

CALCULATIONS OF REDUCED PROBABILITY
FOR E2 TRANSITIONS

CALCULATIONS OF REDUCED PROBABILITY
FOR E2 TRANSITIONS
IN DEFORMED EVEN-EVEN NUCLEI

By

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SCOPE AND CONTENTS:

The reduced probability of E2 transitions between rotational levels built upon γ -vibrational states was calculated for even-even nuclei. General expressions were derived as functions of the spin of the initial state and a parameter γ_{10} . Branching ratios for special cases were obtained, which compare quite favourably with experiment.

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I. Introduction

At a given instant of time the individual motion of the nucleons of a nucleus altogether form a particular pattern, which defines the nuclear surface. As the nucleons change their paths, the pattern changes consequently. There can be a change in the orientation as well as in the shape of the pattern. These are respectively described as the rotation and vibration of the nucleus. Clearly, such motion must be attributed to all the nucleons and are therefore termed collective. Thus there is a definite correlation between the particle and the collective motions.

The nuclear surface may be specified quite generally by

$$R = R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} a_{\lambda\mu} Y_{\lambda}^{\mu}(\theta, \psi) \right] \quad (I-1)$$

The expansion coefficients $a_{\lambda\mu}$ are actually normal coordinates, i.e. the kinetic and the potential energy of the surface are given by

$$T = \frac{1}{2} \sum_{\lambda,\mu} B_{\lambda} |\dot{a}_{\lambda\mu}|^2$$

and
$$V = \frac{1}{2} \sum_{\lambda,\mu} C_{\lambda} |a_{\lambda\mu}|^2$$

respectively. The Lagrangian

$$L = T - V$$

is then a sum of terms, each corresponding to a simple harmonic oscillator with frequency

$$\omega_{\lambda} = \left(\frac{C_{\lambda}}{B_{\lambda}} \right)^{\frac{1}{2}}$$

For $\lambda=0$, α_0 is associated with a change in volume. Since nuclear matter is highly incompressible, such vibration only occurs at very high energy. The terms in $\lambda=1$ merely describe the translational motion of the centre of mass and have no effect on the internal structure of the nucleus. Therefore usually only the $\lambda=2$ terms are of interest, and only such quadrupole vibrations will be considered in the following.

It is convenient to use a body-fixed coordinate system. In such a system the nuclear surface is specified by

$$R = R_0 \left[1 + \sum_{\mu} A_{2\mu} Y_2^{\mu}(\theta', \phi') \right] \quad (\text{I-2})$$

The connection between equations (I-1) and (I-2) is determined by the transformation

$$\alpha_{2\mu} = \sum_{\nu} A_{2\nu} (D_{\mu\nu}^2)^*$$

and

$$Y_{\lambda}^{\mu}(\theta, \phi) = \sum_{\rho} Y_{\lambda}^{\rho}(\theta', \phi') D_{\mu\rho}^{\lambda}(\theta_i)$$

where the $D_{\mu\rho}^{\lambda}$ are the usual D-functions and θ_i the Eulerian angles.

If one chooses the principal axes for the body-fixed coordinate system,

then simplification results in that $A_{21} = A_{2-1} = 0$, and $A_{22} = A_{2-2}$.

The coefficients A_{20} and A_{22} together with the Eulerian angles are the collective coordinates. They specify the nuclear orientation as well as its shape.

It is desirable, however, to make the further substitution

$$A_{20} = \beta \cos \gamma$$

and
$$A_{22} = \frac{1}{\sqrt{2}} \beta \sin \gamma$$

Then the deviation of the nuclear surface from a sphere of radius R_0 along the k -axis is given by

$$\delta R_k = \sqrt{\frac{5}{4\pi}} R_0 \beta \cos \left(\gamma - \frac{2\pi k}{3} \right)$$

Thus γ is a shape parameter. The range $0 \leq \gamma \leq \frac{\pi}{3}$ exhausts all possible shapes. For larger values of γ it merely amounts to a different labeling of the axes. Both β and γ change with respect to time. In β -vibration the nuclear symmetry, if any, is preserved whereas it is not so in γ -vibration.

With reference to the body-fixed coordinate system, the Hamiltonian $H = T + V$, when expressed in terms of the new set of coordinates, becomes

$$H = \frac{1}{2} B (\dot{\beta}^2 + \beta^2 \dot{\gamma}^2) + \frac{1}{2} \sum_k I_k \omega_k^2 + \frac{1}{2} C \beta^2$$

where
$$I_k = 4B\beta^2 \sin^2 \left(\gamma - \frac{2\pi k}{3} \right)$$

is the moment of inertia about the k -axis, and ω_k is the angular velocity of rotation. To construct the wave equation, one transforms the Hamiltonian into (Bohr, 1952)

$$H = -\frac{\hbar^2}{2B} \left[\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2} \frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} \right] + \frac{1}{2} \sum_k I_k \omega_k^2 + \frac{1}{2} C \beta^2$$

This equation, as it stands, represents the Hamiltonian of a nucleus with a spherical equilibrium shape. With additional nucleons outside the closed shell the nucleus will attain a permanent deformation about $\beta = \beta_1$ and $\gamma = \gamma_1$. The potential energy will also accordingly change from $\frac{1}{2}C\beta^2$ to $\frac{1}{2}C\beta^2 + \psi(\beta, \gamma)$. If one restricts to even-even nuclei, and also to low-lying states only, then the Hamiltonian may be written as

$$H = H_0 + U_3$$

$$\text{where } H_0 = -\frac{\hbar^2}{2B} \left[\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2} \frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} \right] \\ + \left(\frac{\hbar^2}{4I_1} + \frac{\hbar^2}{4I_2} \right) [J(J+1) - 2K^2] + \frac{\hbar^2}{2I_3} K^2 \\ + W(\beta, \gamma),$$

$$\text{and } U_3 = \left(\frac{\hbar^2}{4I_1} - \frac{\hbar^2}{4I_2} \right) (J_1^2 - J_2^2),$$

which is small compared with H_0 and can be interpreted as the interaction between rotation and γ -vibration. The purpose is obviously to treat U_3 as perturbation. For small oscillations one can expand

$$W(\beta, \gamma) = W(\beta_1, \gamma_1) + \left(\frac{\partial^2 W}{\partial \beta^2} \right)_{\beta_1, \gamma_1} \frac{(\beta - \beta_1)^2}{2} + \left(\frac{\partial^2 W}{\partial \gamma^2} \right)_{\beta_1, \gamma_1} \frac{(\gamma - \gamma_1)^2}{2} + \dots$$

One then sets to solve

$$H_0 \psi(\beta, \gamma) = E^{(0)} \psi(\beta, \gamma)$$

by substituting

$$\psi(\beta, \gamma) = \xi(x) g(y)$$

with $\beta, x = \beta - \beta_1$, and $y = \gamma - \gamma_1$. Two separate equations result, which can be denoted as

$$H_x \xi(x) = E_{\beta} \xi(x) \quad (I-3)$$

and $H_y g(y) = E_{\gamma} g(y) \quad (I-4)$

II. The γ -vibration Equation for Axially Symmetrical Nuclei

For nuclei with axial symmetry the equilibrium point is given by $(\beta_1, \gamma_1 = 0)$. The explicit form of equation (I-4) is then

$$\left[-\frac{\hbar^2}{2B\beta_1^2} \frac{1}{\gamma} \frac{\partial}{\partial \gamma} \gamma \frac{\partial}{\partial \gamma} + \frac{1}{2} \ell \gamma^2 + \frac{\hbar^2}{8B\beta_1^2} \frac{K^2}{\gamma^2} \right] g(\gamma) = E_\gamma g(\gamma) \quad (\text{II-1})$$

where

$$\ell \equiv \left(\frac{\partial^2 W}{\partial \gamma^2} \right)_{\substack{\beta = \beta \\ \gamma = 0}}$$

This resembles the radial equation of a two dimensional oscillator, with eigenvalues

$$E_{\gamma, n} = \hbar \omega_\gamma (n_\gamma + 1) \quad n_\gamma = 0, 1, 2, \dots$$

There are, however, two essential differences. While the variable

ρ in a two dimensional oscillator problem varies only from zero to infinity, here the variable γ can take up negative values as well.

Secondly the differential element corresponding to the variable ρ is $\rho d\rho$ in the oscillator case, whereas that corresponds to γ is $|\sin 3\gamma| d\gamma$. For small oscillation the latter can be replaced by $3|\gamma| d\gamma$, or by $|\gamma| d\gamma$ if one adjusts the normalization factors of the g -functions.

To solve equation (II-1), one tries a series solution

$$g(\gamma) = f(\gamma) e^{-x^2}$$

where

$$f(\gamma) = \sum_{r=0}^{\infty} A_r x^{r+s}$$

$$\text{and } x^2 \equiv \frac{\beta_1 \sqrt{B_1}}{2\hbar} \gamma^2.$$

Using the usual series solution procedure, one finds that the proper choice for s is $s = \frac{K}{2}$ and the recurrence formula is

$$A_{r+2} = \frac{4(r-n_r + K/2)}{(r+2)(r+2+K)} A_r$$

The series f will terminate at $r = n_r - K/2$. The solutions $g(\gamma)$ depend on both n_r and K , and therefore will be denoted by $g_{n_r}^{(K)}(\gamma)$. For example,

$$g_0^{(0)}(\gamma) = A_0 e^{-x^2}$$

The coefficient A_2 must be determined by normalizing the corresponding g -function to unity, integrating over the element $|\gamma| d\gamma$ from $-\infty$ to $+\infty$. The first few normalized functions are given here for later uses:

$$g_0^{(0)}(\gamma) = \sqrt{2t} e^{-t\gamma^2}$$

$$g_1^{(2)}(\gamma) = 2t\gamma e^{-t\gamma^2}$$

$$g_2^{(0)}(\gamma) = \sqrt{2t} (1 - 2t\gamma^2) e^{-t\gamma^2} \quad (\text{II-2})$$

$$g_2^{(4)}(\gamma) = \sqrt{8t} t\gamma^2 e^{-t\gamma^2}$$

$$g_3^{(2)}(\gamma) = \sqrt{8} t\gamma(1-t\gamma^2) e^{-t\gamma^2}$$

$$g_3^{(6)}(\gamma) = \sqrt{\frac{8}{3}} t^2 \gamma^3 e^{-t\gamma^2},$$

with

$$t \equiv \frac{\beta_1}{2\hbar} \sqrt{kB}$$

III. Interaction between γ -vibration and Rotation

In the present work it is intended to examine the radiative transition E2 between rotational bands built upon different γ -vibrational states. Therefore only the rotational and γ -vibrational energy spectra are of concern. Furthermore, only low-lying states will be considered. In transitions it is the change in energy that really matters. With these in mind, the energy level can be denoted by

$$E_{n_\gamma K J} = (n_\gamma + 1) \hbar \omega_\gamma + \frac{\hbar^2}{6B\beta_1^2} [J(J+1) - 2K^2]$$

The interaction between rotation and γ -vibration has been designated by U_3 , and for small vibrations one can expand the I_K in U_3 to get

$$U_3 \approx - \frac{\hbar^2}{3B\beta_1^2} \frac{\gamma}{\sqrt{3}} (J_1^2 - J_2^2)$$

The operator $(J_1^2 - J_2^2)$ has non-vanishing matrix elements only between states of same J , but with K differing by ± 2 :

$$(J, K | J_1^2 - J_2^2 | J, K \pm 2) = \frac{1}{2} [(J \pm K)(J \mp K - 1)(J \mp K + 1)(J \pm K + 2)]^{1/2} \quad (\text{III-1})$$

and γ only connects states differing in n_γ by ± 1 . Some explicit expressions for the matrix elements of γ are

$$(g_0^{(0)} | \gamma | g_1^{(2)}) = \sqrt{\frac{1}{2t}} = \sqrt{\frac{\hbar^2}{B\beta_1^2}} \frac{1}{\hbar \omega_\gamma} \equiv \gamma_{10}$$

$$(g_1^{(2)} | \gamma | g_2^{(0)}) = -\gamma_{10}$$

$$(g_1^{(2)} | \gamma | g_2^{(4)}) = 2\gamma_{10}$$

$$(g_2^{(0)} | \gamma | g_3^{(2)}) = \sqrt{2} \gamma_{10} \quad (\text{III-2})$$

$$(g_2^{(4)} | \gamma | g_3^{(2)}) = -\sqrt{2} \gamma_{10}$$

and $(g_2^{(4)} | \gamma | g_3^{(6)}) = \sqrt{6} \gamma_{10}$

Thus the interaction U_3 connects the state (n_γ, K, J) with $(n_\gamma \pm 1, K \pm 2, J)$. Figure 1 represents a schematic energy level diagram.

This will serve as a visual aid in the following calculation.

Allowed Energy Levels

K	0	2	4	6
J	0, 2, 4, ---	2, 3, 4, ---	4, 5, 6, ---	6, 7, 8, ---
n_γ	0, 2, 4, --	1, 3, 5, --	2, 4, 6, ---	3, 5, 7, ---

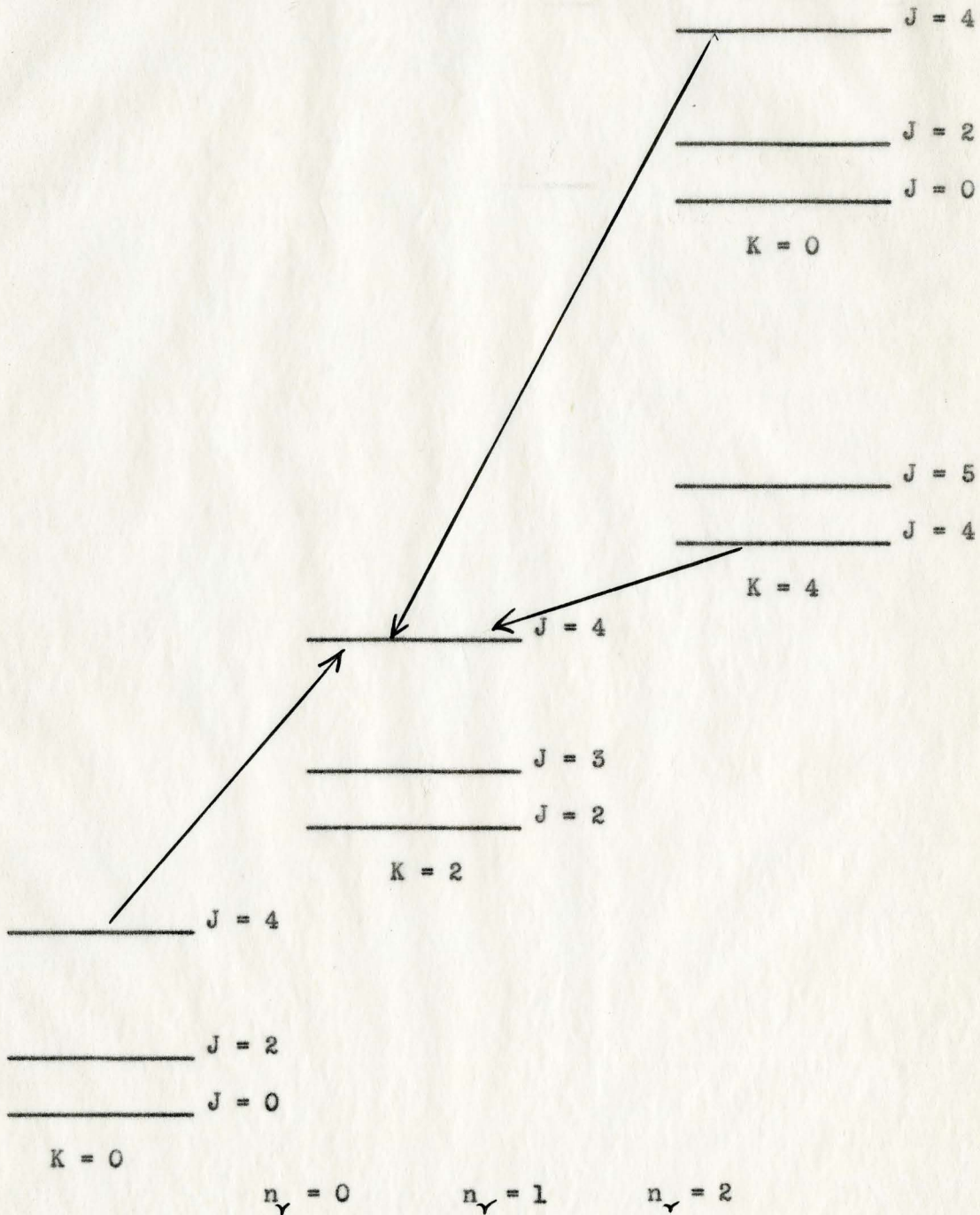


Fig. 1. Schematic Energy Level Diagram. Arrows illustrate admixture added to the state ($n_\gamma = 1, K = 2, J = 4$).

IV. The Reduced Probability of E2 Transitions

The reduced probability of E2 transitions are here classified into five groups according to where the initial and final states are.

1. Transitions from $(n_\gamma = 1, K = 2, J_i)$ to $(n_\gamma = 0, K = 0, J_f)$

As already said, only the rotational and γ -vibrational parts of the total wave function will be of concern. Let the unperturbed wave functions be denoted by $\phi_{n_\gamma K J}(\gamma, \theta_i)$ while perturbed ones by $\Psi_{n_\gamma K J}(\gamma, \theta_i)$. No distinction will be made between wave functions and state vectors, as ambiguity is unlikely to occur. Owing to the interaction U_3 , the initial state $(n_\gamma = 1, K = 2, J_i)$ is in general mixed with three other levels, namely, $(n_\gamma = 0, K = 0, J_i)$, $(n_\gamma = 2, K = 0, J_i)$, and $(n_\gamma = 2, K = 4, J_i)$. (See illustration in fig. 1.) Applying time-independent perturbation theory with the aid of equations (III-1) and (III-2), one gets

$$\Psi_{12J_i} = \phi_{12J_i} - \frac{[1+(-1)^{J_i}]}{2} (a_- \phi_{00J_i} + a_+ \phi_{20J_i}) - b \phi_{24J_i}$$

where

$$a_{\pm} = \frac{1}{3} \sqrt{\frac{(J_i - 1)J_i(J_i + 1)(J_i + 2)}{6}} \frac{\gamma_{10}^3}{(1 \pm \frac{2}{3}\gamma_{10}^2)}$$

and

$$b = -\frac{1}{3} \sqrt{\frac{(J_i - 3)(J_i - 2)(J_i + 3)(J_i + 4)}{3}} \frac{\gamma_{10}^3}{(1 - 2\gamma_{10}^2)}$$

The unperturbed wave functions can be written as the product of $g_{n_\gamma}^{(K)}(\gamma)$ and the D-functions which describe the rotational part:

$$\psi_{n_r K J}(\gamma, \theta_i) = g_{n_r}^{(K)}(\gamma) \sqrt{\frac{2J+1}{(16\pi^2(1+\delta_{K0}))}} \left[D_{MK}^J + (-1)^J D_{M-K}^J \right]$$

The D-functions here are those used by Kerman (1959).

The final states in the present case are ($n_r = 0, K = 0, J_f$), and are described by

$$\Psi_{00J_f} = \phi_{00J_f} + d \phi_{12J_f}$$

with

$$d = \frac{1}{3} \sqrt{\frac{(J_f-1)J_f(J_f+1)(J_f+2)}{6}} \frac{\gamma 10^3}{(1-\frac{2}{3}\gamma 10^2)}$$

The E2 transition operator in the body-fixed system is given explicitly by Davydov and Filippov (1958)

$$Q_{2\mu} = \sqrt{\frac{5}{16\pi}} e Q_0 \left[D_{\mu 0}^2 \cos \gamma + \frac{D_{\mu 2}^2 + D_{\mu, -2}^2}{\sqrt{2}} \sin \gamma \right]$$

where

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta_1$$

is the intrinsic quadrupole moment. As the vibration is assumed to be small, the operator $Q_{2\mu}$ is approximated by

$$Q_{2\mu} = \sqrt{\frac{5}{16\pi}} e Q_0 \left[D_{\mu 0}^2 (1 - \frac{\gamma^2}{2}) + \frac{D_{\mu 2}^2 + D_{\mu, -2}^2}{\sqrt{2}} \gamma \right]$$

The operator γ^2 has non-vanishing elements for $\Delta n_r = 0$ as well as for $\Delta n_r = \pm 2$. In fact

$$\begin{aligned}
(g_0^{(0)} | \gamma^2 | g_0^{(0)}) &= \gamma_{10}^2 \\
(g_1^{(2)} | \gamma^2 | g_1^{(2)}) &= 2\gamma_{10}^2 \\
(g_2^{(0)} | \gamma^2 | g_2^{(0)}) &= 3\gamma_{10}^2 \\
(g_2^{(4)} | \gamma^2 | g_2^{(4)}) &= 6\gamma_{10}^2 && \text{(IV-1)} \\
(g_0^{(0)} | \gamma^2 | g_2^{(0)}) &= -\gamma_{10}^2 \\
(g_0^{(0)} | \gamma^2 | g_2^{(4)}) &= 2\gamma_{10}^2 \\
(g_1^{(2)} | \gamma^2 | g_3^{(2)}) &= -\sqrt{2}\gamma_{10}^2
\end{aligned}$$

The reduced probability of E2 transition is defined as

$$B(E2; i \rightarrow f) = \frac{1}{(2J_i + 1)} \sum_{\mu, M_i, M_f} | (f | Q_{2\mu} | i) |^2$$

This is the definition used by Kerman (1959) and is equivalent to that used by Davydov and Filippov (1958). This is related to the transition probability by

$$T(E2; i \rightarrow f) = \frac{4\pi}{75} \frac{1}{\hbar^6} \left(\frac{\Delta E}{c} \right)^5 B(E2; i \rightarrow f)$$

where ΔE is the transition energy. The transition probability itself is directly proportional to the intensity of transition, which is what experimentally measured.

To calculate $B(E2)$, equations (III-1), (III-2), (IV-1) and the following will be useful:

$$\left(D_{M_f K_f}^{J_f} \mid D_{\mu \nu}^2 \mid D_{M K}^J \right) = \frac{8\pi^2}{2J_f+1} (J_i 2K_i \nu \mid J_f K_f) (J_i 2M_i \mu \mid J_f M_f)$$

$$(J_i 2M_i \mu \mid J_f M_f) = (-1)^{J_i - M_i} (J_i 2-M_i -\mu \mid J_f -M_f),$$

and

$$\sum_{\mu, M_i, M_f} (J_i 2M_i \mu \mid J_i 2J_f M_f)^2 = (2J_f+1)$$

After a rather lengthy calculation, one finds

$$B(E2; n_\nu = 1, K = 2, J_i \rightarrow n_\nu = 0, K = 0, J_f) = \frac{5}{16\pi} e^2 Q_0^2 \gamma_{10}^2 \left[p^2 + 2pq \gamma_{10}^2 + \left(\frac{8pq}{3} + 2pr + q^2 \right) \gamma_{10}^4 \right], \quad (IV-2)$$

where

$$p = (J_i 22-2 \mid J_f 0),$$

$$q = \frac{1}{3} \sqrt{\frac{(J_f-1)J_f(J_f+1)(J_f+2)}{6}} (J_i 220 \mid J_f 2) - \frac{1}{3} \left[\frac{1+(-1)^{J_i}}{2} \right] \sqrt{\frac{(J-1)J(J+1)(J+2)}{6}} (J_i 200 \mid J_f 0),$$

and

$$r = \frac{1}{3} \sqrt{\frac{(J_f-1)J_f(J_f+1)(J_f+2)}{6}} (J_i 220 \mid J_f 2)$$

Since usually γ_{10}^2 is of the order of 10^{-1} , terms higher than γ_{10}^6 have been dropped in the above calculation. Even the γ_{10}^6 term is not meaningful here, for additional contribution to this term may arise if higher terms are included in the expansion of U_3 and $Q_{2\mu}$. As γ_{10}^6 is often of the order of 10^{-3} , and its coefficient in equation (IV-2) is not large enough to make this term comparable to the other two leading terms, one can usually neglect this term entirely in most cases.

Another point that should be mentioned here is that the mixing of $(n_\gamma = 2, K = 4)$ band with $(n_\gamma = 1, K = 2)$ need not be considered. It is found that such a mixing will only introduce terms of order γ_{10}^8 and higher.

The single expression (IV-2) can be applied to E2 transitions between any level in $(n_\gamma = 1, K = 2)$ band to any permissible level in $(n_\gamma = 0, K = 0)$ band. To put it into a more convenient form, one separates it into five expressions:

$$\begin{aligned}
 & B(E2; n_\gamma = 1, K = 2, J_i \rightarrow n_\gamma = 0, K = 0, J_f = J_i + 2) \\
 &= \frac{(J_i - 1)J_i}{4(2J_i + 1)(2J_i + 3)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{4}{3}(2J_i + 5)\gamma_{10}^2 \right], \quad (IV-3a)
 \end{aligned}$$

$$\begin{aligned}
 & B(E2; n_\gamma = 1, K = 2, J_i \rightarrow n_\gamma = 0, K = 0, J_f = J_i + 1) \\
 &= \frac{(J_i - 1)}{2(2J_i + 1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{4}{3}(J_i + 3)\gamma_{10}^2 \right], \quad (IV-3b)
 \end{aligned}$$

$$\begin{aligned}
 & B(E2; n_{\gamma} = 1, K = 2, J_i \rightarrow n_{\gamma} = 0, K = 0, J_f = J_i) \\
 &= \frac{3(J_i - 1)(J_i + 2)}{2(2J_i - 1)(2J_i + 3)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{8}{3} \gamma_{10}^2 \right], \quad (IV-3c)
 \end{aligned}$$

$$\begin{aligned}
 & B(E2; n_{\gamma} = 1, K = 2, J_i \rightarrow n_{\gamma} = 0, K = 0, J_f = J_i - 1) \\
 &= \frac{(J_i + 2)}{2(2J_i + 1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{4}{3} (J_i - 2) \gamma_{10}^2 \right], \quad (IV-3d)
 \end{aligned}$$

$$\begin{aligned}
 & B(E2; n_{\gamma} = 1, K = 2, J_i \rightarrow n_{\gamma} = 0, K = 0, J_f = J_i - 2) \\
 &= \frac{(J_i + 1)(J_i + 2)}{4(2J_i - 1)(2J_i + 1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{4}{3} (2J_i - 3) \gamma_{10}^2 \right], \quad (IV-3e)
 \end{aligned}$$

As an illustration, one finds the branching ratio of $J_i = 2$ to $J_f = 2$ and $J_i = 2$ to $J_f = 0$, to the fourth order in γ_{10} , to be

$$\begin{aligned}
 R_1(B) &\equiv \frac{B(E2; n_{\gamma}=1, K=2, J_i=2 \rightarrow n_{\gamma}=0, K=0, J_f=2)}{B(E2; n_{\gamma}=1, K=2, J_i=2 \rightarrow n_{\gamma}=0, K=0, J_f=0)} \\
 &= \frac{10}{7} (1 + 4\gamma_{10}^2 + \frac{32}{9} \gamma_{10}^4) \quad (IV-4)
 \end{aligned}$$

One recalls that

$$\gamma_{10}^2 = \frac{\hbar^2}{B\beta_1^2 \hbar\omega_{\gamma}}$$

Using the data listed by Van Patter (1959), $R(B)$ is calculated and shown in Table 1 together with the experimental value $R_1(B)$.

Table 1

	E_{002} (keV)	E_{122} (keV)	γ_{10}^2	R_1^{cal} (B)	R_1^{exp} (B)	R_1^{DF} (B)
Se ⁷⁶	560.5	1220	0.46	5.1	26 \pm 7	68
Te ¹²²	563.9	1257	0.45	5.0	91 \pm 14	66
Sm ¹⁵²	121.9	1087	0.11	2.1	2.2 \pm 0.3	1.9
Gd ¹⁵⁴	123.1	966.6	0.13	2.2	\geq 1.8	2.6
Dy ¹⁶⁰	86.6	965.7	0.09	2.0	2.0 \pm 0.1	1.9
Er ¹⁶⁸	79.8	822.0	0.10	2.0	1.8 \pm 0.3	-
W ¹⁸⁴	111.1	904	0.12	2.2	1.8 \pm 0.3	3.0
Os ¹⁹⁰	187.0	557	0.34	3.9	6.8 \pm 1.2	7.1
Hg ¹⁹⁸	411.8	1088	0.38	4.3	34 \pm 4	13
Th ²³²	52.8	790	0.07	1.8	2.1 \pm 0.3	1.7
Pu ²³⁸	44.0	1031	0.04	1.7	1.5 \pm 0.2	1.2

Columns 2, 3 and 6 taken from Van Patter (1959),

Column 7 from Davydov and colleagues (1958)

using an asymmetric rotor model.

$$\gamma_{10}^2 = \frac{\hbar^2}{B_1 B_2} \approx \frac{E_{002}}{E_{122}}$$

The following branching ratios can also be readily obtained by means of equations (IV-3).

$$R_2(B) \equiv \frac{B(E2; n_r=1, K=2, J_i=2 \rightarrow n_r=0, K=0, J_f=4)}{B(E2; n_r=1, K=2, J_i=2 \rightarrow n_r=0, K=0, J_f=0)}$$

$$= \frac{1}{14} (1 + \frac{40}{3} \gamma_{10}^2 + 16 \gamma_{10}^4),$$

and

$$R_3(B) \equiv \frac{B(E2; n_r=1, K=2, J_i=3 \rightarrow n_r=0, K=0, J_f=4)}{B(E2; n_r=1, K=2, J_i=3 \rightarrow n_r=0, K=0, J_f=2)}$$

$$= \frac{2}{5} (1 + \frac{28}{3} \gamma_{10}^2 + \frac{32}{3} \gamma_{10}^4)$$

Mallman (1961) lists the experimental ratios of intensities corresponding to $R_2(B)$ and $R_3(B)$. For comparison the ratios $R_2(B)$ and $R_3(B)$ are therefore converted to intensity ratios, denoted by $R_2(I)$ and $R_3(I)$ respectively, assuming pure E2 transitions. Undoubtedly this assumption is not always valid. The transition energies used in converting $R_2(B)$ to $R_2(I)$ are based on data collected by Van Patter, while those for $R_3(I)$ of Gd^{156} , Dy^{160} and W^{184} are from nuclear data sheets. Recently the compilation of energy levels by Hellwege and Hellwege (1961) became available and the required levels for Th^{228} and Pu^{238} are taken from this book. In the following tables the energy levels are denoted by $n_{\gamma}KJ$.

Table 2

	$R_2(I) = \frac{I(122 \rightarrow 004)}{I(122 \rightarrow 000)}$	exp $R_2(I)$
Gd ¹⁵⁶	4.0×10^{-2}	5.2×10^{-2}
Dy ¹⁶⁰	3.3×10^{-2}	4.2×10^{-2}
W ¹⁸⁴	1.7×10^{-2}	$.67 \times 10^{-2}$
Th ²²⁸	5.3×10^{-2}	7.7×10^{-2}
Pu ²³⁸	6.0×10^{-2}	8.417×10^{-2}

Table 3

	$R_3(I) = \frac{I(123 \rightarrow 004)}{I(123 \rightarrow 002)}$	exp $R_3(I)$
Gd ¹⁵⁶	2.9×10^{-1}	2.2×10^{-1}
Dy ¹⁶⁰	2.4×10^{-1}	4.2×10^{-1}
W ¹⁸⁴	1.8×10^{-1}	0.3×10^{-1}
Th ²²⁸	3.3×10^{-1}	3.4×10^{-1}
Pu ²³⁸	3.3×10^{-1}	6.6×10^{-1}

Recalling that pure E2 transitions have been assumed, it is seen that agreement is reasonably good except for W¹⁸⁴.

2. Transitions from $(n_\gamma = 2, K = 0, J_i)$ to $(n_\gamma = 1, K = 2, J_f)$

In this group the initial state $(n_\gamma = 2, K = 0, J_i)$ is affected by the $(n_\gamma = 1, K = 2, J_i)$ level and also by the $(n_\gamma = 3, K = 2, J_i)$ level. The perturbed wave function may be written as

$$|i\rangle = g_2^{(0)}(\gamma) \sqrt{\frac{(2J_i+1)}{8\pi^2}} D_{M_i, 0}^{J_i} + a g_1^{(2)}(\gamma) \sqrt{\frac{2J_i+1}{16\pi^2}} \left[D_{M_i, 2}^{J_i} + D_{M_i, -2}^{J_i} \right] \\ + b g_3^{(2)}(\gamma) \sqrt{\frac{(2J_i+1)}{16\pi^2}} \left[D_{M_i, 2}^{J_i} + D_{M_i, -2}^{J_i} \right]$$

where

$$a = \frac{1}{3} \sqrt{\frac{(J_i-1)J_i(J_i+1)(J_i+2)}{6}} \frac{\gamma 10^3}{(1 + \frac{2}{3}\gamma^2)}$$

and

$$b = \frac{1}{3} \sqrt{\frac{(J_i - 1)J_i(J_i + 1)(J_i + 2)}{3}} \frac{\gamma_{10}^3}{(1 - \frac{2}{3}\gamma_{10}^2)}$$

It should be remembered that the J_i are always even in this case. The final state ($n_\gamma = 1, K = 2, J_f$) is given by

$$\begin{aligned} |1f\rangle = & g_1^{(2)}(\gamma) \sqrt{\frac{2J_f+1}{16\pi^2}} \left[D_{M_f 2}^{J_f} + (-1)^{J_f} D_{M_f -2}^{J_f} \right] \\ & - \frac{[1+(-1)^{J_f}]}{2} (d_- g_0^{(0)}(\gamma) + d_+ g_2^{(0)}(\gamma)) \sqrt{\frac{2J_f+1}{8\pi^2}} D_{M_f 0}^{J_f} \\ & + f \psi_{24J_f} \end{aligned}$$

where

$$d_{\pm} = \frac{1}{3} \sqrt{\frac{(J_f - 1)J_f(J_f + 1)(J_f + 2)}{6}} \frac{\gamma_{10}^3}{(1 - \frac{2}{3}\gamma_{10}^2)}$$

The mixing with the state ($24J_f$) only gives negligible contribution to the reduced probability of E2 transitions. By a similar procedure as before one obtains

$$\begin{aligned} B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 1, K = 2, J_f) \\ = \frac{5e^2 Q_0^2}{16\pi} \left[p^2 \gamma_{10}^2 + 2pq \gamma_{10}^4 + \left(\frac{8}{3}pq + 2pr + q^2\right) \gamma_{10}^6 \right] \quad (IV-5) \end{aligned}$$

where now

$$\begin{aligned} p = & -(J_i 202 | J_f 2) \\ q = & \frac{1}{3} \sqrt{\frac{(J_i - 1)J_i(J_i + 1)(J_i + 2)}{6}} (J_i 220 | J_f 2) \\ & - \frac{[1+(-1)^{J_f}]}{2} \frac{1}{3} \sqrt{\frac{(J_f - 1)J_f(J_f + 1)(J_f + 2)}{6}} (J_i 220 | J_f 0) \end{aligned}$$

and

$$r = \frac{[1+(-1)^{J_f}]}{2} \frac{1}{3} \sqrt{\frac{(J_f-1)J_f(J_f+1)(J_f+2)}{6}} \quad (J_i 200 | J_f 0)$$

Comparison of equations (IV-2) and (IV-5) indicates they are similar, as one would also have guessed since both transitions involve a change of $\Delta n_\gamma = 1$ and $|\Delta K| = 2$. Again, to the fourth order in γ_{10} , one obtains five separate expressions from equation (IV-5).

$$\begin{aligned} & B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i + 2) \\ &= \frac{1}{4} \frac{(J_i + 3)(J_i + 4)}{(2J_i + 1)(2J_i + 3)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{4}{3}(2J_i + 1)\gamma_{10}^2 \right] \quad (\text{IV-6a}) \end{aligned}$$

$$\begin{aligned} & B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i + 1) \\ &= \frac{(J_i + 3)}{2(2J_i + 1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{4}{3}(J_i - 1)\gamma_{10}^2 \right] \quad (\text{IV-6b}) \end{aligned}$$

$$\begin{aligned} & B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i) \\ &= \frac{3(J_i - 1)(J_i + 2)}{2(2J_i - 1)(2J_i + 3)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{8}{3}\gamma_{10}^2 \right] \quad (\text{IV-6c}) \end{aligned}$$

$$\begin{aligned} & B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i - 1) \\ &= \frac{(J_i - 2)}{2(2J_i + 1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{4}{3}(J_i + 2)\gamma_{10}^2 \right] \quad (\text{IV-6d}) \end{aligned}$$

$$\begin{aligned} & B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i - 2) \\ &= \frac{1}{4} \frac{(J_i - 3)(J_i - 2)}{(2J_i - 1)(2J_i + 1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{4}{3}(2J_i + 1)\gamma_{10}^2 \right] \quad (\text{IV-6e}) \end{aligned}$$

3. Transitions from $(n_\gamma = 2, K = 0, J_i)$ to $(n_\gamma = 0, K = 0, J_f)$

In this case a change of $\Delta n_\gamma = 2$ occurs. If one uses unperturbed wave functions and uses $\cos \gamma \approx 1$ in $Q_{2\mu}$, then $B(E2)$ vanishes. Therefore, though using perturbed wave functions as well as higher order expansion for $\cos \gamma$ in $Q_{2\mu}$, one will not be surprised to find that the $B(E2)$ in case 3 is much smaller than those in cases 1 and 2.

The reduced probability is found to be

$$\begin{aligned}
 B(E2; n_\gamma = 2, K = 0, J_i \rightarrow n_\gamma = 0, K = 0, J_f) \\
 = \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^4 (p^2 + 2pq\gamma_{10}^2) \quad (IV-7)
 \end{aligned}$$

where now

$$p = \frac{1}{2} (J_i 200 | J_f 0)$$

and

$$\begin{aligned}
 q = \frac{1}{3} \sqrt{\frac{(J_i - 1)J_i(J_i + 1)(J_i + 2)}{6}} \quad (J_i 2 - 22 | J_f 0) \\
 - \frac{1}{3} \sqrt{\frac{(J_f - 1)J_f(J_f + 1)(J_f + 2)}{6}} \quad (J_i 202 | J_f 2)
 \end{aligned}$$

As equation (IV-7) is relatively simple, no attempt is made to separate it into five equations. For convenience in using it, however, some Clebsch-Gordan coefficients are listed here:

$$\begin{aligned}
 (0200|20) = 1, & \quad (02-22|20) = 0, & \quad (0202|22) = 1, \\
 (2200|00) = \sqrt{\frac{1}{5}}, & \quad (22-22|00) = \sqrt{\frac{1}{5}}, & \quad (2202|00) = \sqrt{\frac{1}{5}},
 \end{aligned}$$

$$(2200|20) = -2\sqrt{\frac{1}{14}}, \quad (22-22|20) = \sqrt{\frac{2}{7}}, \quad (2202|22) = \sqrt{\frac{2}{7}},$$

$$(2200|40) = 3\sqrt{\frac{2}{35}}, \quad (22-22|40) = \sqrt{\frac{1}{70}}, \quad (2202|42) = \sqrt{\frac{3}{14}},$$

Thus, for instance, for $J_i = 0$ and $J_f = 2$, one readily obtains

$$p = \frac{1}{2}, \quad q = -\frac{2}{3}$$

and hence

$$B(E2; n_\gamma = 2, K = 0, J_i = 0 \rightarrow n_\gamma = 0, K = 0, J_f = 2)$$

$$= \frac{1}{4} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^4 \left(1 - \frac{8}{3} \gamma_{10}^2\right) \quad (IV-8)$$

It is well known that transition from $J_i = 0$ to $J_f = 0$ is absolutely forbidden. This is indeed the case here as $(0200|00)$ vanishes.

The branching ratio of the E2 transitions from $n_\gamma = 2$ to $n_\gamma = 1$ and from $n_\gamma = 2$ to $n_\gamma = 0$ can also be easily obtained by means of equations (IV-5) and (IV-6). For example,

$$\frac{B(E2; n_\gamma = 2, K = 0, J_i = 0 \rightarrow n_\gamma = 0, K = 0, J_f = 2)}{B(E2; n_\gamma = 2, K = 0, J_i = 0 \rightarrow n_\gamma = 1, K = 2, J_f = 2)}$$

$$= \frac{1}{4} \gamma_{10}^2 \left(1 + \frac{8}{3} \gamma_{10}^2\right) \quad (IV-9)$$

4. Transitions from $(n_\gamma = 2, K = 4, J_i)$ to $(n_\gamma = 1, K = 2, J_f)$

For this group of transitions the perturbed wave function of the initial state is given by

$$\Psi_{24J_i} = \phi_{24J_i} + a \phi_{12J_i} + b \phi_{32J_i} + c \phi_{36J_i}$$

where

$$a = -\frac{1}{3} \sqrt{\frac{(J_i-3)(J_i-2)(J_i+3)(J_i+4)}{3}} \frac{\gamma 10^3}{(1-2\gamma 10^2)},$$

$$b = -\frac{1}{3} \sqrt{\frac{(J_i-3)(J_i-2)(J_i+3)(J_i+4)}{6}} \frac{\gamma 10^3}{(1+2\gamma 10^2)},$$

and

$$c = \frac{1}{3} \sqrt{\frac{(J_i-5)(J_i-4)(J_i+5)(J_i+6)}{2}} \frac{\gamma 10^3}{(1-\frac{10}{3}\gamma 10^2)},$$

The final state is given by

$$\psi_{12J_f} = \phi_{12J_f} - \frac{[1+(-1)^{J_f}]}{2} (d_+ (\phi_{00J_f} + d_+ \phi_{20J_f}) - f \phi_{24J_f})$$

with

$$d_+ = \frac{1}{3} \sqrt{\frac{(J_f-1)J_f(J_f+1)(J_f+2)}{6}} \frac{\gamma 10^3}{(1+\frac{2}{3}\gamma 10^2)}$$

and

$$f = -\frac{1}{3} \sqrt{\frac{(J_f-3)(J_f-2)(J_f+3)(J_f+4)}{3}} \frac{\gamma 10^3}{(1-2\gamma 10^2)}$$

The calculated result is

$$B(E2; n_\gamma = 2, K = 4, J_i \rightarrow n_\gamma = 1, K = 2, J_f) \\ = \frac{5e^2 Q_0^2}{16\pi} \left[p^2 \gamma_{10}^2 + 2pq \gamma_{10}^4 + (8pq + 2pr + q^2) \gamma_{10}^6 \right] \quad (\text{IV-10})$$

where the coefficients

$$p = \sqrt{2} (J_i 24 - 2|J_f 2),$$

$$q = -\frac{1}{3} \sqrt{\frac{(J_i-3)(J_i-2)(J_i+3)(J_i+4)}{3}} \quad (J_i 220|J_f 2)$$

$$+ \frac{1}{3} \sqrt{\frac{(J_f-3)(J_f-2)(J_f+3)(J_f+4)}{3}} \quad (J_i 240 | J_f 4),$$

and

$$r = \frac{1}{2} \sqrt{\frac{(J_i-3)(J_i-2)(J_i+3)(J_i+4)}{3}} \quad (J_i 220 | J_f 2)$$

$$- \sqrt{\frac{(J_f-3)(J_f-2)(J_f+3)(J_f+4)}{3}} \quad (J_i 240 | J_f 4)$$

As before, this is separated into five equations, to the fourth order in γ_{10} .

$$B(E2; n_\gamma = 2, K = 4, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i + 2)$$

$$= \frac{(J_i-3)(J_i-2)(J_i-1)J_i}{2(J_i+1)(J_i+2)(2J_i+1)(2J_i+3)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{4}{3}(2J_i+9)\gamma_{10}^2 \right] \quad (IV-11a)$$

$$B(E2; n_\gamma = 2, K = 4, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i + 1)$$

$$= \frac{(J_i-3)(J_i-2)(J_i-1)(J_i+4)}{J_i(J_i+1)(J_i+2)(2J_i+1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + \frac{4}{3}(J_i+7)\gamma_{10}^2 \right] \quad (IV-11b)$$

$$B(E2; n_\gamma = 2, K = 4, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i)$$

$$= \frac{3(J_i-3)(J_i-2)(J_i+3)(J_i+4)}{(2J_i-1)J_i(J_i+1)(2J_i+3)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 + 8\gamma_{10}^2 \right] \quad (IV-11c)$$

$$B(E2; n_\gamma = 2, K = 4, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i - 1)$$

$$= \frac{(J_i-3)(J_i+2)(J_i+3)(J_i+4)}{(J_i-1)J_i(J_i+1)(2J_i+1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{4}{3}(J_i-6)\gamma_{10}^2 \right] \quad (IV-11d)$$

$$B(E2; n_\gamma = 2, K = 4, J_i \rightarrow n_\gamma = 1, K = 2, J_f = J_i - 2)$$

$$= \frac{(J_i+1)(J_i+2)(J_i+3)(J_i+4)}{2(2J_i-1)(J_i-1)J_i(2J_i+1)} \frac{5e^2 Q_0^2}{16\pi} \gamma_{10}^2 \left[1 - \frac{4}{3}(2J_i-7)\gamma_{10}^2 \right] \quad (IV-11e)$$

5. Transitions from $(n_{\gamma} = 2, K = 4, J_i)$ to $(n_{\gamma} = 0, K = 0, J_f)$

It is found that the reduced probability of E2 transitions for this group is extremely small. The result is

$$\begin{aligned}
 & B(E2; n_{\gamma} = 2, K = 4, J_i \rightarrow n_{\gamma} = 0, K = 0, J_f) \\
 & = p^2 \frac{5e^2 Q_0^2}{16\pi} \Upsilon 10^8 \quad (\text{IV-12})
 \end{aligned}$$

where

$$\begin{aligned}
 p^2 = \frac{1}{9} & \sqrt{\frac{(J_f-1)J_f(J_f+1)(J_f+2)}{3}} \quad (J_i 24-2 | J_f 2) \\
 & - \sqrt{\frac{(J_i-3)(J_i-2)(J_i+3)(J_i+4)}{3}} \quad (J_i 22-2 | J_f 0)^2
 \end{aligned}$$

Thus it is seen that the reduced probability for this group is even less than that from $(n_{\gamma} = 2, K = 0, J_i)$ to $(n_{\gamma} = 0, K = 0, J_f)$. This is probably due to the large change in K values here.

V. Discussion

The expressions derived for the reduced probability depend on the parameter γ_{10} , which is essentially the square root of the ratio of the first rotational energy level to the first vibrational level. The variation of γ_{10}^2 over even-even nuclei with $34 < A < 238$ is roughly within the interval of 0.05 to 0.5. The variation of the branching ratios of the reduced probability, on the other hand, is much larger. Thus, for instance, the ratio $R_1(B)$ can vary from 1.5 to 1800 for the above variation of nuclei. This thus limits the range of application of the derived expressions for the reduced probability of E2 transitions.

The original rotor model of Davydov and Filippov (1958) has remarkable success in calculating the transition probability, especially in its capability of accounting for a large range of nuclei. In this model the nucleus is pictured as non-axial in general with a fixed form of the nuclear surface. A parameter γ is introduced to describe the non-axiality. Adiabatic condition is assumed to hold, i.e. the rotation of the nucleus has no effect on the intrinsic state of the nucleus. Starting from the Hamiltonian

$$H = \sum_{k=1}^3 \frac{A J_k^2}{2 \sin^2(\gamma - \frac{2\pi k}{3})}$$

with

$$A = \frac{\hbar^2}{4B\beta_1^2}$$

energy levels are obtained in terms of the parameter γ . From experimental data of energy spectra the parameter γ is determined. This value of γ is then used in the calculation of radiative transitions. Some of their calculated results are included in Table 1.

Recently, Davydov (1961) also includes surface vibrations and discusses non-axial nuclei as well. Thus one may consider this as a generalization of Bohr's model (1952). For axial nuclei, Davydov gives

$$\psi_{JK}(\theta_i, \gamma) = |JK\rangle g_K^{(J)}(\gamma) \quad (V-1)$$

with

$$g_K^{(J)}(\gamma) = F(-\lambda, \frac{1}{2}K + 1; \sqrt{D}\gamma^2) (\sqrt{D}\gamma^2)^{\frac{K}{4}} \exp(-\frac{1}{2}\sqrt{D}\gamma^2) \quad (V-2)$$

where F is a confluent hypergeometric function, and λ is a non-negative integer. The quantity \sqrt{D} is equal to $\frac{B\beta_1^2}{\hbar} \omega_\gamma$. With suitable choice of λ , equation (V-2) reduces to the g -functions used in the present work. Though both λ and n_γ come into the coefficient of $\hbar\omega_\gamma$, there is no restriction on λ apart from being a non-negative integer, whereas n_γ is restricted by the K value (see Fig. 1). The relation between λ and n_γ is

$$\lambda = \frac{2n_\gamma - K}{4}$$

According to the calculation of Davydov, Rostovsky, and Chaban (1961),

$$\frac{B(E2; 22 \rightarrow 21)}{B(E2; 22 \rightarrow 0)} = \frac{10}{7} (1 + 8\gamma^2) \quad (V-3)$$

and

$$\frac{B(E2; 0_{\gamma} \rightarrow 21)}{B(E2; 0_{\gamma} \rightarrow 22)} = \frac{1}{2} \Gamma^2 \left(1 - \frac{20}{3} \Gamma^2 \right) \quad (V-4)$$

where $22 \rightarrow 21$ in the present notation means $(n_{\gamma} = 1, K = 2, J = 2)$ to $(n_{\gamma} = 0, K = 0, J = 2)$, and the 0_{γ} state corresponds to $n_{\gamma} = 2, K = 0,$ and $J = 0$. The parameter Γ is equal to $\gamma_{10}/\sqrt{2}$. It is therefore not surprising that, to the second order in γ_{10} , these are the same as equations (IV-4) and (IV-9), respectively. The difference in the coefficients of the fourth order terms in equations (IV-9) and (V-4) may be traced to the fact that Davydov, et al. use unperturbed wave function for the initial state.

VI. Conclusion

From the comparison with experimental data it seems that the derived expressions for the reduced probability of E2 transitions are in general quite adequate for greatly deformed nuclei, namely, for those with $150 < A < 190$ and $A > 220$. As already mentioned, in the present calculation only approximated expressions are used for U_3 , the interaction between rotation and vibration, and Q_2 , the electric quadrupole operator. It must also be realized that other interactions, such as particle-vibration interaction, have been completely neglected. Furthermore, it has been implicitly assumed that ω_v is the same for all states of a particular nucleus, whereas in reality it is a function of K and J. Such considerations must be taken into account in a more refined calculation.

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