UNCERTAINTY ANALYSIS OF A TWO-DIMENSIONAL HYDRODYNAMIC MODEL

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OF

A TWO-DIMENSIONAL HYDRODYNAMIC MODEL

By

AARON THOMPSON, B.ENG.

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AUTHOR: Aaron F. Thompson, B.Eng. (McMaster University)

SUPERVISORS: Professor Y. Guo & Professor S. Moin

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Abstract

The objective of this thesis was to undertake an uncertainty analysis on the outputs from a two-dimensional hydrodynamic model. The analysis utilized an application of the Resource Management Associates' RMA2 model for the Upper St. Lawrence River in Ontario, Canada. Two uncertainty analysis methods, First-Order Second Moment (FOSM) and Monte Carlo analysis, are applied to calculate the uncertainty in water levels and velocities computed by the model.

Both uncertainty analysis methods can be applied together with two-dimensional hydrodynamic modelling, but based on the findings of this work, the FOSM method is preferred. First, FOSM estimates of uncertainty are slightly larger than those obtained using Monte Carlo analysis. Thus, FOSM provides a conservative estimate of the uncertainty, a positive characteristic. Second, the FOSM method is simpler to apply than Monte Carlo analysis, requiring less information to describe the model inputs, fewer model executions and computations to calculate the uncertainty. Third, FOSM provides an immediate indication of the primary contributors to the uncertainty in the output, where Monte Carlo analysis requires additional effort to do the same.

The model input that contributed the most to the uncertainty in the model outputs is the bottom resistance represented in RMA2 using Manning's n. The uncertainty in Manning's n is large and the model is sensitive to the parameter. As a result, a significant amount of uncertainty in the model outputs is contributed by this parameter.

Uncertainty analysis is a practical addition to the two-dimensional hydrodynamic modelling process. The effort required to complete an uncertainty analysis using the FOSM method is minimal and the resulting insight is meaningful. It provides information to the model developer, quantifying how good the model actually is. It also provides a measure of the accuracy of the model for future model users or clients using hydrodynamic modelling outputs.

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Chapter 1 - Introduction

1.1 General Introduction

Two-dimensional hydrodynamic modelling is used widely in water resources engineering and other related disciplines. Water resource managers make operational decisions based on modelling outputs. Academics use models to understand complex hydraulic processes and to investigate how water bodies influence the environment. Consultants and permitting agencies rely on models to quantify the impacts of water related development projects. There has not been research undertaken to quantify the uncertainty in hydrodynamic modelling outputs and to identify the sources of the uncertainty. This thesis will address the shortcomings of research completed to date with respect to this subject.

Typically, two-dimensional hydrodynamic models are developed to help solve a water resource related problem. They provide information that is otherwise unknown due to inabilities to measure, predict or understand natural hydraulic processes. Model development usually consists of data collection, the selection and creation of an appropriate model, calibration, and verification. The calibration and verification process ensures that the model reproduces the physical system within a prescribed tolerance level. Once calibration and verification are complete, the model is applied to the problem. In most cases, the model is run deterministically with one set of inputs for the problem. The resulting outputs provide a solution for that problem. The deterministic outputs are treated as correct and the modelling process is complete. The outputs are considered correct because of the calibration and verification process, yet there is, in fact, uncertainty or doubt that the values are absolutely correct. The uncertainty in the output stems from our inability to precisely specify the model inputs and from the simplifications inherent in the hydrodynamic model itself.

Two-dimensional hydrodynamic modelling refers to the calculation of water levels and depth- or horizontally-averaged velocities. Most hydrodynamic models are depth-averaged models. The models solve the depth-averaged equations of mass and momentum continuity in two horizontal directions. The simplified nature of the depth-averaged equations utilized by hydrodynamic models creates uncertainty. Hydrodynamic models also require extensive input data to first build the model, and second, to provide the necessary boundary conditions. Our inability to precisely define the input data creates uncertainty in the model inputs. The outputs of two-dimensional, depth-averaged, hydrodynamic models are spatially distributed water levels and velocities in the two horizontal directions. A competent water resources engineer will treat the model outputs as an approximate estimate and use the outputs with caution.

However, a decision maker or other user with little knowledge of hydrodynamic modelling may not be cognisant of the uncertainty which exists in the model output. As a result, the outputs may be interpreted or applied incorrectly.

It is favourable to the engineer, decision maker or other user of the hydrodynamic modelling outputs to have the uncertainty in the model output quantified. The engineer can use the uncertainty to determine the quality of the model and improve the model before it is applied to the problem. If the uncertainty in the model output is known, a decision maker would be more likely to properly utilize the output to make their decision. For example, suppose a water resource manager asked an engineer for a prediction in water level following a change in outflow through a hydroelectric dam. If the engineer not only provided a prediction of the water level but also an estimate of the uncertainty in that prediction, then the decision maker would be able to make a more informed decision. The quantified uncertainty in the model output, in addition to the result itself, increases the overall value of the model. As another example, the outputs from hydrodynamic models are often used as inputs into other models studying pollutant or sediment transport. In these models, river velocities are used to calculate the advective movements of the pollutant or sediment. Since the velocities calculated by the hydrodynamic model have uncertainty that uncertainty should be identified, quantified, and taken into account in the pollutant or sediment transport calculations. It is obvious that a quantification of the uncertainty in hydrodynamic modelling would be beneficial.

The concept of uncertainty is not new and consequently there has been research undertaken to develop mathematical methods to quantify uncertainties in numerical models (Bobba et al. 1995). The methods described in the literature are generally not model specific. They can be applied to many different types of models. Methods such as First-Order Second Moment (FOSM) and Monte Carlo analysis have been applied successfully in a wide range of fields including water resources engineering. Up until now, in open channel hydraulics, uncertainty analysis has been limited to models that are not as complex as two-dimensional hydrodynamic models. Huang (1986) and Cesare (1991) both computed the uncertainty of hydraulic models. The models used in these analyses were one-dimensional applications of Manning's equation. Hall et al. (2005) used Monte Carlo simulation to calculate and attribute the uncertainty of the flood water levels calculated by a one-dimensional flood inundation model. These examples prove the applicability of these methods on simpler hydraulic models. However, a two-dimensional hydrodynamic model is considerably more complex. The simulations solve the more complex equations of mass continuity and conservation of momentum in two-dimensions using numerical methods. The data requirements are more extensive and have greater potential to create uncertainty in the output. There are methods capable of quantifying the uncertainty in two-dimensional hydrodynamic modelling but research in this particular area has not been completed as of yet.

A closely related subject to uncertainty analysis is sensitivity analysis. The two are related due to the processes involved in the analysis but the purposes of the analyses are different. Uncertainty analysis determines the uncertainty in a model output from the uncertainty in model inputs. Sensitivity analysis investigates the flow of data into and out of a model to determine which inputs are most influential on the model output. Sensitivity analysis can also be used to attribute the uncertainty in the model outputs to individual or groups of model inputs. This information is of value to model developers because it allows efforts to be directed towards the inputs that cause the most uncertainty in the model outputs. Focussed data collection, literature review and/or analysis can be conducted based on this knowledge. The type of sensitivity analysis required to attribute the uncertainty in model outputs to particular model inputs has not been widely applied in water resources engineering.

1.2 Thesis Objectives

The objective of this thesis is to perform an uncertainty analysis on a specific two-dimensional hydrodynamic model. The thesis will explore what is involved in uncertainty and sensitivity analysis; what methods are suitable for application in two-dimensional hydrodynamic modelling; what are the outputs and findings from uncertainty analysis; and how can the findings be utilized by the water resources engineering community.

1.3 Background

To accomplish these objectives, an application of the Resource Management Associates' two-dimensional hydrodynamic model (RMA2) of the Upper St. Lawrence River was selected and employed. The RMA2 model is maintained by the United States Army Corps of Engineers (Donnell et al. 2005). The Upper St. Lawrence River application of the model was developed by the author of this thesis while employed by Environment Canada (Thompson and Moin 2003). The original modelling was undertaken to generate water level and velocity data for the Upper St. Lawrence River for use in a larger scale study of the regulation of Lake Ontario and the St. Lawrence River (International Joint Commission (IJC) 2005).

The model covered a 150 km section of the St. Lawrence River from the outlet of Lake Ontario at Kingston / Cape Vincent areas to the control structure at

Cornwall / Massena. The model's grid network consisted of approximately 32000 elements measuring an average of 150 metres by 150 metres. The model used water levels at Kingston and Cape Vincent as the upstream boundary condition. At the downstream boundary the flowrate at the Moses-Saunders Hydroelectric Dam in Cornwall was specified. The model was calibrated and verified using a network of water level gauges and discharge data collected in the St. Lawrence River. The model has been applied for several purposes including the prediction of circulation of water in the river, the prediction of velocities in the river during periods of ice formation and the determination of the effects of flow changes at the Moses-Saunders hydroelectric dam on water levels and velocities in the river. In all of these applications no quantification of uncertainty in the model outputs was made, nor was there any sensitivity analysis conducted.

The uncertainty and sensitivity analysis of the model required the clear definition of a problem to be solved. The model was executed to predict water levels and velocities at three locations in the river for a specific, steady, Lake Ontario level of 74.98 metres and a river flowrate of 7023 m^3/s. The reason three locations were selected for analysis is as follows. The model calculates water levels and velocities at over 69,000 nodes in the model. Nodes define the corners of the 32, 000 elements in the model. Performing an uncertainty analysis on all nodes is unreasonable because of the computational requirements, so three locations were selected and used in the analysis. The three locations are areas of interest in water management and commercial navigation. They are located upstream and downstream of the Iroquois Lock and just upstream of the Moses-Saunders hydroelectric dam. The model is frequently executed to estimate the water level and local velocity at a specified location under specific hydrological conditions. This thesis quantifies the uncertainty in the calculations of water level and velocities for this problem and presents a sensitivity analysis that identifies the sources of the uncertainty.

The choice of steady flow versus unsteady flow calculations warrants further explanation. The RMA2 model of the St. Lawrence River is capable of simulating gradually varied, sub-critical, steady or unsteady flow conditions. The flow in this section of the St. Lawrence River, stretching from the outlet of Lake Ontario at Kingston to Cornwall, Ontario, is classified as gradually varying, sub-critical, and unsteady. It is unsteady due to the changes in outflow made hourly at the main control structure for the river, the Moses-Saunders Hydroelectric Dam. While the model is capable of simulating unsteady flow conditions and the river is in reality unsteady, the sensitivity and uncertainty analysis conducted in this thesis used a steady flow simulation. As explained previously, most applications of the model are addressed using steady flow conditions as boundary conditions. Further, conducting an uncertainty and sensitivity analysis using unsteady flow conditions would increase the computational requirements of the analysis dramatically. The length of time required to complete the model executions would increase as would the length of time required to perform the uncertainty and sensitivity analysis computations. There are also additional complexities and data demands involved with unsteady flow modelling that make uncertainty analysis complicated and impractical. The work presented herein consists of an uncertainty analysis of the steady flow model.

1.4 Thesis Overview

The thesis has been structured so that there are a total of five chapters to present the work. This first chapter is the introductory chapter containing the setting and motivation for undertaking this work. The problem is described in this chapter and the objectives are identified. This chapter also provides the reader with an indication of how the work will progress.

The second chapter contains the literature review. The chapter provides a relevant background to the reader and a justification of the methods selected for application in this thesis. The chapter first describes the concepts of hydrodynamic modelling and presents recent applications of hydrodynamic modelling. The chapter then provides a summary of uncertainty analysis principles and techniques. Two methods, the FOSM method and Monte Carlo method are described in detail and several applications of these methods in water resources engineering cited in literature are examined. Similarly, sensitivity analysis principles and methods are presented. From the literature, a benchmark is established to define what has been undertaken thus far in this area and frame what will be accomplished with this thesis.

The third chapter briefly describes the St. Lawrence River application of the RMA2 model. It begins by introducing the section of the St. Lawrence River that is modelled including its geographical, hydrological and regulatory characteristics. Next, a discussion of how and why the RMA2 model was selected, the data requirements for the RMA2 model, and the sources used to obtain the data for the St. Lawrence River are presented. A brief description of how the RMA2 model of the St. Lawrence River was developed using the Surface Water Modelling System (SMS) is provided. The methods used to establish the model parameters including calibration and literature sources are also summarized in this chapter. The chapter concludes with a brief discussion of the limitations of the RMA2 model of the St. Lawrence River.

The fourth chapter focuses on the uncertainty and sensitivity analysis. To begin the chapter, the uncertainties inherent in the input data required by the hydrodynamic model are discussed. The inputs are separated into two categories: those that can and cannot be measured. The first category, the measurable data, includes the bathymetry, water levels, river discharge, winds and water temperature. The second category, the un-measurable data, includes the RMA2 wetting and drying parameters, Manning's *n*, and Eddy Viscosity parameters. For each of these types of data, the uncertainty is quantified. Next, the uncertainty in the model including the governing equations, model grid, numerical solution method, and convergence criterion is determined. The uncertainty of the model outputs is then quantified using two methods, FOSM and Monte Carlo analysis. A local sensitivity analysis was completed as a component of the FOSM analysis. The results of both the uncertainty analysis and sensitivity analysis are presented in Chapter four along with dialogue.

Chapter five presents conclusions resulting from this work and suggests recommendations for future research.

Chapter 2 – Literature Review

2.1 Two-Dimensional Hydrodynamic Modelling

In the Great Lakes Connecting Channels, a specific type of modelling, two-dimensional hydrodynamic modelling has proven to be useful to regulatory agencies. Environment Canada, the United States Army Corps of Engineers, the United States Geological Survey (USGS), and the Essex Region Conservation Authority have all utilized this type of modelling. It has been used to investigate the hydraulic impacts of shoreline encroachments, dredging, bridges, and other developments. Hydrodynamic models have been developed for the St. Clair and Detroit River Waterway (Holtschlag and Koschik 2001), the St. Lawrence River (Thompson and Moin 2003), and the St. Marys River (Eric Tauriainen, Corps of Engineers, personal communication, April 6, 2004).

The success of these models in the Great Lakes Basin has lead to increased demand for this type of modelling and further applications. A two-dimensional hydrodynamic model was developed as part of the study investigating the regulation of Lake Ontario and the St. Lawrence River. The Lake Ontario – St. Lawrence River Study (IJC 2005) considered the impacts of water levels and flows on all interests within the system including coastal, commercial navigation, environmental, etc. In order for the interests to be examined Technical Working Groups (TWG) were formed and conducted studies to relate the impacts of water level changes on the particular interest. The Environmental TWG determined the impact of water level regulation on fish habitat, using water levels and velocity data calculated by the hydrodynamic model. The commercial navigation TWG utilized modelled velocity data to compute transit times and fuel consumption for vessels heading through the St. Lawrence Seaway. Another application is planned to produce estimates of velocities for ice formation studies for the river. Although the model cannot handle ice covered conditions explicitly, it can be used to predict the velocity in the river which can be used to infer ice formation locations.

The USGS and the Michigan Department of Environmental Quality (MDEQ) developed and applied a two-dimensional model of the St. Clair and Detroit River Waterway to study the sources of water at public water intakes in the system (Holtschlag and Koschik 2001). Two-dimensional velocities were modelled and used to develop a particle tracking application that was used to determine the originating location and travel times for water in the system. The information was given to local water utility mangers and is incorporated into emergency operation plans.

The United States Army Corps of Engineers developed a two-dimensional model of the St. Marys River in 2003 to evaluate the impact of a proposed dredging project on the water levels in the St. Marys River and on the flow split around an international island. The model was used to quantify the expected impacts of the project and allowed the governments of Canada and the United States to reach an agreement on the dredging (Eric Tauriainen, Corps of Engineers, personal communication, April 6, 2004).

These examples of hydrodynamic modelling are not exhaustive. There are more applications for modeling being currently developed. In each case, there is an underlying need for high quality information. The products of hydrodynamic modelling are sometimes the final output of interest (water levels, velocities, etc.), but frequently the products are then taken and used as inputs into other models (i.e. fish models, transport models etc.). The outputs are generally considered to be deterministic, when in reality we as engineers know that the outputs of our models are really only educated estimates of the true values. We know that there is some degree of uncertainty, or doubt, in the predictions. The origin of the uncertainty is in the simplifying nature of models and our inability to precisely define the required input quantities for the models.

2.2 Uncertainty and Sensitivity Analysis

Uncertainty quantification in modelling is sometimes referred to as risk analysis or reliability modelling, where reliability and risk refer to the probability of failure of a system. Reliability or risk based hydraulic modelling is more common today primarily due to advances in computing power. Engineers have always recognized that there is uncertainty in model predictions but up until recently the methods available to quantify the uncertainty in complex models were unpractical due to computing limitations. However, with cheap, efficient computing power readily available, uncertainty analysis of complex models is now possible. Probabilistic approaches to modelling are now more common. The probabilistic approach to modelling is more suitable in many cases (Johnson 1996). For example, the determination of a flood stage requires the collection and assemblage of many types of data to determine the flood discharge. Other types of data are required to evaluate the flood stage with a hydraulic model. Each type of data has associated uncertainty. An analysis of the probability of the flood stage occurring could assist the engineer or other decision maker in determining future land use or mitigation strategies. Other examples of the use of reliability based analysis include the evaluation of scour around bridge piers, life expectancy of hydraulic structures such as dams and levees, and reliability of river restoration projects (Johnson 1996).

The concept of uncertainty has been recognized for some time and there are proven methods available to quantify uncertainties (Bobba et al. 1995). The available methods can be divided into two categories, analytical methods or approximate methods (Tung 1996). The selection of the correct method depends on the availability of data, the level of complexity of the model, and the required accuracy of the method. Analytical techniques are often limited in application for two reasons. The first is the requirement for precise probability distribution functions for the input quantities. The second is the mathematics required to analytically evaluate most complex models is either very difficult or impossible to perform. Analytical methods include the derived distribution technique and integral transformation techniques. Hydrodynamic models involve differential equations that are too complex to solve analytically and therefore analytical uncertainty analysis methods are not applicable. When using complex models or functions, approximate uncertainty analysis methods are practical. Two frequently used approximate uncertainty methods are the FOSM and Monte Carlo analysis. These two methods will be discussed in more detail.

The basic idea of the FOSM method (Yen et al. 1986) is to approximate a model Y involving several input variables by Taylor Series expansion. The model is represented by a function $f(X)$, where X is the set of input variables $(x_1,x_2,\ldots,x_n)$ required to evaluate the model. The input variables are stochastic, i.e., they can be described in terms of their mean and variance. The Taylor Series expansion is used to approximate the mean, variance, and/or higher moments of the outputs of the model. The evaluation of the Taylor Series expansion requires only the mean and variance of each input variable, sensitivity coefficients, and an estimate of the correlation between the input variables. Sensitivity coefficients are partial derivatives, measuring the influence of each input variable on the model output. There is one sensitivity coefficient for each model input with respect to each model output. If non-zero correlation exists between input variables, it is expressed as a correlation coefficient or co-variance. If no correlation exists, the evaluation of the Taylor Series expansion is simplified.

Researchers refer to the FOSM method using various naming conventions. Yeh and Tung (1990) depict the FOSM method as First-Order variance estimation (FOVE), while Bobba et al. (1995) refer to it as functional analysis or First-Order error analysis (FOEA). Scavia et al. (1981) also use the FOEA terminology but additionally use First-Order variance propagation (FOVP) to describe their work. There are additional variations of the FOSM technique in literature (Cesare 1991; International Standards Organization 1995). All of these methods utilize the Taylor Series expansion to approximate the mean and variance of the respective model applications. Therefore, although the naming conventions are different, the applications all use the same basic technique. The method will be referred to as the FOSM in this thesis.

The FOSM method is versatile and relatively simple in formulation when compared to other uncertainty analysis methods. The method is not data intensive. It provides the ability to relate uncertainty in the output of the model Y to individual inputs or groups of inputs and calculates the uncertainties for each variable explicitly. The inputs that contribute the most to uncertainty in the output can be identified and targeted for subsequent study. Non-influential inputs can be de-emphasized or ignored all together. The FOSM method is relatively easy to execute with the bulk of the effort required to utilize the method spent computing the sensitivity coefficients. Sensitivity coefficients can be calculated using forward or central difference methods. Central difference methods are considered more accurate than forward differencing but require additional executions of the model to compute (Poeter and Hill 1998). Once the sensitivity coefficients are computed the subsequent effort required to compute the uncertainty is trivial.

The main disadvantage of the FOSM method is the technique will not be accurate if the variance is not a good descriptor of the variable. The reason is the FOSM does not take into account the distribution of the input parameters. It relies only on the mean and the variance to describe the parameter. The FOSM method will not be accurate when distributions are highly skewed or vary significantly from the normal distribution. Also, the FOSM may not accurately calculate the uncertainty of a model that is highly non-linear, due to the linearization of the function via the Taylor series expansion (Yen et al. 1986). Some of the shortcomings of the method arise from the reality that the FOSM is an approximate uncertainty analysis method. It only considers the first and second-order moments of the input parameters. It is possible to achieve more precision with the method through the inclusion of the higher order moments but the mathematics required are complex. This is especially the case if the input parameters are correlated requiring the computation of covariance for each pair of variables and inclusion of the co-variances in the uncertainty calculations (Tung 1996).

An alternative uncertainty analysis method is Monte Carlo analysis. Monte Carlo analysis is a conceptually simple process for evaluating uncertainty, but it requires the definition of probability distribution functions (PDF) for the input variables. The PDF's and a random number generator are used to develop sets of input parameters that are then evaluated in the model. The set of model output values define the distribution function for the output. The model outputs are also used to compute the mean, variance, and confidence limits.

Monte Carlo analysis is considered a more exact method than the FOSM method. It does not have the same difficulties with non-linear functions and models. It can take into account not only the mean and variance of the input but also the distribution type. It also provides the entire distribution function for the output where the FOSM method only provides the mean and variance. The main drawback of Monte Carlo analysis is that a PDF is required for each input variable. For most problems, a PDF for each variable is unknown, so assumptions on the distribution must be made, introducing uncertainty into the process. The other disadvantage of Monte Carlo analysis is that it can be computationally demanding. This can be an issue for complex models that require a significant amount of time to compute because one run of the model is required for each set of input parameters. The required number of simulations is unknown at the outset of the analysis but is normally on the order of 100 to 2000. A sufficient number of simulations are required to achieve convergence for the computed mean and variance of the output and to obtain a smooth output distribution.

Both the FOSM method and the Monte Carlo analysis have been applied in hydraulic engineering. Huang (1986) computed the probability of failure for the design of a sluice gate in trapezoidal channel first using FOSM and then the Monte Carlo analysis. The results were comparable with the author stating his preference for the Monte Carlo approach with no further explanation. The model investigated by the author was a simple open channel flow model of a trapezoidal channel using Manning's equation. Evaluation of the probability of failure using the Monte Carlo method in this paper would have been quick given the simplicity of the model. The author executed 2000 simulations to achieve convergence in his answer.

Cesare (1991) used the first-order analysis method to determine the influence of uncertain Manning's n on the return period of a depth of flow computed using traditional analysis techniques. The author found that inclusion of the uncertainty in Manning's n resulted in a smaller return period for a given depth of flow than computed using traditional approaches. The changes in return period were considered significant by the author. For example, for large storms (return periods of 100 years or greater), the return periods for a given depth of flow were cut in half when a coefficient of variation of 0.15 in Manning's n was evaluated.

In the field of water quality modelling, Bobba et al. (1996) applied the FOSM method and Monte Carlo analysis methods to quantify the uncertainty in a water quality model. They also compared the results achieved using the two methods. The authors built both methods into the code for a non-point source water quality model. The FOSM and Monte Carlo estimates of uncertainty were outputs of the model. The authors found that if the random input variables were linearly related to the outputs the FOSM analysis provided an exact solution for the errors in the output. Monte Carlo analysis is the preferred method if the random input and output relationships are non-linear. The FOSM method was advantageous in determining the one or two random variables that contributed to the majority of the uncertainty in the model output. In another water quality application, Scavia et al. (1981) compared the FOSM method and Monte Carlo method in the estimation of the variance in outputs from a Lake Eutrophication model of Saginaw Bay, Lake Huron.

Any uncertainty analysis method depends on the ability to quantify the uncertainty in the model inputs. There are two types of model inputs, quantities that can be measured in the field and quantities that cannot be measured. The second category includes many model parameters that must be established by other means. Both types of input data are known to contain uncertainty. The estimation of uncertainty in measured quantities is relatively straightforward, but estimating quantitatively the uncertainty in un-measurable hydraulic parameters such as channel roughness is difficult. The accuracy of the prediction of the uncertainty in the model outputs depends largely on the ability to estimate the uncertainty in the inputs (Johnson 1996). Johnson estimated the uncertainty in key hydraulic parameters using coefficients of variation and probability distributions based on field experiments and many literature sources. The paper provides a helpful summary.

A closely related subject to uncertainty analysis is sensitivity analysis. The two are related in the steps that are undertaken in the analyses but the purpose of the analyses is different. Uncertainty analysis determines the uncertainty in a model output given the uncertainty in model inputs. Sensitivity analysis determines how important individual input elements are with respect to the uncertainty in the outputs. In most cases it is simply performed to determine what inputs have the most influence on the outputs, so as to focus attention to those inputs.

Traditional sensitivity analysis only provides a partial indication of the role of each input with respect to the uncertainty in the output. It does not consider the variance in model inputs. An input can have a highly sensitive effect on the model output but still be a small contributor to overall model uncertainty if the uncertainty in the input is small. Conversely, an input with a small sensitivity may be a large contributor to overall model uncertainty if its uncertainty is significant. In the FOSM method, the sensitivity of the model input must be determined in order to calculate model uncertainty. The sensitivity is also used to attribute the overall uncertainty to individual or groups of model inputs.

Saltelli et al. (2000) define three classes of sensitivity analysis screening, local and global sensitivity analysis. Screening sensitivity analysis aims to qualitatively identify the inputs that influence the outputs the greatest and determine those inputs that are the least sensitive. They are computationally easy to complete but only provide a qualitative not a quantitative estimate of the sensitivity. Local sensitivity analysis concentrates on the individual impact of model inputs on the output. Sensitivity coefficients are calculated separately for each model input through the evaluation of the partial derivative of the model input with respect to each model output. The sensitivity coefficient is a quantitative depiction of the model inputs influence on the model output. Local sensitivity analyses provide more information than screening level sensitivity analysis but are still limited for several reasons. For input parameters that have large variability the sensitivity coefficient may vary depending on where the calculation is made in the input parameter space. Also, if the model is highly nonlinear and various model inputs are affected by uncertainties of different orders of magnitude, a local sensitivity analysis is not recommended. In these situations a global sensitivity analysis method should be used.

Global sensitivity analysis evaluates the effect of varying all model inputs simultaneously. It quantifies the total effect of the input on the model output. It accounts for the variables individual sensitivity and also the combined effect of the variable interacting with the other model inputs. A global sensitivity analysis can evaluate the influence of the model input over its entire range. This knowledge is more valuable than simply its influence at a specific point in the parameter space. Global sensitivity analyses are complex but provide key information as results. Global sensitivity analyses are not limited to linear models or models with inputs that have large parameter uncertainties.

A global sensitivity analysis on a hydrodynamic model was undertaken by Hall et al. (2005). The analysis quantified the sensitivity and subsequently the contribution to model uncertainty from inflow, geometrical data and roughness coefficients. The global sensitivity analysis provided a quantitative measure of the contribution that uncertain factors make individually or collectively to the variance in modelling results. The analysis concluded that channel roughness, described using Manning's *n*, was the dominant factor in determining overall model uncertainty. This conclusion was based on the evaluation of sensitivity indices for the model inputs and a Monte Carlo evaluation of uncertainty. The sensitivity indices were calculated using the method of Sobol (1993) when the parameters were considered uncorrelated. When the parameters were correlated, a replicated Latin Hypercube approach described by Mckay (1995) was utilized. The calculation of sensitivity indices provides a precise quantification of the model input influence. However, sensitivity indices are difficult to evaluate.

In summary, the concept of uncertainty in models has been recognized for some time and methods are available to quantify the doubt. In water resources engineering and specifically hydraulic engineering there have been successful applications of the FOSM and Monte Carlo analysis techniques on relatively simple models. Uncertainty analysis of a more complicated model using a global sensitivity analysis approach was conducted on a one-dimensional hydrodynamic model by Hall et al. (2005). However, there has not been a quantification of overall uncertainty and the identification of the sources of uncertainty for a two- or three-dimensional hydrodynamic model.

This thesis will attempt to quantify the overall uncertainty in the two-dimensional model of the St. Lawrence River developed in 2003. The analysis will use the FOSM method and Monte Carlo methods because they are proven methods suitable for this application. Both uncertainty analysis methods will be employed and the process and results will be compared. The FOSM method effectively involves a local sensitivity analysis and the results of the sensitivity analysis will be discussed. The uncertainty analysis will quantify the level of confidence in computed water levels and velocities and the sensitivity analysis will attribute the uncertainty to specific model inputs. The information gained may be of use to decision makers and future users of modelling outputs. Future model developments, data collection programs and other related research may benefit from the findings of this thesis.

Chapter 3 – Background, Model Development and Calibration

3.1 Introduction

Before undertaking the uncertainty analysis, the model and study area must be introduced. This chapter will introduce the St. Lawrence River study area including its physical attributes, major user groups and regulatory framework. A brief background explaining why a hydrodynamic model of the river was required and the rational for the selection of the RMA2 model will be discussed. As the literature review and introduction have highlighted, uncertainties in input data must be accurately quantified if the overall uncertainty in the model is to be determined. As a preamble to a full discussion of data uncertainties, this chapter will describe the data requirements for the RMA2 model and the sources that were used to obtain the necessary data. The development of the model with the Surface Water Modelling System will then be briefly discussed.

3.2 Study Area

3.2.1 St. Lawrence River

The St. Lawrence River drains the Great Lakes into the Atlantic Ocean. It stretches over 1200 km (720 miles) from the eastern end of Lake Ontario to the Gulf of St. Lawrence. The river can be divided up into several different reaches. The first 180 kilometres of the St. Lawrence River is known as the international section or Upper St. Lawrence River. Unlike downstream reaches of the St. Lawrence River, there is no tidal influence in the international section. The downstream control point of the Upper St. Lawrence River is the Moses-Saunders hydroelectric dam at Cornwall, Ontario. The boundary between Canada and the United States of America lies within the St. Lawrence River and thus the river is shared by both countries over this length. The Upper St. Lawrence River is the focus of this study. A map of Lake Ontario and the St. Lawrence River is shown in Figure 1 and a more detailed map of the international section or Upper St. Lawrence River is shown in Figure 2.

The depth of the River in this reach varies from less than one metre to over 70 metres, averaging just over 10 metres deep. The flow is gradually varied and sub-critical throughout the reach. The upstream end of the river is known as the Thousand Islands region. It is generally characterized as a wide, lake-like section of the river with very minor gradient. Flow through this section is complex with the many islands both large and small, creating a braided channel system. The many islands in this area are formed by the hard rock of the Canadian Shield. The maximum depth in this portion of the river is in excess of 30 metres with one section being over 70 metres deep. Velocities in this section range from less than 0.05 m/s to a maximum of 0.8 m/s in a narrow section of the river known as the American Narrows, adjacent to Alexandria Bay (see Figure 2).

The middle section of the river, starting upstream of Brockville, Ontario is narrower than the Thousands Islands Region. This reach has increased gradient and the average velocity is greater than the upper reach. The number of islands decreases and the river is more uniform in cross section. The sill of Lake Ontario is located within this section of the river at Galop Island. Prior to the dredging of the river in the late 1950s for the construction of the St. Lawrence Seaway there were rapids at Galop Island and this was the control section of the River. The dredging removed the rapids and enlarged the channel in this area to permit commercial navigation in and out of the Great Lakes. The river gradient remains steep through this section and velocities are quite high, approximately 1.3 m/s or 2.7 knots.

Today, the control point in the river is in the lower section of the Upper St. Lawrence River at the hydroelectric dam at Cornwall, Ontario. The Moses-Saunders hydroelectric dam was constructed as part of the St. Lawrence Seaway Project in the 1950's. In front of the dam is a storage reservoir known as Lake St. Lawrence that is surrounded by a retaining dyke protecting the City of Cornwall. There is a spillway at Long Sault which is used during periods of high supplies or in case of an emergency at the dam. A navigation canal, the Wiley-Dondero canal, was constructed to let commercial navigation occur around the hydroelectric dam. There is another control structure and navigation lock approximately 50 kilometres upstream at the village of Iroquois. The Iroquois Dam is utilized during winter ice formation and during periods of extremely high water levels in the lower end of the St. Lawrence River.

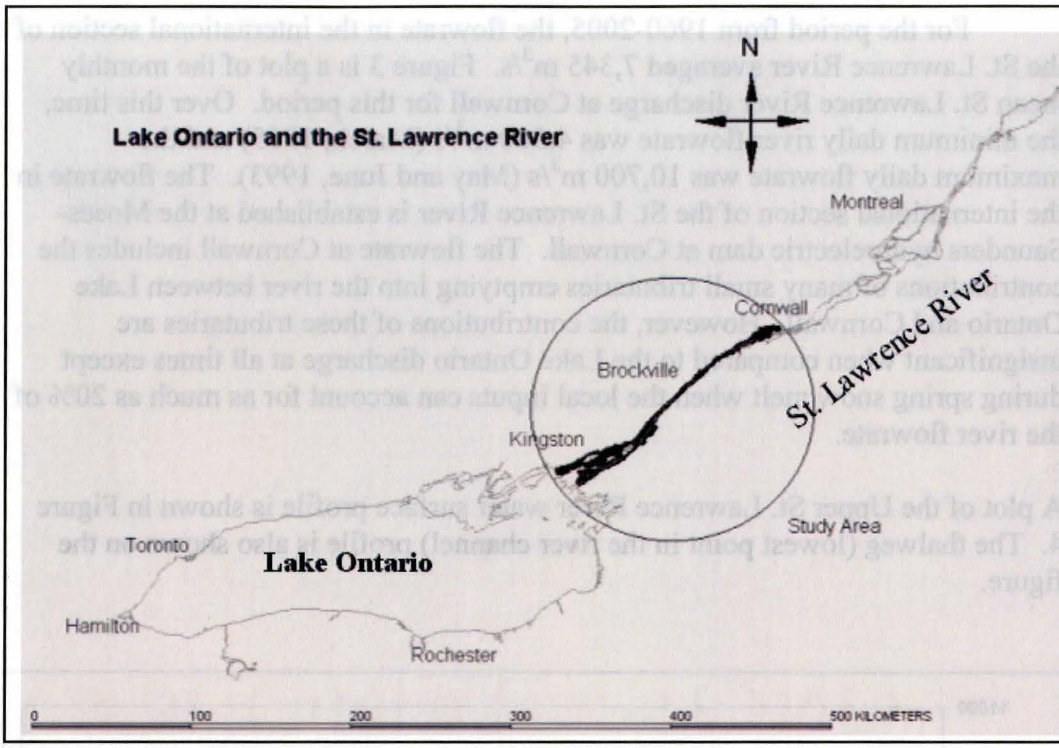

Figure 1 – Map of Lake Ontario and the St. Lawrence River

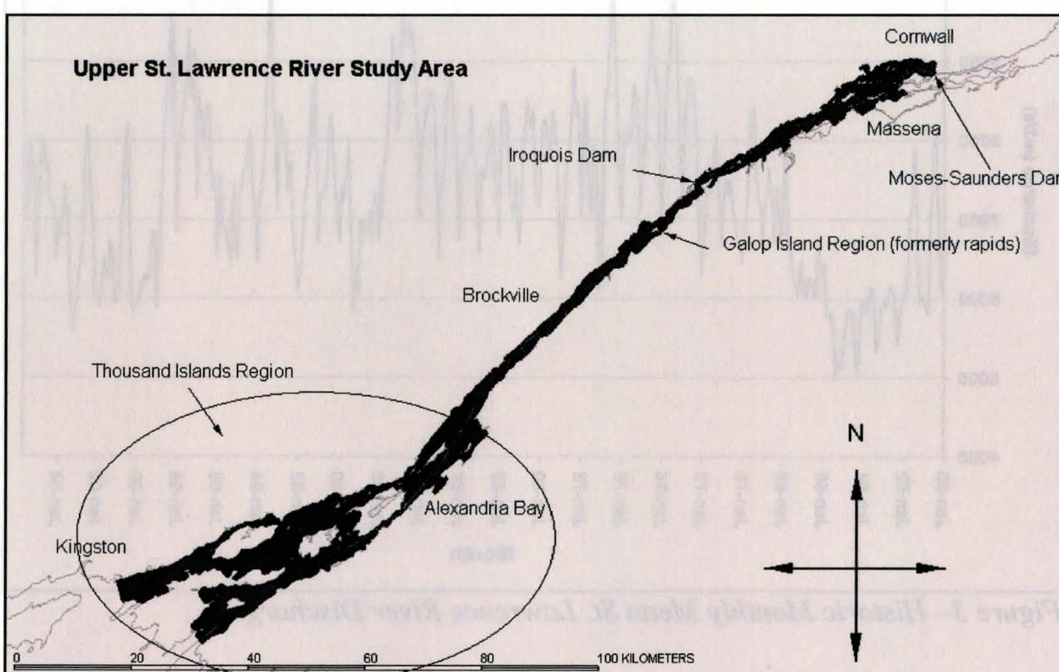

Figure 2 – Map of the Upper St. Lawrence River

3.2.2 Hydrologic Attributes

For the period from 1960-2005, the flowrate in the international section of the St. Lawrence River averaged 7,345 m^3/s. Figure 3 is a plot of the monthly mean St. Lawrence River discharge at Cornwall for this period. Over this time, the minimum daily river flowrate was 4,500 m^3/s (March, 1965) and the maximum daily flowrate was 10,700 m^3/s (May and June, 1993). The flowrate in the international section of the St. Lawrence River is established at the Moses-Saunders hydroelectric dam at Cornwall. The flowrate at Cornwall includes the contributions of many small tributaries emptying into the river between Lake Ontario and Cornwall. However, the contributions of these tributaries are insignificant when compared to the Lake Ontario discharge at all times except during spring snow melt when the local inputs can account for as much as 20% of the river flowrate.

A plot of the Upper St. Lawrence River water surface profile is shown in Figure 4. The thalweg (lowest point in the river channel) profile is also shown on the figure.

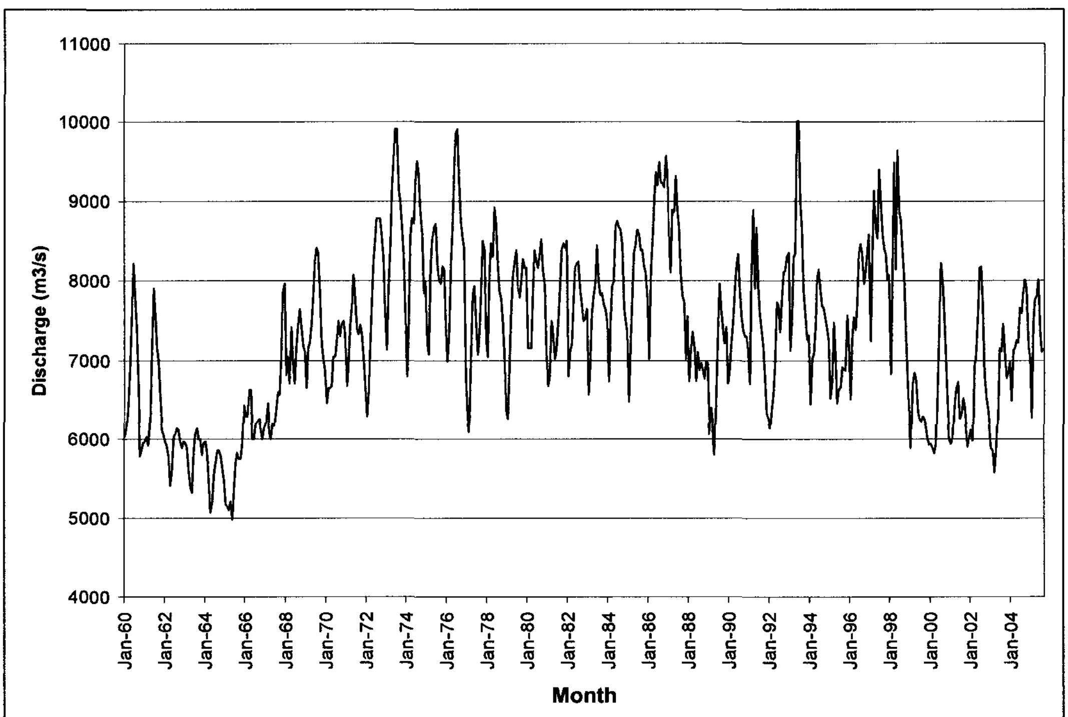

Figure 3– Historic Monthly Mean St. Lawrence River Discharge

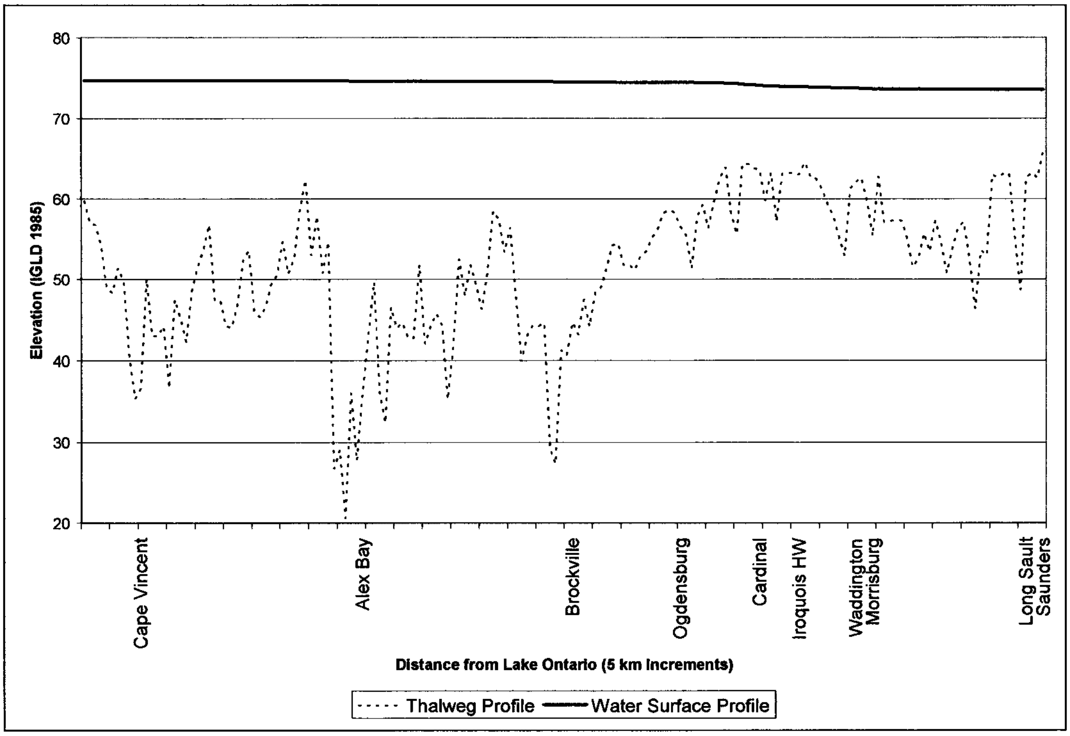

Figure 4 – Upper St. Lawrence River Thalweg and Water Surface Profiles (Lake Ontario Level 74.75m Flowrate of 7000 m3/s)

3.2.3 Interest Groups

The St. Lawrence River is crucial to the local economy providing commercial navigation, hydroelectric power generation, and many recreational benefits. The St. Lawrence Seaway is the conduit between the Great Lakes and the Atlantic Ocean. It allows cargo to flow in and out of the Great Lakes. The Moses-Saunders hydroelectric dam generates electricity for use in Ontario and New York State. The river is also a home to for many permanent residents, seasonal cottagers, and industry connected to the river. The river contains many wetlands providing habitat for migratory birds, mammals, and fish. The Thousand Islands area is a popular recreational boating destination creating significant annual revenues. In the winter, the St. Lawrence River generally freezes over and commercial navigation ceases. However, the river is still utilized for recreational purposes including ice fishing and snowmobiling creating continuous forms of revenue for local businesses.

3.2.4 Regulatory Structure

The flowrate in the St. Lawrence River is regulated. It is set by the International Joint Commission (IJC) through its St. Lawrence River Board of Control (IJC 2005). The IJC is a body established by the Boundary Waters Treaty of 1909 (IJC 1998) for the purpose of resolving disputes over boundary waters between Canada and the United States. The IJC approved the construction of the hydroelectric project spanning the St. Lawrence River between Cornwall and Massena in 1952. To oversee the operation of the dam the IJC established a board of control called the St. Lawrence River Board of Control.

The outflow from Lake Ontario (St. Lawrence River flowrate at Cornwall) is established by the Board of Control using guidelines set forth in regulation plan 1958 D. The regulation plan institutes rules on the release of water from Lake Ontario. It also establishes criteria to adhere to relating to water levels and flows within the system. The regulation plan follows the order of precedence for water use written in the Boundary Waters Treaty:

(1) Uses for domestic and sanitary purposes;
(2) Uses for navigation, including the service of canals for purposes of navigation.
(3) Uses for power and for irrigation purposes.

The St. Lawrence River Board of Control created an Operations Advisory Group (OAG) to handle the day-to-day operations of the regulation of the releases from Lake Ontario. The OAG consists of navigation, hydropower and government officials who recommend weekly outflows for approval by the St. Lawrence River Board of Control. The outflow of the river is set on a weekly time step. There is also variation from the weekly outflow, referred to as peaking and ponding, to optimize hydroelectric power generation. As a result, the actual outflow varies on an hourly basis. However, the peaking and ponding are conducted so that over the duration of each week the cumulative outflow is equal to the weekly outflow approved by the St. Lawrence River Board of Control.

3.2.5 Why a Two-Dimensional Hydrodynamic Model was Required

In recent years, the Board of Control and the IJC have been lobbied to consider more interests in the regulation of Lake Ontario, namely recreational boating interests and the environment. The popularity of recreational boating has steadily increased to become a multi-million dollar activity on Lake Ontario and the St. Lawrence River. Since water levels within the channels and harbours in the system are influenced by regulation, recreational boaters want to have a say in how the outflows are set. Environmental interests have also come to the forefront in recent years with degradation of wetlands, bird and mammal habitat, and fish spawning areas. Science has been developed linking health and biodiversity to water levels within wetlands. Environmental interests would like to see an increase in the variation of water levels within the Lake Ontario and St. Lawrence River system to maintain and improve wetlands. However, increased variability in water levels counteracts other aims of the regulation plan such as flood prevention, limiting shoreline erosion, and reducing occurrences of low water levels that effect commercial navigation.

In recognition of the shortcomings of Plan 1958 D, the IJC initiated a study, the Lake Ontario and St. Lawrence River Study, to review the current criteria for regulating Lake Ontario and develop a new regulation plan. The study is a five year, multi-agency, multi-million dollar effort. The anticipated result of the study is a new plan that takes into account the existing interests, the interests of recreational boaters and the environment. The Lake Ontario and St. Lawrence River Study established a hydrology and hydraulics technical working group to develop models and assemble the base hydrologic data for the study. The hydrology and hydraulics technical working group requested the development of a two-dimensional hydrodynamic model of the river. The model was to be capable of spatially simulating the water levels and flows in the international section of the St. Lawrence River. The model was used to evaluate the water levels and flows in the Upper St. Lawrence River given Lake Ontario water levels and varying releases from the Moses-Saunders hydroelectric dam. The water level and velocity information would then be utilized by other technical working groups carrying out studies relating to the impact of water levels, for example, commercial navigation and the environment.

Following the study, the hydrodynamic model was also used by the Operations Advisory Group (OAG) of the St. Lawrence River Board of Control. The OAG oversees activities such as peaking and ponding for hydroelectric power generation. Hydrodynamic modelling was useful to this group in evaluating the impacts of peaking and ponding on local water levels and velocities. Hydrodynamic modelling could also be used by the group to quantify the effects of temporary changes in outflow for commercial navigation purposes or ice formation.

3.3 Selection of a Model

The international portion of the St. Lawrence River can be described as a braided channel containing many islands both large and small. Man-made changes to the river bottom to construct the St. Lawrence Seaway have altered the natural state of the river significantly. The resulting non-uniform geometry of the river creates strong cross-currents in some areas that pose problems for commercial navigation. Due to the complex geometry and flow conditions present a two-dimensional hydrodynamic model was selected. A two-dimensional, depth-averaged model would be able to capture the effects of the cross-currents and the complex configuration of the river. Two-dimensional models are good for display purposes and are easy to visually validate. The mind can easily absorb and reflect on the information when it is displayed in two dimensions as compared to sometimes difficult to interpret one-dimensional plots or very complicated three-dimensional model outputs.

There have been several two-dimensional models developed including RMA2 (Donnell 2005), River 2D (Steffler and Blackburn 2002) and Mike 21. The model selected for application in the St. Lawrence River was the United States Army Corps of Engineer's RMA2 model. RMA2 was originally developed by the Ian P. King of the Resource Management Associates (RMA). It is currently maintained by the U.S. Army Corps of Engineers Waterways Experimentation Station (WES). The model was selected because of its numerous worldwide applications and in particular application in other Great Lakes Connecting Channels.

RMA2 is a generalized computer code for two-dimensional hydrodynamic simulation. It computes depth-averaged horizontal velocity components and water levels for sub-critical, free surface (ice-free) flow. RMA2 uses a finite-element solution of the Reynolds form of the Navier-Stokes equation for turbulent flows. The program solves the depth-integrated equations of mass and momentum conservation in two horizontal directions. The forms of the governing equations used in RMA2 are:

$$h\frac{\partial u}{\partial t} + hu\frac{\partial u}{\partial x} + hv\frac{\partial u}{\partial y} - \frac{h}{\rho}\left[E_{xx}\frac{\partial^2 u}{\partial x^2} + E_{xy}\frac{\partial^2 u}{\partial y^2}\right] +$$
$$gh\left[\frac{\partial a}{\partial x} + \frac{\partial h}{\partial x}\right] + \frac{gun^2}{\left(1.486h^{1/6}\right)^2}\left(u^2 + v^2\right)^{1/2} \tag{1}$$
$$-\zeta V_a^{\,2}\cos\psi - 2hv\varpi\sin\Phi = 0$$

$$h\frac{\partial v}{\partial t}+hu\frac{\partial v}{\partial x}+hv\frac{\partial v}{\partial y}-\frac{h}{\rho}\left[E_{yx}\frac{\partial^2 v}{\partial x^2}+E_{yy}\frac{\partial^2 v}{\partial y^2}\right]+$$
$$gh\left[\frac{\partial a}{\partial y}+\frac{\partial h}{\partial y}\right]+\frac{gvn^2}{\left(1.486h^{1/6}\right)^2}\left(u^2+v^2\right)^{1/2} \tag{2}$$
$$-\zeta V_a^{\,2}\sin\psi-2hu\varpi\sin\Phi=0$$

$$\frac{\partial h}{\partial t}+h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+u\frac{\partial u}{\partial x}+v\frac{\partial h}{\partial y}=0 \tag{3}$$

where

h	= Water depth
u,v	= Velocities in the Cartesian directions
x,y,t	= Cartesian coordinates and time
ρ	= Density of fluid
E	= Eddy viscosity coefficient, For xx = normal direction on x axis surface For yy = normal direction on y axis surface For xy and yx = shear direction on each surface
g	= Acceleration due to gravity
a	= Elevation of bottom
n	= Manning's roughness n-value
1.486	= Conversion from System International (metric) to non-SI units
ζ	= Empirical wind shear coefficient
V_a	= Wind speed
Ψ	= Wind direction
ω	= Rate of earth's angular rotation
Φ	= Local latitude

RMA2 supports linear and quadratic elements, quadrilateral or triangular in shape. Friction is calculated with the use of a Manning's *n* value. Eddy viscosity parameters are used to control numeric stability and describe energy losses associated with viscosity and turbulence. RMA2 is capable of handling both steady-state and unsteady-state problems. Control structures can be accommodated within the model grid.

The RMA2 model operates under the hydrostatic assumption, that is, accelerations in the vertical direction are negligible and pressure increases uniformly. RMA2 is two-dimensional in the horizontal plane and computes depth-averaged directional velocities. RMA2 is a free-surface model, i.e. the pressure at the surface is atmospheric, so it is not to be used to simulate ice-covered conditions.

Pre- and post-processing of the RMA2 model is accomplished using the Surface Water Modelling System (SMS). This commercial software was developed by the USACE and is currently maintained by the Environmental Modelling Research Lab (EMRL) at Brigham Young University.

3.4 Data Requirements and Sources for the RMA2 Model of the St. Lawrence River

Developing an RMA2 model and making it operational requires several steps. The first is the collection of several types of data to build the model, including bathymetric data, shoreline positioning data, vegetation and bottom type data, boundary condition data, and information on any hydraulic structures within the model domain. It is also necessary to obtain observation data to calibrate and verify the model with. Water level, velocity and discharge data are all useful in the calibration of a hydrodynamic model. Data collection for hydrodynamic modelling is laborious. This section will introduce the types of data that are required to develop a two-dimensional hydrodynamic model and the sources that were utilized to obtain the data for the St. Lawrence River model. Discussion on the quantification of uncertainty in the data will be provided in Chapter four.

3.4.1 Bathymetry Data

Bathymetric information describes the geometry of a river or lakebed. It is gathered through marine radar sounding surveys. In a bathymetric survey, a vessel makes successive passes over the survey area while a depth sounder records the exact position of the river or lake bottom. The location of each bathymetric sounding is defined by its easting, northing, and depth either digitally or on a hard copy field sheets. An elevation for the point is determined by referencing the depth to a measured water level in a specified reference plane at the time of the measurement. The reference plane typically used in surveys conducted in this area is called the low water datum referenced to International Great Lakes Datum (IGLD) of 1985. The zero elevation level of IGLD 85 is at Rimouski, Quebec (Coordinating Committee 1995).

Bathymetry for the Upper St. Lawrence River was obtained from a variety of sources. The Upper St. Lawrence River is an International River with the Canadian and U.S. boundary bisecting the river. As a result, responsibilities for surveying and maintaining bathymetric information for the river are shared between Canada and the United States. In Canada, the agency responsible for collecting and maintaining bathymetric information is the Department of Fisheries and Oceans, Canadian Hydrographic Service (CHS). In the United States two agencies collect and maintain bathymetric data, the Department of Commerce, National Oceanic and Atmospheric Administration (NOAA) and the United States Army Corps of Engineers (USACE). The USACE maintains bathymetric records for harbours and shipping channels, while NOAA maintains databases with nautical data for North America including the Great Lakes - St. Lawrence Region.

Data obtained from the Canadian Hydrographic Service accounts for the largest portion of bathymetry used in this model. Bathymetry for the St. Lawrence River existed in the Canadian waters from Kingston to Chippewa Bay and the entire river downstream of Chippewa Bay to Cornwall. The bathymetry represents 44 hydrographic surveys of the river conducted from 1968 to 1987. The earliest surveys were conducted near Lake Ontario with the most recent surveys representing the downstream sections of the river near Cornwall. Data for the U.S. waters from Cape Vincent to Chippewa Bay was obtained from NOAA. Recent bathymetric data was obtained from the United States Army Corps of Engineers, for Ogdensburg harbour, which is located approximately half way in between Kingston/Cape Vincent and Cornwall/Massena.

All soundings were converted into the same vertical datum, IGLD 85. The data was projected into the Universal Transverse Mercator (UTM) projection system (Richardus and Adler 1972). The UTM zone for the entire Upper St. Lawrence River is Zone 18. All calculations were conducted using SI units, with distance being measured in metres. The soundings were assembled into a single file containing more than 669,000 data points covering an area of approximately 625 square kilometres.

3.4.2 Shoreline Positioning Information

In order to determine the limits of the model domain, shoreline information is needed. The sources of data used in this project were the CHS and NOAA. For all areas where CHS bathymetry was used, there was also a high water line defined on the bathymetric field sheets. This line is defined as 1.3 metres above the low water datum for the region of the survey. This line was used as an initial estimate of the shoreline position.

To verify the shoreline information digital hydrographic charts from the CHS and NOAA were utilized. The hydrographic charts were digitally aligned with the shoreline data and a manual inspection was performed to ensure the quality of the data. Recently, digital aerial photography for the river has been made available to the public by the State of New York. The aerial photography was collected within the past five years and is of significantly higher resolution than the previously available shoreline information. The aerial photography was also digitally aligned with the shoreline data and used to identify any potential gross errors in the data.

3.4.3 Bottom Classification and Vegetation Data

In hydraulic modelling knowledge of the bottom material is an asset to the estimation of a bottom roughness, commonly described using a Manning's *n* value. Relationships between Manning's *n* and mean particle size have been developed and utilized in hydrodynamic modelling (Leclerc et al. 1995). The difficulty in using this approach is in locating sufficient bottom type data. For the international section of the St. Lawrence River no such data could be located. Therefore, bottom type could not be used to estimate Manning's *n* values for this model. The methods used to estimate the Manning's *n* values used in this model will be discussed in detail later in Chapter four of this report.

The field sheet data obtained from the CHS and NOAA did include some description of vegetation, namely locations of marshes and other thick vegetation. Generally, this information was transferred to the hydrographic charts which were used in the creation of Manning's *n* zones during initial model calibration. No field studies were undertaken to collect data on vegetation.

3.4.4 Boundary Condition Data

Boundary condition data required by the model consisted of water levels at the upstream boundaries and discharge at the downstream boundary. The upstream water levels are measured at the Kingston and Cape Vincent water level gauges. Downstream discharge is defined as the estimated Moses-Saunders hydroelectric plant discharge.

Boundary condition data were obtained from various sources. Water level data was obtained from the CHS, NOAA and the Ontario Power Generation (OPG). The Moses-Saunders hydroelectric plant discharge was obtained from OPG.

The time step that the data is available on varies by type. Water level data is available from the CHS at a 15 minute, hourly, daily and monthly time step. NOAA collects and makes available via the internet six minute, hourly, daily and monthly time step data. OPG and New York Power Authority (NYPA) collect and archive hourly, daily, and monthly time step water level data. OPG and NYPA monitor instantaneous discharge at the Moses-Saunders Dam but archive at an hourly and daily time step. ADCP discharge and velocity data are instantaneous measurements and are only valid for one particular period in time. The lowest common time step for all types of data is hourly, with the exception of

ADCP data as previously noted, therefore an hourly time step was selected. Data for the years of 2002 and 2003 were gathered and organized into a database.

An optional type of boundary condition data for the RMA2 program is wind. Although wind friction creates currents that are three-dimensional in nature, the program does allow wind friction to be incorporated in the model. Wind is de-emphasized and generally not used with RMA2 but may be important in the St. Lawrence River application. This is due to the alignment of the river, which is in line with the prevailing winds; and also due to the relative shallowness of many parts of the river that may be affected by wind setup. In order to account for the effects of wind shear in the river a wind field can be specified as a boundary condition to the model. Sources for wind data for the St. Lawrence River were investigated. There are four wind monitoring stations on the Upper St. Lawrence River from buoys located at Alexandria Bay and Superior Shoals, and land based stations at Grenadier Island and the Moses-Saunders Power Dam. The first three stations are located at the upstream end of the river and are all within a close proximity of each other. Surprisingly, the data for these three stations does not show significant correlation, suggesting wind distribution over the river varies spatially. This could be due to many reasons including local topography and orographic effects, wind measurement height and exposure. Given the nature of the data and the options for specifying the wind data as a boundary condition in the RMA2 model, the use of this data was considered infeasible at the current time. Wind was therefore not specified as a boundary condition. However, the impact of wind to the flow conditions in this reach of the St. Lawrence River should be recognized.

Water temperature can be specified in the model to match conditions observed in the field. The physical characteristics of water including its density and viscosity are influenced by its temperature. Water temperature for the river is obtained from several sources including the Kingston water treatment plant, NOAA weather buoys, and OPG.

3.5 Development and Execution of the St. Lawrence River RMA2 Model

The development of a model within the SMS modelling system involves many steps and decisions that although important are not crucial to the focus of this thesis. For more details regarding the development of the model within SMS, the reader can refer to Thompson and Moin (2003) and the SMS users manual (Brigham Young University 2001). The St. Lawrence River model developed for this analysis contains 32,000 elements and roughly 69,000 nodes. The size of the elements varies from less than 10 metres at the Iroquois Dam to more the 200 metres at the upstream limits of the model at Lake Ontario. A sample of the model grid density is shown in Figure 5. There were a total of eight roughness zones defined in the model. The reasoning behind the definition of eight roughness zones will be discussed in section 3.6.4 dealing with model calibration. The model takes approximately two minutes in providing a steady-state solution on a Pentium four computer with a processor of 3.2 GHz and three Gigabytes of RAM.

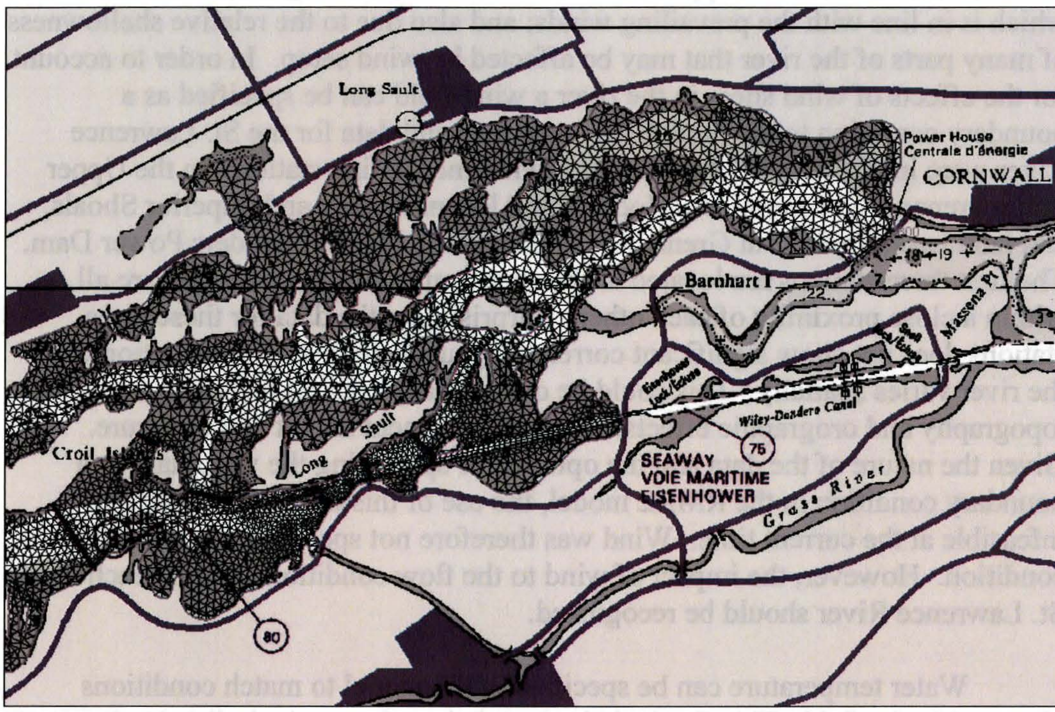

Figure 5 - St. Lawrence River Model Grid at Cornwall, Ontario

3.6 Establishment of Un-measurable Model Parameters (Calibration)

This section will discuss how the model parameters that cannot be measured were established. In general, these parameters were either established from available literature or through calibration. Those parameters established through literature will be dealt with first and then the parameters requiring calibration will be discussed. Consideration of the uncertainty in these parameters will be dealt with in the following chapter.

3.6.1 Wetting and Drying Parameters

There may be times when portions of the mesh will become devoid of water, and later become wet again. This is especially true for the St. Lawrence River, where the flowrate and water levels can vary significantly under the range of hydrologic conditions possible. This process of flooding and drying out is referred to as "wetting and drying" in modelling. There are two options to simulate these events; elemental elimination and marsh porosity. Both of these methods were used in combination for the Upper St. Lawrence River model.

Elimination of elements reduces the conveyance area available in the model by removing elements that are not needed during low water level periods. As water levels drop, the elements along the lateral boundaries of the model are inactivated so that the flow is contained in the interior of the model. An element becomes inactive when the elevation of all of the nodes defining the element is higher than the water level in the river. When the water level becomes higher than the nodal elevation of one or more of the nodes defining the element, the element becomes active and the element is used in the conveyance in the river. This process mimics the decrease or increase in river width as water levels fall or rise. To prevent excessive wetting and drying during the solution process, the RMA2 model requires the user to specify a depth-range in which the nodes are defined as either wet or dry. The range used for this application of the RMA2 model was defined as 0.02 metres depth to 0.2 metres depth. The first number refers to the depth below which the node is considered dry and the second number refers to the depth above which the node is considered wet. These values were determined using guidance provided in the RMA2 reference manual (Donnell et al. 2005). Nodes are checked to see if there is a requirement to add or remove any elements using the elemental elimination method once every four iterations during the solution process.

The marsh porosity option in RMA2 allows elements to transit gradually between wet and dry states. The technique, allows RMA2 to lower the ability of the element to hold water; like squeezing a sponge. The residual water volume existing on a partially wet element is calculated by vertically integrating a wetted area curve for each node of the element. The wetted area curve represents the micro-scale bathymetry variations that occur surrounding each node. Partially wet elements are retained in the calculations until all nodes become dry. Once dry the element is eliminated from the solution using the elemental elimination process as described previously. Dry elements re-enter the computations as soon as one node is re-wet.

The wetting and drying and marsh porosity options within RMA2 allow the model to reach convergence for a range of water levels and flows without making manual modifications to the mesh. The options increase the resilience of the model to varying hydrological conditions and increase the ease of application. The parameters are project specific and were developed with guidance from the RMA2 manual and trial and error with the model.

3.6.2 Manning's n

Robert Manning introduced the Manning's *n* coefficient when he proposed the Manning formula for uniform flow (Chow 1959)

$$V = (1/n)\, R^{2/3} S^{1/2} \qquad (4)$$

where V is the mean velocity in m/s, R is the hydraulic radius in metres, S is the slope of the energy grade line, and *n* is the coefficient of roughness, specifically known as Manning's *n*.

The Manning's *n* coefficient is used by RMA2 to calculate the bed friction (bottom shear stress) that is applied to the flow. Changing the bed friction provides some control of the flow velocity and direction. Normally, without detailed information about the river bed, the same Manning's *n* coefficient is assigned to an entire region of the river. These regions are represented in the model by a finite number of elements that are all assigned the same Manning's *n* coefficient. This was the approach taken for assigning Manning's *n* values to the Upper St. Lawrence River model. The regions were defined as material (roughness) zones in the map module of SMS and the appropriate material zone property was transferred to the mesh when it was generated. The configuration and definition of the material zones evolved over time through the process of model calibration.

3.6.3 Eddy Viscosity

The Eddy Viscosity is used to represent the strength of turbulence within the flow domain. Turbulence is caused by temporal variations in velocity at a very small time step and also by momentum exchange between adjacent regions possessing varying velocities. In other words turbulence is caused by changes in velocity over time or velocity gradients across sections of a river. Fluids moving at different speeds will transfer momentum at their interfaces creating turbulence exchange.

Turbulence exchange is an important phenomenon in two-dimensional modelling. Some numerical methods used in modelling require the addition of a minimal amount of artificial diffusion in order to achieve a "stable" solution that converges while using the Newton-Raphson iteration scheme. The Galerkin method used in RMA2 is one of these methods because the basic numerical procedure includes no artificial diffusion other than the Eddy Viscosity term. The Eddy viscosity term specifies the amount of turbulent diffusion that occurs at the intersection of elements with differing velocities.

In order to achieve numerical stability, a minimal amount of turbulence is required. The Eddy Viscosity should not be too high however or the uniqueness of velocities at neighbouring elements will be diminished and the horizontal velocity variation desired from a two-dimensional model will be lost.

Eddy Viscosity can be assigned to individual elements, material zones or the entire mesh. For the St. Lawrence River model Eddy Viscosity was assigned dynamically to each element in the model using a single Peclet Number for the entire mesh. The Peclet Number relates to the Eddy Viscosity through equation 5.

$$P = \frac{\rho u dx}{E} \qquad (5)$$

where, ρ is the fluid density, u is the average elemental velocity, dx is the length of the element in the stream-wise direction, and E is the Eddy Viscosity with units of Pascal-seconds.

The eddy viscosity varies in relation to the velocity and the stream-wise length of the element. The eddy viscosity values were considered isotropic. The value of the Peclet Number was established through a calibration process.

3.6.4 Calibration of Manning's n and Eddy Viscosity

Calibration is a process in which parameters are adjusted so that computed values match or reasonably match observed data. It is important to quantify the purpose of the modelling and gear the calibration towards these goals. The modeller can choose to calibrate using a wide spectrum of observations covering the entire hydrologic range so that the model will be applicable for a wide range of hydrologic conditions. The other option is to choose a specific smaller range of observations in hopes of obtaining a better match and higher model performance over this smaller range. There are trade offs in both approaches with regard to model suitability and accuracy. A conscience decision must be made at the outset of the calibration process.

For this thesis, the calibration was performed to meet the goal of the problem of the estimation of the water levels and velocities in the river at three key locations in the river and the quantification of the uncertainty in those estimates. The historic range (1960-2005) in water levels at the upstream boundary of the St. Lawrence River (Lake Ontario) is large, varying from 73.75 to 75.75 metres. River discharge has ranged from 4,500 to 10,900 m^3/s of over this same period. One set of Manning's *n* and Eddy Viscosity values while being optimal over the entire hydrologic range would not at the same time provide the optimal fit over a much smaller hydrologic range. Also, the amount of vegetation present in the river over the course of the year varies affecting channel roughness. Vegetation is at a minimum in the late winter and early spring and a maximum in late summer and early fall. Environment Canada's engineers responsible for the regulation of Lake Ontario and the St. Lawrence River have undertaken studies and conclude the vegetation in the Upper St. Lawrence River creates a seasonal effect on the channel roughness (David Fay, Environment Canada, personal communication, May 10, 2001). Therefore, to complete the most precise calibration possible, observations spanning a small, relevant hydrologic range should be selected and the time of year should also be taken into account so as to account for seasonality in roughness.

In this thesis, the model was calibrated using hourly water level observations collected on April 16th, 2002. The average Lake Ontario water level and river discharge on this day were 74.98 metres and 7023 m^3/s respectively. There was an Acoustic Doppler Current Profiler (ADCP) survey of river velocities and discharge conducted in the river on this day providing further calibration data. Using this one day allows a precise set of parameters possible to be determined for a small period in time. During April, there would be little vegetation in the river with the ice having melted. Channel roughness due to vegetative effects would be minimal.

Velocity data was not explicitly used in the calibration but was used to verify and spot check the computed values. The reason velocity data was not used in the calibration was due to the complexities of processing of ADCP data both spatially and temporally to a scale appropriate for calibration of the St. Lawrence River hydrodynamic model. ADCP data collected during transects for the estimation of discharge is not readily usable for estimating the mean velocity over the depth (Muste et al. 2004). Specific operational precautions must be taken during the surveys and post-processing procedures must be followed. These procedures are not trivial. Future research on the RMA2 model of the St. Lawrence may utilize ADCP more explicitly than the present work.

Traditional calibration of a model is completed by manually adjusting the parameters until a suitable match to observation data is achieved. However, in this thesis, a universal inverse modelling code, UCODE, (Poeter and Hill 1998) was used to make the Eddy Viscosity and Roughness coefficient model parameter adjustments systematically and automatically. Manning's *n* values and the Eddy Viscosity were estimated for the model material zones by the use of UCODE, a procedure that applies a nonlinear regression technique to minimize the sum of squared residuals. The sum of squared residuals is calculated as the sum of the differences between the computed and observed water levels at all water level gauges within the model domain. There are a total of ten water level gauges on the St. Lawrence River as shown in Figure 6 but only nine were used in the calibration. Data for the gauge at Waddington was not available at the time of the work.

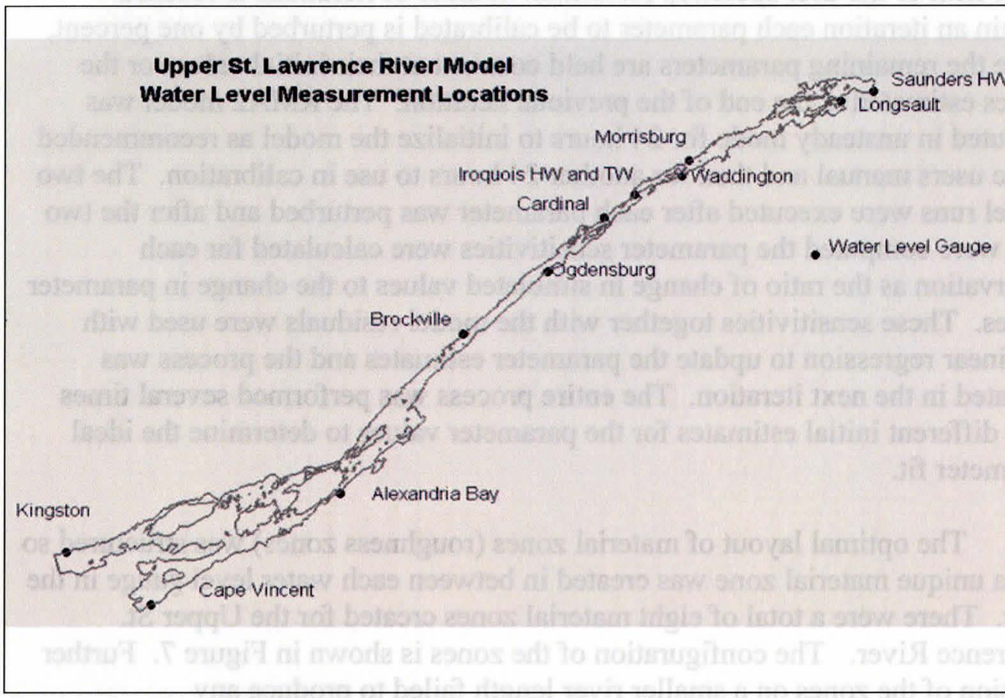

Figure 6 – Upper St. Lawrence River Water Level Measurement Locations

UCODE is a universal code for parameter estimation written in the programming language PERL (Practical Extraction and Report Language). UCODE was used to provide the following functions:

- Manipulate RMA2 input files and read values from output files
- Execute RMA2 in batch mode with different parameter sets
- Compare simulated with expected values using a weighted sum of squared residuals objective function
- Apply a non-linear regression code to adjust parameter values in response to the comparison
- Report the estimated parameters
- Calculate and print statistics to be used to
  - Diagnose inadequate data or identify parameters that probably cannot be estimated
  - Evaluate estimated parameters
  - Evaluate how accurately the model represents the actual processes

The parameter estimation problem was solved by non-linear regression. The sensitivities of parameters and observations to the overall sum of squared residuals were calculated using the central differencing procedure in UCODE. The parameter estimation problem was deemed to have converged when the sum of squared weighted residuals changed by less than two percent from one iteration to the next or the user specified maximum number of iterations is reached. Within an iteration each parameter to be calibrated is perturbed by one percent, while the remaining parameters are held constant at their initial values or the values estimated at the end of the previous iteration. The RMA2 model was executed in unsteady mode for 24 hours to initialize the model as recommended in the users manual and then for another 24 hours to use in calibration. The two model runs were executed after each parameter was perturbed and after the two runs were computed the parameter sensitivities were calculated for each observation as the ratio of change in simulated values to the change in parameter values. These sensitivities together with the model residuals were used with nonlinear regression to update the parameter estimates and the process was repeated in the next iteration. The entire process was performed several times with different initial estimates for the parameter values to determine the ideal parameter fit.

The optimal layout of material zones (roughness zones) was structured so that a unique material zone was created in between each water level gauge in the river. There were a total of eight material zones created for the Upper St. Lawrence River. The configuration of the zones is shown in Figure 7. Further revision of the zones on a smaller river length failed to produce any improvements. This configuration was found to be superior to the other configurations based on the total sum of squared residuals and also the lowest individual water level gauge residual. A summary of the results of the calibration is shown in Table 1 and a plot of the final computed versus observed water levels at the Saunders Headwater Gauge is shown in Figure 8.

The final Manning's *n* values actually account for more than just the bed friction. The Manning's *n* parameters in this model are considered to be lumped parameters representing several forces resisting the flow in the river reaches. In addition to the bed friction, Manning's *n* accounts for resistance due to vegetation and irregular bed geometry. The presence of plants decreases the actual conveyance area in the river, from that defined using the bathymetric data alone. Vegetation also tends to impede the flow due to its irregular shape and roughness. Another force is created by geometrical inaccuracies present in the model grid. The nodes and elements defining the model grid are spaced as close together as is reasonable, but do not necessarily reflect the irregular shape of the river bed exactly. The differences in the shape of the bottom of the river in the model versus the actual river represent a force that will resist the flow. These forces are not explicitly specified in the St. Lawrence River model and are therefore lumped into the Manning's *n* values. It is not possible to separate the influence of these forces from the bed friction. The optimized Manning's *n* values account for these forces in addition to the bottom friction. Turbulent forces are accounted for separately by specifying the Eddy Viscosity.

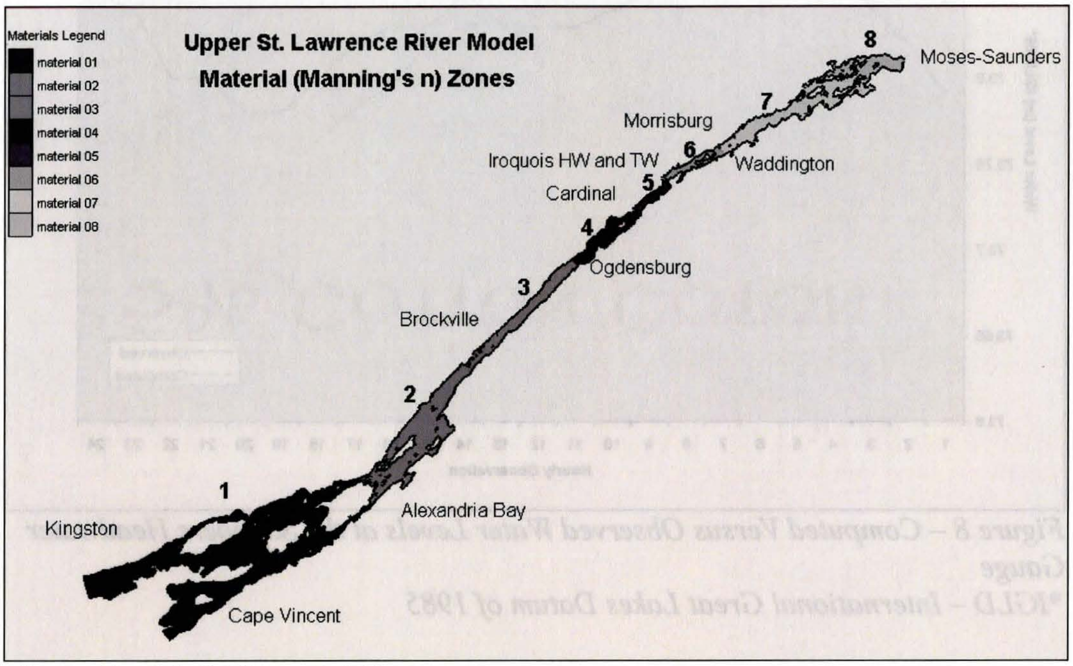

Figure 7 – Model Material Zones

Table 1 – Model Calibration Results

Quantity	Value
Sum of Squared Residuals	1355
Correlation Coefficient	0.998
Minimum Residual	-6.2 cm
Maximum Residual	5.9 cm
Peclet Number (Eddy Viscosity)	17.7
Material Zone 1 Manning's *n* value	0.0309
Material Zone 2 Manning's *n* value	0.0288
Material Zone 3 Manning's *n* value	0.0274
Material Zone 4 Manning's *n* value	0.0288
Material Zone 5 Manning's *n* value	0.0270
Material Zone 6 Manning's *n* value	0.0276
Material Zone 7 Manning's *n* value	0.0326
Material Zone 8 Manning's *n* value	0.0308

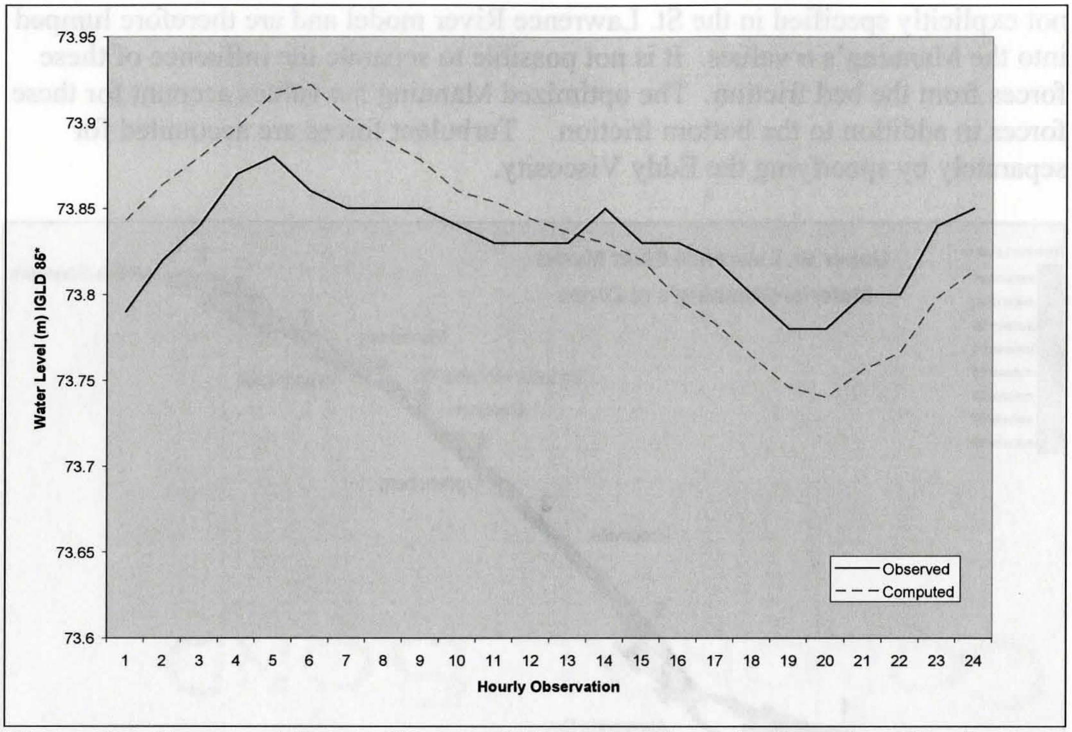

Figure 8 – Computed Versus Observed Water Levels at the Saunders Headwater Gauge
**IGLD – International Great Lakes Datum of 1985*

3.7 Limitations of the RMA2 Model of the St. Lawrence River

The RMA2 model is a free-surface hydrodynamic model, therefore, it cannot account for ice cover. During the months of January to March and early April, the Upper St. Lawrence River is covered by ice and as a result, cannot be simulated with the model. There are other hydraulic models available that can take into account ice formation and transport in the river (Shen, Su, and Liu 2000).

In addition to not being able to account for ice cover the RMA2 model of the St. Lawrence River, at this point, cannot take into account varying gate settings at the Iroquois Dam. The Iroquois dam is located in the village of Iroquois, roughly 40 km upstream of Cornwall. The Iroquois dam is used when the water level between the Iroquois Dam and the Moses Saunders hydroelectric dam, referred to as Lake St. Lawrence, is too high. The gates on the dam are lowered into the water, creating additional head loss at Iroquois and therefore lowering the water level in Lake St. Lawrence downstream. Normally, the Iroquois dam is not needed and all of the gates are fully out of the water. The other instance in which the Iroquois Dam is used is during ice formation. When ice is just about ready to form in the river, the gates are dipped to allow a stable ice cover to form in the section of river immediately downstream of the dam, adjacent to Ogden Island. Once the ice cover is formed back up to the dam, the gates are raised and water can pass underneath the ice.

The St. Lawrence River model as currently developed depicts the Iroquois dam as a series of 31 rectangular piers. The piers are constructed from very small elements and are treated as land/island by the model. In this configuration, it is not possible to simulate a "gates dipped" condition at the Iroquois Dam. In order to be able to simulate a "gates dipped" condition the model would have to be reconstructed by replacing the piers and openings at the dam with control structure elements and a rating curve. A model such as this was constructed and tested but it was found that the depiction of the Iroquois Dam as a control structure created numerical instabilities that proved difficult to overcome. At some point in the future, the model with control structures included will be revisited but for the present analysis the model without the Iroquois Dam explicitly being incorporated was utilized. Therefore, only time periods when the dam gates were not dropped were used calibrate the model and evaluate uncertainties. During these time periods the Iroquois dam will be most accurately depicted and represented by the model.

3.8 Chapter Summary

In this chapter the study area and regulatory framework for the Upper St. Lawrence River were introduced. The reason for development of a model of this portion of the St. Lawrence was explained and a justification for the selection of the RMA2 model was provided. The data requirements for the model and the sources used to obtain the data were then presented. The quality of the data was not mentioned in this chapter but will be discussed in Chapter four. The methods used to establish the model's un-measurable parameters, including the Manning's *n* and Eddy viscosity values were discussed and finally, two important limitations of the model were discussed.

Chapter 4 – Uncertainty Analysis

4.1 Chapter Introduction

The purpose of this chapter is to investigate the uncertainties inherent in the hydrodynamic model of the Upper St. Lawrence River. The investigation will include an estimate of the combined uncertainty in the hydrodynamic model outputs.

The source of the uncertainty can be attributed to different aspects of the modelling process. In this application, there are three broad potential sources of uncertainty: uncertainty from measured data, uncertainty from a model parameter derived through calibration and uncertainty from the model structure itself. These broad groupings were developed to organize the analysis.

The measured data used in the hydrodynamic model include the bathymetry, measured water levels used as boundary conditions and calibration data, the river discharge used as a boundary condition, wind and water temperature also potentially used as boundary conditions. The model parameters derived through calibration include the Manning's *n* and Eddy Viscosity parameters. The model structure refers to the simplifications of the model itself and the decisions made in hydrodynamic model development.

These potential sources of uncertainty will be examined in detail to physically quantify the uncertainty introduced into the hydrodynamic model. The sources will be examined individually and the uncertainty in each will be described qualitatively and if possible quantitatively. The origins of the uncertainty will also be described as well as any relevant literature. It will be evident once later sections of this chapter are read, that this is a very important step in the uncertainty analysis because a precise quantification of the uncertainty in the model outputs requires a precise quantification of the uncertainties in the inputs.

Once the sources have been identified and quantified, the combined uncertainty in the hydrodynamic model outputs will be calculated using the two different methods, the FOSM method and Monte Carlo Analysis. Each of the methods will be described in detail in the context of the uncertainty analysis of the St. Lawrence River hydrodynamic model. The results of the analyses will be compared. In addition to uncertainty analysis, sensitivity analysis will be completed to identify the largest sources of uncertainty in the model outputs. A local sensitivity analysis method will be used as a component of the FOSM method to decompose the uncertainty in the model outputs into its contributions from the model inputs.

4.2 Measurable Data Uncertainties

4.2.1 Bathymetry

Bathymetric data contains two independent measurements, the horizontal position (x-y) and the underwater water depth position or vertical position (z). The two measurements are conducted simultaneously and are subject to independent uncertainty. Horizontal position was historically established through visual observation but is now normally obtained through electronic means using satellite based, global positioning system technology. The vertical positioning of the bottom elevation is measured relative to the position of the boat performing the survey historically using mechanical means but now normally using acoustic technologies. The vertical position of the boat is established relative to the water surface elevation which is also subject to uncertainty. Bathymetric uncertainty is then a function of the combined uncertainty in the two independent measurements used to collect the data. The quantification of the uncertainty in bathymetric data is therefore challenging. The accuracy of a survey point can be represented as an ellipsoid as shown in Figure 9.

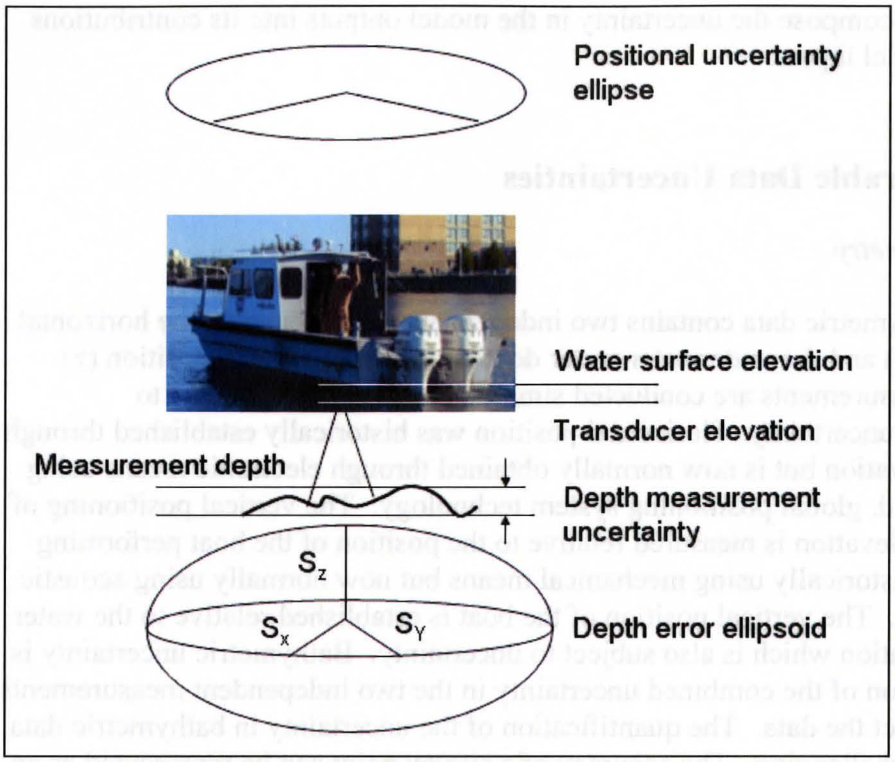

Figure 9 – Three-Dimensional Uncertainty of a Measured Depth.

Given the sources for uncertainty in bathymetric data and hydrographic charting, the agencies responsible for the collection of bathymetric data have developed standards to which the collection of bathymetric data must adhere to. These standards provide a means to estimating the uncertainty in bathymetric data. In Canada, the standard for accuracy in data collection is published by the Canadian Hydrographic Service.

The uncertainty in bathymetry is also dependent on the currency of the data. This is due to the nature of river systems and the erosion/deposition processes that occur. Depending on the nature of the river geomorphology bathymetry may only be valid for a short period in time in an erosion/deposition area or it could be accurate for a long period of time in an erosion resistant area. The other reason the currency of the data is important is that the standards used in the past were less stringent than what is used today due to evolving measurement and plotting technologies. The United States Army Corps of Engineers (USACE) estimates the uncertainty in bathymetric survey data collected for the late 1800s and early 1900s at +/- 3 to 4 feet (0.9 to 1.2 m) (Byrnes et al. 2002). For data collected during the mid 1900s the uncertainties were +/- 2 to 3 feet (0.6 to 0.9 m). Using current USACE and National Ocean Service (a component of NOAA) standards uncertainties are estimated to be approximately +/- 0.5 to 1 ft (0.15 to 0.3 m) for surveys conducted in less than 100 ft (30 m) of water.

Bathymetry data for the model of the Upper St. Lawrence River was obtained from a variety of sources as previously described. The bathymetry data was collected during numerous surveys conducted from 1936 to the late 1980's by NOAA, USACE and CHS. We can estimate the uncertainty for each bathymetry sounding to be +/- 2 to 3 feet (0.6 to 0.9 m) from the work of Byrnes, Baker and Li (2002). One would expect the error to be greater for deeper sections of the river and smaller for shallower sections of the river. If the soundings were to have been collected using modern technology and standards the uncertainties would range from +/- 0.5 feet (0.15m) to more than one foot (0.3 m) since the depth of the St. Lawrence River ranges from 0 to 220 feet (67 metres) but averages approximately 30 feet (9.1 metres). There were more than 600,000 soundings gathered and utilized to build the model.

The uncertainty in bathymetry can be considered to be random over a large geographical area unless the cause for the uncertainty is systematic in nature, in which case it could be attributed to a specific region or transect within the dataset. The agencies responsible for the collection of this data have stringent quality control and quality assurance procedures that are completed following the completion of a hydrographic survey. It is reasonable to assume that there would not be any systematic error within the data.

To import the bathymetry into the model, the individual soundings were linearly interpolated onto the finite element grid nodes as shown in Figure 10. The interpolation is necessary to reduce the number of soundings to a reasonable number for computation. The interpolation of the soundings reduces the amount of uncertainty in the bathymetry in the model. The uncertainty in the bathymetry is assumed to be negligible due to the fact that the random uncertainty associated with the bathymetry points would be cancelled through the process of interpolation. This would not be true if the uncertainty in the data were due to a systematic cause.

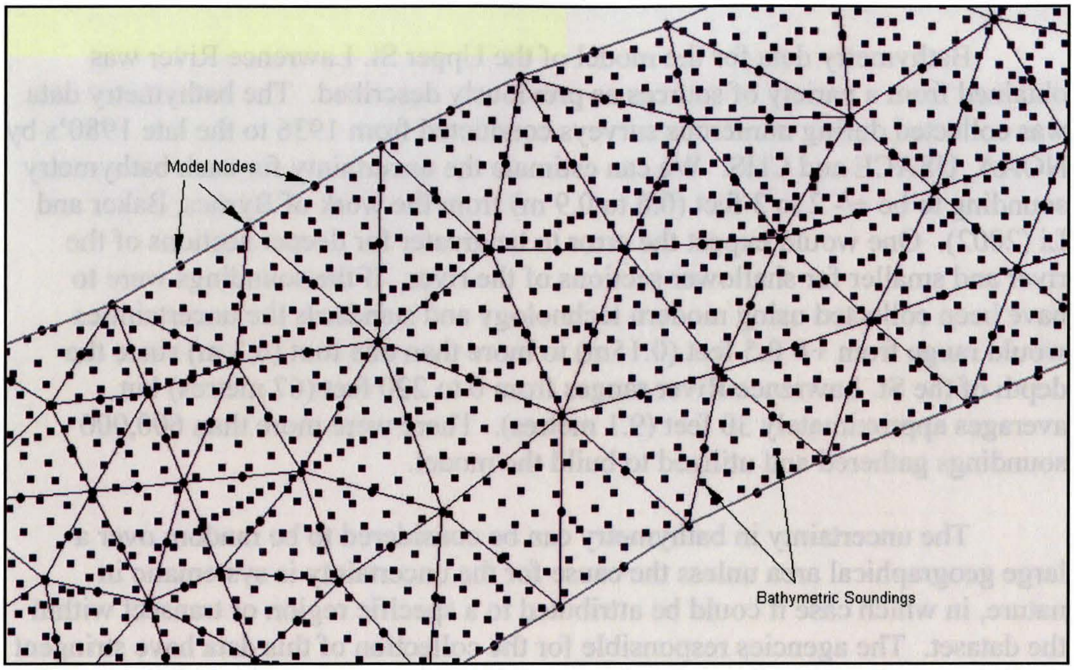

Figure 10 – Visualization of the Interpolation of Bathymetric Soundings to Model Nodes

4.2.2 Water Levels

Water levels are a crucial type of data required for hydrodynamic modelling. Water levels are used as boundary conditions and also as observation data in the calibration of a model. Water levels are collected at 10 locations within the river by the CHS, NOAA and the power entities. Although all locations are used at some point in this analysis, the uncertainty analysis of the model is focussed on the water levels that are used as boundary conditions for the model.

The Upper St. Lawrence River model's upstream boundary conditions are the water levels at the Kingston gauge on the Canadian Side of the river and the Cape Vincent gauge on the U.S. side of the river. The water levels at these two gauges are normally very close to each other except during times when there are high winds inducing as much as 10 centimetres difference between the two gauges. However, those instances are rare and generally the water levels are equal.

The water levels measured at these gauges are considered to be of good quality due to the rigorous procedures followed by the agencies to establish and construct the gauge site, operate the gauge site, archive, quality check, and disseminate the data. However, the data is still subject to a degree of uncertainty originating from levelling and benchmarking uncertainty, ongoing influence of glacial isostatic rebound, finite instrument resolution, and potential recording errors.

Uncertainty is also an issue due to the sampling of data. Water level data for the Kingston gauge from the CHS is recorded instantaneously at the gauge site at three-minute intervals and then archived and made available on the web. Hourly data is also collected instantaneously at the gauge. CHS hourly data is not an average of the three-minute data, it is an instantaneous observation made on the hour. The Cape Vincent gauge is operated by NOAA and they utilize a slightly different measuring strategy. The data for the Cape Vincent gauge is recorded at 6 minute intervals. The six-minute data is an average of 181 one-second water level samples centred on each tenth of an hour. NOAA makes the six-minute data available on the web. In addition they also measure hourly interval data. The hourly data is again not an average of the six-minute observations but it is an observation made on the hour. The hourly observation is made by averaging 181 one-second water level samples centred on the hour. The hourly data does not capture the variability in water levels on a sub-hourly time scale. If a longer time step of data is required, daily for example, the hourly or sub-hourly data must be processed to obtain data on this time step. This processing of data creates a degree of uncertainty that is exacerbated when the data is put into the model. Typically, the model is run in unsteady-state mode using an hourly time step. But if a longer period simulation or a steady-state simulation is desired, then additional statistical techniques must be applied to the data and uncertainties may result.

To estimate the uncertainty, it may be considered as either, static or dynamic. The static component represents the uncertainty caused by levelling and the determination of the absolute datum of the gauge. The dynamic component refers to the uncertainty resulting from the variability in water levels over a short time period. Water level data is processed by the collecting agencies at hourly and daily intervals. This processing of data would introduce dynamic uncertainty into the data. An hourly water level measurement would contain dynamic uncertainty due to the original shorter time step used for data collection. For the current problem, the dynamic component is not considered because the uncertainty introduced due to temporal variations is excluded from the analysis. This thesis assumes that the river is in steady state and the input data can be obtained such that all dynamic or temporal sources of uncertainty are eliminated. The static component of uncertainty in water levels caused by levelling and datum uncertainty is estimated at +/- 1 centimetre. This estimate is reasonable considering the work of Holtschlag and Koschik (2001) in the report describing the calibration of a model of the St. Clair and Detroit River in the Great Lakes Basin. In that report, the author estimated the static component of uncertainty at water level gauges as +/- 0.6 centimetres. NOAA estimates the uncertainty in instantaneous water levels at its Great Lakes water level gauges as +/- 0.6

centimetres (Jeff Oyler, United States National Oceanic and Atmospheric Administration, personal communication, November 16, 2005). Neff and Nicholas (2005) report that the uncertainty in individual Great Lakes gauge measurements is between 0.5 and 1.0 centimetres.

4.2.3 River Discharge

The discharge in the Upper St. Lawrence River is controlled at Cornwall/Massena through the use of three structures. The main control is the Moses-Saunders hydroelectric dam. The dam is roughly 900 metres long and contains 32 turbines, 16 on the Canadian side of the dam and 16 on the U.S. side. Flow passes through the dam through its turbines. The capacity of the dam is roughly 9400 m^3/s and when the supply exceeds that amount flow can pass through the Longsault Dam. Normally, the dam at Longsault is not used. A negligible amount of flow passes through the Eisenhower locks to provide for commercial navigation around the dam. A map of the area is shown in Figure 2 in Chapter three.

The discharge through the power plant is the sum of the discharges through the individual turbines. The discharge through the turbines is established through performance testing conducted after the turbines are initially installed, after turbine upgrades, and at other times periodically to monitor for deterioration. Turbine performance testing involves the measurement of specific hydraulic energy, power output, turbine flow and other quantities. From these measurements rating tables are established to relate the power output at the generating unit attached to the turbine to the amount of flow through the turbine as a function of available specific hydraulic energy which is the head difference between upstream and downstream head ponds. These rating tables are used by the power entities to compute the flow through the turbines and report it to the regulating agencies. The performance testing is witnessed and approved by the regulating agencies including Environment Canada and the United States Army Corps of Engineers.

During the performance testing, turbine flow is measured using the velocity area method in the pen stock and at the intake. The intake is located at the headwater pond and the pen stock is the concrete tube that the water flows through from the intake to the turbine. This method involves the division of the pen stock or intake to be measured into small panels. A frame supporting several 10-15 current metres is lowered into the penstock and the velocities in each of the small panels are measured as shown in Figure 11. The velocities are multiplied by the cross sectional area of the panel to obtain the discharge Q. Velocities are measured at 200-300 locations panels within the measurement section and the sum of the discharges through all of the panels is the turbine flow. The overall uncertainty in the flow measured during the performance testing is estimated at +/- 1.7 % at the 95 % confidence level using the velocity area method (Ontario Power Generation 2003). This published uncertainty is the basis for the estimate of uncertainty in the river discharge.

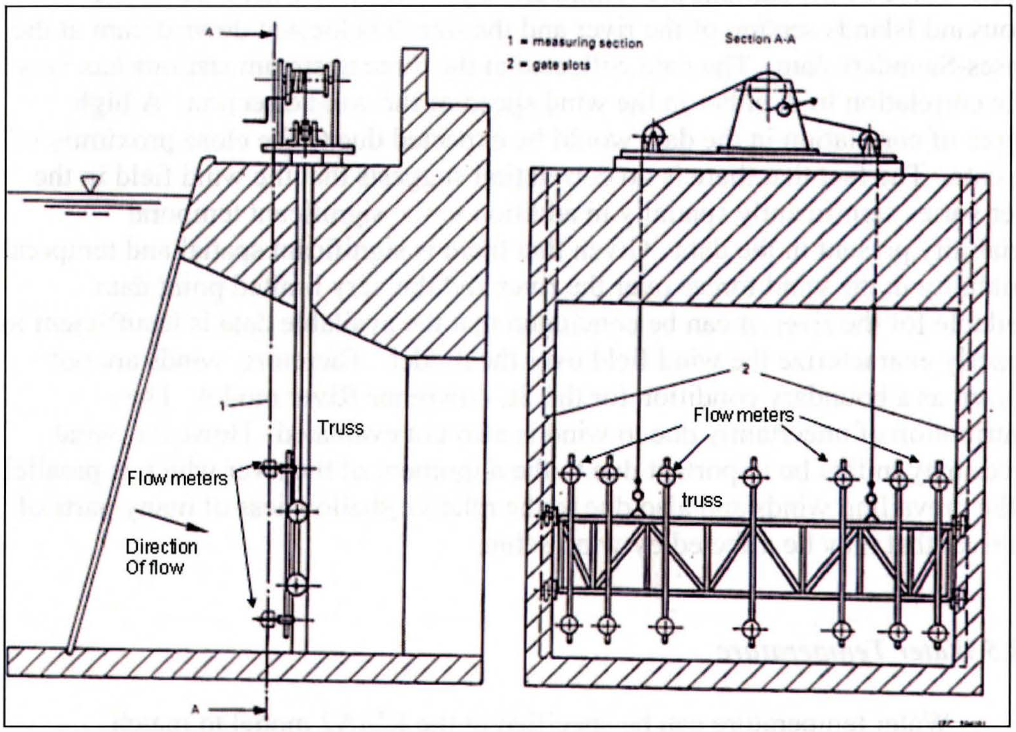

Figure 11 – Velocity/Area Apparatus Used to Measure Flow at the Moses-Saunders Power Plant, With Permission, Ontario Power Generation, 2006.

4.2.4 Wind

The effect of the wind force on the river is not evaluated in the uncertainty and sensitivity analysis of model outputs. This is because there is insufficient information to describe the uncertainty in wind data available for the St. Lawrence River. The agencies that collect the wind data, Environment Canada, the National Weather Service and Ontario Power Generation do not have estimates of the uncertainty in their wind data.

Wind is generally not used in RMA2 models because wind forces are three-dimensional in nature and are therefore de-emphasized in two-dimensional modelling (Donnell et al. 2005). In addition to this fact, wind forces are not specified as a boundary condition with the St. Lawrence River model because the wind data available for the river is insufficient and inconsistent to meet the requirements of the RMA2 model.

The RMA2 model requires the wind stress be applied over the entire surface of the model or individual sections of the model. The wind data for the St. Lawrence River is point data, collected at four wind monitoring stations on the River. Three of the stations are within close proximity of each other in the Thousand Islands section of the river and the fourth is located downstream at the Moses-Saunders dam. The data collected at the three upstream stations has very little correlation in it, either in the wind speed or the wind direction. A high degree of correlation in the data would be expected due to the close proximity of the data. The fact that there is no correlation suggests that the wind field in the river varies significantly spatially in addition to the significant temporal variability present in the data. Given that there is significant spatial and temporal variability in the wind forces over the river and the very limited point data available for the river, it can be concluded that the available data is insufficient to properly characterize the wind field over the model. Therefore, winds are not utilized as a boundary condition for the St. Lawrence River model. The contribution of uncertainty due to wind is also not evaluated. However, wind forces may in fact be important due to the alignment of the river which is parallel to the prevailing winds and also due to the relative shallowness of many parts of the river that may be affected by wind setup.

4.2.5 Water Temperature

Water temperature can be specified in the RMA2 model to match conditions observed in the field. The physical characteristics of water including its density and viscosity are influenced by its temperature. Water temperature data for the river was obtained from NOAA weather buoys.

NOAA estimates the uncertainty in instantaneous water temperature measurements at +/- 0.2 Degrees Celsius (Jeff Oyler, United States National Oceanic and Atmospheric Association, personal communication, November 16, 2005). However, the uncertainty in water temperature data for the St. Lawrence River would be much larger than this quoted uncertainty because the measurements are unrepresentative of the spatial variation in temperature throughout the river. The water temperature in the river varies along its length. The river water temperature is related to the Lake Ontario temperature and is either heated or cooled by the air temperature and solar radiation as it travels down the river (Foltyn and Shen 1986). Water temperatures in the river are measured at a few locations and are surface water temperatures.

The RMA2 model does not allow for spatial variation in water temperature and can only handle one value for the water temperature. A representative average temperature for the river is not trivial to compute given the spatial variation in temperature over the surface of the river and throughout the water column. In running the model, the water temperature is estimated by obtaining data from one of the weather buoys and that measurement is treated as representative of the average temperature in the entire river. From an analysis of water temperature data in the river the temperature can vary by as much as five degrees Celsius from Lake Ontario to the downstream end of the study area at Cornwall. As a conservative estimate, the uncertainty in water temperature was set at +/- five degrees Celsius. The importance of water temperature will be explored in the sensitivity analysis portion of this thesis and the estimate of the uncertainty will be given additional attention if it is warranted.

4.3 Un-measurable Data Uncertainties

4.3.1 Manning's n

The Manning's *n* coefficient is used by RMA2 to calculate the amount of bed friction (bottom shear stress) that is applied to the flow. Changing the bed friction provides some control of the flow velocity and direction. Manning's *n* cannot be measured definitively and in this problem, values for Manning's *n* were determined through the calibration process. The values of Manning's *n* for the eight material zones within the model were optimized so that the computed water levels matched the observed water levels to the highest degree possible. The optimized Manning's *n* values are assumed to be the mean values for the purposes of uncertainty and sensitivity analysis.

The variance in Manning's *n* value was estimated using a reference that investigated the uncertainty in hydraulic parameters (Johnson 1996). The author conducted a literature search for coefficients of variation and distribution functions for hydraulic parameters including Manning's *n*. The author also conducted experiments to obtain uncertainty data for hydraulic variables and made field observations to obtain information on uncertainty. From this work, the author concluded that coefficients of variation for Manning's *n* can range from 0.1-0.3 and the distributions that can be assumed are the normal, triangular, or lognormal distributions.

The fact that in this study, the Manning's *n* values were established through calibration and not through field measurements or observations makes the estimation of the variance more difficult. It is reasonable to assume that the calibration process reduces the level of uncertainty in the Manning's *n* values because the calibrated parameters are validated by comparing observed and computed water levels. The Manning's *n* values determined from the calibration process result in a model that adequately reproduces the water levels in the river given known boundary conditions. However, there is still some level of doubt in the values of the Manning's *n* estimates. The Manning's *n* values were determined through an optimization process that aims to minimize the sum of squared residuals between computed and observed water levels within the model domain. The optimal set of parameters was determined by selecting the calibration run that had the smallest sum of squared residuals. Still, there were other calibration runs that resulted in alternate Manning's *n* parameterizations with only modestly higher sum of squared residuals. These alternate parameterizations may indeed be valid but were not the optimal solution. A coefficient of variation of 0.1 would account for the alternate Manning's *n* values that are possible.

To explore the impact of this assumption, a second estimate of the uncertainty in Manning's *n* was investigated to evaluate the influence on model output uncertainty. The second estimate was made under the assumption that no calibration was performed and the Manning's *n* values were simply estimated from literature. In this case, it is reasonable to assume the coefficient of variation would be larger than that of Manning's *n* determined through calibration. The higher coefficient of variation of 0.3 suggested by Johnson (1996) would be reasonable.

For the first case, the uncertainty in Manning's *n* is assumed to be normally distributed about the mean value determined through calibration. The mean Manning's *n* values and the resulting standard deviations obtained using a coefficient of variation of 0.1 is shown in Table 2. For the second case, the uncertainty in Manning's *n* is again assumed to be normally distributed about the mean value but the uncertainty is estimated using the coefficient of variation of 0.3.

Table 2 – Estimate of Uncertainty in Manning's *n*

Parameter	Mean	Standard Deviation
Material Zone 1 Manning's *n* value	0.0309	.0031
Material Zone 2 Manning's *n* value	0.0288	.0028
Material Zone 3 Manning's *n* value	0.0274	.0027
Material Zone 4 Manning's *n* value	0.0288	.0028
Material Zone 5 Manning's *n* value	0.0270	.0027
Material Zone 6 Manning's *n* value	0.0276	.0028
Material Zone 7 Manning's *n* value	0.0326	.0033
Material Zone 8 Manning's *n* value	0.0308	.0031

4.3.2 Eddy Viscosity

The Eddy Viscosity is used to represent the strength of turbulence within the flow domain. For the St. Lawrence River model, the Eddy Viscosity was assigned dynamically to the model using a Peclet Number. The value of the Peclet Number was established through the calibration process simultaneously with the Manning's *n* values.

There is no literature discussing the uncertainty in Eddy Viscosity values for the RMA2 model. In this thesis, the Eddy Viscosity is assumed to have a level of uncertainty similar to that of the Manning's *n*, using a coefficient of variation of 0.1 of the Peclet number. The mean Peclet number was determined through calibration to be 17 resulting in a standard deviation of 1.7. The distribution of Peclet number was assumed to be normal. There is little concrete information to base these assumptions on. The importance of the Peclet number on the uncertainty in hydrodynamic model outputs will be assessed in the sensitivity analysis described later in this chapter. Should the Peclet number turn out to be an important source of uncertainty in the hydrodynamic model outputs, additional effort will need to be made to refine the estimate of the uncertainty.

One consideration that was important is that the model stability is sensitive to the Peclet number. If the Peclet number is higher than 23 or lower than 13, the model will not consistently reach convergence. The coefficient of variation for the Peclet number was checked to ensure that the range of Peclet numbers resulting from the distribution was within this range.

4.4 Model Uncertainty

The uncertainties in the model inputs have been defined to the greatest extent possible. Before evaluating the combined effect of those uncertainties in the model, the uncertainty that originates within the model itself must be identified. A model is an abstraction from reality. It is a simplification of the actual physical processes involved in the system through the representation of key processes with mathematical equations. In this problem, the complex hydraulics of the St. Lawrence River are represented with a depth averaged, hydrodynamic model. The model of the St. Lawrence is, by necessity, a simplification of the actual river. The RMA2 model is a two-dimensional, depth-averaged model, meaning that the velocities are averaged over the water column. This averaging introduces uncertainty. However, the RMA2 model has been applied widely in water resources engineering by governments, consultants and academia (Donnell et al. 2005). The many successful applications of RMA2 prove that the model is reliable in reproducing the basic hydrodynamics of shallow water systems.

In the RMA2 model, the geometry of the river is represented using a finite number of elements and nodes. The nodes define the boundaries of the elements. The elements in the St. Lawrence River model range in size from five metres by five metres in the vicinity of the Iroquois Dam to over 150 metres by 150 metres in the Upper River near Lake Ontario. The processes that occur in the river within a scale smaller than the size of the elements are not explicitly accounted for and introduce uncertainty. The layout of the model grid was developed over time through the trial and error process. In general, a model should be constructed with the highest number of elements and nodes possible to obtain the best representation of the river. However, as the number of elements and nodes increases, so does the required computation time. The size of the data files generated by the model also increases as the number of elements and nodes increase. The spacing of the nodes determines the overall number of elements and nodes in the model. Figure 12 shows the relationship between nodal spacing and the resulting number of elements and nodes that result in the model grid.

The model grid used in this analysis, is of sufficient resolution that increasing the number of elements and nodes any further will not create any better results than are achieved with this mesh. This conclusion is based on three attempts at calibration of the model. Using the same calibration approach as described in section 3.7.4 of this thesis, with higher resolutions, models did not reduce the sum of squared residuals objective function over that achieved with the current mesh.

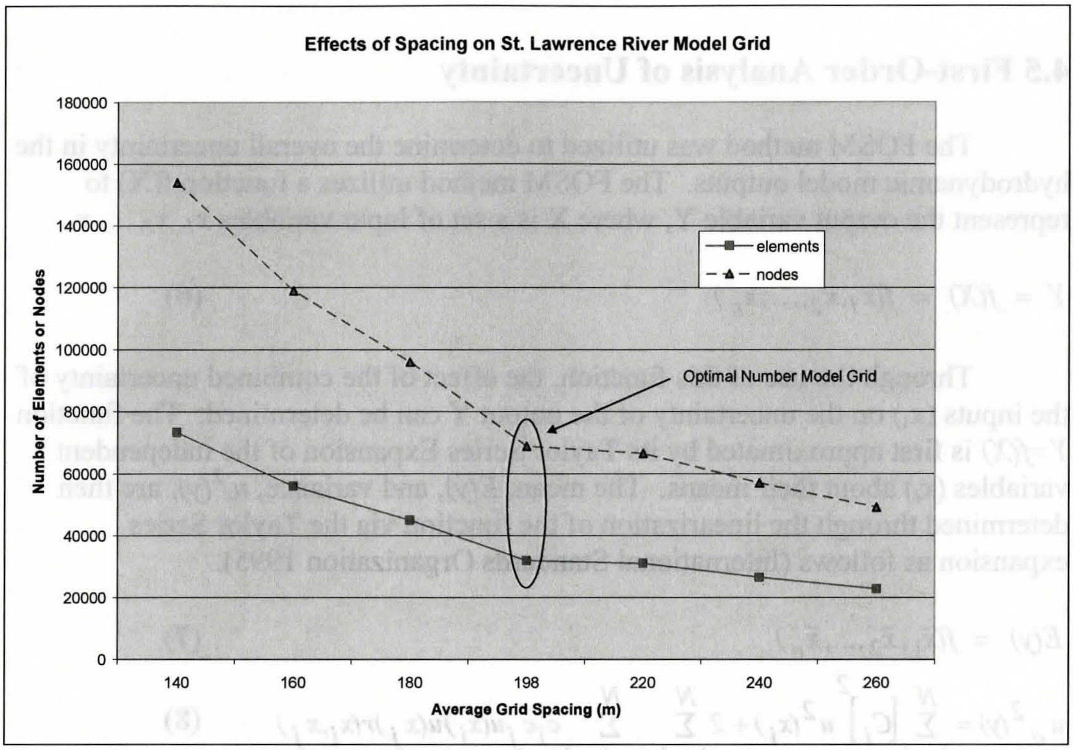

Figure 12 – Grid Spacing vs Number of Element and Nodes

Lastly, the calculations performed by the RMA2 model are of an iterative nature and continue until the computed water levels and velocities do not change from one iteration to the next within a user specified convergence criterion. The convergence criterion used in the St. Lawrence River model was very stringent. The computed water level from one iteration to the next at any node in the model could not change by more than 0.001 metres or else the calculations continued. This level of convergence ensures that the hydrodynamic model provides outputs precise enough to perform the uncertainty analysis with. Since the RMA2 model of the St. Lawrence can perform calculations to this degree of precision, the uncertainty in the model itself may be considered negligible.

All of these considerations regarding model selection, development and execution aim to minimize the uncertainty in the model structure itself. Regardless, model uncertainty is present and recognized. However, it cannot be explicitly quantified for the purposes of calculating the uncertainty in the hydrodynamic modelling outputs that will be performed in the following sections of this chapter.

4.5 First-Order Analysis of Uncertainty

The FOSM method was utilized to determine the overall uncertainty in the hydrodynamic model outputs. The FOSM method utilizes a function f(X) to represent the output variable Y, where X is a set of input variables $x_1, x_2, \ldots, x_n$.

$$Y = f(X) = f(x_1, x_2, \ldots, x_n) \tag{6}$$

Through the use of this function, the effect of the combined uncertainty of the inputs (x_i) on the uncertainty of the output Y can be determined. The function $Y=f(X)$ is first approximated by its Taylor Series Expansion of the independent variables (x_i) about their means. The mean, $E(y)$, and variance, $u_c^2(y)$, are then determined through the linearization of the function via the Taylor Series expansion as follows (International Standards Organization 1995).

$$E(y) = f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \tag{7}$$

$$u_c^2(y) = \sum_{i=1}^{N} \left[C_i\right]^2 u^2(x_i) + 2\sum_{i=1}^{N} \sum_{j=i+1}^{N} c_i c_j u(x_i) u(x_j) r(x_i, x_j) \tag{8}$$

$$C_i = \left[\frac{\partial f}{\partial x_i}\right] \tag{9}$$

where C_i, are the sensitivity coefficients, $x_1, x_2, \ldots, x_n$ are the input variables, and $u^2(x_i)$ are the variances of the individual input variables, and $r(x_i, x_j)$ are the correlation coefficients between pairs of input variables. The sensitivity coefficients are partial derivatives, evaluated at the mean values of x_i. They describe how the output Y varies with changes in x_i or the effect of a very small change in x_i on the output Y. The variances of the input variables (parameters in this case) were discussed in section 4.2. The combined effect of the sensitivity coefficients and the variances in the inputs plus the influence of parameter correlation then determine the uncertainty, u_c^2, in the output.

If the input variables are uncorrelated, i.e., $r(x_i, x_j) = 0$, equation 8 is simplified to:

$$u_c^2(y) = \sum_{i=1}^{N} \left[C_i\right]^2 u^2(x_i) \tag{8}$$

In this application, the RMA2 model is described by a function $r_i = f(S)$, where r_i is the model output $r_i = (r_1, r_2, r_3)$ for location i within the model grid and S are the model inputs $S = (s_1, s_2, \ldots, s_n)$. The model outputs r_1, r_2 and r_3

correspond to the water level, *x* component of the depth averaged velocity, and *y* component of the depth averaged velocity respectively at one point of interest (node) within the model domain. The locations *i* are defined as individual nodes within the model grid. The model inputs S correspond to the upstream boundary condition (water level *Wl)*, the Peclet number *PE*, the eight Manning's *n* values n_i, and the downstream boundary condition (river discharge *Q)*. The influence of wind forces are not considered in this analysis for the reasons presented in 4.2.4. Equations 11, 12, and 13 describe the formulation for the computed water level, x component of the depth averaged velocity, and the y component of the depth averaged velocity respectively:

$$Wl_i = f(S) = f(Wl, PE, n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, Q, Wt) \qquad (11)$$

$$XVel_i = f(S) = f(Wl, PE, n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, Q, Wt) \qquad (12)$$

$$YVel_i = f(S) = f(Wl, PE, n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, Q, Wt) \qquad (13)$$

The expected value, or mean value for the outputs is determined by evaluating the function *f(S)* about the mean values of the input parameters *S.*

$$E[Wl_i] = E[f(S)] = f(\overline{Wl}, \overline{PE}, \overline{n_1}, \overline{n_2}, \overline{n_3}, \overline{n_4}, \overline{n_5}, \overline{n_6}, \overline{n_7}, \overline{n_8}, \overline{Q}, \overline{Wt}) \qquad (14)$$

$$E[XVel_i] = E[f(S)] = f(\overline{Wl}, \overline{PE}, \overline{n_1}, \overline{n_2}, \overline{n_3}, \overline{n_4}, \overline{n_5}, \overline{n_6}, \overline{n_7}, \overline{n_8}, \overline{Q}, \overline{Wt}) \qquad (15)$$

$$E[YVel_i] = E[f(S)] = f(\overline{Wl}, \overline{PE}, \overline{n_1}, \overline{n_2}, \overline{n_3}, \overline{n_4}, \overline{n_5}, \overline{n_6}, \overline{n_7}, \overline{n_8}, \overline{Q}, \overline{Wt}) \qquad (16)$$

Equations 17 through 19 were utilized to calculate the variances in the outputs.

$$Var(Wl_i) = u_c^2 Wl = c_{Wli}^2 u_{Wl}^2 + c_{PEi}^2 u_{PE}^2 + \sum_{j=1}^{8} c_{nj}^2 u_{nj}^2 + c_{Qi}^2 u_Q^2 + c_{Wti}^2 u_{Wt}^2$$
$$+ 2\sum_{j=1}^{12} \sum_{k=j+1}^{12} c_j c_k u(s_j) u(s_k) r(s_j, s_k)$$
$$(17)$$

$$Var(XVel) = u_c^{\,2} XVel = c_{Wl_i}^{\,2} u_{Wl}^{\,2} + c_{PEi}^{\,2} u_{PE}^{\,2} + \sum_{j=1}^{8} c_{nj}^{\,2} u_{nj}^{\,2} + c_{Qi}^{\,2} u_{Q}^{\,2} + c_{Wti}^{\,2} u_{Wt}^{\,2}$$
$$+ 2\sum_{j=1}^{12} \sum_{k=j+1}^{12} c_j c_k u(s_j) u(s_k) r(s_j, s_k) \tag{18}$$

$$Var(Yvel) = u_c^{\,2} Wl = c_{Wl_i}^{\,2} u_{Wl}^{\,2} + c_{PEi}^{\,2} u_{PE}^{\,2} + \sum_{j=1}^{8} c_{nj}^{\,2} u_{nj}^{\,2} + c_{Qi}^{\,2} u_{Q}^{\,2} + c_{Wti}^{\,2} u_{Wt}^{\,2}$$
$$+ 2\sum_{j=1}^{12} \sum_{k=j+1}^{12} c_j c_k u(s_j) u(s_k) r(s_j, s_k) \tag{19}$$

where, j = 1 to 12 are the input parameters (*Wl, PE,* n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, *Q, Wt*).

Unique sensitivity coefficients were calculated for each parameter for the computed water level, x-velocity component and y-velocity component. For example, the sensitivity coefficient for the computed water level due to the influence of Manning's *n* zone one is different than the sensitivity coefficient for the y-velocity component due to the influence of Manning's *n* zone one. The term $r(s_i, s_j)$ refers to the correlation between parameters s_i and s_j. Correlation between individual Manning's *n* zones, the Eddy Viscosity value for the model, and the boundary condition inputs was taken to be zero. The basis for this assumption is discussed in the next few paragraphs.

Individual Manning's *n* values for specific reaches or zones in the model express the resistance to flow due to bottom friction in those specific reaches or zones. Physically, the resistance is a function of the type and size of the material on the bottom of the river and the presence of vegetation. The Manning's *n* values for each individual zone vary according to the type of material present in the zone itself and would not be related to the composition of the bed material in another zone. For this reason, the Manning's *n* zones are assumed to be independent from each other and the correlation is zero as a result. The Eddy Viscosity for the river is an expression of the resistance to flow resulting from turbulence in the river. Turbulence is caused by temporal variations in velocity at a very small time step and also by momentum exchange between adjacent regions possessing varying velocity. In the formation of RMA2 for this application, turbulence is not related to the value of the Manning's *n* in a particular zone in the river and is therefore treated as independent or uncorrelated.

The correlation coefficients for the boundary condition water level and the river discharge Q were not calculated and are assumed to be zero. This assumption is reasonable considering the inputs themselves and the purpose of the uncertainty analysis.

Over a long time step, daily or greater, the water level and discharge are highly correlated as the river discharge must increase or decrease in response to a rising or falling Lake Ontario water level. However, over a short time step, hourly or less, the water level and river discharge are not correlated. The river discharge is determined to meet hydroelectric generation needs over the course of each day and can vary by +/- 850 m^3/s over the course of a day while the Lake Ontario water level remains constant or very close to constant. The upstream water level and river discharge are clearly uncorrelated over a time period less than one day. This analysis is utilizing a steady-state application of the model to evaluate the uncertainty and sensitivity of the model outputs. A steady-state application of the model does not take into account any temporal variation in water levels or discharge. A steady-state application of the model can be executed for any reasonable combination of Lake Ontario water levels and downstream discharge. There need only be a small degree of correlation between Lake Ontario water level and river discharge to ensure the discharge is reasonable for the Lake Ontario water level supplied.

The uncertainty and sensitivity analysis conducted in this thesis is investigating the influence of the uncertainty in the upstream boundary condition (water level W) and the downstream boundary condition (river discharge Q). The uncertainty in each quantity is not correlated with the uncertainty in the other quantity. Therefore, the correlation between the upstream and downstream boundary conditions is considered to be negligible for the purposes of this investigation.

In order to utilize equations 14 through 19 to calculate the expected value and variances for the model, the problem had to be simplified. The RMA2 model of the St. Lawrence River calculates values for equations 11, 12, and 13 at 86667 nodes within the model domain. It is obvious that there are far too many nodes to analyze completely, so to evaluate the uncertainty in the model, three locations were selected. The locations were selected because they are areas of interest within the river. The three locations were at the upstream and downstream approaches to the Iroquois locks and the Saunders headwater gauge. Aerial views of the locations are shown in Figures 13 and 15 and views of the RMA2 grid in these locations are shown in Figures 14 and 16. The approaches to the Iroquois locks are of interest because the currents in the area can make it difficult for commercial ships to enter and exit the locks, especially during high flow conditions. The Saunders headwater gauge is the furthest location from the upstream boundary where the water level is specified. Consequently, the water level at this location is expected to have the greatest uncertainty. The water level at this location is a key indicator of how well the model is performing overall. At these three locations, the sensitivity coefficients and then the uncertainty in the computed water level and velocity were determined. The velocity is broken down into its two horizontal components in the X and Y directions. The velocity magnitude can be easily computed from these two components.

Figure 13 – Aerial View of the Iroquois Dam and Lock Area

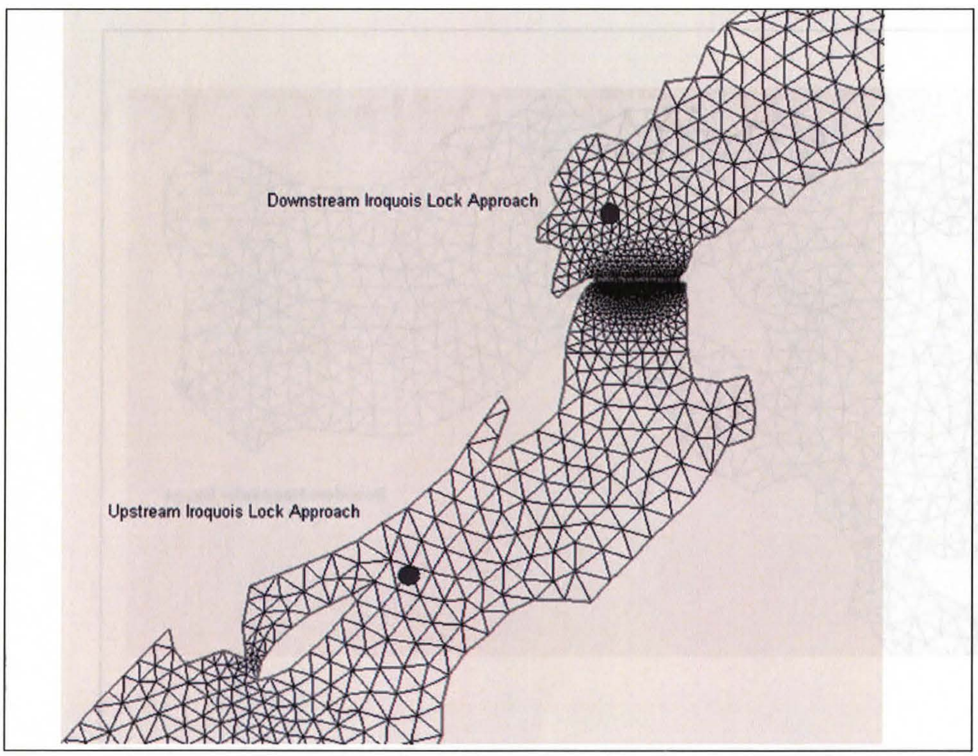

Figure 14 – RMA2 Model Grid of the Iroquois Dam and Lock Area

Figure 15 – Aerial View of the Moses-Saunders Dam Area

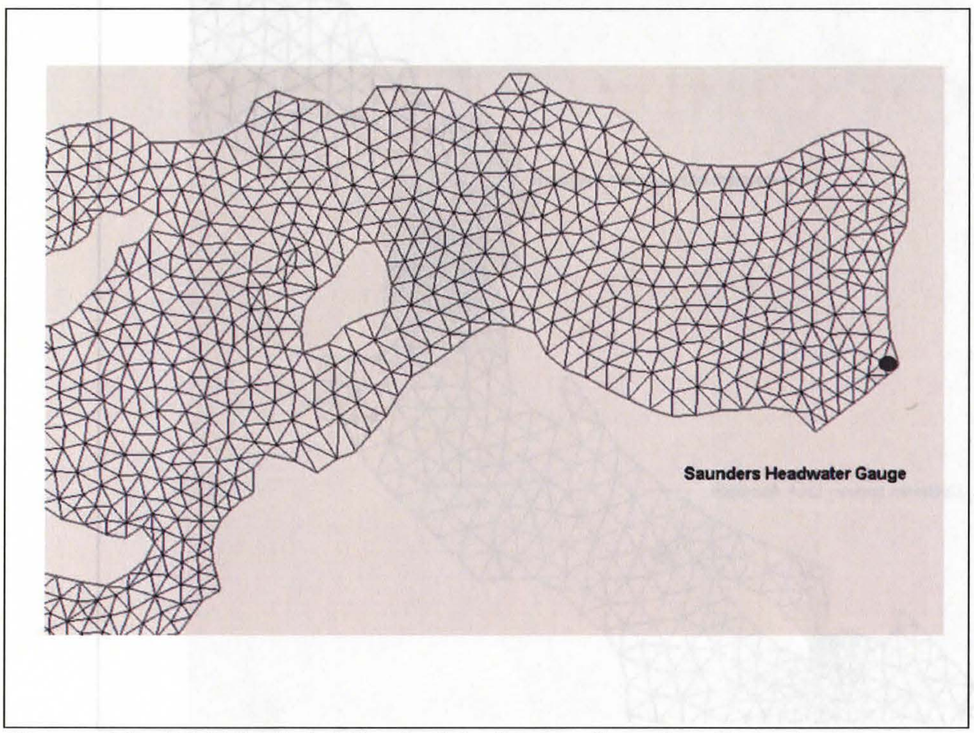

Figure 16 – RMA2 Model Grid of the Moses-Saunders Dam Area

4.5.1 Sensitivity Coefficient Calculations

The sensitivity coefficients (partial derivatives) for all parameters were evaluated using the central differencing technique. The model inputs were each perturbed in turn by an amount Δ to calculate the model outputs necessary to calculate the sensitivity coefficients. While each model input was perturbed the other inputs were held at their mean values. The value of Δ used in this thesis was equal to one standard deviation of the model input.

The size of Δ was found to be insignificant, with one exception, that is, Δ had to be large enough to cause a change in the computed output value. If the value of Δ is too small, the model output does not change leading to the calculation of a false sensitivity coefficient equal to zero. The value of Δ cannot be too large as long as the model input is still reasonable because the sensitivity coefficient is divided by Δ. The same sensitivity coefficients were obtained if the perturbation was set at one or two standard deviations.

$$C_{Wl} = \frac{\partial Wl_i}{\partial x_j} = \frac{Wl(x_j + \Delta x_j) - Wl(x_j - \Delta x_j)}{(x_j + \Delta x_j) - (x_j - \Delta x_j)} \tag{20}$$

$$C_{Xvel} = \frac{\partial XVel_i}{\partial x_j} = \frac{XVel(x_j + \Delta x_j) - XVel(x_j - \Delta x_j)}{(x_j + \Delta x_j) - (x_j - \Delta x_j)} \tag{21}$$

$$C_{Yvel} = \frac{\partial YVel_i}{\partial x_j} = \frac{YVel(x_j + \Delta x_j) - YVel(x_j - \Delta x_j)}{(x_j + \Delta x_j) - (x_j - \Delta x_j)} \tag{22}$$

The calculated sensitivity coefficients are shown in Table 3. There are three observations to be made from the sensitivity coefficients. The first relates to the sign of the sensitivity coefficient. If the sensitivity coefficient is positive that means the computed value, either a water level or velocity, changes in the same direction as the model input. For example, consider the sensitivity coefficient for the computed water level with respect to the boundary condition water level. As the boundary condition water level (at the upstream end of the model) increases so do all computed water levels at other locations in the river. Therefore, the sensitivity coefficient is positive. For another example, consider the sensitivity coefficient for the computed water level at the Saunders headwater gauge with respect to any of the Manning's *n* coefficients. All of these sensitivity coefficients are negative. This makes sense because as the Manning's *n* value increases, so does the head loss in the river, resulting in a lower water level at the Saunders headwater gauge (at the downstream end of the model). Similar statements can be made about the other sensitivity coefficients.

Table 3 – Summary of Sensitivity Coefficients

	CWl			CXvel			CYvel		
	UAI	DAI	SHW	UAI	DAI	SHW	UAI	DAI	SHW
BC WL	1.3E+00	1.3E+00	1.5E+00	0.0E+00	0.0E+00	0.0E+00	-5.0E-02	-5.0E-02	0.0E+00
PE	4.2E-03	4.9E-03	7.3E-03	5.0E-04	7.5E-04	0.0E+00	2.5E-04	5.0E-04	2.5E-04
n1	-5.3E+00	-5.4E+00	-6.1E+00	1.6E-01	0.0E+00	0.0E+00	1.6E-01	3.2E-01	0.0E+00
n2	-5.8E+00	-5.9E+00	-6.6E+00	1.7E-01	0.0E+00	0.0E+00	1.7E-01	3.5E-01	0.0E+00
n3	-2.8E+00	-2.8E+00	-3.2E+00	0.0E+00	0.0E+00	0.0E+00	1.8E-01	1.8E-01	0.0E+00
n4	-2.5E+01	-2.5E+01	-2.8E+01	6.9E-01	3.5E-01	0.0E+00	1.0E+00	1.6E+00	-3.5E-01
n5	-5.3E+00	-8.3E+00	-9.4E+00	-2.8E+00	9.3E-01	0.0E+00	0.0E+00	3.3E+00	0.0E+00
n6	0.0E+00	-2.0E-01	-1.2E+01	0.0E+00	-2.7E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00
n7	0.0E+00	0.0E+00	-6.7E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00
n8	0.0E+00	0.0E+00	-2.1E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00
Q	-2.3E-04	-2.5E-04	-3.9E-04	6.3E-05	3.4E-05	1.3E-05	8.0E-05	1.1E-04	-4.2E-05
Wt	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00	0.0E+00

Note: SHW – Saunders Head Water Gauge, UAI – Upstream Approach to Iroquois Lock, DAI- Downstream Approach to Iroquois Lock

The second observation to be made from the sensitivity coefficients relates to the magnitude of the coefficient. The larger the magnitude of the coefficient, the larger the influence the parameter has on the model output. The sensitivity coefficients for the Manning's *n* values are the largest and would indicate they are influential on both the computed water levels and the computed velocities, with a greater influence on the computed water levels. The model input with the largest individual sensitivity coefficient is the Manning's *n* value for zone four of the model, indicating this is a very important section of the river. The boundary condition water level has a smaller influence on the computed water levels and a little to no influence on the computed velocities. The Peclet Number and water temperature have very little or no influence on the computed water levels and velocities.

The third observation is the magnitude of the sensitivity coefficients for the computed water levels, with respect to all input parameters, increase as the location progresses from the upstream to downstream end of the model. The downstream locations are influenced by more Manning's *n* zones than the upstream locations. The same trend does not appear in the computed velocity sensitivity coefficients.

The sensitivity coefficients only reflect the degree of influence of the model inputs on the model outputs and do not indicate how much uncertainty the model input contributes. Recall from equations 13 and 15, the sensitivity coefficient accounts for only a portion of the uncertainty in the model output with the remainder of the uncertainty originating from the uncertainty (variance) in the model inputs. To briefly illustrate this point, consider the sensitivity coefficient for the river discharge. The sensitivity coefficient for the river discharge is very small but the river discharge does result in a notable amount of the uncertainty in the computed water levels and velocity due to the size of the variance of the river discharge. Additional examples and discussion will be provided in the next section pertaining to the uncertainty in the model outputs and the sources of the uncertainty.

4.5.2 Uncertainty in Model Outputs

After the computation of the sensitivity coefficients, the uncertainty analysis calculations were performed. The calculations were first performed assuming the uncertainty in Manning's *n* and Peclet number is estimated using a coefficient of variation of 0.1 and secondly using a coefficient of variation of 0.3, refer to section 4.3.1 and 4.3.2 for further explanation.

The results are shown in Tables 4, 5, and 6. The expected value or mean of the outputs was determined by running the model with the mean input parameter values. The variance u_c^2, the standard deviation u_c, and the 95 percent confidence intervals, and the coefficient of variation for the model outputs are shown.

Table 4 – FOSM Results for Upstream of Iroquois Lock

	Water Levels		Xvel		Yvel	
	cov = 0.1	cov = 0.3	cov = 0.1	cov = 0.3	cov = 0.1	cov = 0.3
Mean	74.277	74.277	0.387	0.387	0.459	0.459
u_c^2	0.007	0.060	0.000	0.001	0.000	0.000
u_c	0.083	0.244	0.011	0.029	0.010	0.014
95 % ci	0.136	0.400	0.018	0.047	0.016	0.023
c.o.v.	0.001	0.003	0.028	0.074	0.022	0.031

Table 5 – FOSM Results for Downstream of Iroquois Lock

	Water Levels		Xvel		Yvel	
	cov = 0.1	cov = 0.3	cov = 0.1	cov = 0.3	cov = 0.1	cov = 0.3
Mean	74.211	74.211	0.175	0.175	0.642	0.642
u_c^2	0.007	0.065	0.000	0.001	0.000	0.001
u_c	0.085	0.255	0.009	0.027	0.017	0.035
95 % ci	0.140	0.419	0.015	0.045	0.028	0.057
c.o.v.	0.001	0.003	0.052	0.156	0.026	0.054

Table 6 – FOSM Results for Saunders HW Gauge

	Water Levels		Xvel		Yvel	
	cov = 0.1	cov = 0.3	cov = 0.1	cov = 0.3	cov = 0.1	cov = 0.3
Mean	73.835	73.835	0.086	0.086	-0.271	-0.271
u_c^2	0.012	0.101	0.000	0.000	0.000	0.000
u_c	0.111	0.318	0.001	0.002	0.005	0.006
95 % ci	0.182	0.522	0.002	0.003	0.009	0.010
c.o.v.	0.002	0.004	0.017	0.021	-0.019	-0.022

It is obvious that the coefficient of variation of Manning's *n*, which determines the uncertainty in this parameter, is very influential on the uncertainty in the model outputs. The standard deviation in computed water level at all three locations evaluated increased by a factor of roughly three when the uncertainty in the Manning's *n* increased by a factor of three. The increase in uncertainty in computed water level is directly proportional to the increase in uncertainty in these model parameters. This finding is expected due to the equation used for calculating the uncertainty (equation 17). It should be noted that the sensitivity coefficients remain the same no matter what coefficient of variation for Manning's *n* and Peclet number is used. The increase in uncertainty is a result of the increased variance in these input parameters.

The amount of increase in uncertainty in water velocity caused by the increased coefficient of variation is roughly a factor of three for the X-component of the velocity at the downstream approach to the Iroquois Lock location. The increase in uncertainty in the Y-component of the velocity at this location and the velocities at the other locations is less than a factor of three. The main reason for this difference is that Manning's *n* contributes less uncertainty in the computed water velocities than other input quantities such as river discharge. The relative contributions of model inputs will be discussed further in this section.

There are other observations that can be made from the computed uncertainties. The standard deviation in the computed water level at the Saunders headwater gauge is 0.11 metres which results in a 95 % confidence interval of 0.18 metres. The standard deviation in the water levels at the downstream and upstream approaches to the Iroquois lock are smaller than at the Saunders headwater gauge, calculated as 0.85 metres and 0.83 metres respectively. The uncertainty in the computed water levels increases from the upstream end of the model to the downstream end of the model. The reason is the sensitivity coefficients increase as the location progresses further downstream. The resulting coefficient of variation for the uncertainty in the computed water levels is very small. The reason is the magnitude of the water level is far greater than the standard deviation in the computed water level. The coefficient of variation is not a good indicator of the magnitude of the uncertainty. A more appropriate measure would express the amount of uncertainty compared to the range of the computed water level at a given location. For example, the range of the daily water levels at the Saunders headwater gauge is 3.3 metres. If the standard deviation is divided by the range, it results in a value of 0.03. This shows the uncertainty in water level is more significant than the coefficient of variation would suggest. Similar calculations could be conducted for the other locations.

The standard deviations in the computed velocities are very small but result in coefficients of variation that are much larger than those for the computed water levels. The velocities at the upstream and downstream of Iroquois lock locations have larger uncertainties both in absolute terms (standard deviation) and in relative terms (coefficient of variation) than the velocities at the Saunders headwater location. A relationship between the uncertainty in the computed velocities and the location in the river is not evident.

A beneficial aspect of the FOSM approach is that the method allows for the attribution of the uncertainty in the model outputs to individual or groups of model inputs. As mentioned previously, the sensitivity coefficients do not provide complete information on the uncertainty contributed by individual parameters because the effect of the variance is not included. When the variance is multiplied by the square of the sensitivity coefficient, the uncertainty due to the individual parameter is determined. To illustrate this point, the relative contributions on the computed water level at the Saunders head water gauge and the computed velocities at the Downstream Approach to the Iroquois lock (DAI) are shown in Table 7. Water level is the model output of main interest at Saunders headwater gauge and velocities are of primary interest at the approaches to the Iroquois Lock. These tables show results for calculations performed when the uncertainty in Manning's *n* and Peclet Number were determined using a coefficient of variation of 0.1.

The uncertainty in the computed water level at Saunders head water is primarily caused by the Manning's *n* coefficients and specifically zone four. The river discharge Q also contributes significantly to the uncertainty. The Peclet Number and boundary condition water level contribute minimally to the uncertainty in computed water level at Saunders. The sources of uncertainty in computed X- and Y-velocities at the downstream approach to the Iroquois lock are not the same. Manning's *n* in zone six contributes the most uncertainty in X velocity and the river discharge contributes the most uncertainty in Y velocity. The river discharge influences the velocity component aligned longitudinally with the river more than the velocity component aligned transverse with the river (for this simulation the longitudinal velocity is 0.64 m/s while the transverse velocity is 0.18 m/s at this location).

Table 7 – Percent Contribution to the Uncertainty

Model Input	Computed Water Level at Saunders HW	Computed X Velocity at DAI	Computed Y Velocity at DAI
BC WL	2%	0%	0%
PE	1%	1%	0%
n1	3%	0%	0%
n2	3%	0%	0%
n3	1%	0%	0%
n4	55%	1%	8%
n5	5%	10%	31%
n6	9%	70%	0%
n7	4%	0%	0%
n8	0%	0%	0%
Q	17%	18%	60%
Total	100%	100%	100%

Similarly, tables could be generated for the other locations but instead some basic summary observations will be provided. The model inputs that contribute the most uncertainty in the model outputs are the Manning's *n* in zones in the middle section of the river, zones four to six, and the river discharge. Manning's *n* zone four contributes the most uncertainty in computed water levels and the other two zones contribute significantly to the uncertainty in computed velocities. The river discharge contributes the most to the uncertainty in computed velocities. The remaining Manning's *n* zones, one, two, three, seven, and eight do not contribute significantly to the uncertainty in the model outputs. The Peclet number and boundary condition water level do not contribute significantly to the uncertainty in the model outputs. The absolute influence of the individual model inputs is dependent on which coefficient of variation is selected for the Manning's *n* and Peclet Number but the general statements made in this paragraph on the relative contributions of uncertainty from the model inputs do not change.

The FOSM analysis technique provided plausible estimates of the uncertainty in the model outputs and an indication of the sensitivity of the model and the relative contributions of the individual parameters to the uncertainty in the model outputs. The uncertainty in the model outputs is dependent on the uncertainty in the model inputs with the Manning's *n* parameters and river discharge contributing the most to the uncertainty in the model outputs. The inability to accurately define the uncertainty in the Manning's *n* parameter results in a considerable range in the uncertainty values calculated using the FOSM method. The uncertainty in the Peclet Number does not have as significant an influence.

4.6 Monte-Carlo Analysis of Uncertainty

The FOSM method is an approximate uncertainty analysis technique. It is relatively simple to undertake and requires only the mean and variance of the model inputs to be specified. It is known to be inaccurate when the model analyzed is non-linear (Bobba et al. 1996). To confirm the FOSM analysis, a Monte Carlo evaluation of the uncertainty was undertaken. The Monte Carlo analysis method is considered a more exact uncertainty analysis method than the FOSM and does not have the same difficulties with non-linear functions and models. It takes into account not only the mean and variance of the input but also the distribution type. It provides the entire distribution function for the output where the FOSM method only provides the mean and variance.

This section explains how the Monte Carlo analysis was completed. There were several components to this analysis. The first was the determination of probability distribution functions for the input parameters. From these probability distributions, samples may be generated using random numbers generated in Microsoft Excel from the standard normal distribution. To improve efficiency of convergence in the solution a Latin Hypercube Sampling (LHS) method (Saltelli et al. 2000) was used rather than random sampling. The uncertainty of the model was then determined through execution of the model for all of the samples and evaluating the combined uncertainty in the outputs.

Monte Carlo analysis requires significantly more simulations of the hydrodynamic model than the FOSM method. Additionally, the post-simulation processing requirements are greater than the FOSM. For this reason, the issue of the uncertainty in Manning's *n* and Peclet number explored in the FOSM method analysis will not be evaluated in the Monte Carlo analysis. Instead, the coefficient of variation for the Manning's *n* and Peclet number is assumed to be 0.1. The Monte Carlo analysis results will be compared to the FOSM method results using the coefficient of variation of 0.1 to explore the advantages and disadvantages of the methods when applied in hydrodynamic modelling.

4.6.1 Probability Distribution Functions of Input Parameters

Probability distribution functions are a key component of a Monte Carlo analysis. It is from the PDFs that the random samples of model inputs used to evaluate the uncertainty in the model are generated. Unfortunately, determining the appropriate probability distribution function for measured data and model parameters is difficult due to a lack of information. In the absence of information on which distribution to use, uniform or log-uniform distributions may be assumed. It has been shown that the range of the parameter has more of an influence on the uncertainty and sensitivity contribution of a parameter than the distribution type used (Saltelli et al. 2000). If sufficient data exists, statistical methods can be used to estimate the distributions of certain parameters. Alternatively, there may be literature available to characterize what distribution to use for a particular parameter.

In this thesis, probability distribution functions were developed for the upstream water level, the downstream discharge, the Peclet Number, and the eight Manning's *n* roughness values. Based on the results of screening level sensitivity analysis and the FOSM results, the water temperature was not evaluated with the Monte Carlo analysis because of its very negligible impact. The probability distribution functions for each of the model inputs were specified by assuming the uncertainty in each model input is normally distributed. The normal distribution has been utilized in other similar studies (Hall et al. 2005) and, in the absence of information to contradict the normal assumption, it is a reasonable approach. For the Manning's *n* parameter, Johnson (1996) suggests using the normal distribution to approximate the uncertainty in the parameter based on research the research of Cesare (1991) and others.

The normal distribution is defined in equation 23.

$$f(x)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (23)$$

In order to create the normal distribution curve for each parameter using equation 23, the mean and variance of each parameter must be determined. For the Peclet Number and each of the eight Manning's *n* zones, the mean was defined as the value determined through the calibration of the model explained in section 3.6.4 of this thesis. Since these two parameters cannot be measured the values must be determined through calibration. The variance of the parameters was determined using a coefficient of variation of 0.1 as determined by Johnson (1996). The FOSM analysis utilized two coefficients of variations 0.1 and 0.3 and determined that the selection of the coefficient of variation is very influential on the computed uncertainties. To limit the scope of the analysis, due to the computational requirements of Monte Carlo analysis, only one coefficient of variation, 0.1, will be used. Monte Carlo analysis will be compared with the FOSM results that used the coefficient of variation of 0.1. The resulting means and variances for each model parameter calculated using the coefficient of variation of 0.1 are shown in Table 8. The cumulative distribution function for the Peclet number is plotted in Figure 17 and one cumulative distribution function for Manning's *n*, zone number two is shown in Figure 18. These cumulative distribution functions were used to generate the random samples of the inputs for use in the Monte Carlo simulation.

Table 8 – Uncertainty Estimates for RMA2 Model Parameters

	PE	N1	N2	N3	N4	N5	N6	N7	N8
μ	18	0.0309	0.0288	0.0274	0.0288	0.027	0.0276	0.0326	0.0308
σ	2	0.0031	0.0029	0.0027	0.0029	0.0027	0.0028	0.0033	0.0031

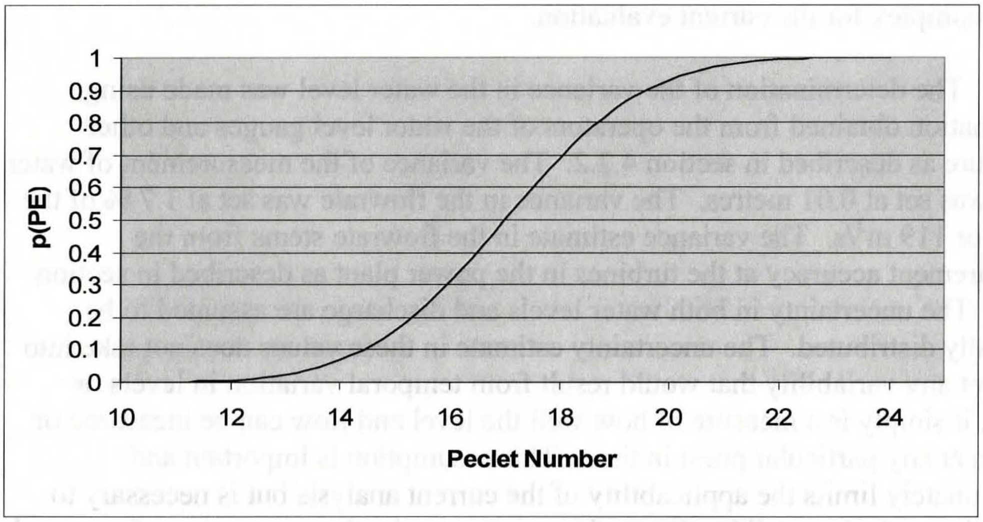

Figure 17 – Cumulative Distribution Function for Peclet Number

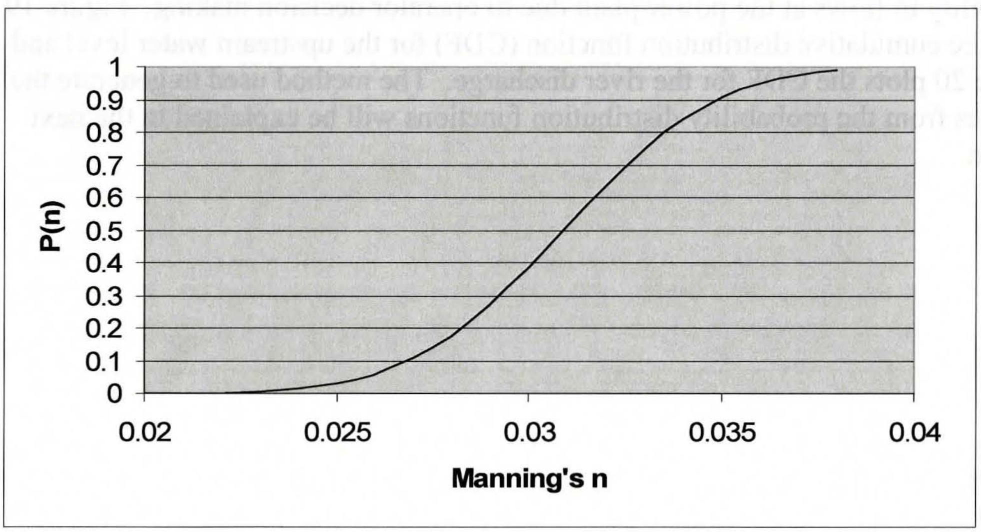

Figure 18 – Cumulative Distribution Function for Manning's n Zone two

The mean and variance for the boundary condition data, the upstream water level and the downstream discharge, were established as follows. The water level or flow specified as a boundary condition is considered as the mean.

In this analysis, a water level of 74.98 metres and a flowrate of 7023 m^3/s were specified.

In a real application of the model, these values would be determined through statistical analysis of measured water level and flow data for the period of interest. In this exercise, the boundary conditions are used to evaluate the steady-state water levels and velocities that would result if these boundary conditions exist. The problem of the evaluation of uncertainty using an unsteady-state model is too complex for the current evaluation.

The determination of the variance in the water level was made using information obtained from the operators of the water level gauges and other literature as described in section 4.2.2. The variance of the measurement of water level was set at 0.01 metres. The variance in the flowrate was set at 1.7 % of the mean or 119 m^3/s. The variance estimate in the flowrate stems from the measurement accuracy at the turbines in the power plant as described in section 4.2.3. The uncertainty in both water levels and discharge are assumed to be normally distributed. The uncertainty estimate in these values does not take into account any variability that would result from temporal variation in levels or flows, it simply is a measure of how well the level and flow can be measured or known at any particular point in time. This assumption is important and unfortunately limits the applicability of the current analysis but is necessary to make the analysis possible. Over a longer time scale, the assumption of a normal distribution for these quantities may not be valid given the nature of the variability in flows at the power plant due to operator decision making. Figure 19 plots the cumulative distribution function (CDF) for the upstream water level and Figure 20 plots the CDF for the river discharge. The method used to generate the samples from the probability distribution functions will be explained in the next section.

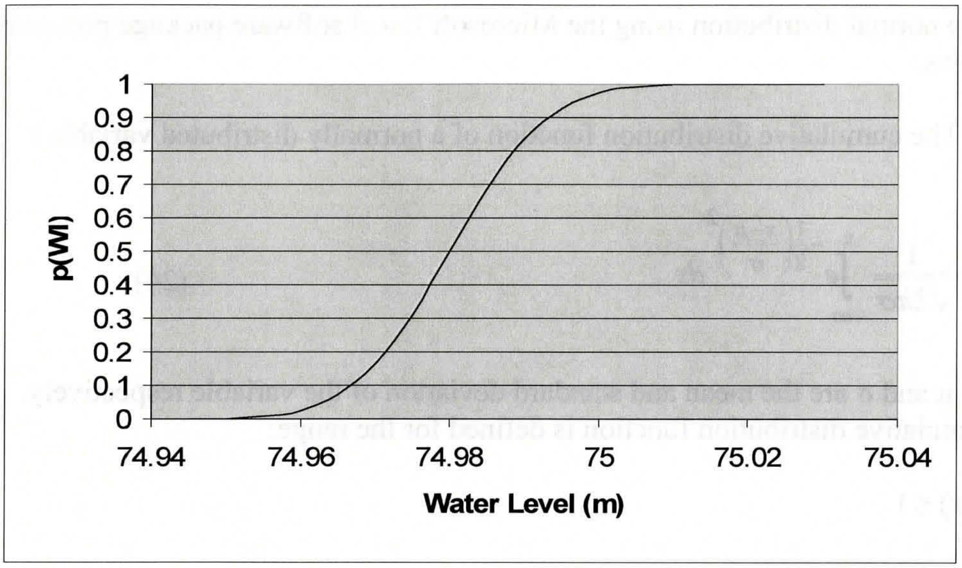

Figure 19 – Cumulative Distribution Function for Lake Ontario Water Level

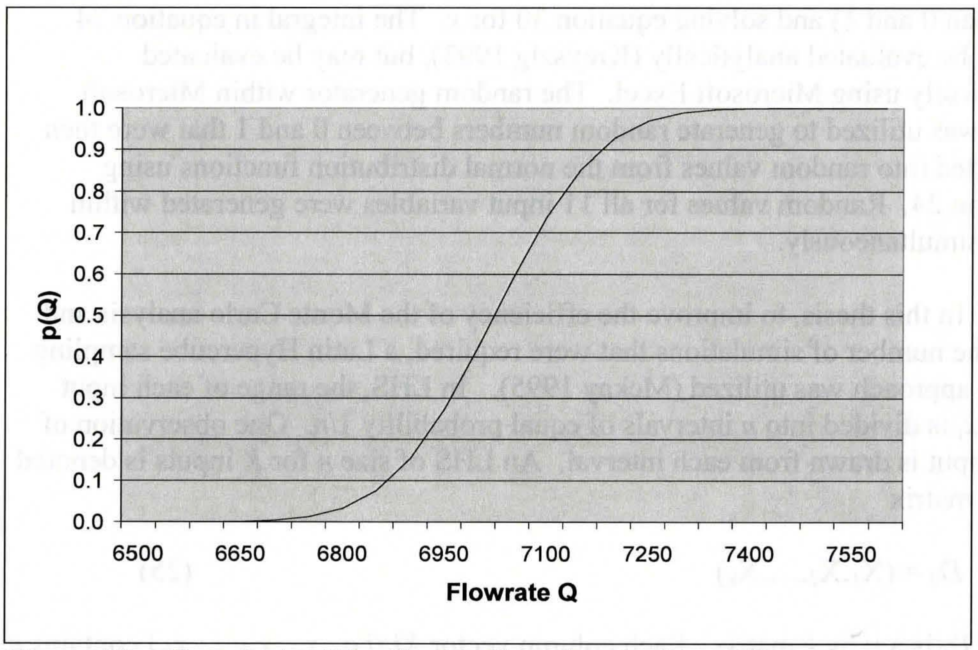

Figure 20 – Cumulative Distribution Function for River Flowrate

4.6.2 Generation of Samples from the Probability Distribution Functions

From the probability distribution functions, values from each distribution can be established for Monte Carlo Analysis. The generation of random samples from the normal distribution using the Microsoft Excel software package proceeds as follows.

The cumulative distribution function of a normally distributed variable x is:

$$F(x) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \tag{24}$$

Where, μ and σ are the mean and standard deviation of the variable respectively. The cumulative distribution function is defined for the range:

$$0 \le F(x) \le 1$$

A sample from the normal distribution function can be generated by setting the cumulative distribution function $F(x)$ equal to the value occurring (between 0 and 1) and solving equation 30 for x. The integral in equation 24 cannot be evaluated analytically (Kreyszig 1993), but may be evaluated numerically using Microsoft Excel. The random generator within Microsoft Excel was utilized to generate random numbers between 0 and 1 that were then converted into random values from the normal distribution functions using equation 24. Random values for all 11 input variables were generated within Excel simultaneously.

In this thesis, to improve the efficiency of the Monte Carlo analysis and limit the number of simulations that were required, a Latin Hypercube sampling (LHS) approach was utilized (Mckay 1995). In LHS, the range of each input factor x_i is divided into n intervals of equal probability $1/n$. One observation of each input is drawn from each interval. An LHS of size n for K inputs is denoted by the matrix

$$D_0 = (X_1, X_2, \ldots, X_k) \tag{25}$$

where D_0 is a n by k matrix. Each column vector Xi (*i.e.*, $x_{i1}, x_{i2}, \ldots, x_{in}$) contains n values of x_{ij} sampled from equal-probability intervals and randomized as to the position in the vector.

LHS ensures that the entire distribution representing each of the input variables is included in the sample to be evaluated (Saltelli et al. 2000). The same is not true when random sampling is used and consequently a large number of samples are needed to ensure that all regions of the input factors are evaluated. For this reason, LHS was employed in this thesis.

In this thesis, the ranges of the variables were divided up into 95 different intervals. Values for the variables were determined using the probability density functions for probabilities at 0.01 sized intervals ranging from 0.03-0.97 (i.e. 0.03, 0.04, 0.05,…, 0.95, 0.96, 0.97). As a result there were a total of 95 possible values for each of the 11 input variables. Using the LHS method 95 samples of the 11 variables were then created. The result was a matrix of input variables with dimensions of 11 columns (K) by 95 rows (n).

4.6.3 Monte Carlo Estimate of Uncertainty

The uncertainty in the computed water levels and velocities was determined by evaluating the 95 samples of input values with the RMA2 model. Determining the number of simulations is an issue to be dealt with in Monte Carlo analysis. The number of simulations must be great enough to be sure that the statistical estimates calculated from the samples have converged. If random sampling is used, hundreds or thousands of simulations may be required to reach sufficient convergence. For this analysis, because the Latin Hypercube method was utilized, the number of simulations required was small. The statistical estimates (mean and variance) converged to three decimal places with 95 samples executed. This level of convergence was appropriate for the purposes of this analysis; therefore no further simulations were required.

The execution of the RMA2 model in steady-state for the St. Lawrence River required approximately two minutes on a 3.2 Gigahertz, Pentium four computer with three Gigabytes of RAM. It took approximately four hours to evaluate 95 samples. The number of executions of the model required the development of an additional piece of software to extract the pertinent model outputs from the very large output files that are created by RMA2. If the number of runs is limited, this extraction can be accomplished manually, but as the number of runs increased it became apparent that a program was required to extract the required data from the model. A visual basic application was developed and utilized to accomplish this task. The development of this software required extra effort to code, debug and compile over and above what was required to complete the FOSM analysis of uncertainty.

From the results of the Monte Carlo runs of the hydrodynamic model the mean, variance, and confidence intervals of the computed water levels and velocities at the three locations of interest were determined. Tables 9, 10, and 11 show the results of the simulations. For comparison purposes, the results from the FOSM analysis have also been included in these tables.

Table 9 – Monte Carlo Results Upstream of Iroquois Lock

	Water Levels		Xvel		Yvel	
	MC	FOSM	MC	FOSM	MC	FOSM
Mean	74.272	74.277	0.387	0.387	0.459	0.459
u_c^2	0.005	0.007	0.000	0.000	0.000	0.000
u_c	0.072	0.083	0.008	0.011	0.008	0.01
95 % ci	0.118	0.136	0.013	0.018	0.013	0.016
99% ci	0.168	0.193	0.019	0.025	0.019	0.023
c.o.v.	0.001	0.001	0.021	0.028	0.018	0.022

Table 10 – Monte Carlo Results Downstream of Iroquois Lock

	Water Levels		Xvel		Yvel	
	MC	FOSM	MC	FOSM	MC	FOSM
Mean	74.205	74.211	0.175	0.175	0.643	0.642
u_c^2	0.006	0.007	0.000	0.000	0.000	0.000
u_c	0.076	0.085	0.008	0.009	0.016	0.017
95 % ci	0.125	0.14	0.013	0.015	0.026	0.028
99% ci	0.177	0.199	0.019	0.021	0.036	0.039
c.o.v.	0.001	0.001	0.046	0.052	0.024	0.026

Table 11 – Monte Carlo Results Saunders Head Water Gauge

	Water Levels		Xvel		Yvel	
	MC	FOSM	MC	FOSM	MC	FOSM
Mean	73.825	73.835	0.086	0.086	-0.271	-0.271
u_c^2	0.009	0.012	0.000	0.000	0.000	0.000
u_c	0.096	0.111	0.001	0.001	0.004	0.005
95 % ci	0.158	0.182	0.002	0.002	0.007	0.009
99% ci	0.224	0.259	0.003	0.003	0.01	0.012
c.o.v.	0.001	0.002	0.017	0.017	-0.016	-0.019

The uncertainties calculated using the Monte Carlo method are slightly smaller then those calculated using the FOSM method but the results are very similar. The reason the Monte Carlo estimates of uncertainty are smaller than the FOSM estimates is likely the inclusion of the entire probability distribution functions in the Monte Carlo analysis, while the FOSM method evaluates the uncertainty using only the mean and variance of the model inputs. The use of normal probability distribution functions to describe model inputs may lead to reduced uncertainty. The conservative nature of FOSM versus Monte Carlo analysis has been demonstrated by others (Lansey 1996).

The computational requirements for the Monte Carlo analysis were much greater than the FOSM method but it is not obvious that the extra effort was justified given the results from both methods were so similar.

4.7 Discussion of the Results of the Uncertainty Analysis

The undertaking of the uncertainty and sensitivity analysis of the St. Lawrence River hydrodynamic model has brought to light several points that warrant discussion.

4.7.1 Comparison of FOSM and Monte Carlo Methods

The application of two uncertainty analysis methods to the same hydrodynamic modelling problem allows a comparison of the methods to be made. Observations can be drawn from the experience of undertaking each approach and the results of the analyses can be compared. Both the FOSM and Monte Carlo analysis methods are suitable for use with hydrodynamic modelling. However, there are also advantages and disadvantages to each method.

To start, the FOSM analysis of uncertainty was easier to undertake than the Monte Carlo analysis. The FOSM method required only the mean and variance of the model inputs in order to complete the analysis. The most difficult part of the analysis was the calculation of the sensitivity coefficients for the model inputs. To calculate each sensitivity coefficient the model had to be executed two times using the central difference method. That meant that two runs of the model for each of the 12 sensitivity coefficients were performed. After the sensitivity coefficients were calculated the mathematics required to calculate the uncertainty in the model outputs was trivial. The Monte Carlo analysis was more complex than the FOSM.

The Monte Carlo analysis required the probability distribution functions for each of the model inputs. The exact probability distribution function is not known with certainty so the Gaussian distribution was assumed for each input parameter based on available literature (Saltelli et al. 2000). From the probability distribution functions, samples of the model inputs were generated using a LHS strategy. A total of 95 samples for each model input were generated. If the LHS strategy was not utilized, far more samples would have been required (on the order of 500) to achieve the same level of convergence. The model was executed once for each of these 95 samples and the results were then analyzed statistically to obtain the estimate of the uncertainty in the model outputs. An additional piece of software developed as a component of this thesis was utilized to extract the pertinent model outputs from the large output files created by RMA2. The software was also utilized to generate the appropriate RMA2 input files required for the Monte Carlo analysis. This software was not required or utilized in the FOSM analysis.

The results of the two analyses were very similar. The uncertainties calculated using the Monte Carlo method were slightly smaller than those calculated using the FOSM method. For one example in Table 11, the uncertainty as measured in standard deviation in the computed water level at Saunders headwater was +/- 0.096 m calculated using the Monte Carlo analysis and +/- 0.111 m using the FOSM method. For another example, from Table 9, the uncertainty in the computed x velocity is +/- 0.008 m/s calculated using the Monte Carlo analysis and +/- 0.011 m/s calculated using the FOSM method.

The reason that the Monte Carlo estimates of uncertainty are smaller than the FOSM estimates is perhaps due to the inclusion of the probability distribution function in the Monte Carlo analysis. Model inputs from the tails of the distributions, with low probabilities of occurrence, are included in the Monte Carlo analysis. The FOSM method evaluates the uncertainty using only the mean value of the model inputs and the variance. The effect of the inclusion of values of the model inputs with both high and low probabilities may lead to reduced uncertainty. The other possible reason for the small difference is that the Monte Carlo analysis takes into account the non-linear aspects of the hydrodynamic model. The effect of the uncertainties in all model inputs may combine to form a smaller total uncertainty than simply the linear sum of the model input uncertainties as computed using the FOSM method. In the FOSM method, the individual sensitivity coefficients multiplied by the model input variances are summed to determine the combined uncertainty in the output. This straight summation may not take into account some of the effect that the combinations of the individual parameters may have on the model outputs. By evaluating the combined effect of all model inputs at the same time, the Monte Carlo analysis takes into account the effects of the uncertainties of the individual parameters in combination. The combined effect of the uncertainties in the model inputs may be less because of cancellation of some uncertainty due to negative and positive contributions to the overall uncertainty.

Whatever the reason is for the differences between the methods, the differences between the uncertainties calculated are small, suggesting that the FOSM method can be effective in estimating the uncertainty in a hydrodynamic model. The calculated uncertainty using the FOSM is larger or more conservative when compared to Monte Carlo analysis. Having the estimate of uncertainty being conservative is good. There is a greater chance the actual value of the model output will fall within the bounds of the uncertainty if the estimate of the uncertainty is conservative. Though, the uncertainty should not be excessively conservative so as to diminish its significance.

A positive characteristic of the FOSM is it requires less input data and fewer executions of the hydrodynamic model to compute the uncertainty than the Monte Carlo analysis. Another beneficial aspect of the FOSM method is that the decomposition of the total uncertainty in the model outputs into the contributions from individual or groups of model inputs is simple and straightforward. The individual contributions of uncertainty from each model input are calculated at the same time the total combined uncertainty in the model outputs is calculated. This provides a quick indication of the model inputs that contribute the most uncertainty in the model outputs. In this analysis, Manning's *n* in zone numbers four, five and six, and the river discharge were quickly identified as being influential on the model and contributing significantly to the uncertainty in various outputs. Other model inputs such as the water temperature, upstream water levels and eddy viscosity were identified as having little or no influence on the uncertainty in certain model outputs.

Decomposition of the total uncertainty in the model outputs using Monte Carlo analysis is not as simple. General sensitivity of model inputs on model outputs can be gleaned from simple regression and correlation analysis but additional efforts involving advanced global sensitivity analysis methods need to be employed to dissolve the uncertainty into its source components. These methods involve strategic sampling strategies and the execution of the model many times in order to achieve convergence. The St. Lawrence River hydrodynamic model is computationally demanding so these advanced sensitivity analysis methods are not practical. The sensitivity indices resulting from global sensitivity analysis are more accurate than sensitivity coefficients calculated in the FOSM method, because they measure not only the individual effect of each model input but also the combined effect of the model inputs interacting with the other model inputs. However, the additional effort is not warranted here. The purpose of this analysis is to discern where the uncertainty in the hydrodynamic modelling inputs originates from and to obtain an estimate of that contribution. The decomposition of uncertainty provided by the FOSM method is accurate enough for this analysis.

4.7.2 Importance of Estimates of Uncertainty in Model Inputs

The second issue is the critical importance of obtaining good information to quantify the uncertainty in the inputs of the hydrodynamic model. Independent of what method is utilized to determine the uncertainty in model outputs, the uncertainty in the model inputs must be accurately estimated or else the uncertainty estimate will be weakened. The model output uncertainty is a function of the model input uncertainties. Thus, the uncertainty in the model inputs is of fundamental importance in both methods.

The inability to certifiably define the uncertainty in the Manning's *n* parameters resulted in a vague estimate of the uncertainty in the model outputs. Literature provided guidelines for estimating the uncertainties in Manning's *n*. However, the literature estimates of the uncertainty span a large range. Johnson (1996) suggests a coefficient of variation between 0.1 and 0.3 to quantify the uncertainty in Manning's *n*. This range creates uncertainty in itself in trying to select a representative estimate of the uncertainty as required by the FOSM and Monte Carlo analysis methods. The effect of using a coefficient of variation of 0.1 and 0.3 to describe Manning's *n* was evaluated in section 4.5.2 and there is clearly a large effect. For example, the uncertainty in computed water level at the Saunders head water gauge location, expressed using the standard deviation, is 0.11 m and 0.33 m using a coefficient of variation of 0.1 and 0.3 respectively for the uncertainty in Manning's *n*. There is a direct relation between the uncertainty in the Manning's *n* values and the computed model outputs.

The amount of uncertainty in the Manning's *n* values used in this thesis is likely on the smaller end of the range suggested by Johnson (1996) and may be less than the amount specified in that paper. The estimates of uncertainty in that paper were established from literature searches of relevant papers and from experiments conducted by the author. Manning's *n* values for test river channels were estimated by a number of hydraulic engineers and engineering students with the results tabulated and analyzed statistically. The Manning's *n* values for the St. Lawrence River model were estimated through calibration. The uncertainty in the Manning's *n* after calibration is likely smaller than the predictions made by Johnson (1996) but still exists.

The reason that the uncertainty in Manning's *n* values used in this thesis are at the lower end of the range proposed by Johnson (1996) is the calibration process which makes use of river water levels to indirectly determine the appropriate Manning's *n* values. In a way, these values are measured, albeit indirectly. The calibration process determines the optimal Manning's *n* values by iteratively adjusting the parameters so that the computed water levels agree with the observed water levels. The validation of the parameter values using the observed water levels provides an indirect means to measure the Manning's *n* values that cannot be explicitly measured. Still, the calibration process is not exact and even though these parameters are optimally determined through calibration, they could still be incorrect or have some degree of uncertainty in them.

There are alternative parameterizations that perform almost as well as the optimal Manning's *n* values. There is also the possibility that the Manning's *n* parameters may be compensating for errors in other aspects of the model such as channel bathymetry. Effort was made to minimize potential errors in bathymetry, but there is a possibility that errors still exist. These errors could result in channel conveyance areas in the model that are different than the actual conveyance areas. There is no easy way to determine whether there are small errors in the bathymetry. The best that can be done is to use the best available bathymetry and assume that it is correct. If there are errors in the bathymetry, the Manning's n values that are determined through calibration to water levels would compensate for these errors. There is no easy way of knowing if this is occurring.

The uncertainty in the other model inputs is less of a problem either because the uncertainty can be accurately defined or because the model input is not influential on the computed model outputs. Good estimates of the uncertainty were located and utilized for the boundary condition water level and boundary condition river discharge. The uncertainty estimates are precise and defendable because they are cited by the agencies responsible for collecting the data. The uncertainty in eddy viscosity is difficult to estimate because there is no literature available and the parameter must also be determined through calibration. However, within its plausible range, as defined in the RMA2 model documentation (Donnell et al. 2005), the eddy viscosity was not significantly influential on the computed model outputs as proven in section 4.3.2 of this thesis. Likewise, the water temperature does not impact the computed water levels and velocities in this model, so its uncertainty is not important to this analysis. The uncertainty in wind was not considered in this analysis because it is not a feasible boundary condition for this model as discussed in section 4.2.4.

Estimating the uncertainties in the model inputs is challenging as discovered in this thesis, but it is one of the most important components of an uncertainty analysis of any model, including a hydrodynamic model.

4.7.3 Discussion about the Extra Effort of Conducting UA and SA in Modelling

From the analysis conducted in this thesis, insight has been gained on the usefulness of uncertainty and sensitivity analysis in the hydrodynamic modelling context.

Typically, hydrodynamic model development is conducted in stages. The first is the identification of the problem. The second is the selection of a model capable of answering questions related to the problem. The third is the collection of data required by the model. The fourth is the inputting of the data into the model. The fifth is the calibration and verification of the model using observed data. Following the calibration and verification, the model is often applied to the problem to generate solutions. A sixth step that is sometimes but not always completed is sensitivity analysis. The purpose of the sensitivity analysis is to discern the importance of individual inputs on the model outputs. However, sensitivity analysis is often not completed. Uncertainty analysis would be a seventh step in the modelling process.

Experience was gained through developing the hydrodynamic model of the St. Lawrence River, completing calibration, verification, sensitivity analysis and uncertainty analysis of the model. It is evident that it is beneficial to undertake sensitivity and uncertainty analyses. The earlier the sensitivity analysis is completed, the earlier the indication of how important each piece of input data or model parameter is learned. This knowledge can be helpful in project planning because increased effort can be placed on accurately defining those model inputs that are most influential on the model outputs. A simple sensitivity analysis, such as the calculation of the sensitivity coefficients can provide a great deal of information to the engineer from a minimal amount of effort. Uncertainty analysis of a hydrodynamic model can be easily completed using the FOSM method if there is information available to describe the uncertainty in the model inputs. Increased attention is being given to uncertainty considerations in measurements and modelling, so it is likely that estimates of uncertainty for data used as model inputs will be more readily available in the future.

The additional information obtained from a sound uncertainty analysis adds further value to the computations performed by the model. The value of the hydrodynamic modelling outputs is increased by the description of the uncertainty in those outputs. The outputs are no longer deterministic estimates of the water levels and velocities in the river, they are probabilistic. It is possible to place a confidence interval around the computed water level or velocity. The confidence interval describes how much confidence can be placed in the computed values. For example, the deterministic computed Saunders headwater water level is 73.83 metres and the probabilistic estimate of the uncertainty says that we are 95 % certain that the computed water level is within +/- 0.16 metres of the computed value. The probabilistic information adds value to the hydrodynamic modelling output and summarizes all of knowledge the engineer has on the model output. Better decisions can be made based on the information provided by uncertainty analysis. As an example, suppose the flood water level for the Saunders headwater is 73.9 metres and the deterministic computed Saunders headwater level is 73.83 metres. However, the 95 % confidence interval, probabilistic water level ranges from 73.68 and 73.98 metres, with a possibility of the flood stage being breached. By accounting for the uncertainty, the engineer/water manager would recognize the possibility that the flood stage might be surpassed and could take corrective action. Treating the model outputs as deterministic may have lead to a different decision.

4.7.4 Shortcomings of the Analysis

This analysis accomplishes the objectives set of this thesis but there are unfortunately several shortcomings with the analysis. These shortcomings could form the basis for additional work in the future, or they could simply be recognized as limitations of an analysis of this type.

The inability to accurately define the uncertainty in Manning's *n* is a major limitation of uncertainty analysis of hydrodynamic modelling. This point has already been discussed in detail in previous sections.

Uncertainties in the bathymetric data were not explicitly dealt with in this thesis. As discussed in section 4.2.1, there was information gathered to describe the uncertainty in the bathymetric data for the model (Byrnes et al. 2002). This uncertainty was assumed to be eliminated during the development of the hydrodynamic model. The 660,000 bathymetric soundings were inputted into the model grid through interpolation. This procedure resulted in the river's geometry being defined in the model using 80,000 model nodes, whose elevation is defined through a linear interpolation of the 660,000 bathymetric soundings. The interpolation process was assumed to eliminate the random uncertainty in the elevation of each individual sounding. This method would not eliminate the uncertainty in the bathymetric data if the cause of the uncertainty was systematic in nature. A systematic uncertainty in the bathymetric data could be caused by error that is introduced over a specific region within the bathymetry or across a survey transect within the dataset. The agencies responsible for the collection of this data have stringent quality control and quality assurance procedures that are completed following the completion of a hydrographic survey. It is reasonable to assume that there would not be any systematic error within the data but an evaluation of the potential impact of a systematic error in the data would be informative. Evaluating such uncertainties in the bathymetry would require additional problem design and evaluation over and above what was accomplished in this thesis.

Another shortcoming of this uncertainty analysis is that the analysis was conducted using a steady-state application of the St. Lawrence River hydrodynamic model, when the model is often applied in unsteady-state mode. Running the model using unsteady water levels and flows as boundary conditions introduces additional complications over and above those evaluated in this thesis. The time step of computation must be given careful consideration, the duration of time required to initialize the model must be determined, and the short term boundary conditions such as wind become more important. Because the model was run in steady-state in this thesis, these issues were not considered but would need to be included in an uncertainty analysis of an unsteady-state hydrodynamic model.

Chapter 5 – Conclusions and Recommendations

5.1 Conclusions

In this thesis, the uncertainty in a specific two-dimensional hydrodynamic model was quantified. The analysis considered the contributions of uncertainties in measurable model inputs, un-measurable model parameters and the model itself. The combined uncertainty was calculated using FOSM and Monte Carlo analysis techniques. Based on the work and discussion presented, several conclusions can be made.

Both FOSM and Monte Carlo analysis can be applied with two-dimensional hydrodynamic modelling, but FOSM is the preferred method. There are several reasons for this conclusion. First, FOSM estimates of uncertainty are slightly larger than those obtained using Monte Carlo analysis resulting in a more conservative answer. There is a greater chance the actual value of the model output will fall within the bounds of the uncertainty if the estimate is conservative. The second reason the FOSM method is preferred, when compared to Monte Carlo analysis, is that FOSM requires less information to describe the model inputs, fewer model executions and computations to calculate the uncertainty. This makes FOSM easier to apply than Monte Carlo analysis. Third, FOSM provides an immediate indication of the primary contributors to the uncertainty in the output, where Monte Carlo analysis requires additional effort to do the same. Sensitivity coefficients are calculated as an interim step of the FOSM method and can be utilized to proportion the uncertainty to individual or groups of model inputs. Based on these considerations, the FOSM method is favoured.

Given the sources of uncertainty in the model inputs investigated, the input that contributes the most to the uncertainty in the model outputs is the bottom resistance described in RMA2 using Manning's *n*. The bottom resistance is caused not only by bed roughness but also by irregular bed geometry and seasonal vegetation. Manning's *n* is used to account for resistances caused by all of these characteristics on the flow. In this particular St. Lawrence River application of the model, the Manning's *n* for one particular reach of the river had the largest influence on computed water levels and velocities. Unfortunately, there is not a precise definition of the uncertainty in Manning's *n* in the literature. As a result, the combined effect of a large uncertainty in Manning's *n* and a model that is highly sensitive to changes in Manning's *n*, results in a significant amount of uncertainty in the outputs of the hydrodynamic model.

The additional effort required to complete an uncertainty analysis of a hydrodynamic model using the FOSM method is minimal and the resulting knowledge obtained is worth the extra effort. Uncertainty analysis is a practical addition to the two-dimensional hydrodynamic modelling process. It provides helpful information to the model developer, quantifying how good the model actually is. The model outputs become probabilistic rather than deterministic. A confidence interval can be placed around the computed water level or velocity, relaying everything the engineer knows about the model output. Better decisions can be made in water resources management by taking into account the uncertainties in hydrodynamic modelling.

5.2 Recommendations

This thesis illustrates that the FOSM uncertainty analysis method can be applied to hydrodynamic modelling with a minimum of effort if estimates of the uncertainty in the model inputs are known. However, determining the uncertainty in the model inputs can be difficult. Based on available literature, the uncertainty in Manning's *n* is estimated to be between a coefficient of variation of 0.1 and 0.3. This is not a precise estimate. In this analysis, Manning's *n* results in the largest proportion of uncertainty of any of the model inputs evaluated. Additional research is needed to quantify the uncertainty in Manning's *n* because of the large influence of the parameter on hydrodynamic models.

Using the FOSM method to quantify uncertainty in two-dimensional hydrodynamic modelling resulted in a conservative estimate of the uncertainty when compared to Monte Carlo analysis. However, the probability distributions for all model inputs were assumed to be normally distributed for the Monte Carlo analysis. It is possible that the selection of distribution for the model inputs may influence the size of the uncertainty calculated using Monte Carlo analysis. Further research should be conducted to determine the effect the choice of distribution has on the results obtained using Monte Carlo analysis.

The uncertainty in model outputs due to wind boundary conditions was not evaluated in this thesis because there was insufficient data available to use wind as a boundary condition. However, should reliable wind data be available for a study area, it should be included as a boundary condition in the model and its contribution to the uncertainty in the model should be evaluated. Uncertainties introduced by unsteady-flow phenomena were also not evaluated in this thesis due to the complexities that would be introduced. Hydrodynamic models are often used to simulate unsteady-flow conditions and therefore, an analysis of the key contributors to the uncertainty would be beneficial. Such an analysis might consider factors such as time step of computation, the effect of initial conditions and boundary conditions of a highly transient nature such as wind.

Finally, it is recommended that uncertainty analysis be incorporated into the development of hydrodynamic modelling. Uncertainty analysis is a practical, meaningful process, and is worth the extra effort it takes to complete. The results of uncertainty analysis should be assessed by the model developer and passed on to future users of the model or modelling outputs for their consideration.

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