ON ASSEMBLE-TO-ORDER SYSTEMS WITH STOCHASTIC LEAD TIMES

By

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ABSTRACT

We present and study the current research trends and advances for optimizing assemble-to-order systems with stochastic lead times. Assemble-to-order systems are widely used in several industries, where end products consist of multiple components and are customizable. Manufacturers can offer greater flexibility to their customers by delaying the assembly of the components until after the order is placed. This means they do not keep an inventory of end-products. Only components can be kept in the inventory and assembled upon demand. We compare and contrast different formulations and the features of assemble-to-order systems studied in the literature including system configuration, replenishment policy, lead time type, and demand distribution. Single-period systems, periodic-review systems, and continuous-review systems are considered. Current assumptions, formulations, solutions, and challenges are discussed.

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LIST OF ABBREVIATIONS

1. ATO: assemble-to-order
2. CBRCD: coordinated base-stock with rationing and compound demand
3. FBFR: fixed base-stock and rationing policies
4. FCFS: first-come-first-serve
5. GI: general independent
6. i.i.d: independent and identically distributed
7. IBRCD: independent base-stock with rationing with compound demands
8. LBLR: lattice-dependent base-stock and rationing policies
9. MDP: Markov decision process
10. MPIBR: multi-product independent base-stock with rationing
11. M-system: a configuration with two products, one common component, and two product-specific components
12. NHB: no-hold-back
13. NP-Hard: non-deterministic polynomial-time hard
14. N-system: a configuration with two products, one common component, and one product-specific component
15. SBSR: state-dependent base-stock and rationing policies
16. W-system: a configuration with three products and two common components

I. INTRODUCTION

In this paper, we review the literature on assemble-to-order systems with stochastic lead times. Assemble-to-order systems are widely used in several industries, where end products consist of multiple components and are customizable. Manufacturers can offer greater flexibility to their customers by delaying the assembly of the components until after the order is placed. This means they do not keep an inventory of end-products. Only components can be kept in the inventory and assembled upon demand. However, this mass-customization method causes many operational complications. Manufacturers need to either keep a base-stock[1] of components in advance to reduce the lead times[2] or they will have to batch-order[3] components, which will bring the supply uncertainty into equation. They also need to deal with uncertainty in demand which will affect their inventory and planning process. These issues create the need for optimal solutions that would guarantee a certain service level[4] and cost-efficient operations at the same time. [1]

1. SIMILAR STUDIES

For decades, several researchers have looked into issues arising in assemble-to-order systems and have tried to optimally solve them. The vast volume of research on the subject has led to the need for a comprehensive study so that the researchers in the field can gain an overall view of the subject and identify the directions for future studies. Several prominent researchers have done a full review of assemble-

[1] A predefined level of component inventory that is replenished one unit at a time.
[2] The time between the order placement and the release the end-product.
[3] In batch-ordering, "whenever the inventory position (net inventory plus inventory on order) drops to or below the reorder point r, an order of size nQ is placed to raise the inventory position up to the smallest integer above r. Clearly, a base-stock policy is a special case of the batch ordering policies with a batch size (Q=1)." [6]
[4] The probability of fulfilling orders without delay. [24]

to-order systems. One of these teams are Jing-Sheng Song and Paul Zipkin, both of whom have several publications on this topic. They have written a chapter on assemble-to-order systems, covering some of the literature on the subject up to January 2003. Zumbul Atan along with a team of four researchers from University of Eindhoven, have covered the rest of literature on the subject between January 2003 and January 2016. Nevertheless, their literature review is not exhaustive, and therefore, there is not a complete overlap between their work and our study. There are several other literature reviews on the subject, each focusing on a different aspect of assemble-to-order systems. However, these two reviews combined will provide the readers with a comprehensive view of the research history so far. [2] [1]

2. Research Objective and Scope

Here, we are focusing on a specific subcategory of assemble-to-order systems that specifically mention stochastic lead times as an assumption; therefore, we will avoid mentioning papers on assemble-to-order systems that do not fall in that category. Although, many of papers written on stochastic inventory optimization and general assemble-to-order systems will be useful to anyone who wishes to conduct research on assemble-to-order systems with stochastic lead times. Thus, we refer the interested reader to the above mentioned reviews for a broader view of the subject.

The objective of this thesis is to focus on the studies on assemble-to-order systems with stochastic lead times, that have been published in English, up to December 2016, and categorizes them based on their assumptions and objectives. To our knowledge, there is no similar comprehensive literature review solely focused on ATO systems with stochastic lead times which justifies the need for a comprehensive review of the subject.

The thesis is organized as follows: Chapter II outlines the research methodology, chapter III describes the overall view of the studies published on the topic and chapter IV contains a detailed summary of the papers. Chapter V concludes the research.

II. Literature Review Research Method

The research method introduced in this chapter has been developed using the guidelines of Kofod-Petersen on systematic literature review [3]. The SLR protocol detailed here describes research questions, the search strategy, the selection criteria, and the analysis and synthesis of data, in accordance with our research objective. Each of these subsections will be explored below.

1. Research Questions

Based on the objective of research, we would like to find the answer to the following research questions:

- RQ1: Which studies have focused on ATO systems with stochastic lead times?
- RQ2: What are the objectives of these studies?
- RQ3: What review period (single period, periodic, or continuous-time) does the study assume in the formulations?
- RQ4: How do these studies differ in their assumptions?

2. Search Strategy

In this section, we will outline the search terms, the sources searched, and the documentation of search results.

2.1. Search Terms

Based on prior experience and familiarity with the subject, we considered the following search terms. Any study included in the final result has to contain all of the 3 terms, or their synonyms listed in the

Table 1, simultaneously.

Table 1 : Search Terms

Term 1	Lead time	Processing time	Production time	Assembly time
Term 2	Stochastic	Probabilistic	Exponential	
Term 3	Assemble to Order	ATO	Assemble-to-order	

2.2. SOURCES

In this study we relied on four different search strategies:

- Prior collection of studies suggested by Dr. Kai Huang
- Online search and
- Reference checking
- Checking authors' list of publications

The primary search engines and databases used for online search were:

- Google
- Bing
- Research Gate
- Science Direct
- McMaster Library
- Wiley
- Jstor
- Informs
- Springer

After obtaining a primary list of studies, we went through all of the references of the studies to single out those that matched the search term criteria. Also, we conducted a thorough search of publications of the authors who had published a study on the same subject. Those studies that contained the search terms were added to the list and downloaded individually. It is notable that a handful of studies were not accessible to us due to not having a paid registration or not being available online; therefore, they were excluded from the search results.

2.3. Documentation of Search Results

The search results were documented in an excel file that contained the following information:

- Title
- Publication date
- Journal
- Authors
- Web address
- Review period
- System configuration
- Replenishment policy
- Demand distribution
- Lead time type
- Objectives of study

These documentation criteria were chosen based on the advice of the supervisors, whose student had already developed a primary list of studies on a closely related subject.

3. Study Selection

In order to filter the relevant studies out of the pool of all studies found using the search terms, we defined the following inclusion criteria.

The inclusion criteria were:

- The appearance of search terms 1, 2, and 3 in the main body of the research
- Definition of the problem in an ATO system
- Assumption of stochastic lead times (or alternatively, production, assembly, or processing times)

In the first search round, 146 studies were found that contained the search terms and when in doubt, we chose to include the study. However, many of these search terms appeared in the references and were not part of the main body of the study; hence, excluding the study from the final results in the second round of selection. On the other hand, some studies contained only some of the terms in the main body and the rest in the reference section. Those studies were excluded as well. The final list was developed after careful examination of the abstract and if necessary, the main body of each study. It contained 30 results, two of which were literature reviews mentioned in the introduction.

Selected studies were documented in a new excel file that contained the same information as the search results.

4. Study Analysis

In order to assess the quality of each paper, we have developed a series of questions that will be answered with a yes (1 point) or a no (0 point). The total cumulative score will show the comparative quality of the papers. The quality criteria for assessment are:

1. QC1: Does the study consider a general multiproduct, multicomponent ATO system configuration?
2. QC2: Does the study measure or optimize more than one performance aspect of the system?
3. QC3: Is the solution clear and relevant to ATO systems with stochastic lead times?
4. QC4: Does the study make the replenishment policy clear?

5. QC5: Does the study make the allocation policy clear?
6. QC6: Is the replenishment policy state-dependent?
7. QC7: Is the allocation policy state-dependent?

The quality assessment summary will be presented in chapter III.

5. DATA SYNTHESIS

For every research question defined, we have a corresponding section that answers that question in detail. RQ1 is answered in detail in chapter IV that contains the summary, characteristics, assumptions, and objectives of every paper. RQ2, RQ3, and RQ4 are answered in chapter III that gives an overall view of the reviewed papers, and lists their objectives, assumptions, and characteristics in one place. Each research question is included ahead of the section where it is answered.

III. CLASSIFICATION OF ASSEMBLE-TO-ORDER SYSTEMS

An assemble-to-order system is a combination of a distribution system and an assembly system. A distribution system has one component, and its major issue is component allocation; while an assembly system has one product, and its major issue is component coordination. An ATO has to solve both of these issues which makes its problems more computationally complex.

1. OBJECTIVES OF STUDIES

RQ2: What are the objectives of these studies?

Most studies are focused on measuring or optimizing the performance of a given system, such as minimizing costs. Another group of studies are focused on designing a better system like the effects of utilizing a common component on ATO system. [1] One can view in Table 2 that in our study, most of papers fall in the former category.

Table 2 : Objectives of Studies

Reference	Objective
Yano 1986	Minimizing the sum of inventory holding costs and tardiness costs
Glasserman 1998	Measuring the sensitivity of lead times – inventory trade-off at a fixed fill rate
Song 1999	Obtaining the exact performance measures for the system (item-based, order-based, system-based)
Lu 2001	Measuring the fill rate
Song 2002	Measuring the optimal trade-off between inventory and service
Lu 2003	Measuring the response-time based order fill-rate

Iravani 2003	Measuring system performance based on customer satisfaction
Dayanik 2003	Measuring order fill-rate and service level
Mohebbi 2004	Measuring "the average total inventory of components per period, the average proportion of products' demand orders per period that were fully satisfied on time (service level), and the average total backorder of finished products per period."
Lu 2005	Optimizing the allocation of budget to inventories to minimize a weighted average of backorders over product types
Zhao 2006	Measuring mean and variance of delivery lead times, the inventory holding costs of components, and order fill-rate for a single product system, and characterizing the dependence between stock-out delays of the components
Benjaafar 2006	Determining optimal allocation and replenishment policies under total discounted cost and average cost
Lu 2007	Measuring average backorders, minimizing total operational costs (inventory costs and backorder penalty costs)
Elhafsi 2008	Determining optimal production and inventory allocation policies
Elhafsi 2009	Determining optimal production and inventory allocation policies
Zhao2009	Calculating expected delivery lead times and order-based fill-rates
Gao 2010	Measuring order-based and item-based the fill rate within a time window and the service level
Lu 2010	Minimizing the average component inventory, total product backorders and maximize fill-rate
Xiong 2011	Minimizing the total expected cost

Nadar 2011	Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost
Nadar 2012	Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost
Zhi 2013	Determining optimal allocation and replenishment policies, to minimize operating costs while maintaining order fulfillment
Nadar 2014	Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost
Elhafsi 2014	Determining optimal production and inventory allocation policies
Nadar 2015	Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost
Nadar Apr 2016	Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost
Nadar 2016	Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

2. Trend Analysis

RQ3: What review period (single period, periodic, or continuous-time) does the study assume in the formulations?

Assemble-to-Order systems can be categorized based on several different characteristics like their product-component configuration, review period, replenishment policy, lead time type, etc. In this study, we focus on the categories based on the systems' review period. Further subcategories based on other factors can be found in Table 3.

ATO systems can be divided into three major categories based on their review periods: single period, periodic, and continuous-time systems.

2.1. Single-Period Systems

These systems' operations take place in one period. The components are ordered at the beginning of the period and the demand is satisfied at the end of period. These models analyse characteristics of an ATO system in a static fashion, hoping that the results can be extended to multi-period models. These studies were quite common in 1980s; however, here, we only have one short study that fits this criterion. [2]

Xiong [4] studies a single-period system with stochastic lead times. The goal is to minimize the total costs.

2.2. Periodic-Review Systems

Periodic review systems look at the decisions made at the beginning of each period and the results at the end of the period. However, in a multi-period model, every period's ending state becomes the next period's beginning state. This means that the backorders[5], inventory levels, replenishment decisions, and component lead times in one period will affect the decisions in the next period. [1]

Mohebbi and Choobineh [5] consider a periodic review ATO system with common component, measuring its average total inventory, fill-rate, and backorders in every period. Both demand and procurement lead times are random variables. They conclude that the effects of component commonality are more significant in a system with both supply and demand uncertainty.

2.3. Continuous-Review Systems

In a continuous-time ATO system, the manufacturer processes the orders as they are placed and can make allocation and replenishment decisions immediately. [2] This model is closer to real-world situation in many manufacturing plants that deal with the orders placed by the end-consumers. In reality, in consumer products,

[5] An order that has not been fulfilled because at least one of components was not available, and is placed in a queue until all of its components become available.

there is no certainty in demand time and volume. Even the most accurate predictions fail to model the demand perfectly, making the case for studies with random demand. All of the studies we see in this section model demand uncertainty, mostly using Poisson distribution. Considering the stochastic nature of the lead times here, that adds another level of complexity to the problem.

In order to ease the computational burden, most researchers have resorted to a base-stock replenishment policy assumption. However, Zhao and Simchi-Levi [6] and Zhao [7] assume batch-ordering replenishment policy and try to measure fill rate and calculate delivery lead times. Both develop numerical methods to assess the optimality of their solutions and use Monte Carlo simulation method for large samples.

Among studies that assume a base-stock policy, Glasserman and Wang [8] and Song and Yao [9] focus on measuring the trade-off between inventory and lead times, and inventory and service level respectively. Most other studies focus on measuring performance indicators like the fill-rate [6], costs, lead times, and component inventory levels. The rest, are concerned with minimizing costs and backorders and maximizing the fill-rate. Iravani et al. [10] show that relying solely on service level as a performance measure might mask the deeper problems with customer satisfaction. They propose a new way of measuring customer satisfaction based on differentiating between key and non-key items. Many researchers have struggled to come up with algorithms leading to global optimums for large-scale systems, and use heuristics, bounds and approximations instead.

Benjaafar and Elhafsi [11] have analyzed a case of different customer classes in combination with multiple components and production facilities, for the first time in the literature. Elhafsi et al. [12] argue that the optimal allocation and base-stock policies should be state-dependent. Elhafsi [13] extends the results to the case of

[6] Number of products that were assembled divided by the total number of products in a specific order.

compound Poisson demand, and offers heuristics for easing the computational burden of the solution.

Nadar and a team of researchers have conducted a series of studies on M-system and general ATO systems, to determine the optimal allocation and replenishment policies minimizing the expected discounted total costs. They have come up with lattice-dependent policies using MDP, and have proven their optimality for different configurations. [14] [15] [16] [17] [18] [19]

3. Study Characteristics and Assumptions

RQ4: How do these studies differ in their assumptions?

The following tables list the characteristics of all of the studies that have been summarized in this thesis:

Table 3 : Characteristics of Studies

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
Yano 1986	-	Two level assembly system with two components	-	-	Stochastic, independent, continuous, and twice differentiable
Glasserman 1998	Continuous	Multiproduct, multicomponent	Base-stock	Poisson Process	Stochastic, with exponentially distributed production times

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
Song 1999	Continuous	Multiproduct, multicomponent	Base-stock, independent	Multivariate Poisson process	Stochastic, i.i.d. exponentially distributed random variables
Lu 2001	Continuous	Multiproduct, multicomponent	Base-stock	i.i.d. random variables denote inter-arrival times	Stochastic, i.i.d. random variables
Song 2002	Continuous	Single product, multicomponent	Base-stock, independent	Poisson Process	Stochastic, i.i.d. random variables (replenishment lead times)
Lu 2003	Continuous	Multiproduct, multicomponent with component commonality	Base-stock, independent	Multiclass batch Poisson processes	Stochastic, i.i.d. random variables
Iravani 2003	Continuous	Multiproduct, multicomponent	Base-stock	Stationary Poisson process	Stochastic, component processing times are

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
					exponential random variables
Dayanik 2003	Continuous	Multiproduct, multicomponent	Base-stock	Poisson Process	Stochastic, exponentially distributed production times
Mohebbi 2004	Periodic	Three products, multicomponent	-	Random Variable	Assembly lead times takes one period; procurement lead times are random variables
Lu 2005	Continuous	Multiproduct, multicomponent	Base-stock	Batch Poisson process	Stochastic
Zhao 2006	Continuous	Multiproduct, multicomponent	Base-stock (single product) or batch-ordering (multiproduct)	Renewal demand arrivals for single product system, and independe	Stochastic, sequential, exogenous random variables (component lead times)

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
				nt Poisson process for multiproduct system	
Benjaafar 2006	Continuous	Single product, multicomponent, multiple customer classes	Base-stock (state-dependent)	A Poisson process for each class	Exponentially distributed production times for components
Lu 2007	Continuous	Multiproduct, multicomponent	Base-stock	Compound Poisson processes	Stochastic, i.i.d. random variables (replenishment lead times)
Elhafsi 2008	Continuous	Multiproduct, multicomponent, modular nested design	Base-stock, state-dependent	Poisson process	Stochastic, exponentially distributed component production times
Elhafsi 2009	Continuous	Multiproduct, multicomponent	Base-stock, state-dependent	Independent compound Poisson processes, with multiple	Stochastic, independent and exponentially distributed

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
				demand classes	
Zhao 2009	Continuous	Multiproduct, multicomponent	Batch-ordering	Independent compound Poisson processes	Stochastic, sequential, and exogenous replenishment lead times
Gao 2010	Continuous	Multiproduct, multicomponent	Base-stock, independent with finite capacity	Independent Poisson processes, multiple demand classes	Stochastic, exponential distribution during up times and repair times
Lu 2010	Continuous	Multiproduct, multicomponent	Base-stock, independent	Poisson Process	Non-identical constant or stochastic
Xiong 2011	Single Period	Single product, multicomponent	No Base-stock	Stochastic, constant probability of cancellation	Stochastic, i.i.d. component lead times
Nadar 2011	Continuous	Multiproduct, multicomponent M-system	Made to stock in fixed-sized batches,	Independent Poisson process	Stochastic, independent and exponentially

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
			lattice-based		distributed component production times
Nadar 2012	Continuous	Multiproduct, multicomponent	Made to stock in fixed-sized batches	Independent Poisson process	Stochastic
Zhi 2013	Continuous	Multiproduct, multicomponent	Base-stock, state-dependent	Independent Poisson processes	Stochastic, exponentially distributed component production times
Nadar 2014	Continuous	Multiproduct, multicomponent M-system	Made to stock in fixed-sized batches	Independent Poisson process	Stochastic, independent and exponentially distributed component production times
Elhafsi 2014	Continuous	Single product, multicomponent	Base-stock, state-dependent	Independent Poisson processes for product and	Stochastic, non-identical and exponentially distributed

Reference	Review period	System configuration	Replenishment policy	Demand distribution	Lead time type
				component demand	production times
Nadar Oct 2015	Continuous	Multiproduct, multicomponent	Made to stock in fixed-sized batches	Independent Poisson process	Stochastic
Nadar Apr 2016	Continuous	Multiproduct, multicomponent M-system	Made to stock in fixed-sized batches	Independent Poisson process	Stochastic
Nadar 2016	Continuous	Multiproduct, multicomponent	Made to stock in fixed-sized batches	Independent Poisson process	Stochastic

3. Quality Assessment of The Studies

Based on the quality criteria defined in the chapter II, we have conducted a thorough review of the studies to determine their relative quality. The results are presented in the following Table 4.

TABLE 4: QUALITY ASSESSMENT

STUDY	QC1	QC2	QC3	QC4	QC5	QC6	QC7	TOTAL
YANO 86	0	0	0	0	0	0	0	**0**
GLASSERMAN 98	1	1	1	1	0	0	0	**4**

SONG 99	1	1	1	1	1	0	0	**5**
LU 2001	1	0	1	1	0	0	0	**3**
SONG 2002	0	1	1	1	0	0	0	**3**
LU 2003	1	0	1	1	1	0	0	**4**
IRAVANI 2003	1	0	1	1	0	0	0	**3**
DAYANIK 2003	1	1	1	1	1	0	0	**5**
MOHEBBI 2004	0	1	1	0	0	0	0	**2**
LU 2005	1	1	1	1	1	0	0	**5**
ZHAO 2006	1	1	1	1	1	0	0	**5**
BENJAAFAR 2006	0	1	1	1	1	1	1	**6**
LU 2007	1	1	1	1	1	0	0	**5**
ELHAFSI 2008	1	1	1	1	1	1	1	**7**
ELHAFSI 2009	1	1	1	1	1	1	1	**7**
ZHAO 2009	1	1	1	1	1	0	0	**5**
GAO 2010	1	1	1	1	1	0	0	**5**
LU 2010	1	1	1	1	1	0	1	**6**
NADAR 2011	0	1	1	1	1	1	1	**6**

XIONG 2011	0	0	1	0	0	0	0	**1**
NADAR 2012	1	1	1	1	1	1	1	**7**
ZHI 2013	1	1	1	1	1	1	1	**7**
NADAR 2014	0	1	1	1	1	1	1	**6**
ELHAFSI 2014	0	1	1	1	1	1	1	**6**
NADAR 2015	1	1	1	1	1	1	1	**7**
NADAR APR 2016	0	1	1	1	1	1	1	**6**
NADAR 2016	1	1	1	1	1	1	1	**7**

IV. ASSEMBLE-TO-ORDER SYSTEMS WITH STOCHASTIC LEAD TIMES: BODY OF KNOWLEDGE

In this chapter, we will review the some of the most in papers written on assemble-to-order systems with stochastic lead times. Papers are categorized based on their subject of study, and ordered chronologically. We look into their review period, assemble-to-order system setting, demand distribution, lead time type, and objective. A short summary of the paper follows this information. This will give the interested reader a detailed view of the previous works done on the topic.

RQ1: Which studies have focused on ATO systems with stochastic lead times?

1. STOCHASTIC LEAD TIME IN TWO-LEVEL ASSEMBLY SYSTEMS

Date: April 1986

Author: Yano, Candace Arai

Journal: IIE Transactions, Vol. 19

Assumptions:

Review period: -

System configuration: Two level assembly system with two components

Demand distribution: -

Lead time type: Stochastic, statistically independent, continuous, and twice differentiable

Objective:

Minimizing the sum of inventory holding costs and tardiness costs

Evaluation Method: Numerical analysis

Summary:

The paper focuses on determining the planned lead times in two-level assembly systems, when lead times are stochastic. The author emphasizes safety time rather than safety stock because of the uncertainty in timing. The problem is solved optimally using an algorithm.

Evaluation Results:

According to computational results, in the optimal solution, usually at least one of the components will have negative safety times and the rest will have significant safety times in the assembly phase. The paper discusses the conditions under which the solution will have negative safety times. It also discusses the effects on lead times variance on the solution. It states that optimal safety times and therefore inventory holding costs will increase as the number of components go high. Longer average seems to be a better strategy than higher variance for planning the lead times. [20]

2. LEAD TIME-INVENTORY TRADE-OFFS IN ASSEMBLE-TO-ORDER SYSTEMS

Date: November 1998

Author: Paul Glasserman, Yashan Wang

Journal: Operations Research, Vol. 46, No. 6

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Poisson

Lead time type: Stochastic, with exponentially distributed production times

Objective:

Measuring the sensitivity of lead times – inventory trade-off at a fixed fill rate

Evaluation Method: Numerical analysis

Summary:

When discussing a service level target, inventory becomes one of the most important factors, as there is always a trade-off between inventory levels and delivery lead times. The paper shows that under certain circumstances, with a base-stock policy, the trade-off between inventory and service level, measured by the fill rate, is linear and approximates the marginal rate. It also investigates the effect of order combination in terms of diversity of the items on this relation. The assembly operation is presumed to be uncapacitated.

Evaluation Results:

The paper discusses the formulation and offers a numerical analysis with several different distributions and classes of production. They conclude that the trade-off in the single-item systems is the sharpest. In multiproduct systems however, the trade-off depends on the effect of components on constraining the fill rate. [8]

3. ORDER FULFILLMENT PERFORMANCE MEASURES IN AN ASSEMBLE-TO-ORDER SYSTEM WITH STOCHASTIC LEAD TIME

Date: January 1999

Author: Jing-Sheng Song, Susan H. Xu and Bin Liu

Journal: Operations Research, Vol. 47, No. 1

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Multivariate Poisson process

Lead time type: Stochastic, i.i.d. exponentially distributed random variables

Objective:

Obtaining the exact performance measures for the system (item-based, order-based, system-based)

Evaluation Method: Numerical analysis

Summary:

This paper studies order-fulfillment performance measures for ATO systems with stochastic lead times. It assumes a multiproduct and multicomponent system with sequential, capacitated stochastic production processes. There is an independent base-stock policy for each component and demand allocation follows an FCFS rule with an infinite planning horizon. There is a backlog queue with capacity from zero (lost sale) to infinity. Several performance measures including waiting-time distribution, fill rate, and service level are calculated. Two subcategories are considered: partial and total order service; and multi-dimensional Markov chains are used for analysis.

Evaluation Results:

Numerical results are discussed in details. They investigate the effects of system-based parameters on the system-based, order-based and item-based performance, and compare the output for POS and TOS, to find the similarity in the behavior of two models. and come to conclusion that in general, the order-based measures behave similar to item-based ones. They also conclude that the congestion level in POS system is higher than TOS, which has a negative effect of POS fill rate and waiting time. This is the first paper that has done an exact numerical analysis of its kind. [21]

4. Performance Analysis of Assemble-to-Order Systems through Strong Approximations

Date: December 2001

Author: Yingdong Lu, Jing-Sheng Song, Weian Zheng

Journal: ACM SIGMETRICS Performance Evaluation Review, Vol. 29, I. 3

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: i.i.d. random variables denote inter-arrival times

Lead time type: Stochastic, i.i.d. random variables

Objective:

Measuring the fill rate

Summary:

This is a very short paper on approximation techniques which applies the “diffusion approximation schemes” used in analyzing queues, to ATO systems. It considers a multiproduct, multicomponent ATO with base-stock policy, and models it as a GI/GI/1 queueing system[7]. The performance measures that are of interest here are fill rate and "fill rate with response windows." The diffusion approximation is used to calculate their steady-state distribution.

Evaluation Results:

The paper discusses the mathematical proof of their proposed formula; however, no numerical analysis is provided. [22]

5. Performance Analysis and Optimization of Assemble-to-Order Systems with Random Lead Times

Date: September 2002

Author: Jing-Sheng Song and David D. Yao

Journal: Operations Research, Vol. 50, No. 5

Assumptions:

Review period Continuous

System configuration: Single product, multicomponent

Demand distribution: Poisson

Lead time type: Stochastic replenishment lead times for each component, i.i.d. random variables

Objective:

Measuring the optimal trade-off between inventory and service

Evaluation Method: Numerical analysis

[7] Single-server queue with general independent distribution of interarrival times

Summary:

Inventory is a sensitive subject as running out of components can lengthen the lead times, and stocking too many units will increase the costs. The paper assumes an independent base-stock policy and an infinite-server model, with an uncapacitated supply mechanism. The optimal level of base-stocks for each component is assessed through two optimization problems: minimizing "the expected number of backorders" and "average component inventory" under certain conditions. Due to the exponential order of complexity, the authors have developed lower and upper bounds for the performance measures to create surrogate problems and solved those problems using greedy algorithms.

Evaluation Results:

Numerical examples show the performance of solutions for different distributions of lead times (deterministic, uniform, Erlang, and Exponential). The solutions are near-optimal for the actual problem; however, they cannot easily be extended to multiproduct problem. The shape of the trade-off curve is almost linear and rather insensitive to lead time distribution. [9]

6. ORDER FILL RATE, LEAD TIME VARIABILITY, AND ADVANCE DEMAND INFORMATION IN AN ASSEMBLE-TO-ORDER SYSTEM

Date: March 2003

Author: Yingdong Lu, Jing-Sheng Song, David D. Yao

Journal: Operations Research, Vol. 51, No. 2

Assumptions:

Review period: Continuous

System configuration: Multiproduct, multicomponent with component commonality

Demand distribution: Multiclass batch Poisson processes

Lead time type: Stochastic, i.i.d. random variables

Objective:

Measuring the response-time based order fill-rate

Evaluation Method: Numerical analysis

Summary:

This paper analyses an ATO system with multiple products and components, an independent base-stock replenishment policy, FCFS allocation rule, backorders, and exogenous suppliers with infinite capacity. Since there is a correlation between component demands, an accurate optimal solution is not known, and that is where base-stock policies come into play. Off-the-shelf order fill rate (based on the components that are currently available) is of primary interest and it is calculated using "the steady-state joint distribution of outstanding orders." The paper further analyses formulations, the impact of lead times variability and product structure, and approximation methods. It also discusses the role of advanced demand information.

Evaluation Results:

The paper discusses the effectiveness of approximation methods using numerical results, with exponential and Erlang distributions. They conclude that "the factorized normal approximation overestimates the fill rates in all cases", and the pairwise normal approximation overestimates it in most cases, outperforming the factored normal approximation in all cases. "The pairwise lower bound outperforms the marginal lower bound in all cases," and in high fill rates (+0.95), outperforms other three methods. For fill rate 0.8 to 0.85, the pairwise normal approximation is the best with a reliable marginal lower bound. The other two methods outperform between 0.85 and 0.95 fill rates. Factorized normal approximation and marginal lower bound can be used for practical estimates, as they perform well without a significant computational burden. [23]

7. On Assemble-to-Order Systems with Flexible Customers

Date: May 2003

Author: S.M.R. Iravani, K.L. Luangkesorn and D. Simchi-Levi

Journal: IIE Transactions, Vol. 35

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Stationary Poisson process

Lead time type: Stochastic, component processing times are exponential random variables

Objective:

Measuring system performance based on customer satisfaction

Evaluation Method: Numerical analysis

Summary:

The paper aims to evaluate the quality of service and considers an ATO in which items in each order are divided in two groups of key and non-key items. Key items are those functionally essential items without which the selective customer would be lost. The authors assume that some selective customers will accept substitution of non-key items, and flexible customers are willing to compromise even key items. They build on the assumptions in the Song et al. [21] and assume that each demand will not contain more than one of each item. They also assume base-stock policy and define a Markov chain with finite state space, incorporating the substitution probabilities. These probabilities depend on manufacturer's strategy, items' availability, and customers' preferences. They compare the customer satisfaction with traditional service level measure defined in the literature and conclude that high service level can – under certain circumstances – conceal customers' dissatisfaction, jeopardizing the long-term profit of the firm.

Evaluation Results:

The paper examines several scenarios in its numerical analysis to measure the effects on base-stock levels and customer satisfaction on profit. The numerical results indicate that base-stock increase can in some cases lead to lower total profits, and relying on customers' flexibility can erode profits in the long run. Also, they show the importance of factoring in customer satisfaction in the decision making process. [10]

8. The Effectiveness of Several Performance Bounds for Capacitated Production, Partial Order Service, Assemble-to-Order Systems

Date: July 2003

Author: S. Dayanik, Jing-Sheng Song and Susan H. Xu

Journal: Manufacturing & Service Operations Management, Vol. 5, I. 3

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent, possible common components

Demand distribution: Poisson

Lead time type: Stochastic, exponentially distributed production times

Objective:

Measuring order fill-rate and service level

Evaluation Method: Numerical analysis

Summary:

The paper assumes an ATO with multiple products and components, FCFS allocation rule, possibility of partial order service, backorders, lost sales, and a base-stock policy. Exponential distribution is assumed for interarrival and production times, which leads to a Markovian model. But this paper tries to develop a less complex approach through performance estimates to approximate service level in different ranges. The authors develop setwise lower and upper bounds for fill rate using greedy heuristics, and extend the results to other metrics. They also develop signal and setwise-Bonferroni lower bounds using another algorithm, and Frechet upper bounds; and analyze and compare the results. Setwise bounds are tighter than the other bounds developed. They further discuss the capacitated and uncapacitated ATO models and how the bounds apply to each one. In general, in systems with above 80% fill rate, except for the signal bound, all lower bounds perform well. But the upper bound performance is highly dependent on the fill rate above 95%.

Evaluation Results:

The paper discusses the numerical results for both backlogging and lost sales, under the same service levels. For a system with nine components, the signal lower bound is generally unsatisfactory, and the Bonferroni bound underperforms the combination bound due to possibility of degeneracy. The first order lower bounds are less accurate than the second order ones. The best lower bound is the second order exact setwise lower bound. The setwise upper bounds are also better than the Frechet bounds. [24]

9. The Impact of Component Commonality in an Assemble-to-Order Environment under Supply and Demand Uncertainty

Date: September 2004

Author: E. Mohebbi and F. Choobineh

Journal: Omega, Vol. 33

Assumptions:

> **Review period** Periodic
>
> **System configuration:** Three products, multicomponent
>
> **Demand distribution:** Random variable
>
> **Lead time type:** Assembly lead times takes one period; procurement lead times is a random variable

Objective:

Measuring "the average total inventory of components per period, the average proportion of products' demand orders per period that were fully satisfied on time (service level), and the average total backorder of finished products per period."

Evaluation Method: Simulation and Numerical analysis

Summary:

The paper focuses on assessing the benefits of component commonality in ATO systems in presence of demand and lead times uncertainty. The authors have designed a simulation in which backordering is possible and there are no safety stocks. Smaller orders have rationing priority, and backorders are filled after the current orders. Four factors are subject of this experiment: product structure, forecast and actual demand for finished products, components' procurement lead times, and initial inventories.

Evaluation Results:

The paper concludes that if there are demand and supply lead times uncertainty at the same time, the average total inventory for each component per period decreases. Demand and supply uncertainty lower the service level. Also, lead times uncertainty and higher number of components increase the average backorders per product in each period. On the other hand, in each period, component commonality increases the service levels. The common component's average total inventory level is lower than the components it replaces. The effects of common component are more notable in systems with both lead times and demand uncertainty. [5]

10. Backorder Minimization in Multi-Product Assemble-to-Order Systems

Date: 2005

Author: Yingdong Lu, Jing-Sheng Song, David D. Yao

Journal: IIE Transactions, Vol. 37

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Batch Poisson Process

Lead time type: Stochastic

Objective:

Optimizing the allocation of budget to inventories to minimize a weighted average of backorders over product types

Evaluation Method: Numerical analysis

Summary:

The paper assumes a multiproduct, multicomponent ATO with base-stock policy where demands are filled based on an FCFS allocation rule. Backorders are possible, and will be filled based on FCFS rule. The complexity of solution depends on how many sets of common components the products have. The authors develop two surrogate problems for the objective function and solve them using approximation techniques. First they relax the integer requirement and develop a lower bound using greedy heuristics. Then they develop an upper bound that can be solved greedily.

Evaluation Results:

The authors perform a numerical analysis on the examples. Based on the results, they claim that using their approach will lead to close-to-optimal results for the original problem in all cases. [25]

11. Performance Analysis and Evaluation of Assemble-to-Order Systems with Stochastic Sequential Lead Times

Date: July 2006

Author: Yao Zhao and David Simchi-Levi

Journal: Operations Research, Vol. 54, No. 4

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Renewal demand arrivals for single product system, and independent Poisson process for multiproduct system

Lead time type: Stochastic, sequential, exogenous random variables (for components)

Evaluation Method: Numerical analysis

Objective:

Measuring mean and variance of delivery lead times, the inventory holding costs of components, and order fill-rate for a single product system, and characterizing the dependence between stock-out delays of the components

Summary:

The paper focuses on continuous-time "ATO systems with uncapacitated component production" and backlogging with either base-stock (single product) or batch-ordering policy (multiproduct) for inventory management. Orders are filled based on the FCFS allocation rule, and lead times are stochastic and sequential (transit times). The paper focuses on the joint probability distribution of component delays which are the difference between the time an order is placed and the time its components become available, and derives the production lead times distribution based on that. For multiproduct systems, the authors decompose the system into multiple single product systems and analyze them. This analysis can be used to evaluate optimization algorithms developed by other researchers.

Evaluation Results:

Numerical analysis is done using the backward method, Clark's two-moment approximation (for small to medium size problems) and Monte Carlo simulation (for medium to large problems). The complexity of the last two methods is linear in the number of products. [6]

12. PRODUCTION AND INVENTORY CONTROL OF A SINGLE PRODUCT ASSEMBLE-TO-ORDER SYSTEM WITH MULTIPLE CUSTOMER CLASSES

Date: December 2006

Author: Saif Benjaafar and Mohsen Elhafsi

Journal: Management Science, Vol. 52, No. 12

Assumptions:

Review period Continuous

System configuration: Single product, multicomponent, n customer classes

Demand distribution: A Poisson process for each class

Lead time type: Exponentially distributed production times for components

Objective:

Determining optimal allocation and replenishment policies under total discounted cost and average cost

Evaluation Method: Numerical analysis

Summary:

The paper models an ATO system with multiple customer classes and instantaneous assembly as a Markov decision process. An order will either be fulfilled, backordered, or lost, potentially incurring shortage cost and component holding cost. These cost determine whether it is best to reject or fulfill a demand to serve a future demand from a more important customer class. Each component is produced at a separate facility with an independent and exponentially distributed production times, and its production can be either initiated or interrupted at every state. The continuous-time problem is transformed into a discrete one using uniform transition rates. The authors prove that components' optimal production policy is a "state-dependent base-stock policy", and the state is defined based on the components' inventory level. Increasing one component's inventory level leads to a non-decreasing change in other components' base-stock, and interruption in a component's production is never optimal. The optimal component rationing levels of each customer class are multilevel and state-dependent and "non-increasing in inventory level of other components".

Evaluation Results:

The paper includes a numerical analysis of optimal policies with lost sales and backorders, in which the proposed policy is compared to a fixed base-stock policy and a heuristic where the difference between two components' inventory is measures against a threshold. The results are obtained by applying a value iteration algorithm to the dynamic programming equation, for the average cost case. The complexity of this algorithm is exponential to the number of components. In the case of backorders, heuristics are less effective, and there is greater difference between different heuristics. Furthermore, coordination between component levels has a more significant effect as compared to the case with lost sales; since in the latter the component level difference is bounded. [11]

13. ESTIMATION OF AVERAGE BACKORDERS FOR AN ASSEMBLE-TO-ORDER SYSTEM WITH RANDOM BATCH DEMANDS THROUGH EXTREME STATISTICS

Date: February 2007

Author: Yingdong Lu

Journal: Naval Research Logistics, Vol 54, I. 1

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Compound Poisson processes

Lead time type: Replenishment lead times are i.i.d. random variables

Objective:

Measuring average backorders, minimizing total operational costs (inventory costs and backorder penalty costs)

Evaluation Method: Numerical analysis

Summary:

The paper considers an ATO system with batched customer orders, uncapacitated replenishment, backordering, base-stock policy, FCFS allocation rule and priority policy. The author discusses the mathematical shortcomings of the previous solutions and approximations, and for single item systems proposes an alternative solution through a random walk approach to calculate the mean and variance of backorders, focusing on the stationary behavior of the system. Then he uses extreme statistics to extend the results to multiproduct ATO systems, and shows that the results improve as the system size grows. The method is similar to normal approximation, using central limit theorem.

Evaluation Results:

Numerical analysis is done using examples. The paper also includes a cost minimization problem, in order to obtain an optimal base-stock level. They solve the problem using interior point nonlinear optimization tool IPOPT and then the closed form approximation and objective value are estimated through simulation. The errors decrease as the system grows larger. [26]

14. Optimal Control of a Nested-Multiple-Product Assemble-to-Order System

Date: November 2008

Author: Mohsen ElHafsi, Herve Camus and Etienne Craye

Journal: International Journal of Production Research, Vol. 46, I. 19

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent, modular nested design

Demand distribution: Poisson processes

Lead time type: Stochastic, exponentially distributed component production times

Objective:

Determining optimal production and inventory allocation policies

Evaluation Method: Numerical analysis

Summary:

This paper studies an ATO with nested product design, where components are produced in separate facilities, one unit at a time, before demand arrival. There is a holding cost associated with component inventory and non-identical lost sale cost associated with unfulfilled demand. This means that sometimes the future demand might take precedent over the current demand. The authors use a MDP to model the problem, where the state is defined by components' inventory level. They conclude that for each component, the optimal inventory policy is a base-stock policy and the optimal allocation policy is multi-level rationing policy, and they are state-dependent.

Evaluation Results:

The authors run a numerical analysis comparing the performance of the proposed optimal policies to fixed base-stock and fixed allocation heuristics ("Multi-Product Independent Base-stock with Rationing or MPIBR"). The arising dynamic programming problem is solved using "the relative value iteration algorithm". The results obtained for the heuristic are suboptimal but effective if parameters are chosen correctly. [12]

15. Optimal Integrated Production and Inventory Control of an Assemble-to-Order System with Multiple Non-Unitary Demand Classes

Date: April 2009

Author: Mohsen Elhafsi

Journal: European Journal of Operational Research, Vol. 194, I. 1

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent compound Poisson processes

Lead time type: Stochastic, independent and exponentially distributed

Objective:

Determining optimal production and inventory allocation policies

Evaluation Method: Numerical analysis

Summary:

The paper uses the same model as Benjaafar and Elhafsi's [11], with multiple demand classes. The author uses an MDP for modeling the problem and considers the similar state-dependent allocation and base-stock policy proven by Elhafsi et al. [12] to be optimal. Several aspects of the system including order size, order size variability, and inventory rationing benefits are considered.

Evaluation Results:

A numerical experiment is done, using dynamic programming with the average cost rate criterion. The complexity is exponential to the number of components and heuristics are used to replace the optimal policy. The results show that the optimal average cost increases with size variability and order size. The author concludes that the order size variability is more detrimental than the order size in determining the optimal average cost rate. He shows that FCFS allocation rule may lead to higher total costs, compared to the proposed rule when demand classes are accounted for, making the case for inventory rationing. Since the optimal policy computation is NP-hard, two heuristics are proposed: Coordinated Base-stock with Rationing and Compound Demand (CBRCD), and Independent Base-stock with Rationing with Compound Demands (IBRCD) policy. They both offer suboptimal but effective performance. [13]

16. Analysis and Evaluation of an Assemble-to-Order System with Batch Ordering Policy and Compound Poisson Demand

Date: 2009

Author: Yao Zhao

Journal: European Journal of Operational Research, Vol. 198

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent compound Poisson processes

Lead time type: Stochastic, sequential, and exogenous replenishment lead times

Objective:

Calculating expected delivery lead times and order-based fill-rates

Evaluation Method: Numerical analysis

Summary:

The paper assumes an ATO with FCFS allocation rule, continuous-time batch ordering policy for component inventory replenishment, and assumption of both split and non-split orders. This is a new step in the ATO systems research as it combines batch-ordering and compound Poisson demand. The author shows exact expressions for measuring performance metrics for the case of a single product with two components and develops algorithms to evaluate them for large ATO systems. He concludes that delivery lead times are overestimated when the dependence among demand sizes or interarrival times is ignored.

Evaluation Results:

A Monte Carlo simulation is used to develop a numerical method for estimation of fill-rate and delivery lead times. An example is detailed to show that fill-rates are always smaller in the case of split orders, and the difference between split and non-split order cases is significant (almost 20%) which makes order-splitting worthwhile. [7]

17. Order-Fulfillment Performance Analysis of an Assemble-to-Order System with Unreliable Machines

Date: April 2010

Author: Chunyan Gao, Houcai Shen, T.C.E. Cheng

Journal: Int. J. Production Economics, Vol. 126

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent Poisson processes, multiple demand classes

Lead time type: Stochastic, general distribution, exponential distribution during uptime

Objective:

Measuring order-based and item-based the fill rate within a time window and the service level

Evaluation Method: Numerical analysis

Summary:

The paper assumes an ATO similar to the one in [21] with backlogging and finite capacity base-stock policy, where components are replenished by independent unreliable machines based on FCFS allocation rule. The processing time, up time and repair time of every machine follow an exponential distribution, and are mutually independent. Partial and total order of service are considered here. The authors model the problem as a Markov decision process. They derive the "exact joint stationary distribution of on-hand inventories" by "matrix-geometric solution approach" and calculate key system measures. They also develop a new method for approximation in large scale systems and test it in a numerical experiment to show the importance of considering machine failures.

Evaluation Results:

The numerical analysis tries to examine the item-based and order-based performance measures in TOS and POS models. Order-based and item-based fill rates and service level for an item decrease as its failure rate increases. Decrease of the failure rate and increase of repair rate of a machine have the same effect. In the POS model, items are used more quickly, which means the machines will work more and consequently be more likely to break down. Increasing the initial inventory level will neutralize the effect of base-stock level, by increasing the probability of break down. [27]

18. No-Holdback Allocation Rules for Continuous Time Assemble-to-Order Systems

Date: June 2010

Author: Yingdong Lu, Jing-Sheng Song and Yao Zhao

Journal: Operations Research, Vol. 58, No. 3

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Poisson

Lead time type: Non-identical constant or stochastic

Objective:

Minimizing the average component inventory, total product backorders and maximize fill-rate

Evaluation Method: Numerical analysis

Summary:

The paper considers an ATO system with multiple products and components, and possibility of one or more common components, with an independent base-stock replenishment policy and complete backlogging. It analyses the performance of ATO systems with No-Hold-Back allocation rules with those that have FCFS allocation rule. With NHB which is a product-based state-dependent allocation rule, if there is a demand with at least one unavailable component, it is placed at the end of the backorder list. This lowers the waiting time for products whose components are available by not assigning those components to other products that cannot be assembled at the time. The performance measures mentioned here are the component inventory, average cost, product backorders, backorder waiting time, and product fill rates. "Flow conservation equations" and "sample path approach" are used for analysis and a W-system is broken down to single-item systems.

Evaluation Results:

The authors demonstrate that the NHB rules perform better than FCFS on immediate fill-rate, but not in fill-rate with a time window, and with symmetric cost structures, they globally minimize the total inventory and backorder costs. The total backorder minimization results hold true for a W-System, M-Systems and N-Systems under certain circumstances. The authors also present approximations based on bounds for backorders obtained using linear programming, and test the accuracy of their approximation through a numerical example; then they compare the effectiveness of NHB with the FCFS rules. The results can be applied to systems with stochastic lead times as well. [28]

19. Coordinating Order Decisions for Assemble-to-Order Enterprise with Uncertainty in Purchase Lead Times and Demand Order

Date: 2011

Author: Yanhua Xiong

Journal: IEEE

Assumptions:

Review period Single period

System configuration: Single product, multicomponent

Demand distribution: Stochastic, with a constant probability of cancellation

Lead time type: Stochastic and i.i.d. component lead times

Objective:

Minimizing the total expected cost

Evaluation Method: Numerical analysis

Summary:

The paper analyses purchasing policy in an ATO with negligible assembly time, no base-stock, and perishable components. The customer communicates an initial order volume forecast to the manufacturer, and places the actual order (or cancels it) a while after that. The manufacturer should decide about the component procurement time and volume, if he wants to minimize the total costs. The author calculates the optimal solution mathematically.

Evaluation Results:

The author uses an example to illustrate the results. He concludes that in case of high cancellation probability, one should delay the component procurement order until the actual end-product order has been placed. [4]

20. NEW FUNCTIONAL CHARACTERIZATIONS AND OPTIMAL STRUCTURAL RESULTS FOR ASSEMBLE-TO-ORDER GENERALIZED M-SYSTEMS

Date: September 2011

Author: Emre Nadar, Mustafa Akan and Alan Scheller-Wolf

Journal: Carnegie Mellon University Research Showcase @CMU (online)

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent M-system

Demand distribution: Independent Poisson process

Lead time type: Stochastic, independent and exponentially distributed component production times

Objective:

Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

Evaluation Method: -

Summary:

The paper models an ATO with M-system setting and lost sales possibility as an "infinite-horizon Markov decision process." Components are made to stock in fixed-sized batches, with independent and exponentially distributed production times and demand intervals. There can be multiple unit of each component in every end-product. The convexity of the cost function is not guaranteed; therefore, the authors use products' component requirements to partition the state space into disjoint lattices, in order to make the optimal cost function convex. Thus, they transform a continuous-time problem to a discrete-time problem. They also prove that this function is "submodular on each of the multiple disjoint lattices of the state space." They conclude that "lattice-dependent base-stock production policy" and "lattice-dependent rationing policy" are conditionally optimal. In the optimal solution, the component replenishment batch sizes should correspond to individual product sizes. Under this condition, producing a batch of a component is optimal if and only if its base-stock level is more than the state vector. The optimality of fulfilling a product's demand depends on its rationing level being less than or equal to the state vector. Once a component is replenished, the optimal base-stock level of all other components, and the optimal rationing level of all non-corresponding products increase (in a non-strict sense). The master product's optimal rationing level decreases, increasing the incentive to fulfill its demand. The authors analyze the results for N-systems, multiple demand classes, variable replenishment quantities, and compound Poisson demand.

Evaluation Results:

Nadar et al. [15] compare the results of a numerical experiment for LBLR, SBSR, and FBFR policies. [14]

This paper and similar studies that will follow are related to a PhD dissertation on the same subject that can be found in the references. [29]

21. PERFORMANCE EVALUATION OF LATTICE-DEPENDENT BASE-STOCK AND RATIONING POLICIES FOR ASSEMBLE-TO-ORDER SYSTEMS

Date: February 2012

Author: Emre Nadar, Mustafa Akan and Alan Scheller-Wolf

Journal: Carnegie Mellon University Research Showcase @CMU (online)

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent Poisson process

Lead time type: Stochastic

Objective:

Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

Evaluation Method: Numerical analysis

Summary:

The paper discusses an ATO with batch-ordering policy and lost sales, and possibility of ordering products with multiple units of the same component. This paper, focusing on general ATO system, is a continuation of another study published by Nadar et al. [14] that analyses a Markov decision process in the M-system. It compares the lattice-dependent base-stock and rationing policies (LBLR) with state-dependent (SBSR) and fixed (FBFR) policies through numerical studies, using these policies as heuristics for a general ATO system, and focusing on the measurement of average cost rate.

Evaluation Results:

The authors conduct numerical experiments for general ATO systems to evaluate the performance of an LBLR policy as a heuristic, comparing it with two other heuristics: a FBFR and SBSR. Linear programming is used to calculate a global optimal cost, and mixed integer programming is utilized for calculation of optimal cost in each heuristic class. LBLR policies outperform SBSR and FBFR by a maximum of 2.7% and 4.8% margin, giving a global optimal cost in all samples. FBFR policies underperform the other two sets, due to the lack of coordination between the reordering batch sizes and component usage rates in assembly stage. Submodularity and supermodularity are not necessarily true for a general ATO system, so the optimality of LBLR should be proven through a different method. [15]

22. INTEGRATED PRODUCTION AND INVENTORY CONTROL OF ASSEMBLE-TO-ORDER SYSTEMS WITH INDIVIDUAL COMPONENTS DEMAND

Date: September 2013

Author: Li Zhi

Journal: Laboratoire d'Automatique, Génie Informatique et Signal (LAGIS) in École Centrale de Lille

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent Poisson process

Lead time type: Stochastic production times

Objective:

Determining optimal allocation and replenishment policies, to minimize operating costs while maintaining order fulfillment

Evaluation Method: Numerical analysis

Summary:

This is a PhD dissertation focusing on an ATO system with stochastic production and customer interarrival times. There is a cost associated with lost sales, backordering, and holding inventory. Similar to the works of Benjaafar and Elhafsi [11] and Elhafsi et al. [12], the author uses MDP to find the optimal policy "under the total expected discounted cost and the average cost per period criteria." The author concludes that the optimal policies are state-dependent base-stock and rationing policies. Several different heuristics are developed and their performance is evaluated through numerical experiments.

Evaluation Results:

The performance of heuristics is measured through the percentage cost difference between the heuristic and the optimal policy. H1 is very close to the optimal policy in the pure lost sales case. The best heuristic is H4 which is the one that considers all the base-stock levels and the rationing levels. But this heuristic is computationally intensive. Therefore, H5 and H7 that are computationally more efficient and in most cases have very close results are recommended. [30]

23. TECHNICAL NOTE - OPTIMAL STRUCTURAL RESULTS FOR ASSEMBLE-TO-ORDER GENERALIZED M-SYSTEMS

Date: May 2014

Author: Emre Nadar, Mustafa Akan and Alan Scheller-Wolf

Journal: Operations Research, Vol. 62, No. 3

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent M-system

Demand distribution: Independent Poisson process

Lead time type: Stochastic

Objective:

Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

Summary:

This paper is a part of studies by Nadar et al. on ATO systems that focuses on an M-system with lost sales and batch ordering, using Markov decision process. It further discusses the functional characterization and proofs of [14]. [18]

24. AN ASSEMBLE-TO-ORDER SYSTEM WITH PRODUCT AND COMPONENTS DEMAND WITH LOST SALES

Date: June 2014

Author: Mohsen ElHafsi, Li Zhi, Herve Camus, Etienne Craye

Journal: International Journal of Production Research, Vol. 53, I. 3

Assumptions:

Review period Continuous

System configuration: Single product, multicomponent

Demand distribution: Independent Poisson processes for product and component

Lead time type: Stochastic, non-identical and exponentially distributed production times

Objective:

Determining optimal production and inventory allocation policies

Evaluation Method: Numerical analysis

Summary:

The paper assumes a single product ATO with similarities to that of Benjaafar and Elhafsi [11]. The problem is solved using MDP and dynamic programming. A state-dependent threshold or base-stock policy and “state-dependent threshold or rationing type” are optimal. The optimal policies are then compared in the case of lost sales. The authors show that the optimal rationing policy is not static and depends on the inventory levels of components. In addition, an increase in the components inventory level decreases the desirability of fulfilling individual components’ demand.

Evaluation Results:

Performance evaluation of three heuristics is done through comparing their average cost rate (as a relative percentage difference) with that of the optimal policy. All three heuristics perform well in comparison to the optimal policy. The two more exhaustive heuristics give more accurate results. Therefore, the less time-consuming heuristics can be used to obtain the control threshold values and then the other two heuristics can be employed to refine the results using those values. [31]

25. REPLENISHMENT AND FULFILLMENT BASED AGGREGATION FOR GENERAL ASSEMBLE-TO-ORDER SYSTEMS

Date: October 2015

Author: Emre Nadar, Alp Akcay, Mustafa Akan, Alan Scheller-Wolf

Journal: Research gate (online)

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent Poisson process

Lead time type: Stochastic

Objective:

Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

Evaluation Method: Numerical analysis

Summary:

The paper considers the same problem formulation as Nadar et al. [14] for a general ATO system with a “state space consisting of component inventory levels”. To reduce the complexity and state space size, the authors develop a state space aggregation method that uses information about component requirements and batch sizes to form an aggregate state. This is a dynamic programming method that approximates the cost function by partitioning the state space into disjoint subsets (lattices) of the same size, with optimal rationing and base-stock policies. The authors define a disaggregation rule that reduces computation time of the aggregate problem by an average of 95.8%. They also prove the optimality of the LBLR policy discussed in their previous papers, for the general ATO system. Then they use the optimal policy structure to improve computational complexity of the value iteration algorithm in the aggregate state space. Furthermore, they analyze the aggregate problem in a case where one product has fulfilment priority, and calculate a finite error bound for the cost function. The bound is used to develop a new algorithm that would converge to the optimal solution for the original problem. This algorithm is proven to be computationally efficient.

Evaluation Results:

The numerical experiment examines the LBLR policy in detail. It considers different rules and compares their performance including “disaggregating each aggregate state into its two extreme original states (the smallest and the largest states)”, and a uniform rule that assigns equal probabilities to all original states. The results, measured by the the percentage deviation from the optimal cost, is on average 17.05% and 26.85% respectively, which proves the superiority of the first rule. The computational time of this rule is also 95.8% lower when using the value iteration algorithm. [16]

26. Experimental Results Indicating Lattice-Dependent Policies May Be Optimal for General Assemble-to-Order Systems

Date: April 2016

Author: Emre Nadar, Mustafa Akan, Alan Scheller-Wolf

Journal: Production and Operations Management, Vol. 25, I. 4

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent M-system

Demand distribution: Independent Poisson process

Lead time type: Stochastic

Objective:

Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

Evaluation Method: Numerical analysis

Summary:

This paper conducts another numerical experiment on the M-system introduced in Nadar et al. [14], building on the results presented in Nadar et al. [15]. It concludes that LBLR is better than SBSR in terms of computational complexity, as it performs much faster. In addition, it shows through a regression study that when product differentiation is significant and the fill rate should remain high, LBLR again outperforms SBSR. The results do not necessarily hold for a general ATO system.

Evaluation Results:

The numerical experiments evaluate the LBLR policy against SBSR and FBFR policies, over 22500 instances. LBLR finds the globally optimal cost, outperforming SBSR and FBFR both on the results and the computational time. [17]

27. The Benefits of State Aggregation with Extreme-Point Weighting for Assemble-to-Order Systems

Date: September 2016

Author: Emre Nadar, Alp Akcay, Mustafa Akan, Alan Scheller-Wolf

Journal: Research gate (Online)

Assumptions:

Review period Continuous

System configuration: Multiproduct, multicomponent

Demand distribution: Independent Poisson process

Lead time type: Stochastic

Objective:

Determining optimal allocation and replenishment policies, by minimizing the expected discounted total cost

Evaluation Method: Numerical analysis

Summary:

This paper builds on the results of Nadar et al. [17] , studying a general ATO system under the same conditions. The main focus is on state aggregation method and lattice-dependent policies.

Evaluation Results:

The authors conduct numerical studies to demonstrate the results obtained in this paper and Nadar et al. [16]. The proposed LBLR policy leads to an average of 96.4% gain in computation time, and an average of 6.32% distance from the optimal solution in the test cases. [19]

V. CONCLUDING REMARKS

In this thesis we reviewed the literature on assemble-to-order systems with stochastic lead times. Despite the extensive efforts in the operations management research community to model assemble-to-order systems, we are yet to see a general optimal model of these systems that could account for all of the complexities and uncertainties that the manufacturers face. The focus is mostly on optimizing the operations of a system under certain assumptions, and to a limited extent, on designing a better system with optimal policies. The performance measures that are mostly used are those related to customer satisfaction like fill-rate, or focused on cost-related measures. Modeling all of these complexities is no easy job; as the researcher has to consider the demand fluctuations, lead time (including production time) uncertainty, and uncertainty on the supplier side. Although the literature on stochastic lead time in inventory management systems is substantial, there is a relatively limited research history on stochastic lead time in assemble-to-order systems.

Most researchers working on this topic have opted for modeling the demand uncertainty in addition to the probabilistic lead times, usually with Poisson and exponential distributions, respectively. Nevertheless, they have mostly chosen to simplify the computational complexity of the problems by assuming a base-stock policy, and in many cases, a first-come-first-serve allocation rule. Lu, Song and Zhao [28] challenge the optimality of FCFS rule, proving the optimality of no-holdback-rules for certain ATO configurations.

Most works done prior to 2006 focus on state-independent policies. However, in a series of studies, starting in 2006, a team of researchers including Benjaafar, Elhafsi, Camus, Zhi, and Craye, have come up with optimal state-dependent replenishment and allocation policies, assuming compound Poisson processes for demand distribution. This takes us a step closer to finding a solution for the real-world manufacturing problems, as the demand for one component in many cases is correlated with the demand for other components; hence, their inventory levels will be inevitably correlated in an optimal policy. [11] [31] [13] [12] [30]

Markov decision processes have gained some ground in the recent years, leading to further development of state-dependent and lattice-dependent solutions that factor in the dependencies of the components and demand classes. But the algorithms developed so far, remain too complex to be applicable to large scale problems, and that calls for approximation methods and heuristics. Many of the papers we have listed have included bounds and approximations as well as heuristics; however, these methods are not always theoretically verified, and in some cases, the difference between upper and lower bounds are substantial. There is still room for improvement when it comes to approximations for ATO systems. Lu has made some effort to incorporate extreme statistics into his performance analysis of the ATO systems. [26] Other papers rely on different methods but their assumptions limit the practicality of their methods for a general ATO system.

A relatively new team of researchers including Nadar, Akan, Scheller-Wolf, and Akcay, are focusing their efforts on lattice-dependent optimal policies. These policies seem promising, but some of their results are yet to be verified by the research community. [19] [16] [29] [17] [14] [15] [18] Their latest work extends their earlier results to general ATO systems, using state space aggregation, and it claims to reduce the computation time by an average of 96.4%, with an average distance of 6.32% from the optimal solution. This allows their solution to be applied to systems with a high number of components and general configuration, which is a desirable outcome. Their results are yet to be applied to systems with variable replenishment and backorder quantities, that so far have poses a great challenge in measurement of optimal policies and performance metrics. [26] [19]

There is still a lot to be done to reach a theoretically correct, yet computationally manageable solution for an optimal assemble-to-order system with stochastic lead times. There are many possible directions for future research. Optimization algorithms are in many cases exponential and hence impractical for large-scale problems. Approximation methods and bounds have room for improvement as they are not accurate enough in many solutions. Furthermore, some simplifying assumptions like system configuration, allocation and replenishment policies can be reconsidered, to extend the current optimal solutions to a general ATO system under demand, production, and supply uncertainty.

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