BLIND CHANNEL EQUALIZATION FOR SISO AND SIMO CHANNELS USING SECOND ORDER STATISTICS

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To my family

Abstract

In this thesis we develop several approaches to the problem of blind channel equalization based on second-order statistics (SOS). We consider the single-input single-output (SISO) system with minimum phase channel where the received signal is sampled at the symbol rate (T-spaced equalizer). We formulate the equalizer design criterion as a simple convex optimization problem, where the equalizer can be obtained efficiently avoiding the local minima problem.

We also extend the problem to the single-input multiple-output (SIMO) systems where the received signal is sampled at an integer multiple of the symbol rate. We formulate the problem as a convex optimization problem using the features existing in the channel matrix structure. The problem can be solved efficiently to obtain the equalizer where a global minima is guaranteed. Moreover, we modify this formulation and deduce a closed form solution to the equalizer. Although both methods are sensitive to the channel order as well as existing subspace methods, they perform better than the subspace methods when the channel matrix is close to being singular.

Furthermore, we propose an efficient direct minimum mean square error (MMSE) approach to estimate the equalizer. The method does not rely on the channel order and utilizes the channel matrix structure in SIMO systems. Therefore, it outperforms existing algorithms including the previously proposed methods. However, due to the large amount of computations involved in this method we present a new algorithm that belongs to the same class with moderate computational complexity and acceptable performance loss with respect to the latter algorithm.

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Chapter 1

Introduction

In communication systems it is required to reliably transmit data from the transmitter to the receiver through a channel. In the case of ideal transmission/reception scenarios the channel does not impose any kind of distortions to the transmitted data. However, transmission through a practical channel causes distortion to the original transmitted signal. One of the practical problems in digital communications is intersymbol interference (ISI), which causes a given transmitted symbol to be distorted by other transmitted symbols. The ISI is imposed on the transmitted signal due to the band limiting and the multipath effects of practical channels. The effect of ISI can be reduced by passing the received signal through a filter at the receiver end. This process is known as channel equalization, and the filter is usually denoted as equalizer. If the channel is known, the equalizer is designed based on the information available at the receiver end about the channel. If the channel is unknown, two methods are used to obtain this design. Training based method depends on transmitting a short sequence of data (known at the receiver end) to the receiver which in turn uses this sequence to design an equalizer that minimizes the error between the final output (estimated data) and the known transmitted sequence. However, inclusion of this training sequence with the transmitted information reduces the throughput of the system. The second method is based on estimating the equalizer blindly based on information about the statistics of the channel input. This method is, thus, denoted as blind channel equalization. Moreover, the structure of the channel may be used as an additional information (if known) in the equalization procedure.

1.1 Communication System

A simple block diagram for a communication system is shown in Figure 1.1, where a transmitter processes an input data sequence to a suitable format to be transmitted through the channel. In digital communication system the transmitter usually consists of a source encoder, a channel encoder, a pulse shaping filter and a modulator. The source encoder compresses the transmitted data by removing the redundancy found in the original data. The channel encoder adds controlled redundancy to the data to improve the reliability of the transmission. In most communication systems the pulse shaping filter is a root raised cosine filter which has the effect of reducing the resulting ISI at the receiver end; where the output signal from the matched filter (root raised cosine filter) has nulls at multiple of the sampling period. After that, the signal is passed through the modulator which maps the signal to a certain constellation and modulates it to the required frequency band. The final signal is then

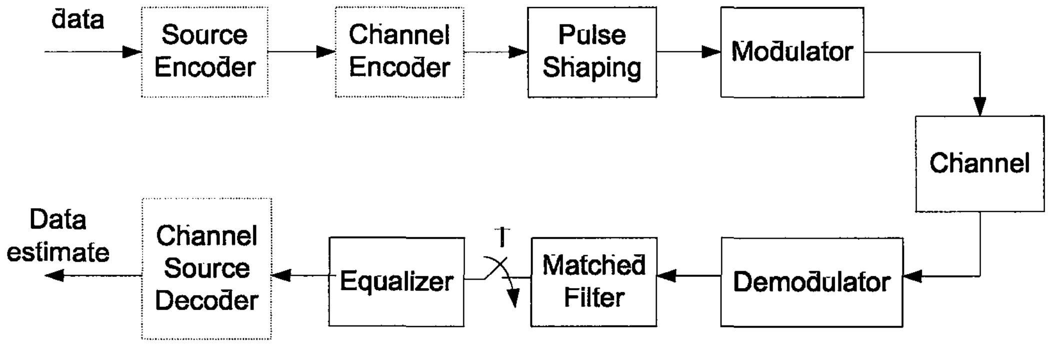

Figure 1.1: Communication System.

transmitted through the channel to the receiver. After demodulation and matched filtering with the receiver filter, the analog output is sampled with the symbol time T (symbol spaced receiver). Propagation through practical channels may cause severe distortion to the matched filter output. Therefore, an equalizer will be required to reduce this effect. In the following, the transmission system shown in Figure 1.2 will be considered for the discrete time baseband representation. The equalizer is the same as Figure 1.1 while the remaining blocks are combined into an equivalent discrete time baseband channel with impulse response denoted as $h(n)$ which is responsible for the ISI effect. In subsequent chapters, the channel impulse response $h(n)$ is assumed to be finite and time invariant. Although in general the channel is time varying, however, when the variations are slow such that a channel estimate can be obtained from a data sequence shorter than channel variation time then we can consider it as a time invariant channel.

1.2 Characterization of Propagation Channels

As the signal propagates from the transmitter to the receiver it is subjected to different kinds of distortion. The concept of modelling a propagation channel as a tapped delay filter was first introduced in [1]. Choosing the tap spacing to be less than the inverse bandwidth of the transmitted signal is sufficient to satisfy the requirements for accurate modelling. Therefore, it is common to model the channel as a finite impulse response (FIR) filter with taps chosen at the signal's sampling interval with complex valued weights. Moreover, the delay spread and the Doppler spread were introduced as characteristics for the propagation channel. The delay spread of a channel measures how much the transmitted pulse is elongated by passing through the channel. The Doppler spread measures the amount that the channel widen the spectrum of a transmitted signal. The ISI effect arises when the symbol interval is in the order of the channel delay spread which commonly occurs in high data rate transmission. Consider the discrete baseband representation of the communication system shown in Figure 1.2, where the channel is considered to be a linear time invariant (LTI) channel. The received sequence is the discrete time convolution between the transmitted

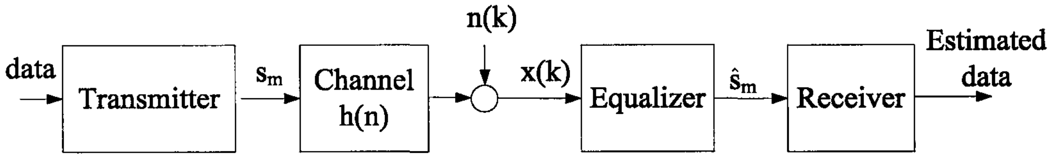

Figure 1.2: Baseband Communication System.

symbols s_m and the channel impulse response $h(n)$. This can be written as

$$\begin{aligned} x(k) &= \sum_m s_m h(k-m) + n(k), \\ &= s_k h(0) + \sum_{m \neq k} s_m h(k-m) + n(k). \end{aligned} \tag{1.1}$$

The second term represents the ISI caused by the channel. This effect arises due to the existence of a channel bandwidth limitation causing the transmitted pulse to be elongated in time domain. The other factor that introduces the ISI in wireless communication is the multipath problem where the channel consists of multiple propagation paths. In order to simplify the analysis, it is usually assumed that finite number of propagation paths exist and that each path differs from the other in amplitude (attenuation/gain) and by a constant time delay. Assuming that the direct line of sight channel impulse response is $p(t)$, and n multipaths, the total channel impulse response can be written as,

$$h(t) = p(t) + \alpha_1 p(t-t_1) + \cdots + \alpha_n p(t-t_n),$$

where α_i and t_i are the amplitude and the time delay of the i-th multipath, respectively. As we mentioned before, using equalizer at the receiver end is essential to mitigate the effect of ISI resulting from the multipath channel.

1.3 Equalization

As both the signal and the noise pass through the equalizer, the latter should be designed such that it mitigates the ISI and avoids the noise amplification. Most equalizers design criteria aim to minimize the output noise along with ISI mitigation, however, a tradeoff between these needs and the complexity always exists. Equalization techniques can be classified into two main categories, linear and nonlinear equalizers. Linear equalizers are simple to implement, but their abilities to reduce the output noise are limited compared to nonlinear equalizers [2]. On the other hand, nonlinear equalizers can efficiently reduce the noise at the output and this is achieved with relatively higher complexity than linear equalizers. In this thesis we focus on linear equalizers due to lower implementation complexity. An attractive set of linear equalizers, from the perspective of implementation, are implemented using a transversal structure which is a typical FIR filter. FIR filters consist of a series of delay elements and taps with adjustable weights to adjust the impulse response. The transversal structure of an FIR filter is shown in Figure 1.3. In particular, linear

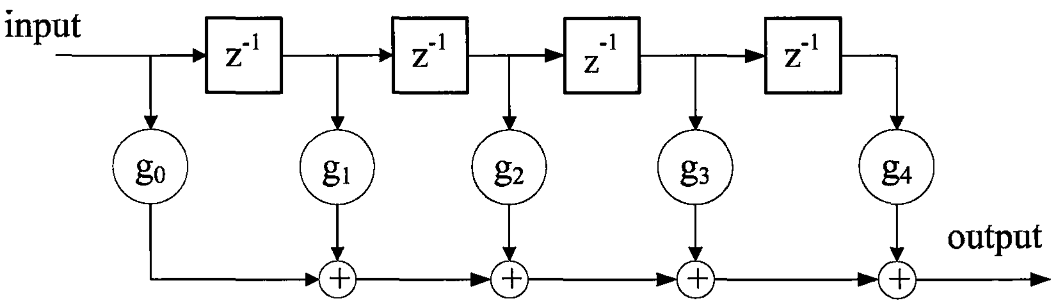

Figure 1.3: Transversal FIR Filter Structure

transversal equalizer can be expressed as an FIR filter as follows,

$$\mathbf{g}(z) = \sum_{i=0}^{L} g_i z^{-1} \tag{1.2}$$

where g_i is the i-th tap weight of the equalizer and L is the equalizer order. The weights are obtained by minimizing a certain cost function that guarantees the mitigation of the ISI and the decrease of the output noise as much as possible. The most popular linear equalizers are, the zero-forcing (ZF) equalizer and the minimum mean square error (MMSE) equalizer.

1.3.1 Zero-Forcing Equalizer ZF

Consider the system shown in Figure 1.2, the received signal can be written as,

$$x(k) = \sum_{m} s_m h(k-m) + n(k), \tag{1.3}$$

which can be expressed in the z-domain as follows,

$$x(z) = h(z)s(z) + n(z). \tag{1.4}$$

The zero forcing equalizer is the inverse channel filter $g(z) = 1/h(z)$ and hence the equalizer output can be expressed as,

$$\begin{aligned} y(z) &= g(z)h(z)s(z) + g(z)n(z), \\ &= s(z) + \tilde{n}(z), \end{aligned}$$

where $\tilde{n}(z)$ is the output noise. Note that, it may not be applicable to implement $g(z)$ as an FIR filter when there does not exist such a finite weights g_i's satisfying that,

$$\sum_{i=0}^{L} g_i z^{-1} = \frac{1}{h(z)}. \tag{1.5}$$

In this case it may be implemented as an infinite impulse response (IIR) filter. Although zero forcing equalizer has the ability to cancel the ISI, it deals poorly with the noise, as a result, the output signal-to-noise ratio (SNR) decreases. Moreover, if the channel has a spectral null or is deeply attenuated at certain frequencies within the band of interest then the output noise power will be increased dramatically. Therefore, zero forcing equalizers perform well at high SNR while for noisy channels a performance degradation occurs over other equalizers (those have the capability of reducing the output noise power) due to decreased output SNR even though they remove the ISI.

1.3.2 Minimum Mean Square Error Equalizer

The minimum mean square error (MMSE) equalizer is designed such that the mean square error between the transmitted symbol s_m and its estimate $\hat{s}_m$ is minimized. Therefore, MMSE equalizers perform better than ZF equalizers for noisy channels because the effect of the noise is considered in the equalizers design. Consider the linear equalizer $\mathbf{g} = [g_0, g_1, \cdots, g_L]^T$, where the weights g_i's are obtained by minimizing the following cost function,

$$\min_{\mathbf{g}} \mathrm{E}\{|s_m - \hat{s}_m|^2\}. \tag{1.6}$$

Substituting by the equalizer coefficients $\mathbf{g}$ and the received data vector $\mathbf{x}$ we get the following cost function,

$$\min_{\mathbf{g}} \mathrm{E}\{|s_m - \mathbf{g}^T\mathbf{x}|^2\},$$
$$\min_{\mathbf{g}} \mathrm{E}\{|s_m|^2 - \mathbf{g}^T\mathbf{x}s_m^* - \mathbf{g}^H\mathbf{x}^*s_m + \mathbf{g}^T\mathbf{x}\mathbf{x}^H\mathbf{g}^*\}$$

where $(.)^H$ and '$*$' denote the hermitian transpose and the conjugate operators respectively. Define $\mathbf{R}_x = \mathrm{E}\{\mathbf{x}\mathbf{x}^H\}$ to be the covariance matrix of the received signal and $\mathbf{r}_d = \mathrm{E}\{\mathbf{x}s_m^*\}$ the cross covariance vector, the MMSE equalizer $\mathbf{g}$ that minimize the above cost function is given as,

$$\mathbf{g} = \mathbf{R}_x^{-1}\mathbf{r}_d. \tag{1.7}$$

1.4 Blind Equalization

Blind channel equalization has attracted many researchers in the last decades [3], [4], [5], [6], [7], [8], [9] and [10]. In such a technique, while both the channel and the transmitted symbols are unknown to the receiver, it utilizes the observed signal to estimate the propagation channel and/or the transmitted symbols. Although the transmitted symbols and the channel are unknown, the receiver should have a starting point for estimation. The blind equalization is mainly built on the exploitation of the statistics of the transmitted symbols and in some cases the structure of the channel matrix may be used as an additional information.

Conventional design of the equalizers require either the knowledge of the channel or the transmitted symbols. The latter is known as a training sequence where, most of the communication systems depend on having an access to the transmitted symbols in equalizer design. Although training sequence is efficient, transmitting a training sequence decreases the system throughput. For time invariant channels the loss is insignificant while for time varying channels the time used for training is comparable to the transmission time. An example where a training sequence is not favorable is the computer network where links between terminals and central computers need to be established in an asynchronous way, in some situation, training is impossible.

Many approaches to the problem of channel estimation and equalization use the higher order statistics (HOS) of the received signal to solve the problem [11], [12] and [13]. HOS contain not only the magnitude but also the phase information about the process, that is why they are efficiently employed to estimate/equalize nonminimum phase channels [14]. Moreover, HOS of Gaussian process are equal to zero, hence, they are capable of handling colored Gaussian noise [15]. However, although

HOS based techniques solve the problem efficiently, they suffer from many drawbacks. HOS are computationally very expensive and require large data record to estimate the statistics. Moreover, for time varying channels this method may fail to be fast enough to track the channel changes.

In the recent years most of the channel estimation and equalization algorithms were directed to employ the second-order statistics (SOS) instead of HOS. The first SOS approach to blind channel estimation in multichannel system was proposed in [16] where the transmitted signal is assumed to be white and the channel matrix is estimated through a recursive method. It requires the estimation of the channel order and the noise variance. However, the algorithm does not consider the special structure of the channel matrix as well as it is affected by the covariance matrix estimation accuracy due to the recursion procedure used in estimation. The extension for colored signal is presented in [17]. The subspace method proposed by Moulines *et al.*[18], [19] and [20] exploits the structure of the channel matrix and due to this feature the algorithm performs better than some existing algorithms. It should be noticed that, the perfect knowledge of the channel order is required as order estimation error leads to a performance degradation. Also it may be not robust when the channel matrix is close to being singular. The cross relation (CR) approach presented in [21] is also a type of subspace estimation where the channel order is an important issue in estimation. The method is very effective for small data samples at high SNR.

A new algorithm that does not rely on the signal and noise subspace separation and consequently robust against channel order estimation error was proposed by Ding [22]. Also a closely related approach was introduced by Prakriya [23]. Although these algorithms are not sensitive to the channel order estimation error, they do not include the channel matrix structure in the estimation procedure. The moment matching technique proposed in [24] is also robust against channel order estimation error and channel condition yet it suffers from the problem of local minima in the estimation of the channel.

It can be noticed that, most of the proposed algorithms for channel equalization using SOS suffer from at least one of the following:

- Channel order is a critical parameter in formulation.
- Performance degradation when the channel matrix approaches singularity.
- Error propagation due to recursion.
- Error accumulation when multiple estimated covariance matrices are utilized.
- Not considering the channel matrix structure in estimation.

In this thesis we propose algorithms that avoid and overcome most of these problems.

1.5 Contributions and Outline

In what follows we give a brief overview of our work. Detailed definitions and background information are given in individual chapters as appropriate. In this thesis we present different approaches to blind channel estimation and equalization problem. We consider two systems, the single-input single-output (SISO) system and the single-input multiple-output (SIMO) system. First we formulate the problem of blind channel equalization for SISO minimum phase channels as a convex optimization problem in the autocorrelation sequence of the equalizer. Then using the spectral factorization technique the equalizer can be obtained efficiently. The two main advantages of convex formulations are, they can be efficiently solved using interior point methods and they guarantee a global minima.

Furthermore, a convex formulation for channel equalization is also presented for SIMO system. Quadratic objective function and linear constraints are employed to estimate a zero-forcing (ZF) equalizer. In addition, we deduce a closed form solution to the equalizer. The proposed algorithms are robust when the channel matrix approaches singularity. Moreover, the channel matrix structure is employed in estimation.

Finally a new direct MMSE method for blind SIMO channel is presented where the structure of the channel matrix and the statistics of the transmitted signal are jointly employed to estimate the channel. This method outperforms other SIMO equalization methods where it does not require the perfect knowledge of the channel order. Moreover, it is robust against channel matrix singularity.

This thesis is organized as follows. In Chapter 2 we present the SISO minimum phase channel equalization. In Chapter 3 we develop a convex formulation for SIMO channel equalization as well as a closed form solution to the problem. In Chapter 4 robust direct methods for SIMO channel equalization are proposed which outperform existing algorithms. Each chapter includes simulation results to demonstrate the performance of the proposed algorithms and comparisons with existing algorithms.

Chapter 2

SISO Minimum Phase Channel Equalization

In this chapter we consider the blind channel equalization problem for single-input single-output (SISO) minimum phase channels. We formulate the problem as a convex optimization problem where the optimization variable is the autocorrelation sequence of the equalizer. The equalizer is obtained by applying the well known spectral factorization technique to the autocorrelation sequence that results from the solution of the optimization problem. The proposed method is simple and can efficiently estimate the equalizer where a global minima is guaranteed.

2.1 Introduction

Several approaches have been proposed to solve the channel estimation and equalization problem. Applying higher order statistics (HOS) results in efficient but computationally expensive methods to equalize the channel. Recently, most of the blind channel equalization algorithms utilize the second-order statistics (SOS). In this chapter, we present a method for baud rate (T-spaced) channel equalization via simple convex optimization problem. We estimate the equalizer using the SOS of the channel output. The proposed approach depends on matching the equalizer output autocorrelation and the channel input autocorrelation, (which is ideally assumed to be a delta function). We will show that minimizing the ℓ_2-norm (respectively the ℓ_∞-norm) of the output autocorrelation will lead to a semi-definite programming (SDP) (respectively linear programming (LP)) problem in the autocorrelation sequence of the equalizer. Applying the spectral factorization technique to the equalizer autocorrelation sequence obtained by solving the optimization problem yields the FIR equalization filter.

It is important to mention that, SDP and LP problems are both convex optimization problems. The main advantage of convex optimization problems is that, they do not suffer from local minima. Moreover, well developed and highly efficient interior point algorithms are available to solve these problems [25].

2.2 Problem Analysis

Consider the baseband communication system shown in Figure2.1. The channel input

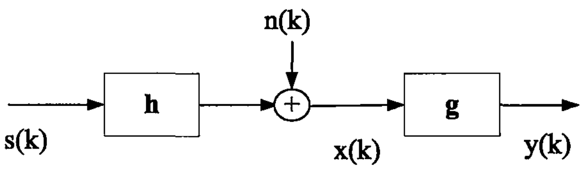

Figure 2.1: System Model

s is assumed to be unknown at the receiver end. However, its statistics in the second order sense are known and considered to be white (uncorrelated signal). Moreover, the channel impulse response $\mathbf{h}(z)$ and its order p are unknown and expressed as,

$$\mathbf{h}(z) = h_0 + h_1 z^{-1} + \cdots + h_p z^{-p}.$$

In the following we will consider the noise free case. The received signal can be written as,

$$\mathbf{x}(k) = \sum_j \mathbf{h}(k-j)\mathbf{s}(j) = \mathbf{h} \otimes \mathbf{s}, \tag{2.1}$$

where $\mathbf{s} = (s_1, \cdots, s_n)^T$ is the transmitted signal and '$\otimes$' denotes the convolution operator. The equalizer output can also be expressed as,

$$\mathbf{y}(k) = \sum_j \mathbf{g}(k-j)\mathbf{x}(j), \tag{2.2}$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_{n+p})^T$ is the received data vector, $\mathbf{y}$ is the equalizer output and $\mathbf{g}$ is the FIR equalizer that is required to be estimated with an order q. In other words,

$$\mathbf{g}(z) = g_0 + g_1 z^{-1} + \cdots + g_q z^{-q}.$$

The basic idea in our approach is to estimate an equalizer $\mathbf{g}(z)$ that minimizes the error between the actual output autocorrelation $\mathbf{r}_y$ and the ideal output autocorrelation $\mathbf{r}_{ideal} = (0\ 0 \cdots 0\ 1\ 0\ \cdots 0\ 0)^T$, where we assumed a white input data vector $\mathbf{s}$. The output autocorrelation sequence can be expressed as follows,

$$\begin{aligned}
\mathbf{r}_y(m) &= \mathrm{E}\{\mathbf{y}^*(k)\ \mathbf{y}(k+m)\} \\
&= \mathrm{E}\left\{\left(\sum_j \mathbf{g}^*(k-j)\mathbf{x}^*(j)\right)\left(\sum_\ell \mathbf{g}(k+m-\ell)\mathbf{x}(\ell)\right)\right\} \\
&= \sum_j \sum_\ell \mathbf{g}^*(k-j)\mathbf{g}(k+m-\ell)\ \mathrm{E}\{\mathbf{x}^*(j)\mathbf{x}(\ell)\} \\
&= \sum_j \sum_\ell \mathbf{g}^*(k-j)\mathbf{g}(k+m-\ell)\ \mathbf{r}_x(\ell-j) \\
&= \sum_j \sum_\ell \mathbf{g}^*(-j)\mathbf{g}(m-\ell)\ \mathbf{r}_x(\ell-j).
\end{aligned}$$

substituting $i = \ell - j$ we get,

$$\begin{aligned}\mathbf{r}_y(m) &= \sum_j \sum_i \mathbf{g}^*(-j)\mathbf{g}(m-i-j)\ \mathbf{r}_x(i) \\ &= \sum_i \mathbf{r}_g(m-i)\ \mathbf{r}_x(i). \end{aligned} \tag{2.3}$$

It is obvious that, the output autocorrelation sequence $\mathbf{r}_y$ is simply the linear convolution of the autocorrelation $\mathbf{r}_x$ of the received vector and the autocorrelation $\mathbf{r}_g$ of the equalizer.

2.3 Problem Formulation

In this section we provide two optimization formulations for estimating the equalizer $\mathbf{g}(z)$ based on the ℓ_2-norm and the ℓ_∞-norm of the vector $\mathbf{r}_y$, and discuss the frequency domain characteristics of the autocorrelation $\mathbf{r}_g$. The autocorrelation $\mathbf{r}_x$ of the received vector $\mathbf{x}$ can be estimated from N data vectors as,

$$\hat{\mathbf{r}}_x(k) = \frac{1}{N}\sum_{n=1}^{N}\sum_i \mathbf{x}_n(i)\mathbf{x}_n(i+k),$$

where $\mathbf{x}_n$ is a truncated subsequence of the received signal. The equalizer output autocorrelation (2.3) can be written in a matrix form as follows,

$$\mathbf{r}_y = \mathbf{R}_x\mathbf{r}_g, \quad \text{where} \quad \mathbf{R}_x = \begin{pmatrix} \mathbf{r}_{x(L)} & 0 & 0 \\ \mathbf{r}_{x(L-1)} & \ddots & 0 \\ \vdots & & \mathbf{r}_{x(L)} \\ \mathbf{r}_{x(L-1)} & & \mathbf{r}_{x(L-1)} \\ \mathbf{r}_{x(L)} & & \vdots \\ 0 & \ddots & \mathbf{r}_{x(L-1)} \\ 0 & 0 & \mathbf{r}_{x(L)} \end{pmatrix}$$

In the following sections we will formulate the problem of estimating the equalizer autocorrelation in two convex optimization forms, ℓ_2-norm and ℓ_∞-norm. These convex formulations can be solved efficiently and result in the equalizer autocorrelation sequence. Also the frequency domain characteristics of the equalizer autocorrelation are discussed and introduced in the optimization problem as a linear inequality. Finally, the spectral factorization technique is utilized to obtain the equalizer coefficients from the autocorrelation sequence.

2.3.1 ℓ_2-norm Approach

The assumption that the transmitted signal is white implies that the autocorrelation sequence of the transmitted signal resemble a delta function (i.e. $\mathbf{r}_s(m) \simeq \delta(m)$). Therefore, the equalizer can be estimated by minimizing the cost function $||(\mathbf{r}_y - \mathbf{r}_s)||^2$. Moreover, the Fourier transform of the equalizer autocorrelation should be greater than or equal to zero for all frequency points. Based on these facts, the problem of finding the equalizer autocorrelation can be formulated as,

$$\begin{aligned} \underset{\mathbf{r}_g}{\text{minimize}} \quad & \sum_{m \neq 0} \mathbf{r}_y^2(m) \quad \Leftrightarrow \quad \min_{\mathbf{r}_g} \quad (\mathbf{r}_y^T \mathbf{r}_y - 1) \\ \text{subject to} \quad & \mathbf{r}_y(0) = 1 \\ & \mathbf{r}_g(e^{j\omega}) \geq 0 \quad \forall \omega. \end{aligned} \tag{2.4}$$

The above formulation can be rewritten in terms of the equalizer autocorrelation vector and the received autocorrelation vector in a matrix form as minimizing a quadratic objective function with linear equality and inequality constraints as,

$$\begin{aligned} \underset{\mathbf{r}_g}{\text{minimize}} \quad & \mathbf{r}_g^T \mathbf{R}_x^T \mathbf{R}_x \mathbf{r}_g \\ \text{subject to} \quad & \mathbf{r}_x^T \mathbf{r}_g = 1 \\ & \mathbf{r}_g(e^{j\omega}) \geq 0 \quad \forall \omega. \end{aligned} \tag{2.5}$$

Moreover, the quadratic objective function can be replaced by a linear function and the constraint $\mathbf{r}_g^T\mathbf{R}_x^T\mathbf{R}_x\mathbf{r}_g \leq t$, which can be formulated using the Schur complement [26] as a positive semi-definite (PSD) constraint as follows,

$$\begin{aligned} \underset{\mathbf{r}_g,t}{\text{minimize}} \quad & t \\ \text{subject to} \quad & \mathbf{r}_x^T\mathbf{r}_g = 1 \\ & \mathbf{r}_g(e^{j\omega}) \geq 0 \quad \forall\omega \\ & \begin{bmatrix} (\mathbf{R}_x^T\mathbf{R}_x)^{-1} & \mathbf{r}_g \\ \mathbf{r}_g^T & t \end{bmatrix} \geq 0. \end{aligned} \tag{2.6}$$

This convex optimization problem can be solved efficiently using the interior point methods [25] to obtain the equalizer autocorrelation vector.

2.3.2 ℓ_∞-norm Approach

Considering the infinity norm will reduce the complexity of the problem from an SDP problem to an LP one. This can be written as,

$$\begin{aligned} \underset{\mathbf{r}_g}{\text{minimize}} \quad & \max_{m\neq 0} \quad \mathbf{r}_y(m) \\ \text{subject to} \quad & \mathbf{r}_y(0) = 1 \\ & \mathbf{r}_g(e^{j\omega}) \geq 0 \quad \forall\omega. \end{aligned} \tag{2.7}$$

The formulation will differ only in the objective function. Following the same previous steps, this objective function can be replaced by a linear function associated with linear inequality constraint instead of a PSD constraint as follows,

$$\begin{aligned} \underset{\mathbf{r}_g,t}{\text{minimize}} \quad & t \\ \text{subject to} \quad & \mathbf{r}_y(0) = 1 \\ & ||\mathbf{R}_x\mathbf{r}_g||_\infty \leq t \\ & \mathbf{r}_g(e^{j\omega}) \geq 0 \quad \forall\omega. \end{aligned} \tag{2.8}$$

The problem is transformed to an LP problem with linear equality and inequality constraints.

2.3.3 Frequency Domain Constraint

The Fourier transform of the equalizer autocorrelation sequence $\mathbf{r}_g$ is defined as,

$$\mathbf{r}_g(e^{j\omega}) = \mathbf{r}_g(0) + 2\sum_{k=1}^{q} \mathbf{r}_g(k) \cos k\omega. \tag{2.9}$$

Since $\mathbf{r}_g$ is an autocorrelation sequence then $\mathbf{r}_g(e^{j\omega})$ is nonnegative for all values of ω, i.e.,

$$\mathbf{r}_g(e^{j\omega}) \geq 0, \qquad \omega \in [0, \pi]. \tag{2.10}$$

From this frequency representation we can note that for fixed ω the inequality (2.10) is a linear inequality in the equalizer autocorrelation coefficients, i.e., it defines a closed hyper-halfspace. Therefore (2.10) is an intersection of infinitely many hyper-halfspaces parameterized by ω and consequently it defines a closed convex set. A popular method of handling the linear constraint $\mathbf{r}_g(e^{j\omega}) \geq 0$ is to sample it at certain frequencies $\omega_n, n = 1, \ldots, N.$ in the interval $[0, \pi]$. The infinite constraints (2.10) are replaced by N linear inequalities [26].

$$\mathbf{r}_g(0) + 2\sum_{k=1}^{q} \mathbf{r}_g(k) \cos k\omega_n \geq 0 \qquad n = 1, \ldots, N. \tag{2.11}$$

To this point, the optimization problem is complete and its solution yields the equalizer autocorrelation sequence. In the following section we will show how the equalizer coefficients can be obtained from this autocorrelation sequence.

2.4 Spectral Factorization

The autocorrelation sequence $\mathbf{r}_g$ of a given vector $\tilde{\mathbf{g}}$ is obtained as follows,

$$\mathbf{r}_g(k) = \sum_{i=0}^{q-k} \tilde{\mathbf{g}}(i)\tilde{\mathbf{g}}(i+k). \tag{2.12}$$

It is important to mention that, starting from $\mathbf{r}_g$ and working towards finding a vector $\tilde{\mathbf{g}}$ whose its correlation match $\mathbf{r}_g$ will not yield a unique solution. Where there are many possible solutions $\tilde{\mathbf{g}}_j$ (where j is related to the number of zeros of $\mathbf{r}_g$ in z-plane) having the same autocorrelation sequence $\mathbf{r}_g$. The spectral factorization method [26] computes the unique minimum-phase spectral factor $\mathbf{g} = (g_0, g_1, \ldots\ldots\ldots, g_{q-1}, g_q)$ which satisfies the property that,

$$g_0 + g_1 z^{-1} + g_2 z^{-2} + \cdots\cdots + g_q z^{-q}, \tag{2.13}$$

is nonzero for $|z| > 1$ and

$$\mathbf{r}_g(k) = \sum_{i=0}^{q-k} \mathbf{g}(i)\mathbf{g}(i+k). \tag{2.14}$$

In our proposed method, we apply the spectral factorization technique to obtain the polynomial $\mathbf{g}(z)$ from the autocorrelation sequence.

2.5 Simulation Results

In this section numerical examples are presented to evaluate the performance of the proposed method.

2.5.1 Noise Free Channel Equalization

In this example we consider the minimum phase channel $h(n)$ = [1, 0.5, 0.2, 0.1]. A transmitted data vector s of length 1000 is drawn from BPSK constellation. The autocorrelation of the received signal is estimated by segmenting the received vector $\mathbf{x}$ into subvectors each of length $L_x = 10$. In this simulation the equalizer order is adjusted to $q = 10$ and consequently the equalizer autocorrelation sequence length is set to $L_{r_g} = 21$ in the optimization problem. We consider the ℓ_∞-norm approach where it is computationally less expensive than the ℓ_2-norm approach. The constraint $\mathbf{r}_g(e^{j\omega}) \geq 0$ is discretized in the frequency domain to $N = 512$ frequency points [26]. Figure. 2.2 illustrates the combined channel impulse response where the equalizer $\mathbf{g}(z)$ perfectly equalizes the channel.

2.5.2 BER versus SNR

The second evaluation for the proposed method will focus on the ability of the proposed algorithm to estimate the equalizer $\mathbf{g}(z)$ in the presence of additive white Gaussian noise (AWGN) at different signal-to-noise ratios (SNRs). A transmitted data vector $\mathbf{s}$ of length 1000 is drawn from 4-QAM constellation. We consider the following minimum phase channel,

$$\mathbf{h}(z) = \frac{1 + 0.2z^{-1}}{1 + 0.8z^{-1}}.$$

The transmitted data is convolved with the channel impulse response and AWGN is added to form the received vector $\mathbf{x}$ at different SNRs. For each SNR the equalizer $\mathbf{g}(z)$ is estimated and utilized to equalize the channel output. For the purpose of comparison we simulate the (BER) for the zero-forcing (ZF) equalizer (when the channel is perfectly known), where $\mathbf{g}(z) = 1/\mathbf{h}(z)$. Figure 2.3 shows the performance of the proposed method compared to ZF equalizer. The proposed method has the same performance of ZF equalizer at low SNR as the dominant factor in this case is the noise, whereas, for relatively high SNR the dominant factor that affects the performance is the inter-symbol interference terms appearing in the combined channel.

2.6 Conclusion

We present a new approach to channel equalization problem based on second order statistics for minimum phase channels. In particular, we formulate the problem as a convex optimization problem which can be solved efficiently using interior point methods. Therefore, our approach does not suffer from local minima. The method we provide is computationally less expensive than the correlation fitting method which involves nonlinear optimization over the channel parameters. The performance in the presence of white Gaussian noise is very close to the ZF equalizer with known channel at the receiver end. This intuitively mean that, the channel was perfectly equalized even at low SNR.

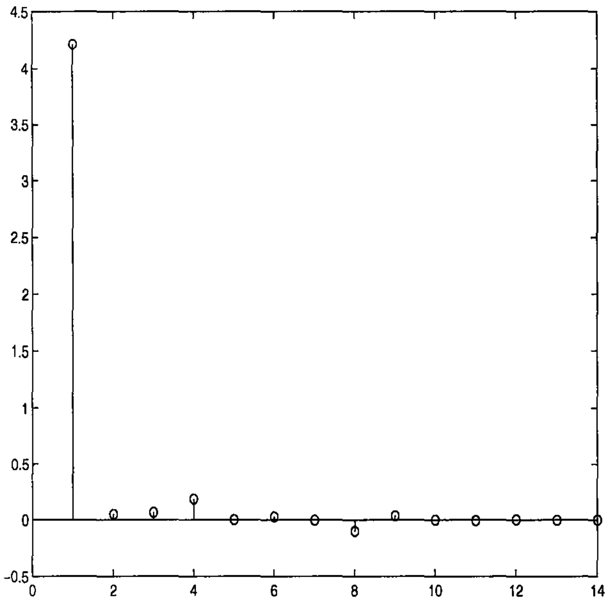

Figure 2.2: Combined channel impulse response

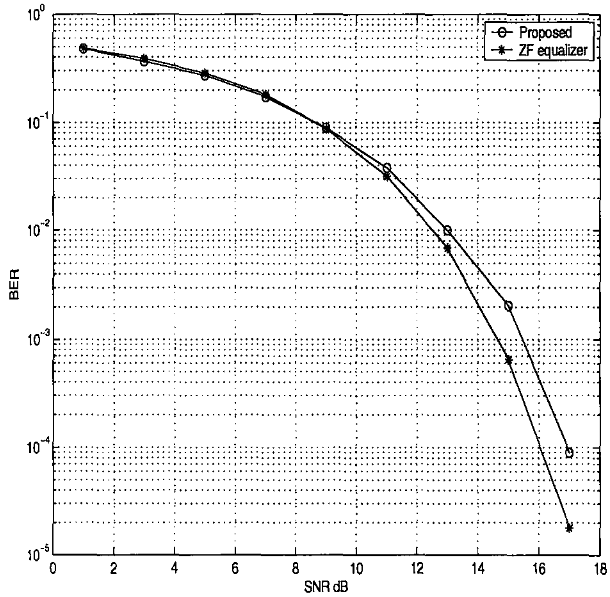

Figure 2.3: Bit error rate for different SNR.

Chapter 3

SIMO Equalization Via Convex Optimization

3.1 Introduction

In this chapter we present a new approach to blind channel equalization for single-input multiple-output (SIMO) systems. We estimate the equalizer via simple convex optimization problem with linear and quadratic constraints using the second-order statistics (SOS) of the received signal and the structure of the channel matrix in SIMO systems. The main advantage of the proposed method is that, it is robust when the channel matrix approaches singularity. Moreover, convex optimization problems do not suffer from the local minima problem that exists in many algorithms [27]. Furthermore, we show that this formulation can be modified to obtain a closed form solution to the equalizer. In the next section we will present a brief description for the fractionally spaced equalizers and then we will discuss in details the proposed algorithm.

3.2 Fractionally Spaced Equalizer

Most of the recent algorithms are directed towards the fractionally spaced equalizers instead of the traditional T-spaced equalizers, where the equalizer taps are closer in time than the symbol interval. Equivalently, the channel output is sampled at an integer multiple of the symbol rate. This model is equivalent to a multichannel of single-input and multiple-output where the output of each subchannel is still at the symbol rate and the estimate of the transmitted symbols is obtained by combining the equalizers output. In the following section we introduce a brief description to the fractionally spaced equalizers (FSEs) highlighting its advantage over the traditional symbol spaced equalizers (SSEs).

3.2.1 T-Spaced to Fractionally Spaced Model

A block diagram explaining the process of oversampling and the equivalent multichannel model is shown in Figure 3.1. In Figure 3.1(a) pulse shaping filter, channel propagation effects, and receiver input filter are modelled in one block. The transmitted signal is considered to be a sequence of T-spaced delta functions. The channel output $x(t)$ is sampled at a rate of p/T and passed through the fractionally spaced equalizer. The equalizer output $y(n)$ is constructed by combining p output samples to produce T-spaced output $y(k)$.

An equivalent discrete system is shown in Figure 3.1(b) where the input signal is represented by samples drawn from a certain constellation (QAM,PSK,..), and spaced by $p-1$ zeros. The discrete channel model is simply the original continuous channel (including transmitter filter, channel propagation, and receiver filter) sampled at a rate p/T. The system can be represented as a multichannel structure shown in Figure 3.1(c). The discrete channel is divided into p subchannels each of length $L+1$ and formed by decimating the discrete channel impulse response by p.

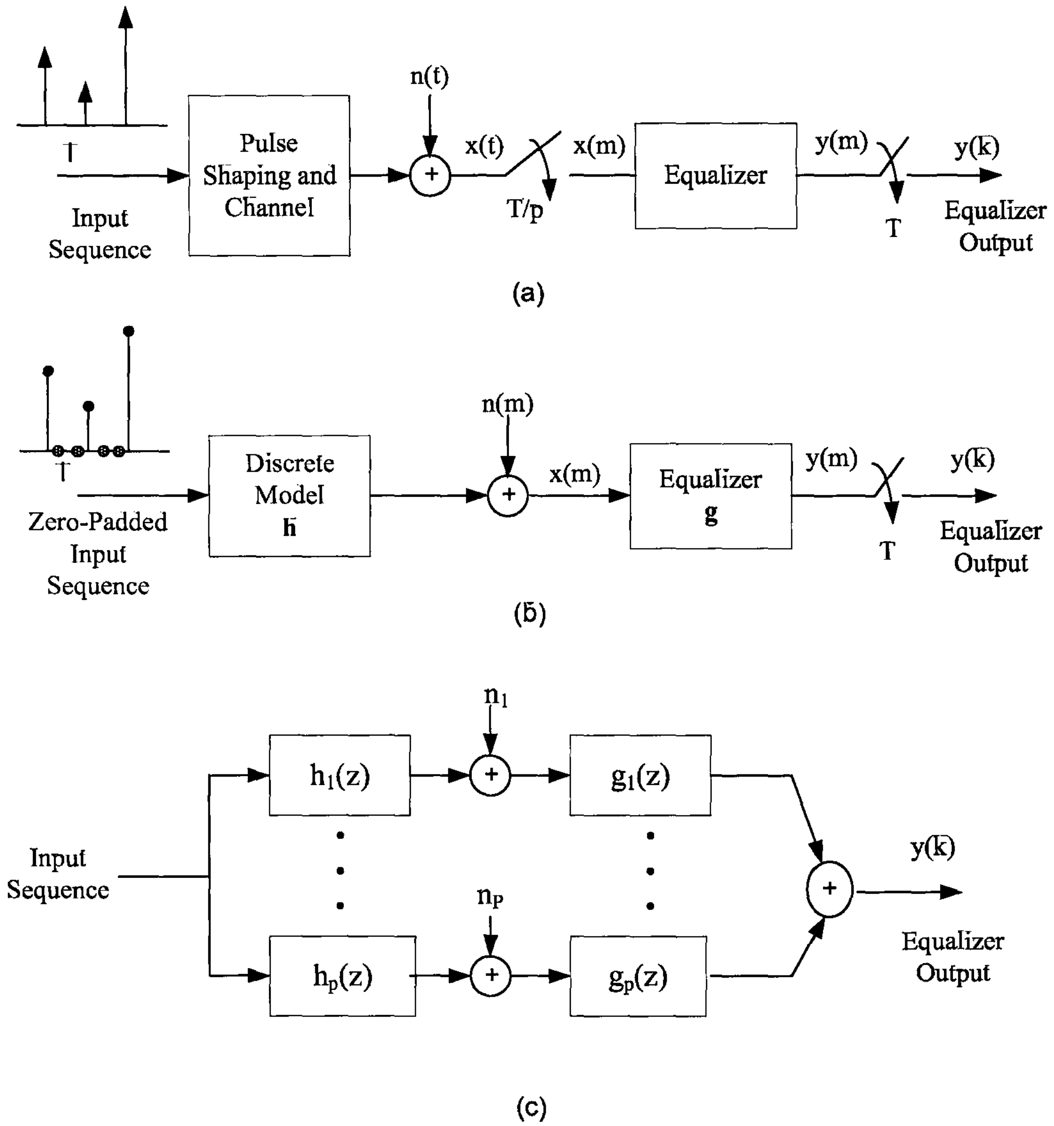

Figure 3.1: Fractionally spaced system. (a) Oversampling by factor p. (b) Equivalent discrete system. (c) Equivalent multichannel model.

3.2.2 Advantages of Fractionally Spaced over T-spaced Equalizers

Although it seems that FSEs are computationally more expensive than the T-spaced equalizers, they overcome many problems that exist in the traditional TSEs. In particular, sampling the received signal at higher rate make it insensitive to timing phase error and a matched filter can be build in the discrete domain. As a result, in practice sampling at a rate $1/T$ is not enough to satisfy the Nyquist rate. Moreover, the equalizer length in the fractionally spaced system is much smaller than that of the symbol spaced case. An important advantage in FSEs is that they provide FIR solution. Recently most of the channel equalization algorithms are directed to the FSEs due to their capability to equalize the channel using only the SOS of the received signal instead of using the higher order statistics (HOS) which need more computations and large data record to estimate the signal statistics [28].

3.3 Channel Condition for FSEs

In the fractionally spaced system shown in Figure 3.1(c), the combined channel impulse response between the transmitted signal and the final equalizer output is given as,

$$\mathbf{f}(n) = \mathbf{h}_1(n) \otimes \mathbf{g}_1(n) + \cdots + \mathbf{h}_i(n) \otimes \mathbf{g}_i(n) + \cdots + \mathbf{h}_p(n) \otimes \mathbf{g}_p(n),$$

where $'\otimes'$ denotes the linear convolution operator, $\mathbf{h}_i$ is the i-th subchannel and $\mathbf{g}_i$ is the i-th subequalizer baud rate impulse response. Considering the z-transform of both sides we get,

$$\mathbf{f}(z) = \mathbf{h}_1(z)\mathbf{g}_1(z) + \cdots + \mathbf{h}_p(z)\mathbf{g}_p(z).$$

For perfect equalization the combined channel impulse response should be zero everywhere except at certain delay $d > 0$, i.e.,

$$z^{-d} = \mathbf{h}_1(z)\mathbf{g}_1(z) + \cdots + \mathbf{h}_p(z)\mathbf{g}_p(z).$$

The above equation (which is known as Bezout relation [29]) leads to the perfect equalization requirement concerning the subchannels roots. Specifically, for the existence of a finite length equalizer, all the subchannel polynomials must not share a common root. To illustrate this condition, let us consider the existence of a common zero between the subchannels, and let this common zero be represented by the polynomial $\mathbf{h}_0(z)$. The i-th subchannel can be factorized as,

$$\mathbf{h}_i(z) = \mathbf{h}_0(z)\tilde{\mathbf{h}}_i(z).$$

Then the Bezout relation will be written as,

$$\begin{aligned} z^{-d} &= \mathbf{h}_0(z)(\tilde{\mathbf{h}}_1(z)\mathbf{g}_1(z) + \cdots + \mathbf{h}_p(z)\mathbf{g}_p(z)) \\ &= \mathbf{h}_0(z)\tilde{\mathbf{f}}(z). \end{aligned}$$

From the above equation it is clear that there is no FIR filter $\tilde{\mathbf{f}}(z)$ which when multiplied by $\mathbf{h}_0(z)$ results in a delay z^{-d}.

3.4 Problem Formulation

In a linear time invariant system, the received signal is given by,

$$x(t) = \sum_m s_m h(t - mT) + n(t), \tag{3.1}$$

where s_m is transmitted data, T is the symbol baud duration, $h(t)$ is the channel impulse response and $n(t)$ is additive white noise independent of the input sequence. In the fractionally spaced scenario the received signal is sampled p times its original baud rate. The resulting i-th subchannel $h_i(n) = h(nT - (i-1)\Delta)$, $i = 1, \ldots, p$ and $\Delta = T/p$, is modelled as an FIR filter of order L. The fractionally spaced system is shown in Figure 3.2.

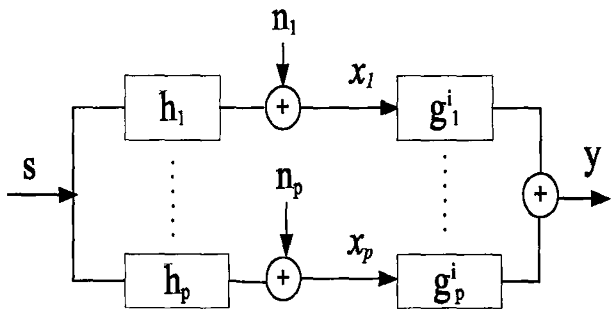

Figure 3.2: Multichannel model

The $p \times 1$ received vector $\boldsymbol{x}(k)$ can be expressed as,

$$\boldsymbol{x}(k) = \sum_m s_m \mathbf{h}(k-m) + \boldsymbol{n}(k), \tag{3.2}$$

where $\mathbf{h}(k) = [h_1(k) \cdots h_p(k)]^T$ and $\boldsymbol{n}(k) = [n_1(k), \ldots, n_p(k)]^T$ is the oversampled noise vector. Collecting M received vectors (where M is the length of each subequalizer), the model can be expressed in a matrix form as follows,

$$\mathbf{x}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \tag{3.3}$$

where,

- $\mathbf{x}(k) = [\boldsymbol{x}(k)^T, \cdots, \boldsymbol{x}(k-M+1)^T]^T$ is $pM \times 1$ received vector,
- $\mathbf{s}(k) = [s_k, \cdots, s_{k-M-L+1}]^T$ is $(M+L)\times 1$ transmitted sequence of (i.i.d) symbols with zero mean and covariance matrix $E\{\mathbf{s}(k)\mathbf{s}^H(k)\} = \mathbf{I}_s$,
- $\mathbf{n}(k) = [\boldsymbol{n}(k)^T, \cdots, \boldsymbol{n}(k-M+1)^T]^T$ is $pM \times 1$ white Gaussian noise with zero mean and covariance matrix, $E\{\mathbf{n}(k)\mathbf{n}^H(k)\} = \sigma^2\mathbf{I}_n$,

- $\mathbf{H}$ is $pM \times (M+L)$ Toeplitz channel matrix,

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(L) & \cdots & 0 \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

Let $\mathbf{h}_i$ denotes the i-th column of the channel matrix $\mathbf{H}$ and h_i^j is the j-th element of the i-th column. The condition for identifying the channel matrix $\mathbf{H}$ is given in [20], [27] and [30] where the channel matrix $\mathbf{H}$ should have a full column rank.

3.5 Covariance Matrix

The covariance matrix $\mathbf{R}_i$ of the received vector $\mathbf{x}$ at delay i is given by,

$$\begin{aligned} \mathbf{R}_i &= \mathrm{E}\{\mathbf{x}(k)\mathbf{x}^H(k-i)\}, \\ &= \mathbf{H}\mathrm{E}\{\mathbf{s}(k)\mathbf{s}^H(k-i)\}\mathbf{H}^H + \mathrm{E}\{\mathbf{n}(k)\mathbf{n}^H(k-i)\}, \\ &= \mathbf{H}\mathbf{J}_s^i\mathbf{H}^H + \sigma^2\mathbf{J}_n^{ip}, \end{aligned}$$

where the transmitted signal power is assumed to be unity and the transmitted signal and noise are assumed to be white, with zero mean and independent. The matrix $\mathbf{J}$ is the Jordan matrix that contains zeros everywhere and ones in the first sub-diagonal below the main diagonal, $\mathbf{J}_s$ is $(M+L) \times (M+L)$ and $\mathbf{J}_n$ is $pM \times pM$ Jordan matrices for signal and noise respectively. For the special case when $i = 0$, the covariance matrix has the following form,

$$\mathbf{R}_0 = \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_n. \tag{3.4}$$

In practical scenarios, the covariance matrix $\mathbf{R}_i$ is not available and is replaced by $\tilde{\mathbf{R}}_i$, which is the sampled covariance matrix at delay i, that is defined as follows for

K received vectors,

$$\tilde{\mathbf{R}}_i = \frac{1}{K-i}\sum_{k=1}^{K-i}\mathbf{x}(k)\mathbf{x}^H(k-i).$$

The noise variance $\hat{\sigma}^2$ can be estimated from the covariance matrix $\tilde{\mathbf{R}}_0$ by applying the eigenvalue decomposition technique to separate signal and noise subspaces. Then the noise updated covariance matrix at delay i is given by,

$$\hat{\mathbf{R}}_i = \tilde{\mathbf{R}}_i - \hat{\sigma}^2\mathbf{J}_n^{ip}.$$

In the next sections we will formulate the problem considering the ideal covariance matrix $\mathbf{R}_i$ to emphasize the idea of the proposed algorithm. However, after formulating the optimization problem, $\mathbf{R}_i$ will be replaced by the sampled covariance matrix $\hat{\mathbf{R}}_i$ and minor modifications will be introduced to guarantee a feasible and effective solution to the optimization problem.

3.6 Blind Zero Forcing Equalizer

Consider the equalizers bank shown in Figure 3.2 for SIMO system. Each subequalizer $\mathbf{g}_j^i$ for $j = 1, \ldots, p$ is an FIR filter with length M. The estimated symbol at delay i is given as,

$$\hat{s}_i = \hat{\mathbf{g}}_i^H\mathbf{x}(k), \qquad 1 \le i \le M+L, \tag{3.5}$$

where $\hat{\mathbf{g}}_i = \mathrm{vec}([\mathbf{g}_1^i, \cdots, \mathbf{g}_p^i]^T)$, and 'vec' denotes the vector operator that generates a new vector by stacking the columns of a given matrix. Considering all the available delays, the equalization matrix can be written as $\hat{\mathbf{G}} = [\hat{\mathbf{g}}_1, \cdots, \hat{\mathbf{g}}_{M+L}]^T$. For zero forcing equalization at delay i where $i = 1, \ldots, M+L$, the equalizer should satisfy the following condition for perfect equalization,

$$\mathbf{H}^H\hat{\mathbf{g}}_i = \mathbf{e}_i, \tag{3.6}$$

where $\mathbf{e}_i$ is the zero vector with the i-th entry equals to one. If $\mathbf{H}$ is known and the condition $\mathbf{H}$ is a full column rank matrix is satisfied, then $\hat{\mathbf{g}}_i^H$ is simply the i-th row of the pseudo inverse of $\mathbf{H}$. For unknown channel matrix the equalizer $\hat{\mathbf{g}}_i$ can be estimated blindly without any knowledge about the channel or the transmitted signal. In the following section, we will present a simple convex optimization formulation to estimate the equalizer $\hat{\mathbf{g}}_i$ using the second-order statistics of the received signal while taking into account the structure of the channel matrix. Moreover, We will show that the zero padding in the channel matrix columns in a fractionally spaced system and the channel matrix full column rank property (independent columns) can be utilized to efficiently estimate the equalizer at different delays.

3.7 Equalizer Constraints

In this section, we will consider the zero forcing equalizer. We will start by finding the constraints that should be satisfied by the equalizer $\hat{\mathbf{g}}_i$. Let us assume that the delay $i = d$ is the one of interest. Multiplying both sides of (3.6) by the channel matrix $\mathbf{H}$ results in,

$$\mathbf{H}\mathbf{H}^H\hat{\mathbf{g}}_d = \mathbf{H}\mathbf{e}_d = \mathbf{h}_d, \tag{3.7}$$

where $\mathbf{h}_d$ is the d-th column of the channel matrix $\mathbf{H}$. Substituting the covariance matrix of the received signal (3.4) (Assuming $\sigma^2 = 0$) in the above expression will lead to,

$$\mathbf{R}_0\hat{\mathbf{g}}_d = \mathbf{h}_d. \tag{3.8}$$

From the columns structure of the channel matrix $\mathbf{H}$, it can be noted that the last $p(M-d)$ elements of the channel column vector $\mathbf{h}_d$ are zeros and this number of zeros is independent of the sub-channels order L. It only depends on the delay d that is assigned according to the desired delay in the equalization procedure. In general, the j-th entry in the i-th column of the channel matrix is $h_i^j = 0$, for $i = 1, \ldots, M$. and

$pi < j \leq pM$. By partitioning the covariance matrix $\mathbf{R}_0$ into two sub-matrices $\mathbf{R}_{01}$ and $\mathbf{R}_{02}$ of sizes $pd \times pM$ and $p(M-d) \times pM$ respectively and denoting the first pd elements in $\mathbf{h}_d$ by the vector $\tilde{\mathbf{h}}_d$, equation (3.8) can be written as,

$$\begin{bmatrix} \mathbf{R}_{01} \\ \mathbf{R}_{02} \end{bmatrix} \hat{\mathbf{g}}_d = \begin{bmatrix} \tilde{\mathbf{h}}_d \\ \mathbf{0} \end{bmatrix}, \tag{3.9}$$

this results in,

$$\mathbf{R}_{02}\hat{\mathbf{g}}_d = \mathbf{0}, \tag{3.10}$$

which implies that $\hat{\mathbf{g}}_d$ belongs to the null space of $\mathbf{R}_{02}$. It is necessary that the equalizer $\hat{\mathbf{g}}_d$ satisfies (3.10), however, it is not sufficient to obtain the equalizer at any delay except at $d = 1$. It is of crucial importance to mention that although $\hat{\mathbf{g}}_1$ can be utilized to equalize the channel, it would amplify the noise. Therefore, the output signal-to-noise ratio is decreased and the performance of the channel equalization is degraded.

The following proposition illustrates how the constraint (3.10) can force the equalizer $\hat{\mathbf{g}}_d$ to partially satisfy the perfect equalization condition in (3.6).

Proposition 1: Assume that there exists a vector $\hat{\mathbf{g}}_d$ where

$$\mathbf{H}^H\hat{\mathbf{g}}_d = \boldsymbol{\alpha}, \tag{3.11}$$

and $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_{M+L})^T$. If $\hat{\mathbf{g}}_d$ satisfies (3.10), then $\boldsymbol{\alpha}$ has the following structure,

$$\alpha_i = \begin{cases} \alpha_i & i = 1, \ldots, d \\ 0 & i = d+1, \ldots, M+L \end{cases} \tag{3.12}$$

Proof: Starting from (3.11) and multiply both sides by $\mathbf{H}$ from the right we get,

$$\begin{aligned} \mathbf{H}\mathbf{H}^H\hat{\mathbf{g}}_d &= \mathbf{H}\boldsymbol{\alpha}, \\ &= \sum_{i=1}^{M+L} \alpha_i \mathbf{h}_i, \\ &= \sum_{i=1}^{d-1} \alpha_i \mathbf{h}_i + \alpha_d \mathbf{h}_d + \sum_{i=d+1}^{M+L} \alpha_i \mathbf{h}_i. \end{aligned} \tag{3.13}$$

Note that, since the elements h_i^j for $i \leq d$ (first and second term in (3.13)) and $j > pd$ are zeros then the last $p(M-d)$ elements of any linear combination of the first d columns of the channel matrix $\sum_{i=1}^{d} \alpha_i \mathbf{h}_i$ will be zeros, and independent of the values of α_i for $i = 1, \ldots, d$. In the last term of (3.13), the elements h_i^j for $i > d$ and $pd < j \leq pi$ are nonzero and since these vectors are independent (full column rank assumption) then any linear combination $\sum_{i=d+1}^{M+L} \alpha_i \mathbf{h}_i$ can not lead to zeros in the last $p(M-d)$ positions unless $\alpha_i = 0$ for $i = d+1, \ldots, M+L$. Q.E.D.

To continue the equalization procedure we have to ensure that $\alpha_i = 0$, for $i = 1, \ldots, d-1$. This can be achieved by considering the following constraint,

$$\mathbf{R}_{d_1} \hat{\mathbf{g}}_d = \mathbf{0}, \tag{3.14}$$

where $\mathbf{R}_{d_1}$ is the covariance matrix at delay $d_1 = M + L - d + 1$. This can be proven as follows,

$$\begin{aligned} \mathbf{R}_{d_1} \hat{\mathbf{g}}_d &= \mathbf{H}\mathbf{J}_s^{d_1}\mathbf{H}^H \hat{\mathbf{g}}_d, \\ &= \mathbf{H}\mathbf{J}_s^{d_1} \boldsymbol{\alpha}, \\ &= \mathbf{H}(0, \ldots, 0, \alpha_1, \ldots, \alpha_{d-1}), \\ &= \sum_{i=1}^{d-1} \alpha_i \mathbf{h}_{d_1 - i}, \\ &= \mathbf{0}. \end{aligned} \tag{3.15}$$

It is clear that if the condition $\mathbf{R}_{d_1} \hat{\mathbf{g}}_d = \mathbf{0}$ is satisfied, this will lead directly to $\alpha_i = 0$ for $i = 1, \ldots, d-1$. It is important to remark that the trivial solution $\hat{\mathbf{g}}_d = \mathbf{0}$ satisfies (3.10) and (3.15), however, in order to ensure that $\hat{\mathbf{g}}_d$ is the zero-forcing equalizer in (3.6), we have to add the following linear constraint to the equalizer,

$$\mathbf{e}_{pd-1}^H \mathbf{R}_0 \hat{\mathbf{g}}_d = \mathbf{r}^H \hat{\mathbf{g}}_d = 1, \tag{3.16}$$

where $\mathbf{e}_{pd-1}$ is a $pM \times 1$ vector with all its entries equal to zeros except one in the position $(pd-1)$ and $\mathbf{r}$ is the $(pd-1)$-th column of the covariance matrix $\mathbf{R}_0$. This constraint can be expressed in terms of the channel matrix entries as $h_d^{pd-1} = 1$.

3.8 Equalization Via Convex Optimization

In this section, we formulate the problem of finding the equalizer $\hat{\mathbf{g}}_d$ for the fractionally spaced system as a convex optimization problem. It is required to minimize a quadratic objective function $\hat{\mathbf{g}}_d^H \mathbf{R}_0 \hat{\mathbf{g}}_d$ subject to the linear constraints derived in the previous section as,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d}{\text{minimize}} \quad & \hat{\mathbf{g}}_d^H \mathbf{R}_0 \hat{\mathbf{g}}_d \\ \text{subject to} \quad & \mathbf{r}^H \hat{\mathbf{g}}_d = 1, \quad \mathbf{R}_{02} \hat{\mathbf{g}}_d = \mathbf{0}, \quad \mathbf{R}_{d_1} \hat{\mathbf{g}}_d = \mathbf{0}. \end{aligned} \tag{3.17}$$

It is well known that this objective function can be replaced by a linear function τ subject to the constraint $\tau - \hat{\mathbf{g}}_d^H \mathbf{R}_0 \hat{\mathbf{g}}_d \geq 0$, which can be reformulated as a positive semi-definite constraint using the Schur-complement [25] as follows,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d, \tau}{\text{minimize}} \quad & \tau \\ \text{subject to} \quad & \mathbf{r}^H \hat{\mathbf{g}}_d = 1, \quad \mathbf{R}_{02} \hat{\mathbf{g}}_d = \mathbf{0}, \quad \mathbf{R}_{d_1} \hat{\mathbf{g}}_d = \mathbf{0}, \\ & \begin{bmatrix} \mathbf{R}_0^{\dagger} & \hat{\mathbf{g}}_d^H \\ \hat{\mathbf{g}}_d & \tau \end{bmatrix} \succeq 0, \end{aligned} \tag{3.18}$$

where '†' denotes the matrix pseudo inverse operator. For ideal estimation of the covariance matrices (infinity number of samples), this optimization problem will lead directly to a perfect equalizer $\hat{\mathbf{g}}_d^H \mathbf{H} = \mathbf{e}_d^H$.

3.8.1 Constraints Relaxation

In practice, the covariance matrices are estimated from finite number of samples as well as the noise variance. Therefore, it is not necessary that the equality constraints

$\hat{\mathbf{R}}_{02}\hat{\mathbf{g}}_d = \mathbf{0}$ and $\hat{\mathbf{R}}_{d_1}\hat{\mathbf{g}}_d = \mathbf{0}$ in (3.18) lead to the required perfect equalizer or even a feasible solution. In order to count the finite samples estimation effect, we will modify the constraints by introducing the constants τ_1 and τ_2 as a relaxation to the original problem as follows,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d, \tau}{\text{minimize}} \quad & \tau \\ \text{subject to} \quad & \hat{\mathbf{r}}^H \hat{\mathbf{g}}_d = 1, \quad \|\hat{\mathbf{R}}_{02}\hat{\mathbf{g}}_d\|_\infty \leq \tau_1, \quad \|\hat{\mathbf{R}}_{d_1}\hat{\mathbf{g}}_d\|_\infty \leq \tau_2, \\ & \begin{bmatrix} \hat{\mathbf{R}}_0^\dagger & \hat{\mathbf{g}}_d^H \\ \hat{\mathbf{g}}_d & \tau \end{bmatrix} \succeq 0, \end{aligned} \tag{3.19}$$

where the covariance matrices are replaced by the estimated matrices. Moreover, the last two constraints are relaxed such that the maximum absolute value of each entry in the resulting vector remains less than a certain value τ_1 and τ_2. Although the above optimization is complete, the solution will be affected by the values assigned to the constants τ_1 and τ_2. In fact, small values for τ_1 and τ_2 can turn the problem infeasible, while large values will make the constraints ineffective. In order to overcome this problem we will treat τ_1 and τ_2 as variables and instead of assigning fixed values for τ_1 and τ_2, we will add these two variables to the objective function and the minimization will be carried out over the new objective function $\tau + \tau_1 + \tau_2$. This will guarantee that the minimum values for τ_1 and τ_2 are achieved and hence improve the performance of the channel equalization over the method of assigning fixed values. The problem can therefore be written as,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d, \tau, \tau_1, \tau_2}{\text{minimize}} \quad & \tau + \tau_1 + \tau_2 \\ \text{subject to} \quad & \hat{\mathbf{r}}^H \hat{\mathbf{g}}_d = 1, \quad \|\hat{\mathbf{R}}_{02}\hat{\mathbf{g}}_d\|_\infty \leq \tau_1, \quad \|\hat{\mathbf{R}}_{d_1}\hat{\mathbf{g}}_d\|_\infty \leq \tau_2, \\ & \begin{bmatrix} \hat{\mathbf{R}}_0^\dagger & \hat{\mathbf{g}}_d^H \\ \hat{\mathbf{g}}_d & \tau \end{bmatrix} \succeq 0. \end{aligned} \tag{3.20}$$

It is important to mention that, the constraint $\hat{\mathbf{R}}_{d_1}\hat{\mathbf{g}}_d = \mathbf{0}$ is highly affected by the estimation quality of the covariance matrix $\hat{\mathbf{R}}_{d_1}$. This can be inferred from (3.15), where we assume that the signal covariance matrix at delay d_1 is perfectly estimated as $\mathbf{J}_s^{d_1}$ and hence we have $\mathbf{J}_s^{d_1}\mathbf{H}^H\hat{\mathbf{g}}_d = (0, \ldots, 0, \alpha_1, \ldots, \alpha_{d-1})$. These d_1 leading zeros ensure that there is no contribution from the first d_1 columns of the channel matrix in the constraint. However, the estimated covariance matrix $\hat{\mathbf{R}}_{d_1}$ will not satisfy these leading zeros anymore and in order to improve the performance of the proposed algorithm, we will modify the covariance matrix $\hat{\mathbf{R}}_{d_1}$ by introducing blocks of zeros in the previously known position of the original matrix $\mathbf{H}\mathbf{J}_s^{d_1}\mathbf{H}^H$ (These positions depend only on d_1 and p which are known to the receiver end) as follows,

$$\hat{\mathbf{R}}_{d_{1m}} = \hat{\mathbf{R}}_{d_1} \odot (\sum_{k=1}^{d-1} \mathbf{J}^{M-k} \otimes \mathbf{1}_{p\times p}), \tag{3.21}$$

where $\mathbf{1}_{p\times p}$ is a $p \times p$ matrix with all its entries equal to 1 and $\hat{\mathbf{R}}_{d_{1m}}$ is the modified covariance matrix at delay d_1. The modified problem can be written as,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d,\tau,\tau_1,\tau_2}{\text{minimize}} \quad & \tau + \tau_1 + \tau_2 \\ \text{subject to} \quad & \hat{\mathbf{r}}^H\hat{\mathbf{g}}_d = 1 \quad \|\hat{\mathbf{R}}_{02}\hat{\mathbf{g}}_d\|_\infty \leq \tau_1 \\ & \|(\hat{\mathbf{R}}_{d_1} \odot (\sum_{k=1}^{d-1} \mathbf{J}^{M-k} \otimes \mathbf{1}_{P\times P}))\hat{\mathbf{g}}_d\|_\infty \leq \tau_2, \\ & \begin{bmatrix} \hat{\mathbf{R}}_0^\dagger & \hat{\mathbf{g}}_d^H \\ \hat{\mathbf{g}}_d & \tau \end{bmatrix} \succeq 0. \end{aligned} \tag{3.22}$$

This optimization problem consists of a linear objective function with linear equality, linear inequalities and positive semi-definite (PSD) constraints which can be efficiently solved using interior point methods to find the equalizer $\hat{\mathbf{g}}_d$. In the next section we will show how the original optimization problem (3.17) can be reformulated by considering the ℓ_2-norm to obtain a closed form solution of the equalizer $\hat{\mathbf{g}}_d$.

3.9 Closed Form Solution

In this section, we present a closed form solution of the equalizer $\hat{\mathbf{g}}_d$. Considering the original problem formulation in (3.17), the equality constraints can be reformulated using the ℓ_2-norm instead of ℓ_∞-norm which results in the following problem,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d}{\text{minimize}} \quad & \hat{\mathbf{g}}_d^T \mathbf{R}_0 \hat{\mathbf{g}}_d \\ \text{subject to} \quad & \mathbf{r}^T \hat{\mathbf{g}}_d = 1, \quad \hat{\mathbf{g}}_d^H \mathbf{R}_{02}^H \mathbf{R}_{02} \hat{\mathbf{g}}_d = 0, \quad \hat{\mathbf{g}}_d^H \mathbf{R}_{d_1}^H \mathbf{R}_{d_1} \hat{\mathbf{g}}_d = \mathbf{0}. \end{aligned} \tag{3.23}$$

By following the same steps considered in the previous section (relaxation), these two constraints can be added to the objective function. Define a matrix $\mathbf{Q} = \mathbf{R}_0 + \mathbf{R}_{02}^H \mathbf{R}_{02} + \mathbf{R}_{d_1}^H \mathbf{R}_{d_1}$, the problem can be written now as follows,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d}{\text{minimize}} \quad & \hat{\mathbf{g}}_d^H \mathbf{Q} \hat{\mathbf{g}}_d \\ \text{subject to} \quad & \mathbf{r}^H \hat{\mathbf{g}}_d = 1. \end{aligned} \tag{3.24}$$

Considering the estimated finite samples covariance matrices and replacing the covariance matrix $\hat{\mathbf{R}}_0$ by $\tilde{\mathbf{R}}_0$ (i.e. before noise suppression) in order to minimize the noise power at the output, The problem can be modified as follows,

$$\begin{aligned} \underset{\hat{\mathbf{g}}_d}{\text{minimize}} \quad & \hat{\mathbf{g}}_d^H \hat{\mathbf{Q}} \hat{\mathbf{g}}_d \\ \text{subject to} \quad & \hat{\mathbf{r}}^H \hat{\mathbf{g}}_d = 1, \end{aligned} \tag{3.25}$$

where $\hat{\mathbf{Q}} = \tilde{\mathbf{R}}_0 + \hat{\mathbf{R}}_{02}^H \hat{\mathbf{R}}_{02} + \hat{\mathbf{R}}_{d_1}^H \hat{\mathbf{R}}_{d_1}$. This problem can be solved analytically and results in the equalizer,

$$\hat{\mathbf{g}}_d = \frac{\hat{\mathbf{Q}}^{-1} \hat{\mathbf{r}}}{\hat{\mathbf{r}}^H \hat{\mathbf{Q}}^{-1} \hat{\mathbf{r}}}. \tag{3.26}$$

Although the above optimization problem is complete, it should be noticed that the solution is affected by the signal power and the noise power individually and not by the signal-to-noise ratio. This can be illustrated by assuming that the received signal is multiplied by a factor γ such that,

$$\mathbf{x}(k) = \gamma(\mathbf{Hs}(k) + \mathbf{n}(k)),$$

and let us denote the covariance matrices in this case by a subscript w. The following relations are easy to be deduced,

$$\begin{aligned}\tilde{\mathbf{R}}_{0w} &= \gamma^2\tilde{\mathbf{R}}_0,\\ \hat{\mathbf{R}}_{02w} &= \gamma^2\hat{\mathbf{R}}_{02},\\ \hat{\mathbf{R}}_{d_1w} &= \gamma^2\hat{\mathbf{R}}_{d_1}.\end{aligned}$$

The new matrix $\hat{\mathbf{Q}}_w = \tilde{\mathbf{R}}_{0w} + \hat{\mathbf{R}}_{02w}^H\hat{\mathbf{R}}_{02w} + \hat{\mathbf{R}}_{d_1w}^H\hat{\mathbf{R}}_{d_1w}$ can be expressed in terms of the original covariance matrices as follows,

$$\hat{\mathbf{Q}}_w = \gamma^2\tilde{\mathbf{R}}_0 + \gamma^4\hat{\mathbf{R}}_{02}^H\hat{\mathbf{R}}_{02} + \gamma^4\hat{\mathbf{R}}_{d_1}^H\hat{\mathbf{R}}_{d_1}.$$

Although the signal-to-noise ratio does not change, it can be easily noticed that, the matrix $\hat{\mathbf{Q}}_w$ is affected by the weighting factor γ. For small values of γ the last two terms will be vanished and become ineffective in the process of equalizer estimation, while for large values the last two terms will dominate. In both cases, this will lead to estimation errors.

We conjecture, without proof of optimality, that by introducing a new factor β as given below we can overcome this problem and adjust the effect of this weighting problem on the matrix $\hat{\mathbf{Q}}$ as follows,

$$\hat{\mathbf{Q}} = \tilde{\mathbf{R}}_0 + \frac{\hat{\mathbf{R}}_{02}^H\hat{\mathbf{R}}_{02}}{\beta} + \frac{\hat{\mathbf{R}}_{d_1}^H\hat{\mathbf{R}}_{d_1}}{\beta}, \qquad \text{where,} \quad \beta = \frac{||\tilde{\mathbf{R}}_0||_F}{(pM)^2}.$$

One of the important issue in this formulation is the choice of β as it affects the performance of the equalizer. More investigation is required to find out an optimal (or suboptimal) value for β.

3.10 Simulation Results

In this section we present three examples to evaluate the performance of the proposed algorithms. The oversampling factor $p = 4$ and the noise is white, Gaussian and with zero mean. We will examine the ability of the algorithm to remove the inter symbol interference (ISI), as well as the bit error rate (BER) for different Signal-to-Noise Ratios (SNRs).

3.10.1 Example 1: ISI and Eye Diagram

we consider the following channel,

$$h = \begin{bmatrix} .04 & -0.05 & 0.07 & -.2 & -0.5 & 0.72 & 0.36 & 0.21 & 0.03 & 0.07 & .03 & -.01 \end{bmatrix},$$

which is a typical telephone channel [12]. The sub-channel order is $L = 2$, the other parameters are adjusted such that, the delay $d = 6$, the smoothing lag (length of each sub-equalizer) $M = 10$ and $d_1 = M + L - d + 1 = 7$. The data symbols are i.i.d and drawn from 16-QAM constellation and the covariance matrix is estimated over 1000 symbols. The simulation is carried out at signal-to-noise ratio SNR=20dB. Figure 3.3(a) shows the channel impulse response $h(n)$ and Figure 3.3(b) present the received signal constellation before equalization. In Figure 3.3(c) and (d) the impulse response of the combined channel (channel and equalizer) and the equalized signal constellation using the ℓ_∞ optimization problem are plotted. Finally in Figure 3.3(e) and (f), the combined channel response and the equalized signal constellation using the closed form solution are shown. The proposed algorithms efficiently equalize the channel and open the eye diagram.

3.10.2 Example 2: Channel Close to Singularity

We consider a minimum phase channel with the following impulse response,

$$h(z) = \frac{1 + 0.2z^{-1}}{1 - 0.8z^{-1}}.$$

This channel is selected such that, the channel matrix which is formed by oversampling $h(n)$ approaches singularity. The subchannel order is $L = 3$, each subequalizer length is $M = 11$ and the delay $d = 7$. The transmitted signal is i.i.d drawn from a 4-QAM constellation. The covariance matrix is estimated over 1000 symbols at SNR=18 dB. The simulation results are shown in Figure 3.4 where the channel impulse response $h(n)$ is plotted in Figure 3.4(a) and the received signal constellations is shown in Figure 3.4(b). The resulting combined channel impulse response $f = \sum_{i=1}^{P} h_i \otimes g_i$ and the equalized signal constellation (for ℓ_∞-norm convex optimization problem) are shown in Figure 3.4(c) and Figure 3.4(d) respectively. Also the combined channel impulse response and the equalized signal constellation for the closed form solution are shown in Figure 3.4(e) and Figure 3.4(f) respectively. From this example we conclude that, the proposed methods can efficiently equalize the channel even though when the channel matrix is close to being singular.

3.10.3 Example 3: BER versus SNR

In this example, we use the channel given in example 1. To compare the proposed method (ℓ_∞-norm) with other existing algorithms. We simulate two methods, the subspace method (SSM) [20] and the constrained optimization method (COM) [30] for channel equalization. The transmitted signal is i.i.d and drawn from BPSK constellation. Each sub-equalizer is an FIR filter of length $M = 9$, the delay is adjusted to $d = 6$ and $d_1 = 6$. The covariance matrices are estimated over 500 symbols. The BER comparison is shown in Figure 3.5 where it can be inferred that the proposed algorithm results in lower BER compared to [20] and [30]. It should be mentioned that in both methods [20] and [30], the sub-channel order is assumed to be known where both methods depend on the perfect estimation of the sub-channel order L. However, a performance degradation will occur in the case of incorrect estimation. Moreover, while the SSM method utilizes an estimated channel column vector with zeros are placed in the previously known positions (depending on the channel order) along with the finite covariance matrix to estimate the equalizer, the behavior of the SSM method at high SNR is affected as shown in Figure 3.5.

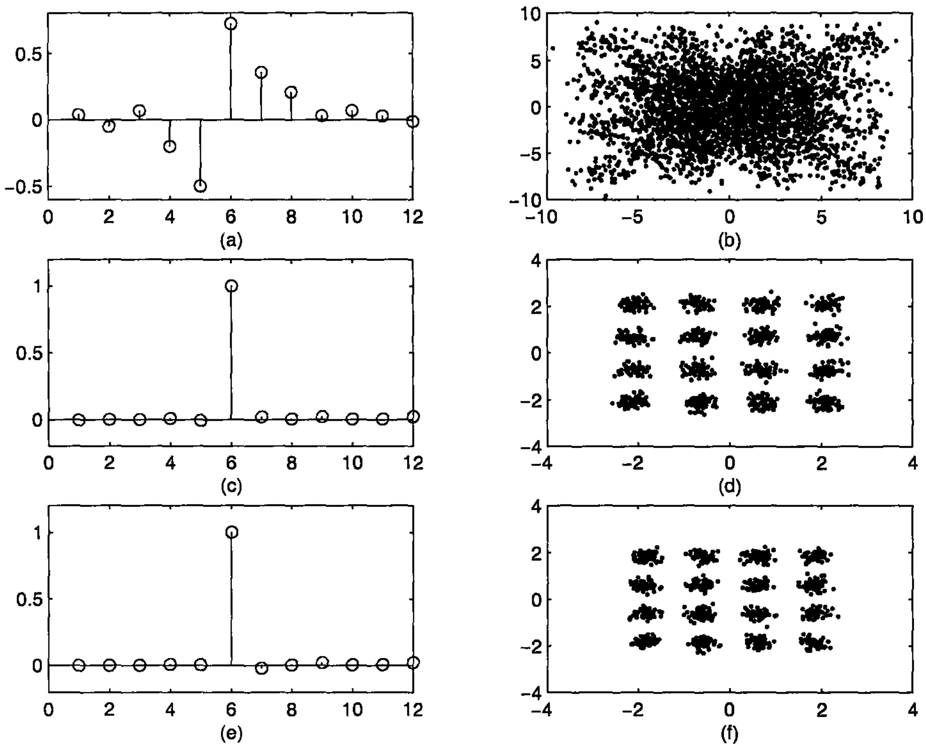

Figure 3.3: (a) Channel $h(n)$. (b) Received signal constellation. (c) and (d) Combined channel impulse response and equalized signal constellation: Convex Optimization. (e) and (f) Combined channel impulse response and equalized signal constellation: Closed form.

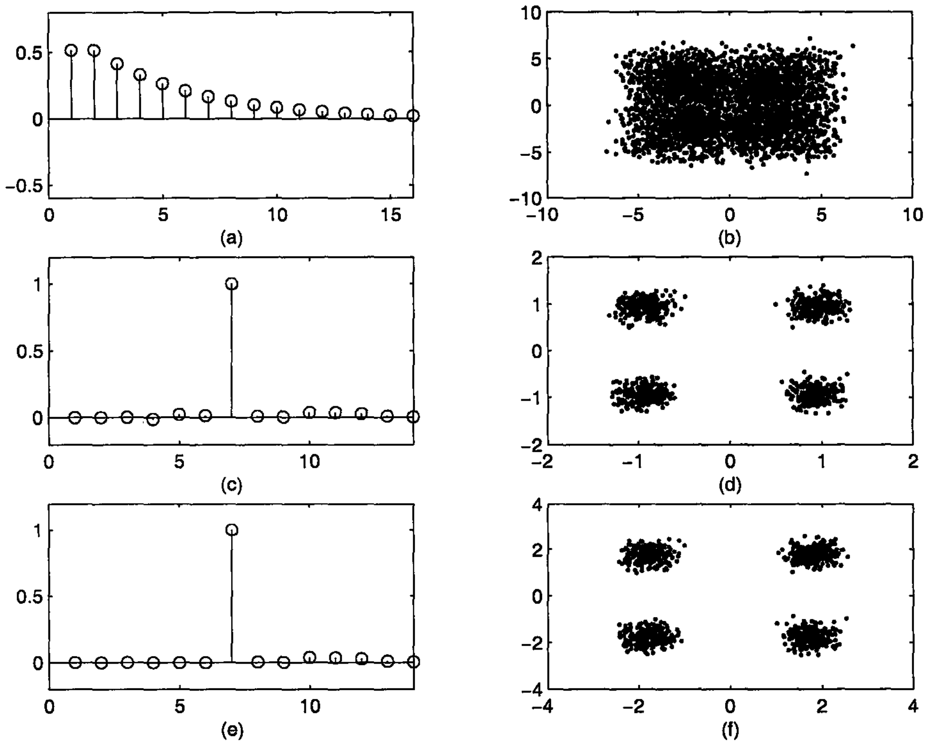

Figure 3.4: (a) Channel $h(n)$. (b) Received signal constellation. (c) and (d) Combined channel and equalized signal constellation: Convex Optimization. (e) and (f) Combined channel and equalized signal constellation: Closed form.

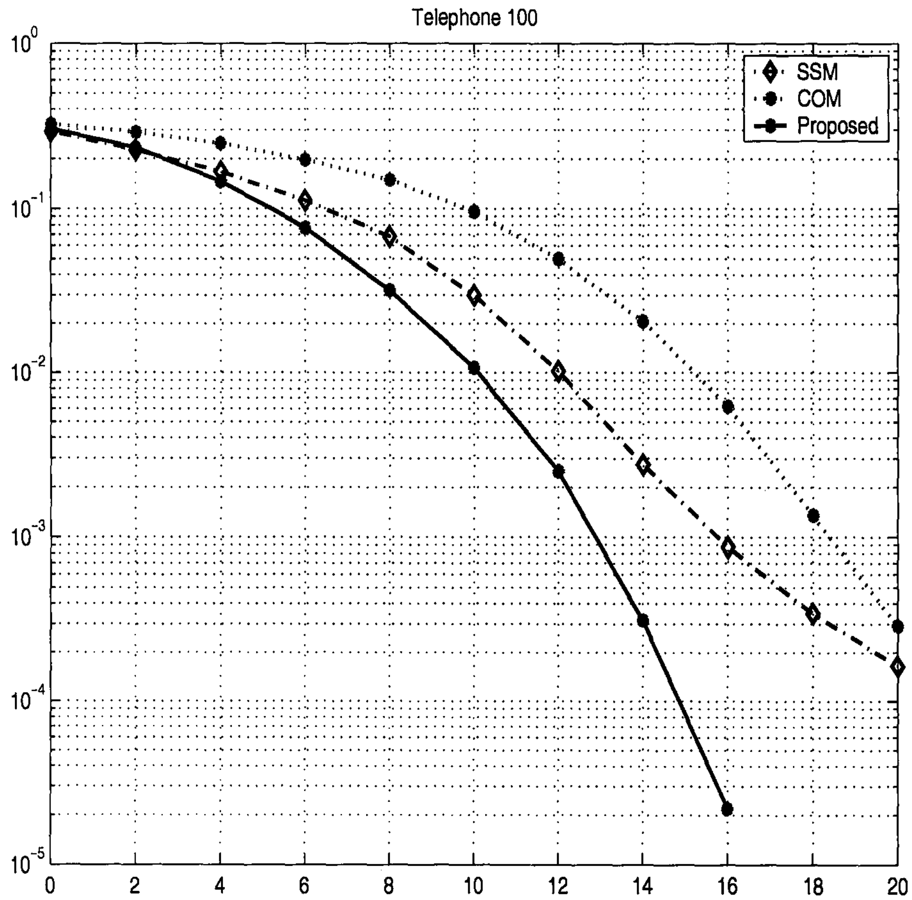

Figure 3.5: BER vs SNR

Chapter 4

Direct MMSE Channel Equalization

In this chapter, we present two approaches to blind channel equalization based on second-order statistics for SIMO systems. Due to the oversampling associated with FS equalization, the equivalent channel matrix possesses a particular structure enabling us to estimate the channel based only on information exists in the covariance matrix at lag zero. Simulation examples are provided to demonstrate the performance advantage of the proposed algorithms compared to existing techniques.

4.1 Introduction

Recently, blind channel equalization algorithms have been directed to utilizing the second-order statistics of the received signal to estimate the equalizer. The algorithm presented in [16], [31] is one of the first algorithms utilizing the second-order statistics to equalize the channel in the case of fractionally sampled channel output. It uses a recursive method to estimate the channel using estimated covariance matrices at different delays, therefore, error propagation may occur. Moreover, the method in

[16] does not include the channel structure while estimating the channel. The subspace algorithm proposed in [20] utilizes the channel matrix structure to estimate the channel where it relies on the fact that, the noise subspace is orthogonal to the space spanned by the columns of the channel matrix. However, this method is sensitive to channel order estimation error and performance degradation is noticed when the channel matrix approaches singularity. Another subspace approach is presented in [30] where the analogy between the Capon direction of arrival estimation technique and blind channel estimation is analyzed. Although [30] presents a closed form solution, it requires prior knowledge of the channel order. Furthermore, [30] does not utilize the channel matrix structure in estimation. The methods presented in [22] and [23] do not require knowledge of the channel order and do not depend on the signal and noise subspace separation. However, they estimate the signal subspace in order to remove the noise component from the estimated covariance matrix at lag zero. Moreover, they require additional covariance matrices at different delays and the channel matrix structure is not employed in the estimation.

In this chapter, we present two methods A and B for channel estimation/equalization in fractionally spaced system. Unlike [20], our proposed method-A is less sensitive to channel order estimation error and performs well when the channel is close to being singular. Although method-A depends on recursion while estimating the channel, it performs better than [16] as it makes use of the channel structure. Moreover, method-A uses only the covariance matrix at delay zero to equalize the channel. Therefore, it has an advantage over methods that employ multiple covariance matrices computed at different delays, e.g. [23], [22] and [16], where the covariance matrix estimation accuracy affects the equalization performance. On the other hand, method-B is computationally less expensive than the methods from [23] and [22] as well as method-A. However, Method-B partially uses the channel structure and no matrix inversion is needed as in [23] and [22]. Furthermore, we will show that the covariance matrix at different delays can be obtained directly from the zero lag covariance matrix and can be used to estimate the channel with low computational complexity and good performance.

4.2 Problem Formulation

In this section the same definitions and formulations for the fractionally spaced system in chapter 3 will be used. We highlight some formulations and assumptions needed such that the flow of the proposed algorithm becomes consistent. The received signal is given as,

$$\mathbf{x}(k) = \mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \tag{4.1}$$

where $\mathbf{H}$ is $pM \times (M+L)$ Toeplitz channel matrix,

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(L) & \cdots & 0 \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

Before we proceed to channel estimation we summarize the assumptions made regarding the signal, noise, channel matrix and the subchannels as well.

- Transmitted symbols are (i.i.d) with zero mean and $\mathrm{E}\{\mathbf{s}(k)\mathbf{s}^H(k)\} = \mathbf{I}_s$,
- The noise is Gaussian with zero mean and $\mathrm{E}\{\mathbf{n}(k)\mathbf{n}^H(k)\} = \sigma^2\mathbf{I}_n$,
- The channel matrix $\mathbf{H}$ is full column rank ([20], [22], [16] and [27]),
- Equivalently, the subchannels h_i have no common zeros.

4.3 Fractionally Spaced Equalizer

4.3.1 Blind MMSE Equalizer

Consider the equalizers bank shown in Figure 3.2 where $\mathbf{g}_i, i = 1, \cdots, p$ is FIR of order $M - 1$. The estimated symbol at delay i is given by,

$$\hat{s}_i = \tilde{\mathbf{g}}_i^H \mathbf{x}(k), \qquad 1 \leq i \leq M + L, \tag{4.2}$$

where $\tilde{\mathbf{g}}_i = \text{vec}([\mathbf{g}_1^i, \cdots, \mathbf{g}_p^i]^T)$ and the notation 'vec' denotes the vector operator. Considering all the available delays, the equalization matrix can be written as $\tilde{\mathbf{G}} = [\tilde{\mathbf{g}}_1, \cdots, \tilde{\mathbf{g}}_{M+L}]^T$. For the minimum mean square equalizer MMSE the equalization matrix is given by,

$$\begin{aligned}
\mathbf{G} &= \arg\min_{\mathbf{G}} \text{E}\{\| \mathbf{G}\,\mathbf{x}(k) - \hat{\mathbf{s}}(k)\|^2\} \\
&= \arg\min_{\mathbf{G}} \text{E}\{\mathbf{x}^H\mathbf{G}^H\mathbf{G}\mathbf{x} - 2\mathbf{x}^H\mathbf{G}^H\hat{\mathbf{s}} - \hat{\mathbf{s}}^H\hat{\mathbf{s}}\} \\
&= \arg\min_{\mathbf{G}} \text{E}\{\text{trace}\{\mathbf{G}\mathbf{x}\mathbf{x}^H\mathbf{G}^H - 2\hat{\mathbf{s}}\mathbf{x}^H\mathbf{G}^H\}\} \\
&= \arg\min_{\mathbf{G}} \text{trace}\{\mathbf{G}\mathbf{R}_0\mathbf{G}^H - 2\mathbf{H}^H\mathbf{G}^H\}
\end{aligned}$$

where $\hat{\mathbf{s}}(k)$ is the estimated sequence. The equalizer can be obtained from the above optimization problem in closed form as,

$$\mathbf{G} = \mathbf{H}^H\mathbf{R}_0^{\dagger}, \tag{4.3}$$

where '$\dagger$' denotes a matrix pseudo inverse operator. Since $\mathbf{R}_0$ can be estimated from the received signal, it remains to estimate the columns of the channel matrix $\mathbf{H}$ corresponding to different delay equalizers. The channel equalization quality is affected by the delay selection. It was noticed in many algorithms that, the delay i should be selected such that the corresponding channel column vector contains all the channel coefficients.

4.3.2 Covariance Matrix

The covariance matrix $\mathbf{R}_i$ of the received signal $\mathbf{x}$ at delay i is given by,

$$\begin{aligned}\mathbf{R}_i &= \mathrm{E}\{\mathbf{x}(k)\mathbf{x}^H(k-i)\},\\ &= \mathbf{H}\mathbf{J}_s^i\mathbf{H}^H + \sigma^2\mathbf{J}_n^{ip},\end{aligned} \tag{4.4}$$

where $\mathbf{J}_s$ is $(M+L)\times(M+L)$ and $\mathbf{J}_n$ is $pM \times pM$ Jordan matrices (matrix with ones in the subdiagonal below the main diagonal) defined as,

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The covariance matrix at delay $i = 0$ (which is a special case from (4.4)) is the one of interest in all algorithms as it provides information about signal and noise subspaces. The covariance matrix $\mathbf{R}_0$ and its eigen decomposition are given as follows,

$$\begin{aligned}\mathbf{R}_0 &= \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_n,\\ &= [\mathbf{U}_1\mathbf{U}_2]\begin{bmatrix} \mathbf{\Lambda}+\sigma^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{U}_1^H \\ \mathbf{U}_2^H\end{bmatrix},\end{aligned} \tag{4.5}$$

where the transmitted data and noise are assumed white, independent of each other and with zero mean. Separating signal and noise subspaces, we can express $\mathbf{H}\mathbf{H}^H$ as follows,

$$\mathbf{H}\mathbf{H}^H = \mathbf{U}_1\mathbf{\Lambda}\mathbf{U}_1^H.$$

The channel matrix $\mathbf{H}$ can be obtained up to an unknown orthonormal matrix $\mathbf{V}^H$ (where $\mathbf{V}$ is the $(M+L)\times(M+L)$ right singular matrix of $\mathbf{H}$) as,

$$\begin{aligned}\mathbf{H} &= \mathbf{U}_1\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}^H\\ &= \mathbf{A}\mathbf{V}^H,\end{aligned} \tag{4.6}$$

where $\mathbf{A} = \mathbf{U}_1\mathbf{\Lambda}^{\frac{1}{2}}$ and $\mathbf{V}^H = [\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{M+L}]$. In the next section we will present two methods (A and B) for blind channel equalization. The key step in method-A is to find $\boldsymbol{v}_1$ and then apply a recursive method to estimate the j-th vector $\boldsymbol{v}_j$ from which we can obtain the j-th channel matrix column containing all the channel coefficients. While in method-B we avoid the recursion procedure in method-A and compromise between the computations and the performance. Note that, it is not necessary to estimate the equalizer at all the $(M+L)$ delays (i.e $\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{M+L}$). However, estimating a channel column vector that has all the channel coefficients results in a performance that is very close to the best delay equalizer.

4.4 Channel Estimation

4.4.1 Method A

In this section we present a robust and efficient algorithm to estimate a channel column vector that contains all the channel coefficients. This will result in an equalizer with performance very close to the best delay equalizer. We define a matrix $\mathbf{H}_1$ of size $p(M-1) \times (M+L)$, containing the last $p(M-1)$ rows of the original matrix $\mathbf{H}$ with the following structure,

$$\mathbf{H}_1 = \begin{bmatrix} 0 & \mathbf{h}(0) & \cdots & \mathbf{h}(L) & \cdots & 0 \\ \vdots & & \ddots & & \ddots & \vdots \\ 0 & & & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

Since $\mathbf{H}$ is a full column rank matrix with rank($\mathbf{H}$) $= M + L$ then it is clear that rank($\mathbf{H}_1$) $= M + L - 1$ where the first column in $\mathbf{H}_1$ induces the rank deficiency. The matrix $\mathbf{H}$ can be divided into two sub-matrices $\tilde{\mathbf{H}}_1$ and $\mathbf{H}_1$ of sizes $(M-1) \times (M+L)$ and $p(M-1) \times (M+L)$ respectively (the same procedure is applied to matrix $\mathbf{A}$ and results in $\tilde{\mathbf{A}}_1$ and $\mathbf{A}_1$ respectively). Then (4.6) can be written as,

$$\begin{bmatrix} \tilde{\mathbf{H}}_1 \\ \mathbf{H}_1 \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}}_1 \\ \mathbf{A}_1 \end{bmatrix} \mathbf{V}^H. \tag{4.7}$$

Equating the last $p(M-1)$ rows it yields,

$$\mathbf{H}_1 = \mathbf{A}_1 \mathbf{V}^H. \tag{4.8}$$

Comparing the first column in both sides of (4.8) we get,

$$\mathbf{A}_1 \boldsymbol{v}_1 = \mathbf{0}, \tag{4.9}$$

meaning that $\mathbf{v}_1$ belongs to the null space of $\mathbf{A}_1$. Since $\mathbf{V}^H$ is a full rank matrix and $\text{rank}(\mathbf{H}_1) = M + L - 1$, from (4.8) it yields that $\text{rank}(\mathbf{A}_1) = M + L - 1$ and $\mathbf{A}_1^H \mathbf{A}_1$ has a unique zero eigenvalue. Since $\boldsymbol{v}_1$ is a unit norm vector, then $\boldsymbol{v}_1$ is the solution to the following problem.

$$\underset{\boldsymbol{v}_1}{\text{minimize}} \qquad \boldsymbol{v}_1^H \mathbf{A}_1^H \mathbf{A}_1 \boldsymbol{v}_1 \qquad\qquad \text{subject to} \quad \|\boldsymbol{v}_1\| = 1. \tag{4.10}$$

Therefore, $\boldsymbol{v}_1$ is the eigenvector corresponding to the unique zero eigenvalue of $\mathbf{A}_1^H \mathbf{A}_1$. Starting from $\boldsymbol{v}_1$ we can estimate $\boldsymbol{v}_2$ and from $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ we can estimate $\boldsymbol{v}_3$ and so on. Applying this recursive method we can obtain $\boldsymbol{v}_j$ where $j > 1$. In this method we make use from the orthogonality property existing between $\boldsymbol{v}_j$ and $\boldsymbol{v}_i$ for $i = 1, \ldots, j-1$. To illustrate the recursion procedure, let us consider that $\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{j-1}$ are already estimated and the vector $\boldsymbol{v}_j$ is the one of interest at the moment.

To find the j-th column of $\mathbf{V}^H$ $(j > 1)$, we define a matrix $\mathbf{H}_j$ of size $p(M-j) \times (M+L)$ that has the last $p(M-j)$ rows of the channel matrix $\mathbf{H}$ with the condition $p(M-j) > (M+L)$. This condition is necessary to ensure that, $\mathbf{H}_j$ is a tall matrix and the rank deficiency arises from the first j zero columns. The structure of $\mathbf{H}_j$ is shown below,

$$\mathbf{H}_j = \begin{bmatrix} 0 & \cdots & 0 & \mathbf{h}(0) & \cdots & \mathbf{h}(L) & \cdots & 0 \\ \vdots & & \vdots & & \ddots & & \ddots & \vdots \\ 0 & \cdots & 0 & & & \mathbf{h}(0) & \cdots & \mathbf{h}(L) \end{bmatrix}.$$

Following the same steps, the matrices $\mathbf{H}$ and $\mathbf{A}$ in (4.6) can be partitioned and considering the last $p(M-j)$ rows of the channel matrix $\mathbf{H}$ we obtain,

$$\mathbf{H}_j = \mathbf{A}_j \mathbf{V}^H, \tag{4.11}$$

where $\text{rank}(\mathbf{A}_j) = M + L - j$ and the matrix $\mathbf{A}_j^H \mathbf{A}_j$ has j zero eigenvalues. Equating the j-th column in each side we obtain, $\mathbf{A}_j \boldsymbol{v}_j = \mathbf{0}$. which can be rewritten as,

$$\boldsymbol{v}_j^H \mathbf{A}_j^H \mathbf{A}_j \boldsymbol{v}_j = \mathbf{0}. \tag{4.12}$$

Since the null space $\mathcal{N}(\mathbf{A}_j^H \mathbf{A}_j)$ has dimension j then $\boldsymbol{v}_j$ can not be obtained directly from (4.12) as many solutions exist for $\boldsymbol{v}_j$ and satisfy (4.12). Let $\mathcal{N}(\mathbf{A}_j^H \mathbf{A}_j) = \text{Range}(\mathbf{U}_j) = \text{span}(\boldsymbol{u}_1, \cdots, \boldsymbol{u}_j)$ where, $\boldsymbol{u}_i$ is the i-th eigenvector of $\mathbf{A}_j^H \mathbf{A}_j$ corresponding to the i-th zero eigenvalue ($i = 1, \ldots, j$). Since $\boldsymbol{v}_j \in \text{span}(\boldsymbol{u}_1, \cdots, \boldsymbol{u}_j)$ then $\boldsymbol{v}_j$ can be expressed as a linear combination of these vectors i.e.,

$$\boldsymbol{v}_j = \mathbf{U}_j \mathbf{a}, \tag{4.13}$$

where $\mathbf{a} = (a_1, \cdots, a_j)^T$ and $\|\mathbf{a}\|^2 = 1$. This condition guarantees that $\|\boldsymbol{v}_j\|^2 = 1$. At this stage, it is required to find a vector $\mathbf{a}$ such that $\boldsymbol{v}_j$ is the j-th column of the matrix $\mathbf{V}^H$. However, $\boldsymbol{v}_j$ should also belong to the null space of the matrix $\mathbf{W}_j$ whose columns are the first $(j-1)$-th columns of $\mathbf{V}^H$ (previously estimated) defined as, $\mathbf{W}_j = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{j-1}]$. This can be formulated as,

$$\mathbf{W}_j^H \boldsymbol{v}_j = \mathbf{W}_j^H \mathbf{U}_j \mathbf{a} = \mathbf{B}\mathbf{a} = \mathbf{0}, \tag{4.14}$$

meaning that, the vector $\mathbf{a}$ lies in the null space of $\mathbf{B}$. Since $\mathbf{W}_j^H$ is $(j-1) \times (M+L)$ full row rank matrix and $\mathbf{U}_j$ is $(M+L) \times j$ full column rank matrix, it follows that, the matrix $\mathbf{B}$ is $j - 1 \times j$ full row rank matrix and $\mathbf{B}^H\mathbf{B}$ has a unique zero eigenvalue. Therefore, $\mathbf{a}$ is the eigenvector corresponding to the unique zero eigenvalue of $\mathbf{B}^H\mathbf{B}$. Starting from $\boldsymbol{v}_1$ and applying this recursive algorithm we can estimate the first j columns of $\mathbf{V}^H$. The advantage of method-A appears in the estimation of $\boldsymbol{v}_j$ where it lies in the intersection of two sets $\mathcal{N}(\mathbf{W}_j)$ (orthogonal to previous $\boldsymbol{v}_i$, $i < j$) and $\mathcal{N}(\mathbf{A}_j^H\mathbf{A}_j)$ (due to the channel matrix structure). Therefore, utilizing the orthogonality property and the channel matrix structure leads to a better estimates to the channel. In comparison to the methods proposed in [16], [20] the computational complexity associated with our proposed method is higher due to the number of eigen decompositions required. However, our method outperforms existing methods as can be noticed from Figure 4.2 where method-A has a gain of 1.5 dB and 3 dB over the Direct [22] and the TXK [16] methods respectively at BER=10^{-4}. Moreover, the proposed method can efficiently estimate the channel when the channel matrix approaches singularity where the method utilizes the eigenvectors and the associated eigenvalues of the covariance matrix to estimate the equalizer while the SSM method [20] fails at moderate SNR because it utilizes only the eigenvectors to estimate the equalizer as shown in Figure 4.3.

Algorithm Steps

The following steps summarize the procedure of estimating the equalizer $\mathbf{g}$.

1. Apply eigen decomposition to $\hat{\mathbf{R}}_0$ to obtain $\mathbf{U}_1$, $\mathbf{\Lambda}$ and noise variance $\hat{\sigma}^2$.

2. Form the matrix $\mathbf{A} = \mathbf{U}_1\mathbf{\Lambda}^{\frac{1}{2}}$.

3. Start a loop for j=1 to J where $L + 1 \leq J \leq \lfloor (M + L)/2 \rfloor$.

   - Form the matrix $\mathbf{A}_j$ as the last $p(M - j)$ rows of the matrix $\mathbf{A}$.
   - Construct the matrix $\mathbf{U}_j$ that contains the j eigen vectors of $\mathbf{A}_j^H\mathbf{A}_j$ corresponding to the j zero eigenvalues.
   - Form the matrix $\mathbf{W}_j = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{j-1}]$, where $j > 1$ and $\mathbf{W}_1 = \mathbf{I}$.

- The vector $\mathbf{a}$ is obtained as the eigen vector corresponding to the unique zero eigenvalue of $\mathbf{W}_j^H \mathbf{U}_j \mathbf{U}_j^H \mathbf{W}_j$.
- Substitute to obtain $\boldsymbol{v}_j = \mathbf{U}_j \mathbf{a}$.
- Go to step 4.

4. The equalizer at delay i is obtained as $\mathbf{g}_i = \mathbf{R}_0^{-1}\mathbf{h}_i = \mathbf{R}_0^{-1}\mathbf{U}_1 \mathbf{\Lambda}^{\frac{1}{2}} \boldsymbol{v}_i$.

4.4.2 Method B

In this method the equalizer is estimated from the vector $\boldsymbol{v}_1$ obtained in the previous section and the covariance matrix at delay i. We will show that this covariance matrix can be obtained directly from the covariance matrix at delay zero using the structure of the channel matrix. Moreover, the computational complexity is much less than method-A, [22] and [32]. Considering the covariance matrix at delay i, (4.4) can be written as,

$$\mathbf{R}_i - \sigma^2 \mathbf{J}_n^{ip} = \mathbf{H}\mathbf{J}_s^i \mathbf{H}^H.$$

In practice, the noise variance and the signal subspace order are estimated from the covariance matrix at delay zero using the eigen decomposition technique. Substituting the channel matrix $\mathbf{H}$ by its singular value decomposition (4.6) we obtain,

$$\mathbf{R}_i - \sigma^2 \mathbf{J}_n^{ip} = \mathbf{U}_1 \mathbf{\Lambda}^{\frac{1}{2}} \mathbf{V}^H \mathbf{J}_s^i \mathbf{V} \mathbf{\Lambda}^{\frac{1}{2}} \mathbf{U}_1^H.$$

Multiplying the above relation by $\mathbf{\Lambda}^{\frac{1}{2}} \mathbf{U}_1^H$ from the left and by its Hermitian transpose from the right side getting,

$$\mathbf{\Lambda}^{-\frac{1}{2}} \mathbf{U}_1^H (\mathbf{R}_i - \sigma^2 \mathbf{J}_n^{ip}) \mathbf{U}_1 \mathbf{\Lambda}^{-\frac{1}{2}} = \mathbf{V}^H \mathbf{J}_s^i \mathbf{V}. \tag{4.15}$$

Defining a new matrix $\mathbf{C}_i$ associated with the delay i as $\mathbf{C}_i = \mathbf{\Lambda}^{-\frac{1}{2}} \mathbf{U}_1^H (\mathbf{R}_i - \sigma^2 \mathbf{J}_n^{ip}) \mathbf{U}_1 \mathbf{\Lambda}^{-\frac{1}{2}}$ where $\mathbf{C}_i$ can be obtained from the covariance matrices at delay zeros and i. Substituting in (4.15) results in,

$$\mathbf{C}_i = \mathbf{V}^H \mathbf{J}_s^i \mathbf{V}. \tag{4.16}$$

Multiplying $\mathbf{C}_i$ by $\mathbf{V}$ and $\mathbf{V}^H$ from left and right respectively,

$$\mathbf{V}\mathbf{C}_i\mathbf{V}^H = \mathbf{J}_s^i = \begin{bmatrix} \mathbf{0}_{\mathbf{i}\times\mathbf{M+L-i}} & \mathbf{0} \\ \mathbf{I}_{\mathbf{M+L-i}} & \mathbf{0} \end{bmatrix}.$$

By equating the $(i+1, 1)$ entry in both sides we get,

$$\boldsymbol{v}_{i+1}^H \mathbf{C}_i \boldsymbol{v}_1 = 1. \tag{4.17}$$

As $\boldsymbol{v}_1$ and $\mathbf{C}_i$ are already estimated, it remains to solve (4.17) to obtain $\boldsymbol{v}_{i+1}$. To find $\boldsymbol{v}_{i+1}$ from (4.17), Let us start by multiplying each term in (4.16) by its Hermitian transpose as,

$$\mathbf{C}_i^H\mathbf{C}_i = \mathbf{V}^H\mathbf{J}_s^{i^H}\mathbf{J}_s^i\mathbf{V}.$$

Multiplying the above equation by $\mathbf{V}$ and $\mathbf{V}^H$ from left and right respectively results in,

$$\mathbf{V}\mathbf{C}_i^H\mathbf{C}_i\mathbf{V}^H = \mathbf{J}_s^{i^H}\mathbf{J}_s^i = \begin{bmatrix} \mathbf{I}_{\mathbf{M+L-i}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_{\mathbf{i}} \end{bmatrix}, \tag{4.18}$$

and recalling that $\boldsymbol{v}_1$ is the first column of $\mathbf{V}^H$ and comparing the top left entry in each matrix in (4.18) we have,

$$\boldsymbol{v}_1^H\mathbf{C}_i^H\mathbf{C}_i\boldsymbol{v}_1 = 1,$$

meaning that $||\mathbf{C}_i\boldsymbol{v}_1|| = 1$. Since $\mathbf{V}$ is an orthonormal matrix i.e., $||\boldsymbol{v}_{i+1}|| = 1$ then (4.17) can be written as,

$$\boldsymbol{v}_{i+1}^H\mathbf{C}_i\boldsymbol{v}_1 = ||\boldsymbol{v}_{i+1}||\;||\mathbf{C}_i\boldsymbol{v}_1||\cos\theta = 1,$$

which implies that $\theta = 0$, and hence,

$$\begin{aligned} \boldsymbol{v}_{i+1} &= \mathbf{C}_i\boldsymbol{v}_1, \\ \boldsymbol{v}_i &= \mathbf{C}_{i-1}\boldsymbol{v}_1. \end{aligned} \tag{4.19}$$

Therefore, the equalizer at delay $d = i$ $(i \geq 2)$ can be expressed as,

$$\begin{aligned}\mathbf{g}_i &= \mathbf{R}_0^{\dagger}\mathbf{h}_i,\\ &= (\mathbf{U}_1\mathbf{\Lambda}^{-1}\mathbf{U}_1^H)(\mathbf{U}_1\mathbf{\Lambda}^{\frac{1}{2}}\boldsymbol{v}_i), &\quad (4.20a)\\ &= \mathbf{U}_1\mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{C}_{i-1}\boldsymbol{v}_1,\\ &= \mathbf{U}_1\mathbf{\Lambda}^{-\frac{1}{2}}(\mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{U}_1^H(\mathbf{R}_{i-1} - \sigma^2\mathbf{J}_n^{p(i-1)})\mathbf{U}_1\mathbf{\Lambda}^{-\frac{1}{2}})\boldsymbol{v}_1, &\quad (4.20b)\\ &= \mathbf{R}_0^{\dagger}(\mathbf{R}_{i-1} - \sigma^2\mathbf{J}_n^{p(i-1)})\mathbf{U}_1\mathbf{\Lambda}^{-\frac{1}{2}}\boldsymbol{v}_1, &\quad (4.20c)\end{aligned}$$

where (4.20a) is obtained from (4.6), and (4.20b) follows directly from the definition of $\mathbf{C}_i$.

Algorithm Steps

The following steps summarize the procedure of estimating the equalizer $\mathbf{g}$ and highlight the processes of estimations and decompositions for different matrices in the proposed method.

1. Estimate the covariance matrices $\hat{\mathbf{R}}_0$ and $\hat{\mathbf{R}}_{i-1}$, $i \geq 2$.
2. Apply eigen decomposition to $\hat{\mathbf{R}}_0$ to obtain $\mathbf{U}_1$, $\mathbf{\Lambda}$ and noise variance $\hat{\sigma}^2$.
3. Form the matrix $\mathbf{A}_1 = \mathbf{U}_1\mathbf{\Lambda}^{\frac{1}{2}}$.
4. Obtain $\boldsymbol{v}_1$ from the eigen decomposition of $\mathbf{A}_1^H\mathbf{A}_1$.
5. Substitute $\hat{\mathbf{R}}_0$, $\hat{\mathbf{R}}_{i-1}$, $\mathbf{U}_1$, $\mathbf{\Lambda}$ and $\boldsymbol{v}_1$ in (4.20c) to obtain the equalizer $\mathbf{g}_i$.

We can directly obtain $\mathbf{R}_i$ from $\mathbf{R}_0$ in a recursive method as follows. Let us partition the covariance matrix at delay i to three disjoint blocks as $\mathbf{R}_i = [\mathbf{Z}\ \mathbf{P}\ \mathbf{Q}]$ where $\mathbf{Z}$ is $pM \times p$, $\mathbf{P}$ is $pM \times p(M-2)$ and $\mathbf{Q}$ is $pM \times p$. Using the channel matrix structure, the covariance matrix at delay $i+1$ can be obtained from the covariance matrix at delay i by the following relation,

$$\mathbf{R}_{i+1} = \mathcal{F}\{\mathbf{R}_i\} = [\mathbf{P}\ \ \mathbf{Q}\ \ \mathbf{J}^p\mathbf{Q}], \quad (4.21)$$

where $\mathcal{F}$ is an operator acting on the sub matrices $\mathbf{Z}, \mathbf{P}, \mathbf{Q}$. To illustrate this operation, by using the channel structure it can be shown that $\mathbf{HJ}_s\mathbf{H}^H = \mathcal{F}\{\mathbf{HH}^H\}$. Starting from $\mathbf{R}_0$, the covariance matrix at delay i can be obtained through the recursion method (4.21). Obviously, the computation complexity in method-B is much less than method-A. The algorithm is less sensitive to the over-estimation of the signal subspace order. Furthermore, the algorithm avoids the error propagation associated with the recursive channel estimation in [16], as well as the error accumulation due to the multiplication of the estimated covariance matrices at different delays in [22] and [23].

4.5 Simulation Results

In this section three examples are presented to evaluate the performance of the proposed algorithms. For each case, the covariance matrix is estimated using 1000 samples.

4.5.1 Example 1: ISI and Eye Diagram

In this example, the performance of methods A and B to equalize the channel are evaluated. The channel impulse response is shown in Figure4.1(a) which consists of two delayed raised cosine pulses as follows,

$$h(t) = p(t, \alpha)W(t) - 0.6p(t - t_0, \alpha)W(t),$$

where $\alpha = 0.1$ is the roll-off factor, $t_0 = \frac{T}{4}$ is the pulse delay and $W(t)$ is a rectangular window of length $6T$. The sub-channels order is $L = 5$, the delay $i = 6$, oversampling factor $p = 4$ and the sub-equalizer length $M = 10$. The input sequence is drawn from 16-QAM constellation. Figure 4.1(b) shows the received signal constellation while Figures 4.1(c) and (d) show the equalized signal constellation with method-A and method-B respectively at SNR= 25 dB. As with all second-order statistics based algorithms, an intrinsic phase angle ambiguity may exist in the estimated signal. This ambiguity can be easily removed through the use of a single initial training symbol.

4.5.2 Example 2: BER versus SNR

In this example, we illustrate the performance of the proposed methods A and B regarding the BER for different SNRs. The channel impulse response utilized in this example is,

$$h = \begin{bmatrix} 0.04 & -0.05 & 0.07 & -0.2 & -0.5 & 0.72 & 0.36 & 0.21 & 0.03 & 0.07 & 0.03 & -0.01 \end{bmatrix}$$

The parameters are adjusted as, $M = 6$, $p = 4$, $i = 3$ and $L = 2$. The input sequence is drawn from BPSK constellation and the channel order is assumed to be known. The covariance matrix is estimated over 1000 transmitted symbols. The BER curve is averaged over 500 Monte Carlo runs. The performance is compared with [16] (TXK) and [22] (Direct) and is shown in Figure 4.2. Method-A has a lower BER than the other methods, this can be attributed to the strategy used to estimate the columns of $\mathbf{V}^H$ (or equivalently the columns of the channel matrix) where a restriction is imposed on $\boldsymbol{v}_i$ such that it lies in the intersection of $\mathcal{N}(\mathbf{W}_i)$ and $\mathcal{N}(\mathbf{A}_i^H\mathbf{A}_i)$.

The improvement in performance introduced by method-B over [16] and [22] results from avoiding the recursive estimation in [16] and covariance matrices inversion and multiplication in [22]. Moreover, the dependence on a single covariance matrix at delay i instead of multiplication of estimated covariance matrices [22], [23] reduces the error. In addition, method-B offers a reduction in the computational complexity.

4.5.3 Example 3: Channel Matrix Approaches Singularity

In this example we consider a channel matrix that is close to being singular. This particular case is considered to compare our proposed methods with the subspace methods in [20] and [30] which are not robust against singularity although the channel order is perfectly known. The channel impulse response is given as,

$$h(z) = \frac{0.7 - z^{-1}}{1 - 0.7z^{-1}}$$

The channel is truncated such that the subchannels order $L = 2$. The delay is chosen as $i = 6$, oversampling factor $p = 4$ and the length of each subequalizer is $M = 9$. The input sequence is an i.i.d 16-QAM signal and the noise is AWGN. We compare the performance of the proposed methods with the Subspace method [20] and Capon method [30] for different SNRs. As shown in Figure 4.3 the proposed methods efficiently equalize the channel and open the eye diagram while the methods presented in [30] and [20] failed to do so at SNR=20 dB. Increasing the SNR to 28 dB, the later methods become capable of equalizing the channel and opening the eye diagram. This illustrates that, our proposed methods A and B have an advantage over the subspace methods when the channel matrix approaches singularity even when the channel order in known.

4.6 Conclusion

In this chapter, we have addressed the problem of blind channel equalization for fractionally spaced systems. We have exploited the channel structure induced by oversampling the received signal in order to develop equalization techniques that outperform existing schemes. The proposed methods can efficiently equalize the channel even when the channel matrix is close to being singular.

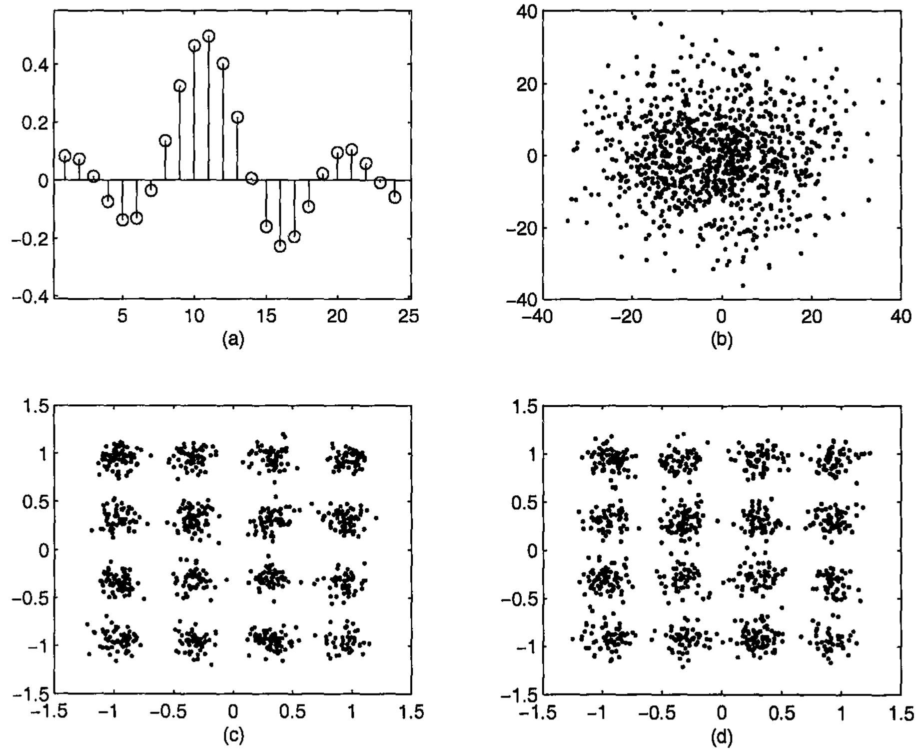

Figure 4.1: (a) Channel coefficients. (b) Received signal constellation. (c) Output signal constellation method-A. and (d)Equalized signal method-B

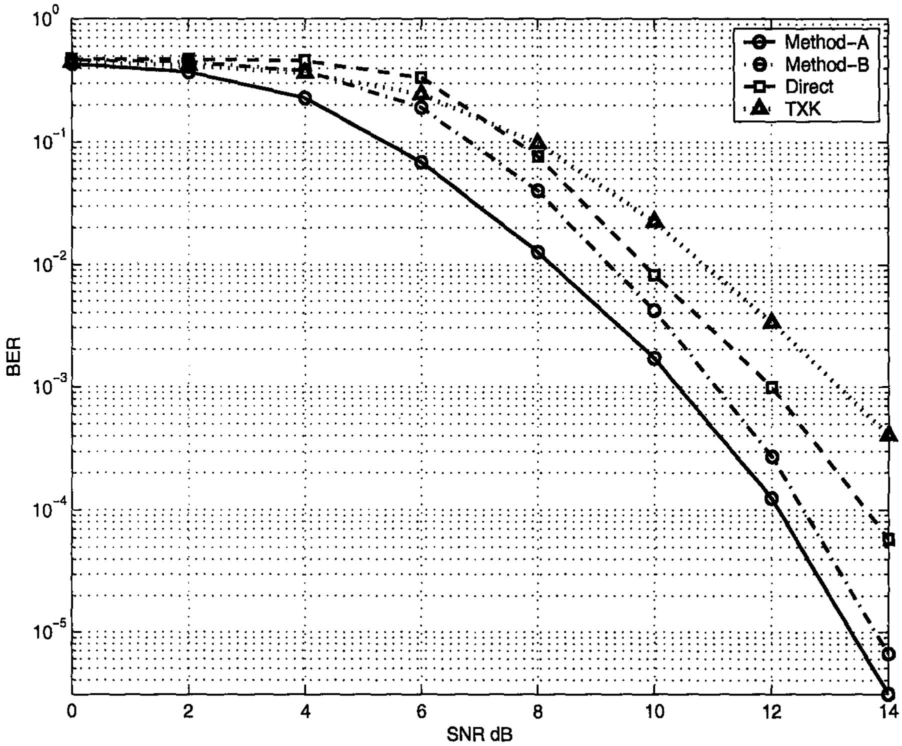

Figure 4.2: BER v.s. SNR

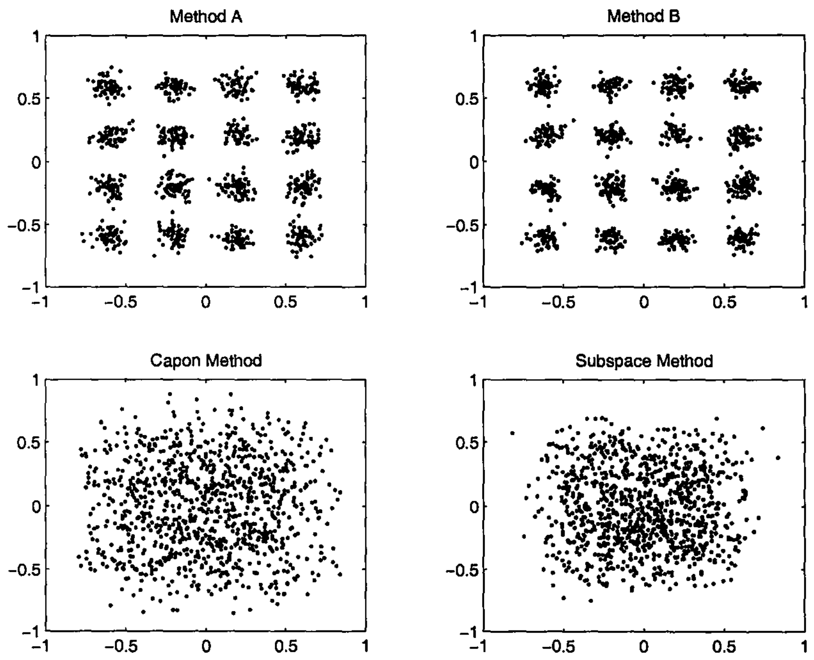

Figure 4.3: Equalized signal constellation at 20 dB

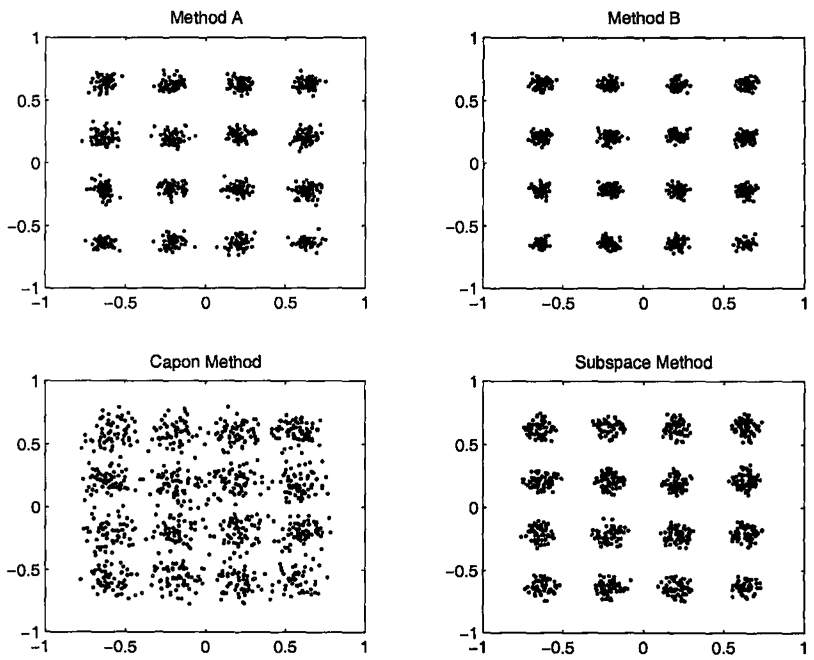

Figure 4.4: Equalized signal constellation at 28 dB

Chapter 5

Discussions and Conclusions

Blind channel equalization is a powerful technique in communication systems to remove the distortion introduced by the channel. Many approaches and algorithms were presented in the last few years to estimate/equalize the channel blindly based on higher order statistics (HOS). However, HOS are more computationally expensive and require larger data record than second-order statistics (SOS). Therefore, recent approaches were directed to SOS based blind equalization algorithms. Specifically, when Tong *el. al* [16] exploited the capability of SOS to equalize the channel for single-input multiple-output (SIMO) systems (Fractionally Spaced Equalizer FSE).

In this thesis we have shown that, for minimum phase channel the T-spaced equalizer could be estimated using the SOS via convex optimization problem where the equalizer autocorrelation sequence is the variable of interest in the problem. Factorizing the estimated autocorrelation sequence (obtained by solving the optimization problem) using the spectral factorization technique results in the equalizer coefficients. Although this method is simple, efficient and global minima is guaranteed, the approach is not applicable to nonminimum phase channels. Therefore, the extension to the fractionally spaced equalizers was presented. We have formulated the problem as a convex optimization problem which was solved efficiently providing an estimate to the equalizer coefficients directly. Moreover, simple modifications to the original formulation have been introduced which enabled us to obtain a closed form solution to the problem. However, both approaches require the knowledge of the channel order as it is a critical parameter in the formulation. In addition, the proposed algorithms outperform the subspace methods when the channel matrix approaches singularity even though the channel order is perfectly known. This advantage arises as the proposed methods utilize the left singular vectors of the channel matrix and the associated singular values to equalize the channel, while the subspace methods rely only on the left singular vectors obtained from the eigenvalue decomposition of the covariance matrix.

Furthermore, we have proposed an algorithm that does not rely on the channel order to avoid the problem encountered in the previous algorithms. The method outperforms existing algorithms as it utilizes the channel matrix structure as well as the transmitted signal statistics. The new algorithm is complex since it requires a large number of eigenvalue decompositions. To mitigate this shortcoming, we presented a simplified algorithm with substantially reduced complexity and only a small performance degradation.

Our techniques address many of the difficulties traditionally found in most blind equalization algorithms (see section 1.4). The proposed algorithms are insensitive to channel order over estimation and utilize the channel matrix structure as well. Moreover, they are robust against error propagation in recursive estimation and they are robust when the channel matrix approaches singularity.

The proposed algorithms are based on batch processing procedure, as a future direction this could be extended to the recursive implementation scenario where online equalization is required. However, more investigations and analysis are required for the sensitivity of the estimation in method-A due to the recursion procedure. We expect that the error propagation will be less than other algorithms as the equalizer is estimated through the intersection of subspaces which guarantees that the equalizer should satisfy many constraints simultaneously.

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