

**REACH ENHANCEMENT IN BOTH DIRECT-DETECTION AND  
COHERENT DETECTION OPTICAL FIBER COMMUNICATION  
SYSTEMS**

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DETECTION OPTICAL FIBER COMMUNICATION SYSTEMS**

**By**

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**Dedications**

To my Dad, Mom  
and siblings

# Abstract

Early methods of optical fiber communication systems haven't been much promising in terms of efficiency. The presence of various impairments in the fiber channel has forced researchers to uncover solutions in order to minimize those effects. With the advancement of technology, optical solutions were finally easier to implement in the system. To this day, optical compensation methods are still found to be as the best way to minimize fiber impairments. However, such technique does introduce enormous complexity to the system, in addition to a large cost. For that reason, the main focus had to shift to an alternative method. Electrical compensation techniques have provided the factor of simplicity to the optical communication system, not to mention that they are relatively cheaper than optical compensators. Furthermore, electrical schemes were found to handle fiber impairments in a relatively efficient manner.

In this thesis, an optical fiber communication scheme using the direct-detection method is simulated. A frequency shifter in the optical domain will be used for the system to have a coherent like detection. At the receiver's side, a linear equalizer is realized to offset the linear effects caused by the fiber. To our knowledge, this will mark the first direct-detection transmission system to pass the one thousand kilometre mark in fiber length. Furthermore, we simulate another optical fiber communication design using the coherent detection. A nonlinear compensator adapting the Volterra approach will be used to offset

nonlinear impairments. Such performance will be compared to that of a linear compensator. Design trade-offs will be analyzed, and the nonlinear compensator is found to improve performance when a dispersion compensation fiber (DCF) is introduced in the optical domain.

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# Introduction

The purpose of the introductory chapter is to give a background on optical fiber communication systems, provide a literature review on what has been done in this field and state the contribution that this research would provide to this area.

## 1.1 Background

Optical fiber communication systems could be broadly defined as a process of transmitting light from one end (transmitter side) to the other (receiver side) using optical fibers. The field of fiber optics was born in the 1950's [1], but started to develop rapidly in the 1960's mainly for image transmission purposes. Back then, fiber glasses were extremely lossy (loss  $>1000$  dB/Km). Nevertheless, the 1970's brought a major change to the field when the silica fiber losses were brought down to less than 20 dB/Km. Furthermore, technology breakthroughs have made possible the invention of a low-loss silica fiber (0.2 dB/km). Such a revolution, coupled with the invention of the laser in 1960 [2], finally made a high-capacity optical communication system possible. One main advantage of such a system is that light could propagate in the fiber on a long distance with minimum attenuation. For that, optical fibers have replaced the copper wire cables for long transmission uses in many places. Nowadays, optical fiber is used for many

communication purposes such as internet, telephone and television, hence making it an essential aspect in today's technology.

## **1.2 Benefiting from the optical fiber**

Communication systems have long relied on the use of electrical cables and copper wire. Existing disadvantages included short distance transmission and high intensity loss. For that, a new alternative had to be established. Optical fiber permitted the transmission of a signal on a much longer distance, while maintaining a relatively low attenuation of the light. Perhaps the biggest advantage that we could benefit from using such a medium is the fact that one can transmit a signal at a really high bit rate, i.e. there exists optical fibers that can allow a signal to be modulated at a rate of 111 Gb/s/channel over 2100 Km of fiber length [28]. Though, most commercial systems operate at a bit rate of 10 Gb/s/channel and the 40 Gb/s/channel. On a short distance use, it was found hard for the fiber optic systems to replace the existing electric cables since the process will be costly. However, new buildings are starting to adapt to the optical networking [5]. One optical fiber can carry much more data than a copper wire and therefore it can substitute several cables. Furthermore, optical cables are resistant to interference between one another thus eliminating the cross-talk problem that is present between the copper wires. Telephone communications were revolutionized in 1988 when the first transatlantic submarine optical fiber system started to operate [3]. Ever since, the latter invention has occupied most major long distance communications. Nowadays, countries such as Japan have even

started to replace the digital subscriber line (DSL) cables [5] with optical fibers for internet use, and several companies have managed to carry high speed internet, TV and telephone straight to homes [4].

### **1.3 Optical fiber drawbacks**

Optical fiber communication systems have gained so much importance in the past decade. The fact that one can transmit a signal over a long distance with great efficiency has made such a discovery an important field for study. However, researchers till this day are trying to overcome obstacles that appear when using such a system. First, the different losses that are present in the fiber stand for one of the problems. Material absorptions (due to the glass in the fiber) cause a power loss in the optical output. The Rayleigh scattering effect in such a medium does provide another reason for the light to scatter to the fiber cladding, thus diminishing the output power [6], [7]. Other power reductions may occur when two optical fibers are joined together. This may be known as the splice loss [8]. For all reasons mentioned, an optical amplifier is usually introduced after the fiber in order to compensate for the losses. Consequently, such action will bring in amplified noise to the channel, thus establishing another disadvantage. Other noises may include the shot noise (due to random generation of electrons) and thermal noise (in photo-detectors) [9]. An additional obstacle that one might add is that the optical channel will lead to broadening the signal due to the dispersive effects that are present in the fiber. Both chromatic dispersion (CD) and polarized mode dispersion (PMD) are major causes to expanding the



signal pulse [10]. Other than altering the output, those dispersive factors cause one optical pulse to interfere with its neighbouring pulse, thus the intersymbol interference effect. A significant drawback that optical fiber communication system possesses is the fact that it's a nonlinear medium. Self-phase modulation (SPM) and Cross-phase modulation (XPM) take place in the fiber due to the Kerr effect [11]. Chapter 2 of this thesis will include more physical and mathematical details about the fiber properties and downsides.

## 1.4 Optical Compensation

Since the introduction of the optical fiber communication systems, various methods have been designed to overcome obstacles that the system introduces. Researchers have tried to offset effects such as dispersion and nonlinearities, thus maximizing the reach of the fiber to the receiver's end. Methods of compensation could be merely divided into two parts: (i) optical compensation and (ii) electrical compensation. To this day, the compensation of the signal at the optical domain has proved to be more efficient than using electrical methods [12]. On the other hand, electrical components are more commercially available, not to mention that they are much cheaper to build and operate. Therefore when designing a system, in most cases, one has to compromise between efficiency (optical compensation) and cost (electrical compensation).

In a single mode optical fiber, there exists a wavelength in which the fiber would have minimal dispersive effects [24]. In case of a silica based fiber, this effect would occur when the wavelength is  $\lambda = 1.3\mu m$ . This is called the zero-dispersion wavelength. Then

again, the fiber does operate at minimum attenuation when the wavelength is  $\lambda = 1.55 \mu\text{m}$ . As a result, it would be a good idea to work around the latter wavelength. Nowadays, dispersion-shifted fibers were designed such that they have zero dispersion at  $\lambda = 1.55 \mu\text{m}$ . But it turned out that the fiber nonlinear effects such as forward mixing (FWM) are greatly enhanced in such fibers. Therefore, the standard single-mode fibers (SMF) with  $D = 17 \text{ ps/km/nm}$  at  $\lambda = 1.55 \mu\text{m}$  or non-zero dispersion shifted fiber (NZ-DSF) with  $D \approx 6 \text{ ps/km/nm}$  are commonly used [42]. For a moderate fiber length, the fiber dispersion does worsen the error rate of the transmitted signal due to the pulse broadening. For that reason, a dispersion compensation fiber (DCF) is usually introduced after the fiber in order to undo the broadening effects [13], [23], [24].

#### 1.4.1 Dispersion Compensation Fiber

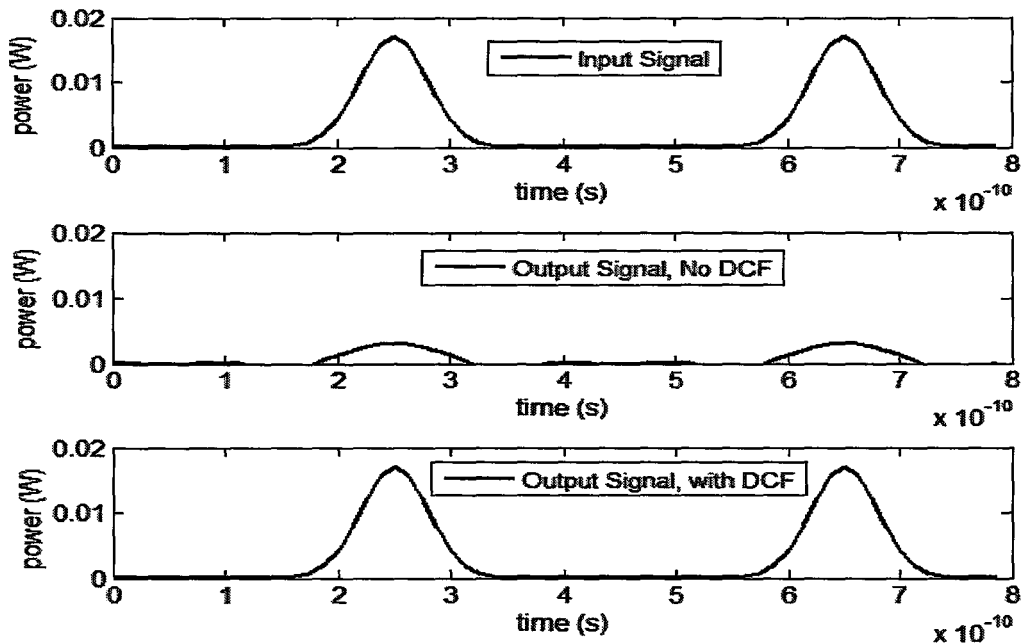


Figure 1. 1: DCF benefit towards the dispersive signal

Figure 1.1 depicts the propagation of a signal along an 80 Km single mode fiber for two systems. The fiber was assumed to be noise free and with no nonlinearity. The chromatic dispersion coefficient of the channel was  $\beta_2 = -21ps^2 / km$ . When the DCF is not introduced, the output of the channel is altered by the dispersive effects. Although with a proper DCF design, it is possible to offset the effects of the dispersion and hence having an output signal almost identical to the original input signal. The choice of the DCF parameters will explained at a later section but in this case,  $\beta_2^{DCF} = +130ps^2 / km$  and  $L^{DCF} \approx 12.9Km$ .

Other than offsetting the presence of chromatic dispersion in a fiber, the DCF does have the benefit of narrowing the effect of Cross-phase modulation (XPM). When the DCF is not introduced in the system, the optical signal transmitted along the fiber broadens along its propagation. Therefore, the nonlinear effects of the broadened pulse can affect its neighbouring pulses. But with the introduction of the DCF in the system, the broadening of the pulse will end and hence cancelling the effect of the XPM. In that case, nonlinear impairments will only be caused by the SPM effect.

Other than the chromatic dispersion, imperfections in a fiber design do introduce the polarized mode dispersion (PMD) effect. When a light is emitted into a fiber, the x and y polarizations of the signal travel at different speeds due to the non-ideal conditions of the medium. Several methods have been made available to compensate for the PMD in the optical domain [22]. PMD can never be entirely eliminated, although methods of

compensation have proven to eliminate a majority of its effect. PMD coefficients can be really high in a fiber-optic link that is old in nature. Moreover, such effect can be threatening in a high bit rate transmission system. However, only transmissions at a rate of 10 Gb/s will be used in this research. PMD effects will be assumed to be negligible. For that reason, further details about this type of fiber dispersion will not be provided.

### 1.4.2 Planar lightwave circuit (PLC) optical Equalizer

Other methods used to compensate for the fiber offsets include an optical equalizer [25],[26],[27]. Such device is achieved by cascading several Mach-Zender interferometers on a single silica device.

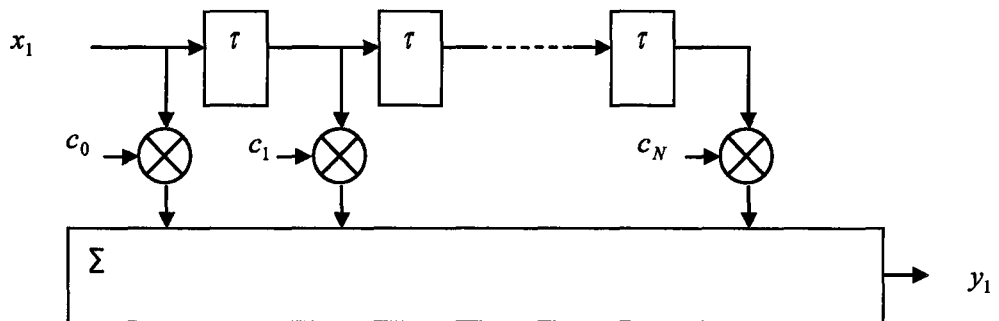


Figure 1. 2: Equivalent Tapped delay line (TDL) representation of a PLC [27]

An easy way to understand the PLC equalizer is to represent it using a tapped delay line schematic, as shown in Figure 1.2 [27]. The coefficients  $c_0, c_1 \dots c_N$  are complex coefficients depending on the PLC control parameters, and the output is given by:

$$y_1(t) = \sum_{k=0}^N c_k x_1(t - k\tau) \quad (1.1)$$

It was proposed in [27] to calculate the control parameters adaptively, and that's by using the least mean square (LMS) algorithm to minimize the mean square error. More on the LMS algorithm will be explained at a later section. However, once converged, the PLC coefficients will play the role of offsetting impairments such as the chromatic dispersion (CD) effect.

Other optical compensation methods include the optical phase conjugation (OPC) [14], which is a process of reversing the direction and phase variation of the light beam emitted. Such device has helped compensate nonlinearity effects such as the SPM. In general, offsetting fiber impairments in the optical domain have been proved to be very efficient, yet they bring along negative aspects such as increase in system cost as well as raising its complexity. However, in this research, the focus will be mainly on electrical equalization. Only devices such as optical amplifiers and DFC's will be used in term of research objectives.

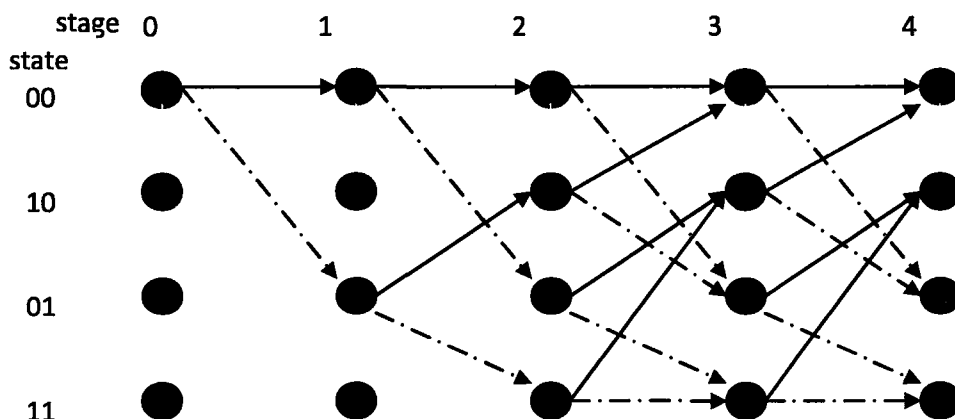
## 1.5 Electrical Compensation

Optical compensation does offer a wide range of solutions when it comes to dealing with fiber impairments. For instance, such compensation provides great efficiency because it deals with the drawbacks in the domain where they actually occur (optical domain). However, complexity arises when designing optical devices, not to mention the necessity of precision when dealing with signals such as lasers. And with high complexity comes a major boost in the cost. The fact the one can deal with such problems at the receiver end has shifted main focus to the electrical side. Electrical compensation has been necessary in various communication fields such as radio, wireless, telephone, television etc... [15],[16],[17]. For this reason, it was found to be a good idea to mimic solutions for problems that have already challenged researchers in those areas. Therefore after photo-detection, one could employ techniques that have been already established to the resulting electrical signal. Electrical compensation would not provide the same efficiency that could be acquired in the optical domain. In a non-coherent system, after the process of photo-detection, the complex optical field envelope  $x(t)$  coming from the fiber end becomes real, i.e.  $y(t) = |x(t)|^2$ , where  $y(t)$  is the electrical signal, hence losing some of its information. This procedure is known as the direct-detection method. One could overcome losing the complex elements of the signal by building a coherent receiver [29],[32], in which the electrical output would impersonate the optical signal in its behaviour in an ideal case. However, such receiver does raise a significant amount of complexity and cost to the system. Coherent detection will be explained thoroughly at a

later chapter. Another major problem that might arise from compensating a signal in the electrical domain is the fact that one is trying to deal with fiber impairments (nonlinearity, dispersion, noise etc...) that have already happened in the optical domain, making the process of estimating the transmitted signal a much harder task. All in all, many electrical methods can be realized at the receivers end, and we here will state the different compensation approaches that have made this technique a popular one. Thereafter, various realizations of electrical systems will be provided and compared.

### 1.5.1 Maximum Likelihood Sequence Estimation (MLSE)

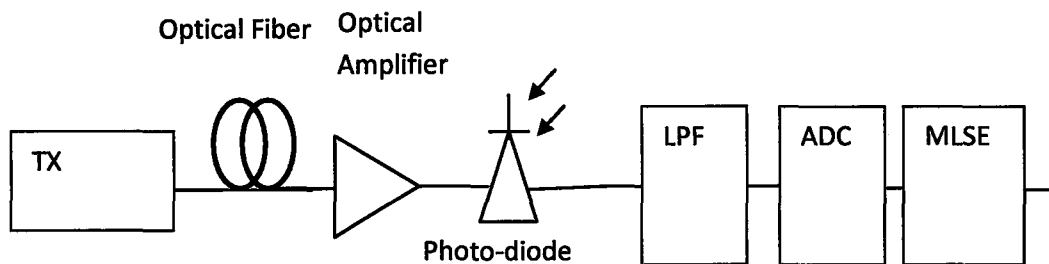
Intersymbol interference, resulting mainly from fiber dispersion, is whenever the effect of one transmitted pulse does not fade away completely before the transmission of a next one. As stated earlier, chromatic dispersion could be offset by introducing a dispersion compensation fiber (DCF) in the optical domain. They are usually introduced in between the fiber end and the optical amplifier. But in case of a long distance transmission fiber (usually in the hundreds or even thousands of kilometres in range), such system requires the installation of numerous amount of DCF's at every amplifier spacing, hence boosting the cost. For that reason, one could implement a Maximum Likelihood Sequence Estimation (MLSE) device at the receivers end [18],[19]. The MLSE method estimates the most probable transmitted bit using the maximum-likelihood criterion. Meaning, when numerous bits are transmitted, several combinations of that bit stream are considered, and the sequence with the highest probability (most likely to have occurred) is assumed to be the transmitted signal [39] .



**Figure 1. 3:** State diagram of a 4 state trellis tree along four stages . Dark arrows (branch metrics) indicate that a bit “0” was transmitted. Dashed arrows indicate that a bit “1” is transmitted.

The MLSE can be implemented using the Viterbi algorithm. The branch metrics in Figure 1.3 are computed, and the received signal will be estimated regarding the shortest path that it will take through the trellis tree. In general, the efficiency of the MLSE is proportional to the memory size that it possesses [39]. If an MLSE system has a memory of  $n=2$ , it means that it can only estimate the path of two bits over  $2^n=4$  states. However, the light accumulates dispersion while traveling through the fiber, and at some point, the broadening of a transmitted pulse will increase and affect more than one neighbour. In that case, a memory of  $n=2$  will not be enough to estimate the transmitted signal. Therefore, the memory has to be increased. One major drawback that could result from such action is a boost in computational time.





**Figure 1. 4:** Diagram depicting a direct detection system with use of a MLSE

A schematic of a direct-detection system using a MLSE receiver is shown in Figure 1.4. After photo-detection, the electrical signal passes through low-pass-filter (LPF) to limit the noise introduced by the amplifier. Afterwards, the filtered current gets sampled using an analog-to-digital converter (ADC), whereas the sampled bits will be estimated using the MLSE receiver. Other than compensation of chromatic dispersion (CD), MLSE has been shown to offset some of the Polarized-mode dispersion (PMD) impairments [38]. More on the behaviour of the MLSE with regard to PMD and CD will be discussed later on when compared with other systems.

The MLSE performance does also depend on the resolution of the ADC. Furthermore, it was shown in [31] that the efficiency of a system is relatively proportional to the number of samples that can represent a bit interval. In [31], a non-return-to-zero (NRZ) on-off-keying (OOK) signal at a rate of 10 Gb/s was transmitted over a Corning® Leaf fiber with a dispersion of  $D=4.25$  ps/nm/km and a nonlinear coefficient of  $\gamma=1.2949/W/Km$ . The input optical power was chosen to be 6 dBm. A 16 state MLSE was used at the receiver's side to estimate the original transmitted signal.

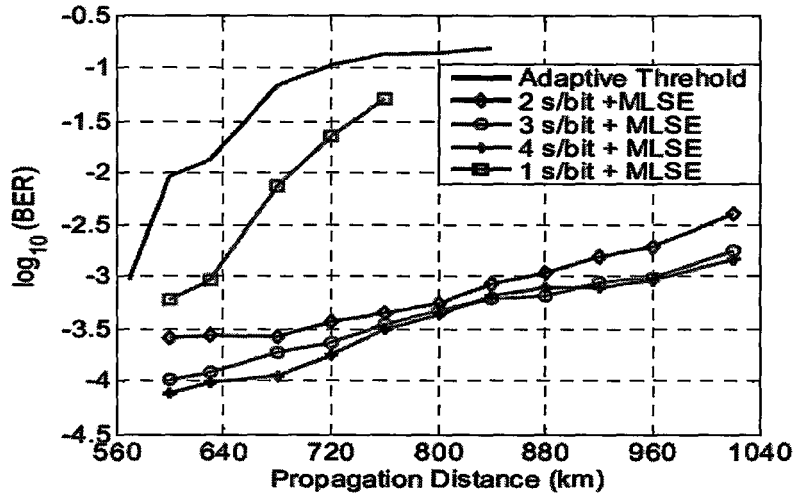
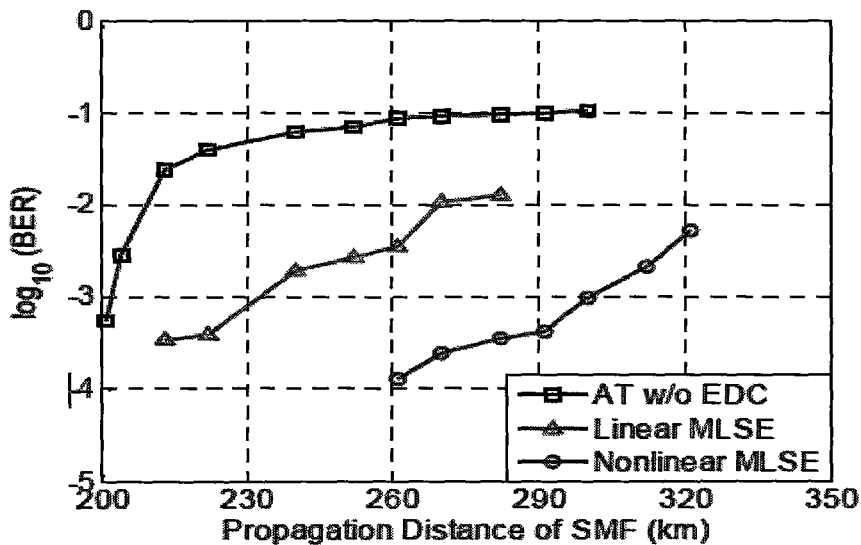


Figure 1. 5: BER for MLSE receiver with 4-bit ADC resolution and different oversampling factors. The figure is a copyright of [31] and was used with permission.

The best MLSE performance was found when the system used a 4-bit ADC. Further increase in the resolution did not show any improvement in the system. But in case of number of samples in a bit (oversampling factor), the system did perform poorly when “1” or even “2” samples per bit were used (As shown in Figure 1.5). The ideal fit was chosen when 3 samples per bit was performed under an ADC with a resolution of 4 bits. The transmission on the Leaf fiber was extended to 960 Km for a BER of  $10^{-3}$ .

MLSE can also be used to equalize nonlinear impairments caused by the fiber. Such system can use the Volterra approach to cancel the effects caused by the SPM and XPM. In [35], a raised-cosine NRZ-OOK signal at a rate of 10 Gb/s was used to propagate along a single mode fiber with a dispersion coefficient of  $D=17$  ps/nm/km and a nonlinear coefficient of  $\gamma = 2.4432/W/Km$ . Again, a 16 state MLSE device employing the Volterra approach was used at the receiver’s side to offset ISI and nonlinearity.



**Figure 1. 6: BER for MLSE receiver with linear channel estimator and nonlinear channel estimator based on 3<sup>rd</sup> order Volterra theory. The figure is a copyright of [35] and was used with permission.**

In Figure 1.6, three different receiver's performances are compared. When it comes to the classical adaptive threshold (AT) method, the maximum fiber length that could be used while operating at the  $10^{-3}$  penalty error was approximately 205 Km. The linear MLSE performance was capable to extend the fiber to an approximate length of 235 Km. After that point, nonlinear impairments became visible as the linear MLSE was not able to estimate the signal any further. However, when applying the nonlinear MLSE (Volterra approach), such system proved to somehow compensate drawbacks caused by the Kerr effects (XPM and SPM). While obeying the  $10^{-3}$  error penalty, the SMF was able to reach the 300 Km mark; thus giving a remarkable benefit over the linear MLSE with a 65 Km advantage.

### 1.5.2 Decision-Feedback Equalizer and Feed-Forward Equalizer

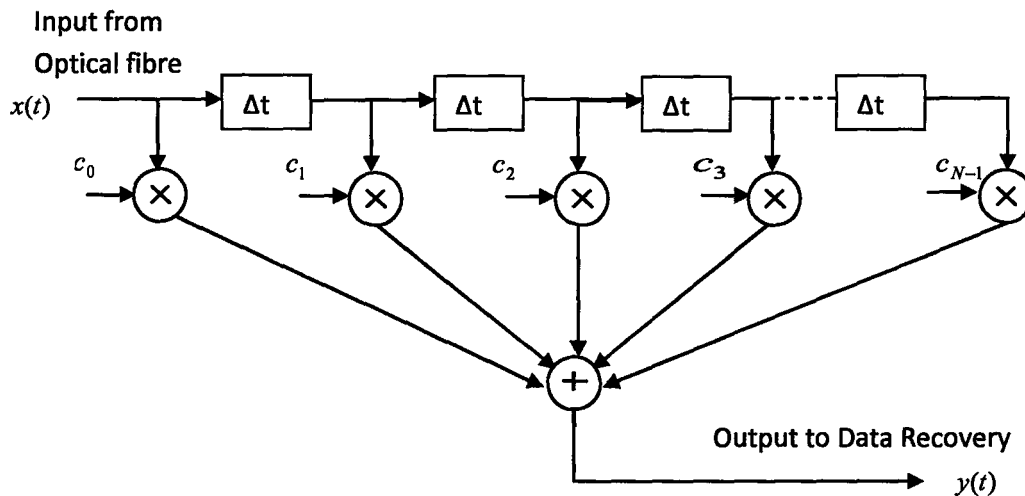


Figure 1. 7: A schematic representing a  $k$ -Tap Feed-Forward Equalizer

At a transmission rate of 10 Gb/s, the propagation of a signal along an optical fiber would force significant penalties for links over 60 Km in length [36], that's when assuming the existence of fiber impairments (nonlinearity, noise, dispersion...) and no accessibility of compensation methods. Alternative methods can be used to offset fiber impairments, in which one is showed in Figure 1.7. Such a compensation scheme is called a Feed-Forward equalizer (FFE) [30],[38],[41] and the input output relationship of this device is given by:

$$y(t) = \sum_{k=0}^{N-1} c_k x(t - k\Delta t) \quad (1.2)$$

where  $c_k$  represents the tap-coefficient of the equalizer,  $\Delta t$  represents the delay between the taps, while  $N$  denotes the number of taps available in the FFE. The diagram shown in Figure 1.7 represents a general schematic of a linear channel. With the right choice of number of taps and the tap coefficients, one can use this equalizer as a replica to represent any linear channel. Furthermore, one can choose tap coefficients for the equalizer to act like the inverse of a specified channel. In our case, the FFE can be used to undo the linear dispersion effect caused by the fiber. Now several algorithms can be applied to ensure convergence of the tap-coefficients for the required behaviour, where one could state the least mean-square (LMS) which is a method aimed at producing the least square error difference between the desired and the actual signal. Or the recursive least squares (RLS) algorithm, in which one would find the minimum of the sum of the absolute square of the error signal recursively.

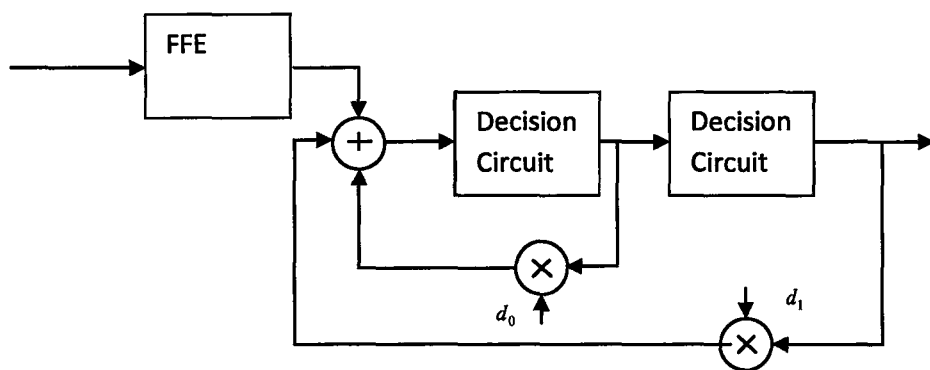


Figure 1. 8: A two-stage Decision-Feedback equalizer trailed by an FFE [36].

An additional electrical compensator that one could use is the decision-feedback equalizer (DFE) [20],[21]. It is capable to handle distortions drawn by the fiber. It consists of a decision circuit within the forward branch, and depending on the previous bit emitted, the DFE applies on the current bit an amount controlled by  $d_i$  (analog control voltage) which helps eliminate postcursors drawn by the ISI. In some cases, having just an FFE as an equalizer is enough to ensure chromatic dispersion reduction. However, studies have confirmed that joining the DFE and the FFE (as shown in Figure 1.8) have helped reducing the fiber impairments even more. In [36], analyses were performed in which the behaviours of both FFE and FF-DFE (Feed-forward decision-feedback equalizer) were compared. The signal was a non-return-to-zero (NRZ) pseudorandom binary sequence (PRBS) with a length of  $2^{31}-1$ , transmitted at a rate of 10 Gb/s along a single mode fiber with a dispersion factor of  $D=16\text{ps/km.nm}$ . The input launch power was fixed to a value ensuring minimum nonlinear/dispersion impairments. When using only an FFE (13 taps) and maintaining a  $10^{-9}$  error penalty, the results have shown that the required optical signal to noise ratio (OSNR) was found to be 27 dB for the system to reach a 140 Km of fiber length. Meanwhile when using a 5 tap FFE and one stage DFE, the OSNR was found to be 23 dB while maintaining the same length. Furthermore, the system was able to reach the 160 Km mark of fiber length by maintaining an OSNR of around 26 dB.

### 1.5.3 Performance comparison of the electrical compensators

Due to the different impairments that the optical fiber presents, it would be reasonable to compare the performances of the MLSE on one side, and the FFE-DFE on the other. Chromatic dispersion, Polarized-Mode dispersion and nonlinearities (SPM, XPM) will be taken into consideration separately and/or jointly. In [38], a pseudorandom bit sequence of length  $2^{31}-1$ , NRZ-OOK modulated, was transmitted at a rate of 10.664 Gb/s through a single mode fiber. The dispersion coefficient was  $D=17\text{ps/km.nm}$ , and the optical launch power into the fiber was kept at 0 dBm in order to avoid fiber nonlinearities. Two stages of band pass filters were used to minimize the Amplified spontaneous emission (ASE) noise caused by the amplifier. At the fiber output, a photodiode (direct-detection) will be used to convert the optical signal into electrical, after which the outputted current will proceed to the electrical compensator to get estimated. The MLSE module consists of a 4 state Viterbi decoder and it uses the blind channel estimation, which means that no training sequence is required. A 3 bit A/D converter is employed to the MLSE, and it uses twofold oversampling (2 bits/symbol) of the received signal. On the other hand, a 10 Gb/s multi-tap FFE-DFE receiver is employed and it uses the blind equalization technique. The number of taps and stages in the FFE-DFE were chosen to provide a best fit to the system.

In the case of chromatic dispersion, the performance of the two equalizers were matching for the first 60 Km of fiber length, all while maintaining an error below  $10^{-3}$ . However,

for a fiber length of 105 Km, the MLSE outperformed the FF-DFE equalizer with an OSNR penalty of approximately 3.2 dB.

Although the PMD effect will not be taken into consideration in this thesis, it would be convenient to mention the performance of the equalizers with respect to the differential group delay (DGD), which is an effect that directly relates to the PMD. The MLSE receiver gave the best performance by tolerating 63 ps of DGD with an OSNR penalty of 2 dB. Furthermore, the authors show how the presence of CD could even increase the tolerance PMD. In summary, the MLSE was found to behave more efficiently when compared to the FF-DFE model for the given direct-detection system, in the case of CD and PMD.

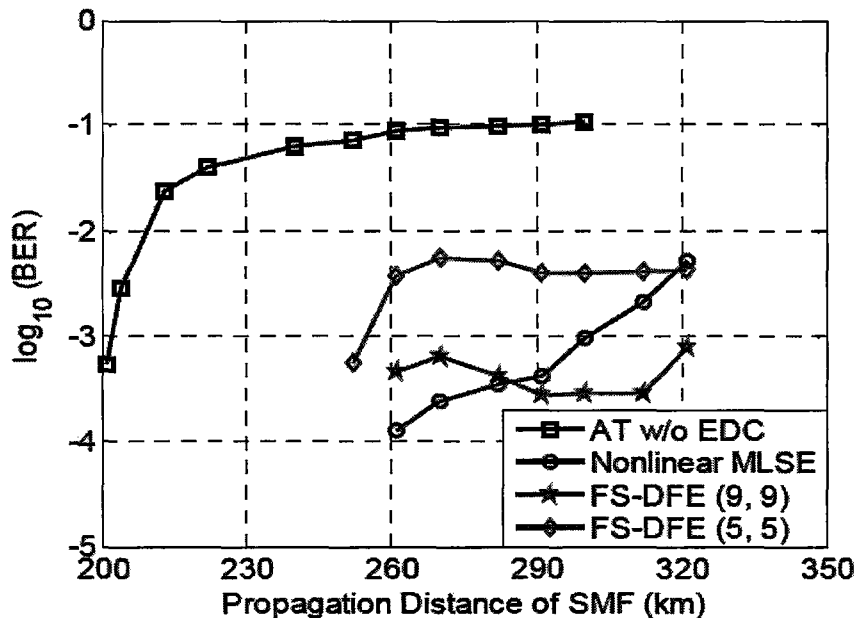


Figure 1. 9: BER for MLSE receiver with nonlinear channel estimator and FS-DFE with nonlinear Volterra canceller. The figure is a copyright of [35] and was used with permission.



We go on to compare the nonlinear cancellation applied by the Volterra approach given in [35] for the two different equalizers. The parameters of the signal and channel were mentioned previously. We have a 16 state nonlinear MLSE with blind channel estimation, connected with a 4-bit resolution ADC to digitize the current with two samples per bit. We also have a Fractionally-spaced decision feedback equalizer (FS-DFE) with blind channel estimation (same as FFE-DFE), it has an oversampling factor of two.

When transmitted through a single mode fiber with dispersion coefficient of  $D = 17 \text{ ps/km.nm}$  and nonlinear coefficient of  $\gamma = 2.4432/W/\text{Km}$ , the nonlinear MLSE was able to reach the 300 Km mark while staying below a  $10^{-3}$  error penalty (Figure 1.9). In case of an FS-DFE with an FFE delay of 5 taps and a Volterra canceller memory of 5, the fiber transmission reached an approximate 255 Km of fiber length. However when increasing both the delay and memory of the FS-DFE to 9, it leads to a fiber transmission of 320 Km.

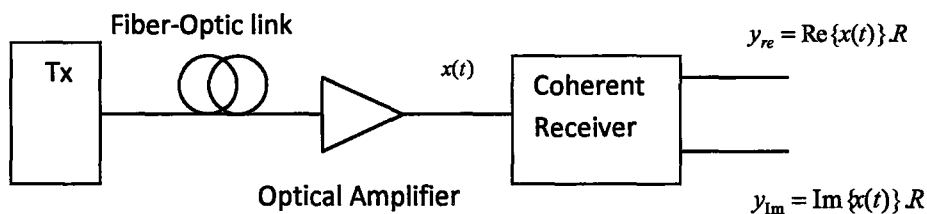
In conclusion, the MLSE receiver outperformed the FFE-DFE system in case of chromatic dispersion and PMD. However, in the presence of nonlinearity and with the appropriate choice of taps for the FFE and the Volterra canceller, the FFE-DFE proved to handle the Kerr effects much more efficiently. One might add that the MLSE system has the disadvantage of being computationally extensive, especially in the case of long haul transmissions.

## 1.6 Coherent Detection

In case of a direct-detection system, the optical transmitted signal loses some of its information after photo-detection. For instance, let the optical field envelope at the output of the fiber be  $x(t)$ . The corresponding photo-current  $y(t)$  at the receiver's side is:

$$y(t) = |x(t)|^2 \cdot R \quad (1.3)$$

where  $R$  is the responsivity of the photo-detector. Since the phase of the optical signal is lost in a direct-detection system, it is not possible to transmit phase modulated signals such as phase-shift keying (PSK) or quadrature amplitude modulation (QAM). Also, the direct detection process introduces nonlinearity in (1.3). Therefore, linear dispersive effects cannot be undone by the simple linear equalizers in the electrical domain. Instead, one can implement a coherent detector [29],[33],[37] at the receiver's side such that the electrical signal is directly proportional to the complex optical field envelope.



**Figure 1. 10: Schematic depicting Coherent Detection in a fiber-optic system**

Ideally, a coherent detector has the capability to output both the real and imaginary part of the optical signal, and that is done with the help of a local oscillator operating at a frequency matching the carrier frequency of the signal. In Figure 1.10, the output optical signal is  $x(t)$ . Assuming an Ideal coherent receiver, the output electrical signal  $y(t)$  is given by:

$$y(t) = R(\text{Re}(x(t)) + i\text{Im}(x(t))) = Rx(t) \quad (1.4)$$

where  $R$  is the responsivity of the photo-diode. From (1.4), we find that the output electrical signal is directly proportional to the optical field envelope. Coherent detection will be explained in much more detail at a later chapter.

## 1.7 Modulation Format

The efficiency of an optical communication system is highly dependable on the way the signal is usually modulated. We here will discuss two modulation formats: on-off-keying (OOK) and Phase shift keying (PSK).

OOK transmission is a classical method of modulating a signal, and is considered to have the simplest generator. In case of a binary OOK, such modulation is carried out by usually assigning a pulse (Gaussian, raised-cosine etc...) when a bit “1” is transmitted, meanwhile assigning no pulse when a “0” is transmitted. OOK is similar to the standard amplitude modulation used in commercial broadcast systems and therefore, a simple envelope

detector or square-law detector can be used at the receiver. A drawback of the system of the scheme is that 50 % of transmitter power is wasted as the carrier power.

Alternatively, the Phase-shift keying (PSK) technique cut some of the drawbacks that the OOK system possesses. It is usually carried away by changing the phase of the carrier signal. In case of a Binary PSK, such modulation could be applied by applying a  $0^\circ$  phase shift to the carrier in case a “1” bit was transmitted, or a  $180^\circ$  phase shift in case of a “0”. For system based PSK, the SNR required to reach a fixed BER is 3 dB less than that for ASK systems. However, a more complex coherent receiver has to be used to retrieve the phase information.

The authors in [34] have compared the performance of two systems using different equalizers, where one was using a non-return-to-zero (NRZ) OOK modulation, meanwhile the other was using an NRZ binary differential-phase-shift keying signal (BDPSK). BPSK has a slight advantage over BDPSK (0.5 dB penalty) at a BER of  $5.10^{-4}$ , for that one might consider such comparison. Various electrical compensators were used to study the effect of CD and PMD, however we will only state the results of the MLSE receiver (Best case). Both signals were transmitted at a rate of 10 Gb/s through a single mode fiber with a dispersion coefficient of  $D=17$  ps/nm/km. While maintaining an error penalty below  $5.10^{-4}$ , the required OSNR for an OOK system was approximately 14 dB for the system to reach 240 Km of fiber length. Meanwhile, it only took an approximate 11.6 dB for the DPSK modulated signal to reach the same fiber length.

Moreover, the OOK modulated system could tolerate an approximate 75 ps of DGD (PMD) at an OSNR rate of 12 dB. However, the DPSK modulated signal was able to tolerate the same amount of DGD with an OSNR rate of 10 dB.

Different other modulation formats could also be used in an optical communication system (QAM, Duobinary etc...). However, a PSK modulated signal is ideal for our purposes, as it will be the modulation to be used.

## **1.8 Thesis Contribution**

Various topics have been discussed up to this point. The history of fiber-optic communication was briefly stated, in addition to the improvements that it has given to the general field of communications. The impairments that are present in the fiber channel cause a serious threat to the signal. For this reason, different methods of compensations were stated, in which we divided into two parts. The optical compensation method was discussed, wherein several compensation schemes were presented. Nevertheless, such a technique does introduce a matter of complexity and cost increase to the system. As a result, one can switch his main interest to the electrical compensation methods. They provide a factor of simplicity to the optical system, and with the fact they're rather cheaper than optical devices, one can rely on those techniques to build a system that is relatively efficient.

In case of a direct-detection system, a variety of electrical compensators were presented. However, best case scenario schemes could only propagate the signal to up to just a few hundred kilometres in fiber length. In this thesis, a direct-detection method using a frequency-shifter and a linear equalizer is proposed. To our best knowledge, such system will be the first to pass the one thousand kilometre mark in fiber length. In the proposed scheme, the detection process becomes linear with the help of the frequency shifter. As a result, a simple linear equalizer in the electrical domain can undo the distortion due to the linear dispersive effects.

In case of a coherent detection, many researchers lately have started adopting such method due to the many benefits that it provides. A scheme using the Volterra approach is proposed. A nonlinear equalizer is used at the receiver's side, and the trade-offs will be discussed with regard to compensating nonlinearity. When compared to the linear equalizer, the nonlinear compensator will not provide an advantage. For that, a DCF will be introduced to limit some of the nonlinearity effect caused by the XPM. Moreover, we will show that a linear equalizer could actually offset nonlinear impairments caused in the channel.

## **1.9 Thesis Outline**

The introductory chapter was devoted to discuss a general aspect of the optical fiber communication systems. The optical fiber advantages and disadvantages were stated, and

the different methods of compensation were given. A literature review of the field so far was presented, compared and discussed; where in the thesis contributions to the field were stated.

Chapter 2 will be devoted to give a theoretical background of the optical communication system. The constraints on transmitting an optical signal through a fiber will be examined. All the fiber impairments that were mentioned previously will be discussed thoroughly in that section. Mathematical models for the optical fiber will be given, and in some cases derived.

Chapter 3 of this thesis will present the main contribution of the research accomplished. A direct-detection system model using a frequency shifter will be given, as both the transmitter and the receiver part will be thoroughly discussed. Assumptions will be made and stated. Design trade-offs will be analyzed, as this would mark to our knowledge a first direct-detection system that could exceed the one thousand kilometre mark in fiber length, therefore passing older schemes by a wide margin.

Chapter 4 will be dedicated to offer a second and final part of the research contribution. A coherent detection system model is given, and comprehensively examined throughout different stages. The receiver equalizer will adapt the Volterra approach that was performed by [35] on a direct-detection system, and the performance will be compared to that of a linear equalizer. Optical compensators will be used when needed, and once again, design trade-offs will be examined regarding nonlinear compensation.

Chapter 5 will conclude the research done in this thesis, as possible future works will be proposed.



# Chapter 2

## Channel model

The purpose of this chapter is to give a background on optical fiber communication systems, provide the various properties when using such a channel, as well as state the constraints and drawbacks that it includes

### 2.1 Light Propagation in Fiber

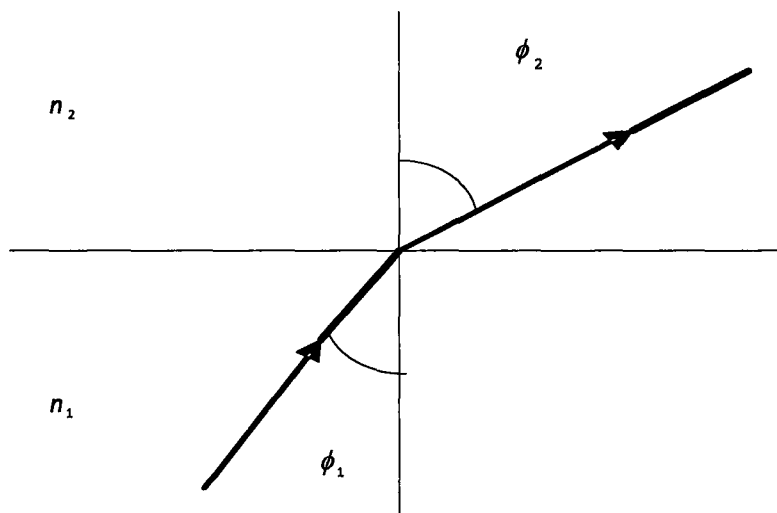


Figure 2. 1: Interface between two dielectrics

Let the interface between two dielectrics be as shown in Figure 2.1. From snell's law we have:

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 \quad (2.1)$$

where  $n_1$  and  $n_2$  are the refractive indexes of the dielectrics. Let  $n_1 > n_2$ , from (2.1) it follows that  $\phi_1 > \phi_2$ . As the angle of incidence  $\phi_1$  increases, the angle of refraction  $\phi_2$  increases as well. For a particular incident angle  $\phi_1 = \phi_c$ , the refractive angle becomes  $\phi_2 = 90^\circ$ . (2.1) becomes:

$$\sin \phi_c = n_2 / n_1 \quad (2.2)$$

$\phi_c$  is called the critical angle, and if the angle of incidence is increased beyond that point, the incident ray is completely reflected. This phenomenon is called "Total internal reflection".

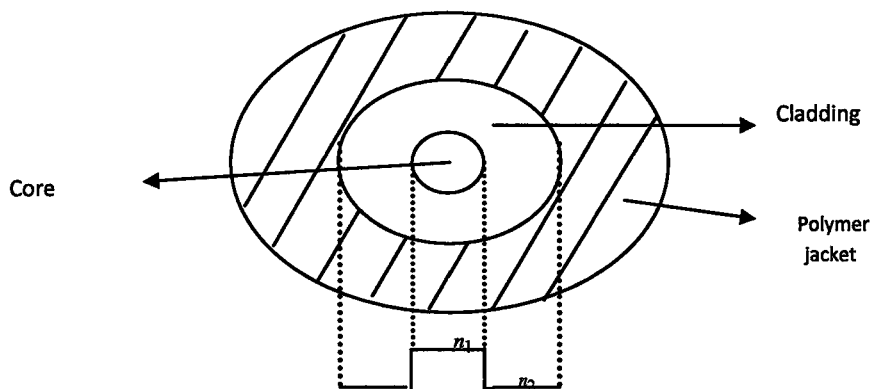
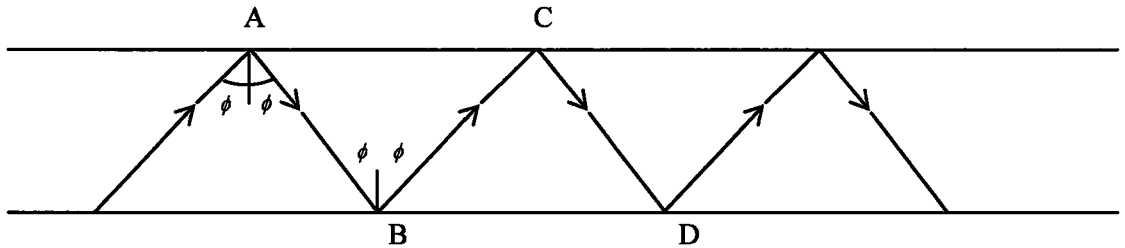


Figure 2. 2: Step Index Fiber



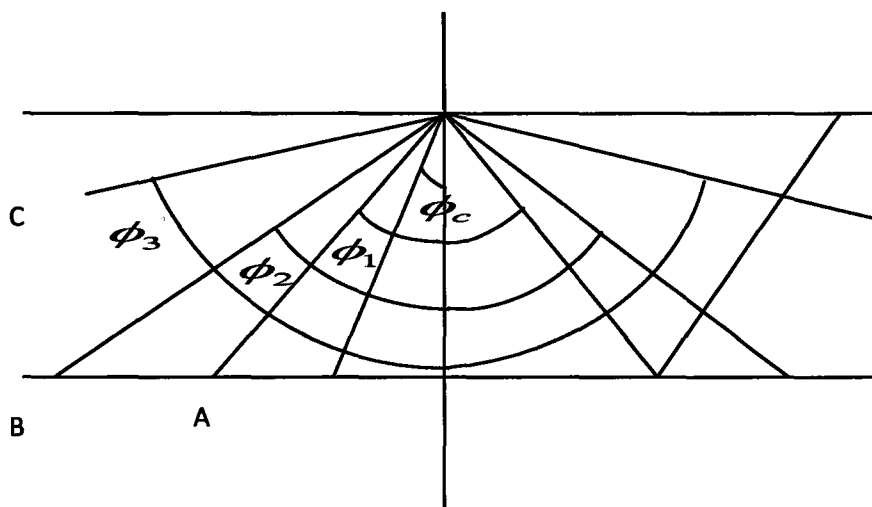
**Figure 2.3: Ray Propagation in Fibers**

Now consider a step index fiber as in Figure 2.2. The core index  $n_1$  is larger than the cladding index  $n_2$ . If a ray is with an angle  $\phi > \phi_c$  is transmitted, Figure 2.3 shows that it will undergo total internal reflection at A. The reflected ray will also undergo total internal reflection at B and so on. The process carries on until the ray is propagated till the end of the fiber. This process is an efficient way to transmit a signal, since the efficiency of the reflection is nearly 100%. Although if  $\phi < \phi_c$ , this case is always associated with a refraction, making the efficiency of the reflection way smaller.

## 2.2 Fiber Modes

### 2.2.1 Multi-Mode Fibers

As we saw earlier, a ray would undergo total internal reflection if it was transmitted within the range of  $\phi_c < \phi < \pi/2$ . If  $\phi = \pi/2$ , this just means that the ray will not experience any sort of reflection, thus explaining the upper bound of the range. One could transmit infinite amount of rays at the same time through a fiber. Nevertheless, the wave optics theory states that only discrete angles in the range of  $\phi_c < \phi < \pi/2$  are allowed.



**Figure 2. 4: Multiple rays propagating in a Fiber**

Consider the sketch in Figure 2.4.  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  are all discrete angles and they are larger than the critical angle  $\phi_c$ . Each of those angles corresponds to a guided mode. A multi mode fiber (MMF) is a fiber that supports multiple modes, or in ray-optics language, multiple discrete angles. They are mainly used to transmit signals over a short distance (just a few kilometers). Their main drawback is that they cause modal dispersion leading to the pulse broadening. Meaning, suppose that two Gaussian pulses A and B are launched, as shown in Figure 2.4. The Gaussian pulse at A will take the longer route to reach the end of the fiber, reflecting on the boundaries of the core way more than the pulse emitted at B. At the output, different components of the signal might arrive at different times for both A and B; thus altering the shape of the output.

### 2.2.2 Single Mode Fiber

As mentioned before, multi-mode fibers are used for small distance optical communication purposes. To overcome the modal dispersion that is created by those fibers, one could transmit only one signal through the fiber. In this case, such optical systems could have a significant reach in transmission length. As a matter of fact, a fiber could be designed in order to support only one ray. A  $V$  parameter is defined  $V$  as

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2} \quad (2.3)$$

with  $\lambda$  being the wavelength of the ray,  $a$  being the core diameter, and  $n_1$  and  $n_2$  are respectively the refractive indexes of the core and the cladding of the fiber. If  $V < 2.405$ , then the fiber supports only one mode (i.e. single mode fiber).

In this thesis, our only focus will be on long distance transmission systems. As a result, we will not be dealing with multi-mode fibers as they don't acquire a far fiber reach.

## 2.3 Fiber Losses

When a ray is transmitted through an optical fiber, the signal power could suffer some losses due to many reasons. In one case, if the emitted beam has an angle that is below the critical angle  $\phi_c$ , it will be absorbed by cladding of the fiber. Another explanation of the

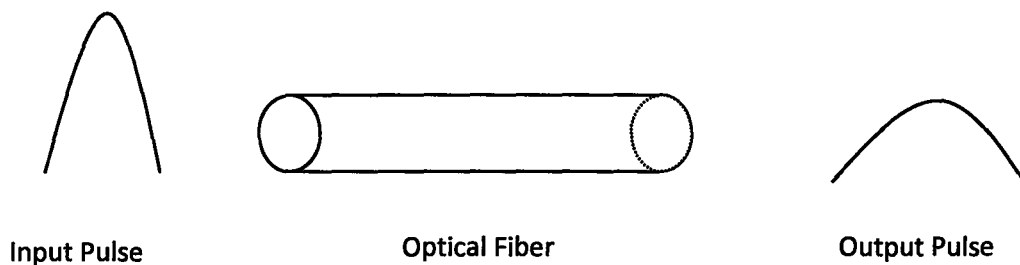
loss of the fiber would be due to the ‘Rayleigh scattering’. When manufacturing a fiber, some density fluctuations freeze into the silica fiber, making the optical light to scatter in all directions. Other causes would be because of material absorption due to glass (found in pure silica), or water vapors and metal (found in non pure silica). Let an optical signal of power  $P_{in}$  be the input of a fiber of length  $L$ . The power at the output is given by:

$$P_{out} = P_{in} \exp(-\alpha L) \quad (2.4)$$

where  $\alpha$  is the attenuation constant measuring the total fiber losses from all possible causes. Expressing (2.4) in dB yields:

$$\alpha_{dB} = -\frac{10}{L} \log\left(\frac{P_T}{P_0}\right) = 10\alpha \log(e) = 4.343 \times \alpha \quad (2.5)$$

## 2.4 Fiber dispersion



**Figure 2. 5: Pulse broadening due to Dispersion**

Let a Gaussian pulse be transmitted through an optical fiber, as shown in Figure 2.5. At the output, the pulse witnesses some broadening. In case of a single mode fiber, this can be explained as a wavelength dependence of the group velocity  $v_g$  on the optical

frequency  $\omega$ . This is known as group velocity dispersion (GVD), and such effect causes components of the optical signal to travel at different group velocities. We here define the propagation constant  $\beta$ , and is related to the group velocity  $v_g$  using

$$\frac{1}{v_g} = \frac{d\beta}{d\omega} \quad (2.6)$$

After the propagation of the signal along a length of fiber  $z$ , the broadening of the signal will be given by

$$\Delta T = \frac{d^2\beta}{d\omega^2} z \Delta\omega = \beta_2 z \Delta\omega \quad (2.7)$$

where  $\beta_2$  is the dispersion coefficient and  $\Delta\omega$  is the spectral width of the signal. Finally, the dispersion parameter  $D$  is given by

$$D = -(2\pi/\lambda^2)\beta_2 \quad (2.8)$$

where  $\lambda$  is the wavelength of the signal.  $D$  is the chromatic dispersion (CD) parameter, and is usually used to define the amount of dispersion in the fiber. For a standard single-mode fiber,  $D = 17$  ps/km/nm. Fiber dispersion plays a big role in altering the input pulses. Another problem that arises from CD is the intersymbol interference (ISI), meaning that if two pulses were to be transmitted, the effect of one broadened pulse does not fade away completely before the transmission of a next one. One would want to

mention the effects of the polarized-mode dispersion (PMD) in the fiber. It could be briefly defined as dispersion caused when the light travels through a fiber with a core that is not perfectly circular. In that case, the  $x$  and  $y$  polarizations of the light would travel at different speeds, mainly due to the imperfections that are present in the fiber. However, the PMD effects in the research will not be taken into consideration, as they will be neglected.

## 2.5 Nonlinearities

In a response to an electrical field emitted, a medium changes its refractive index due to the occurrence of nonlinear polarization. This is known as the Kerr Effect. In case of an optical fiber, self-Phase modulation (SPM) occurs when a light pulse provokes a nonlinear phase delay due to change in the refractive index. That change is directly related to the optical intensity given by

$$n(\omega, I) = n_o(\omega) + n_2 I(t) \quad (2.9)$$

with  $I$  being the intensity of the pulse shape,  $n_o$  is the linear refractive index and  $n_2$  is the second order nonlinear refractive index of the fiber. Consequently, the nonlinear phase shift  $\phi_{NL}(t)$  is given by the following:

$$\phi_{NL}(t) = (2\pi / \lambda) n_2 I(t) L \quad (2.10)$$



where  $L$  is the length of the fiber and  $\lambda$  is the wavelength of the field. (2.10) tells us the nonlinear phase shift depends mainly on the pulse shape and the fiber length. Since the phase shift varies with time, optical pulses become chirped. Not to mention that the spectrum of the pulse could change when propagating along a fiber. In the case of signals transmitted successively, a pulse could be responsible of a nonlinear phase shift of a neighboring optical pulse. This is known as the Cross-phase modulation (XPM). Other than altering the shape of the signal, this effect could lead to a channel cross-talk in wavelength-division multiplexed (WDM) systems.

## 2.6 Fiber transfer function

When the output of the signal generator that is operating at frequency  $\omega$  is incident to a linear single mode fiber channel, the optical field distribution can be written in the form:

$$E(x, y, z, t) = \phi(x, y)F(t, z) = \phi(x, y) \int_{-\infty}^{\infty} \tilde{X}(\omega) \exp[-j(\omega t - \beta(\omega)z)] d\omega \quad (2.11)$$

with

- $\phi(x, y)$  being the transverse field distribution,
- $\beta(\omega)$  is the propagation constant,
- $x(t)$  is the input optical temporal distribution,

-  $\tilde{X}(\omega)$  is the Fourier transform of the signal, its defined as

$$\tilde{X}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) \exp(+j\omega t) dt ,$$

-  $F(t, z)$  will yield the incident pulse shape  $x(t)$  at distance  $z$ , where

$$F(t, 0) = x(t) .$$

The propagation constant  $\beta$  is a function of the frequency  $\omega$ , however it is usually hard to calculate it in practice. Therefore, it would be convenient if we expand  $\beta(\omega)$  in a Taylor series around a carrier frequency  $\omega_0$

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \beta_2(\omega - \omega_0)^2 / 2 + \beta_3(\omega - \omega_0)^3 / 6 + \dots \quad (2.12)$$

with

- $\beta_0$  is the propagation constant at  $\omega_0$
- $\beta_1$  is the inverse group velocity.
- $\beta_2$  is the dispersion coefficient.

-  $\beta_{3,4,\dots}$  are higher order dispersive terms.

In this research, the higher order dispersive terms are disregarded. In essence, only the first three terms of (2.12) are used.

Let  $\Omega = \omega - \omega_0$ , therefore :

$$\begin{aligned} F(t, z) &= \int_{-\infty}^{\infty} \tilde{X}(\Omega) e^{-j(\omega_0 t - \beta_0 z) + j\beta_1 \Omega z + j\beta_2 \Omega^2 z / 2} e^{-j\Omega t} d\Omega \\ &= e^{-j(\omega_0 t - \beta_0 z)} \int_{-\infty}^{\infty} \tilde{X}(\Omega) e^{j\beta_1 \Omega z + j\beta_2 \Omega^2 z / 2} e^{-j\Omega t} d\Omega \\ &= e^{-j(\omega_0 t - \beta_0 z)} \int_{-\infty}^{\infty} \tilde{X}(\Omega) H_1(\Omega) e^{-j\Omega t} d\Omega \end{aligned}$$

with  $e^{-j(\omega_0 t - \beta_0 z)}$  being the carrier wave, and  $\int_{-\infty}^{\infty} \tilde{X}(\Omega) H_1(\Omega) e^{-j\Omega t} d\Omega$  being the envelope.

The linear fiber transfer function is given by:

$$H_{linear}(\Omega) = \exp(j\Omega\beta_1 z + j\Omega^2\beta_2 z / 2) \quad (2.13)$$

Setting  $\beta_2 = 0$  in (2.13),  $F(t, z)$  becomes:

$$F(t, z) = e^{-j(\omega_0 t - \beta_0 z)} \int_{-\infty}^{\infty} \tilde{X}(\Omega) e^{j\beta_1 \Omega z} e^{-j\Omega t} d\Omega = e^{-j(\omega_0 t - \beta_0 z)} x(t - \beta_1 z) \quad (2.14)$$

(2.14) tells us that when the fiber is dispersion free ( $\beta_2 = 0$ ), the optical output would be just a delay of  $\beta_1 z$  of the input pulse.

In reality, the fiber channel is considered to be nonlinear. Adding the SPM and XPM nonlinear effects to the channel, the output  $X_{out}(\Omega)$  to a signal  $X_{in}(\Omega)$  transmitted into the nonlinear SMF would be [40]:

$$\begin{aligned}
 X_{out}(\Omega) = X_{in}(\Omega, z) = & \\
 X_{in}(\Omega)H_{linear}(\Omega, z) & \\
 + \iint X_{in}(\Omega_1)X_{in}^*(\Omega_2)X_{in}(\Omega - \Omega_1 + \Omega_2)H_{nonlinear}(\Omega_1, \Omega_2, \Omega - \Omega_1 + \Omega_2, z)d\Omega_1d\Omega_2 & \quad (2.15)
 \end{aligned}$$

where  $H_{nonlinear}$  is the third order nonlinear transfer function of the fiber.

## 2.7 Photodetectors

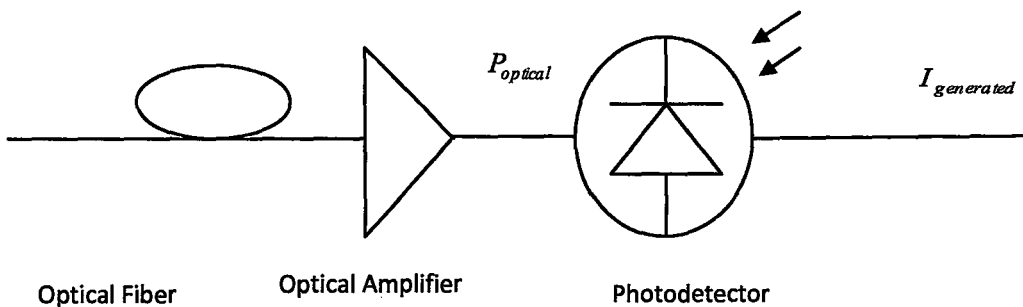


Figure 2. 6: Photodetector in an optical communication system

Following the fiber link, the optical output is then passed through a photodiode converting it into an electrical signal. The responsivity of a photodiode is defined as the ratio of the current generated over the incident optical power, i.e.:

$$R = \frac{I_{generated}}{P_{optical}} \Leftrightarrow I_{generated} = R.P_{optical} \quad (2.16)$$

After photo-detection, the current will head to the receivers side. Nonlinearity and dispersion compensation in the optical side have been found to be really complex, in addition to the fact that they are costly. Using photo-detection, cheap electrical systems at the receivers side could be used to repair the signal that was altered because the many factors (noise, nonlinearity, dispersion, power loss). Although not as efficient as optical compensation, electrical systems have proved to be a really good alternative when it comes to cost and outcome.

## 2.8 Noise

When an optical ray is emitted through a fiber, the signal suffers a power loss at the output. For that reason, an optical amplifier is usually introduced at the output end to compensate for such a loss. This action comes at a cost. Once the signal is compensated, amplified spontaneous emission (ASE) noise is introduced to the signal. We assume that the ASE is modeled as additive white Gaussian noise (AWGN) with the following power spectral density:

$$PSD_{noise} = n_{sp}(G-1)hf$$

where

$n_{sp}$  is the spontaneous emission parameter.

$G$  is the Gain of the amplifier.

$hf$  is the photon energy .

$h$  is Planck's constant ,  $h = 6.625e^{-34}$  J.s .

$f$  is the frequency of the carrier .

Other factors contribute to the noise in the communication system. At the fiber end, the optical output heads to the photo-detector for it to get converted into an electrical signal. Consequently, shot noise is introduced to the current due to random generation of electrons. Let the current output of the photo-detector be:

$$I(t) = I_1 + i_s(t) \quad (2.17)$$

The first term of (2.17) represents the deterministic part of the signal (proportional to the output power of the optical signal). The second term is the shot noise current, and it's a random variable with zero mean and an average noise power of:

$$N_s = \sigma_s^2 \cdot R_L \quad (2.18)$$

where  $\sigma_s^2$  is the shot noise variance

$R_L$  is the load resistance.

In a real life optical communication system, the output current could even rely on the nature of its surrounding. More electron current could be generated with a temperature increase. Since the motion of electrons is random, another type of current is added to the system and it's called the thermal noise. Its average noise power is given by:

$$N_T = 4k_B T \Delta f / R_L \quad (2.19)$$

with  $k_B$  is the Boltzmann's constant.

$T$  is the absolute temperature.

$\Delta f$  is the bandwidth of the signal.

In this thesis, we will assume that the amplified spontaneous emission (ASE) noise will be the dominant factor in the channel. In essence, shot noise and thermal noise will be considered to be negligible.

# Chapter 3

## Significant reach enhancement for a direct-detection system

This chapter will include a scheme utilizing an offset optical carrier transmission and adaptive linear equalizer in electrical domain to enhance the transmission reach of a direct detection optical transmission system. In the proposed scheme, the detection process becomes linear and thereby, a simple linear equalizer in electrical domain can undo the distortion due to linear dispersive effects.

### 3.1 Introduction

Various methods have been proposed in order to compensate the dispersion of the optical fiber in the electrical domain for direct detection systems [32],[43]. Direct detection receivers are known for their simplicity to build and operate, not to mention a lower cost. To compensate for fiber dispersion in direct detection systems, linear adaptive equalizers can be used. As mentioned before, a linear equalizer consists of delay and add circuits and

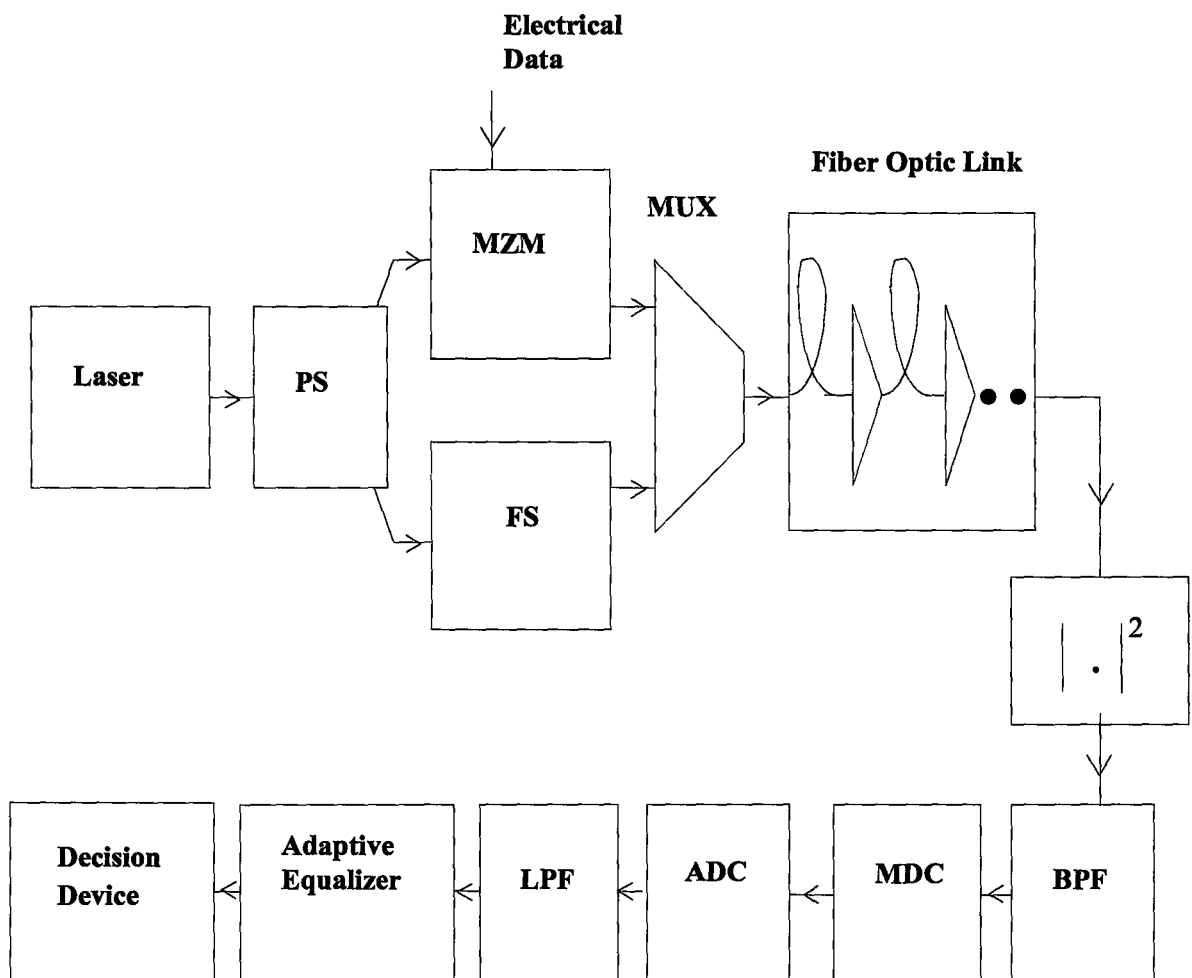


is simple to implement, but it is not very effective in compensating fiber dispersion because of the nonlinearity introduced by the detection process. In contrast, Maximum Likelihood Sequence Estimation (MLSE) was found to be quite effective, but computationally extensive. Commercially available 4-state MLSE can compensate for the dispersion of 114 Km of standard single-mode fiber (SMF) using non- return to zero (NRZ) format and 214 Km of standard SMF using duo-binary format [44]. To extend the transmission distance further, a large number of states would be required and online computation would necessitate a large amount of computational resources. In this chapter, a scheme to compensate for the fiber dispersion in a direct detection system using a simple linear equalizer is proposed. An optical carrier with its frequency shifted by a certain amount from the mean frequency of the message signal is added at the transmitter so that the detection process at the receiver becomes linear after the electrical band pass filter.

### **3.2 System Design**

The output of a laser is split into two parts using a power splitter, as shown in Figure 3.1. The optical carrier at the upper arm is modulated by 10 Gb/s NRZ data using a Mach-Zehnder modulator (MZM) and the frequency  $f_c$  of the optical carrier at the lower arm is shifted by acousto-optic or electro-optic frequency shifter. The output of the MZM and the frequency shifter are combined using a wavelength division multiplexer (MUX) and the combined signal is transmitted through a fiber-optic link. The output of the fiber is

converted into photo-current using a direct detection receiver. The modulated signal and offset carrier beating leads to a signal band centered around the offset frequency  $\Delta f$ . Therefore, a band pass filter centered around  $\Delta f$  is used to obtain the message signal modulating the microwave carrier. After microwave down conversion, the signal passes through the electrical signal processing unit in which filtering and adaptive equalization of dispersion are performed in digital domain. The receiver architecture is similar to that



**Figure 3. 1: System Model.** PS: Power Splitter, MZM: Mach-Zehnder Modulator, FS: Frequency shifter, MUX: Multiplexer, BPF: Band Pass Filter, MDC: Microwave Down Converter, ADC: Analog to Digital Converter, LPF: Low Pass filter

used in direct-detection systems based on OFDM [45]. In conventional direct detection systems, the photocurrent is proportional to the absolute square of the optical field distribution and therefore, the phase information is lost and the detection process is nonlinear. As a result, the linear equalizer in electrical domain that operates linearly on the received photo-current cannot undo the fiber dispersion effectively. In contrast, in the scheme of Figure 1, the output signal after the microwave down converter is directly proportional to the message signal in the absence of noise and distortion. In this case, linear equalizers can completely compensate for fiber dispersion. Let the optical field of the laser be

$$q_c = A_c \exp(i2\pi f_c t) \quad (3.1)$$

where  $f_c$  is the angular frequency of the laser and the optical field after the MZM be

$$q_{MZM} = A_c \sqrt{x} m(t) \exp(i2\pi f_c t) \quad (3.2)$$

where  $m(t)$  is the message signal and  $x$  is the power splitting ratio. The frequency-shifter shifts the laser frequency by  $\Delta f$  and therefore, the combined signal entering the fiber optic link is

$$q_{in} = A_c \exp(i2\pi f_c t) \left[ \sqrt{1-x} \exp(i2\pi \Delta f t) + m(t) \sqrt{x} \right] \quad (3.3)$$

In the absence of distortion and noise, the photocurrent of the receiver can be written as

$$I(t) = A_c^2 \left\{ (1-x) + x|m(t)|^2 + 2\sqrt{x(1-x)} \operatorname{Re}[m(t) \exp(i2\pi\Delta f t)] \right\} \quad (3.4)$$

The first two terms in (3.4) correspond to baseband and the last term leads to a scaled copy of the message spectrum centered around  $\Delta f$ . Let the bandwidth of the message be  $W$ . If the offset frequency  $\Delta f$  is larger than  $2W$ , a band pass filter (shown in Figure 3. 2)

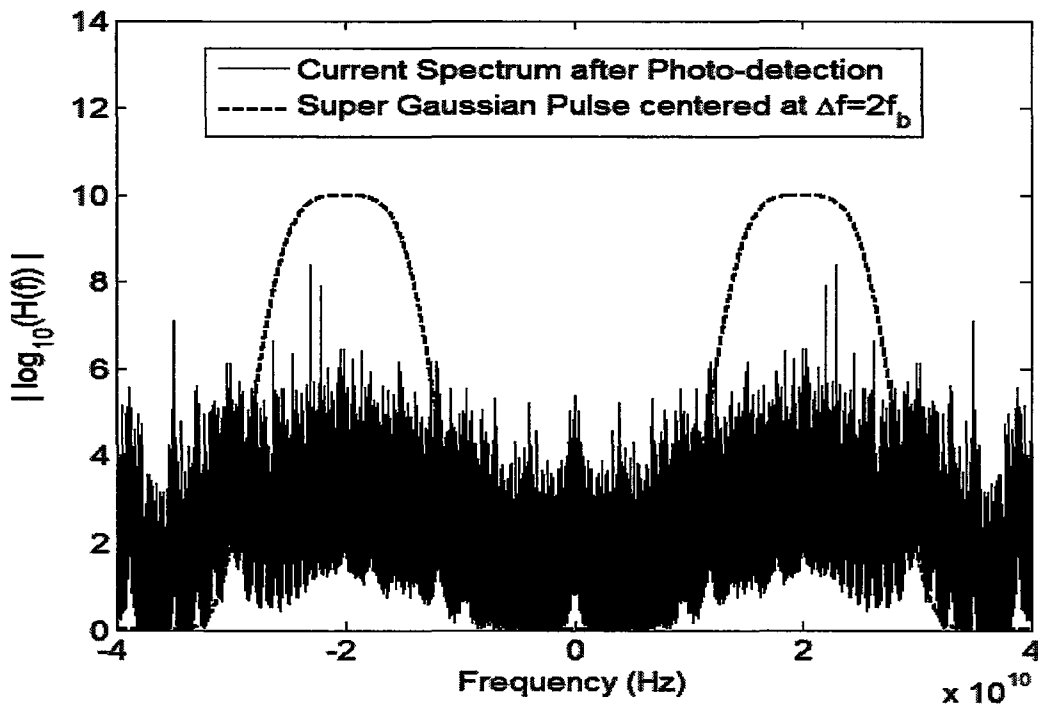


Figure 3. 2: Current spectrum after photodetection passed through a Super Gaussian band pass filter centered at  $\Delta f = 2f_b$ .  $f_b$  is the bit rate

centered around  $\Delta f$  rejects the baseband and with a further microwave down conversion, we obtain the message signal  $m(t)$ . If  $\Delta f$  is smaller than or comparable to  $W$ , it leads to significant intermodulation cross-talk between the second and third terms of (3.4).

If  $\Delta f \gg 2W$ , the offset carrier will interfere with other channels if wavelength division multiplexing (WDM) is used. Therefore, we optimize  $\Delta f$  through numerical simulations. The output of the microwave down converter is converted to the digital domain and is passed through a low pass filter (LPF) to limit the noise. The filtered signal is processed by a linear adaptive equalizer to compensate for fiber dispersion (FFE). The equalizer uses the least mean square (LMS) algorithm [46]. At the equalizer, we have the following

$$y(n) = w^H(n)u(n) \quad (3.5)$$

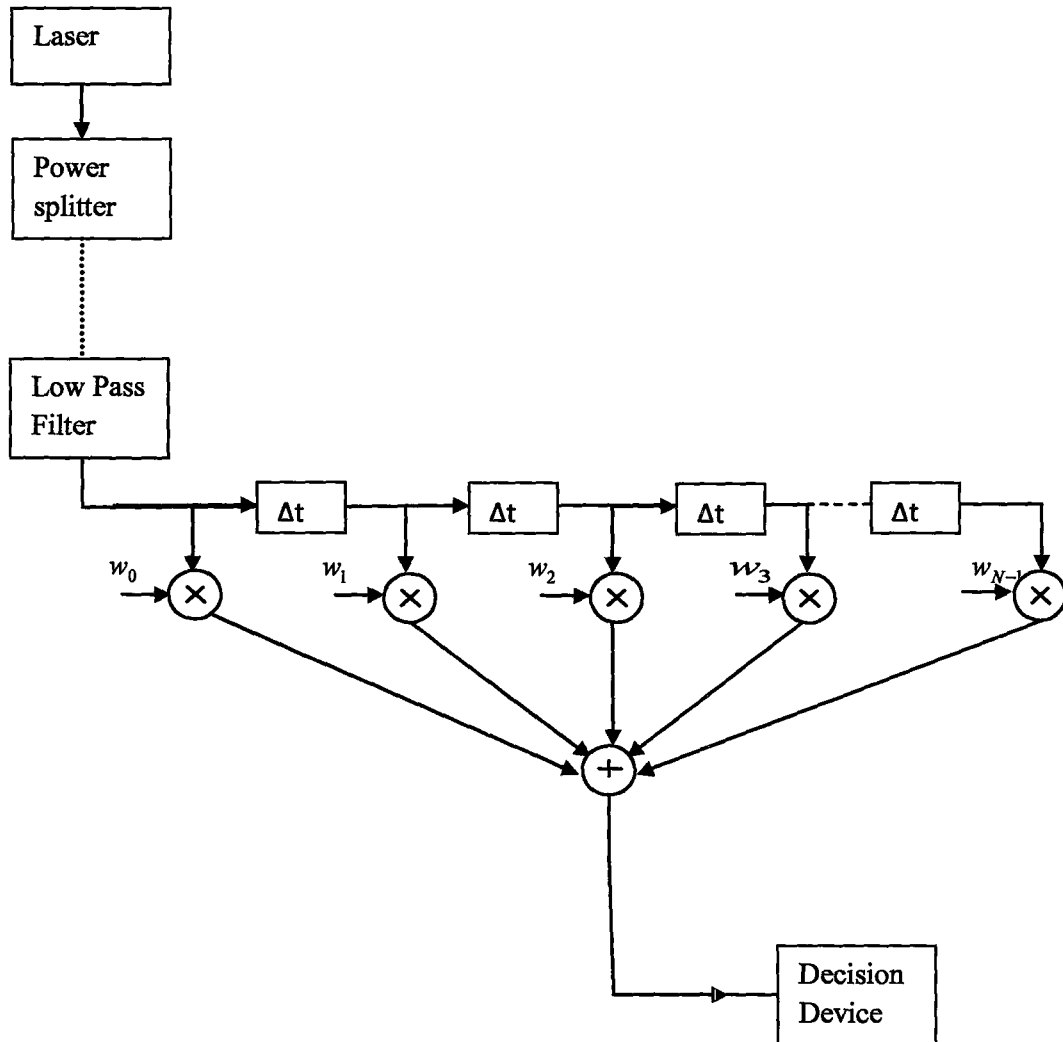
where  $y(n)$  is the output of the equalizer,  $u(n)$  is the input and  $w(n)$  are the tap-weights that represent the linear channel. The superscript  $H$  denotes the Hermitian transpose of  $w(n)$ . Before the system goes online, a training sequence of bits  $d(n)$  known by the receiver will be transmitted. This procedure is implemented so that the tap-weights  $w(n)$  of the equalizer are adjusted accordingly to the channel. The error signal is given by

$$e(n) = d(n) - y(n) \quad (3.6)$$

The output  $y(n)$  of the equalizer will be subtracted from  $d(n)$ , and the error signal  $e(n)$  will be minimized accordingly using the tap weight adaptation equation

$$w(n+1) = w(n) + u(n)e^*(n)\mu \quad (3.7)$$

with  $\mu$  being the step size of the iterations.



**Figure 3. 3: Schematic depicting the linear equalizer in the system model.**

After ensuring the convergence of the tap weights using the least-mean square (LMS) algorithm, the linear equalizer will be setup at the receivers side (as shown in Figure 3.3) to undo the linear effects caused by the fiber.

### 3.3 Simulation Results

We performed the Monte Carlo simulation of a fiber optic link using the well known split-step Fourier technique. The following parameters are used unless otherwise specified: A 80 km long standard single mode fiber (SMF) of dispersion coefficient  $D=17 \text{ ps/nm.km}$  is used between inline amplifiers. Fiber loss is fully compensated by the inline amplifiers. The fiber loss is 0.2 dB/km, the nonlinear coefficient is  $\gamma=1.3/\text{W/km}$  and the spontaneous noise factor  $n_{sp}=1.5$ . Shot and thermal noise are assumed to be negligible, i.e. they will be dominated by the amplified spontaneous noise.

A Mach-Zehnder modulator is used to generate a 10 Gb/s NRZ-PSK signal of  $2^{16}$  bits with a computational bandwidth of 80 GHz. No dispersion compensation is done in the optical domain. At the receiver, 6 GHz electrical low pass filter is used to limit the noise. The number of taps,  $n$ , used in the equalizer was 1000. The major impairments are caused by intermodulation cross-talk, fiber dispersion and nonlinearity. To investigate the impact of intermodulation crosstalk, we first turned off the fiber nonlinearity ( $\gamma=0$ ) and calculated the bit error rate (BER) as a function of transmission distance for various offset frequencies.

When the frequency offset is  $2f_b$  ( $f_b$  being the bit rate), the maximum transmission distance to keep the BER below  $10^{-3}$  is about 500 km, as shown in Figure 3.4. Note that the BER shows an oscillatory behaviour. This is because the intermodulation cross-talk is the dominant penalty in this case and the interference of the second term of (3.4) with the

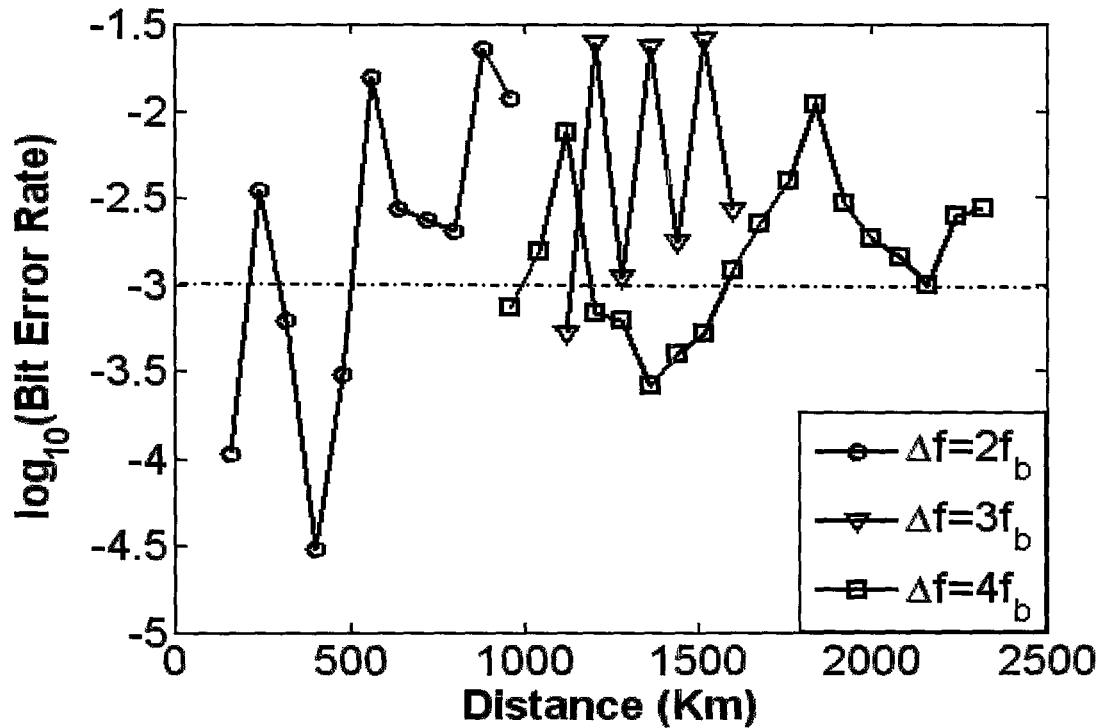


Figure 3.4: Bit error rate as a function of fiber length for three different values of the frequency offset  $\Delta f$ ,  $x = 0.5$ ,  $\gamma = 0$ , laser launch power = 2 dBm.

last term could be constructive at some distance and destructive at some other distance. The bit error rate performance enhances with  $\Delta f = 3f_b$ , reaching 1480 Km of fiber length. Finally when  $\Delta f = 4f_b$ , the transmission distance can be extended to 2160 km. The higher the offset frequency is, the less interference from the baseband signal  $|m(t)|^2$ .



Figure 3.5 shows the bit error rate as a function of power splitting ratio. As can be seen, when  $\gamma=0$  (linear channel), the optimum power splitting ratio is 0.5. This is because the maximum value of  $\sqrt{x(1-x)}$  (term 3 of (3.4)) occurs at  $x=0.5$ . The offset frequency was chosen to be centered at  $\Delta f = 4f_b$  since it gives the best case scenario. The launch power at the input was 6 dBm, and the fiber length was 2880 Km.

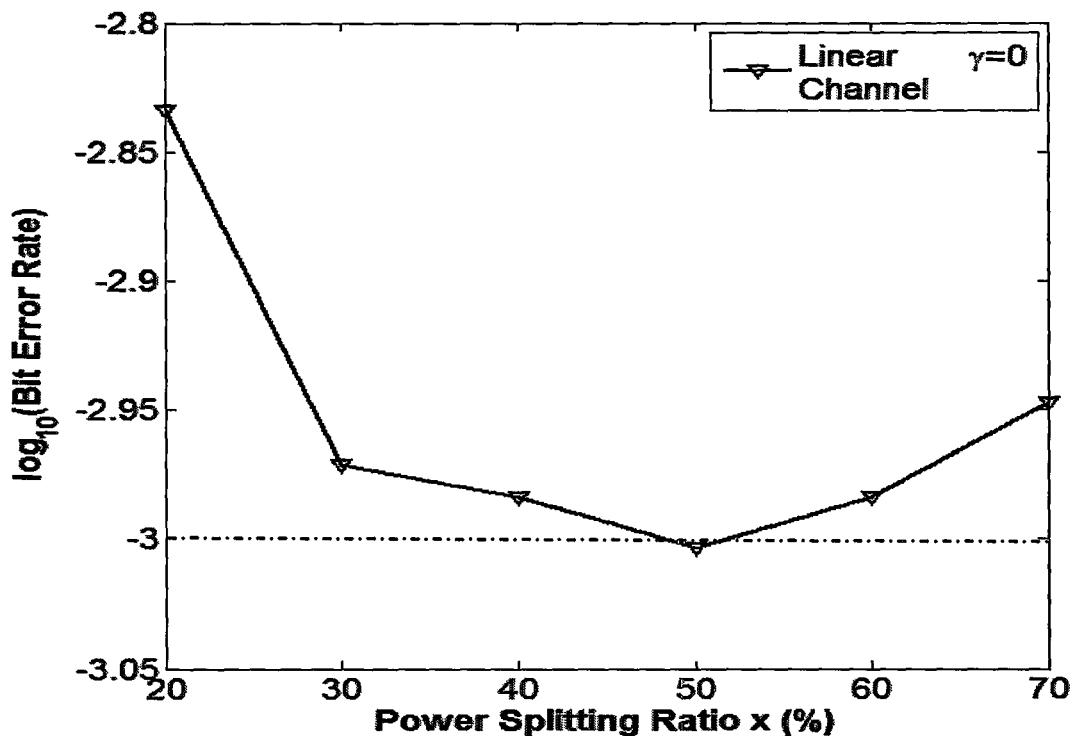


Figure 3. 5: Bit error rate as a function of the power splitting ratio  $x$ , laser launch power = 6 dBm ,  $\Delta f = 4f_b$  , Fiber Length in linear Channel=2880 Km.

In reality, the fiber channel is considered to be nonlinear. For that reason, when  $\gamma \neq 0$ , as the message signal power increases, distortion due to intra-channel four (IFWM) wave mixing increases too. Therefore, to minimize the BER, one would expect the message signal power to be smaller than the off-set carrier power. Various simulations were performed to find the best power splitting ratio performance in presence of nonlinearity. Figure 3.6 shows that BER is minimum when the message signal power is 5% ( $x = 0.05$ ) of the total power. The input launch power was again 6 dBm, and the fiber length was found to be 1120 Km all while maintaining a bit error rate less than  $10^{-3}$ .

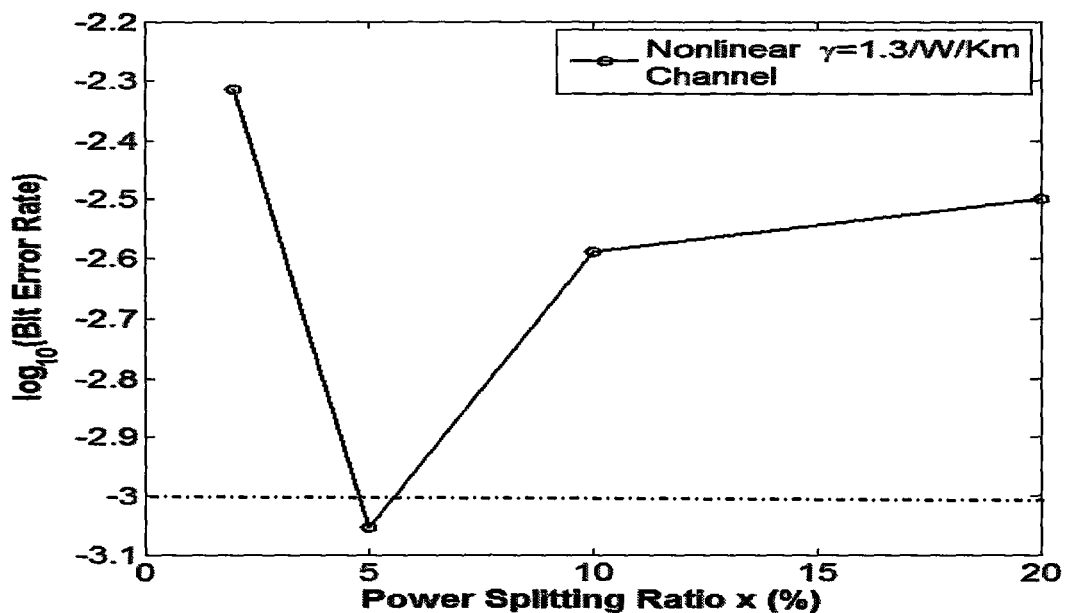


Figure 3. 6: Bit error rate as a function of the power splitting ratio  $x$ , laser launch power = 6 dBm ,  $\Delta f = 4f_b$ , Fiber Length in nonlinear Channel=1120 Km.

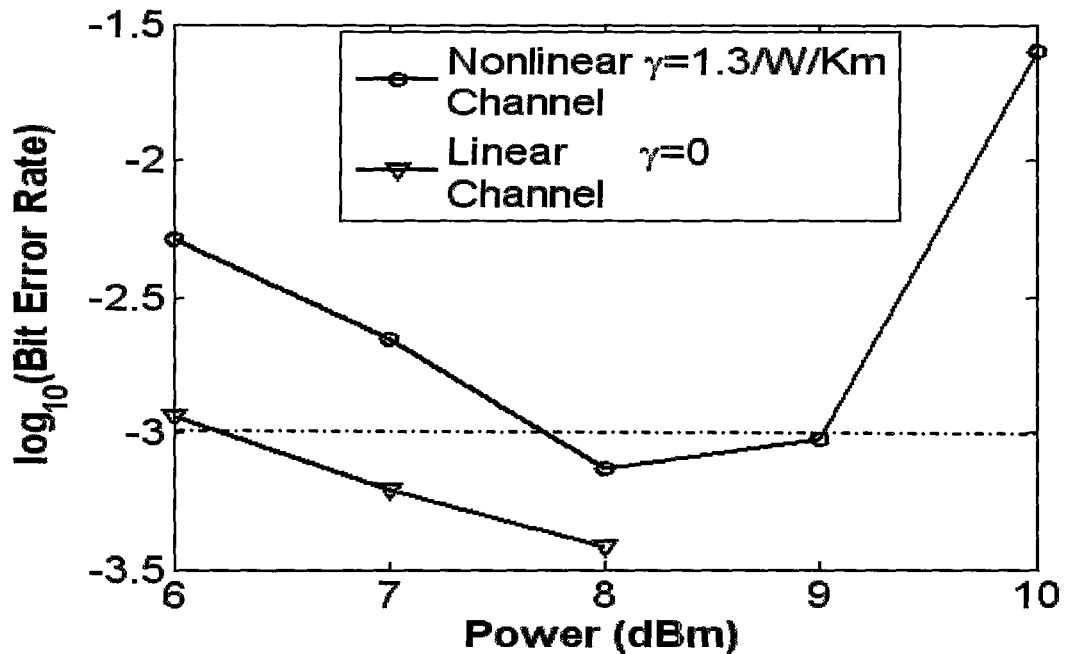


Figure 3. 7: Bit error rate as a function of the laser launch power,  $x = 0.05$ ,  $\gamma = 1.3/w/km$ ,  $\Delta f = 4f_b$ , Fiber Length= 1360 Km.

One could employ  $x = 0.05$  as the ideal power splitting ratio. Therefore, it would be a good idea to find the maximum fiber length reach that one could acquire. Figure 3.7 shows the BER as a function of the launch power at a transmission distance of 1360 km which corresponds to an accumulated dispersion of 23120 ps/nm. When  $\gamma = 0$ , BER can be decreased by increasing the launch power. But, when  $\gamma \neq 0$ , BER decreases initially as the launch power increases, but eventually the performance gets worse because the impairments due to IFWM and nonlinear phase noise dominate. The advantage of the proposed scheme is that the detection process can be made linear without requiring the local oscillator at the receiver and the laser line width need not be very small as required

for coherent systems. But the drawback is that the offset frequency should be about four times the spectral width,  $W$ , to avoid the intermodulation cross-talk and therefore, dense WDM may not be possible with this technique. Nevertheless, using more spectrally efficient formats such as duo-binary or quadrature amplitude modulation, it may be possible to suppress the intermodulation cross-talk which would be the subject of future investigation.

### **3.4 Conclusion**

In conclusion, we have proposed a scheme to enhance the reach of a dispersion limited direct detection system using a simple linear equalizer. An optical carrier with its frequency offset from the signal is transmitted to make the detection process linear. The numerical simulation results show that transmission distance can be extended to 1360 *Km* corresponding to accumulated dispersion of 23120 *ps/nm* using the proposed scheme.

# Chapter 4

## Linear and nonlinear electrical compensation in a coherent detection system

This chapter will include a nonlinear equalizer using the Volterra approach employed in a coherent detection system. Such compensator will be compared to a linear equalizer, and a threshold detector. Moreover, a dispersion compensation fiber (DCF) will be set up in the optical domain to engage in the performance of the system.

### 4.1 Introduction

Optical communication systems in a single-mode fiber have long employed the direct-detection method, i.e. one would try and estimate the transmitted bits by analyzing the current signal after photo-detection. Although this technique is easy to implement, one major disadvantage of such method is that the signal loses its phase (complex nature) after passing the photodiode. Many compensators at the receiver's side have been utilized

to extend the reach in transmission. However, none of those methods have seemed to be promising for one to achieve a long transmission system (thousands of kilometres). At the output of the photo-detector, the resulting current  $I(t)$  is proportional to the magnitude squared of the signal  $s(t)$  via:

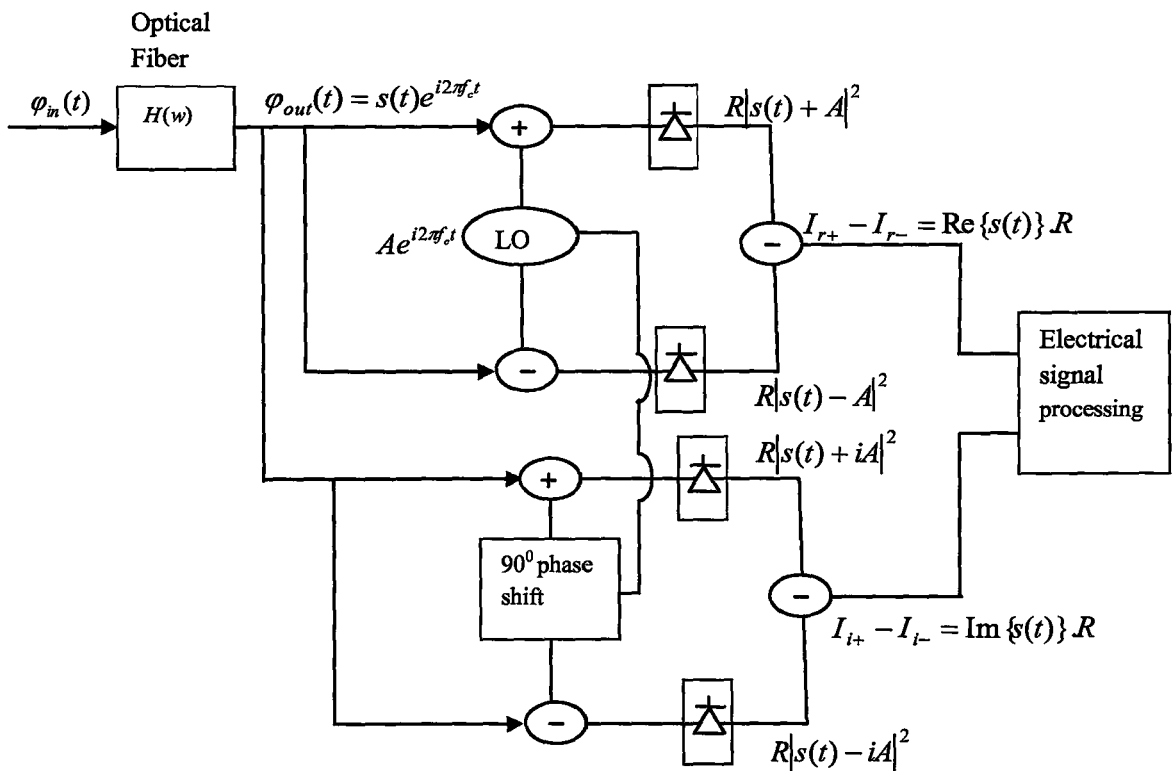
$$I(t) = R|s(t)|^2 \quad (4.1)$$

where  $R$  is the responsivity of the diode. Major problems and constraints would arise when using a direct-detection receiver. First of all, by losing the phase of the signal, one would not be able to use signal formats that modulate the phase (PSK). Phase-shift keying does have a major advantage over OOK and DPSK modulated signals, and with that, maximum efficiency in the system would not be achieved. Second of all, when employing (4.1) to an optical signal, a nonlinear term caused by the square law detection will appear at the output of the photo-detector; such occurrence will add more noise, nonlinearity and ISI to the transmitted signal. With all that being said, compensators at the electrical domain will have a hard time to offset the major impairments caused by the fiber. As a result, signals will be estimated with less efficiency, hence leading to a smaller fiber length transmission.

For those reasons mentioned, one would use a coherent detection system. In an ideal case, the output current  $I(t)$  of the coherent detector will be directly proportional to the optical signal  $s(t)$ . The phase of the signal will be retrieved with no nonlinear terms added to it. Formats that are relying on phase modulation could be used, and electrical compensators

employed at the receiver’s side will perform more efficiently; thus leading to reach enhancement in fiber length. However, disadvantages would include increase in system cost, not to mention the complexity that would result from employing a coherent receiver.

### 4.2 Coherent Detection



**Figure 4. 1:** Schematic depicting a coherent employed at the output of an optical fiber channel. LO: Local oscillator. R: Responsivity of the photo-diode

The schematic of a homodyne balanced coherent detector is illustrated in Figure 4.1. The optical field of the laser incident to the fiber is given by

$$\varphi_{in}(t) = s_{in}(t)e^{i2\pi f_c t} \tag{4.2}$$

where  $s_m(t)$  is the transmitted optical signal. At the output of the fiber, the optical field is given by

$$\varphi_{out}(t) = s(t)e^{i2\pi f_c t} \quad (4.3)$$

where  $s(t)$  is the output signal of the fiber. Note the signal is affected by fiber impairments. In the upper part of the schematic, a local oscillator (LO) is present and is operating at the frequency  $f_c$  of the laser, i.e.

$$\varphi_{lo} = Ae^{i2\pi f_c t} \quad (4.4)$$

where  $A$  is the amplitude of the oscillating frequency. In the first stage of the process (upper arm of the upper part of the schematic), the output field of the fiber (from (4.3)) is added to the local oscillator frequency (from (4.4)) using a 3-dB coupler, and the combined signal is passed through a photo-diode. The resulting current  $I_{r+}$  is

$$I_{r+} = R|s(t) + A|^2 \quad (4.5)$$

On the lower arm of the upper part of the schematic, the local oscillating field is subtracted from (4.3) using an MZ interferometer and passed through a photo-diode. The resulting current  $I_{r-}$  is

$$I_{r-} = R|s(t) - A|^2 \quad (4.6)$$



Finally, subtracting (4.6) from (4.5) yields the real part of the signal  $s(t)$

$$I_{r+} - I_{r-} = \text{Re}\{s(t)\}R \quad (4.7)$$

Coincidentally, one could extract the imaginary part of the signal using the same method.

However, the local oscillating frequency would be phase shifted by  $90^\circ$ . We have:

$$\varphi'_{lo} = Ae^{i2\pi f_c t + \pi/2} \quad (4.8)$$

From there onwards, one would apply the same procedure mentioned earlier to the lower part of Figure 4.1. The imaginary part of the signal  $s(t)$  is given by:

$$I_{i+} - I_{i-} = R|s(t) + iA|^2 - R|s(t) - iA|^2 = \text{Im}\{s(t)\}R \quad (4.9)$$

Finally, by preserving both the real and imaginary part of  $s(t)$ , one would gain the phase characteristics of the transmitted signal. Afterwards, electrical compensating methods will be applied to the current signal, in order to get estimated.

### 4.3 Nonlinear electrical equalizer using the Volterra approach

After coherent detection, one would want to estimate the signal damaged by the fiber impairments. We have shown earlier that the fiber dispersive parameters could be offset by employing the linear equalizer used previously. However, such equalizer does not compensate for the nonlinear parameters caused by the Kerr effects (SPM, XPM). For that, one could employ the Volterra approach for nonlinear cancellation. As mentioned

earlier, this method was used by the authors of [35] for a direct-detection receiver. The results have shown that a nonlinear MLSE enhanced the reach of the fiber by 65 Km, when it was compared to a linear MLSE equalizer. For that, it would be reasonable to do such a comparison in a coherent system.

Let an optical signal  $x(t)$  be incident towards a nonlinear fiber channel  $h(t)$ , the resulting signal  $s(t)$  at the fiber end is given by [35]

$$s(t) = \int_{-\infty}^{\infty} x(t-t')h_{linear}(t')dt' + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_{nonlinear}(t_1, t_2, t_3)x(t-t_1)x^*(t-t_2)x(t-t_3)dt_1dt_2dt_3 \quad (4.10)$$

Let  $z$  be the sampled current after the electrical filter, a discrete form of (4.10) would look like

$$s_k = \sum_{l=0}^{L-1} a_{k-l} \underbrace{h_{linear}(l)}_{\text{Linear channel coefficients}} + \sum_{i_1=0}^{N-1} \sum_{i_2=0}^{N-1} \sum_{i_3=0}^{N-1} \underbrace{a_{k-i_1} a_{k-i_2}^* a_{k-i_3}}_u \underbrace{h_{nonlinear}(i_1, i_2, i_3)}_{\text{nonlinear channel coefficients}} \quad (4.11)$$

where  $N$  represent the number of taps of the nonlinear equalizer,  $a$  is the estimated transmitted bit, and  $u$  is a vector representing the input signal with memory  $N$ . The linear channel coefficients are given by  $h_{linear}(l)$ , where the tap weight coefficients can be computed using the least mean square algorithm mentioned earlier. The second term of (4.11) is  $h_{nonlinear}(i_1, i_2, i_3)$ , and it represents the nonlinear channel coefficients of the fiber

(Volterra Kernels). Again, those coefficients can be estimated using the LMS algorithm.

At first, the nonlinear coefficient with the size of  $N^3$  is initialized,

$$h_{nl} = \underbrace{[0, 0, \dots, 0]}_{\text{Size} = N^3} \quad (4.12)$$

We go on by calculating the initialised current  $r$  using

$$r = \underbrace{h_{linear}^H a_k}_{\text{linear current}} + \underbrace{\left( \sum_{i_1=0}^{N-1} \sum_{i_2=0}^{N-1} \sum_{i_3=0}^{N-1} a_{k-i_1} a_{k-i_2} a_{k-i_3} \right) h_{nonlinear}}_{\text{nonlinear current}} \quad (4.13)$$

Afterwards, one would want to calculate the error signal between the received current  $z_k$  and the new estimated current  $r$ , we have

$$e = z_k - r \quad (4.14)$$

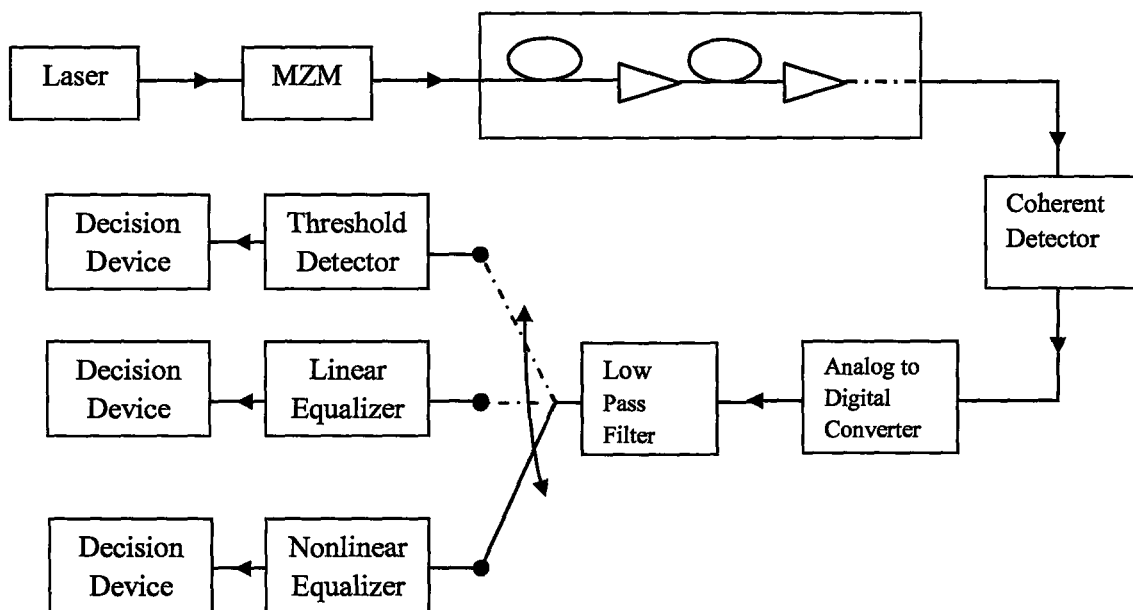
We finally update the new channel coefficients using

$$h_{nl} = h_{nl} + \mu e^* u \quad (4.15)$$

with  $\mu$  being the step size of the iteration. The procedure would repeat until  $e$  is minimized, and  $h_{nl}$  coefficients would converge to the new estimated value.

#### 4.4 System Design and Simulation results

The system model is depicted in Figure 4.2. We again performed the Monte Carlo simulation of a fiber optic link using the well known split-step Fourier technique..A Mach-Zender modulator is used to generate an RZ-PSK signal of length  $2^{16}$  at a rate of 10 Gb/s. The computational bandwidth is 80 GHz. The optical signal will propagate along a single mode fiber (SMF) with a dispersion coefficient of  $D=17 \text{ ps/nm/km}$  and a nonlinear coefficient  $\gamma=1.3/W/km$ . An optical amplifier with a spontaneous emission factor of  $n_{sp}=1.5$  is placed at every 80 Km of fiber length. It will be used to fully compensate for the fiber loss which is given at  $0.2 \text{ dB/km}$ . Shot and Thermal noise will be dominated by the amplifier noise, hence considered negligible. For the moment, no dispersion compensation is done at the optical domain.



**Figure 4. 2:** Schematic representing the design of a coherent system. Three different methods of detection will be used: Threshold Detector, Linear Equalizer and Nonlinear Equalizer.

The optical signal at the fiber output will pass through the coherent detector mentioned previously. At the receiver's side, an ADC will be used to digitize the signal. Unless stated otherwise, a 7 GHz electrical low pass filter is employed to limit the noise introduced by the optical amplifier. Three different methods of detection will be compared to decide the transmitted bits.

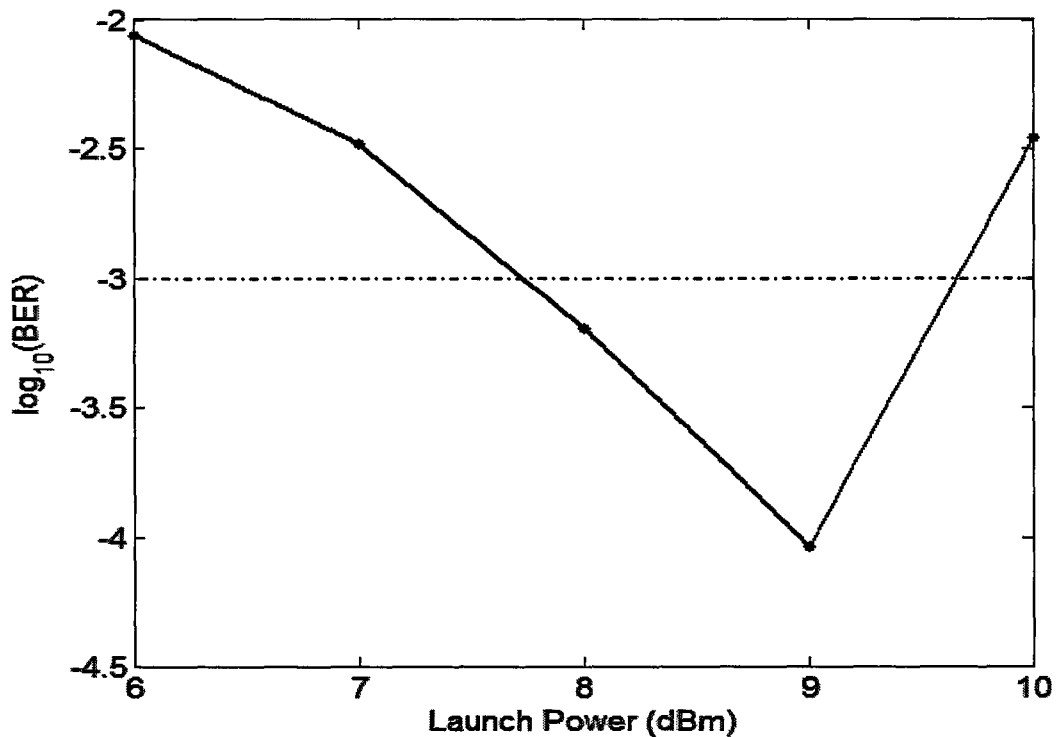


Figure 4.3: Bit error rate as a function of launch power for a threshold detector. Fiber length=240 Km

We start off with the classical threshold detector method. It is employed by comparing the threshold current of the received bit with a reference threshold current  $I_{th}$ . Since the signal is modulated in a binary PSK format, one would choose  $I_{th} = 0$  as the reference threshold. In essence, if the received sample current is larger than  $I_{th}$ , the detector would

output bit “1”. On the other hand, if the received current was smaller than  $I_{th}$ , it would output the bit “0” as being transmitted. Figure 4.3 depicts a bit error rate performance of a threshold detector. The fiber length used was 240 Km, and the electrical filter 3 dB bandwidth is 4.5 GHz. Note in the figure that the BER decreases when increasing the launch power. However when the input power of the signal is 10 dBm, the bit error rate of the signal increases. This marks the point when the signal is dominated by nonlinear impairments. Further increase in fiber length (320 Km) lead to an error performance higher than  $10^{-3}$ . Which is explains the error penalty of  $10^{-4}$  used in this case.

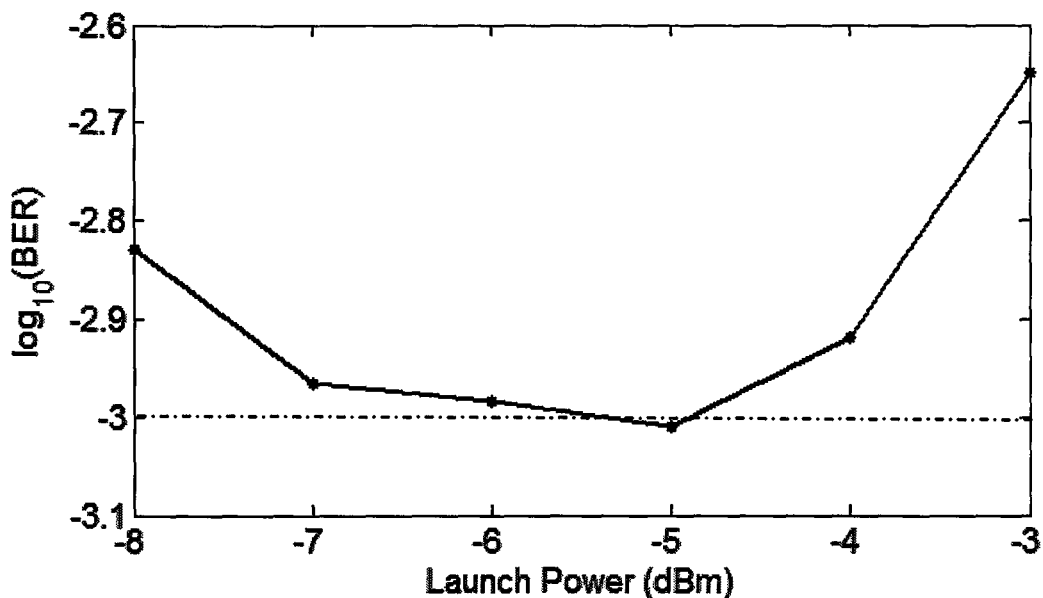


Figure 4. 4: Bit error rate as a function of launch power for a linear equalizer. Fiber length=3520 Km

For our second detector, a linear equalizer was used in order to offset the fiber dispersion. The number of taps was  $n=300$ , and the coefficients of the equalizer were calculated when the system was offline. The fiber was able to be extended to a length of 3520 Km. This is one of the major advantages of using an equalizer in a coherent detection. By preserving

the phase of the signal, one could extend the fiber to a length greatly larger than one would get when using a direct-detection method. Note that a launch power of -5 dBm is sufficient for the system to operate with maximum performance. In contrast, a 9 dBm launch power was required for a transmission distance of just 240 Km for systems based on threshold detection.

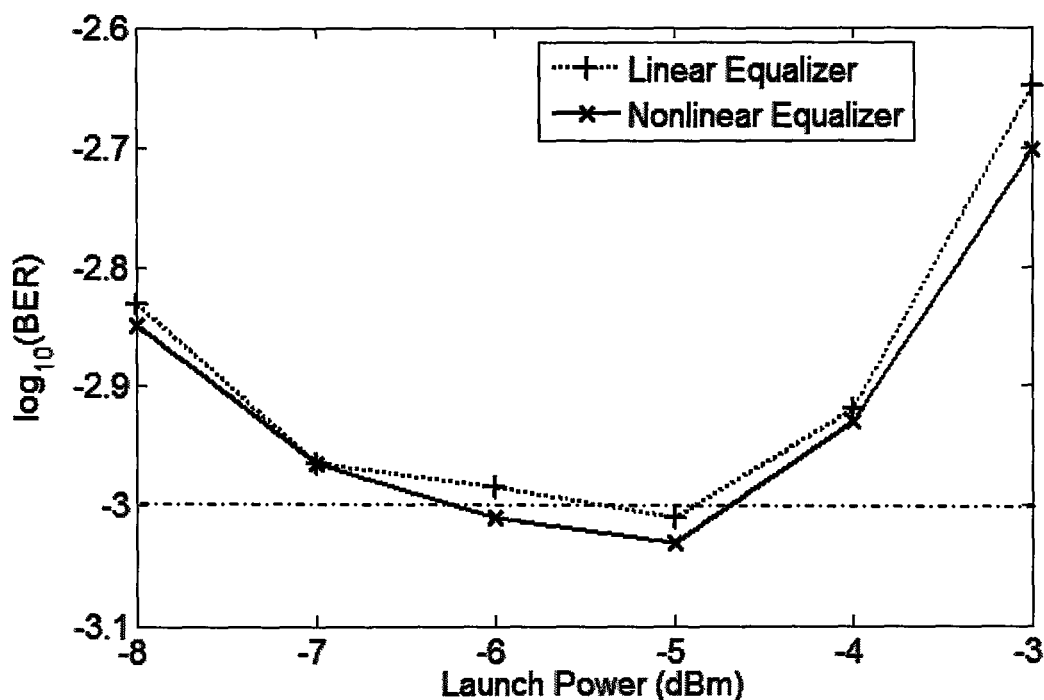


Figure 4. 5: Bit error rate as a function of launch power for both nonlinear and linear equalizer. Fiber length=3520 Km

Figure 4.5 shows the bit error rate of a nonlinear equalizer as a function of launch power. The fiber length used was 3520 Km. Superimposed is the performance of the linear equalizer that was generated previously. The number of taps of the nonlinear equalizer is  $n_{\text{taps}}=4$ . The goal is here is to compare is to compare the performances of the two

equalizers. As shown in the figure, adding a nonlinear canceller with the equalizer did not benefit the system much. In essence, the fiber reach could not be extended any further. This could be explained due to the large presence of intra-channel Cross-Phase modulation (IXPM). When an optical bit is transmitted, the pulse broadens along its propagation through the fiber. The more the length of the fiber is, the more nonlinear interference (IXPM) a bit would have on its neighbours. After 3520 Km of propagation, the system is dominated by SPM and IXPM factors, and therefore a nonlinear equalizer would not benefit the system

#### 4.5 System Including DCF

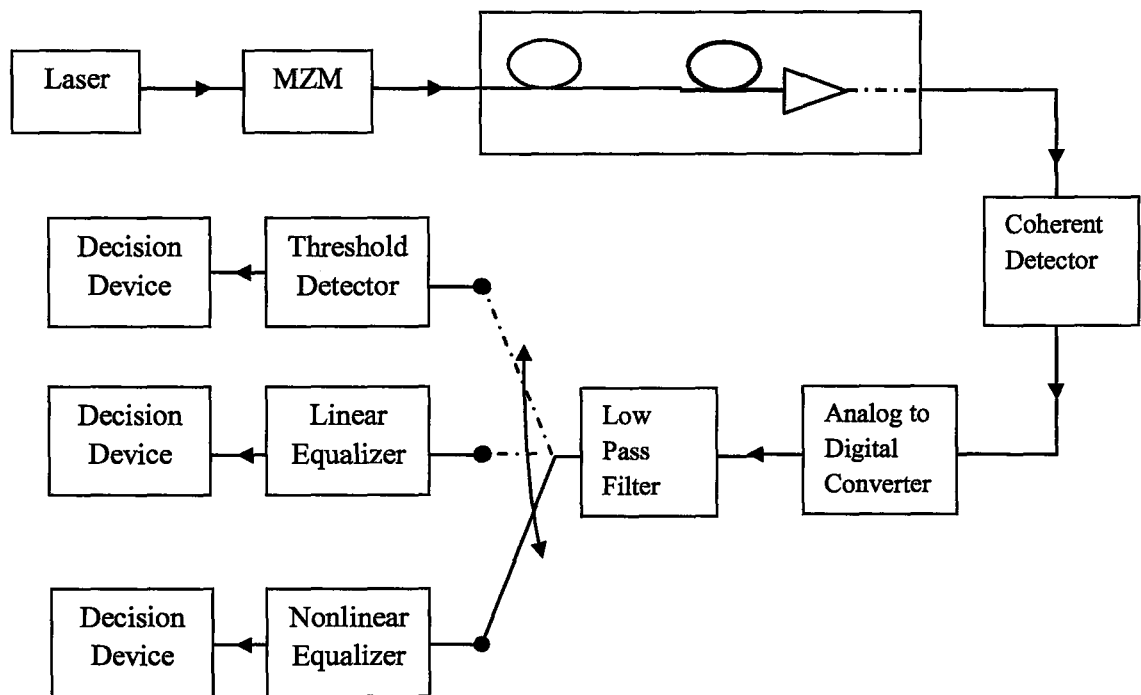


Figure 4. 6: System model including a dispersion compensation fiber (DCF)



Optical transmission using coherent detection has proved that it can reach a much longer distance than a direct-detection system. The nonlinear equalizer used earlier did not prove to have an advantage over linear compensation mainly because of the presence of IXPM. To limit such effect, one could include a dispersion compensation fiber (DCF) after every fiber span. The task of the DCF is to cancel the accumulated dispersion that is present in the fiber. The main purpose of the equalizer in this case is not to compensate for the dispersion, but for the nonlinear effects. In essence, one would choose the parameters of the DCF using the following

$$D_{SMF}L_{SMF} = -D_{DCF}L_{DCF} \quad (4.16)$$

We here will operate the system with the same parameters used earlier. However, for equation (4.16) to hold,  $L_{DCF} = 12.92 \text{ Km}$  and  $D_{DCF} = -105.23 \text{ ps/nm/km}$ . Moreover, the nonlinear DCF coefficient is  $\gamma_{DCF} = 5.2/W/km$ .

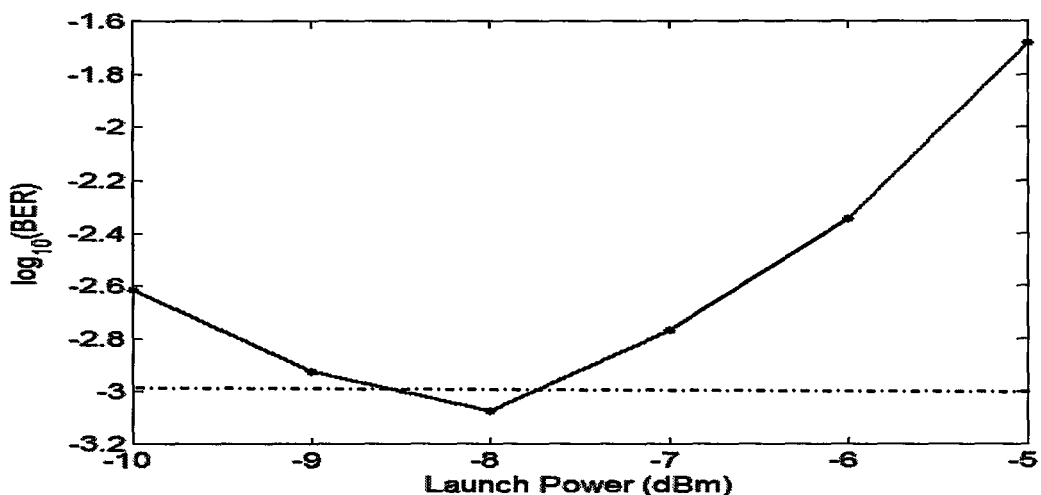


Figure 4. 7: BER as a function of launch power for a threshold detector (DCF included). Fiber Length=5280 Km

First of all, we test the outcome of the system when using a threshold detector at the receiver's side (Figure 4.7). The fiber length was 5280 Km. As expected, optical compensation has proved to be much more efficient than the electrical one. By only including a DCF, one could extend the reach of the system to a length much larger than when only using an electrical equalizer.

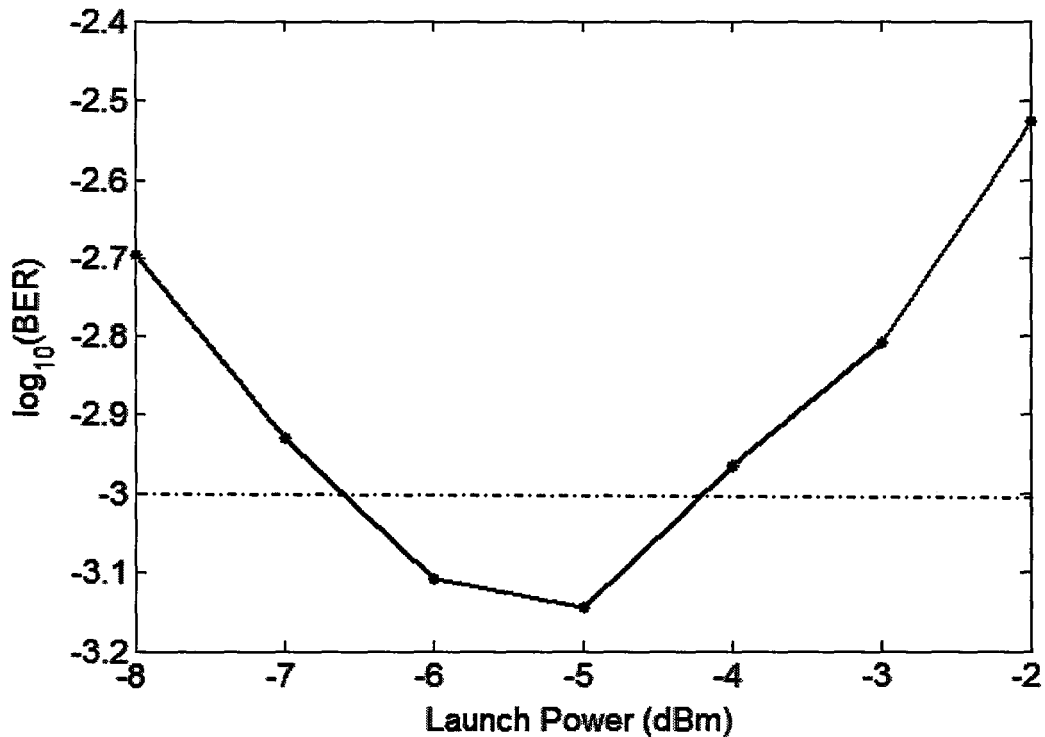


Figure 4. 8: BER as a function of launch power for a linear equalizer (DCF included). Fiber Length=8800 Km

Generally speaking, the introduction of a DCF in the system is enough to compensate for the accumulated dispersion. Moreover, the linear equalizer is usually used to offset the same linear dispersive effects caused by the fiber impairments. For that, one would expect the DCF included system to not certainly need a linear equalizer at the receiver's

side. However, Figure 4.8 shows that the fiber reach could be extended to a length of 8800 Km when a 50 tap linear equalizer is added. Again, the tap coefficients were computed when the system was offline. This behaviour is due to the fact that the computed tap coefficients represent a linear characterization of the nonlinear fiber channel. In essence, the linear equalizer is partially compensating for the nonlinear effects, and this was shown by extending the reach of the fiber by 3520 Km.

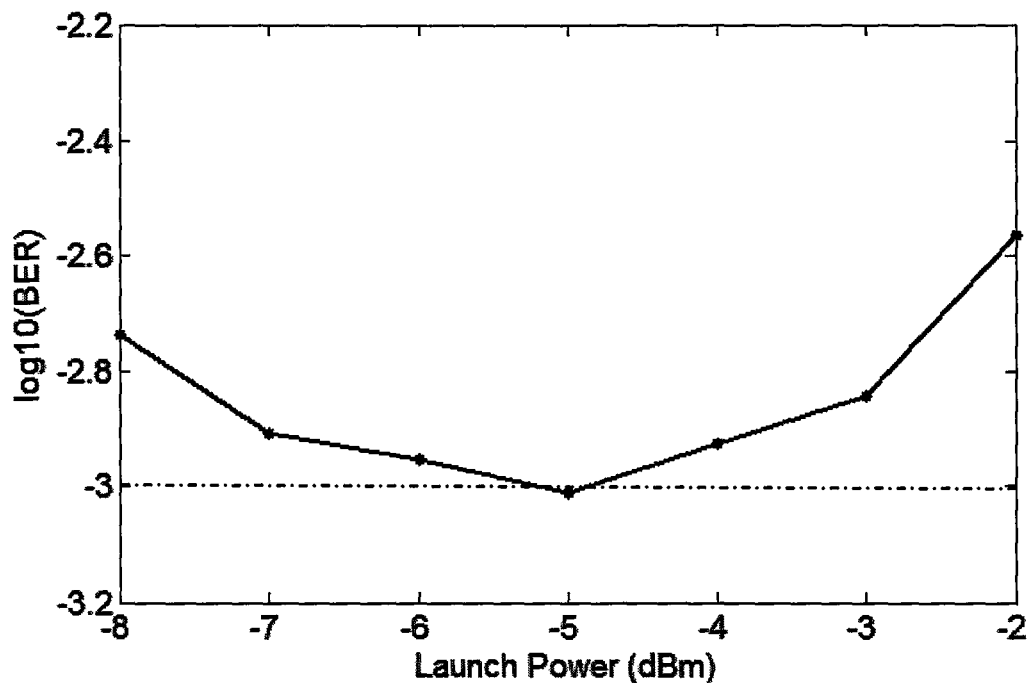


Figure 4. 9: BER as a function of launch power for a nonlinear equalizer (DCF included). Fiber Length=9200 Km

One would want to know how the system performs if a nonlinear equalizer were to be included in the DCF embedded system. Simulations were carried out for a fiber length of 9200 Km. The nonlinear equalizer only showed a marginal improvement compared to when using a linear compensator. The fiber reach was extended by 400 Km. The IXPM

effects were definitely minimized when using a DCF. However, one would have expected the nonlinear equalizer to enhance the reach of the system by a wide margin, as was in the case of a direct-detection system. Nevertheless, it was found earlier that the linear equalizer compensated for self-phase modulation (SPM) effects. For that, the nonlinear equalizer used is only compensating some nonlinear effects caused by the SPM that the linear equalizer couldn't handle.

## **4.6 Conclusion**

A coherent detection scheme was presented in this section. The system showed that it could carry a much longer transmission than the direct-detection method would provide. Three different receivers were tested. The first case was just the regular threshold comparator method with no electrical compensation. That system still performed better than a direct-detection method would generally grant. The second receiver included a linear equalizer, as it proved to perform very efficiently when the phase of the signal was saved via coherent detection. However, the nonlinear equalizer using the Volterra approach did not rather give any advantages to the system. The wide dominance of the XPM made it hard for the equalizer to compensate for nonlinearity. Therefore, we decided to implement a DCF in the system, hence limiting the presence of IXPM. The results showed that the linear equalizer is in fact compensating for nonlinearity. Moreover, the introduction of a DCF did in fact limit the presence of IXPM. The nonlinear equalizer did have a slight advantage over the linear compensator, as it was able to compensate some of SPM effects that the linear equalizer could not handle.

# Chapter 5

## Conclusion and Future work

### 5.1 Conclusion

Different methods of compensation were presented in this thesis. The main goal was to find a reliable system that would allow efficient compensation of fiber impairments in the electrical domain. We first proposed a scheme involving a direct-detection method. In the design, an offset optical carrier transmission was implemented in the optical domain in order to have a coherent like detection, while not using a local oscillator. At the receiver's side, a simple tap-delay line equalizer was used to offset the fiber dispersive impairments. When compared to other direct-detection methods, the scheme was able to enhance the reach of the fiber by a very wide margin. However, a main drawback that should be mentioned is that the offset optical carrier should be set at 40 GHz for the 10 Gb/s system to have good efficiency all while avoiding channel cross-talk for a transmission system of 1360 Km. This fact could cause difficulties in dense wavelength-division multiplexing (DWDM) systems, which operate at multiple optical carriers of the laser. When the offset

carrier is set at 20 GHz, the transmission distance was reduced to about 500 Km. In this case, WDM is possible with a channel spacing of 40 GHz.

The second design involved a coherent detection method. In the scheme, a nonlinear equalizer using the Volterra approach is applied at the receiver's side. The purpose of the compensator was to analyze its performance towards minimizing the nonlinear effects introduced by the fiber. For that, we compared the results to a receiver implementing a linear equalizer. Coherent detection does have the advantage of maintaining the phase of the signal after photo-detection. As a result, one would expect the system to operate rather better than in case of a direct-detection method. When using a direct detection receiver, it was found in [36] that the fiber reach was extended to 138 Km for a launch power of 0 dBm when using a linear compensator. In case of a coherent detection, the fiber reach was able to be extended to a length of 3520 Km with a launch power of only -5 dBm. However, the nonlinear equalizer did not show any advantages in the coherent scheme. A transmission system of that long triggers nonlinear impairments (IXPM and IFWM) to dominate the channel. Hence, a nonlinear equalizer using the Volterra approach would not be suited for this system. For this reason, one would implement a dispersion compensation fiber in the optical domain to limit the effect of IXPM and IFWM. Furthermore, results did show that the linear equalizer was in fact compensating for some the nonlinear impairments (SPM) caused by the Kerr effect. Moreover, the nonlinear equalizer did in fact show an advantage over the linear compensator. This was finally achieved by reducing the IXPM effects in the system. However, the advantage was only

marginal, as the nonlinear equalizer compensated some the SPM effects that the linear equalizer could not offset.

## 5.2 Future Work

The direct-detection system mentioned in chapter 3 did suffer intermodulation cross-talk when transmitting the signal through the channel. Future work in the design would include modulating the input signal using the duo-binary format. Such modulation does have benefit of being spectrally efficient, hence minimizing the effect of the crosstalk. Moreover, quadrature-amplitude modulation (QAM) could be used for the same purpose. One constraint we had to follow which was increasing the offset frequency to  $\Delta f = 4f_b$ , when only using a 10 Gb/s transmission rate system. As mentioned before, such action would cause problems to DWDM systems. However, duo-binary or QAM formats could significantly decrease  $\Delta f$  and therefore, DWDM can be realized using the proposed approach.

In case of the coherent system mentioned in chapter 4, future work would include transmitting the signal at rate of 40 Gb/s. Coherent transmission has proven to reach the longer distance (thousands of kilometres in fiber length). However, when using a 10 Gb/s transmission, the output of the signal would be dominated with nonlinearities. For that reason, the nonlinear equalizer did not show any efficiency. Nevertheless, operating with the mentioned rate would decrease the reach of the fiber, which results to less nonlinearity

in the system. Consequently, the nonlinear equalizer using the Volterra approach would have a better chance in cancelling the nonlinearity caused by the IXPM and the IFWM.



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