HOMOMORPHIC PROCESSING OF SURFACE
RECORDED EMG SIGNALS
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RECORDED EMG SIGNALS

by

DANIEL WILLIAM STASHUK, B.Sc.(Elec. Eng.)

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AUTHOR: Daniel William Stashuk B.Sc.(Elec. Eng.)
        (Waterloo)

SUPERVISOR: Dr. H. de Bruin

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ABSTRACT

Electromyographic (EMG) signals contain both neural and muscle information. Consequently, EMG signals can be modelled as the composition of two component signals, one of these being a low frequency neural input, the other a relatively high frequency, constant spectrally shaped, stationary, unitary muscle response. Utilizing this model and homomorphic processing estimates of the two component signals can be obtained. These estimates contain neural and muscle information respectively.

This thesis establishes the basis for the use of this multiplicative model. It also outlines the application of multiplicative homomorphic processing to EMG signals. The results of this processing are shown to be valid and to contain useful information.

The thesis concludes that the model is both appropriate and useful. It also points out that the use of this model and homomorphic processing allows the simultaneous extraction of both neural and muscle information from the EMG signal, a result which is not possible with other currently used processing techniques.
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CHAPTER 1
INTRODUCTION

It has been known for many years that muscle contraction is initiated and controlled by neural inputs. It has also been known that the contraction of muscle is accompanied by substantial electrical activity. This electrical activity can therefore be considered as an information source of both muscle activity and neural control. By the use of suitable electrodes and amplification, the electrical activity of muscles can be measured. This measured activity is called an electromyographic (EMG) signal. EMG signals then contain information about both the neural control and response during contraction of the muscle from which they were recorded. The work presented in this thesis was initiated in an effort to determine a signal processing technique to extract both the neural and muscle information present in EMG signals.

To this end the physiology and physical parameters which are the basis of recorded EMG signals were investigated and are summarized in Chapter 2. This chapter concludes with a discussion of the stochastic nature of recorded EMG signals.
Chapter 3 outlines the present uses of EMG signals as well as existing processing techniques and models used to analyse them. It also introduces a multiplicative model with new conceptualizations of the two component signals, one being a low frequency neural input, the other a relatively high frequency unitary muscle response (UMR). The fundamental assumption on which this model is based is that, independent of the contraction level, the spectral shape of the EMG signal recorded during constant contractions does not change. The chapter concludes with the results of a study which investigated this assumption.

Multiplicative processes are especially suited to the use of homomorphic processing, for the determination of their component signals. For this reason Chapter 4 summarizes the general theory of homomorphic processing and discusses its application to multiplicative systems. Homomorphic processing is then compared to other techniques of processing EMG signals. The results of a comparison of the performances of homomorphic processing and rectification followed by low pass filtering applied to simulated EMG signals are reported.

In an effort to substantiate the proposed model and to establish the information content of the component signals real EMG signals were homomorphically processed. The results are presented in Chapter 5. EMG signals recorded
during constant muscle contractions were processed and spectral parameters for the raw EMG data and their corresponding UMRs were compared. The chapter continues with a discussion of the stationarity of these signals and their corresponding UMRs as assessed by the run test. Signals collected during phasic muscle contractions were also processed and the information content of their component signal estimates is then addressed. It is shown that the extracted neural input estimate represents the phasic control of the muscle contraction. Further a relationship between UMR spectral parameters and the muscle being studied is established. Finally the stationarity of the corresponding UMRs as determined by the run test is also assessed.

The thesis ends with a concluding chapter outlining the suitability of the proposed model and summarizing the significance of the results. Future research directions for the application of the proposed model and homomorphic processing to EMG signals are also discussed.
CHAPTER 2
THE EMG SIGNAL

2.1 Introduction
Electromyographic (EMG) signals are signals collected by suitable electrodes, which represent the electrical activity associated with muscle contraction. To understand the EMG signals collected and to use them as a source of information it is essential to have a knowledge of the basic structure and function of nerves and muscles. This chapter briefly summarizes the physiology of nerves and muscles. More comprehensive treatments of these topics can be found in standard texts such as Guyton (1977) and Katz (1966). It then relates the physiology to the EMG signals recorded from different electrode types and configurations. Finally it discusses the stochastic nature of the recorded EMG signal.

2.2 Physiology of Nerve and Muscle
2.2.1 The Nerve Cell
Nerve cells or neurons are single specialized cells which perform the function of information transmission and processing. Processing of information is confined to the Central Nervous System (CNS), which is made up of the brain
and the spinal cord. The transmission of information is primarily carried out by the Peripheral Nervous System (PNS), although some transmission obviously takes place in the processing action of the CNS. The PNS is further divided into two functional groups, that of afferent or sensory fibres (neurons) and that of efferent, or motor fibres. The sensory fibres relay the information from our sensory organs to the CNS. The motor fibres relay to muscles the activation intent of the CNS.

Neurons have evolved into specialized cells capable of transmitting electrical signals which they use to communicate with each other. Anatomically, the nerve cell appears as in Figure 2.1. As can be seen from the figure, the neuron has a main cell body complete with nucleus, as do many other biological cells. However, the neuron's intricate system of dendrites and long axon projection, (up to 1 meter in length) clearly show its structural adaptation. The axons of human nerve cells are about 1-10 microns in diameter.

Functionally, the neuron is also quite different from other cells. Although the membrane of the nerve cell is a standard bilipid layer it is also capable of drastic ionic permeability changes which allow it to transmit electrical signals, as will be presently explained. The bilipid layer of the membrane acts also as a biological capacitor, approximately 1 μF/CM² and is capable of withstanding extremely high electrical fields without breakdown and thus sustaining a potential across it.
Figure 2.1 The Nerve Cell
(Katz 1966)
The nerve cells, due to concentration gradients of specific ions across the membrane, primarily high relative sodium (Na+) concentration outside the cell and high relatively potassium (K+) concentration inside the cell, do possess a membrane potential \( (V_m) \). The value of the membrane potential is determined by the membrane relative conductivity to the specific ions present on both sides of the membrane and to the electro-chemical driving force acting on each ion species. The ions of main concern are again Na⁺ and K⁺. The equilibrium potential of Na⁺, \( E_{Na^+} \) and \( E_{K^+} \), are about +40 mV and -120 mV respectively. Subsequently the driving force for these ions are the respective differences between the membrane potential and the equilibrium potential for the ion of interest. \( V_m - E_{Na^+} \) or \( V_m - E_{K^+} \).

2.2.2 The Action Potential

At rest due to a high K⁺ conductivity \( (g_{K^+}) \) the cell membrane potential is approximately -85 mV (inside relative to outside). However, the Na⁺ and K⁺ conductivities, \( g_{Na^+}, g_{K^+} \) of the membrane are sensitive to membrane voltage changes. As the membrane is depolarized (brought toward zero potential) the \( g_{Na^+} \) is significantly increased, as well the \( g_{K^+} \) is increased to a lesser degree. If the depolarization is sufficient for the net current to cause further depolarization, for more Na⁺ to move out of the
cell then $K^+$ in, a positive feedback situation develops, in which a further increase in $g_{Na}^+$ and $g_K^+$ results in further depolarization. The membrane rapidly approaches $E_{Na}^+$, the equilibrium potential for Na$^+$, at which point Na$^+$'s driving force goes to zero. Also as time progresses $g_{Na}^+$, despite the membrane potential rapidly returns to its resting state while $g_K^+$ remains high. This forces the membrane potential back to rest. The rapid transmembrane potential excursion just described is called the nerve action potential (AP). This all or none phenomenon reaches the same peak voltage (determined by $E_{Na}^+$) and has the same time course (approximately 1 msec. dependent on nerve cell diameter and temperature) each time a depolarization above threshold is reached. Threshold is defined as the minimal depolarization required to elicit an AP. The production of an AP obviously requires energy. This energy is supplied by the existing transmembrane concentration gradients. If many AP's are produced without recharging the system these chemical concentrations run down. To prevent this a complex metabolically driven process, the sodium potassium pump operates to maintain the concentration gradients.

Once an AP has been elicited at a point on the nerve it will propagate in both directions along the fibre. This propagation is brought about by the depolarization, beyond threshold, of membrane sections, adjacent to site of the
initial AP, resulting from the local ionic current flows associated with creating the initial AP. The propagation velocity of the AP increases with cell radius. The approximate velocity of transmission for bare neurons is in the 1-5 m/s range.

To speed up AP transmission, nature has provided nerve cells with insulating cells, i.e. myelinating Schwann or Glia cells. These cells wrap themselves around the axon only exposing it to the interstitial fluid at regular 1 mm intervals. These Nodes-of-Ranvier as they are called are the only points new AP's can be created. This causes the AP to be passively conducted, as through an electrical cable from node to node. Passive conduction is much faster, subsequently myelinated nerves conduct AP's at higher propagation velocities (50 m/s). The AP transmission in myelinated nerves appears to have the AP's jumping from node to node, thus the term saltatory conduction has been used for this mode of transmission. Myelination allows nerves to have high conduction velocities without becoming massively large or expending great amounts of metabolic energy to keep the chemical gradients intact.

Neurons communicate with each other and muscle via synaptic connections. At a synapse, either electrically or chemically the information of a presynaptic A.P. is transmitted by a depolarization or hyperpolarization of the post
synaptic membrane. Depolarizations are generally considered as excitatory (i.e. tending to elicit an AP). Hyperpolarizations are generally considered as inhibitory.

The dendritic system, with its many synaptic connections to other nerve cells, is constantly integrating a barrage of excitatory and inhibitory signals. When the excitatory inputs are sufficient to excite the cell (i.e. depolarization past threshold) an action potential is created at the axon hillock. The AP propagates down the axon to synapses at other neuronal dendrites or muscle fibres thus communicating with these structures.

2.2.3 Muscle

Muscle is a complex collection of individual muscle fibres and connective tissue. Muscle fibres are grouped into bundles (fasicles). The fasicles are grouped into whole muscles by connective tissue. Muscles are connected to bone by connective tissue called tendons. Each muscle fibre consists of a chain of sacromeres, of about 10-80 microns in diameter. Each sacromere consists of several hundred to several thousand myofibrils. The myofibrils are the basic contraction elements. The exact chemical and physical mechanism of myofibril contraction is beyond the scope of this discussion, but further description can be found in Guyton (1977). The membrane of the muscle fibre like the
neuron is excitable. The membrane capacitance of the muscle is greater than that of the neuron at about 6 \( \mu F/cm^2 \). The increase is due to the muscles excitation contraction coupling mechanism. Depolarizations beyond threshold result in muscle action potentials (MAP's) which like AP's propagate in both directions from the initiation point. Muscle fibres are not myelinated and thus have conduction velocities of about 5 m/s.

The initiation of muscle contraction is usually linked to the MAP. The depolarization of the muscle fibre membrane results in the release of calcium ions (\( Ca^{++} \)) in the fibre. This results, through an excitation contraction coupling (ECC) mechanism, (Guyton 1977) in muscle contraction. Although the ECC is an extremely complex non-linear mechanism, in general, as the frequency of depolarizations increases the amount of contraction is increased. The complexity of the ECC, under dynamic muscle contraction makes attempts to relate muscle electrical activity and contraction state difficult.

Muscle has associated with it a series and parallel mechanical compliance resulting from the connective tissue components, between muscle and bone and around the muscle fibre groups (fasicles). Any contraction level that is to be measured externally, must first stretch or energize this compliant component. This is similar to voltage readings
across series or parallel capacitors.

2.2.4 Nerve-Muscle Interaction

Since EMG signals result from muscle activity and muscles are activated by motor neurons of the PNS, the motor neuron muscle interactions is of primary interest in this discussion. The motor neuron is the sole PNS efferent path from the CNS to the muscle. The motor neuron cell body (anterior horn cell), located in the spinal cord, processes via integration the activity of the synaptic connections to its dendrites. Upon suitable depolarization (past threshold) an AP is created which propagates along the axon and its distal branches to neuro-muscular junctions, one for each axon branch. At the neuro-muscular junction the axon forms a synapse with the muscle fibre. This is called the motor point. Here, via chemical transmission the post synaptic muscle fibre membrane is depolarized past threshold by suitable permeability changes, resulting in a muscle action potential (MAP) being created. The created MAP then propagates in both directions from the motor point resulting in muscle fibre contraction at each point along the fibre, subsequent to the MAP passing. The net effect is a smooth muscle fibre contraction.

Chemical rather than electrical transmission is used at the neuro-muscular junction because the nerve cannot
supply the required current directly, to depolarize the muscle membrane beyond threshold. This is due to the muscle membrane increased capacitance per square area and its increased size. The neuro-muscular junction chemical transmitter mechanism acts as a pulse transformer whose output provides sufficient electrical current to drive the low impedance muscle fibre membrane.

The single motor neuron does branch as it reaches the muscle. Therefore, a single motor axon terminates at neuro-muscular junctions on a number of muscle fibres, one neuro-muscular junction per fibre. This means that a single motor neuron firing results in the essentially simultaneous creation of a number of MAP's on involved muscle fibres and in the 'synchronous' contraction of these muscle fibres. This group of muscle fibres 'synchronously' contracting and the motor neuron exciting them is called the motor unit.

The individual MAP's of the muscle fibres of the motor unit summate to create what is called a motor unit action potential (MUAP). All the muscle fibres of a given motor unit are not actually synchronously excited because of different distances from the motor neuron cell body to specific fibres and due to different conduction velocities in the distal branches of the motor axon. These effects, however, are minimal and the motor unit fibre activity can be considered as synchronous.
2.2.5 **Muscle Contraction**

The response to the single firing of a motor neuron is a brief contraction followed by relaxation called a muscle twitch. Twitch durations are measured as the time for the muscle to contract and relax. Muscle fibres are basically of two types. A muscle fibre may be a fast twitch (or phasic muscle) with twitch times of 10-20 ms (Guyton 1977). Muscle with mostly phasic fibres are for fast response movements, but fatigue quickly. A muscle fibre may also be a slow twitch (or tonic muscle) with twitch times of 80-120 ms (Guyton 1977). Muscle with mostly tonic fibres are usually postural muscle, slowly responding, but slow to fatigue. The muscles of man are usually of mixed fibre type and have intermediate total muscle twitch times. A motor unit has a common fibre type.

It is important to differentiate between the muscle's electrical activity and its mechanical activity. The electrical impulse, (MAP) has a time duration at any one site on the fibre, depending on the fibre diameter, in the range of several milliseconds. Therefore, the time constants of the electrical responses are at least an order of magnitude less than the time constants of the possible mechanical muscle responses. As well electrically there is no difference in fibre type responses.
2.2.6 Control of Muscle Contraction

When a train of neuronal spikes are incident to a muscle or motor unit and the inter spike interval is less than the twitch duration greater muscle force than for a single twitch will be created. If the incoming spike train is above a critical frequency, the twitches are fused and tetanus or maximum muscle force will be created. This ability to produce greater force subsequent to closely following impulses is due to muscle compliance charging and to ECC potentiation due to increased Ca$^{++}$ release.

Another method of increasing muscle tension produced is to simultaneously or within the same twitch times, stimulate more motor units. This is called recruitment of additional motor units or simply recruitment. The incremental increase in maximum force produced will depend on the recruited unit's size, its relative synchrony with the other active units and its impulse frequency upon activation.

Dynamic control and maintenance of different levels of contraction is then effected by the two processes of recruitment of new motor units and/or the altering of the firing rates of the motor units already firing. Firing rates range from zero to greater than 50 Hz. Henneman et al (De Luca 1979) observed that recruitment usually starts with the smaller units. Milner-Brown et al (De Luca 1979) found that in some contractions recruitment order and impulse
frequency upon activation is such that the incremental force change $\Delta F$ versus the total force $F$ is constant. That is $\Delta F/F = K$. The exact interplay of these two force variables, firing rate and recruitment is not exactly known. It is thought to change with different force rates of contraction, that is different velocity, accelerations and length of shortening muscle (De Luca 1979). However, the literature does seem to indicate the following interplay between firing rate and recruitment for force-varying isometric contractions. Recruitment plays the major role for contraction levels up to 30% of maximum voluntary contraction (MVC). From 30 to 75% MVC the increase of firing rate plays a larger role in force increase, with the recruitment effect diminishing. For contraction above 75% MVC firing rate increase has the primary role in force increase (De Luca 1979).

2.3 Basis of Electromyographic Signal

2.3.1 Signal Source

The source of the electromyographic (EMG) signal is the individual MAP's of the active muscle fibres. Since all the fibres of a given unit are 'synchronously' active the source of the EMG signal can be considered to be the MUAP's of active motor units. Although the potentials of the MAP's are transmembrane potentials, the currents that
flow to create the MAP's produce perceptable electric fields in the medium external to the muscle fibres. Suitable electrodes placed in these fields will have potentials impressed upon them. The potential difference between electrodes can then be differentially amplified to result in the electromyographic signal. The AP produced by nerves do not contribute significantly to the EMG signal due to the relatively small current involved. However, with special electrodes and recording sites very close to the nerve, electroneurographic signals can also be recorded.

2.3.2 Volume Conduction

As just previously stated, the EMG signal is the difference in the measured electric field at two electrode sites. The electric fields are created by the currents that flow to produce MAP's. The current involved to create a MAP flows in a circuit composed of the muscle membrane, the fibre internal solution, and the external medium. The membrane and internal current are not perceptable with standard EMG electrodes. Therefore the current of interest is that flowing in the external medium.

The external medium is usually modelled as a volume conductor of homogeneous composition. The external medium is not purely resistive but has time constants that are extremely short compared to active fibre conduction veloci-
ties. This means that phase delays due to distances to the recording site from MAP locations are negligible and that conduction speeds of travelling waves, are much greater in the external medium than in active fibres.

The externally flowing current as it leaves the muscle fibre membrane disperses throughout the volume of the external medium. As the distance from the fibre increases so does the current dispersion. This means that as you move away from the fibre the current density decreases. The electric field in a homogeneous medium is proportional to the current density. Therefore, as the radial distance d from the fibre increases the electric field decreases. This means that the voltage difference across a given electrode pair will decrease as the distance from the active fibre to the recording site increases. Buchthal et al (De Luca 1979) reports that it is an approximately inversely linear (i.e. \( 1/d \)) relationship.

During muscle activity many fibres are active at any one time. The external field at any one spot will then be the integration of the effects of all the active fibres at the recording site. The net external field is then dependent on

i) The number of active fibres.

ii) The size of the active fibres. MAP amplitude and hence external field effects increase with the size of the fibre such that \( V \propto k a^{1.7} \) where
k is a constant and a is the fibre radius as reported by Rosenfalck (De Luca 1979).

iii) The distances from the active fibres to the recording site. Distances may be such that no perceptable field effects occur at the recording site even though a fibre is active.

iv) The amount of synchronization between active fibre MAP's.

2.3.3 Recorded Signal Factors

The actual EMG signal recorded depends upon how the external fields created by the integration of the active fibre contributions, is sensed. The main factors affecting the characteristics of the signals recorded are: distance to recording site; size of electrodes used; spacing between electrodes; and electrode-external medium interface transfer function or filtering effects.

The distance to the active muscle fibres has two main effects. As previously stated the first effect is to decrease the amplitude of the recorded signals due to reduced electric field strengths at the recording site. The second effect is that of low pass filtering. The impedance of the external medium is such that high frequency signals are more severely attenuated than low frequency signals. As the distance from the active muscle fibres increases the bandwidth of the low pass filter decreases (Lindstrom and Magnusson 1977).
The size of the recording electrodes will determine the electrode impedance and its effective field pickup area. Larger electrodes of similar physical composition will have reduced impedance due to increased surface area. Due to high electrode-external medium interface impedance, adjacent points along the electrode essentially see different electric field strengths, that is the electrode does not short out the electric field created. The net field the electrode detects is then, the spatial integration of the field adjacent to it, over its whole surface area. This means that larger electrodes record over larger areas and their net effect is to detect the average field over their surface area. With travelling potential waves in space (i.e. time varying fields), as are created by the propagation of MAP's down active fibres, the amount of spatial integration will affect the frequency components of the detected signal. Spatial integration reduces the high frequency components of travelling field waves.

With time varying electric fields, fields containing propagating wave fronts, the effect of electrode spacing is that of differentiating. As the spacing decreases the recorded signal becomes closer and closer to being the derivative of the travelling wave. Consequently, reduced spacings increases recorded signal bandwidth. Generally reduced spacing causes reduced recorded signal amplitude. This is because it is the potential difference between
electrodes which is amplified. As the electrodes are moved closer together the potential difference between electrodes generally reduces.

The electrode-external medium interface is very important in determining the electrodes impedance per unit area and subsequently its filtering effects on the recorded signal. The type of materials used for the electrode and the electrolyte interfacing the tissue with the electrode determines the impedance per unit area of the electrode-external medium interface.

2.3.4 Instrumentation

Since the EMG signal is of the order of, at most several millivolts, differential amplification is necessary to remove common mode noise such as 60 Hz power line signals. The common mode rejection ratio should be at least 80 dB. For EMG signal recording the differential amplifiers are A.C. coupled. The main reasons for this are as follows:

i) No EMG signal activity at D.C. The EMG signal is essentially zero mean (Hogan 1980).

ii) To remove instrumentation D.C. bias levels and offsets.

iii) To remove electrode interface polarization potentials if they exist.

The amount of amplification is determined by the recording situation. Any signal bandpass shaping is also determined
by the recording situation and output requirements. For example, in surface recorded EMG from dynamic movement a high pass filter is included to remove signal components below 15-20 Hz. This removes motion artifact. A low pass filter is also used to remove the signal components above 250 Hz. Experiments have shown that little signal energy resides at these high frequencies, for this recording situation. Such band pass filtering effectively reduces the bandwidth of recorded noise and thus reduces its power.

2.3.5 Recording Electrodes and Their Effects

The most popular EMG recording electrode types and their effects on the signal recorded will now be addressed. The main types of electrodes are:

i) Monopolar needle.
ii) Coaxial needle.
iii) Bipolar needle.
iv) Bipolar fine wire.
v) Bipolar surface.

Monopolar Needle Electrode

The monopolar needle electrode is .2 mm in diameter and is insulated but for the last .5 mm at its tip. The monopolar needle uses a surface electrode, some distance away usually over inactive tissue, as a reference electrode and measures all potentials relative to it. This results in
relatively large amplitude signals. The exposed needle area is small .3 mm$^2$, resulting in very little spatial averaging. Therefore, high frequency components of travelling waves are recorded. The small area of the recording surface allows for precise single motor unit measurements.

Co-axial Needle Electrode

The co-axial needle electrode like the co-axial cable is composed of an outer cylindrical sheath conductor with a central inner conducting wire. The outer and inner conductor are insulated from each other. The co-axial needle is .3-.7 mm in diameter with the inner conductor (electrode) exposed only at its tip. The outer conductor (electrode) the cannula of the needle, is exposed the length of the needle. The inner electrode has very little surface area and does little spatial averaging. The outer electrode spatially averages over the length of the cannula inserted into the tissue. If this length is substantial the outer electrode acts like a reference electrode and results, similar to monopolar recordings are obtained (Lindstrom 1977).

Bipolar Needle Electrode

Bipolar needle electrodes have two insulated conductors permanently fixed within the cannula of a needle. The cannula is about .7 mm in diameter. Signals are thus recorded over small areas very close together. This elec-
trode type perform signal differentiation with low amplitude high frequency signals being recorded.

**Bipolar Fine Wire Electrode**

Like the bipolar needle electrode, fine wire electrodes have two insulated wires inserted into the cannula of a needle. The fine wires of this electrode however, are not permanently fixed to the cannula. After inserting the wires with the needle, the needle is withdrawn from the tissue. The fine wires, .003" in diameter are exposed at the ends and bent to stick to the muscle mass as the needle is withdrawn. The surface areas of the fine wire electrodes are small and the electrodes are close together. Although similar to bipolar needle electrodes, they have larger inter-electrode spacing and recording areas, making them less selective and performing less signal differentiation.

**Bipolar Surface Electrodes**

Bipolar surface electrodes as their name suggests are applied to the skin surface, dry or with a conductive gel, and collect signals in a differential mode from different muscle areas. The area of surface electrodes ranges from 7 - 110 mm². The size of the electrodes determines the amount of spatial averaging done and thus affects the bandwidth of the recorded signals. Spacing of electrodes determines the amount of differentiating of the detected fields, the volume of muscle mass recorded from and the bandwidth
of the recorded signal. Since distances between the surface electrodes and the muscle generator are relatively large, tissue filtering effects are significant, and affect the bandwidths of the signals recorded.

In summary, surface electrodes record far away from the muscle source, perform substantial amounts of spatial averaging produce signals of lower bandwidths, and record from large muscle volumes. In contrast, needle electrodes are close to the signal generators, perform little spatial averaging, produce signals of higher bandwidth, and record from small muscle volumes.

2.4 Stochastic Nature of Signals Recorded

EMG signals recorded during muscle activity, are stochastic in nature. The signals are stochastic because they are the result or the sensing of the activity of a large number of muscle fibres or motor units, firing independently.

The recording of a single MUAP is also a stochastic event. The rate of firing of motor neurons is a random variable and thus the rate of MUAP creation is random in nature (Clamann, 1969). Also, any change in the relative positions of the recording electrodes and source will change both the shape and size of the MUAP measured, in a random way.

When MUAP's from more than one motor unit are
detected, the signal becomes more stochastic in nature. The individual motor units fire not only at a random rate but they fire independently of other motor units (Cramman, 1969). This results in MUAP's from different motor units arriving or contributing at the recording site in an asynchronous fashion. Random distances of the fibres in a motor unit to the recording site leads to random MUAP shapes being recorded. Also, random distances from the motor unit to the recording site result in random MUAP sizes. These combined with the temporal asynchrony of the motor unit discharges leads to the recording of completely stochastic EMG signals.

The stochastic nature of the EMG signal as explained above is due to the physiology and anatomy of nerve and muscle and the way the signals are recorded. It is not due to instrumentation noise. Although instrumentation does introduce noise which is broadband and covers the signal bandwidth, its power is low and insignificant compared to the EMG signals collected and good signal to noise ratios are obtained. Motion artifact and other biological signals (e.g. electrocardiogram) may have significant power in the EMG bandwidth and must be accounted for by other methods.

For moderate muscle activity, the EMG signal is the integrated result of many independent simultaneously occurr-
ing events. As such, it can be assumed to have a Gaussian probability density function by the Central Limit Theorem (Hogan 1980a; Parker 1977). The EMG signal is zero mean both by instrumentation constraints being A.C. coupled, and by experimental findings. (Hogan 1980a; Parker 1977). The variance of the recorded EMG signal is related to the muscle's level of activation (Hogan 1980a; Parker 1977). The EMG signal does not have a white spectrum and successive samples of EMG signal are correlated. A Gaussian distribution is completely described by its mean and variance. Since the mean is zero, the EMG signal is completely described by knowing its variance and its intersample correlation.

The autocorrelation function of sampled EMG signals and consequently the power spectral density has been found to be of constant shape for a given subject, muscle and recording situation for both needle (Parker 1977) and surface (Hogan 1980a) recordings for contraction levels above 5% of maximum voluntary contraction (MVC). Constant shape means no statistically significant change when the variance of the autocorrelation or power spectral density estimates and the variance of the EMG process itself are considered. The large numbers of asynchronously active muscle fibres and motor units contributing to the EMG signal result in an electric field which changes significantly in amplitude statistics only, as the level of activation changes.
Frequency changes are expected within subjects for different muscles; when recording from different subjects; with different clinical states of the neuro-muscular system; and for different recording situations.
CHAPTER 3
MULTIPLICATIVE MODEL FOR EMG SIGNALS

3.1 Introduction

The electromyographic (EMG) signal contains much information about the neuro-muscular system from which it was recorded. EMG signals are used for a wide variety of clinical, research and rehabilitative purposes. The type and amount of information extracted from the EMG is dependent upon the processing techniques used to assess the recorded signals. This chapter summarizes briefly the major uses of EMG signals. It then discusses existing processing techniques and conceptual models of the recorded EMG signals, used to maximize the amount of information obtained. An existing two component multiplicative model is discussed and an alternate multiplicative model with new components is proposed. One of these new components being related to neural input while the other is related to the unitary activity of the muscle(s) being recorded from. The chapter concludes with the results and conclusions of a study performed to assess the constancy, of the shape of the frequency spectra of EMG signals recorded with surface electrodes during isometric-isotonic muscle contractions at selected percentages of maximum force. This study confirmed an essential assumption of the multiplicative model.
3.2 Uses of EMG

The EMG signal, as an information source of muscle activity, is used for different reasons by different users. In myoelectrics, the signal is used to try to extract force information. Changes in the EMG are related to changes in the level of force being produced by the muscle. Kinesiologists and Human Locomotion (Gait) researchers use the EMG to try to determine phasic activity patterns of the muscles and how they relate to the neural control of body movements. They use the EMG as an indicator of relative muscle activity and neural input. Clinical Electromyographers are interested in the specifics of the EMG signals recorded. The characteristics of the signals recorded, both amplitude and frequency, are used along with other clinical tests, in assessing the clinical state of the neuro-muscular system. Clinical electromyographers are interested in assessing the source of clinical problems, whether these are nerve or muscle related. Some neural problems are related to partial or complete conduction block of action potentials or demyelination of the motor neuron axons. Muscle problems are related to the number of motor units, the size of the motor units, the distribution of the motor units within the muscle and firing rates of the motor units for the muscle tested, compared to clinical normal values. These parameters can indicate muscle denervation whether chronic or acute, the amount of
lateral nerve sprouting that has taken place and whether muscle wasting has occurred.

Clinical electromyographers quite commonly use needle electrodes to extract their information from small muscle volumes. This requires that many areas of the muscle must be sampled and averaged parameters considered. EMG for myoelectric controllers and kinesiologic EMG are usually collected with surface electrodes to obtain a representative signal of the whole muscle.

3.3 Existing Processing Techniques and Models

Processing or quantification of EMG signals for the purpose of information extraction has been investigated by many researchers. In clinical electromyography the attempt has been to extract signal parameters which are related to the clinical state of the neuro-muscular system. Buchthal and his co-worker, (1941, 1952, 1953a, 1953b, 1954a, 1954b, 1955) have proposed methods of qualifying the recorded motor unit action potentials (MUAPs) and have examined physical and physiological factors responsible for the changes of the parameters of the MUAPs. Buchthal's parameters, while exact, are laborious to obtain and have been used only for special clinical diagnoses. Willison (1963, 1964) suggested quantifying the EMG from moderate contractions, producing full interference patterns, by counting the frequency of the change of
direction of the signal (number of turns) and the average peak to peak amplitude. The process was automated by Fitch (1967) and shows some clinical promise. A different approach was taken by Kopec et al (1973). They have developed a specialized digital instrument which quantifies recorded signals by estimating the duration of the MUAPs and the number of phases in the MUAP per unit time. The technique uses minimal contraction levels and requires multiple sampling sites, but yet is not time exhaustive and produces immediate results for interpretation. The quantification of EMG signals by frequency spectrum parameters has been studied by Richardson (1951), Walton (1952) and more recently Larsson (1968, 1975) and Lindstrom (1977). As suggested by these researchers the frequency spectrum holds much information about the EMG signal and the underlying neuro-muscular state. As frequency spectrum estimation techniques improve and obtained spectra are better understood, frequency analysis of EMG signals will become more useful in clinical electromyography.

Much work has been done to identify the best method of extracting the level of force production from the EMG signal, e.g. Evans et al (1980); Hogan (1980a, 1980b); Kreifeldt (1974). In an effort to determine an optimum processor for extracting muscle force from the EMG, they have modelled the EMG signal as an amplitude modulation system. This model was based on the observation, that the
shape of the frequency spectra of the recorded signals varied little with level of muscle contraction. They suggest the output EMG is the result of the multiplication of two signals. One signal, the modulating signal, being the force signal while the other is white noise passed through a linear system. The transfer function of the linear system represents the tissue and electrode filtering effects.

The resulting processors all used changes in signal amplitude statistics to represent the changes in the force levels. The processors most commonly used are all based on some form of amplitude demodulation. Amplitude demodulation is a process of extracting the modulating signal (low pass signal) from the composite signal. Demodulation is effected by processing the composite signal with a non-linear function and low pass filtering (LPF) the output. The output of the low pass filter, following relinearization, is the estimate of the modulating signal. Hogan suggested using the square of the signal followed by LPF and square root transformation (relinearization) as the optimum choice for force level extraction. However, he also reported identical performance for a processor which uses the full wave rectified signal followed by LPF as the force estimate. Kreifeldt tested processing with higher and lower powers of the signal such as 1/2, 1/4, 2nd and 4th powers prior to LPF without any significant improvements. Relinearization, which is the
multiplying of the non-linear function output after LPF by the inverse of the non-linear function was tested by Kriefeldt and Hogan. They both report that if the relinearization process overcompensates, that is multiplies by a higher order inverse function, smooth force estimates are obtained. This technique creates artificially smooth force records but reduces the system sensitivity to force changes.

The results of this past research indicate:

i) spectral shape changes with contraction level are insignificant,

ii) amplitude statistics, particularly changes in variance, are sensitive to changing activation levels,

iii) the multiplicative (amplitude modulation) model for the EMG signal is a reasonable one.

3.4 New Model

As reported by Parker et al (1977) and Hogan (1980a, b), the autocorrelation function of EMG signals is of constant shape for contractions above 5% MVC. This means that the frequency spectrum of these signals is of constant shape over this contraction range. This results from a "constant" interference pattern at the recording electrode sites. Only the amplitude of the interference pattern changes with muscle activation level. Thus, it is proposed that the output EMG signal is the result of a muscle activation or neural input signal modulating (multiplying) a unitary muscle response (UMR) or interference pattern. The proposed model is further described in Figure 3.1 and can be
expressed mathematically as follows:

\[ E(t) = N(t) \cdot I(t) \] (3.1)

where \( E(t) \) is the recorded EMG signal
\( N(t) \) is the neural input
\( I(t) \) is the unitary muscle response

The neural input or muscle activation level, \( N(t) \), is by physical definition always greater than zero and less than some maximum tetanic level. The unitary muscle response, \( I(t) \), is a Gaussian distributed zero mean stochastic process based on muscle physiology, structure and the recording situation, as discussed in Section 2.5. It has a variance dependent on the recording situation and the muscle being studied.

The neural input is a low frequency signal since it relates to the muscle state of activation which as described in Section 2.2.2 is slowly changing. The unitary muscle response is a relatively high frequency signal. It is related to the high frequency changes in the electric field in the external medium of the active muscle fibres. These rapid changes occur as active motor units randomly fire and are dependent on the amount of spatial-temporal averaging of motor unit activity which is occurring to create the electric field in the external medium. These frequency components of \( I(t) \) may change from muscle to muscle and from subject to subject. They may also change with different
Figure 3.1 Multiplicative Model.

\[ E(t) = N(t) \cdot I(t) \]

UMR EMG \( I(t) \)

Neural input \( N(t) \)

Simulated Product \( E(t) \)

Real Recorded Signal \( E(t) \)
clinical states of the muscle. Changes in this signal's frequency spectrum might be used as an indicator of the number of active motor units and their size. The frequency content of this signal component can also be related to the type of electrodes used and their spacing, that is to the amount of spatial averaging and/or differentiation being performed by the recording electrodes.

The model differs from those previously proposed in that it does not attempt to relate the level of muscle activity, as determined by its electrical activity, to the force being produced by the muscle. The relationship between muscle electrical activity and its level of force for isometric contractions has been proposed to be both linear Milner-Brown and Stein (1975) and non-linear Vredenbregt and Rau (1973). This relationship, for dynamic, contractions, is certainly non-linear due to the complexities of the excitation contraction coupling mechanism, and the compliance properties of the muscle. Therefore, any estimates of muscle force derived from muscle EMG activity recorded from dynamic contractions are destined to contain errors. Instead, the model suggests this low frequency component of the composite signal is related to the net neural input or is representative of the effective control. The model is thus applicable for analyzing EMG signals recorded during dynamic contractions such as gait. The model proposes that the high frequency
signal component, I(t), also contains useful information which was not previously considered.

Stationarity is a property of stochastic systems and signals that requires that their statistics do not change over time. Stationarity in the wide sense states that the mean and autocorrelation function of a signal do not change with time (Papoulis, 1965). Strictly speaking, as the muscle fatigues, or its physiologic state changes the EMG signal will not be stationary even for isometric-iso-tonic (constant) contractions. However, over periods of time where the muscle is in a constant physiological state, the EMG signal for constant contractions will at least be stationary in the wide sense. It is assumed that the unitary muscle response will satisfy the same stationarity conditions as the EMG signals recorded from constant contractions. This assumption will be tested later in this manuscript.

3.4 EMG Power Spectra of Constant Contractions

The assumption, that the frequency spectra of surface recorded EMG signals are of constant shape, independent of the contraction level was tested. EMG signals were recorded for isometric contractions at four different contraction levels for three subjects, sampled and stored on a digital computer. A Fortran IV program, FREQP5, was written which constructed estimates of the power spectrum of the sampled
data, based on Fast Fourier Transforms (FFT) calculated for overlapping windows of the data. The final spectrum was an average over the windows taken. The program allows for a variable number of overlapping data segments (windows) to be chosen and variable length FFTs to be calculated. A more complete program description and Fortran listing are included in Appendix I.

FREQ5 also calculated the median frequency as:

$$FM = \frac{\sum_{i=1}^{N/2+1} p_i \Delta f(i-1) \Delta f}{\sum_{i=1}^{N/2+1} p_i \Delta f}$$

the statistical band width as:

$$SB = \frac{\left[ \sum_{i=1}^{N/2+1} p_i \Delta f \right]^2}{\sum_{i=1}^{N/2+1} p_i^2 \Delta f}$$

where:

$$\Delta f = \text{frequency resolution}$$

$$p_i = \text{ith spectral coefficient}$$

$$N = \text{number of data samples in the record}$$

and the percentage of total power in three selectable frequency bands.

These parameters were used to determine the homogeneity of the spectral shapes at different contraction levels.
The EMG signals were sampled at a 500 Hz sampling rate and collected in 2000 sample data records. This allowed 4 seconds of the EMG signal to be sampled and collected at a time. FREQP5 was used to read the collected records and process the data as follows. The data records were divided into three fifty percent overlapping 1000 point segments. The overlapping segments were weighted by a Hanning window to reduce frequency dispersion as per Brigham (1974) and Bergland (1969). Each segment was augmented with 24 zeros and 1024 point FFTs, with .49 Hz frequency resolution, were calculated. The three spectra were averaged to achieve the final spectral estimate from which the median frequency, statistical band width and the percent power in prescribed frequency bands (0-50 Hz, 50-125 Hz and 125-250 Hz) were calculated. Results can be found in Table 3-1.

As can be seen from Table 3-1 there is a relatively small change in FM and SB with change in contraction level within subjects. Also note that there is more variation in FM and SB from subject to subject than from contraction level to contraction level within subjects. The range of variation observed in FM and SB agree with those reported by Hogan, (1980b) and Petrofsky (1980) respectively. The percent power in the selected frequency bands show similar variation patterns as those just described for FM and SB. The center frequency band 50-125 Hz was especially stable, even from
subject to subject. It is important to note that the changes from contraction level to contraction level within a subject, that did occur in the measured frequency spectrum parameters appeared to happen at random. These changes were probably due to the stochastic nature of the signals being recorded and showed no deterministic trend with contraction level. From these results it can be concluded that the frequency spectrum of the EMG signals recorded had a homogeneous shape independent of force level. The increasing signal power measured with increasing force level indicates that the EMG signal amplitude increases with force level. These two conclusions support the multiplicative model proposed for surface recorded EMG signals.
### TABLE 3-1

<table>
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<tr>
<th>SUBJECT</th>
<th>CONTRACTION LEVEL % MVC</th>
<th>FM Hz</th>
<th>SB Hz</th>
<th>% POWER IN FREQ. BANDS 0-50 Hz</th>
<th>% POWER IN FREQ. BANDS 50-100 Hz</th>
<th>% POWER IN FREQ. BANDS 125-250 Hz</th>
<th>PWR $\mu V^2$</th>
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<td>75.1</td>
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<td>32.9</td>
<td>55.5</td>
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<td>26.3</td>
<td>60.0</td>
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<td>45.5</td>
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SPECTRAL ANALYSIS OF EMG SIGNALS RECORDED FROM RECTUS FEMORIS DURING ISOMETRIC CONTRACTIONS WITH THE KNEE AT 120°.
CHAPTER 4
HOMOMORPHIC PROCESSING OF SURFACE RECORDED EMG SIGNALS

4.1 Introduction

The multiplicative model just proposed for electromyographic (EMG) signals is essentially a non-linear model. As such, it does not lend itself to the direct application of linear filtering techniques for component signal estimation or extraction. Therefore non-linear techniques must be utilized. One such technique is homomorphic signal processing. This chapter discusses the generalized theory of homomorphic processing, multiplicative homomorphic processing and homomorphic processing of surface recorded EMG signals. The chapter concludes with a comparison of homomorphic processing to other techniques, complete with theoretical discussions and simulated tests.

4.2 Homomorphic Processing

The following discussion on homomorphic processing is essentially based on Chapter 10, Homomorphic Signal Processing, of Oppenheim and Schafer's book, Digital Signal Processing (1975). An attempt will be made here to summarize the important points involved in homomorphic processing as outlined in that chapter. The reader is invited to
Homomorphic processing is a technique used to analyze non-linear systems based on the use of transformations. The transformations convert the non-linear system such that in the transformed space, the system acts as a generally linear system. This is to say that in the transformation space, the theory of generalized superposition holds. The theory of generalized superposition can be stated as follows for linear systems:

\[ T[x_1(n) + x_2(n)] = T[x_1(n)] + T[x_2(n)] \]
and

\[ T[Cx_1(n)] = CT[x_1(n)] \]

where \( T \) is the system transformation and \( x_1(n) \) and \( x_2(n) \) are any two system inputs. \( C \) is any scalar.

Generalizing this, let \( \Lambda \) symbolize a rule for combining inputs with each other, and \( \varnothing \) a rule for combining inputs with scalars. Similarly, let \( \zeta \) be a rule for combining system outputs with each other and \( \subseteq \) be a rule for joining system outputs with scalars. With \( H \) representing the system transformation generalized superposition can be stated as:

\[ H[x_1(n) \Lambda x_2(n)] = H[x_1(n)] \zeta H[x_2(n)] \]
and

\[ H[C \varnothing x_1(n)] = C \subseteq H[x_1(n)] \]
Linear systems are special cases of the above with $\Lambda$ and $\zeta$ being addition and $\xi$ and $\sim$ being multiplication.

For the theory of linear vector spaces to be used, the input and output operations must be able to satisfy the algebraic rules of vector addition and scalar multiplication. Therefore the input and output operations $\Lambda$ and $\zeta$ must be both commutative and associative. That is,

$$x_1(n) \Lambda x_2(n) = x_2(n) \Lambda x_1(n)$$
$$y_1(n) \zeta y_2(n) = y_2(n) \zeta y_1(n)$$

and

$$x_1(n) \Lambda [x_2(n) \Lambda x_3(n)] = [x_1(n) \Lambda x_2(n)] \Lambda x_3(n)$$
$$y_1(n) \zeta [y_2(n) \zeta y_3(n)] = [y_1(n) \zeta y_2(n)] \zeta y_3(n)$$

Other such rules must also be satisfied if a suitable vector space and transformation are to be defined.

If the system inputs can be represented in a linear vector space where $\Lambda$ and $\sim$ correspond to vector addition and scalar multiplication and the system outputs can be represented in a linear vector space with $\zeta$ and $\xi$ corresponding to vector addition and scalar multiplication, then the system can be represented in canonic form as shown in Figure 4-1. In this figure $D_{\Lambda}$ has the property that:

$$D_{\Lambda}[x_1(n) \Lambda x_2(n)] = D_{\Lambda}[x_1(n)] + D_{\Lambda}[x_2(n)] = \hat{x}_1(n) + \hat{x}_2(n)$$
$$D_{\Lambda}[c \sim x_1(n)] = c \cdot D_{\Lambda}[x_1(n)] = c \hat{x}_1(n)$$
Canonic Representation of Homomorphic Systems

\[ \text{Figure 4-1} \]

\( D_\Lambda \) satisfies the theory of generalized superposition with an input operation of \( \Lambda \) and an output operation of addition. \( D_\Lambda \) transforms the two signals combined by the rule \( \Lambda \) to a conventional linear space where the signals \( D_\Lambda(x_1(n)) \) and \( D_\Lambda(x_2(n)) \) are combined by addition. The system \( L \) is a standard linear system such that:

\[
L[\hat{x}_1(n) + \hat{x}_2(n)] = L[\hat{x}_1(n)] + L[\hat{x}_2(n)] = \hat{y}_1(n) + \hat{y}_2(n)
\]

\[
L[c\hat{x}_1(n)] = cL[\hat{x}_1(n)] = cy_1(n)
\]

The system \( D^{-1}_\zeta \) transforms from the additive domain to the \( \zeta \) domain so:

\[
D^{-1}_\zeta[\hat{y}_1(n) + \hat{y}_2(n)] = D^{-1}_\zeta[\hat{y}_1(n)] \zeta D^{-1}_\zeta[\hat{y}_2(n)] = y_1(n) \zeta y_2(n)
\]

\[
D^{-1}_\zeta[c\hat{y}_1(n)] = c \zeta D^{-1}_\zeta[\hat{y}_1(n)] = c \zeta y_1(n)
\]

\( D_\Lambda \) and \( D_\zeta \) are fixed by their operations \( \Lambda, \zeta \) and \( \zeta, \zeta \) respectively and are characteristic of their class of systems. \( D_\Lambda \) and \( D_\zeta \) are therefore called characteristic systems.
All homomorphic systems with the same input and output operations (i.e. $D_A = D_\zeta$) differ only in their linear part. This is because $D_A$ and $D_\zeta$ are fixed by the input and output operations. Therefore, once the characteristic system for the class has been determined (i.e. $D_A$ or $D_\zeta$) the remaining system determination becomes simply a linear filtering problem.

Example: If $x_1(n)$ is needed from the composite signal

$$x(n) = x_1(n) \land x_2(n)$$

The following can be considered:

$$D_A[x(n)] = D_A[x_1(n) \land x_2(n)] = D_A[x_1(n)] + D_A[x_2(n)] = \hat{x}_1(n) + \hat{x}_2(n)$$

we now choose the linear system whose output is

$$L[\hat{x}_1(n) + \hat{x}_2(n)] = \hat{x}_1(n) = D_A[x_1(n)] = \hat{y}(n)$$

Then with $D_\zeta = D_A$; $D_\zeta^{-1} = D_A^{-1}$

$$y(n) = D_\zeta^{-1}[\hat{y}(n)] = D_A^{-1}[D_A[x_1(n)]] = x_1(n)$$

To achieve perfect separation of $x_1(n)$ and $x_2(n)$ we must be able to perfectly separate $\hat{x}_1(n)$ and $\hat{x}_2(n)$ by linear filtering. How well this separation can be achieved depends on the $\land$ operation and the properties of $x_1(n)$ and $x_2(n)$.

In practice, homomorphic systems usually have equal input and output operations. These operations are usually multiplication or convolution.
4.3 Multiplicative Homomorphic Processing

The class of homomorphic systems that satisfy the generalized superposition rules in which the operation for joining inputs, $\Lambda$, is multiplication and the operation for joining inputs with scalars, $\odot$, is exponentiation, has signals which can be expressed as

$$X(n) = [X_1(n)]^\alpha \cdot [X_2(n)]^\beta$$  \hspace{1cm} (4-1)

The characteristic system for multiplication must have the property that

$$D_\star [[X_1(n)]^\alpha \cdot [X_2(n)]^\beta] = \alpha D_\star [X_1(n)] + \beta D_\star [X_2(n)]$$

A transform or system which formally has these properties is the logarithm function. For Equation (4-1)

$$\log[X(n)] = \alpha \log[X_1(n)] + \beta \log[X_2(n)]$$

where $X_1(n)$ and $X_2(n) > 0$ for all $n$

$X(n)$ may however not always be greater than zero. This forces the utilization of complex signal representation. In such cases, complex logarithms must be used. This leads to the general canonic form of homomorphic systems with multiplication as the input and output operation shown in Figure 4-2. In this figure $X(n)$, $\hat{X}(n)$, $\hat{Y}(n)$ and $Y(n)$ are in general complex.

When using multiplicative homomorphic processing to separate input signal components of composite input signals a suitable choice for the linear system must be made. The
extent to which $X_1(n)$ and $X_2(n)$ can be separated from the composite input signal $X(n) = X_1(n) \cdot X_2(n)$ depends on the amount of spectral overlap of $\hat{X}_1(n)$ and $\hat{X}_2(n)$ the characteristic system's (i.e. logarithmic transform's) outputs. If the spectral overlap of these signals is not significant, the signals can be separated with minimum error.

4.4 Homomorphic Processing of Surface Recorded EMG Signals

As postulated in Section 3.4, for moderate to maximum muscle contraction levels, surface recorded EMG signals can be modelled as a multiplicative process. This is due to the consistent shape of the frequency spectrum as shown in Section 3.5. The sampled EMG signals can then be mathematically expressed as:

$$E(n) = N(n) \cdot I(n)$$
where \(E(n)\) is the recorded EMG signal,
\(N(n)\) is the modulating neural signal,
\(I(n)\) is the unitary muscle response,
and \(n\) is the sample number.

Note: Amplifier gain and electrical noise are not included in the model. The gain is omitted since it is an arbitrary scalar and the noise is omitted because under most recording conditions, it is small compared to the EMG signal being recorded.

It has been shown experimentally that EMG signals recorded by suitable surface electrodes, suitably spaced, during isometric-isotonic (constant) muscle contractions are limited to a bandwidth of 10 to 250 Hz (Shein, 1980). This bandwidth, however, is determined by the type of electrodes used, their configuration, and the filters employed to remove noise as stated in Chapter 2. Since constant muscle contractions can be represented by \(I(n)\) multiplied by a constant neural input a similar bandwidth for \(I(n)\) can be assumed.

\(N(n)\) which represents the neural intent (control) of the contraction level is limited in its frequency component make up by the contraction time constants of the muscle. For this reason, \(N(n)\) has little signal power above 10 Hz. \(N(n)\) is a positive only signal, since it represents neural input, which varies from minimal to maximal, but is always positive. The component signals \(I(n)\) and
N(n) as explained above occupy separate frequency bands. Therefore, they can be individually estimated from the composite signal, the recorded EMG, by homomorphic processing.

The model is transformed from the multiplicative domain into the additive domain by taking the complex logarithm of E(n). The complex logarithm is defined as:

\[
\log[X(n)] = \log[|X(n)|] + j \arg[X(n)]
\]

where \(X(n)\) is some complex number.

The complex logarithm of \(E(n)\) then becomes

\[
\log[E(n)] = \log[|E(n)|] + j \arg[E(n)]
\]

where \(|E(n)| = |N(n) \cdot I(n)|\).

General superposition as outlined in Section 4.1 holds since \(N(n)\) is always positive and real.

that is

\[
|E(n)| = N(n) \cdot |I(n)|
\]

and \(\arg[E(n)] = \arg[N(n)] + \arg[I(n)]\)

\[= \arg[I(n)]\]

since \(\arg[N(n)] = 0\).

This results in

\[
\log[E(n)] = \log[N(n)] + \log[|I(n)|] + j \arg[I(n)] \quad (4-2)
\]

Since \(I(n)\) is either a positive or negative real number,

\[\arg[I(n)] = 0 \text{ if } I(n), \text{ that is } E(n) \text{ is positive}\]

\[= \pi \text{ if } I(n), \text{ that is } E(n) \text{ is negative.}\]

Separation of \(E(n)\) into its component signals \(N(n)\) and \(I(n)\)
is now the linear filtering problem of separating $\log[E(n)]$ into appropriate component signals. Assuming the logarithm transformation has not significantly altered the component signals frequency bands, Equation 4-2 can be separated into:

$$\log [\hat{N}(n)] \text{ and } \log [|\hat{I}(n)|] + j \arg [\hat{I}(n)]$$

by low pass filtering $\log [E(n)]$ with the appropriate cut-off frequency.

The $\hat{\cdot}$ notation is used, which represents estimate, because the logarithm transformation, mainly the absolute value operation on $E(n)$, causes the spectra of $\log [N(n)]$ and $\log [|I(n)|]$ to overlap. The overlap is not significant however and estimates $\log [\hat{N}(n)]$ and $\log [|\hat{I}(n)|]$ with small errors can be obtained. The imaginary part of $\log [E(n)]$, namely $j \arg[I(n)]$, is included with $\log [|I(n)|]$ upon filtering. This is because $\arg [N(n)] = 0$ as stated above and the imaginary part of $\log [E(n)]$ is entirely due to $I(n)$.

The component signals $\log [N(n)]$ and $\log [|I(n)|] + j \arg (I(n))$ are transformed from the additive domain back into the multiplicative domain by complex exponentiation.

This results in

$$\hat{N}(n) = \exp(\log [\hat{N}(n)]) = \hat{N}(n) \quad (4-3)$$

$$\hat{I}(n) = \exp(\log [|\hat{I}(n)|] + j \arg [I(n)]) \quad (4-4)$$

$$= |\hat{I}(n)| \cdot \exp(j \arg[I(n)])$$

Since $\arg [I(n)] = 0$ if $E(n)$ is positive

$$= \pi \text{ if } E(n) \text{ is negative}$$
Equation 4-4 becomes
\[\hat{I}(n) = +|\hat{I}(n)| \quad \text{if } E(n) \text{ is positive}\]
\[= -|\hat{I}(n)| \quad \text{if } E(n) \text{ is negative}\]
or
\[\hat{I}(n) = |\hat{I}(n)| \cdot \text{sign}[E(n)]\]
where \(\text{sign}[E(n)]\) is the algebraic sign of \(E(n)\).
The above result allows the total homomorphic processing to be done using real arithmetic as shown in Figure 4-3.

4.5 Comparison of Homomorphic Processing to Other Techniques

4.5.1 Theoretical Considerations

Homomorphic processors are similar to other EMG processors, mean rectified EMG (MRE), or root mean squared (RMS) processors, in that they process the composite recorded EMG signal in a non-linear fashion. This non-linear processing followed by appropriate low pass filtering is similar to envelope demodulation techniques, used with amplitude modulated (AM) radio waves.

The multiplication of two signals in the time domain or amplitude modulation results in the convolution of the frequency spectra of the component signals. When the component signals do not overlap in the frequency domain, the lower frequency signal, the modulating signal, is essentially shifted up into the frequency range of the modulated or
Figure 4.3 Realization of Homomorphic Processing Using Real Arithmetic Components.
carrier signal. With AM radio waves the carrier is a single frequency and is represented in the frequency domain as an impulse at the carrier frequency and the modulating signal is a band of low frequency components. Thus multiplication in the time domain results in the convolution in the frequency domain of the carrier impulse with the low pass modulating signal. The resultant spectrum is that of the low pass signal, unchanged in shape but relocated or shifted in frequency to appear on either side of the carrier impulse. (See Figure 4-4.) The situation when conceptualizing the EMG mechanism as a multiplicative process is fundamentally the same. However, the carrier is now I(n) a high pass random signal composed of a band of high frequencies which are modulated by a low pass random signal N(n) composed of a band of low frequencies. The resultant convolved frequency spectrum is essentially located where the spectrum of I(n) is, but it is not simply the spectrum of N(n) shifted in frequency. The resultant spectral shape depends on the spectral shapes of N(n) and I(n). If N(n) is an impulse at zero frequency, which is approximately what is expected for constant muscle contractions, the convolved spectrum will be identical in shape to I(n) but of greater power. Figure 4-5 depicts the component one-sided spectra and the resultant convolved spectra, for EMG signals, surface recorded during normal human gait from the quadriceps
Figure 4.4 Frequency Effects of Multiplication and Square Transformation.
Figure 4.5 Frequency Effects of Multiplying EMG Component Signals.
muscles. The changed shape of the convolved spectra, 
(changed from the spectral shape of N(n)) makes perfect 
determination of the component signals from the composite, 
impossible.

Envelope demodulation of AM radio waves is effected 
by passing the composite signal through a non-linear device 
and then an appropriate low pass filter to extract the 
desired modulating signal. The carrier is known and there­
fore extraction of it adds no information. For signals 
created using single frequency component carriers the non­
linear processing shifts the convolved spectra both up in 
frequency and back to the frequency origin, without a change 
in shape. This results in the modulating signal, times a 
scalar, existing in its original frequency band. The modu­
lating signal times the above described scalar can then be 
extracted from the non-linear processed original composite 
signal by appropriate low pass filtering and subsequent 
relinearization. These events for square transformation 
are depicted in Figure 4-4c. Therefore, with AM radio waves 
or multiplication with a single frequency carrier, demodu­
lation is only limited by the ambient noise levels and the 
low pass filter characteristics.

When a composite signal is created with a carrier 
consisting of a band of frequencies such as with I(n) of the 
EMG process, non-linear processing still results in shifting
of a component spectrum to the frequency origin. However the spectral shape of the shifted spectrum is not that of the modulating signal, but it is a spectral shape which has been distorted by the original convolution, as just discussed, as well as the subsequent non-linear processing. This distortion of the shifted spectrum is the source of errors when extraction of the modulating signal is attempted by low pass filtering and appropriate relinearization. Figure 4-6 shows the resulting problems when a carrier of two frequency components is modulated by a low pass signal and subsequently squared. As can be seen in Figure 4.6(b) the convolved spectrum is distorted. The further distortion resulting from non-linear processing can be seen in Figure 4.6(c). Subsequent low pass filtering and square root transformation will not extract the desired signal \( A(f) \) but the estimate \( \hat{A}(f) \) contaminated by noise terms centred at \( \pm \Delta w \) where \( \Delta w = |w_{c_1} - w_{c_2}| \). Thus modulating signals, such as \( N(n) \), when convolved with carriers composed of frequency bands, such as \( I(n) \) will not be able to be uniquely retrieved due to the spectral distortion resulting from the convolution and subsequent envelope detection methods. The signal to noise response of the demodulation process will then depend on the bandwidth of the low pass filter used to extract the desired modulating signal as well as the amount of spectral distortion which has occurred. This spectral distortion
Figure 4.6 Frequency Effects of Two Frequency Component Carrier and Square Transformation
which occurs with EMG signals explains the poor signal to noise ratio's reported in the literature for force extraction as compared to signal to noise ratio's for conventional AM radio waves. A more rigorous analysis of the expected noise is beyond the scope of this thesis. Homomorphic processing is not free of these spectral distortions and as such does not produce significantly better results, than other demodulation techniques. Homomorphic processing does however, allow the carrier I(n) to be estimated.

Homomorphic processing of EMG signals, like the MRE demodulation technique, involves rectification and subsequent low pass filtering. However, the transformation of the data into the logarithmic domain before linear filtering allows both the modulating signal and the carrier to be extracted. The frequency spectra of the logarithms of the modulating signal, (N(n)), and the absolute value of the carrier, (I(n)), do overlap. This is due mainly to the spectral spreading effect of rectifying the carrier, (I(n)), but it is also due to the spectral spreading effect of the logarithmic transformation. This spectral overlap of the two component signals determines the amount of error to be expected in \( \hat{N}(n) \) and \( \hat{I}(n) \) defined in Equation 4.3 and 4.4.

4.5.2 **Simulation Tests**

It was decided to investigate the extent of this
spectral overlap and to compare homomorphic signal processing with the more common technique of MRE for modulating signal \( N(n) \) extraction. To accomplish this, two Fortran IV programs HOMTST and RECTST were written. The programs simulate EMG signals collected from phasic activities. Each program reads a file containing raw EMG signals collected during controlled constant contractions and multiplies this signal by a simulated modulating signal. The modulating signal chosen is a sinusoid of selectable frequency which is offset by a D.C. bias to have values from zero to two. Such a modulating signal of 1 to 2 Hz frequency is quite similar to the phasic neural patterns seen in human gait for the quadriceps muscles. The composite signal is then processed, homomorphically in HOMTST, and by rectification and low pass filtering (MRE) in RECTST, to extract the modulating signal. The extracted signals are multiplied by a constant such that they have mean values equal to those of the corresponding input modulating signals. The root mean square difference or root mean square error (RMSE) between the input and extracted signals is then calculated. The ratio of root mean square (RMS) of the input signal to RMSE is defined as the signal to noise ratio (SNR).

The cut-off frequency of the low pass filters (FC) was varied from 3 to 10 Hz for various modulating signal frequencies from 1 to 5 Hz. Figures 4.7 and 4.8 show typical
Figure 4.7 Typical Results From Homomorphically Processed Simulated EMG Signal.
Figure 4.8 Typical Results From MRE Processing Simulated EMG Signals.
results obtained for homomorphic and MRE processing respectively. A summary of the results is shown in Table 4-1. A complete listing of HOMTST and RECTST can be found in Appendix II.

<table>
<thead>
<tr>
<th>MODULATING FREQUENCY Hz</th>
<th>HOMOMORPHIC SNR</th>
<th>MRE SNR</th>
</tr>
</thead>
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<tr>
<td>1.0</td>
<td>6.4 5.1 4.4 3.4 2.9</td>
<td>5.1 4.3 3.9 3.4 3.0</td>
</tr>
<tr>
<td>1.5</td>
<td>4.3 3.9 3.7 3.3 2.9</td>
<td>4.3 3.7 3.5 3.3 3.0</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0 3.9 3.8 3.2 2.8</td>
<td>3.9 4.4 4.1 3.5 3.0</td>
</tr>
<tr>
<td>5.0</td>
<td>2.9 3.7 3.2 3.1 3.0</td>
<td>2.1 3.5 3.8 3.6 3.3</td>
</tr>
</tbody>
</table>

**TABLE 4-1**

**SIGNAL TO NOISE RATIOS FOR SIMULATED SIGNALS**

As can be seen in this table, the amount of spectral overlap of the component signals in the logarithmic domain is not significant and signal to noise ratios comparable to those obtained by rectification and low pass filtering are obtained. As would be expected, based on the component signal spectra, the closer FC was to the actual modulating signal frequency, the better the obtained SNR was. In cases where FC was lower than the modulating signal frequency poor
SNRs resulted. This is as would be predicted from the previous theoretical discussion.
5.1 **Introduction**

In this chapter the results of the application of homomorphic processing to surface recorded EMG signals will be discussed. To substantiate the model proposed in Section 3.4 the results of homomorphic processing of EMG signals recorded during isometric-isotonic (constant) muscle contractions are reported. The results of stationarity testing of both EMG signals recorded during constant muscle contractions and their corresponding processed unitary muscle responses (UMRs) are also presented. The results of homomorphic processing applied to EMG signals from phasic muscle contractions concludes the chapter. This includes the depiction of the processing of an example record giving the estimates of the component signals, the neural input and the UMR. This is followed by a comparison of the frequency spectra and stationarity testing of, UMRs obtained from homomorphic processing of EMG signals recorded bilaterally from four lower limb muscles, of a subject, walking at two different speeds.
5.2 Homomorphic Processing of EMG Signals From Constant Contractions

5.2.1 Power Spectra Comparisons

The proposed model (Section 3.4) suggests that any EMG signal recorded during a constant muscle contraction is composed of a UMR, \( I(n) \), multiplied by a constant neural input signal, \( N(n) \). In the frequency domain the equivalent conceptualization is the spectrum of the UMR convolved with an impulse located at the frequency origin. The magnitude of the impulse is dependent on the level of contraction, or level of constant neural input. The spectrum resulting from this convolution is simply the UMR spectrum times a constant. This means that the spectra of an EMG signal recorded during a constant contraction and of its corresponding UMR should be identical but for this constant. The two spectra should have identical shapes but different powers. One way of testing this is to compare the median frequencies, statistical bandwidths and powers of the two spectra. The results of such a comparison for four different contraction levels and three different subjects are shown in Table 5-1.

Large disposable surface electrodes 1.2 cm in diameter with a constant spacing of 3.5 cm were used over the rectus femoris for signal collection. The raw EMG data were collected and the raw EMG spectra estimated as in
<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>CONTRACTION LEVEL % MVC</th>
<th>RAW EMG</th>
<th>UNITARY MUSCLE RESPONSE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CONTRACTION LEVEL % MVC</td>
<td>MEDIAN FREQ. Hz</td>
<td>STATISTICAL BANDWIDTH Hz</td>
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<tr>
<td>F.S.</td>
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<td>75.1</td>
<td>90.6</td>
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<td>75.1</td>
<td>84.2</td>
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<td>101.6</td>
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<td></td>
<td>25</td>
<td>77.0</td>
<td>87.4</td>
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**TABLE 5-1**

POWER SPECTRA COMPARISON BETWEEN EMG SIGNALS RECORDED DURING CONSTANT CONTRACTIONS AND THEIR CORRESPONDING UMRs. KNEE JOINT AT 120°.
Section 3.5. Homomorphic processing was carried out by the Fortran IV program HOMFIL (Appendix III). The cut-off frequency of the low pass filter used for homomorphic processing was chosen to be 3 Hz. An example of the results of the processing for typical EMG data collected during constant muscle contraction are shown in Figure 5-1. The ripple in the neural input estimate is due to the spectral overlap of \( \log[N(n)] \) and \( \log[|I(n)|] \) as described in Section 4.4, and is reduced with a decrease in the cut-off frequency of the processing low pass filter. The spectra of the processed UMR's were calculated using FREQR (Appendix I) as follows. The 1500 data point UMR files created by HOMFIL were divided into two 50% over-lapping 1000 point segments and weighted by a Hanning window. Each segment was then augmented by 24 zero valued samples and 1024 point power spectra were calculated for each data window. The final spectrum, from which the power spectral parameters were calculated, was the average of these two spectra.

As can be seen in Table 5-1 the spectra for a given subject and contraction level are quite similar with differences only in the total signal powers. This substantiates the proposed multiplicative model. Homomorphic processing separates the constant level of contraction component, \( N(n) \), and the UMR component, \( I(n) \), from the composite signal.
Figure 5.1 Typical Results From Homomorphically Processing EMG Signals From Constant Contractions.
The power of the UMR has little variance, within a subject for the different levels of contraction and the variance in signal power from subject to subject for any level of contraction is only slightly larger. This is to be expected for normal subjects, when recording from the same muscle with the same electrode configuration.

5.2.2 Stationarity Testing of EMG Signals From Constant Contractions

The stationarity of EMG signals recorded during constant muscle contraction and their corresponding UMRs were tested using Bendat and Piersol's (1971) run test as implemented in the Fortran IV program RUNTST. For a description and a listing of RUNTST see Appendix IV. As suggested by Bendat and Piersol (1971) the segment lengths chosen for the run test should be such that they contain enough data to span many periods of the lowest frequency component of the signal being tested. By assumption and recording hardware constraints the minimum frequency component of the EMG signals recorded is 10 Hz as described in Section 4.4. Since the data was collected at 500 Hz and was analysed in 2000 sample records, 4 second long data windows resulted. The run test program was therefore run with 20, 16, 12 and 8 data segments per window corresponding to data segment lengths of .20, .25, .33 and .50 seconds. The results at
the .050 level of significance, of the stationarity tests, for the different segment lengths, of the raw EMG and UMR data used for Table 5-1 are tabulated in Table 5-2.

It can be seen from this table that the majority of the EMG and UMR data can be considered stationary. This agrees with results obtained for EMG signals recorded using similar sized electrodes, from the biceps brachii during constant contraction by Abdel Azim (1975). Visual inspection of data records which by the run test were deemed non-stationary appeared quite stationary. This apparent inconsistency may result from the low power of the run test under certain conditions. This is further discussed in Section 5.3.3.

5.3 Homomorphic Processing of EMG Signals

From Phasic Contractions

5.3.1 Example Record

To demonstrate the effects of homomorphic processing on EMG signals recorded during phasic muscle contractions a typical record from a gait study was chosen. The processing was performed by the Fortran IV program, HOMFIL, described in Appendix III, with a low pass filter cut-off frequency of 5 Hz. The record chosen is one which contains a sampled EMG signal, collected as in Section 5.2.1., from the quadriceps muscle, of a 16-year old normal male, during a normal gait of 1. m/sec. The input and processed signals are shown in
<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>CONTR. LEVEL % MVC</th>
<th>NO. OF SECT.</th>
<th>CONST. CONTR. RESULT</th>
<th>UMR RESULT</th>
<th>SUBJECT</th>
<th>CONTR. LEVEL % MVC</th>
<th>NO. OF SECT.</th>
<th>CONST. CONTR. RESULT</th>
<th>UMR RESULT</th>
</tr>
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</tr>
<tr>
<td></td>
<td>25</td>
<td>20</td>
<td>S</td>
<td>S</td>
<td>25</td>
<td>20</td>
<td>NS\textsubscript{1}^-</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td></td>
<td>16</td>
<td>NS\textsubscript{1}^-</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td></td>
<td>12</td>
<td>NS\textsubscript{1}^-</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td></td>
<td>8</td>
<td>NS\textsubscript{1}^-</td>
<td>S</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE SYMBOLS**

- **S** - Data is stationary.
- **NS\textsubscript{1}^-** - Data means not stationary.
- **NS\textsubscript{2}^-** - Data mean squares not stationary.
- **NS\textsubscript{3}^-** - Data mean and means square not stationary.

Superscript - : too few runs.
Superscript + : too many runs.

**TABLE 5-2**

RUN TEST STATIONARITY RESULTS FOR EMG DATA RECORD DURING CONSTANT MUSCLE CONTRACTION AND THE CORRESPONDING UMRs AT A .050 SIGNIFICANCE LEVEL.
Figure 5.2.

As can be seen in this figure, homomorphic processing separates the composite signal into a neural input estimate and an estimate of the UMR. The neural input signal estimates the neural control of the muscle activity. This signal is comparable to that obtained by rectification and low pass filtering of the composite EMG signal. As was determined in section 4.5.2, the errors in the estimates of the neural input, are of similar magnitude, for the two processing techniques. The UMR component is determined by both the electrode configuration and the muscle being examined. Therefore the UMR should change for different muscles. Conversely it should remain the same for particular muscles even though the phasic activity patterns are altered. This point is addressed in the next section.

5.3.2 UMR Power Spectra Comparisons

EMG signals were collected bilaterally from four lower limb muscles during normal gait at 1.0 m/s and 1.5 m/s, of the subject mentioned in section 5.3.1. The four muscles studied were the quadriceps, biceps femoris, tibialis anterior and gastrocnemius. Although the same electrode configuration as in section 5.2.1 was used for the quadriceps muscle group, miniature Beckman electrodes with a diameter of .3 cm and a spacing of .9 cm. were used for the
Figure 5.2  Typical Results From Homomorphically Processing EMG Signals From Phasic Contractions.
other muscles. Processing of these EMG signals was then performed by HOMFIL, with a 5 Hz low pass filter cut-off frequency. The UMR data files created by HOMFIL were then processed by FREQR as in section 5.2.1. The power spectral parameters calculated can be found in Table 5-3.

This table shows that the UMR spectra changed from muscle to muscle. Further the UMR spectra for specific muscles were essentially constant for the two walking speeds. The table also shows that the UMR spectra for the same muscle for both limbs were similar. This consistency of the UMR spectra, for a given muscle, for different phasic contractions, demonstrates a definite relationship between a muscle and its UMR. This relationship is further demonstrated by the increased median frequency of the UMR from the tibialis anterior (TA) muscle compared to the median frequency of the UMR from the gastrocnemius muscle (GAST). The increased median frequency of the UMR for the TA muscle is expected, since it has fewer muscle fibres per motor unit on average (Desmedt 1981), fewer total number of motor units and is less affected by the tissue filtering effects since it is more superficial, than the gastrocnemius muscle. The total power of the processed UMRs also varied from muscle to muscle and remained relatively similar, for specific muscles, for the two walking speeds. The overall variance in the power measurement, however, was small and clearcut
<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>MUSCLE</th>
<th>F.C. Hz</th>
<th>S.B. Hz</th>
<th>PWR. μV²</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.A. L1</td>
<td>R.F.</td>
<td>93.7</td>
<td>106.2</td>
<td>.267E-04</td>
</tr>
<tr>
<td></td>
<td>B.F.</td>
<td>110.1</td>
<td>141.0</td>
<td>.321E-04</td>
</tr>
<tr>
<td></td>
<td>T.A.</td>
<td>128.0</td>
<td>140.7</td>
<td>.284E-04</td>
</tr>
<tr>
<td></td>
<td>GAST</td>
<td>98.6</td>
<td>115.8</td>
<td>.241E-04</td>
</tr>
<tr>
<td>L.A. L2</td>
<td>R.F.</td>
<td>92.5</td>
<td>93.6</td>
<td>.237E-04</td>
</tr>
<tr>
<td></td>
<td>B.F.</td>
<td>111.3</td>
<td>138.8</td>
<td>.297E-04</td>
</tr>
<tr>
<td></td>
<td>T.A.</td>
<td>128.6</td>
<td>145.1</td>
<td>.274E-04</td>
</tr>
<tr>
<td></td>
<td>GAST</td>
<td>102.9</td>
<td>130.0</td>
<td>.251E-04</td>
</tr>
<tr>
<td>L.A. R1</td>
<td>R.F.</td>
<td>89.2</td>
<td>98.9</td>
<td>.236E-04</td>
</tr>
<tr>
<td></td>
<td>B.F.</td>
<td>110.1</td>
<td>141.0</td>
<td>.394E-04</td>
</tr>
<tr>
<td></td>
<td>T.A.</td>
<td>121.5</td>
<td>151.9</td>
<td>.304E-04</td>
</tr>
<tr>
<td></td>
<td>GAST</td>
<td>93.3</td>
<td>114.5</td>
<td>.235E-04</td>
</tr>
<tr>
<td>L.A. R2</td>
<td>R.F.</td>
<td>90.5</td>
<td>108.3</td>
<td>.258E-04</td>
</tr>
<tr>
<td></td>
<td>B.F.</td>
<td>104.9</td>
<td>121.3</td>
<td>.270E-04</td>
</tr>
<tr>
<td></td>
<td>T.A.</td>
<td>125.8</td>
<td>154.6</td>
<td>.242E-04</td>
</tr>
<tr>
<td></td>
<td>GAST</td>
<td>94.7</td>
<td>112.3</td>
<td>.260E-04</td>
</tr>
</tbody>
</table>

**TABLE SYMBOLS**

- L1 - left side 1 m/s
- L2 - left side 1.5 m/s
- R1 - right side 1 m/s
- R2 - right side 1.5 m/s
- R.F. - Rectus Femorus
- B.F. - Biceps Femorus
- T.A. - Tibialis Anterior
- GAST - Gastrocnemius

**TABLE 5-3**

POWER SPECTRA COMPARISONS OF UMRs PROCESSED FROM EMG DATA RECORDED DURING PHASIC CONTRACTION.
power ranges for specific muscles were not evident with the limited amount of data analysed. Further studies which determine mean values together with expected variances, of the frequency parameters considered here, including total power, for specific muscles, for both normals and different pathological cases might establish a clinical use of the UMR.

5.3.3 Stationarity Testing of the UMRs From Phasic Contractions

The UMRs resulting from the homomorphic processing of the EMG data described in section 5.3.1 were tested for stationarity using the program RUNTST, see Appendix IV. The number of segments was varied as in section 5.2.2 and the results are tabulated in Table 5-4.

As with the UMRs tested in section 5.2.2 this data as summarized in Table 5-4 can also be considered stationary. This provides further evidence of the suitability of the proposed multiplicative model which states that a phasic EMG signal is the result of the multiplication of a non-stationary neural input signal by a stationary constant muscle response.

As was also found in section 5.2.1 the run test concluded that some data records were non-stationary and this was not apparent with visual inspection of these records.
<table>
<thead>
<tr>
<th>SIDE</th>
<th>MUSCLE</th>
<th>NO. OF SECT.</th>
<th>RESULT</th>
<th>SIDE</th>
<th>MUSCLE</th>
<th>NO. OF SECT.</th>
<th>RESULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>RF</td>
<td>20 16 12 8</td>
<td>S S NS₃⁺</td>
<td>Right</td>
<td>RF</td>
<td>20 16 12 8</td>
<td>S S</td>
</tr>
<tr>
<td></td>
<td>BF</td>
<td>20 16 12 8</td>
<td>S S</td>
<td>BF</td>
<td>20 16 12 8</td>
<td>S S</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TA</td>
<td>20 16 12 8</td>
<td>S NS₃⁺</td>
<td>TA</td>
<td>20 16 12 8</td>
<td>S S</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GAST</td>
<td>20 16 12 8</td>
<td>S S S S</td>
<td>GAST</td>
<td>20 16 12 8</td>
<td>S S</td>
<td></td>
</tr>
</tbody>
</table>

TABLE SYMBOLS AS PER TABLE 5-2 AND TABLE 5-3.

1-1.0 m/sec  2-1.5 m/sec

TABLE 5-4
RUN TEST STATIONARITY RESULTS FOR UMRs PROCESSED FROM EMG DATA
RECORDED DURING PHASIC CONTRACTIONS FOR SUBJECT L.A. AT A
.050 LEVEL OF SIGNIFICANCE.
This might be due to an inherent weakness of the run test. This weakness being the apparent low power of the test under certain circumstances. Specifically, this results in arbitrary results for the run test when the segment lengths become too short or too few segments are used. The weakness of segments which are too short is addressed by Bendat and Piersol (1971) and as suggested these authors can be overcome by taking segment lengths, corresponding to sufficient time periods, which are many times longer than the periods of the lowest frequency components of the data. Segment lengths chosen in this fashion can restrict the total number of segments from fixed length data records, resulting in too few segments, which also reduces the power of the test. Therefore, it is important to realize that to properly test the stationarity of data, data segments of suitable length and number must be used.
CHAPTER 6
CONCLUSIONS

EMG signals result from the measurement of neuromuscular electrical activity and consequently contain information about both neural control and muscle state. The type of information extracted from EMG signals depends on the signal processing technique used.

In an effort to maximize the amount of information that could be obtained from EMG data a multiplicative model for EMG signals was proposed. The model conceptualized the EMG signals as the product of two component signals, a low frequency neural input and a relatively high frequency unitary muscle response (UMR). The model is based on the assumption that EMG signals recorded during isometric-isotonic (constant) muscle contractions have constant spectral shapes independent of contraction level. This assumption was proved to be true, by the consistency found in the power spectral parameters, median frequency and statistical bandwidth, of EMG signals recorded during constant muscle contractions at different levels of contraction.

The particular nature of the multiplicative model required the consideration of non-linear processing algorithms. The multiplicative process, when the component
signals are of essentially different frequency content as with the proposed EMG model readily avails itself to homomorphic processing techniques. Homomorphic processing was therefore applied to both simulated EMG data and data recorded during various muscle activities. The performance of homomorphic processing in extracting the neural input component was compared to that of the common EMG processing technique of rectification and low pass filtering (MRE). The comparison was performed using simulated EMG signals of known neural input and the parameter of interest was the signal to noise ratio of the respective processing algorithm. The two processing methods had similar signal to noise ratios with homomorphic processing having slightly improved ratios for low frequency neural inputs. Homomorphic processing performance might be further improved by the application of Kalman filtering techniques in the logarithmic domain, as suggested by Evans et al (1980) to better separate the component signals.

Homomorphic processing was also applied to EMG data recorded during constant muscle contractions and spectral parameters of the raw data and their corresponding UMRs were compared. The high degree of similarity found in these spectral parameters further substantiates the proposed multiplicative model.

Additional homomorphic processing was performed on
EMG signals recorded from various lower limb muscles during human gait. The power spectra, as characterized by several spectral parameters, for the resulting UMRs were revealed to be somewhat unique for each muscle and constant for a specific muscle for different phasic contractions. These results again support the proposed multiplicative model and also suggest a possible clinical use for the unitary muscle response.

The variations discovered in the spectral parameters for different muscles, suggests a relationship between the UMR and the corresponding muscle state. Further studies which determine expected values and variances for these spectral parameters of the UMR for specific muscles under normal and various pathological states, may establish the UMR as a source of clinical information.

The stationarity of EMG signals recorded during constant muscle contractions and their corresponding processed UMRs were tested using the run test for several different numbers and lengths of data segments. The majority of these EMG signals were found to be stationary. All the UMRs were found to be stationary, even those resulting from non-stationary EMG data. This confirms the model prediction. Stationarity tests were also performed on the processed UMRs corresponding to EMG data recorded from several lower limb muscles during human gait. All of these UMRs
were found to be stationary confirming this model assumption. The stationarity testing revealed an inherent weakness in the run test. This weakness being a loss of test power if too few or too short data segments are used. Thus long data records are required to assess stationarity with confidence if the data contains low frequency components.

The results of this work suggest that the multiplicative model for EMG signals is a reasonable one for both phasic and constant contractions and that homomorphic processing of EMG signals produces useful estimates of both the neural input and the muscle response. This means that with the application of homomorphic processing information about both neural control and muscle state can be extracted simultaneously, for both phasic and constant muscle contractions. The simultaneous extraction of both neural and muscle information from EMG signals is not possible with currently used processing techniques.
APPENDIX I

POWER SPECTRUM ESTIMATION

The estimation of power spectra and the calculation of power spectral parameters for the EMG data analysed in this work were performed by the Fortran IV programs FREQP5 and FREQR. This appendix describes the theoretical basis of these programs. The slight differences between the two programs is stated and a listing of FREQP5 is included.

The programs calculate the power spectrum of the input data, by the method of averaging periodograms. This algorithm is based on a program suggested by Rabiner et al (1979). This technique of power spectral estimation was first proposed by Welch (1967).

For a sampled data sequence X(n), the modified periodogram spectrum estimate is obtained by dividing X(n) into K overlapping segments of length L. This algorithm used an overlap of L/2 so that for an N length data sequence:

\[ K = \left\lfloor \frac{N - L/2}{L/2} \right\rfloor \]

where the square brackets represent integer truncation. After appropriate weighting the ith data segment can be expressed as

\[ x_i(n) = x((i-1)L/2 + n)w_d(n) \]

\[ 1 \leq n \leq L, \quad 1 \leq i \leq K \]
where \( w_d(n) \) is an \( L \) point Hamming or rectangular data weight­ing window. The \( M \) point (\( M \leq L \)) discrete Fourier transforms (DFT) of the weighted segments \( x_i(n) \) are defined as:

\[
X_i(n) = \sum_{n=0}^{m-1} x_i(n)e^{-j\frac{2\pi kn}{M}}
\]

\( 1 \leq k \leq M, \quad 1 \leq i \leq K \)

These DFTs are calculated using an FFT routine where \( M \) must be an integral power of 2, (see listing following FREQP5 listing). When \( L < M \) the sequence \( x_i(n) \) is augmented with \( M - L \) zero valued samples. The power spectrum of the \( i \)th segment is:

\[
S_i(k) = |X_i(k)|^2 \quad 1 \leq k \leq M, \quad 1 \leq i \leq K
\]

The final spectrum estimate \( S_{xx}(2\pi k/M) \) at normalized radian frequency \( (2\pi k/M) \) is then obtained by averaging the individual \( S_i(k) \).

\[
S_{xx}(2\pi k/M) = \frac{1}{ku} \sum_{i=1}^{K} S_i(k), \quad 1 < k \leq M
\]

where \( u = \sum_{n=1}^{L} w^2_d(n) \) is included to achieve an unbiased spectral estimate.

The segment spectra \( |X_i(k)|^2 \) are computed two at a time by suitably arranging the sequence \( X(n) \) into complex vectors as:

\[
x(n) = x_i(n) + jx_{i+1}(n)
\]
then \( X(k)X^*(k) + X(m-k)X^*(m-k) = 2\left[|X_i(k)|^2 + |X_{i+1}(k)|^2\right] \)
\( 0 \leq k \leq M/2 \)

where \( X(k) \) is the M-point DFT of the complex sequence, \( x(n) \) and \( X_i(k) \) and \( X_{i+1}(k) \) are the M-point DFT's of \( x_i(n) \) and \( x_{i+1}(n) \). This procedure halves the number of FFTs required.

These programs also calculate the following spectral parameters, median frequency, statistical bandwidth, percent of total power in selectable frequency bands, ratio of powers in separately selectable high and low frequency bands and total power. The program also calculates the signal mean rectified value and root mean square value (RMS) and allows specific subsets of data points in a record to be analysed.

FREQR is identical to FREQP5 but it processes real data files instead of integer files and it processes a complete data record not allowing any specific subset selection. A listing of FREQP5 follows.
FREQP5 SOURCE LISTING

C......FREQP5, FOR PROGRAM TO ESTIMATE AND PLOT THE POWER
C......SPECTRUM OF INPUT DATA FILES, BASED ON AVERAGED
C......PERIODOGRAMS, OF OVERLAPPING WINDOWS, OF THE INPUT
C......DATA. THE WINDOWS CAN BE WEIGHTED BY A HANNING
C......OR RECTANGULAR DATA WINDOW. THE PROGRAM ALSO ESTIMATES
C......THE FOLLOWING POWER SPECTRUM PARAMETERS:
C......MEDIAN FREQUENCY
C......STATISTICAL BANDWIDTH
C......PERCENT POWER IN THREE SELECTABLE FREQ. BANDS
C......HIGH-LOW RATIO; RATIO OF POWER IN HIGH BAND TO LOW BAND
C......TOTAL POWER IN SIGNAL
C......
C......THE FOLLOWING AMPLITUDE STATISTICS ARE ALSO CALCULATED:
C......MEAN RECTIFIED EMG VALUE (MRE)
C......ROOT MEAN SQUARE VALUE (RMS)
C......
C......THE LENGTH OF THE WINDOWS CHOSEN AND THE FFT'S
C......CALCULATED IS SELECTABLE. THIS ALLOWS VARIABLE
C......FREQUENCY RESOLUTIONS AND STATISTICAL VARIANCES OF
C......RESULTING SPECTRUM ESTIMATES. THE PROGRAM ALSO ALLOWS
C......THE SPECIFIC RANGE OF DATA POINTS DESIRED FROM
C......THE INPUT TO BE CHOSEN.
C......
C......NOTE:
C......FREQR.FOR IS IDENTICAL TO FREQP5.FOR EXCEPT
C......THAT IT READS REAL UNFORMATTED SEQUENTIAL ACESS
C......FILES RATHER THAN INTEGER DIRECT ACESS FILES.
C......IT ALSO DOES NOT ALLOW ANY DATA SELECTION
C......WITHIN CHOSEN FILES.

DIMENSION IDAT(2000), IX(1024), SPEC(1024)
DIMENSION JWIN(2, 2), XA(2048), AXIS (4, 4)
DIMENSION IEL(3, 2), IBFREQ(3, 2), IDAT1(2000)
DIMENSION ABPTOT(3), IFILE(7), IHLB(4), IILHLB(4)
COMPLEX X(1024), XMN
DATA JWIN(1, 1), JWIN(1, 2) /'RE', 'CT'/
DATA JWIN(2, 1), JWIN(2, 2) /'HA', 'MG'/
DATA NN: IYES /'NO', 'YE'/
MAXM = 2048
LHM = MAXM / 2 + 1
NTOT = 60
N = 2000
DEFINE FILE 1 (NTOT, N, U, JREC)
WRITE(7,101)
101  FORMAT( ' WHAT IS DATA FILE NAME?'/***:********:****'/ )
READ(5,201)(IFILE(J),J=1,7)
CALL ASSIGN(1,IFILE,14,'RDO')
201  FORMAT(7A2)
WRITE(7,102)
102  FORMAT( ' WHAT IS THE SAMPLE FREQ AND BAND WIDTH?'/
+  1X,'***:',1X,'***' )
READ(5,202)ISAMP,IBAND
202  FORMAT(I4,1X,I4)
WRITE(7,103)
103  FORMAT( ' WHAT ARE FREQ BANDS (HZ)?'/1X,'***-***' )
DO 300 I=1,3
300  READ(5,203)(IBFREQ(I,J),J=1,2)
203  FORMAT(2(I3,1X,))
WRITE(7,104)
104  FORMAT( ' WHAT ARE FREQ BANDS FOR H/L RATIO (HZ)?'/
+  1X,'LOW BAND',2X,'HIGH BAND'/1X,'*** *** *** ***' )
READ(5,204)(IIHLB(J),J=1,4)
204  FORMAT(4(I3,2X,))
WRITE(7,777)
777  FORMAT( ' DO YOU HAVE A ZERO MEAN SIGNAL?$/ )
READ(5,201)IZERO
C
C READ IN ANALYSIS PARAMETERS M,IWIN,L
C
WRITE(7,9999)
9999  FORMAT( ' FFT LENGTH ='
  1,5X,' MUST BE A POWER OF 2'/***' )
READ(5,9997) M
4
IF(M.GT.MAXM) WRITE(7,9998)
9998  FORMAT( ' M TOO LARGE -- REENTER VALUE '/ )
IF(M.GT.MAXM) GO TO 4
9997  FORMAT(I4)
WRITE(7,9996)
9996  FORMAT( ' WINDOW TYPE 1=RECTANGULAR, 2=HAMMING'/'*')
READ(5,9995) IWIN
9995  FORMAT(I1)
5
WRITE(7,9994)
9994  FORMAT( ' WINDOW LENGTH ='
  1,5X,' MUST BE LESS THAN 1025'/***' )
READ(5,9997) L
5
IF(L.GT.M) GO TO 5
WRITE(6,4000)(IFILE(J),J=3,7)
4000  FORMAT(5X,'FILE:','5A2,'/
WRITE(6,4100)
4100  FORMAT(2X,'REC',4X,'SRE',4X,'RMS',4X,'FC',4X,
  1'SB',4X,'H/L',6X,'L',6X,'M',6X,'H',8X,'PWR'/)
304  WRITE(7,106)
106  FORMAT( ' WHAT IS THE REC. NO.?'/** )
READ(5,206)IREC
206  FORMAT(I3)
SCAL=(17./19.05)*1023.
DELTAF=ISAMP/(1.0*M)
INUM=((IBAND*1.0)/DELTAF)+1
DO 306 I=1,3
DO 305 J=1,2
305 IEL(I,J)=((IBFREQ(I,J)*1.0)/DELTAF)+1
306 CONTINUE
READ(1/IREC) (IDAT1(J),J=1,N)
WRITE(7,7510)
7510 FORMAT( 'WHAT DATA POINTS ARE DESIRED?'/ '**** ****' )
READ(5,7511) N1,N2
7511 FORMAT(I4,1X,I4)
IF(N1.EQ.0)N1=1
IF(N2.EQ.0)N2=2000
N=N2-N1+1

C
C NSECT = THE TOTAL NUMBER OF ANALYSIS SECTIONS
C NP = THE TOTAL NUMBER OF SAMPLES ACTUALLY USED
C OVERLAP OF 2 TO 1 IS USED ON ADJACENT ANALYSIS SECTIONS
C NP = N IF(N-L/2)/(L/2) = AN INTEGER

MHLF1 =M/2+1
NSECT = (N-L/2)/(L/2)
NP = NSECT*(L/2) + L/2

C READ IN DESIRED DATA.
C
DO 10 J=1,N
K=J+N1-1
IDAT(J)=IDAT1(K)
XA(J)=FLOAT(IDAT1(K))/409.6
10 CONTINUE
XSUM=0.

C CALCULATE DATA MEAN.
C
DO 20 J=1,N
XSUM=XSUM+XA(J)
20 CONTINUE
XMEAN=XSUM/N

C ONLY FOR CALCULATION OF MRE AND RMS
C
IF(IZERO.EQ.0) GO TO 31
DO 30 J=1,N
XA(J)=XA(J)-XMEAN
30 CONTINUE

C MAKE SIGNAL ZERO MEAN IF DESIRED
C SET XMN FOR LATER PROCESSING
C
XMN=CMPLX(XMEAN,XMEAN)
GO TO 32
C SET XMN FOR LATER PROCESSING.
C
31 XMN=CMPLX(0.0,0.0)
C
C CALCULATE MRE AND RMS.
C
32 MRE=0.
RMS=0.
DO 40 J=1,N
MRE=MRE+ABS(XA(J))
RMS=RMS+XA(J)**2
40 CONTINUE
MRE=MRE/N
RMS=SQRT(RMS/N)
C
C RESET N FOR REPEAT RUNS.
C
N=2000
C
C GENERATE WINDOW
C
U=FLOAT(L)
IF (IWIN.NE.2) GO TO 60
U=0.
FL=FLOAT(L-1)
TWOPH=8.*ATAN(1.0)
DO 50 I=1,L
FI=FLOAT(I-1)
WD=.54-.46*COS(TWOPH*FI/FL)
U=U+WD*WD
50 CONTINUE
60 CONTINUE
C
C LOOP TO ACCUMULATE SPECTRA 2 AT A TIME
C
SS=1.
DO 70 I=1,MHLFl
   SPEC(I)=0.
70 CONTINUE
C
C READ L/2 SAMPLES TO INITIALIZE BUFFER
C
L1 = L/2
NRD=L/2
L2=L/2
CALL GETX(XA,L2,IDAT,NRD,SS)
SS=SS+FLOAT(NRD)
IMN=L/2+1
KMX=(NSECT+1)/2
NSECTP=((NSECT+1)/2)**2
NRD=L
DO 190 K=1,KMX
C MOVE DOWN UPPER HALF OF XA BUFFER

C

DO 80 I=1,L1  
   J=L1+I  
   X(I)=CMPLX(XA(J),0.)  
80 CONTINUE

IF(K.NE.KMX .OR.NSECTP.EQ.NSECT) GO TO 95  
DO 90 I=IMN,NRD  
   XA(I)= 0.0  
90 CONTINUE

NRD=L/2

L2 = 0
CALL GETXCXA,L2,IDAT,NRD,SS)

DO 90 I=1,L1  
   J= I+L1  
   X(J)=CMPLX(XA(I),XA(J))-XMN  
   X(I)=CMPLX(REAL(X(I)),XA(I))-XMN  
90 CONTINUE

IF(K.NE.KMX .OR.NSECTP.EQ.NSECT) GO TO 130

C AN ODD NUMBER OF SECTIONS -- ZERO OUT THE SECOND PART
C

DO 120 I=1,L  
   X(I)= CMPLX(REAL(X(I)),0.)  
120 CONTINUE

130 CONTINUE

SS=SS+FLOAT(NRD)

IF (IWIN.NE.2) GO TO 150

FL = FLOAT(L-1)
DO 140 I=1,L  
   FI = FLOAT(I-1)  
   X(I)=X(I)*(.54 -.46 * COS(TWOPI*FI/FL))  
140 CONTINUE

150 CONTINUE

IF (L.LE.M) GO TO 170

LP1=L+1  
DO 160 I=LP1,M  
   X(I)=(0.,0.)  
160 CONTINUE

170 CONTINUE

CALL FFT(X,M,0)

DO 180 I=2,MHLF1  
   J=M+2-I  
   SPEC(I) = SPEC(I) + REAL(X(I)*CONJG(X(I))) + X(J)*CONJG(X(J)))
180 CONTINUE

SPEC(1) = SPEC(1) + REAL(X(1)*CONJG(X(1)))**2

190 CONTINUE
C NORMALIZE SPECTRAL ESTIMATE

FNORM = 2. * U * NSECT
DO 210 I=1,MHLF1
   SPEC(I) = SPEC(I)/FNORM
210 CONTINUE
IF(IZERO.EQ.NNO) GO TO 41
SPEC(1) = 0

C CALCULATE TOTAL POWER AND STATISTICAL BANDWIDTH

41 PTOT=0
PTOT2=0
DO 311 J=2,MHLF1
   PTOT=PTOT+SPEC(J)*(1/(1.0*M))
   PTOT2=PTOT2+SPEC(J)**2*(1/(1.0*M))
311 CONTINUE
PTOT=PTOT*2+SPEC(1)*(1/C1.0*M)
PTOT2=PTOT2*4+(SPEC(1)**2*(1/(1.0*M)))
SB=PTOT2*PTOT/ISAMP

C CALCULATE PERCENT POWER IN SELECTED FREQ. BANDS.

DO 312 J=1,J
   ABPTOT(J) = 0.
   BPTOT=0.
   IF(IEL(J,1).LE.1.AND.IEL(J,2).LE.1.) GO TO 312
   IST=IEL(J,1)
   IET=IEL(J,2)
   IBTOT=IET-IST+1
   DO 313 JJ=IST,JET
      BPTOT=BPTOT+SPEC(JJ)*(1/(1.0*M))
313 BPTOT=BPTOT*2
   IF(IST.EQ.1)BPTOT=BPTOT-SPEC(1)*(1/(1.0*M))
   ABPTOT(J)=BPTOT/PTOT*100.
312 CONTINUE

C CALCULATE MEDIAN FREQ.

SUM=0.0
DO 315 I=2,MHLF1
315 SUM=SUM+2*SPEC(I)*(1/(1.0*M))+(I-1)*(1/(1.0*M))
FMED=SUM/PTOT*ISAMP

C CALCULATE HIGH/LOW RATIO

DO 316 J=1,4
316 IHLB(J)=(IIHLB(J)*1.)/DELTAF+1
RATIO = 0.
HBPTOT=0.
IF(IHLB(1).LE.1.AND.IHLB(2).LE.1.) GO TO 42
BBPTOT=0.
IL1=IHLB(1)-1
IL2=IHLB(2)-IHLB(1)+1
IH1=IHLB(3)-1
IH2=IHLB(4)-IHLB(3)+1
DO 317 I=1,IL2
317 BBPTOT=BBPTOT+SPEC(IL1+I)*(1/(1.0*M))
BBPTOT=BBPTOT*2
IF(IL1.EQ.0)BBPTOT=BBPTOT-SPEC(1)*(1/(1.0*M))
DO 318 I=1,IL2
318 HBPTOT=HBPTOT+SPEC(IH1+I)*(1/(1.0*M))
HBPTOT=HBPTOT*2
IF(IH1.EQ.0)HBPTOT=HBPTOT-SPEC(1)*(1/(1.0*M))
RATIO=HBPTOT/BBPTOT
IMULT=1.
WRITE(6,4101)IREC,SRE,RMS,FMED,SB,RATIO,
1(ABPTOT(J),J=1,3),PTOT
4101 FORMAT(2X,I3,2X,F5.3,2X,F5.3,2X,F5.1,2X,
1F5.1,2X,F5.3,3(2X,F5.1),4X,E9.3,/)
REWIND 6
DO 350 I=2,MH1F1
   SPEC(I)=SPEC(I)*2
350 CONTINUE
SCALD=(6.0/19.05)*1023
CALL REMAX(SPEC,2,MH1F1,XMAX)
SCAL2=SCALD/XMAX
GO TO 326
325 CONTINUE
WRITE(7,7500)
7500 FORMAT(' WHAT IS THE AMP. MULT? /* *** ')
READ(5,7501)IMULT
7501 FORMAT(I3)
326 CONTINUE
C
C PLOT POWER SPECTRUM
C
IX(1)=0
XTEMP=0.
DO 307 I=2,INUM
   XTEMP=XTEMP+DELTAF
C MULT. BY 15 TO AVOID INT.TRUNC.
   IX(I)=XTEMP*15.
307 CONTINUE
SMULT=SCAL/IX(INUM)
C HASH MARK 5HZ FOR 1KHZ SAMPLE RATE
C 10HZ FOR 2KHZ SAMPLE RATE
IHASH=(ISAMP*15)/100
C
AXIS(1,1)=0.
AXIS(1,2)=17.
AXIS(1,3)=0.0
AXIS(1,4)=0.
XD=1.
C

SUBROUTINE TO LOAD WORKING VECTOR WITH DESIRED POINTS.

C

SUBROUTINE GETX(X,L2,IDAT,NRD,SS)
DIMENSION X(1),IDAT(1)
DO 10 I=1,NRD
   X(I+L2) = FLOAT(IDAT(I+SS-1))/409.6
10   CONTINUE

RETURN
END
SUBROUTINE FFT
D.JIM COOLEY'S SIMPLE FFT PROGRAM USES DECIMATION IN TIME ALGORITHM
D.X IS AN N=2**M POINT COMPLEX ARRAY THAT INITIALLY CONTAINS THE INPUT
D.AND ON OUTPUT CONTAINS THE TRANSFORM
D.THE PARAMETER INV SPECIFIED DIRECT TRANSFORM IF 0 AND INVERSE IF 1
D

SUBROUTINE FFT(X,N,INV)
COMPLEX X(1),U,W,T,CMPLX
D.X = COMPLEX ARRAY OF SIZE N -- ON INPUT X CONTAINS
D.THE SEQUENCE TO BE TRANSFORMED
D.N = SIZE OF FFT TO BE COMPUTED -- N = 2**M FOR 1.LE.M.LE.15
D.INV = PARAMETERS TO DETERMINE WHETHER TO DO A DIRECT TRANSFORM (INV=0)
D.OR AN INVERSE TRANSFORM (INV=1)
D

M=ALOG(REAL(N))/ALOG(2.)+.1
NV2 = N/2
NM1 = N - 1
J = 1
DO 40 I=1,NM1
  IF(I.GE.J) GO TO 10
  T=X(J)
  X(J)=X(I)
  X(I)=T
  K=NV2
10  IF(K.GE.J) GO TO 30
  J=J-K
  K=K/2
  GO TO 20
30  J=J+K
40  CONTINUE
PI = 4.*ATAN(1.,0)
DO 70 L=1,N
  LE = 2**L
  LE1 = LE/2
  U = (1.,0,0,0)
  W = CMPLX(COS(PI/REAL(LE1)),-SIN(PI/REAL(LE1))
  IF(INV.NE.0) W = CONJG(W)
DO 60 J=1,LE1
  DO 50 I=J,N,LE
    IP = I + LE1
    T = X(IP)*U
    X(IP) = X(I) - T
    X(I) = X(I) + T
50  CONTINUE
U = U*W
60  CONTINUE
70  CONTINUE
IF(INV.EQ.0) RETURN
DO 80 I=1,N
  X(I) = X(I)/CMPLX(REAL(N),0.)
80  CONTINUE
RETURN
END
APPENDIX II
HOMTST SOURCE LISTING

C......HOMTST,FOR PROGRAM TO TEST THE PERFORMANCE OF
C......HOMOMORPHIC PROCESSING FOR THE EXTRACTION OF
C......NERUAL INPUTS FROM COMPOSITE EMG SIGNALS.
C......
C......
DIMENSION XDAT(1500),ISGN(1500),YDAT(1500)
DIMENSION IDAT(1500),IFILE(7),X2DAT(1500)
DIMENSION RMDAT(1500),IWORK(1500),IX(1500)
EQUIVALENCE (ISGN,IDAT)
DATA NNO,IYES/’NO’,’YE’/
C
C......INPUT SUITABLE UMR FILE.
C
WRITE(7,9997)
9999 FORMAT(’ ENTER UMRFILE NAME/1X,’***:********:***’)
READ(5,9998)(IFILE(J),J=1,7)
9998 FORMAT(7A2)
WRITE(6,6699)(IFILE(J),J=3,7)
6699 FORMAT(’ FILE:’ ,2X,5A2//)
WRITE(6,6999)
6999 FORMAT(’ FREQ’,6X,’RMSE’,6X,’RMSI’,8X,’RMSO’,7X,’SNR’
19X,’XMI’,7X,’XMO’)
NSM=1500
CALL ASSIGN(1,IFILE,14,’R0’)
110 WRITE(7,7997)
7997 FORMAT(’ WHAT IS SAMPLING FREQ?’/’ ***’/)
READ(5,7796)IFS
7796 FORMAT(14)
WRITE(7,7960)
7960 FORMAT(’ DO YOU WANT GRAPHICS DISPLAY OF DATA? YES? OR NO?’)
READ(5,9959)IANS4
9959 FORMAT(A2)
IFLAG2=0
IF(IANS4.EQ.IYES) IFLAG2=1
C
C......INPUT LPF CUTOFF FREQ TO BE USED FOR PROCESSING.
C
WRITE(7,7795)
7795 FORMAT(’ WHAT IS THE LPF CUTOFF FREQ?’/’ **,**’)
READ(5,7794)FC
7794 FORMAT(F4,1)
C
C......READ UMR FILE
C
READ(1)(X2DAT(J),J=1,NSM)

98
C
C INPUT FREQ. OF SIMULATED NEURAL CONTROL.
C
333 WRITE(7,8999)
8999 FORMAT(1,A15)
READ(5,8999)FREQ
8999 FORMAT(F5.1)
C
C CREATE SIMULATED NEURAL INPUT OF DESIRED
C FREQ. OFFSET SINE WAVE.
C
SPCYC=500./FREQ
ADD=360/SPCYC
FACT=180./3.14159
DEGREE=0.0
X=0.0
DO 10 I=1,NSM
   YDAT(I)=SIN(X)+1.01
   X=DEGREE/FACT
10 CONTINUE
IF(IFLAG2.EQ.0)GO TO 815
C
C PLOT UMR FILE AND OFFSET SINE WAVE.
C
GSCALE=1023/19.05
SCALN=1.0*GSCALE
XTEMP=0.0
DO 400 I=1,1500
   IX(I)=XTEMP
   XTEMP=XTEMP+.5
   IDAT(I)=YDAT(I)*100
   IWOR(1)=X2DAT(I)
400 CONTINUE
CALL MIN(IWORK,NSM,NSM,IMIN)
CALL MAX(IWORK,NSM,NSM,IMAX)
IF(IMAX.GE.IABS(IMIN)) GO TO 3
YMULT=SCALN/(1.*IMIN)
GO TO 4
3 YMULT=SCALN/(1.*IMAX)
4 CALL SCALE(IWORK,NSM,1,NSM,11.5,YMULT)
CALL PLOTEK(IX(1),IWORK(1),NSM,1,0,1)
CALL MAX(IDAT,NSM,NSM,IMAX)
YMULT=SCALN/(1.*IMAX)
CALL SCALE(IDAT,NSM,1,NSM,9,0,YMULT)
CALL PLOTEK(IX(1),IDAT(1),NSM,1,0,0)
815 CONTINUE
C
C CREATE SIMULATED EMG SIGNAL.
C
DO 11 I=1,NSM
   XDAT(I)=YDAT(I)*X2DAT(I)
   IWOR(I)=XDAT(I)
11 CONTINUE
IF(IFLAG2.NE.1) GO TO 816

CALL MIN(IWORK,NSM,NSM,IMIN)
CALL MAX(IWORK,NSM,NSM,IMAX)
IF(IMAX.GE.IABS(IMIN)) GO TO 1
YMULT=SCALED/(1.*IMIN)
GO TO 2
1 YMULT=SCALED/(1.*IMAX)
2 CALL SCALE(IWORK,NSM,1,NSM,7,0,YMULT)
CALL PLOTEK(IX(1),IWORK(1),NSM,1,0,0)

C......PERFORM HOMOMORPHIC PROCESSING

816 DO 20 I=1,NSM
ISGN(I)=1
IF(XDAT(I).EQ.0)XDAT(I)=XDAT(I)+.01
IF(XDAT(I).LT.0.0) ISGN(I)=-1
XDAT(I)=ABS(XDAT(I))
XDAT(I)=ALOG(XDAT(I))
RMDAT(I)=XDAT(I)
20 CONTINUE
FS=IFS

C......CALL LOW PASS FILTER
C......LOW PASS FILTER USED IS A DIGITALLY IMPLEMENTED 2ND ORDER BUTTERWORTH. DATA IS PASSED TWICE THROUGH FILTER, ONCE FORWARDS AND ONCE BACKWARDS. THIS RESULTS IN AN EQUIVALENT 4TH ORDER RESPONSE WITH NO PHASE DELAY. SEE LISTING FOLLOWING THIS PROGRAM.

CALL LFF(RMDAT,NSM,FC,FS)
DO 21 I=1,NSM
XDAT(I)=(XDAT(I)-RMDAT(I))
XDAT(I)=EXP(XDAT(I))*ISGN(I)
RMDAT(I)=EXP(RMDAT(I))
21 CONTINUE

C
C
C

......NOTE: RECTST.FOR IS IDENTICAL TO HOMTST.FOR BUT FOR THE LAST 15 LINES WHICH ARE REPLACED BY THE FOLLOWING 10 LINES.
C
C......PERFORM RECFL PROCESSING
C
C 816 DO 20 I=1,NSM
C XDAT(I)=ABS(XDAT(I))
C RMDAT(I)=XDAT(I)
C 20 CONTINUE
C FS=IFS
C......LOW PASS FILTER EMG SIGNAL
C
C CALL LPF(RMDAT,NSM,FC,FS)
C DO 21 I=1,NSM
XDAT(I)=XDAT(I)-RMDAT(I)
RMDAT(I)=RMDAT(I)
C 21 CONTINUE
C
C
C......CALCULATE SIGNAL TO NOISE RATIO.
C
SMI=0.0
SMO=0.0
SSI=0.0
SSO=0.0
DO 23 I=251,(NSM-100)
SMI=YDAT(I)+SMI
SMO=RMDAT(I)+SMO
SSI=YDAT(I)**2+SSI
SSO=RMDAT(I)**2+SSO
23 CONTINUE
XM=SMI/(NSM-350.)
XMO=SMO/(NSM-350.)
RMSI=SQRT(SSI/(NSM-350.))
RMSO=SQRT(SSO/(NSM-350.))
C
C......CORRECT FOR MEAN VALUE DIFFERENCES.
C
XM=XMI/XMO
RK=RMSI/RMSO
ES=0.0
DO 24 I=251,(NSM-100)
ES=(YDAT(I)-XM*RMDAT(I))**2+ES
24 CONTINUE
RMSE=SQRT(ES/(NSM-350.))
SNR=RMSI/RMSE
IF(IFLAG2.NE.1)GO TO 640
C
C......PLOT OUTPUT FILES.
C
DO 22 I=1,250
XDAT(I)=0.0
RMDAT(I)=0.0
22 CONTINUE
DO 27 I=1,NSM
IWORK(I)=100*RMDAT(I)
IDAT(I)=100*XDAT(I)
27 CONTINUE
DO 28 I=NSM-100,NSM
IWORK(I)=0
IDAT(I)=0
28 CONTINUE
CALL MAX(IDAT,NSM,NSM,IMAX)
CALL MIN(IDAT,NSM,NSM,IMIN)
IF(IMAX,GE,10*(IMIN))GO TO 5
YMULT=SCALED/(1.*IMIN)
GO TO 6
5 IF(IMAX,EQ,0) IMAX=1
YMULT=SCALED/(1.*IMAX)
6 CALL SCALE(IDAT,NSM,1,NSM,4.0,YMULT)
CALL PLOTEK(IX(1),IDAT(1),NSM,1,0,0)
CALL MAX(IWORK,NSM,NSM,IMAX)
YMULT=SCALED/(1.*IMAX)
CALL SCALE(IWORK,NSM,1,NSM,1.0,YMULT)
CALL PLOTEK(IX(1),IWORK(1),NSM,1,0,0)
CALL PLOTEK(0,730,1,1,0,0)
CALL HOME
WRITE(9,6979)(IFILE(J),J=3,7),FC,FREQ
6979 FORMAT(' FILE:','SA2,' LPFCF IS','F5.1,3X,
1'INPUT FREQ ','F5.1)
WRITE(9,5999)
5999 FORMAT(' /// /// /// 40X,'UMR EMG INPUT')
WRITE(9,6976)
6976 FORMAT(' /// /// /// 40X,'NEURAL INPUT')
WRITE(9,5998)
5998 FORMAT(' /// /// /// 40X,'PRODUCT')
WRITE(9,6975)
6975 FORMAT(' /// /// /// 40X,'UMR OUTPUT ')
802 WRITE(9,6973)
6973 FORMAT(' /// /// /// 40X,'NEURAL OUTPUT')
804 CONTINUE
REWIND 9
840 CONTINUE
WRITE(6,6998)FREQ,RMSE,RMSI,RMSO,SNR,XMI,XMO
6998 FORMAT(1X,F5.1,1X,6(3X,E8.2))
REWIND 6
WRITE(7,7500)
7500 FORMAT(' DO YOU WANT ANOTHER RUN? YES OR NO ')
READ(S,7501)IANS8
7501 FORMAT(A2)
IF(IANS8,EQ,'YES') GO TO 333
END
C...... LOW PASS FILTER ROUTINE.
C...... THIS ROUTINE IS A LOW PASS DIGITAL FILTER THROUGH WHICH THE
C...... DATA IS PASSED TWICE, ONCE IN THE FORWARD DIRECTION AND
C...... ONCE IN THE BACKWARD DIRECTION TO GET RID OF ANY TIME DELAY
C...... INTRODUCED BY THE FILTER.
C
C...... THE FILTER SIMULATES A SECOND ORDER BUTTERWORTH FILTER.
C
C...... USAGE:
CALL LPF(X,N,FC,FS)
C
X(N) = DATA TO BE FILTERED
C
FC = CUTOFF FREQUENCY OF THE FILTER
C
FS = SAMPLING RATE OF THE SIGNAL
C
-------------------------------------
SUBROUTINE LPF(X,N,FC,FS)
DIMENSION X(N),TEMP(3)
FC1=FC/.S0224
FI=4.*ATAN(1.)
FI=(FI*FC1)/FS
F=SIN(FI)/COS(FI)
F2=F*F
FR2=F*SQRT(2.)
FAC=F2+FR2+1.
A=F2/FAC
B=2.*A
C=-(2.*F2-2.)/FAC
D=-(F2+1.-FR2)/FAC
DO10 I=1,3
10 TEMP(I)=X(I)
DO1I=3,N
X(I)=A*TEMP(3)+B*TEMP(2)+A*TEMP(1)+C*X(I-1)+D*X(I-2)
IF(I.EQ.N)GO TO 1
TEMP(1)=TEMP(2)
TEMP(2)=TEMP(3)
TEMP(3)=X(I+1)
1 CONTINUE
DO20 I=1,3
20 TEMP(I)=X(N-I+1)
DO2I=3,N
K=N-I+1
X(K)=A*TEMP(3)+B*TEMP(2)+A*TEMP(1)+C*X(K+1)+D*X(K+2)
IF(K.EQ.1)GO TO 2
TEMP(1)=TEMP(2)
TEMP(2)=TEMP(3)
TEMP(3)=X(K-1)
2 CONTINUE
RETURN
END
APPENDIX III
HOMFIL SOURCE LISTING

C........HOMFIL PROGRAM TO HOMOMORPHICALLY PROCESS INPUT
C........DATA FILES.
C........
C........
C........
C........

DIMENSION XDAT(2000),SGN(2000),I3FILE(7)
DIMENSION IDAT(2000),IFILE(7),I2FILE(7)
DIMENSION IHSTRK(20),LAST(20),NSAMP(20)
EQUIVALENCE (SGN(1),IDAT),(SGN(1001),IWORK)
DATA NNO,YES/’NO’,’YE’/
WRITE(7,9999)
9999 FORMAT(‘ENTER DATA FILE NAME’/1X,’*’/10X,10X,10X)
READ(5,9998)(IFILE(J),J=1,7)
9998 FORMAT(7A2)
WRITE(6,6699)(IFILE(J),J=3,7)
6699 FORMAT(‘FILE’/1X,2X,5A2//)
WRITE(6,6699)
6999 FORMAT(1X,’RECS’,1X,’RAWVAR’,2X,’ISOVAR’,2X,
1,’MEAN’,1X,’NEUVAR’,5X,’SNR’,5X,’NMAX’,5X,’NMIN’/)
NTOT=64
N=2000
CALL ASSIGN(1,IFILE,14,’RDO’)
DEFINE FILE 1 (NTOT,N,NREC)
10 WRITE(7,7997)
7997 FORMAT(’WHAT IS SAMPLING FREQ?’/1X,’*’/)
READ(5,7796)IFS
7796 FORMAT(14)
RPER=1.0/IFS
WRITE(7,7960)
7960 FORMAT(‘DO YOU WANT GRAPHICS DISPLAY OF DATA? YES OR NO?’)
READ(5,9959)IANS4
9959 FORMAT(A2)
IFLAG2=0
IF(IANS4.EQ.IYES)IFLAG2=1
WRITE(7,7795)
7795 FORMAT(’WHAT IS THE LPF CUTOFF FREQ?’/1X,’*’/)
READ(5,7794)FC
7794 FORMAT(F4.1)
WRITE(7,2607)
2607 FORMAT(‘DO YOU WANT OUTPUT FILES? YES OR NO’)
READ(5,2608)IANS2
2608 FORMAT(A2)
IFLAG1=1
IF (IANS2.EQ.NNO) IFLAG1=0
500 WRITE(7,9900)
9900 FORMAT(' WHAT REC. NO. '/**')
READ(5,9899) JREC
9899 FORMAT(12)
C
READ IN DATA FILE
C
READ(1',JREC) (IDAT(J),J=1,2000)
NSM=N
DO 10 I=1,N
   IF (IDAT(I).LT.2047) GO TO 10
   NSM=I-1
10 CONTINUE
DO 11 I=1,NSM
   XDAT(I)=IDAT(I)
11 CONTINUE
IF (IFLAG2.EQ.0) GO TO 815
DO 12 I=1,250
   IDAT(I)=0
12 CONTINUE
GS SCALE=1023/19.05
SCALED=1.5*GS SCALE
XTEMP=0.0
DO 400 I=1,2000
   IX(I)=XTEMP
   XTEMP=XTEMP+.5
400 CONTINUE
CALL MAX(IDAT,NSM,NSM,IMAX)
YMULT=SCALED/(1.*IMAX)
CALL SCALE(IDAT,NSM+1,NSM+9.0,YMULT)
CALL FLOTEK(IX(1),IDAT(1),NSM+1,0,1)
815 XSUM=0.
DO 60 I=1,NSM
   XSUM=XSUM+XDAT(I)
60 CONTINUE
XMN=XSUM/FLOAT(NSM)
DO 99 I=1,NSM
   XDAT(I)=XDAT(I)-XMN
99 CONTINUE
PWRT=0.0
DO 26 I=250,NSM
   PWRT=PWRT+XDAT(I)**2
26 CONTINUE
PWRT=PWRT/(NSM-250)/409.6**2
IF (IFLAG1.EQ.0) GO TO 811
WRITE(7,2605)
2605 FORMAT(' WHAT IS UMRENG FILE NAME? '/**')
READ(5,2606)(I2FILE(I),I=1,7)
2606 FORMAT(7'A2')
CALL ASSIGN(2,I2FILE,14,'NEW')
WRITE(7,2609)
2609 FORMAT(‘ WHAT IS THE NEMUFG FILE NAME? ’/’************’)
READ(5,2610)(I3FILE(I),I=1,7)
2610 FORMAT(7A2)
CALL ASSIGN(3,I3FILE,14,’NEW’)
81 CONTINUE
C
C DO HOMOMORPHIC PROCESSING
C
DO 20 I=1,NSM
  SGN(I)=1.0
  IF(XDAT(I),EQ.0.) XDAT(I)=XDAT(I)+.01
  IF(XDAT(I),LT.0.0) SGN(I)=-1.0
  XDAT(I)=ABS(XDAT(I))
  XDAT(I)=ALOG(XDAT(I))
  RMDAT(I)=XDAT(I)
20 CONTINUE
FS=IFS
C
C LOW PASS FILTER AS IN HOMTST, FOR SEE APPENDIX II
C
CALL LPF(RMDAT,NSM,FC,FS)
DO 21 I=1,NSM
  XDAT(I)=XDAT(I)-RMDAT(I)
  XDAT(I)=100*EXP(XDAT(I))*SGN(I)
  RMDAT(I)=100*EXP(RMDAT(I))
21 CONTINUE
C
CNOTE:
C  RECFIL. FOR IS IDENTICAL TO HOMFIL. FOR BUT FOR THE
C  LAST 15 LINES WHICH ARE REPLACED BY THE FOLLOWING
C  10 LINES:
C
DO 20 I=1,NSM
  RMDAT(I)=ABS(XDAT(I))
  XDAT(I)=RMDAT(I)
20 CONTINUE
FS=IFS
C
CALL LPF(RMDAT,NSM,FC,FS)
DO 21 I=1,NSM
  XDAT(I)=100*XDAT(I)
  RMDAT(I)=100*RMDAT(I)
21 CONTINUE
C
C
PWRI=0.
FWRN=0.
XDATM=0.
RMDATM=0.
C
C CALCULATE POWERS AND VARIANCES
C
DO 25 I=250,NSM
   PWRI=XDAT(I)**2+PWRI
   PWRN=RMDAT(I)**2+PWRN
   XDATH=XDAT+XDAT(I)
   RMDATH=RMDATM+RMDAT(I)
   CONTINUE
   PWRI=PWR/10000./((NSM-250)/409.6)**2
   PWRN=PWR/10000./((NSM-250)/100.
   XDATH=XDATM/FLOAT(NSM-250)/100.
   RMDATH=RMDATM/100./((NSM-250.)
   RNEVAR=PWRN-RMDATM**2
   SNR=RMDATM/(SQRT(RNEVAR))
   DO 22 I=1,250
      XDAT(I)=0.0
      RMDAT(I)=0.0
   CONTINUE
   CALL REMAX(RMDAT,251,NSM,XMAX)
   CALL REMIN(RMDAT,251,NSM,XMIN)
   XMIN=XMIN/100.
   XMAX=XMAX/100.
   IF(IFLAG2.EQ.0) GO TO 812
   DO 50 I=1,NSM
      RMDAT(I)=RMDAT(I)/100.
      XDAT(I)=XDAT(I)/100.
   CONTINUE
   WRITE APPROPRIATE FILES
   WRITE(3)(RMDAT(I),I=250,NSM)
   WRITE(2)(XDAT(I),I=250,NSM)
   DO 51 I=1,NSM
      RMDAT(I)=RMDAT(I)*100.
      XDAT(I)=XDAT(I)*100.
   CONTINUE
   812 CONTINUE
   IF(IFLAG2.NE.1) GO TO 640

DO 20 I=1,NSM
   IWORK(I)=RMDAT(I)
   IDAT(I)=XDAT(I)
   CONTINUE
   CALL MAX(IDAT,NSM,NSM,IMAX)
   CALL MIN(IDAT,NSM,NSM,IMIN)
   IDMAX=IMAX
   IDMIN=IMIN
   CALL MAX(IWORK,NSM,NSM,IMAX)
   CALL MIN(IWORK,NSM,NSM,IMIN)
   MMAX=IMAX
   MMIN=IMIN
   SCALED=1.5*GSSCALE
APPENDIX IV
STATIONARITY TESTING

Methods of assessing the stationarity of signals were investigated. These efforts lead to the conclusion that the non-parametric run test was the most commonly used, (Bendat and Piersol, 1971, Sugimoto et al 1978, 1977 Wang and Vagnucci, 1980, Cohen 1977).

The run test checks for non-random trends in the data. It involves dividing available records into M segments. A suitable parameter is then estimated for each segment. The parameter estimate is then compared to (subtracted from) some constant value and a + or - sign is then assigned to that segment based on the algebraic outcome of the comparison. The number of runs is then determined for the resulting sequence of + and - signs. A run is defined as a string of identical observations (signs) that is followed and preceded by a different observation or no observation at all. The number of runs in this sequence is an indication of the existence of a trend in the original data. If the sequence is composed of independent observations of the same random variable the number of runs will be a random variable $r$ with mean $\mu_r$ and variance $\sigma_r^2$ given as: (Bendat and Piersol 1971).
\[ r = \frac{2N_1 N_2}{M} + 1 \]

\[ \sigma_r^2 = \frac{2N_1 N_2 (2N_1 N_2 - N)}{M^2 (M - 1)} \]

where \( M \) is the number of segments (sequence length)
\( N_1 \) is the number of + signs
\( N_2 \) is the number of - signs

The number of runs compared to its expected value and variance leads to the determination of the existence of a trend at a selected level of significance. That is, it determines the rejection or acceptance of the hypothesis of independent observations of the same random variable in the normal statistical hypothesis testing fashion. Thus, too few or too many runs will lead to the rejection of the independent observations hypothesis and therefore to the acceptance of the existence of a trend in the data.

The wide sense stationarity of data can be assessed by testing for trends in the data's mean and mean square values (Bendat and Piersol 1971). This allows the run test to be used to test for stationarity. The run test for this work was implemented by the Fortran IV program RUNTST. A listing of this program is found later in this Appendix. RUNTST divides the input data records into \( M \) segments, where \( M \) is restricted to being an even number. The mean and mean square values of each segment are then estimated and compared
to their corresponding median values of all the M segments with a + or - sign being assigned accordingly. The two resulting sequences are of length M and each have an equal number of + and - signs. For this special case the following run probabilities exist, (Sugimoto, 1977, 1978). If r is the number of runs

for \( r = 2k \)

\[
Pr = \frac{(M/2-1)(M/2-1)}{\binom{M}{k-1} \binom{M}{k-1}}
\]

for \( r = 2k+1 \)

\[
Pr = \frac{2(M/2-1)(M/2-1)}{\binom{M}{k} \binom{M}{k}}
\]

The number of runs \( R \) is then determined for these sequences and using these probabilities for specific numbers of runs the probability of at least \( R \) number of runs is calculated using

\[
P_R = \sum_{r=1}^{R} Pr
\]

The program then accepts the stationarity hypothesis if \( P_R \) is greater than \( \alpha/2 \) and less than \( 1 - \alpha/2 \) for each sequence, where \( \alpha \) is the level of significance. A listing of RUNTST follows.
RUNTST SOURCE LISTING

C........RUNTST.FOR PROGRAM TO TEST STATIONARITY OF DATA RECORDS
C........ USING TWO SIDED RUN TEST
D DIMENSION IDAT(2000),IFILE(7),XMN(100),XMS(100)
  DATA NNO,iYES/'NO','YE'/
  WRITE(7,9999)
  9999 FORMAT(' ENTER DATA FILE NAME'/1X,'***',***')
     READ(5,9998)(IFILE(J),J=1,7)
  9998 FORMAT(7A2)
     WRITE(7,9997)
  9997 FORMAT(' TOT. NO. OF REC. AND NO. OF SAMPLES PER REC.?'/1X,
      1 '***',1X,'***')
     READ(5,9996)(NTOT,N)
  9996 FORMAT(14,1X,I4)
     CALL ASSIGN(1,IFILE,14,'RDO')
     DEFINE FILE 1(NTOT,N,U,JREC)
     WRITE(7,9995)
  9995 FORMAT(' HOW MANY DATA SEGMENTS? MUST BE EVEN'/1X,'**')
     READ(5,9994)NSEG
  9994 FORMAT(I2)
     WRITE(7,9993)
  9993 FORMAT(' WHAT SIGNIFICANCE LEVEL?'/1X,'***')
     READ(7,9992)ALPHA
  9992 FORMAT(F5.3)
     WRITE(7,9991)
  9991 FORMAT(' WHAT REC. NO.?'/1X,'***')
     READ(5,9990)JREC
  9990 FORMAT(I3)
     WRITE(5,9978)(IFILE(J),J=3,7)JREC
  9978 FORMAT(5A2,5X,'RECORD NO.',I3,/)
     READ(1,JREC) (IDAT(J),J=1,N)
     WRITE(7,9999)
  9999 FORMAT(' DO YOU WANT A ZERO MEAN SIGNAL? YES OR NO?')
     READ(5,9998)IANS5
  9998 FORMAT(A2)
     IF(IANS5.NE.IYES) GO TO 555
  555 CONTINUE
     SUM=0.0
     DO 4 I=1,N
        SUM=SUM+IDAT(I)
  4 CONTINUE
     XMEAN=SUM/N
     DO 6 I=1,N
        IDAT(I)=IDAT(I)-XMEAN
  6 CONTINUE
C........DIVIDE DATA RECORD IN TO NSEG SEGMENTS
     DO 5 I=1,NSEG
        XMN(I)=0.0
        XMS(I)=0.0
  5 CONTINUE
$\text{NSSEG}=N/\text{NSEG}$
$\text{NP}=$NSSEG*NDSEG
$\text{T01}=0.0$
$\text{T02}=0.0$

**C..**  **CALCULATE THE MEAN AND MEAN SQUARE VALUES FOR EACH SEGMENT**

**C..**  **SECTION AND STORE IN SEPARATE VECTORS**

DO 10 I=1,NSSEG
   DO 15 J=1,NSSEG
      \[ K=(I-1)*\text{NSSEG}+J \]
      \[ \text{XMN}(I)=\text{XMN}(I)+\text{IABS}(	ext{IDAT}(K)) \]
      \[ \text{XMS}(I)=\text{XMS}(I)+\text{FLOAT}(	ext{IDAT}(K))^{\times 2} \]
   15 CONTINUE
   \[ \text{XMN}(I)=\text{XMN}(I)/\text{NSSEG} \]
   \[ \text{XMS}(I)=\text{XMS}(I)/\text{NSSEG} \]
10 CONTINUE

**C..**  **CALL RUNSUB SUBROUTINE TO CALCULATE PROBABILITIES**

CALL RUNSUB(XMN,NSSEG,IRUN1,PROBT1,IND1,ALPHA)
CALL RUNSUB(XMS,NSSEG,IRUN2,PROBT2,IND2,ALPHA)

IF(IND1.NE.0) GO TO 104
WRITE(6,9989)
9989 FORMAT('DATA MEAN SQUARE VALUES SHOW TREND-TOO FEW RUNS')
104 IF(IND1.NE.1) GO TO 105
WRITE(6,9988)
9988 FORMAT('DATA MEAN SQUARE VALUES SHOW TREND-TOO MANY RUNS')
105 IF(IND2.NE.0) GO TO 106
WRITE(6,9987)
9987 FORMAT('DATA MEAN SQUARE VALUES SHOW TREND-TOO FEW RUNS')
106 IF(IND2.NE.1) GO TO 107
WRITE(6,9986)
9986 FORMAT('DATA MEAN SQUARE VALUES SHOW TREND-TOO MANY RUNS')
107 IF(IND1.EQ.2.AND.IND2.EQ.2) GO TO 108
GO TO 112
108 WRITE(6,9985)
9985 FORMAT('DATA MEANS AND MEAN SQUARES SHOW NO TREND')
1 DATA IS STATIONARY/
112 CONTINUE
WRITE(6,9984)IRUN1,IRUN2
9984 FORMAT('/5X,'NO. OF RUNS OF MEANS',2X,I3/
15X,'NO. OF RUNS OF MEAN SQUARES',2X,I3)
WRITE(6,9983)ALPHA,NSSEG,N,NP
9983 FORMAT('/5X,'LEVEL OF SIGNIFICANCE',2X,F5.3,/'
15X,'NO. OF SEGMENTS',2X,I3,/5X,'NO. OF DATA POINTS',2X,I4
1,/5X,'NO. OF DATA POINTS USED',2X,I4)
WRITE(6,9979)IRUN1,PROBT1,IRUN2,PROBT2
9979 FORMAT('/1 PROB. OF LESS THAN',I3,' RUNS OF MEANS IS ',E9.2,/
1'/ PROB. OF LESS THAN',I3,' RUNS OF MEAN SQUARES IS ',E9.2,/
RENEW 6
WRITE(7,9982)
9982 FORMAT('DO YOU WANT ANOTHER RUN?')
READ(5,9981)IANS
9981 FORMAT(A2)
IF(IANS.EQ.N0)GO TO 109
SUBROUTINE RUNSUB(VECT,N,IRUN,PROBT,IND,ALPHA)

TO TEST FOR TREND IN A VECTOR ABOUT ITS MEAN VALUE

BY USING THE RUN TEST.

VECT INPUT VECTOR TO BE TESTED FOR TREND.
N NO. OF DATA POINTS IN THE INPUT VECTOR.
IRUN NO. OF RUNS ABOUT THE MEDIAN VALUE OF VECT.
PROBT PROBABILITY OF LESS THAN IRUN NO. OF RUNS.
IND INDICATOR OF TREND
0 VECT HAS TREND TOO FEW RUNS.
1 VECT HAS TREND TOO MANY RUNS.
2 VECT HAS NO TREND.

LEVEL OF SIGNIFICANCE OF TEST.

SUBROUTINE RUNSUB(VECT,N,IRUN,PROBT,IND,ALPHA)
DIMENSION VECT(1),DIF(500),PROB(500),SORT(500)
DATA NNO,IYES/'NO','Y'/'
N=2*(N/2)

CALCULATE THE MEDIAN VALUE OF VECT

DO 20 I=1,N
   SORT(I)=VECT(I)
20 CONTINUE
DO 21 I=1,N-1
   DO 22 J=1+I,N
      IF(SORT(I),LE,SORT(J)) GO TO 22
      TEMP=SORT(I)
      SORT(I)=SORT(J)
      SORT(J)=TEMP
22 CONTINUE
21 CONTINUE
XMED1=(SORT(N/2)+SORT(N/2+1))/2

DETERMINE THE NO. OF RUNS ABOUT THE MEDIAN.
IRUN=0
DO 25 I=1,N-1
   DIF(I)=VECT(I)-XMED1
   DIF(I+1)=VECT(I+1)-XMED1
   IF(DIF(I),GE,0) GO TO 100
   IF(DIF(I+1),LT,0) GO TO 101
   IRUN=IRUN+1
25 CONTINUE
GO TO 101
100 IF(DIF(I+1),GE,0) GO TO 101
    IRUN=IRUN+1
101 CONTINUE
25 CONTINUE
IRUN=IRUN+1
C......DETERMINE IF THE NO. OF RUNS INDICATES TREND IN THE DATA.
C......I.E. NONSTATIONARITY:
  PROBT=0.0
  M=N/2
  IF(IRUN.EQ.N) GO TO 107
C......CALCULATE EACH PROBABILITY FOR ODD EXACT NO. OF RUNS.
  DO 45 K=5,IRUN+2
     IF(K.GT.IRUN) GO TO 45
     I=(K-1)/2
     XTEMP=1.0/(2*I)*(M-I)
     XTEMP=ALOG10(XTEMP)
     DO 47 JJ=1,M-1
         A=ALOG10(1.0*(M-JJ))
         B=ALOG10(1.0*(I-JJ))
         C=ALOG10(1.0*(2*M-JJ))
         XTEMP=XTEMP+3*A-2*B-C
  47 CONTINUE
  DO 50 JJ=1,I-1
     A=ALOG10(1.0*(M-JJ))
     B=ALOG10(1.0*(I-JJ))
     C=ALOG10(1.0*(2*M-JJ))
     XTEMP=XTEMP+3*A-2*B-C
  50 CONTINUE
C......CALCULATE EACH PROBABILITY OF EVEN EXACT NO. OF RUNS.
  DO 60 K=4,IRUN+2
     IF(K.GT.IRUN) GO TO 60
     I=K/2
     XTEMP=.5
     XTEMP=ALOG10(1.0*XTEMP)
     DO 65 JJ=1,I-1
         A=ALOG10(1.0*(M-JJ))
         B=ALOG10(1.0*(I-JJ))
         C=ALOG10(1.0*(2*M-JJ))
         XTEMP=XTEMP+3*A-2*B-C
  65 CONTINUE
  DO 70 JJ=1,M-1
         A=ALOG10(1.0*(M-JJ))
         C=ALOG10(1.0*(2*M-JJ))
         XTEMP=XTEMP+A-C
  70 CONTINUE
C......CALCULATE THE PROBABILITIES FOR 1, 2 OR 3 EXACT RUNS.
  PROB(1)=0.0
  C1=1.0
  DO 75 JJ=1,M
      C1=C1*(M-JJ+1)/(2*M-JJ+1)
  75 CONTINUE
  PROB(2)=2*C1
  PROB(3)=2*(M-1)*C1
C...........CALCULATE THE TOTAL OF THE PROBABILITIES FOR EACH EXACT
C...........NO. OF RUNS TO DETERMINE PROBABILITY OF THERE BEING LESS
C...........THAN OR EQUAL TO IRUN NO. OF RUNS.
DO 80 JJ=1,IRUN
   PROBT=PROBT+PROB(JJ)
80 CONTINUE
   IND=3
   IF(PROBT.GE.ALPHA)GO TO 104
   IND=0
104 IF(PROBT.LE.1-ALPHA) GO TO 105
   IND=1
105 IF(IND.EQ.0.OR.IND.EQ.1) GO TO 106
   IND=2
106 CONTINUE
   GO TO 108
107 PROBT=1.0
   IND=1
108 CONTINUE
   RETURN
END
REFERENCES


