An Efficient Design for Robust Downlink Power Control Using Worst-case Performance Optimization

AN EFFICIENT DESIGN FOR ROBUST DOWNLINK POWER CONTROL USING WORST-CASE PERFORMANCE OPTIMIZATION

By

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A Thesis
Submitted to the School of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Applied Science

McMaster University
February 2009

MASTER OF APPLIED SCIENCE (2009)	MCMASTER UNIVERSITY
(Electrical and Computer Engineering)	Hamilton, Ontario

TITLE: **An Efficient Design for Robust Downlink Power Control Using Worst-case Performance Optimization**

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SUPERVISOR: Dr. Kon M. Wong

NUMBER OF PAGES: ix, 69

Abstract

Downlink power control and beamforming designs in wireless system have been a recent research focus. To achieve reliable and efficient designs, good estimation of wireless channel knowledge is desired. However, the presence of uncertain channel knowledge due to constant changing radio environment will cause performance degradation in system designs. Thus the mismatches between the actual and presumed channel state information (CSI) may frequently occur in practical situations. Robust power control and beamforming were introduced considering the channel uncertainty. In this thesis, a new robust downlink power control solution based on worst-case performance optimization is developed. Our approach explicitly models uncertainties in the downlink channel correlation (DCC) matrices, uses worst-case performance optimization and guarantees that the quality of service (QoS) constraints are satisfied for all users using minimum amount of power. An iterative algorithm to find the optimum power allocation is proposed. The key in the iteration is the step to solve an originally non-convex problem to obtain worst-case uncertainty matrices. When the uncertainty is small enough to guarantee that the DCC matrices are positive semidefinite, we obtain a closed-form solution of this problem. When the uncertainty is large, we transform this intractable problem into a convex problem. Simulation

results show that our proposed robust downlink power control using the approach of worst-case performance optimization converges in a few iterations and reduces the transmission power effectively under imperfect knowledge of the channel condition.

Acknowledgements

I would like to first express my greatest appreciation to Dr. Wong for supervising and supporting my work. His enthusiasm, encouragement and patience were essential to the completion of this project.

Also I would like to thank Dr. Jiankang Zhang for his insightful and patient discussion and guide for the progress of this project. Both Dr. Wong and Dr. Zhang's devotion to their work is a great encouragement and example for my future career.

I want to thank Dr. Davidson for his tutoring in leading me to optimization theory which becomes the tool for solving the core problem in my project.

I also want to thank staff in the department, in particular Cheryl, Terry and Cosmin whose efficient work helps me out a lot of trouble.

I want to thank my friends around me in McMaster University. Their encouragement and kindness are good help in my study and life. Last but not least, I would like to thank my family for their love and support which played an instrumental role in supporting my overseas study.

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Chapter 1

Introduction

1.1 Introduction

Application of antenna arrays in wireless system can increase system capacity by making use of spatial diversity [9–11, 13, 14]. Particularly, the application of antenna arrays mounted on base stations has been intensively studied in mobile communications [1, 2, 8, 35]. As a result, beamforming technology based on antenna array has been a well focused topic. Power control usually goes with beamforming technique. Many contributions have been achieved on uplink beamforming(receiver beamforming), while downlink beamforming(transmit beamforming) as well as power control is relatively new due to the difficulty to obtain accurate downlink channel information, mobile hardware design complexity etc. [26,29]. Several downlink power control and beamforming techniques have been proposed in the literature [11, 23, 33]. Most of these designs assume that the second order statistics of the channel (which are represented by the Downlink Channel Correlation (DCC) matrix) are exactly known.

However, this assumption is difficult to satisfy in practice because of the channel variability, mismatch of the DCC matrices and imperfect array calibration, etc. [18, 38]. As a result, the performance of such techniques can degrade dramatically because the Quality of Service (QoS) cannot be guaranteed when the Tx CSI is not perfect. Therefore, robust design with the inaccuracy of channel estimate taken into account is important in power control and beamforming. Robust uplink beamforming in the presence of imperfect receiver (Rx) channel state information (CSI) has been studied in the literature [19–22, 39].

Recently, several robust downlink power control and beamforming designs with the existence of imperfect channel estimates have been addressed in order to improve the robustness against the uncertainties in the channel estimates. The robust downlink beamforming method is discussed in [1]. It improves the robustness by constraining the worst-case QoS constraint represented by signal-to-interference-plus-noise ratio(SINR)to be satisfied for a lower bound SINR. It guarantees that the received SINR is sufficiently high for all possible values of the DCC matrices that fall within the upper and lower bounds. However, the deficiency of this method is the computational cost of solving the semidefinite programming (SDP) problem. Besides, the DCC matrix is required in all channel links in order to compute the transmission powers and this requires high communication rate between base stations and the central unit. In order to overcome the communication problem, it is more efficient to determine the beamforming weight vectors in advance and then to satisfy the QoS by adjusting the transmission powers. The robust downlink power control method discussed in [35] determines the beamforming weight vectors locally in a way which only requires DCC matrices in channel links between the base station(BS) and its allocated users. To compute the transmission powers, this robust downlink power control algorithm requires only the knowledge of the channel link gains, a scalar, for all channel links. As a result, the communication cost is reduced as compared to the cost of transmitting the DCC *matrices* in the robust downlink beamforming method. Unfortunately, the QoS constraints are tightened by also replacing the worst-case SINR with its *lower bound* in each of them. Since the worst-case SINR is replaced by its lower bound in both [1] and [35] for computing the transmission power, both designs improve the robustness by guaranteeing the *lower bound* SINR rather than the *worst-case* SINR. Although these strategies guarantees the QoS, it is a conservative guarantee in that more power may be spent than necessary.

Our goal in this thesis is to efficiently design a robust power control algorithm for downlink transmission while complying with worst-case QoS constraint. We develop an algorithm that determines the minimum transmission power required to ensure that the QoS constraints for all users are satisfied. An iterative procedure to jointly determine the power allocation and worst case uncertainties is proposed. In the iteration, although the natural formulation of the step to solve worst case uncertainty matrices is a complicated non-convex optimization problem, it can be separated into two subproblems, the first one of which has a closed form solution. However, the original formulation of the second subproblem is still non-convex. To solve the second subproblem, we consider two cases where uncertainty is either small or large. For uncertainty smaller than a specific threshold, we obtain an analytic global optimal solution to the subproblem. For uncertainty larger than this threshold, we transform the problem into a convex one and solve it efficiently using interior point method [45]. Although we use the iterative procedure, our algorithm is run-time efficient for the joint solution of the worst case downlink beamforming power allocation and the uncertainty matrices. Computer simulations verify that the iterative procedure converges in a few iterations and show that our proposed algorithm reduces the transmission power more effectively than other existing robust power control designs under imperfect knowledge of the channel condition.

1.2 Scope of the thesis

The objective of this thesis is to efficiently design a new downlink power control scheme to guarantee certain QoS requirement considering the existence of norm-bounded channel uncertainty. Towards this goal, the thesis is organized in chapters as the following:

In chapter 2, we first provide an overview of wireless cellular system, particularly with the focus on system structure and wireless radio links. Then power control and beamforming technologies as important methods to improve system performance are introduced with a discussion on special consideration and difficulty in downlink designs. We then present mathematical models of downlink signal transmission. In the last section of this chapter, we briefly review on some existing downlink power control and beamforming designs. Generalized eigenvalue based beamforming [41] algorithm is introduced as the fundamental building block to design beamformers in our work. Some existing work on non-robust downlink beamforming and power control [29,30,35], robust downlink beamforming and power control follows [1,34] in the last two subsections of the chapter.

In chapter 3, our robust downlink power control design is presented. Since the original formulation of the problem is a non-convex problem which cannot be solved using convex optimization tools, we make use of an iterative algorithm to determine the transmission powers and worst-case channel uncertainties jointly. Then, the problems of finding the transmission powers and uncertainties are formulated respectively. It is straightforward to calculate powers given a set of uncertainty matrices. However, it is much more difficult to find out the uncertainty matrices given a set of powers since the formulation of uncertainty matrices problem is non-convex either. Our main contributions lie in resolving of the uncertainty matrices problem. Both analytic solution and convex transformation solution are given in detail under different conditions.

In chapter 4, we demonstrate our results through various simulations by tuning different parameters. We also make comparison with other designs. Convergence of our algorithm is also illustrated through simulation.

Chapter 5 comes to an conclusion of this thesis and points out possible future work as extension of our work.

Chapter 2

Power Control and Beamforming in Wireless System

2.1 Introduction

This chapter paves the background knowledge for this thesis. Cellular wireless system is introduced in the first section with emphasis on cell partition and frequency reuse. We then present a brief review on wireless radio transmission link. Various environmental factors affect the transmission of signal through wireless links, the result of which are large scale power path loss and small scale fast fading. Noise is introduced due to these two reasons. Specifically, the small scale fluctuation of radio channel will make accurate estimate of channel information very difficult in practice. In a multiuser system, inter-user interference is another source affecting service quality. Power control and beamforming techniques are described in the second section as methods to increase system performance. Downlink or transmit beamforming is compared with uplink beamforming regarding the complexity in practical implementation and difficulty in channel estimation. Downlink system signal model is formulated in subsequent section. Since our work focuses on robust downlink power control and beamforming, in the third section, we review some existing achievements on non-robust and robust downlink power control and beamforming.

2.2 Cellular Wireless System

2.2.1 System Structure

In earliest wireless mobile systems, a high powered transmitter with a single ubiquitous antenna was employed on a base station to achieve satisfactory large celled coverage. However, due to the fact that this method uses single frequency to cover a large area, it prohibited the reuse of radio frequencies while radio spectrum resource is limited. Therefore, cellular system design scheme was introduced to facilitate frequency reuse and increase system capacity. The resulting smaller cells use base stations armed with low powered antenna(s). Within a specific geographic area, different cells are allocated with different part of the total available channels using a given set of frequencies. Outside this area, those frequencies can be reused for other sets of cells. Each cell contains a Base Station(BS) which is connected to a switching center and communicates with mobiles through radio links. Ideally, the cell shape is hexagon and BS transmitter is located in the center of the hexagon. Mobile users with single antenna inside each cell are served by the BS belonging to this cell. Frequency reuse introduced through this cell structure can improve system capacity effectively. With the increasing service requirement, some other techniques such as cell splitting,

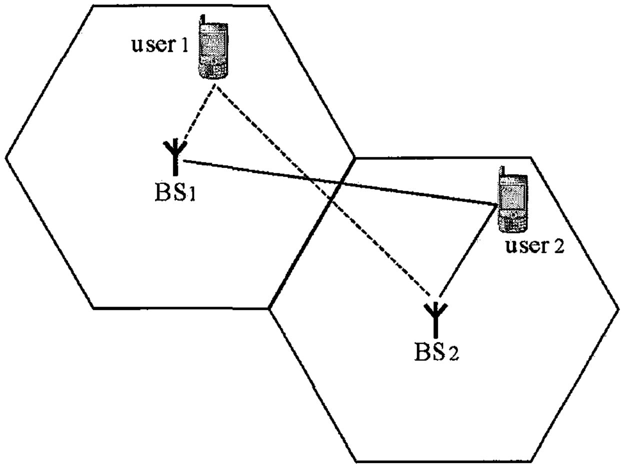


Figure 2.1: Wireless Radio Link

sectoring [26] and antenna arrays etc. are also introduced to increase system capacity.

2.2.2 Radio Channel Link

The wireless connection between a user and its BS is the channel link through which radio signals are transmitted. This can be noted from FIG. 2.1. However, in practical situations, the links are in one way not necessarily Line of Sight(LOS), and in another, are statistically changing due to non-ideal environment and users' movement. Free-space propagation model will be introduced first. Using Friss free space equation, the received power of a user from a transmitter in a given distance away is represented as

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \tag{2.1}$$

where P_t is the transmitted power, d is the (transmitter and receiver)T-R separation distance in meters, $P_r(d)$ is the received power which is a function of the T-R separation, G_t is the transmitting antenna gain, G_r is the receiver antenna gain, L is

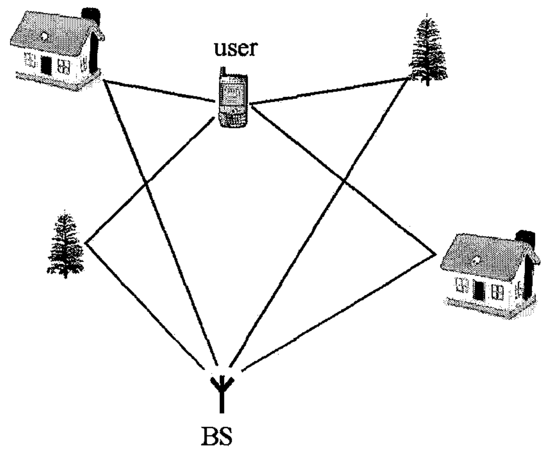


Figure 2.2: Radio Propagation Environment

the system loss factor not related to propagation and λ is the wavelength in meters. Therefore, the large scale path loss represented in dB is obtained as below:

$$PL(\text{dB}) = 10\log\frac{P_t}{P_r} = -10\log\frac{c}{d^2} \tag{2.2}$$

As we can see, in an ideal environment, the received power is reversely proportional to second power of the BS user distance.Thus the path loss exponent or the attenuation factor is 2. However, in real environment, the propagation is affected by three main mechanisms, which are reflection, diffraction and scattering [26]. FIG. 2.2 demonstrates the basic idea of the propagation in real environment. The impact of these three mechanisms will worsen the path loss, so that the path loss exponent is usually larger than 2. Therefore there are some other large scale propagation models [3, 26] for more precise estimation of the average received power in the mobile end. Usually, for urban areas with dense buildings, the path loss between BS and users within the cell can be the as high as power of the distance between the two [5]. In this thesis, for simulation purpose, the path loss exponent will be chosen as 4.

Corresponding to large scale path loss, there is small-scale fading, which makes the channel model a time variant system. Small-scale fading describes rapid fluctuation of radio signal strength over very short time. The factors affecting this fading includes multi-path propagation, mobile's movement and surrounding objects' movement(Doppler shift will be introduced in these two cases). The direct impact of this small-scale channel variation is the mathematical model of the radio channel, which will be formulated as time variant impulse response with some statistics property. This will make precise estimation of uplink or downlink Channel State Information(CSI) almost impossible.

2.3 Power Control and Beamforming Introduction

2.3.1 Power Control

In a communications system, one of the most widely used measures of system performance is signal-to-noise ratio(SNR) [12], which is the ratio of between desired signal power and the noise. Accordingly, in a system with inter-user interference, this Quality of Service(QoS)criterion is modified as signal-to-interference-noise ratio, i.e. SINR. Usually there will be a minimum acceptable SINR threshold to guarantee the service quality. Thus it is essential that a user receives a power strength high enough but not that high as to affect other users. Power control is the technology to lever transmitting power to a proper level. Both the BS and user require power control. A BS can send control information to the user to tune its power. Vice versa, the user can send control signal to BS to lower or strengthen the transmitted power. Particularly, in a CDMA system, power control also helps mitigate the near-far problem and

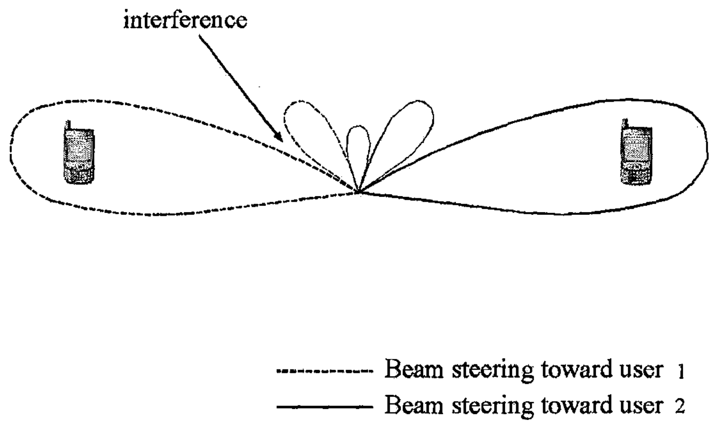


Figure 2.3: Beamforming Illustration

increases system capacity [27].

2.3.2 SDMA and Beamforming

Multiple access technology is widely used in communications system to provide high system capacity. It refers to the technology that allows a number of users to simultaneously communicate through a shared resource, frequency spectrum for example. Besides Frequency Division Multiple Access(FDMA), Time Division Multiple Access(TDMA) and Code Division Multiple Access(CDMA), Space Division Multiple Access(SDMA) is another commonly used multiple access scheme in wireless communications systems. SDMA provides spatial diversity to improve system performance and is realized in three forms [4]. The first one is closely related with the concept of *cell* in section 2.2. In this primitive form of SDMA, frequency reuse is realized by transmit signals in the same frequency in different cells with prescribed distances away from each other [26]. The second form is sectoring i.e. the cells are partitioned into 120^o or 60^o sectors. The third form of SDMA is realized by employing antenna array in either transmit mode or reception mode so that directional beams can be generated to serve different areas or receive signals from users within these areas. By default, this is the commonly referred SDMA in many publications [9]. The antenna array adopted for beamforming is usually Uniformed Linear Array(ULA) [4] in which antenna elements are evenly separated. Those antenna elements can be simple omnidirectional antennas [9].

The signals generated from different elements of an array are synthesized to form a single output of the array, with beams towards their targeted users. The process of combining the output of the antenna elements in an array to form specific beam patterns is beamforming. FIG. 2.3 illustrates the basic idea of beamforming. Conventionally, beam forming was realized by adjusting only the phase of signals from each antenna element. In this case, the shape of the beams are fixed. To make beamforming more adaptive, both the gain and phase can be modified. The gain and phase applied on each antenna element is equivalently to a complex weight coefficient. Those weight coefficients constitute a weight vector in terms of the antenna array. Therefore, the essential part of beamforming is the design of weight vectors.

There are two kinds of beamforming, say, receiver beamforming and transmitter or downlink beamforming. It appears that beamforming designs for transmission and receiving are equivalent, due to the reciprocal property of the radio channel. However, many previous works have focused on receiver beamforming, while transmitter beamforming is relatively new and difficult [7–9], the reasons of which are as follows. Firstly, transmit beamforming is a concern of the entire system while receiver beamforming only impacts the signal quality for the specific user. This is because transmitted signals from each beamformer will not only be received by the desired users as useful information, but also will be received by other users as interference. The goal to guarantee a certain quality level for desired users meanwhile to keep the interference to other users low will have to be a joint design concerning all users in the whole system. Secondly, it is harder to gain a good estimate of the channel knowledge in transmitter mode. In receiver beamforming, a trained serial of symbols can be used to estimate the channel(uplink) information by analyzing the local feedback from the receiver output. Whereas in transmission beamformer, downlink channel information estimate can only be done at the receiver and this will require another uplink channel feedback to the BS beamformer. Thus, the required signal processing and hardware techniques will be more challenging for transmit beamforming. Efficient designs for downlink beamforming is highly demanded.

2.3.3 Downlink Signal Model

Consider a cellular communication system in which K co-channel users are served by M base stations (BS). Each BS is equipped with an array of N antennas while a mobile user employs a single antenna. Each BS transmits a narrow-band composite signal consisting of several co-channel signals each targeted at one of the users. We assume that all users are sharing the same time and frequency slots and the channels are stationary and flat Rayleigh fading. Also, we assume that the users are incoherently located. Let P_ℓ be the transmission power for the ℓth user. The downlink beamforming structure corresponding to the signal model used here can be seen from

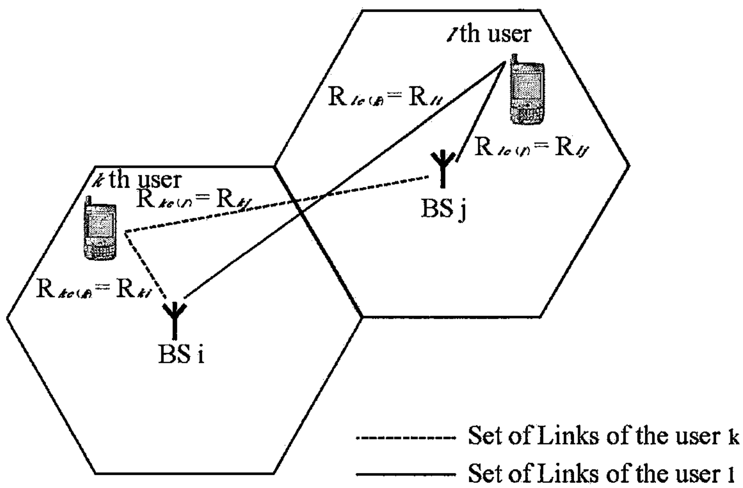


Figure 2.4: Downlink Channel Correlation Matrix Model

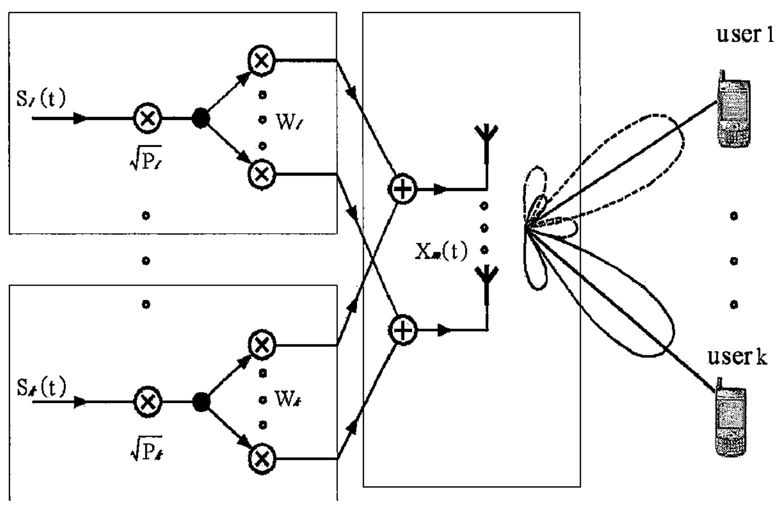


Figure 2.5: Downlink Beamforming Structure Illustation

FIG. 2.5. The transmission signal from the mth BS can be written as

$$\mathbf{x}_m(t) = \sum_{\ell \in \mathcal{S}_m} \sqrt{P_\ell}\mathbf{w}_\ell s_\ell(t) \tag{2.3}$$

where $\mathbf{x}_m(t)$ is an $N \times 1$ vector, $s_\ell(t)$ is the signal which targets the ℓth user at time instant t, $\mathbf{w}_\ell$ is an $N \times 1$ vector of normalized beamformer weight (i.e. $\|\mathbf{w}_\ell\|_2 = 1$) designed for the ℓth user, and $\mathcal{S}_m$ denotes the set of users indices that are served by the mth BS. We assume the signal set $\{s_\ell(t)\}_{\ell=1}^{K}$ is orthonormal. Assume that the kth and ℓth users are assigned to ith and jth BS, respectively. The assigned BS for ℓth user is represented as $c(\ell)$. The pair $(kc(\ell))$ represents the channel between kth user and $c(\ell)$th base station. Thus, $\mathbf{h}_{km}$ is the $N \times 1$ vector of random complex channel coefficients between the mth BS and the kth user. Here, $(\cdot)^H$ denotes Hermitian transpose. The base-band signal received at the kth user is given by

$$\begin{aligned} y_k(t) &= \sum_{m=1}^{M} \mathbf{h}_{km}^H \mathbf{x}_m(dt) + n_k(t) \\ &= \sum_{\ell=1}^{K} \sqrt{P_\ell} s_\ell(t) \mathbf{w}_\ell^H \mathbf{h}_{kc(\ell)} + n_k(t) \\ &= \sqrt{P}_k s_k(t) \mathbf{w}_k^H \mathbf{h}_{kc(k)} + \sum_{\ell=1;\ell\neq k}^{K} \sqrt{P_\ell} s_\ell(t) \mathbf{w}_\ell^H \mathbf{h}_{kc(\ell)} + n_k(t) \end{aligned} \tag{2.4}$$

The three parts of the received signal are respectively, the desired signal: $y_k^d(t) = \sqrt{P}_k s_k(t)\mathbf{w}_k^H \mathbf{h}_{kc(k)}$, the interference: $y_k^i(t) = \sum_{\ell=1;\ell\neq k}^{K} \sqrt{P}_\ell s_\ell(t)\mathbf{w}_\ell^H \mathbf{h}_{kc(\ell)}$ and the zero-mean noise component $n_k(t)$. The DCC matrix between the mth BS and the kth user is defined as

$$\mathbf{R}_{km} = \mathbf{E}\{\mathbf{h}_{km}\mathbf{h}_{km}^H\} \tag{2.5}$$

Since there are K users and M cells, there are in total $K \times M$ DCC matrices. The illustration of DCC matrices is shown in FIG. 2.4. In this figure, the desired signal for the kth user is transmitted from its assigned BS, with the index $c(k) = i$. Meanwhile, the signal from the BS(with index $c(\ell) = j$) assigned to ℓth user is partly received on kth user as interference, through the channel link $(k, c(\ell))$. The average SINR of the kth user is expressed as

$$\text{SINR}_k = \frac{\text{desired user power}}{\text{noise+interference power}} = \frac{P_k \mathbf{w}_k^H \mathbf{R}_{kc(k)} \mathbf{w}_k}{\sigma_k^2 + \sum_{\ell=1;\ell\neq k}^{K} P_\ell \mathbf{w}_\ell^H \mathbf{R}_{kc(\ell)} \mathbf{w}_\ell} \quad (2.6)$$

where σ_k^2 denotes the noise variance of the kth user and $\mathbf{R}_{kc(\ell)}$ is the DCC matrix between the BS with the index $c(\ell)$ and the kth user.

As mentioned in section 2.3.1, SINR is chosen as the measurement of service quality. A minimum threshold γ_k of SINR will be given for each user as the key constraint of power control and beamforming problems. Thus, the goal of power control is to design a set of powers $\{P_k\}_{k=1}^K$ to satisfy SINR criterion and the goal of beamforming is to design a set of weight vectors $\{\mathbf{w}_k\}_{k=1}^K$ to meet the criterion. Overall, the model described in this section explicitly and separately express powers $\{P_k\}_{k=1}^K$ and beamforming vectors $\{\mathbf{w}_k\}_{k=1}^K$ as system parameters. We will be using this model in our work. There are also some other works [28–30] which absorb powers into weight vectors to jointly design these two set of parameters.

2.4 Downlink Power Control and Beamforming Design

2.4.1 Generalized Eigenvalue-Based Beamformer

Generalized eigenvalue based beamforming is a widely used design scheme [31–33,39] to attain desired directional beams. A generalized eigenvalue-based beamformer is obtained by maximizing the ratio of the power for the desired user to the power of total interference from other users. The beamforming weight is computed by pointing the main beam to the direction of the intended user and producing nulls in the direction of other users within the same cell. Here, the users out of the cell are not considered, the reason of which is to reduce the cost of transmitting channel information. The beamformer of kth user is written as:

$$\mathbf{w}_k = arg \max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{R}_{kc(k)} \mathbf{w}}{\mathbf{w}^H (\sum_{\ell \in S_k; \ell \neq k} \mathbf{R}_{\ell c(k)}) \mathbf{w}} \tag{2.7}$$

This problem is solved as a generalized eigenvalue problem. Define $\mathbf{R}_k$ as $\sum_{\ell \in S_k; \ell \neq k} \mathbf{R}_{\ell c(k)}$, Equation (2.7) can be solved by solving the following optimization problem

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^H \mathbf{R}_k \mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{R}_{kc(k)} \mathbf{w} = 1 \end{aligned} \tag{2.8}$$

Using Lagrangian multiplier method [39], the optimal weight vector of the kth user is attained as

$$\mathbf{w}_k^{opt} = P\{\mathbf{R}_{kc(k)} (\mathbf{R}_k)^{-1}\} \tag{2.9}$$

where $P\{.\}$ is the operator which gives the principle vector of a matrix, i.e. the eigenvector corresponding to the maximum eigenvalue of the matrix. Here, $\mathbf{w}_k$ is a normalized vector, i.e. $\|\mathbf{w}_k\|_2 = 1$. This method needs the knowledge of the channel correlation matrices only from those users within the same cell. If it happens that there is only one user in the cell, the solution will be simply

$$\mathbf{w}_k^{opt} = P\{\mathbf{R}_{kc(k)}\} \tag{2.10}$$

Since $\|\mathbf{w}_k\|_2$ is normalized, a set of powers $\{P_k\}_{k=1}^K$ need to be calculated to satisfy QoS requirement. We will be using this method to compute beamforming weight vectors in our work in later chapters. Also, as mentioned in subsection , the scale factor $\sqrt{P}_k$ can be integrated into weight vector, through which the transmitted power can be expressed as $\mathbf{w}_k^H \mathbf{w}_k$.

2.4.2 Non-robust Downlink Power Control and Beamforming

In [29,30], optimal beamforming scheme which makes use of semidefinite programming optimization method is proposed. In this work, transmission power is implicitly incorporated into weight vectors so that the powers and weights are determined jointly. The problem is thus formulated as:

$$\begin{aligned} \min_{\mathbf{w}_k} \quad & \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{w}_k \\ \text{subject to} \quad & \frac{\mathbf{w}_k^H \mathbf{R}_{kc(k)} \mathbf{w}_k}{\sigma_k^2 + \sum_{\ell=1;\ell\neq k}^{K} \mathbf{w}_\ell^H \mathbf{R}_{kc(\ell)} \mathbf{w}_\ell} \geq \gamma_k, k = 1, ..., K \end{aligned} \tag{2.11}$$

This is equivalently transformed as

$$\begin{aligned} \min_{\mathbf{w}_k} \quad & \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{w}_k \\ \text{subject to} \quad & \mathbf{w}_k^H \mathbf{R}_{kc(k)} \mathbf{w}_k - \gamma_k \sum_{\ell=1;\ell\neq k}^{K} \mathbf{w}_\ell^H \mathbf{R}_{kc(\ell)} \mathbf{w}_\ell \geq \gamma_k \sigma_k^2 \qquad k = 1, \dots, K, \end{aligned} \tag{2.12}$$

where the objective, i.e. the total transmit power is minimized when the constraints become equality, i.e. the received SINR of all users are equal to the prescribed threshold. Problem (2.12) is a quadratic optimization problem with non-convex quadratic constraints. Thus it needs to be transformed into convex form to be solved by standard optimization tools. In [29], the authors make use of semidefinite relaxation to give the solution. Define $\mathbf{W}_i = \mathbf{w}_i\mathbf{w}_i^H$, the semidefinite relaxation form of Problem is written as

$$\begin{aligned} \min_{\mathbf{W}_k} \quad & \sum_{k=1}^{K} \text{Tr}[\mathbf{W}_k] \\ \text{subject to} \quad & \text{Tr}[\mathbf{R}_{kc(k)}\mathbf{W}_k] - \gamma_k \sum_{\ell=1;\ell\neq k}^{K} \text{Tr}[\mathbf{R}_{kc(\ell)}\mathbf{W}_\ell] = \gamma_k\sigma_k^2 \\ & \mathbf{W}_k = \mathbf{W}_k^H \\ & \mathbf{W}_k \succeq 0 \qquad k = 1, \dots, K, \end{aligned} \tag{2.13}$$

This relaxation transformation can only yield a lower bound of the real optimum solution of original Problem , because the resulting solution of matrices $\{\mathbf{W}_k\}_{k=1}^K$ will possibly have larger than 1 rank. However, from the definition that $\mathbf{W}_k = \mathbf{w}_k\mathbf{w}_k^H$, the rank of the obtained the matrix $\mathbf{W}_k$ should be 1. In this case, those vectors $\mathbf{u}_j = \text{span}[\mathbf{w}_j](\|\mathbf{u}_j\|_2 = 1, j = 1, ..., K)$ will be chosen as the desired weight vector, through which powers can be obtained [29].

We can see that this method treats the matrices $\{\mathbf{W}_k\}_{k=1}^K$ as optimization variables which involve $N \times N \times K$(N is the number of antenna elements, K is the number of users) complex numbers to design. Also, the approximation of rank 1 matrix will result in inaccurate solution.

If weight vectors are calculated and normalized in advance through 2.8 and powers are represented as $\{P_k\}_{k=1}^K$. Under this condition, we can consider a power control optimization problem w.r.t. $\{P_k\}_{k=1}^K$. The goal of power control is to find the transmission powers $\{P_k\}_{k=1}^K$ ($P_k > 0$) that minimize the total transmission power while a certain required QoS is guaranteed for each user. The formulation of the power control problem is written as:

$$\begin{aligned} \min_{P_k} \quad & \sum_{k}^{K} P_k \\ \text{subject to} \quad & \text{SINR}_k \geq \gamma_k \\ & P_k > 0, \qquad k = 1, \ldots, K, \end{aligned} \tag{2.14}$$

where SINR_k refers to 2.6 and γ_k is a prescribed QoS threshold for the kth user. In [35] it is proved that the total transmission power is minimized when the inequality constraints (2.14) become equalities . The minimum transmission power can be obtained by balancing all the received SINR's to the prescribed QoS threshold. Substituting 2.6 into the QoS constraint in (2.14), the constraint can be rewritten as

$$P_k \frac{\mathbf{w}_k^H \tilde{\mathbf{R}}_{k,c(k)} \mathbf{w}_k}{\gamma_k} - \sum_{\ell=1;\ell \neq K}^{K} P_l \mathbf{w}_l^H \tilde{\mathbf{R}}_{k,c(\ell)} \mathbf{w}_\ell = \sigma_k^2 \qquad \text{for} \quad k = 1, \ldots, K \tag{2.15}$$

and (2.15) can be expressed as the following matrix equation

$$\mathbf{\Omega} \mathbf{p}_t = \mathbf{n}, \tag{2.16}$$

where $\mathbf{p}_t = [P_1, \ldots, P_K]^T$ and $\mathbf{n} = [\sigma_1^2, \ldots, \sigma_K^2]^T$ are the $K \times 1$ vectors of the transmission and the noise powers, respectively, and the transition matrix $\mathbf{\Omega}$ is defined as

$$[\mathbf{\Omega}]_{i,j} = \begin{cases} \mathbf{w}_i^H \tilde{\mathbf{R}}_{i,c(i)} \mathbf{w}_i / \gamma_i & \text{for} \quad i = j \\ -\mathbf{w}_j^H \tilde{\mathbf{R}}_{i,c(j)} \mathbf{w}_j & \text{for} \quad i \neq j \end{cases} \tag{2.17}$$

where $[\cdot]_{i,j}$ denotes the (i,j)th element of a matrix. The notation $(\cdot)^T$ stands for the transpose. Using (2.16), the transmission powers can be obtained by

$$\mathbf{p}_t = \mathbf{\Omega}^{-1} \mathbf{n}. \tag{2.18}$$

Since all transmission power P_k for $k = 1, \ldots, K$ must be positive, the positivity of all P_k has to be checked. In the case that $P_k \leq 0$ for some values of k, the underlying problem is then infeasible. In order to make the problem feasible, we should decrease the required QoS (i.e. decrease γ_k), or reduce the number of users (i.e. remove some users) in the network.

Comparing with the method in problem 2.13, the second method from equation 2.15 is more computationally efficient, since a close form solution can be obtained. However, both methods assume accurate estimate of channel information, i.e. channel correlation matrices $\{\mathbf{R}_{kc(\ell)}\}_{k=1;\ell=1}^{K}$. It is impractical in reality to obtain precise downlink channel information as we have discussed in subsection 2.2.2 and subsection 2.3.2. Thus the obtained power allocation solution may not be good enough to satisfy QoS requirement.

2.4.3 Robust Downlink Power Control and Beamforming

Obtaining an accurate estimate of the downlink channel information can be very expensive in practice. The downlink channel estimate is inaccurate due to local scattering around the users [43,44], limited duration of training [18,37], and/or outdated channel knowledge, etc. [38,39]. Owing to the fact that the performance of downlink power control techniques is highly dependent on the channel information, a serious problem associated with downlink power control techniques is that their performance can degrade substantially when even slight mismatch exists between the actual and presumed channel knowledge. As a result, downlink power control designs that improve the robustness against the channel uncertainty are desirable [1,34,38].

In [1], the robust downlink beamforming is introduced to improve the robustness against uncertainties in the channel estimates. It is assumed that the *actual* DCC matrix $\mathbf{R}_{km}$ is bounded as

$$0 \preceq \mathbf{R}^{\mathrm{l}}_{km} \preceq \mathbf{R}_{km} \preceq \mathbf{R}^{\mathrm{u}}_{km} \tag{2.19}$$

where $\mathbf{R}^{\mathrm{l}}_{km}$ and $\mathbf{R}^{\mathrm{u}}_{km}$ are the *upper* and *lower bounds* of the actual DCC matrix $\mathbf{R}_{km}$. The received SINR for the kth user is then defined as

$$\mathrm{SINR}^{\mathrm{LB1}}_k = \frac{\mathbf{w}_k^H \mathbf{R}^{\mathrm{l}}_{kc(k)} \mathbf{w}_k}{\sigma_k^2 + \sum_{\ell=1;\ell\neq k}^{K} \mathbf{w}_\ell^H \mathbf{R}^{\mathrm{u}}_{kc(\ell)} \mathbf{w}_\ell}. \tag{2.20}$$

Note that (2.20) is referred as a *lower bound* received SINR because (2.20) is the ratio of the minimum signal power (because the lower bound of the DCC matrix $\mathbf{R}^{\mathrm{l}}_{kc(k)}$ is in the numerator) to the maximum power of interference (and because upper bounds of the DCC matrices $\mathbf{R}^{\mathrm{u}}_{k,c(\ell)}$ are in the denomiator). The optimal transmission power which guarantees the worst-case (lower bound) SINR to be above a certain threshold γ_k is obtained by solving the following optimization problem

$$\begin{aligned} \min_{\mathbf{w}_k} \quad & \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{w}_k \\ \text{subject to} \quad & \mathbf{w}_k^H \mathbf{R}^{\mathrm{l}}_{kc(k)} \mathbf{w}_k - \gamma_k \sum_{\ell=1;\ell\neq k}^{K} \mathbf{w}_\ell^H \mathbf{R}^{\mathrm{u}}_{k,c(\ell)} \mathbf{w}_\ell \geq \gamma_k \sigma_k^2 \qquad k = 1, \ldots, K, \end{aligned} \tag{2.21}$$

which can be converted to a SDP problem similarly as Problem 2.13. Note that (2.21) can be used to determine the transmission powers which guarantee the lower bound SINR. This method will cost more power than necessary and is also computing demanding by solving the formulated SDP problem. Moreover, this robust downlink beamforming requires the information of all the DCC matrices by solving Problem (2.21. As a consequence, a high communication rate between the system BSs and mobile users is needed in order to solve (2.21).

In order to design more efficient robust techniques, the uncertainty in DCC matrices should be explicitly considered as a parameter during the design process. Another approach to robust downlink power control is proposed in [35]. For practical situations in which the presumed DCC matrix is uncertain, the actual DCC matrix can be written as as [1, 25, 38]

$$\mathbf{R}_{km} = \tilde{\mathbf{R}}_{km} + \mathbf{E}_{km} \tag{2.22}$$

for $k = 1, ..., K$ and $m = 1, ..., M$,where $\tilde{\mathbf{R}}_{km}$ is the presumed DCC matrix between the mth BS and the kth user and $\{\mathbf{E}_{km}\}_{k=1,m=1}^{K}, M$ is the unknown DCC matrix uncertainty. There are totally $K \times M$ uncertainty matrices $\mathbf{E}_{km}$. Using the idea similar to that in [39, 41], the Frobenius norm of the uncertainty matrix $\mathbf{E}_{km}$ is bounded by some known constant $\bar{\varepsilon} > 0$ such that

$$\|\mathbf{E}_{km}\|_F \leq \bar{\varepsilon}\|\tilde{\mathbf{R}}_{km}\|_F \triangleq \varepsilon_{km} \tag{2.23}$$

where ε_{km} determines the maximal expected amount of the uncertainty matrix. Also, different from previous method, this method explicitly considers individual power and normalized beamforming weight vectors as in (2.14). In the presence of downlink channel errors, the received SINR of the kth user is defined by

$$\xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}) \triangleq \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)})\mathbf{w}_k}{\sigma_k^2 + \sum_{\ell=1;\ell\neq k}^{K} P_\ell \mathbf{w}_\ell^H(\tilde{\mathbf{R}}_{kc(\ell)} + \mathbf{E}_{kc(\ell)})\mathbf{w}_\ell}$$

In order to improve the robustness against unknown but norm-bounded DCC matrix uncertainties, the QoS constraints are required to be satisfied for the worst-case SINR for each user i.e. for $k = 1, \ldots, K$, the minimum value of SINR is no less than the required threshold:

$$\min_{\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}} \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{k,c(k)} + \mathbf{E}_{kc(k)})\mathbf{w}_k}{\sigma_k^2 + \sum_{\ell=1;\ell\neq k}^{K} P_l \mathbf{w}_l^H(\tilde{\mathbf{R}}_{kc(\ell)} + \mathbf{E}_{kc(\ell)})\mathbf{w}_\ell} \geq \gamma_k \tag{2.24}$$

where the norms of all $\mathbf{E}_{kc(j)}$ $(j = 1, \ldots, K)$ are bounded according to (2.23). Notice in the case that user k and ℓ are in the same cell, $c(k) = c(\ell)$ and $\mathbf{E}_{kc(k)} = \mathbf{E}_{kc(\ell)}$. This means the numerator and the denominator of left hand side of (2.24) will share the same optimization variable $\mathbf{E}_{kc(k)} or \mathbf{E}_{kc(\ell)}$ for any $k \neq \ell$.This fraction form means that (2.24) is a *non-convex* problem. However, independently maximizing and minimizing the denominator and numerator, respectively, may only yield a lower bound of the worst case SINR instead of the worst-case SINR. Then the QoS constraints will be strengthened by replacing the worst-case SINR by its lower bound in each of them [35,36],

$$\mathrm{SINR}_k^{\mathrm{LB2}} = \frac{P_k \mathbf{w}_k^H (\tilde{\mathbf{R}}_{kc(k)} - \varepsilon_{kc(k)} \mathbf{I}) \mathbf{w}_k}{\sigma_k^2 + \sum_{l=1; l \neq k}^{K} P_l \mathbf{w}_l^H (\tilde{\mathbf{R}}_{kc(l)} + \varepsilon_{kc(l)} \mathbf{I}) \mathbf{w}_l}. \tag{2.25}$$

Note that (2.24) is now lower-bounded by (2.25) and it is called the lower bound SINR. The transmission power required for the lower bound SINR to satisfy the QoS constraint can be obtained by modifying (2.14) by replacing the SINR_k by $\mathrm{SINR}_k^{\mathrm{LB2}}$. However, this method which replaces the worst-case SINR by its lower bound for computing the transmission powers may give an unnecessarily pessimistic solution because the lower bound is extremely unlikely in practice. Furthermore, another deficiency associated with the method lies in the fact that the constraint $\tilde{\mathbf{R}}_{k,m} + \mathbf{E}_{k,m} \succeq 0$ has not been included in (2.24). As a result, the DCC matrices in (2.25) may not result in PSD DCC matrices.

Chapter 3

An Efficient Design for Robust Downlink Power Control

3.1 Introduction and Problem Formulation

As we discussed in the last two sections of Chapter 1, non-robust downlink power control and beamforming do not consider the uncertainty of channel information, which will result in inaccurate power designs. Thus robust designs taking channel uncertainty into account is required. However, in some existing robust designs [1,35, 36], constraining lower bounds of SINR to be no less than the QoS threshold, the robust design schemes cost more power than necessary. Besides, SDP formulation in [35] is not computationally efficient by treating symmetric matrices as optimization variables. Therefore, a more efficient and accurate downlink power control scheme is needed. In this chapter, a novel robust downlink power control algorithm based on worst-case SINR optimization is developed. The goal is to determine the minimum amount of total transmission power required to ensure that the QoS constraints are satisfied for all users under the uncertain channel conditions. This original problem involves both uncertainty matrices and powers as variables, thus it is complicated and hard to transform into convex form. To get around the difficulty of convex transformation, an iterative procedure treating uncertainty matrices and powers in separate steps is introduced first. The core of the iteration is the step to obtain worst-case uncertainty matrices, which is still a non-convex complicated optimization problem. Through our analysis, both analytic solution and convex transformation are given for this worst-case uncertainty problem under different conditions.

We formulate the channel uncertainty in the same way as in (2.22) and (2.23). Therefore, it requires the received SINR to be greater than the QoS threshold in the presence of an arbitrary, but bounded in norm, unknown channel uncertainty. That is, in the presence of downlink channel errors, the received SINR of the kth user is required to satisfy

$$\xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}) \triangleq \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)})\mathbf{w}_k}{\sigma_k^2 + \sum_{l=1;l\neq k}^{K} P_l \mathbf{w}_l^H(\tilde{\mathbf{R}}_{kc(l)} + \mathbf{E}_{kc(l)})\mathbf{w}_l} \geq \gamma_k \tag{3.26a}$$

$$\text{for all} \quad \|\mathbf{E}_{kc(j)}\|_F \leq \varepsilon_{kc(j)} \tag{3.26b}$$

$$\text{such that} \quad \tilde{\mathbf{R}}_{kc(j)} + \mathbf{E}_{kc(j)} \succeq 0, \quad j = 1, \ldots, K \tag{3.26c}$$

where $\mathbf{E}_{km}$ is the uncertainty in DCC matrix. Note that DCC constraints (3.26c) are explicitly imposed to ensure that the resulting DCC matrices (i.e., with the uncertainty) are PSD. Note that to satisfy (3.26a) , the minimum of RHS must satisfy the constraint, therefore (3.26a) is equivalent to the worst-case QoS constraint i.e. $\min_{\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}} \xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}) \geq \gamma_k$. Therefore, our formulation of robust downlink power control problem is written as:

Problem 1

$$\begin{aligned} \min_{\{P_k\}_{k=1}^{K}, \{\mathbf{E}_{kc(j)}\}_{k,j=1}^{K}} \quad & \sum_{k=1}^{K} P_k \\ \textit{subject to} \quad & \min_{\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}} \xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}) \geq \gamma_k \\ & \|\mathbf{E}_{kc(j)}\|_F \leq \varepsilon_{kc(j)} \\ & \tilde{\mathbf{R}}_{kc(j)} + \mathbf{E}_{kc(j)} \succeq 0 \end{aligned}$$

where $j, k = 1, ..., K$ and we assume that the normalized beamforming weight vectors $\{\mathbf{w}\}_{k=1}^{K}$ are calculated in advance. Note that, both the uncertainty matrices $\{\mathbf{E}_{kc(j)}\}_{k,j=1}^{K}$ and powers $\{P_k\}_{k=1}^{K}$ are the optimization variables to obtain. Our main task in the following section is to solve optimization Problem 1.

3.2 Iterative Algorithm

Problem 1 involves two groups of variables, the user power $\{P_k\}_{k=1}^{K}$ and uncertainty matrices $\{\mathbf{E}_{k,c(j)}\}_{j=1}^{K}$. Particularly, the worst-case QoS constraint in this problem requires to get the representation of minimum SINR, i.e. $\min_{\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}} \xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^{K}) \geq \gamma_k$ first, which is hard to achieve. Moreover, the fraction form is hard to transform to convex form too. Due to this obstacle presented by the QoS constraint, Problem 1 is difficult to be straightforwardly solved by dealing with two groups of variables. However, notice that in the QoS constraint, if we assume, for some reason, uncertainty matrices have been obtained in advance, power allocation could be linearly computed similarly with Equation 2.18. And if powers have been obtained in advance, uncertainty matrices could also be calculated in an easier way. This broaches the idea of iteratively solving the optimization Problem 1. To overcome this, we propose an algorithm to determine transmission power allocation $\{P_k\}_{k=1}^K$ and worst-case uncertainty matrices $\{\mathbf{E}_{kc(j)}\}_{j=1}^K$ iteratively, which captures the following two steps:

1. *Find the worst-case uncertainty matrices.* Given the transmission powers $\{P_k\}_{k=1}^K$, the worst-case uncertainty matrices of the kth user can be found by solving

   Problem 2

$$\min_{\{\mathbf{E}_{kc(j)}\}_{j=1}^K} \quad \xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^K) \tag{3.27}$$

$$\textit{subject to} \quad \|\mathbf{E}_{kc(j)}\|_F \leq \varepsilon_{kc(j)} \tag{3.28}$$

$$\tilde{\mathbf{R}}_{kc(j)} + \mathbf{E}_{kc(j)} \succeq 0, \qquad j = 1, \ldots, K. \tag{3.29}$$

2. *Find the transmission powers.* Once we have obtained the worst-case uncertainty matrices in Step 1, we make use of the relationship between the transmitted power and the uncertainty matrices from equation (2.16)

$$\mathbf{p} = \mathbf{\Phi}^{-1}\boldsymbol{\sigma} \tag{3.30}$$

   This is obtained by equating the worst-case SINR to the prescribed QoS threshold $\text{SINR}_k^{\text{wc}} \triangleq \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{kc(k)}+\mathbf{E}_{kc(k)}^{\text{wc}})\mathbf{w}_k}{\sigma_k^2+\sum_{\ell=1;\ell\neq k}^K P_\ell \mathbf{w}_\ell^H(\tilde{\mathbf{R}}_{kc(\ell)}+\mathbf{E}_{kc(\ell)}^{\text{wc}})\mathbf{w}_\ell} = \gamma_k$ for $k = 1, ..., K$, where $\mathbf{p} = [P_1, \ldots, P_K]^T$, $\boldsymbol{\sigma} = [\sigma_1^2, \ldots, \sigma_K^2]^T$ and the (ij)th entry of matrix $\mathbf{\Phi}$ is determined by

$$[\mathbf{\Phi}]_{ij} = \begin{cases} \mathbf{w}_i^H(\tilde{\mathbf{R}}_{ic(i)} + \mathbf{E}_{ic(i)}^{\text{wc}})\mathbf{w}_i/\gamma_i & \text{for} \quad i = j \\ -\mathbf{w}_j^H(\tilde{\mathbf{R}}_{ic(j)} + \mathbf{E}_{ic(j)}^{\text{wc}})\mathbf{w}_j & \text{for} \quad i \neq j \end{cases} \tag{3.31}$$

Therefore, the above iterative method for alternatively finding the worst-case uncertainty matrices and the transmission powers can be summarized as the following algorithm:

Algorithm 1 *(robust downlink power control algorithm)*

1. *Initialize power vector* $\mathbf{p}$ *using (3.30) with* $\mathbf{E}^{\mathrm{wc}}_{kc(j)}(1) = \mathbf{0}$ *for* $i, j = 1, \cdots, K$.

2. *Compute the worst-case uncertainty matrices* $\{\mathbf{E}^{\mathrm{wc}}_{kc(j)}(n+1)\}_{j,k=1}^{K}$ *with* $\{P_k\}_{k=1}^{K} = \{P_k(n)\}_{k=1}^{K}$ *by solving optimization Problem 2.*

3. *Update the transmission powers* $\{P_k(n+1)\}_{k=1}^{K}$ *using Eq. (3.30) with* $\{\mathbf{E}_{kc(j)}\}_{j,k=1}^{K}$ *being equal to the optimal* $\{\mathbf{E}^{\mathrm{wc}}_{kc(j)}(n+1)\}_{j,k=1}^{K}$ *obtained in step 2).*

4. *The iteration stops if* $|\sum_{k=1}^{K} P_k(n) - P_k(n+1)| \leq \delta$ *is satisfied, where* δ *is some prescribed small positive number. Otherwise, return to Step 2).*

Problem 2 has a fractional function(3.27) as the objective while subject to second-order cone(SOC)(3.28) and linear matrix inequality(LMI)(3.29) constraints. Thus Problem 2 is still not convex. In the following subsections, we will discuss how to efficiently solve this non-convex problem.

3.3 Solution to Worst-Case Uncertainty Problem

As we have noticed, Problem 2 is specifically for kth user, i.e., k is given. Thus we need to solve K such problems for $k = 1, ..., K$ using the same method. Suppose that there are L_k users indexed by $j_{k_1}(j_{k_1} \neq k,\ k_1 = 1, ..., L_k)$ belonging to the same BS $c(k)$ as kth user. Then, there are $J_k = K - L_k - 1$ users indexed by $j_{k_2}, k_2 = 1, ..., J_k$ belonging to the other BSs. Therefore, $\mathbf{E}_{kc(j_{k_1})} = \mathbf{E}_{kc(k)}$. Accordingly, we can partition the summation in the denominator in two parts i.e. the received SINR_k can be rewritten as

$$\begin{aligned}\xi_k(\{\mathbf{E}_{kc(j)}\}_{j=1}^K) &= \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)})\mathbf{w}_k}{\sigma_k^2 + \Sigma_{\ell=1;\ell\neq k}^K P_\ell \mathbf{w}_\ell^H(\tilde{\mathbf{R}}_{kc(\ell)} + \mathbf{E}_{kc(\ell)})\mathbf{w}_\ell} \\ &= \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{k,c(k)})\mathbf{w}_k}{\sigma_k^2 + \eta_1 + \eta_2}\end{aligned}$$

where $\eta_1 = \Sigma_{k_1=1}^{L_k} P_{j_{k_1}} \mathbf{w}_{j_{k_1}}^H (\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)})\mathbf{w}_{j_{k_1}}$ which only involves uncertainty matrix $\mathbf{E}_{kc(k)}$ causing intra-cell interference and $\eta_2 = \Sigma_{k_2=1}^{J_k} P_{j_{k_2}} \mathbf{w}_{j_{k_2}}^H (\tilde{\mathbf{R}}_{kc(j_{k_2})} + \mathbf{E}_{kc(j_{k_2})})\mathbf{w}_{j_{k_2}}$ which involves uncertainty matrices $\mathbf{E}_{kc(j_{k_2})}(k_2 = 1, ..., J_k)$ causing inter-cell interference. Since the variables related to η_1 and η_2 originate from two different channel sets $\{(kc(k))\}$ and $\{(kc(j_{k2}))\}_{k2=1;j_{k2}\neq k}^{J_k}$, η_1 and η_2 can be separately considered without affecting each other. Notice that for the special case that there is single user in a cell, i.e., L_k =0 and $\eta_1 = 0$. Problem 2 of minimizing SINR_k can be solved by firstly maximizing η_2 and then minimizing SINR_k with η_2 replaced by its maximum value. This is realized by solving the following two subproblems.

Subproblem 1: Solve the following optimization problem:

$$\begin{aligned}\max_{\{\mathbf{E}_{kc(j_{k_2})}\}_{k_2=1}^{J_k}} \eta_2 &= \sum_{k_2=1}^{J_k} P_{j_{k_2}} \mathbf{w}_{j_{k_2}}^H (\tilde{\mathbf{R}}_{kc(j_{k_2})} + \mathbf{E}_{kc(j_{k_2})})\mathbf{w}_{j_{k_2}} \\ \text{subject to} \quad & \|\mathbf{E}_{kc(j_{k_2})}\|_F \leq \varepsilon_{kc(j_{k_2})} \\ & \tilde{\mathbf{R}}_{kc(j_{k_2})} + \mathbf{E}_{kc(j_{k_2})} \succeq 0\end{aligned} \tag{3.32}$$

for any fixed integer k $(1 \leq k \leq K)$. This is again equivalent to solving the following J_k problems treating each item in the summation of the objective function (3.32) :

Problem 3

$$\begin{aligned} \max_{\mathbf{E}_{kc(j_{k_2})}} \quad & \mathbf{w}_{j_{k_2}}^H(\tilde{\mathbf{R}}_{kc(j_{k_2})} + \mathbf{E}_{kc(j_{k_2})})\mathbf{w}_{j_{k_2}} \\ \textit{subject to} \quad & \|\mathbf{E}_{kc(j_{k_2})}\|_F \leq \varepsilon_{kc(j_{k_2})} \\ & \tilde{\mathbf{R}}_{kc(j_{k_2})} + \mathbf{E}_{kc(j_{k_2})} \succeq 0 \end{aligned}$$

for $k_2 = 1, ..., J_k$.

The reason the power $P_{j_{k_2}}$ is taken out of the objective function is that it is a constant when only considering matrices $\mathbf{E}_{kc(j_{k_2})}$ as optimization variable. Using similar method as in [34], we consider the following problem:

$$\max_{\mathbf{E}} \mathbf{w}^H(\mathbf{R} + \mathbf{E})\mathbf{w} \text{ subject to } \|\mathbf{E}\|_F \leq \varepsilon \tag{3.33}$$

Employing Lagrange multiplier method, the maximum value of (3.33), $\mathbf{w}^H(\mathbf{R} + \varepsilon\mathbf{I})\mathbf{w}$ is attained when

$$\mathbf{E} = \varepsilon\frac{\mathbf{w}\mathbf{w}^H}{\|\mathbf{w}\|^2} = \varepsilon\mathbf{w}\mathbf{w}^H \tag{3.34}$$

The solution of (3.33) applies to Problem 3. Therefore, the closed form solution to Problem 3 is given by $\mathbf{E}_{kc(j_{k_2})} = \varepsilon_{kc(j_{k_2})}\mathbf{w}_{j_{k_2}}\mathbf{w}_{j_{k_2}}^H$. Define

$$\beta_k = \max_{\{\mathbf{E}_{kc(j_{k_2})}\}_{k_2=1}^{J_k}} \eta_2 = \Sigma_{k_2=1}^{J_k} P_{j_{k_2}}\mathbf{w}_{j_{k_2}}^H(\tilde{\mathbf{R}}_{kc(j_{k_2})} + \varepsilon_{kc(j_{k_2})}\mathbf{I})\mathbf{w}_{j_{k_2}} \tag{3.35}$$

Attention should be taken here that the solutions of $\mathbf{E}_{kc(j_{k_2})}$ are independent of $\{P_k\}_{k=1}^K$ and will remain the same in each iteration of the problem. Considering all of the subproblems, there are in total $K \times (M-1)$ such uncertainty matrices the solutions of which remain unchanged during each iteration(this could be part of reason of the quick convergence of the iteration in later chapter).

Subproblem 2: For *the single user case*, Subproblem 2 as an equivalent transformation of Problem 2:

Problem 4

$$\begin{aligned} \min_{\mathbf{E}_{kc(k)}} \quad & \frac{P_k \mathbf{w}_k^H (\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)}) \mathbf{w}_k}{\sigma_k^2 + \beta_k} \\ \textit{subject to} \quad & \|\mathbf{E}_{kc(k)}\|_F \leq \varepsilon_{kc(k)} \\ & \tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)} \succeq 0 \end{aligned}$$

This is equivalent to solving the following problem:

$$\begin{aligned} \min_{\mathbf{E}_{kc(k)}} \quad & \mathbf{w}_k^H (\tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)}) \mathbf{w}_k \\ \text{subject to} \quad & \|\mathbf{E}_{kc(k)}\|_F \leq \varepsilon_{kc(k)} \\ & \tilde{\mathbf{R}}_{kc(k)} + \mathbf{E}_{kc(k)} \succeq 0 \end{aligned}$$

This is a reciprocal problem of Problem 3 and therefore the same method applies. The solution of $\mathbf{E}_{kc(k)}$ to Problem (4) is

$$\mathbf{E}_{kc(k)} = -\varepsilon_{kc(k)} \frac{\mathbf{w}_k \mathbf{w}_k^H}{\|\mathbf{w}_k\|^2} = -\varepsilon_{kc(k)} \mathbf{w}_k \mathbf{w}_k^H \tag{3.36}$$

This results in the objective of Problem (4) equal to:

$$\frac{P_k \mathbf{w}_k^H (\tilde{\mathbf{R}}_{kc(k)} - \varepsilon_{kc(k)} \mathbf{I}) \mathbf{w_k}}{\sigma_k^2 + \beta_k}$$

In conclusion, for single user case, through solving subproblem 1 and Problem (4), the solution of uncertainty matrices Problem (2) i.e. $\mathbf{E}_{kc(j_{k_2})} = \varepsilon_{kc(j_{k_2})} \mathbf{w}_{j_{k_2}} \mathbf{w}_{j_{k_2}}^H (k_2 = 1, ..., J_k)$ and $\mathbf{E}_{kc(k)} = -\varepsilon_{kc(k)} \mathbf{w}_k \mathbf{w}_k^H$.

For *more than one user case*, Subproblem 2 as an equivalent transformation of Problem 2 turns into:

Problem 5

$$\begin{aligned} \min_{\mathbf{E}_{kc(k)}} \quad & \frac{P_k \mathbf{w}_k^H(\tilde{\mathbf{R}}_{kc(k)}+\mathbf{E}_{kc(k)})\mathbf{w}_k}{\sigma_k^2+\beta_k+\Sigma_{k_1=1}^{L_k} P_{jk_1}\mathbf{w}_{jk_1}^H(\tilde{\mathbf{R}}_{kc(k)}+\mathbf{E}_{kc(k)})\mathbf{w}_{jk_1}} \\ \textit{subject to} \quad & \|\mathbf{E}_{kc(k)}\|_F \leq \varepsilon_{kc(k)} \\ & \tilde{\mathbf{R}}_{kc(k)}+\mathbf{E}_{kc(k)} \succeq 0 \end{aligned}$$

Making use of the property that $\mathbf{w}_k^H\mathbf{E}_{kc(k)}\mathbf{w}_k = \mathrm{Tr}(\mathbf{E}_{kc(k)}(\mathbf{w}_k\mathbf{w}_k^H))$, Problem 5 is equivalent to the following optimization problem:

$$\begin{aligned} \min_{\mathbf{E}_{kc(k)}} \quad & \frac{c_{1k}+\mathrm{Tr}(\mathbf{E}_{kc(k)}\mathbf{A}_k)}{c_{2k}+\mathrm{Tr}(\mathbf{E}_{kc(k)}\mathbf{B}_k)} \\ \text{subject to} \quad & \|\mathbf{E}_{kc(k)}\|_F \leq \varepsilon_{kc(k)} \\ & \tilde{\mathbf{R}}_{kc(k)}+\mathbf{E}_{kc(k)} \succeq 0 \end{aligned} \tag{3.37}$$

where $\mathbf{A}_k = \mathbf{w}_k\mathbf{w}_k^H$, $\mathbf{B}_k = \Sigma_{k_1=1}^{L_k} P_{jk_1}\mathbf{w}_{jk_1}\mathbf{w}_{jk_1}^H$, $c_{1k} = \mathrm{Tr}(\tilde{\mathbf{R}}_{kc(k)}\mathbf{A}_k)$, $c_{2k} = \mathrm{Tr}(\tilde{\mathbf{R}}_{kc(k)}\mathbf{B}_k)+$ $\sigma_k^2+\beta_k$. Then we will introduce real matrices $\hat{\mathbf{E}}_{kc(k)}$, $\hat{\mathbf{R}}_{kc(k)}$, $\hat{\mathbf{A}}_k$ and $\hat{\mathbf{B}}_k$ to get around the operation with complex vectors and matrices. The presumed DCC matrix $\tilde{\mathbf{R}}_{k,c(k)}$ is correspondingly mapped to:

$$\hat{\mathbf{E}}_{k,c(k)} = \frac{1}{\sqrt{2}}\begin{bmatrix} \Re(\mathbf{E}_{k,c(k)}) & -\Im(\mathbf{E}_{k,c(k)}) \\ \Im(\mathbf{E}_{k,c(k)}) & \Re(\mathbf{E}_{k,c(k)}) \end{bmatrix} \tag{3.38}$$

where $\Re(\mathbf{E}_{k,c(k)})$ is the real part of matrix $\hat{E}_{k,c(k)}$ and $\Im(\mathbf{E}_{k,c(k)})$ is the imaginative part of matrix $\hat{E}_{k,c(k)}$. Similarly we get $\hat{\mathbf{R}}_{k,c(k)}$, $\hat{\mathbf{A}}_k$ and $\hat{\mathbf{B}}_k$. It is straightforward to obtain that $\mathrm{Tr}(\hat{\mathbf{E}}_{k,c(k)}\hat{\mathbf{A}}_k) = \mathrm{Tr}(\mathbf{E}_{k,c(k)}\mathbf{A}_k)$. And $\tilde{\mathbf{R}}_{k,c(j)}+\mathbf{E}_{k,c(j)} \succeq 0$ is equivalent to $\hat{\mathbf{R}}_{k,c(k)}+\hat{\mathbf{E}}_{k,c(k)} \succeq 0$. Since $\|\mathbf{E}\|_F = (\sum_{i=1}^N\sum_{j=1}^N |e_{ij}|^2)^{1/2}$, it is ready to get that

$\left\|\hat{\mathbf{E}}\right\|_F = \|\mathbf{E}\|_F$. Thus $\left\|\mathbf{E}_{kc(j)}\right\|_F \leq \varepsilon_{kc(j)}$ is equivalent to $\left\|\hat{\mathbf{E}}_{kc(j)}\right\|_F \leq \varepsilon_{kc(j)}$. Then (3.37) turns into:

Problem 6

$$\begin{aligned} \min_{\hat{\mathbf{E}}_{kc(k)}} \quad & \frac{c_{1k}+Tr(\hat{\mathbf{E}}_{kc(k)}\hat{\mathbf{A}}_k)}{c_{2k}+Tr(\hat{\mathbf{E}}_{kc(k)}\hat{\mathbf{B}}_k)} \\ \textit{subject to} \quad & \left\|\hat{\mathbf{E}}_{kc(k)}\right\|_F \leq \varepsilon_{kc(k)} \\ & \hat{\mathbf{R}}_{kc(k)} + \hat{\mathbf{E}}_{kc(k)} \succeq 0 \end{aligned}$$

Below we introduce notation $vec()$ and notation $mat()$ to simplify (6). Assume $M^2 \times 1$ vector $\mathbf{x} = [x_{11}, ...x_{M1}, x_{12}...x_{M2}...x_{MM}]$ and $M \times M$ matrix $\mathbf{X}$,

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{M1} & x_{M2} & \cdots & x_{MM} \end{bmatrix} \tag{3.39}$$

Then notation $vec(\mathbf{X})$ denotes a vector by stacking the columns matrix $\mathbf{X}$ and $mat(\mathbf{x})$ denotes an $M \times M$ square matrix by filling it with the entries of the vector $\mathbf{x}$. Define $\mathbf{a} = vec(\hat{\mathbf{A}}_k)$,$\mathbf{b} = vec(\hat{\mathbf{B}}_k)$ and $\mathbf{e} = vec(\hat{\mathbf{E}}_{kc(k)})$. Notice that we have the property that $\mathbf{a}^H\mathbf{e} = \mathrm{Tr}(\hat{\mathbf{E}}_{k,c(k)}\hat{\mathbf{A}})$. For notational simplicity, we drop the subscripts and rewrite Problem 6 in terms of vectors as

Problem 7

$$\begin{aligned} \min_{\mathbf{e}} \quad & \frac{c_1+\mathbf{a}^H\mathbf{e}}{c_2+\mathbf{b}^H\mathbf{e}} \\ \textit{subject to} \quad & \mathbf{e}^H\mathbf{e} \leq \varepsilon^2 \\ & \hat{\mathbf{R}} + mat(\mathbf{e}) \succeq 0 \end{aligned}$$

With a fractional objective, the optimization Problem 7 is not convex. In the following we will reformulate it so that its solution can be efficiently obtained.

3.3.1 Closed Form Solution of Problem 7

When the presumed DCC matrix $\hat{\mathbf{R}}_{kc(k)}$ is positive definite and ε is small enough, the PSD constraint in Problem 7 will be naturally met. In the lemma below, we will give a sufficient condition regarding ε to ensure the PSD constraint is satisfied and thus can be excluded from Problem 7.

Lemma 1 *When $\hat{\mathbf{R}}_{kc(k)}$ is positive definite, define $\lambda_{\min}(\mathbf{R})$ as the smallest eigenvalue of $\hat{\mathbf{R}}_{kc(k)}$, then $\varepsilon \leq \lambda_{\min}(\mathbf{R})$ is a sufficient condition to ensure the PSD constraint is satisfied.*

Proof: Assume the eigenvalues of $\hat{\mathbf{E}}_{kc(k)}$ are $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_N$, where N is the number of antenna elements in the antenna array. It is straightforward that $\lambda_1 \geq -\max|\lambda_i|(i = 1, ..., N)$

Therefore we have

$$\hat{\mathbf{R}} + \hat{\mathbf{E}}_{kc(k)} \succeq \hat{\mathbf{R}} + \lambda_1 \mathbf{I} \succeq \hat{\mathbf{R}} - \max|\lambda_i|\mathbf{I} \tag{3.40}$$

Due to the fact that $\max|\lambda_i| \leq \sqrt{\mathrm{tr}(\hat{\mathbf{E}}_{kc(k)}^H \hat{\mathbf{E}}_{kc(k)})}$ and $\mathrm{Tr}(\hat{\mathbf{E}}_{kc(k)}^H \hat{\mathbf{E}}_{kc(k)}) \leq \varepsilon^2$ we have

$$\hat{\mathbf{R}}\mathbf{I} - \max\mathbf{I}|\lambda_i| \succeq \hat{\mathbf{R}} - \sqrt{\mathrm{tr}(\hat{\mathbf{E}}_{kc(k)}^H \hat{\mathbf{E}}_{kc(k)})}\mathbf{I} \succeq \hat{\mathbf{R}} - \varepsilon \mathbf{I} \tag{3.41}$$

Therefore it is seen from above that $\hat{\mathbf{R}} - \varepsilon\mathbf{I} \succeq 0$ is sufficient to guarantee $\hat{\mathbf{R}} + \hat{\mathbf{E}}_{kc(k)} \succeq 0$, and on the other hand the condition $\varepsilon \leq \lambda_{\min}(\mathbf{R})$ is sufficient to deduce $\hat{\mathbf{R}} - \varepsilon\mathbf{I} \succeq 0$. Hereby, the above lemma is verified.

Below, we assume Lemma 1 is satisfied and that the condition $c_2^2 - \varepsilon^2\|\mathbf{b}\|_2^2 \neq 0$ is satisfied. Then, Problem 7 can be reduced to

Problem 8

$$\begin{aligned} \min_{\mathbf{e}} \quad & \tfrac{c_1+\mathbf{a}^H\mathbf{e}}{c_2+\mathbf{b}^H\mathbf{e}} \\ \textit{subject to} \quad & \mathbf{e}^H\mathbf{e} \leq \varepsilon^2 \end{aligned}$$

We first consider the case where $\mathbf{a}$ and $\mathbf{b}$ are linearly dependent, i.e., there exists a pair of complex numbers μ_1 and μ_2 one of which is not zero such that $\mu_1\mathbf{a} + \mu_2\mathbf{b} = 0$.

1. If $\mu_1 = 0$ and $\mu_2 \neq 0$, then, we have $\mathbf{b} = \mathbf{0}$, which contradicts with the condition that $\mathbf{B}_k \neq \mathbf{0}$.

2. If $\mu_1 \neq 0$ and $\mu_2 = 0$, then, we obtain $\mathbf{a} = \mathbf{0}$, which contradicts with the condition that $\mathbf{A}_k \neq \mathbf{0}$.

3. If $\mu_1 \neq 0$ and $\mu_2 \neq 0$, then, we have $\mathbf{a} = \mu\mathbf{b}$, where $\mu = -\frac{\mu_2}{\mu_1}$. Substituting this into the objective in Problem 8 leads to $\frac{c_1+\mathbf{a}^H\mathbf{e}}{c_2+\mathbf{b}^H\mathbf{e}} = \mu^* + \frac{c_1-\mu^*c_2}{c_2+\mathbf{b}^H\mathbf{e}}$($\mu^(*)$ is the conjugate value of μ). Therefore, the minimum value is $\mu^* + \frac{c_1-\mu^*c_2}{c_2+\varepsilon\|\mathbf{b}\|_2}$ when $c_1 - \mu^*c_2 \geq 0$ and $\mathbf{e} = \varepsilon\mathbf{b}/\|\mathbf{b}\|_2$, and is $\mu^* + \frac{c_1-\mu^*c_2}{c_2-\varepsilon\|\mathbf{b}\|_2}$when $c_1 - \mu^*c_2 < 0$ and $\mathbf{e} = -\varepsilon\mathbf{b}/\|\mathbf{b}\|_2$.

Therefore, in the following we only need to consider the case where $\mathbf{a}$ and $\mathbf{b}$ are linearly independent. Replacing variables $\mathbf{y}$ and z as $\mathbf{y} = \frac{\mathbf{e}}{c_2+\mathbf{b}^H\mathbf{e}}$ and $z = \frac{1}{c_2+\mathbf{b}^H\mathbf{e}}$. Using the Cauchy-Schwartz Inequality , we have $|\mathbf{b}^H\mathbf{e}|^2 \leq \|\mathbf{b}\|_2^2\|\mathbf{e}\|_2^2 \leq \varepsilon^2\|\mathbf{b}\|_2^2$. Therefore, $\frac{1}{c_2+\varepsilon\|\mathbf{b}\|_2} \leq z \leq \frac{1}{c_2-\varepsilon\|\mathbf{b}\|_2}$. Now, Problem 8 is reformulated into

Problem 9

$$\begin{aligned} \min_{\mathbf{y},z} \quad & c_1 z + \mathbf{a}^H \mathbf{y} \\ \textit{subject to} \quad & c_2 z + \mathbf{b}^H \mathbf{y} = 1 \\ & \mathbf{y}^H \mathbf{y} \leq z^2 \varepsilon^2 \\ & d_1 \leq z \leq d_2 \end{aligned}$$

where $\frac{1}{c_2+\varepsilon\|\mathbf{b}\|_2} = d_1$, $\frac{1}{c_2-\varepsilon\|\mathbf{b}\|_2} = d_2$, $\mathbf{a}$ *and* $\mathbf{b}$ *are linearly independent.*

Although with linear objective and two linear constraints, Problem 9 is still not convex due to the second non-convex quadratic constraint with both $\mathbf{y}$ and z as optimization variables.

Thus it is hard to analytically obtain the optimal solution. However, we can first assume z is fixed, only $\mathbf{y}$ being the variable to be considered. By so doing, we start from the following problem:

$$\begin{aligned} \min_{\mathbf{y}} \quad & c_1 z + \mathbf{a}^T \mathbf{y} \\ \text{subject to} \quad & c_2 z + \mathbf{b}^T \mathbf{y} = 1 \\ & \mathbf{y}^T \mathbf{y} \leq z^2 \varepsilon^2 \end{aligned} \tag{3.42}$$

(3.42) is a simple convex problem with linear objective, one linear equality constraint and one convex quadratic inequality constraint. Geometrically, this is equivalent to finding the minimum inner product between an unknown and a known vector, where the unknown vector belongs to the intersection (a solid round plane) of a plane and a solid ball. Strong duality and Slater's conditions hold for it and thus KKT optimality conditions are satisfied [45]. The Lagrangian of (3.42) is:

$$\begin{aligned} L(\mathbf{y}, v, \lambda)_{\lambda \geq 0} &= (c_1 z + \mathbf{a}^T \mathbf{y}) + v(c_2 z + \mathbf{b}^T \mathbf{y} - 1) + \lambda(\mathbf{y}^T \mathbf{y} - z^2 \varepsilon^2) \\ &= [\lambda \mathbf{y}^T \mathbf{y} + (v\mathbf{b}^T + \mathbf{a}^T)\mathbf{y}] + (v c_2 z - v - \lambda z^2 \varepsilon^2 + c_1 z) \end{aligned} \tag{3.43}$$

where λ and v are Lagrangian multipliers. KKT conditions are:

$$\nabla_{\mathbf{y}} L = 2\lambda \mathbf{y} + (v\mathbf{b} + \mathbf{a}) = 0 \tag{3.44}$$

$$c_2 z + \mathbf{b}^T \mathbf{y} - 1 = 0 \tag{3.45}$$

$$\lambda(\mathbf{y}^T \mathbf{y} - z^2 \varepsilon^2) = 0 \tag{3.46}$$

$$\lambda \geq 0 \tag{3.47}$$

For 3.47, if $\lambda = 0$, combining (3.44), it is required that $v\mathbf{b} + \mathbf{a} = 0$. However, we only consider the case that $\mathbf{b}$ and $\mathbf{a}$ are linear independent. Therefore, $\mathbf{y}^T \mathbf{y} - z^2 \varepsilon^2 = 0$ and $\lambda > 0$, which turns the inequality constraints into equality constraint. Therefore Problem 9 can be equivalently transformed into:

Problem 10

$$\begin{aligned} \min_{\mathbf{y}} \quad & c_1 z + \mathbf{a}^H \mathbf{y} \\ \textit{subject to} \quad & c_2 z + \mathbf{b}^H \mathbf{y} = 1 \\ & \mathbf{y}^H \mathbf{y} = z^2 \varepsilon^2 \end{aligned}$$

Now, we can solve Problem 10 by eliminating z first. Since $c_2 > 0$, we have

$$z = (1 - \mathbf{b}^H \mathbf{y})/c_2 = (1 - \mathbf{b}^H \mathbf{y})\tilde{c_2} \tag{3.48}$$

with $\tilde{c_2} = \frac{1}{c_2}$ Hence, Problem 10 is reduced to

$$\begin{aligned} \min_{\mathbf{y}} \quad & \tfrac{c_1}{c_2} + \mathbf{f}^H\mathbf{y} \\ \text{subject to} \quad & \mathbf{y}^H\mathbf{Q}\mathbf{y} + \mathbf{q}^H\mathbf{y} - c \ = 0 \end{aligned} \tag{3.49}$$

where $\mathbf{Q} = (\mathbf{I} - \tilde{c_2}^2\varepsilon^2\mathbf{b}\mathbf{b}^T)$, $\mathbf{q} = 2\tilde{c_2}^2\varepsilon^2\mathbf{b}$, $c = \tilde{c_2}^2\varepsilon^2$ and $\mathbf{f} = (\mathbf{a} - c_1\tilde{c_2}\mathbf{b}) = (\mathbf{a} - \frac{c_1}{c_2}\mathbf{b})$. In this case, the Lagrangian function is

$$L_2(\lambda, \mathbf{y}) = c_1\tilde{c_2} + \mathbf{f}^H\mathbf{y} + \frac{1}{\lambda}(\mathbf{y}^H\mathbf{Q}\mathbf{y} + \mathbf{q}^H\mathbf{y} - c). \tag{3.50}$$

Requiring that its gradient with respect to $\mathbf{y}$ vanish i.e.,

$$\nabla_{\mathbf{y}} L_2 = \frac{2\mathbf{Q}\mathbf{y}}{\lambda} + \mathbf{f} + \frac{\mathbf{q}}{\lambda} = 0 \tag{3.51}$$

we arrive at

$$\mathbf{y} = -\frac{1}{2}\mathbf{Q}^{-1}(\lambda\mathbf{f} + \mathbf{q}) \tag{3.52}$$

Substituting this into the equality constraint in (3.49) and using the symmetry property of $\mathbf{Q}$ result in

$$\lambda^2\mathbf{f}^H\mathbf{Q}^{-1}\mathbf{f} - \mathbf{q}^H\mathbf{Q}^{-1}\mathbf{q} - 4c = 0 \tag{3.53}$$

Since we have assumed that $c_2^2 - \varepsilon^2\|\mathbf{b}\|_2^2 \neq 0$, we have $\mathbf{Q}^{-1} = \mathbf{I} + \beta\mathbf{b}\mathbf{b}^H$, where $\beta = \frac{\varepsilon^2}{c_2^2 - \varepsilon^2\|\mathbf{b}\|_2^2}$.

Substituting this into (3.53) yields $\lambda = \sqrt{\frac{\mathbf{q}^H\mathbf{Q}^{-1}\mathbf{q} + 4c}{\mathbf{f}^H\mathbf{Q}^{-1}\mathbf{f}}} = \frac{2\varepsilon}{\|c_2\mathbf{a} - c_1\mathbf{b}\|}$. Then, we get the solution to Problem 7 as:

$$\mathbf{e} = \frac{\mathbf{y}}{z} = \frac{-\varepsilon[c_2(c_2\mathbf{a} - c_1\mathbf{b}) + \varepsilon^2(\mathbf{b}\mathbf{b}^H\mathbf{a} - \mathbf{b}^H\mathbf{b}\mathbf{a}) + \varepsilon c_2\mathbf{b}\,\|c_2\mathbf{a} - c_1\mathbf{b}\|_2]}{\varepsilon\mathbf{b}^H(c_2\mathbf{a} - c_1\mathbf{b}) + c_2^2\,\|c_2\mathbf{a} - c_1\mathbf{b}\|} \tag{3.54}$$

Then we can obtain the solution of $\mathbf{y}$, z and $\mathbf{e}$ from (3.52) and (3.48). Hereby, making use of inverse transformation of $\mathbf{e} = \mathbf{vec}(\hat{\mathbf{E}}_{\mathbf{kc(k)}})$ and Equation (3.39), we arrive at the optimal solution to Problem 10 as

$$\mathbf{E}_{kc(k)} = \frac{-\varepsilon[c_2\mathbf{D}_k + \varepsilon^2\mathbf{F}_k + \varepsilon\mathbf{B}_k \|\mathbf{D}_k\|_F]}{\varepsilon(c_2 tr(\mathbf{B}_k\mathbf{A}_k) - c_1\|\mathbf{B}_k\|_F^2) + c_2\|\mathbf{D}_k\|_F} \quad (3.55)$$

where $\mathbf{D}_k = c_2\mathbf{A}_k - c_1\mathbf{B}_k$ and $\mathbf{F}_k = \mathbf{B}_k tr(\mathbf{B}_k\mathbf{A}_k) - \mathbf{A}_k\|\mathbf{B}_k\|_F^2$. Traversing k from 1 to K, all of the intra-cell uncertainty matrices can be computed as in (3.55). So far, we have obtained the closed form solution of uncertainty matrix $\mathbf{E}_{kc(k)}$ causing intra-cell interference in Problem 2 under the condition that uncertainty is small enough to guarantee the semi-positiveness of DCC matrices $\mathbf{R}_{c(k)}$.

3.3.2 Convex transformation of Problem 7

When ε exceeds certain limit, i.e., $\varepsilon > \lambda_{\min}(\mathbf{R})$, we must include the PSD constraint of matrix $\tilde{\mathbf{R}} + \text{mat}(\mathbf{e})$. In this case, it is not easy to obtain a closed form solution. However, we can transform Problem 7 into a convex problem. To do this, again let $\mathbf{y} = \frac{\mathbf{e}}{c_2+\mathbf{b}^H\mathbf{e}}$ and $z = \frac{1}{c_2+\mathbf{b}^H\mathbf{e}}$. Notice that $z > 0$ because its denominator is a summation of noise power and interference power, therefore, the quadratic inequality constraint in Problem 7 is equivalent to

$$\begin{bmatrix} z\mathbf{I} & \mathbf{y} \\ \mathbf{y}^H & \varepsilon^2 z \end{bmatrix} \succeq 0 \quad (3.56)$$

With this, Problem 7 is now turned into the following convex problem:

Problem 11

$$\begin{aligned} \min_{\mathbf{y},z} \quad & c_1 z + \mathbf{a}^H \mathbf{y} \\ \textit{subject to} \quad & c_2 z + \mathbf{b}^H \mathbf{y} = 1 \\ & \begin{bmatrix} z\mathbf{I} & \mathbf{y} \\ \mathbf{y}^H & \varepsilon^2 z \end{bmatrix} \succeq 0 \\ & d_1 \le z \le d_2 \\ & z\tilde{\mathbf{R}} + \mathrm{mat}(\mathbf{y}) \succeq 0 \end{aligned}$$

It can be efficiently solved for $\mathbf{E}_{k,c(k)}$ using interior point method [45]. Finally, our robust power control method to solve the optimization Problem 1 can be summarized as the following theorem by solving Problem 2.

Theorem 1 *The optimization Problem 2 can be efficiently solved as follows:*

1. *The solutions to Problem 2 with respect to uncertainty matrices contributing to inter-cell interference are determined as* $\mathbf{E}_{kc(j_{k_2})} = \varepsilon_{kc(j_{k_2})} \mathbf{w}_{j_{k_2}} \mathbf{w}_{j_{k_2}}^H$ *for* $k_2 = 1, ..., J_k$ *by solving Problem 3.*

2. *When* $\mathbf{R}_{kc(k)}$ *is positive definite and* $\varepsilon_{kc(j_k)} \le \lambda_{\min}(\mathbf{R}_{kc(k)})$*, the closed form solution for uncertainty matrices* $\mathbf{E}_{kc(k)} (k = 1, ..., K)$ *contributing to intra-cell interference to Problem 2 is given as follows:*

   (a) *If* $\mathbf{a}$ *and* $\mathbf{b}$ *are linearly dependent such that* $\mathbf{a} = \mu \mathbf{b}$*, solution is* $\mathbf{E}_{kc(k)} = \varepsilon \frac{\mathbf{B}_k}{\mathbf{B}_k \|_2}$ *when* $c_1 - \mu^* c_2 \ge 0$*, and* $\mathbf{E}_{kc(k)} = -\varepsilon \frac{\mathbf{B}_k}{\mathbf{B}_k \|_2}$ *when* $c_1 - \mu^* c_2 < 0$.

   (b) *If* $\mathbf{a}$ *and* $\mathbf{b}$ *are linearly independent, the solution is given by 3.55.*

3. *When uncertainty is large, i.e.,* $\varepsilon_{kc(j_k)} > \lambda_{\min}(\mathbf{R}_{kc(k)})$*, the nonconvex Problem 2 is transformed into a convex Problem 11 which can be efficiently solved using numerical method.*

4. *Substitute* $\{\mathbf{E}^{\text{wc}}_{k,c(j)}(n+1)\}_{j,k=1}^{K}$ *obtained in the above three steps into the third step of Algorithm 1 and the iteration continues until the algorithm stops.*

3.4 Conclusion

In this chapter, a novel robust downlink power control algorithm based on worst-case SINR optimization is proposed and developed. The goal is to determine the minimum amount of total transmission power required to ensure that the QoS constraints are satisfied for all users under the uncertain channel conditions. This original problem involves both uncertainty matrices and powers as variables, thus it is complicated and hard to transform into convex form. To get around direct solving of the problem, an iterative procedure treating uncertainty matrices and powers jointly in different steps is introduced first. In the iteration, although the natural formulation of the step to solve worst case uncertainty matrices is a complicated non-convex optimization problem, it can be separated into two subproblems, the first one of which has a closed form solution. But the original formulation of the second subproblem is still non-convex. To solve the second subproblem, we consider two cases where uncertainty is either small or large. For uncertainty smaller than a specific threshold, we obtain an analytic global optimal solution to the subproblem. For uncertainty larger than this threshold, we transform the problem into a convex one and solve it efficiently using interior point method [45].

Chapter 4

Simulation Results

In this chapter, we demonstrate some of our simulation results. We first provide a simulation experiment in order to examine the convergence of our iterative algorithm.The convergence of our iterative algorithm is shown with different choices of the initial powers. All the results show that our algorithm converges quickly for different initial powers within several iterations. Second, the performance of the proposed worst-case robust downlink power control is demonstrated by means of computer simulations. We compare our algorithm, i.e. Worst Case Robust Downlink Power Control(WCRDPC) with the Lower Bound Robust Downlink Power Control(LBRDPC) in [35]. Simulation results clearly show that our method uses less power than the LBRDPC method for the same given SINR constraint. We also demonstrate the comparison between our analytic method and convex transformation method in section 3.3. In the case when PSD constraint cannot be excluded, we cannot apply analytic method. However, we can still use analytic solution even if it is not the true solution, as a good indication for the numerical value.

We list our experiment results from two of our simulation scenarios. One is a system with $M = 2$ cells and $K = 3$ users. The other is a system with $M = 2$ cells and $K = 5$ users. We assume that each user is incoherent locally scatter sources [40, 43] with uniform angular distribution, characterized by the central angle θ and angular spread ϕ which are randomly selected from the interval of $[0°, 6°]$. Each BS is assumed to have a transmission uniform linear array (ULA) of N omnidirectional sensors (different N are used in the experiments in order to validate the convergence) which are spaced half a wavelength apart. Furthermore, this model assumes signal attenuation proportional to ν^{-4} as mentioned in 2.2.2, where ν is the BS-user distance. Noise powers, σ_k^2, are assumed to be the same for each user, i.e. $\sigma_k^2 = \sigma_0^2 = 0.1$ for $k = 1, \ldots, K$. Also, we assume that the required SINR QoS minimum level is identical for each user so that $\gamma_k = \gamma_0 = \gamma$ for $k = 1, \ldots, K$. The SeDuMi convex optimization MATLAB toolbox [46] has been used to numerically compute the worst-case channel uncertainties for robust downlink power control using worst-case performance optimization.

4.1 Convergence Simulation

Different parameters, antenna array size $N = 4, 6, 8$, SINR threshold $\gamma = 6, 8$, are used in order to validate the convergence. For each experiment with certain combination of N and γ, 1000 simulation runs with uniform distributed random initial powers are used to validate the convergence of our iterative procedure. Also, in the simulations, the uncertainty factor ε is the same value 0.005.

Fig. 4.6 to Fig. 4.11 show the transmission power versus number of iterations for our algorithm. The y axis represents total power while x axis represents the iteration number. To demonstrate the convergence more efficiently, for each set of parameters, we plot all results with different initial powers in one single figure. If the total power difference between adjacent iterations is less than a prescribed small threshold, we treat the iteration convergent. In our experiment, the threshold is 0.001. The detail of each figure is listed as below:

1. Fig. 4.6(a) and (b) show the convergence for 3 users, $N = 4$, $\gamma = 6dB$ and $\gamma = 8dB$, initial power $P_k, k = 1,2,3$ between 0 and 0.5. Total power converges to around 0.15 and 0.2 for $\gamma = 6dB$ and $\gamma = 8dB$ after few iterations.

2. Fig. 4.7(a) and (b) show the convergence for 3 users, $N = 6$, $\gamma = 6dB$ and $\gamma = 8dB$, initial power $P_k, k = 1,2,3$ between 0 and 0.5. Total power converges to 0.1 and 0.15 for $\gamma = 6dB$ and $\gamma = 8dB$ after a few iterations.

3. Fig. 4.8(a) and (b) show the convergence for 3 users, $N = 8$, $\gamma = 6dB$ and $\gamma = 8dB$, initial power $P_k, k = 1,2,3$ between 0 and 0.5. Total power converges to 0.75 and 1.3 for $\gamma = 6dB$ and $\gamma = 8dB$ after a few iterations.

4. Fig. 4.9(a) and (b) show the convergence for 5 users, $N = 4$, $\gamma = 6dB$ and $\gamma = 8dB$, initial power $P_k, k = 1,2,3,4,5$ between 0 and 0.5. Total power converges to 0.6 and 2.2 for $\gamma = 6dB$ and $\gamma = 8dB$ after a few iterations.

5. Fig. 4.10(a) and (b) show the convergence for 5 users, $N = 4$, $\gamma = 6dB$ and $\gamma = 8dB$, initial power $P_k, k = 1,2,3,4,5$ between 0 and 0.5. Total power converges to 0.4 and 1.0 for $\gamma = 6dB$ and $\gamma = 8dB$ after a few iterations.

6. Fig. 4.11(a) and (b) show the convergence for 5 users, $N = 4$, $\gamma = 6dB$ and $\gamma = 8dB$, initial power $P_k, k = 1,2,3,4,5$ between 0 and 0.5. Total power converges to 0.2 and 0.5 for $\gamma = 6dB$ and $\gamma = 8dB$ after two or three iterations.

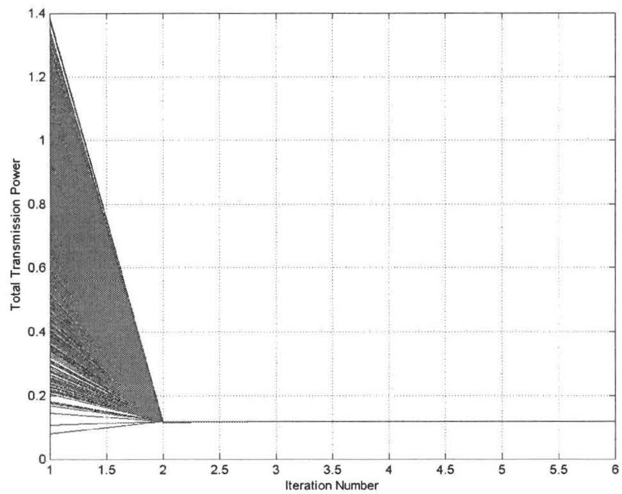
Observing the convergence figures, we can come up with the following:

1. Simulation results clearly demonstrate that in all experiments, our iterative algorithm converges quickly after even two or three iterations, regardless the size of the antenna arrays, the required QoS level, the initial power. This quick convergence could be due to the fact that most of the uncertainty matrices, $K \times (M-1)$ inter-cell matrices have been obtained through solving Subproblem 1 as in section 3.3 and then remain the same in each iteration.

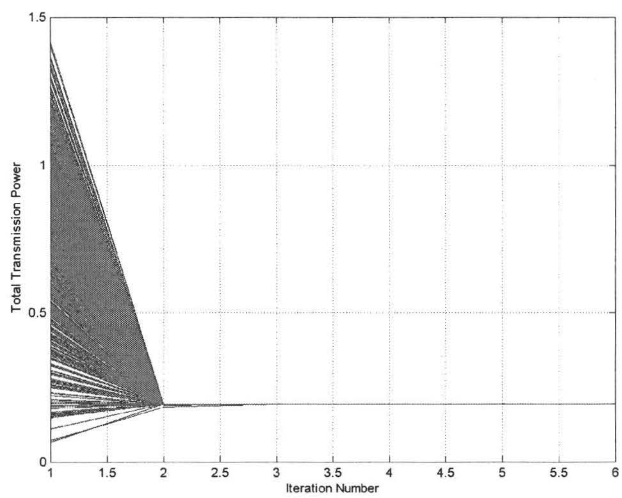
2. Naturally, the higher the QoS quality is required, the more power is needed given the same antenna array size and noise level.

4.2 Comparison with Existing Method

We will compare our method, i.e. Worst Case Robust Downlink Power Control(WCRDPC), with other existing method, i.e. Lower Bound Robust Downlink Power Control(LBRDPC), in [35] in this section. The same fixed transmission weight vectors $\mathbf{w}_k$ for $k = 1, \ldots, K$ are used in all algorithms. Generalized eigenvalue-based beamformer [38, 42] is used to compute the fixed transmission weight vectors in the simulations. We provide comparison results for different combinations of antenna array size$N = 4, 6, 8$ and QoS level $\gamma = 6, 8$. 3 users case is chosen for the comparison. Then in each comparison

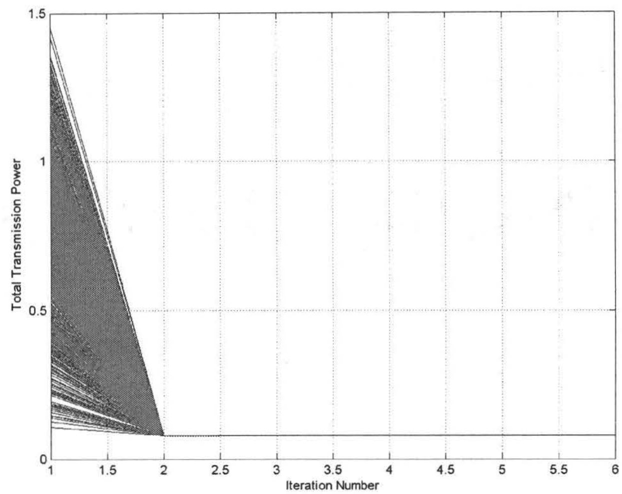


(a) Convergence: 3 users, N=4, $\gamma = 6dB$

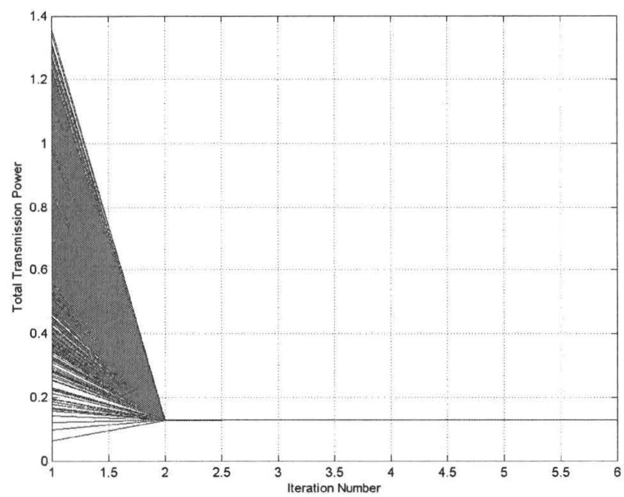


(b) Convergence: 3 users, N=4, $\gamma = 8dB$

Figure 4.6: Convergence: 3 users, N=4

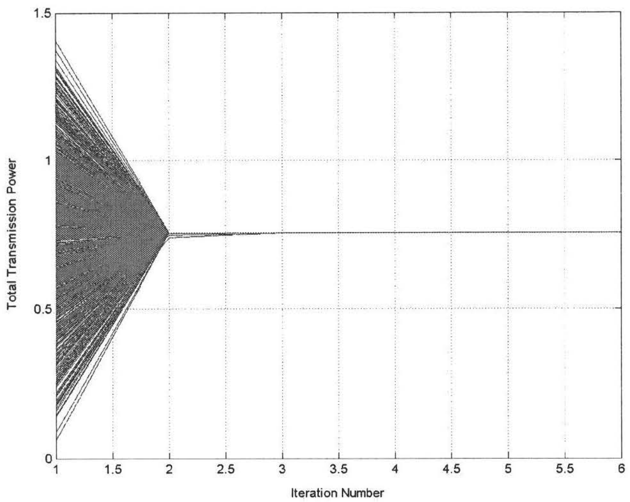


(a) Convergence: 3 users, N=6, $\gamma = 6dB$

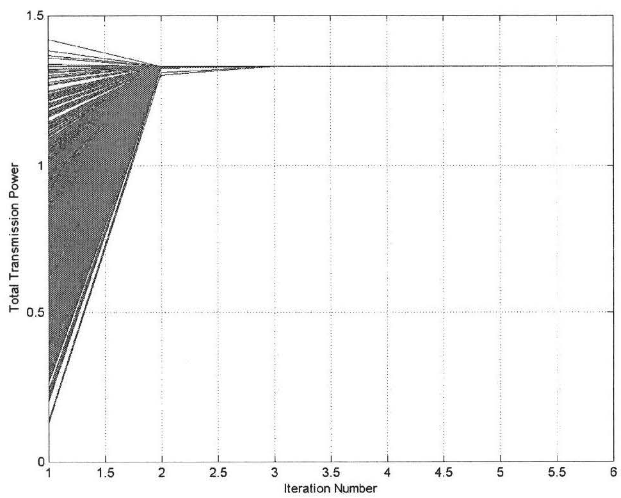


(b) Convergence: 3 users, N=6, $\gamma = 8dB$

Figure 4.7: Convergence: 3 users, N=6

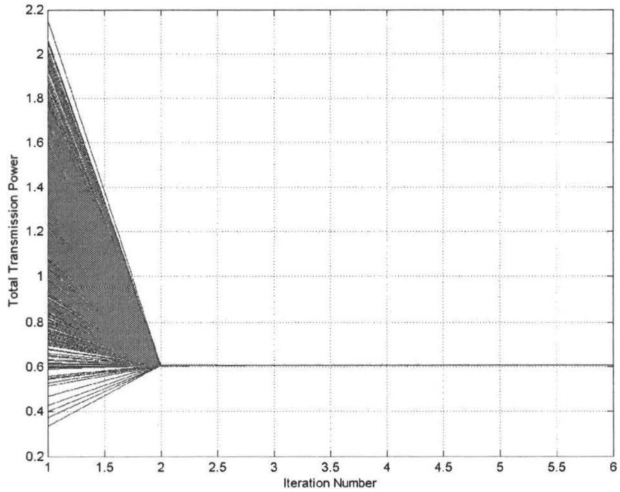


(a) Convergence: 3 users, N=8, $\gamma = 6dB$

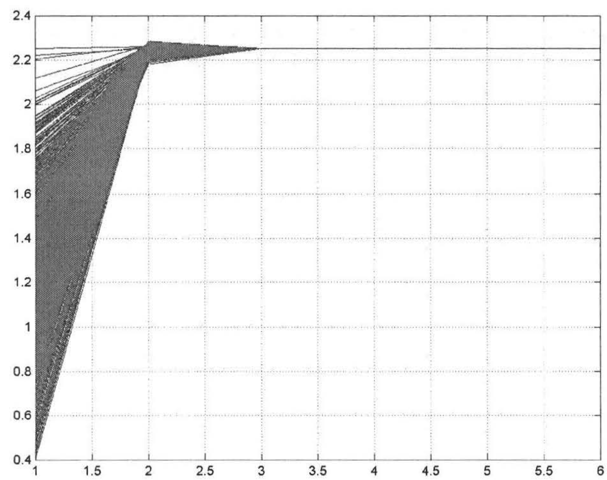


(b) Convergence: 3 users, N=8, $\gamma = 8dB$

Figure 4.8: Convergence: 3 users, N=8

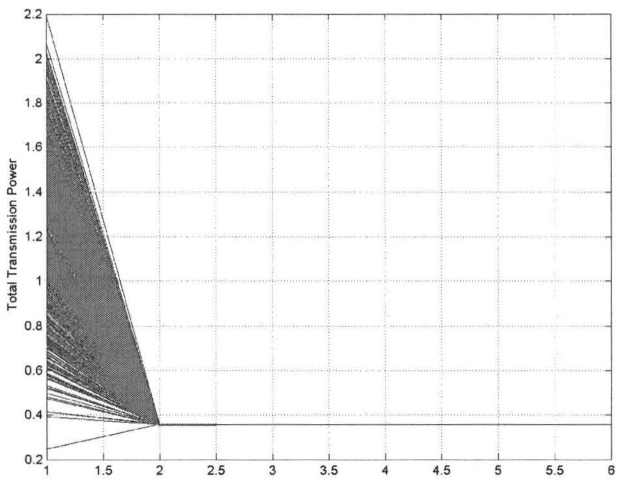


(a) Convergence: 5 users, N=4, $\gamma = 6dB$

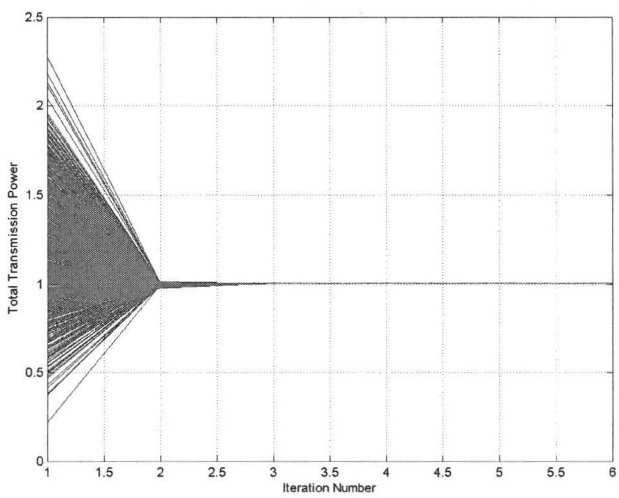


(b) Convergence: 5 users, N=4, $\gamma = 8dB$

Figure 4.9: Convergence: 5 users, N=4

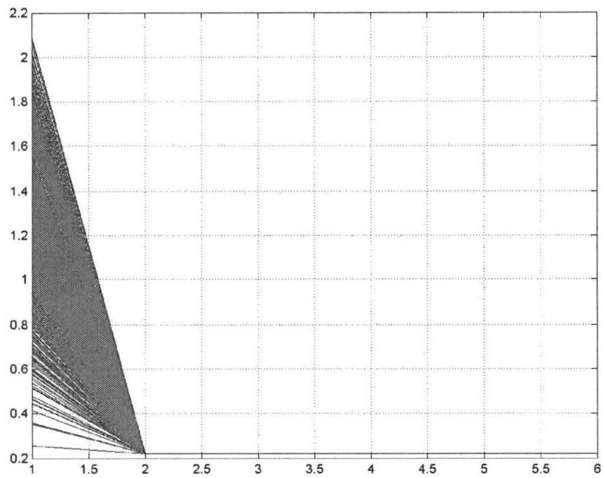


(a) Convergence: 5 users, N=6, $\gamma = 6dB$

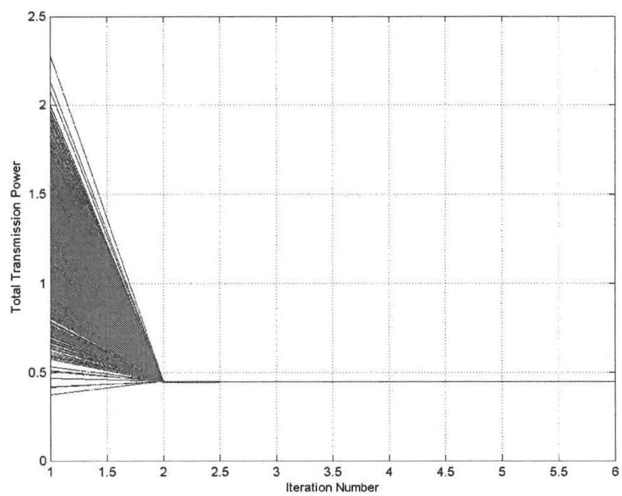


(b) Convergence: 5 users, N=6, $\gamma = 8dB$

Figure 4.10: Convergence: 5 users, N=6



(a) Convergence: 5 users, N=8, $\gamma = 6dB$



(b) Convergence: 5 users, N=8, $\gamma = 8dB$

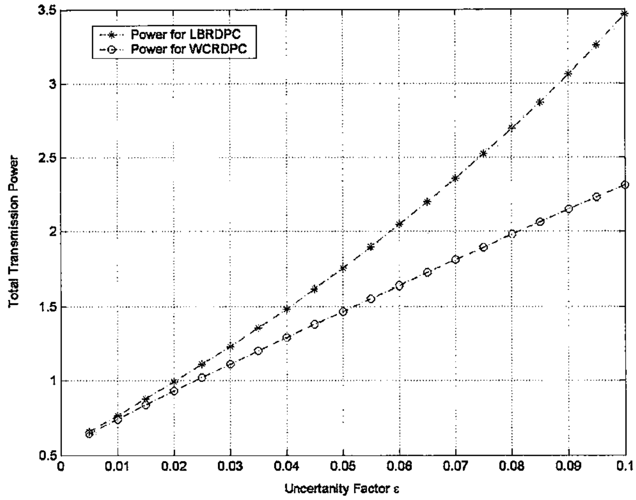
Figure 4.11: Convergence: 5 users, N=8

experiment, we choose a se of values for the angels and distances between users and BSs, while the uncertainty factor ε is between 0.005 and 0.1.

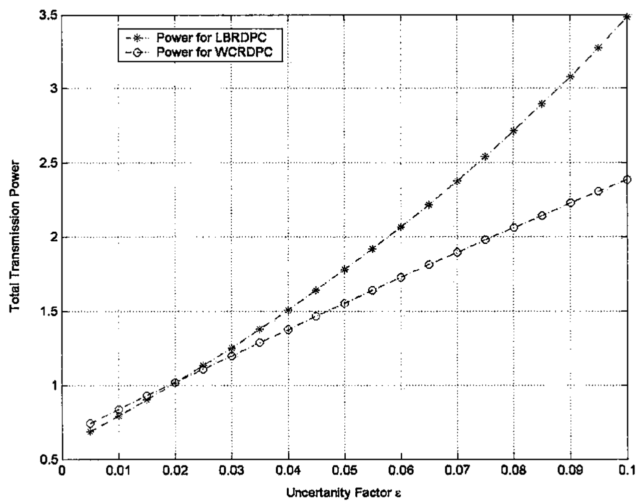
From Fig. 4.12 to Fig. 4.17, each of them contains two typical comparison results with the given antenna array size N, QoS threshold γ. From all of the simulation results in Fig. 4.12 until Fig. 4.17, we can see that our method uses less power than LBRDPC in whichever case. Also, with the increase of uncertainty factor ε, the saved power using our method is also increasing.

4.3 Comparison between Analytic Solution and Numerical Method

One other thing worth noting is that our analytic solution can be a good approximate of the numerical solution even if in some cases the condition of analytic solution does not apply. For example, as in section 3.3.2, when ε exceeds certain limit, i.e.,$\varepsilon > \lambda_{\min}(\mathbf{R})$, we must include the PSD constraint of matrix $\tilde{\mathbf{R}}+\text{mat}(\mathbf{e})$ and use numerical tool to solve the worst case uncertainty Subproblem 2. This can take much longer time than closed form solution. Therefore, if the results for both solutions are quite close with each other, we could simply use analytic method to calculate the power allocation without much deviation from the true value. In this way, using analytic method to calculate the power allocation can be much more efficient comparing with other numerical methods [1]. Fig. ?? demonstrates two examples regarding analytic solution in section 3.3.1 and numerical solution in section 3.3.2.

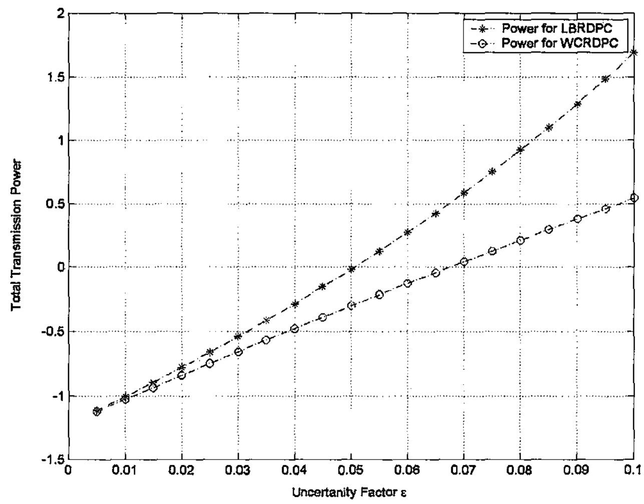


(a)

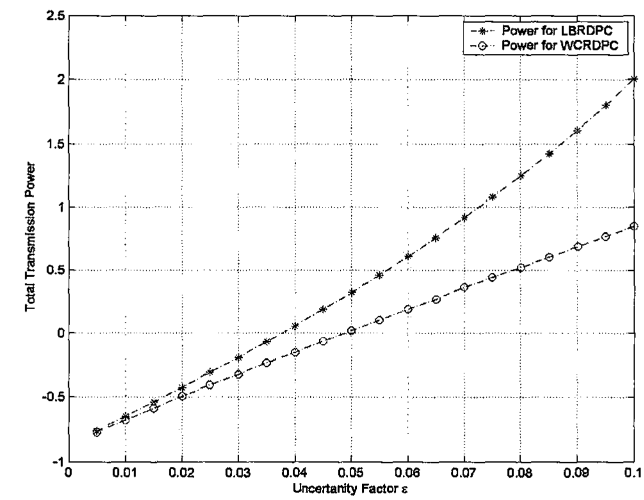


(b)

Figure 4.12: Total Transmission Power(dB) vs. ε: 3users, N=4, γ = 6dB

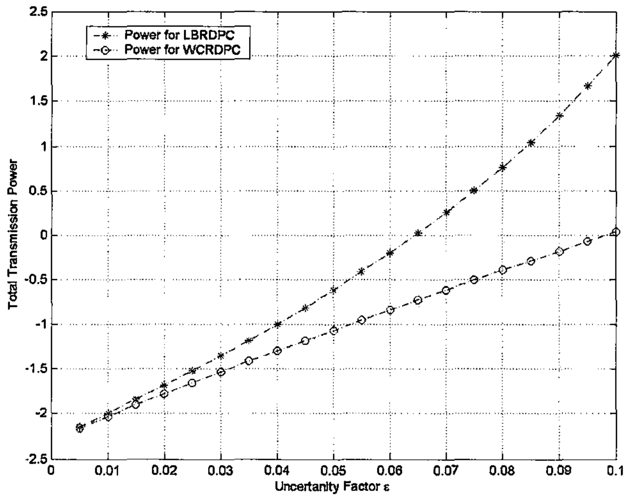


(a)

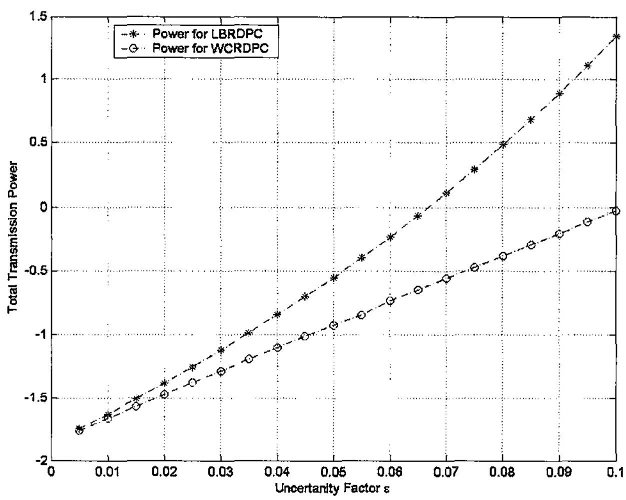


(b)

Figure 4.13: Total Transmission Power(dB) vs. ε: 3users, N=6, $\gamma = 6$dB

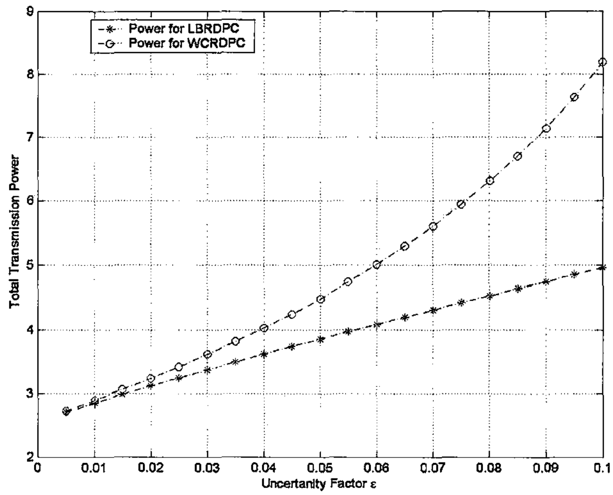


(a)

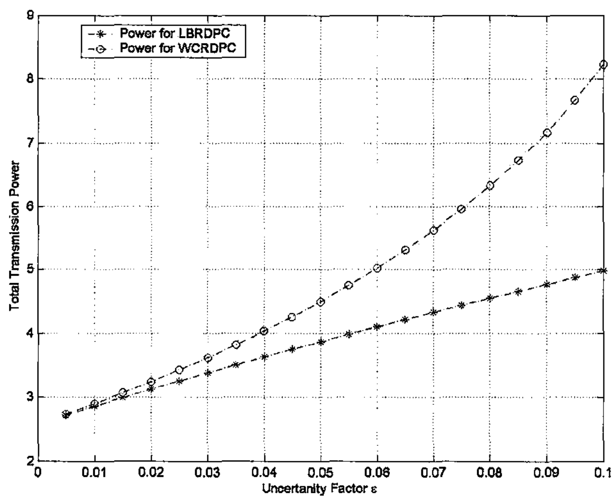


(b)

Figure 4.14: Total Transmission Power(dB) vs. ε: 3users, N=8, $\gamma = 6$dB

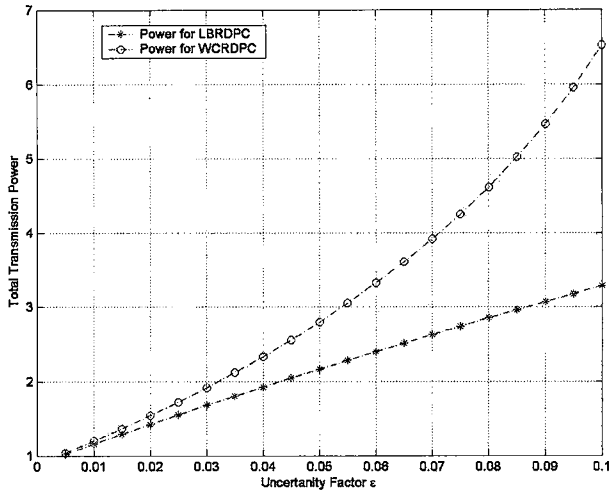


(a)

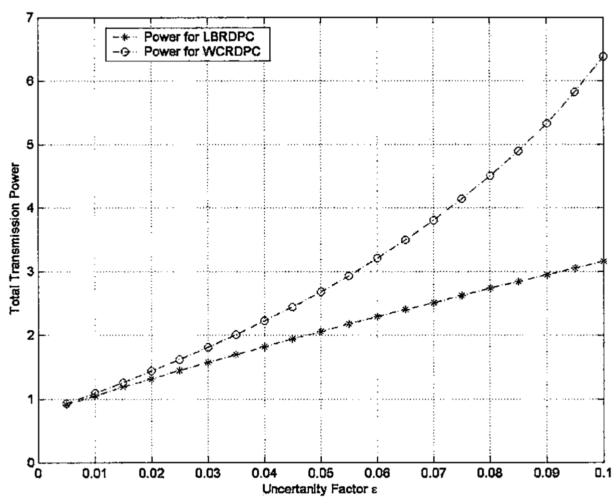


(b)

Figure 4.15: Total Transmission Power(dB) vs. ε: 3users, N=4, $\gamma = 8$dB

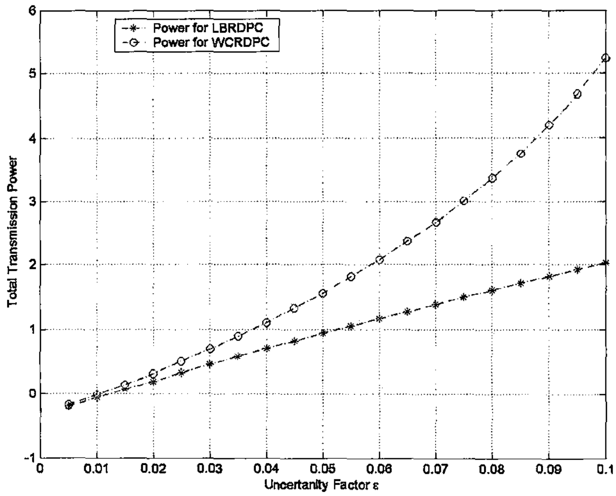


(a)

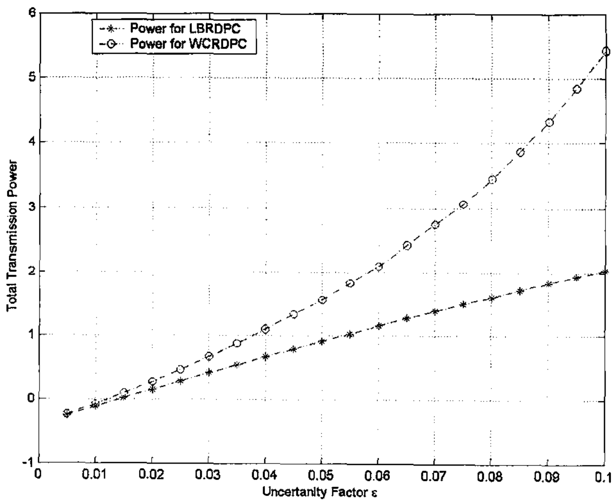


(b)

Figure 4.16: Total Transmission Power(dB) vs. ε: 3users, N=6, $\gamma = 8$dB



(a)



(b)

Figure 4.17: Total Transmission Power(dB) vs. ε: 3users, N=8, $\gamma = 8$dB

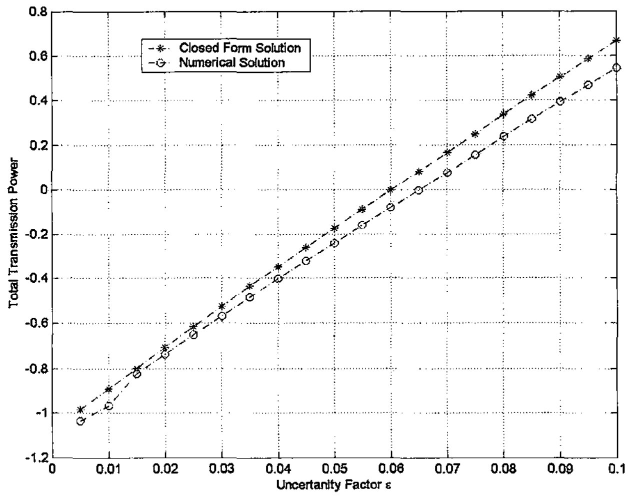


Figure 4.18: Closed Form Solution vs. Numerical Solution: 3 users, γ = 6dB,mse=7.53e-004

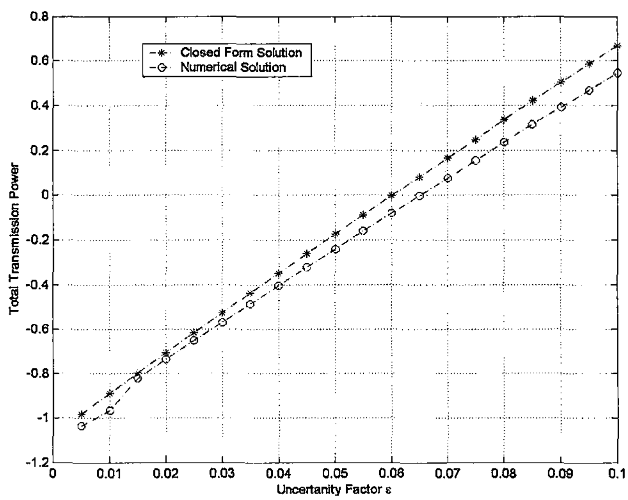


Figure 4.19: Closed Form Solution vs. Numerical Solution: 3 users, γ = 6dB,mse= 0.0062

Chapter 5

Conclusions

In this thesis , we have designed an efficient robust power control algorithm for downlink transmission while complying with worst-case QoS constraint. We develop an algorithm that determines the minimum transmission power required to ensure that the QoS constraints for all users are satisfied. In conclusion, our contributions in this thesis can be summarized as below:

1. An iterative procedure to jointly determine the power allocation and worst case uncertainties is proposed.

2. The original non-convex worst-case uncertainty Problem 2 is simplified into two subproblems as in section 3.3.

3. Closed-form solution for Subproblem 2 is given as in 3.55.

4. Numerical solution for Subproblem 2 is given as in Problem 11.

5. Computer simulations verify that the iterative procedure converges in a few iterations.

6. Our proposed algorithm reduces the transmission power more effectively than other existing robust power control designs under uncertain channel conditions.

The following two issues remain unresolved and will be our future work:

1. In spite of the fact that our simulations verify that the proposed iterative Algorithm 1 is convergent very quickly, thus far, the analytic proof of this convergence will be one future topic.

2. Even if the Algorithm is convergent, we still need to verify how close the solution is to the true optimal solution.

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