

1           **Connectivity of phases and growth**  
2           **mechanisms in peritectic alloys solidified at**  
3           **low speed: An X-ray tomography study of**  
4                           **Cu-Sn**

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12 The variety of microstructures that form at low solidification speed  
 13 in peritectic alloys, -bands, islands or even coupled (or coopera-  
 14 tive) growth of the primary  $\alpha$ - and peritectic  $\beta$ -phases-, have been  
 15 previously explained by nucleation-growth mechanisms. In a recent  
 16 investigation on Cu-Sn, a new growth mechanism was conjectured  
 17 on the basis of 2-dimensional optical microscopy and Electron Back-  
 18 Scattered Diffraction (EBSD) observations made in longitudinal sec-  
 19 tions. In the present contribution, X-ray tomography has been used  
 20 to confirm this mechanism:  $\alpha$ - and  $\beta$ -phases can be totally inter-  
 21 connected in 3 dimensions and bands (or islands) can result from an  
 22 overlay mechanism, rather than from a nucleation events sequence.  
 23 When the lateral growth of a new layer is too fast, an instability can  
 24 lead to the formation of a lamellar structure as for eutectic alloys.

Steel, bronze, brass and many other commercially important alloys exhibit  
 peritectic transitions during solidification [1]. This invariant reaction in bi-  
 nary alloys is characterized by the transformation of two phases, liquid ( $\ell$ )  
 and primary phase ( $\alpha$ ), into the peritectic phase ( $\beta$ ), at temperature  $T_p$   
 and peritectic composition  $C_p$  (see Fig. 1). In the hypo-peritectic region, i.e., for  
 a nominal composition of the alloy  $C_0$  such that  $C_\alpha < C_0 < C_p$ , a number of  
 different microstructures have been observed during directional solidification  
 at low growth rate in an imposed thermal gradient  $G$  [2]. By low growth rate,  
 we mean a growth velocity  $V$  such that a planar front of the  $\alpha$ -phase would  
 be stable, should the peritectic phase not be present. This condition is met

when

$$V < V_{crit} = \frac{GD_\ell}{\Delta T_{0\alpha}} \quad (1)$$

where  $G$  is the thermal gradient,  $D_\ell$  is the diffusion coefficient of the solute element in the liquid phase and  $\Delta T_{0\alpha}$  the solidification interval of the  $\alpha$ -phase.

[Fig. 1 about here.]

The diversity of microstructures obtained when  $C_\alpha < C_0 < C_p$  and  $V < V_{crit}$  is due to an inherent instability of both the  $\alpha$ - and the  $\beta$ -planar fronts. Figure 1 illustrates schematically why neither of these phases are stable under such conditions. Neglecting any nucleation undercooling, the first solid to form under directional solidification is  $\alpha$  with a composition  $k_{0\alpha}C_0$ , where  $k_{0\alpha}$  is the partition coefficient for the  $\alpha$  phase. In the transient to steady-state, the  $\alpha$ -solid grows with a planar front morphology while the interfacial liquid composition  $C_\ell^{*\alpha}$  increases from  $C_0$  to  $C_0/k_{0\alpha}$  (points labeled (1) to (3) in Fig. 1). Concurrently, the interface temperature moves from the liquidus,  $T_{liq}^\alpha(C_0)$ , to the solidus of  $\alpha$ ,  $T_{sol}^\alpha(C_0) = T_{liq}^\alpha(C_0/k_{0\alpha})$ . However, as soon as  $C_\ell^{*\alpha}$  goes beyond  $C_\ell = C_\ell(T_p)$ , the liquid becomes undercooled with respect to  $\beta$  (grey area in Fig. 1(a)), since the liquidus of  $\alpha$  is below that of  $\beta$ . Therefore, the peritectic phase can nucleate and grow ahead of the primary phase [3–5]. If the  $\beta$ -phase entirely covers the primary phase and prevents it from further growth, this gives rise to a *band* of  $\beta$  covering the previous  $\alpha$  solid.

Considering now the growth of a  $\beta$ -planar front starting at point (4) (Fig. 1(b)), the solid composition  $C_\beta^*$  is larger than  $C_0$ . Therefore, the interfacial composition has to decrease in order to reach a steady-state  $\beta$ -planar front growth (points (4) to (5)). At some point, the liquid interfacial composition falls below  $C_\ell(T_p)$ . This liquid is now undercooled with respect to  $\alpha$  (grey area in Fig.

1(b)), allowing then the  $\alpha$ -phase to nucleate and grow ahead of the  $\beta$  phase (point labeled (6)). As the loop shown in Fig. 1 (points (1) to (6)) can start again, this sequence of  $\alpha$ - and  $\beta$ -phases nucleation-growth events leads to an alternation of  $\alpha$ - and  $\beta$ - bands perpendicular to the thermal gradient  $G$ .

Such bands have been observed in longitudinal sections of Fe-Ni alloys [3,12], and more recently in the Cu-Sn system [13]. A Nucleation and Constitutional Undercooling (NCU) criterion has also been developed, by Hunziker *et al* [14], to predict the appearance of these bands based on the critical undercoolings for the nucleation of the  $\alpha$ - and  $\beta$ -phases. However, when the lateral spreading of a new phase in a direction perpendicular to  $G$  over the pre-existing front growing in the direction of  $G$  is considered, together with the solute field interactions near the triple junction  $\alpha - \ell - \beta$  [7,5,10], incomplete coverage of one phase over the other may occur. This leads to the formation of *islands*, which are therefore like incomplete bands.

Finally, another interesting microstructure that can form in peritectic alloys at low growth rate is the simultaneous growth of  $\alpha$ - and  $\beta$ -lamellae in the direction of the thermal gradient  $G$ . Although similar to the well-known coupled growth of eutectic alloys, the  $\alpha$ - and  $\beta$ -lamellae of peritectic alloys both reject solute elements ahead of the interface. Less solute is rejected by the  $\beta$ -phase since  $k_{0\beta} > k_{0\alpha}$ . Predicted in 1959 by Chalmers [16], coupled (or cooperative) growth in peritectic alloys was first observed in Ni-Al, by Lee and Verhoeven [18], and then in Fe-Ni by Kurz and co-workers [19,12]. As shown by directional solidification experiments and multi-phase field simulations [9–12,20], lamellae structures can start growing from islands, providing the distance separating the islands falls within a range of stable lamellar spacings [10].

73 The observations of bands, islands and cooperative lamellae growth in peri-  
74 tectics was limited up to now to small solidification interval alloys, typically  
75 with  $\Delta T_{0\alpha} \lesssim 10$  K. Surprisingly, none of the previous investigations tried to  
76 determine if the bands, islands and lamellae of the two phases had a crys-  
77 tallographic relationships, e.g., by using Electron Back-Scattered Diffraction  
78 (EBSD). In a recent contribution, Kohler *et al*[13] investigated peritectic so-  
79 lidification in the Cu-Sn system, an alloy with a solidification interval  $\Delta T_{0\alpha}$   
80 nearly fifty times that of Fe-Ni in the hypo-peritectic region (around 120 K  
81 near  $C_p$ ). Using a Bridgman solidification setup with quench, in which so-  
82 lutional convection was minimized by having small specimen dimensions, these  
83 authors observed all three of the peritectic microstructures discussed above -  
84 lamellae, bands, and islands. The crystallographic orientations of the resulting  
85 microstructures were also investigated.

86 An example micrograph from the work of Kohler *et al*[13] is shown in Fig. 2  
87 for a Cu-21wt%Sn specimen solidified at  $0.58 \mu\text{m/s}$ . In the top view of this  
88 longitudinal section micrograph, the  $\alpha$ -phase appears in light brown while the  
89  $\beta$ -phase is colored in blueish brown. The solidification direction goes from  
90 left to right. As can be seen, there is a transition in growth from lamellae to  
91 bands and then back to lamellae. The transition from bands to lamellae on the  
92 right-hand side clearly goes through an intermediate stage of semi-continuity  
93 and then partial bands or islands. In the bottom view, a reconstructed false  
94 color micrograph obtained from the EBSD measurements is shown. Due to  
95 a solid-state transformation of  $\beta$  either into  $(\delta + \alpha)$  via a eutectoid reaction  
96 or into a martensitic phase, only the  $\alpha$ -phase could be indexed. As can be  
97 seen, the  $\alpha$ -phase in this section is made of two grains, yellow and blue. More  
98 importantly, within a grain, the same orientation is measured for the lamellae

99 on the left and the right sides of the micrograph, and is not lost when discrete  
100 bands form in the middle of the longitudinal section.

101 [Fig. 2 about here.]

102 These observations can be explained by one of two mechanisms: (i) The whole  
103 structure is in fact continuous in 3 dimensions (3D) but may appear as discon-  
104 tinuous discrete bands in 2D metallographic sections; (ii) When a new phase  
105 nucleates ahead of an existing one, there is a systematic coherency relation-  
106 ship. Of these two mechanisms, the second mechanism can be immediately  
107 ruled out, although a Kurdjumov-Sachs crystallographic relationship was ob-  
108 served between the fcc- $\alpha$  and bcc- $\beta$  phases in another region of the specimen  
109 where the  $\beta$ -phase did not transform and could be indexed [13]. There are 24  
110 such relationships between the  $\langle 100 \rangle_\alpha$  and  $\langle 100 \rangle_\beta$  reference systems. In the  
111 first mechanism, a new band of a phase, say  $\beta$ , does not require any nucle-  
112 ation event at the  $\alpha - \ell$  interface if it exists already in another region of the  
113 specimen. The new phase  $\beta$  can then simply grow laterally from this region  
114 since the liquid ahead of the  $\alpha - \ell$  exhibits the largest supersaturation. This  
115 “*overgrowth*” or “*overlay mechanism*”, which is of course not possible in 2D,  
116 is shown schematically in Fig. 3. As lateral growth of one phase over the other  
117 occurs in a nearly nil (horizontal) thermal gradient and at a velocity which can  
118 be much higher than the pulling velocity  $V$  of the specimen, it can be unstable  
119 (see Fig. 3). It then develops  $\beta$ -cells. Since the  $\alpha$ -phase is still growing in be-  
120 tween the spaces left by the  $\beta$ -cells, an alternate sequence of the primary and  
121 peritectic phases is formed and a cooperative growth is thus initiated between  
122 the  $\alpha$ - and  $\beta$ -phases leading to the formation of a  $\alpha$ - $\beta$  lamellar structure. A  
123 similar situation can occur during the lateral propagation of an  $\alpha$ -layer over  
124 a  $\beta$ -liquid interface. This mechanism is similar to that observed by Akamatsu

125 *et al* [21] on organic eutectic alloys at the onset of coupled growth.

126 [Fig. 3 about here.]

127 In order to check the continuity of the  $\alpha$ - and  $\beta$ -phases in 3D, and hence the  
128 validity of this growth mechanism, X-ray tomography experiments were car-  
129 ried out at the TOMCAT beam-line of the Swiss Light Source (Paul Scherrer  
130 Institute, Villigen, Switzerland) on Cu-Sn specimens 300  $\mu\text{m}$  in thickness. The  
131 energy of the X-ray beam was 30 keV, the detector resolution was 1.4  $\mu\text{m}$  per  
132 voxel side, and the specimen-to-screen distance was 2 mm. The combination  
133 of high X-ray energy and small sample dimensions was necessary since both  
134 Cu and Sn are strong X-ray absorbers, with Sn having a higher Z-number and  
135 hence is more absorbing.

136 Figure 4 shows eight 3D views of the microstructure from a sub-section of  
137 the specimen shown in Fig. 2, in a region where lateral overgrowth gave rise  
138 to the lamellar structure. The images are shown in reverse contract with the  
139  $\alpha$ -phase appears in dark brown since it is leaner in Sn ( $C_\alpha \approx 13.5 \text{ wt}\%$ ), while  
140 the  $\beta$ -phase appears brighter since  $C_\beta \approx 22 \text{ wt}\%$ . In this series of images, the  
141 thermal gradient  $G$  is vertical; the evolution in microstructure is shown by two  
142 fixed vertical sections and one horizontal section which “moves” in the solid-  
143 ification direction from image (a) to image (h). Although this sequence was  
144 recorded post-mortem, and not in-situ during growth of the microstructure,  
145 it shows the complexity of the phase interconnections. In some regions, e.g.,  
146 at mid-height, the two longitudinal sections exhibit a band-type morphology.  
147 In others, e.g., near the top of the left section, it is clearly lamellar. As shown  
148 by the moving horizontal section, the  $\alpha$ -phase is interconnected. While lateral  
149 growth of both phases appears to be not as smoothed as illustrated in Fig. 3,

150 it finally gives rise to a fairly regular arrangement of lamellae near the top of  
151 the specimen. As mentioned in [13,22], solutal convection is probably respon-  
152 sible for the irregularities of the microstructure, especially during overgrowth.  
153 Convection not only gradually increases the average composition of the liquid  
154 during growth, due to macrosegregation, it also affects the local composition.  
155 Numerical simulations [22] revealed a complex, unsteady streamline pattern,  
156 with solute-enriched and solute-lean plumes of liquid going upward and down-  
157 ward, respectively, with a non-zero horizontal component of the velocity (i.e,  
158 helicoidal movements).

159 [Fig. 4 about here.]

160 The connectivity of the phases can also be seen in the 3D image shown in  
161 Figure 5. In this figure, the  $\alpha$  phase has been removed from the drawing,  
162 leaving only the interconnected  $\beta$  phase. The thermal gradient  $G$  is vertical  
163 and points up. As can be seen in the figure, the  $\beta$  phase goes through two  
164 distinct transitions. Firstly, in the lower half of the image there is a transition  
165 from a lamellae to a bands -type structure. Secondly, in the upper half of  
166 the image, there is the reverse transition, from a bands to a lamellae -type  
167 structure. These transitions, 3D in nature, provide clear evidence supporting  
168 the growth overlay mechanism as a means of phase propagation. Although it  
169 might appear that the upper  $\beta$  region is separate from the lower region, and  
170 hence the result of a new nucleation event, we believe that the two regions are  
171 in fact continuous and that the link between these two regions occurs outside  
172 the bounding box of the X-ray tomographic scan.

173 [Fig. 5 about here.]

174 Despite convection effects and solid state transformations that make the analy-

175 sis of the post-mortem microstructure more difficult, X-ray tomographic obser-  
176 vations have clearly shown the 3D connectivity of the  $\alpha$ - and  $\beta$ -phases during  
177 low speed growth of Cu-Sn alloys. These observations explain the continu-  
178 ity of the grains measured previously by EBSD in 2D sections and confirm  
179 that bands might form by an overgrowth mechanism of one phase over the  
180 other and do not require necessarily re-nucleation. If lateral growth is unsta-  
181 ble, bands can give rise to lamellae in a similar fashion to the initial state  
182 of coupled eutectic growth. X-ray tomography also opens new opportunities  
183 for the study of peritectic solidification at low growth rate. In a first step,  
184 we plan to grow Cu-Sn alloys in capillaries of smaller dimensions (typically  
185 about 400  $\mu\text{m}$  diameter) in order to further reduce convection and allow for 3D  
186 characterization of the whole specimen. In a second step, we hope to perform  
187 direct in-situ X-ray tomography observations of Cu-Sn growth at low speed in  
188 a specifically designed furnace.

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238 **List of Figures**

239	1	Schematic diagram of (a) $\alpha$ -planar front growth leading to	
240		an undercooled liquid with respect to the $\beta$ -phase, and (b)	
241		$\beta$ -planar front growth leading to an undercooled liquid with	
242		respect to the $\alpha$ -phase.	13
243	2	Longitudinal section of a Cu-21wt%Sn specimen solidified at	
244		$V = 0.58 \mu\text{m/s}$ in a thermal gradient $G = 200 \text{ K/cm}$ . The top	
245		view shows the microstructure with $\alpha$ and $\beta$ in light brown and	
246		blueish brown, respectively. The bottom view shows an EBSD	
247		false color reconstructed map based on points measured in	
248		the $\alpha$ -phase. (Enlarge view from [13,22]).	14
249	3	Schematic of the overlay growth mechanism of one phase (in	
250		this case $\beta$ ) over a planar front of $\alpha$ . (From [13,23]).	15
251	4	3D reconstructed X-ray tomography views of the $\alpha$ (dark	
252		brown) and $\beta$ (brown-yellow) microstructure in a region of the	
253		specimen shown in Fig. 2, where bands (in (b)) are destabilized	
254		and give rise to a lamellae structure (in (h)). The thermal	
255		gradient $G$ is vertical and up.	16
256	5	3D segmented view of the $\beta$ phase in a region of the specimen	
257		shown in Fig. 2. The transition from lamellae to bands is	
258		evident. The thermal gradient $G$ vertical and up.	17

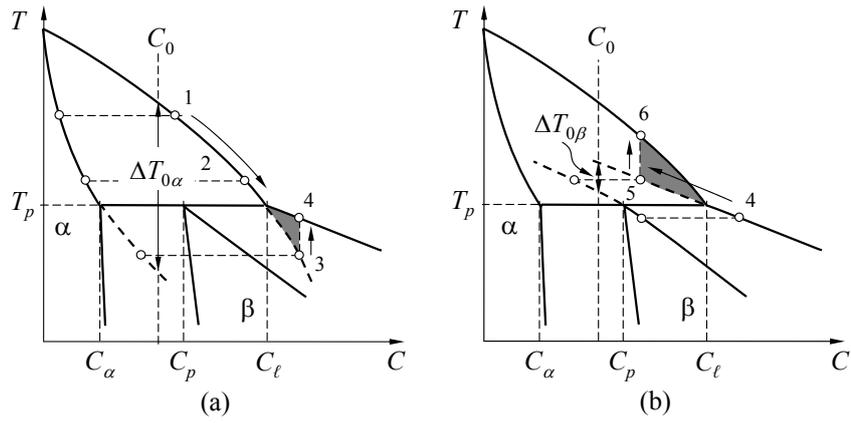


Fig. 1. Schematic diagram of (a)  $\alpha$ -planar front growth leading to an undercooled liquid with respect to the  $\beta$ -phase, and (b)  $\beta$ -planar front growth leading to an undercooled liquid with respect to the  $\alpha$ -phase.

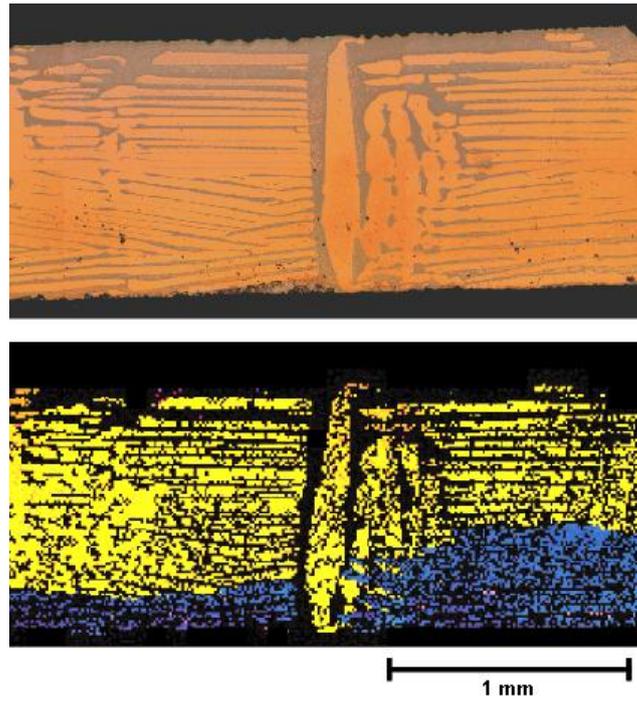


Fig. 2. Longitudinal section of a Cu-21wt%Sn specimen solidified at  $V = 0.58 \mu\text{m/s}$  in a thermal gradient  $G = 200 \text{ K/cm}$ . The top view shows the microstructure with  $\alpha$  and  $\beta$  in light brown and blueish brown, respectively. The bottom view shows an EBSD false color reconstructed map based on points measured in the  $\alpha$ -phase. (Enlarge view from [13,22]).

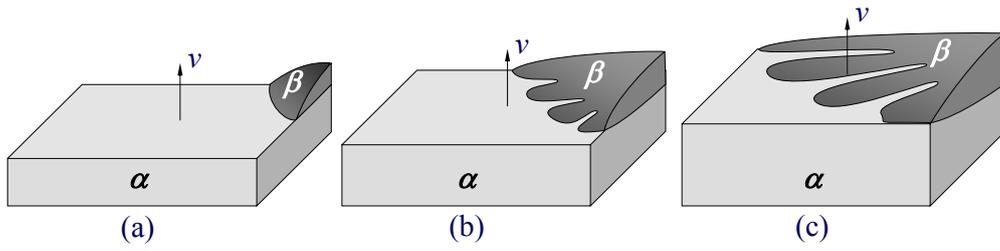


Fig. 3. Schematic of the overlay growth mechanism of one phase (in this case  $\beta$ ) over a planar front of  $\alpha$ . (From [13,23]).

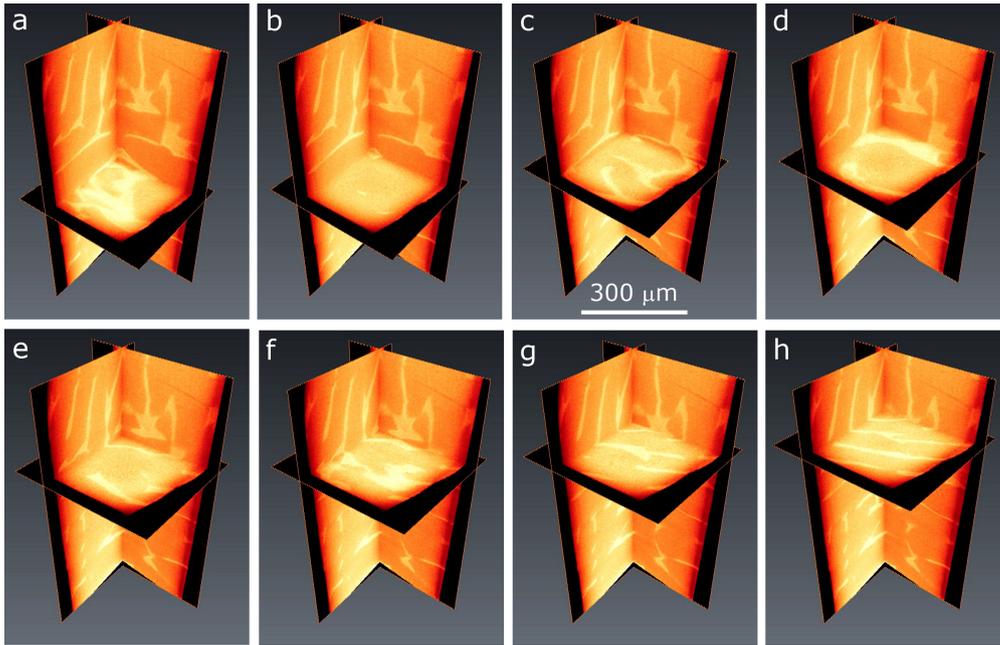


Fig. 4. 3D reconstructed X-ray tomography views of the  $\alpha$  (dark brown) and  $\beta$  (brown-yellow) microstructure in a region of the specimen shown in Fig. 2, where bands (in (b)) are destabilized and give rise to a lamellae structure (in (h)). The thermal gradient  $G$  is vertical and up.

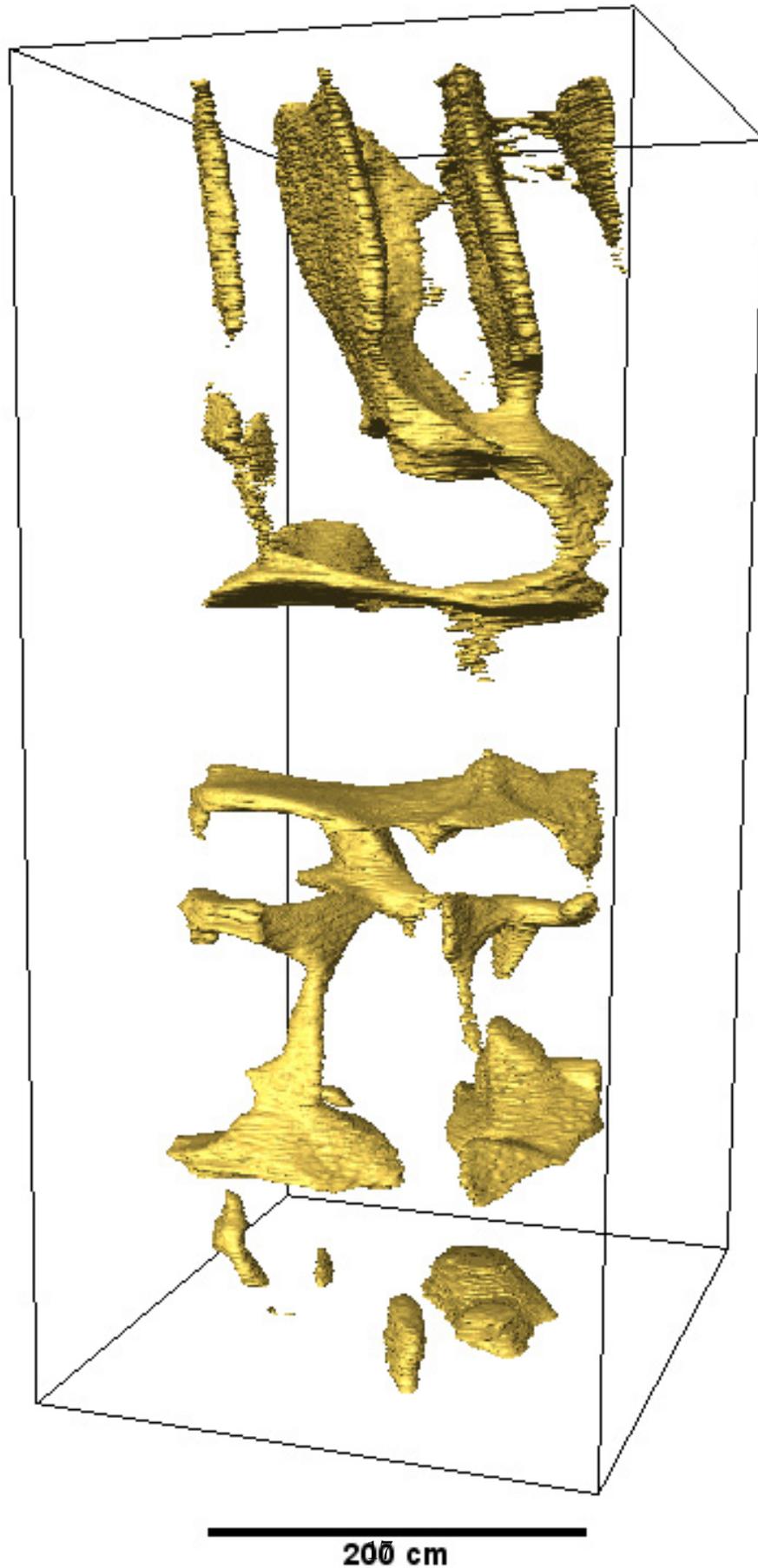


Fig. 5. 3D segmented view of the  $\beta$  phase in a region of the specimen shown in Fig. 2. The transition from lamellae to bands is evident. The thermal gradient  $G$  vertical and up.