MIXING STUDIES ON A FULL SCALE AERATION TANK

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AERATION TANK

By

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SCOPE AND CONTENTS:

The dispersion model was used to study mixing levels in a full scale aeration tank. The effect of air flow rate, water flow rate and diffuser type was investigated. The peak time technique proved satisfactory in predicting the theoretical tracer response curve generated using the dispersion model.

The dispersion model adequately described the longitudinal mixing that occurred in a full scale aeration tank equipped with fine and coarse bubble air diffusers.

Response curves from two tanks-in-series were also obtained.

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TABLE OF CONTENTS

TABLE OF CONTENTS (Continued) Page

INTRODUCTION

The activated sludge processes comprise a variety of biological treatment systems. These processes all feature a reactor vessel, or series of vessels, commonly known as an aeration tank. Here, suspended, colloidal and dissolved organic waste material is subjected to intimate contact with an optimum quantity of active sludge organisms in an aerobic environment. After an appropriate reaction time, the clarified reactor effluent should have considerably lower organic content than the influent waste water.

For the satisfactory design of the reaction vessel, the kinetics of the removal reaction must be known. However, with the kinetic information only, prediction of the performance of such a design is restricted to one of two idealized flow systems, plug flow or backmix (completely mixed) flow. If the reactor design closely approximates one of these ideals, then it may be considered, with negligible error, that the system has an idealized flow pattern. In some cases, deviation from ideality may be considerable thus requiring knowledge about the flow characteristics of the reactor contents, e.g. the extent of longitudinal mixing within the reactor. The residence time distribution of fluid in a reactor may be used to estimate these characteristics. For a first order reaction, the exact performance of the vessel as a reactor may be predicted only if the degree of longitudinal mixing can be quantitatively determined. For reactions independent of substrate concentration, i.e. zero order, the level of mixing has no effect on the degree of conversion. For higher order reactions, only the limits of conversion may be determined when the degree of longitudinal mixing is known.

Previous work by Timpany (1) indicated that the dispersed plug flow (dispersion) model adequately described the longitudinal mixing conditions of a laboratory model spiral flow aeration tank. The dispersion model is based on a backmixing equation analogous to Fick's second law of molecular diffusion. The solution of this equation for a pulse tracer input gives a family of response curves with the intensity of dispersion as the parameter. A characteristic feature, the peak time, of the experimental response curve was used to predict the theoretical response curve generated from the dispersion model.

The present study was carried out in order to determine if the dispersion model adequately described mixing conditions in a full-scale aeration tank. The applicability of the model for use in predicting aeration tank performance was evaluated through a comparison of theoretical and experimental response curves for several conditions of varying air and water flow rates. In addition, the effect of the diffuser type on the mixing level was investigated. The variation in the response curve due to the partitioning the test tank into two equal sized tanks-in-series was also investigated.

Currently, the concept of a completely mixed aeration system for biological wastewater treatment systems is enjoying immense popularity. Frequently, such a system is claimed for a particular design; yet quantitative knowledge as to the degree of mixing within the aeration system is not ascertained.

This study attempted to quantitatively determine the degree of longitudinal mixing taking place in aeration tanks of length to width ratios larger than those traditionally encountered. Another object of the study was to reveal whether, for given operating conditions, neglect of the longitudinal dispersion coefficient will lead to a significant error in calculating the extent of conversion of the reactor.

THEORETICAL DISCUSSION

In an attempt to characterize nonideal flow patterns within reactor vessels, several model types have been proposed. "Mixed models" have been used to describe a system (reactor) that can be visualized as being subdivided into various flow regimes connected in series and/or parallel. Such a model has a finite number of stages, e.g. stirred tank, plug flow, stagnant volume, recycle flow, by-pass flow and cross-flow. Some authors, Milbury _et al_ (1965), Chollette _et al_ (1960), and Silveston (1966), regard the multi-model approach as satisfactory for describing nonideal flow systems. While the mixed model can be used in predicting conversions of higher order reactions more readily if the order of components is known, it is difficult to visualize a model that has little if any resemblance to the actual flow situation within the reactor. Response studies do not indicate the sequence of components of the mixed model. For non-linear kinetics, this sequence of components, or time schedule of mixing, affects reactor performance.

The results of extensive studies concerning reactor flow characteristics have been published in the chemical engineering literature. Several authors, Danckwerts (1953), Levenspiel (1957) (1962), Bischoff (1962), and Van der Laan (1958), consider that longitudinal mixing in reactors may be described as a diffusion process. The "Dispersion model" (dispersed plug flow model) draws an analogy between mixing in actual flow and molecular diffusion. Hence, backmixing of fluid flowing in the x-direction may be represented by an equation similar to Fick's second law of molecular diffusion. With a uniform intensity of backmixing, the equation is:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \qquad (1)$$

where the parameter D is the longitudinal dispersion coefficient, uniquely characterizing the degree of backmixing during flow. The development of this approach has previously been considered to be limited to flow through tubular and packed bed reactors. In such considerations, the term "dispersion" includes molecular as well as turbulent or eddy diffusion. The term "longitudinal" is used to distinguish mixing in this direction from mixing in the radial direction, which is not considered. In the application of the dispersion model, lateral dispersion is considered to be great enough to ensure a uniform tracer concentration at any given cross section. The advantages of a one-parameter (D) model are considerable, yet Silveston (1966) considered that the one-parameter model is difficult to fit to experimental data.

A mass balance over an elemental cross section of tank yields the following:

$$D \frac{\partial^2 C}{\partial x^2} - u \frac{\partial C}{\partial x} - \frac{\partial C}{\partial t} = 0 \qquad (2)$$

where: D = Longitudinal dispersion coefficient (ft^2/hr),
C = concentration of tracer (mg/l),
x = distance along tank length (ft),
u = bulk velocity along tank length (ft/hr), and
t = time.

The solution of this second order differential equation for a tracer pulse input to a closed vessel is then required. Timpany (1966) presented the solution for a pulse input response curve ("C" curve), for a reactor with fixed end boundaries, as determined by both Miyauchi (1953) and Thomas and McKee (1944). This specific solution is:

$$\frac{C}{C_o} = 2 \sum_{n=1}^{\infty} \frac{\mu_n (U \sin \mu_n + \mu_n \cos \mu_n)}{(U^2 + 2U + \mu_n^2)} \exp\left[U - \frac{U^2 + \mu_n^2}{2U} \theta\right] \qquad (3)$$

where: C = exit concentration of tracer at time t (mg/l),

C_o = mass of tracer added divided by nominal tank volume (mg/l),

$U = \frac{uL}{2D}$,

L = tank length (ft),

$\mu_n = \cot^{-1}\left[\frac{1}{2} \left(\frac{\mu_n}{U} - \frac{U}{\mu_n}\right)\right]$, and

θ = time divided by the theoretical detention time calculated from tank volume and flow rate.

This expression may be used to generate a family of curves with D/uL, the dispersion number, as the parameter.

The applicability of a particular model in predicting residence time distributions may be confirmed by comparing the experimental tracer response curve to the theoretical response curve. This theoretical curve is generated using a characteristic feature of the experimental curve. Previously, the variance of the experimental response curve was used to determine the value of the dispersion number, D/uL. Van der Laan (1958) has shown that:

$$\sigma^2 = 2\left(\frac{D}{uL}\right) - 2\left(\frac{D}{uL}\right)^2 \left(1-\exp\left[-\frac{uL}{D}\right]\right) \qquad (4)$$

where: σ^2 = variance or second moment of a curve about its mean.

Timpany (1966) found that, with high degrees of dispersion, the variance technique was inadequate as values of C_o/4000, and less, had to be accurately determined using common laboratory equipment. In addition, tests would have to be continued for at least ten detention times to accurately calculate the variance of the response curve; the long tail becomes dominant in calculating the second moment of the curve. For the long detention times involved in full scale testing (4 to 15 hr), an extensive test program would be difficult to carry out.

In analyzing experimental response curves obtained, Timpany (1966) used the time of peak exit concentration to predict the D/uL value for the theoretical dispersion model. Equation (3) was differentiated, set equal to zero, and solved for the θ, or $t_p/\bar{t}$, value associated with each D/uL. This peak time technique was found to provide an adequate fit of the theoretical to the experimental response curve. The following approximate empirical relationships as determined by Timpany (1966) were used:

$$\frac{D}{uL} = 0.2\left(\frac{t_p}{\bar{t}}\right)^{-1.34} \quad \text{for } 0.03 < \frac{t_p}{\bar{t}} < 0.3 \qquad (5)$$

and $$\frac{D}{uL} = 4.027(10)^{-2.09(t_p/\bar{t})} \quad \text{for } 0.3 < \frac{t_p}{\bar{t}} < 0.8 \qquad (6)$$

where: t_p = time of peak exit concentration (hr), and

$\bar{t}$ = theoretical detention time (hr).

Timpany and Murphy (1967) considered that the dispersion model appeared to be an adequate model to describe spiral flow aeration tanks. These investigators obtained response curves from both a full scale prototype tank and a 1/13th geometric scale laboratory model. Their preliminary results suggested that the axial dispersion coefficient, D, for the prototype equaled that of the model divided by the square of the scale factor. A similar factor was proposed by Danckwerts (1953) when considering scale-up of rotating stirrers.

The residence time distribution (RTD) does not give the exact flow pattern in the reactor. As indicated by Dawkins (1963), it is possible to have an infinite number of different flow models all yielding identical response curves. In using RTD's, the time schedule of mixing of reactor contents is not known. Prediction of reactor performance by the use of the RTD and kinetic rate constant is then limited to reactions unaffected by this time schedule of mixing, i.e. first order reactions. Such predictions are also limited to isothermal reactions. For second order reactions, or non-isothermal reactions, only the limits of the extent of reaction may be calculated from the RTD and rate constant information.

In predicting reactor performance for an isothermal constant volume first order reaction, a mass balance on an elemental section of reactor length yields the following differential equation (Danckwerts (1953)):

$$\frac{d^2C}{dx^2} - \frac{u}{D}\frac{dC}{dx} - \frac{kC}{D} = 0 \qquad (7)$$

where: C is the concentration of reactant at cross section x, and

k is the reaction constant in time $^{-1}$.

This equation has been solved analytically by both Danckwerts (1953) and Wehner and Wilhelm (1956). The solution obtained is:

$$\frac{C}{C_o} = \frac{4a \exp\left[\frac{1}{2}\frac{uL}{D}\right]}{(1+a)^2 \exp\left[\frac{auL}{2D}\right] - (1-a^2) \exp\left[-\frac{a}{2}\frac{uL}{D}\right]} \qquad (8)$$

where: $a = \sqrt{1 + 4k\bar{t}\,(D/uL)}$,

$\frac{C}{C_o}$ = fraction of reactant unconverted.

When $D = 0$, equation (8) becomes

$$\frac{C}{C_o} = e^{-k\bar{t}} \qquad (9)$$

which is the solution for plug flow, and when $D = \infty$, equation (8) takes the form:

$$\frac{C}{C_o} = \frac{1}{1 + k\bar{t}} \ , \qquad (10)$$

the solution for a completely mixed reactor. Levenspiel and Bischoff (1959) have solved equation (7) numerically for second order reactions.

Although the completely mixed system has considerable advantages, e.g. reduced effect of shock loads, more stable operation and more uniform oxygen requirements, the major disadvantage in such a system is the reduced substrate removal as compared to that obtained by a plug flow system. This becomes readily apparent when comparing equations (9) and (10). The plug flow system is somewhat unstable in its operation as small changes in operating conditions, not damped as in the completely mixed system, may severely affect reactor performance.

Milbury _et al_ (1965) presented laboratory results indicating increased substrate removal obtained by operating a tank-in-series vessel as compared to a completely mixed system. This data was questioned by McKinney (1965) who claimed better performance from three parallel tanks as opposed to three tanks-in-series. However, under the experimental conditions reported by McKinney, the oxygen demand in the first tank exceeded the oxygen transfer capacity. This severely limited any significant conclusions that could be drawn from such a study.

McKinney (1962) also claims that when the aeration tank contents are completely mixed, the resulting activated sludge process is distinctly different from the conventional activated sludge process. Unfortunately, the "conventional activated sludge process" was not defined with respect to the parameters defining mixing levels.

A thorough review of all aspects of flow patterns in chemical reactors may be found in the discussion by Levenspiel and Bischoff (1963). This complete discussion covers the concept of residence time distributions, dispersion, mixed and tanks-in-series models, and the application of nonideal patterns of flow to chemical reactors. Over 300 references are included.

Extensive laboratory studies by Thomas and McKee (1944) indicated that the tank tended to greater mixing with increased air flow rates and with distributed hydraulic loading. An excellent review of the sanitary engineering literature dealing with mixing considerations was presented by Timpany (1966). Among others, he reviewed the previous work of Calvert and Bloodgood (1934), Hasseltine (1932), Kessener (1935), and Kehr (1936), who all found considerable mixing of aeration tank contents taking place. Although many investigations have been carried out, the present situation was adequately described by Timpany, "A better understanding of the mechanism of removal and the flow characteristics is essential for a more rational design of activated sludge aeration tanks".

EXPERIMENTAL EQUIPMENT AND TECHNIQUES

A. Description of System

The aeration tank used for the test program was one of six aeration tanks at the Burlington Skyway Water Pollution Control Plant, Burlington, Ontario. Each aeration tank, 270 ft x 27 ft x 15 ft S.W.D., contains a 14 in. diameter air header, the centre line located 24 in. above the tank floor and 36 in. from the wall. Diffusers may be installed at 18 in. centres on each side of the header. All aeration tanks were initially equipped with fine bubble diffusers. Process air is supplied by three centrifugal compressors having a maximum discharge of 4000 scfm each. A 36 in. diameter main header distributes air to the six branch headers, each equipped with a butterfly valve capable of throttling the air flow to that particular aeration tank.

Raw water drawn from Hamilton Harbour was discharged into the test tank through a 12 in. inlet located at the vertical centre line of the tank and 7 ft below the liquid surface. Effluent was discharged over a standard weir covering one-half the width of the tank. A 2 1/2 in.

foam control spray line was located along the length of the test tank. This line contained 50 spray nozzles spaced at 5 ft intervals.

Water flow measurements were made using a venturi flow nozzle installed in the water feed line. This nozzle was supplied as having an accuracy of ± 3 percent uncalibrated.

Air flows to the test tank were measured using a 5/16 in. diameter standard steel pitot tube. This was the only possible method of measuring the air flow to the test section without interfering with normal plant operation. Using the orifice flow meter installed on the 36 in. main air header and by shutting off the air supplied to the rest of the plant, it was found that the pitot tube gave results ranging from 10 to 30 percent lower than the actual flow rate. The extreme error in air flow measurement occurred when the butterfly valve was throttled considerably; this was required when coarse bubble diffusers were installed. These lower flow readings were attributed to the non-ideal location of the pitot as:

1. The pitot was installed in the 12 in. air header only 36 in. downstream from the connection to the 36 in. air header, and

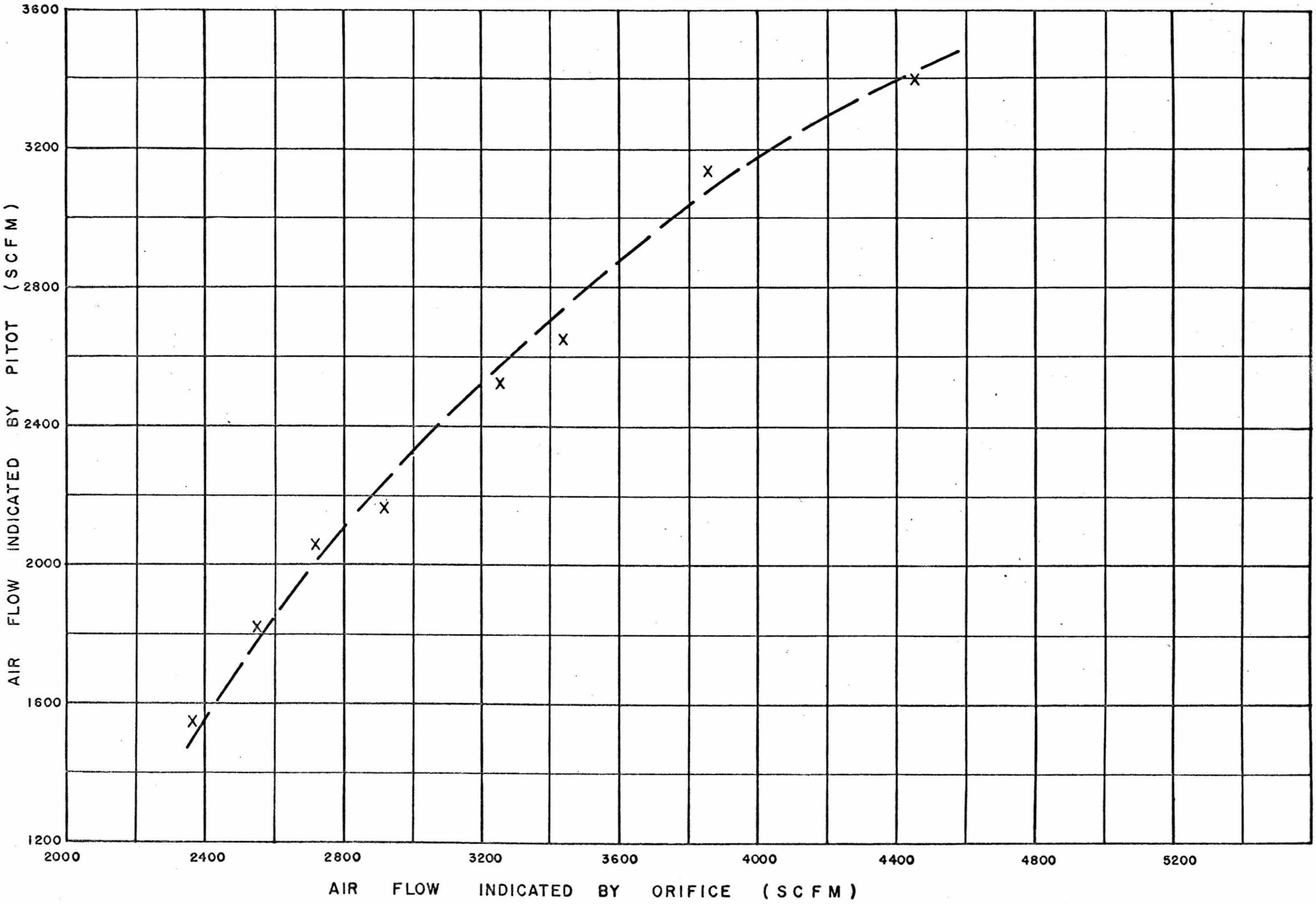
2. The butterfly control valve was located approximately 12 in. downstream of the pitot.

These extreme errors in air flow measurement were overcome by preparing a curve of flow rates indicated by the pitot against those indicated by the orifice meter. This "correction curve" is shown in Fig. 1. For the tests using the coarse bubble diffusers, the air flow rates recorded were those obtained using the "correction curve", Fig. 1. As previously mentioned, the pitot tube was the only means of determining air flows to the test section. Process air could not be shut off, even on an intermittent basis, during the test program, a period of approximately four months. Process air temperature and pressure were recorded continuously.

A shelter located at the effluent end of the aeration tank housed the tracer monitoring equipment. This consisted of a G.K. Turner & Associates Model 111 recording fluorometer and a Bausch and Lomb V.O.M.-5 strip chart recorder. Fluorescence measurements were made using primary filters 1-60, and 58 and a secondary filter, 23A. A laboratory pump was used to continuously sample the

FIGURE I

PLOT OF AIR FLOW MEASUREMENT BY PITOT
Vs
AIR FLOW MEASUREMENT BY ORIFICE



effluent at a rate of 100 ml/min. This sample was taken at a location 5 ft from the side wall opposite the air header, 2 ft below the liquid surface and 3 in. from the tank end. After passing through the fluorometer, the temperature of the effluent sample was determined at periodic intervals during the course of a test.

Two types of diffusers were used in this study. One was a fine bubble device described as a "Precision Tube" by the manufacturer, Chicago Pump Company of Chicago, Illinois. This diffuser is a 3 in. perforated ribbed metal cannister, 24 in. long, that is wrapped in Saran cord. The other diffuser used was a coarse bubble device, a "Sparjer", supplied by Walker Process Equipment Incorporated, of Aurora, Illinois. The Sparjer orifices were 7/32 in. in diameter. Even air distribution along the length of the header was assumed as the diffusers were evenly spaced throughout the tank length.

In order to determine the response of two equal tanks-in-series, a partition was installed across the entire tank width at the 135 ft tank length. This partition was made of 1/2 in. plywood supported by 2 in. x 4 in. frames.

B. Description of Experimental Procedure

The tracer used in this study was Commercial Rhodamine B Extra, supplied by DuPont of Canada Limited. As a wide range of fluid temperatures would be encountered, and since fluorescence intensity increases with decreasing temperature, it was necessary to determine the relationship between fluorescence intensity and temperature. The fluorescence-temperature curves determining this relationship are shown in Appendix A. Fluorescence readings obtained during the tests were converted to standard fluorescence readings at 20°C. The fluorescence-concentration calibration curve at 20°C is also shown in Appendix A. From the work of Feuerstein and Selleck (2) it was noted that Rhodamine B has a photochemical decay half-life of 31 hr. While this decay rate determination was a qualitative measure only, such a half-life was not thought to yield erroneous results in residence time studies of short duration. The effect of suspended solids at the concentrations encountered was considered negligible. Samples of Hamilton Harbour water were analyzed as follows:

TABLE I

HAMILTON HARBOUR* WATER QUALITY

Date	pH	Total Alkalinity as $CaCO_3$ mg/1	BOD mg/1	Turbidity as SiO_2 mg/1	Total Kjeldahl as N mg/1
1967 May 17	7.4	105	--	14	4.8
June 6	7.6	106	--	11	--
July 4	7.0	86	--	8	1.1
July 18	7.3	102	--	11	--
Aug. 22	7.6	109	--	12	0.5
Aug. 25	8.1	92	3.9	11	--
Sept. 9	7.5	110	--	12	--

* Samples taken in vicinity of WPCP raw water intake.

No background fluroescence in the Hamilton Harbour water was measurable at any time, using the fluorometer range sensitivity (x10) that was used throughout the tests.

In studying the mixing characteristics of the diffusers, statistical techniques were used in planning the experimental program and in interpreting the results.

An experimental design test program was used to determine the main and interaction effects of the detention time and air flow rate on the dispersion number. A description of the rotatable central composite experimental design that was used is given in Appendix B.

Prior to a test, a quantity of tracer was weighed (usually 300-400 gm) and dissolved in 5 gal of water. Air and water flow rates were adjusted and one hour later the tracer solution was added to the influent end of the tank. It was considered that this method of dye input approximated an ideal pulse function. An experimental tracer response curve was then obtained.

The tracer measurement was discontinued after the well-defined peak of the response curve was obtained. This resulted in a response curve that was truncated, usually at less than one nominal detention time. Response curves of several detention times in duration were not obtained due to the test time required (24-48 hr) coupled with the additional time required to washout the tracer in preparation for the following test. Only two or three tests could then be carried out in a week.

As Timpany (1966) has pointed out, after one detention time, the tanks-in-series, completely mixed, and dispersion models all predict essentially the same response curve for the relatively high mixing levels considered.

For the first series of tests, 344 "Precision Tubes" were installed in the test tank. These diffusers were evenly distributed along both sides of the air header. Since there was a tendency for the dye to cause foaming when using the fine bubble diffusers, it was necessary to use the foam control spray line. Water delivered by this line was 0.8 IGPM per nozzle or 40 IGPM. In analyzing the results, this extra water flow was neglected.

The second series of tests were carried out using 178 Sparjers installed on 18 in. centres on the wall side of the air header. The spray line for foam control was not in use during these tests. Due to the lower pressure drop across the diffusers compared to the fine bubble devices in the other tanks, it was necessary to almost completely close the butterfly valve controlling the air supply to the test section. This resulted in the extreme errors in the air flow measurements using the pitot tube.

The tests on two tanks-in-series were performed using "Sparjers" as the aeration device.

C. Sources of Error

The prime consideration of this program was to determine whether or not the dispersion model could adequately describe the longitudinal mixing taking place in a full-scale aeration tank. Lack of fit between the theoretical and the experimental response curves should be accounted for. Slight discrepancies between the two curves could be attributed to the following: 3 percent error in water flow measurement, fluorescent tracer decay and/or adsorption by suspended solids or concrete walls, error in determining temperature at which the tracer concentration is determined, and neglecting spray water flow. These errors were considered minimal.

As previously mentioned, air flow measurements when using the fine bubble diffusers indicated slightly lower flows than actual. This minor systematic error should still allow a quantitative relationship between the dispersion coefficient and the air flow rate to be determined. An expression could be derived and, although not absolute, such an expression would indcate the effect of increased air flow on the dispersion number. The degree of longitudinal mixing associated with each type of diffuser could also be compared.

The two principal assumptions made for this study were:

1. The method of dye input approximated a pulse function, and
2. There was uniform air distribution to the diffusers along the length of the air header.

The physical magnitude of the experimental system precluded obtaining an ideal test situation. Such conditions as poor test media, foaming of tracer, tracer adsorption by concrete, and difficulty in air flow measurement were encountered.

EXPERIMENTAL RESULTS

A. Data Analysis

The data from the tracer response curves obtained during the tests were transferred to data cards by taking fluorometer readings at equal time intervals. A computer programme was written to analyze the individual test results.

Using the calibration curves in Appendix A, fluorometer readings were converted to readings at 20°C and then tracer concentrations were calculated. The time of peak exit concentration was determined from the response curve and the dispersion number D/uL was calculated using equation (5) or (6).

In order to compare the experimental response curves to the theoretical curves generated using equation (3), the experimental curves were reduced to dimensionless plots of C/C_o vs $t/\bar{t}$. A plotting routine was incorporated into the programme to allow both the theoretical and experimental response curves to be printed simultaneously.

The programme used for these calculations and curve plotting is reproduced in Appendix C.

B. Experimental Results Using Fine Bubble Diffusers

The dispersion coefficient (D) was determined using "Precision Tube" diffusers for air rates of 700 to 2770 scfm or 6.4 to 25.3 scfm/1000 cu ft tank volume. These correspond to unit air rates of 2.0 to 8.0 scfm per diffuser. In addition, response curves were obtained over the range of liquid detention times of 4.75 hr to 13.20 hr. The experimental results obtained using the "Precision Tubes" are summarized in Table II.

Typical experimental response curves obtained are shown in Figs. 2 and 3. The theoretical response curves calculated from the experimental peak times are also shown in Figs. 2 and 3.

The analysis of variance (as outlined in Appendix B) yielded the following results. Only the first order effect of air flow on the dispersion coefficient was significant. Second order and interaction effects of air and water flow rates were insignificant. The effect of the water flow rate (detention time) on the dispersion coefficient was also insignificant. From the analysis of variance, an expression describing the effect of the air flow rate on the dispersion

TABLE II

EXPERIMENTAL RESULTS USING FINE BUBBLE DIFFUSERS

Test No.	Air Flow scfm	Water Flow IGPM	Theoretical Det^{n} Time hr	Peak Time hr	Associated D/uL	D ft^2/hr
4	1740	1260	9.01	2.85	0.879	7110
5	1750	860	13.20	3.61	1.130	6240
8	990	1890	6.01	2.77	0.437	5310
10	2530	1260	12.01	3.56	1.014	6150
11	2770	1260	9.01	3.06	0.786	6360
12	930	950	12.01	3.75	0.897	5440
13	700	1260	9.01	3.60	0.587	4750
14	1790	2390	4.75	2.25	0.412	6320
15	1730	1260	9.01	3.00	0.811	6560
16	1730	1260	9.01	3.08	0.777	6290
18	1730	1260	9.01	3.20	0.729	5900
19	1730	1260	9.01	3.06	0.786	6360
20	2360	1910	5.94	2.58	0.497	6100

FIGURE 2

EXPERIMENTAL AND THEORETICAL RESPONSE CURVES

TEST No. 13

FINE BUBBLE DIFFUSERS

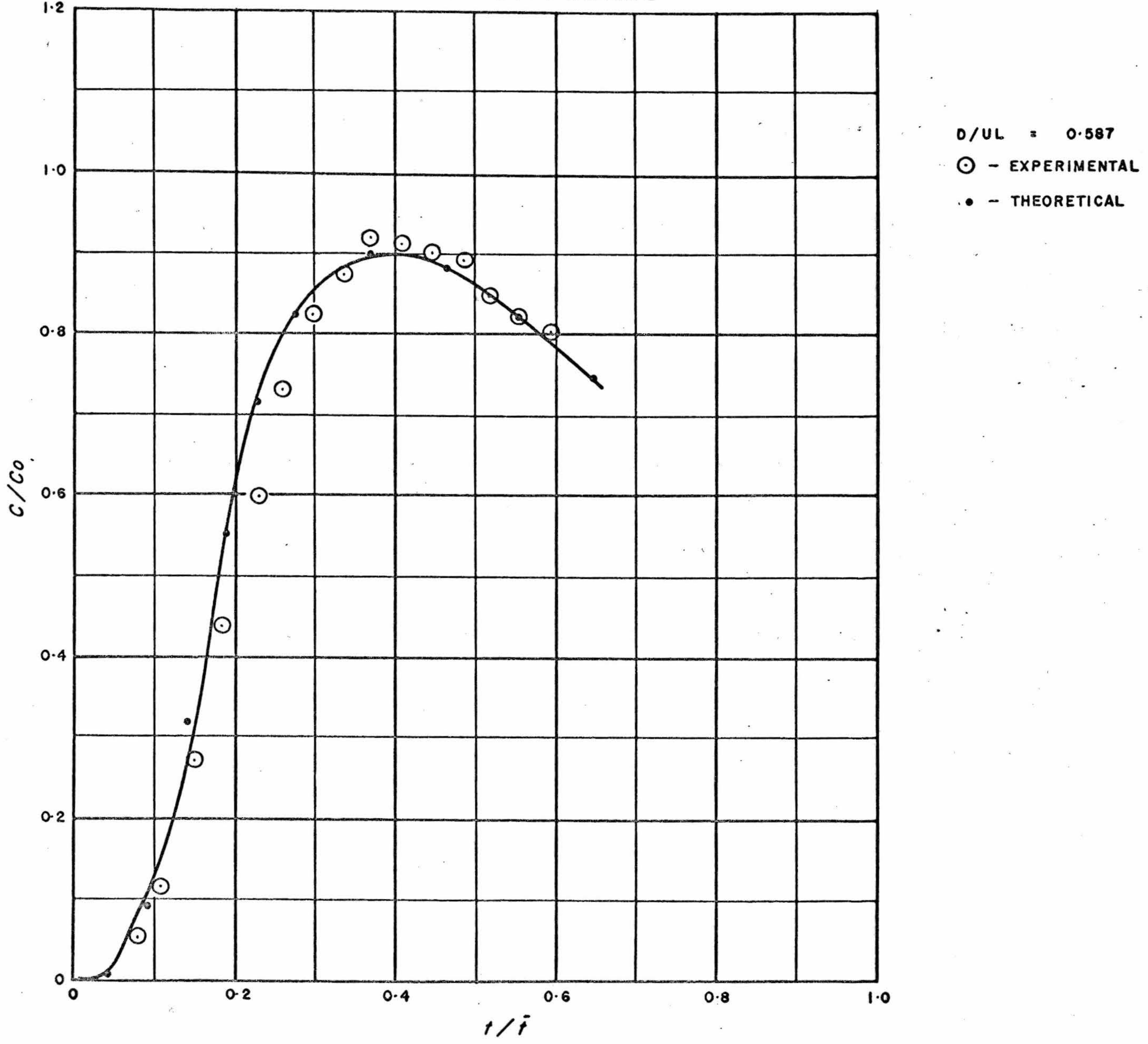
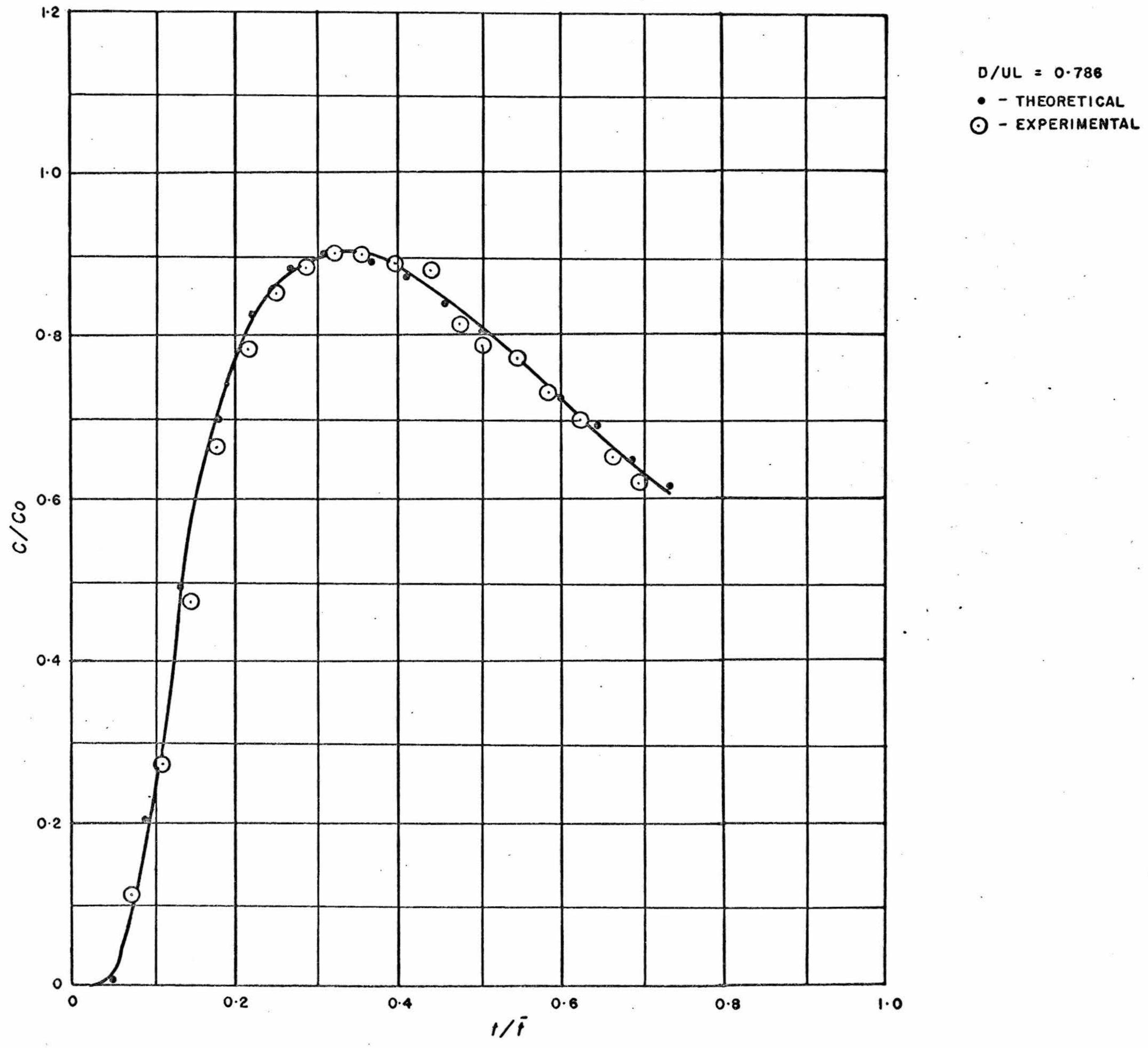


FIGURE 3

EXPERIMENTAL AND THEORETICAL RESPONSE CURVES

TEST No.19

FINE BUBBLE DIFFUSERS



coefficient over the range of applied air flows of 700 scfm to 2770 scfm was obtained. This expression is:

$$D = 5340 + 0.64\, Q_A$$

where: D = longitudinal dispersion coefficient (ft^2/hr), and

Q_A = applied air flow rate (scfm)

This expression was obtained from the analysis of variance that is part of the experimental design that was used.

C. Experimental Results Using Coarse Bubble Diffusers

With Sparjer diffusers installed in the aeration tank, response curves were obtained over the range of air flows of 2350 scfm to 2960 scfm (21.6 to 27.2 scfm/1000 cu ft tank volume) and over the range of liquid detention times of 4.75 hr to 13.28 hr. The air flow rates applied correspond to unit air rates of 13.1 scfm to 16.5 scfm per diffuser. Table III summarizes the experimental results obtained using Sparjer coarse bubble diffusers.

Typical response curves obtained using Sparjers are shown in Fig. 4 and 5. From the analysis of variance,

TABLE III

EXPERIMENTAL RESULTS USING COARSE BUBBLE DIFFUSERS

Test No.	Air* Flow scfm	Water Flow IGPM	Theoretical Det^n Time hr	Peak Time hr	Associated D/uL	D ft^2/hr
102	2700	1270	8.94	2.96	0.818	6670
103	2600	1270	8.94	2.75	0.916	7470
104	2970	1890	6.01	2.33	0.623	7560
106	2350	1260	9.01	3.09	0.773	6250
107	2380	1910	5.94	2.33	0.611	7490
108	2640	2390	4.75	2.17	0.447	6860
111	2960	1260	9.01	2.74	0.932	7540
112	2970	945	12.01	3.04	1.253	7610
113	2690	1260	9.01	2.92	0.847	6850
114	2690	1290	8.80	2.92	0.816	6760
115	2690	1260	9.01	3.00	0.811	6560
116	2690	860	13.28	3.58	1.152	6320
117	2420	945	12.01	3.25	1.146	6960

*Corrected using Figure 1.

FIGURE 4
EXPERIMENTAL AND THEORETICAL RESPONSE CURVES
TEST No. III
COARSE BUBBLE DIFFUSERS

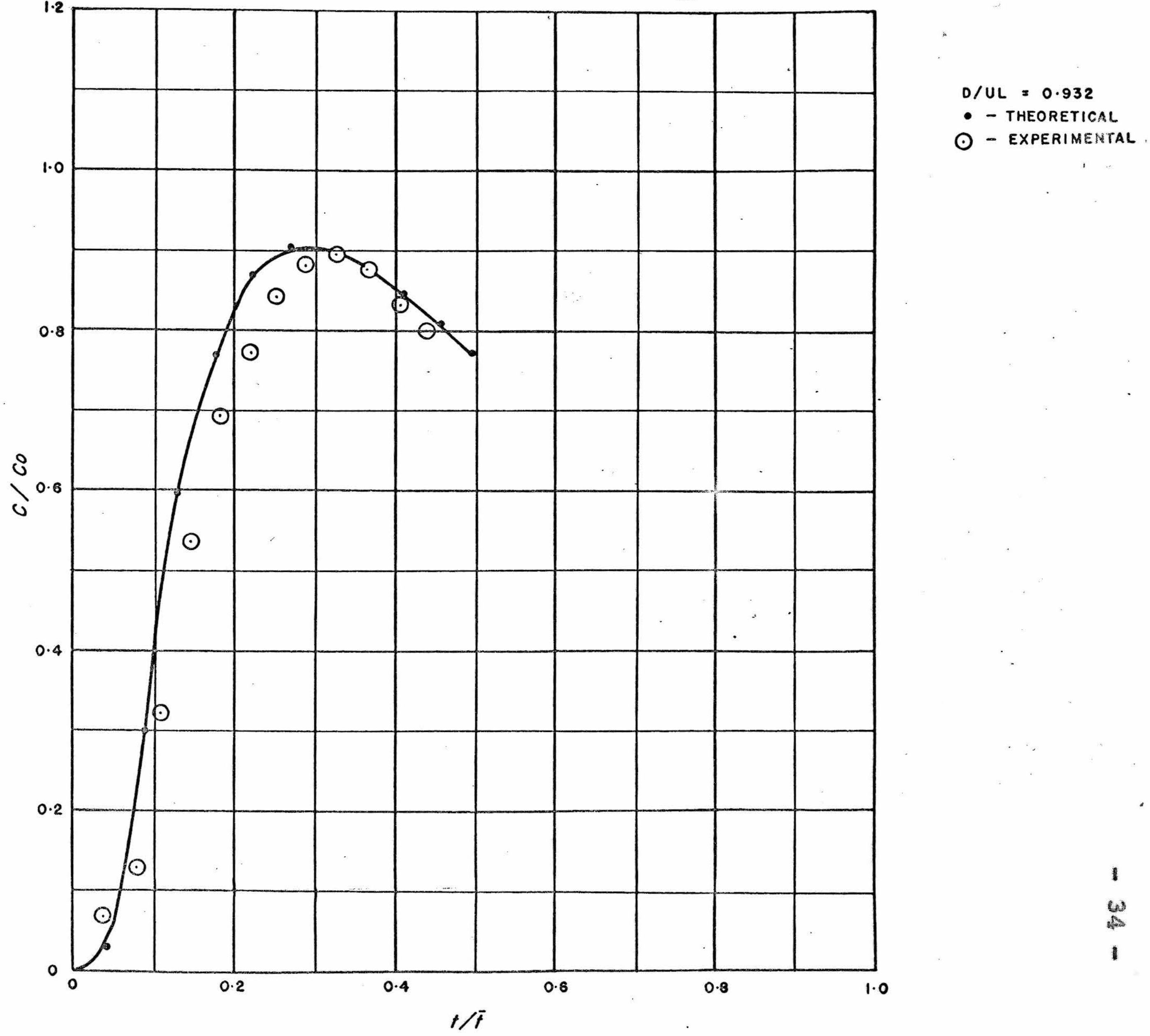
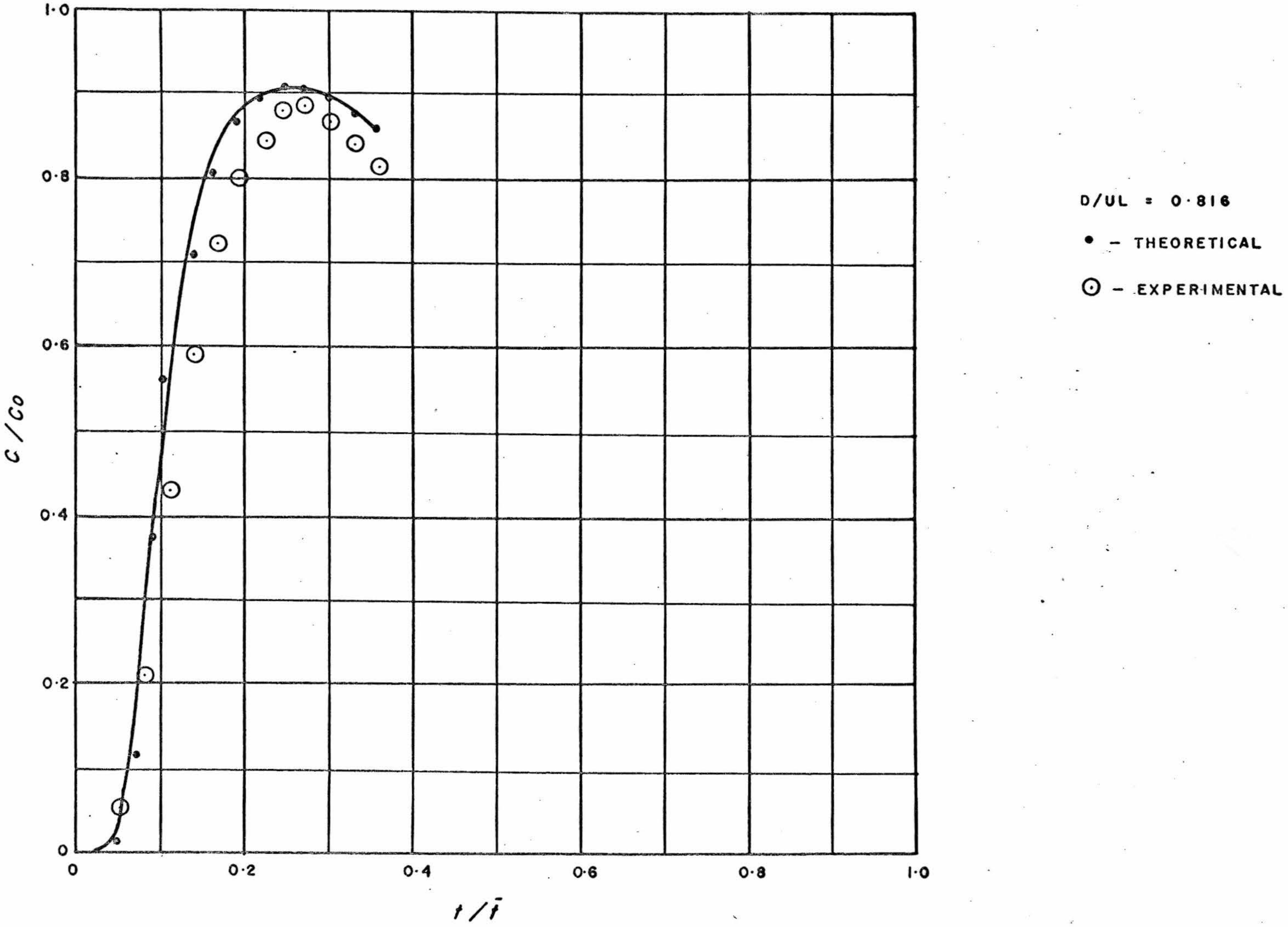


FIGURE 5
EXPERIMENTAL AND THEORETICAL RESPONSE CURVES
TEST No.114
COARSE BUBBLE DIFFUSERS



the following results were obtained. Over the range of air and water flow rates studied, the effects of both the air and water flow rates were insignificant, i.e. first order, second order and interaction effects of air and water flow rates were insignificant. The significant value of the dispersion coefficient obtained was 6760 ft^2/hr.

D. Experimental Results Using 135 ft Length Tank

Two response curves were obtained using the 135 ft length tank at a liquid detention time of 4.5 hr and an applied air rate of 1350 scfm (24.6 scfm/1000 cu ft tank volume). (Air was applied at a rate of 2690 scfm to the 270 ft long tank (24.6 scfm/1000 cu ft tank volume). Equal air distribution between the two 135 ft tanks was assumed.)

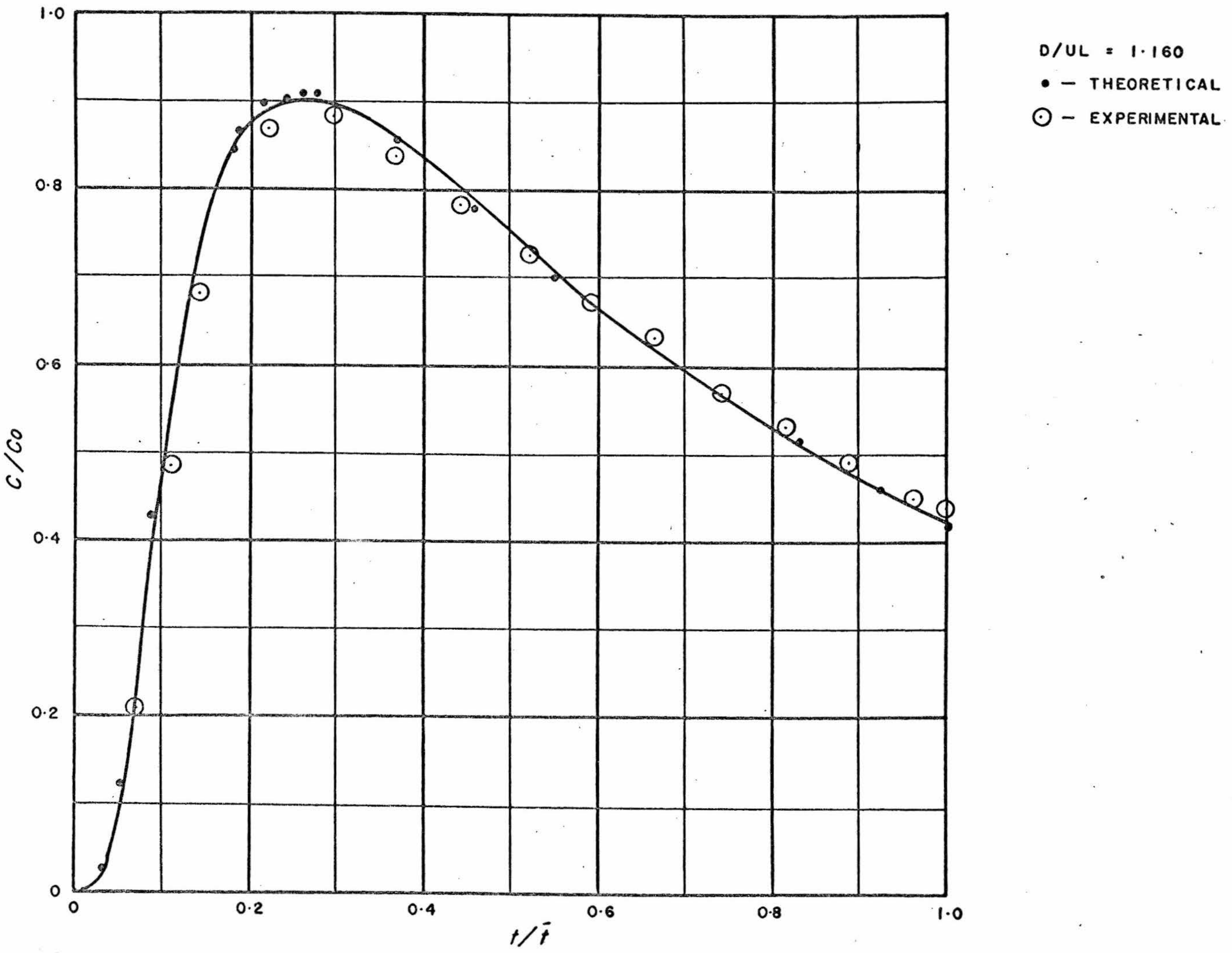
The dispersion numbers obtained for these two tests were 1.18 and 1.16. The corresponding dispersion coefficients were 4790 ft^2/hr and 4690 ft^2/hr respectively. One of the response curves obtained is shown in Fig. 6.

FIGURE 6

EXPERIMENTAL AND THEORETICAL RESPONSE CURVES

TEST No. 300

135 FT. TANK LENGTH, COARSE BUBBLES DIFFUSERS



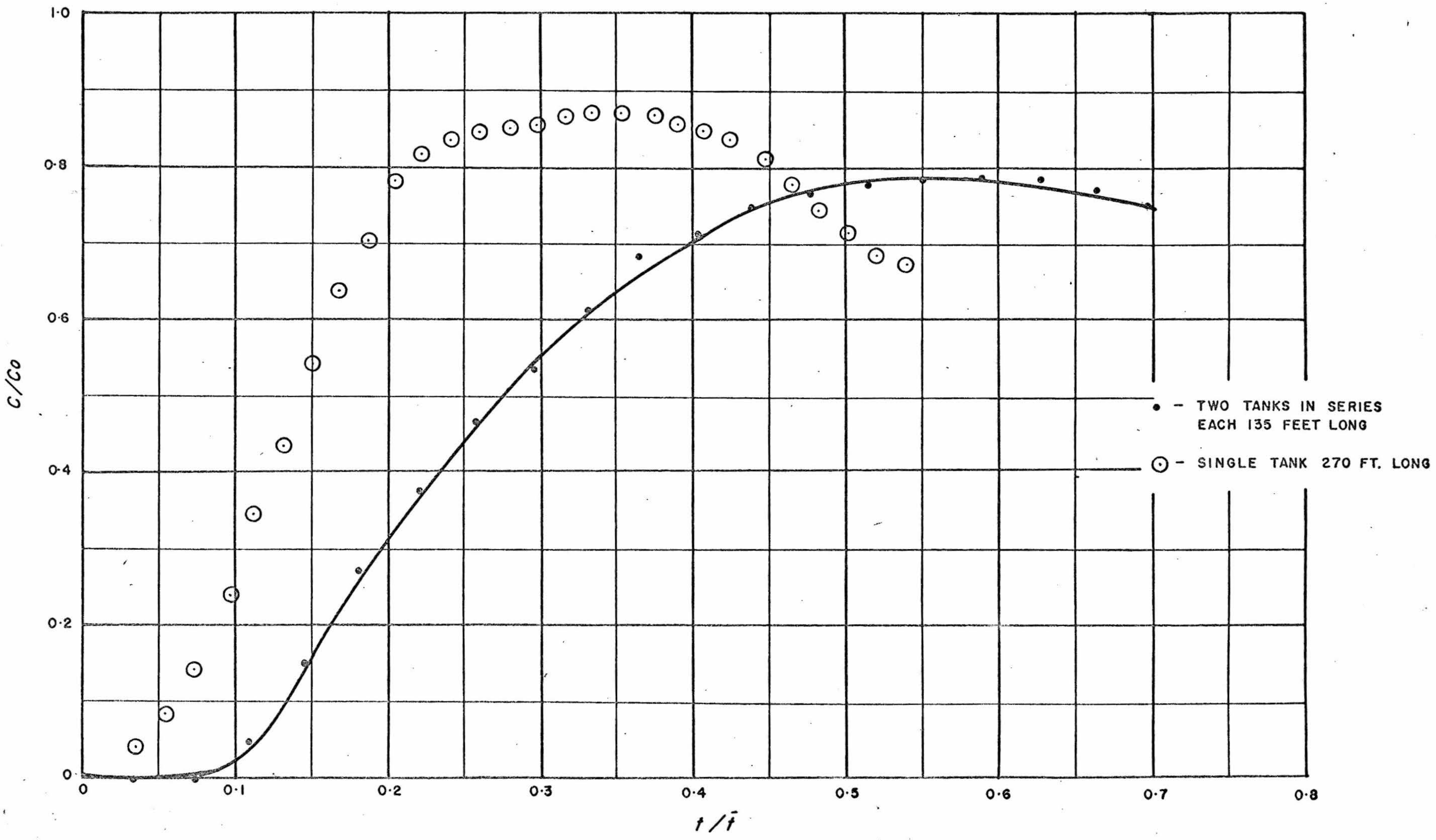
E. Experimental Results Using Two 135 ft Tanks in Series

The experimental response curve obtained from two tanks-in-series, equipped with coarse bubble diffusers, is shown in Fig. 7. Also shown in Fig. 7 is the response curve obtained under the same conditions of air and water flows but without the partition in the tank.

FIGURE 7

EFFECT OF PARTITIONING TEST TANK

DIMENSIONLESS RESPONSE CURVES



DISCUSSION OF RESULTS

A. Use of Dispersion Model

For all the experimental response curves obtained under a variety of test conditions, the use of the peak time technique, using equations (5) or (6), proved effective in predicting the associated theoretical response curve, generated using the dispersion model.

When considering the use of the dispersion model to describe the mixing taking place, the prominent features of an experimental response curve must be recognized. For all the experimental curves obtained, there was a considerable lag time until a significant amount of tracer reached the exit. In some tests, the lag time was as great as 30 min. This time lag is significant as the normalized response curve also represents the exit age distribution function. Thus sufficient time is allowed for adsorption of most of the sewage pollutants with no raw sewage leaving the aeration tank immediately. The actual response curves all featured a lag time, an initial positive curvature followed by an inflection point and a negative curvature. Even though a finite stage model can be used to predict a time lag, these other typical features of the experimental response curve cannot be readily predicted with any model other than the dispersion model.

This agreement between the experimental and calculated response curves as shown in Figs. 2, 3, 4, 5, and 6, indicates that the dispersion model describes the longitudinal mixing taking place in a full-scale aeration tank equipped with air diffusers.

B. Effects of Water Flow, Air Flow, and Diffuser Type

Using the fine bubble diffusers, Fig. 8 shows the effect of liquid detention time (water flow rate) on the dispersion coefficient, D, and on the dispersion number, D/uL. Similarly, Fig. 9 illustrates the effect of liquid detention time on D and D/uL, using coarse bubble diffusers.

For both diffusers studied, changing the water flow rate had no effect on the dispersion coefficient, D. These results are in agreement with those found by Timpany (1966). Changing the water flow rate affects only the longitudinal bulk velocity, u, thus altering the dispersion number, D/uL. As the liquid detention time increases, the "u" term decreases

FIGURE 8
EFFECT OF DETENTION TIME ON D AND D/UL USING FINE BUBBLE DIFFUSERS

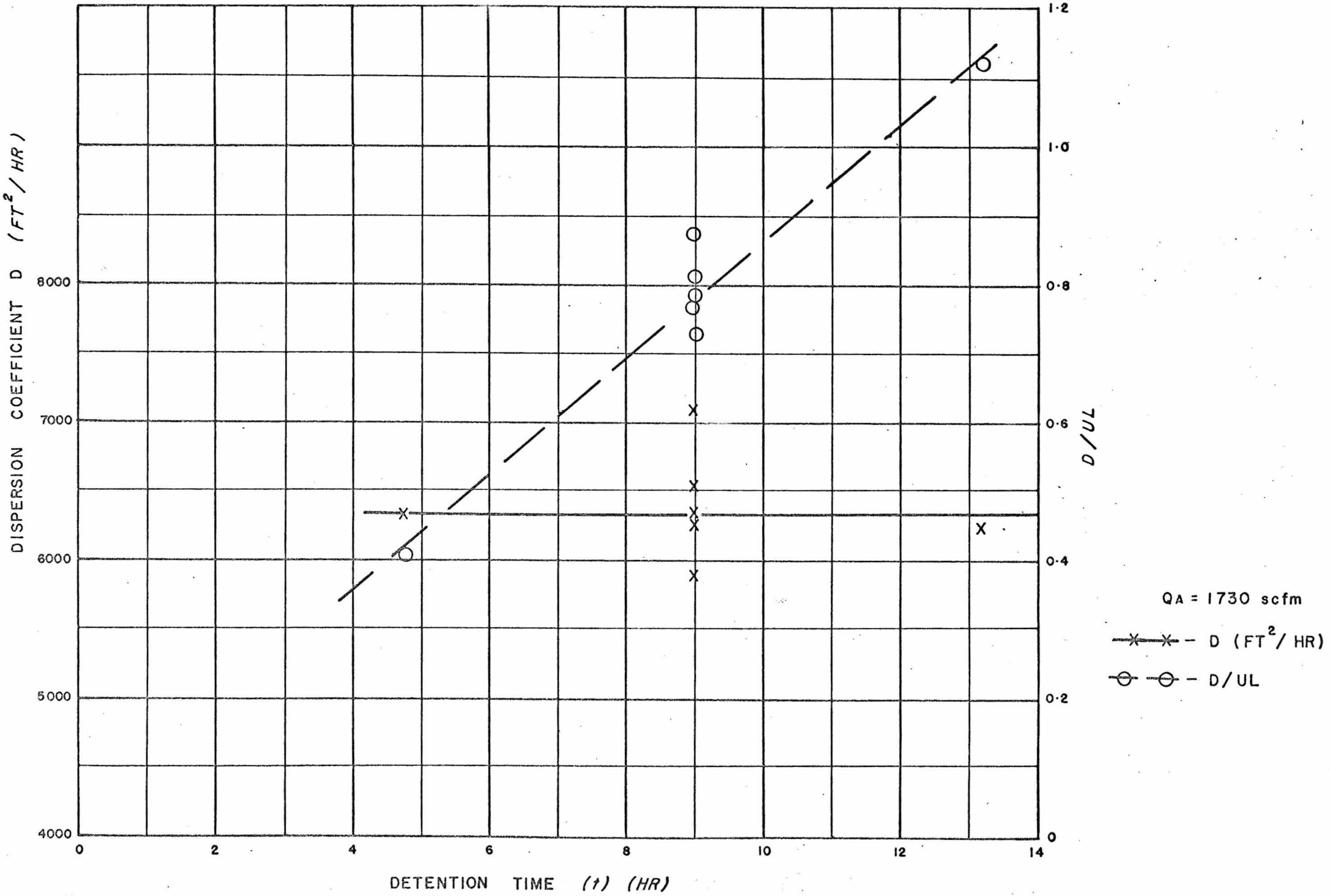
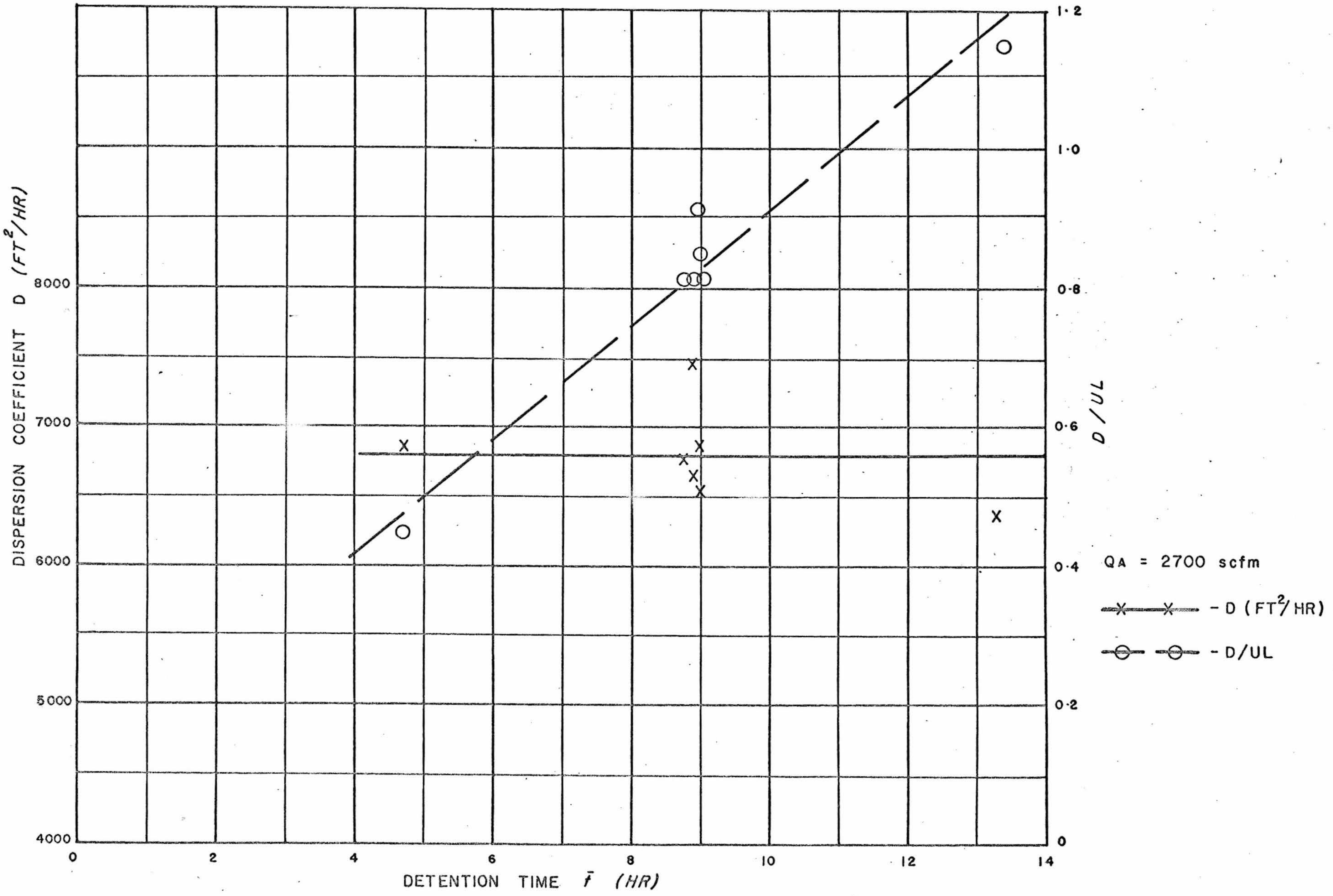


FIGURE 9
EFFECT OF DETENTION TIME ON D AND D/UL USING COARSE BUBBLE DIFFUSERS



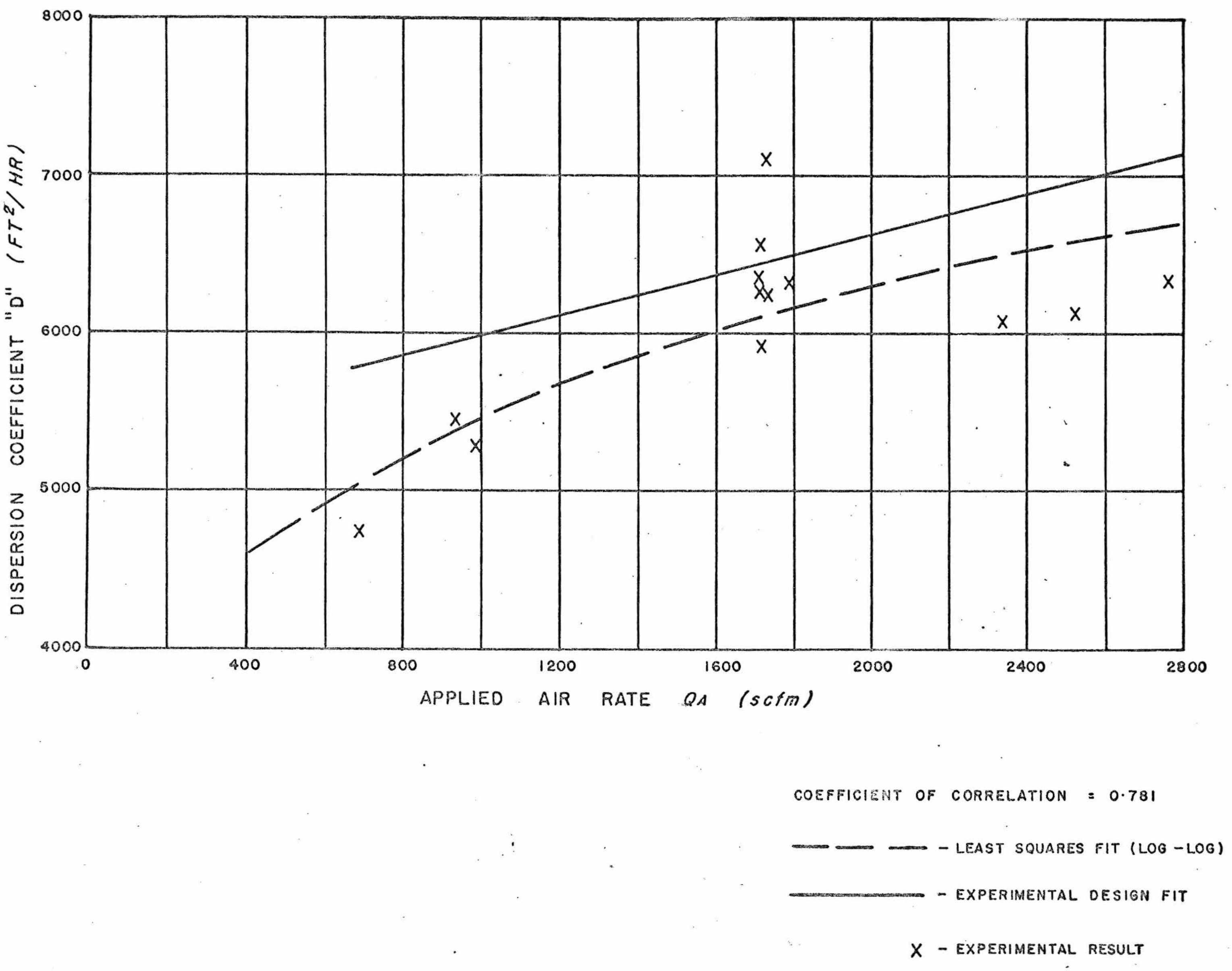
and complete mixing is approached, i.e. D/uL $\rightarrow \infty$. Timpany (1966) found that for a constant air flow and detention time, D/uL can be considered a constant over the range of 12 to 30 degrees Centigrade.

The results obtained when using the Precision Tubes indicated that only the first order effect of air flow on the dispersion coefficient was significant. In the statistical analysis of the results, significance was tested at the 95% confidence level.

A plot of the dispersion coefficients obtained for various applied air rates is shown in Fig. 10. These results were obtained over a range of applied air rates of 2 to 8 scfm per diffuser. The design air rate for these diffusers is 4 to 8 scfm/diffuser. When using this particular experimental design, the results obtained apply only within the limits of experimental conditions studied. Fig. 10 indicates that a diminishing increase in mixing is obtained for increasing applied air rates. This result is similar to that obtained by Timpany (1966) when he studied fine bubble ceramic diffusers.

FIGURE 10

DISPERSION COEFFICIENT "D" Vs APPLIED AIR RATE "QA"
USING FINE BUBBLE DIFFUSERS



This effect is also confirmed if significance is tested at the 99% confidence level. The second order effect of air flow rate then becomes significant and the coefficient of this second order term is negative.

As previously stated, the dispersion coefficient is described as: $D = 5340 + 0.64 Q_A$. This expression was derived from the experimental design used. However, the least squares expression relating D to Q_A is: $D = 1349(Q_A)^{0.203}$. The correlation coefficient for this log-log regression was 0.781.

With the Sparjers installed, the test results indicated that the applied air rate had no significant effect on the dispersion coefficient within the range of applied air rates studied (13.1 to 16.5 scfm per diffuser). The recommended applied air rates for these Sparjers, as installed, are 8 to 11 scfm/diffuser. Although the narrow range of applied air rates studied may have led to the result of insignificant effect of air rate, the Sparjers could not be studied at lower air flow rates due to plant limitations. As fine bubble diffusers were installed in the other five aeration tanks, the butterfly valve could not be throttled sufficiently to overcome the difference in head losses between the two types of diffusers.

In Fig. 11, the dispersion coefficients, obtained using Sparjers, are plotted against applied air rates. The least squares expression (log-log) relating D to Q_A was found to be: $D = 3980Q_A^{0.364}$. However, the correlation coefficient for this regression was 0.378. It can then be assumed that the least squares line has no significance; D is then independent of Q_A as was indicated by the experimental design.

In Fig. 12, the dispersion coefficients obtained when using the Sparjers (Table III) and the Precision Tubes (Table II) are plotted against the applied air rate. A log-log regression on this data gave an expression relating D to Q_A as follows: $D = 1000Q_A^{0.246}$. The correlation coefficient for this regression was 0.820. The degree of mixing appears to be governed only by the quantity of air applied and is independent of the means of applying the air.

When comparing the results obtained with the Sparjers and with the Precision Tubes, similar applied air rates must be considered. For the Sparjers then, the significant value of the dispersion coefficient obtained was 6760 ft^2/hr. With the Precision Tubes, at this same applied air rate,

FIGURE 11

DISPERSION COEFFICIENT "D" Vs APPLIED AIR RATE "QA" USING COARSE BUBBLE DIFFUSERS

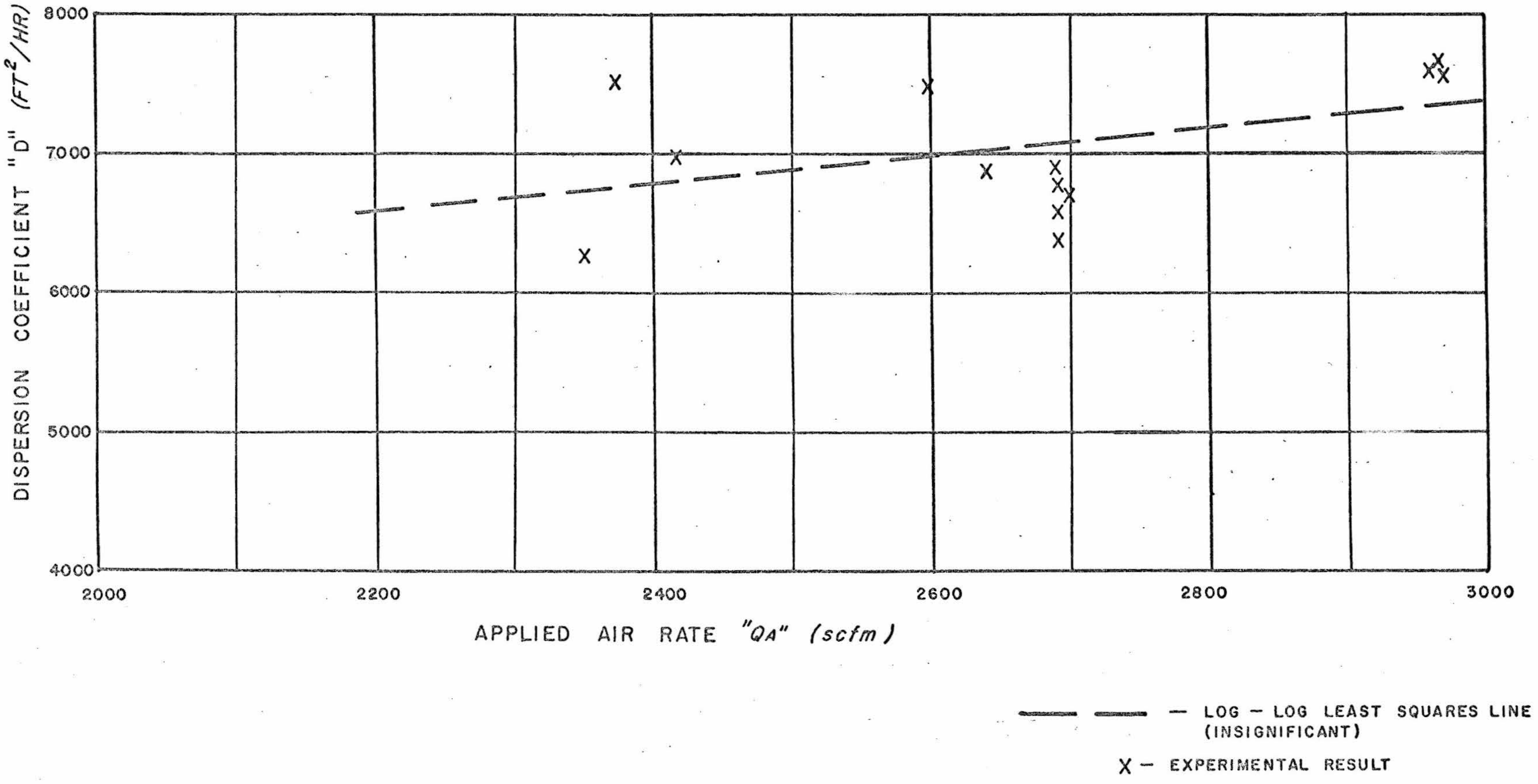
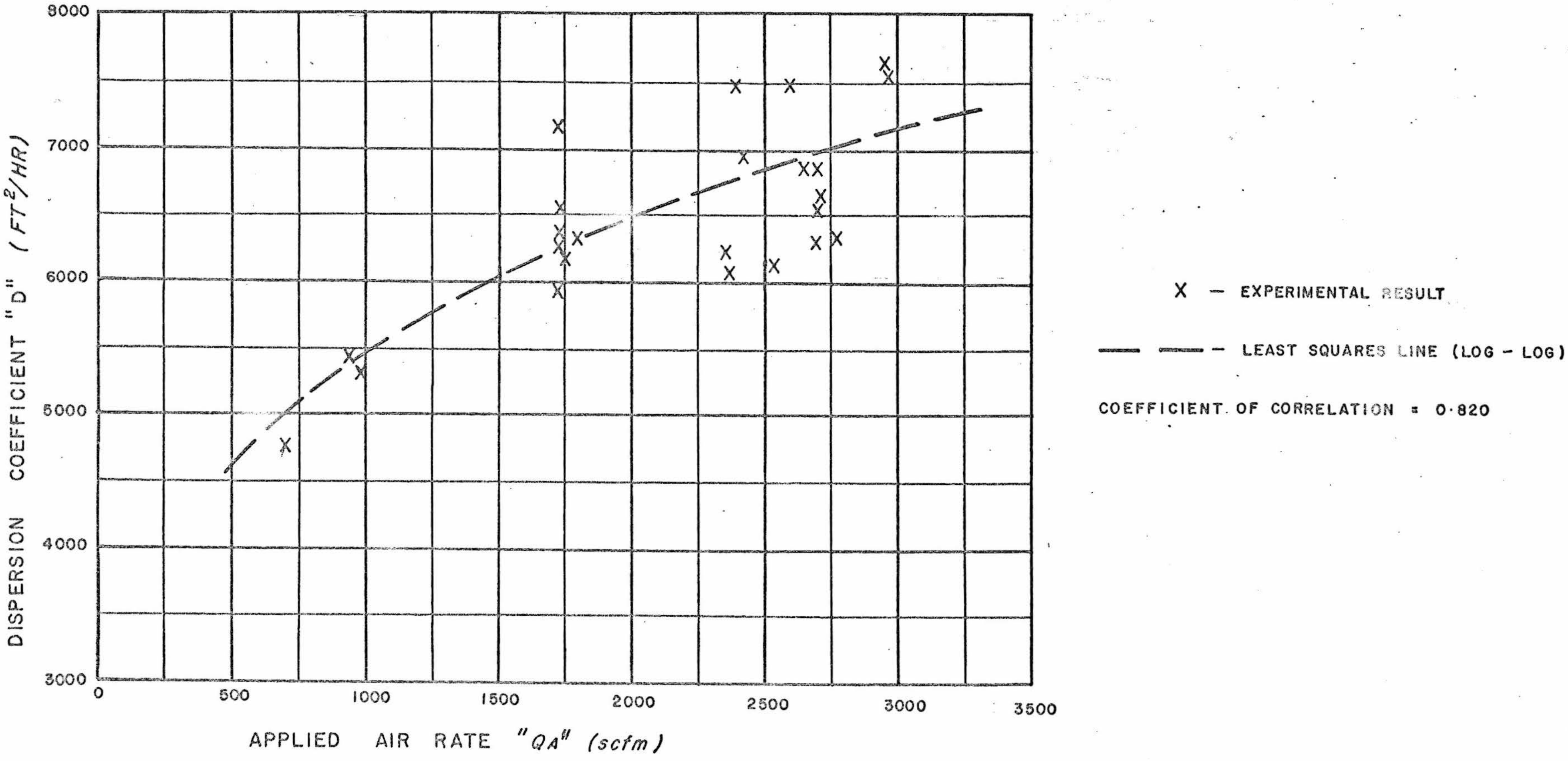


FIGURE 12

DISPERSION COEFFICIENT "D" Vs APPLIED AIR RATE "QA"
USING BOTH FINE AND COARSE BUBBLE DIFFUSERS



(25.7 scfm/1000 cu ft tank volume) the dispersion coefficient that would be obtained is 6700 ft^2/hr. Considering the repeatability error involved in these tests, these two dispersion coefficients obtained may be considered identical.

In the field tests carried out by Timpany (1966), response curves were obtained using a 66 ft long, 30 ft wide, and 15 ft deep aeration tank equipped with Sparjer diffusers. With an applied air rate of 22.2 scfm/1000 cu ft tank volume, dispersion coefficients of 6310 and 5150 ft^2/hr were obtained. These "D" values obtained compare well with the significant value of D (6760 ft^2/hr) obtained using Sparjers in this study.

C. Effect of Altering Tank Geometry

For the two tests using the 135 ft tank, the applied air rate per unit tank volume was the same as that used for several of the tests with the 270 ft tank. With identical liquid bulk velocities, the expected dispersion coefficient would then be equal to that obtained on the 270 ft tank. The dispersion coefficients obtained were approximately 0.7 times those obtained in the full length tank. i.e. 4750 ft^2/hr compared with 6630 ft^2/hr. As insufficient tests were carried out on the 135 ft tank, it could not be concluded whether the altered tank length or possible unequal air distribution (greater air applied at the influent end) led to the unexpected results. Turbulence from the liquid entering the first tank may have also provided an additional degree of mixing.

The effect of partitioning the 270 ft tank into two 135 ft tanks-in-series is shown in Fig. 7. As expected, the response curve of the partitioned tank shifted towards the plug flow system. An unsuccessful attempt was made to predict the response curve for the two tanks-in-series using the dispersion model and the previously obtained dispersion number of the 135 ft tank.

D. Effect of Longitudinal Mixing on Performance as a Reactor

It was stated in the Introduction that this study was carried out in order to determine if neglect of the longitudinal dispersion coefficient would significantly affect the predicted reactor performance. The use of the dispersion model to predict reactor performance is limited to first order reactions since the time schedule of mixing is not known. The solution of conversion for such first order reactions as given in equation (8) is a function of D/uL and $k\bar{t}$. A computer programme was written to determine the extent of reaction (first order) for various mixing levels, ranging from plug flow (D/uL = 0) to complete mixing (D/uL = 1000), using equation (8).

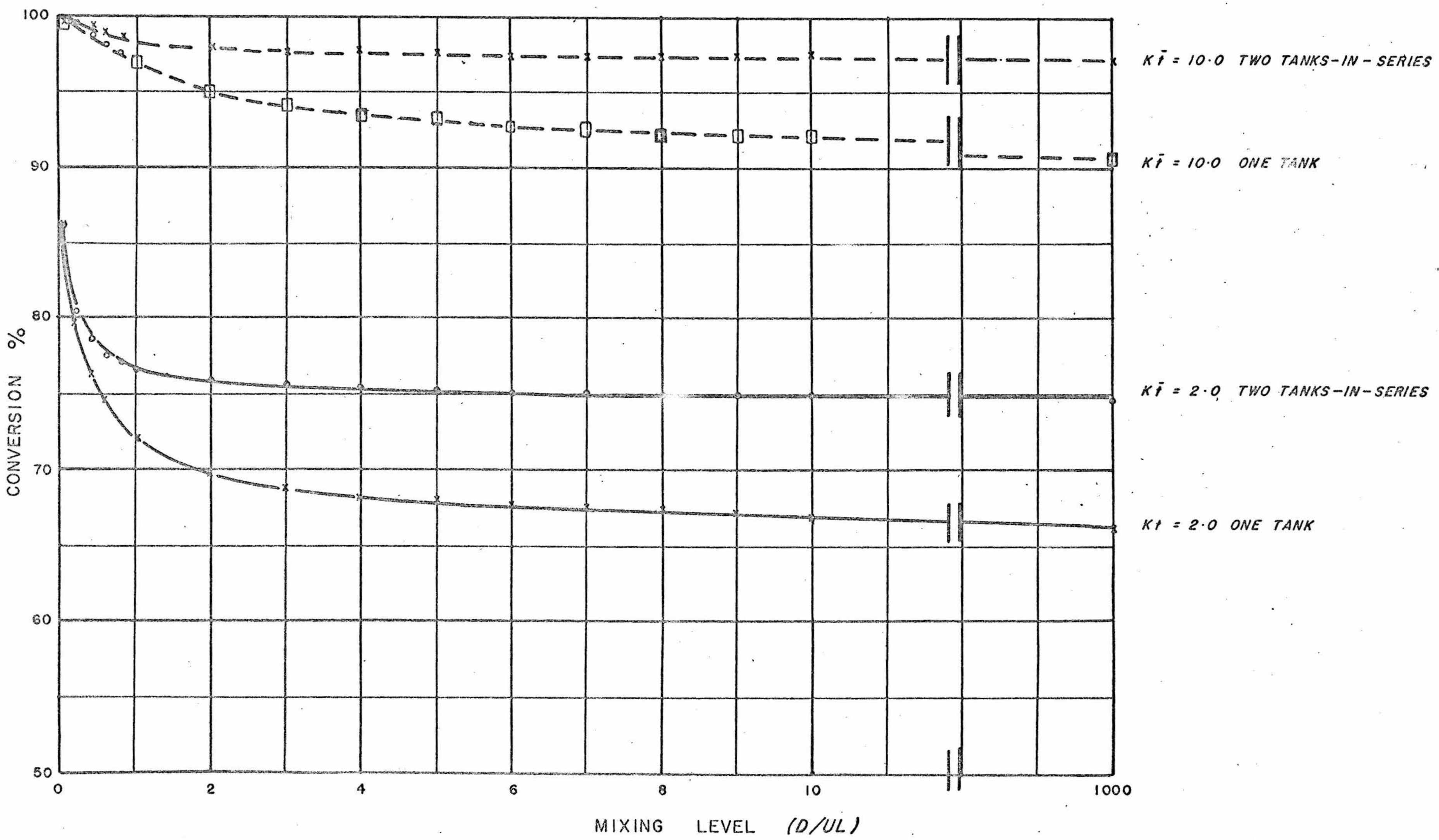
In Fig. 13, the effect of mixing level on conversion is shown for $k\bar{t}$'s of 2.0 and 10.0. These two values were arbitrarily chosen although a $k\bar{t}$ figure of 10.0 approximates that value obtained in a conventional activated sludge aeration system. (90% removal of substrate for a completely mixed system gives a $k\bar{t}$ value of 9.0.)

From Fig. 13 and using a $k\bar{t}$ of 10.0, it can be seen that if complete mixing is assumed, the conversion expected would be 90.8%. For the test tank studied (L:W::10:1), the D/uL values approximated 1.0. The actual conversion would then be 97%; the error in predicting reactor performance is 6.4%. As the performance error is on the conservative side, it appears that neglect of the dispersion coefficient in predicting reactor performance amounts only to automatic inclusion of a relatively small safety factor in reactor sizing.

FIGURE 13

EFFECT OF MIXING LEVEL ON CONVERSION FOR VARIOUS $K\bar{t}$'s

(FOR FIRST ORDER REACTION ONLY)



Also shown in Fig. 13 is the effect of partitioning the reactor into two tanks-in-series. The maximum benefit that could be derived from partitioning occurs when the single tank is completely mixed. Again, for a $k\bar{t}$ of 10.0, the increase in conversion by using two tanks-in-series is from 90.8% to 97.3%. This effectively increases the performance of the reactor by 7.2%, assuming a first order reaction ((97.3-90.8)/90.8 x 100%). However, if a $k\bar{t}$ of 2.0 is assumed, then the increase in conversion is from 66.6% to 75.0%, an increase of 12.6%. It is apparent that shifting towards a plug flow system yields greater advantages for reactions featuring very low rate constants, short detention times, and relatively low degrees of conversion.

In light of the previous discussion, it appears that neglect of the longitudinal dispersion coefficient (degree of mixing) does not lead to significant error in calculating the output of the reactor. The assumption of complete mixing provides a relatively small safety factor in reactor design yet does not lead to gross over-design of biological reactors for waste water treatment systems.

E. Significance of Mixing Levels Determined

The values of the dispersion coefficients obtained during this study ranged from 4750 to 7610 ft^2/hr. Timpany (1966), in his tests involving a 66 ft x 30 ft x 15 ft aeration tank, obtained dispersion coefficients of 6310 and 5150 ft^2/hr. If a minimum D value of 5000 ft^2/hr is assumed, when considering conventional applied air rates, then it can be concluded that most activated sludge process aeration tanks could be considered to be completely mixed. This does not consider systems wherein the aeration tanks are physically compartmentalized into tanks-in-series systems.

The outcome of this quantitative investigation into mixing indicates that much of the current literature favouring completely mixed systems appears to be over-emphasized and perhaps misleading. This could be especially true where "completely-mixed" systems are claimed to be designed. The advantages of complete mixing are well known but it appears that most activated sludge systems inherently take advantage of extensive mixing. In the case of diffused air systems, the considerable mixing taking place results from the large quantities of air being injected to satisfy the oxygen requirements of the process.

As a result of these high mixing levels encountered, perhaps a re-evaluation of the general applicability of tapered and step aeration is required. These concepts are based on the assumption that a plug flow system is frequently encountered in aeration tanks. If most aeration tanks tend towards a completely mixed system, then the modifications of tapered and step aeration are perhaps redundant.

A more illustrative example of the degree of mixing could be obtained if the suspended solids concentration profile along the tank length could be determined for various values of D/uL.

The mixing levels encountered in aeration tanks equipped with mechanical aerators should be determined. This would include aeration tanks equipped with one aerator and those featuring a series of mechanical aerators in adjacent bays. For the system featuring multiple aerators in one tank, the question of whether separate zones of influence exist at each aerator could be resolved.

CONCLUSIONS

1. The dispersion model, plug flow of a fluid with some degree of backmixing superimposed, accurately describes the extent of longitudinal mixing in a full scale aeration tank equipped with air diffusers.
2. For an aeration tank equipped with air diffusers, the peak time technique may be used to predict the theoretical response curve that agrees with the experimental response curve. Using the dispersion model, the theoretical D/uL value is obtained from a correlation with the time of peak exit concentration.
3. The water flow rate has no effect on the dispersion coefficient, D, when using fine or coarse bubble diffusers, over the range of liquid detention times of 4.8 to 13.3 hr.
4. Using coarse bubble diffusers, the air flow rate had no effect on the dispersion coefficient, D, over the range of applied air rates of 13 to 16 scfm per diffuser.
5. With the fine bubble diffusers, increasing the applied air rate resulted in a diminishing increase in the dispersion coefficient, for the range of applied air rates of 2 to 8 scfm per diffuser.

6. At identical applied air rates (scfm/1000 ft^3 tank volume), there was no significant difference in the dispersion coefficient obtained with the coarse bubble diffusers compared to that obtained using the fine bubble diffusers.
7. For design of activated sludge process aeration tanks of length to width ratios of 10:1 or less, neglect of the longitudinal dispersion coefficient will not lead to significant error in calculating the output of the reactor if complete mixing is assumed.
8. A re-evaluation of the concept and applicability of tapered and step aeration is required when considering mixing levels such as those found in this study.

RECOMMENDATIONS

1. Tracer studies should be carried out on aeration tanks equipped with mechanical aerators. This would include aeration tanks equipped with one aerator and those featuring a series of mechanical aerators in adjacent bays. For multiple aerators in adjacent bays, it could then be determined if separate zones of influence exist at each aerator.
2. The effect of physical compartmentalization of aeration tanks on liquid flow patterns should be studied.

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APPENDIX A

Fluorometer Calibration Curves

APPENDIX "A"

In Figure 1, fluorescence (plotted as fluorometer scale reading) is shown as a function of temperature for various concentrations of Rhodamine B Extra. Over the temperature range of 10° to 25°C, temperature coefficients were obtained for the concentrations shown. The following relationship between fluorescence intensity and temperature was determined.

$$\text{Log}_{10} \frac{F}{F_{20}} = 0.373 - 0.0187\ T$$

where: F = fluorescence reading in units at temperature T°C,

F_{20} = corresponding fluorescence reading at 20°C, and

T = temperature in °C.

In Figure 2, fluorescence at 20°C is plotted against concentration of Rhodamine B Extra. The resulting expression used to calculate tracer concentration was:

$$C = 1.322F_{20} + 4.945$$

where: C = tracer concentration in ppb, and

F_{20} = fluorescence reading at 20°C.

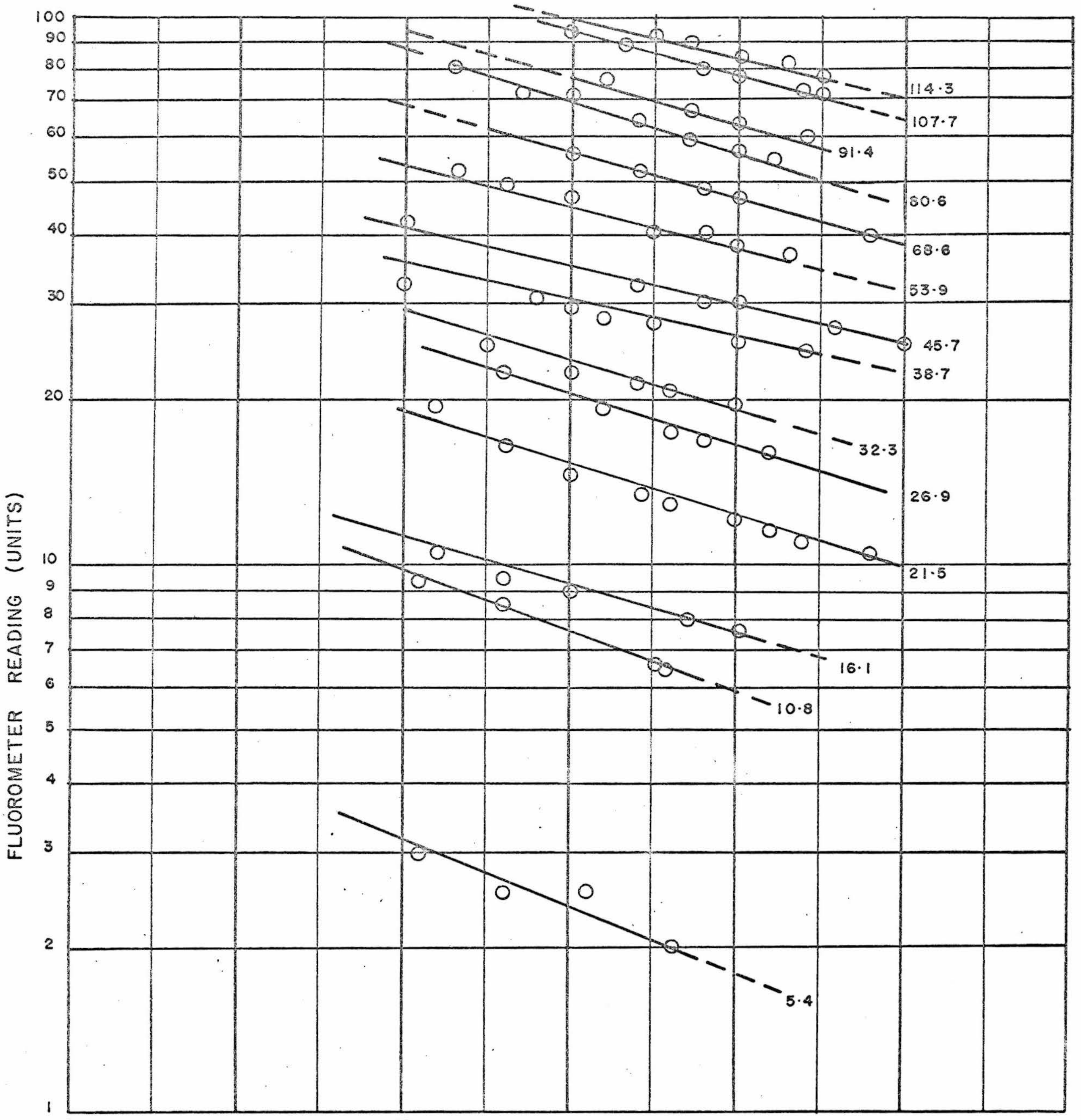
All fluorescence readings were made at a range scale sensitivity of 10X.

APPENDIX "A"

FIGURE I

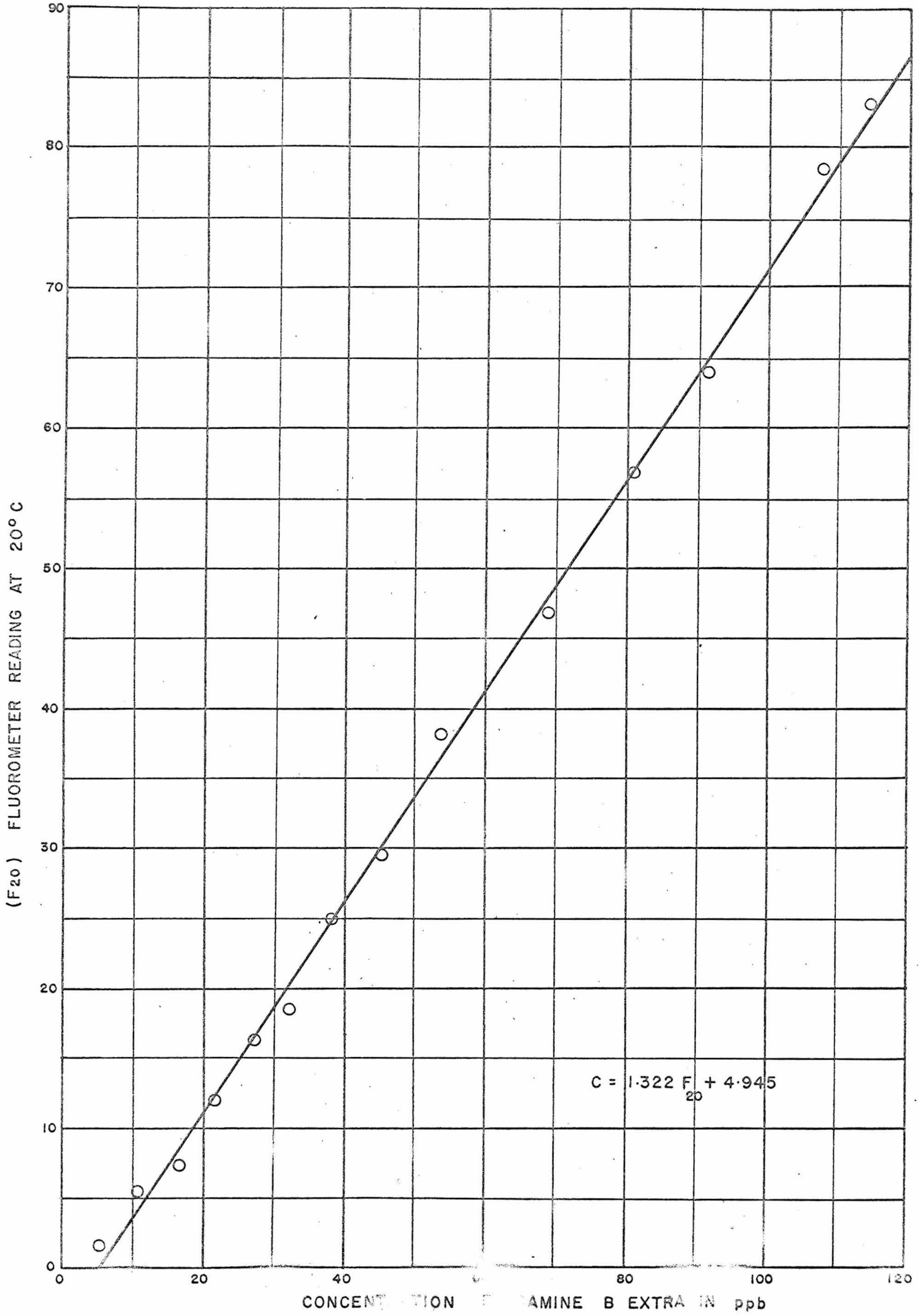
FLUORESCENCE Vs TEMPERATURE FOR VARIOUS CONCENTRATIONS OF RHODAMINE B EXTRA

(CONCENTRATIONS IN PPB)



APPENDIX "A" FIGURE 2

PLOT OF FLUORESCENCE AT 20° C
Vs
RHODAMINE B EXTRA CONCENTRATION



APPENDIX B

Experimental Design

APPENDIX B

Central Composite Rotatable Experimental Design

The purpose of an experimental design is to extract the required information with the minimum of experimental effort. The procedure is to choose a two level factorial design so that all effects of first order and all interaction effects of second order can be estimated. This design is then supplemented with further points which allow the estimation of quadratic effects. Such a design avoids unnecessary replications at each point.

A quadratic model involving two x variables (water and air flow rates) and the measured dependent variable, y, (the dispersion coefficient D) is used to model the response surface, i.e.

$$y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{12} x_1 x_2 + \beta_{22} x_2^2$$

This expression contains linear terms in x_1 and x_2, squared terms in x_1^2, and x_2^2, and the cross-product term $x_1 x_2$. The regression coefficients in this model are then estimated.

The statistical analysis of the results is performed using the analysis of variance. The sums of squares are partitioned into the contribution due to a first order equation, the additional contribution due to the second order terms, a "lack of fit" component measuring the deviations of the responses from the fitted surface, and a measure of the experimental error.

The computer programme that was written to facilitate the analysis of variance is shown in Table V.

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TABLE IV

COMPUTER NOMENCLATURE

Programme Symbol	Meaning or Equivalent
BO	β_0
B1	β_1
B2	β_2
B11	β_{11}
B12	β_{12}
B22	β_{22}
I	Counter
RB1	β_1 ratio
RB2	β_2 ratio
R2ND	Second order terms ratio
RFRST	First order terms ratio
RLF	Lack of fit ratio
SST	$\sum (x_i - \bar{x})^2$
X1	Coded water flow rate
X2	Coded air flow rate
Y	Dispersion Coefficient D
Y1BAR	Mean Y value for replicates

TABLE V

EXPERIMENTAL DESIGN

ANALYSIS OF VARIANCE

```
C     ANALYSIS OF VARIANCE FOR ROTATABLE CENTRAL COMPOSITE EXPERIMENTAL
C     DESIGN
C     REPLICATE TEST DATA INTRODUCED FIRST
C     SECOND ORDER MODEL
C     Y=B0+B1X1+B2X2+B11(X1**2)+B12X1X2+B22(X2**2)
      DIMENSION  X1(20),X2(20),Y(20)
      READ (5,10)   (X1(I),X2(I),Y(I), I=1,13)
   10 FORMAT (3F20.5)
C     CALCULATION OF VARIOUS SUMS
C     P=SUMX1X1 , Q=SUMX2X2 , R=SUMX1Y , S=SUMX2Y , T=SUMX1X2Y ,
C     U=SUMX1X1X2X2 , V=SUMX1X1Y , W=SUMX1X1X1X1 , Z=SUMX2X2X2X2 ,
C     ZZ=SUMY , VV=SUMX2X2Y , PP=SUMYY
      P=0.0
      PP=0.0
      Q=0.0
      R=0.0
      S=0.0
      T=0.0
      U=0.0
      V=0.0
      VV=0.0
      W=0.0
      Z=0.0
      ZZ=0.0
      DO 20 I=1,13
      P=P+X1(I)**2
      PP=PP+Y(I)**2
      Q=Q+X2(I)**2
      R=R+X1(I)*Y(I)
      S=S+X2(I)*Y(I)
      T=T+X1(I)*X2(I)*Y(I)
      U=U+X1(I)**2*X2(I)**2
      VV=VV+X2(I)**2*Y(I)
      V=V+X1(I)**2*Y(I)
      W=W+(X1(I)**2)*(X1(I)**2)
      Z=Z+(X2(I)**2)*(X2(I)**2)
   20 ZZ=ZZ+Y(I)
C     CALCULATION OF COEFFICIENTS
      B0=((ZZ*W-V*P)*(Z*W-U*U)+(V*U-VV*W)*(Q*W-U*P))/((W*13.0-P*P)*
     1(Z*W-U*U)+(P*U-Q*W)*(W*Q-U*P))
      B22=(B0*(W*13.0-P*P)-ZZ*W+V*P)/(U*P-Q*W)
      B11=(ZZ-13.0*B0-B22*Q)/P
      B1=R/P
      B2=S/Q
      B12=T/U
C     CALCULATION OF SUMS OF SQUARES FOR VARIOUS SOURCES
C
C     D=SUM OF SQUARES FOR FIRST ORDER TERMS
C     DEGREES OF FREEDOM = 2.0
      D=B1*R+B2*S
```

```
C     E=SUM OF SQUARES FOR SECOND ORDER TERMS
C     DEGREES OF FREEDOM = 3.0
      E=B0*ZZ+B11*V+B22*VV+B12*T-(ZZ**2)/13.0
C     TOTAL SUM OF SQUARES
C     DEGREES OF FREEDOM = 12.0
      SST=PP-(ZZ**2)/13.0
C     F=EXPERIMENTAL ERROR SUM OF SQUARES
C     DEGREES OF FREEDOM = 4.0
      Y1BAR=(Y(1)+Y(2)+Y(3)+Y(4)+Y(5))/5.0
      F=0.0
      DO 30 I=1,5
   30 F=F+(Y(I)-Y1BAR)**2
      FF=F/4.0
C     G=LACK OF FIT SUM OF SQUARES
C     DEGREES OF FREEDOM = 3.0
      G=SST-F-D-E
C
C     RATIOS VS GIVEN F VALUES
C
C     HYPOTHESIS THAT LACK OF FIT CONTAINS NO MORE THAN REPEATABILITY
C    1ERROR
      RLF=(G/3.0)/FF
      WRITE (6,2) RLF
    2 FORMAT (1H1,21H LACK OF FIT RATIO = ,F7.2,5X,59H COMPARED WITH F(3
     1,4,0.05) = 6.59 AND F(3,4,0.01) = 16.69   /)
      IF (RLF.GT.6.59) GO TO 4
      WRITE (6,3)
    3 FORMAT (1H0,55H LACK OF FIT CONTAINS NO MORE THAN REPEATABILITY ER
     1ROR //)
C     IF RLF INSIGNIFICANT,LUMP TOGETHER WITH REPEATABILITY ERROR
      FF=(G+F)/7.0
    4 CONTINUE
C     SIGNIFICANCE OF FIRST ORDER TERMS
      RFRST=(D/2.0)/FF
      WRITE (6,21) RFRST
   21 FORMAT (1H0,27H FIRST ORDER TERMS RATIO = ,F7.2,5X,35H COMPARED WI
     1TH F(2,7,0.05) = 4.74  /)
C     SIGNIFICANCE OF B1 AND B2
      IF (RFRST.LT.4.74) GO TO 29
      RB1=B1*R/FF
      RB2=B2*S/FF
      WRITE (6,22) RB1,RB2
   22 FORMAT (1H0,12H B1 RATIO = ,F7.2,12H B2 RATIO = ,F7.2,5X,34H COMPA
     1RED WITH F(1,7,0.05) = 5.59 /)
   29 CONTINUE
C     SIGNIFICANCE OF SECOND ORDER TERMS
      R2ND=(E/3.0)/FF
      WRITE (6,31) R2ND
   31 FORMAT (1H0,28H SECOND ORDER TERMS RATIO = ,F7.2,5X,34H COMPARED W
     1ITH F(3,7,0.05) = 4.35 /)
C NOTE--  GIVE F VALUES ASSUME LACK OF FIT MEAN SQUARE CONTAINS ONLY
C       EXPERIMENTAL ERROR
      WRITE (6,32) B0,B1,B2,B12,B11,B22
   32 FORMAT (6F20.3)
      STOP
      END
```

APPENDIX C

Computer Programme used for Response Curve Analysis

TABLE VI

COMPUTER NOMENCLATURE

Programme Symbol	Meaning or Equivalent
AT	Counter
C(I)	Theoretical C/Co Summation Term
CO	Co
DNPKT	D/uL by peak time
DPKT	D by peak time
F/F20	Fluorescence at WT^{o}C/fluorescence at 20^{o}C
FC(I)	Experimental C
FR	Function
FR(I)	Fluorometer Reading
FRO(I)	Fluorometer Reading at 20^{o}C
I,K,L,	Counters
N	Number of data points
O	Theoretical $t/\bar{t}$
P	D/uL by peak time
PCREC	Experimental percent of tracer recovery
PKT	Experimental peak time
QA	Q_A
QW	Q_W

TABLE VI (Continued)

Programme Symbol	Meaning or Equivalent
R(I)	μ_i
RC(I)	Experimental C/Co
REC	Theoretical percent tracer recovery
RPKT	$t_p/\bar{t}$
RT(I)	Experimental $t/\bar{t}$
RTEND	Last value of experimental reduced time
RTINC	Experimental reduced time interval
SC(K)	Theoretical C/Co
SUMC	C
TBAR	Theoretical Detention time
TINC	Experimental time increment
TKD	Tank depth
TKL	Tank length
TKW	Tank width
TN	Test number
TRACIN	Mass of tracer dosed
TVALUE(I)	Theoretical C/Co
U	Bulk velocity
WT	Water temperature
XYZ	Theoretical C/Co

TABLE VII

EXPERIMENTAL AND THEORETICAL RESPONSE CURVE ANALYSIS

```
C     EXPERIMENTAL RESPONSE CURVE ANALYSIS FOR PEAK TIME TESTS ONLY
      DIMENSION FR(1000),FRO(1000), FC(1000), RT(1000)
      COMMON TVALUE(2000) , RC(1000)
C     READ TEST CONDITIONS
    1 READ (5,10) TN,TKL,TKW,TKD,QA,QW,WT
   10 FORMAT (7F8.3)
      IF(TN.EQ.999.9) GO TO 990
C     READ ANALYSIS CONDITIONS
      READ (5,11) TRACIN,PKT,TINC,N
   11 FORMAT (3F10.5,I10)
C     DATA INPUT
      READ (5,12) (FR(I), I=1,N)
   12 FORMAT (16F5.1)
      TBAR=TKL*TKW*TKD*6.229/QW/60.0
C     CONVERSION TO STANDARD FLUORESCENCE CONCENTRATION
C     Z=LOGF/F20,ZZ=F/F20
      Z=0.37245-0.01867*WT
      ZZ=0.0
   13 ZZ=0.1+ZZ
      IF(ALOG10(ZZ).LT.Z) GO TO 13
   14 ZZ=ZZ-0.01
      IF (ALOG10(ZZ).GT.Z) GO TO 14
   15 ZZ=0.001 + ZZ
      IF (ALOG10(ZZ).LT.Z) GO TO 15
      DO 20 I=1,N
      FRO(I)=FR(I)/ZZ
   20 FC(I)=(FRO(I)+3.73629)/0.75647
C     DISPERSION NUMBER CALCULATION FROM PEAK TIME
      RPKT=PKT/TBAR
      IF (RPKT.LE.0.3) DNPKT=0.2*RPKT**(-1.3355)
      IF (RPKT.GT.0.3) DNPKT=4.027*10.0**(-2.09*RPKT)
      U=TKL/TBAR
      DPKT=DNPKT*U*TKL
C     PRINT PARAMETERS WITH IDENTIFICATION
      WRITE (6,50) TN,TKL,TKW,TKD,QA,QW,WT,TINC,N,TBAR,DNPKT,DPKT
   50 FORMAT(1H1,10H TEST NO. ,F6.1,3X,13H TANK LENGTH ,F6.1,5H FEET,3X,
     112H TANK WIDTH ,F5.1,5H FEET,3X,11HTANK DEPTH ,F5.1,5H FEET,3X,9HA
     2IR FLOW ,F7.0,4H CFM//12H  WATER FLOW,F7.0,4HIGPM,3X,13HWATER  TEM
     3P. ,F5.1, 14H DEGREES CENT.,3X, 14HTIME INCREMENT ,F6.2, 5H MIN.,
     43X, 17H NO. OF DATA PTS.,I5,//1X,19H THEOR. DETN. TIME  ,F6.2,
     53H HR,3X,27HDISPERSION NO. BY PEAK TIME    ,
     6  ,F8.4,3X,35HDISPERSION COEFFICIENT BY PEAK TIME  ,F9.2////)
C
C     CALCULATION OF PERCENT RECOVERY
      SUMC=0.0
      DO 300 I=1,N
  300 SUMC=SUMC+FC(I)
      PCREC=SUMC*TINC*QW*4.5459*1.0E-6/TRACIN*100.0
      WRITE (6,301)  PCREC
  301 FORMAT (1H0,34H EXPERIMENTAL PER CENT RECOVERY = ,F7.2//)
      CALCULATION OF NORMALIZED DISPERSION CURVE
      CO=TRACIN/(TKL*TKW*TKD*28.316 ) * 1.0E6
      RTINC=TINC/TBAR/60.0
```

```
      DO 60 I=1,N
      AT=I
      RC(I)=FC(I)/CO
   60 RT(I) = AT * RTINC
      RTEND = RT(N)
C     PRINT NORMALIZED DISPERSION CURVE DATA
      WRITE (6,70) (RC(I),RT(I),I=1,N)
   70 FORMAT (1H0,///30X,29H NORMALIZED DISPERSION CURVE //8(16H C/CO
     1T/BART   )//(8(F6.3,F9.4,1X)))
      P=DNPKT
      CALL THEORY ( P, RTINC, RTEND )
      CALL HPLOT (RTINC,N)
      GO TO 1
  990 WRITE(6,999)
  999 FORMAT(1H1)
      STOP
      END
$IBFTC THEORY
      SUBROUTINE THEORY ( P, RTINC, RTEND )
C     THEORETICAL DISPERSION CURVE ANALYSIS FOR PULSE INPUT
      DIMENSION          R(1000),SC(100),C(100),DT(10)
      COMMON TVALUE(2000)
      WRITE (6,7) P
    7 FORMAT(1H1,6H D/UL=,F9.4,10X,38HDIMENSIONLESS CURVE AT RTIM INTERV
     1ALS  ,53X,6H TIME    )
C     P=D/UL,  NOTE - U IS BULK VELOCITY
      I=1
C     NOTE - FOLLOWING U TERM REPRESENTS 1/2 THE PECLET NUMBER
      U=0.5/P
      R(I)=1.4
      O IS REDUCED PEAK TIME, O=TP/TBAR
      O=0.0
C     DETERMINING R(I) = UN FOR N = 1 TO 50
   10 R(I)=R(I)-0.001
      FR=COS(R(I))/SIN(R(I))-R(I)*P+0.25/(R(I)*P)
      IF (FR) 10,10,20
   20 R(I)=R(I)+0.00001
      FR=COS(R(I))/SIN(R(I))-R(I)*P+0.25/(R(I)*P)
      IF (FR) 25,20,20
   25 R(I)=R(I)-0.0000001
      FR=COS(R(I))/SIN(R(I))-R(I)*P+0.25/(R(I)*P)
      IF (FR) 25,28,28
   28 I=I+1
      R(I)=R(I-1)+3.1416
      IF(I.LE.49 )   GO TO 10
      WRITE (6,29) (R(I),I=1,50)
   29 FORMAT (1H ,10F10.5)
C     DETERMINING VALUE OF C(I) = C/CO FOR VALUES OF REDUCED PEAK TIME,
      L=0
      SUMC=0.0
   30 DO 46 K=1,10
      L = L+1
      O = O+RTINC
      SC(K)=0.0
      DO 40 I=1,50
      C(I)=2.*R(I)*(U*SIN(R(I))+R(I)*COS(R(I)))*EXP(U-((U**2+R(I)**2)/(2
     1.0*U))*O)/(U**2+2.0*U+R(I)**2)
C     SC(K) = FINAL VALUE OF C/CO
```

```
      SC(K)=SC(K)+C(I)
      IF(I.EQ.1) GO TO 40
      IF(ABS(C(I)+C(I-1)).LT.0.001)GOTO45
   40 CONTINUE
   45 CONTINUE
      XYZ = SC(K)
      TVALUE(L) = XYZ
      SUMC=SUMC+SC(K)
   46 CONTINUE
      WRITE(6,50) (SC(K),K=1,10),O
   50 FORMAT(1H0,10F10.4,15X,F10.4)
   55 IF(O.LE.RTEND) GO TO 30
      REC=SUMC*RTINC*100.0
      WRITE (6,301) REC
  301 FORMAT (1H0,24H THEORETICAL RECOVERY =    , F7.2//)
      RETURN
      END
$IBFTC HPLOT
      SUBROUTINE HPLOT (RTINC,N)
      DIMENSION LINE(130)
      COMMON TVALUE(2000) , RC(1000)
      INTEGER BLANK, DOT, X, STAR, PLUS
      DATA BLANK, DOT, X, STAR, PLUS / 1H , 1H., 1HX, 1H*, 1H+ /
C     SETTIN UP X AXIS WITH GRID
      NSBHL = 10
      NSBHLN = NSBHL
      DO 910 L=1,130
  910 LINE(L) = DOT
      DO 920 L=11,121,10
  920 LINE(L) = PLUS
      WRITE(6,930) RTINC, LINE
  930 FORMAT( 1H1, 7HRTINC =, F8.4 /// 1X, 130A1 )
C     PRINTING DATA POINTS
      DO 940 L=2,129
  940 LINE(L) = BLANK
      III=0
C     FOLLOWING STATEMENT CONTROLS NUMBER OF DATA POINTS TO BE PRINTED
      DO 960  I=1,N
      III=III+1
      LINE(1) = DOT
      LINE(130) = DOT
C     BACKGROUND GRID MARKINGS
      IF (III.NE.NSBHLN )  GO TO 950
      DO 945 II=1,121,10
  945 LINE(II) = PLUS
      NSBHLN = NSBHLN + NSBHL
  950 L= ABS(RC(I)*100.0 + 0.5 )
      IF(L.LE.0) L=1
      LINE(L) = X
      LT= ABS(TVALUE(I)*100.0 + 0.5 )
      IF(LT.LE.0) LT=1
      LINE(LT) = STAR
      WRITE(6,955) LINE
  955 FORMAT(1H , 130A1 )
      DO 960 L=1,130
  960 LINE(L) = BLANK
C     ENCLOSING PLOT WITH ANOTHER Y AXIS
      DO 970 L=1,130
```

```
970 LINE(L) = DOT
    DO 980 L=11,121,10
980 LINE(L) = PLUS
      WRITE(6,955) LINE
    RETURN
    END
```

APPENDIX D

Nomenclature

TABLE VIII

Nomenclature

a = $\sqrt{1 + 4k\bar{t}\,(D/uL)}$

C = Exit concentration of tracer at time t (mg/l)

C_o = mass of tracer added divided by nominal tank volume (mg/l)

D = axial dispersion coefficient (ft^2/hr)

k = first order rate constant (hr^{-1})

L = tank length (ft)

Q_A = applied air flow rate (ft^3/min)

t = time (hr)

$\bar{t}$ = theoretical detention time (hr)

t_p = time of peak exit concentration (hr)

u = bulk velocity along tank length (ft/hr)

U = $uL/2D$

x = distance along tank length (ft)

Greek Symbols

θ = time divided by the theoretical detention time calculated from tank volume and flow rate.

$\mu_n = \mu_1, \mu_2, \mu_3, \ldots . \mu_n = \cot^{-1}\left[1/2\left(\frac{\mu_n}{U} - \frac{U}{\mu_n}\right)\right]$

σ^2 = variance or second moment of a curve about its mean