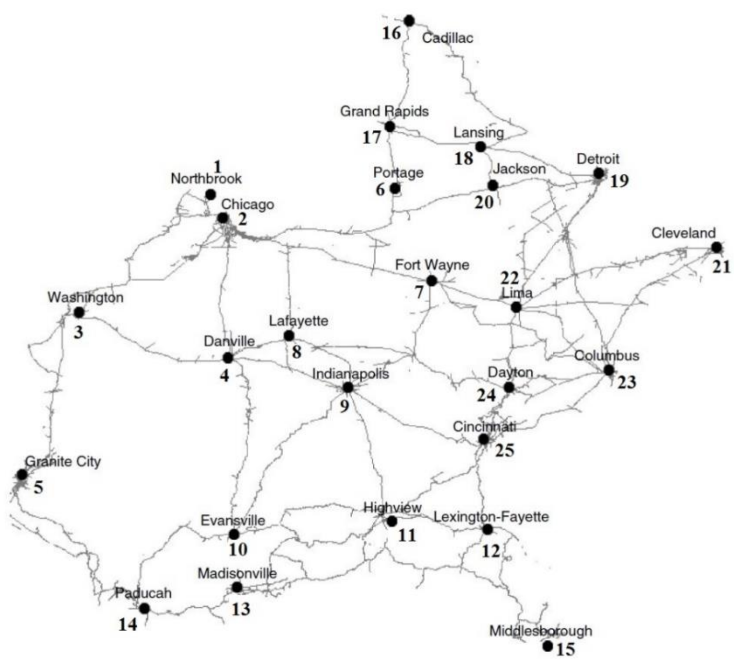


Figure 5.8. Railroad network in the Midwest United States [Source: Verma et al., (2011)]

As explained in Section 5.2.1, accident probabilities in yards and arcs, p_k^v and p_{ij}^v, depend on the number of hazmat railcars in shipment v, $N(v)$. In addition, p_{ij}^v is dependent on the length of the arc as well. They are computed as follows: $p_{ij}^v = \text{arc } (i,j)\text{'s length (mile)} \times 7.35 \times 10^{-11} \times N(v)$ and $p_k^v = 6.42 \times$

$10^{-10} \times N(v)$. To calculate accident consequences in yards and arcs, c_k^v and c_{ij}^v, ArcGIS (ESRI, 2007) was used to estimate population exposure which is utilized as the measure of consequence in this research. The techniques used to estimate these parameters are borrowed from other peer-reviewed works. For the sake of brevity, we do not repeat the details here, and refer the reader to Verma and Verter (2007), Verma (2011), and Hosseini and Verma (2017).

5.4.1. Role of Train Service Design

Our first intention was to wholly comply with the train services used in Verma et al. (2011), but we realized that the current design does not allow us to obtain adequate results by conducting the k-minimal CVaR paths algorithm. This derived us to re-design them for this computational study and also made us discern how important the role of train service design in ensuring risk equity is. By employing the current design of the train services, all shipments from yards 16 and 17 to other yards are forced to be transported only through service leg 16 -> 18 (see Figure 5.2). The total hazmat volume for these shipments is equal to 346 railcars, therefore the risk load on arc (16,18), i.e. the LHS of (5.10), is always greater than or equal to $(346) \times 161 \times 0.735 \times 10^{-10} \times 1940 = 0.007943$, where 161 is the length of arc (16,18) in miles, and 1940 shows the population exposure around this arc.

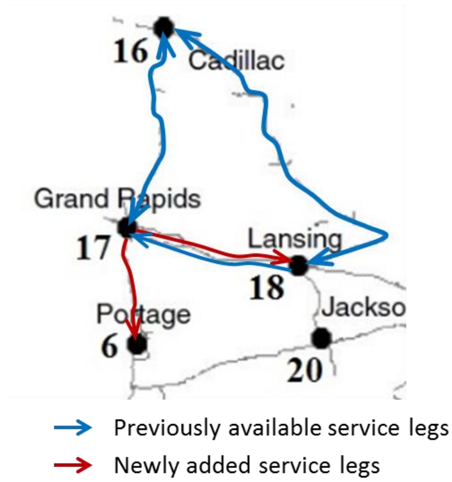


Figure 5.9. Re-designing the train services to ensure risk equity

Therefore, it is not possible to reduce the risk load on this overloaded arc (just by rerouting the shipments) unless new service legs are provided. In fact, when arc (16,18) is removed from the network by the k-minimal CVaR paths algorithm, no alternative service legs are provided by the train services to be used for routing shipments like (O = 17, D = 14) and (O = 16, D = 14), hence Dijkstra's shortest path algorithm becomes infeasible too soon and the algorithm stops. We resolved this issue by re-designing the two of current train services in the network, namely, train service {4} and {13}, and changed them from 2 -> 6 -> 8 -> 0 and 9 -> 7 -> 17 -> 16 to **17** -> 6 -> 8 -> 0 and 9 -> 7 -> 17 -> **18**, respectively. This way, two new service legs 17-> 6 and 17-> 18 will be added to the network (see Figure 5.2), which help to prevent overloading on arc (16,18) and consequently enhance risk equity in the whole network.

5.4.2. Risk Equity Analysis

In this section, we analyze how imposing risk equity constraints on transferring yard and arcs, namely, (5.9) and (5.10), respectively, affect optimal CVaR routing of the rail hazmat shipments throughout the network. To do so, we initially solve problem P without enforcing risk equity constraints, where the objective is just to do the routings in such a way that the total CVaR risk generated along the routes of all hazmat shipments is minimized. It turns out that the maximum risk loads occur on arc (2,6) and transferring yard 2 with the associated values of 242×10^{-5} and 21×10^{-5}, respectively. In the next step, we first enter the risk equity constraint of arcs, (5.10), into the model, then remove it and only enter the transferring yard risk equity, (5.9), into the model, and finally consider the whole problem P, i.e. with both risk equity constraints. The k-minimal CVaR paths algorithm is then applied to each of these three scenarios while δ_{ij} and δ_k values are set to gradually decrease from the maximum risk load numbers found above (when there are no risk equity constraints in the problem). Reducing δ values continues until the algorithm reports an error resulting from the infeasibility of Dijkstra's shortest path algorithm since no alternative route can be found after all overloaded arcs and/or transferring yards are removed from the network.

In the first scenario, where only the risk equity of arcs is considered in problem P, as the max risk allowed on the arcs declines gradually, the transferring yard risks increase. Figure 5.3 depicts how making the δ_{ij} values tighter causes the risk load at the riskiest transferring yards of the network to enlarge. It means that ensuring a lower level of risk on all the arcs of the network accompanies by the higher level of risk at the transferring yards. Figure 5.4 provides an example of step-by-step rerouting done by the k-minimal CVaR paths algorithm in order to finally find a route for the 468th shipment with O=14, D=1, and N=3, while the risk equity of 187×10^{-5} is met on all the arc of the network. In every step, the arcs which violated the risk equity constraint in the previous step and are removed for rerouting are shown by dotted lines. The transferring yards are enclosed by a rectangle and different colors are used to distinguish the various train services in the network. In addition, the CVaR along the generated route and the risk load on the arcs which exceed the risk equity are indicated in each step as well. It is intuitive to predict that, in every step, the optimal CVaR value of the route which will be generated in the next step, after that the overloaded arcs are removed from the network, will increase.

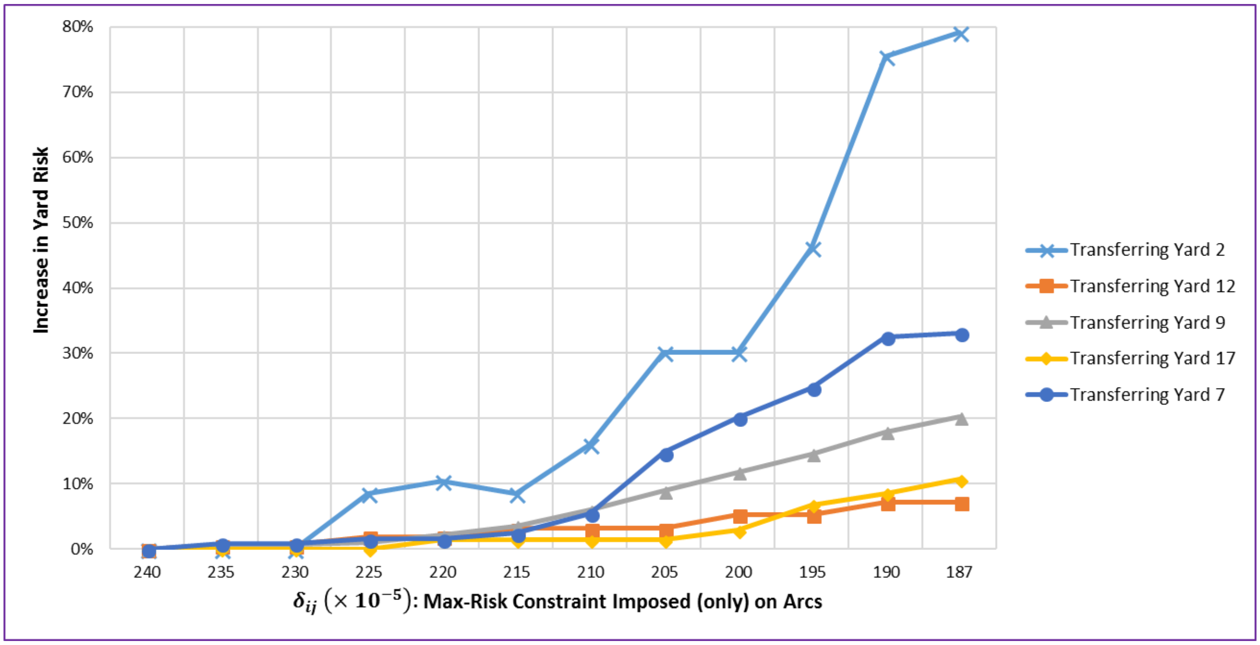
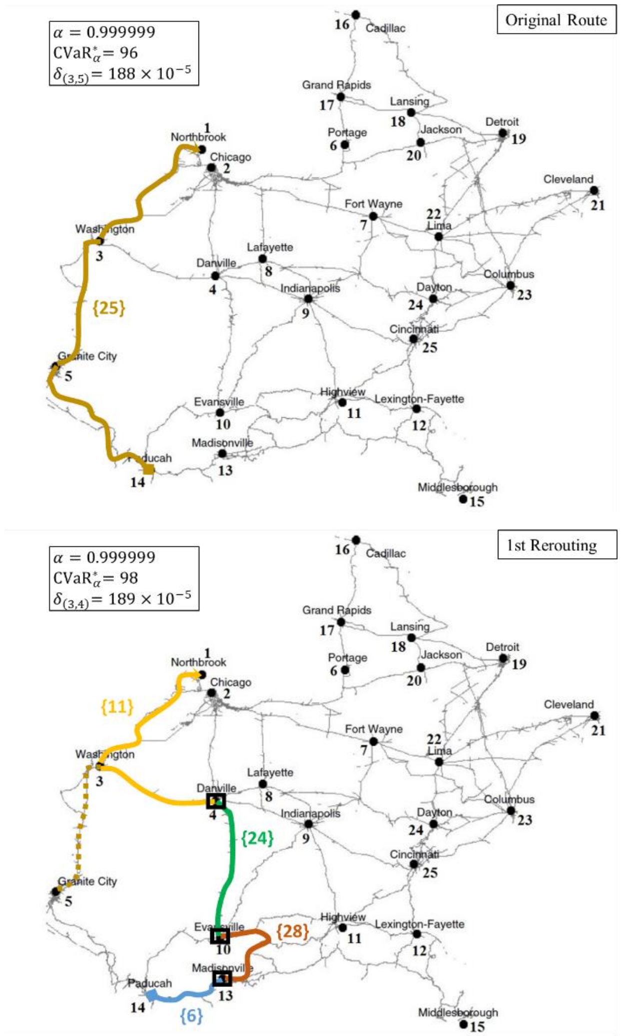


Figure 5.10. Increase in transferring yard risks as risk equity becomes tighter on arcs

Likewise, in the second scenario, we impose risk equity constraints only at the transferring yards (decrease in δ_k) and analyze its impact on the risk loads of the eight riskiest arcs of the network. The result is represented in Figure 5.5. The details of applying the k-minimal CVaR paths algorithm to the 547th shipment with O=15, D=18, and N=2, while the risk equity of 10×10^{-5} is guaranteed at all the transferring yards of the network is described in Figure 5.6. The transferring yards which violated the risk equity constraint in the previous steps and are removed for rerouting of this shipment are enclosed by dotted red lines. Note that a yard may be identified as an overloaded transferring yard and so not be used for the new routes of the shipment, but it can still be used as a transshipment yard (along a single train service) for the rerouting of the shipment since no transfer operation (between two train services) will be implemented at this point.

Finally, in the last scenario, we solve the whole problem P using the k-minimal CVaR paths algorithm, where both risk equity constraint on arcs and transferring yards are considered. Figure 5.7 demonstrates the steps have been taken by the algorithm for rerouting and consequently building a route for the 497th shipment with O=7, D=3, and N=2, while maximum risk load $\delta_{ij} = 220 \times 10^{-5}$ and $\delta_k = 16 \times 10^{-5}$ are imposed on all the arcs and transferring yards of the network, respectively. This time, the algorithm performs the rerouting in such a way that both overloaded arcs and transferring yards are avoided simultaneously. The number of shipments rerouted by the algorithm to ensure risk equity throughout the network in all the three scenarios are depicted in Figure 5.8. Furthermore, as discussed above, CVaR value along with the generated route for each hazmat shipment increases as the risk equity constraints on arcs and transferring yards become tighter. The reason is clear; the limitation imposed by the risk equity causes the shipments to undergo possible several re-routings. This means deviation from the original routes with the corresponding minimum CVaR values which could be achieved in the absence of risk equity constraints. Figure 5.9 elucidates how the total optimal CVaR value, gained by the summation of optimal CVaR values of all the shipments in the network (after the re-routings), rise by imposing the maximum risk enforcement on the arcs and transferring yards in all the three scenarios.


Original Route
$\alpha = 0.999999$
$\mathrm{CVaR}^*_\alpha = 96$
$\delta_{(3,5)} = 188 \times 10^{-5}$
{25}
1 Northbrook
Chicago 2
Washington 3
Danville 4
Granite City 5
Portage 6
Fort Wayne 7
Lafayette 8
Indianapolis 9
Evansville 10
Highview 11
Lexington-Fayette 12
Madisonville 13
Paducah 14
Middlesborough 15
16 Cadillac
Grand Rapids 17
Lansing 18
Detroit 19
Jackson 20
Cleveland 21
22 Lima
Columbus 23
Dayton 24
Cincinnati 25
1st Rerouting
$\alpha = 0.999999$
$\mathrm{CVaR}^*_\alpha = 98$
$\delta_{(3,4)} = 189 \times 10^{-5}$
{11}
{24}
{28}
{6}

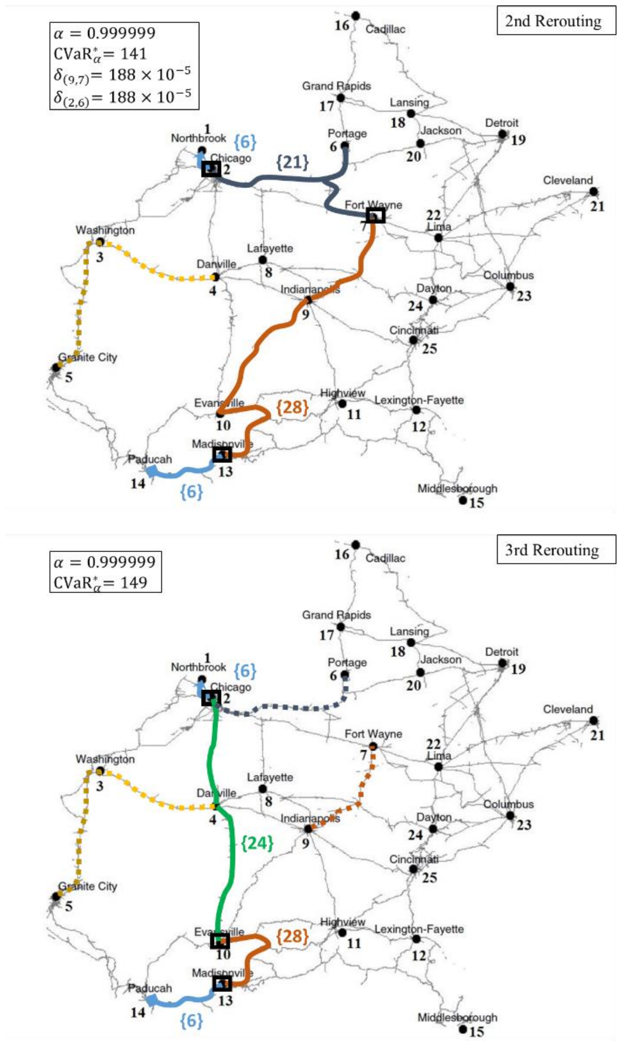


Figure 5.11. $\boldsymbol{k}$**-minimal CVaR paths determination for the 468th shipment by imposing risk equity constraint (only) on arcs:** $\boldsymbol{\delta_{ij} = 187 \times 10^{-5}}$

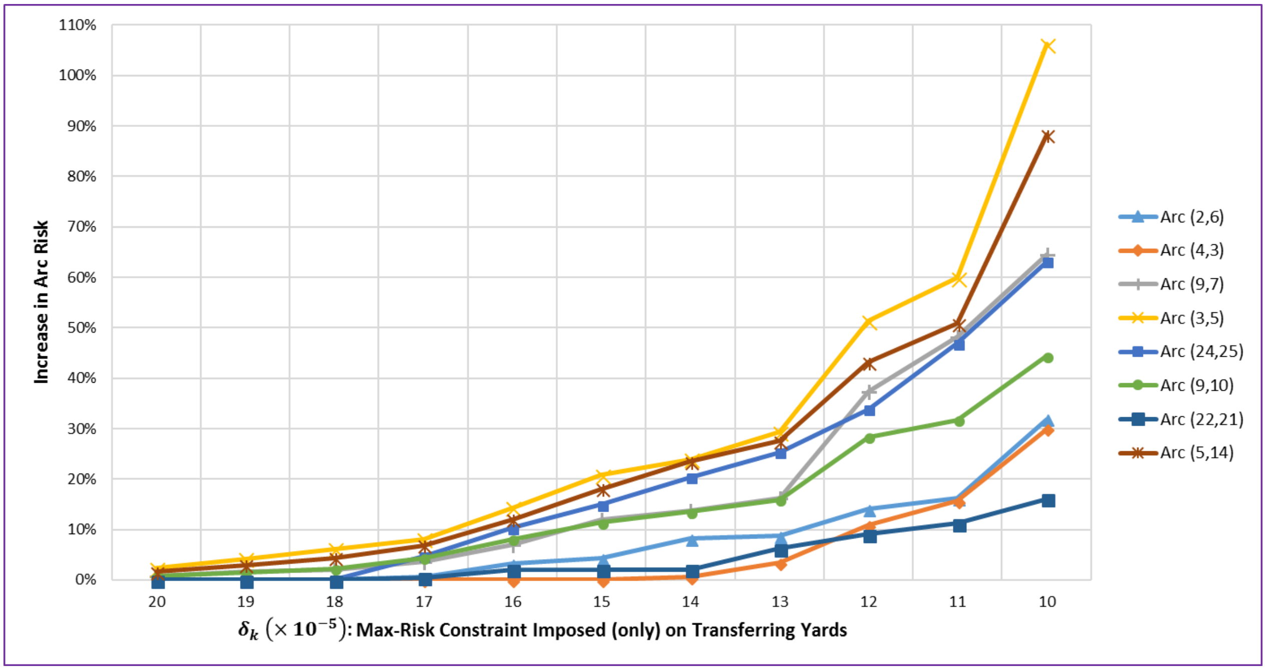
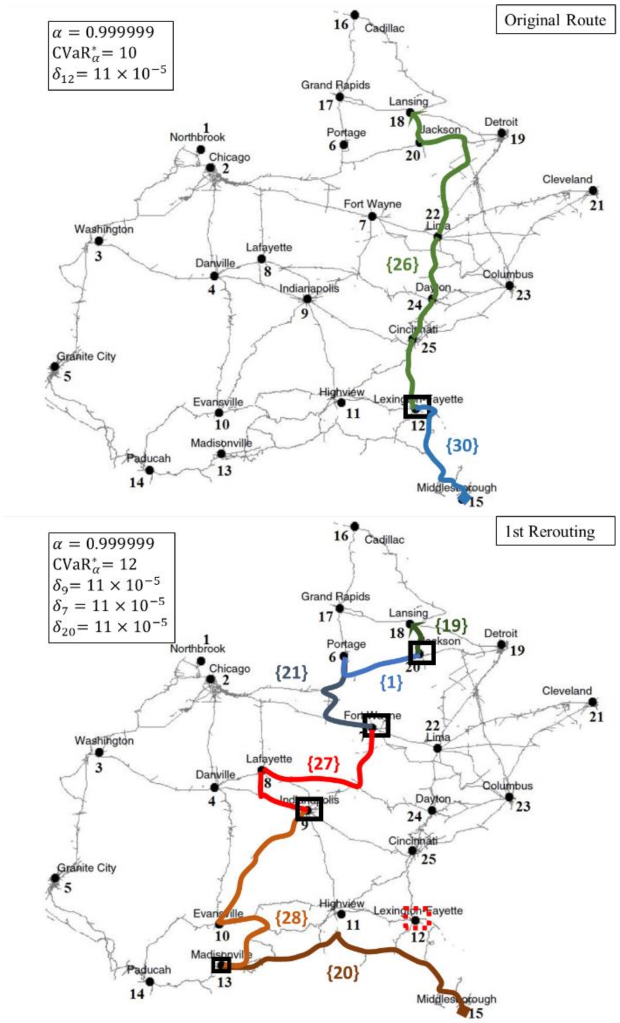
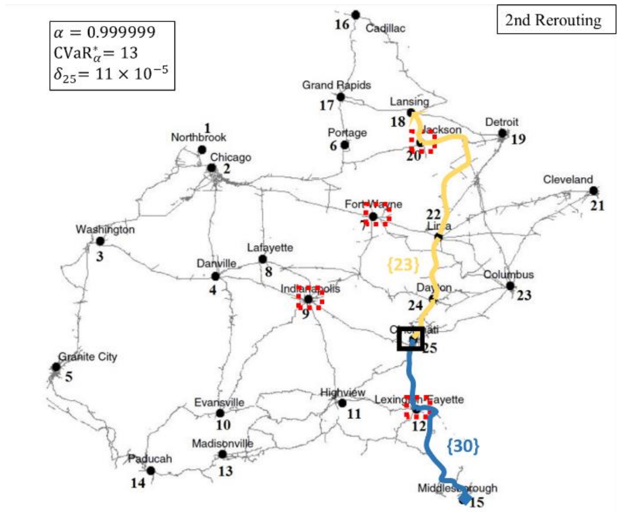
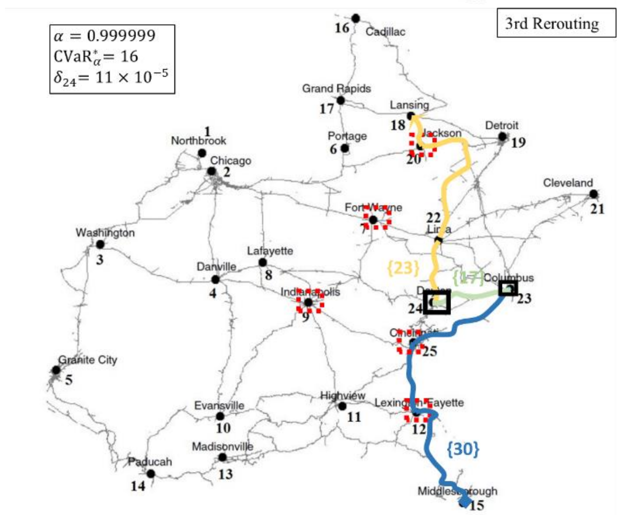


Figure 5.12. Increase in the risk load of arcs as risk equity becomes tighter on transferring yards



Original Route
$\alpha = 0.999999$
$\text{CVaR}_{\alpha}^{*} = 10$
$\delta_{12} = 11 \times 10^{-5}$
16 Cadillac
Grand Rapids 17
Lansing 18
Jackson 20
Detroit 19
Portage 6
Northbrook 1
Chicago 2
Cleveland 21
Fort Wayne 7
22 Lima
Washington 3
Lafayette 8
Danville 4
{26}
Columbus 23
Indianapolis 9
24
25
Granite City 5
Highview 11
Evansville 10
12
{30}
Madisonville 13
Paducah 14
15
1st Rerouting
$\alpha = 0.999999$
$\text{CVaR}_{\alpha}^{*} = 12$
$\delta_{9} = 11 \times 10^{-5}$
$\delta_{7} = 11 \times 10^{-5}$
$\delta_{20} = 11 \times 10^{-5}$
{19}
{21}
{1}
{27}
{28}
{20}


2nd Rerouting
$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 13$
$\delta_{25} = 11 \times 10^{-5}$
16 Cadillac
Grand Rapids
17
Lansing
18
Jackson
20
Detroit
19
1
Northbrook
Chicago
2
Portage
6
Cleveland
21
Fort Wayne
7
22
Lima
Washington
3
Lafayette
8
Danville
4
{23}
Columbus
23
Indianapolis
9
Dayton
24
25
Granite City
5
Highview
11
Lexington Fayette
12
Evansville
10
{30}
Madisonville
13
Paducah
14
Middlesborough
15


3rd Rerouting
$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 16$
$\delta_{24} = 11 \times 10^{-5}$
16 Cadillac
Grand Rapids
17
Lansing
18
Jackson
20
Detroit
19
1
Northbrook
Chicago
2
Portage
6
Cleveland
21
Fort Wayne
7
22
Lima
Washington
3
Lafayette
8
Danville
4
{23}
{17}
Columbus
23
Indianapolis
9
24
25
Granite City
5
Highview
11
Lexington Fayette
12
Evansville
10
{30}
Madisonville
13
Paducah
14
Middlesborough
15

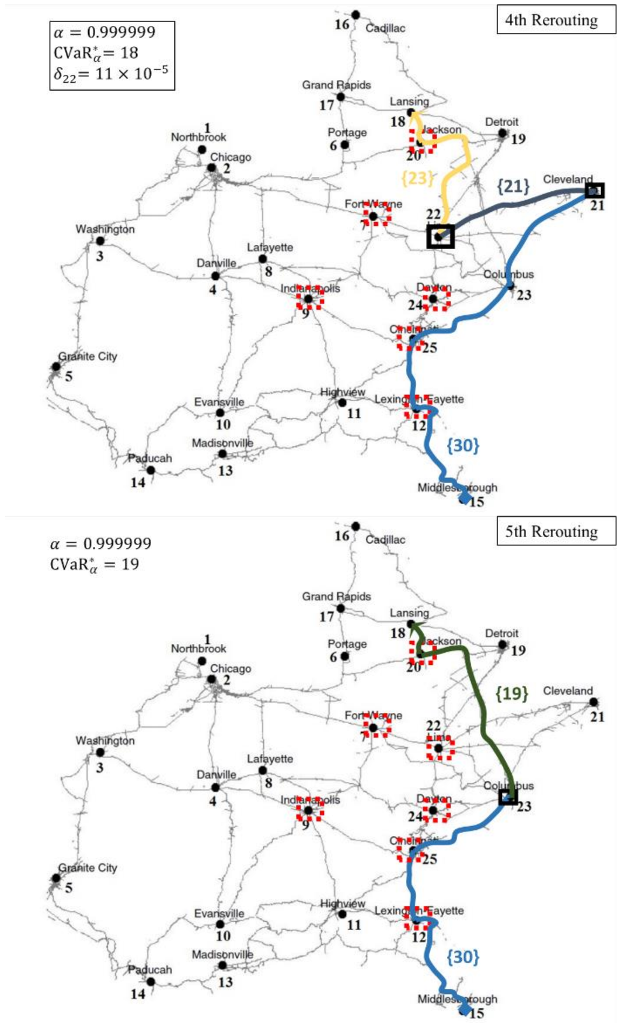
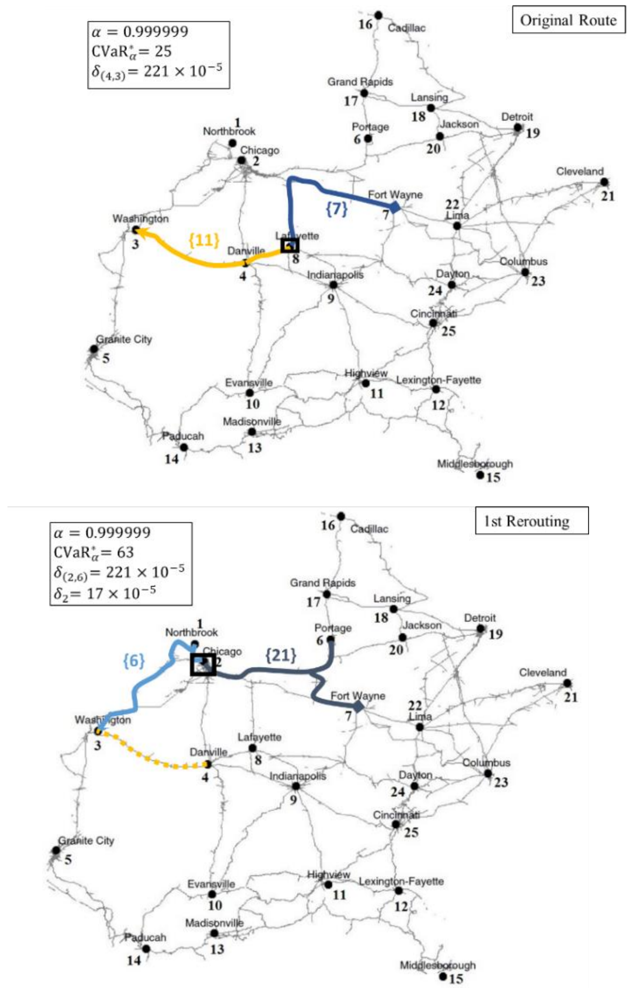
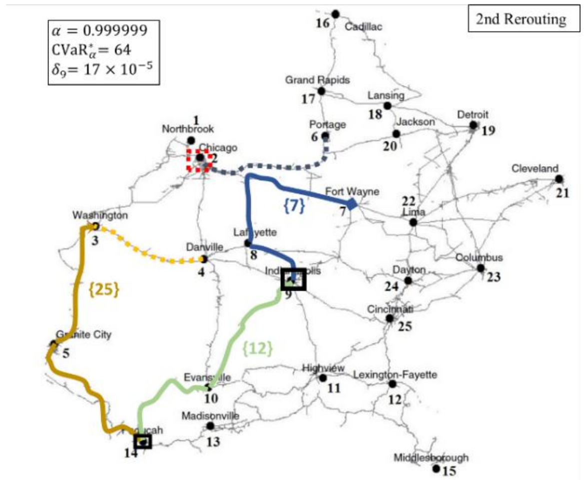


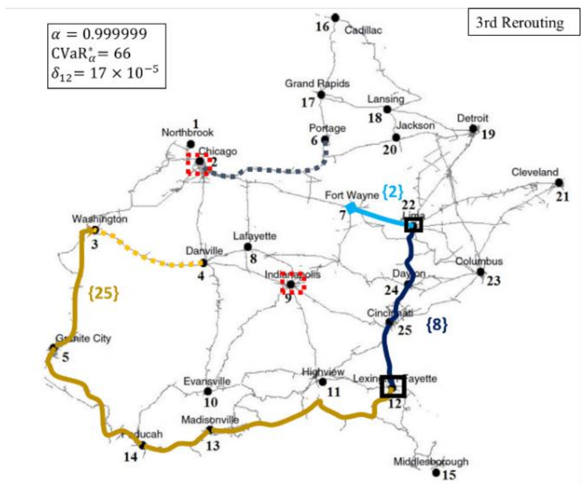
Figure 5.13. k-minimal CVaR paths determination for the 547th shipment by imposing risk equity constraint (only) on transferring yards: $\delta_k = 10 \times 10^{-5}$



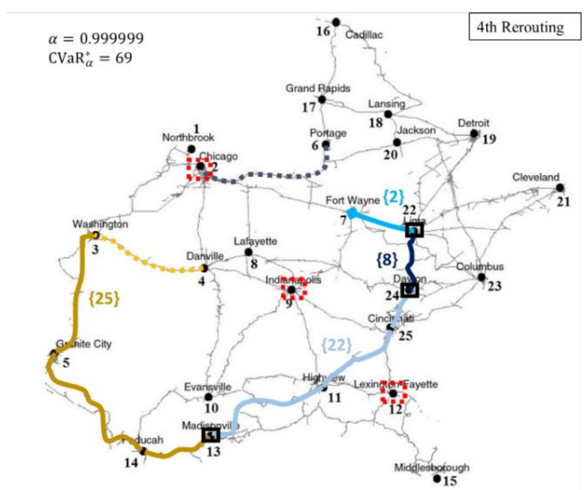
Original Route
$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 25$
$\delta_{(4,3)} = 221 \times 10^{-5}$
{7}
{11}
1st Rerouting
$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 63$
$\delta_{(2,6)} = 221 \times 10^{-5}$
$\delta_{2} = 17 \times 10^{-5}$
{6}
{21}
1 Northbrook
2 Chicago
3 Washington
4 Danville
5 Granite City
6 Portage
7 Fort Wayne
8 Lafayette
9 Indianapolis
10 Evansville
11 Highview
12 Lexington-Fayette
13 Madisonville
14 Paducah
15 Middlesborough
16 Cadillac
17 Grand Rapids
18 Lansing
19 Detroit
20 Jackson
21 Cleveland
22 Lima
23 Columbus
24 Dayton
25 Cincinnati



$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 64$
$\delta_9 = 17 \times 10^{-5}$
2nd Rerouting
16 Cadillac
Grand Rapids 17
Lansing 18
Portage 6
Jackson 20
Detroit 19
Northbrook 1
Chicago 2
Cleveland 21
Fort Wayne 7
22 Lima
{7}
Washington 3
Lafayette 8
Danville 4
Columbus 23
Dayton 24
9
{25}
Cincinnati 25
Granite City 5
{12}
Highview 11
Lexington-Fayette 12
10
Madisonville 13
14
Middlesborough 15

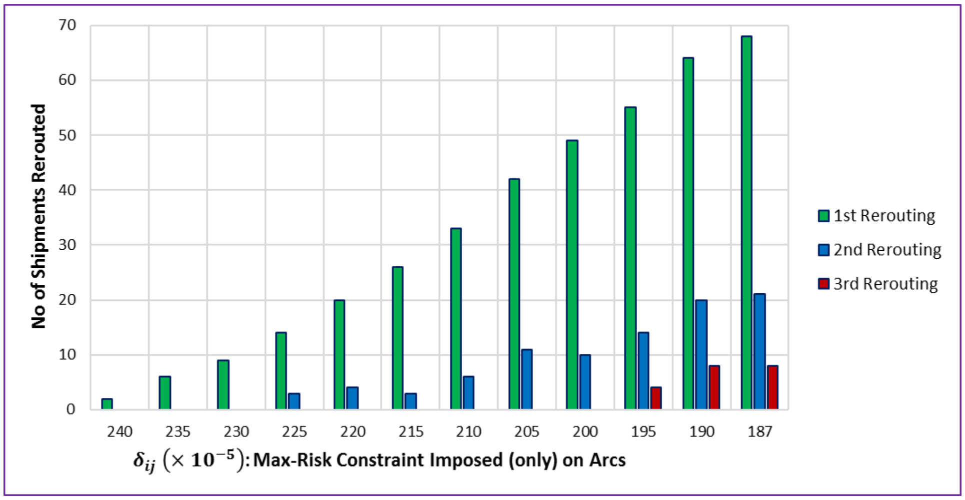


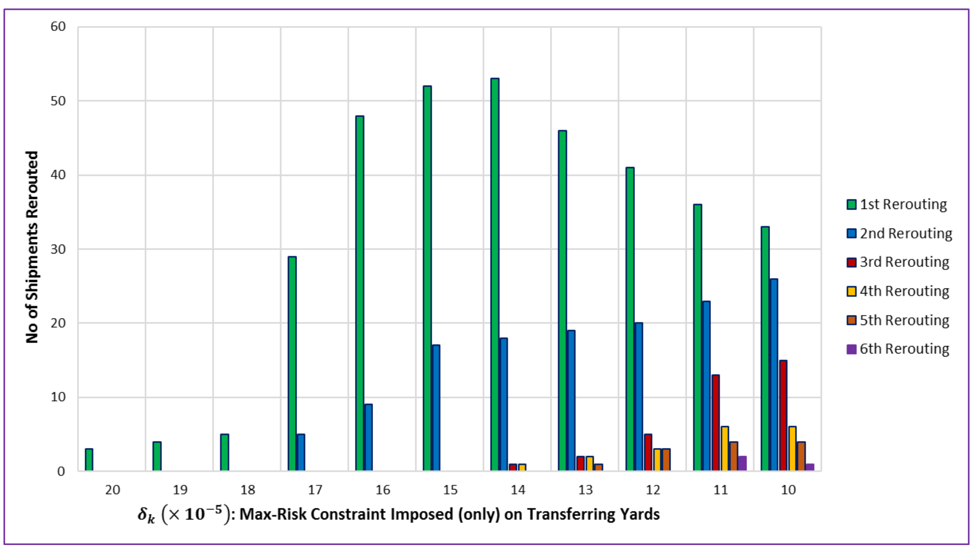
$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 66$
$\delta_{12} = 17 \times 10^{-5}$
3rd Rerouting
16 Cadillac
Grand Rapids 17
Lansing 18
Portage 6
Jackson 20
Detroit 19
Northbrook 1
Chicago 2
Cleveland 21
Fort Wayne {2}
7
22
Washington 3
Lafayette 8
Danville 4
Columbus 23
24
9
{25}
25
{8}
Granite City 5
Evansville 10
Highview 11
12
Madisonville 13
14
Middlesborough 15



$\alpha = 0.999999$
$\mathrm{CVaR}_{\alpha}^{*} = 69$
4th Rerouting
16 Cadillac
Grand Rapids 17
Lansing 18
Portage 6
Jackson 20
Detroit 19
Northbrook 1
Chicago 2
Cleveland 21
Fort Wayne {2}
7
22
Washington 3
Lafayette 8
Danville 4
{8}
Columbus 23
24
9
{25}
25
{22}
Granite City 5
Evansville 10
11
12
13
14
Middlesborough 15

Figure 5.14. k-minimal CVaR paths determination for the 497th shipment by imposing risk equity constraint on arcs and transferring yards: $\delta_{ij} = 220 \times 10^{-5}$ and $\delta_k = 16 \times 10^{-5}$, respectively





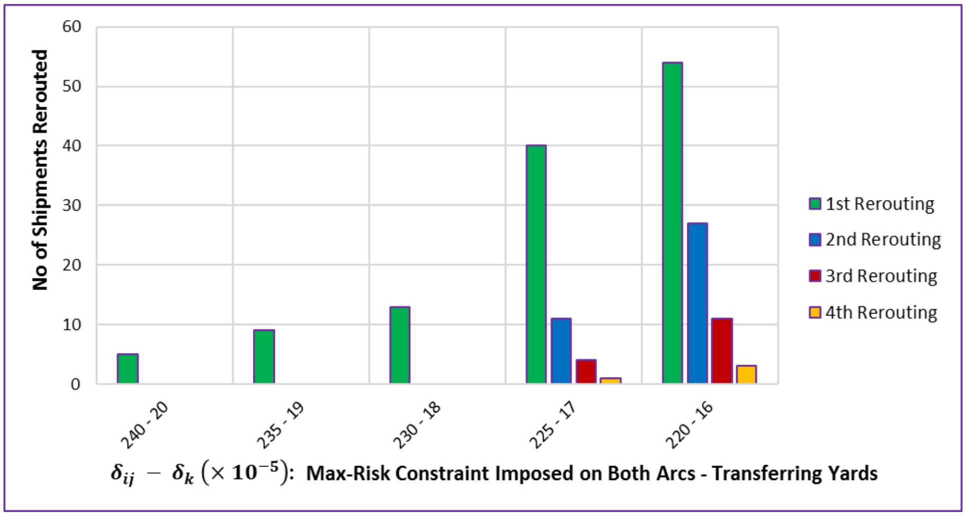


Figure 5.15. No of shipments rerouted by the *k*-minimal CVaR paths algorithm to ensure risk equity constraints

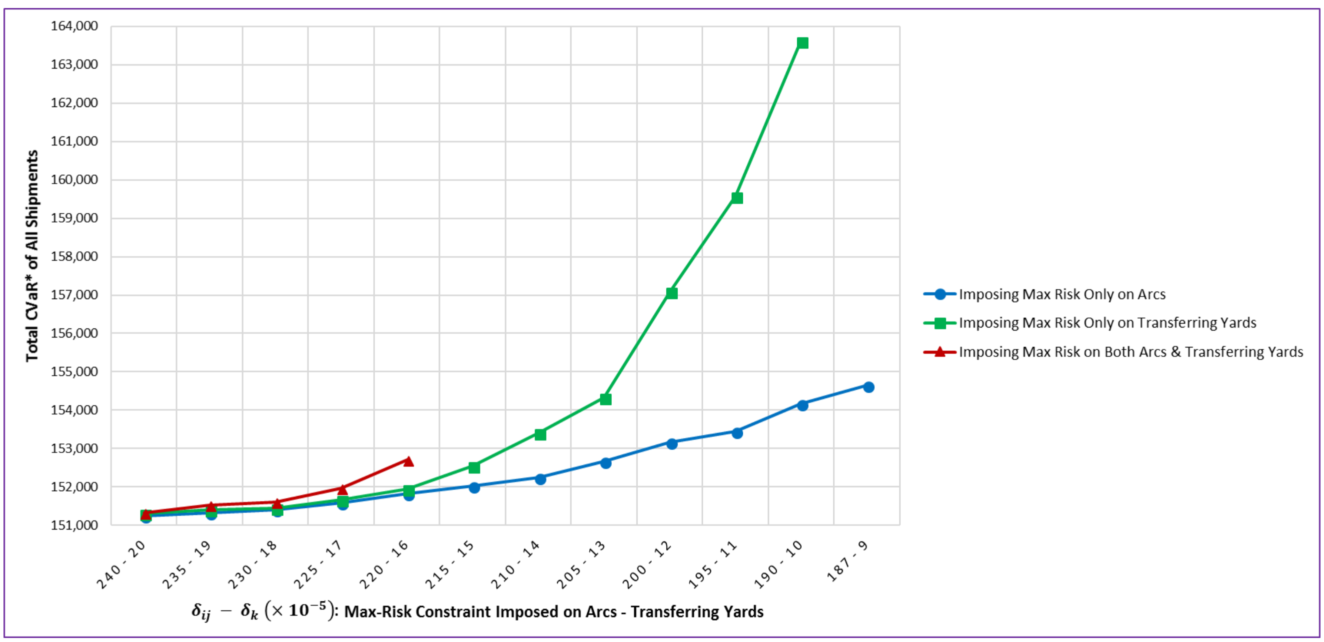


Figure 5.16. Increase in the total optimal CVaR value by imposing risk equity constraints

5.4.3. Optimality Evaluation of Solution Values

In this section, we attempt to solve the Lagrangian dual problem $\max\{L(u) \mid u \geq 0\}$ and so achieve an effective lower bound on the optimal solution values for the scenarios studied above. To do so, we plug the values of $\sum_{\bar{v}} \mathrm{CVaR}_{\alpha}^{r^*}(\bar{v})$, which are gained by the k-minimal CVaR paths algorithm and used to build the above discussions in Sections 5.4.1 and 5.4.2, into the Subgradient algorithm, where at each step, they are used as L^{ub}, an upper bound on the optimal value $L(u)$, in the step size t_q calculation. At each iteration q of the Subgradient algorithm, we first calculate $L(u^q)$ using the Lagrangian function algorithm, which is then utilized to compute the subgradients γ^q. Consequently, we update Lagrangian multipliers u^q by means of the step size $t_q = \frac{\theta_q\left(L^{ub} - L(u^q)\right)}{\|\gamma^q\|^2}$. Note that at iteration $q = 0$, we start with all the multipliers $u_k^0, \forall k$ and $u_{ij}^0, \forall (i,j)$ being equal to zero.

It turns out that $L(u)$ fails to increase for some fixed number of iterations for all the scenarios. An example is provided in Figure 5.10 for the case in which we impose the maximum risk of 240×10^{-5} only on the arcs and get $\sum_{\bar{v}} \mathrm{CVaR}_{\alpha}^{r^*}(\bar{v}) = 151{,}244.00$. In Table 5.1, we compare the solution values $\sum_{\bar{v}} \mathrm{CVaR}_{\alpha}^{r^*}(\bar{v})$ (i.e. the values of total CVaR for all the shipments in the network) gained by the k-minimal CVaR paths algorithm for all the different scenarios with the Lagrangian lower bound *LB*. It lists the values of the relative *gap* (in percentage) of the solution values with respect to the lower bound, i.e. $\left(\left(\sum_{\bar{v}} \mathrm{CVaR}_{\alpha}^{r^*}(\bar{v}) - LB\right)/LB\right)\%$. Comparing to the lower bound, we conclude that the quality of the solution values is high with an average relative gap of 1.40% and a maximum value of 8.19%.

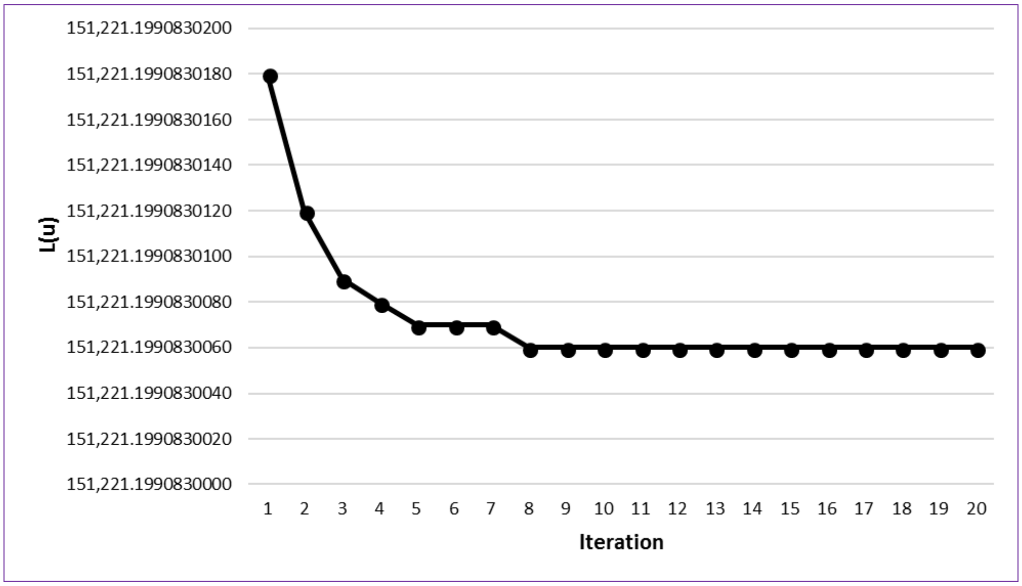


Figure 5.17. Change in Lagrangian function by iterating Subgradient algorithm for risk equity $\delta_{ij} = 240 \times 10^{-5}$

Max-Risk Imposed Only on Arcs $\boldsymbol{\delta_{ij}}$ $(\times \mathbf{10^{-5}})$	**Gap**	**Max-Risk Imposed Only on Yards** $\boldsymbol{\delta_k}$ $(\times \mathbf{10^{-5}})$	**Gap**	**Max-Risk Imposed on Both Arcs and Yards** $\boldsymbol{\delta_{ij}}$ & $\boldsymbol{\delta_k}$ $(\times \mathbf{10^{-5}})$	**Gap**
235	0.08%	19	0.13%	235 & 19	0.20%
230	0.12%	18	0.15%	230 & 18	0.26%
225	0.24%	17	0.30%	225 & 17	0.50%
220	0.40%	16	0.48%	220 & 16	0.98%
215	0.53%	15	0.88%		
210	0.68%	14	1.45%		
205	0.95%	13	2.05%		
200	1.29%	2	3.88%		
195	1.47%	11	5.52%		
190	1.94%	10	8.19%		
187	2.26%				

Table 5.15. $\boldsymbol{k}$-minimal CVaR paths algorithm gap from the Lagrangian lower bound

5.5 Conclusion

In this chapter, we proposed a CVaR-based routing plan with risk equity considerations for multiple hazmat shipments each of with is specified by origin-destination pair and the number of hazmat railcars to be shipped throughout the railroad network. The primary objective is to find the best route for each rail shipment with the aim of minimizing the summation of the risks generated along the routes as measured by CVaR. The maximum risk limitations, however, have been also added to the model in order to prevent particular arcs and transferring yards of the network being too much overloaded. A k-minimal CVaR paths algorithm has been developed and experimentally evaluated on a US railroad network, the efficiency and effectiveness of which is also demonstrated by the lower bound provided by a Lagrangian method.

As described, the itineraries of the train services used in the network to transport the hazmat shipments play an important role in reducing the total risk throughout the network. We assumed that the train services are predefined in this chapter, but an interesting extension to this work is to study the problem from a more strategic perspective where minimizing the total CVaR risk and risk equity consideration are taken into account beside other affecting factors when the train services are designed.

In addition to the safe and equitable route selection, there is another issue in the context of multiple rail hazmat shipments that deserves adequate consideration. When there are many rail hazmat shipments in the network, which are usually executed simultaneously, the safe scheduling of all these shipments becomes critical. One way to address this issue is to integrate routing and scheduling decisions, although it increases the complexity of the model dramatically. For example, a safe hazmat shipment scheduling should avoid doing a transfer operation at a yard when the risk is high because of another transfer operation being carried out in the yard at the same time.

Another direction for future research is to relax this assumption in the chapter that all arc and yard attributes in the network including the accident probabilities and consequences are known. Robust optimization methods can be employed to help provide us with routing strategies which are less sensitive to changes in arc and yard attributes and also are robust to the inaccuracy of data.

Chapter 6. Conclusion and Future Research

Railroad is one of the safest modes for transporting hazmat, although the possibility of catastrophic events because of the involvement of multiple railcars, however small, does exist. Low probability-high consequence nature of rail hazmat incidents necessitates the development of a risk-averse routing methodology for routing rail hazmat shipments, while the efficacy of the risk assessment methodologies developed in the last two decades has been limited, since they either are risk neutral, hence unable to prevent high consequence events, or yield a single route between a given origin-destination pair, regardless of the risk preference of decision makers.

To fill this gap in the literature, we made the first attempt to develop a Value-at-Risk (VaR) assessment methodology to facilitate risk-averse and flexible routing of rail hazmat shipments. The proposed VaR methodology was then used to study 232 problem instances generated using the realistic infrastructure of a railroad operator in Midwest United States. The further analysis showed that by focusing on the tail of the distribution, VaR is a more suitable measure of hazmat risk than the three most popular measures (i.e. traditional risk (TR), incident probability (IP), and population exposure (PE)), although it still cuts off and thus ignores the adverse tail of the risk distribution, i.e. the region where catastrophic outcomes reside. This crucial limitation plus optimal VaR value of zero, which implies that complete risk measure for the corresponding route could not be generated, does create an incentive to search for alternative risk measures that could quantify the affected populace beyond the threshold VaR.

One such alternate measure is conditional value-at-risk (CVaR) which is an extension of VaR. We have made the first attempt to propose a CVaR-based risk assessment methodology for rail hazmat shipments. The methodology development entailed: providing a clear definition of CVaR for hazmat shipments, which was missing in the existing literature; delineating several relevant properties; and, an optimization program. The validation of the proposed methodology was accomplished by applying it to study several realistic size problem instances, which were further analyzed to conclude: *first*, CVaR is both distinct and superior to both TR and VaR measures; *second*, CVaR provides risk-averse routing of hazmat shipments; and *third*, CVaR generates different optimal routes based on the confidence-level of the decision makers.

The analyses conducted on the case study, i.e. the realistic infrastructure of a railroad operator in Midwest United States, show that for a given origin-destination pair, both optimal VaR and CVaR values have a positive relationship with both the hazmat volume, and the confidence level (or risk-preference) of the decision maker. Also, at higher confidence levels (i.e., risk-averse), safer but longer VaR and CVaR routes that make use of the service legs and yards at the periphery of the network are generated. More specifically, as confidence level goes from the theoretical minimum value of zero to the theoretical maximum value of one, the decision maker moves away from being risk-neutral and towards becoming risk-averse. Finally, for a given route, longer trains would result in lower VaR and CVaR values because of the potential to exploit the decile-based configurations more appropriately.

It is important that developing such routing plans using VaR and CVaR methodologies in a railroad setting is more complex than in highway transportation because of three reasons: *first*, the characteristics of railroad operations need to be taken into consideration; *second*, one could only work with the given set of pre-defined train services, which would entail transfer operations at yards and thus the corresponding risk; and *third*, decision about optimal train configuration should also be taken into consideration.

While in Chapters 3 and 4, the problem focuses on a single shipment and a single origin-destination hazmat routing plan, Chapter 5 involves multiple shipments and multiple origin–destination hazmat routing decisions. We developed an optimization framework to minimize the total CVaR throughout the whole network, which takes into account risk equity as well, i.e. dispersing hazmat traffic flows to reduce the accumulative consequences brought by busy rail links and transferring yards. This way, not only the total risk over the population is controlled, but also the equity distribution of this risk over population zones, which can only be achieved by simultaneously considering all of the local route planning problems, i.e. through global route planning, is guaranteed as well. An effective algorithm has been then developed to determine the k-minimal CVaR paths by conducting several re-routings for each shipment until all maximum risk limitations are satisfied. The superior quality of the proposed algorithm has been demonstrated by comparing its results with the lower bound produced by the Lagrangian and subgradient methods. Finally, the same network utilized in the previous two chapters has been studied to provide several experimental analyses.

This work can be expanded in a significant direction towards a more realistic multi-trip framework: One could develop a bi-objective optimization framework which takes into account both risk and cost. This aspect leads to a harder class of problem that can address the interests of two stakeholders in rail hazmat transportation, i.e. regulatory agencies and railroad companies. The proposed framework would contain both railroad blocking problem, with the objective of minimizing classification and flow cost, in order to take into consideration the perspective of the railroad company, and CVaR risk methodology, to control the risk induced by rail hazmat transportation over the population and the environment, which is the primary concern for the regulator. This way, significant managerial insights with regard to trade-offs between cost and risk could be gained. In fact, the main aim would be to develop a Pareto frontier, which could be used by the two primary stakeholders to make judicious decisions in an effort to achieve significant reductions in population exposure (or avoid catastrophe) without incurring unacceptable increases in operational costs.

There are several other directions for future research. The proposed VaR and CVaR approaches assume a static setting, when in fact, a time-dependent transportation network would be more realistic. Both the accident probabilities and consequences associated with the service legs and rail-yards would vary based on a pre-defined time period such a day, week or month. Hence, an interesting extension could be the adaptation of the proposed approaches within a dynamic or stochastic setting. In addition to the dependence on time, both accident probabilities and consequences could have some inherent imprecision, which may impact the analysis. Hence, one other related area of research could involve applying a robust optimization technique to overcome data uncertainty when preparing risk-averse routing plans using VaR and CVaR.

Another future research direction includes applying the proposed CVaR risk measure in the tactical planning problem for railroad transportation of hazmat. The problem is to determine the number of trains of different types needed in the network, the number and makeup of each type of train service, and the itineraries for each shipment such that the transport cost and the transport risk are minimized for the given set of demand for mixed freight. The application of the proposed CVaR methodology can also be built on rail–truck intermodal transportation (RTIM), which exploits the positive attributes of both trains and trucks. The problem then would be to determine the best shipment plan for both hazardous and non-hazardous freight in an RTIM network, wherein a set of pre-defined lead times must be satisfied in choosing the truck routes and the intermodal train services to be used. The objective is to minimize the total cost as well as the total CVaR risk associated with intermodal hazmat shipments.

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