OPTIMUM COMPUTER DESIGN

OF

EXTERNAL SPUR GEARS

OPTIMUM COMPUTER DESIGN

OF

EXTERNAL SPUR GEARS

by

JOHN DAVID STRATTON, B. ENG.

A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfillment of the Requirements

for the Degree

Master of Engineering

McMaster University

April 1972

MASTER OF ENGINEERING (1972)
(Mechanical Engineering)

McMaster University
Hamilton, Ontario

TITLE: Optimum Computer Design of External Spur Gears

AUTHOR: John David Stratton, B. Eng. (McMaster University)

SUPERVISOR: Professor J. N. Siddall

NUMBER OF PAGES: vi, 162, A 159

SCOPE AND CONTENT:

Using established spur gear design practice, a user-oriented computer-aided design package is created by which a user requires minimal knowledge of, or experience in either FORTRAN, optimization or gear design, although this practice is risky since the designer's judgement should be employed to the same extent as in a manual design. The package is essentially material independent with options to specify the design variables as constant, standard or variable. The great flexibility incorporated in the routine enables the designer a full range of design features from control to power gearing. The structure of the package enables implementation of new theory or new optimization criteria with relative ease.

ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to the following people:

Professor J. N. Siddall for his guidance and supervision,

Mr. G. B. Collin of Spar Aerospace Products Limited, Toronto,

Mr. F. G. East of Hamilton Gear and Machine Company, Toronto,

Mr. J. Miura of Canadian Acme Screw and Gear, Limited, Toronto, for their technical assistance and plant tours,

Messrs. Yosri Diab, Moosa Mahomed and Richie Peppin and Ms. Lora Minaker for their able assistance and encouragement,

The McMaster Computer Centre personnel for their help,

Ms. Marg Hruboska and Ms. Sheila Wright for their excellent typing of the manuscript,

The financial support of this work by the National Research Council of Canada is gratefully acknowledged.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

As technology advances, the designer becomes burdened with more complex analysis requiring a higher degree of accuracy. Each element in a machine design demands such special treatment that it is almost beyond the capabilities of an average designer to maintain an adequate background necessary to successfully complete a competent, optimum design. In an effort to increase design efficiency and accuracy, computer-aided design has come into widespread use.

Gear design is no exception in regard to the complexity of the design effort. A delicate balance between art and science, gear design has become increasingly suited to computer analysis. Many of the larger gear manufacturing firms have developed computer-aided designs to ensure their competitive position in the open market. Empirical techniques developed from their design experience have been incorporated in their programs to yield a final design. However, these programs are generally not distributed in the gearing industry, for obvious reasons. Thus, a computer program package is required which will provide the "part-time" gear designer adequate information for optimum machine design as well as yielding the small gear manufacturing concern a "base" program in which further sophistications could be implemented easily.

The aim of this thesis is to develop a FORTRAN computer program package to optimize external spur gear design with the least possible restrictions to the designer. The program will be capable of standard or non-standard design practice without any loss of flexibility with regard to program usage or modification. The input is such that the designer can bring his judgement to bear to the same extent as he would in a manual design.

CHAPTER 2

SPUR GEAR DESIGN

2.1 INTRODUCTION

The subsequent sections of this chapter present the generally accepted external spur gear design procedure which has become prevalent in North America. Drawing theory from various sources, a design criterion is developed to access the feasibility of a design for implementation in a formal optimization technique to achieve the best possible design subject to the designer's specifications. In areas of the design where sufficient data is lacking, intuitive approximations are incorporated with existing formulations to give greater flexibility to the procedure.

2.2 TERMINOLOGY AND NOMENCLATURE

Although gear nomenclature is relatively standardized there is still a wide usage of terms and definitions. AGMA standards [19, 31] have been summarized to include variations utilized in this thesis employed to aid the reading of both the presented theory and subsequent computer programs. The following presentation will include definitions followed by a list of symbols. In the theoretical and computer analysis, the pinion is represented by subscripts p or 1 and the gear by subscript g or 2, unless otherwise specified. All definitions have been adapted to refer specifically to external spur gears.

Addendum is the height or radial distance tooth projects beyond the pitch line or pitch circle.

Addendum Coefficient for the purposes of this discussion is the product of the addendum size and the diametral pitch.

Addendum Circle coincides with the tops of the teeth in a cross section.

Arc of Action is the arc of the pitch circle through which a tooth profile moves from the beginning to the end of contact with a mating profile.

Arc of Approach is the arc of the pitch circle through which a tooth profile moves from its beginning of contact until the point of contact arrives at the pitch point.

Arc of Recess is the arc of the pitch circle through which a tooth profile moves from contact at the pitch point until contact ends.

Note: In the computer program, the length of approach and the length of recess are employed instead of the arc. The definitions remain similar except that the lengths are measured along the line of contact.

Backlash is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the operating pitch circles.

Base Circle is the circle from which involute tooth profiles are derived.

Base Pitch in an involute gear is the pitch on the base circle or along the line of action. Corresponding sides of involute gear teeth are parallel curves, and the base pitch is the constant and fundamental distance between them along a common normal in a plane of rotation.

Back Rack is a gear with teeth spaced along a straight line, and suitable for straight line motion, adopted as the basis of a system of interchangeable gears.

Centre Distance is the distance between the parallel axes of a spur gear; the distance between the centres of the pitch circles.

Chordal Tooth Thickness is the length of the chord subtending a circular thickness arc.

Circular Tooth Thickness is the length of arc between the two sides of a gear tooth, on the pitch circle unless otherwise specified.

Circular Pitch is the distance along the pitch circle or pitch line between corresponding profiles of adjacent teeth.

Clearance is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

Composite Error is an error of gear action composed from a number of contributory sources detected by rotating the gear with another gear or a master gear.

Contact Ratio is the number of angular base pitches through which a tooth surface rotates from the beginning to the end of contact.

Crowned Teeth have surfaces modified in the lengthwise direction to produce localized contact.

Dedendum is the depth or radial distance of a tooth space below the pitch line or pitch circle.

Dedendum Coefficient for the purposes of this discussion is the product of the dedendum size and the diametral pitch.

Dedendum (root) circle is tangent to the bottom of the tooth spaces in a cross section.

Diametral Pitch is the ratio of the number of teeth to the pitch circle diameter in inches.

Eccentricity is the distance between the axis of a surface of revolution and its axis of rotation in a given plane.

Equal-addendum Teeth are engaging gear teeth having equal addendums.

External Gear is a gear with teeth formed on the outer surface of a cylinder while internal gear teeth are formed on the inner surface of a cylinder.

Face Width is the length of the teeth in an axial plane.

Fillet Curve is the concave portion of the tooth profile where it joins the bottom of the tooth.

Fillet Radius is the radius of a circular arc approximating the fillet curve. In generated teeth the fillet curve has a varying radius of curvature.

Full Depth Teeth are those in which the working depth equals 2.0 divided by the diametral pitch.

Gear is any machine part with gear teeth. Of two gears that run together, the one with the larger number of teeth is called the gear and is generally referenced by a subscript g or 2.

Gear Ratio is the ratio of the larger to the smaller number of teeth in a pair of gears.

Hob is a milling cutter in the form of a screw thread which forms the gear teeth by generation.

Involute is the locus described by the end of a line unwound from the circumference of a circle.

Involute Function is a trigonometrical function equal to the tangent of an angle minus its value in radians.

Length of Action is the distance along the line of action which the point of contact moves during the action of the tooth profile.

Line of Action is the imaginary line along which contact occurs during the engagement of two tooth profiles. It is a straight line passing through the pitch point and tangent to the base circle.

Line of Contact is the line along which two tooth surfaces are tangent to each other.

Line of Centres connects the centres of the pitch circles of two engaging gears.

Long and Short Addendum Teeth are engaging gear teeth having unequal addendums.

Master Gear is an accurately made gear used for measuring the error in action of product gears.

Number of Teeth is the number of teeth contained in the whole circumference of the pitch circle.

Pinion is the smaller of a pair of gears and generally referenced by subscripts p or 1.

Out-of-Roundness is the irregular radial variation from a surface of revolution in a given plane of rotation, exclusive of eccentricity.

Pitch Circle is the curve of intersection of a pitch surface of revolution and a plane of rotation. According to theory, it is the imaginary circle that rolls without slipping with a pitch circle of the mating gear.

Pitch Line corresponds in the cross section of a rack to the pitch circle in the cross section of a gear.

Pitch Point is the point of tangency of two pitch circles on the line of centres.

Pressure Angle is the angle at a pitch point between the line of action which is normal to the tooth surface and the plane tangent to the pitch surface.

Profile Radius of Curvature is the radius of curvature of a tooth profile, usually at the pitch point or a point of contact.

Radial Runout is the total variation in a direction perpendicular to the axis of rotation of an indicated surface from a plane surface of revolution.

Spacing is the measured distance between corresponding points on adjacent teeth.

Spur Gears are gears which are cylindrical in form and operate on parallel axes with straight teeth parallel to the axis.

Standard Centre Distance is the centre distance on which two gear mesh such that the sum of the circular tooth thicknesses at the pitch circles and the design backlash equal the circular pitch.

Stub Teeth are those in which the working depth is less than 2.000 divided by the diametral pitch.

Surface of Revolution is a surface generated by translating a line about an axis at a given distance. In the case of a spur gear, the surface of revolution is cylindrical.

Tip Radius is the radius of the circular arc used to join a side-cutting edge and an end-cutting edge in gear-cutting tools.

Tip Relief is an arbitrary modification of a tooth profile whereby a small amount of material is removed near the tip of the gear tooth.

Tooth Profile is one side of a tooth in a cross section between the outside circle and the root circle.

Tolerance is the specified range between limits equal to the algebraic difference of allowable variations.

Undercut is a condition in generated gear teeth when any part of the fillet curve lies inside of a line drawn tangent to the working profile at its point of junction with the fillet.

Variation is the amount of deviation from a specified value.

Whole Depth is the total depth of the tooth space equal to addendum plus dedendum.

Working Depth is the depth of engagement of two gears and is the sum of their addendums.

Computer Symbols	Theory Symbols	Description
ADD	a	Addendum (inches)
ADDK	a_k	Addendum coefficient $(a=\frac{a_k}{D_p})$
ADDL		Addendum limit to tooth point (inches)
ANGC	θ_C	Angle between origin of fillet on dedendum circle and gear tooth centre-line (radians)
ANGL	θ_L	Load angle (radians)
BBA	b_a	Distance between pitch line and end of straight profile on generating hob tooth flank (inches)
BBX	b_x	Distance between tooth centreline and centre of rounded corner on generating hob tooth (inches)
BBY	b_y	Distance between pitch line and centre of rounded corner of generating hob tooth (inches)
BHN	Bhn	Brinell hardness
BL	B_L	Backlash including tooth thinning and machining tolerance (inches)
BLMIN	B_L^{min}	Theoretical minimum backlash on standard centre distances
BLMINT		Actual minimum backlash on standard centre distances

BLMAX	B_L^{max}	Theoretical maximum backlash on standard centre distance
BLMAXT		Actual maximum backlash on standard centre distance
BLMAXU		Actual maximum backlash on extended centre distance due to tolerance
BP	B_p	Base pitch
CCC	c	Clearance (inches)
CD	C_d	Centre distance (inches)
CP	C_p	Circular pitch
CRATIO	m_c	Contact ratio
DED	b	Dedendum (inches)
DEDK	b_k	Dedendum coefficient $(b=\frac{b_k}{D_p})$
DELBL	ΔB_L	Minimum-maximum backlash range (inches)
DP	D_p	Diametral Pitch
E	E	Modulus of elasticity (psi)
EFF		Efficiency
ERR		Error in action (inches)
FW	F_w	Face width (inches)
HP	hp	Horsepower
HUBL		Hub length (inches)
HUBR		Outer hub radius (inches)
PAB	P_{AB}	Allowable power in bending (HP)
PAD	ϕ	Pressure angle (degrees)
PAR		Pressure angle (radians)
PAW	P_{AW}	Allowable power in wear (HP)

PLV	PLV	Pitch line velocity (fpm)
PR	R	Pitch circle radius (inches)
PTOL		Profile tolerance (inches)
RATIO	m_g	Gear Ratio
RB	R_b	Base circle radius (inches)
RHO		Density (lb/cubic inch)
RI	R_I	Dedendum circle radius (inches)
RL	R_L	Load radius on tooth centreline (inches)
RIM	RIM	Radius to inner portion of gear blank rim (inches)
RM	R_M	Interference limit radius (inches)
RO	R_O	Addendum circle radius (inches)
RPM	n	Shaft speed (revolutions per minute)
RU	R_U	Undercut limit radius (inches)
SAC	σ_C	Allowable contact stress (psi)
SAF	σ_F	Allowable fatigue stress (psi)
SB	σ_b	Actual bending stress (psi)
SBM	σ_b^{max}	Maximum allowable bending stress (psi)
SHAFT		Shaft diameter (inches)
SS	σ_W	Actual wear stress (psi)
SSM	σ_w^{max}	Maximum allowable wear stress (psi)
TCT	e_{TCT}	Total composite tolerance (inches)
TEETH	N	Number of teeth
TO	T_0	Circular tooth thickness at addendum circle
TOLL	TOL_L	Lead tolerance is a measure of the parallelism of the tooth faces in a spur gear

TOLP	TOL_p	Pitch tolerance (inches)
TOLR	TOL_R	Runout tolerance (inches)
TP	T_p	Circular tooth thickness at pitch circle (inches)
TPTL		Upper and lower limits of tooth thickness tolerance at the pitch circle (inches) by caliper measurement
TPTE		Actual amount of tooth thickness tolerance including runout and tooth thickness variation determined from composite action analysis (inches)
TPTV		Actual tooth thickness tolerance determined by composite action analysis (inches)
U	ν	Poisson's ratio
WEB	WEB	Thickness of gear blank web (inches)
WA	W_a	Axial tooth load (lb)
WN	W_n	Normal tooth load (lb)
WR	W_r	Radial tooth load (lb)
WT	W_t	Tangential tooth load (lb)

Other variables not listed here will be defined when they are used in either the theory or computer program.

2.3 CONJUGATE ACTION

The essential purpose of a gear-tooth system is to transmit uniform rotary motion between shafts, and an almost infinite number of tooth profiles exist to fulfil this requirement. Kinematically, gear teeth act against each other in a similar manner to cams. If the tooth profiles produce rotary motion coupled with a constant angular-velocity ratio during meshing, the surfaces are said to be conjugate. It must be remembered, however, that any discussion of this conjugate action unrealistically assumes that the teeth are perfectly formed, perfectly smooth and absolutely rigid.

The basic law of conjugate gear-tooth action states that to transmit uniform rotary motion between shafts, the normals to the tooth profiles at all points of contact must pass through a fixed point, the pitch point, on the line of centres of the shafts. This law is illustrated mathematically and graphically in Equation (2.3.1) and Figure 2.3.1.

$$\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} \tag{2.3.1}$$

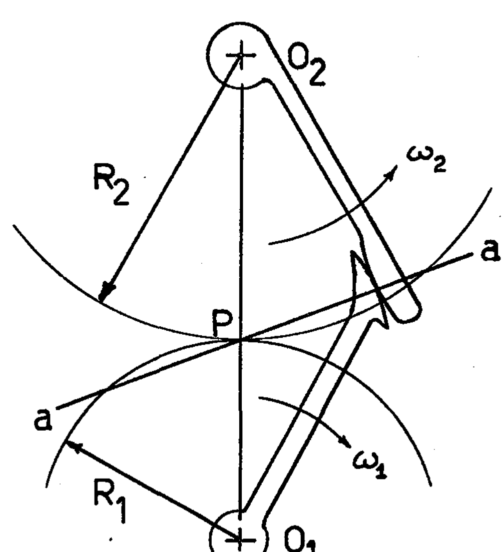


FIGURE 2.3.1 Conjugate Gear-Tooth Action

Point P is the pitch point on the line of centres O_1O_2 where the common normal, or line of action aa intersects. Thus, for every instantaneous point of contact, the line of action will pass through the fixed point P. The locus of the points of contact, or path of action, however, may not necessarily pass along the line of action, although some profiles have this property.

By specifying one gear-tooth form or a particular path of contact, a mating profile could be realized which would develop conjugate action. However, problems arise which virtually eliminate the vast majority of possible profiles. For example, although two profiles may mate and run together properly, two other gears of the form of either original may not run together correctly. If there is to be interchangeability between gears, enabling all gears of all sizes conjugate to the same basic rack form* to mesh properly, the path of contact must be symmetrical about the pitch point. With this condition satisfied, then the basic rack profile of the system will also be symmetrical in relation to the pitch point. Therefore, all gears with any number of teeth that are conjugate to this basic rack will be conjugate with each other.

Another practical problem that exists is the reproduction of these curves economically on existing machinery. In addition, changes in shaft centres due to misalignment and large forces may seriously effect the capabilities of the profile to maintain conjugate action.

---

* For every set of conjugate gear-tooth profiles, a basic rack form exists with infinite diameter conjugate to the set.

One of the most widely used gear-tooth profiles is the involute with properties highly suited for gearing applications.

2.4 INVOLUTE AND FILLET PROPERTIES

2.4A INVOLUTE PROFILE

The involute profile with its many unique and valuable properties is, with few exceptions, in universal use for gear teeth. References [1, 2, 3, 4] offer in-depth presentations of these properties as well as the mathematical development of the involute surfaces. Buckingham [2] summarizes the properties as follows:

1. The shape of the involute curve is dependent only upon the size of the base circle.

2. If one involute, rotating at a uniform rate of motion, acts against another involute, it will transmit a uniform angular motion to the second regardless of the distance between the centres of the two base circles.

3. The rate of motion transmitted from one involute to another depends only upon the relative sizes of the base circles of the two involute and is inversely proportional to those sizes.

4. The common tangent to the two base circles is both the path of contact and the line of action.

5. The path of contact of an involute is a straight line. Any point on this line may therefore be taken as a pitch point, and the path of contact will remain symmetrical in relation to this pitch point.

6. The intersection of the common tangent to the two base circles with their common centre line establishes the radii of the pitch circles of the mating involutes. No involute has a pitch circle until it is brought into contact with another involute, or with a straight line constrained to move in a fixed direction.

7. The pitch diameters of two involutes acting together are directly proportional to the diameters of their base circles.

8. The pressure angle of two involutes acting together is the angle between the common tangent to the two base circles and a line perpendicular to their common centre line. No involute has a pressure angle until it is brought into contact with another involute, or with a straight line constrained to move in a fixed direction.

9. The form of the basic rack of the involute is a straight line. The pressure angle of an involute acting against such a rack is the angle between the line of action and a line representing the direction in which this rack moves.

10. The pitch radius of an involute acting against a straight line rack form is the length of the radial line, perpendicular to the direction of motion of the rack, measured from the centre of the base circle to its point of intersection with the line of action.

Although no formal proof will be given for the fore-mentioned summary, the following mathematical development of the involute profile will give some insight to the authenticity of these properties.

An involute curve is the locus described by the end of a line unwound from the circumference of a circle as illustrated in Figure 2.4.1.

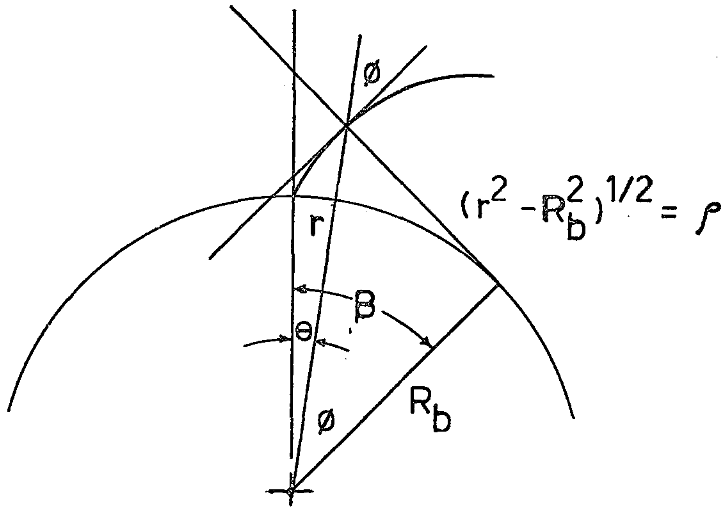


FIGURE 2.4.1 Involute Generation

where

R_b = base circle radius

r = radius to any point on involute

ρ = radius of curvature of involute at radius r

By the method of generation of the involute curve, the length of the generating line $(r^2-R_b^2)^{\frac{1}{2}}$ is also the length of the circumference of the base circle subtended by angle β. Thus

$$\beta = \theta+\phi$$

$$\beta = \frac{(r^2-R_b{}^2)^{\frac{1}{2}}}{R_b}$$

but

$$\tan \phi = \frac{(r^2-R_b{}^2)^{\frac{1}{2}}}{R_b}$$

therefore

$$\theta = \beta-\phi$$

$$\theta = \frac{(r^2-R_b^2)^{\frac{1}{2}}}{R_b} - \tan^{-1}\frac{(r^2-R_b^2)^{\frac{1}{2}}}{R_b} \tag{2.4.1}$$

$$\theta = \mathrm{INV}\phi \tag{2.4.2}$$

$$= \tan\phi-\phi \tag{2.4.3}$$

$$r = R_b/\cos\phi \tag{2.4.4}$$

From the above geometrical conditions, it can be readily seen that the radius of curvature of any point on the involute profile is the length of generating line to that point.

Figure 2.4.2 illustrates the involute action of a mating pair of gears, or twin involute generation.

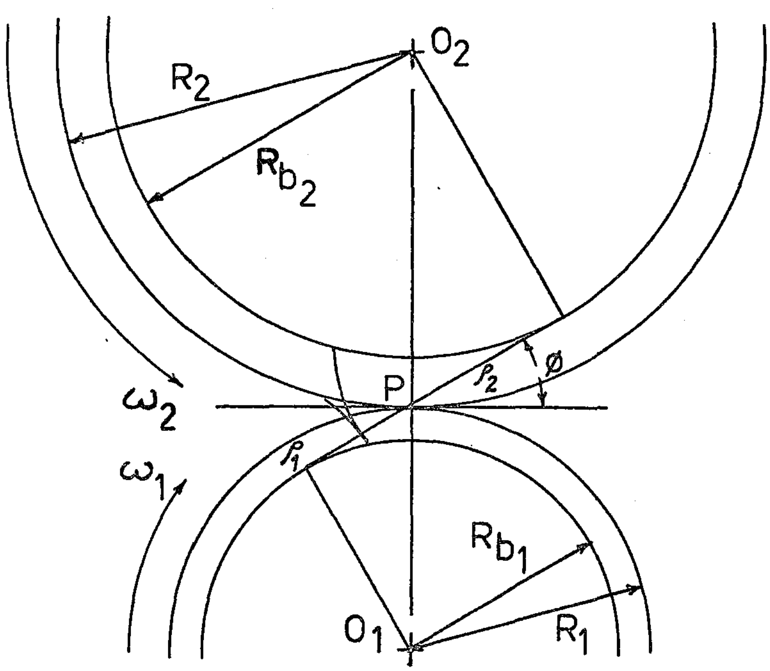


FIGURE 2.4.2 Involute Action of Mating Pair of Gears

The point of contact moves along the generating line, which does not change position because it is always tangent to the base circles, and, since the generating line is always normal to the involutes at the point of contact, the requirement of uniform motion is satisfied.

A complete gear tooth profile includes both the involute profile and the trochoidal fillet, which will be discussed later in this section, usually symmetrically generated about the centre line of the tooth. With the involute equations as a foundation, the following analysis describes the tooth properties which will clarify further development of the gear design.

Beginning with the involute profile, treating the fillet as a separate problem, it is possible to define the coordinates of the tooth involute profile once the arc tooth thickness and pressure angle at a definite radius have been specified. Referring to Figure 2.4.3 where

T_1 = given arc tooth thickness,
R_1 = given radius of profile,
ϕ_1 = given pressure angle at radius R_1, radians
r = any radius of profile,
T = arc tooth thickness at r,
ϕ = pressure angle at r, radians,
R_b = base circle radius of involute.

equation (2.4.4) transposed becomes

$$R_b = R_1 \cos\phi_1$$

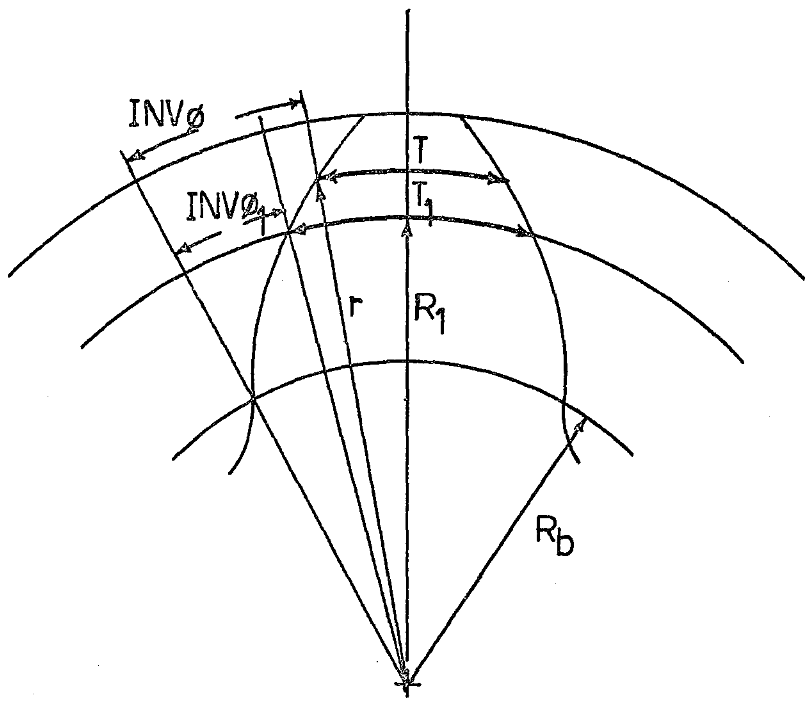


FIGURE 2.4.3 Tooth Thickness Determination

Dealing with the half thickness of the tooth, since the teeth form is symmetrical, the angle of the half thickness at R_1 in radians is equal to $T_1/2R_1$. With the angle between the involute origin at the base circle and any radius r specified by θ or the involute function INV , the half thickness of the tooth at the base circle is equal to $(T_1/2R_1) + INV\phi_1$. As T/2r represents the angle between the tooth centreline and any given point r on the involute profile, the half tooth thickness at any radius r is equal to the half thickness at the base circle, minus the involute function of the pressure angle at the specified radius r which results in

$$T/2r = \theta_r = (T_1/2R_1) + INV\phi_1 - INV\phi \qquad (2.4.5)$$

or a tooth thickness at any radius r of

$$T = 2r[(T_1/2R_1) + INV\phi_1 - INV\phi]$$

All angular measure is made in radians.

A physical limitation of the profile generation occurs at a radius which produces a pointed tooth. At this radius the thickness becomes zero, reducing equation (2.4.5) to

$$INV\phi = (T_1/2R_1) + INV\phi_1$$

An iterative solution of the angle ϕ coupled with equation (2.4.4) yields the limiting radius of the involute profile.

In general practice the values of T_1, R_1 and ϕ_1 are specified as the properties of the involute at the design pitch circle radius. Further reference to these values at the pitch circle will be specified as T_p, R and ϕ.

2.4B FILLET PROFILE

Although mathematically the involute and trochoidal fillet may be considered separately, the physical generation of these curves produce a single profile. By knowing certain characteristics of the generating cutter, the resultant fillet profile on the gear may be determined analytically. The fillet coordinates are very dependent on the type of cutting method employed for tooth generating. Two methods considered here are hobbing and basic rack generation which yield a similar mathematical analysis. For the purpose of clarity and simplicity, future reference to hob will specify either the hob or rack cutter.

Figure 2.4.4 illustrates the tooth section of a hob where

b = dedendum of gear and addendum of hob

b_a = distance from pitch line of hob to point of tangency of rounded corner with straight line form

b_x = distance from centre line of hob tooth to centre of rounded corner

b_y = distance from pitch line of hob to centre of rounded corner

c = clearance at bottom of tooth space

C_p = circular pitch

ϕ = pressure angle at R and one-half included angle of hob tooth

R = gear pitch circle radius

r_T = radius of rounded corner of hob tooth

T_S = arc tooth thickness of gear tooth at pitch circle radius R, and hob tooth space width at pitch line.

from which the following relationships may be derived:

$$c = r_T(1-\sin\phi) \qquad (2.4.6)$$

$$b_y = b-r_T \qquad (2.4.7)$$

$$b_x = \frac{(C_p-T_S)}{2} - (b\tan\phi_1 + \frac{r_T}{\cos\phi}) \qquad (2.4.8)$$

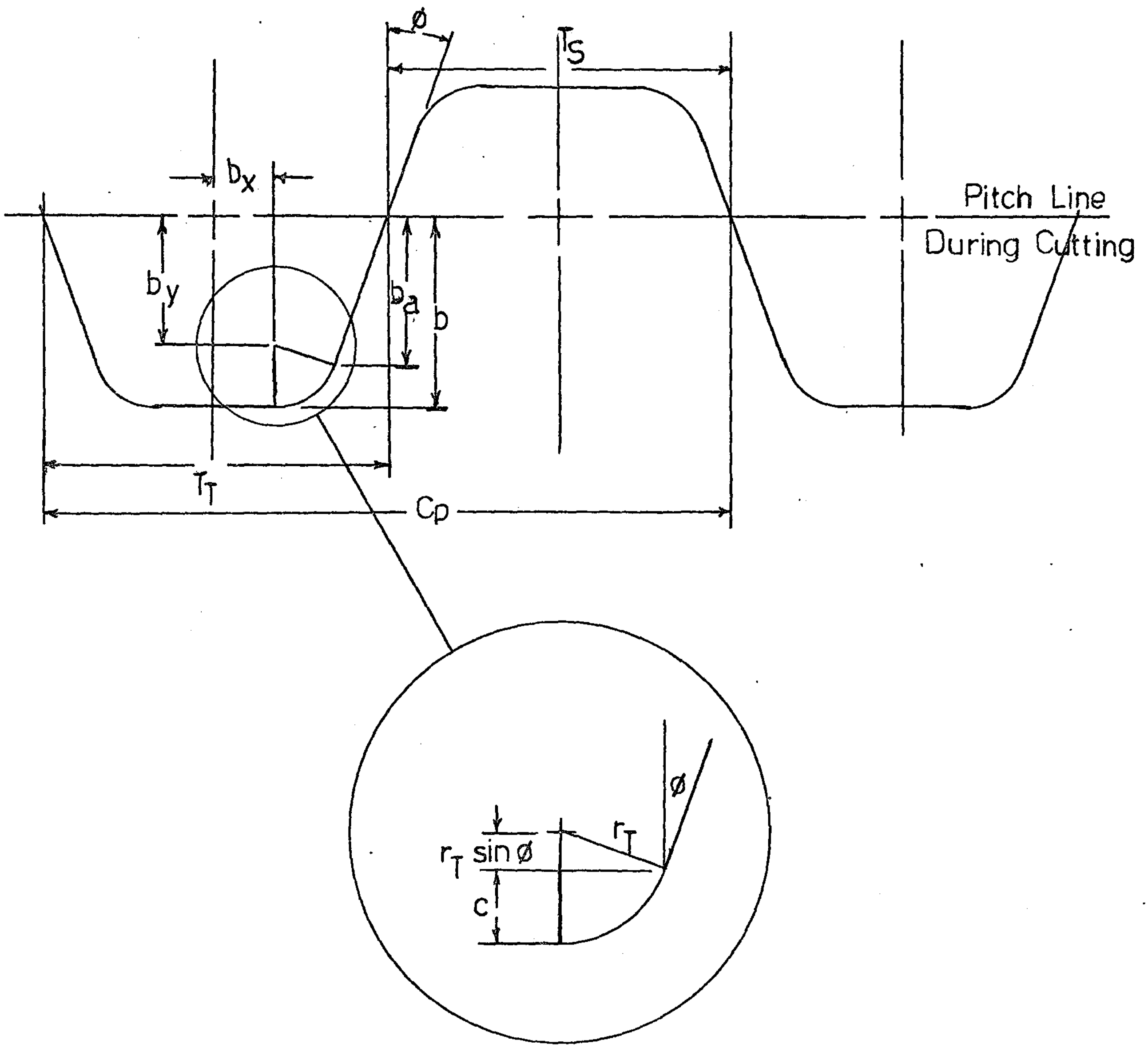


FIGURE 2.4.4 Hob Tooth Properties

The above equations are developed about a pitch line tangent to the design pitch circle of the gear. Thus, the angle between the centre-line of gear tooth and the origin of the trochoid may be defined by the arc length swept out on the hob pitch line between the centre of the hob tooth space and the centre of the rounded corner, or mathematically

$$\theta_c = \frac{(C_p/2)-b_x}{R} \qquad (2.4.9)$$

For a standard hob the tooth space width T_S equals the tooth thickness T_T at the standard pitch line which, therefore, would generate a pitch circle tooth thickness of $C_p/2.0$ on the gear. However, if the hob

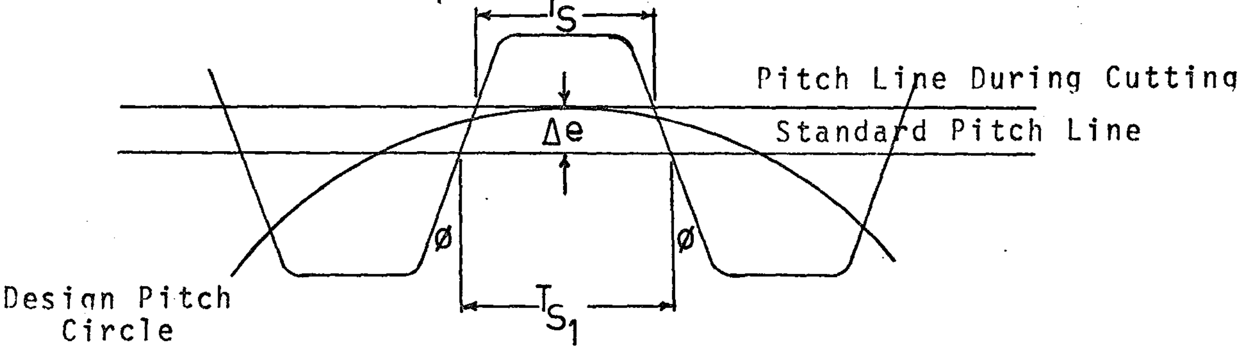


FIGURE 2.4.5 Variation in Tooth Thickness from Hob Offset

were advanced or withdrawn from the gear blank by some offset distance Δe, from this standard pitch line, as in Figure 2.4.5, the resultant tooth thickness on the gear pitch circle would be proportionately smaller or larger to the amount of offset by a value

$$T_S - T_{S_1} = 2\Delta e\tan\phi$$

$$\Delta T = 2\Delta e\tan\phi \qquad (2.4.10)$$

Here the cutter has straight sides, but the similar equation evolves approximately for the situation of one gear advanced to another, to relate tooth thickness change to centre distance variation.

An offset advancing the hob into the gear blank is defined by a negative distance.

For an involute profile, conjugate action cannot take place below the radius R_b which would form a tangent circle drawn from the centre of the gear to the path of contact. Also, if the mating profile projects beyond this point of tangency, a cusp will exist in the theoretical form of the tooth profile since two points of contact should exist for the same radial distance on the gear. With this occurrence, the corner of the mating gear will interfere, making improper contact with the incomplete profile. In the case when the interfering member is the generating tool, an undercut tooth form will result, as part of the conjugate profile will be removed by the corner of the generating tooth which travels in a trochoidal path in relation to the generated gear. A trochoid is the locus of a fixed point on the mating member moving in relation to the centre of the gear. Usually, for ease of analysis, this fixed point on the generating hob is the centre of the rounded corner of the tooth, or the corner of a sharp cornered tooth when the radius of the tooth corner round is considered zero. Analyzing these trochoids, the form of the fillet may then be determined for both undercut and non-undercut conditions, including the inter-relationship of the involute and fillet forms.

For initial consideration, Figure 2.4.6 represents a hob with a sharp cornered tooth (i.e. $r_T = 0$) where

R = gear pitch circle radius

b_y = distance from pitch line of hob to sharp corner of hob tooth. The addendum of the hob becomes the dedendum of the gear.

r_t = any trochoid radius

θ_t = vectorial angle of trochoid

ψ_t = angle between tangent to trochoid and radius vector

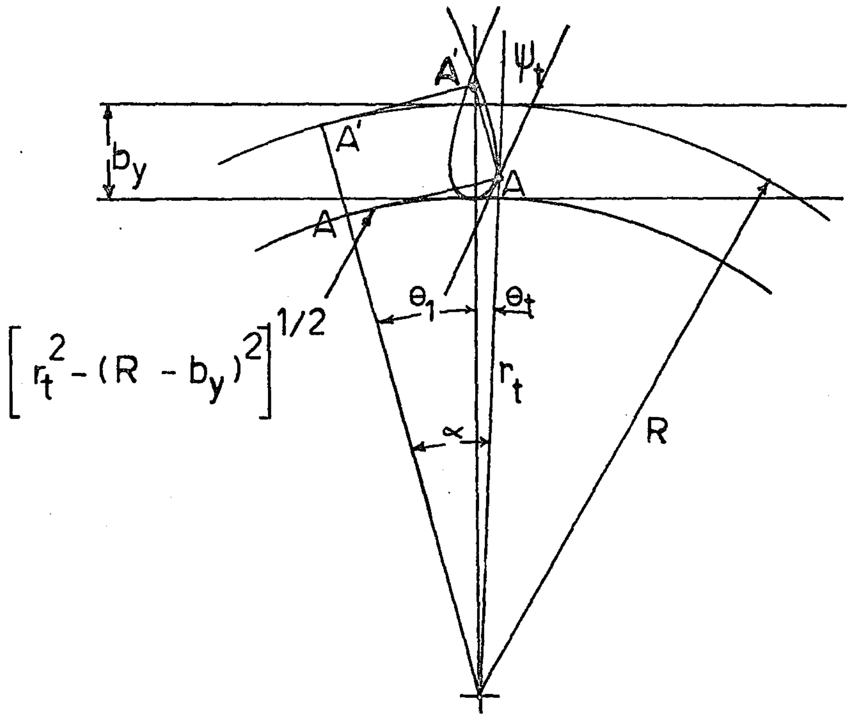


FIGURE 2.4.6 Trochoid Generation for Sharp Cornered Hob Tooth

The geometry of Figure (2.4.6) yields the following equation:

$$\tan(\theta_1+\theta_t) = \frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{R-b_y}$$

$$\theta_t = \tan^{-1}\left(\frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{R-b_y}\right) - \theta_1$$

Since AA=A'A' and A'A' equals the arc length A'A", we know that

$$\theta_1 = \frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{R}$$

and thus

$$\theta_t = \tan^{-1}\left(\frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{R-b_y}\right) - \frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{R} \tag{2.4.11}$$

but

$$\alpha = \theta_1+\theta_t$$

$$\alpha = \cos^{-1}\left(\frac{R-b_y}{r_t}\right) \tag{2.4.12}$$

and

$$\tan\alpha = \frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{R-b_y}$$

or

$$(r_t^2-(R-b_y)^2)^{\frac{1}{2}} = (R-b_y)\tan\alpha$$

Therefore, from equation (2.4.10)

$$\theta_t = \tan^{-1}(\tan\alpha) - (\tan\alpha - \frac{b}{R}\tan\alpha)$$

$$\theta_t = \frac{b}{R}\tan\alpha - (\tan\alpha-\alpha)$$

$$\theta_t = \frac{b_y}{R}\tan\alpha - \mathrm{INV}\alpha \tag{2.4.13}$$

At the same time, the tangent angle to the trochoid becomes, in radial coordinates,

$$\tan\psi_t = \frac{r_t d\theta_t}{dr_t}$$

and from Equation (2.4.10)

$$\frac{d\theta_t}{dr_t} = \frac{d}{dr_t}\left(\tan^{-1}\left(\frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{(R-b_y)}\right) - \frac{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}{r}\right)$$

$$\frac{d\theta_t}{dr_t} = \frac{(R-b_y)}{r_t(r_t^2-(R-b_y)^2)^{\frac{1}{2}}} - \frac{r_t}{R(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}$$

which results in

$$\tan\psi_t = \frac{R(R-b_y)-r_t^2}{R(r_t^2-(R-b_y)^2)^{\frac{1}{2}}}$$

$$\tan\psi_t = \frac{(R-b_y)-(r_t^2/R)}{(r_t^2-(R-b_y)^2)^{\frac{1}{2}}} \quad (2.4.14)$$

Expanding Equation (2.4.14) further

$$\tan\psi_t = \frac{1-(r_t^2/(R(R-b_y)))}{R[(r_t^2-(R-b_y)^2]^{\frac{1}{2}}/R(R-b_y)}$$

$$\tan\psi_t = \frac{1-\frac{r_t^2}{R(R-b_y)}}{\tan\alpha}$$

$$\psi_t = \tan^{-1}\left(\frac{1-\frac{r_t^2}{R(R-b_y)}}{\tan\alpha}\right) \quad (2.4.15)$$

When a hob is employed as a cutting tool, however, the corner of the hob tooth is rounded, either purposefully or by wear. Considering the hob corner to be rounded with a radius r_T, the trochoidal fillet produced will be the envelope of the family of circles whose centres are moving along the trochoidal path of the centre of the round as in Figure 2.4.7.

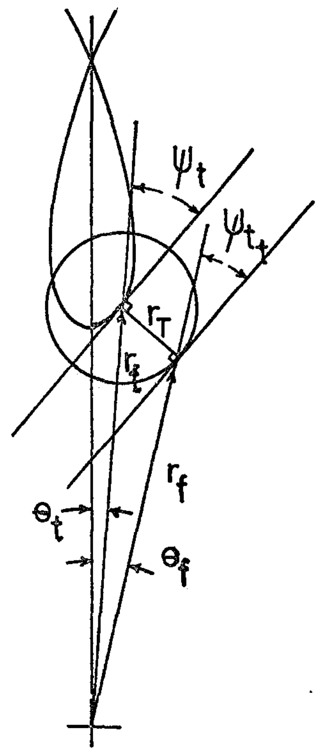


FIGURE 2.4.7 Fillet Generation for Rounded Corner Hob Tooth

where

r_t = any radius of trochoid

θ_t = vectorial angle of trochoid

r_f = any radius of fillet form

θ_f = vectorial angle of fillet form

r_T = radius of corner rounding

b_y = distance from pitch line to centre of rounding

Applying the law of cosines

$$r_f^2 = r_t^2 + r_T^2 - 2r_t r_T \cos(90° - \psi_t)$$

$$r_f = (r_t^2 + r_T^2 - 2r_t r_T \sin\psi_t)^{\frac{1}{2}} \qquad (2.4.16)$$

and

$$\cos(\theta_f-\theta_t) = (r_t-r_T\sin\psi_t)/r_f$$

$$\theta_f=\theta_t+\cos^{-1}[(r_t-r_T\sin\psi_t)/r_f] \qquad (2.4.17)$$

From Figure 2.4.7, the tangent angle ψ_{tt} between the radial vector and the tangent line to the fillet becomes

$$\psi_{tt} = \psi_t-\cos^{-1}\left(\frac{r_t-r_T\sin\psi_t}{r_f}\right) \qquad (2.4.18)$$

or
$$\psi_{tt} = \psi_t-(\theta_f-\theta_t)$$
$$= \psi_t+\theta_t-\theta_f$$

Although the trochoidal fillet and involute profile have been specified mathematically, equations to link both curves together as a physical unit must be derived. Since the involute curve begins at the base circle, no conjugate gear tooth action can take place below this radius as was mentioned before. If any portion of the straight side of a basic rack flank extends below the base circle during operation, interference will occur. A similar situation occurring at the time of generation would result in removal of a part of the involute profile. In order to avoid this undercutting, the straight portion of the hob tooth flank must not extend below the line where the base circle becomes tangent to the line of action, as in Figure 2.4.8.

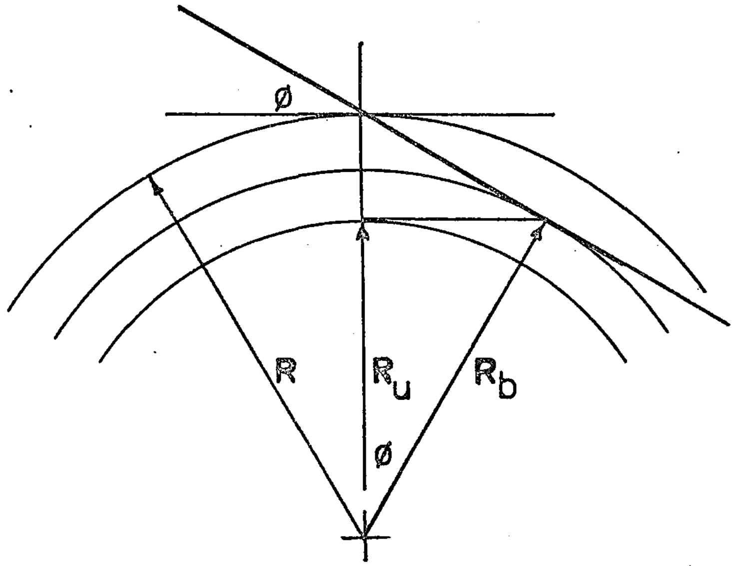


FIGURE 2.4.8 Undercut Limit Radius

Thus the undercut limit becomes

$$R_u = R_b\cos\phi$$
$$= R\cos^2\phi$$

Relating this undercut limit radius to the generated dedendum circle of the gear, for no undercutting

$$R_I \geq R_u \qquad (2.4.19)$$

$$R_I \geq R_b\cos\phi \qquad (2.4.20)$$

for sharp cornered hob teeth, or

$$R_I \geq R_b\cos\phi - r_T(1-\sin\phi) \qquad (2.4.21)$$
$$\geq R_b\cos\phi - c$$

for the rounded corner hob teeth, where R_I represents the dedendum circle radius of the gear.

If no undercut is present, the trochoidal fillet will be tangent to the involute profile at a radius where the end of the straight flank of the hob crosses the path of contact, as illustrated in Figure 2.4.9, which yields

$$r_f = [(R\sin\phi-(b_a/\sin\phi))^2 + R_b^2]^{\frac{1}{2}} \qquad (2.4.22)$$

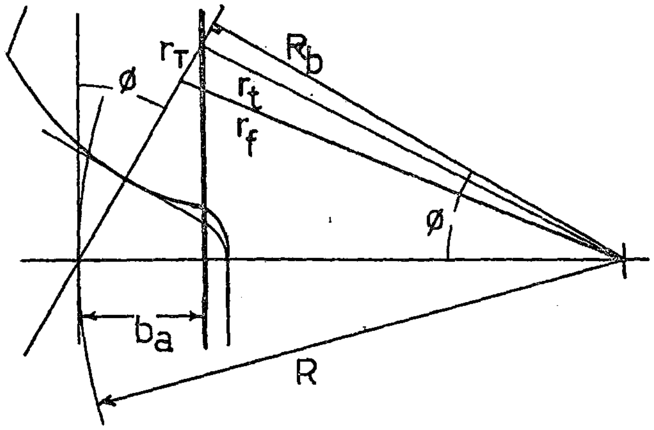


FIGURE 2.4.9 Radius to Involute-Trochoidal Fillet Intersection During Non Undercut Conditions

from the geometry. The corresponding trochoid radius, from similar geometrical analysis, becomes

$$r_t = (R_b^2+((r_f^2-R_b^2)^{\frac{1}{2}}+r_T)^2)^{\frac{1}{2}}$$
$$r_t = (r_f^2+r_T^2+2r_T(r_f^2-R_b^2)^{\frac{1}{2}})^{\frac{1}{2}} \qquad (2.4.23)$$

If the hob has sharp cornered teeth, this corner becomes the straight portion limit of the flank and the tangency radius a point on the trochoid.

In the case of undercutting, the radius to involute-trochoidal fillet intersection cannot be determined geometrical as in the no undercut condition, but must be computed using iterative techniques. Physically, the angle between the centre line of gear tooth and the point on the involute, plus the angle between the centre line of the trochoid and the point on the trochoidal fillet, must equal the angle between the trochoid centre line and the tooth centre line. Combining equations (2.4.5), (2.4.9) and (2.4.17), the result becomes

$$\theta_c = \theta_f + \theta_r \tag{2.4.24}$$

at the intersection radius.

When a sharp cornered hob tooth generates the undercut curves, the trochoid of the corner becomes the actual fillet so that the base circle radius may be taken as the initial point for the iterative process. Buckingham [8] suggests an approximate solution [Equation (2.4.25)] for the undercut radius which may be used as a second point in the process.

$$R_2 = R_b + \frac{(R_u - R_I)/\sin^2\phi}{6R\cos\phi} \tag{2.4.25}$$

For the rounded corner hob tooth, the trochoid radius producing a fillet radius of R_b must be determined before following the above iterative procedure. Further development of these properties will be discussed later under the appropriate sections required for the final design analysis.

Thus far, reference to the undercutting due to cutting has been developed with no mention of a similar problem arising when two involute gears mesh. If the outer circles of the gears (i.e. the addendum circle) extend beyond the point of tangency of the line of action with the base circle of the mating gear, similar interference as with the cutter will result. Therefore, employing Figure 2.4.10, the maximum addendum

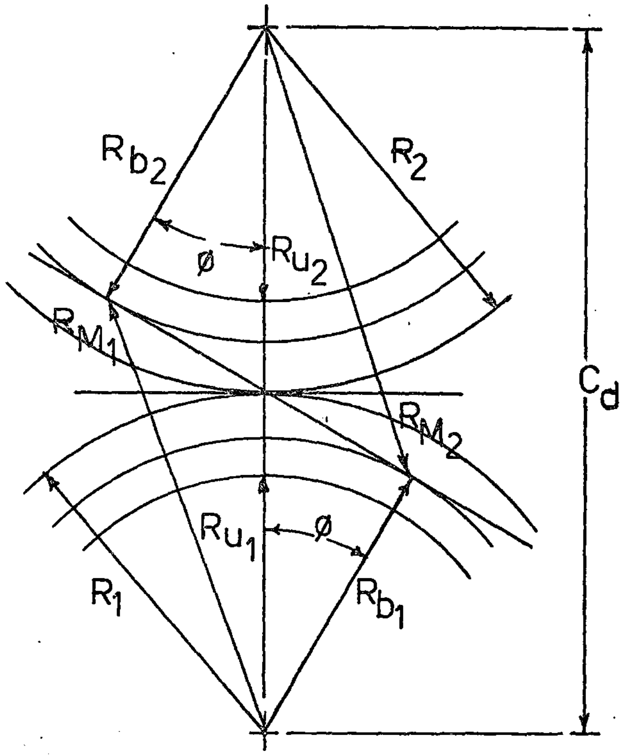


FIGURE 2.4.10 Maximum Addendum Circle Radius to Avoid Interference

circle to avoid interference becomes

$$R_{M_1} = (R_{b_1}{}^2 + (C_d \sin\phi)^2)^{\frac{1}{2}}$$

$$R_{M_2} = (R_{b_2}{}^2 + (C_d \sin\phi)^2)^{\frac{1}{2}}$$

$$R_{U_1} = C_d - R_{M_2}$$

$$R_{U_1} = C_d - R_{M_1}$$

or generally

$$R_M = (R_b{}^2 + (C_d \sin\phi)^2)^{\frac{1}{2}} \qquad (2.4.26)$$

The foregoing equations have been incorporated basically in three subroutines: Subroutine CUTTER, Subroutine FILLET and Subroutine

ADDEND, Appendices [A.1], [A.2] and [A.3]. However, many of these equations have also been employed as FORTRAN Statement Functions in subroutines requiring their use; these functions will be discussed when the appropriate routines are developed later in the thesis.

SUBROUTINE CUTTER(ANGC,BL,CCC,CD,CP,DED,NCUT,PAR,PR,RB,RM,RU,TP,BBA,BBX,
BBY,RT)

Use: This routine determines the undercut limit radius, the maximum addendum circle radius to prevent interference, the angle between the tooth centre line and fillet origin on the dedendum circle, and some geometric characteristics of the cutting tool teeth.

Calling Sequence: Once the various gear and pinion geometrical properties are specified in Subroutine UREAL this routine is called in Subroutine UREAL to develop the above properties.

SUBROUTINE FILLET(ANGC, NCUT,PAR, PR,RAD,RB,RI,RU,RRTL,RRTU,TP,BBA,
BBX,BBY,RT)

Use: This routine determines the radius to the point of intersection of the involute and fillet profiles.

Calling Sequence: To determine the contact ratio during undercut conditions, this routine is called from Subroutine LENGTH which is called from Subroutine CONRAT.

Special Features: The subroutine handles the non-undercut tangency point exactly as the theory in this chapter was developed. However, the iterative solution required to return the undercut intersection point has a few features to enable a convergent solution to be found. Using a linear search followed by a false position root determination Appendix [B], the intersection radius of the involute-fillet profile is determined.

In the case of the sharp cornered hob tooth, the trochoid also becomes the fillet profile. Therefore, half of the step suggested by Buckingham was used for the linear search.

However, the rounded corner hob tooth cutter presents an added problem since the fillet and trochoid are separate entities. A similar process was carried out, only this time the trochoid radius yielding a fillet radius equal to the base circle radius was determined before continuing the intersection radius determination. The linear search step length used in calculating the fillet-base circle was 10% of the functional difference between the fillet radius with trochoid radius set at R_b and the actual base circle radius R_b. Once this trochoid radius was determined, the process continued in a manner similar to the sharp cornered tooth case.

SUBROUTINE ADDEND(ADDL,PAR,PR,RB,RO,TO,TP)

Use: This routine determines the addendum length of pointed teeth and tooth thickness at the addendum circle.

Calling Sequence: Having specified a pitch circle tooth thickness in Subroutine THICK, this routine is called to find the pointed tooth radius. These routines are both found in Subroutine SPUR.

Special Features: Employing the linear search and false position technique described in Appendix [B], the required radius is determined, thus defining the maximum possible addendum size. If the pointed tooth radius is less than the addendum circle radius, the tooth thickness at the addendum circle is set to zero. In the linear search, difference between the addendum circle and pitch circle radii specify the step length. If, during the optimization, this difference becomes zero, then an arbitrary step length of 0.1 inches is assumed.

2.5 FUNDAMENTALS

As a consequence of choosing the involute profile to produce the desired conjugate action, the properties of this curve and its generation present a foundation of fundamental equations used in the design procedure. These equations provide a reference point from which more sophisticated theory may be developed to include stress constraints, contact ratio, etc. With a specified centre distance C_d and a desired angular velocity ratio, the law of conjugate gear tooth action mathematically states that

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{R_1} \tag{2.5.1}$$

or

$$\frac{n_1}{n_2} = \frac{R_2}{R_1} \tag{2.5.1A}$$

where ω represents the angular velocity; r, the pitch circle radii and n, the shaft speeds in rpm. If the two pitch circles were considered rolling against each other without slipping at the above uniform angular velocities then the linear pitch line velocity is

$$\begin{aligned} PLV &= \omega_1 R_1 = \omega_2 R_2 \\ &= 2\pi n_1 R_1 = 2\pi n_2 R_2 \end{aligned} \tag{2.5.2}$$

The pitch circle radii of an involute gear are not fixed but are dependent on the centre distance and the magnitude of the base circle radii which are constant for a particular gear as pointed out in the summary of the involute properties in Chapter [2.4].

Figure (2.4.2) can be used to illustrate many of the geometrical relationships existing between the various radii and angles. As examples:

$$C_d = R_1 + R_2 \tag{2.5.3}$$

$$R_b = R\cos\phi \tag{2.5.4}$$

$$M_g = \frac{\omega_1}{\omega_2}$$

$$= \frac{R_2}{R_1} \qquad (2.5.5)$$

where M_g represents the gear ratio.

If several involutes were developed on the same base circle, the profiles of several teeth would result. Evenly spacing these teeth, and considering only one side of the tooth as in Figure (2.5.1), the distance between the teeth equals the circumference of the base circle divided by the number of teeth of the gear from which the base pitch is defined.

$$B_p = \frac{2\pi R_b}{N} \qquad (2.5.6)$$

with N specifying the number of teeth.

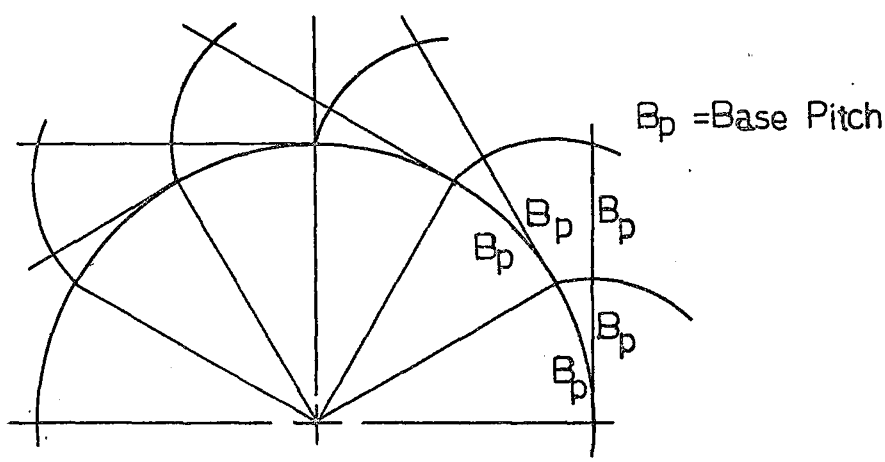


FIGURE 2.5.1 Geometrical Representation of Base Pitch

In similar fashion, the circular pitch defines the spacing between tooth profiles on the pitch circle giving

$$C_p = \frac{2\pi R}{N} \qquad (2.5.7)$$

A third "pitch" known as diametral pitch, having no physical meaning, is defined as the number of teeth on the gear per inch of pitch diameter.

$$D_P = \frac{N}{2R} \tag{2.5.8}$$

Combining Equations (2.5.6) and (2.5.7) a useful relation

$$C_P D_P = \pi \tag{2.5.9}$$

is obtained.

The addendum (dedendum) circles are defined by adding (subtracting) the addendum (dedendum) lengths to the pitch radii, as in the following equations:

$$R+a = R_0 \tag{2.5.10}$$

$$R-b = R_I \tag{2.5.11}$$

where a = addendum and b = dedendum. The clearance between the mating teeth is defined here as

$$C = b_m - a \tag{2.5.12}$$

where the subscript m refers to the mating gear.

Having defined or developed the foregoing equations, further analyses may now be undertaken. These equations are employed in the computer program in Subroutine PITCH, Appendix [A.4].

SUBROUTINE PITCH(RATIO,CD,TEETH1,TEETH2,RPM1,PAR,PI,PR1,PR2,RB1,RB2,BP,
CP,DP,PLV)

Use: This routine determines the various geometrical relationships developed in this chapter.

Calling Sequence: Subroutine VARY calls this routine after each new set of variables is generated through Subroutine UREAL. The routine is also used initially in Subroutine VARY1 to define the argument values if the package is used for analysis only, with no variables, or to aid in defining starting values for some variables (i.e. addendum-dedendum factors) when required for optimization.

2.6 LOADING

When two gears act against each other, the resultant load W_n, is directed along the line of action. In Figure 2.6.1 the pinion and gear are separated with the loads and reactions directed, respectively, at the pitch point and the shaft centre of each gear to constitute a couple.

Using the pinion as an example, the forces may be resolved into components, illustrated in Figure 2.6.2.

Employing the subscripts r and t to indicate the radial and tangential directions with respect to the pitch circle, the moment of the couple W_t and F_t represents the torque application required to drive the gear set.

$$T = W_t R_1 \tag{2.6.1}$$

Defining horsepower as

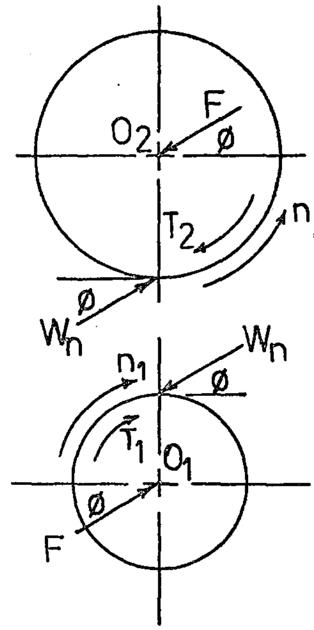
$$hp = \frac{2\pi Tn}{33000} \tag{2.6.2}$$

with hp = horsepower

T = torque, ft. lbs.

n = shaft speed, rpm

Equation (2.6.1) combined with the pitch line velocity, Equation (2.5.2) results in



F reaction force of shaft

W_n load along line of action

T torque

FIGURE 2.6.1 Forces on Mating Gear Set

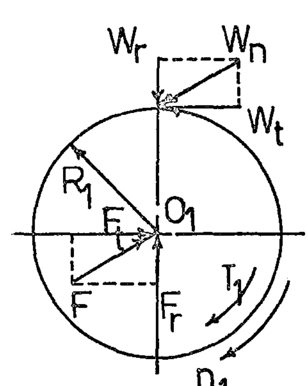


FIGURE 2.6.2 Component Forces Acting on Pinion

$$hp = \frac{W_t(PLV)}{33,000} \tag{2.6.3}$$

From the geometry of Figure 2.6.2, the following relationships can be derived:

$$W_t = W_n \cos\phi \tag{2.6.4}$$

$$W_r = W_n \sin\phi \tag{2.6.5}$$

$$W_r = W_t \tan\phi \tag{2.6.6}$$

Since the pitch surface of a spur gear sweeps out a cylinder, not a cone, no axial component is generated during loading, thus

$$W_a = 0.0 \tag{2.6.7}$$

These equations are used in Subroutine TORQUE and Subroutine TLOAD as part of the design analysis, Appendices [A.5] and [A.6].

SUBROUTINE TORQUE(HP,PI,RPM,TORQ)

Use: To determine the torque on the gear.

Calling Sequence: Subroutine SPUR calls this routine after the initiation of the design analysis.

SUBROUTINE TLOAD(HP,PLV,PAR,WA,WR,WT,WN)

Use: To determine the magnitude of tooth loading.

Calling Sequence: This is the first routine called after the variables have been specified in Subroutine UREAL.

2.7 TOOTH STRESSES

The most important design factors limiting the load carrying capacity of any gear set are the bending and wear stresses developed on a tooth. Buckingham [2, Chapter 18 and 22] presents one of the best historical accounts of the search for an accurate bending analysis over the last 200 years. Lewis' formula [36]for the beam strength of a gear tooth combined with Dolan and Broghamer's [38] fillet stress concentration factor has been the foundation for bending stress analysis in North America for the last thirty years. Various authors mentioned by Buckingham set forth numerous equations to modify the static case of the Lewis Formula to implement a realistic dynamic load analysis. Finally, in 1931, Buckingham [9],as chairman of an ASME special research committee,proposed a method of dynamic load determination. This same committee also initiated wear analysis employing Hertzian theory [37] of rolling cylinders to simulate the action of gear tooth profiles during wear conditions.

The American Gear Manufacturer's Association (AGMA), an organization of manufacturers and academics in the gearing industry, in its interest in achieving a certain degree of uniformity in the design and manufacture of gears, has presented general stress formulas which are modifications of the original bending and wear analysis equations. To build up to these equations, the following sections will develop the original theory as a foundation for discussion of the AGMA formulas.

2.7A BENDING STRESS

With an initial assumption of a gear tooth as a stubby cantilever beam stressed at the base of the beam, Lewis conceived the idea of inscribing a parabola of uniform strength inside the gear tooth. Appendix [C.1] offers proof that, if a parabola is made into a cantilever beam, the stress is constant along the surface of the parabola.

Inscribing the largest parabola that will fit in the gear-tooth shape, the most critically stressed position on the gear tooth is located at the point of tangency of the parabola and the tooth profile. Deriving the bending moment for a rectangular cantilever beam on thickness t, unit width, and length h from the base of the beam to the uniformly applied load W_t, the following stress equations for elementary beam theory evolve

$$\sigma = \frac{M_y}{I}$$

$$= \frac{(W_t h)(t/2)}{\frac{t^3}{12}}$$

$$= \frac{6W_t h}{t^2} \quad \text{for unit width} \qquad (2.7.1)$$

With the line of action crossing the tooth at different load angles θ_L during rotation, as in Figure 2.7.1, the above theory may be generally applied as follows.

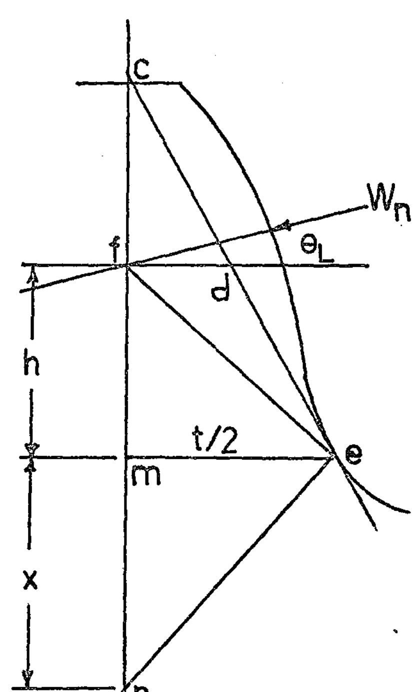


FIGURE 2.7.1 Gear Tooth as Parabolic Cantilever Beam

If the load acts at the tooth centre line at point f, then the parabola of uniform strength becomes tangent to the tooth profile at point e determined by having tangent line ce located such that cf=fm on the fillet. Appendix [C.2] illustrates the proof that point e lies on the parabola and may be considered the point of maximum bending stress on the tooth profile. Applying Equation (2.7.1) the bending stress becomes

$$\sigma_b = \left(\frac{W_n}{F_w}\right) \cos\theta_L \left(\frac{6h}{t^2}\right) \tag{2.7.2}$$

with the direct compressive stress becoming

$$\sigma_c = \frac{\left(\frac{W_n}{F_w}\right) \sin\theta_L}{t} \tag{2.7.3}$$

again assuming unit face width. The total normal load is divided by the face width so that unit width analysis may be used. The total tensile and compressive stresses thus become

$$|\sigma_t| = |\sigma_b| - |\sigma_c|$$

$$|\sigma_{comp}| = |\sigma_b| + |\sigma_c|$$

Numerically, the maximum resultant stress is the compressive stress found on the non-loaded side of the tooth, but in practice, fatigue failures in gear teeth generally begin at the fillet under tensile stress on the loaded side of the tooth. Taking the latter case as the criterion for stress analysis combining Equations (2.7.2) and (2.7.3)

$$|\sigma_{total}| = |\sigma_b| - |\sigma_c|$$

$$= \left(\frac{W_n}{F_w}\right) \cos\theta_L \frac{6h}{t^2} - \frac{\left(\frac{W_n}{F_w}\right) \sin\theta_L}{t}$$

$$= \frac{W_n}{F_w} \left(\frac{6h\cos\theta_L - t\sin\theta_L}{t^2}\right)$$

However, by Equation (2.6.4), the total stress in bending may be modified using the transmitted load, so that

$$|\sigma_{total}| = \frac{W_t}{F_w \cos\theta} \left(\frac{6h\cos\theta_L - t\sin\theta_L}{t^2}\right)$$

where θ represents the pressure angle. At the same time, from the geometry of Figure 2.7.1

$$x = \frac{t^2}{4h}$$

which enables the equation to be further altered to

$$|\sigma_{total}| = \frac{W_t}{F_w} \left[\frac{\cos\theta_L}{\cos\theta} \frac{1.5}{x} - \frac{\tan\theta_L}{t} \right] \tag{2.7.4}$$

Expanding Equation (2.7.4) further by multiplying by D_p/D_p gives

$$|\sigma_{total}| = \frac{W_t \, D_p}{F_w \, y} \tag{2.7.5}$$

where y is the modified Lewis Tooth Form Factor and is given by

$$y = \frac{D_p}{\frac{\cos\theta_L}{\cos\theta}\left(\frac{1.5}{x} - \frac{\tan\theta_L}{t}\right)} \tag{2.7.6}$$

A photoelastic investigation by Dolan and Broghamer [38] established expressions for stress concentration correction factors thus bringing Equation (2.7.6) closer to reality.

Further discussion of the tooth form factor and the stress correction factor will be continued in chapter 2.8A concerning geometry modifications factors. For the present, Equation (2.7.5) may be modified to take in a new geometry factor Q_j.

$$|\sigma_{total}| = \frac{W_t \, D_p}{F_w \, Q_j} \tag{2.7.7}$$

where

$$Q_j = \frac{y}{Q_f \, M_n} \tag{2.7.8}$$

and

Q_f = stress concentration factor

M_n = load sharing ratio

With this formulation as a basis, the AGMA has introduced modification factors Q_o, Q_s, Q_m and Q_r, to bring the analysis closer to reality. These factors, discussed in later sections, enable the designer to establish the properties of the final design for special cases of loading. Knowing the allowable fatigue stress of the gear material, a feasible design within this limit may be found from the following, where $|\sigma_{total}|$ now becomes σ_b. The predicted stress is

$$\sigma_b = \frac{W_t \, D_p}{F_w \, Q_j} \left(\frac{Q_o \, Q_s \, Q_m}{Q_v} \right) \tag{2.7.9}$$

and the critical stress is

$$\sigma_b^{max} = \sigma_F(Q_L/(Q_r Q_T)) \tag{2.7.10}$$

where

σ_F = allowable fatigue stress of the gear material

For no failure,

$$\sigma_b \leq \sigma_b^{max} \tag{2.7.11}$$

Since either empirical results or analytic solutions may be used to yield the stress factors, this type of formulation is much more flexible than any "true" theory developed which may be disproven in the future. Any stress given by the basic theory may form a ratio with any future analytic or experimental analysis to develop one of these modification factors for "exact" design.

Equations (2.7.9) and (2.7.10) are employed in the computer program in Subroutine BEND, Appendix [A.9].

SUBROUTINE BEND(WT,DP,FW,QOD,QODL1,QODL2,QJ1,QJ2,SB1,SB2,SBM1,SBM2,SAF1,
SAF2)

Use: To determine the actual and allowable bending stress.

Calling Sequence: This routine is called from Subroutine UREAL after the various modification factors are determined.

2.7A.1 TOOTH LOADING

It must be remembered, however, that the magnitude of bending stress is highly dependent on the load location on the tooth. The worst possible case of loading would occur with one tooth bearing all the load at its tip. This situation occurs when the addendum circle of the gear intersects with the line of action as illustrated in Figure 2.7.2.

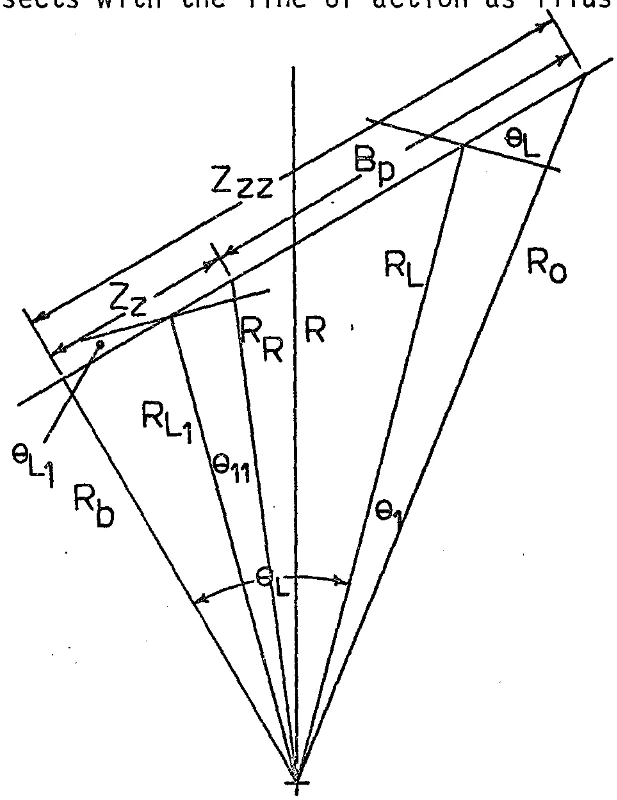


FIGURE 2.7.2 Geometry of Tooth-Tip Loading

Using Equation (2.4.5) to determine the angle between the tooth centre line and the radial vector to the contact point on the tooth profile, the load angle can be evaluated as follows:

$$\theta_L = \cos^{-1}(R_b/r)-\theta_1 \qquad (2.7.12)$$

$$R_L = R_b/\cos(\theta_L) \qquad (2.7.13)$$

where r = radius to point of tooth contact

R_L = load radius at tooth centre line

θ_L = load angle

θ_1 = angle between tooth centre line and radial vector to load point on tooth profile

Thus, by knowing the radius vector to the load point, the load radius and load angle may be determined using Equations (2.7.12) and (2.7.13). In the case of tip loading, $r=R_o$. To evaluate the load angle and radius of the succeeding tooth of the gear during tip-loading, the load contact point may be determined using

$$Z_{zz} = (R_o{}^2-R_b{}^2)^{\frac{1}{2}} \tag{2.7.14}$$

$$Z_z = Z_{zz}-B_p \tag{2.7.15}$$

$$R_R = (Z_z{}^2+R_b{}^2)^{\frac{1}{2}} \tag{2.7.16}$$

and then employing Equations (2.4.5), (2.7.12) and (2.7.13) to find the load angle and radius.

The only problem existing in this form of analysis is to determine the radius to the point of contact on the line of action. Initial analysis by Lewis assumed that the application of the load at the tip of the tooth represented the worst case of loading. This assumption was not incorrect as even the best gears at that time were not very accurate and it was quite possible for a single tooth to bear all the load. However, as gears became more accurate, enabling load sharing to occur, the tip-load condition was not necessarily the most critical. Higher contact ratios and less error in present day gears enable a second pair of teeth to be in contact when one pair has reached the tip-load condition of one member. This worst load condition occurs when a single pair of teeth carrying full load continue contact to a point where a second pair are ready to come into contact, as pictured in Figure 2.7.3.

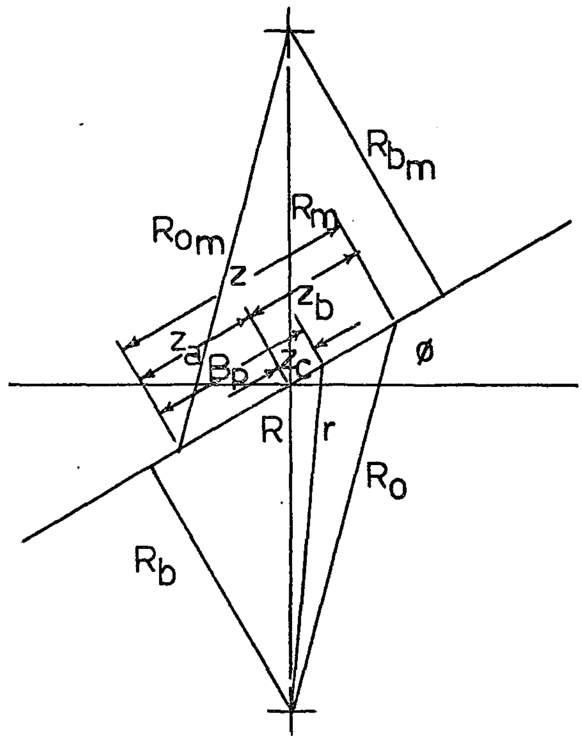


FIGURE 2.7.3 Geometry of Point of Highest Single Tooth Contact Loading

It can be readily seen that the contact point occurs one base pitch away from the initial point of contact. At the same time it must be visualized that tip-loading of one gear represents the mid-loading condition of the mating gear. By determining the radius to the desired point of contact, the foregoing analysis using Equations (2.4.5), (2.7.12) and (2.713) may be used to evaluate the load angle and radius. Thus, expanding the geometry of Figure 2.7.3 mathematically

$$Z_a = (R_{o_m}{}^2 - R_{b_m}{}^2)^{\frac{1}{2}} - (R_m{}^2 - R_{b_m}{}^2)^{\frac{1}{2}} \tag{2.7.17}$$

$$Z_b = (R_o{}^2 - R_b{}^2)^{\frac{1}{2}} - (R^2 - R_b{}^2)^{\frac{1}{2}} \tag{2.7.18}$$

$$Z = Z_a + Z_b \tag{2.7.19}$$

$$Z_c = B_p - Z_a \tag{2.7.20}$$

and applying the law of cosines

$$r = (R^2+Z_c{}^2+(2R)(Z_c\sin\phi))^{\frac{1}{2}} \qquad (2.7.21)$$

yields the contact point radius. Following a similar procedure as for the tip-load case for determining the contact point radius of the succeeding tooth, the following results

$$Z_{zz} = (r^2-R_b{}^2)^{\frac{1}{2}} \qquad (2.7.22)$$

which can be coupled with Equations (2.7.15) and (2.7.16).

Thus, by knowing the contact radius, the load angle and radius may be evaluated using the previous analysis. The highest single tooth contact loading analysis, however, can only be assumed if the deformation of the gear teeth is enough to eliminate the base pitch error in the teeth due to profile and pitch tolerances during machining. Criteria for this will be discussed in the following sections.

The theory of this section is incorporated in the computer program in Subroutine LOAD, Appendix [A.7].

SUBROUTINE LOAD(RL,ANGL,RLL,ANGLL,NLOAD,BP,PAR,PR,PRM,RB,RBM,RO,ROM,TP)

Use: This routine determines the radius to the point of load application at the tooth centre line as well as the load angle for either tip-loading or point of highest single tooth contact loading. The load radius and load angle for the succeeding tooth are also determined.

Calling Sequence: This routine is called in Subroutine UREAL if the mode of loading is assumed by the user. If the user wishes the mode of loading to be determined by the computer program, the routine is called in Subroutine SHARE.

Special Features: If some of the logic statements in this routine seem redundant, it must be remembered that during the optimization search, solutions bordering on the limits of the theory or the limits of physical restrictions may be encountered. These logic statements restrict the analysis to acceptable computations within the limits of the computer. For example, r must be greater than R_b in the statement

$\theta = \cos^{-1}(R_b/r)$.

2.7A.2 TOOTH DEFLECTION AND LOAD SHARING

As gear manufacturing improved, all properly designed gears with low errors had sufficient overlap of successive pairs of teeth to allow possible load sharing in the load zone at the beginning and end of contact. With this possibility arising, many gear designers believed that tip-loading was too drastic a load condition assuming that the highest bending stresses would occur down the face of the tooth below the tip, thus making gears designed by Lewis' assumptions stronger than necessary.

Timoshenko and Baud [48], Walker [40], Weber, and Van Zandt [39] are a few of the published sources of tooth deflection sources for load sharing analysis. Van Zandt's work seems to be the most widely accepted in North America with the AGMA basing a load sharing chart on his findings. In his paper Van Zandt states that his results were about 45% greater than that found by Weber while he says Weber found Walker's results to give 15 to 25% less deflection than according to Weber's calculations. With Van Zandt's results basically from experiment for one pressure angle system with no apparent empirical formulation to generalize for all designs, the writer decided to adapt Walker's empirical formula proportionately by the percentages indicated, to assume a deflection equation close to Van Zandt's results. This seemed to be a correct procedure as Van Zandt stated in his paper that in the absence of more complete data he proportionately increased Weber's deflection curves (which were quite similar in shape to Walker's) to conform to the limited deflection tests Van Zandt had done at the time of the paper. Coupled with load distribution analysis given by Merritt [4] and a double tooth contact analysis by Buckingham [2], the author developed a load sharing analysis technique which hopefully gives reasonable results. Since no experimentation has been done to verify the procedure, no guarantee of the results can be given.

Walker's formula is closely related to the bending analysis used to evaluate the tooth form factor, in that it uses the chordal tooth thickness at the point of highest stress concentration on the fillet, the distance from the load application point on the tooth centre line to the tooth thickness chord, and the load angle at the load point to yield the deflection

$$\delta = KW_n' \left(\frac{h_1}{t_1E_1}\cos\theta_{L_1} + \frac{h}{t_2E_2}\cos\theta_{L_2}\right) \tag{2.7.23}$$

where δ = deflection of both teeth at point of contact

K = constant

W_n' = load per inch face normal to the involute

E = modulus of elasticity (psi)

θ_L = load angle (degrees)

h,t = constant stress parabola properties (Figure 2.7.1) with the subscripts defining the two gears.

However, for linear stiffness ρ_c,

$$\rho_c = \frac{W_n}{\delta} \tag{2.7.24}$$

$$= \frac{F_w \; W_n{}'}{\delta}$$

Thus

$$\rho_c = \frac{F_w}{K\left(\frac{h_1}{t_1 E_1}\cos\theta_{L_1} + \frac{h_2}{t_2 E_2}\cos\theta_{L_2}\right)} \tag{2.7.25}$$

over the whole face width.

If the contact ratio is greater than one, the load is alternately carried by one and two pairs of teeth with the load divided between successive pairs of teeth in proportion to the respective combined stiffnesses at the points of contacts concerned, assuming the profiles and spacing are precise. Thus,

$$W_{n_1} = W_n \; \frac{\rho_{c_1}}{\rho_{c_1}+\rho_{c_2}} \tag{2.7.26}$$

$$W_{n_2} = W_n \; \frac{\rho_{c_2}}{\rho_{c_1}+\rho_{c_2}} \tag{2.7.27}$$

where the subscripts 1 and 2 refer to the two pairs of teeth loaded.

If, however, the second pair of teeth can not come in contact during the no load condition due to involute profile deviations, the initial pair of teeth must deflect by the amount of the error before the second pair of teeth can share the load. If the gap between the contact profiles of the two teeth is ε, the total load must be greater than $\varepsilon \cdot \rho_c$ before the second pair come into contact. Assuming the total load deflects the first pair of teeth, the load sharing is divided as follows:

$$W_{n_1} = \frac{W_n \rho_{c_1} + \varepsilon \cdot \rho_{c_1} \cdot \rho_{c_2}}{\rho_{c_1} + \rho_{c_2}} \tag{2.7.28}$$

$$W_{n_2} = \frac{W_n \rho_{c_2} - \varepsilon \cdot \rho_{c_1} \cdot \rho_{c_2}}{\rho_{c_1} + \rho_{c_2}} \tag{2.7.29}$$

as long as

$$W_n > \varepsilon \cdot \rho_{c_1} \tag{2.7.30}$$

Knowing the base pitch error (i.e. the error in action) along the line of action, the load proportions for tip-loading, and its successive pair, as well as point of highest single tooth contact loading (hereafter described as mid-loading), and its successive pair, are evaluated and compared for the worst stress condition. The actual worst stress may be determined from the Lewis theory directly since the relative magnitudes only are wanted. The worst stress condition, whether tip-loading or mid-loading, then becomes the prime stress criterion exclusive of any load sharing.

From the geometric development of the point of highest single tooth contact loading, it can be readily seen that for "perfect" gears, when one gear is in tip-loading, the mating gear is in mid-loading. It will also become obvious from Equations (2.7.28) and (2.7.29) that the worst load case for either tip-loading or mid-loading occurs when the mating pair under observation must deflect <u>first</u> before the following pair comes into contact. With the stiffnesses of the mating pairs specified in a given position of contact it is obvious that the resultant tip-load, for example,necessary for tip deflection and load sharing, will be larger than the resultant tip-load from load sharing when the mid-loaded tooth deflected to achieve load sharing. Thus the eight possible loading combinations may be reduced with discussion centering on the four cases of tip and mid-loading for both the gear and pinion from the standpoint of initial deflection of each case.

If the bending stress for either pinion or gear tip-loading exceeds the mid-loading bending stress for either case, then the tip-loading geometry factor for both gears will be used with the transmitted load for the actual bending stress analysis. On the other hand, the mid-loading geometry factors would be employed if the bending stress conditions were reversed.

The theory of this chapter has been incorporated in Subroutine SHARE [Appendix A.8] as a method of checking for load sharing. As the method demands more computation, it is suggested that a design be found for tip and mid-point loading and then test the "best" design for load sharing.

```
SUBROUTINE SHARE(ANGC1,ANGC2,ANGL1,ANGL2,BBY1,BBY2,BP,DP,E1,E2,ERR,FW,
                 NCUT1,NCUT2,NNLOAD,PAD,PAR,PI,PR1,PR2,QO,QV,QJ1,QJ2,
                 RB1,RB2,RI1,RI2,RL1,RL2,RLL1,RLL2,RLM1,RLM2,RO1,RO2,
                 RT1,RT2,TP1,TP2,WN)
```

Use: This routine determines if there is load sharing between successive pairs of teeth in a mating gear set. Analysis is made for tip and highest point of single tooth contact loading with the mode of loading producing the highest bending stress chosen for final stress analysis.

Calling Sequence: If the flag NLOAD=0 is called for Subroutine UREAL this analysis is carried out.

Special Features: The constant specified by Walker for Equation (2.7.23) was 14.0 from his experimental analysis. However, to partially conform with Van Zandt's this constant has been raised to 25.0.

2.7B WEAR STRESS

The previous sections have been concerned with the stress and strength of a gear tooth subjected to bending action. However, other modes of tooth failure occur affecting the surface of the tooth to produce failure. For example, pitting is a surface fatigue failure due to many repetitions of high contact stresses; scoring is a surface failure due to lubrication failure; abrasion is a surface failure due to the presence of foreign material. The combination of rolling and sliding motions of the gear tooth surfaces moving across each other cause additional compressive and tensile stresses to develop due to the sliding plus the coefficient of friction. Stress cycles during heavy loading result in both surface cracks and plastic flow on the contacting surface to bring about material failure on the tooth profile. Dudley [6], employing the work of Hertz [37], presented a derivation of an approximate solution to the surface fatigue stress problem, from which the AGMA developed their general formula similar in nature to the bending stress analysis.

From Hertz's analysis of two cylinders with axes parallel, as in Figure (2.7B.1), the width of band of contact resultant from an applied load of F pounds over the length L is

$$B = \left(\frac{16F}{L} \; \frac{(K_1+K_2)r_1r_2}{(r_1+r_2)} \right)^{\frac{1}{2}}$$

where

$$K_1 = \frac{1-\nu_1{}^2}{\pi E_1}$$

$$K_2 = \frac{1-\nu_2{}^2}{\pi E_2}$$

with ν_i = Poisson's ratio

E_i = modules of Elasticity

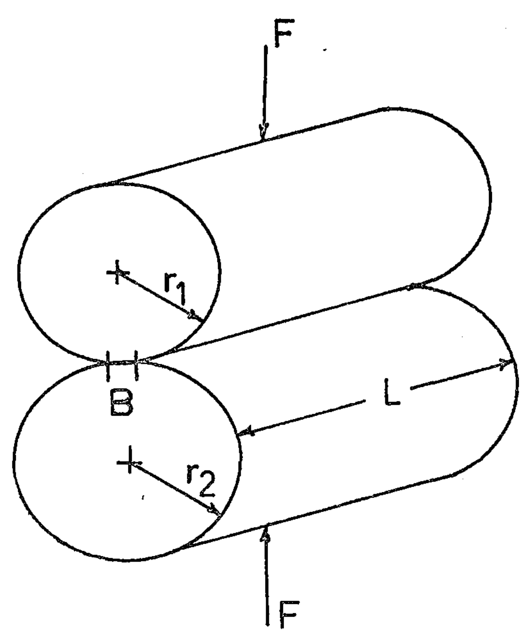


FIGURE 2.7B.1 Hertzian Cylinders in Contact

At any instant of time when the tooth profiles are in contact, the surfaces of the teeth at those points may be considered cylinders with centres on the base circle, as illustrated in Figure 2.4.1 concerning involute development. Thus

$$B = \left(\frac{16F(K_1+K_2)}{\pi L(\frac{1}{r_1}+\frac{1}{r_2})} \right)^{\frac{1}{2}} \qquad (2.7.31)$$

The maximum compressive stress between the contacting surfaces of the cylinders is

$$\sigma_w = \frac{4F}{\pi LB} \qquad (2.7.32)$$

Since the load transmitted acts along the line of action and the width of the tooth equals the face width, Equation (2.7.32) becomes

$$\sigma_w = \frac{4W_n}{\pi F_w B}$$

from which, applying Equation (2.7.31)

$$\sigma_w^{\,2} = \frac{16W_n^{\,2}}{\pi^2 F_w^{\,2} \left(\frac{16W_n}{\pi F_w} \left(\frac{K_1+K_2}{\frac{1}{r_1}+\frac{1}{r_2}} \right) \right)}$$

$$\sigma_w = \left[\frac{W_n \, (\frac{1}{r_1}+\frac{1}{r_1})}{\pi F_w (K_1+K_2)} \right]^{\frac{1}{2}}.$$

but $\quad W_n = \dfrac{W_t}{\cos\phi}$

$$\sigma_w = \left[\frac{W_t}{\pi F_w (K_1+K_2) \left(\frac{r_1 r_2}{r_1+r_2} \right)} \right]^{\frac{1}{2}}$$

By multiplying the denominator of this equation by d/d, where d is the pinion pitch circle diameter

$$\sigma_W = \left[\frac{W_t}{F_W d}\left(\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1} \quad \frac{1-\nu_2^2}{E_2}\right)}\right)\left(\frac{1}{\left(\frac{r_1 r_2}{r_1+r_2}\right) \quad \frac{\cos\phi}{d}}\right)\right]^{\frac{1}{2}} \qquad (2.7.33)$$

By specifying

$$C_E = \left[\frac{1}{\pi\left(\frac{1-\nu_1^2}{E} \quad \frac{1-\nu_2^2}{E}\right)}\right]^{\frac{1}{2}} \qquad (2.7.34)$$

and

$$C_J = \left[\left(\frac{r_1 r_2}{r_1+r_2}\right) \frac{\cos\phi}{d}\right] \qquad (2.7.35)$$

then the basic wear stress equation becomes

$$\sigma_W = C_E \left(\frac{W_t}{F_W d C_J}\right)^{\frac{1}{2}} \qquad (2.7.36)$$

Using the Equation (2.7.33) as a basis, the AGMA developed a generalized surface durability equation using modification factors to establish a realistic design criterion. Knowing the allowable contact stress of the gear material, the following equations allow the designer within the limits of the modification factors to be discussed in later sections, to predict the wear capabilities of his design in regards to destructive pitting. The predicted stress is

$$\sigma_W = C_E \left[\frac{W_t}{F_W d}\left(\frac{C_f \; C_m \; C_o \; C_s}{C_J \; C_V}\right)\right]^{\frac{1}{2}} \qquad (2.7.37)$$

The allowable critical fatigue stress is

$$\sigma_W{}^{max} = \sigma_C \frac{C_L \; C_H}{C_R \; C_T} \qquad (2.7.38)$$

where

σ_C = allowable contact stress of the gear material

The design criterion is

$$\sigma_W \leq \sigma_W^{max} \quad (2.7.39)$$

Similar reasoning for the use of modification factors, as used for the bending equations, may also be applied here. The equations introduced in this section are incorporated in Subroutine WEAR, Appendix [A.10].

SUBROUTINE WEAR(COD,CODL1,CODL2,CE,CJ,FW,FW,PR1,SAC1,SAC2,SS1,SS2,SSM1,
SSM2,WT)

Use: To determine the actual and allowable contact stress on the tooth face.

Calling Sequence: This routine is called in Subroutine UREAL after the bending stress is evaluated.

2.7C ALLOWABLE POWER

With Equations (2.7.9), (2.7.10), (2.7.37), and (2.7.38) specifying the constraining equations of both bending and wear analysis, the maximum allowable horsepower transmitted by a gear set within the conditions specified may be evaluated. Since σ_b^{max} and σ_W^{max} represent the limits of the allowable stresses for bending and wear analysis, the maximum load to achieve this value can be calculated so that

$$\frac{\sigma_F \, Q_L}{Q_R \, Q_T} = \frac{W_t \, Q_o}{Q_V} \frac{D_p}{F_W} \frac{Q_s \, Q_m}{Q_J}$$

and

$$\sigma_c \frac{C_L \; C_H}{C_T \; C_R} = C_E \left(\frac{W_t \; C_o}{C_v} \frac{C_s}{dF_w} \frac{C_m \; C_f}{C_J} \right)^{1/2}$$

but $$h_p = \frac{W_t \; PLV}{33,000} \qquad (2.6.3)$$

Specifying the allowable power in bending as PAB and the allowable power in wear as PAW, the following results

$$PAB = \frac{PLV}{33000} \frac{Q_V \; F_W \; Q_J}{D_p \; Q_o \; Q_s \; Q_m} \frac{Q_L \; \sigma_F}{Q_R \; Q_T} \qquad (2.7.40)$$

$$PAW = \frac{PLV}{33000} \frac{dF_W \; C_V \; C_J}{C_o \; C_s \; C_m \; C_f} \left(\sigma_c \frac{C_L \; C_H}{C_E \; C_R \; C_T} \right)^2 \qquad (2.7.41)$$

However, with the pitch circle radius in inches, the pitch line velocity in feet per minute becomes

$$PLV = (2\pi n_1 R_1)/12 \qquad (2.5.2)$$

which changes Equations (2.7.40) and (2.7.41) to

$$PAB = \frac{\pi n_1 F_W}{396,000} \frac{2R_1 \; Q_V \; Q_J}{D_p \; Q_o \; Q_s \; Q_m} \frac{Q_L \; \sigma_F}{Q_R \; Q_T} \qquad (2.7.42)$$

$$PAW = \frac{\pi n_1 F_W}{396,000} \frac{C_J \; C_V}{C_s \; C_m \; C_f \; C_o} \left[\frac{\sigma_c 2R_1 \; C_L \; C_H}{C_E \; C_T \; C_R} \right]^2 \qquad (2.7.43)$$

Knowing the values of the modification factors as well as gear and material properties, the maximum allowable horsepower transmitted by a gear set may be determined, giving some insight into the capabilities of the design. Subroutine POWER, Appendix [A.11], employs this theory in the computer program.

SUBROUTINE POWER(CE,CJ,COD,CODL1,CODL2,QJ1,QJ2,QOD,QODL1,QODL2,DP,FW,PAB1,
PAB2,PAW1,PAW2,PI,PR1,RPM1,SAC1,SAC2,SAF1,SAF2)

Use: To determine the maximum allowable power that can be transmitted under wear and bending conditions for the pinion and gear.

Calling Sequence: Subroutine UREAL calls this routine after the wear and bending stress routines are called.

2.8 MODIFICATION FACTORS

The AGMA, in an effort to standardize design practice, developed the foregoing stress analysis so that future changes in the art could easily be incorporated in the analysis without a major renovation to the theory. The stress equations of both bending and wear are divided into three groups of terms concerned with the loading, the tooth size and the stress distribution as the following expressions indicate:

$$\sigma_b = \frac{W_t Q_o}{Q_v} \frac{D_p}{F_w} \frac{Q_s Q_m}{Q_J} \tag{2.7.9}$$

$$\sigma_b^{max} = \frac{\sigma_F Q_L}{Q_R Q_T} \tag{2.7.10}$$

$$\sigma_b \lesssim \sigma_b^{max} \tag{2.7.11}$$

$$\sigma_w = C_E \left(\frac{W_t C_o}{C_v} \frac{C_s}{dF_w} \frac{C_m C_f}{C_J} \right)^{\frac{1}{2}} \tag{2.7.37}$$

$$\sigma_w^{max} = \sigma_c \frac{C_L C_H}{C_T C_R} \tag{2.7.38}$$

$$\sigma_w \lesssim \sigma_w^{max} \tag{2.7.39}$$

where

	σ_b	= calculated tensile bending stress at the root of the teeth, psi
	σ_W	= calculated contact stress
	C_E	= elastic coefficient
LOAD	W_t	= transmitted tangential load at operating pitch diameter, lb
	Q_o, C_o	= overload factor
	Q_v, C_v	= dynamic factor
tooth size	D_p	= diametral pitch
	d	= pinion operating pitch diameter, inches
	F_W	= net face width of the narrowest of the mating teeth, inches
	C_s	= size factor
stress distribution	Q_s	= size factor
	Q_m, C_m	= load distribution factor
	Q_J, C_J	= geometry factor
	C_f	= surface finish factor
	σ_F	= allowable fatigue stress, psi
	σ_C	= allowable compressive stress, psi
	R_L, C_L	= life factor
	Q_T, C_T	= temperature factor
	Q_R, C_R	= factor of safety (reliability factor)
	C_H	= hardness ratio factor

The subsequent sections will present the AGMA standards used in the above analysis along with modifications of these standards used in the computer program. It will be readily seen that future stress formulas created from new theory can form ratios with the "base" theory for development of the above factors. The computer programs have the same flexibility, as each factor is developed independently in individual subroutines. Similar factors for wear and bending will be developed in the same sections.

The derivation of the following factors are extracted from various AGMA standards mentioned in the references.

2.8A GEOMETRY FACTORS INCLUDING STRESS CONCENTRATION

From the development of the base bending stress formula in Chapter 2.7, the tooth form factor

$$Y = \frac{D_p}{\frac{\cos\theta_L}{\cos\theta}\left(\frac{1.5}{x} - \frac{\tan\theta_L}{t}\right)} \tag{2.7.6}$$

was derived and modified by Dolan and Broghamer's stress concentration factor [38] to give the bending stress geometry factor

$$Q_J = \frac{Y}{Q_f m_n} \tag{2.7.8}$$

The geometry factor evaluates the shape of the tooth, the position at which the most damaging load is applied, stress concentration due to geometric shape, and the sharing of load. Accurate spur gears develop the most critical stress when the load is applied at the highest point of tooth where a single pair of teeth is carrying all the load. Less accurate spur gears, having errors that prevent two pairs of teeth from sharing the load, may be stressed most heavily when load is applied at the tip. Load sharing was discussed in Section 2.7A.2.

Figure 2.7.1 illustrates the tooth form factor layout used for any general load application on the tooth. Using some of the geometrical relationships of this figure, equation (2.7.6) was derived. The Dolan and Broghamer stress correction factor employs similar relationships for the equation

$$Q_f = c_1 + \left(\frac{t}{r_f}\right)^{c_2}\left(\frac{t}{h}\right)^{c_3} \tag{2.8A.1}$$

where

h = distance fm from Figure 2.7.1

$t/2$ = distance me from Figure 2.7.1

and

$$r_f = r_T + \frac{(b-r_T)^2}{RR_o+(b-r_T)} \tag{2.8A.2}$$

with r_f = radius of curvature of fillet

r_T = edge radius of tool

RR_o = the relative radius of curvature of the pitch circle of the gear and the pitch line of the generating tool. For generation by a rack or hob, RR_o equals the pitch radius R of the gear being generated. For generation by a pinion shaped cutter, $1/RR_o = 1/R + 1/R_c$ where R_c is the pitch radius of the cutter.

b = dedendum of gear

From experimental analysis the constants of Equation (2.8A.1) are tablulated as follows:

TABLE 2.8A.1

Values of C_1, C_2 and C_3 of Equation 2.8A.1

Pressure Angle	C_1	C_2	C_3
14½°	0.22	0.20	0.40
20°	0.18	0.15	0.45
25°	0.14	0.11	0.50

For other pressure angles not presented in the above table, values for the constants may be obtained using linear interpolation or extrapolation.

The stress correction factor actually depends on the effective stress concentration, location of load, plasticity effects, residual stress effects, material composition effects, surface finish resulting from gear production or service, Hertz stress effects, size effects and end of tooth effects. With this many factors affecting the stress concentration, the analytic method presented can only be expected to give approximate results for all situations.

The load sharing ratio, m_n, is influenced by the contact ratio, but may be taken as 1, since the most critical position of spur gear load application normally occurs when only one tooth is in contact.

The geometry values from Figure 2.7.1 can be determined graphically following the procedure outlined in AGMA standard for Rating the Strength of Spur Gear Teeth (AGMA 220.02), or using an iterative method developed for the computer using basic theory. Redrawing Figure 2.7.1 to include the radius to a point on the fillet, Figure 2.8A.1 illustrates an iterative process to determine the highest stress point on the fillet.

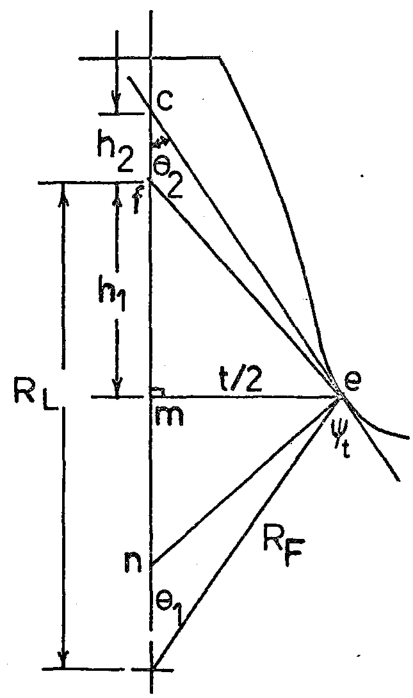


FIGURE 2.8A.1 Geometrical Determination of Highest Stress Point on the Fillet

According to Appendix [C1 and C2], the parabola of constant stress will be tangent to the fillet at point e when $h_1 = h_2$. This relationship may be expanded to give a function

$$F(R_F) = h_1 - h_2 = 0$$

$$= - \left[\frac{t/2}{\tan\theta_2} - (R_L - R_F\cos\theta_1)\right]$$

$$+ (R_L - R_F\cos\theta_1)$$

but $$\frac{t}{2} = R_F\sin\theta_1$$

or $$f = +(2.0R_L - R_F(2.0\cos\theta_1+(\sin\theta_1/\tan\theta_2)))$$

and $$\theta_2 = \psi t-\theta_1$$

therefore

$$F(R_F) = 2R_L-R_R(2\cos\theta_1+(\sin\theta_1/\tan(\psi_t-\theta_1))) \qquad (2.8A.3)$$

Thus, solving for the root of this equation employing the linear search and false position technique outlined in Appendix [B], the geometric relations for the tooth form factor and the stress correction factor can be derived. The profile and tangent angle of the fillet with respect to the centre of the gear may be determined using the equations developed in Chapter 2.4.

From the extended theory of the Lewis technique, the limiting load radius would occur when the constant stress parabola becomes tangent to the fillet at the dedendum circle, at which point the tangent angle ψ_t = 90° and the vectorial angle $\theta_1 = \theta_c$, where θ_c is the angle between the tooth centre line and the origin of the fillet curve at the dedendum circle, with a fillet radial vector equal to the dedendum circle radius. Since $h_1 = h_2$ is a necessary requirement in the theory for the constant stress parabola assumption to exist, then

$$R_L^{min} = R_I(\cos\theta_c+0.5\ \frac{\sin\theta_c}{\tan(\frac{\pi}{2} - \theta_c)}) \qquad (2.8A.4)$$

which results from setting $\theta_1 = \theta_c$, $R_F = R_I$ and $F(R_F) = 0$ Equation (2.8A.3)

This analysis has been used in Subroutine JFACT [Appendix A.12] and Subroutine CWALL [Appendix A.13]for computed geometry factor determination.

SUBROUTINE CWALL(BBY,ANGC,ANGR,NCUT,PI,PR,RI,RL,RRF,RRO,RT)

Use: This subroutine calculates, in radial coordinates with respect to the centre of the gear referenced to the centre line of the tooth, the point on the tooth fillet considered the location of highest stress concentration.

Calling Sequence: Subroutine JFACT calls this routine as part of the development of geometry factor used in bending analysis.

Special Features: Because of the nature of the fillet equations, the function Equation (2.8A.3) is discontinuous thus preventing a gradient solution. The linear search and false position technique enables the desired solution to be found. Since the parabola of the physically limiting case would be tangent to the fillet at the dedendum circle, this was chosen as the first point in the iterative process. The step length from this point was chosen arbitrarily as 10% of the distance between the dedendum cirlce radius and the radius where the tangent angle to the trochoid becomes zero. A check to prevent divergence on the discontinuous portion of the curve was also employed.

In the case of the sharp cornered cutter tooth, the trochoid of the corner represents the fillet, thus the angle between the radial vector and the tangent line to the fillet can be computed directly from the theory. However, for rounded corner cutter teeth the angle between the radial vector and the tangent line to the fillet can not be computed from the trochoid tangent angle but must be altered slightly as in Equation (2.4.18). The program was developed using the above theory and was only modified slightly to incorporate the root determination technique.

SUBROUTINE JFACT(ANGC,ANGL,BBY,DP,NCUT,PAD,PAR,PI,PR,RI,RL,RLM,RT,H,T, QJ,Y)

Use: This subroutine calculates the geometry factor and the tooth form factor for spur gear bending stress analysis. The minimum load application radius possible for tooth form factor analysis by the Lewis technique is also calculated.

Calling Sequence: Depending on the method used to determine the point of load application, this routine can be called from either Subroutine UREAL or Subroutine SHARE. Computer calculated load points call this routine from Subroutine SHARE while user specified load points require this routine in Subroutine UREAL.

Special Features: If the computed load radius during the analysis exceeds the minimum limit, Equation (2.8A.4), then the iterative solution is bypassed, the load point is assumed to be at the limit and the analysis is continued. As an added feature, the Dolan and Broghamer Equation (2.8A.1) with its constants from TABLE (2.8A.1) has been generalized by developing linear equations for the constants using the 14½° and 20° pressure angles as base points for equation determination.

As in the bending stress analysis, the wear geometry factor results from the derivation of contact stress on the tooth profile, Section 2.7B. The geometry factor, Equation (2.7.35), extracted from Equation (2.7.33), acts as the base equation for the general mathematical development. The greatest contact stress occurs at the lowest point of single tooth contact of the pinion where the sliding velocity and friction factor of the tooth profiles would be greatest and the relative radius of curvature of the two involute profile would also be the smallest, thus forcing the stress to be greatest. From Figure (2.8A.2) the geometry factor is

$$C_J = \left(\frac{r_1 r_2}{r_1+r_2}\right)\left(\frac{\cos\phi}{d}\right) \tag{2.7.35}$$

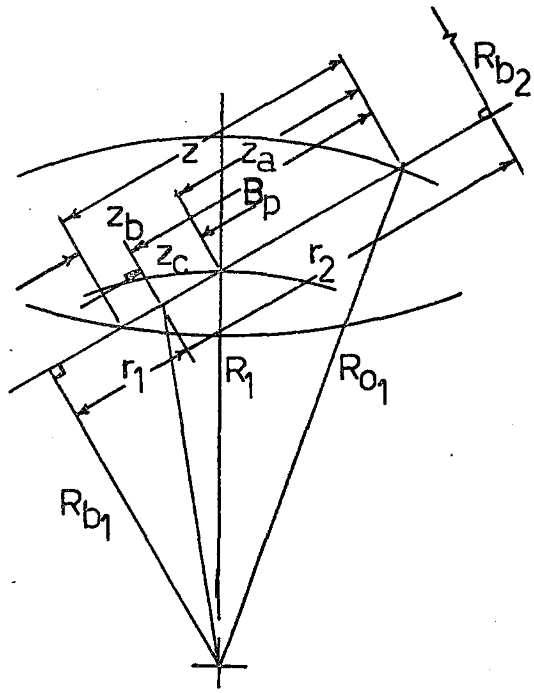


FIGURE 2.8A.2 Lowest Point of Single Tooth Contact

with r_1, r_2 = radii of curvature of pinion and gear involute at point of contact

d = pinion pitch circle diameter

ϕ = pressure angle

This can be changed to

$$C_J = \left[\frac{(R_1 \sin\phi - Z_c)(R_2 \sin\phi + Z_c)}{(R_1 \sin\phi - Z_c) + (R_2 \sin\phi + Z_c)}\right] \frac{\cos\phi}{2R_1}$$

$$= \frac{\cot\phi}{2R_1} \left[\frac{R_1 R_2 \sin^2\phi - Z_c \sin\phi (R_2 - R_1) - Z_c{}^2}{(R_1 + R_2)}\right]$$

$$= R_1 R_2 \frac{\cot\phi}{2R_1} \left(\frac{\sin^2\phi - Z_c \sin\phi \, \frac{1}{R_1} - \frac{1}{R_2} - \frac{Z_c{}^2}{R_1 R_2}}{R_1 + R_2}\right)$$

But $$m_g = \frac{R_2}{R_1} = \text{gear ratio}$$

therefore, expanding the above equation

$$C_J = 0.5\cot\phi \left(\frac{m_g}{m_g+1}\right) \left(\sin + \frac{Z_c}{R_2}\right) \quad \left(\sin\phi - \frac{Z_c}{R_1}\right) \qquad (2.8A.5)$$

and from Figure 2.8A.1

$$Z_a = \sqrt{R_{o_1}{}^2 - R_{b_1}{}^2} - \sqrt{R_1{}^2 - R_{b_1}{}^2} \qquad (2.8A.6)$$

$$Z_b = \sqrt{R_{o_2}{}^2 - R_{b_2}{}^2} - \sqrt{R_2 - R_{b_2}{}^2} \qquad (2.8A.7)$$

$$Z_c = B_p - Z_a \qquad (2.8A.8)$$

In the computer program, the above theory is used in Subroutine IFACT [Appendix A.14] to evaluate the wear geometry factor.

SUBROUTINE IFACT(BP,CJ,PAR,PR1,PR2,RATIO,RB1,K01)

Use: This routine determines the geometry factor for the worst case of surface loading at the point of lowest single single tooth contact.

Calling Sequence: Subroutine UREAL calls this routine when evaluating the other modification factors.

Special Features: As can be seen from the AGMA wear stress equation, the contact stress would tend to infinity when the geometry factor approached zero. To avoid this during the optimization, the geometry factor is set to an arbitrary zero of 10^{-50} if $(\sin\phi - Z_c/R_1) \leq 0$.

2.8B ELASTIC COEFFICIENT

The elastic coefficient term of the wear stress analysis comes from the contact stress in Equation (2.7.33), and is defined by

$$C_E = \left[\frac{1}{\pi\left(\frac{1-\nu_1{}^2}{E_1} + \frac{1-\nu_2{}^2}{E_2}\right)}\right]^{\frac{1}{2}} \qquad (2.7.34)$$

where ν = Poisson's ratio

E = modulus of elasticity

with subscripts 1 and 2 representing the pinion and the gear, respectively.

This analysis of pinion and gear properties is used in Subroutine EFACT [Appendix A.15].

SUBROUTINE EFACT(CE,E1,E2,PI,U1,U2)

Use: To determine the elastic coefficient for the surface stress analysis.

Calling Sequence: This routine is called from Subroutine SPUR with other modification factors not affected by variable changes during optimization.

2.8C DYNAMIC (VELOCITY) FACTOR

The bending and wear stress analysis have both been computed using the average transmitted load, while in actual fact there will be load fluctuations. The dynamic load is due to vibrations in the geared system which produce sudden accelerations of the gears, followed by impact loading when the mating gear teeth come back into mesh. The nature of the tooth vibrations is affected by the inertia and stiffness of all rotating elements, the rotational and pitch line speeds, the tooth spacing and profile errors, the magnitude of transmitted load per inch of face and the tooth stiffness. Shipley [7, Chapter 14] has traced some of the works of the many investigators, and states that they are not in full agreement as to the maner in which dynamic load effects should be evaluated. He suggests that Buckingham's method [2] seems to be the proper approach to use considering the state of the art.

The fundamental Buckingham equation for dynamic load determination is

$$W_d = W_t+(W_a(2W_2-W_a))^{\frac{1}{2}} \qquad (2.8C.1)$$

where

W_d = dynamic load, lbs

W_t = transmitted load, lbs

W_a = acceleration load, lbs

W_2 = force required to deform teeth through amount of effective error, lbs

with the acceleration load defined as

$$W_a = \frac{W_1 W_2}{W_1 + W_2} \tag{2.8C.2}$$

where

W_1 = average force required to accelerate the masses when they are considered as absolutely rigid, lbs

The forces may be defined as

$$W_1 = \left[\frac{\tan\phi(1-\cos\phi)}{150\phi^2}\right]\left(\frac{1}{R_1} + \frac{1}{R_2}\right) m(PLV) \tag{2.8C.3}$$

where

ϕ = pressure angle

R = pitch radius, inches

m = effective mass influence at gear pitch line, slugs

PLV = pitch line velocity, fpm

with

$$m = \frac{m_1 m_2}{m_1 + m_2} \tag{2.8C.4}$$

and m_1 and m_2 = effective masses acting at pitch line of pinion and gear respectively, slugs

while $$W_2 = W_t\left[\frac{e}{d} + 1\right] \tag{2.8C.5}$$

where e = measured error in action, inches

d = deformation of the teeth at the pitch line caused by load W_t, inches

Buckingham then presented deformation formulas determined by Timoshenko and Baud [48] to calculate the amount of deflection for teeth under load conditions. At the same time an approximation for mid-tooth deflection based on experimental results was given for use in the dynamic analysis. Since that time more work has been done on deflection analysis without a concrete analytic or emperical formula being presented. With no dependable formula being given for deformation analysis except for a modified Walker formula (Chapter 2.7A.2 on Tooth Deflection and Load Sharing) coupled with the vast disagreement of dynamic load analysis, as well as the difficulties in specifying the elemental properties of a design, the author decided to use the less formal approach specified by the AGMA [24,26].

The following three formulas are given by the AGMA:

c_v or $Q_v = 1.0$ for high precision shaved or ground spur gears where no appreciable dynamic load is developed

c_v or $Q_v = \left(\frac{78}{78+PLV^{\frac{1}{2}}}\right)^{\frac{1}{2}}$ for high precision shaved or ground spur gears where dynamic load is developed

c_v or $Q_v = \frac{50}{50+PLV^{\frac{1}{2}}}$ for spur gears finished by hobbing or shaping

Initially, the more accurate gears are made, the more precise the mountings will be. On this premise the author felt that dynamic loads would then also be a function of AGMA quality number representative of the profile or tooth spacing errors. Thus six equations of the same form suggested by the AGMA have been used in the program to determine the velocity factor. These equations, illustrated in Figure 2.8C.1, are

$$c_{v_1} = \frac{600}{600+PLV} \qquad NQUAL=3,4,5 \qquad (2.8C.6)$$

$$c_{v_2} = \frac{1200}{1200+PLV} \qquad NQUAL=6,7 \qquad (2.8C.7)$$

$$c_{v_3} = \frac{50}{50+PLV^{\frac{1}{2}}} \qquad NQUAL=8,9 \qquad (2.8C.8)$$

$$c_{v_4} = \frac{78}{78+PLV^{\frac{1}{2}}} \qquad NQUAL=10,11,12 \qquad (2.8C.9)$$

$$c_{v_5} = \left[\frac{78}{78+PLV^{\frac{1}{2}}}\right]^{\frac{1}{2}} \qquad NQUAL=13,14,15 \qquad (2.8C.10)$$

$$c_{v_6} = 1.0 \qquad NQUAL=16 \qquad (2.8C.11)$$

$$Q_v = c_v \qquad (2.8C.12)$$

where NQUAL is the AGMA quality number

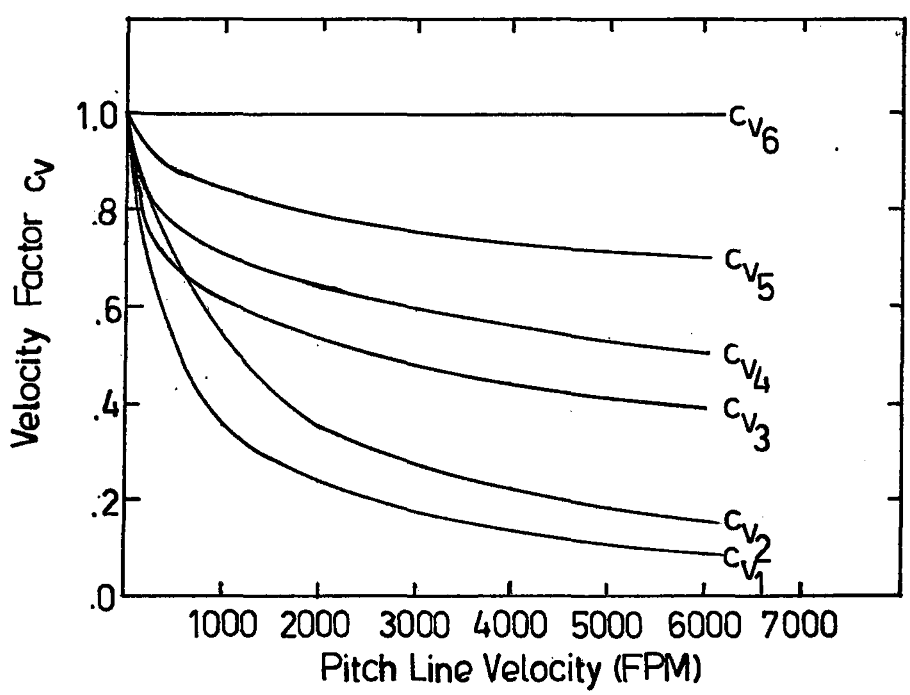


FIGURE 2.8C.1 Velocity Factor C_v

These equations are used in Subroutine VFACT [Appendix A.16] to evaluate the dynamic effect in the computer program.

If in the future a more acceptable method of dynamic load determination becomes available to the user, it can be easily incorporated in the program with the appropriate variables listed in the labelled common blocks and called through the argument list of the new subroutine. By taking the ratio of the transmitted load to the dynamic load, the new velocity factor is determined for use in the stress analysis.

SUBROUTINE VFACT(CV,QV,NQUAL,PLV)

Use:	To determine the velocity (dynamic) factor for the stress analysis.
Calling Sequence:	Subroutine UREAL calls this routine once the pitch line velocity has been specified.

2.8D LOAD DISTRIBUTION FACTOR

When the load distribution across the tooth face does not result in 100% contact due to misalignment of the axes of rotation, cutting errors and elastic deflection of the teeth, gear blank, shafts, bearings and housing, the resulting load concentration raises the stress on the tooth. To compensate for this higher stress the rated strength of the teeth must be increased. The AGMA standards [24,26] present an empirical technique for the load distribution factor determination if the misalignment is known. This technique employs a few equations coupled with some empirical curves, from which the distribution factor may be found. However, in this computer program, it was assumed that a loading analysis, general enough for use in an optimization routine, may not present the flexibility the user requires. An alternative solution, employed in the program, uses an empirical curve also developed in the same AGMA standards, shown in Figure 2.8D.1. This curve, from the AGMA standards [24,26], coupled with Table 2.8D.1 from AGMA standard [27] was the basis for the analysis in the program. Since the factors representing the most accurate conditions on the chart fall along the curve, and assuming that the quality of the gear, expressed by the AGMA quality number, intuitively represents the accuracy of the assembly, a relationship between the load distribution factor from the curve and the gear quality was derived. The curve representing the most accurate conditions was divided into three portions for analysis as follows:

$$F_w \leq 2.0'' \qquad C_m' = 1.3 \tag{2.8D.1}$$

$$\begin{aligned} 2.0''<F_w < 18.0'' \qquad C_m' = {} & -9.6282\times10^{-8}F_w^6 \\ & +6.33757\times10^{-6}F_w^5 \\ & -1.5862\times10^{-4}F_w^4 \\ & +1.82424\times10^{-3}F_w^3 \\ & -9.30188\times10^{-3}F_w^2 \\ & +4.82409\times10^{-2}F_w \\ & +1.22786 \end{aligned} \tag{2.8D.2}$$

$$F_w \geq 18.0'' \qquad C_m' = \frac{F_w}{0.45F_w+2.0} \tag{2.8D.3}$$

Equation (2.8D.2) results from a curve fitting routine using points taken from the curve, while Equation (2.8D.2) is suggested by the AGMA standard.

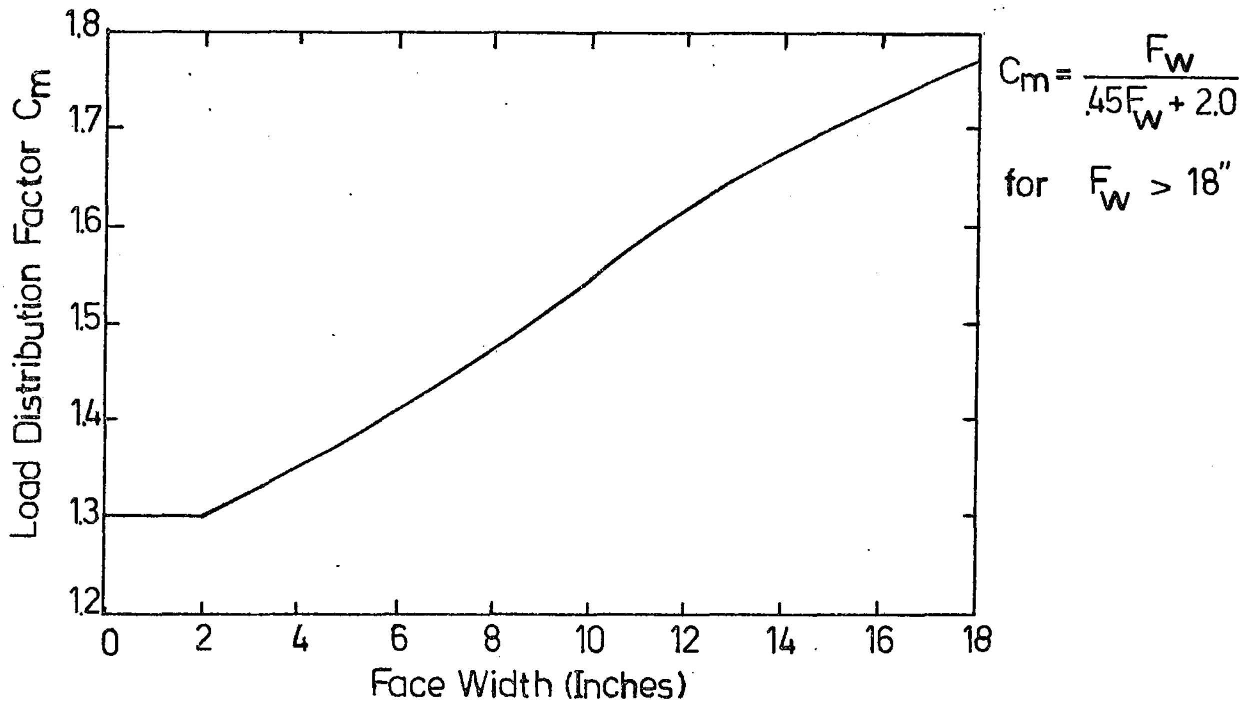


FIGURE 2.8D.1 Load Distribution Factor C_m

TABLE 2.8D.1 Load Distribution Factor Q_m

Condition of Support	Face width, in.			
	2 in and under	6 in.	9 in.	16 in. and under
Accurate mountings, low bearing clearances, minimum elastic deflection, precision gears.	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across face.	1.6	1.7	1.8	2.0
Accuracy and mounting such that less than full face contact exists.	Over 2.0			

Gear qualities greater than the AGMA quality number 14 were assumed perfect and the above values were used unmodified. However, lower gear qualities of AGMA quality numbers 3-14 were assumed to follow the arbitrary formula derived in coordination with Table 2.8D.1, so that

$$Q_m = c_m = c_m' + 0.9^{\frac{(15-NQUAL)}{12}} \qquad (2.8D.4)$$

where NQUAL = AGMA quality number. The Equations (2.8D.1) (2.8D.2) and (2.8D.3) provide a useful guide for the load distribution factor as long as the $\left(\frac{\text{face width}}{\text{pinion pitch diameter}}\right)$ ratio does not exceed 2. Ratios above this limit suggest that a more detailed analysis should be followed. The above formulas are incorporated in the computer analysis in Subroutine MFACT, Appendix [A.17].

SUBROUTINE MFACT(CM,QM,FW,NQUAL)

Use: This routine determines the load distribution factor for the stress analysis.

Calling Sequence: Subroutine UREAL calls this subroutine when the other modification factors are evaluated.

2.8E OVERLOAD FACTOR

The overload factor makes allowances for the roughness or smoothness of operation of both the driving and driven apparatus. Specific overload factors can only be established after considerable field experience is gained in a particular application. In determining the overload factor, consideration should be given to the fact that many prime movers develop momentary overload torques appreciably greater than those determined by the name plate ratings of either the prime mover or the driven apparatus. Since specific overload factors could not be employed in the computer analysis, Table 2.8E.1, extracted from the AGMA standards [24,26] has been incorporated in the program in equation form.

TABLE 2.8E.1

Overload Factors C_o, Q_o

Power Source	Character of Load on Driven Machine		
	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75+
Light Shock	1.25	1.50	2.00+
Medium Shock	1.50	1.75	2.25+

This table is for speed decreasing drives only. For speed increasing drives, the quantity, $0.1 \left|\frac{n_G}{n_p}\right|^2$ is added to the above factors.

Service factors have been established where field data is available for specific applications. These service factors include not only the overload factor, but also the life factor and factor of safety. If a specific service factor is used in place of the overload factor $[c_o, Q_o]$, use a value of 1.0 for C_R, Q_R and C_L, Q_L. The mathematical expressions for Table 2.8E.1 is

$$c_o = \frac{(\text{driven}^2 - \text{driven} + 2\text{driver} + 6)}{8} \qquad (2.8E.1)$$

where driven = 1.0 load on driven machine - uniform

= 2.0 load on driven machine - moderate

= 3.0 load on driven machine - heavy

and

driver = 1.0 power source - uniform

= 2.0 power source - light shock

= 3.0 power source - medium shock

This is developed for the computer in Subroutine OFACT, Appendix [A.18]. Equation (2.8E.1) reproduces Table 2.8E.1 exactly with the above values, while interpolation or extrapolation may be accomplished by using different driven and driver values.

SUBROUTINE OFACT(CO,QO,DRIVEN,DRIVER,NDRIVE,RATIO)

Use:	To determine the overload factors for the stress analysis.
Calling Sequence:	Called in Subroutine UREAL when other modification factors developed.

2.8F SIZE FACTOR

The size factor, which reflects the effect of dimensions on the uniformity of material properties, depends primarily on tooth size, gear diameter, face width, ratio of tooth size to gear diameter, area of contact pattern, ratio of case depth to tooth size and hardenability and heat treatment of materials. Standard size factors for spur gear teeth have not yet been established for cases where there is a detrimental size effect. The size factor may be taken as unity for most spur gears provided a proper choice of steel is made for the size of the parts and the case depth or hardness pattern is adequate. Subroutine SFACT, Appendix [A.19], sets the size factor in the computer program.

SUBROUTINE SFACT(CS,QS)

Use: This routine determines the size factor for the stress analysis.

Calling Sequence: Subroutine SPUR calls this routine during the initial execution of the design. If in the future the size factor becomes dependent on variables in the design, the routine could be altered and placed in Subroutine UREAL to become part of the optimization procedure.

2.8G SURFACE CONDITION FACTOR

The surface condition factor depends on the surface finish as affected by cutting, shaving, lapping, grinding, shot peening, and the like, and also depends on residual stress and plasticity effects from work hardening. It may be taken as unity when a good surface is developed by either processing or run-in. With no other information available, the surface condition factor was set as 1 in Subroutine FFACT, Appendix [A.19] the computer routines.

SUBROUTINE FFACT(CF)

Use: To determine the surface condition factor for wear stress analysis.

Calling Sequence: Called in Subroutine SPUR along with the other modification factors not dependent on variable quantities.

2.8H HARDNESS RATIO FACTOR

The hardness ratio factor depends on the gear ratio and the hardness of the pinion and gear material. Table 2.8H.1 offers some typical hardness combinations used in design.

TABLE 2.8H.1

Typical Hardness Combinations

GEAR (BHN)	PINION (BHN)
180	210
210	245
225	265
245	285
255	300
270	315
285	335
300	350

Figure 2.8H.1, extracted from the AGMA standards [24,26], may be used as a guide for the hardness ratio factor employed in the design process.

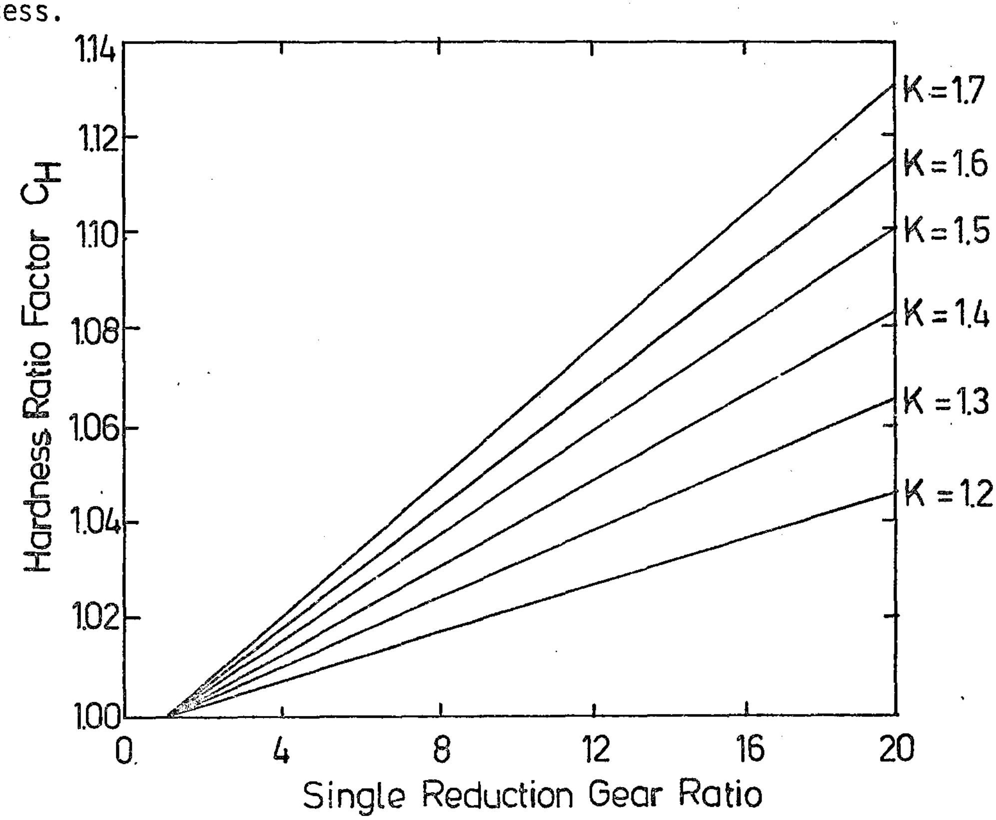


FIGURE 2.8H.1 Hardness Ratio Factor C_H

where $K = \dfrac{\text{Brinell of Pinion}}{\text{Brinell of Gear}}$

and for $K<1.2$ use $C_H = 1.0$

To give the graph in Figure 2.8H.1 a degree of generality, a mathematical expression was fitted to this family of lines so that intermediate points could be extracted.

$$C_H = (0.052808\ K^{0.225683} - 0.052632)\left(\frac{n_G}{n_p} - 1.0\right) + 1.0 \qquad (2.8H.1)$$

Subroutine HFACT, Appendix [A.20], uses Equation (2.8H.1) in the design analysis to evaluate the hardness ratio factor.

SUBROUTINE HFACT(BHN1,BHN2,RATIO,CH)

Use: To determine the hardness ratio factor for surface stress analysis.

Calling Sequence: As this analysis is not dependent on any program related variables, this routine is called by Subroutine SPUR.

2.8I LIFE FACTOR

The life factors Q_L and C_L adjust the allowable loading for the required number of cycles to account for the change in fatigue strength as a function of the loading cycles. The fatigue strength versus life cycles curve known as an S-N diagram has been determined for steel producing curves similar to Figure 2.8I.1

The curve becomes horizontal for steel at the fatigue or endurance limit after a certain number of cycles, indicating that for working stresses below this limit, failure will not occur regardless of the number of stress cycles. By determining the ratio of fatigue strength to fatigue limit at a particular life cycle, the life factor

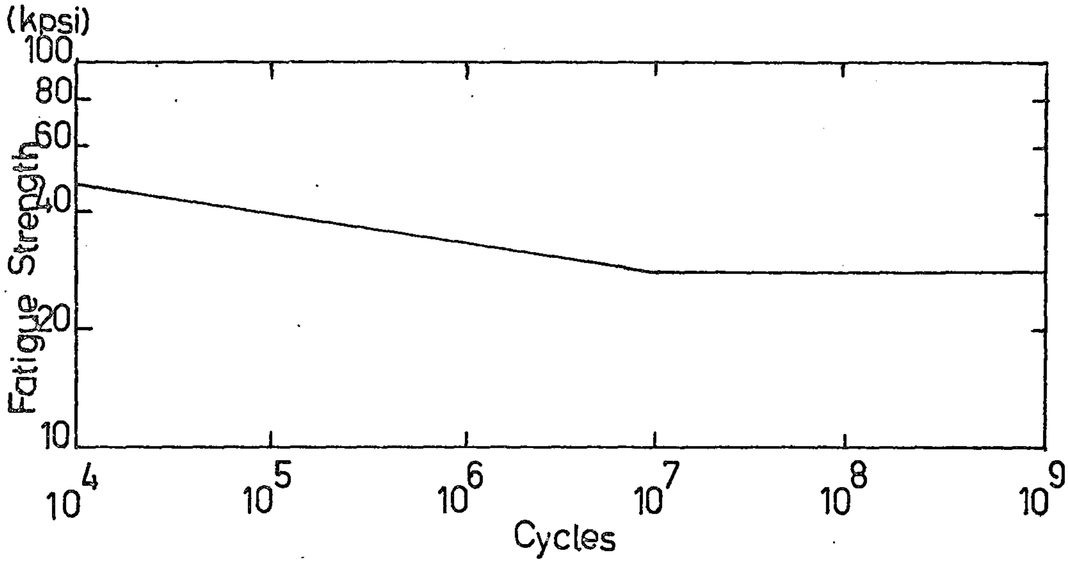


FIGURE 2.8I.1 Typical S-N Diagram for Steel

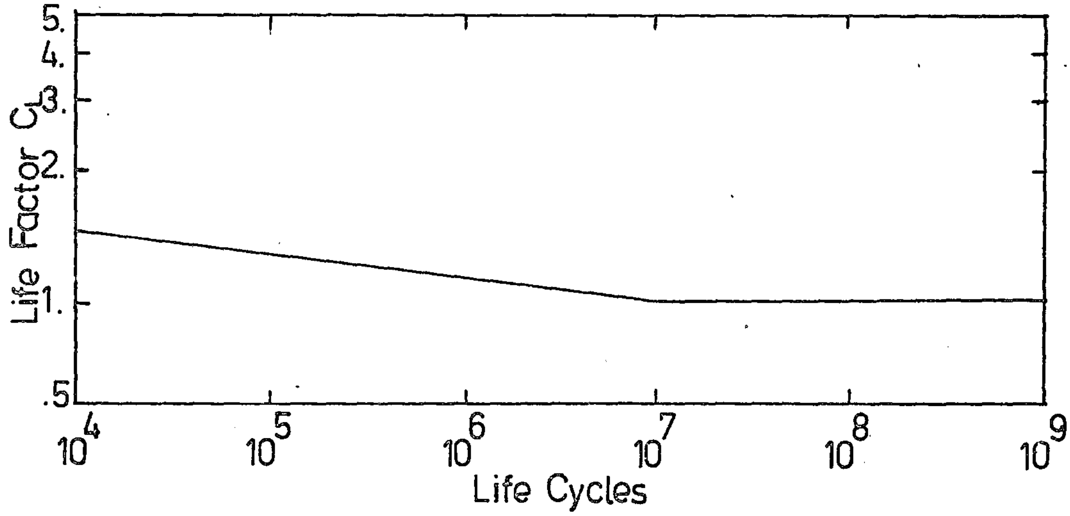


FIGURE 2.8I.2 Life Factor C_L

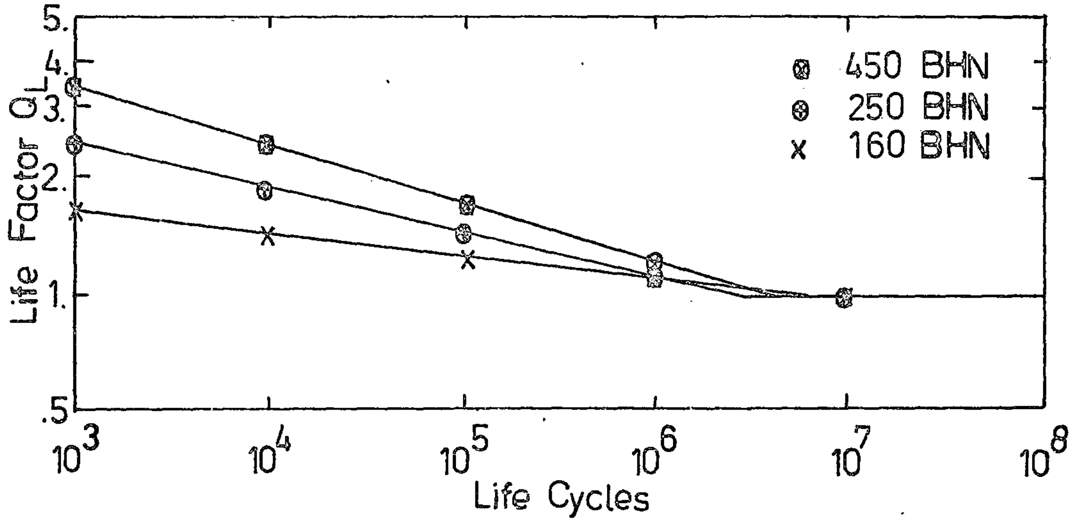


FIGURE 2.8I.3 Life Factor Q_L

may be determined. Since the curve never becomes horizontal for non-ferrous metals and alloys, these materials do not have an endurance limit.

The life factors suggested by the AGMA standards [24,26] are for steel assuming that the endurance limit will occur at 10^7 life cycles for all steels recommended by AGMA in the standards. From the available data the fatigue curve for pitting may be represented by Figure 2.8I.2. Mathematically, this curve has been formulated as

$$C_L = 2.575607 \text{ CYCLE}^{-0.058697} \quad \text{for CYCLE} < 10^7 \qquad (2.8\text{I}.1)$$

$$C_L = 1.0 \quad \text{for CYCLE} \geq 10^7 \qquad (2.8\text{I}.2)$$

The life factors for bending analysis were plotted to give curves as in Figure 2.8I.3 with a mathematical formulation of

$$Q_L{}^{160} = 2.335254 \text{ CYCLE}^{-0.056092} \qquad (2.8\text{I}.3)$$

$$Q_L{}^{250} = 5.236361 \text{ CYCLE}^{-0.112266} \qquad (2.8\text{I}.4)$$

$$Q_L{}^{450} = 9.626709 \text{ CYCLE}^{-0.150709} \qquad (2.8\text{I}.5)$$

Using the three hardness curves to determine the life factor for the required cycles, linear interpolation utilizing the following equations may be used to determine the life factor for the particular material hardness.

$$Q_L = Q_L{}^{160} + \frac{\text{Bhn}-160}{250-160}\left(Q_L{}^{250} - Q_L{}^{160}\right) \qquad (2.8\text{I}.6)$$

$$Q_L = Q_L{}^{250} + \frac{\text{Bhn}-250}{450-250}\left(Q_L{}^{450} - Q_L{}^{250}\right) \qquad (2.8\text{I}.7)$$

When using material other than steel in the computer program, the life cycles factor, CYCLE may be set greater than 10^7 with the actual fatigue strengths of the material at the desired life replacing the endurance limits in the bending and wear stress analysis.

The values for steel are incorporated in the computer program Subroutine LFACT, Appendix [A.21].

SUBROUTINE LFACT(BHN,CYCLE,CL,QL)

Use: To determine the life factor for stress analysis.

Calling Sequence: Subroutine SPUR calls the routine during initial execution when the modification factors of program independent variables are tabulated.

2.8J RELIABILITY FACTOR

The reliability factors, Q_R and C_R were introduced by the AGMA to offer the designer an opportunity to design for a specified reliability. However, the data is rather crude, as shown in Table 2.8J.1 from the AGMA standards [24,26]. Failure in this table does not mean an immediate failure under applied load, but rather a shorter life than the minimum specified.

TABLE 2.8J.1

Reliability Factors C_R and Q_R

Requirement of Application	C_R	Q_R
High reliability	1.25+	1.50+
Fewer than 1 failure in 100	1.00	1.00
Fewer than 1 failure in 3	0.80**	0.70

** At this value, plastic profile deformation might occur rather than pitting.

As the AGMA table does not provide adequate information for a mathematical development, an intuitive expression was developed using the above factors as a base. Arbitrarily, the factor of safety was assumed to increase linearly from 66-2/3% to 99% and then logarithmically above 99%, according to the following equations:

$$\text{Reliability} \leq 0.99,\ C_R = 0.773196\text{RELI}+0.234536 \qquad (2.8J.1)$$

$$\text{Reliability} > 0.99,\ C_R = 0.444444\left(\frac{1}{1-\text{RELI}}\right)^{0.176091} \qquad (2.8J.2)$$

The factor of safety of Equation (2.8J.2) goes to infinity if the reliability becomes 100%, which is intuitively true. Some intermediate values of Equation (2.8J.2) are tabulated in Table 2.8J.2 for reference.

TABLE 2.8J.2

Some Intermediate Values of Equation 2.8J.2

Reliability %	C_R
66.67	0.750
99.00	1.000
99.30	1.065
99.5	1.130
99.7	1.236
99.9	1.50
99.99	2.250

In the computer program these values are evaluated in Subroutine RFACT, Appendix [A.22].

SUBROUTINE RFACT(CR,QR,RELI)

Use: To determine the factor of safety for the stress analysis.

Calling Sequence: Subroutine SPUR calls this routine during the initial execution.

Special Features: If a reliability of greater than or equal to 100% is presented to the routine, the reliability is reset to 99.99%.

2.8K TEMPERATURE FACTOR

From the AGMA standards [24,26] the temperature factors Q_T and C_T can generally be taken as unity when the gears operate with oil or gear blank temperatures not exceeding 250 degrees F. In some instances, it is necessary to use a Q_T and C_T value greater than unity for carburized gears operating at oil temperatures above 180 degrees F for wear analysis factor, C_R or above 160 degrees F for bending analysis factor, Q_T.

The following equations are used in the computer program in Subroutine TFACT, Appendix A.23 in all cases:

$$T_F < 160, \quad Q_T = 1.0 \tag{2.8K.1}$$

$$T_F < 180, \quad C_T = 1.0 \tag{2.8K.2}$$

$$T_F \geq 160, \quad Q_T = \frac{460+T_F}{620} \tag{2.8K.3}$$

$$T_F \geq 180, \quad C_T = \frac{460+T_F}{640} \tag{2.8K.4}$$

SUBROUTINE TFACT(CT,QT,TEMP)

Use: To determine the temperature factor for the stress analysis.

Calling Sequence: Subroutine SPUR calls this routine during the initial execution.

2.8L OVERALL DERATING FACTORS

The various factors thus far developed may be regarded as safety factors to the original Lewis and Hertzian analysis. With these factors as part of the numerator and denominator of the stress equations, an overall feeling for the magnitude of this factor of safety can not be found. Lumping these terms into one factor gives the designer a better insight into the problem than can all the separate values. These following equations do not have any design significance other than a coagulation of many separate terms.

$$Q_{OD} = \frac{Q_m\ Q_o\ Q_s}{Q_v} \qquad (2.8L.1)$$

$$C_{OD} = \frac{C_F\ C_m\ C_o\ C_s}{C_v} \qquad (2.8L.2)$$

$$C_{OD}{}^L = \frac{C_H\ C_L}{C_R\ C_T} \qquad (2.8L.3)$$

$$Q_{OD}{}^L = \frac{Q_L}{Q_R\ Q_T} \qquad (2.8L.4)$$

Subroutine FACTOR, Appendix [A.24], groups all these terms for use in the computer program.

SUBROUTINE FACTOR(CF,CH,CL1,CL2,QL1,QL2,CM,QM,CO,QO,CR,QR,CS,QS,CT,QT,
CV,QV,COD,CODL1,CODL2,QOD,QODL1,QODL2)

Use: This routine groups all the individual modification factors into an overall factor used in the stress analysis.

Calling Sequence: Once all the modification factors are called in Subroutine UREAL, this routine is called.

Special Features: It must be noted that there are two life terms for each type of stress analysis. Since the number of loading cycles for the gear and pinion are different proportional to the gear ratio, the overall life derating factors for the pinion and gear are different.

2.9 UNDERCUTTING AND INTERFERENCE

For an involute profile, conjugate action can not take place below the base circle, which is that formed by a tangent circle drawn from the gear centre to the path of contact. If the initial point of contact of the driven gear is outside this point of tangency, so that the tip of the driven tooth is forced into contact with the flank of the driver below the base circle, then conjugate action is not secured, because this portion of the flank is not of involute shape and interference occurs. When the tooth profiles are generated by the cutting tools there is no interference, because the flank of the driver is undercut. However, this weakens the base of the tooth. Further insight into this problem may be gained from Chapter 2.4B on fillet profiles. This section in the chapter on the tooth profiles--involute and fillet--illustrates how the interference and undercutting phenomenon occurs. Equations given in Chapter 2.4B can also be used to bypass the above problem during the design process.

With part of the involute profile removed by undercutting, the length of contact also decreases, resulting in a lower contact ratio. Also, the thinning of the base of the tooth due to undercutting weakens the tooth. Several methods are available to eliminate undercutting or interference but each is accompanied by detrimental effects which must be weighted in the design. For example, by increasing the number of teeth, interference can be eliminated but if the gears are to transmit a given amount of power, then more teeth can only be used by enlarging the pitch diameter. This increases the gear size and pitch line velocity, which is usually undesirable. At the same time, the increased pitch line velocity results in noisier gears and reduced power transmission. A more acceptable solution is to increase the pressure angle, creating a smaller base circle so that a greater portion of the tooth profile has an involute shape. Although a larger pressure angle means that fewer teeth may be employed with correspondingly smaller gears, the frictional forces and bearing loads are increased.

As outlined at the conclusion of Chapter 2.4B, Subroutine FILLET and Subroutine CUTTER develop the geometry factors which suggest whether interference or undercutting occurs. Equations (2.4.19), (2.4.20) and (2.4.21) act as a basis for constraining the design to non-undercut conditions. To prevent interference in the design, we must have $R_o \leq R_m$, where R_o is addendum circle radius and R_m is defined by Equation (2.4.26). A further constraint prevents the addendum of one gear to be larger than the dedendum of the mate. These inequality constraints are dealt with in Subroutine CONST of the optimization routines.

2.10 CONTACT RATIO

Correct geometry in order to secure smooth and continuous action is a necessary requirement to successfully design a gear set, especially if the number of teeth are reduced. The length of tooth contact must be long enough to ensure an overlap between a successive pair of mating teeth. Care must be taken not to reduce this contact duration below a satisfactory minimum value, even if the teeth are undercut. The length of contact is a relatively simple geometric condition to evaluate analytically for non-undercut gears. However, the shortening of the length of contact due to undercutting adds a further complication to the analysis. M. F. Spotts [35] predicts the effects of undercutting in hobbed spur gear teeth, thus giving an insight into the lost action from undercutting. If the general method of contact ratio evaluation is coupled with Spotts' work, the contact length and contact ratio for both undercut and non-undercut conditions can be determined rapidly by the computer program.

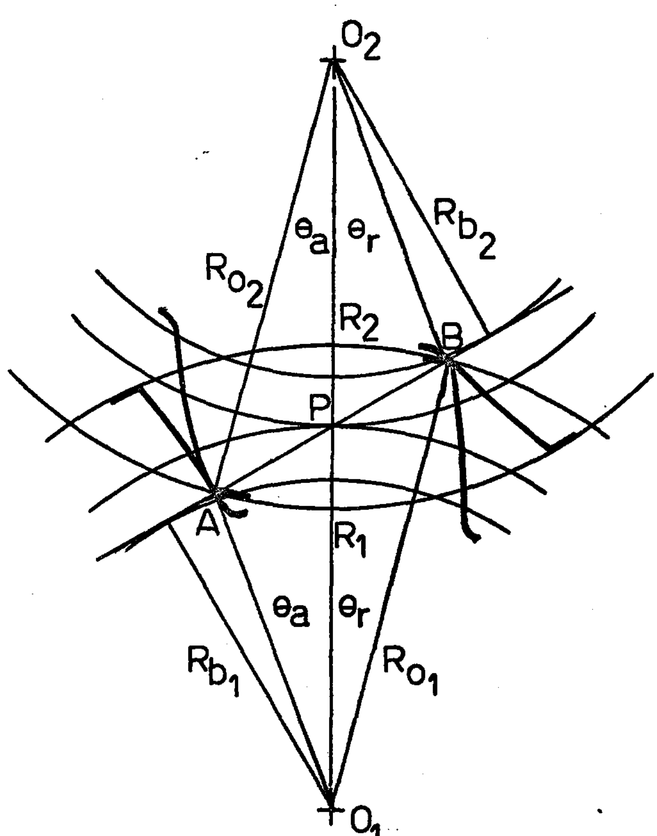


FIGURE 2.10.1 Geometry of Length of Contact Determination

We define the contact ratio as the ratio of the length of contact to the base pitch, since the base pitch is the interval between successive tooth profiles along the path of contact. The contact ratio, m_c becomes

$$m_c = \frac{AB}{B_p}$$

from Figure (2.10.1). The length AP may be defined as the length of approach l_a, while length PB is the length of recess l_r, and the angles subtended at the centre of both gears by these lines are the angle of approach, θ_a, and the angle of recess, θ_r, respectively. Mathematically, these lengths may be expressed as

$$AP = (R_{o_2}{}^2 - R_{b_2}{}^2)^{\frac{1}{2}} - R_2 \sin\phi$$

$$PB = (R_{o_1}{}^2 - R_{b_1}{}^2)^{\frac{1}{2}} - R_1 \sin\phi$$

or in terms of AB

$$AB = (R_{o_1}{}^2 - R_{b_1}{}^2)^{\frac{1}{2}} + (R_{o_2}{}^2 - R_{b_2}{}^2)^{\frac{1}{2}} - C_d \sin\phi$$

since $C_d = R_1 + R_2$. Thus the contact ratio for non-undercut gears becomes

$$m_c = \frac{(R_{o_1}{}^2 - R_{b_1}{}^2)^{\frac{1}{2}} + (R_{o_2}{}^2 - R_{b_2}{}^2)^{\frac{1}{2}} - C_d \sin\phi}{B_p} \qquad (2.10.1)$$

As a point of clarification, Equation (2.10.1) may yield a correct value of contact ratio for undercut gears if the addendum of the mating gear crosses the path of contact at a point closer to the pitch point than the undercut portion of the gear involute. In Spotts' paper, referred to above, he defines the circle passing through the fillet-involute intersection at undercut conditions as the undercut circle, and states that the loss of contact is equal to the distance between the pressure line-base circle tangency and the point where the undercut circle crosses the path of contact. Thus, from Figure 2.10.2, the last action occurs along the line AC which leaves CP as the only contact length. Using Figure 2.4.1 from the involute profile development, and the theory of Chapter 2.4A, we get

$$\alpha = \cos^{-1}(R_b/r_c) \tag{2.10.2}$$

$$CP = R_b(\tan\phi - \tan\alpha) \tag{2.10.3}$$

By knowing the radius to the involute-fillet intersection during undercut, the loss of contact length can be determined using Equations (2.10.2) and (2.10.3). Splitting the contact length into two parts, length of approach and length of recess, the contact ratio of all possible undercut and non-undercut combinations can be evaluated simply, since with a gear set subject to conditions of undercut and non-undercut, only four possible combination sets can exist. The mathematical

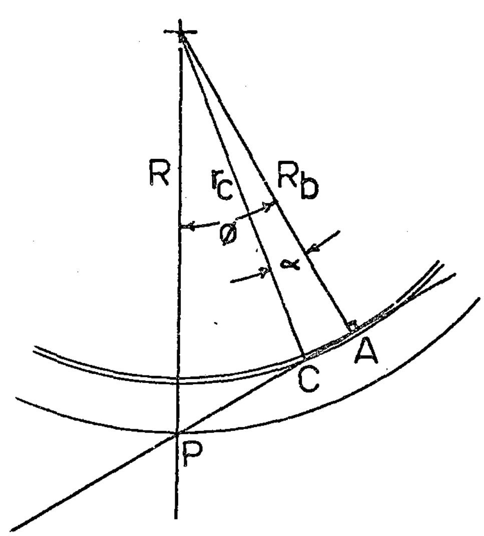


FIGURE 2.10.2 Loss of Contact Due to Undercut

determination of the radius of the undercut circle has been explained in Chapter 2.4B, and the description of Subroutine FILLET to evaluate this radius was also given at the conclusion of Chapter 2.4B. The determination of the contact ratio for all conditions is developed for the computer in Subroutine CONRAT, Appendix [A.25], Subroutine LENGTH, Appendix [A.26] and Subroutine FILLET [A.2].

SUBROUTINE CONRAT(ANGC1,ANGC2,BP, CRATIO,NCUT1,NCUT2,NDRIVE,PAR, ,PR1,PR2,RB1,RB2,RI1,RI2,RO1,RO2,RU1,RU2,TP1,TP2, XLA,XLR,BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2)

Use: To determine the contact ratio for non-undercut and undercut conditions and the length of approach and recess for the gear set.

Calling Sequence: This routine is called in Subroutine UREAL when all the geometric features of the gear are specified. Subroutine LENGTH and Subroutine FILLET are only called if an undercut gear set combination arises, otherwise this routine functions without any other programs.

Special Features: A simple method to determine the undercut non-undercut combination used in the design is employed at the beginning of the routine and is self-explanatory from comments included in the routine. The extra long argument list used in this routine is only necessary for the most part in determining the undercut contact ratio.

SUBROUTINE LENGTH(ANGC, NCUT,PAR, PR,RB,RI,RO,RU,TP,BBA,BBX,BBY,RT, XXX)

Use: This routine determines the length of contact from the undercut circle to the pitch point for undercut conditions.

Calling Sequence: Subroutine CONRAT only calls this routine when undercut conditions arise. Subroutine FILLET is employed by this routine to find the radius of the undercut circle.

2.11 EFFICIENCY

Although spur gears are a very efficient method of transmitting power (in the range of 98 percent or more), designers often require reliable efficiency information because this small friction loss can cause considerable concern since it must be dissipated as heat throughout the gear system. In applications where large amounts of power are being transmitted, the efficiency becomes very important.

Buckingham [2] develops the efficiency equations with the coefficient of friction first assumed constant and then variable.

Actual tests [9] made of the power losses with spur gears indicate that the general form of the curves representing the average coefficients of friction plotted against sliding or pitch line velocities, is similar to graphs representing the performance of plain bearings. Merritt [4] also presents a technique similar to Buckingham's constant friction factor technique in his development of the efficiency of spur gears. Buckingham began with the following rather unrealistic assumptions: perfectly shaped and equally speced involute teeth, a constant normal pressure at all times between the teeth in engagement, when two or more pairs of teeth carry the load simultaneously, the normal pressure is shared equally between them. He then developed the following equations for efficiency:

$$\text{Efficiency} = 1- \left[\frac{1+(1/m_g)}{A_a+A_r}\right] \frac{f}{2} (A_a{}^2+A_r{}^2) \qquad (2.11.1)$$

when the coefficient of friction is assumed as constant

$$\text{Efficiency} = 1- \left[\frac{1-(1/m)}{B_a+B_r}\right] \left(\frac{f_a}{2} A_a{}^2+\frac{f_r}{2} A_r{}^2\right) \qquad (2.11.2)$$

when the average coefficients of friction of approach and recess are different, where

m_g = gear ratio

A_a, A_r = arc of approach and recess on driver, respectively

f = average coefficient of friction

f_a = average coefficient of friction of approach

f_r = average coefficient of friction of recess

and

$$A_a = \frac{(R_{o_2}{}^2 - R_{b_2}{}^2)^{\frac{1}{2}} - R_2 \sin\phi}{R_{b_1}} \tag{2.11.3}$$

$$A_r = \frac{(R_{o_2}{}^2 - R_{b_1}{}^2)^{\frac{1}{2}} - R_1 \sin\phi}{R_{b_1}} \tag{2.11.4}$$

with the subscripts 1 and 2 referring to the driver and driven gear respectively.

For general use, the constant coefficient of friction Equation (2.11.1) is used for simplicity. However, the coefficient of friction is not constant but varies with different loads, speeds, lubricants and gear materials, as well as different types of surface finishes. Actual tests have indicated that, at low speeds, the values of the coefficient of friction are high, reducing rapidly to a minimum with increasing speed, and then rising again slowly with further increases in speed. After pointing out that the nature of the sliding between involute gear teeth consists of sliding in one direction during approach, reducing to zero at the pitch point where the direction of sliding changes and increases again as the contact progresses through the recess action. Buckingham states that, since the direction of sliding changes at the pitch point, the coefficient of friction could never be wholly within the field of perfect film lubrication during the period of engagement of a pair of mating teeth. He also observed that the friction of approach appeared to be about double that of recess on hobbed, milled and shaped gears of cast iron, soft steel, bronze and aluminum, while on hardened and ground steel gears, the friction factors seemed equal for approach and recess at low speed. From the research an empirical formula was suggested (for friction factor in terms of sliding velocity) from the tests on soft steel.

$$f = \frac{0.050}{e^{0.125V_s}} + 0.002\sqrt{V_s} \qquad (2.11.5)$$

Also, from what was mentioned previously

$$f_a = \frac{4}{3} f \qquad (2.11.6)$$

$$f_r = \frac{2}{3} f \qquad (2.11.7)$$

For the lack of adequate general information, Equations (2.11.2) to (2.11.7) have been employed in the computer Subroutine EFFIC to evaluate efficiency. Since the empirical formula used to determine the average coefficient of friction was developed using soft steel, the resultant analysis will not be exact for all applications. As long as steel is used as gear material, however, the efficiencies resultant from this analysis will be slightly lower than the exact values. If future developments produce an average coefficient of friction factor dependent on lubrication and material properties, it can be substituted for the friction analysis already incorporated. Further analysis may also determine the correct proportions of the approach and recess friction factors related to the average.

Subroutine EFFIC, Appendix [A.27] incorporates the foregoing analysis in the computer design.

SUBROUTINE EFFIC(EFF, RB1,RB2, PAR,PLV,RATIO,NDRIVE,XLA, XLR)

Use: To determine the frictional efficiency of the gear set.

Calling Sequence: This routine is called from Subroutine UREAL.

Special Features: Two assumptions are made when using this routine. The average coefficient of friction equation developed by Buckingham [2] represents the friction factor for all gear materials, and the friction of approach is 1-1/3 times the average friction factor and the friction of recess is 2/3 times the average friction factor.

2.12 TOLERANCES

Not unlike any manufactured item, the dimensions of gears are subject to specified permissible variations. These tolerances constitute a complex area of gear specification which directly affects gear performance, materials and finishes, fabrication and inspection techniques and cost.

The American Gear Manufacturers Association (AGMA) handbook [31] recommends gear specifications for quality, material, treatment and measuring methods and practices. For convenience and simplicity, gearing selections are identified by an AGMA class number, consisting of a Quality Number identifying specific tooth element tolerances, a letter indicating tooth thickness tolerance and two letters followed by a number indicating material, treatment and hardness. This particular section of the spur gear design deals only with tooth element and thickness tolerances with no regard to material specifications. The higher the quality number, the more precise the gearing will be and the closer the tolerances. The cost of fabricating a gear set is a direct function of the tolerances specified and this is related to the quality number. A more in-depth discussion of cost will be given in Chapter 2.15A.

Only certain tooth element tolerances and their centre distance tolerance will be evaluated here since a lot information in this area has not been standardized. For this reason many of the dimensions, such as gear blank dimensions, are not toleranced and must be taken as "worst-case" conditions with the user specifying the tolerances desired. References [7, Chapter 9; 10] present a comprehensive appraisal of tolerances in gear design.

The tolerance equations discussed in this chapter represent a standardized method of error determination employed by most manufacturers. Tolerances evolving from these equations can be obtained by the majority of manufacturers operating in conjunction with the AGMA suggested values. Knowing the degree of error obtainable for a particular Quality Number, gear elements such as tooth thickness and backlash requirements may be evaluated as part of further gear design analysis.

The tooth element tolerances -- runout, pitch, profile tooth-to-tooth composite and total composite tolerances -- are all determined from the same basic inspection set up, illustrated in Figure 2.12.1

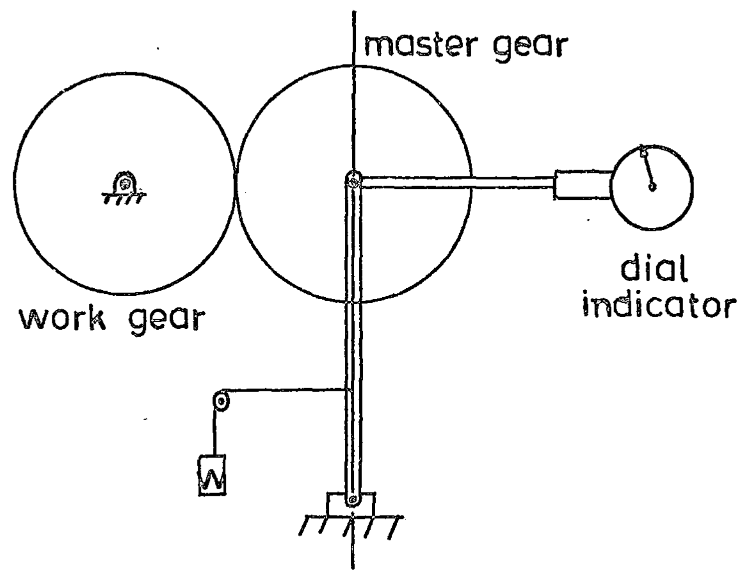


FIGURE 2.12.1 Composite Action Setup

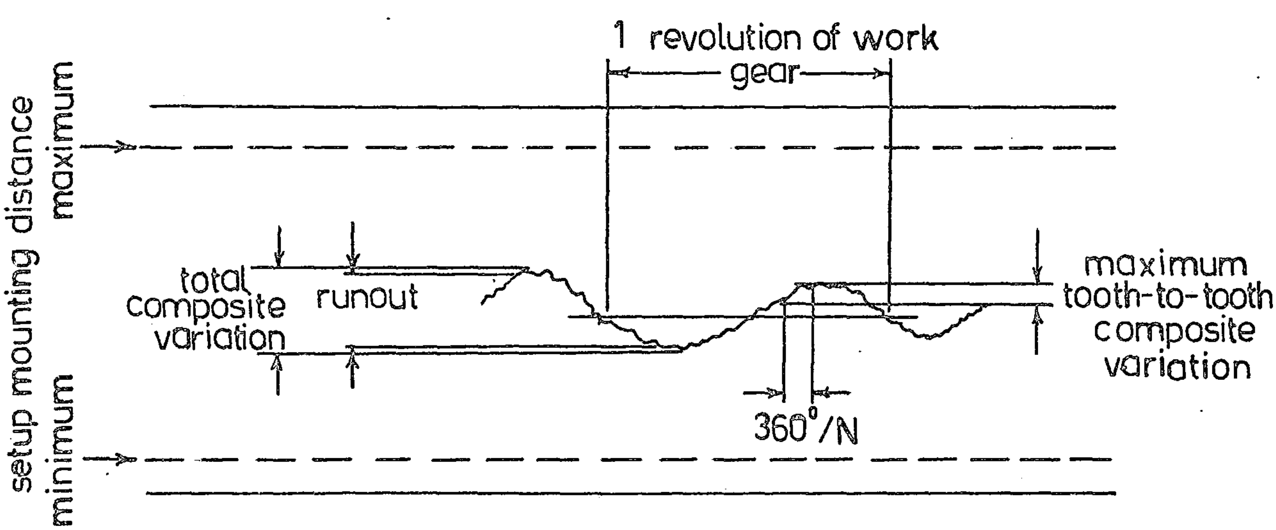


FIGURE 2.12.2 Composite Action Error Plot

When the working gear rolls in tight mesh against the master "perfect" gear, the deviation from the true centre distance, pictured in Figure 2.12.2, will be representative of the errors in the working gear. In this work any reference to a "variation" means the actual amount of error while a "tolerance" refers to the allowable amount of "variation".

The AGMA handbook, previously mentioned, presents typical values for use in gear calculation of the design process. If runout, pitch and profile tolerances are specified, they should be in lieu of the composite action tolerances and vice versa. Although all suggested tolerances are represented by equations, the runout pitch, profile and composite action tolerances may be considered valid only for diametral pitches below 20 D_p while the composite action tolerances are also valid for diametral pitch above 20 D_p. Some ambiguity in this AGMA handbook exists in the diametral pitch range of the tolerance equations for runout, profile and pitch tolerances.

The runout is the total variation of the distance between a surface of revolution and an indicated surface measured perpendicular to the surface of revolution.

Detrimental effects may result from these variations since the teeth may bind during a portion of the mesh if an adequate amount of backlash is not provided.

Runout may include the effects of eccentricity, out of roundness profile variation, spacing and tooth thickness variation. Eccentricity may be due to

a) single eccentricity caused by the difference in centres used during cutting and running, and/or distortions in mounting,

b) multiple eccentricity of a cyclical nature caused by errors in machine tools, cutting tools and lack of rigidity in set up, and

c) irregular runout caused by hardness variation in the gear blank, the cutting tool's inability to cut to a constant depth, or by heat treatment distortions.

From Figure 2.12.1 and 2.12.2 the runout variation on the centre distance between working and master gears is equal to the difference of the total composite tolerance and the tooth-to-tooth composite tolerance.

$$e_{runout} = e_{TC} - e_{TTC} \tag{2.12.1}$$

An AGMA suggested equation for runout tolerance is

$$TOL_R' = 59(2R)^{0.238}(D_p)^{-0.484}(1.4)^{(8-Q_n)} \tag{2.12.2}$$

where

R = pitch radius (inches)

D_p = diametral pitch

Q_N = AGMA quality number

TOL_R = runout tolerance

However, Equation (2.12.2) returns tolerances in ten-thousandths of an inch, which was altered for the computer to

$$TOL_R = TOL_R'(10^{-4}) \tag{2.12.3}$$

These equations reproduce tolerances tables presented in the same AGMA standard and, therefore, must be considered valid for designs of diametral pitch less than $20D_p$.

The pitch tolerance is the allowable amount of pitch variation, which in turn, is the difference between pitch and the measured distance between any two adjacent teeth, as illustrated in Figure 2.12.3.

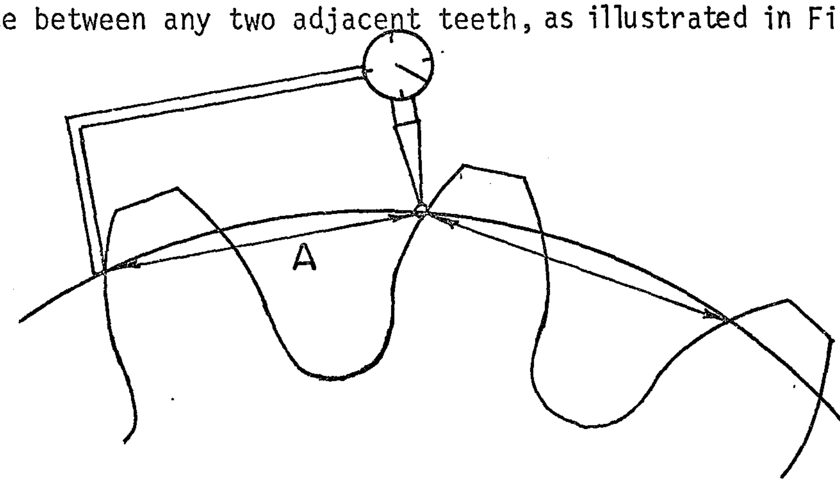


FIGURE 2.12.3 Pitch Variation Measurement

While the pitch for circular gears is the theoretical length of a circular arc, actual checking is accomplished by measuring a chordal dimension shown as A. Tooth-to-tooth spacing is a measurement of three adjacent profiles in the same manner as Figure 2.12.3.

Since these pitch errors indicate the tooth-to-tooth spacing, a potent source of gear noise arises from pitch errors. The frequency and the rate of change of the pitch errors from tooth-to-tooth are important factors in gear noise since more objectionable notes result from higher frequencies, while the rate of change of pitch error coupled with profile errors affects the angular acceleration and impact forces between the teeth. Pitch errors usually represent the departures of the cutting edge position relative to the motion of the member which drives the cutter and the departure of the work from uniform angular velocity relative to the motion of the cutter.

An AGMA suggested equation for pitch tolerance is

$$TOL'_{PITCH} = 10.5(2R)^{(0.177)}(D_p)^{(-0.224)}(1.42)^{(8-Q_n)} \tag{2.12.4}$$

and

$$TOL_{PITCH} = TOL'_{PITCH}(10^{-4}) \tag{2.12.5}$$

Similar to the runout tolerances, the suggested tolerances for pitch are valid for designs of diametral pitch less than $20D_p$.

The profile error is the variation of the shape of a tooth as evaluation from its root to its tip, exclusive of root and tip modifications. Excluding distortion during heat treatment, the principal sources of profile error arise from inaccuracies of the generating cutter tooth profile, errors in setting the generating cutter, and departures from uniformity of the motion between cutter and work. Excess of metal from the true profile represents a positive error while a deficiency of metal, a negative error. The profile tolerance is normally designated as the width of a specified envelope enclosing the positive-negative error as in Figure 2.12.4.

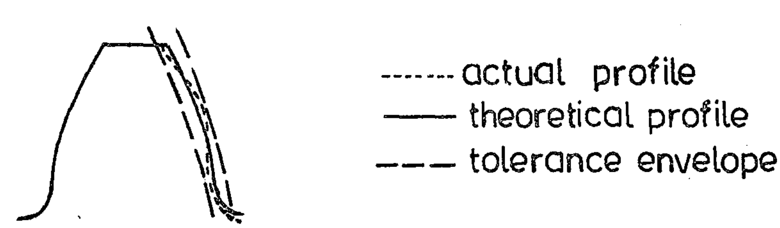


FIGURE 2.12.4 Profile Error

An AGMA suggested equation for profile tolerance is

$$TOL'_{PROFILE} = 21.5(2R)^{0.154}(D_p)^{(-0.435)}(1.4)^{8-Q_n} \qquad (2.12.6)$$

and

$$TOL_{PROFILE} = TOL'_{PROFILE}(10^{-4}) \qquad (2.12.7)$$

Similar to runout and pitch tolerances, the suggested tolerances for pitch are valid for designs of diametral pitch less than $20D_p$.

Another error quite similar to the pitch error would be the base pitch error measured along the line of action. Knowing the magnitude of this error for both gears of a mesh, the error in action would be given directly. The error in action is the amount of error between the contacting faces of the following tooth pair. When the teeth deform, it is the magnitude of the error in action which determines whether the following pair of teeth will share part of the load. Since no formulas or standard measurements have been given to determine the error in action, Dudley [6] gives an approximation for this error as the sum of the pitch error plus half the profile error for each gear. This relationship is employed in Subroutine UREAL before going into the load sharing analysis of Subroutine SHARE.

Composite action is the variation in centre distance when a work gear is rolled in tight mesh with a master "perfect" gear as in Figures 2.12.1 and 2.12.2. The tooth-to-tooth composite variation and the total composite variation can be evaluated by means of master gears which have smaller errors than those expected in the gears to be inspected. The total composite error specification combines the effect of runout, pitch, profile and tooth thickness errors. The tooth-to-tooth composite error also results from the combined effect of the foregoing errors but only reflects variations in successive teeth.

AGMA suggested equations for tooth-to-tooth composite tolerance and total composite tolerance are

1) Tooth-to-tooth composite tolerance (TTCT)

$$TTCT = 54.7(D_p)^{(-0.48)}(2R)^{(-0.24)}(1.4)^{(8-Q_n)} \qquad (2.12.\ )$$

for the number of teeth ≤ 20

$$TTCT = 38.2(D_p)^{(-0.36)}(2R)^{(-0.13)}(1.4)^{(8-Q_n)} \qquad (2.12.9)$$

for $20 <$ number of teeth ≤ 32

and $$TTCT = 25(D_p)^{(-0.24)}(1.4)^{(8-Qn)} \qquad (2.12.10)$$

for number of teeth > 32

2) Total Composite Tolerance (TCT)

$$TCT = 15 \left|\frac{20.2}{D_p}\right| 0.24(D_p)^{(-0.15)} \quad 1.16^{(10-x)}(1.4)^{(8-Q_n)}$$

$$- (0.075)(20D_p)[(20/D_p)-(2R)] \qquad (2.12.11)$$

for number of teeth ≤ 20.2

$$TCT = 14.5[(2R)^{(0.24)}(D_p)^{(-0.15)}](1.16)^{(10-x)}(1.4)^{(8-Q_n)}$$

(2.12.12)

for number of teeth > 20.2

where $$X = [5.0337\log_{10}(D_p)]-0.5153 \quad (2.12.13)$$

This analysis will provide valid tolerances for the diametral pitch range of $0.5D_p$ to $200D_p$.

The tooth thickness tolerances are to be interpreted as the maximum permissible variation of tooth thickness of all of the teeth in all of the gears made in accordance with a specific specification. These values, therefore, are the allowable range in thickness between the thinnest and the thickest teeth of any gear. The theoretical or basic tooth thickness of a gear is customarily equal to one half of its circular pitch on its standard pitch circle. Unless otherwise specified, the actual maximum tooth thickness on an unassembled gear will generally be slightly less than the theoretical value, since the manufacturer usually makes an allowance for some backlash at mesh (discussed in next section). The minimum tooth thickness will be somewhat less than maximum since a machining tolerance on tooth size is required. A table, taken from the AGMA handbook, gives suggested tooth thickness tolerance classes for spur gears from which the following equations were approximated and compared with the discrete values in Figure 2.12.5.

Specifying the tooth thickness tolerance as e_T

$$e_T = 0.015807D_p^{-0.653066} \quad (2.12.14)$$

for $D_p < 10.0$

$$e_T = 0.37423D_p^{-0.978801} \quad (2.12.15)$$

for $D_p \geq 10.0$

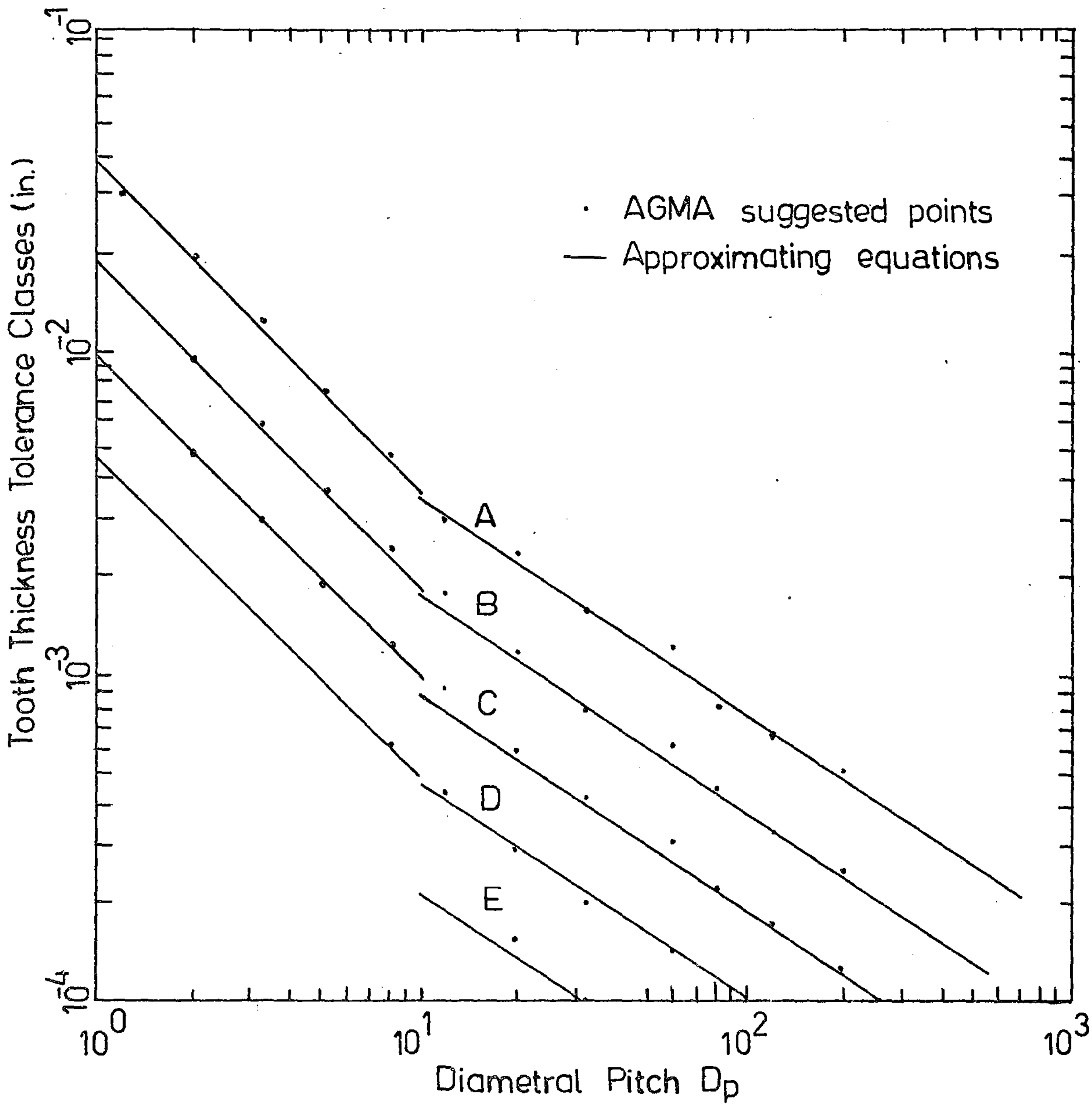


FIGURE 2.12.5 Comparison of AGMA Suggested Tooth Thickness Tolerances with Approximating Equations

The five classes A,B,C,D,E have been represented in the computer employing Equations (2.12.14) and (2.12.15) as class A. Thus, the tooth thickness tolerances may be specified in general as

$$[e_T]_n = \frac{e_T}{2^{(n-1)}} \qquad (2.12.16)$$

where $\quad n = 1,2,3,4,5$ for the five classes A to E.

These tolerances represent the tooth thickness variation at the design pitch circle evaluated by measuring instruments such as calipers.

An alternative method of measuring tooth thickness tolerance is to measure centre distance variations for an intimate meshing of master and test gear as illustrated in Figure 2.12.1 Michalec [10] describes a method of relating this composite action variation to the tooth thickness of the test gear, knowing the various properties of the master gear. However, this technique requires information concerning the master gear which would restrict the usage of the computer program.

As a simplifying approximation [34], a geometrical relationship results from the tooth separation, illustrated in Figure 2.12.6.

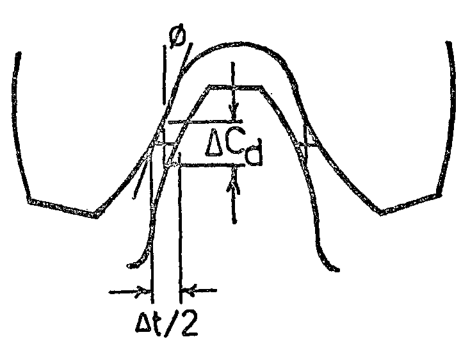


FIGURE 2.12.6 Tooth Thickness Variation with Centre Distance Change

which yields

$$\frac{\Delta t}{2} = \Delta C_d \tan\phi \qquad (2.12.17)$$

Although this is not as accurate as the composite action tooth thickness determination, it is adequate for most gear work. With backlash specified empirically, the need for more accuracy is not generally required. However, if high precision is absolutely necessary in the backlash determination, Michalec's technique should be utilized with known master gear specifications.

Since actual graphs of composite action similar to Figure 2.12.2 produce reasonably smooth variations which could be approximated by a sine function such that, if the total composite error was predominently runout, then the instantaneous centre distance variation would be

$$\Delta C_d = \left(\frac{e_{TC} - e_{TTC}}{2}\right)\sin\theta \qquad (2.12.18)$$

where θ is the gear rotational position with a positive centre distance change indicating thicker teeth. If, however, the total composite error was predominently tooth-to-tooth error, then the instantaneous centre distance change would be

$$\Delta C_d = \left(\frac{e_{TTC}}{2}\right)\sin(n\theta) \qquad (2.12.19)$$

where n is the number of teeth. By superposition the instantaneous centre distance change for both conditions would be

$$\Delta C_d = \left(\frac{e_{TC} - e_{TTC}}{2}\sin\theta + \frac{e_{TTC}}{2}\sin(n\theta)\right) \qquad (2.12.20)$$

To select the proper tooth thickness tolerance class, Equations (2.12.17) and (2.12.19) can be used to find the actual tooth thickness variation. Also, a tooth thickness tolerance can be selected from Equation (2.12.16) so that the class tolerance is equal to or less than the composite action method, since the composite action includes other errors besides the tooth thickness error measured by calipers.

The need to define tooth thickness variations will become more evident in the next section [2.13] when backlash is discussed. For the present the interrelation of many of these tolerances represents the prime concern.

The backlash discussion will present gear element tolerances necessary for consideration of the backlash of the gear set. However, some non-gear element tolerances are important in the analysis for successful gear operation. Due to its affect on backlash and contact ratio, centre distance tolerance becomes a primary concern. This tolerance is a function of the backlash requirement, gear quality, pitch and centre distance magnitude, all of which must be adjusted to avoid excessive backlash, low contact ratio affecting load capacity and smooth operation, and binding conditions. Table 2.12.1 from reference [7, Chapter 9] represent typical centre distance tolerances which are represented in the following equations:

$$3 \leq \text{quality number} \leq 7 \qquad TOL_{C_d} = 0.0100+0.0100\left(\frac{C_d-12}{12}\right)$$

$$\text{with a minimum of} \qquad TOL_{C_d} = 0.0020 \qquad (2.12.21)$$

8 $\leq$ quality number $\leq$ 12 $\quad TOL_{C_d} = 0.0020+0.0020(\frac{C_d-12}{12})$ (2.12.22)

with a minimum of $\quad TOL_{C_d} = 0.0005$

13 $\leq$ quality number $\leq$ 16 $\quad TOL_{C_d} = 0.0005+0.0005(\frac{C_d-12}{12})$ (2.12.23)

with a minimum of $\quad TOL_{C_d} = 0.0001$

These bilateral tolerances are doubled in the computer program to achieve the necessary unilateral tolerance required for analysis.

TABLE 12.1

Suggested Centre Distance Tolerances

Quality	Centre Distance				
	Under 1"	1-6"	6-12"	12-24"	Over 24"
Commercial (3-7)	±0.02	±0.003	±0.005	±0.010	±0.010 per ft
Precision (8-12)	±0.0005	±0.001	±0.002	±0.002	±0.002 per ft
High Precision (13-16)	±0.0001	±0.0002	±0.0002	±0.0003	±0.0005 per ft

In the computer program no centre distance allowance has been made as it has been assumed that backlash may be achieved by tooth thinning. However, the centre distance tolerance is employed so that the maximum amount of backlash for worst tooth thickness-centre distance variation conditions may be obtained.

As no definite criterion for tolerance specification has been presented for universal acceptance, the computer program has been established to consider the worst case of tolerance for many of the variables. A manufacturer employing the program could incorporate his own criterion for tolerance specification. A non-manufacturer user must remember that tolerances affect cost to a great degree and therefore he must try to obtain the tolerance criterion of his supplier so that he may wisely select these tolerances. With proper care, tolerances not mentioned here will not detrimentally affect the gear performance. For example, face width has not been toleranced, so that any values attached to the face width should be of the form $^{+x.xxxx}_{-0.0000}$. Similarly, material hardness or surface finish should have actual values of at least that specified by the program. Variables such as addendum and dedundum, which, along with the pitch circle radius, specify the addendum-dedendum circle radii, must have tolerances carefully checked so that binding will not occur. The outer radius employed in the program must be considered as maximum, including actual error as well as runout. A similar situation occurs with the dedendum circle radius, which must be toleranced so that adequate clearance appears during "worst" conditions of tolerance. The dedendum utilized in the program is the normal size with no regard to either tooth thinning or thickness errors. Since the tooth thickness at the pitch circle represents the minimum value due to both thinning and tolerance, the actual dedendum will be larger than the dedendum value specified. This method was selected to achieve the worst stress condition on the teeth and to guarantee a minimum clearance for all conditions of error including runout.

Tolerancing is an extremely complex subject in any machine element design, but it becomes critical in gear operation.

Further advances in analytic tolerance evaluation or more adequate information should be incorporated into the program once available. Discussion of the subroutines employing the foregoing tolerance equation will be presented in the next section under backlash. Subroutine ERROR, Appendix [A.28] develops the various tooth element and composite action tolerances suggested by the AGMA for stored program computers.

SUBROUTINE ERROR(DP,FW,NQUAL,PR,TEETH,TOLR,TOLP,PTOL,TOLL,TTCT,TCT)

Use:	To determine the various cutting tolerances for the gear.
Calling Sequence:	Subroutine UREAL calls this routine once the gear geometrical conditions are determined.

2.13 BACKLASH ANALYSIS

References [4,7,10] discuss backlash in varying degrees but do not present a definite analytical approach to the analysis. Much of what is presented here is a conglomeration of various reference sources on backlash with the author expressing his interpretation of the design backlash procedure.

Backlash in an assembled gear set is the clearance between the teeth of the meshing gears measured either along the line of action or along the pitch circle. By definition, backlash cannot exist in a single gear. In a mating gear set, the backlash that exists is a function of the actual centre distance at which the gears operate, the teeth thicknesses, the teeth deflection due to loading, temperature changes which may cause differential expansion of the gears and mountings, and possibly other factors. The minimum backlash specified by the user is assumed to take care of temperature differentials and tooth loading deflection. If this method is not satisfactory for the user's conditions, increased analysis in tooth deflection and temperature differentials coupled with appropriate constraints could be incorporated into this base program so that computer calculated backlash requirements could be employed. For the present, however, insufficient analytical background exists to use this technique.

The minimum backlash will occur when all the tolerances react at the same time to give the shortest centre distance and the thickest teeth with the high points of gear runout, while the maximum backlash will occur as these tolerances move in the opposite direction. As presented in Chapter 2.12 on tolerances, the actual tooth thickness is dependent on tooth thinning and tooth element tolerances, while changes in centre distance produce a proportional change in tooth thickness at the new pitch circle.

Design backlash is incorporated into the mesh to ensure that contact will not occur on the non-driving side of the gear teeth. Although backlash may be introduced by increased centre distance, the usual practice is tooth thinning with operation on the standard centre distance.

The total mesh backlash is dependent on the summation of the design backlash for the pinion and the gear, each of which has its constant and variable sources to backlash. Interpreted mathematically,

$$B_{mesh} = B_{gear} + B_{pinion} \tag{2.13.1}$$

$$B_{gear} = B_C + B_V \qquad (2.13.2)$$

where $B_C = \sum$ constant backlash sources

$B_V = \sum$ variable backlash sources

The constant backlash sources would include deliberate tooth thinning to achieve some minimum desired backlash, or operation at a changed centre distance. Variable sources of backlash would include all errors in the tooth dimensions as well as bearing, housing and mounting, which would contribute to the degree of backlash. If the tooth thickness is assumed to be an independent variable, then analysis for tooth deflection due to loading, coupled with various geometrical constraints, can yield an optimum amount of backlash during an optimization search. However, this form of analysis, at present, is too complex since the theory of deflection has not been developed with enough accuracy to successfully predict deflection in all cases. Thus, from experience, acceptable amounts of backlash, as a function of diametral pitch, have been presented from which tooth thickness may be extracted as a dependent variable. Table 2.13.1 from Dudley [7] recommends minimum-maximum backlash for assembled gears, which are plotted in Figure 2.13.1 using the approximating equations

$$B_L^{min} = 0.025(B_L^L)D_p^{-0.903090} \qquad (2.13.3)$$

$$B_L^{max} = 0.040(B_L^U)D_p^{-0.903090} \qquad (2.13.4)$$

where B_L^L and B_L^U are control factors to raise or lower the constant values of the backlash term to give greater flexibility in specifying backlash limits.

TABLE 2.13.1

Suggested Backlash for Power Gearing

DIAMETRAL PITCH	BACKLASH (INCHES)
1	0.025-0.040
1½	0.018-0.027
2	0.014-0.020
2½	0.011-0.016
3	0.009-0.014
4	0.007-0.011
5	0.006-0.009
6	0.005-0.008
7	0.004-0.007
8 and 9	0.004-0.006
10-13	0.003-0.005
14-32	0.002-0.004

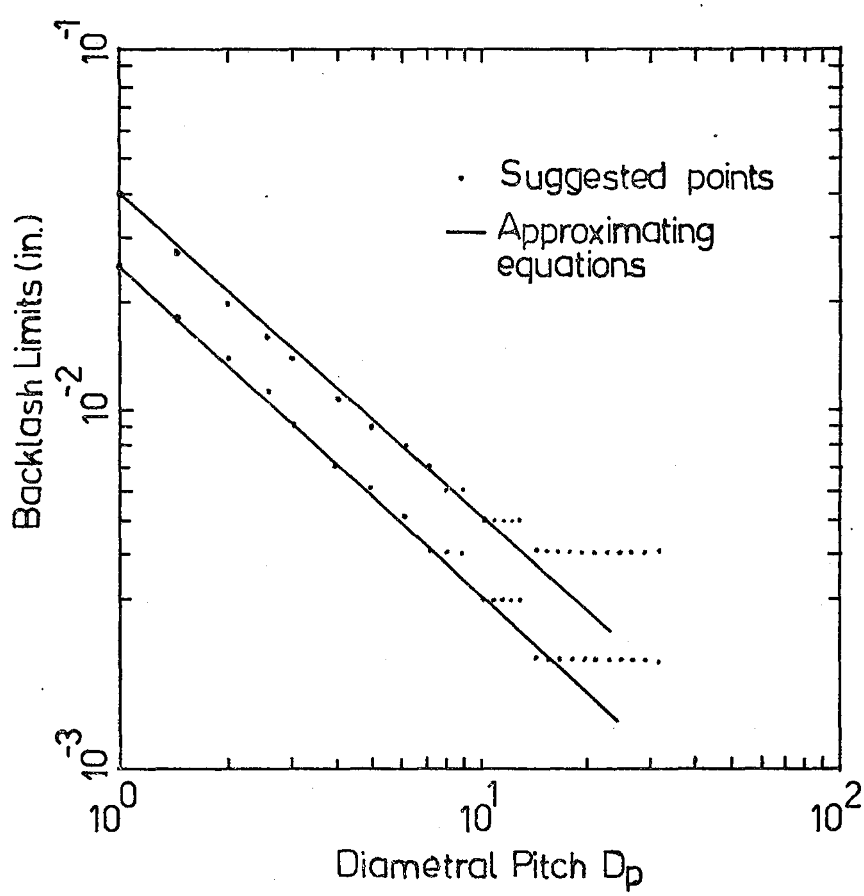


FIGURE 2.13.1 Comparison of Suggested Backlash with Approximating Equations

Having specified the minimum and maximum backlash that the mesh requires, the proportion of this backlash range shared between the pinion and gear may be represented as

$$\frac{B_{pinion}}{B_{mesh}} = B_L{}^R \qquad (2.13.5)$$

or employing Equation (2.13.1)

$$\frac{B_{gear}}{B_{mesh}} = 1-B_L{}^R \qquad (2.13.6)$$

From Chapter 2.12 on tolerances, tooth thickness variations, including tooth thickness tolerance and runout, can be selected to fit within the upper and lower limits of the backlash requirements of the pinion and gear considered separately. Since this tooth thickness variation from the two sources conceptually represents the allowable range in thickness between the thinnest and thickest teeth of a gear, the total amount of design backlash becomes this unilateral tooth thickness variation, plus a tooth thinning allowance to achieve the minimum backlash. It must be remembered that, although the tooth thickness tolerance of the gear may be specified, the runout has the effect of varying the centre distance of the mesh, thus varying the tooth thickness at the operating pitch circle. With the minimum and maximum backlash difference specifying the range of allowable tooth thickness variation from runout and tooth thickness error, constraints must control the optimization search so that the tooth thickness error does not exceed the allowable backlash range. The minimum backlash requirement specifies the amount of tooth thinning from the theoretical tooth thickness, which is usually equal to one half of the circular pitch on its standard pitch circle, and the sum of the tooth thickness variations of the pinion and gear must be less than the backlash range as outlined in Equation (2.13.7).

$$B_L-(e_t^P+e_t^G) \geq 0 \tag{2.13.7}$$

It should be noted that an increase in centre distance either by allowance, tolerance, or negative runout decreases the tooth thickness at the operating pitch circle and thus increases the backlash. Since no centre distance allowance has been used and the tolerance has been specified in this analysis as $^{+x.xxxx}_{-0.0000}$, Equations (2.12.23) to (2.12.25) are utilized in evaluating the backlash for centre distance variations, while Equation (2.12.20) is employed to analyze the effect of runout and tooth thickness tolerances on backlash. The constant source of backlash in this analysis evolves from the minimum desired backlash. For a specific design, the worst extreme values of the backlash contributors may be arithmetically totalled to calculate the maximum backlash.

The backlash analysis procedure may be summarized as follows:

1) Establish the maximum and minimum backlash requirements.
2) Determine the proportion of the minimum backlash to be shared between the pinion and gear in the form of tooth thinning of each component.
3) Knowing the composite action tolerances and the maximum tooth thickness after tooth thinning, the minimum tooth thickness may be determined by adding twice the maximum error evaluated in Equation (2.12.20) to the design pitch circle radius and computing the tooth

thickness at this radius which would now become the minimum tooth thickness at the design pitch circle. A similar procedure will specify the minimum tooth thickness for both gears. This analysis assumes that the tooth machining errors, specifying the range between the maximum and minimum tooth thickness, create a perfect involute envelope at both limits. Thus, by advancing the "minimum" involute radially outward by the total composite tolerance the tooth thickness difference between the envelopes may be approximated by Equation (2.12.7).

4) With the minimum tooth thickness specified, including runout and tooth element errors, the backlash due to machining errors becomes the difference between the maximum and minimum tooth thicknesses at the design pitch circle. This backlash plus the backlash due to tooth thinning specifies the maximum backlash at the design pitch circle.

5) When the centre distance is defined with a positive unilateral tolerance, the maximum overall backlash at this extended centre distance occurs when the minimum design tooth thickness is coupled with the centre distance increase.

By specifying this minimum tooth thickness as the actual operating thickness, the conditions of "worst" stress may be analyzed for the gear. Deviations from this minimum will only tend to strengthen the tooth by thickening it. At the same time, any tooth thickness variations, due to either runout or tooth element errors, will not cause the gears to bind as allowances have been made for this circumstance. For the remainder of the analysis, the gear is assumed perfect at the minimum conditions specified, with the thickness size reductions nullifying the effects of runout and other errors.

In actual fact, the gears may have a lesser maximum backlash since the conditions analyzed for the set were the worst possible combinations contributing to the particular calculation. If some guarantee of the actual magnitude of the error combination could be presented for the program, lesser compensations for error could be taken in the program.

Subroutine BLASH, Appendix [A.29] and Subroutine TOLCD, Appendix [A.30] develop the backlash conditions and centre distance tolerances for the gear set.

SUBROUTINE BLASH(BLMIN,BLMINT,BLMAX,BLMAXT,BL1,BL2,BLL,BLU,BLR,CP,DP,DELBL,
NQUAL,PAR,TPTL1,TPTL2,TPTU1,TPTU2,TPTE1,TPTE2,TPTV1,
TPTV2,TTCT1,TTCT2,TCT1,TCT2).

Use: This routine determines

a) the maximum and minimum backlash desired at the design pitch radius,
b) the actual maximum and minimum backlash at this operating design pitch radius,
c) the maximum tooth thinning for backlash including the machining tolerance,
d) the difference between the minimum and maximum backlash,
e) the tooth thickness tolerance,
f) the actual maximum tooth-to-tooth thickness error from tooth element errors, and
g) the actual maximum tooth-to-tooth errors from runout and tooth element erros.

Calling Subroutine: Subroutine UREAL calls this routine after the suggested tolerances have been specified.

Special Features: With the tooth-to-tooth composite error specifying the tooth thickness error, the tooth thickness tolerance from the various classes is determined utilizing Equation (2.12.16), transposed so that the numerical value of the class may be evaluated using the next smallest tolerance classes (i.e. the next greatest tolerance class number). This required tolerance class is then tested against the permitted tolerance class levels for the particular gear quality. If the tolerance class is larger than required, the constraint requiring the actual tooth thickness error to be greater than the tolerance class value will be violated. This condition must be true since the tooth thickness error from composite action analysis includes further errors which make it slightly larger than the tooth thickness tolerance class value.

SUBROUTINE TOLCD(BLMAXU,CD,CDR,CDTOLL,CDTOLU,NQUAL,PAR,PI,PR1,PR2,

RATIO,RB1,RB2,TEETH1,TP1,TP2).

Use: To determine

a) the centre distance tolerance, and
b) the maximum backlash at the extended limit of the centre distance tolerance.

Calling Sequence: Subroutine SPUR calls this routine after the design analysis is complete and before the analysis is printed out and returned.

2.14 GEAR BLANK DIMENSIONS

In designing a gear blank, [15] illustrated in Figure 2.14.1, the prime consideration almost always is rigidity. The hub must be long enough so that the gear will rotate in a single plane without wobble. It must also have sufficient diameter to provide adequate metal for keyways, to maintain a proper fit with the shaft and to transmit the required torque through the hub to the web without serious stress concentrations. Proper rigidity in the web and rim become another consideration since these dimensions affect the inertial contribution to dynamic loading.

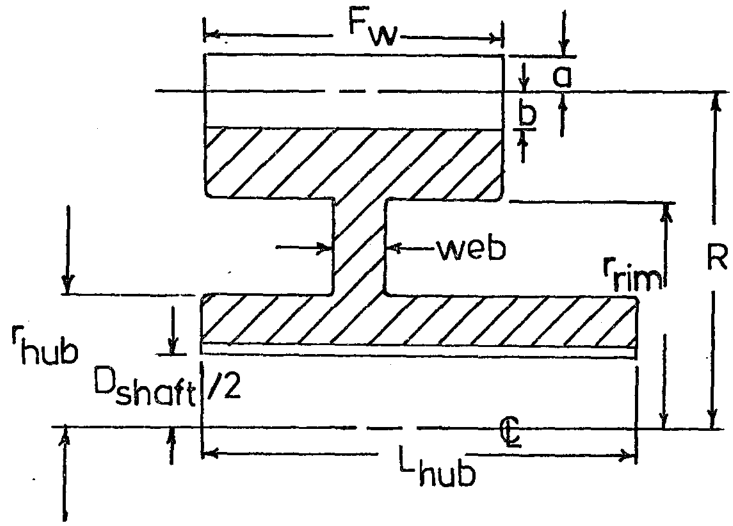


FIGURE 2.14.1 Gear Blank Nomenclature

Since no general rule could be found for the analytic design of the gear blanks, and since the stresses in the blank elements are usually low compared to the tooth stresses, the blank dimensions were generally designed to certain proportions of a specified variable such as face width or shaft diameter. The following equations represent suggested values for the use in the design of gear blanks.

From general practice, keys were chosen with a size one-fourth the shaft diameter, adjusting the hub length (HUBL) and the key length so that the torsional stresses are satisfied. Knowing the torque transmitted through the gear, the force tangential to the shaft at the keyway can be determined by

$$F = \frac{T}{r_{shaft}} \tag{2.14.1}$$

Using a square key of thickness t

$$t = 2r_{shaft} \tag{2.14.2}$$

the shear stress through the key over the length l may be specified as

$$\sigma^{A}_{shear} = \frac{F}{tl} \tag{2.14.3}$$

which must be less than the shear strength sy

$$\sigma_{sy} = 0.577 s_y \tag{2.14.4}$$

where s_y equals the yield strength of the material while the distortion-energy theory for pure torsion provides the modification constant. To resist crushing, the area of one half on the key face is used, with the actual stress required to be less than the yield stress

$$\sigma^{A}_{crushing} = \frac{F}{tl/2} \tag{2.14.5}$$

Thus

$$n\sigma^{A}_{shear} \leq sy \tag{2.14.6}$$

$$n\sigma^{A}_{crushing} \leq s_y \tag{2.14.7}$$

where n is the safety factor. The greater length of key required to support the shear or crushing stress is chosen as the length of the key as well as the length of the hub of the blank. The foregoing equations are the only stress analysis for the gear blank. The face width is specified as being the minimum allowable hub length, although the stresses may indicate that a smaller length is acceptable.

$$L_{HUB} \geq F_W \tag{2.14.8}$$

If the shaft was specified as zero diameter, the shaft and gear blank are considered one piece with no key and a hub length equal to the face width.

The radius to the outer portion of the hub (HUBR) is arbitrarily assumed 75% greater than the radius of the shaft, such that

$$r_{HUB} \quad 1.75(D_{shaft}/2) \tag{2.14.9}$$

At the same time, the thickness of the rim is chosen arbitrarily as equal to the whole depth of the tooth. Thus the inner rim radius (RIM) is equal the dedendum circle minus the working depth,

$$r_{RIM} = R_I-(a+b) \tag{2.14.10}$$

Similarly the web is chosen arbitrarily as being 50% of the face width and located in the mid portion of the blank.

$$WEB = 0.50F_W \tag{2.14.11}$$

Physically, the dimensions are limited so that

$$r_{HUB} \leq r_{RIM} \tag{2.14.12}$$

with the web equal the face width of Equation (2.14.12) is at its limit. Another limitation to the web thickness occurs if the face width is greater than or equal to 0.1 inches and the web thickness is less than 0.1 inches, at which point the web becomes constant at 0.1 inches.

The volume of the blank is determined by summing three discrete parts of the blank - the hub, the web and the rim. Since the teeth are generated from a blank, the metal removed from the tooth space is wasted but must be accounted for as part of the blank material. Thus the addendum circle radius and inner rim radius specify a ring of material from which the teeth are cut. The web may be considered a circular plate of constant thickness while the hub acts as a hollow cylinder of constant thickness. All these elements contribute to the volume.

These equations were only included in the program to give some criterion for choosing a minimum volume design. Since the particular geometry has not been included in any other aspect of the design such as dynamic loading, the values presented here are not necessarily concrete design specifications. If an adequate stress analysis were available, the variables of the blank design could also be incorporated in the optimization process as independent variables. Coupled with this, more accurate dynamic load analysis would specify limiting constraints, giving an optimum gear blank with low dynamic loads. Also, the user may find that the gear blank criterion used by his supplier may be instituted into the design package to offer him more realistic solutions.

To incorporate this analysis in the computer program Subroutine SIZE Appendix [A.31] and Subroutine VOLUME Appendix [A.32] were employed.

SUBROUTINE SIZE(ADD,DED,FW,HUBL,HUBR, RI,RIM,SHAFT,SAF,TORQ,WEB,XKEY)

Use:	To specify the gear blank dimensions for use in the volume determination of the gear set.
Calling Sequence:	Subroutine UREAL calls this routine after the geometric conditions of the gear set have been finalized.
Special Features:	The analysis for determination of the key and hub length utilizes the yield stress of the material from which a shear stress is evaluated. However, only the fatigue stress of the material was provided for the design. For steel, the yield strength is

approximately twice the fatigue strength. This assumption plus a safety factor of 2 were incorporated in the analysis.

Since the results of this subroutine are only employed in the volume calculations and do not affect the design seriously, the resultant values are realistic for present solutions. However, when gear blank analysis becomes necessary for other analysis besides volume, this subroutine should be updated accordingly.

SUBROUTINE VOLUME(FW,HUBL,HUBR,PI,RIM,RO,SHAFT,VOL,WEB)

Use: To determine the gear blank volumes for application in the optimization function.

Calling Sequence: Subroutine UREAL calls this routine once the gear blank dimensions have been specified.

2.15 MISCELLANEOUS ANALYSIS NOT PROGRAMMED

A close examination of this spur gear design package will reveal that certain topics have not been dealt with directly as part of the optimization. There may be some features which the user feels are quite important which the author has not included. This section deals with the most obvious deletions with a qualitative explanation of reasons for the deletions as well as possible methods of solution to incorporate these features.

2.15A COST

One of the most obvious optimization criteria lacking in this design is cost. This would seem to be one of the main features of the whole optimization process for the average user. Cost generally depends on

a) the gear material including heat treatment,

b) the volume of material,

c) the overall gear dimensions such as face width or pitch circle diameter which govern machining costs,

d) the AGMA quality number which affects the tolerances, and

e) the standardization of gear dimensions with the available machine tools.

Although there may be other factors on which cost is dependent, the above list illustrates the main cost factors of individual gears. Except for the volume of material, the cost of obtaining the other features will be highly shop-dependent. The gear manufacturer's ability to achieve the user requirements of gear size, tolerance specification and non-standard design practices will directly affect the cost to varying degrees. For example, the required effort in obtaining finer tolerances increases rapidly as the tolerances approach zero, while relaxation of tolerances beyond some limit may have little affect on cost, as illustrated in Figure 2.15A.1.

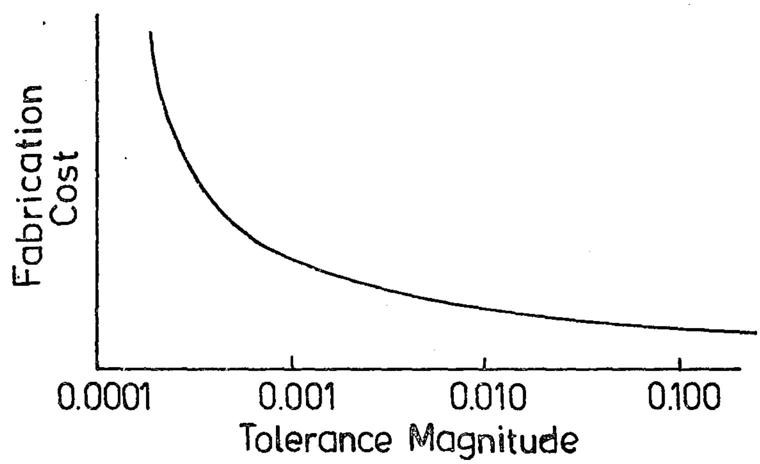


FIGURE 2.15A.1 Cost Versus Tolerance Magnitude

At the same time, as the gear size increases, the ability of the manufacturer's equipment to handle the gear may decrease, thus increasing the cost.

Since cost is closely related to the particular manufacturer's capability, the cost, related to these factors in either equations or charts, has been left to the user to implement in the program. The program has been so arranged that the analysis is incorporated in Subroutine UREAL where all variables, dependent and independent, are created. Additional variables for the design process are easily placed in labelled COMMON blocks while the cost may be installed in the optimization function following the criterion of multifactor optimization featured in Chapter 5. In the present program, volume of material is the only indirect method of cost analysis, with no real value of cost being presented.

2.15B SCORING, LUBRICATION, SURFACE FINISH, TEMPERATURE EFFECTS AND HEAT TRANSFER

Although bending and pitting failure determination has been carried out as the main stress criteria, various other factors may effect gear life as well. Although definite design procedures have not been presented in call cases, Dudley [6,7], Michalec [10], and some AGMA standards [30,33] propose solutions for many of the problems.

Radial scratch lines on the tooth face resulting from the scoring process occur for the following reasons: excessive load coupled with a break down in lubrication, too rough a surface finish, large tooth errors, high coefficients of friction, poor material properties and large sliding velocities. Dudley [6] states that most gear designers can not agree on a formula to guard against scoring, but gives two methods generally accepted by designers at the present state of the art. The PVT formula, using the product of the Hertz contact pressure, the worst sliding velocity during the mesh, and length of the line of action, provides a value which is compared with empirical limits of scoring. This technique could have been easily incorporated into the computer package since the length of contact on the line of action is determined for contact ratio analysis; the sliding velocities, for efficiency analysis and the Hertz contact pressure, as part of the wear stress analysis. However, a more recent method known as the "flash temperature" formula seems to fit closer to test data and field experience. An AGMA Information Sheet [33] explains the use of the formula with test data indicating the results from various lubricants. As laid out by the AGMA, this formula could have been easily implemented into the package, but was not since the user would have to supply information regarding surface finish and lubricants, or corresponding information incorporated into the package, which might be restrictive to the user. The assumption was made that, for the average user, adequate lubrication and surface finish would be supplied, although not specified by the design. If this approach is not satisfactory, the user may supply his own subroutine into Subroutine UREAL to evaluate scoring, and providing comparisons (i.e. constraints) with known lubricant scoring limits in Subroutine CONST. New variable names would be introduced into the labelled COMMON blocks where required in a similar manner to the existing program.

Lubrication failures generally result from inadequate "wetting" of tooth surfaces before meshing, inadequate viscosity to develop suitable film between contacting surfaces under high contact pressure and temperature, inadequate properties to reduce friction below a safe limit between surfaces, inability of lubricant to remove heat developed during contact, and contamination of lubricant with dirt, sand, metal particles, sludge or acids which tend to wear or corrode tooth surfaces. Knowing the various properties of a lubricant and lubricating system, analysis may be made on different wear conditions as well as heat removal from tooth surfaces. If one lubricant is specified, however, the analysis for the package becomes restrictive if the user wishes to utilize a different lubrication technique. On the other hand, a large variety of lubricants may be less restrictive, but the problem arises in the selection of the correct lubricant and system for a particular design. The user may find that it will be to his benefit to specify a lubrication-wear analysis for a few lubricants which will reduce the need for stock-piling a quantity of different lubricants. With a lubricant and lubricating system specified, the properties of the lubricant may be easily incorporated with the "flash temperature" formula to analyze scoring as well as establishing a more valid friction factor analysis for use in the efficiency determination.

Surface finish as seen from the "flash temperature" formula has some affect in the wear characteristics of a tooth surface, since the "flash temperature" increases rapidly if the surface roughness after "run-in" is high. At the same time, the material endurance limit is appreciably reduced if the quality of the surface finish is poor. In an effort to make the computer package material independent, this quantity has not been incorporated as part of the design; it is assumed that surface finish quality will be a function of the AGMA quality number with the gear set being "run-in" before actual use to achieve a good operating surface finish.

Temperature effects and heat transfer have a substantial effect on the operating gear set, since temperature gradients cause thermal expansion of various elements which may be detrimental in terms of increased stress or poor operating conditions outside the tolerance limits. The amount of temperature rise depends largely on the ability of the lubricant and gear housing to absorb and dissipate heat, while the lubricant capabilities depend on lubricant properties, method of application and external cooling sources. The gear housing provides the means of application of the lubricant as well as the external cooling sources, either natural or artificial. These factors are thus beyond the capabilities of the program due to the restrictive nature of their implementation. For a particular design the user may find that this information may be quite relevant and, therefore, may readily implement the pertinent theory into the program as described previously.

215C NOISE

At present, very little data is available to develop a general noise factor to be employed in a design. Generally, gear noise is an indication of the accuracy with which a gear has been produced.

Noise develops as a result of the clashing of loaded teeth related to tooth errors and tooth-load deflection, as well as forced and resonant vibrations of the gears and housing. In all cases, the noise origin is found in the non-uniform tooth load, which is the result of non-uniform angular velocity and the inertia of the rotating masses while secondary causes may be produced by static or dynamic instability or torque reversals due to the drive system or torsional vibrations. Uniformly changing errors may produce vibrations corresponding to the frequency of tooth engagement, while randomly varying errors may create numerous different frequencies and irregular noise patterns. Since it is almost impossible to obtain a set of gears to run without some noise, the elimination of the above causes by having more accurate teeth and less inertia in the rotating masses seems to be the only cure. Shigley [15] presents a curve, Figure 2.15C.1, illustrating the permissible error in action, e for a reasonable noise level, which may be approximated by

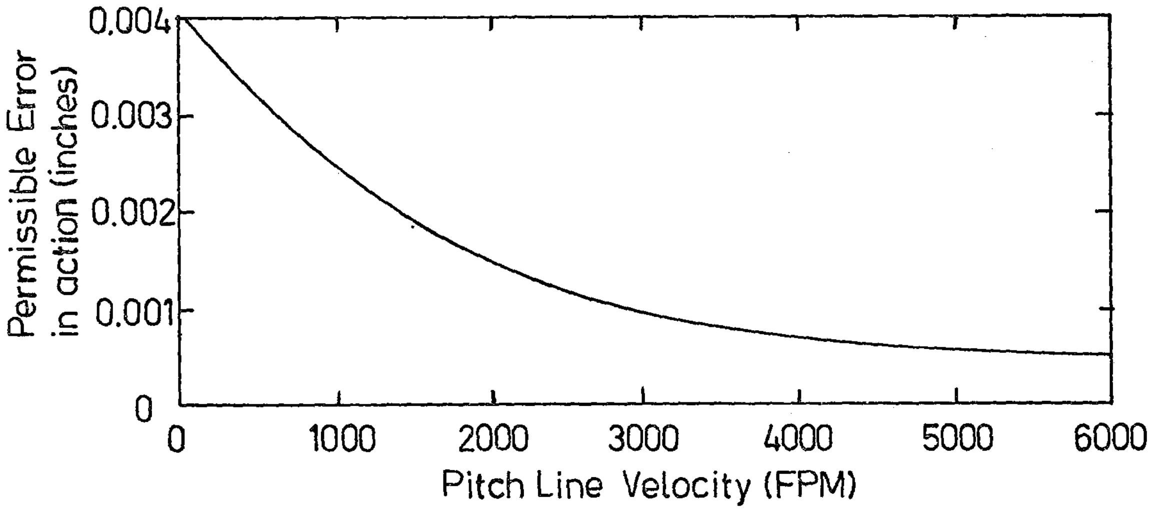


FIGURE 2.12C.1 Suggested Permissible Error for Acceptable Noise Level

$$e_{noise} = 0.082079e^{-0.001230PLV} \quad (2.15C.1)$$

for $PLV \leq 4000$ fpm

$$e_{noise} = 0.0005 \quad (2.15C.2)$$

$PLV > 4000$ fpm

However, since noise is a purely subjective topic which has not been analytically or empirically developed mathematically, this is another topic left to the discretion of the user.

2.15D TOOTH MODIFICATIONS

The theory developed concerning gear teeth has assumed that the teeth mesh in true involute contact. However, errors of manufacturer, deflection of the teeth under load and deflections of mountings under load all combine to prevent theoretical contact. In order to reduce excess tooth loads due to premature tip contact or excessive tip contact pressures, profile modification has become a usual practice. Dean [7, Chapter 5] presents a general criterion for modifying the tooth profiles to allow for errors of gear manufacture as well as deflection under load. Also mentioned is the practice of "crowning" to relieve the ends of the teeth to force contact near the mid-face of the tooth. As this area of gear design remains highly empirical with no definite criterion for determining if modification should be employed or not, this area of design has been left open. If a criterion of profile modification were implemented by the user, the analysis should generally take place after the optimization, when the analysis has returned to Subroutine SPUR.

CHAPTER 3

OPTIMIZATION

3.1 OPTIMIZATION CRITERION

All of the previous sections are concerned with feasibility of a design. This package is also concerned with getting the best possible design, and to do this, we must first establish criteria of desirability.

As the complexity of gear design increases, it becomes increasingly important to weight a number of parameters in the overall optimization criterion. No longer can only one dependent variable (eg. volume of gear material) be optimized, without also attempting to minimize cost, maximize contact ratio, maximize efficiency or minimize dynamic loading and the like. Siddall [46] has summarized various works on the subject of multifactor optimization from which the method employed for this computer package has been extracted. The following paragraphs briefly describe the reasoning behind the selection of the criteria known as the minimization of the sum of the inverted utility functions, which is utilized in this optimization.

Since the different dimensions of the dependent variables does not allow a direct combination for an overall criterion, a method to combine the variables must be found. One solution is to relate the variables through utility functions of the dependent variables. The utility function, scaled between zero and one and properly formed, does not need weighting coefficients to suggest the relative importance between the various dependent variables. Maximizing the summation of the individual utility may go to zero without forcing the total utility to zero. This would make the design undesirable, but may be accepted as an overall optimum by the program. Maximizing the product of the individual utility functions solves this inadequacy since the combined utility goes to zero if an individual utility goes to zero, however, multiplying the utilities may intuitively make the overall optimization criterion too sensitive to small changes in the individual utility functions. By inverting each utility so that the reciprocal of utility may be thought of as undesirability, the combined undesirability may be minimized as follows:

$$u_0^{-1} = \sum_{i=1}^{s} u_i^{-1} \tag{3.1.1}$$

$$= \frac{u_2 u_3 \ldots u_i + u_1 u_3 \ldots u_i + \ldots + u_1 u_2 \ldots u_{i-1}}{u_1 u_2 \ldots u_i} \tag{3.1.2}$$

$$= \text{minimum}$$

Equation (3.1.2) illustrates the combined advantages of pure addition and pure multiplication of the individual utility functions. Any function becoming zero forces the undesirability to infinity while the addition-multiplication condition stabilizes the combined function.

At the present time, with no real data available to accurately define utility functions, the curves are chosen to be linear between maximum and minimum values, which assumes that all the dependent variables optimized affect the overall optimization function equally. Dependent variables to be minimized would have a utility function similar to case 1, Figure 3.1.1 and a utility function of

$$u_j = 1.0 - \frac{y_i - y_i^{min}}{y_i^{max} - y_i^{min}} \tag{3.1.3}$$

while maximized dependent variables similar to case 2, Figure 3.1.1 would have a utility function equation of

$$u_j = \frac{y_i - y_i^{min}}{y_i^{max} - y_i^{min}} \tag{3.1.4}$$

where y_i represents the ith optimized dependent variable.

y_i^{min} and y_i^{max} represent arbitrary limit values on the dependent variable about which a utility function equivalent to the desirability of a design is created. Using case 1 of Figure 3.1.1 as an example, the desirability of a design, which produces a dependent variable y_i, near y_i^{max} is very small while the desirability of a design with dependent variable y_i closer to y_i^{min} is greater. A utility of 1 usually specifies the "ultimate" design although the utility may go higher.

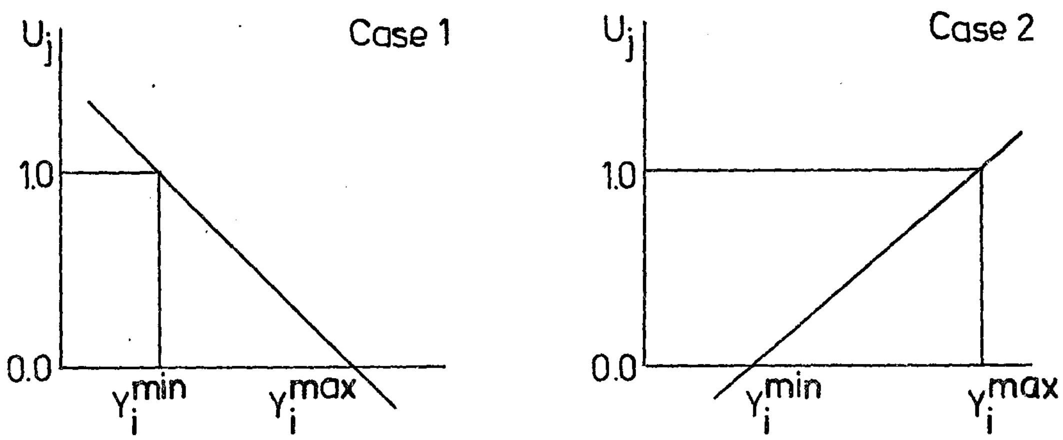


FIGURE 3.1.1 Utility Functions

Again using case 1 of Figure 3.1.1 as an example, the undesirability or reciprocal of utility, sketch in Figure 3.1.2,

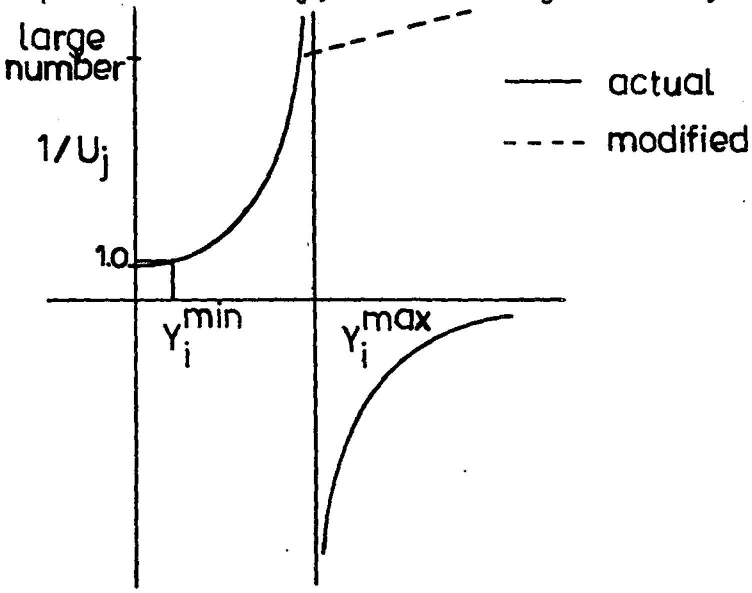


FIGURE 3.1.2 Reciprocal of Utility

becomes discontinuous at the point y_i^{max} where the utility function becomes negative. In both case 1 and 2 the undesirability function becomes discontinuous between positive and negative infinity when the original utility function becomes less than zero. To avoid this, an arbitrary large number has been chosen as infinity with a linear increase from this value for variables y_i exceeding the limits. This configuration will generally force the dependent variables in such a manner to decrease the undesirability.

In this package, the four dependent variables are volume, contact ratio, centre distance and face width, each of which is identified by flags NOF1, NOF2, NOF3, NOF4 respectively, each having a value of 1 or 0. The corresponding flag is set at 1 to indicate that the user wishes this criterion employed in his optimization. This technique is very flexible with additional dependent variables only requiring the specification of NOF5, NOF6, ... etc., and some minor programming changes. See Chapter 4.3. Thus, numerous quantities can be combined to yield an overall optimum of many dependent variables.

3.2 OPTIMIZATION TECHNIQUES

Having formulated the problem in mathematical terms, it now becomes possible to obtain the best design by formal optimization techniques [46, 47]. Any optimization problem may be developed in terms of

$$u = u(x_1, x_2, x_3, \ldots, x_n) = \begin{matrix}\text{maximum}\\ \text{or}\\ \text{minimum}\end{matrix}$$

$$\psi_i(x_1, x_2, x_3, \ldots, x_n) = 0 \text{ where } i=1,m$$

$$c_j \leq \phi_j(x_1, x_2, x_3, \ldots, x_n) \leq C_j$$

where $j = 1,p$ and c_j and C_j are constants. u expresses the optimization function and the remaining equations define the equality and inequality constraints, if they exist, on the optimization function.

This theory has been utilized in the optimization portion of the gear design employing the OPTISEP technique [47] of problem formulation, incorporating various optimization search strategies. Appendix [D] presents part of the OPTISEP presentation. In this method, the user is only required to write 1) a calling program containing required DIMENSION statements and input variable, the optimization technique call statement and a supplied subroutine for standard printed output, all following basic FORTRAN procedure, 2) Subroutine UREAL to define the optimization function, and 3) Subroutines CONST and EQUAL to define the inequality and equality constraints, respectively. All subroutines relevant to the optimization technique used can then be coupled with the user created subroutines.

Two direct search techniques [42, 43, 44], Subroutines SEEK1 and SEEK3 and one gradient technique [45], Subroutine NPFMIN were used in the optimization search of the spur gear design to determine the fastest, most accurate technique for final implementation. After a number of tests, Subroutine SEEK1 gave the least minimum results of the three methods, with subroutine SEEK3 next, and Subroutine NPFMIN third. Although Subroutines SEEK1 and SEEK3 used the same direct search strategy, the artificial optimization function employed by Subroutine SEEK3 is more complicated, thus taking more time. The gradient method, Subroutine NPFMIN, was hampered in its search because the gradients, which were too complicated to determine analytically due to the constraints were determined using the finite difference technique. In addition, the search algorithm utilized by the optimization strategy made certain assumptions which were not all fulfilled by the complex functions created by the gear design and the artificial unconstrained optimization function.

Although SEEK1 seemed the fastest, it is possible for this routine to hang up on constraints. For this reason, all three methods have been incorporated in the program with a flag NTYPE specifying the technique desired for the optimization. The user may find it advantageous to optimize initially using Subroutine SEEK1, then reoptimizing with

Subroutine SEEK3, utilizing the previous optimum as starting values for the new search.

If another optimization technique provided by the user proves more efficient than the methods presented, it may be implemented easily by arranging the new optimization program similar to the OPTISEP construction, Appendix [D], and utilizing the present base routines Subroutine SPUR, UREAL, and CONST to incorporate the spur gear design and constraints.

CHAPTER 4

USER ORIENTATION OF PACKAGE

4.1 CONCEPT OF USER ORIENTATION

Demands for more load capacity at higher speeds, with the conflicting requirement of weight reduction, are only a few of the problems confronting the modern machine designer which require his full knowledge of current design practices. These include not only materials or manufacturing methods, but also accepted methods of analysis and design. The package brings together a conglomeration of this kind of design information, organized into a logical procedure for spur gear design.

The whole concept of this package is to present the gear designer with a rapid tool to successfully complete a feasible computer design with at least the same flexibility and input of judgement offered by a manual solution. At the same time, the package enables future modifications to the design procedure to be incorporated easily, thus allowing the maintenance of a "current design practice", in a well organized and convenient manner. To accomplish this, the formulation has been divided into two parts--the actual gear design procedure, and the optimization procedure. By developing these parts independently, implementation of different design variables by users becomes easier, as illustrated later in this chapter.

Not unlike any design, the problem is made up of independent or design variables and input data, all of which act together to define some design criteria which must be minimized or maximized within certain constraints to achieve a suitable result.

To simplify the computation in this package, the design procedure is developed in three parts involving dependent variables which are problem-oriented, and calculated in Subroutine SPUR, dependent variables which are functions of the design variables, and calculated in Subroutine UREAL, and constraints which are calculated in Subroutine CONST. By utilizing this procedure, greater flexibility is achieved so that modifications are implemented easily into the design. All variables, whether independent or dependent, are represented by names which are independent of the optimization routine. This allows a union between the optimization and design procedures through an intermediate routine which equivalences the pseudonyms of the design procedure with the design variable array name of the optimization routine. For example, if the design variables in an optimization routine are specified in an array X(I),I=1,n where n is the total number of design variables and Fw represents face width, an independent variables in the gear design, then the cross-link between the two procedures may be achieved by a statement

$$Fw = X(J) \tag{4.1.1}$$

which transfers the value of the design variables X(J) to the pseudonym Fw for use in the design procedure. In this manner, numerous values may take on design variable status just by altering the intermediate routine, thus increasing the scope of the design process. Thus the user is able to pick and choose the number of design variables he desires simply by changing a few control values, without the need to alter either the design procedure or the optimization procedure.

Since the design and optimization procedure are independent except through the intermediate routine, the package can thus be utilized in a pure analysis mode with all design variables specified by the user, an optimization mode where the computer program determines the best design variables for given conditions, or some intermediate mode. It was with these features in mind that the user package was developed, giving the user essentially complete control of the design through control factors which allow various changes in the design procedure to be initiated.

4.2 PACKAGE DESIGN

In general, the purpose of gearing is to transmit motion between shafts having the output speed some function of the input. In most applications, an amount of power is to be transferred between the shafts, subject to certain environmental conditions. The ability of a gear to successfully fulfil its operating requirements becomes a function of some dependent variables which in turn depend on a number of independent geometrical and material property variables. The problem arises in differentiating between independent or design variables and input variables, so that the highest degree of flexibility is maintained for most applications.

Due to discreteness of material properties over a range, it seemed more appropriate to have these properties as input variables which would remain constant during an optimization. However, if a material had continuous properties over a range, the program could be modified so as to incorporate these material properties as design variables.

With the material properties described as input variables, the gear geometry remains the only method of specifying a feasible design within the limitations of the operating and environmental requirements. The centre distance, face width, pressure angle, number of teeth and addendum-dedendum values of both gears best represent the independent variables affecting a design criteria.

As was mentioned in Section 4.1, a link can be made between the pseudonyms of the design procedure and the independent variable array of the optimization through an intermediate routine. Thus, it becomes possible to select the desired design variables of a design. Using various logic statements to create a control array indicating what design variables are variable (i.e. computer determined values) or standard (i.e. following some prescribed convention), the correct equivalence of the pseudonyms and the design variable array can be implemented with the aid of the controlling array. The creation of the control array is brought about in Subroutine VARY1, Appendix [E] while the equivalence process during optimization is carried out in Subroutine VARY, Appendix [E]. For example, if centre distance CD is to be a design variable in the program, the user specifies CD = 3HVAR (see Appendix G for explanation. Through logic statements in VARY1 the control array NVAR(I) = 6 indicates that CD is the 6th pseudonym in a list of the eight gear design variables and the Ith design variables of n variables to be optimized in this particular problem.

In Subroutine VARY during the optimization, the appropriate pseudonyms are equivalenced with the optimization design array by advancing through the control array and extracting the relevant locations of the pseudonyms. This method provides a fast cross-link between the optimization and design routines, with a high degree of flexibility. In a similar manner, values specifed as standard, where applicable (see Appendix G), have a control array NSTD to control the extraction of the appropriate specification formula during optimization. When design variables are given numerical values by the user, they remain constant throughout the analysis, thus bypassing use in the control arrays. If all design variables are specified, no optimization is carried out and only the feasibility analysis of the given design is returned.

Having specified how variables are incorporated into the design, an explanation of a few basic design assumptions should be given. Although one can analyze with exact quantities in a paper design, deviation from these values either in manufacture or operation may seriously effect the capacity of the gear to fulfil its requirements. Thus, in this package an attempt has been made to analyze the "worst" conditions of the design so that the resultant gear will always be operating in better conditions. For this reason, the few tolerances used in the program are unilateral in order that analysis can be made for the "worst" case. Items which are not toleranced are taken as the worst limit.

The following list discusses briefly the reasoning behind the selection of a particular design procedure for certain variables.

1) The centre distance should have a positive unilateral tolerance to prevent binding of the teeth.

2) The face width should have a positive unilateral tolerance to prevent stress failure.

3) The addendum circle radius should have a negative unilateral tolerance to prevent interference with the mating gear.

4) The dedendum circle radius should have a negative unilateral tolerance to ensure that adequate clearance is available. This tolerance will be a function of the amount of deliberate tooth thinning and tooth thickness errors which tend to create a larger dedendum in generated gears. Constraints should ensure that the minimum clearance is zero and there is no undercutting even if the tooth thinning effects are considered.

5) The tooth thickness at the pitch circle radius should have a positive unilateral tolerance thus enabling tooth stresses to be evaluated for the thinnest tooth condition.

6) The backlash range required by the design should exceed the backlash introduced to the design by error sources.

Another assumption utilized in the routine requires that the analysis be carried out for a floating point number of teeth, which is integerized before returning to the calling program. In this procedure the closest integers on either side of the floating point solution are tested for the most feasible result. This allows the number of teeth which is a discrete quantity, to be handled as a continuous variable in the optimization.

To fall within certain physical limits the following inequality constraints were implemented in Subroutine CONST.

1) PHI(1) and PHI(2) ensure that the actual bending stress is less than the allowable.

2) PHI(3) and PHI(4) ensure that the actual wear (pitting) stress is less than the allowable.

3) PHI(5) and PHI(6) ensure that undercut will not occur even with a dedendum circle radius variation due to errors, while PHI(7) and PHI(8) enable interference to be eliminated.

4) PHI(9) and PHI(10) prevent the teeth from being pointed, while PHI(11) and PHI(12) ensure adequate clearance is provided.

5) PHI(13) to PHI(18) ensure that the centre distance, face width and pressure angle fall within user specified limits.

6) PHI(19) ensures that the errors of manufacture do not contribute to backlash more than the allowed backlash range, while PHI(20) and PHI(21) place the required tooth thickness tolerance class within the actual machining errors.

7) PHI(22) and PHI(23) prevent the tooth thickness at the addendum circle from being below a limiting value.

8) PHI(24) and PHI(25) prevent the load point on a gear tooth from being below a limiting radius for the bending stress analysis.

Presented in this manner, the gear design is completely flexible for modification. The analysis can also be expected to present a reliable design solution due to the considerations used in the design procedure.

4.3 PACKAGE MODICATIONS

As gear design information reaches the user, he may wish to incorporate new analysis in this program. The following section presents a brief description of the procedure for implementation of modications.

If the new analysis is not dependent on variables which change as a result of variations of the design variables, it may be incorporated in Subroutine SPUR before or after the optimization CALL statements, depending on the requirements. On the other hand, if the new analysis depends either directly on any of the design variables, then it should be evaluated in Subroutine UREAL. New variable names (pseudonyms), whether independent or dependent, should be placed in all the labelled COMMON blocks for data transfer. New design variables should be implemented into Subroutines VARY1 and VARY in the same manner as the present program, so that the same flexibility may be maintained. It will be noted in Subroutines VARY1 and VARY that NN represents the total number of geometric design variables utilized in each optimization. At the present time N, which represents the total number of design variables, is equal to NN. However, future design variable additions will require a larger value of N with NN+1 to N representing the new variables. Changes to the package will be understood more easily after careful examination of Subroutines SPUR, UREAL, VARY1 and VARY. Care should be taken to maintain adequate array sizes for altered parts of the program.

If further dependent variables are to be incorporated as design criteria in the optimization function, utility functions may be made for the new variables following the method outlined in Section 3.1. Each new dependent variable will require the specification of a flag (eg. NOF5, NOF6, ..., etc.), which expresses the user's desire to utilize this criteria in the overall optimization. This desire is represented internally in a controlling array NOF(NOFM), where NOFM is the total possible dependent variables used in the optimization criterion (in the present case NOFM=4). The first NOFN elements of this array contain numbers specifying the locations of the dependent variables utilized in the optimization criterion, where NOFN may equal 1 to NOFM. A FORTRAN DO loop (I=1,NOFN), combined with a computed GO TO statement, extracts the desired criterion from the list of possibilities, as illustrated in Subroutine UREAL. On reading the program listing of Subroutine SPUR (under title of SETUP OF OPTIMIATION CRITERION FLAGS) and Subroutine UREAL (under title of OPTIMIZATION CRITERION) this technique will become clearer.

CHAPTER 5

DISCUSSION AND CONCLUSION

With such a large number of input and design variable options available to the user, a complete test of all combinations would be impossible. Like all computer design packages, a considerable number of trials must be run to ensure that the results can be completely guaranteed. Although a large number of examples were run for extreme cases of spur gear design in an attempt to test the capabilities and limitations of the computer package, more tests will be needed in the future. The results were quite satisfactory, although in some cases, design requirements pushed the solution to a physically undesirable region, although feasible from a theoretical viewpoint. Thus, as has been mentioned previously, the designer's judgement should not be lacking completely in the design process. The following paragraphs illustrate some problems which may be encountered during the optimization.

As the design is optimized, the solution tends towards limiting values in its search for the minimum optimization criterion. In some cases, as the optimum is reached, or being reached, an exploratory search may suggest a design combination which could not physically exist, but must be evaluated to indicate infeasibility. Generated solutions returned from the programmed analysis may be infinite, for example, which may generate fatal computer error messages when this result is utilized in further analysis. This problem occurs only with the optimization routine SEEK1, when the directed random search is being carried out in Subroutine SHOT at the assumed optimum. If these errors occur, the result of the last iteration printed in the intermediate data may be taken as the optimum since the search was indicating an optimum before SHOT was called. The final gear design layout may be produced by inserting the last iteration results in the "analysis-only" mode of the package (see Appendix G). Alternatively, these values could be used as starting values for another optimization technique.

Two other problem areas are the handling of a standard design practice and the discreteness of the number of teeth. Although standard design procedure may be handled in the design (see Appendix G), the optimum solution may not result in the selection of a standard diametral pitch if the number of teeth or the centre distance are design variables. If the gear is to be manufactured using standard tooling, a standardized diametral pitch may also be a requirement besides the specification of the standard addendum-dedendum sizes. To bypass this limitation, the following formula, in FORTRAN notation, may be used to compute various combinations of centre distance and number of teeth for the closest standard diametral pitch.

```
CD = (TEETH1*(RATIO+1.0))/(2.0*DP)            (5.1)
```

In the "analysis-only" mode, the design may be tested for various numbers of teeth close to the optimum solution utilizing the closest standard diametral pitches above and below the optimum solution and keeping the centre distance as a dependent variable. Another approach is to specify centre distance and calculate the number of teeth to keep the diametral pitch standard. Familiarity with the package capabilities for incorporating different options will enable the user to select an optimum standard solution with relative ease.

Through the package all design variables have been employed as floating point quantities (i.e. decimal numbers), so that a continuous optimization function could be obtained for each variable. However, the number of teeth in a gear is a discrete quantity which demands that this variable should be treated as an integer value. To overcome this discrepancy, the number of teeth is used as a floating point number in the package but is integerized after the optimization is complete (see Section 4.2). This creates the problem that a constraint may be violated when the number of teeth is integerized, thus making the design infeasible as in the example problem of Appendix G. The corrected final solution may be obtained by using the "analysis-only" mode close to the optimum until an acceptable solution is reached which eliminates the violation, but yields a discrete number of teeth.

Generally, enough relevant information is printed to give the overall view of the resultant spur gear design. A close examination of this output will enable the designer to make logical changes which will represent more closely his real life situation. This package offers the capability of both "analysis-only" and optimization with a programming structure which can easily be changed with advances in the design process, or to account for analyses not discussed. Most benefit can be gained from this package by employing an interactive optimization, utilizing a teletype or viewing scope which will quickly present the design to the user. Ultimately the package may be used as a base for a plotting routine to yield final gear drawings. The package may also be built into an overall optimization of machines incorporating gears, or combined with other computer packages, directly or indirectly related with gear usage, including their manufacture.

REFERENCES

(1) Buckingham, E., "Spur Gears", McGraw-Hill, New York 1928.

(2) Buckingham, E., "Analytical Mechanics of Gears", McGraw-Hill, New York, 1949.

(3) Candee, A.H., "Introduction to the Kinematic Geometry of Gear Teeth", Chilton, New York, 1961.

(4) Merritt, H.E., "Gears", Third Edition, Pitman, London, 1955.

(5) Davis, W.O., "Gears for Small Mechanisms", N.A.G. Press, London, 1970.

(6) Dudley, D.W., "Practical Gear Design", McGraw-Hill, New York, 1954.

(7) Dudley, D.W., "Gear Handbook", McGraw-Hill, New York, 1962

(8) Buckingham, E., "Manual of Gear Design-Section Two", Industrial Press, Brighton, 1935.

(9) Buckingham, E., "Dynamic Loads on Gear Teeth", (Report of the ASME Special Research Committee on the Strength of Gear Teeth), American Society of Mechanical Engineers, New York, 1931.

(10) Michalec, G.W., "Precision Gearing - Theory and Practice", Wiley, New York, 1966.

(11) Steeds, W., "Involute Gears", Longmans, Green, London, 1948.

(12) Tuplin, W.A., "Gear Design", The Machinery Co. Ltd., London, 1962.

(13) Stokes, A., "High Performance Gear Design", The Machinery Co. Ltd., London, 1970.

(14) Chironis, N.P., "Gear Design and Application", McGraw-Hill, New York, 1967.

(15) Shigley, J.E., "Mechanical Engineering Design", McGraw-Hill, New York, 1963.

(16) Shigley, J.E., "Theory of Machines", McGraw-Hill, New York, 1961.

(17) Johnson, R.C., "Optimum Design of Mechanical Elements", Wiley, New York, 1961.

(18) AGMA 109.11 "Contact Ratio of Hobbed Spur Gear Teeth", June 1957.

(19) AGMA 112.04, "Terms, Definitions, Symbols and Abbreviations", August 1965.

(20) AGMA 115.01, "Reference Information - Basic Gear Geometry", July 1959.

(21) AGMA 150.03, "Application Classification for Spur, Helical, Herringbone and Bevel Gear Gear motors", April 1968.

(22) AGMA 201.02, "Tooth Proportions for Coarse - Pitch Involute Spur Gears (ANSI B6.1 - 1968), August 1964.

(23) AGMA 207.05, "Tooth Proportions for Time-Pitch Involute Spur and Helical Gears", June 1971.

(24) AGMA 210.02, "Surface Durability (Pitting) of Spur Gear Teeth", January 1965.

(25) AGMA 215.01 "Information Sheet for Surface Durability (Pitting) of Spur, Helical, Herringbone and Bevel Gear Teeth, September 1966.

(26) AGMA 220.02, "Rating the Strength of Spur Gear Teeth", August 1966.

(27) AGMA 225.01, "Information Sheet for Strength of Spur, Helical, Herringbone and Bevel Gear Teeth", December 1967.

(28) AGMA 226.01, "Information Sheet - Geometry Factors for the Strength of Spur, Helical, Herringbone and Bevel Gear Teeth", August 1970.

(29) AGMA 229.08, "Computed Geometry Factor for Hobbed Spur Gears", October, 1964.

(30) AGMA 251.01, "Specification - Lubrication of Industrial Gearing", January 1964.

(31) AGMA Gear Handbook - Volume 1 - Gear Classification, Materials and Measuring Methods of Unassembled Gears, January 1971.

(32) AGMA 411.02, "Design Procedure for Aircraft Engine and Power Take-Off Spur and Helical Gears", September 1966.

(33) AGMA 217.01, "Information Sheet - Gear Scoring Design Guide for Aerospace Spur and Helical Power Gears",

(34) Theon, R.L., "How to Find Exact Values of Backlash in Spur Gears for Changes in Tooth Thickness or Centre Distance", Machine Design, January 23, 1958

(35) Spotts, M.F., "How to predict effects of undercutting - Hobbed Spur Gear Teeth", Machine Design, Vol. 28, April 29, 1956.

(36) Lewis, W., Investigation of the Strength of Gear Teeth, Proceedings of the Engineering Club, Philadelphia 1893.

(37) Hertz, H., "On the Contact of Solid Elastic Bodies and on Hardness, Journal of Math., Vol. 92, PP. 156-171, 1831.

(38) Dolan, T.J. and E.I. Broghamer, "A Photoelastic Study of the Stresses in Gear Tooth Fillets", Univ. Illinois Expt. Sta. Bull. 335, March, 1942.

(39) Van Zandt, R.P., "Beam Strength of Spur Gears - Whether to Use the Higher or Lower Strength Factor", SAE Quarterly Transactions, Vol. 6, No. 2, April 1952

(40) Walker, H. "Gear Tooth Deflection and Profile Modification", The Engineer, Vol. 166, pp 409-412, 434-436, 1938; Vol. 170, pp 102-104, 1940.

(41) White, D.D., and J.L. Henderson, "Computer-Aided Spur Gear Design", SAE paper 690564, September 1969.

(42) Hooke, R. and T.A. Jeeves, "Direct Search Solution of Numerical and Statistical Problems", Westinghouse Research Laboratories, Scientific Paper,1960.

(43) Fiacco, A.V. and G.P. McCormick, "Extensions of SUMT for Nonlinear Programming : Equality Constraints and Extrapolation", Management Science, Vol. 12, July 1966, pp 816 -828.

(44) Fiacco, A.V. and G.P. McCormick, "Nonlinear Programming : Sequential Unconstrained Minimization Technique", Wiley, New York, 1968.

(45) Fletcher, R., "A New Approach to Variable Metric Algorithm", Computer Journal, Vol. 13, No. 3, August 1970

(46) Siddall, J.N., Analytical Decision Making in Engineering Design, Prentice Hall, Englewoodcliffs, N.J., 1972.

(47) ----------, OPTISEP - Designers' Optimization Subroutines, McMaster University, Canada, 1971.

(48) Timoshenko, S., and R.V. Baud, Strength of Gear Teeth, Mechanical Engineering, Vol. 48, Nov. 1926, pp. 1105-1109.

```
      SUBROUTINE CUTTER(ANGC,BL,CCC,CD,CP,DED,NCUT,PAR,PR,RB,RM,RU,TP,
     1BBA,BBX,BBY,RT)
C
C     THIS ROUTINE DETERMINES ...
C     A) THE UNDER-CUT LIMIT RADIUS
C     B) THE MAXIMUM ADDENDUM CIRCLE RADIUS TO PREVENT INTERFERENCE
C     C) THE ANGLE BETWEEN TOOTH CENTRELINE AND FILLET ORIGIN ON THE
C        DEDENDUM CIRCLE
C     D) SOME GEOMETRICAL CHARACTERISTICS OF THE CUTTING TOOL TEETH
C     E) TOOTH THICKNESS AT PITCH CIRCLE
C
C     ANGC     = ANGLE BETWEEN THE TOOTH CENTRELINE AND FILLET ORIGIN
C                AT THE DEDENDUM CIRCLE
C     BL       = DESIGN BACKLASH ON TOOTH INCLUDING DELIBERATE TOOTH
C                THINNING AND THICKNESS TOLERANCE (INCHES)
C     CCC      = CLEARANCE (INCHES) ...(DED1-ADD2 OR DED2-ADD1)
C     CD       = CENTRE DISTANCE (INCHES)
C     CP       = CIRCULAR PITCH (INCHES/TOOTH)
C     NCUT     = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C              = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PR       = PITCH RADIUS (INCHES)
C     RB       = RADIUS OF BASE CIRCLE (INCHES)
C     RM       = MAXIMUM RADIUS BEFORE INTERFERENCE OCCURS (INCHES)
C     RU       = UNDERCUT LIMIT RADIUS (INCHES)
C     TP       = TOOTH THICKNESS AT PITCH CIRCLE (INCHES)
C     BBA      = DISTANCE BETWEEN PITCH CIRCLE AND END OF STRAIGHT
C                PROFILE ON GENERATING RACK TOOTH FLANK (INCHES)
C     BBX      = DISTANCE BETWEEN TOOTH CENTERLINE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH (INCHES).
C     BBY      = DISTANCE BETWEEN PITCH CIRCLE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH (INCHES)
C     RT       = ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH (INCHES)
C
C     USING A STANDARD HOB FORM, TOOTH THINNING WILL PRODUCE A LARGER
C     DEDENDUM AS THE HOB IS FED INTO THE GEAR BLANK
C     THE HOB PROPERTIES ARE REFERENCED TO THE CENTRELINE OF THE GEAR
C     TOOTH SPACE AT THE GEAR PITCH CIRCLE.  THESE PSEUDO PROPERTIES
C     REPRESENT THE FEED-IN POSITION OF THE HOB
C     THE RESULTANT TOOTH FILLET PROPERTIES REPRESENT THE WORST GEOMETRY
C     FOR BENDING STRESS ANALYSIS
C
C
      TP=(CP/2.0)-BL
      EEE=(-BL/(2.0*TAN(PAR)))
      DDED=DED-EEE
      GO TO (100,200),NCUT
  100 RU=RB*COS(PAR)
      RT=0.0
      BBA=DDED
      BBY=DDED
      GO TO 300
  200 RU=RB*COS(PAR)-CCC
      RT=CCC/(1.0-SIN(PAR))
      BBA=DDED-CCC
```

```
      BBY=ODED-RT
  300 BBX=((CP-TP)/2.0)-((BBY*TAN(PAR))+(RT/COS(PAR)))
      IF(BBX.LT.0.0)BBX=0.0
      ANGC=((CP/2.0)-BBX)/PR
      RM=SQRT((RB**2)+((CD*SIN(PAR))**2))
      RETURN
      END



      SUBROUTINE FILLET(ANGC,NCUT,PAR,PR,RAD,RB,RI,RU,RRTL,RRTU,
     1TP,BBA,BBX,BBY,RT)
C
C     THIS SUBROUTINE DETERMINES THE RADIUS TO THE POINT OF INTERSECTION
C     OF THE FILLET AND INVOLUTE PROFILES.
C
C     ANGC     = ANGLE BETWEEN THE TOOTH CENTRELINE AND FILLET ORIGIN
C                AT THE DEDENDUM CIRCLE
C     NCUT     = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C              = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     PAR      = PRESSURE ANGLE  (RADIANS)
C     PR       = PITCH RADIUS  (INCHES)
C     RAD      = RADIUS TO THE INTERSECTION OF THE FILLET AND INVOLUTE
C                PROFILES  (INCHES)
C     RB       = RADIUS OF BASE CIRCLE (INCHES)
C     RI       = ROOT CIRCLE RADIUS  (INCHES)  ...  (PR-DED)
C     RU       = UNDERCUT LIMIT RADIUS (INCHES)
C     RRTL     = LOWER LIMIT RADIUS OF TROCHOID TO PRODUCE FILLET TANGENT
C                TO ROOT CIRCLE  (INCHES)
C     RRTU     = UPPER LIMIT RADIUS OF TROCHOID TO PRODUCE FILLET
C                INTERSECTION WITH THE INVOLUTE  (INCHES)
C     TP       = TOOTH THICKNESS AT PITCH CIRCLE  (INCHES)
C     BBA      = DISTANCE BETWEEN PITCH CIRCLE AND END OF STRAIGHT
C                PROFILE ON GENERATING RACK TOOTH FLANK (INCHES)
C     BBX      = DISTANCE BETWEEN TOOTH CENTERLINE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH  (INCHES)
C     BBY      = DISTANCE BETWEEN PITCH CIRCLE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH  (INCHES)
C     RT       = ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH  (INCHES)
C
C
      XINV(ANG)=TAN(ANG)-ANG
      TART(ANG,BBY,PR,RRT)=ATAN((1.0-((RRT**2)/(PR*(PR-BBY))))/TAN(ANG))
      TANG(TP,PR,ANG1,ANG2)=((TP/(2.0*PR))+XINV(ANG1)-XINV(ANG2))
      FRRC(ANG,RRT,RT)=SQRT((RRT**2)+RT*(RT-2.0*RRT*SIN(ANG)))
      FASC(ANG,BBY,PR)=(BBY/PR)*TAN(ANG)-XINV(ANG)
      FARC(ANG,ANG1,BBY,PR,RRF,RRT,RT)=FASC(ANG,BBY,PR)+ACOS((RRT-RT*SIN
     1(ANG1))/RRF)
      RITT(ANG,BBA,PR,RB)=SQRT(((PR*SIN(ANG)-(BBA/SIN(ANG)))**2)+RB**2)
C
      ERROR=1.0E-06
      GO TO (100,200),NCUT
  100 XX=PR-BBY
      IF((XX-RT).LT.RU) GO TO 105
```

A.2

A 2

```
      RRF=RITT(PAR,BBA,PR,RB)
      ANG=ACOS(XX/RRF)
      ANGF=FASC(ANG,BBY,PR)
      RRTL=XX-RT
      RRTU=RRF
      RAD=RRF
      IF(RAD.LT.RB) RAD=RB
      RETURN
  105 R1=RB
      ANG1=ACOS(XX/R1)
      ANG11=ACOS(RB/R1)
      FUNC1=FASC(ANG1,BBY,PR)+TANG(TP,PR,PAR,ANG11)-ANGC
      DELR=(((RU-(XX-RT))/SIN(PAR))**2)/(6.0*PR*COS(PAR))
      IF(DELR.LE.0.0)  GO TO 125
      DELR=DELR/2.0
      R2=R1
      FUNC2=FUNC1
  110 R1=R2
      FUNC1=FUNC2
      R2=R2+DELR
      ANG2=ACOS(XX/R2)
      ANG22=ACOS(RB/R2)
      FUNC2=FASC(ANG2,BBY,PR)+TANG(TP,PR,PAR,ANG22)-ANGC
      IF((FUNC1*FUNC2).GT.0.0)GO TO 110
  115 R3=(R1*FUNC2-R2*FUNC1)/(FUNC2-FUNC1)
      ANG3=ACOS(XX/R3)
      ANG33=ACOS(RB/R3)
      FUNC3=FASC(ANG3,BBY,PR)+TANG(TP,PR,PAR,ANG33)-ANGC
      IF((ABS(FUNC3)).LT.ERROR)  GO TO 130
      IF((FUNC3*FUNC1).GT.0.0)  GO TO 120
      R2=R3
      FUNC2=FUNC3
      GO TO 115
  120 R1=R3
      FUNC1=FUNC3
      GO TO 115
  125 R3=R1
  130 RRTL=XX-RT
      RRTU=R3
      RAD=R3
      IF(RAD.LT.RB) RAD=RB
      RETURN
  200 XX=PR-BBY
      IF((XX-RT).LT.RU)  GO TO 210
      RRF=RITT(PAR,BBA,PR,RB)
      R3=SQRT((RRF**2)+(RT**2)+2.0*RT*SQRT((RRF**2)-(RB**2)))
      ANG3=ACOS(XX/R3)
      ANG31=TART(ANG3,BBY,PR,R3)
      RRTL=XX
      RRTU=R3
      RAD=RRF
      IF(RAD.LT.RB) RAD=RB
      RETURN
  210 R1=RB
      ANG1=ACOS(XX/R1)
```

```
      ANG11=TART(ANG1,BBY,PR,R1)
      RRF1=FRRC(ANG11,R1,RT)
      FUNC1=RRF1-RB
      DELR=-FUNC1/10.0
      IF(DELR.EQ.0.0) GO TO 230
      R2=R1
      FUNC2=FUNC1
  215 R1=R2
      FUNC1=FUNC2
      R2=R2+DELR
      ANG2=ACOS(XX/R2)
      ANG21=TART(ANG2,BBY,PR,R2)
      RRF2=FRRC(ANG21,R2,RT)
      FUNC2=RRF2-RB
      IF((FUNC1*FUNC2).GT.0.0) GO TO 215
  220 R3=(R1*FUNC2-R2*FUNC1)/(FUNC2-FUNC1)
      ANG3=ACOS(XX/R3)
      ANG31=TART(ANG3,BBY,PR,R3)
      RRF3=FRRC(ANG31,R3,RT)
      FUNC3=RRF3-RB
      IF((ABS(FUNC3)).LE.ERROR) GO TO 235
      IF((FUNC3*FUNC1).GT.0.0) GO TO 225
      R2=R3
      FUNC2=FUNC3
      GO TO 220
  225 R1=R3
      FUNC1=FUNC3
      GO TO 220
  230 R3=R1
  235 R1=R3
      ANG1=ACOS(XX/R1)
      ANG11=TART(ANG1,BBY,PR,R1)
      RRF1=FRRC(ANG11,R1,RT)
      ANGF1=FARC(ANG1,ANG11,BBY,PR,RRF1,R1,RT)
      IF(RRF1.LT.RB) RRF1=RB
      ANG12=ACOS(RB/RRF1)
      FUNC1=ANGF1+TANG(TP,PR,PAR,ANG12)-ANGC
      DELR=(((RO-(XX-RT))/SIN(PAR))**2)/(6.0*PR*COS(PAR))
      IF(DELR.LE.0.0) GO TO 255
      DELR=DELR/2.0
      R2=R1
      FUNC2=FUNC1
  240 R1=R2
      FUNC1=FUNC2
      R2=R2+DELR
      ANG2=ACOS(XX/R2)
      ANG21=TART(ANG2,BBY,PR,R2)
      RRF2=FRRC(ANG21,R2,RT)
      ANGF2=FARC(ANG2,ANG21,BBY,PR,RRF2,R2,RT)
      IF(RRF2.LT.RB) RRF2=RB
      ANG22=ACOS(RB/RRF2)
      FUNC2=ANGF2+TANG(TP,PR,PAR,ANG22)-ANGC
      IF((FUNC1*FUNC2).GT.0.0) GO TO 240
  245 R3=(R1*FUNC2-R2*FUNC1)/(FUNC2-FUNC1)
      ANG3=ACOS(XX/R3)
```

```
      ANG31=TART(ANG3,BBY,PR,R3)
      RRF3=FRRC(ANG31,R3,RT)
      ANGF3=FARC(ANG3,ANG31,BBY,PR,RRF3,R3,RT)
      IF(RRF3.LT.RB) RRF3=RB
      ANG32=ACOS(RB/RRF3)
      FUNC3=ANGF3+TANG(TP,PR,PAR,ANG32)-ANGC
      IF((ABS(FUNC3)).LE.ERROR) GO TO 260
      IF((FUNC3*FUNC1).GT.0.0) GO TO 250
      R2=R3
      FUNC2=FUNC3
      GO TO 245
  250 R1=R3
      FUNC1=FUNC3
      GO TO 245
  255 R3=R1
      RRF3=RRF1
  260 RRTL=XX
      RRTU=R3
      RAD=RRF3
      IF(RAD.LT.RB) RAD=RB
      RETURN
      END



      SUBROUTINE ADDEND (ADDL,PAR,PR,RB,RO,TO,TP)
C
C     THIS ROUTINE DETERMINES THE ADDENDUM LENGTH OF POINTED TEETH
C     AND TOOTH THICKNESS AT ADDENDUM CIRCLE
C
C     ADDL     = ADDENDUM LENGTH TO POINT (INCHES)
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     RB       = BASE CIRCLE RADIUS (INCHES)
C     RO       = ADDENDUM CIRCLE RADIUS (INCHES)
C     TO       = TOOTH THICKNESS AT ADDENDUM CIRCLE (INCHES)
C     TP       = TOOTH THICKNESS AT PITCH CIRCLE (INCHES)
C
C
      XINV(ANG)=TAN(ANG)-ANG
      ERROR=1.0E-06
      XXX=(TP/(2.0*PR))+XINV(PAR)
      R1=PR
      ANG1=PAR
      FUNC1=XINV(ANG1)-XXX
      DELR=ABS(RO-PR)
      IF(DELR.EQ.0.0)DELR=0.1
      R2=R1
      FUNC2=FUNC1
    1 R1=R2
      FUNC1=FUNC2
      R2=R2+DELR
      ANG2=ACOS(RB/R2)
      FUNC2=XINV(ANG2)-XXX
```

```
      IF((FUNC1*FUNC2).GT.0.0) GO TO 1
    2 R3=(R1*FUNC2-R2*FUNC1)/(FUNC2-FUNC1)
      ANG3=ACOS(RB/R3)
      FUNC3=XINV(ANG3)-XXX
      IF((ABS(FUNC3)).LT.ERROR) GO TO 4
      IF((FUNC3*FUNC1).GT.0.0) GO TO 3
      R2=R3
      FUNC2=FUNC3
      GO TO 2
    3 R1=R3
      FUNC1=FUNC3
      GO TO 2
    4 ADDL=R3-PR
      ANG=ACOS(RB/RO)
      TO=2.0*RO*(XXX-XINV(ANG))
      IF((PR+ADDL).LT.RO) TO=0.0
      RETURN
      END


      SUBROUTINE PITCH (RATIO,CD,TEETH1,TEETH2,RPM1,PAR,PI,PR1,PR2,RB1,
     1RB2,BP,CP,DP,PLV)
C
C     THIS ROUTINE DETERMINES VARIOUS GEOMETRICAL RELATIONSHIPS
C     FOR A GEAR SET
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     RATIO   = GEAR RATIO (IE. GEAR TEETH / PINION TEETH)
C     CD      = CENTRE DISTANCE (INCHES)
C     TEETH   = NUMBER OF TEETH
C     RPM     = SHAFT SPEED (REVOLUTIONS PER MINUTE)
C     PAR     = PRESSURE ANGLE (RADIANS)
C     PI      = 3.141592........
C     PR      = PITCH RADIUS (INCHES)
C     RB      = BASE CIRCLE RADIUS (INCHES)
C     BP      = BASE PITCH (INCHES)
C     CP      = CIRCULAR PITCH (INCHES)
C     DP      = DIAMETRAL PITCH (TEETH PER INCH)
C     PLV     = PITCH LINE VELOCITY (FPM)
C
      PR1=CD/(RATIO+1.0)
      PR2=CD-PR1
      PD1=2.0*PR1
      RB1=PR1*COS(PAR)
      RB2=PR2*COS(PAR)
      DP=TEETH1/PD1
      CP=(PI*PD1)/TEETH1
      BP=CP*COS(PAR)
      PLV=(PI*PD1*RPM1)/12.0
      TEETH2=RATIO*TEETH1
      RETURN
      END
```

```
SUBROUTINE TORQUE (HP,PI,RPM,TORQ)

THIS ROUTINE DETERMINES THE TORQUE ON THE GEAR

HP       = HORSEPOWER
PI       = 3.141592.....
RPM      = SHAFT SPEED (REVOLUTIONS PER MINUTE)
TORQ     = TORQUE (FT-LBS)

TORQ=(33000.0*HP)/(2.0*PI*RPM)
RETURN
END
```

```
SUBROUTINE TLOAD (HP,PLV,PAR,WA,WR,WT,WN)

THIS ROUTINE DETERMINES THE MAGNITUDE OF LOADING ON THE TEETH

HP       = HORSEPOWER
PLV      = PITCH LINE VELOCITY (FPM)
PAR      = PRESSURE ANGLE  (RADIANS)
WA       = AXIAL LOAD (LBS))
WR       = RADIAL LOAD (LBS)
WT       = TANGENTIAL LOAD (LBS)
WN       = NORMAL LOAD (LBS)

WT=(33000.0*HP)/PLV
WR=WT*TAN(PAR)
WA=0.0
WN=WT/COS(PAR)
RETURN
END
```

```
      SUBROUTINE LOAD(RL,ANGL,RLL,ANGLL,NLOAD,BP,PAR,PR,PRM,RB,RBM,
     1RO,ROM,TP)
C
C     THIS ROUTINE DETERMINES THE RADIUS TO THE POINT OF LOAD
C     APPLICATION AT THE TOOTH CENTRELINE AS WELL AS THE LOAD ANGLE
C     FOR EITHER TIP LOADING OR POINT OF HIGHEST SINGLE TOOTH CONTACT
C     LOADING. THE LOAD RADIUS AND LOAD ANGLE FOR THE FOLLOWING TOOTH
C     ARE ALSO DETERMINED.
C
C     RL       = RADIUS TO POINT OF LOAD APPLICATION ON GEAR TOOTH
C                CENTRELINE (INCHES)
C     ANGL     = LOAD ANGLE (RADIANS)
C     RLL      = RADIUS TO POINT OF LOAD APPLICATION ON FOLLOWING GEAR
C                TOOTH CENTRELINE (INCHES)
C     ANGLL    = FOLLOWING TOOTH LOAD ANGLE (RADIANS)
C     NLOAD    = 1 FOR TIP-LOADING
C              = 2 FOR HIGHEST POINT OF SINGLE TOOTH CONTACT
C     BP       = BASE PITCH (INCHES)
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     PRM      = PITCH CIRCLE RADIUS OF MATING GEAR (INCHES)
C     RB       = BASE CIRCLE RADIUS (INCHES)
C     RBM      = BASE CIRCLE RADIUS OF MATING GEAR (INCHES)
C     RO       = ADDENDUM CIRCLE RADIUS (INCHES)
C     ROM      = ADDENDUM CIRCLE RADIUS OF MATING GEAR (INCHES)
C     TP       = TOOTH THICKNESS AT PITCH CIRCLE (INCHES)
C
C
      XINV(ANG)=TAN(ANG)-ANG
      TANG(TP,PR,ANG1,ANG2)=((TP/(2.0*PR))+XINV(ANG1)-XINV(ANG2))
      IF(NLOAD.EQ.2) GO TO 1
      ANG=ACOS(RB/RO)
      ANGA=TANG(TP,PR,PAR,ANG)
C
C     ANGA     = ANGLE BETWEEN CENTERLINE OF TOOTH TO EDGE OF TOP LAND
C                ON ADDENDUM CIRCLE
C
      ANGL=ANG-ANGA
      RL=RB/COS(ANGL)
      IF(RL.GT.RO) RL=RO
      ZZZ=SQRT((RO**2)-(RB**2))
      ZZ=ZZZ-BP
      IF(ZZ.LT.0.0) ZZ=0.0
      RR=SQRT((RB**2)+(ZZ**2))
      IF(RR.LT.RB) RR=RB
      ANG=ACOS(RB/RR)
      ANG1=TANG(TP,PR,PAR,ANG)
      ANGLL=ANG-ANG1
      RLL=RB/COS(ANGLL)
      RETURN
    1 ZB=SQRT((ROM**2)-(RBM**2))-SQRT((PRM**2)-(RBM**2))
      ZZ=(BP-ZB)
      RR=SQRT((PR**2)+(ZZ**2)+2.0*PR*ZZ*SIN(PAR))
      ANG=ACOS(RB/RR)
      ANG1=TANG(TP,PR,PAR,ANG)
```

```
      ANGL=ANG-ANG1
      RL=RB/COS(ANGL)
      IF(RL.GT.RO) RL=RO
      ZZZ=SQRT((RR**2)-(RB**2))
      ZZ=ZZZ-BP
      IF(ZZ.LT.0.0) ZZ=0.0
      RR=SQRT((RB**2)+(ZZ**2))
      IF(RR.LT.RB) RR=RB
      ANG=ACOS(RB/RR)
      ANG1=TANG(TP,PR,PAR,ANG)
      ANGLL=ANG-ANG1
      RLL=RB/COS(ANGLL)
      RETURN
      END


      SUBROUTINE SHARE(ANGC1,ANGC2,ANGL1,ANGL2,BBY1,BBY2,BP,DP,E1,E2,
     1ERR,FW,NCUT1,NCUT2,NNLOAD,PAD,PAR,PI,PR1,PR2,QO,QV,QJ1,QJ2,
     2RB1,RB2,RI1,RI2,RL1,RL2,RLL1,RLL2,RLM1,RLM2,RO1,RO2,RT1,RT2,
     3TP1,TP2,WN)
C
C     THIS ROUTINE DETERMINES IF THERE IS LOAD SHARING BETWEEN
C     SUCCESSIVE TEETH OF A MATING GEAR SET. ANALYSIS IS MADE FOR TIP
C     AND HIGHEST POINT OF SINGLE TOOTH CONTACT LOADING. THE MODE OF
C     LOADING YIELDING THE HIGHEST BENDING STRESS WILL BE USED FOR
C     FINAL STRESS ANALYSIS.
C
C     SUBSCRIPT  (1) REFERS TO THE PINION
C     SUBSCRIPT  (2) REFERS TO THE GEAR
C     ANGC     = ANGLE BETWEEN THE TOOTH CENTRELINE AND THE FILLET ORIGIN
C                AT THE DEDENDUM CIRCLE (RADIANS)
C     ANGL     = LOAD PRESSURE ANGLE (RADIANS)
C     BBY      = DISTANCE BETWEEN PITCH CIRCLE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH (INCHES)
C     BP       = BASE PITCH (INCHES)
C     DP       = DIAMETRAL PITCH (TEETH PER INCH)
C     E        = MODULUS OF ELASTICITY (PSI)
C     ERR      = ERROR IN ACTION (INCHES)
C     FW       = FACE WIDTH (INCHES)
C     NCUT     = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C              = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     NNLOAD   = 1 FOR TIP LOADING
C              = 2 FOR HIGHEST POINT OF SINGLE TOOTH CONTACT LOADING
C     PAD      = PRESSURE ANGLE (DEGREES)
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PI       = 3.14192.......
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     QO       = OVERLOAD CORRECTION FACTOR
C     QV       = VELOCITY CORRECTION FACTOR
C     QJ       = GEOMETRY FACTOR
C     RB       = BASE CIRCLE RADIUS (INCHES)
C     RI       = DEDENDUM CIRCLE RADIUS (INCHES)
C     RL       = RADIUS TO POINT OF LOAD APPLICATION ON GEAR TOOTH
```

```
C                CENTRELINE (INCHES)
C     RLL      = RADIUS TO POINT OF LOAD APPLICATION ON GEAR TOOTH
C                CENTRELINE OF FOLLOWING TOOTH (INCHES)
C     RLM      = MINIMUM RADIUS TO POINT OF LOAD APPLICATION ON GEAR
C                TOOTH CENTRELINE FOR ANALYSIS USING LEWIS TECHNIQUE
C                OF TOOTH FORM FACTOR DETERMINATION (INCHES)
C     RO       = ADDENDUM CIRCLE RADIUS (INCHES)
C     RT       = ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH (INCHES)
C     TP       = TOOTH THICKNESS AT PITCH CIRCLE (INCHES)
C     WN       = NORMAL LOAD (LBS)
C
C
      DIMENSION RRR(8),AAA(8),QQQ(8),HHH(8),TTT(8),STIF(4)
      STIFF(ANGL1,ANGL2,E1,E2,FW,H1,H2,T1,T2)=FW/(25.0*(((H1/(T1*E1))
     1*COS(ANGL1))+((H2/(T2*E2))*COS(ANGL2))))
      NLOAD1=1
      NLOAD2=2
      DO 1 I=1,3,2
      CALL LOAD(RRR(I),AAA(I),RRR(I+1),AAA(I+1),NLOAD1,BP,PAR,PR1,PR2,
     1RB1,RB2,RO1,RO2,TP1)
      CALL LOAD(RRR(I+4),AAA(I+4),RRR(I+5),AAA(I+5),NLOAD2,BP,PAR,
     1PR2,PR1,RB2,RB1,RO2,RO1,TP2)
      NLOAD1=2
      NLOAD2=1
    1 CONTINUE
      DO 2 I=1,4
      CALL JFACT(ANGC1,AAA(I),BBY1,DP,NCUT1,PAD,PAR,PI,PR1,RI1,RRR(I),
     1RLM1,RT1,HHH(I),TTT(I),QQQ(I),Y1)
      CALL JFACT(ANGC2,AAA(I+4),BBY2,DP,NCUT2,PAD,PAR,PI,PR2,RI2,
     1RRR(I+4),RLM2,RT2,HHH(I+4),TTT(I+4),QQQ(I+4),Y2)
    2 CONTINUE
      DO 3 I=1,3,2
      STIF(I)=STIFF(AAA(I),AAA(I+5),E1,E2,FW,HHH(I),HHH(I+5),TTT(I),
     1TTT(I+5))
      STIF(5-I)=STIFF(AAA(I+1),AAA(I+4),E1,E2,FW,HHH(I+1),HHH(I+4),
     1TTT(I+1),TTT(I+4))
    3 CONTINUE
      DWN=WN*(QO/QV)
      IF(DWN.LT.(ERR*STIF(1)))GO TO 4
      IF(DWN.LT.(ERR*STIF(2)))GO TO 4
      XXX=ERR*STIF(1)*STIF(4)
      YYY=STIF(1)+STIF(4)
      FT1=(DWN*STIF(1)+XXX)/YYY
      FM2=(DWN*STIF(4)+XXX)/YYY
      XXX=ERR*STIF(2)*STIF(3)
      YYY=STIF(2)+STIF(3)
      FT2=(DWN*STIF(2)+XXX)/YYY
      FM1=(DWN*STIF(3)+XXX)/YYY
      BST1=(FT1*AAA(1)*DP)/(FW*QQQ(1))
      BSM1=(FM1*AAA(3)*DP)/(FW*QQQ(3))
      IF(BST1.GT.BSM1) GO TO 4
      BST2=(FT2*AAA(7)*DP)/(FW*QQQ(7))
      BSM2=(FM2*AAA(5)*DP)/(FW*QQQ(5))
      IF(BST2.GT.BSM2) GO TO 4
      RL1=RRR(3)
```

```
      RL2=RRR(5)
      ANGL1=AAA(3)
      ANGL2=AAA(5)
      RLL1=RRR(4)
      RLL2=RRR(6)
      QJ1=QQQ(3)
      QJ2=QQQ(5)
      NNLOAD=2
      RETURN
    4 RL1=RRR(1)
      RL2=RRR(7)
      ANGL1=AAA(1)
      ANGL2=AAA(7)
      RLL1=RRR(2)
      RLL2=RRR(8)
      QJ1=QQQ(1)
      QJ2=QQQ(7)
      NNLOAD=1
      RETURN
      END


      SUBROUTINE BEND(WT,DP,FW,QOD,QODL1,QODL2,QJ1,QJ2,SB1,SB2,
     1SBM1,SBM2,SAF1,SAF2)
C
C     THIS ROUTINE DETERMINES THE ACTUAL AND ALLOWABLE BENDING STRESS
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     WT        = TRANSMITTED LOAD ON TEETH (LBS)
C     DP        = DIAMETRAL PITCH (TEETH PER INCH)
C     FW        = FACE WIDTH (INCHES)
C     QOD       = OVERALL DERATING FACTOR
C               = (QO*QS*QM)/QV
C     QODL      = OVERALL LIFE DERATING FACTOR
C               = QL/(QT*QR)
C     QJ        = GEOMETRY FACTOR
C     QL        = LIFE FACTOR
C     QM        = LOAD DISTRIBUTION CORRECTION FACTOR
C     QO        = OVERLOAD CORRECTION FACTOR
C     QR        = RELIABILITY FACTOR (FACTOR OF SAFETY)
C     QS        = SIZE CORRECTION FACTOR
C     QT        = TEMPERATURE FACTOR
C     QV        = VELOCITY FACTOR
C     SB        = ACTUAL BENDING STRESS (PSI)
C     SBM       = MAXIMUM ALLOWABLE BENDING STRESS (PSI)
C     SAF       = MAXIMUM ALLOWABLE FATIGUE STRESS (PSI)
C
C
      SB1=(WT*DP*QOD)/(FW*QJ1)
      SB2=SB1*(QJ1/QJ2)
      SBM1=SAF1*QODL1
      SBM2=SAF2*QODL2
```

```
      RETURN
      END




      SUBROUTINE WEAR(COD,CODL1,CODL2,CE,CJ,FW,PR1,SAC1,SAC2,SS1,SS2,
     1SSM1,SSM2,WT)
C
C     THIS ROUTINE DETERMINES THE ACTUAL AND ALLOWABLE CONTACT STRESS
C     ON THE TOOTH FACE (PSI)
C
C     SUBSCRIPT  (1) REFERS TO THE PINION
C     SUBSCRIPT  (2) REFERS TO THE GEAR
C     COD        = OVERALL DERATING FACTOR
C                = (CO*CS*CM*CF)/CV
C     CODL       = OVERALL LIFE DERATING FACTOR
C                = (CL*CH)/(CT*CR)
C     CE         = ELASTIC COEFFICIENT FACTOR
C     CF         = SURFACE CONDITION FACTOR
C     CH         = HARDNESS RATIO FACTOR
C     CJ         = GEOMETRY FACTOR
C     CL         = LIFE FACTOR
C     CM         = LOAD-DISTRIBUTION CORRECTION FACTOR
C     CO         = OVERLOAD CORRECTION FACTOR
C     CR         = RELIABILITY FACTOR (FACTOR OF SAFETY)
C     CS         = SIZE CORRECTION FACTOR
C     CT         = TEMPERATURE FACTOR
C     CV         = VELOCITY CORRECTION FACTOR(DYNAMIC FACTOR)
C     FW         = FACE WIDTH (INCHES)
C     PR         = PITCH RADIUS (INCHES)
C     SAC        = ALLOWABLE CONTACT STRESS (PSI)
C     SS         = ACTUAL SURFACE STRESS (PSI)
C     SSM        = MAXIMUM ALLOWABLE SURFACE STRESS (PSI)
C     WT         = TRANSMITTED LOAD ON TEETH (LBS)
C
C
      SS1=CE*SQRT((WT*COD)/(2.0*PR1*FW*CJ))
      SS2=SS1
      SSM1=SAC1*CODL1
      SSM2=SAC2*CODL2
      RETURN
      END
```

```
      SUBROUTINE POWER(CE,CJ,COD,CODL1,CODL2,QJ1,QJ2,QOD,QODL1,QODL2,
     1DP,FW,PAB1,PAB2,PAW1,PAW2,PI,PR1,RPM1,SAC1,SAC2,SAF1,SAF2)

C     THIS ROUTINE DETERMINES THE MAXIMUM ALLOWABLE HORSEPOWER THAT
C     CAN BE TRANSMITTED UNDER WEAR AND BENDING STRESS CONDITIONS

C     CE       = ELASTIC COEFFICIENT FACTOR
C     CJ       = GEOMETRY FACTOR
C     COD      = OVERALL DERATING FACTOR
C              = (CO*CS*CM*CF)/CV
C     CODL     = OVERALL LIFE DERATING FACTOR
C              = (CL*CH)/(CT*CR)
C     QJ       = GEOMETRY FACTOR
C     QOD      = OVERALL DERATING FACTOR
C              = (QO*QS*QM)/QV
C     QODL     = OVERALL LIFE DERATING FACTOR
C              = QL/(QT*QR)
C     DP       = DIAMETRAL PITCH (TEETH PER INCH)
C     FW       = FACE WIDTH (INCHES)
C     PAB      = MAXIMUM ALLOWABLE POWER...BENDING ANALYSIS (HP)
C     PAW      = MAXIMUM ALLOWABLE POWER...WEAR ANALYSIS (HP)
C     PI       = 3.141592.....
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     RPM      = SHAFT SPEED (REVOLUTIONS PER MINUTE)
C     SAC      = ALLOWABLE CONTACT STRESS (PSI)
C     SAF      = MAXIMUM ALLOWABLE FATIGUE STRESS (PSI)

C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     C-FACTORS FOR WEAR ANALYSIS
C     Q-FACTORS FOR BENDING ANALYSIS


      XXX=PI*RPM1*FW/396000.0
      XXX1=XXX*((2.0*PR1)/(QOD*DP))
      XXX2=XXX*(CJ/COD)*(((2.0*PR1)/CE)**2)
      PAB1=XXX1*QJ1*SAF1*QODL1
      PAB2=XXX1*QJ2*SAF2*QODL2
      PAW1=XXX2*((SAC1*CODL1)**2)
      PAW2=XXX2*((SAC2*CODL2)**2)
      RETURN
      END
```

```
      SUBROUTINE JFACT(ANGC,ANGL,BBY,DP,NCUT,PAD,PAR,PI,PR,RI,RL,RLM,
     1RT,H,T,QJ,Y)
C
C     THIS SUBROUTINE CALCULATES THE GEOMETRY FACTOR AND THE TOOTH FORM
C     FACTOR FOR SPUR GEAR BENDING STRESS ANALYSIS
C     THE MINIMUM LOAD APPLICATION RADIUS POSSIBLE FOR TOOTH FORM FACTOR
C     ANALYSIS BY LEWIS TECHNIQUE IS ALSO CALCULATED
C
C     ANGC     = ANGLE BETWEEN THE TOOTH CENTRELINE AND FILLET ORIGIN
C                AT THE DEDENDUM CIRCLE
C     ANGL     = LOAD PRESSURE ANGLE
C     BBY      = DISTANCE BETWEEN PITCH CIRCLE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH (INCHES)
C     DP       = DIAMETRAL PITCH (TEETH PER INCH)
C     NCUT     = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C              = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     PAD      = PRESSURE ANGLE (DEGREES)
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PI       = 3.14159............
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     RI       = ROOT CIRCLE RADIUS (INCHES)
C     RL       = RADIUS TO POINT OF LOAD APPLICATION ON GEAR TOOTH
C                CENTRELINE (INCHES)
C     RLM      = MINIMUM RADIUS TO POINT OF LOAD APPLICATION ON GEAR
C                TOOTH CENTRELINE FOR ANALYSIS USING LEWIS TECHNIQUE
C                OF TOOTH FORM FACTOR DETERMINATION (INCHES)
C     RT       = ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH (INCHES)
C     H        = DISTANCE FROM POINT OF LOAD APPLICATION ON TOOTH CENTRE-
C                LINE TO ASSUMED WALL OF TOOTH CANTILEVER (INCHES)
C     T        = CHORDAL TOOTH THICKNESS AT POINT OF HIGHEST STRESS
C                CONCENTRATION ON TOOTH FILLET (INCHES)
C     QJ       = GEOMETRY FACTOR
C     Y        = TOOTH FORM FACTOR
C
C     THIS SUBROUTINE ALSO REQUIRES **SUBROUTINE CWALL** TO DETERMINE
C     THE POINT OF MAXIMUM STRESS CONCENTRATION ON THE FILLET CURVE.
C
      YYY(ANGL,ANG,DP,X,T)=DP/((COS(ANGL)/COS(ANG))*((1.5/X)-(TAN(ANGL)
     1/T)))
      SCF(PAD,H,RF,T)=(0.325455-0.007273*PAD)+(((T/RF)**(0.331819-0.0090
     191*PAD))*((T/H)**(0.268182+0.009091*PAD)))
      FRM(RRO,BBY,RT)=RT+((BBY**2)/(RRO+BBY))
C
      RLM=RI*(COS(ANGC)+(0.5*SIN(ANGC)/TAN((PI/2.0)-ANGC)))
      IF(RL.LE.RLM) GO TO 1
      CALL CWALL(BBY,ANGC,ANGR,NCUT,PI,PR,RI,RL,RRF,RRO,RT)
      XXX=RL
      GO TO 2
    1 XXX=RLM
      ANGR=ANGC
      RRO=PR
      RRF=RI
    2 T=2.0*RRF*SIN(ANGR)
      H=XXX-RRF*COS(ANGR)
      X=(T**2)/(4.0*H)
```

```
      Y=YYY(ANGL,PAR,DP,X,T)
      RF=FRM(RRO,BBY,RT)
      XKF=SCF(PAD,H,RF,T)
      QJ=Y/XKF
      RETURN
      END



      SUBROUTINE CWALL(BBY,ANGC,ANGR,NCUT,PI,PR,RI,RL,RRF,RRO,RT)
C
C     THIS SUBROUTINE CALCULATES,IN RADIAL COORDINATES WITH RESPECT
C     TO THE CENTRE OF THE GEAR REFERENCED TO CENTRELINE OF THE TOOTH,
C     THE POINT ON THE TOOTH FILLET CONSIDERED THE LOCATION OF HIGHEST
C     STRESS CONCENTRATION TO BE USED IN THE Y- AND J-FACTOR
C     DETERMINATION
C
C     BBY      = DISTANCE FROM PITCH CIRCLE TO CENTRE OF ROUNDED CORNER
C                OF GENERATING TOOTH.   BBY=DED IF THE GENERATING TOOL
C                TOOTH HAS A SHARP CORNER. (INCHES)
C     ANGC     = ANGLE BETWEEN THE TOOTH CENTRE AND THE ORIGIN OF THE
C                FILLET ON THE ROOT CIRCLE
C     NCUT     = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C              = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     PI       = 3.141592.....
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     RI       = DEDENDUM CIRCLE RADIUS (INCHES)
C     RL       = RADIUS TO POINT OF LOAD APPLICATION ON GEAR TOOTH
C                CENTRELINE (INCHES)
C     RRO      = RELATIVE RADIUS OF CURVATURE OF THE PITCH CIRCLE OF THE
C                GEAR AND THE PITCH LINE OR PITCH CIRCLE OF THE
C                GENERATING TOOL
C     RT       = ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH (INCHES)
C
C     ANGR,RRF ... RADIAL COORDINATES OF POINT OF HIGHEST STRESS
C     CONCENTRATION ON FILLET CURVE.
C
C
      WALL(RL,RRT,ANG1,ANG2)=2.0*RL-RRT*(2.0*COS(ANG1)+(SIN(ANG1)/TAN(AN
     1G2-ANG1)))
      XINV(ANG)=TAN(ANG)-ANG
      TART(ANG,BBY,PR,RRT)=ATAN((1.0-((RRT**2)/(PR*(PR-BBY))))/TAN(ANG))
      TANG(TP,PR,ANG1,ANG2)=((TP/(2.0*PR))+XINV(ANG1)-XINV(ANG2))
      FRRC(ANG,RRT,RT)=SQRT((RRT**2)+RT*(RT-2.0*RRT*SIN(ANG)))
      FASC(ANG,BBY,PR)=(BBY/PR)*TAN(ANG)-XINV(ANG)
      FARC(ANG,ANG1,BBY,PR,RRF,RRT,RT)=FASC(ANG,BBY,PR)+ACOS((RRT-RT*SIN
     1(ANG1))/RRF)
C
      ERROR=1.0E-06
      GO TO (100,200),NCUT
  100 XX=PR-BBY
      R1=XX
      ANG1=0.0
      ANG11=ANGC
```

```
      ANG12=PI/2.0
      FUNC1=WALL(RL,R1,ANG11,ANG12)
      DELR=SQRT(PR*XX)-R1
      IF(DELR.LE.0.0) GO TO 125
      DELR=DELR/5.0
  103 DELR=DELR/2.0
      R2=R1
      FUNC2=FUNC1
  105 R1=R2
      FUNC1=FUNC2
      R2=R2+DELR
      ANG2=ACOS(XX/R2)
      ANG21=ANGC-FASC(ANG2,BBY,PR)
      ANG22=TART(ANG2,BBY,PR,R2)
      FUNC2=WALL(RL,R2,ANG21,ANG22)
      IF(FUNC2.GE.FUNC1) GO TO 103
      IF((FUNC1*FUNC2).GT.0.0) GO TO 105
  110 R3=(R1*FUNC2-R2*FUNC1)/(FUNC2-FUNC1)
      ANG3=ACOS(XX/R3)
      ANG31=ANGC-FASC(ANG3,BBY,PR)
      ANG32=TART(ANG3,BBY,PR,R3)
      FUNC3=WALL(RL,R3,ANG31,ANG32)
      IF((ABS(FUNC3)).LE.ERROR) GO TO 120
      IF((FUNC1*FUNC3).GT.0.0) GO TO 115
      R2=R3
      FUNC2=FUNC3
      GO TO 110
  115 R1=R3
      FUNC1=FUNC3
      GO TO 110
  120 RRF=R3
      RRO=PR
      ANGR=ANG31
      RETURN
  125 RRF=R1
      RRO=PR
      ANGR=ANG11
      RETURN
  200 XX=PR-BBY
      R1=XX
      ANG1=0.0
      ANG11=PI/2.0
      RRF1=XX-RT
      ANG12=ANGC
      FUNC1=WALL(RL,RRF1,ANG12,ANG11)
      DELR=SQRT(PR*XX)-R1
      IF(DELR.LE.0.0) GO TO 225
      DELR=DELR/5.0
  203 DELR=DELR/2.0
      R2=R1
      RRF2=RRF1
      ANG22=ANG12
      FUNC2=FUNC1
  205 R1=R2
      RRF1=RRF2
```

```
      ANG12=ANG22
      FUNC1=FUNC2
      R2=R2+DELR
      ANG2=ACOS(XX/R2)
      ANG21=TART(ANG2,BBY,PR,R2)
      RRF2=FRRC(ANG21,R2,RT)
      ANG22=ANGC-FARC(ANG2,ANG21,BBY,PR,RRF2,R2,RT)
      ANG21=ANG21-ACOS((R2-RT*SIN(ANG21))/RRF2)
      FUNC2=WALL(RL,RRF2,ANG22,ANG21)
      IF(ABS(R2-R1).LT.ERROR) GO TO 225
      IF(FUNC2.GE.FUNC1) GO TO 203
      IF((FUNC1*FUNC2).GT.0.0) GO TO 205
210   R3=(R1*FUNC2-R2*FUNC1)/(FUNC2-FUNC1)
      ANG3=ACOS(XX/R3)
      ANG31=TART(ANG3,BBY,PR,R3)
      RRF3=FRRC(ANG31,R3,RT)
      ANG32=ANGC-FARC(ANG3,ANG31,BBY,PR,RRF3,R3,RT)
      ANG31=ANG31-ACOS((R3-RT*SIN(ANG31))/RRF3)
      FUNC3=WALL(RL,RRF3,ANG32,ANG31)
      IF((ABS(FUNC3)).LE.ERROR) GO TO 220
      IF((FUNC1*FUNC3).GT.0.0) GO TO 215
      R2=R3
      FUNC2=FUNC3
      GO TO 210
215   R1=R3
      FUNC1=FUNC3
      GO TO 210
220   RRF=RRF3
      RRO=PR
      ANGR=ANG32
      RETURN
225   RRF=RRF1
      RRO=PR
      ANGR=ANG12
      RETURN
      END
```

```
      SUBROUTINE IFACT(BP,CJ,PAR,PR1,PR2,RATIO,RB1,RO1)
C
C     THIS ROUTINE DETERMINES THE GEOMETRY FACTOR FOR THE WORST CASE
C     OF SURFACE LOADING AT THE POINT OF LOWEST SINGLE TOOTH CONTACT
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     BP       = BASE PITCH (INCHES)
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     RATIO    = GEAR RATIO (IE. GEAR TEETH / PINION TEETH)
C     RB       = BASE CIRCLE RADIUS (INCHES)
C     RO       = ADDENDUM CIRCLE RADIUS (INCHES)
C
C
      XXX=(0.5/TAN(PAR))*(RATIO/(RATIO+1.0))
      ZA1=(SQRT((RO1**2)-(RB1**2))-SQRT((PR1**2)-(RB1**2)))
      ZC1=(BP-ZA1)
      IF(SIN(PAR).LT.(ZC1/PR1)) GO TO 1
      CJ=XXX*(SIN(PAR)+(ZC1/PR2))*(SIN(PAR)-(ZC1/PR1))
      RETURN
    1 CJ=1.0E-50
      RETURN
      END
```

```
      SUBROUTINE EFACT (CE,E1,E2,PI,U1,U2)
C
C     THIS ROUTINE DETERMINES THE ELASTIC COEFFICIENT FOR THE SURFACE
C     STRESS ANALYSIS
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     CE       = ELASTIC COEFFICIENT FACTOR
C     E        = MODULUS OF ELASTICITY (PSI)
C     PI       = 3.141592............
C     U        = POISSONS RATIO
C
C
      CCE=1.0
      CE=SQRT(CCE/(PI*(((1.0-(U1**2))/E1)+((1.0-(U2**2))/E2))))
      RETURN
      END
```

```
      SUBROUTINE VFACT(CV,QV,NQUAL,PLV)
C
C     THIS SUBROUTINE DETERMINES THE VELOCITY (DYNAMIC) FACTOR FOR THE
C     STRESS ANALYSIS
C
C     CV,QV    = VELOCITY OR DYNAMIC FACTOR
C     NQUAL    = A.G.M.A. QUALITY NUMBER
C     PLV      = PITCH LINE VELOCITY (FPM)
C
C
      NNN=NQUAL-2
      GO TO(100,100,100,200,200,300,300,400,400,400,500,500,500,600),NNN
  100 CV=600.0/(600.0+PLV)
      GO TO 700
  200 CV=1200.0/(1200.0+PLV)
      GO TO 700
  300 CV=50.0/(50.0+SQRT(PLV))
      GO TO 700
  400 CV=78.0/(78.0+SQRT(PLV))
      GO TO 700
  500 CV=SQRT(78.0/(78.0+SQRT(PLV)))
      GO TO 700
  600 CV=1.0
  700 QV=CV
      RETURN
      END
```

```
      SUBROUTINE MFACT (CM,QM,FW,NQUAL)
C
C     THIS ROUTINE DETERMINES THE LOAD DISTRIBUTION FACTOR FOR THE
C     STRESS ANALYSIS
C
C     CM,QM    = LOAD DISTRIBUTION FACTOR
C     FW       = FACE WIDTH (INCHES)
C     NQUAL    = A.G.M.A. QUALITY NUMBER
C
C
      IF(FW.GT.2.0) GO TO 1
      CM=1.3
      GO TO 3
    1 IF(FW.GT.18.0) GO TO 2
      CM=(((((-9.62822E-08*FW+6.33757E-06)*FW-1.5862E-04)*FW+1.82424E-03)
     1*FW-9.30188E-03)*FW+4.82409E-02)*FW+1.22786
      GO TO 3
    2 CM=FW/(0.45*FW+2.0)
    3 IF(NQUAL.GT.14) GO TO 4
      CM=CM+(0.9*(FLOAT(15-NQUAL)/12.0))
      QM=CM
      RETURN
    4 QM=CM
      RETURN
      END
```

```
      SUBROUTINE OFACT (CO,QO,DRIVEN,DRIVER,NDRIVE,RATIO)
C
C     THIS ROUTINE DETERMINES THE OVERLOAD FACTORS FOR THE STRESS
C     ANALYSIS
C
C     CO,QO    = OVERLOAD FACTORS
C     DRIVEN   = 1.0 LOAD ON DRIVEN MACHINE - UNIFORM
C              = 2.0 LOAD ON DRIVEN MACHINE - MODERATE
C              = 3.0 LOAD ON DRIVEN MACHINE - HEAVY
C     DRIVER   = 1.0 POWER SOURCE - UNIFORM
C              = 2.0 POWER SOURCE - LIGHT SHOCK
C              = 3.0 POWER SOURCE - MEDIUM SHOCK
C     NDRIVE   = 1 FOR PINION DRIVE
C              = 2 FOR GEAR  DRIVE
C     RATIO    = GEAR RATIO (IE. GEAR TEETH / PINION TEETH)
C
C
      XXX=((DRIVEN-1.0)*DRIVEN+2.0*DRIVER+6.0)/8.0
      GO TO (1,2),NDRIVE
    1 CO=XXX
      GO TO 3
    2 CO=0.01*(RATIO**2)+XXX
    3 QO=CO
      RETURN
      END


      SUBROUTINE SFACT (CS,QS)
C
C     THIS ROUTINE DETERMINES THE SIZE FACTOR FOR THE STRESS ANALYSIS
C
C     CS,QS    = SIZE CORRECTION FACTOR
C
C
      CS=1.0
      QS=CS
      RETURN
      END


      SUBROUTINE FFACT (CF)
C
C     THIS ROUTINE DETERMINES THE SURFACE FINISH CORRECTION FACTOR FOR
C     THE SURFACE STRESS ANALYSIS
C
C     CF       = SURFACE FINISH CORRECTION FACTOR
C
C
      CF=1.0
      RETURN
      END
```

```
      SUBROUTINE HFACT (BHN1,BHN2,RATIO,CH)
C
C     THIS ROUTINE DETERMINES THE HARDNESS RATIO FACTOR FOR SURFACE
C     STRESS ANALYSIS
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     BHN      = BRINELL HARDNESS
C     RATIO    = GEAR RATIO (IE. GEAR TEETH / PINION TEETH)
C     CH       = HARDNESS RATIO FACTOR
C
C
      HR=BHN1/BHN2
      IF(HR.LT.1.2) GO TO 1
      XXX=0.052808*(HR**0.225683)-0.052632
      CH=XXX*(RATIO-1.0)+1.0
      RETURN
    1 CH=1.0
      RETURN
      END



      SUBROUTINE LFACT (BHN,CYCLE,CL,QL)
C
C     THIS ROUTINE DETERMINES THE LIFE FACTORS FOR STRESS ANALYSIS
C
C     BHN      = BRINELL HARDNESS
C     CYCLE    = NUMBER OF LOAD CYCLES FOR PINION
C     CL,QL    = LIFE FACTOR
C
C
      IF(CYCLE.GT.1.0E+07) GO TO 1
      CL=2.575607*(CYCLE**(-0.058697))
      GO TO 2
    1 CL=1.0
    2 QL160=2.335254*(CYCLE**(-0.056092))
      QL250=5.236361*(CYCLE**(-0.112266))
      QL450=9.626709*(CYCLE**(-0.150709))
      IF(QL160.LT.1.0)QL160=1.0
      IF(QL250.LT.1.0)QL250=1.0
      IF(QL450.LT.1.0)QL450=1.0
      IF(BHN.GT.250.0) GO TO 3
      QL=QL160+((BHN-160.0)/90.0)*(QL250-QL160)
      RETURN
    3 QL=QL250+((BHN-250.0)/200.0)*(QL450-QL250)
      RETURN
      END
```

```
      SUBROUTINE RFACT (CR,QR,RELI)
C
C     THIS ROUTINE DETERMINES THE RELIABILITY FACTOR (SAFETY FACTOR) FOR
C     THE STRESS ANALYSIS
C
C     RELI     = RELIABILITY
C     CR,QR    = RELIABILITY FACTOR
C
C
      IF(RELI.GE.1.0) RELI=0.9999
      IF(RELI.LT.0.99) GO TO 1
      CR=0.444444*((1.0/(1.0-RELI))**0.176091)
      GO TO 2
    1 CR=0.773196*RELI-0.234536
    2 QR=CR
      RETURN
      END
```

```
      SUBROUTINE TFACT (CT,QT,TEMP)
C
C     THIS ROUTINE DETERMINES THE TEMPERATURE FACTORS FOR THE STRESS
C     ANALYSIS
C
C     CT,QT    = TEMPERATURE CORRECTION FACTORS
C     TEMP     = TEMPERATURE OF GEAR OIL OR GEAR BLANK
C
C
      CT=1.0
      QT=1.0
      IF(TEMP.GT.160.0) QT=(460.0+TEMP)/620.0
      IF(TEMP.GT.180.0) CT=(460.0+TEMP)/640.0
      RETURN
      END
```

```
      SUBROUTINE FACTOR(CF,CH,CL1,CL2,QL1,QL2,CM,QM,CO,QO,CR,QR,CS,QS,
     1CT,QT,CV,QV,COD,CODL1,CODL2,QOD,QODL1,QODL2)
C
C     THIS ROUTINE DETERMINES THE OVERALL DERATING FACTORS FOR BENDING
C     AND WEAR STRESS ANALYSIS
C
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     CF       = SURFACE CONDITION FACTOR
C     CH       = HARDNESS RATIO FACTOR
C     CL,QL    = LIFE FACTOR
C     CM,QM    = LOAD-DISTRIBUTION CORRECTION FACTOR
C     CO,QO    = OVERLOAD CORRECTION FACTOR
C     CR,QR    = RELIABILITY FACTOR (FACTOR OF SAFETY)
```

```
C     CS,QS     = SIZE CORRECTION FACTOR
C     CT,QT     = TEMPERATURE FACTOR
C     CV,QV     = VELOCITY CORRECTION FACTOR(DYNAMIC FACTOR)
C     COD       = OVERALL DERATING FACTOR
C     QOD       = OVERALL DERATING FACTOR
C     CODL      = OVERALL LIFE DERATING FACTOR
C     QODL      = OVERALL LIFE DERATING FACTOR
C
C     C-FACTORS USED IN WEAR ANALYSIS
C     Q-FACTORS USED IN BENDING ANALYSIS
C
      COD=(CO*CS*CM*CF)/CV
      QOD=(QO*QS*QM)/QV
      XXX=CH/(CR*CT)
      YYY=QR*QT
      CODL1=CL1*XXX
      CODL2=CL2*XXX
      QODL1=QL1/YYY
      QODL2=QL2/YYY
      RETURN
      END


      SUBROUTINE CONRAT(ANGC1,ANGC2,BP,CRATIO,NCUT1,NCUT2,NDRIVE,
     1PAR,PR1,PR2,RB1,RB2,RI1,RI2,RO1,RO2,RU1,RU2,TP1,TP2,XLA,XLR,
     2BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2)
C
C     THIS ROUTINE DETERMINES ...
C     A) THE CONTACT RATIO FOR NON-UNDERCUT AND UNDERCUT CONDITIONS
C     B) THE LENGTH OF APPROACH AND RECESS FOR THE GEAR SET
C
C     THE ROUTINE ALSO REQUIRES **SUBROUTINE LENGTH** TO DETERMINE THE
C     LENGTH OF CONTACT IF UNDERCUTTING IS PRESENT.**SUBROUTINE LENGTH**
C     REQUIRES **SUBROUTINE FILLET**.
C     NOTE...LIMIT TO LENGTH OF APPROACH OR LENGTH OF RECESS SET TO ZERO
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     ANGC      = ANGLE BETWEEN THE TOOTH CENTRELINE AND FILLET ORIGIN
C                 AT THE DEDENDUM CIRCLE
C     BP        = BASE PITCH (INCHES)
C     CRATIO    = CONTACT RATIO
C     NCUT      = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C               = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     NDRIVE    = 1 FOR PINION DRIVE
C               = 2 FOR GEAR   DRIVE
C     PAR       = PRESSURE ANGLE (RADIANS)
C     PR        = PITCH RADIUS (INCHES)
C     RB        = BASE CIRCLE RADIUS (INCHES)
C     RI        = ROOT CIRCLE RADIUS (INCHES) ... (PR-DED)
C     RO        = ADDENDUM CIRCLE RADIUS (INCHES)
C     RU        = UNDERCUT LIMIT RADIUS (INCHES)
```

```
C     TP       =  TOOTH THICKNESS AT PITCH CIRCLE  (INCHES)
C     XLA      =  LENGTH OF APPROACH (INCHES)
C     XLR      =  LENGTH OF RECESS (INCHES)
C     BBA      =  DISTANCE BETWEEN PITCH CIRCLE AND END OF STRAIGHT
C                 PROFILE ON GENERATING RACK TOOTH FLANK (INCHES)
C     BBX      =  DISTANCE BETWEEN TOOTH CENTERLINE AND CENTRE OF ROUNDED
C                 CORNER ON GENERATING RACK TOOTH  (INCHES)
C     BBY      =  DISTANCE BETWEEN PITCH CIRCLE AND CENTRE OF ROUNDED
C                 CORNER ON GENERATING RACK TOOTH  (INCHES)
C     RT       =  ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH (INCHES)
C
C
      NNN=1
      IF(RU1.GT.RI1)  NNN=NNN+1
      IF(RU2.GT.RI2)  NNN=NNN+2
C
C     NNN      =  1   NEITHER GEAR UNDERCUT
C              =  2   PINION UNDERCUT ONLY
C              =  3   GEAR    UNDERCUT ONLY
C              =  4   BOTH GEARS UNDERCUT
C
      GO TO(400,300,200,100),NNN
  100 CALL  LENGTH(ANGC1,NCUT1,PAR,PR1,RB1,RI1,RU1,TP1,BBA1,
     1BBX1,BBY1,RT1,XXX1)
      CALL  LENGTH(ANGC2,NCUT2,PAR,PR2,RB2,RI2,RU2,TP2,BBA2,
     1BBX2,BBY2,RT2,XXX2)
      GO TO 500
  200 CALL  LENGTH(ANGC2,NCUT2,PAR,PR2,RB2,RI2,RU2,TP2,BBA2,
     1BBX2,BBY2,RT2,XXX2)
      XXX1=SQRT((RO2**2)-(RB2**2))-PR2*SIN(PAR)
      GO TO 500
  300 CALL  LENGTH(ANGC1,NCUT1,PAR,PR1,RB1,RI1,RU1,TP1,BBA1,
     1BBX1,BBY1,RT1,XXX1)
      XXX2=SQRT((RO1**2)-(RB1**2))-PR1*SIN(PAR)
      GO TO 500
  400 XXX1=SQRT((RO2**2)-(RB2**2))-PR2*SIN(PAR)
      XXX2=SQRT((RO1**2)-(RB1**2))-PR1*SIN(PAR)
  500 IF(XXX1.LT.0.0)XXX1=0.0
      IF(XXX2.LT.0.0)XXX2=0.0
      CRATIO=(XXX1+XXX2)/BP
      GO TO (600,700),NDRIVE
  600 XLA=XXX1
      XLR=XXX2
      RETURN
  700 XLA=XXX2
      XLR=XXX1
      RETURN
      END
```

```
      SUBROUTINE LENGTH(ANGC,NCUT,PAR,PR,RB,RI,RU,TP,BBA,BBX,BBY,RT,XXX)
C
C     THIS ROUTINE DETERMINES THE LENGTH OF CONTACT FOR A GEAR FROM THE
C     UNDERCUT CIRCLE TO THE PITCH POINT FOR UNDERCUT CONDITIONS
C
C     THE ROUTINE ALSO REQUIRES **SUBROUTINE FILLET** TO DETERMINE THE
C     RADIUS TO THE FILLET-INVOLUTE INTERSECTION
C
C     ANGC     = ANGLE BETWEEN THE TOOTH CENTRELINE AND FILLET ORIGIN
C                AT THE DEDENDUM CIRCLE
C     NCUT     = 1 IF GEAR CUT BY RACK WITH SHARP CORNERED TEETH
C              = 2 IF GEAR CUT BY RACK WITH ROUNDED CORNERS
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PR       = PITCH RADIUS (INCHES)
C     RB       = RADIUS OF BASE CIRCLE (INCHES)
C     RI       = ROOT CIRCLE RADIUS (INCHES) ... (PR-DED)
C     RU       = UNDERCUT LIMIT RADIUS (INCHES)
C     TP       = TOOTH THICKNESS AT PITCH CIRCLE (INCHES)
C     BBA      = DISTANCE BETWEEN PITCH CIRCLE AND END OF STRAIGHT
C                PROFILE ON GENERATING RACK TOOTH FLANK (INCHES)
C     BBX      = DISTANCE BETWEEN TOOTH CENTERLINE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH (INCHES)
C     BBY      = DISTANCE BETWEEN PITCH CIRCLE AND CENTRE OF ROUNDED
C                CORNER ON GENERATING RACK TOOTH (INCHES)
C     RT       = ROUNDED CORNER RADIUS OF GENERATING GEAR TOOTH (INCHES)
C     XXX      = LENGTH OF CONTACT (INCHES)
C
C
      CALL FILLET(ANGC,NCUT,PAR,PR,RAD,RB,RI,RU,RRTL,RRTU,TP,BBA,
     1BBX,BBY,RT)
      ANG=ACOS(RB/RAD)
      XXX=RB*(TAN(PAR)-TAN(ANG))
      RETURN
      END
```

```
      SUBROUTINE EFFIC(EFF,RB1,RB2,PAR,PLV,RATIO,NDRIVE,XLA,XLR)
C
C     THIS ROUTINE DETERMINES THE FRICTIONAL EFFICIENCY OF THE GEAR SET
C
C     SUBSCRIPT (1) REFERS TO THE PINION
C     SUBSCRIPT (2) REFERS TO THE GEAR
C     EFF      = EFFICIENCY
C     RB       = BASE CIRCLE RADIUS (INCHES)
C     PAR      = PRESSURE ANGLE (RADIANS)
C     PLV      = PITCH LINE VELOCITY (FPM)
C     RATIO    = GEAR RATIO (IE. GEAR TEETH / PINION TEETH)
C     NDRIVE   = 1 FOR PINION DRIVE
C              = 2 FOR GEAR   DRIVE
C     XLA      = LENGTH OF APPROACH (INCHES)
C     XLR      = LENGTH OF RECESS (INCHES)
C
C
      FS(VS)=(0.050/EXP(0.125*VS))+0.002*SQRT(VS)
C
C     STATEMENT FUNCTION  FS(VS)  GIVES THE COEFFICIENT OF FRICTION
C     AS A FUNCTION OF SLIDING VELOCITY FOR SOFT STEELS.  EQUATION
C     IS PRESENTED BY BUCKINGHAM IN **ANALYTICAL MECHANICS OF GEARS**
C
      GO TO (10,20),NDRIVE
   10 AA=XLA/RB1
      AR=XLR/RB1
      RRR=1.0/RATIO
      GO TO 30
   20 AA=XLA/RB2
      AR=XLR/RB2
      RRR=RATIO
   30 XXX=(PLV/2.0)*(1.0+RRR)*COS(PAR)
      VSA=XXX*AA
      VSR=XXX*AR
      FA=FS(VSA)
      FR=FS(VSR)
      XXXX=AA+AR
      FA=(4.0/3.0)*FS(VSA)
      FR=(2.0/3.0)*FS(VSR)
      EFF=1.0-((1.0+RRR)/XXXX)*((FA/2.0)*(AA**2)+(FR/2.0)*(AR**2))
      RETURN
      END
```

```
      SUBROUTINE ERROR(DP,FW,NQUAL,PR,TEETH,TOLR,TOLP,PTOL,TOLL,
     1TTCT,TCT)
C
C     THIS ROUTINE DETERMINES THE VARIOUS CUTTING TOLERANCES FOR
C     VARYING GEAR SIZES
C
C     DP       = DIAMETRAL PITCH (TEETH PER INCH)
C     FW       = FACE WIDTH (INCHES)
C     NQUAL    = A.G.M.A. QUALITY NUMBER
C     PR       = PITCH CIRCLE RADIUS (INCHES)
C     TEETH    = NUMBER OF TEETH
C     TOLR     = RUNOUT TOLERANCE
C     TOLP     = PITCH TOLERANCE
C     PTOL     = PROFILE TOLERANCE
C     TOLL     = LEAD TOLERANCE (INCHES)
C     TTCT     = TOOTH TO TOOTH COMPOSITE TOLERANCE
C     TCT      = TOTAL COMPOSITE TOLERANCE
C
C
      QUAL=NQUAL
      TOLR=58.0*((2.0*PR)**0.238)*(DP**(-0.484))*(1.4**(8-NQUAL))
      TOLP=10.5*((2.0*PR)**0.177)*(DP**(-0.224))*(1.42**(8-NQUAL))
      PTOL=21.5*((2.0*PR)**0.154)*(DP**(-0.435))*(1.4**(8-NQUAL))
      TOLL=(((-0.00244*QUAL+0.13638)*QUAL-2.69177)*QUAL+18.995)*
     1(FW**0.72)
      IF(TEETH.LE.20.0) GO TO 1
      IF(TEETH.LT.32.0) GO TO 2
      TTCT=25.0*(DP**(-0.24))*(1.4**(8-NQUAL))
      GO TO 3
    1 TTCT=54.7*(DP**(-0.48))*((2.0*PR)**(-0.24))*(1.4**(8-NQUAL))
      GO TO 3
    2 TTCT=38.2*(DP**(-0.36))*((2.0*PR)**(-0.13))*(1.4**(8-NQUAL))
    3 XXX=(5.0337*ALOG10(DP))-0.5153
      IF(TEETH.LE.20.2) GO TO 4
      TCT=14.5*((2.0*PR)**(0.24*(DP**(-0.15))))*(1.16**(10.0-XXX))*
     1(1.4**(8-NQUAL))
      GO TO 5
    4 TCT=15.0*((20.2/DP)**(0.24*(DP**(-0.15))))*(1.16**(10.0-XXX))*
     1(1.4**(8-NQUAL))-((1.5*DP)*((20.0/DP)-(2.0*PR)))
    5 TOLR=1.0E-04*TOLR
      TOLP=1.0E-04*TOLP
      PTOL=1.0E-04*PTOL
      TOLL=1.0E-04*TOLL
      TTCT=1.0E-04*TTCT
      TCT=1.0E-04*TCT
      RETURN
      END
```

```
      SUBROUTINE BLASH(BLMIN,BLMINT,BLMAX,BLMAXT,BL1,BL2,BLL,BLU,BLR,CP,
     1DP,DELBL,NQUAL,PAR,TPTL1,TPTL2,TPTU1,TPTU2,TPTE1,TPTE2,
     2TPTV1,TPTV2,TICT1,TICT2,TCT1,TCT2)
C
C      THIS ROUTINE DETERMINES...
C      A) MINIMUM AND MAXIMUM BACKLASH DESIRED AT OPERATING PITCH RADIUS
C      B) ACTUAL MINIMUM AND MAXIMUM BACKLASH AT OPERATING PITCH RADIUS
C      C) MAXIMUM TOOTH THINNING FOR BACKLASH INCLUDING MACHINING
C         TOLERANCE
C      D) DIFFERENCE BETWEEN MINIMUM AND MAXIMUM BACKLASH
C      E) TOOTH THICKNESS TOLERANCE CLASS
C      F) ACTUAL MAXIMUM TOOTH THICKNESS VARIATION FROM TOOTH ELEMENT
C      ERRORS
C      G) ACTUAL MAXIMUM TOOTH THICKNESS VARIATION FROM TOOTH ELEMENT
C      ERRORS AND RUNOUT
C
C      SUBSCRIPT (1) REFERS TO THE PINION
C      SUBSCRIPT (2) REFERS TO THE GEAR
C      BLMIN    = DESIRED MINIMUM BACKLASH AT OPERATING PITCH
C                 RADIUS (INCHES)
C      BLMINT   = ACTUAL MINIMUM BACKLASH AT OPERATING PITCH
C                 RADIUS (INCHES)
C      BLMAX    = DESIRED MAXIMUM BACKLASH AT OPERATING PITCH
C                 RADIUS (INCHES)
C      BLMAXT   = ACTUAL MAXIMUM BACKLASH AT OPERATING PITCH
C                 RADIUS (INCHES)
C      BL       = MAXIMUM TOOTH THINNING FOR BACKLASH INCLUDING MACHINING
C                 TOLERANCE (INCHES)
C      BLL      = FACTOR TO CONTROL LOWER BACKLASH LIMIT
C      BLU      = FACTOR TO CONTROL UPPER BACKLASH LIMIT
C      BLR      = RATIO OF PINION BACKLASH TO TOTAL BACKLASH
C      CP       = CIRCULAR PITCH (INCHES/TOOTH)
C      DP       = DIAMETRAL PITCH (TEETH PER INCH)
C      DELBL    = DIFFERENCE BETWEEN DESIRED MAXIMUM AND MINIMUM
C                 BACKLASH (INCHES)
C      NQUAL    = A.G.M.A. QUALITY NUMBER
C      PAR      = PRESSURE ANGLE (RADIANS)
C      TPTL     = LOWER TOOTH THICKNESS TOLERANCE (INCHES)
C      TPTU     = UPPER TOOTH THICKNESS TOLERANCE (INCHES)
C      TPTE     = ACTUAL MAXIMUM TOOTH THICKNESS VARIATION FROM TOOTH
C                 ELEMENT ERRORS AND RUNOUT
C      TPTV     = ACTUAL MAXIMUM TOOTH THICKNESS VARIATION FROM TOOTH
C                 ELEMENT ERRORS
C
C      FOR THE BACKLASH DIFFERENCE REQUIREMENTS THERE IS A CHOICE OF
C      FIVE(5) TOOTH THICKNESS TOLERANCE CLASSES DEPENDING ON THE QUALITY
C      NUMBER. EACH CLASS IS ONE HALF(1/2) THE PREVIOUS CLASS.
C      A MINIMUM BACKLASH IS DIVIDED PROPORTIONATELY BETWEEN THE
C      GEAR AND THE PINION THUS SPECIFYING THE MAXIMUM TOOTH THICKNESS
C      FOR THE TWO GEARS. THE WORKING TOOTH THICKNESS IS DETERMINED BY
C      SUBTRACTING THE TOOTH THICKNESS TOLERANCE (ASSUMED UNILATERAL)
C      FROM THE MAXIMUM TOOTH THICKNESS.
C
C
C      DETERMINE BACKLASH REQUIREMENTS
```

```
      BLMIN=(BLL*0.025)*(DP**(-0.903090))
      BLMAX=(BLU*0.040)*(DP**(-0.903090))
      BLMIN1=BLR*BLMIN
      BLMIN2=BLMIN-BLMIN1
C
C     BLMIN1 AND BLMIN2 EQUAL THE AMOUNT OF DELIBERATE TOOTH THINNING
C     ON THE PINION AND GEAR TO ACHIEVE MINIMUM BACKLASH
C
      DELBL=BLMAX-BLMIN
C
C     DETERMINE LARGEST TOOTH THICKNESS TOLERANCE CLASS
C
      IF(DP.LT.10.0) GO TO 1
      TTOL=0.015807*(DP**(-0.653066))
      GO TO 2
    1 TTOL=0.037423*(DP**(-0.978801))
C
C     DETERMINE REQUIRED TOLERANCE CLASS
C
    2 TPTV1=2.0*TTCT1*TAN(PAR)
      TPTV2=2.0*TTCT2*TAN(PAR)
      TTOL1=TPTV1
      TTOL2=TPTV2
      IF(BLR.EQ.0.0)GO TO 3
      XXX=((ALOG(TTOL/TTOL1)/ALOG(2.0))+1.0)+1.0
      N1=XXX
      IF(N1.LT.1)N1=1
      IF((XXX-AINT(XXX)).EQ.0.0) N1=N1-1
      GO TO 4
    3 N1=5
    4 XXX=((ALOG(TTOL/TTOL2)/ALOG(2.0))+1.0)+1.0
      N2=XXX
      IF(N2.LT.1)N2=1
      IF((XXX-AINT(XXX)).EQ.0.0) N2=N2-1
C
C     DETERMINE UPPER TOLERANCE LIMIT
C
      TPTU1=TTOL/(2.0**(N1-1))
      TPTU2=TTOL/(2.0**(N2-1))
      NNNNN=NQUAL-2
      GO TO (10,10,11,12,13,13,13,13,13,13,13,13,13,13),NNNNN
   10 IF(N1.GT.1)TPTU1=TTOL
      IF(N2.GT.1)TPTU2=TTOL
      GO TO 14
   11 IF(N1.GT.2)TPTU1=TTOL/2.0
      IF(N2.GT.2)TPTU2=TTOL/2.0
      GO TO 14
   12 IF(N1.GT.3)TPTU1=TTOL/4.0
      IF(N2.GT.3)TPTU2=TTOL/4.0
      GO TO 14
   13 IF(N1.GT.5)TPTU1=TTOL/16.0
      IF(N2.GT.5)TPTU2=TTOL/16.0
   14 TPTL1=0.0
      TPTL2=0.0
```

```
C      DETERMINE AMOUNT OF TOOTH THINNING FOR BACKLASH INCLUDING TOOTH
C      THICKNESS TOLERANCE
C
       TPTE1=2.0*TCT1*TAN(PAR)
       TPTE2=2.0*TCT2*TAN(PAR)
       BL1=BLMIN1+TPTE1
       BL2=BLMIN2+TPTE2
       BLMINT=BLMIN
       BLMAXT=BLMIN+(TPTU1+TPTU2)
       RETURN
       END



       SUBROUTINE TOLCD(BLMAXU,CD,CDR,CDTOLL,CDTOLU,NQUAL,PAR,PI,PR1,PR2,
      1RATIO,RB1,RB2,TEETH1,TP1,TP2)
C
C      THIS ROUTINE DETERMINES...
C      A) CENTRE DISTANCE TOLERANCE
C      B) MAXIMUM BACKLASH AT OUTER LIMIT OF CENTRE DISTANCE TOLERANCE
C
C      SUBSCRIPT (1) REFERS TO THE PINION
C      SUBSCRIPT (2) REFERS TO THE GEAR
C      BLMAXU  = MAXIMUM BACKLASH AT UPPER LIMIT OF CENTRE DISTANCE
C                TOLERANCE (INCHES)
C      CD      = CENTRE DISTANCE (INCHES)
C      CDR     = CENTRE DISTANCE TOLERANCE MODIFICATION FACTOR
C      CDTOLL  = LOWER CENTRE DISTANCE TOLERANCE (INCHES)
C      CDTOLU  = UPPER CENTRE DISTANCE TOLERANCE (INCHES)
C      NQUAL   = A.G.M.A. QUALITY NUMBER
C      PAR     = PRESSURE ANGLE (RADIANS)
C      PI      = 3.141592.....
C      PR      = PITCH CIRCLE RADIUS (INCHES)
C      RATIO   = GEAR RATIO (IE. GEAR TEETH/PINION TEETH)
C      RB      = BASE CIRCLE RADIUS (INCHES)
C      TEETH   = NUMBER OF TEETH
C      TP      = TOOTH THICKNESS AT PITCH RADIUS (INCHES)
C
C      MAXIMUM BACKLASH INCLUDES VALUES FOR TOOTH THINNING, EXTENSION OF
C      CENTRE DISTANCE AND MACHINING TOLERANCES
C
C      THE TOOTH THICKNESS IS ALREADY REDUCED BY AN ALLOWANCE TO YIELD A
C      MINIMUM BACKLASH PLUS A UNILATERAL TOOTH THICKNESS TOLERANCE WHICH
C      INCLUDES RUNOUT AND TOOTH ELEMENT ERRORS
C

       XINV(ANG)=TAN(ANG)-ANG
       THICK(TP,PR,RR,ANG1,ANG2)=2.0*RR*((TP/(2.0*PR))+XINV(ANG1)-
      1XINV(ANG2))
C
C      DETERMINE BILATERAL CENTRE DISTANCE TOLERANCE AND THEN CONVERT
C      TO UNILATERAL TOLERANCE
```

```
      NNNNN=NQUAL-2
      GO TO (10,10,10,10,10,11,11,11,11,11,12,12,12,12),NNNNN
   10 CDTOL=0.0100+((CD-12.0)/12.0)*0.0100
      IF(CDTOL.LT.0.0020)CDTOL=0.0020
      GO TO 13
   11 CDTOL=0.0020+((CD-12.0)/12.0)*0.0020
      IF(CDTOL.LT.0.0005)CDTOL=0.0005
      GO TO 13
   12 CDTOL=0.0005+((CD-12.0)/12.0)*0.0005
      IF(CDTOL.LT.0.0001)CDTOL=0.0001
   13 CDTOLU=2.0*COR*CDTOL
      CDTOLL=0.0
C
C     DETERMINE GEAR CHARACTERISTICS AT EXTENDED CENTRE DISTANCE
C
      OCD=CD+CDTOLU
      OPR1=OCD/(RATIO+1.0)
      OPR2=OCD-OPR1
      OCP=(2.0*PI*OPR1)/TEETH1
      ANG1=ACOS(RB1/OPR1)
      ANG2=ACOS(RB2/OPR2)
      OTP1=THICK(TP1,PR1,OPR1,PAR,ANG1)
      OTP2=THICK(TP2,PR2,OPR2,PAR,ANG2)
C
C
C     SINCE THE TOOTH THICKNESS VALUE REPRESENTS THE THEORETICAL TOOTH
C     THICKNESS (IE. CP/2.0) MINUS THE THINNING ALLOWANCE AND THE
C     THICKNESS ERROR DUE TO RUNOUT AND TOOTH ELEMENT ERRORS, THE
C     MAXIMUM BACKLASH AT THE EXTENDED CENTRE DISTANCE INCLUDES ALL
C     DETRIMENTAL SOURCES
C
      BLMAXU=(OCP-(OTP1+OTP2))
      RETURN
      END
```

```
      SUBROUTINE SIZE(ADD,DED,FW,HUBL,HUBR,RI,RIM,SHAFT,SAF,
     1TORQ,WEB,XKEY)
C
C     ADD      = ADDENDUM (INCHES)
C     DED      = DEDENDUM (INCHES)
C     FW       = FACE WIDTH (INCHES)
C     HUBL     = HUB LENGTH (INCHES)
C     HUBR     = OUTER RADIUS OF HUB (INCHES)
C     RI       = DEDENDUM CIRCLE RADIUS (INCHES)
C     RIM      = INNER RADIUS OF RIM (INCHES)
C     SHAFT    = SHAFT DIAMETER (INCHES)
C     SAF      = MAXIMUM ALLOWABLE FATIGUE STRESS (PSI)
C     TORQ     = TORQUE (FT-LBS)
C     WEB      = WEB THICKNESS (INCHES)
C     XKEY     = KEY WIDTH (INCHES) ......ASSUME SQUARE KEY
C
      SSY=0.577*SAF
C
C     DISTORTION ENERGY THEORY
C
      SY=2.0*SAF
      SAFE=2.0
C
C     SAFE     = SAFETY FACTOR
C
      IF(SHAFT.EQ.0.0) GO TO 1
      XKEY=SHAFT/4.0
      FORCE=TORQ/(SHAFT/24.0)
      HUBL=2.0*SAFE*FORCE/(XKEY*SY)
      HUBL1=(SAFE*FORCE)/(XKEY*SSY)
      GO TO 2
    1 XKEY=0.0
      HUBL=FW
      HUBL1=HUBL
    2 RIM=RI-(ADD+DED)
      HUBR=(1.75*SHAFT)/2.0
      WEB=0.5*FW
      IF(HUBL.LT.HUBL1) HUBL=HUBL1
      IF(HUBL.LT.FW) HUBL=FW
      IF(HUBR.GT.RIM) HUBR=RIM
      IF(RIM.EQ.HUBR) WEB=FW
      IF(WEB.LT.0.1.AND.FW.GE.0.1) WEB=0.1
      RETURN
      END
```

```
      SUBROUTINE VOLUME (FW,HUBL,HUBR,PI,RIM,RO,SHAFT,VOL,WEB)
C
C     THIS SUBROUTINE DETERMINES THE VOLUME OF MATERIAL REQUIRED FOR THE
C     GEAR BLANK TO THE ADDENDUM CIRCLE RADIUS
C
C     FW      = FACE WIDTH (INCHES)
C     HUBL    = HUB LENGTH (INCHES)
C     HUBR    = OUTER HUB RADIUS (INCHES)
C     PI      = 3.141592.....
C     RIM     = INNER RADIUS OF RIM (INCHES)
C     RO      = ADDENDUM CIRCLE RADIUS (INCHES)
C     SHAFT   = SHAFT DIAMETER (INCHES)
C     VOL     = VOLUME (CUBIC INCHES)
C     WEB     = WEB THICKNESS (INCHES)
C
C     IF OUTER HUB RADIUS IS LESS THAN SHAFT RADIUS, VOLUME DETERMINED
C     ASSUMING SHAFT RADIUS ZERO. (IE. HUB AND SHAFT BECOME ONE PIECE)
C
C
      XXXXX=SHAFT
      IF(SHAFT.GT.HUBR)XXXXX=0.0
      VOL=PI*((((HUBR**2)-((XXXXX/2.0)**2))*HUBL)+(((RIM**2)-(HUBR**2))*
     1WEB)+(((RO**2)-(RIM**2))*FW))
      RETURN
      END



      SUBROUTINE VARY1
C
C     THIS SUBROUTINE DETERMINES THE VARIABLES IN THE PROBLEM, ORGANIZES
C     ARRAYS TO KEEP TRACK OF THE VARIABLES, DETERMINES INITIAL VALUES
C     AND OUTPUTS INPUT DATA
C
C     THE ROUTINE ALSO REQUIRES **SUBROUTINE PITCH**
C
C
      DIMENSION NSORT(11)
      COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
      COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
      COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
      COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
      COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
      COMMON/BLK3B/BLL,BLU,BLR,CDR
      COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
      COMMON/BLK4A/ISIRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
      COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
      COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
      COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
      COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
      COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
      COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
      COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
      COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
```

```
      COMMON/BLK9 /COD,CODL1,CODL2,QOD,QODL1,QODL2
      COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
      COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
      COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
      COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
      COMMON/BLK12 /X( 8),XSTRT( 8),RMAX( 8),RMIN( 8),PHI(25),PSI( 1)
      COMMON/BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
      COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
      COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
      COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
      COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
      J=0
      K=0
      N=0
      NCD=0
      NFW=0
      NTOOTH=0
      NOPT=0
      NSUM=0
      XXX=3HVAR
      YYY=3HSTD
      ZZZ=3HCON
      IF(IDATA.EQ.0) GO TO 999
      WRITE(6,500)
      WRITE(6,501)
      WRITE(6,502)HP
      WRITE(6,503)RPMI
      WRITE(6,504)RPMO
      WRITE(6,505)
      WRITE(6,506)SAF1,SAF2
      WRITE(6,507)SAC1,SAC2
      WRITE(6,508)E1,E2
      WRITE(6,509)U1,U2
      WRITE(6,510)RHO1,RHO2
      WRITE(6,511)BHN1,BHN2
  999 NSUM=NSUM+1
      NSORT(NSUM)=1
      IF(XXX.NE.PAD) GO TO 1
      N=N+1
      NSUM=NSUM+1
      NVAR(N)=7
      NSORT(NSUM)=3
      GO TO 2
    1 PAR=(PAD/180.0)*PI
      NSUM=NSUM+1
      NSORT(NSUM)=2
    2 IF(XXX.NE.TEETH1) GO TO 3
      N=N+1
      NSUM=NSUM+1
      NTOOTH=N
      K=K+1
      NVAR(N)=8
      NSORT(NSUM)=5
      GO TO 4
    3 TEETH2=RATIO*TEETH1
```

```
      NSUM=NSUM+1
      NSORT(NSUM)=4
    4 IF(XXX.NE.CD)  GO TO 5
      N=N+1
      NSUM=NSUM+1
      NCD=N
      NVAR(N)=6
      NSORT(NSUM)=7
      GO TO 6
    5 NSUM=NSUM+1
      NSORT(NSUM)=6
      IF(NOF3.EQ.1.AND.CDMAX.EQ.CDMIN)  NOF3=0
    6 IF(XXX.NE.FW)  GO TO 7
      N=N+1
      NSUM=NSUM+1
      NFW=N
      NVAR(N)=5
      NSORT(NSUM)=9
      GO TO 8
    7 NSUM=NSUM+1
      NSORT(NSUM)=8
      IF(NOF4.EQ.1.AND.FWMAX.EQ.FWMIN)  NOF4=0
    8 NSUM=NSUM+1
      NSORT(NSUM)=10
      IF(ZZZ.EQ.ADDK1)  GO TO 1010
      IF(YYY.EQ.ADDK1)  GO TO 9
      IF(XXX.NE.ADDK1)  GO TO 10
      N=N+1
      NSUM=NSUM+1
      NVAR(N)=1
      NSORT(NSUM)=12
      GO TO 11
    9 J=J+1
      NSUM=NSUM+1
      NSTD(J)=1
      NSORT(NSUM)=14
      GO TO 11
   10 NSUM=NSUM+1
      NSORT(NSUM)=18
      J=J+1
      NSTD(J)=5
      GO TO 11
 1010 NSUM=NSUM+1
      NSORT(NSUM)=22
   11 IF(ZZZ.EQ.ADDK2)  GO TO 1013
      IF(YYY.EQ.ADDK2)  GO TO 12
      IF(XXX.NE.ADDK2)  GO TO 13
      N=N+1
      NSUM=NSUM+1
      NVAR(N)=2
      NSORT(NSUM)=13
      GO TO 14
   12 J=J+1
      NSUM=NSUM+1
      NSTD(J)=2
```

```
      NSORT(NSUM)=15
      GO TO 14
   13 NSUM=NSUM+1
      NSORT(NSUM)=19
      J=J+1
      NSTD(J)=6
      GO TO 14
 1013 NSUM=NSUM+1
      NSORT(NSUM)=23
   14 NSUM=NSUM+1
      NSORT(NSUM)=11
      IF(ZZZ.EQ.DEDK1) GO TO 1016
      IF(YYY.EQ.DEDK1) GO TO 15
      IF(XXX.NE.DEDK1) GO TO 16
      N=N+1
      NSUM=NSUM+1
      NVAR(N)=3
      NSORT(NSUM)=12
      GO TO 17
   15 J=J+1
      NSUM=NSUM+1
      NSTD(J)=3
      NSORT(NSUM)=16
      GO TO 17
   16 NSUM=NSUM+1
      NSORT(NSUM)=20
      J=J+1
      NSTD(J)=7
      GO TO 17
 1016 NSUM=NSUM+1
      NSORT(NSUM)=24
   17 IF(ZZZ.EQ.DEDK2) GO TO 1019
      IF(YYY.EQ.DEDK2) GO TO 18
      IF(XXX.NE.DEDK2) GO TO 19
      N=N+1
      NSUM=NSUM+1
      NVAR(N)=4
      NSORT(NSUM)=13
      GO TO 20
   18 J=J+1
      NSUM=NSUM+1
      NSTD(J)=4
      NSORT(NSUM)=17
      GO TO 20
   19 NSUM=NSUM+1
      J=J+1
      NSTD(J)=8
      NSORT(NSUM)=21
      GO TO 20
 1019 NSUM=NSUM+1
      NSORT(NSUM)=25
   20 NN=N
      IF(IDATA.EQ.0) GO TO 47
      DO 46 I=1,NSUM
      NNNNN=NSORT(I)
```

```
      GO TO (21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,
     140,41,42,43,44,45),NNNNN
   21 WRITE(6,600)
      GO TO 45
   22 WRITE(6,601)PAD
      GO TO 45
   23 WRITE(6,602)
      GO TO 45
   24 WRITE(6,603)TEETH1,TEETH2
      GO TO 45
   25 WRITE(6,604)
      GO TO 45
   26 WRITE(6,605)CD
      GO TO 45
   27 WRITE(6,606)
      GO TO 45
   28 WRITE(6,607)FW
      GO TO 45
   29 WRITE(6,608)
      GO TO 45
   30 WRITE(6,609)
      GO TO 45
   31 WRITE(6,610)
      GO TO 45
   32 WRITE(6,611)
      GO TO 45
   33 WRITE(6,612)
      GO TO 45
   34 WRITE(6,613)
      GO TO 45
   35 WRITE(6,614)
      GO TO 45
   36 WRITE(6,615)
      GO TO 45
   37 WRITE(6,616)
      GO TO 45
   38 WRITE(6,617)ADDK1
      GO TO 45
   39 WRITE(6,618)ADDK2
      GO TO 45
   40 WRITE(6,617)DEDK1
      GO TO 45
   41 WRITE(6,618)DEDK2
      GO TO 45
   42 WRITE(6,619)ADD1
      GO TO 45
   43 WRITE(6,620)ADD2
      GO TO 45
   44 WRITE(6,619)DED1
      GO TO 45
   45 WRITE(6,620)DED2
   46 CONTINUE
   47 NNN=1
      IF(NN.EQ.0) GO TO 110
      DO 109 I=1,NN
```

```
      NNNNN=NVAR(I)
      IF(NNNNN.LE.5) GO TO 108
   99 GO TO (100,101,102,103,104,105,106,107),NNNNN
  100 ADD1=1.0/DP
      XSTRT(I)=ADD1
      RMAX(I)=2.0*XSTRT(I)
      RMIN(I)=0.0
      GO TO 109
  101 ADD2=1.0/DP
      XSTRT(I)=ADD2
      RMAX(I)=2.0*XSTRT(I)
      RMIN(I)=0.0
      GO TO 109
  102 DED1=1.250/DP
      XSTRT(I)=DED1
      RMAX(I)=2.0*XSTRT(I)
      RMIN(I)=0.0
      GO TO 109
  103 DED2=1.250/DP
      XSTRT(I)=DED2
      RMAX(I)=2.0*XSTRT(I)
      RMIN(I)=0.0
      GO TO 109
  104 FW=2.0*PR1
      XSTRT(I)=FW
      RMAX(I)=5.0*ABS(FWMAX-FWMIN)
      RMIN(I)=0.0
      GO TO 109
  105 XK=(SAC1**2)*(((1.0-(U1**2))/E1)+((1.0-(U2**2))/E2))*((PI*SIN(PAR)
     1)/2.0)
      XQ=(HP*((RATIO+1.0)**3))/(RPM1*RATIO)
      CD=(((15750.0*XQ*(RATIO+1.0))/XK)**(1.0/3.0))
      XSTRT(I)=CD
      RMAX(I)=5.0*ABS(CDMAX-CDMIN)
      RMIN(I)=0.0
      GO TO 109
  106 PAD=20.0
      PAR=(PAD/180.0)*PI
      XSTRT(I)=PAD
      RMAX(I)=100.0
      RMIN(I)=0.0
      GO TO 109
  107 TEETH1=AINT((2.0/(SIN(PAR)**2))+1.0)
      XSTRT(I)=TEETH1
      RMAX(I)=100.0
      RMIN(I)=0.0
      GO TO 109
  108 IF(NNN.EQ.1)CALL PITCH(RATIO,CD,TEETH1,TEETH2,RPM1,PAR,PI,PR1,PR2,
     1RB1,RB2,BP,CP,DP,PLV)
      NNN=0
      GO TO 99
  109 CONTINUE
  110 IF(J.EQ.0) GO TO 212
      IF(NNN.EQ.1)CALL PITCH(RATIO,CD,TEETH1,TEETH2,RPM1,PAR,PI,PR1,PR2,
     1RB1,RB2,BP,CP,DP,PLV)
```

```fortran
      DO 211 I=1,J
      NNNNN=NSTD(I)
      GO TO (201,202,203,205,207,208,209,210),NNNNN
  201 ADD1=1.0/DP
      GO TO 211
  202 ADD2=1.0/DP
      GO TO 211
  203 IF(DP.GT.20.0) GO TO 204
      DED1=1.250/DP
      GO TO 211
  204 DED1=(1.200/DP)+0.002
      GO TO 211
  205 IF(DP.GT.20.0) GO TO 206
      DED2=1.250/DP
      GO TO 211
  206 DED2=(1.200/DP)+0.002
      GO TO 211
  207 ADD1=ADDK1/DP
      GO TO 211
  208 ADD2=ADDK2/DP
      GO TO 211
  209 DED1=DEDK1/DP
      GO TO 211
  210 DED2=DEDK2/DP
  211 CONTINUE
  212 IF(IDATA4.EQ.0) GO TO 310
      WRITE(6,650)
      WRITE(6,654)
      WRITE(6,656)
      WRITE(6,664)
      WRITE(6,651)CYCLE
      WRITE(6,652)RELI
      WRITE(6,653)TEMP
      WRITE(6,655)DRIVEN,DRIVER
      WRITE(6,657)BLR
      WRITE(6,658)BLL
      WRITE(6,659)BLU
      WRITE(6,660)CDR
      WRITE(6,663)NLOAD
      WRITE(6,661)NCUT1,NCUT2
      WRITE(6,662)NQUAL
      WRITE(6,675)
      WRITE(6,676)CDMAX
      WRITE(6,677)CDMIN
      WRITE(6,678)FWMAX
      WRITE(6,679)FWMIN
      WRITE(6,682)PADMAX
      WRITE(6,683)PADMIN
      WRITE(6,680)SHAFTI
      WRITE(6,681)SHAFTO
      IF(NN.EQ.0) GO TO 310
      WRITE(6,700)
      DO 309 I=1,NN
      NNNNN=NVAR(I)
      GO TO (301,302,303,304,305,306,307,308),NNNNN
```

```
301 WRITE(6,701)I
    GO TO 309
302 WRITE(6,702)I
    GO TO 309
303 WRITE(6,703)I
    GO TO 309
304 WRITE(6,704)I
    GO TO 309
305 WRITE(6,706)I
    GO TO 309
306 WRITE(6,705)I
    GO TO 309
307 WRITE(6,707)I
    GO TO 309
308 WRITE(6,708)I
309 CONTINUE
310 IF(NN.EQ.0)NOPT=1
    RETURN
500 FORMAT(1H1,9X,29HSPUR GEAR DESIGN...INPUT DATA/10X,29(1H-)/10X,
   129(1H-))
501 FORMAT(///10X,18HPOWER REQUIREMENTS/10X,18(1H-)//)
502 FORMAT(10X,49HHORSEPOWER TRANSMITTED. . . . . . . . .      HP  =,
   112X,E16.8/)
503 FORMAT(10X,49HINPUT  SPEED (RPM). . . . . . . . . . .    RPMI  =,
   112X,E16.8/)
504 FORMAT(10X,49HOUTPUT SPEED (RPM). . . . . . . . . . .    RPMO  =,
   112X,E16.8/)
505 FORMAT(///10X,24HGEAR MATERIAL PROPERTIES,31X,6HPINION,19X,
   14HGEAR/10X,24(1H-),31X,6(1H-),19X,4(1H-)//)
506 FORMAT(10X,49HMAXIMUM ALLOWABLE FATIGUE STRESS (PSI).   SAF  =
   1E16.8,8X,E16.8/)
507 FORMAT(10X,29HMAXIMUM ALLOWABLE COMPRESSIVE/10X,49HSTRESS (PSI). .
   1 . . . . . . . . . . . . .           SAC  =,E16.8,8X,E16.8/)
508 FORMAT(10X,49HMODULUS OF ELASTICITY (PSI) . . . . . .       E  =,
   1E16.8,8X,E16.8/)
509 FORMAT(10X,49HPOISSONS RATIO. . . . . . . . . . . . .       U  =,
   1E16.8,8X,E16.8/)
510 FORMAT(10X,49HDENSITY (LBS/CU. IN.) . . . . . . . . .     RHO  =,
   1E16.8,8X,E16.8/)
511 FORMAT(10X,49HBRINELL HARDNESS. . . . . . . . . . . .     BHN  =,
   1E16.8,8X,E16.8/)
600 FORMAT(///10X,12HINPUT LAYOUT,43X,6HPINION,19X,4HGEAR/10X,12(1H-),
   143X,6(1H-),19X,4(1H-)//)
601 FORMAT(10X,61HPRESSURE ANGLE (DEGREES). . . . . . . .     PAD  =
   1            ,E16.8/)
602 FORMAT(10X,74HPRESSURE ANGLE (DEGREES). . . . . . . .     PAD  =
   1                 VARIABLE/)
603 FORMAT(10X,49HNUMBER OF TEETH . . . . . . . . . . . .   TEETH  =
   1E16.8,8X,E16.8/)
604 FORMAT(10X,74HNUMBER OF TEETH . . . . . . . . . . . .   TEETH  =
   1                 VARIABLE/)
605 FORMAT(10X,61HCENTRE DISTANCE (INCHES). . . . . . . .      CD  =
   1            ,E16.8/)
606 FORMAT(10X,74HCENTRE DISTANCE (INCHES). . . . . . . .      CD  =
   1                 VARIABLE/)
```

```
607 FORMAT(10X,61HFACE WIDTH (INCHES) . . . . . . . . . .      FW   =
   1              ,E16.8/)
608 FORMAT(10X,74HFACE WIDTH (INCHES) . . . . . . . . . .      FW   =
   1                      VARIABLE/)
609 FORMAT(10X,49HADDENDUM (INCHES) . . . . . . . . . . .     ADD  =)
610 FORMAT(10X,49HDEDENDUM (INCHES) . . . . . . . . . . .     DED  =)
611 FORMAT(1H+,63X,8HVARIABLE)
612 FORMAT(1H+,87X,8HVARIABLE)
613 FORMAT(1H+,64X,6H1.0/DP)
614 FORMAT(1H+,88X,6H1.0/DP)
615 FORMAT(1H+,63X,8H1.250/DP)
616 FORMAT(1H+,87X,8H1.250/DP)
617 FORMAT(1H+,63X,F7.4,3H/DP)
618 FORMAT(1H+,86X,F7.4,3H/DP)
619 FORMAT(1H+,58X,E16.8)
620 FORMAT(1H+,82X,E16.8)
650 FORMAT(1H1,9X,18HINPUT REQUIREMENTS/10X,18(1H-)//10X,88(1H*))
651 FORMAT(10X,49HREQUIRED NUMBER OF LOADING CYCLES . . .  CYCLE  =
   112X,E16.8/)
652 FORMAT(10X,49HREQUIRED RELIABILITY. . . . . . . . . .   RELI   =
   112X,E16.8/)
653 FORMAT(10X,49HGEAR BLANK TEMPERATURE (DEGREES F). . .   TEMP   =
   112X,E16.8/)
654 FORMAT(10X,87HMODE OF LOADING . . . . . . . . . . . . . DRIVEN  =  1
   1.0 LOAD ON DRIVEN MACHINE - UNIFORM/58X,40H=  2.0 LOAD ON DRIVEN M
   2ACHINE - MODERATE/58X,37H=  3.0 LOAD ON DRIVEN MACHINE - HEAVY/10X
   3,77HMODE OF POWER SOURCE. . . . . . . . . . . DRIVER  =  1.0 POWER S
   4OURCE - UNIFORM/58X,33H=  2.0 POWER SOURCE - LIGHT SHOCK/58X,34H=
   5 3.0 POWER SOURCE - MEDIUM SHOCK/)
655 FORMAT(10X,49HACTUAL MODE OF LOADING. . . . . . . . . . DRIVEN  =
   112X,E16.8//10X,49HACTUAL MODE OF POWER SOURCE . . . . . . DRIVER
   2=,12X,E16.8/)
656 FORMAT(10X,83HCUTTING TOOL TYPE . . . . . . . . . . . .  NCUT   =  1
   1 IF GEAR CUT BY RACK WITH SHARP/63X,14HCORNERED TEETH/58X,37H=  2
   2IF GEAR CUT BY RACK WITH ROUNDED/63X,7HCORNERS/)
657 FORMAT(10X,33HRATIO OF PINION TOOTH THINNING TO/10X,49HTOTAL TOOTH
   1 THINNING. . . . . . . . . .      BLR   =,12X,E16.8/)
658 FORMAT(10X,49HMINIMUM BACKLASH MODIFICATION FACTOR. .     BLL    =
   112X,E16.8/)
659 FORMAT(10X,49HMAXIMUM BACKLASH MODIFICATION FACTOR. .     BLU    =
   112X,E16.8/)
660 FORMAT(10X,25HCENTRE DISTANCE TOLERANCE/10X,49HMODIFICATION FACTOR
   1 . . . . . . . . . .        CDR   =,12X,E16.8/)
661 FORMAT(10X,49HCUTTING TOOL TYPE A) PINION . . . . . .  NCUT1   =
   112X,I3,//28X,31HB) GEAR    . . . . . .   NCUT2   =,12X,I3/)
662 FORMAT(10X,49HA.G.M.A. QUALITY NUMBER . . . . . . . .  NQUAL   =
   112X,I3/)
663 FORMAT(10X,49HLOAD LOCATION MODE. . . . . . . . . . .  NLOAD   =
   112X,I3/)
664 FORMAT(10X,75HMODE OF LOAD LOCATION ON TOOTH. . . . .  NLOAD   =  0
   1 FOR PROGRAM DETERMINED,/58X,26H=  1 FOR TOOTH TIP LOADING,/58X,
   238H=  2 FOR POINT OF HIGHEST SINGLE TOOTH,/63X,15HCONTACT LOADING,
   3/10X,88(1H*)//)
675 FORMAT(1H1,9X,21HUSER SIZE LIMITATIONS/10X,21(1H-)//)
676 FORMAT(10X,49HMAXIMUM CENTRE DISTANCE (INCHES). . . .  CDMAX   =
```

```
     112X,E16.8/)
 677 FORMAT(10X,49HMINIMUM CENTRE DISTANCE (INCHES). . . .  COMIN   =
     112X,E16.8/)
 678 FORMAT(10X,49HMAXIMUM FACE WIDTH (INCHES) . . . . . .  FWMAX   =
     112X,E16.8/)
 679 FORMAT(10X,49HMINIMUM FACE WIDTH (INCHES) . . . . . .  FWMIN   =
     112X,E16.8/)
 680 FORMAT(10X,49HINPUT SHAFT DIAMETER. . . . . . . . . . SHAFTI   =
     112X,E16.8/)
 681 FORMAT(10X,49HOUTPUT SHAFT DIAMETER . . . . . . . . . SHAFTO   =
     112X,E16.8/)
 682 FORMAT(10X,49HMAXIMUM PRESSURE ANGLE (DEGREES). . . . PADMAX   =
     112X,E16.8/)
 683 FORMAT(10X,49HMINIMUM PRESSURE ANGLE (DEGREES). . . . PADMIN   =
     112X,E16.8/)
 700 FORMAT(///10X,32HVARIABLE LIST AND INITIAL VALUES/10X,32(1H-)//)
 701 FORMAT(10X,49HPINION ADDENDUM (INCHES). . . . . . . . .   ADD1  =
     114X,2HX(,I2,1H))
 702 FORMAT(10X,49HGEAR   ADDENDUM (INCHES). . . . . . . . .   ADD2  =
     114X,2HX(,I2,1H))
 703 FORMAT(10X,49HPINION DEDENDUM (INCHES). . . . . . . . .   DED1  =
     114X,2HX(,I2,1H))
 704 FORMAT(10X,49HGEAR   DEDENDUM (INCHES). . . . . . . . .   DED2  =
     114X,2HX(,I2,1H))
 705 FORMAT(10X,49HCENTRE DISTANCE (INCHES). . . . . . . . .     CD  =
     114X,2HX(,I2,1H))
 706 FORMAT(10X,49HFACE WIDTH (INCHES) . . . . . . . . . . .     FW  =
     114X,2HX(,I2,1H))
 707 FORMAT(10X,49HPRESSURE ANGLE (DEGREES). . . . . . . . .    PAD  =
     114X,2HX(,I2,1H))
 708 FORMAT(10X,49HNUMBER OF PINION TEETH. . . . . . . . . . TEETH1  =
     114X,2HX(,I2,1H))
      END
```

```
      SUBROUTINE VARY(X)
      DIMENSION X(1)
      COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
      COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
      COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
      COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
      COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
      COMMON/BLK3B/BLL,BLU,BLR,CDR
      COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
      COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
      COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
      COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
      COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
      COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
      COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
      COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
      COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
      COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
```

```
      COMMON/BLK9 /COD,CODL1,CODL2,QCD,QODL1,QODL2
      COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
      COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
      COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
      COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
      COMMON/BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
      COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
      COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
      COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
      COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
      IF(NN.EQ.0) GO TO 99
      DO  9 I=1,NN
      NNNNN=NVAR(I)
      GO TO (1,2,3,4,5,6,7,8),NNNNN
    1 ADD1=ABS(X(I))
      GO TO 9
    2 ADD2=ABS(X(I))
      GO TO 9
    3 DED1=ABS(X(I))
      GO TO 9
    4 DED2=ABS(X(I))
      GO TO 9
    5 FW=ABS(X(I))
      GO TO 9
    6 CD=ABS(X(I))
      GO TO 9
    7 PAD=ABS(X(I))
      PAR=(PAD/180.0)*PI
      GO TO 9
    8 TEETH1=ABS(X(I))
    9 CONTINUE
   99 CALL PITCH (RATIO,CD,TEETH1,TEETH2,RPM1,PAR,PI,PR1,PR2,RB1,RB2,BP,
     1CP,DP,PLV)
      IF(J.EQ.0) GO TO 21
      DO 20 I=1,J
      NNNNN=NSTD(I)
      GO TO (10,11,12,14,16,17,18,19),NNNNN
   10 ADD1=1.0/DP
      GO TO 20
   11 ADD2=1.0/DP
      GO TO 20
   12 IF(DP.GT.20.0) GO TO 13
      DED1=1.250/DP
      GO TO 20
   13 DED1=(1.200/DP)+0.002
      GO TO 20
   14 IF(DP.GT.20.0) GO TO 15
      DED2=1.250/DP
      GO TO 20
   15 DED2=(1.200/DP)+0.002
      GO TO 20
   16 ADD1=ADDK1/DP
      GO TO 20
   17 ADD2=ADDK2/DP
      GO TO 20
```

```
 18 DED1=DEDK1/DP
    GO TO 20
 19 DED2=DEDK2/DP
 20 CONTINUE
 21 IF(ADD1.GT.DED2) ADD1=DED2
    IF(ADD2.GT.DED1) ADD2=DED1
    RI1=PR1-DED1
    RI2=PR2-DED2
    RO1=PR1+ADD1
    RO2=PR2+ADD2
    CCC1=DED1-ADD2
    CCC2=DED2-ADD1
    RETURN
    END
```

```
      SUBROUTINE PRINT
C
C     THIS ROUTINE PRINTS SPUR GEAR DESIGN OUTPUT IN A STANDARD FORMAT
C
      COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
      COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
      COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
      COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
      COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
      COMMON/BLK3B/BLL,BLU,BLR,CDR
      COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
      COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
      COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
      COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
      COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
      COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
      COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
      COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
      COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
      COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
      COMMON/BLK9 /COD,CODL1,CODL2,QOD,QODL1,QODL2
      COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
      COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
      COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
      COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
      COMMON/BLK12 /X( 8),XSTRT( 8),RMAX( 8),RMIN( 8),PHI(25),PSI( 1)
      COMMON/BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
      COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
      COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
      COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
      COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
C
C
      IF(IWRITE.EQ.0)  RETURN
      WRITE(6,800)
      WRITE(6,801)PAD
      WRITE(6,802)CD
      WRITE(6,803)FW
      WRITE(6,804)TEETH1,TEETH2
      WRITE(6,805)ADD1,ADD2
      WRITE(6,806)DED1,DED2
      WRITE(6,842)CCC1,CCC2
      WRITE(6,807)PR1,PR2
      WRITE(6,808)RB1,RB2
      WRITE(6,809)RO1,RO2
      WRITE(6,810)RI1,RI2
      WRITE(6,811)RM1,RM2
      WRITE(6,812)RU1,RU2
      WRITE(6,844)TP1,TP2
      WRITE(6,843)TO1,TO2
      WRITE(6,813)BP
      WRITE(6,814)CP
      WRITE(6,815)DP
      WRITE(6,816)EFF
      WRITE(6,817)RATIO
```

```
      WRITE(6,818)CRATIO
      WRITE(6,820)
      WRITE(6,821)HP
      WRITE(6,845)PAB1,PAB2
      WRITE(6,846)PAW1,PAW2
      WRITE(6,841)WN
      WRITE(6,822)WT
      WRITE(6,823)WR
      WRITE(6,824)WA
      WRITE(6,825)PLV
      WRITE(6,826)RPM1,RPM2
      WRITE(6,827)TORQ1,TORQ2
      WRITE(6,828)ANGL1,ANGL2
      WRITE(6,829)RL1,RL2
      GO TO (1,2),NNLOAD
    1 WRITE(6,850)
      GO TO 3
    2 WRITE(6,851)
    3 WRITE(6,830)
      WRITE(6,831)E1,E2
      WRITE(6,832)U1,U2
      WRITE(6,833)RHO1,RHO2
      WRITE(6,834)BHN1,BHN2
      WRITE(6,835)SAF1,SAF2
      WRITE(6,836)SBM1,SBM2
      WRITE(6,837)SB1,SB2
      WRITE(6,838)SAC1,SAC2
      WRITE(6,839)SSM1,SSM2
      WRITE(6,840)SS1,SS2
      WRITE(6,900)
      WRITE(6,901)QJ1
      WRITE(6,902)QJ2
      WRITE(6,903)CJ
      WRITE(6,904)CE
      WRITE(6,905)CF
      WRITE(6,906)CM,QM
      WRITE(6,907)CO,QO
      WRITE(6,908)CS,QS
      WRITE(6,909)CV,QV
      WRITE(6,910)COD,QOD
      WRITE(6,911)CH
      WRITE(6,912)CL1,QL1
      WRITE(6,913)CL2,QL2
      WRITE(6,914)CR,QR
      WRITE(6,915)CT,QT
      WRITE(6,916)CODL1,QODL1
      WRITE(6,917)CODL2,QODL2
      WRITE(6,870)
      WRITE(6,871)BLMIN
      WRITE(6,872)BLMAX
      WRITE(6,873)BLMINT
      WRITE(6,874)BLMAXT
      WRITE(6,875)BLMAXU
      WRITE(6,876)BL1,BL2
      WRITE(6,1000)
```

```
      WRITE(6,1001)TOLR1,TOLR2
      WRITE(6,1002)TOLP1,TOLP2
      WRITE(6,1003)PTOL1,PTOL2
      WRITE(6,1009)TOLL1,TOLL2
      WRITE(6,1004)TTCT1,TTCT2
      WRITE(6,1005)TCT1,TCT2
      WRITE(6,1006)TPTU1,TPTU2,TPTL1,TPTL2
      WRITE(6,1010)TPTV1,TPTV2
      WRITE(6,1011)TPTE1,TPTE2
      WRITE(6,1007)CDTOLU,CDTOLL
      WRITE(6,1008)ERR
      WRITE(6,860)
      WRITE(6,861)HUBL1,HUBL2
      WRITE(6,862)HUBR1,HUBR2
      WRITE(6,863)RIM1,RIM2
      WRITE(6,864)WEB1,WEB2
      WRITE(6,865)VOL1,VOL2
C
      RETURN
  800 FORMAT(1H0,9X,11HGEAR LAYOUT,44X,6HPINION,19X,4HGEAR,/10X,11(1H-),
     144X,6(1H-),19X,4(1H-)//)
  801 FORMAT(10X,49HPRESSURE ANGLE (DEGREES). . . . . . . . .    PAD   =
     112X,E16.8/)
  802 FORMAT(10X,49HCENTRE DISTANCE (INCHES). . . . . . . . .     CD   =
     112X,E16.8/)
  803 FORMAT(10X,49HFACE WIDTH (INCHES) . . . . . . . . . . .     FW   =
     112X,E16.8/)
  804 FORMAT(10X,49HNUMBER OF TEETH . . . . . . . . . . . . .  TEETH   =
     1E16.8,8X,E16.8/)
  805 FORMAT(10X,49HADDENDUM (INCHES) . . . . . . . . . . . .    ADD   =
     1E16.8,8X,E16.8/)
  806 FORMAT(10X,49HDEDENDUM (INCHES) . . . . . . . . . . . .    DED   =
     1E16.8,8X,E16.8/)
  807 FORMAT(10X,49HPITCH CIRCLE RADIUS (INCHES). . . . . .       PR   =
     1E16.8,8X,E16.8/)
  808 FORMAT(10X,49HBASE CIRCLE RADIUS (INCHES) . . . . . .       RB   =
     1E16.8,8X,E16.8/)
  809 FORMAT(10X,49HADDENDUM CIRCLE RADIUS (INCHES) . . . .       RO   =
     1E16.8,8X,E16.8/)
  810 FORMAT(10X,49HDEDENDUM CIRCLE RADIUS (INCHES) . . . .       RI   =
     1E16.8,8X,E16.8/)
  811 FORMAT(10X,30HMAXIMUM ADDENDUM CIRCLE RADIUS/10X,49HBEFORE INTERFE
     1RENCE OCCURS (INCHES) . .         RM   =,E16.8,8X,E16.8/)
  812 FORMAT(10X,49HUNDERCUT LIMIT RADIUS (INCHES). . . . .       RU   =
     1E16.8,8X,E16.8/)
  813 FORMAT(///10X,49HBASE PITCH (INCHES) . . . . . . . . . . .      BP
     1=,12X,E16.8/)
  814 FORMAT(10X,49HCIRCULAR PITCH (INCHES) . . . . . . . . .     CP   =
     112X,E16.8/)
  815 FORMAT(10X,49HDIAMETRAL PITCH (TEETH/INCH). . . . . . .     DP   =
     112X,E16.8/)
  816 FORMAT(10X,49HEFFICIENCY. . . . . . . . . . . . . . . .    EFF   =
     112X,E16.8/)
  817 FORMAT(10X,49HGEAR RATIO (GEAR TEETH /PINION TEETH) .  RATIO   =
     112X,E16.8/)
```

```
818 FORMAT(10X,49HCONTACT RATIO . . . . . . . . . . . . . . CRATIO  =
   112X,E16.8/)
820 FORMAT(1H1,9X,16HLOADING ANALYSIS,39X,6HPINION,19X,4HGEAR,/10X,
   116(1H-),39X,6(1H-),19X,4(1H-)//)
821 FORMAT(10X,49HHORSEPOWER TRANSMITTED. . . . . . . . .      HP  =
   112X,E16.8/)
822 FORMAT(10X,49HTANGENTIAL LOAD (LBS) . . . . . . . . .      WT  =
   112X,E16.8/)
823 FORMAT(10X,49HRADIAL LOAD (LBS) . . . . . . . . . . .      WR  =
   112X,E16.8/)
824 FORMAT(10X,49HAXIAL LOAD (LBS). . . . . . . . . . . .      WA  =
   112X,E16.8/)
825 FORMAT(10X,49HPITCH LINE VELOCITY (FPM) . . . . . . .     PLV  =
   112X,E16.8/)
826 FORMAT(10X,49HSHAFT SPEED (RPM) . . . . . . . . . . .     RPM  =
   1E16.8,8X,E16.8/)
827 FORMAT(10X,49HTORQUE (FT-LBS) . . . . . . . . . . . .    TORQ  =
   1E16.8,8X,E16.8/)
828 FORMAT(10X,49HLOAD ANGLE (RADIANS). . . . . . . . . .    ANGL  =
   1E16.8,8X,E16.8/)
829 FORMAT(10X,23HRADIUS TO LOAD ON TOOTH/10X,49HCENTRELINE (INCHES) .
   1 . . . . . . . . .          RL   =,E16.8,8X,E16.8//)
830 FORMAT  (//10X,39HMATERIAL PROPERTIES AND STRESS ANALYSIS,16X,6HPI
   1NION,19X,4HGEAR,/10X,39(1H-),16X,6(1H-),19X,4(1H-)//)
831 FORMAT(10X,49HMODULUS OF ELASTICITY (PSI) . . . . . .       E  =
   1E16.8,8X,E16.8/)
832 FORMAT(10X,49HPOISSONS RATIO. . . . . . . . . . . . .       U  =
   1E16.8,8X,E16.8/)
833 FORMAT(10X,49HDENSITY (LBS/CU. IN.) . . . . . . . . .     RHO  =
   1E16.8,8X,E16.8/)
834 FORMAT(10X,49HBRINELL HARDNESS. . . . . . . . . . . .     BHN  =
   1E16.8,8X,E16.8/)
835 FORMAT(10X,49HMAXIMUM ALLOWABLE FATIGUE STRESS (PSI).     SAF  =
   1E16.8,8X,E16.8/)
836 FORMAT(10X,49HMAXIMUM ALLOWABLE BENDING STRESS (PSI).     SBM  =
   1E16.8,8X,E16.8/)
837 FORMAT(10X,49HACTUAL BENDING STRESS (PSI) . . . . . .      SB  =
   1E16.8,8X,E16.8/)
838 FORMAT(10X,29HMAXIMUM ALLOWABLE COMPRESSIVE/10X,49HSTRESS (PSI). .
   1 . . . . . . . . . . . .        SAC  =,E16.8,8X,E16.8/)
839 FORMAT(10X,49HMAXIMUM ALLOWABLE WEAR STRESS (PSI) . .     SSM  =
   1E16.8,8X,E16.8/)
840 FORMAT(10X,49HACTUAL WEAR STRESS (PSI). . . . . . . .      SS  =
   1E16.8,8X,E16.8/)
841 FORMAT(10X,49HNORMAL LOAD (LBS) . . . . . . . . . . .      WN  =
   112X,E16.8/)
842 FORMAT(10X,49HCLEARANCE (INCHES). . . . . . . . . . .     CCC  =
   1E16.8,8X,E16.8/)
843 FORMAT(10X,27HTOOTH THICKNESS AT ADDENDUM/10X,49HCIRCLE (INCHES) .
   1 . . . . . . . . . . .       TO  =,E16.8,8X,E16.8/)
844 FORMAT(10X,32HMINIMUM CIRCULAR TOOTH THICKNESS/10X,49HAT PITCH CIR
   1CLE (INCHES). . . . . . . . .        TP  =,E16.8,8X,E16.8/)
845 FORMAT(10X,49HMAXIMUM ALLOWABLE POWER...BENDING (HP).     PAB  =
   1E16.8,8X,E16.8/)
846 FORMAT(10X,49HMAXIMUM ALLOWABLE POWER...WEAR (HP) . .     PAW  =
```

```
     1E16.8,8X,E16.8/)
850  FORMAT(10X,38H***LOADING ANALYSIS FOR TIP LOADING***/)
851  FORMAT(10X,72H***LOADING ANALYSIS FOR POINT OF HIGHEST SINGLE TOOT
     1H CONTACT LOADING***/)
860  FORMAT(1H1,9X,21HGEAR BLANK DIMENSIONS,34X,6HPINION,19X,4HGEAR,
     1/10X,21(1H-),34X,6(1H-),19X,4(1H-)//)
861  FORMAT(10X,49HHUB LENGTH (INCHES) . . . . . . . . . . .   HUBL   =
     1E16.8,8X,E16.8/)
862  FORMAT(10X,49HOUTER HUB RADIUS (INCHES) . . . . . . . .   HUBR   =
     1E16.8,8X,E16.8/)
863  FORMAT(10X,49HINNER RIM RADIUS (INCHES) . . . . . . . .    RIM   =
     1E16.8,8X,E16.8/)
864  FORMAT(10X,49HWEB THICKNESS (INCHES). . . . . . . . . .    WEB   =
     1E16.8,8X,E16.8/)
865  FORMAT(10X,49HGEAR BLANK VOLUME (CUBIC INCHES). . . .      VOL   =
     1E16.8,8X,E16.8/)
870  FORMAT(1H1,9X,17HBACKLASH ANALYSIS/10X,17(1H-)/)
871  FORMAT(10X,36HDESIRED MINIMUM BACKLASH AT STANDARD/10X,49HCENTRE D
     1ISTANCE (INCHES). . . . . . . . .   BLMIN  =,12X,E16.8/)
872  FORMAT(10X,36HDESIRED MAXIMUM BACKLASH AT STANDARD/10X,49HCENTRE D
     1ISTANCE (INCHES). . . . . . . . .   BLMAX  =,12X,E16.8/)
873  FORMAT(10X,35HACTUAL MINIMUM BACKLASH AT STANDARD/10X,49HCENTRE DI
     1STANCE (INCHES). . . . . . . . BLMINT  =,12X,E16.8/)
874  FORMAT(10X,35HACTUAL MAXIMUM BACKLASH AT STANDARD/10X,49HCENTRE DI
     1STANCE (INCHES). . . . . . . . BLMAXT  =,12X,E16.8/)
875  FORMAT(10X,35HMAXIMUM BACKLASH AT CENTRE DISTANCE/10X,49HTOLERANCE
     1 LIMIT (INCHES). . . . . . . . BLMAXU  =,12X,E16.8/)
876  FORMAT(10X,35HMAXIMUM TOOTH THINNING FOR BACKLASH/10X,49HINCLUDING
     1 MACHINING TOLERANCE (INCHES).      BL  =,2X,E16.8,6X,E16.8/)
900  FORMAT(1H1,9X,20HMODIFICATION FACTORS/10X,20(1H-),//,10X,42HC-FACT
     1ORS EMPLOYED IN WEAR STRESS ANALYSIS/10X,45HQ-FACTORS EMPLOYED IN
     2BENDING STRESS ANALYSIS///)
901  FORMAT(10X,32HBENDING ANALYSIS GEOMETRY FACTOR/10X,49HFOR THE PINI
     1ON. . . . . . . . . . . . . . .     QJ1  =,12X,E16.8/)
902  FORMAT(10X,32HBENDING ANALYSIS GEOMETRY FACTOR/10X,49HFOR THE GEAR
     1. . . . . . . . . . . . . . . .     QJ2  =,12X,E16.8/)
903  FORMAT(10X,49HWEAR ANALYSIS GEOMETRY FACTOR . . . . . .      CJ   =
     112X,E16.8/)
904  FORMAT(10X,49HELASTIC COEFFICIENT FACTOR. . . . . . . .      CE   =
     112X,E16.8/)
905  FORMAT(10X,49HSURFACE CONDITION FACTOR. . . . . . . . .      CF   =
     112X,E16.8/)
906  FORMAT(10X,49HLOAD DISTRIBUTION CORRECTION FACTOR . .    CM,QM   =
     1E16.8,8X,E16.8/)
907  FORMAT(10X,49HOVERLOAD CORRECTION FACTOR. . . . . . . .  CO,QO   =
     1E16.8,8X,E16.8/)
908  FORMAT(10X,49HSIZE CORRECTION FACTOR. . . . . . . . . .  CS,QS   =
     1E16.8,8X,E16.8/)
909  FORMAT(10X,49HVELOCITY CORRECTION FACTOR. . . . . . . .  CV,QV   =
     1E16.8,8X,E16.8//)
910  FORMAT(10X,49HOVERALL DERATING FACTOR . . . . . . . . .COD,QOD   =
     1E16.8,8X,E16.8///)
911  FORMAT(10X,49HHARDNESS RATIO FACTOR . . . . . . . . . .      CH   =
     112X,E16.8/)
912  FORMAT(10X,22HLIFE CORRECTION FACTOR/10X,49HFOR THE PINION. . . .
```

```
     1. . . . . . . . .CL1,QL1  =,E16.8,8X,E16.8/)
 913 FORMAT(10X,22HLIFE CORRECTION FACTOR/10X,49HFOR THE GEAR. . . .
     1. . . . . . . . .CL2,QL2  =,E16.8,8X,E16.8/)
 914 FORMAT(10X,49HRELIABILITY CORRECTION FACTOR . . . . .  CR,QR   =
     1E16.8,8X,E16.8/)
 915 FORMAT(10X,49HTEMPERATURE CORRECTION FACTOR . . . . .  CT,QT   =
     1E16.8,8X,E16.8//)
 916 FORMAT(10X,28HOVERALL LIFE DERATING FACTOR/10X,49HFOR THE PINION.
     1. . . . . . . . . .CODL1,QODL1  =,E16.8,8X,E16.8/)
 917 FORMAT(10X,28HOVERALL LIFE DERATING FACTOR/10X,49HFOR THE GEAR. .
     1. . . . . . . . . .CODL2,QODL2  =,E16.8,8X,E16.8/)
1000 FORMAT(1H0,9X,10HTOLERANCES,47X,6HPINION,17X,4HGEAR,/10X,10(1H-),
     147X,6(1H-),17X,4(1H-)//)
1001 FORMAT(10X,51HRUNOUT TOLERANCE (INCHES) . . . . . . .   TOLR   =
     1E16.8,6X,E16.8/)
1002 FORMAT(10X,51HPITCH TOLERANCE (INCHES). . . . . . . .   TOLP   =
     1E16.8,6X,E16.8/)
1003 FORMAT(10X,51HPROFILE TOLERANCE (INCHES). . . . . . .   PTOL   =
     1E16.8,6X,E16.8/)
1004 FORMAT(10X,24HTOOTH TO TOOTH COMPOSITE/10X,51HTOLERANCE (INCHES).
     1. . . . . . . . . .    TTCT   =   ,E16.8,6X,E16.8/)
1005 FORMAT(10X,51HTOTAL COMPOSITE TOLERANCE (INCHES). . .    TCT   =
     1E16.8,6X,E16.8/)
1006 FORMAT(10X,51HTOOTH THICKNESS TOLERANCE (INCHES). . .   TPTU   = +
     1E16.8,5X,1H+,E16.8//52X,9HTPTL   = -,E16.8,5X,1H-,E16.8/)
1007 FORMAT(10X,49HCENTRE DISTANCE TOLERANCE (INCHES). . . CDTOLU   =
     112X,1H+,E16.8//50X,9HCDTOLL   =,12X,1H-,E16.8/)
1008 FORMAT(10X,49HERROR IN ACTION . . . . . . . . . . . .    ERR   =
     113X,E16.8/)
1009 FORMAT(10X,51HLEAD TOLERANCE (INCHES) . . . . . . . .   TOLL   =
     1E16.8,6X,E16.8/)
1010 FORMAT(10X,36HTOOTH THICKNESS VARIATION FROM TOOTH/10X,51HELEMENT
     1ERRORS (INCHES) . . . . . . . .    TPTV   =   E16.8,6X,E16.8/)
1011 FORMAT(10X,36HTOOTH THICKNESS VARIATION FROM TOOTH/10X,51HELEMENT
     1ERRORS AND RUNOUT (INCHES). . .    TPTE   =   E16.8,6X,E16.8/)
      END


      SUBROUTINE HINT(PHI)
C
C     THIS ROUTINE PRINTS OUT SUGGESTED REMEDIES FOR VIOLATED
C     CONSTRAINTS
C     IF CONSTRAINT VIOLATED NNN=1 AND SUGGESTIONS WILL BE PRINTED AFTER
C     EACH SET OF CONSTRAINTS CHECKED
C
      DIMENSION PHI(1)
C
      WRITE(6,1)
      NNN=0
      DO 50 I=1,4
      IF(PHI(I).GE.0.0) GO TO 50
      NNN=1
      NNNNN=1
```

```
      GO TO (10,20,30,40),NNNNN
   10 WRITE(6,11)
      GO TO 50
   20 WRITE(6,21)
      GO TO 50
   30 WRITE(6,31)
      GO TO 50
   40 WRITE(6,41)
   50 CONTINUE
      IF(NNN.EQ.1) WRITE(6,51)
      NNN=0
      DO 80 I=5,6
      IF(PHI(I).GE.0.0) GO TO 80
      NNN=1
      NNNNN=I-4
      GO TO (60,70),NNNNN
   60 WRITE(6,61)
      GO TO 80
   70 WRITE(6,71)
   80 CONTINUE
      IF(NNN.EQ.1) WRITE(6,81)
      NNN=0
      DO 110 I=7,8
      IF(PHI(I).GE.0.0) GO TO 110
      NNN=1
      NNNNN=I-6
      GO TO (90,100),NNNNN
   90 WRITE(6,91)
      GO TO 110
  100 WRITE(6,101)
  110 CONTINUE
      IF(NNN.EQ.1) WRITE(6,111)
      NNN=0
      DO 140 I=9,10
      IF(PHI(I).GE.0.0) GO TO 140
      NNN=1
      NNNNN=I-8
      GO TO (120,130),NNNNN
  120 WRITE(6,121)
      GO TO 140
  130 WRITE(6,131)
  140 CONTINUE
      IF(NNN.EQ.1) WRITE(6,141)
      NNN=0
      DO 170 I=11,12
      IF(PHI(I).GE.0.0) GO TO 170
      NNN=1
      NNNNN=I-10
      GO TO (150,160),NNNNN
  150 WRITE(6,151)
      GO TO 170
  160 WRITE(6,161)
  170 CONTINUE
      IF(NNN.EQ.1) WRITE(6,171)
      NNN=0
```

```
      DO 220 I=13,16
      IF(PHI(I).GE.0.0) GO TO 220
      NNN=1
      NNNNN=I-12
      GO TO (180,190,200,210),NNNNN
  180 WRITE(6,181)
      GO TO 220
  190 WRITE(6,191)
      GO TO 220
  200 WRITE(6,201)
      GO TO 220
  210 WRITE(6,211)
  220 CONTINUE
      IF(NNN.EQ.1) WRITE(6,221)
      NNN=0
      DO 250 I=18,19
      IF(PHI(I).GE.0.0) GO TO 250
      NNN=1
      NNNNN=I-17
      GO TO (230,240),NNNNN
  230 WRITE(6,231)
      GO TO 250
  240 WRITE(6,241)
  250 CONTINUE
      IF(NNN.EQ.1) WRITE(6,251)
      NNN=0
      DO 290 I=19,21
      IF(PHI(I).GE.0.0) GO TO 290
      NNN=1
      NNNNN=I-18
      GO TO (260,270,280),NNNNN
  260 WRITE(6,261)
      GO TO 290
  270 WRITE(6,271)
      GO TO 290
  280 WRITE(6,281)
  290 CONTINUE
      IF(NNN.EQ.1) WRITE(6,291)
      NNN=0
      DO 320 I=22,23
      IF(PHI(I).GE.0.0) GO TO 320
      NNN=1
      NNNNN=I-21
      GO TO (300,310),NNNNN
  300 WRITE(6, 301)
      GO TO 320
  310 WRITE(6,311)
  320 CONTINUE
      IF(NNN.EQ.1) WRITE(6,321)
      NNN=0
      DO 350 I=24,25
      IF(PHI(I).GE.0.0) GO TO 350
      NNN=1
      NNNNN=I-23
      GO TO (330,340),NNNNN
```

```
330 WRITE(6,331)
    GO TO 350
340 WRITE(6,341)
350 CONTINUE
    IF(NNN.EQ.1) WRITE(6,351)
    NNN=0
    RETURN
  1 FORMAT(///10X,30HVIOLATED CONSTRAINT EVALUATION/10X,30(1H-)//10X,
   142HFOR MORE COMPLETE EXPLANATION...SEE MANUAL//)
 11 FORMAT(10X,50HPHI( 1)  PINION BENDING STRESS CONSTRAINT VIOLATED/)
 21 FORMAT(10X,50HPHI( 2)  GEAR   BENDING STRESS CONSTRAINT VIOLATED/)
 31 FORMAT(10X,47HPHI( 3)  PINION WEAR STRESS CONSTRAINT VIOLATED/)
 41 FORMAT(10X,47HPHI( 4)  GEAR   WEAR STRESS CONSTRAINT VIOLATED/)
 51 FORMAT(/19X,40HHORSEPOWER TOO LARGE FOR TRANSMISSION ON/19X,41HGIV
   1EN CENTRE DISTANCE AND FACE WIDTH WITH/19X,25HGIVEN MATERIAL PROPE
   2RTIES.//19X,42HTRY  A) LARGER CENTRE DISTANCE AND/OR FACE/27X,35HW
   3IDTH...IF VALUES ALREADY VARIABLE,/27X,30HINCREASE THEIR LIMITING
   4VALUES/24X,30HB) IMPROVE MATERIAL PROPERTIES//)
 61 FORMAT(10X,24HPHI( 5)  PINION UNDERCUT/)
 71 FORMAT(10X,24HPHI( 6)  GEAR   UNDERCUT/)
 81 FORMAT(/19X,39HTRY  A) LARGER RATIO OF NUMBER OF TEETH/27X,22HVERS
   1US CENTRE DISTANCE/24X,34HB) SMALLER DEDENDUM SIZE CRITERION//)
 91 FORMAT(10X,44HPHI( 7)  PINION TEETH INTERFERENCE WITH GEAR/)
101 FORMAT(10X,46HPHI( 8)  GEAR   TEETH INTERFERENCE WITH PINION/)
111 FORMAT(/19X,39HTRY  A) LARGER RATIO OF NUMBER OF TEETH/27X,22HVERS
   1US CENTRE DISTANCE/24X,34HB) SMALLER ADDENDUM SIZE CRITERION/24X,
   233HC) LARGER DEDENDUM SIZE CRITERION/27X,15HFOR MATING GEAR//)
121 FORMAT(10X,43HPHI( 9)  PINION TEETH BEYOND POINTING LIMIT/)
131 FORMAT(10X,43HPHI(10)  GEAR   TEETH BEYOND POINTING LIMIT/)
141 FORMAT(/19X,39HTRY  A) LARGER RATIO OF NUMBER OF TEETH/27X,22HVERS
   1US CENTRE DISTANCE/24X,34HB) SMALLER ADDENDUM SIZE CRITERION//)
151 FORMAT(10X,53HPHI(11)  PINION DEDENDUM SMALLER THAN GEAR   ADDENDU
   1M/)
161 FORMAT(10X,53HPHI(12)  GEAR   DEDENDUM SMALLER THAN PINION ADDENDU
   1M/)
171 FORMAT(/19X,39HTRY  A) CHANGING ADDENDUM-DEDENDUM SIZE/27X,
   19HCRITERION//)
181 FORMAT(10X,45HPHI(13)  UPPER CENTRE DISTANCE LIMIT EXCEEDED/)
191 FORMAT(10X,45HPHI(14)  LOWER CENTRE DISTANCE LIMIT EXCEEDED/)
201 FORMAT(10X,40HPHI(15)  UPPER FACE WIDTH LIMIT EXCEEDED/)
211 FORMAT(10X,40HPHI(16)  LOWER FACE WIDTH LIMIT EXCEEDED/)
221 FORMAT(/19X,37HTRY  A) HOLDING VARIABLES CONSTANT AT/27X,39HUPPER
   1AND LOWER LIMITS, TEST FOR STRESS/27X,21HCONSTRAINT VIOLATIONS/
   224X,35HB) EXTEND RANGE OF LIMITS IF STRESS/27X,25HCONSTRAINTS NOT
   3VIOLATED.//24X,36HSTRESS CONSTRAINTS PROBABLY VIOLATED/24X,30HIF T
   4HESE CONSTRAINTS VIOLATED.//)
231 FORMAT(10X,46HPHI(17)  MAXIMUM PRESSURE ANGLE LIMIT EXCEEDED/)
241 FORMAT(10X,46HPHI(18)  MINIMUM PRESSURE ANGLE LIMIT EXCEEDED/)
251 FORMAT(/19X,38HTRY  A) CHANGING PRESSURE ANGLE LIMITS//)
261 FORMAT(10X,32HPHI(19)  BACKLASH RANGE EXCEEDED/)
271 FORMAT(10X,56HPHI(20)  PINION TOOTH THICKNESS TOLERANCE RANGE EXCE
   1EDED/)
281 FORMAT(10X,56HPHI(21)  GEAR   TOOTH THICKNESS TOLERANCE RANGE EXCE
   1EDED/)
291 FORMAT(19X,33HTRY  A) INCREASING BACKLASH RANGE/24X,37HB) INCREASI
```

```
   1NG A.G.M.A. QUALITY NUMBER//)
301 FORMAT(10X,55HPHI(22)  MINIMUM TIP THICKNESS OF PINION TOOTH EXCEE
   1DED/)
311 FORMAT(10X,55HPHI(23)  MINIMUM TIP THICKNESS OF GEAR   TOOTH EXCEE
   1DED/)
321 FORMAT(/19X,39HTRY  A) CHANGING ADDENDUM-DEDENDUM SIZE/27X,9HCRITE
   1RION/24X,35HB) SMALLER RATIO OF NUMBER OF TEETH/27X,22HVERSUS CENT
   2RL DISTANCE//)
331 FORMAT(10X,52HPHI(24)  MINIMUM RADIUS TO POINT OF LOAD APPLICATION
   1/19X,35HON PINION TOOTH CENTRELINE EXCEEDED/)
341 FORMAT(10X,52HPHI(25)  MINIMUM RADIUS TO POINT OF LOAD APPLICATION
   1/19X,35HON GEAR   TOOTH CENTRELINE EXCEEDED/)
351 FORMAT(/19X,38HTRY  A) CHANGING PRESSURE ANGLE LIMITS/24X,34HB) CH
   1ANGING ADDENDUM-DEDENDUM SIZE/27X,9HCRITERION//)
    END
```

```
    SUBROUTINE SPUR
    DIMENSION WORK1( 8),WORK2( 8),WORK3( 8),WORK4( 8),WORK5(60)
    COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
    COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
    COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
    COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
    COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
    COMMON/BLK3B/BLL,BLU,BLR,CDR
    COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
    COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
    COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
    COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
    COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
    COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
    COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
    COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
    COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
    COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
    COMMON/BLK9 /COD,CODL1,CODL2,QOD,QODL1,QODL2
    COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
    COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
    COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
    COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
    COMMON/BLK12 /X( 8),XSTRT( 8),RMAX( 8),RMIN( 8),PHI(25),PSI( 1)
    COMMON/BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
    COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
    COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
    COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
    COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
    COMMON/OPTI/KO,NNDEX
    DATA F,G,R,REDUCE,NSHOT,NTEST,MAXM/0.01,0.01,1.0,0.05,2,100,300/
    DATA NEQUS,NCONS/ 0,25/
    PI=4.0*ATAN(1.0)
    IF(RPMI.GE.RPMO) GO TO 1
    NDRIVE=2
    RPM1=RPMO
```

```
      RPM2=RPMI
      SHAFT1=SHAFTO
      SHAFT2=SHAFTI
      GO TO 2
    1 NDRIVE=1
      RPM1=RPMI
      RPM2=RPMO
      SHAFT1=SHAFTI
      SHAFT2=SHAFTO
    2 RATIO=RPM1/RPM2
      CALL VARY1
      KO=1
C
C     SETUP OF OPTIMIZATION CRITERION FLAGS
C
      NOFN=0
      RRR=((RATIO**2)+1.0)/((RATIO+1.0)**2)
C
C     VOLUME LIMIT DETERMINED AT PITCH CIRCLE RADIUS INSTEAD OF ADDENDUM
C     CIRCLE RADIUS AS IN SUBROUTINE **VOLUME**. THIS WILL NOT AFFECT
C     THE OPTIMIZATION TO ANY GREAT EXTENT.
C
      VOLMAX=PI*FWMAX*(CDMAX**2)*RRR
      VOLMIN=PI*FWMIN*(CDMIN**2)*RRR
      IF(VOLMAX.EQ.VOLMIN)NOF1=0
      IF(NOF1.EQ.0) GO TO 3
      IF(NCD.NE.0)NOF3=1
      IF(NFW.NE.0)NOF4=1
      NOFN=NOFN+1
      NOF(NOFN)=1
    3 IF(NOF2.EQ.0) GO TO 4
      NOFN=NOFN+1
      NOF(NOFN)=2
    4 IF(NOF3.EQ.0)GO TO 5
      NOFN=NOFN+1
      NOF(NOFN)=3
    5 IF(NOF4.EQ.0)GO TO 6
      NOFN=NOFN+1
      NOF(NOFN)=4
    6 IF(NOFN.EQ.0)NOPT=1
C
C
      CALL TORQUE (HP,PI,RPM1,TORQ1)
      CALL TORQUE (HP,PI,RPM2,TORQ2)
      CALL EFACT (CE,E1,E2,PI,U1,U2)
      CALL FFACT (CF)
      CALL HFACT (BHN1,BHN2,RATIO,CH)
      CYCLE1=CYCLE
      CYCLE2=CYCLE1/RATIO
      CALL LFACT (BHN1,CYCLE1,CL1,QL1)
      CALL LFACT (BHN2,CYCLE2,CL2,QL2)
      CALL OFACT (CO,QO,DRIVEN,DRIVER,NDRIVE,RATIO)
      CALL RFACT (CR,QR,RELI)
      CALL SFACT (CS,QS)
      CALL TFACT (CT,QT,TEMP)
```

```
      IF(NOPT.EQ.1) GO TO 50
      IF(NCD.EQ.0) GO TO 10
      GO TO ( 7, 8, 8),NTYPE
    7 CALL OPTIF1 (XSTRT,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      GO TO 9
    8 CALL OPTIF2 (XSTRT,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
    9 CD=CD*SQRT(COD/CODL1)
      XSTRT(NCD)=CD
   10 IF(IDATA.EQ.0) GO TO 20
      I=1
      WRITE(6,1010)I,XSTRT(I)
      IF(N.LT.2) GO TO 20
      DO 15 I=2,N
      WRITE(6,1011)I,XSTRT(I)
   15 CONTINUE
   20 IF(NCD.EQ.0) GO TO 21
      IF(XSTRT(NCD).GT.CDMAX)  XSTRT(NCD)=CDMAX
   21 IF(NFW.EQ.0) GO TO 22
      IF(XSTRT(NFW).GT.FWMAX)  XSTRT(NFW)=FWMAX
   22 IF(IDATA.EQ.0) GO TO 30
      I=1
      WRITE(6,1014)I,XSTRT(I)
      IF(N.LT.2) GO TO 30
      DO 25 I=2,N
      WRITE(6,1015)I,XSTRT(I)
   25 CONTINUE
   30 IF(ISTRT.EQ.0) GO TO 40
      DO 35 I=1,N
      XSTRT(I)=STRT(I)
   35 CONTINUE
      IF(IDATA.EQ.0) GO TO 40
      I=1
      WRITE(6,1012)I,STRT(I)
      IF(N.LT.2) GO TO 40
      DO 36 I=2,N
      WRITE(6,1013)I,STRT(I)
   36 CONTINUE

   40 GO TO (42,44,46),NTYPE
   42 CALL SEEK1(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,NSHOT,NTEST,MAXM,F,G,U,WORK1,WORK2,WORK3,WORK4)
      GO TO 48
   44 CALL SEEK3(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,MAXM,INDEX,NVIOL,F,G,R,REDUCE,U,WORK1,WORK2,WORK3,WORK4)
      GO TO 48
   46 MAXM=100
      F=1.0E-04
      G=1.0E-04
      CALL NPFMIN(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,MAXM,F,G,R,REDUCE,U,WORK1,WORK2,WORK3,WORK4,WORK5)

   48 IF(NTOOTH.EQ.0) GO TO 50
      IF(IWRITE.NE.0) CALL ANSWER(U,X,PHI,PSI,N,NCONS,NEQUS)
   50 GO TO (55,65,65),NTYPE
   55 IF(NTOOTH.EQ.0) GO TO 60
```

```
      X(NTOOTH)=AINT(TEETH1)
      CALL OPTIF1 (X,UU1,UARTF1,PHI,PSI,NCONS,NEQUS,NVIOL)
      X(NTOOTH)=X(NTOOTH)+1.0
      CALL OPTIF1 (X,UU2,UARTF2,PHI,PSI,NCONS,NEQUS,NVIOL)
      IF(UARTF1.LT.UARTF2) X(NTOOTH)=X(NTOOTH)-1.0
   60 CALL OPTIF1 (X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      GO TO 75
   65 IF(NTOOTH.EQ.0) GO TO 70
      X(NTOOTH)=AINT(TEETH1)
      CALL OPTIF2 (X,UU1,UARTF1,PHI,PSI,NCONS,NEQUS,NVIOL,R)
      X(NTOOTH)=X(NTOOTH)+1.0
      CALL OPTIF2 (X,UU2,UARTF2,PHI,PSI,NCONS,NEQUS,NVIOL,R)
      IF(UARTF1.LT.UARTF2) X(NTOOTH)=X(NTOOTH)-1.0
   70 CALL OPTIF2 (X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
   75 IF(NVIOL.NE.0)KO=1
      CALL TOLCD(BLMAXU,CD,CDR,CDTOLL,CDTOLU,NQUAL,PAR,PI,PR1,PR2,
     1RATIO,RB1,RB2,TEETH1,TP1,TP2)
      IF(IWRITE.EQ.0) RETURN
      CALL ANSWER(U,X,PHI,PSI,N,NCONS,NEQUS)
      IF(KO.EQ.0.AND.NVIOL.EQ.0) GO TO 80
      WRITE(6,1001)
      IF(NVIOL.EQ.0) GO TO 90
      CALL HINT(PHI)
      WRITE(6,1000)
      GO TO 90
   80 WRITE(6,1002)
   90 CALL PRINT
      WRITE(6,1003)
      RETURN
 1000 FORMAT(1H1)
 1001 FORMAT(1H1,9X,44HSPUR GEAR DESIGN...RESULTS OF LAST ITERATION,
     1/10X,44(1H-)/10X,44(1H-)//)
 1002 FORMAT(1H1,9X,35HSPUR GEAR DESIGN...OPTIMUM SOLUTION,
     1/10X,35(1H-)/10X,35(1H-)//)
 1003 FORMAT(/////10X,27HSPUR GEAR DESIGN...COMPLETE,
     1/10X,27(1H-)/10X,27(1H-)//)
 1010 FORMAT(//10X,43HCOMPUTED STARTING VALUES. . . . . . .XSTRT(,I2,
     14H)   =,12X,E16.8)
 1011 FORMAT(47X,6HXSTRT(,I2,4H)   =,12X,E16.8)
 1012 FORMAT(//10X,43HUSER STARTING VALUES. . . . . . . . . STRT(,I2,
     14H)   =,12X,E16.8)
 1013 FORMAT(48X,5HSTRT(,I2,4H)   =,12X,E16.8)
 1014 FORMAT(//10X,43HMODIFIED STARTING VALUES. . . . . . .XSTRT(,I2,
     14H)   =,12X,E16.8)
 1015 FORMAT(47X,6HXSTRT(,I2,4H)   =,12X,E16.8)
      END
```

```
      SUBROUTINE UREAL (X,U)
      DIMENSION   X(1)
      COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
      COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
      COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
      COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
      COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
      COMMON/BLK3B/BLL,BLU,BLR,CDR
      COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
      COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
      COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
      COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
      COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
      COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
      COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
      COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
      COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
      COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
      COMMON/BLK9 /COD,CODL1,CODL2,QOD,QODL1,QODL2
      COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
      COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
      COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
      COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
      COMMON/BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
      COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
      COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
      COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
      COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
      CALL VARY(X)
      CALL TLOAD (HP,PLV,PAR,WA,WR,WT,WN)
      CALL ERROR(DP,FW,NQUAL,PR1,TEETH1,TOLR1,TOLP1,PTOL1,TOLL1,TTCT1,
     1TCT1)
      CALL ERROR(DP,FW,NQUAL,PR2,TEETH2,TOLR2,TOLP2,PTOL2,TOLL2,TTCT2,
     1TCT2)
      CALL BLASH(BLMIN,BLMINT,BLMAX,BLMAXT,BL1,BL2,BLL,BLU,BLR,CP,
     1DP,DELBL,NQUAL,PAR,TPTL1,TPTL2,TPTU1,TPTU2,TPTE1,TPTE2,
     2TPTV1,TPTV2,TTCT1,TTCT2,TCT1,TCT2)
      CALL CUTTER(ANGC1,BL1,CCC1,CD,CP,DED1,NCUT1,PAR,PR1,RB1,RM1,RU1,
     1TP1,BBA1,BBX1,BBY1,RT1)
      CALL CUTTER(ANGC2,BL2,CCC2,CD,CP,DED2,NCUT2,PAR,PR2,RB2,RM2,RU2,
     1TP2,BBA2,BBX2,BBY2,RT2)
      CALL ADDEND (ADDL1,PAR,PR1,RB1,RO1,TO1,TP1)
      CALL ADDEND (ADDL2,PAR,PR2,RB2,RO2,TO2,TP2)
      CALL CONRAT(ANGC1,ANGC2,BP,CRATIO,NCUT1,NCUT2,NDRIVE,
     1PAR,PR1,PR2,RB1,RB2,RI1,RI2,RO1,RO2,RU1,RU2,TP1,TP2,XLA,XLR,
     2BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2)
      CALL EFFIC(EFF,RB1,RB2,PAR,PLV,RATIO,NDRIVE,XLA,XLR)
      CALL SIZE(ADD1,DED1,FW,HUBL1,HUBR1,RI1,RIM1,SHAFT1,SAF1,
     1TORQ1,WEB1,XKEY1)
      CALL SIZE(ADD2,DED2,FW,HUBL2,HUBR2,RI2,RIM2,SHAFT2,SAF2,
     1TORQ2,WEB2,XKEY2)
      CALL VOLUME(FW,HUBL1,HUBR1,PI,RIM1,RO1,SHAFT1,VOL1,WEB1)
      CALL VOLUME(FW,HUBL2,HUBR2,PI,RIM2,RO2,SHAFT2,VOL2,WEB2)
      CALL IFACT(BP,CJ,PAR,PR1,PR2,RATIO,RB1,RO1)
      CALL MFACT (CM,QM,FW,NQUAL)
```

```
      CALL VFACT(CV,QV,NQUAL,PLV)
      CALL FACTOR(CF,CH,CL1,CL2,QL1,QL2,CM,QM,CO,QO,CR,QR,CS,QS,CT,QT,
     1CV,QV,COD,CODL1,CODL2,QOD,QODL1,QODL2)
C
C     LOADING ANALYSIS
C
      ERR=TOLP1+TOLP2+((PTOL1+PTOL2)/2.0)
      IF(NLOAD.EQ.0) GO TO 1
      NNLOAD=NLOAD
      CALL LOAD(RL1,ANGL1,RLL1,ANGLL1,NNLOAD,BP,PAR,PR1,PR2,RB1,RB2,
     1RO1,RO2,TP1)
      CALL LOAD(RL2,ANGL2,RLL2,ANGLL2,NNLOAD,BP,PAR,PR2,PR1,RB2,RB1,
     1RO2,RO1,TP2)
      CALL JFACT(ANGC1,ANGL1,BBY1,DP,NCUT1,PAD,PAR,PI,PR1,RI1,RL1,RLM1,
     1RT1,H1,T1,QJ1,Y1)
      CALL JFACT(ANGC2,ANGL2,BBY2,DP,NCUT2,PAD,PAR,PI,PR2,RI2,RL2,RLM2,
     1RT2,H2,T2,QJ2,Y2)
      GO TO 2
C
C     DETERMINE IF LOAD SHARING EXISTS
C
    1 CALL SHARE(ANGC1,ANGC2,ANGL1,ANGL2,BBY1,BBY2,BP,DP,E1,E2,
     1ERR,FW,NCUT1,NCUT2,NNLOAD,PAD,PAR,PI,PR1,PR2,QO,QV,QJ1,QJ2,
     2RB1,RB2,RI1,RI2,RL1,RL2,RLL1,RLL2,RLM1,RLM2,RO1,RO2,RT1,RT2,
     3TP1,TP2,WN)
    2 CALL BEND(WT,DP,FW,QOD,QODL1,QODL2,QJ1,QJ2,SB1,SB2,SBM1,SBM2,
     1SAF1,SAF2)
      CALL WEAR(COD,CODL1,CODL2,CE,CJ,FW,PR1,SAC1,SAC2,SS1,SS2,
     1SSM1,SSM2,WT)
      CALL POWER(CE,CJ,COD,CODL1,CODL2,QJ1,QJ2,QOD,QODL1,QODL2,
     1DP,FW,PAB1,PAB2,PAW1,PAW2,PI,PR1,RPM1,SAC1,SAC2,SAF1,SAF2)
C
C     OPTIMIZATION CRITERION
C
      U=0.0
      ZERO=1.0E-51
      IF(NOFN.EQ.0)GO TO 101
      DO 100 I=1,NOFN
      NNNNN=NOF(I)
      GO TO (10,20,30,40),NNNNN
   10 VOL=VOL1+VOL2
      UUU=1.0-((VOL-VOLMIN)/(VOLMAX-VOLMIN))
      IF(UUU.LE.ZERO) GO TO 99
      UU=1.0/UUU
      U=U+UU
      GO TO 100
   20 UUU=CRATIO-1.0
      IF(UUU.LE.ZERO) GO TO 99
      UU=1.0/UUU
      U=U+UU
      GO TO 100
   30 UUU=1.0-((CD-CDMIN)/(CDMAX-CDMIN))
      IF(UUU.LE.ZERO) GO TO 99
      UU=1.0/UUU
      U=U+UU
```

```
      GO TO 100
   40 UUU=1.0-((FW-FWMIN)/(FWMAX-FWMIN))
      IF(UUU.LE.ZERO) GO TO 99
      UU=1.0/UUU
      U=U+UU
      GO TO 100
   99 UU=(1.0+ABS(UUU))*10.0E+50
      U=U+UU
  100 CONTINUE
  101 CONTINUE
C
C
      RETURN
      END



      SUBROUTINE CONST(X,PHI,NCONS)
      DIMENSION X(1),PHI(1)
      COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
      COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
      COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
      COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
      COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
      COMMON/BLK3B/BLL,BLU,BLR,CDR
      COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
      COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
      COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
      COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
      COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
      COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
      COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
      COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
      COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
      COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
      COMMON/BLK9 /COD,CODL1,CODL2,QOD,QODL1,QODL2
      COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
      COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
      COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
      COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
      COMMON/BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
      COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
      COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
      COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
      COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
C
C     BENDING STRESS
C
      PHI( 1)=SBM1-SB1
      PHI( 2)=SBM2-SB2
C
C     SURFACE OR CONTACT STRESS
C
      PHI( 3)=SSM1-SS1
```

```
      PHI( 4)=SSM2-SS2
C
C     UNDERCUTTING AND INTERFERENCE
C
      PHI( 5)=(RI1-(BL1/(2.0*TAN(PAR))))-RU1
      PHI( 6)=(RI2-(BL2/(2.0*TAN(PAR))))-RU2
      PHI( 7)=RM1-RO1
      PHI( 8)=RM2-RO2
C
C     GEOMETRICAL
C
      PHI( 9)=ADDL1-ADD1
      PHI(10)=ADDL2-ADD2
      PHI(11)=DED1-ADD2
      PHI(12)=DED2-ADD1
C
C     USER LIMITATIONS
C
      PHI(13)=CDMAX-CD
      PHI(14)=CD-CDMIN
      PHI(15)=FWMAX-FW
      PHI(16)=FW-FWMIN
      PHI(17)=PADMAX-PAD
      PHI(18)=PAD-PADMIN
C
C     BACKLASH AND ERROR ANALYSIS
C
      PHI(19)=DELBL-(TPTE1+TPTE2)
      PHI(20)=TPTV1-TPTU1
      PHI(21)=TPTV2-TPTU2
C
C     SIZE LIMITATIONS
C
      PHI(22)=TO1-(0.25/DP)
      PHI(23)=TO2-(0.25/DP)
      PHI(24)=RLL1-RLM1
      PHI(25)=RLL2-RLM2
C
      RETURN
      END
```

```
      SUBROUTINE SEEK1(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,NSHOT,NTEST,MAXM,F,G,U,WORK1,WORK2,WORK3,WORK4)
C
C     THIS ROUTINE IS A DIRECT SEARCH OPTIMIZATION TECHNIQUE USING
C     THE HOOKE AND JEEVES ALGORITHM
C
C
      DIMENSION X(1),PHI(1),PSI(1),RMAX(1),RMIN(1),XSTRT(1)
      DIMENSION WORK1(1),WORK2(1),WORK3(1),WORK4(1)
      COMMON/OPTI/KO,NNDEX
C
      IF(IDATA.EQ.0) GO TO 1
      WRITE(6,200)
      WRITE(6,201)
      WRITE(6,202)N
      WRITE(6,203)NCONS
      WRITE(6,204)NEQUS
      WRITE(6,205)IDATA
      WRITE(6,206)IPRINT
      WRITE(6,207)NSHOT
      WRITE(6,208)NTEST
      WRITE(6,209)MAXM
      WRITE(6,210)F
      WRITE(6,211)G
      WRITE(6,212) (RMAX(I),I=1,N)
      WRITE(6,213) (RMIN(I),I=1,N)
      WRITE(6,214) (XSTRT(I),I=1,N)
      IF(IPRINT.EQ.0) GO TO 2
      WRITE(6,215)
      WRITE(6,216)
      GO TO 2
    1 IF(IPRINT.EQ.0) GO TO 2
      WRITE(6,200)
      WRITE(6,216)
    2 KO=0
      KOUNT=0
      INDEX=1
      NNDEX=1
C
C     INDEX = 1  IF **SUBROUTINE SEARCH** CALLED BY EITHER **SUBROUTINE
C                SEEK1** OR **SUBROUTINE SEEK3**
C           = 0  IF **SUBROUTINE SEARCH** CALLED BY **SUBROUTINE
C                FEASBL**
C
      DO 3 I=1,N
      X(I)=XSTRT(I)
    3 CONTINUE
    4 CALL SEARCH(X,PHI,PSI,RMAX,RMIN,X,N,NCONS,NEQUS,IPRINT,INDEX,
     1NVIOL,MAXM,F,G,R,U,WORK1,WORK2,WORK3,WORK4)
      CALL SHOT(X,PHI,PSI,RMAX,RMIN,N,NCONS,NEQUS,NSHOT,NTEST,KK,F,G,U,
     1WORK1,WORK2,WORK3)
      IF(KK.EQ.1) GO TO 5
      IF(KO.EQ.0) RETURN
C
C     SINCE KO CANNOT BE RESET IN **SUBROUTINE SHOT**, IF SEARCH IS
```

```
C     INFEASIBLE (IE. KO=1) AT THIS STAGE, THEN **SUBROUTINE SEARCH**
C     FAILED AND **SUBROUTINE SHOT** FOUND NO IMPROVEMENT
C
      WRITE(6,217)
      RETURN
    5 IF(IPRINT.EQ.0) GO TO 6
      CALL OPTIF1(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      WRITE(6,218)U,UART,(X(I),I=1,N)
    6 KOUNT=KOUNT+1
      IF(KOUNT.LE.NSHOT) GO TO 4
      WRITE(6,219)NSHOT
      KO=1
      RETURN
  200 FORMAT(1H1,47HOPTIMIZATION USING DIRECT SEARCH METHOD...SEEK1/1X,
     147(1H-)//)
  201 FORMAT(1X,10HDATA INPUT/1X,10(1H-)//)
  202 FORMAT(1X,60HNUMBER OF INDEPENDENT VARIABLES . . . . . . . . . .
     1    N  =,I6/)
  203 FORMAT(1X,60HNUMBER OF INEQUALITY (.GE.0.0) CONSTRAINTS. . . .   N
     1CONS  =,I6/)
  204 FORMAT(1X,60HNUMBER OF EQUALITY CONSTRAINTS. . . . . . . . . .   N
     1EQUS  =,I6/)
  205 FORMAT(1X,60HINPUT DATA PRINTED OUT FOR (IDATA.NE.0) . . . . .   I
     1DATA  =,I6/)
  206 FORMAT(1X,60HINTERMEDIATE OUTPUT EVERY IPRINT ITERATIONS . . .  IP
     1RINT  =,I6/)
  207 FORMAT(1X,60HNUMBER OF DIRECTED RANDOM SEARCHES PERMITTED. . .   N
     1SHOT  =,I6/)
  208 FORMAT(1X,60HNUMBER OF TEST POINTS IN DIRECTED RANDOM SEARCH .   N
     1TEST  =,I6/)
  209 FORMAT(1X,60HMAXIMUM NUMBER OF ITERATIONS. . . . . . . . . . . .
     1MAXM  =,I6/)
  210 FORMAT(1X,60HFRACTION OF RANGE USED AS STEP SIZE . . . . . . . .
     1    F  =,3X,E16.8/)
  211 FORMAT(1X,60HFRACTION OF RANGE USED FOR CONVERGENCE CRITERION.
     1    G  =,3X,E16.8/)
  212 FORMAT  (61H0ESTIMATED UPPER BOUND ON RANGE OF X(I). . . . . . . RMA
     1X(I)  =,//(5E16.8))
  213 FORMAT  (61H0ESTIMATED LOWER BOUND ON RANGE OF X(I). . . . . . . RMI
     1N(I)  =,//(5E16.8))
  214 FORMAT  (61H0STARTING VALUES OF X(I) . . . . . . . . . . . . . .XSTR
     1T(I)  =,//(5E16.8))
  215 FORMAT(1H1)
  216 FORMAT(21H0INTERMEDIATE RESULTS/1X,20(1H-)//17X,1HU,13X,4HUART,
     17X,27HINDEPENDENT VARIABLES  X(I)//)
  217 FORMAT(80H0DIRECT SEARCH HAS HUNG UP AND DIRECTED RANDOM SEARCH CA
     1NNOT FIND A BETTER POINT/ 1X,44HTRY  A) ENSURING THAT FEASIBLE REG
     2ION EXISTS/6X,32HB) MORE FEASIBLE STARTING VALUES/6X,29HC) CHANGIN
     3G VALUES OF F AND G//)
  218 FORMAT(8H0.SHOT.  ,6E16.8/(40X,4E16.8))
  219 FORMAT(56H0DIRECTED RANDOM SEARCH FOUND AN IMPROVEMENT BUT NSHOT =
     1,I6,18H HAS BEEN EXCEEDED/1X,45HTRY  A) USING LAST RESULTS AS STAR
     2TING VALUES/6X,28HB) INCREASING VALUE OF NSHOT//)
      END
```

```
      SUBROUTINE SEEK3(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,MAXM,INDEX,NVIOL,F,G,R,REDUCE,U,WORK1,WORK2,WORK3,WORK4)
C
C     THIS ROUTINE IS A DIRECT SEARCH OPTIMIZATION TECHNIQUE USING
C     THE HOOKE AND JEEVES ALGORITHM
C
C     IF INDEX=0 THEN **SUBROUTINE SEEK3** CALLED BY **SUBROUTINE
C     FEASBL**
C
C
      DIMENSION X(1),PHI(1),PSI(1),RMAX(1),RMIN(1),XSTRT(1)
      DIMENSION WORK1(1),WORK2(1),WORK3(1),WORK4(1)
      COMMON/OPTI/KO,NNDEX
C
      IF(IDATA.EQ.0) GO TO 1
      IF(INDEX.NE.0) WRITE(6,200)
      WRITE(6,201)
      WRITE(6,202)N
      WRITE(6,203)NCONS
      WRITE(6,204)NEQUS
      WRITE(6,205)IDATA
      WRITE(6,206)IPRINT
      WRITE(6,207)MAXM
      WRITE(6,208)F
      WRITE(6,209)G
      WRITE(6,210)R
      WRITE(6,211)REDUCE
      WRITE(6,212) (RMAX(I),I=1,N)
      WRITE(6,213) (RMIN(I),I=1,N)
      WRITE(6,214) (XSTRT(I),I=1,N)
      IF(IPRINT.EQ.0) GO TO 2
      WRITE(6,215)
      WRITE(6,216)
      GO TO 2
    1 IF(IPRINT.EQ.0) GO TO 2
      IF(INDEX.NE.0) WRITE(6,200)
      WRITE(6,216)
    2 DO 3 I=1,N
      X(I)=XSTRT(I)
    3 CONTINUE
      KO=0
      KOUNT=0
      NNDEX=2
      RR=R
      ULAST=10.0E+40
    4 CALL SEARCH(X,PHI,PSI,RMAX,RMIN,X,N,NCONS,NEQUS,IPRINT,
     1INDEX,NVIOL,MAXM,F,G,RR,U,WORK1,WORK2,WORK3,WORK4)
      IF(INDEX.EQ.0) RETURN
      IF(KO.EQ.1) GO TO 7
      KOUNT=KOUNT+1
      IF(IPRINT.EQ.0) GO TO 5
      IF(MOD(KOUNT,IPRINT).NE.0) GO TO 5
      CALL OPTIF2(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,RR)
      WRITE(6,217)RR
      WRITE(6,218)U,UART,(X(I),I=1,N)
```

```
    5 IF(ABS(U-ULAST).LT.1.0E-07*ABS(ULAST)) GO TO 8
      IF(RR.LT.1.0E-20) GO TO 6
      ULAST=U
      RR=RR*REDUCE
      GO TO 4
    6 WRITE(6,219)RR
      KO=1
      GO TO 8
    7 WRITE(6,220)
      KO=1
    8 CALL OPTIF2(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,RR)
      RETURN
  200 FORMAT(1H1,47HOPTIMIZATION USING DIRECT SEARCH METHOD...SEEK3/1X,
     147(1H-)//)
  201 FORMAT(1X,10HDATA INPUT/1X,10(1H-)//)
  202 FORMAT(1X,60HNUMBER OF INDEPENDENT VARIABLES . . . . . . . . . .
     1    N   =,I6/)
  203 FORMAT(1X,60HNUMBER OF INEQUALITY (.GE.0.0) CONSTRAINTS. . . .   N
     1CONS   =,I6/)
  204 FORMAT(1X,60HNUMBER OF EQUALITY CONSTRAINTS. . . . . . . . . .   N
     1EQUS   =,I6/)
  205 FORMAT(1X,60HINPUT DATA PRINTED OUT FOR (IDATA.NE.0) . . . . .   I
     1DATA   =,I6/)
  206 FORMAT(1X,60HINTERMEDIATE OUTPUT EVERY IPRINT ITERATIONS . . .  IP
     1RINT   =,I6/)
  207 FORMAT(1X,60HMAXIMUM NUMBER OF ITERATIONS. . . . . . . . . . .
     1MAXM   =,I6/)
  208 FORMAT(1X,60HFRACTION OF RANGE USED AS STEP SIZE . . . . . . .
     1    F   =,3X,E16.8/)
  209 FORMAT(1X,60HFRACTION OF RANGE USED FOR CONVERGENCE CRITERION.
     1    G   =,3X,E16.8/)
  210 FORMAT(1X,60HINITIAL PENALTY MULTIPLIER FOR CONSTRAINTS. . . .
     1    R   =,3X,E16.8/)
  211 FORMAT(1X,45HREDUCTION FACTOR FOR PENALTY MULTIPLIER AFTER/1X,
     160HEACH MINIMIZATION . . . . . . . . . . . . . . . .   REDUCE  =,
     23X,E16.8)
  212 FORMAT  (61H0ESTIMATED UPPER BOUND ON RANGE OF X(I). . . . . . . RMA
     1X(I)   =,//(5E16.8))
  213 FORMAT  (61H0ESTIMATED LOWER BOUND ON RANGE OF X(I). . . . . . . RMI
     1N(I)   =,//(5E16.8))
  214 FORMAT  (61H0STARTING VALUES OF X(I) . . . . . . . . . . . . .XSTR
     1T(I)   =,//(5E16.8))
  215 FORMAT(1H1)
  216 FORMAT(21H0INTERMEDIATE RESULTS/1X,20(1H-)//17X,1HU,13X,4HUART,
     17X,27HINDEPENDENT VARIABLES  X(I)//)
  217 FORMAT(4X,4HRR =,E16.8)
  218 FORMAT(3X,6E16.8/(40X,4E16.8))
  219 FORMAT(24H0NO CONVERGENCE WITH R =,E16.8/)
  220 FORMAT(47H0SEEK3 UNABLE TO FIND A FEASIBLE STARTING POINT/1X,44HTR
     1Y  A) ENSURING THAT FEASIBLE REGION EXISTS/6X,32HB) MORE FEASIBLE
     2STARTING VALUES/6X,29HC) CHANGING VALUES OF F AND G//)
      END
```

```
      SUBROUTINE NFFMIN(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,MAXM,F,G,R,REDUCE,U,DD,EE,FF,GG,HH)
C
C     THIS ROUTINE WAS ADAPTED FOR OPTISEP FROM FLETCHERS PROGRAM.
C     THEORY FOR THE METHOD CAN BE FOUND IN ** THE COMPUTER JOURNAL
C     VOLUME 13 NUMBER 3 AUGUST 1970 **
C
      DIMENSION X(1),XSTRT(1),RMAX(1),RMIN(1),PHI(1),PSI(1)
      DIMENSION DD(1),EE(1),FF(1),GG(1),HH(1)
      LOGICAL CONV
      COMMON/OPTI/KO,NNDEX
      IF(IDATA.EQ.0) GO TO 1
      WRITE(6,200)
      WRITE(6,201)
      WRITE(6,202)N
      WRITE(6,203)NCONS
      WRITE(6,204)NEQUS
      WRITE(6,205)IPRINT
      WRITE(6,206)IDATA
      WRITE(6,207)MAXM
      WRITE(6,208)F
      WRITE(6,209)G
      WRITE(6,210)R
      WRITE(6,211)REDUCE
      WRITE(6,212) (RMAX(I),I=1,N)
      WRITE(6,213) (RMIN(I),I=1,N)
      WRITE(6,214) (XSTRT(I),I=1,N)
      IF(IPRINT.EQ.0) GO TO 996
      WRITE(6,215)
      WRITE(6,300)
      GO TO 996
    1 IF(IPRINT.EQ.0) GO TO 996
      WRITE(6,200)
      WRITE(6,300)
  996 KO=0
      ITN=0
      NFNS=1
      NSTRT=1
      RR=R
      ULAST=1.0E+40
      DO 2 I=1,N
      X(I)=XSTRT(I)
      XXXXX=ABS(RMAX(I)-RMIN(I))
      DD(I)=F*XXXXX
      EE(I)=G*XXXXX
    2 CONTINUE
  997 CALL OPTIF2(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,RR)
      CALL SLOPE(X,PHI,PSI,N,NCONS,NEQUS,GG,DD,RR)
      KO=0
      STEP=1.0
      IDX=N
      IDG=N+N
      IH=IDG+N
  998 IJ=IH+1
      DO 3 I=1,N
```

```
      DO 3 J=I,N
      HH(IJ)=0.0
      IF(I.EQ.J)HH(IJ)=1.0
      IJ=IJ+1
    3 CONTINUE
  999 CONV=.TRUE.
      GDX=0.0
      DO 6 I=1,N
      Z=0.0
      IJ=IH+1
      IF(I.EQ.1) GO TO 41
      II=I-1
      DO 4 J=1,II
      Z=Z-HH(IJ)*GG(J)
      IJ=IJ+N-J
    4 CONTINUE
   41 DO 5 J=I,N
      Z=Z-HH(IJ)*GG(J)
      IJ=IJ+1
    5 CONTINUE
      IF(ABS(Z).GT.EE(I)) CONV=.FALSE.
      HH(IDX+I)=Z
      GDX=GDX+GG(I)*Z
    6 CONTINUE
      IF(IPRINT.EQ.0) GO TO 7
      IF(MOD(ITN,IPRINT).NE.0) GO TO 7
      WRITE(6,301)RR
      WRITE(6,302)ITN,NFNS,U,UART,(X(I),I=1,N)
    7 IEXIT=1
      IF(CONV) GO TO 24
      IEXIT=3
      IF(GDX.GE.0.0) GO TO 24
      Z=1.0
      IF(ITN.LT.N) Z=STEP
      W=2.0*(U-UART)/GDX
      IF(W.EQ.0)W=Z
      IF(W.LT.Z) Z=W
      STEP=Z
    8 GDX=GDX*Z
      DO 9 I=1,N
      HH(IDX+I)=HH(IDX+I)*Z
      FF(I)=X(I)+HH(IDX+I)
    9 CONTINUE
      CALL OPTIF2(FF,U1,UART1,PHI,PSI,NCONS,NEQUS,NVIOL,RR)
      CALL SLOPE(FF,PHI,PSI,N,NCONS,NEQUS,HH,DD,RR)
      NFNS=NFNS+1
      IEXIT=2
      IF(ABS(UART1-UART).LT.1.0E-07*ABS(UART)) GO TO 24
      IEXIT=5
      IF(ITN.EQ.MAXM) GO TO 24
      GPDX=0.0
      DO 10 I=1,N
      HH(IDG+I)=HH(I)-GG(I)
      GPDX=GPDX+HH(I)*HH(IDX+I)
   10 CONTINUE
```

```
      DGDX=GPDX-GDX
      IF(UART.GT.(UART1-0.0001*GDX)) GO TO 11
      IEXIT=4
      IF(GPDX.LT.0.0.AND.ITN.GT.N) GO TO 24
      Z=3.0*(UART-UART1)+GPDX+GDX
      W=SQRT(1.0-(GDX/Z)*(GPDX/Z))*ABS(Z)
      Z=1.0-(GPDX+W-Z)/(DGDX+2.0*W)
      IF(Z.LT.0.1) Z=0.1
      GO TO 14
   11 UART=UART1
      U=U1
      DO 12 I=1,N
      GG(I)=HH(I)
      X(I)=FF(I)
   12 CONTINUE
      IF(DGDX.GT.0.0) GO TO 15
      GDX=GPDX
      Z=4.0
   14 STEP=Z*STEP
      GO TO 8
   15 IF(GPDX.LT.0.5*GDX) STEP=2.0*STEP
      DGHDG=0.0
      DO 19 I=1,N
      Z=0.0
      IJ=IH+1
      IF(I.EQ.1) GO TO 17
      II=I-1
      DO 16 J=1,II
      Z=Z+HH(IJ)*HH(IDG+J)
      IJ=IJ+N-J
   16 CONTINUE
   17 DO 18 J=1,N
      Z=Z+HH(IJ)*HH(IDG+J)
      IJ=IJ+1
   18 CONTINUE
      DGHDG=DGHDG+Z*HH(IDG+1)
      HH(I)=Z
   19 CONTINUE
      IF(DGHDG.LT.0.0) DGHDG=DGDX*0.01
      IF(DGDX.LT.DGHDG) GO TO 21
      W=1.0+DGHDG/DGDX
      DO 20 I=1,N
      HH(IDX+I)=W*HH(IDX+I)-HH(I)
   20 CONTINUE
      DGDX=DGDX+DGHDG
      DGHDG=DGDX
   21 IJ=IH
      DO 22 I=1,N
      W=HH(IDX+I)/DGDX
      Z=HH(I)/DGHDG
      DO 22 J=I,N
      IJ=IJ+1
      HH(IJ)=HH(IJ)+W*HH(IDX+J)-Z*HH(J)
   22 CONTINUE
      ITN=ITN+1
```

```
      GO TO 999
   24 GO TO (25,26,28,28,29),IEXIT
   25 IF(ABS(UART-ULAST).LT.1.0E-07*ABS(ULAST)) GO TO 27
      ULAST=UART
      NSTRT=1
      IF(RR.GT.1.0E-20) GO TO 30
      KO=1
      WRITE(6,405)RR
      RETURN
   26 IF(ABS(UART1-ULAST).LT.G*ABS(UART1)) GO TO 27
      ULAST=UART1
      NSTRT=1
      IF(RR.GT.1.0E-20) GO TO 30
      KO=1
      WRITE(6,405)RR
      RETURN
   27 KO=0
      CALL OPTIF2(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,RR)
      IF(NVIOL.NE.0) KO=1
      IF(KO.EQ.1) GO TO 31
      RETURN
   28 IF(NSTRT.EQ.5) GO TO 32
      NSTRT=NSTRT+1
      ULAST=UART
      GO TO 30
   29 WRITE(6,404)MAXM
      KO=1
      RETURN
   30 RR=RR*REDUCE
      GO TO 997
   31 WRITE(6,401)IEXIT,RR
      WRITE(6,402)
      RETURN
   32 KO=1
      WRITE(6,403)
      RETURN
  200 FORMAT(1H1,52HOPTIMIZATION USING THE NEW FLETCHER POWELL TECHNIQUE
     1/1X,52(1H-)//)
  201 FORMAT(1X,10HDATA INPUT/1X,10(1H-)//)
  202 FORMAT(1X,60HNUMBER OF INDEPENDENT VARIABLES . . . . . . . . . .
     1   N  =,I6/)
  203 FORMAT(1X,60HNUMBER OF INEQUALITY (.GE.0.0) CONSTRAINTS. . . .   N
     1CONS  =,I6/)
  204 FORMAT(1X,60HNUMBER OF EQUALITY CONSTRAINTS. . . . . . . . . .   N
     1EQUS  =,I6/)
  205 FORMAT(1X,60HINTERMEDIATE OUTPUT EVERY IPRINT ITERATIONS . . .  IP
     1RINT  =,I6/)
  206 FORMAT(1X,60HINPUT DATA PRINTED OUT FOR (IDATA.NE.0) . . . . .   I
     1DATA  =,I6/)
  207 FORMAT(1X,60HMAXIMUM NUMBER OF ITERATIONS. . . . . . . . . . .
     1MAXM  =,I6/)
  208 FORMAT(1X,49HFRACTION OF RANGE USED FOR GRADIENT DETERMINATION/1X,
     160HBY FINITE DIFFERENCE. . . . . . . . . . . . . . .         F  =,
     23X,E16.8/)
  209 FORMAT(1X,60HFRACTION OF RANGE USED FOR CONVERGENCE CRITERION.
```

```
    1    G   =,3X,E16.8/)
210 FORMAT(1X,60HINITIAL PENALTY MULTIPLIER FOR CONSTRAINTS. . . .
    1    R   =,3X,E16.8/)
211 FORMAT(1X,45HREDUCTION FACTOR FOR PENALTY MULTIPLIER AFTER/1X,
    160HEACH MINIMIZATION . . . . . . . . . . . . . . . .  REDUCE  =,
    23X,E16.8)
212 FORMAT  (61H0ESTIMATED UPPER BOUND ON RANGE OF X(I). . . . . . RMA
    1X(I)   =,//(5E16.8))
213 FORMAT  (61H0ESTIMATED LOWER BOUND ON RANGE OF X(I). . . . . . RMI
    1N(I)   =,//(5E16.8))
214 FORMAT  (61H0STARTING VALUES OF X(I) . . . . . . . . . . . . .XSTR
    1T(I)   =,//(5E16.8))
215 FORMAT(1H1)
300 FORMAT(1X,10HITERATIONS,2X,10HFUNCTIONAL,10X,1HU,13X,4HUART,7X,
    126HINDEPENDENT VARIABLES X(I)/13X,11HEVALUATIONS//)
301 FORMAT(20X,4HRR =,E16.8)
302 FORMAT(4X,I3,10X,I3,4X,6E16.8/(56X,4E16.8))
401 FORMAT (64H0PROGRAM HAS CONVERGED TO AREA IN INFEASIBLE REGION WIT
    1H IEXIT =, I3,/61X,3HR =,E16.8//1X,36HTRY A) A MORE OPTIMUM STARTI
    2NG POINT,/5X,28HB) SMALLER VALUES OF F AND G,/)
402 FORMAT(51H0WHERE IEXIT = 1  STEP LENGTH CONVERGENCE CRITERION/7X,
    150HIEXIT = 2  FUNCTIONAL CHANGE CONVERGENCE CRITERION//)
403 FORMAT(54H0PROGRAM ALGORITHM HAS FAILED DUE TO MATRIX HH BECOMING
    1SINGULAR,/1X,36HTRY A) A MORE OPTIMUM STARTING POINT,/5X,28HB) SMA
    2LLER VALUES OF F AND G,/5X,37HC) A DIFFERENT OPTIMIZATION TECHNIQU
    3E,//1X,47HNOTE ... LAST ITERATION MAY BE CLOSE TO OPTIMUM/)
404 FORMAT(49H0MAXIMUM NUMBER OF ITERATIONS EXCEEDED ... MAXM =,I6/)
405 FORMAT(///10X,23HNO CONVERGENCE WITH R =,E16.8)
    END
```

```
      SUBROUTINE SEARCH(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IPRINT,
     1INDEX,NVIOL,MAXM,F,G,R,U,XA,XB,DX,TX)

      THIS ROUTINE IS THE DIRECT SEARCH ALGORITHM OF HOOKE AND JEEVES
      THIS SEARCH TECHNIQUE IS EMPLOYED IN **SUBROUTINE SEEK1** AND
      **SUBROUTINE SEEK3**

      INDEX     = 0  WHEN **SUBROUTINE SEARCH** CALLED BY **SUBROUTINE
                     FEASBL**
      NNDEX     = 1  WHEN **SUBROUTINE SEARCH** CALLED BY **SUBROUTINE
                     SEEK1**
      NNDEX     = 2  WHEN **SUBROUTINE SEARCH** CALLED BY **SUBROUTINE
                     SEEK3**

      XA        = INITIAL BASE POINT OF X(I)
      XB        = SECONDARY BASE POINT OF X(I)
      DX        = FRACTION OF RANGE USED AS STEP SIZE
      TX        = FRACTION OF RANGE USED AS CONVERGENCE CRITERION


      DIMENSION X(1),PHI(1),PSI(1),RMAX(1),RMIN(1),XSTRT(1)
      DIMENSION XA(1),XB(1),DX(1),TX(1)
      COMMON/OPTI/KO,NNDEX

      KO=0
      M1=0
      NVIOL1=1
      DO 1 I=1,N
      X(I)=XSTRT(I)
      XA(I)=X(I)
      DX(I)=F*ABS(RMAX(I)-RMIN(I))
      TX(I)=G*DX(I)
    1 CONTINUE
      NCALL=1
    2 GO TO ( 3, 4),NNDEX
    3 CALL OPTIF1 (X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
      GO TO 5
    4 CALL OPTIF2 (X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
    5 IF(NCALL.EQ.1) UARTO=UART
      IF(NVIOL.EQ.0)NVIOL1=0
      IF(INDEX.EQ.0) GO TO 25
    6 GO TO ( 7, 8, 9,21),NCALL
    7 NFAIL=0
      DO 11 I=1,N
      X(I)=X(I)+DX(I)
      NCALL=2
      GO TO 2
    8 IF(UART.LT.UARTO) GO TO 10
      X(I)=X(I)-2.0*DX(I)
      NCALL=3
      GO TO 2
    9 IF(UART.LT.UARTO) GO TO 10
      NFAIL=NFAIL+1
      X(I)=X(I)+DX(I)
      GO TO 11
```

```
10 UARTO = UART
11 CONTINUE
   IF(NFAIL.EQ.N) GO TO 17
12 DO 13 I=1,N
   XB(I)=X(I)
13 CONTINUE
   M1=M1+1
   GO TO (14,15),NNDEX
14 IF(MOD(M1,IPRINT).NE.0) GO TO 15
   CALL OPTIF1(X,UL,UARTL,PHI,PSI,NCONS,NEQUS,NVIOL)
   WRITE(6,30)M1,UL,UARTL,(X(I),I=1,N)
15 IF(M1.GT.MAXM) GO TO 26
   DO 16 I=1,N
   X(I)=X(I)+(X(I)-XA(I))
16 CONTINUE
   NCALL=4
   GO TO 2
17 DO 18 I=1,N
   IF(DX(I).GT.TX(I)) GO TO 19
18 CONTINUE
   GO TO 25
19 DO 20 I=1,N
   DX(I)=DX(I)/2.0
20 CONTINUE
   GO TO 7
21 IF(UART.LT.UARTO) GO TO 23
   DO 22 I=1,N
   XA(I)=XB(I)
   X(I)=XA(I)
22 CONTINUE
   GO TO 7
23 DO 24 I=1,N
   XA(I)=XB(I)
24 CONTINUE
   UARTO=UART
   GO TO 7
25 IF(NVIOL1.NE.0) GO TO 6
26 GO TO (27,28),NNDEX
27 CALL OPTIF1(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
   GO TO 29
28 CALL OPTIF2(X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
29 IF(NVIOL.EQ.0) RETURN
   IF(M1.GT.MAXM) WRITE(6,31)MAXM
   KO=1
   RETURN
30 FORMAT(1H ,I4,3X,6E16.8/(40X,4E16.8))
31 FORMAT(63HONO FEASIBLE SOLUTION AFTER ALLOWABLE NUMBER OF MOVES...
  1MAXM  =,I6/1X,38HTRY  A) A MORE FEASIBLE STARTING POINT/6X,
  229HB) INCREASING NUMBER OF MOVES//)
   END
```

```
      SUBROUTINE SHOT(X,PHI,PSI,RMAX,RMIN,N,NCONS,NEQUS,NSHOT,NTEST,KK,
     1F,G,U,WORK1,WORK2,WORK3)
C
C     THIS ROUTINE PRODUCES A DIRECTED RANDOM SEARCH INTENDED TO MOVE
C     THE DIRECT SEARCH AWAY FROM CONSTRAINT BOUNDARIES, THUS PREVENTING
C     THE SEARCH FROM MOVING SLOWLY ALONG A CONSTRAINT BOUNDARY
C     LARGE STEPS EQUAL TO 10.0 TIMES THE INITIAL STEP SIZE IN
C     **SUBROUTINE SEARCH** ARE IMPLEMENTED
C
C     U        = OPTIMUM OBTAINED BY DIRECT SEARCH
C                IF DIRECTED RANDOM SEARCH IMPROVES OPTIMUM, U IS CHANGED
C                TO THE IMPROVED VALUE
C     KK       = INDICATOR OF IMPROVEMENT IN U
C     WORK1    = RANDOM NUMBERS BETWEEN 0.0 AND 1.0
C     WORK2    = TRIAL VALUES OF X(I) FROM DIRECTED RANDOM SEARCH
C     WORK3    = FRACTION OF RANGE USED IN DIRECTED RANDOM SEARCH
C
C
      DIMENSION X(1),PHI(1),PSI(1),RMAX(1),RMIN(1)
      DIMENSION WORK1(1),WORK2(1),WORK3(1)
C
      KK=0
      CALL FRANDN(WORK1,N,N)
      CALL OPTIF1(X,UU,UTEST,PHI,PSI,NCONS,NEQUS,NVIOL)
      UMIN=UTEST
      U=UU
      DO 1 I=1,N
      RANGE=ABS(RMAX(I)-RMIN(I))
      WORK3(I)=10.0*F*RANGE
      IF(WORK3(I).GT.(0.1*RANGE))  WORK3(I)=0.1*RANGE
    1 CONTINUE
      DO 4 J=1,NTEST
      CALL FRANDN(WORK1,N,0)
      DO 2 I=1,N
      WORK2(I)=X(I)-WORK3(I)*(1.0-2.0*WORK1(I))
    2 CONTINUE
      CALL OPTIF1(WORK2,UU,UTEST,PHI,PSI,NCONS,NEQUS,NVIOL)
      IF(NVIOL.NE.0) GO TO 4
      IF(UTEST.GE.UMIN) GO TO 4
      UMIN=UTEST
      U=UU
      DO 3 I=1,N
      X(I)=WORK2(I)
    3 CONTINUE
      KK=1
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SLOPE(X,PHI,PSI,N,NCONS,NEQUS,GRAD,DELX,R)
C
C     THIS ROUTINE DETERMINES THE FINITE DIFFERENCE GRADIENT OF THE
C     ARTIFICIAL OBJECTIVE FUNCTION CREATED IN **SUBROUTINE OPTIF2**
C
C     THE GRADIENT OF EACH VARIABLE IS DIVIDED BY THE MAXIMUM ABSOLUTE
C     GRADIENT DETERMINED IN **SUBROUTINE MAXMUM**
C
C
      DIMENSION X(1),PHI(1),PSI(1)
      DIMENSION GRAD(1),DELX(1)
C
      CALL OPTIF2(X,U1,FUNC1,PHI,PSI,NCONS,NEQUS,NVIOL1,R)
      DO 1 I=1,N
      X(I)=X(I)+DELX(I)
      CALL OPTIF2(X,U2,FUNC2,PHI,PSI,NCONS,NEQUS,NVIOL2,R)
      X(I)=X(I)-DELX(I)
      GRAD(I)=(FUNC2-FUNC1)/DELX(I)
    1 CONTINUE
      CALL MAXMUM(N,GRAD,GRADM)
      IF(GRADM.EQ.0.0) GO TO 3
      DO 2 I=1,N
      GRAD(I)=GRAD(I)/GRADM
    2 CONTINUE
      RETURN
    3 DO 4 I=1,N
      GRAD(I)=0.0
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MAXMUM(N,GRAD,GRADM)
C
C     THIS ROUTINE DETERMINES THE MAXIMUM ABSOLUTE VALUE IN AN ARRAY
C
C
      DIMENSION GRAD(1)
C
      GRADM=ABS(GRAD(1))
      DO 1 I=1,N
      IF(GRADM.LT.ABS(GRAD(I))) GRADM=ABS(GRAD(I))
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FEASBL(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,MAXM,F,G,U,STEPP,WORK1,WORK2,WORK3,WORK4)
C
C     THIS ROUTINE USES **SUBROUTINE SEEK3** TO DRIVE ALL PHI(I)
C     INEQUALITY CONSTRAINTS FEASIBLE AND THEN REDUCES THE PSI(I)
C     EQUALITY CONSTRAINTS BY MINIMIZING THE SUM OF ABS(PSI(I)) VALUES
C     SUBJECT TO THE CONDITION THAT ALL PHI(I) VALUES REMAIN
C     FEASIBLE (.GE.0.0)
C
C
      DIMENSION X(1),PHI(1),PSI(1),RMAX(1),RMIN(1),XSTRT(1),STEPP(1)
      DIMENSION WORK1(1),WORK2(1),WORK3(1),WORK4(1)
      COMMON/OPTI/KO,NNDEX
C
      WRITE(6,100)
      KO=0
      KOT=0
      INDEX=0
      REDUCE=0.05
      R=1.0
      DO 1 I=1,N
      X(I)=XSTRT(I)
    1 CONTINUE
      IF(NCONS.EQ.0) GO TO 4
      CALL UREAL(X,U)
      CALL CONST(X,PHI,NCONS)
      DO 2 I=1,N
      IF(PHI(I).LT.0.0) GO TO 3
    2 CONTINUE
      GO TO 4
    3 CALL SEEK3(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,IDATA,
     1IPRINT,MAXM,INDEX,NVIOL,F,G,R,REDUCE,U,WORK1,WORK2,WORK3,WORK4)
      IF(NVIOL.NE.0) GO TO 14
C
C     IF **SUBROUTINE COULD NOT DRIVE ALL PHI(I).GE.0.0 THEN
C     **SUBROUTINE FEASBL** CANNOT OBTAIN A FEASIBLE POINT
C
    4 IF(NEQUS.EQ.0) RETURN
C
C     NOTE...THE FRACTION OF THE RANGE USED AS STEP SIZE SHOULD NOT
C     EXCEED 5 PERCENT.
C     FOR A VERY FEASIBLE POINT (IE. ALL PSI(I) VERY SMALL) CHOOSE F
C     VERY SMALL
C
      PERCNT=0.05
      IF(ABS(F).LT.0.05)PERCNT=F
      DO 5 I=1,N
      STEPP(I)=PERCNT*ABS(RMAX(I)-RMIN(I))
    5 CONTINUE
      CALL SUMPSI(X,PSI,NEQUS,SUMO)
    6 NFAIL=0
      IF(NFAIL.EQ.N) GO TO 12
      DO 11 I=1,N
      X(I)=X(I)+STEPP(I)
      CALL UREAL(X,U)
```

```
      CALL CONST(X,PHI,NCONS)
      DO  7 J=1,NCONS
C
C     IGNORE A MOVE WHICH MAKES ANY PHI(I).LT.0.0
C
      IF(PHI(J).LT.0.0) GO TO 8
    7 CONTINUE
      CALL SUMPSI(X,PSI,NEQUS,SUM1)
      IF(SUM1.GE.SUM0) GO TO 8
      SUM0=SUM1
      GO TO 11
    8 X(I)=X(I)-2.0*STEPP(I)
      CALL UREAL(X,U)
      CALL CONST(X,PHI,NCONS)
      DO  9 L=1,NCONS
      IF(PHI(L).LT.0.0) GO TO 10
    9 CONTINUE
      CALL SUMPSI(X,PSI,NEQUS,SUM2)
      IF(SUM2.GE.SUM0) GO TO 10
      SUM0=SUM2
      GO TO 11
   10 X(I)=X(I)+STEPP(I)
      NFAIL=NFAIL+1
   11 CONTINUE
      GO TO 6
C
C     REDUCE STEPP(I) BY A FACTOR OF 4.0 UP TO 4 TIMES. THIS MEANS STEPP
C     REDUCES TO  LESS THAN .0002*(RMAX(I)-RMIN(I)),  OR IF  F.LT.0.05
C     THEN MINIMUM STEPP(I)=(F/256)*(RMAX(I)-RMIN(I)). THEREFORE THE
C     PSI(I) VALUES MAY BE DRIVE AS SMALL AS DESIRED BY ENTERING VERY
C     SMALL VALUES OF F
C
   12 KUT=KUT+1
      IF(KUT.GT.4) GO TO 15
      DO 13 I=1,N
      STEPP(I)=STEPP(I)/4.0
   13 CONTINUE
      GO TO 6
   14 WRITE(6,101)
      KO=1
   15 CALL UREAL(X,U)
      CALL EQUAL(X,PSI,NEQUS)
      IF(NCONS.NE.0)CALL CONST(X,PHI,NCONS)
      RETURN
  100 FORMAT(1H1,25HFEASIBLE SEARCH....FEASBL/1X,25(1H-)//)
  101 FORMAT(38H0FEASBL COULD NOT FIND FEASIBLE REGION/)
      END
```

```
      SUBROUTINE SUMPSI(X,PSI,NEQUS,SUM)
C
C     THIS ROUTINE DETERMINES THE SUM OF THE ABSOLUTE VALUE OF THE
C     EQUALITY CONSTRAINTS
C
C
      DIMENSION X(1),PSI(1)
C
      CALL EQUAL(X,PSI,NEQUS)
      SUM=0.0
      DO 1 I=1,NEQUS
      SUM=SUM + ABS(PSI(I))
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FRANDN(A,N,M)
C
C     THIS ROUTINE GENERATES RANDOM NUMBERS BETWEEN 0.0 AND 1.0
C
C     B  IS A MACHINE-DEPENDENT CONSTANT AND  B=2.0**(I/2+1)+3.0
C     WHERE  I  = NUMBER OF BITS IN AN INTEGER WORD (I=47 FOR CDC6400)
C
C
      DIMENSION A(1)
C
      B=16777219.
      X=M
      X=X/0.8719467
      IF(X.NE.0.0)Y=AMOD(ABS(X),3.18967)
      DO  2 K=1,N
      DO  1 J=1,2
      Y=AMOD(B*Y,1.0)
    1 CONTINUE
      A(K)=Y
C
C     AVOID Y=0. AND Y=1. TO PREVENT DIVIDING INTO ZERO
C
      IF(Y.EQ.0.0.OR.Y.EQ.1.0)Y=0.182818285
    2 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OPTIF1 (X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL)
C
C     THIS ROUTINE INCORPORATES EQUALITY AND INEQUALITY CONSTRAINTS
C     IN AN ARTIFICIAL OBJECTIVE FUNCTION OF THE FORM...
C
C     UART = U + CCC*SUM(ABS(PHI(I))) + CCC*SUM(ABS(PSI(I)))
C
C     WHERE...PHI(I) = INEQUALITY CONSTRAINTS
C             PSI(I) =   EQUALITY CONSTRAINTS
C                CCC = PENALTY FACTOR
C                    = 10.0E+20
C
C
C      TO PREVENT VERY MINOR VIOLATIONS OF INEQUALITY CONSTRAINTS
C     ASSUME ZERO=-1.0E-10
C      ONLY THE VIOLATED CONSTRAINTS ARE MULTIPLIED BY THE PENALTY
C     FACTOR IN THE ARTIFICIAL OBJECTIVE FUNCTION
C
      DIMENSION X(1),PHI(1),PSI(1)
C
      NVIOL=0
      SUM1=0.0
      SUM2=0.0
      CCC=10.0E+20
      ZERO=-1.0E-10
      CALL UREAL(X,U)
      IF(NCONS.EQ.0) GO TO 2
      CALL CONST(X,PHI,NCONS)
      DO 1 I=1,NCONS
      IF(PHI(I).GE.ZERO) GO TO 1
      SUM1=SUM1 + ABS(PHI(I))
      NVIOL=NVIOL + 1
    1 CONTINUE
      SUM1=CCC*SUM1
    2 IF(NEQUS.EQ.0) GO TO 4
      CALL EQUAL(X,PSI,NEQUS)
      DO 3 I=1,NEQUS
      SUM2=SUM2 + ABS(PSI(I))
    3 CONTINUE
      SUM2=CCC*SUM2
    4 UART=U+SUM1+SUM2
      RETURN
      END
```

```
      SUBROUTINE OPTIF2 (X,U,UART,PHI,PSI,NCONS,NEQUS,NVIOL,R)
C
C     THIS ROUTINE INCORPORATES EQUALITY AND INEQUALITY CONSTRAINTS
C     IN AN ARTIFICIAL OBJECTIVE FUNCTION OF THE FORM...
C
C     UART = U + R*SUM(1.0/PHI(I)) + SUM((PSI(J)**2)/SQRT(R))
C
C     WHERE...PHI(I) = INEQUALITY CONSTRAINTS
C             PSI(I) =   EQUALITY CONSTRAINTS
C
C
C      TO PREVENT VERY MINOR VIOLATIONS OF INEQUALITY CONSTRAINTS ASSUME
C     ZERO=-1.0E-10
C      TO AVOID DIVIDING BY APPROXIMATELY ZERO IN THE INEQUALITY
C     PORTION OF THE ARTIFICIAL OBJECTIVE FUNCTION, ASSUME THAT
C     CONSTRAINTS (ABS(PHI(I)).LT.-ZERO) ARE NOT TO BE PENALIZED.
C      VIOLATIONS OF INEQUALITY CONSTRAINTS ARE MULTIPLIED BY A LARGE
C     CONSTANT ASSUMED HERE TO BE 10.0E+20
C      THIS FORM OF ARTIFICIAL OBJECTIVE FUNCTION KEEPS THE SEARCH AWAY
C     FROM INFEASIBLE REGIONS DURING THE INITIAL SEARCH
C
      DIMENSION X(1),PHI(1),PSI(1)
C
      NVIOL=0
      SUM1=0.0
      SUM2=0.0
      ZERO=-1.0E-10
      CALL UREAL(X,U)
      IF(NCONS.EQ.0) GO TO 3
      CALL CONST(X,PHI,NCONS)
      DO 2 I=1,NCONS
      IF(PHI(I).GT.-ZERO) GO TO 1
      IF(PHI(I).GT.ZERO) GO TO 2
      NVIOL=NVIOL+1
      SUM1=SUM1 + 10.0E+20*ABS(PHI(I))
      GO TO 2
    1 SUM1=SUM1+R/ABS(PHI(I))
    2 CONTINUE
    3 IF(NEQUS.EQ.0) GO TO 5
      DIV=SQRT(R)
      CALL EQUAL(X,PSI,NEQUS)
      DO 4 J=1,NEQUS
      SUM2=SUM2+(ABS(PSI(J))**2)/DIV
    4 CONTINUE
    5 UART=U+SUM1+SUM2
      RETURN
      END
```

```
      SUBROUTINE ANSWER(U,X,PHI,PSI,N,NCONS,NEQUS)
C
C     THIS ROUTINE OUTPUTS THE FINAL SOLUTION IN A
C     STANDARD FORM.
C     IF OPTIMUM IS NOT REACHED (IE. KO =1), THEN
C     THE RESULTS OF THE LAST ITERATION ARE PRINTED.
C
      DIMENSION X(1),PHI(1),PSI(1)
      COMMON/OPTI/KO,NNDEX
C
      CALL UREAL(X,U)
      IF(KO.EQ.0) GO TO 1
      WRITE(6,5)
      WRITE(6,6)U
      GO TO 2
    1 WRITE(6,7)
      WRITE(6,8)U
    2 IF(N.EQ.0) GO TO 3
      WRITE(6,9)(I,X(I),I=1,N)
    3 IF(NCONS.EQ.0) GO TO 4
      CALL CONST(X,PHI,NCONS)
      WRITE(6,10)
      WRITE(6,11)(I,PHI(I),I=1,NCONS)
    4 IF(NEQUS.EQ.0) RETURN
      CALL EQUAL(X,PSI,NEQUS)
      WRITE(6,12)
      WRITE(6,13)(I,PSI(I),I=1,NEQUS)
      RETURN
    5 FORMAT(1H1,22X,25HRESULTS AT LAST ITERATION,/)
    6 FORMAT(29X,3HU =,E16.8/)
    7 FORMAT(1H1,22X,22HOPTIMUM SOLUTION FOUND,/)
    8 FORMAT(20X,12HMINIMUM  U =,E16.8/)
    9 FORMAT(25X,2HX(,I2,3H) =,E16.8)
   10 FORMAT(//20X,22HINEQUALITY CONSTRAINTS/)
   11 FORMAT(23X,4HPHI(,I2,3H) =,E16.8)
   12 FORMAT(//20X,22H  EQUALITY CONSTRAINTS/)
   13 FORMAT(23X,4HPSI(,I2,3H) =,E16.8)
      END
```

APPENDIX B

Step by Step Search Plus False Position Method of Root Determination

In some cases of the gear design analysis it was found that over a certain range used to determine the roots of an equation, the equation would become discontinuous. Application of gradient methods could not guarantee convergence in these cases. To overcome this problem of discontinuity, a step by step search technique followed by a false-position technique was utilized in root determination.

Figure B.1 represents a functional possibility that arises in the gear analysis.

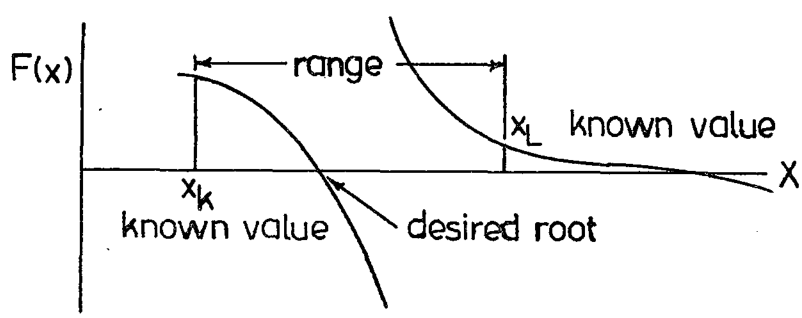


FIGURE B.1 Example Discontinuous Function

Two points were usually known from limits of physical conditions. With these limits specifying a range, a step size of approximately 10% of this range furnished the means for a step-by-step search by increasing the lower point by this increment.

$$X_{K+1} = X_K + \Delta RANGE \tag{B.1}$$

Although this technique does not ensure convergence for all cases, the method seems to work quite well for gear design in the package.

APPENDIX C.1

Parabola as Constant Stress Beam

The whole premise of the Lewis technique depends on the assumption that a parabola represents a constant stress beam. The following presentation is offered as proof of this premise.

We assume a beam, Figure C.1.1 with constant stress on its top and bottom surface. With the bending moment specified as

$$m = wx \tag{C.1.1}$$

the stress is given as

$$\sigma = \frac{mc}{I} = \text{constant} \tag{C.1.2}$$

$$= \frac{(wx)\ (y/2)}{\left(\frac{bY^3}{12}\right)}$$

$$= \frac{6wx}{bY^2}$$

which transposed yields

$$X = \underbrace{\left(\frac{b\sigma}{6w}\right)}_{\text{constant}} Y^2 \qquad \text{(C.1.2.)}$$

This represents a parabola.

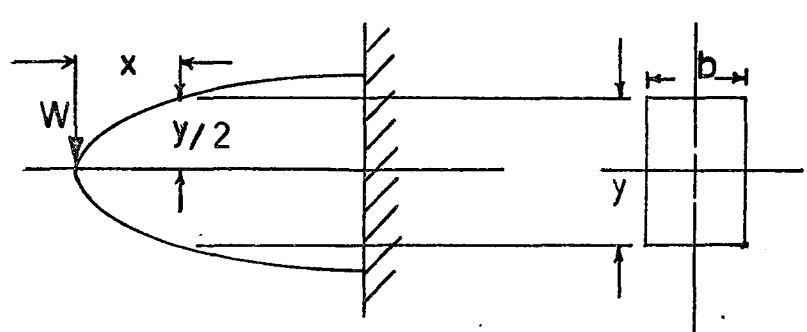


FIGURE C.1.1. Constant Stress Beam

APPENDIX C.2

Method of Establishing Size of Parabola Representing Constant Stress Beam Inside Tooth Profile

Knowing the parabola represents a constant stress beam, a geometric relationship between gear tooth dimensions and this parabola aids in determining the points of maximum stress on the tooth profiles. A stress equivalent may be made to the tooth by inscribing the largest possible parabola in the tooth for analysis. Figure C.2.1 illustrates that stress points A and B may be found with the relation that DE = EB or DC = CF,

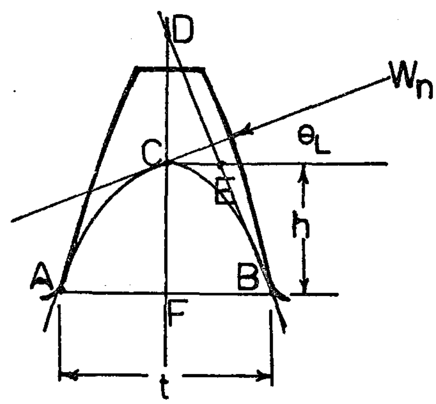


FIGURE C.2.1. Layout of Parabola in Tooth

when the correct parabola is inscribed in the tooth.

In general the formula of a parabola illustrated in Figure C.2.2 is

$$y = ax^2 \tag{C.2.1}$$

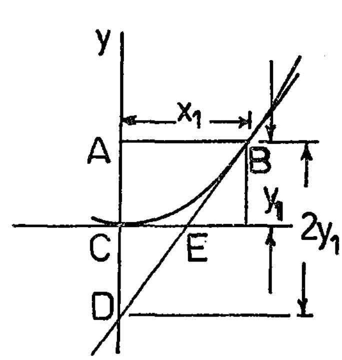


FIGURE C.2.2 Properties of a Parabola

With the function specified for two points, the following conditions must be satisfied

$$|f(X_{K+1})| < |f(X_K)| \qquad (B.2)$$

and $$F(X_K).f(X_{K+1}) < 0 \qquad (B.3)$$

Making the initial test (B.2) prevents the search from diverging if the step length puts the variable beyond the discontinuity, as long as the functional value beyond the discontinuity is greater than the functional value of the variable before. If this test is not satisfied the step length is reduced by half and the process repeated. In most cases the step length of 10% of the range proved suitable for the search. Once the variable straddled the root (B.3), then the false position technique, Figure B.2, was employed.

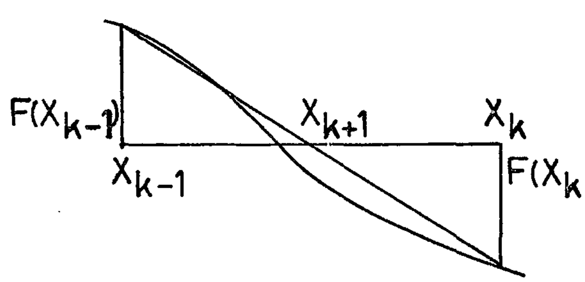


FIGURE B.2 False Position Technique

This process may be iterated by using the approximation to the root of

$$X_{K+1} = \frac{X_{K-1}\ F(X_K) - X_K F(X_{K-1})}{F(X_K) - F(X_{K-1})} \qquad (B.4)$$

and at the same time requiring that

$$F(X_K).F(X_{K-1}) < 0 \qquad (B.3)$$

at each step.

with the gradient or slope B equal to

$$\left(\frac{dy}{dx}\right)_B = 2a\ X_1 \tag{C.2.2}$$

But at B

$$y_1 = a\ X_1{}^2 \tag{C.2.3}$$

or

$$\frac{2y_1}{X_1} = \frac{2a\ X_1{}^2}{X_1} \tag{C.2.4}$$

$$= 2a\ X_1$$

Therefore,

$$\left(\frac{dy}{dX}\right)_B = \frac{2y_1}{X_1} \tag{C.2.5}$$

which proves by similar triangle in Figure C.2.2 that

DE = EB or DC = CF

Thus, the point of tangency on the fillet profile fulfilling this property represents the point of highest stress concentration on the tooth during bending.

APPENDIX D

Extracts from the OPTISEP Manual

The information presented in the appendix depicts the general arrangement of the OPTISEP [47] technique of optimization, with the user description of three optimization methods.

The following layout is typical of an OPTISEP ontimization. The PROGRAM MAIN card is a CDC 6400 computer control card.

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION X(N),PHI(NCONS),PSI(NEQUS),RMAX(N),RMIN(N),XSTRT(N)
      DIMENSION WORK1(N),WORK2(N),WORK3(N),WORK4(N)
      DATA F,G,NSHOT,NTEST,MAXM/0.01,0.01,2,100,300/
      DATA N,NCONS,NEQUS/    ,    ,    /
      READ (5,1)(RMAX(I),RMIN(I),XSTRT(I),I=1,N)
1     FORMAT(3E16.8)
        ...
        ...
        ...
      CALL SEEK1(X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,NEQUS,
     1IDATA,IPRINT,NSHOT,NTEST,MAXM,F,G,U,WORK1,WORK2,WORK2,WORK4)
      CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)
        ...
        ...
        ...
      STOP
      END

      SUBROUTINE UREAL(X,U)
      DIMENSION X(1)
        ...
        ...
        ...
      U =
      RETURN
      END

      SUBROUTINE CONST (X,PHI,NCONS)
      DIMENSION X(1),PHI(1)
      PHI(1) =
      PHI(2) =
        ...
        ...
        ...
      PHI(NCONS) =
      RETURN
      END
```

```
SUBROUTINE EQUAL(X,PSI,NEQUS)
DIMENSION X(1),PSI(1)
PSI(1) =
PSI(2) =
 ...
 ...
 ...
PSI(NEQUS) =
RETURN
END
```

These user inserted routines would then be followed by the necessary auxiliary routines to operate Subroutine SEEK1, or these routines could be stored in binary on permanent files inserted into the program with the use of control cards.

SUBROUTINE UREAL(X,U)

Purpose

To calculate the value of the objective function at a point

$$U = U(X_1, X_2, \ldots, X_n)$$

where U = minimum at the optimum

Method

The objective function may be defined by

(a) a simple FORTRAN arithmetic assignment statement

(b) a complex analysis coded to include any legal FORTRAN statements and/or CALL's to one or more auxiliary subroutines.

Whatever the method of analysis, the final value of the objective function must be placed in U.

Input Variables

X(I) the current values of the independent variables

Output Variables

U the value of the objective function corresponding to the input values X(I)

How to Set up Subroutine UREAL

The following cards must be punched by the user

```
SUBROUTINE UREAL(X,U)
DIMENSION X(1)
 ...
 ...
 ...
U = results of objective function analysis.
RETURN
END
```

Miscellaneous

If additional data is required to perform the analysis, the necessary information should be transformed from the MAIN program or the appropriate subroutine to UREAL through labelled COMMON blocks.

Where possible, the user should include conditional STOP's or logical by-passes of erroneous analysis in his coding to prevent invalid results from being returned to the optimization procedure.

SUBROUTINE EQUAL(X,PSI,NEQUS)

Purpose

To calculate the values of the equality constraints at a point

$$\psi_j = \psi_j(X_1,X_2,\ldots,X_n) \quad j = 1,m$$

where $\psi_j = 0$ at a feasible point.

Method

The equality constraint functions may be defined by

(a) simple FORTRAN arithmetic assignment statements

(b) a complex analysis coded to include any legal FORTRAN statements and/or CALL's to one or more auxiliary subroutines.

Whatever the method of analysis, the final values of the constraints must be stored in the PSI(I) array.

Note: If the user's problem has no equality constraints, then Subroutine EQUAL may be omitted altogether.

Input Variables

X(I) the current values of the independent variables

NEQUS the number of equality constraints.

Output Variables

PSI(I) the value of the equality constraints corresponding to the input values X(I).

How to Set Up Subroutine EQUAL

The following cards must be punched by the user

```
SUBROUTINE EQUAL (X,PSI,NEQUS)
DIMENSION X(1),PSI(1)
PSI(1) = result of equality constraint (1) analysis
PSI(2) = result of equality constraint (2) analysis
```

```
  ...
  ...
  ...
PSI(NEQUS) = result of equality constraint (NEQUS) analysis
RETURN
END
```

Miscellaneous

If additional data is required to perform the analysis, the necessary information should be transferred from the MAIN program or the appropriate subroutine to EQUAL through labelled COMMON blocks.

Where possible, the user should include conditional STOP's or logical by-passes of erroneous analysis in his coding to prevent invalid results from being returned to the optimization procedure.

SUBROUTINE CONST(X,PHI,NCONS)

Purpose

To calculate the values of the inequality constraints at a point

$$\phi_K = \phi_K\ (X_1, X_2, \ldots, X_n) \quad k = 1,p$$

where $\phi_k \geq 0$ at a feasible point.

Method

The inequality constraint functions may be defined by

(a) simple FORTRAN arithmetic assignment statements

(b) a complex analysis coded to include any legal FORTRAN statements and/or CALL's to one or more auxiliary subroutines.

Whatever the method of analysis, the final values of the constraints must be stored in the PHI(I) array.

Note: If the user's problem has no inequality constraints, then Subroutine CONST may be omitted altogether.

Input Variables

X(I)	the current values of the independent variables
NCONS	the number of inequality constraints

Output Values

PHI(I)	the value of the inequality constraints corresponding to the input values X(I).

How to Set up Subroutine CONST

The following cards must be punched by the user.

```
SUBROUTINE CONST(X,PHI,NCONS)
DIMENSION X(1),PHI(1)
PHI(1) = result of inequality constraint (1) analysis
PHI(2) = result of inequality constraint (2) analysis
```

```
  ...
  ...
  ...
PHI(NCONS) = result of inequality constraint (NCONS) analysis
RETURN
END
```

Miscellaneous

If additional data is required to perform the analysis, the necessary information should be transfered from the MAIN program or the appropriate subroutine to CONST through labelled COMMON blocks.

Where possible, the user should include conditional STOP's or logical by-passes of erroneous analysis in his coding to prevent invalid results from being returned to the optimization procedure.

SUBROUTINE SEEK1 (X,PHI,PSI,RMAX,RMIN,XSTRT
N,NCONS,NEQUS,IDATA,IPRINT,
NSHOT,NTEST,MAXM,F,G,U,
WORK1,WORK2,WORK3,WORK4)

Purpose

To minimize $U = U(X_1,X_2,\ldots,X_n)$

subject to $\psi_j(X_1,X_2,\ldots,X_n) = 0 \quad j = 1,m$

$\phi_k(X_1,X_2,\ldots,X_n) \geq 0 \quad k = 1,p$

Method

A direct search method [42] followed by a directed random search is used with the constraints incorporated in an unconstrained artificial objective function of the form:

$$UART = U(X_1,X_2,\ldots,X_n) + 10^{20} \sum |\phi_K(X_1,X_2,\ldots,X_n)| + 10^{20} \sum_{j=1}^{m} |\psi_j(X_1,X_2,\ldots X_n)|$$

[for $\phi_K(X_1,X_2,\ldots,X_n) < 0$]

Starting with an initial base point, an exploratory search is made by incrementing a variable by a small amount. The incremented value of the variable is retained, if UART is improved. However, if the move does not improve UART, then a negative step is tried. If this also fails then the variable is returned to its base point value. Each variable is checked in this manner and if no move improves UART, the step lengths are halfed and the search repeated. If all step lengths are already less than their user-specified minimum values, then an optimum is assumed.

If the search yields a lower value of UART, then a new base point is established and a pattern move equal to the vector joining the original and new base points is attempted. If the pattern move is successful, the new search is started from that point. Otherwise, the search starts from the new base point. This procedure is repeated until MAXM cycles have been exceeded, or until no improvement can be found with all step lengths less than their user specified minima.

It is possible for this type of method to hang up on constraints, or to achieve a local rather than global optimum. For this reason, the final optimum is checked by generating NTEST random values of the artificial objective function in the vicinity of the assumed optimum. If an improved point is found, it becomes the new base point for another search procedure. Only NSHOT complete iterations through the search and directed random check are permitted.

<u>Input Variables</u>

The following program parameters must be set by the user.

N	number of design or independent variables
NCONS	number of inequality constraints
NEQUS	number of equality constraints
IDATA	= 0 input data is not printed out
	= 1 input data is printed out
IPRINT	= 0 intermediate results not printed out
	= integer value to print out intermediate results for every IPRINT cycles.
NSHOT	maximum number of complete cycles through search and directed random search.
NTEST	number of random points to be generated in directed random search.

MAXM maximum number of search cycles

F fraction of range used as initial step size

G fraction of initial step size used as minimum step length

RMAX(I) estimated upper bounds on X(I), dimensioned with value of N

RMIN(I) estimated lower bounds on X(I), dimensioned with value of N

XSTRT(I) input starting values of X(I), dimensioned with value of N

Output Variables

U optimum value of the objective function, evaluated in UREAL

X(I) optimum values of the independent variables, dimensioned with value of N

PHI(I) inequality constraint functions, evaluated in CONST, dimensioned with value of NCONS

PSI(I) equality constraint functions, evaluated in EQUAL, dimensioned with value of NEQUS

Working Arrays

WORK1 dimensioned with value of N

WORK2 dimensioned with value of N

WORK3 dimensioned with N

WORK4 dimensioned with value of N

Programming Information

Generally adequate values for the input variables are

NSHOT = 2

NTEST = 100

MAXM = 300

F = .01

G = .01

XSTRT(I) = (RMAX(I) + RMIN(I))/2.0; a known feasible start is preferable.

If the input value of NCONS or NEQUS is zero, the argument value of PHI or PSI in the calling program dimension statement must be set to one.

SEEK1 is a very fast method but tends to hang up, especially with equality constraints.

SEEK1 has full variable dimensioning. The calling program must provide dimensioning as given above.

If printout of the optimum is desired directly from SEEK1, then the statement CALL SEEK1 in the calling program may be followed immediately by

CALL ANSWER(U,X,PHI,PSI,N,NCONS,NEQUS)

This prints the optimum point and the value of the ϕ's and ψ's. However, there is no way of knowing if SEEK1 has hung up on a constraint or valley and is indicating a false optimum.

If the method has not converged, an appropriate error message is printed and SEEK1 returns to the calling program. The labelled COMMON block [COMMON/OPTI/KO,NNDEX] may be placed in the calling program to detect KO = 1 (i.e. non-optimum solution) if the standard printout of subroutine ANSWER is not desired.

Subroutines called are SEARCH,SHOT,FRANDN,OPTIF1,UREAL,CONST EQUAL,ANSWER.

SUBROUTINE SEEK3 (X,PHI,PSI,RMAX,RMIN,XSTRT,N,NCONS,
NEQUS,IDATA,IPPINT,MAXM,INDEX,
NVIOL,F,G,R,REDUCE,U,WORK1,WORK2,
WORK3,WORK4)

Purpose

To minimize $U = U(X_1 X_2, \ldots, X_n)$

subject to $\psi_j(X_1, X_2, \ldots, X_n) = 0 \quad j = 1,m$

$\phi_k(X_1, X_2, \ldots, X_n) \geq 0 \quad k = 1,p$

Method

SEEK3 employs the same search procedure as SEEK1 except for the artificial, unconstrained objective function defined as

[for $\phi_k(X_1, X_2, \ldots, X_n) \geq$

$$U(X_1, X_2, \ldots, X_n, r_i) = U(X_1, X_2, \ldots, X_n) + \sum \frac{r_i}{\phi_k(X_1, X_2, \ldots, X_n)}$$

$$+ 10^{20} \sum |\phi_k(X_1, X_2, \ldots, X_n)| + \sum_{j=1}^{m} \frac{\psi_j(X_1, X_2, \ldots, X_n)^2}{(r_i)^{1/2}}$$

[for $\phi_k(X_1, X_2, \ldots, X_n) < 0$]

The inequality constraint portion of the function is broken into two parts to account feasible or infeasible starting point conditions. To permit an infeasible starting point (i.e. $\phi_k(X_1, X_2, \ldots, X_n) < 0$), the absolute value of the violated constraint is multiplied by a large number to drive the solution feasible rapidly. In the feasible region (i.e. $\phi_k(X_1, X_2, \ldots, X_n) \geq 0$) the first inequality constraint term controls the function for the K = 1,p inequality constraints. The minimization is controlled by the constant r_i which reduces after each minimization by a constant factor "REDUCE" (i.e. r_{i+1} = REDUCE * r_i where $0 < r_{i+1} < r_i$).

If a feasible point is not found during the first minimization (i.e. min. $U(X_1,X_2,\ldots,X_n, {}_1)$ where $r_1 = 1.0$ is reasonable starting value), then it is assumed that no feasible solution exists for the problem and an error message is printed out. With the optimization forced feasible initially, a feasible starting point for minimizing $U(X_1,X_2,\ldots,X_n, r_{i+1})$ should be obtained from the feasible solution for $U(X_1,X_2,\ldots,X_n,r_i)$.

<u>Input Variables</u>

The following program parameters must be defined by the calling program.

N	number of design or independent variables
NCONS	number of inequality constraints
NEQUS	number of equality constraints
IDATA	= 0 input data is not printed out
	= 1 input data is printed out
IPRINT	= 0 intermediate results not printed out
	= integer value to print out intermediate results for every IPRINT cycles.
MAXM	maximum number of search cycles.
INDEX	set equal to one
F	fraction of range used as initial step size
G	fraction of initial step size used as minimum step length
R	penalty multiplier for constraints
REDUCE	reduction factor for penalty multiplier after each minimization.
RMAX(I)	estimated upper bounds on X(I), dimensioned with value of N

RMIN(I)	estimated lower bounds on X(I), dimensioned with value of N
XSTRT(I)	input starting values of X(I), dimensioned with value of N

<u>Output Variables</u>

NVIOL	counter of the number of inequality constraints violated
RR	current value of the penalty function multiplier (RR printed out only when IPRINT > 0)
U	minimum value of the objective function, evaluated in UREAL
X(I)	optimum values of the independent variables, dimensioned with value of N
PHI(I)	inequality constraint functions, evaluated in CONST, dimensioned with the value of NCONS
PSI(I)	equality constraint functions, evaluated in EQUAL, dimensioned with the value of NEQUS

<u>Working Arrays</u>

WORK1	dimensioned with value of N
WORK2	dimensioned with value of N
WORK3	dimensioned with value of N
WORK4	dimensioned with value of N

<u>Programming Information</u>

R,REDUCE - The values used for R and REDUCE can affect the rate of convergence but are otherwise fairly problem - independent. A detailed discussion of criteria for choosing R and REDUCE is given in reference [44],section 8.5.

SEEK3 terminates when either

(a) $$\frac{U_{i+1}(X_1,X_2,\ldots,X_n) - U_i(X_1,X_2,\ldots,X_n)}{U_i(X_1,X_2,\ldots,X_n)} < 10^{-7}$$ or

(b) $RR < 10^{-20}$

Generally adequate values for the input variables are

F	=	.01
G	=	.01
R	=	1.0
REDUCE	=	.05
MAXM	=	300
XSTRT(I)	=	(RMAX(I) + RMIN(I)/2.0; a known feasible start is preferable.

If the input value of NCONS or NEQUS is zero, the argument value of PHI or PSI in the calling program dimension statement must be set to one.

SEEK3 has full variable dimensioning. The calling program must provide dimensioning as given above.

If printout of the optimum is desired directly from SEEK3, then the statement CALL SEEK3 in the calling program must be followed immediately by

CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)

This prints out the optimum point and the value of the ϕ's and ψ's. However, there is no way of knowing if SEEK3 has hung up on a constraint or valley and is indicating a false optimum.

If the method has not converged, an appropriate error message is printed out and SEEK3 returns to the calling program.

The labelled common block [COMMON/OPTI/KO,NNDEX] may be placed in the calling program to detect KO = 1 (i.e. non-optimum solution) if the standard printout of subroutine ANSWER is not desired.

Subroutines called are SEARCH,OPTIF2,ANSWER,UREAL,CONST and EQUAL.

SUBROUTINE NPFMIN (X,PHI,PSI,RMAX,RMIN,XSTRT, N,NCONS,NEQUS,IDATA,IPRINT, MAXM,F,G,R,REDUCE,U,DD,EE,FF, GG,HH)

Purpose

To minimize $U = U(X_1, X_2, \ldots, X_n)$

subject to $\psi_j(X_1, X_2, \ldots, X_n) = 0 \quad j = 1,m$

$\phi_k(X_1, X_2, \ldots, X_n) \geq 0 \quad k = 1,p$

Method

NPFMIN employs the same artificial unconstrainted objective function as SEEK3 coupled with a gradient search alogarithm [45]. This method of search evaluates the new value of an independent variable at the K+1 step as

$$X_i^{k+1} = X_i^k + h_i^k$$

where h_i^k defines a step length which is a function of the partial derivatives at X_i^k and all the derivatives at the previous steps.

The search is considered optimum if the value of U does not change significantly in two successive steps.

Subroutine SLOPE, uses the finite difference technique to evaluate the gradients which are then normalized to prevent the wide disparity in gradients magnitudes created by the artificial objective function.

Input Variables

The following program parameters must be defined by the calling program.

N number of design or independent variables

NCONS	number of inequality constraints
NEQUS	number of equality constraints
IDATA	= 0 input data is not printed out
	= 1 input data is printed out
IPRINT	= 0 intermediate results not printed out
	= integer value to print out intermediate results for every IPRINT iterations.
MAXM	maximum number of search iterations
F	fraction of range used a step length for finite difference gradient determination
G	fraction of range used as step size minima for optimum termination. Should not be less than value of F.
R	penalty multiplier for constraints
REDUCE	reduction factor for penalty multiplier after each minimization.
RMAX(I)	estimated upper bounds on X(I), dimensioned with value of N
RMIN(I)	estimated lower bounds X(I), dimensioned with value of N
XSTRT(I)	initial starting values of X(I), dimensioned with value of N

<u>Output Variables</u>

RR	current value of the penalty function multiplier (RR printed out only when IPRINT>0)
U	minimum value of the objective function, evaluated in UREAL
X(I)	optimum values of the independent variables, dimensioned with value of N
PHI(I)	inequality constraint functions, evaluated in CONST, dimensioned with the value of NCONS
PSI(I)	equality constraint functions, evaluated in EQUAL dimensioned with the value of NEQUS

Working Arrays

DD	dimensioned with value of N
EE	dimensioned with value of N
FF	dimensioned with value of N
GG	dimensioned with value of N
HH	dimensioned with value of N(N+7)/2

Programming Information

R,REDUCE - The values used for R and REDUCE can affect the rate of convergence but are otherwise fairly problem - independent. A detailed discussion of criteria for choosing R and REDUCE is given in reference [44], section 8.5.

NPFMIN terminates when either

(a) $$\frac{U_{i+1}(X_1,X_2,\ldots,X_n) - U_i(X_1,X_2,\ldots,X_n)}{U_i(X_1,X_2,\ldots,X_n)}$$

(b) $RR < 10^{-20}$

Generally adequate values for the input variables are

F = .0001

G = .0001

R = 1.0

REDUCE = .05

MAXM = 100

XSTRT(I) = (RMAX(I) + RMIN(I))/2.0; a known feasible start is preferable

If the input value of NCONS or NEQUS is zero, the argument value of PHI or PSI in the calling program dimension statement must be set to one.

NPFMIN has full variable dimensioning. The calling program must provide dimensioning as given above.

If printout of the optimum is desired directly from NPFMIN, then the statement CALL NPFMIN in the calling program must be followed immediately by

CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)

This prints out the optimum point and the value of the ϕ's and ψ's. However, there is no way of knowing if NPFMIN has hung up on a constraint or valley and is indicating a false optimum.

If the method has not converged, an appropriate error message is printed out and NPFMIN returns to the calling program. The labelled COMMON block [COMMON/OPTI/KO,NNDEX] may be placed in the calling program to detect KO = 1 (i.e. non optimum solution) if the standard printout of Subroutine ANSWER is not desired.

Subroutine called are SLOPE,MAXMUM,OPTIF2,ANSWER,UREAL, CONST and EQUAL.

```
SUBROUTINE FEASBL (X,PHI,PSI,RMAX,RMIN,XSTRT,
                   N,NCONS,NEQUS,IDATA,IPRINT,
                   MAXM,F,G,U,STEPP,WORK1,WORK2,
                   WORK3,WORK4)
```

Purpose

Subroutine FEASBL is used to obtain a feasible starting point. Initially subroutine SEEK3 is employed to drive the inequality constraints feasible, without continuing to an optimum after which a direct search in the feasible region drives the equality constraints feasible, using the sum of the equality constraints as the objective function.

Input Variables

The following program parameters must be defined by the calling program.

N	number of design or independent variables
NCONS	number of inequality constraints
NEQUS	number of equality constraints
IDATA	= 0 input data is not printed out
	= 1 input data is printed out
IPRINT	= 0 intermediate results not printed out
	= integer value to print out intermediate results after every IPRINT cycles.
MAXM	maximum number of search cycles
F	fraction of range used as initial step size
G	fraction of initial step size used as minimum step length
RMAX(I)	estimated upper bounds on X(I), dimensioned with value of N

RMIN(I)	estimated lower bounds on X(I), dimensioned with value of N
XSTRT(I)	inout starting values of X(I), dimensioned with value of N

Outout Variables

U	feasible value of objective function
X(I)	feasilbe values of independent variables, dimensioned with value of N
PHI(I)	inequality constraint function, evaluated in CONST, dimensioned with value of NCONS
PSI(I)	equality constraint function, evaluated in EQUAL, dimensioned with value of NEQUS

Working Arrays

STEPP	dimensioned with value of N
WORK1	dimensioned with value of N
WORK2	dimensioned with value of N
WORK3	dimensioned with value of N
WORK4	dimensioned with value of N

Programming Information

Generally useful input values are

MAXM = 300

F = .01

G = .01

XSTRT(I) = (RMAX(I) + RMIN(I))/2.0

If the input value of NCONS or NEOUS is zero, the arguments of PHI or PSI in the calling program dimension statement must contain one.

FEASBL has full variable dimensioning. The calling program must provide dimensioning as given above.

If printout of the feasible solution is desired directly from FEASBL, then the statement CALL FEASBL in the calling program may be followed immediately by

```
CALL ANSWER (U,X,PHI,PSI,N,NCONS,NEQUS)
```

This prints out the feasible point and the values of the ϕ's and ψ's.

Subroutines called are SEEK3, SEARCH,OPTIF2,SUMPSI, UREAL,EQUAL,CONST,ANSWER.

APPENDIX E

AUXILIARY ROUTINES

A number of subroutines have been incorporated into the computer which are not related to either the optimization technique or the actual spur gear design. These routines have been developed to arrange the independent and dependent variables, to write out appropriate data and error messages and generally provide the controlling mechanism of the overall package. The following presents a brief summary of the purpose and operation of these routines.

SUBROUTINE SPUR

Use	:	Main calling program for the gear design from which the optimization routines are called
Calling Sequence	:	The user's calling routine made of the appropriate labelled common blocks, the input variable list and the CALL SPUR statement call this routine.
Special Features	:	(1) It determines if system is pinion or gear drive, (2) It uses Subroutine VARY1 to suggest starting values for design variables. (3) It defines optimization criteria control array. (4) It defines constant dependent variables of design. (5) It redefines starting values if user specified. (6) It calls optimization routine, if required. (7) It has output results and message printed if requested.

SUBROUTINE VARY1

Use	:	To determine the design variables in the problem, organize control arrays, establish initial starting values and print input data if required.
Calling Sequence	:	Subroutine SPUR calls this routine

Special Features : (1) It prints out input data, if requested. (2) It determines status of design variables (i.e. constant, standard or variable). (3) It prints out status, if requested. (4) It initializes values of variables and standard quantities. (5) It prints out relative positions of design variable pseudonyms in optimization independent variable array X(I).

SUBROUTINE VARY (X)

Use : To equivalence pseudonyms of gear design with variable array X(I) during optimization search.

Calling Sequence : First routine called in Subroutine UREAL

Special Features : To keep all variables positive the gear design pseudonyms are equated to the absolute value of the variable array X(I). In effect the objective function of a negative variable becomes the mirror images of its positive variable equivalent.
The addendum and dedendum values specify the addendum dedendum circle radii and the clearance at the end of this routine ensuring that clearance will never be negative.

SUBROUTINE PRINT

Use : To print out the spur gear design output in a standard format.

Calling Sequence : Subroutine SPUR calls this program after the analysis is complete.

Special Features : The format for all printing has not been established for terminal typewriter or scope output.

SUBROUTINE HINT (PHI)

Use : To print out suggested remedies for violated constraints for non-optimum results

Calling Sequence : Subroutine SPUR calls this routine before entering Subroutine PRINT.

Special Features : If violations occur in the constraints each constraint is checked, with error message and suggested remedy printed for each violation. These suggested remedies are not the only method of achieving a feasible solution. Close examination of the output results may indicate an easier solution.

APPENDIX F

INDEX OF SUBROUTINES

The following information enables the reader to find the related theory and the program listing for a particular subroutine. After a brief description of each subroutine, brackets [ ] will enclose the section containing the appropriate theory and parenthesis ( ) will indicate the program listing location in Appendix A.

FFACT	determines surface finish factors for stress analysis [2.8G] (A.20)
FILLET	determines radius to point of intersection of the fillet and involute profiles [2.4] (A.2)
FRANDN	random number generator (A.50)
HFACT	determines hardness ratio factor for stress analysis [2.84] (A.21)
HINT	prints suggested remedies for violated constraints [Appendix E] (A.37)
IFACT	determines geometry factor for wear stress analysis [2.8A] (A.14)
JFACT	determines geometry factor for bending stress analysis [2.8A] (A.12)
LENGTH	determines length of approach and length of recess for undercut conditions [2.10] (A.27)
LFACT	determines life factors for stress analysis [2.8I] (A.22)
LOAD	determines radius to load application on the gear tooth centre line [2.7A.1] (A.7)
MAXMUM	determines maximum value of an array (A.47)
MFACT	determines the load distribution factors for stress analysis [2.8D] (A.17)
NPFMIN	gradient method optimization routine [Appendix D] (A.43)
OFACT	determines onverload factor for stress analysis [2.8E] (A.18)
OPTIF1 OPTIF2	determine two different artificial unconstrained optimization functions (A.51, A.52)
PITCH	determines some fundamental geometric relations in the gear design [2.5] (A.4)
POWER	determines the maximum permissible power transmission for bending and wear analysis [2.7C] (A.11)
PRINT	prints design output in standard format [Appendix E] (A.36)

RFACT	determines reliability factor for stress analysis [2.8J] (A.23)
SEARCH	direct search optimization technique (A.44)
SEEK1 } SEEK3 }	direct search optimization techniques with artificial unconstrained optimization functions [Appendix D] (A.41, A.42)
SFACT	determines size connection factor for stress analysis [2.8F] (A.19)
SHARE	determines if load sharing exists between successive pairs of mating teeth [2.7A.] (A.8)
SHOT	a directed random search technique employed with Subroutine SEEK1 (A.45)
SIZE	determines dimensions of gear blank [2.14] (A.32)
SLOPE	determines gradients by finite difference technique (A.46)
SPUR	main calling program for the gear design [Appendix E] (A.38)
SUMPSI	determines sum of equality constraint violations in Subroutine FEASBL (A.49)
TFACT	determines temperature correction factor for stress analysis [2.8K] (A.24)
TLOAD	determines load components of tooth loading [2.6] (A.6)
TOLCD	determines centre distance tolerance and maximum backlash at worst case [2.12, 2.13] (A.31)
TORQUE	determines the applied torque on the gear [2.6] (A.5)
VARY	provides link between design variables and optimization routine [Appendix E] (A.35)
VARY1	organizes various control arrays and suggests values to be used in optimization routine [Appendix E] (A.34)
UREAL	determines the optimization criterion as well as computing analysis of gear design [Appendix D] (A.39)
VFACT	determines velocity correction factor for stress analyses [2.8C] (A.16)

VOLUME	determines volume of gear blank [2.14] (A.33)
WEAR	determines the actual and allowable wear stresses [2.7B] (A.10)

APPENDIX G

USER'S GUIDE

1.1 INTRODUCTION

This package is one of a series for the automatic optimum design of engineering components or devices. Its purpose is the optimum design of external spur gears. The package is user oriented and requires no knowledge of computer programming (FORTRAN) or optimization. While the package can be used with a minimal knowledge of, or experience in, gear design, this practice is risky. Judgement cannot be completely removed from any design, nor can a successful design be guaranteed by any theoretical analysis if the design assumptions do not match the real life situation. The user exercises his judgement by options available in the input coding.

The program is basically self-sufficient, offering the gear designer the conglomeration of information from various accepted design sources brought together into one package. To assist in the event of difficulties with the package or as a reference for further development, the user should see reference [1] which develops all aspects of the package.

1.2 CONFIGURATION

Figure 1.1 illustrates the basic gear geometry on which the independent or design variables are listed.

1.3 OPTIMIZATION CRITERION

The user can select some or all of the following as the optimization criterion, in any combination:

(a) minimum volume of gear set

(b) maximum contact ratio

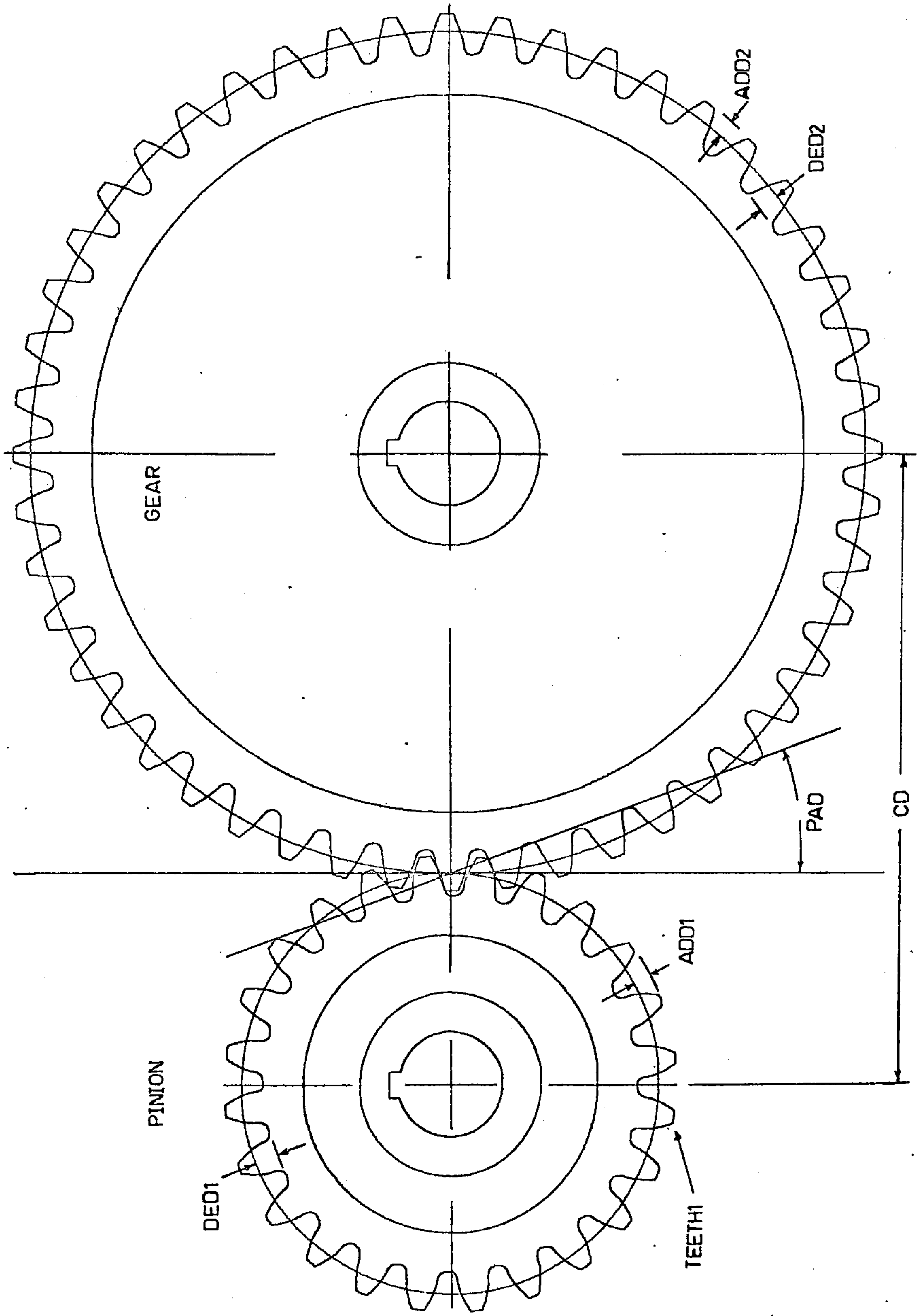


FIGURE 1.1 Gear Layout.

(c) minimum centre distance

(d) minimum face width

If the volume is optimized, then the centre distance and face width will be optimized also if they have been defined as design variables.

Since the above criteria may be used in any combination, multifactor optimization was implemented to achieve the overall criterion. Each criteria is developed in terms of a linear utility function between zero and one which expresses the importance of dependent variables over the range of its minimum and maximum values. For example, high desirability or utility (e.g. utility = 1) occurs for minimum volume specified as function of face width and centre distance minima, for maximum contact ratio arbitrarily chosen as 2, and for minimum face width and minimum centre distance specified by the user. On the other hand, low utility (e.g. utility = 0) occurs for maximum volume expressed as function of face width-centre distance maxima, for minimum contact ratio arbitrarily chosen as 1, and for maximum face width and maximum centre distance specified by the user. (See Figure 1.3.1 and 1.3.2)

The overall criterion is achieved by minimizing the sum of the reciprocals of these individual utilities. No attempt has been made to make one criteria more important in the program. However, the user may partially weight the importance of face width and centre distance and, therefore, volume, by employing a large minimum-maximum range for either variable. This approach tends to give less importance to the variable as it approaches

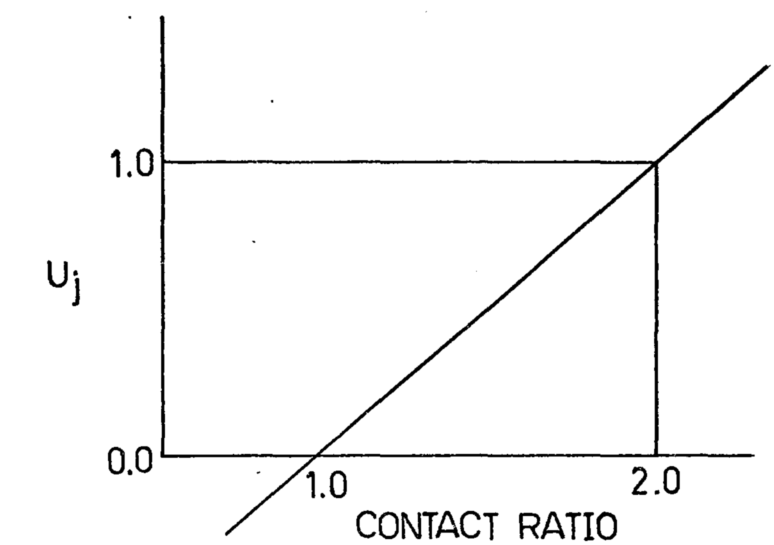


FIGURE 1.3.1 Utility Function (Contact Ratio)

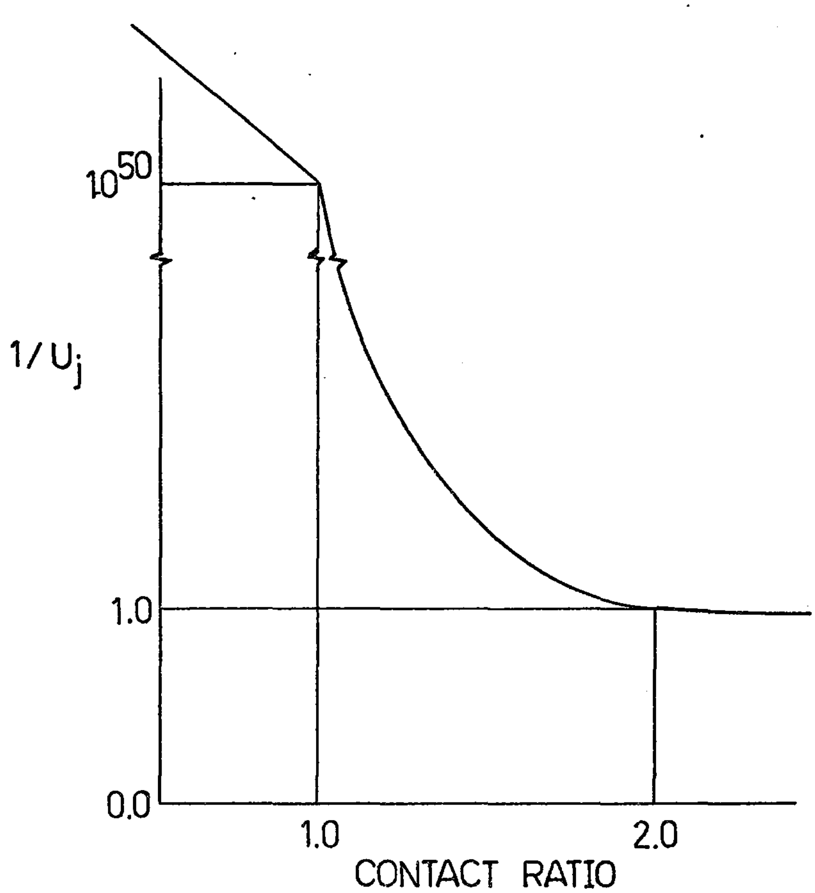


FIGURE 1.3.2 Modified Inverse Utility (Contact Ratio)

a feasible region near the lower portion of its range. However, by compressing the range, the variable takes on greater importance over most values, especially if the variable is near the maximum limit. Note that these limits also specify step lengths for the direct search optimization techniques and, therefore, their values should be chosen with discretion. See reference [1] for further details on multifactor optimization.

1.4 INPUT DATA

The user must define quantities for all the following input variables utilizing FORTRAN DATA statements, READ statements, or individual arithmetic assignment statements. The format of the numbers (i.e. integer or floating point) must be specified correctly where noted. An integer number is expressed with no decimal point (e.g. 21), while floating point numbers are expressed with a decimal point (e.g. 21.6 or 2.16E+01). Note that the number of teeth in a gear is an integer value, but must be specified in floating point notation (e.g. 21.0).

Printout Control (integer numbers)

```
IDATA      = 1  input data printed out
           = 0  input data not printed out

IPRINT     = integer value for printing every IPRINT cycles of
             intermediate results.
           = 0  intermediate results not printed out

IWRITE     = 1  final results printed out
           = 0  final results not printed out

NTYPE      = 1  optimization by Subroutine SEEK1
           = 2  optimization by Subroutine SEEK3
           = 3  optimization by Subroutine NPFMIN
                NOTE:  A value for NTYPE must be specified even
                when no optimization will occur.  For consistency
                set NTYPE = 1 in these cases.
```

Material Properties (floating point numbers)

BHN1 BHN2	Pinion Gear	Brinell Hardness

Typical Gear-Pinion Hardness Combinations[2,3]

Gear Bhn	Pinion Bhn
180	210
210	245
225	265
245	285
255	300
270	315
285	335
300	350

E1 E2	Pinion Gear	modulus of elasticity (psi)
RHO1 RHO2	Pinion Gear	material density (lb/in^3)
U1 U2	Pinion Gear	Poisson's ratio
SAC1 SAC2	Pinion Gear	allowable contact stress number, or surface endurance limit (psi)

NOTE: An allowable contact stress number for 10 million cycles of load application is determined by field experience, for each material and condition of that material. This number varies considerably with heat treatment. The designer should consult any new American Gear Manufacturers Association (AGMA) rating practices as they come available, and use these contact stress numbers whenever applicable. The contact stress numbers listed in Table 1.1 may be used as a guide, with the lower values of the chart suggested for general design purposes. The upper values may be used when high quality material is used, when section size and design allows maximum response to heat treatment, and when proper control is effected by adequate inspection.

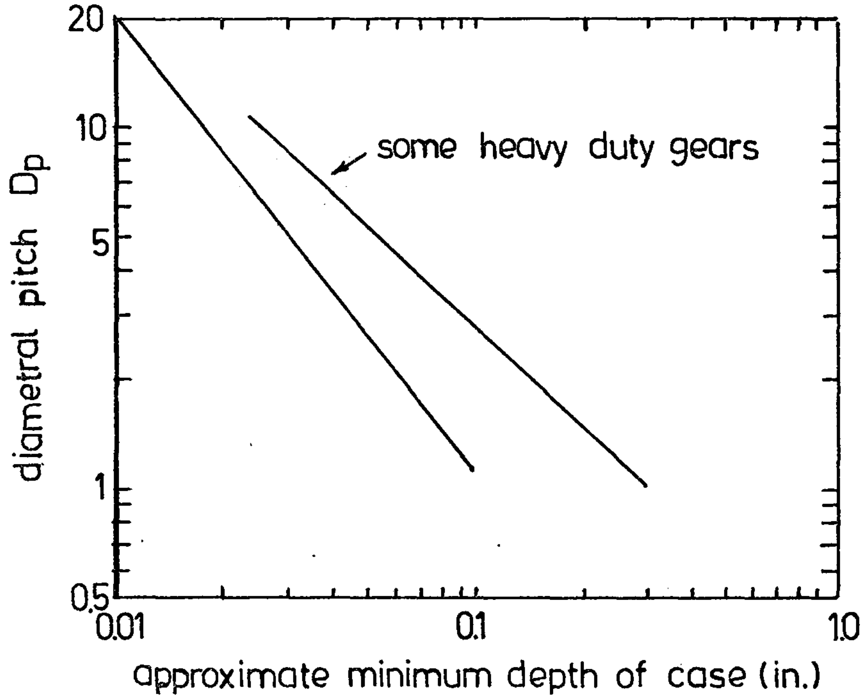
TABLE 1.1 Allowable Contact Stress Number - S_{ac}

Material	Surface Hardness, Minimum	s_{ac}	Material	Surface Hardness, Minimum	s_{ac}
Steel	Through Hardened		Cast Iron		
	180 Bhn	85-95,000	AGMA Grade 20		50-60,000
	240 Bhn	105-115,000	AGMA Grade 30	175 Bhn	65-75,000
			AGMA Grade 40	200 Bhn	75-85,000
	300 Bhn	120-135,000	Nodular Iron		90-100% of the s_{ac} value of steel with the same hardness
	360 Bhn	145-160,000	Annealed	165 Bhn	
	440 Bhn	170-190,000	Normalized	210 Bhn	
			Oil Quench and Temper	255 Bhn	
	Case Carburized (see Note 1)				
	55 R_c	180-200,00	Bronze	Tensile Strength psi (min)	s_{ac}
	60 R_c	200-225,000	Tin Bronze AGMA 2C(10-12% Tin)	40,000	30,000
	Flame or Induction Hardened		Aluminum Bronze ASTM B 148-52 (Alloy 9C-H.T.)	90,000	65,000
	50 R_c	170-190,000			

Note 1. For minimum case depths at the pitch diameter as shown in Figure 1.2

Material	Material hardness Min.	s_{af}-psi
Steel		
Case Carburized and Hardened	55 R_c	55-65,000
Induction or Flame Hardened		
Hard Root	300 Bhn	use values from Figure 1.3
Unhardened Root	-	22,000
Cast Iron		
AGMA Grade 20	-	5,000
AGMA Grade 30	175 Bhn	8,500
AGMA Grade 40	200 Bhn	13,000

TABLE 1.2 Allowable Fatigue Design Stress - S_{af}



NOTE: Total case depth is the depth at which the carbon content of the case and core are equal.

FIGURE 1.2 Depth of Case at Pitch Line

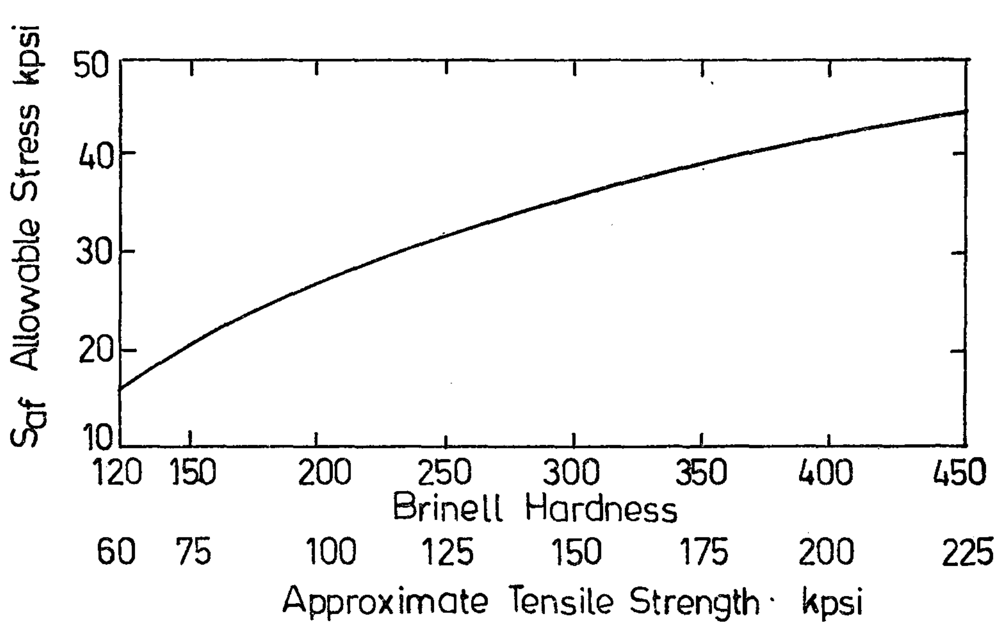


FIGURE 1.3 Allowable Fatigue Stress for Steel Gears - S_{af}

SAF1	Pinion	allowable fatique stress, or
SAF2	Gear	allowable design bending stress (psi)

NOTE: An allowable design bending (fatigue) stress for 10 million cycles of load application is determined by field experience, for each, material and condition of that material. This stress varies considerably with heat treatment, forging or casting practice, material composition and with various surface treatments. Frequently, shot peening permits a higher allowable stress to be used. The allowable fatigue design stresses for steel, shown in Figure 1.3 are values suggested for general design purposes, while values for surface hardened steel and other materials are shown in Table 1.2. Use 70 percent of the allowable fatigue design stress values for idler gears and other gears where the teeth are loaded in both directions.

Design Requirements (floating point numbers)

HP — Horsepower transmitted

RPMI — Input shaft speed (RPM)

RPMO — Output shaft speed (RPM)

SHAFTI — Input shaft diameter (inches)

SHAFTO — Output shaft diameter (inches)
NOTE: The shaft diameters are only required as part of the gear blank volume determination. No constraints affecting gear size or radial loading as a function of shaft diameter have been incorporated. If the final solution results in a gear of smaller diameter than the shaft, the gear and shaft must be made as one, or the shaft has been overdesigned.

INDEPENDENT or DESIGN VARIABLES

The package has been arranged so that each design variables may be incorporated into the routine in three possible modes of operation: constant, variable or standard, where applicable. If all design variables are given constant values by the user, only a feasibility analysis will be carried out. Variables specified as standard will become internal functions of other variables. When a design variable is specified as variable, the optimization routine chooses values which enable a minimum optimization criterion to be achieved, if possible.

CD = floating point number for constant centre distance (inches)

= 3HVAR for variable centre distance

FW = floating point number for constant face width (inches)

= 3HVAR for variable face width

PAD = floating point number for constant pressure angle (degrees)

= 3HVAR for variable pressure angle

TEETH1 = floating point number for constant number of teeth

= 3HVAR for variable number of teeth

NOTE: The number of gear teeth results automatically from the presentation of the number of pinion teeth through the gear ratio. $\left(N^G = m_g\ N^P\right)$

ADDK1 Pinion
ADDK2 Gear = floating point number for addendum coefficient, defined by
ADDK1 = Dp x addendum of pinion
ADDK2 = Dp x addendum of gear

= 3HSTD for AGMA standard addendum specified as 1.0/Dp

NOTE: This addendum system is used in conjunction with the 20° and 25° pressure angle systems.

= 3HVAR for variable addendum determination. In this case the addendum values are determined directly without reference to the diametral pitch Dp

= 3HCON for a constant addendum. This facility is utilized only when analysis is desired for a given gear design (i.e. all design constants) variables are user specified. The actual floating point value of the addendum must be specified in input variables ADD1 and ADD2 for the pinion and gear respectively.

NOTE: The addendum coefficient for pinion and gear may be selected independently of each other from the above possibilities.

DEDK1 DEDK2	Pinion Gear }	= floating point number for dedendum coefficient defined by DEDK1 = Dp x dedendum of pinion DEDK2 = Dp x dedendum of gear
	= 3HSTD	for AGMA standard dedendum specified as $1.25/D_p$ for $D_p \leq 20$, or $\frac{1.200}{D_p} + 0.002$ for $D_p > 20$ NOTE: This dedendum system is used in conjunction with the 20° pressure angle system. The 25° pressure angle is specified as $1.25/D_p$ for the whole diametral pitch range. Thus for a 25° pressure angle system set DEDK1 = 1.25 or DEDK2 = 1.25.
	= 3HVAR	for a variable dedendum determination. In this case the dedendum values are determined directly without reference to the diametral pitch D_p.
	= 3HCON	for constant dedendum. This facility is utilized only when analysis is desired for a given gear design (i.e. all design variables user specified constants). The actual floating point value of the dedendum must be specified in input variables DEDK1 and DEDK2 for the pinion and gear respectively.

NOTE: The dedendum coefficient for pinion and gear may be selected independently of each other from the above possibilities.

ADD1 ADD2	Pinion Gear }	addendum

= floating point number for constant addendum (inches)

NOTE: In this case we must have

$$\left.\begin{array}{l}\text{ADDK1}\\\text{ADDK2}\end{array}\right\} = \text{3HCON}$$

This value need not be specified if the addendum coefficient has a notation other than 3HCON.

DED1 Pinion } dedendum
DED2 Gear }

= floating point number for constant dedendum (inches)
NOTE: In this case we must have
DEDK1 }
DEDK2 } = 3HCON
This value need not be specified if the dedendum coefficient has a notation other than 3HCON.

NOTE: Variable in the sense used above (i.e. 3HVAR) indicates that the values are computer determined.

SIZE LIMITS (floating point numbers)

CDMAX Upper centre distance limit (inches)

CDMIN Lower centre distance limit (inches)

FWMAX Upper face width limit (inches)

FWMIN Lower face width limit (inches)

PADMAX Upper pressure angle limit (degrees)

PADMIN Lower pressure angle limit (degrees)

NOTE: Since these limit values are used to evaluate the initial step size (i.e. 10% of range) for the direct search optimization technique, an extremely large range of the limits should not be taken even though the user does not care about the limiting values. If a feasible answer is known for the design, the values may be set to some proportion above and below the feasible solution so that the initial search will be in the middle of the range. On the other hand, if a feasible answer is not known, values of 0 and 100 inches for the lower-upper centre distance limits and 0° and 45° for the angular limits are reasonable values. Effective face width limits may be determined by multiplying the centre distance limits by (2.0/(RATIO + 1.0)) where RATIO is the ratio of the faster shaft speed. More reasonable design limits will become evident after the initial optimization run. Centre distance may be given a larger weight relative to the face width in the optimization criterion (see section 1.3) by lowering the maximum centre distance limit and increasing the maximum face width limit.

The pressure angle step size has been arbitarily chosen as 1° so that the limits will not alter the search in the same manner as the length limits.

The final value of face width should not exceed the final centre distance value times (2.0/(RATIO + 1.0)), so as to remain within the load distribution analysis of the program.

DESIGN CONTROL FACTORS (floating point numbers)

BLL Lower backlash limit modification factor

BLU Upper backlash limit modification factor

BLR Ratio of pinion tooth thinning to total tooth thinning

CDR Centre distance tolerance modification factor

NOTE: Suggested values for minimum and maximum backlash have been incorporated in algebraic functions ($y = ax^b$) with the modification factors increasing or decreasing the "a" term. For normal power gearing, BLL = BLU = 1.0 is suggested, while for control gearing BLL = BLU = 0.0 results in zero delibrate tooth thinning, but an equivalent tooth thinning to represent an error envelope over the gear tooth is produced.

The tooth thinning ratio is normally BLR = 0.5, with each gear accepting half of the tooth thinning allowance for minimum backlash. If the pinion has a small number of teeth which have been enlarged to avoid problems of undercut (i.e. smaller ratio of the number of teeth to pitch circle diameter), the gear may take a greater portion of this backlash allowance in order to avoid the absurdity of increasing the tooth thickness to avoid undercut and then thinning the tooth to introduce backlash. In such cases the value of BLR may range between 0.0 and 0.5.

The suggested centre distance tolerances for the various AGMA quality classes have been incorporated in linear equations ($y = ax + b$), with the resultant tolerance directly proportional to the linear result times the modification factor. The CDR factor has been introduced to provide a degree of flexibility, and it expresses the ratio of the desired centre distance tolerance to the computer suggested centre distance tolerance. Since the computer values reflect AGMA suggestions, CDR = 1.0 should be generally utilized.

CYCLE Required number of life cycles of operation.

NOTE: The fatigue strength of a material varies as a function of the number of loading cycles. For steel, this strength decreases with increased loading cycles until the endurance limit is reached, after which the fatigue strength basically remains constant for increased loading cycles. The AGMA has found that, for steels, the endurance limit occurs generally at 10^7 cycles, with the fatigue strength at lesser cycles determined by a general life factor function. Table 1.1 and 1.2 along with Figure 1.3 represent suggested fatigue strength or endurance limits for various steels.

If the fatigue limits are being used for materials not listed in these tables, the actual fatigue strength of the material at the required number of life cycles of operation should be specified in SAF1, SAF2, SAC1 and SAC2 while the value of CYCLE should be set greater than 10^7. This procedure bypasses the life function specified in the program for the steels mentioned in the previous charts.

DRIVEN — Driven machine loading mode

= 1.0 load on driven machine - uniform
= 2.0 load on driven machine - moderate
= 3.0 load on driven machine - heavy

NOTE: Table 1.3 may aid in the selection of the loading mode of the driven machinery.

DRIVER — Power source loading mode

= 1.0 power source - uniform
= 2.0 power source - light shock
= 3.0 power source - medium shock

NOTE: Table 1.4 may aid in the selection of the loading mode of the driven machinery.

RELI — Reliability of design

NOTE: For general applications set RELI = 0.99 while for more reliable designs set RELI = 0.999

TEMP — Oil or gear blank temperatures (degrees Fahrenheit)

<u>DESIGN CONTROL FACTORS</u> (integer numbers)

NCUT1 — Pinion $\big\}$ Hob cutter type
NCUT2 — Gear

= 1 if gear cut by rack or hob with sharp cornered teeth

= 2 if gear cut by rack or hob with rounded corners.

TABLE 1.3 Load Classifications for Various Applications.[4]

The following table lists and classifies the character of the load in various applications for gearmotors. A gearmotor is defined as the combination of an enclosed gear drive and an elective motor with the frame of one component supporting the other, and with the motor shaft common with or directly coupled to the input pinion shaft. Thus, these values should only be taken as minimum suggestions in classifying an application. The pitch line velocity during operation should not exceed 5000 fpm.

Nomenclature :

U = uniform

M = medium shock

H = heavy shock

L.C. = load classification

Application	L.C.	Application	L.C.
AGITORS		Scale Hopper	
Pure Liquids	U	Frequent Starts	M
Liquids and Solids	M	CAN FILLING MACHINES	U
Liquids -- Variable		CANE KNIVES	M
Density	M	CAR DUMPERS	H
Semi-liquids		CAR PULLERS -- Intermittent	
--Variable Density	M	Duty	M
BLOWERS		CLARIFIERS	U
Centrifugal		CLASSIFIERS	M
Lobe		CLAY WORKING MACHINERY	
Vane		Brick Press	H
BREWING and DISTILLING		Briquette Machine	H
Bottling Machinery	U	Clay working Machinery	M
Brew Kettles		Pug Mill	M
-- Continuous Duty	U	COMPRESSORS	
Cookers -- Continuous		Centrifugal	U
Duty	U	Lobe	M
Mash Tubs		Reciprocating	
-- Continuous Duty	U	Multi-Cylinder	M
		Single-Cylinder	H

Application	L.C.
CONVEYORS -- UNIFORMLY LOADED OR FED	
Apron	U
Assembly	U
Belt	U
Bucket	U
Chain	U
Flight	U
Oven	U
Screw	U
CONVEYORS -- HEAVY DUTY NOT UNIFORMLY FED	
Apron	M
Assembly	M
Belt	M
Bucket	M
Chain	M
Flight	M
Live Roll (Package)	U
Oven	M
Reciprocating	H
Screw	M
Shaker	H
CRANES and HOISTS	
Main Hoists	
Heave Duty	H
Medium Duty	M
Reversing	M
Skip Hoists	M
Trolley Drive	M
Bridge Drive	M
CRUSHERS	
Ore	H
Stone	H
DREDGES	
Cable Reels	M
Conveyors	M
Cutter Head Drives	H
Jig Drives	H
Maneuvering Winches	M
Pumps	M
Screen Drive	H
Stackers	M
Utility Winches	M
ELEVATORS	
Bucket -- Uniform Load	U
Heavy Load	M
Continuous	U
Centrifugal Discharge	U
Escalators	U
Freight	M
Gravity Discharge	U

Application	L.C.
Man Lifts	
Passenger	
Service -- Hand Life	H
FANS	
Centrifugal	U
Cooling Towers	
Induced Draft	M
Forced Draft	
Induced Draft	M
Large (Mine, etc.)	M
Large Industrial	M
Light (Small Diameter)	U
FEEDERS	
Apron	M
Belt	M
Disc	U
Reciprocating	H
Screw	M
FOOD INDUSTRY	
Beet Slicer	M
Cereal Cooker	U
Dough Mixer	M
Meat Grinders	M
GENERATORS -- (Not Welding)	U
HAMMER MILLS	H
LAUNDRY WASHERS	
Reversing	M
LAUNDRY TUMBLERS	M
LINE SHAFTS	
Heavy Shock Load	H
Moderate Shock Load	M
Uniform Load	U
LUMBER INDUSTRY	
Barkers -- Spindle Feed	M
Barkers -- Main Drive	H
Carriage Drive	
Conveyors -- Burner	M
Main or Heavy Duty	M
Main Log	H
Merry-go-Round	M
Slab	H
Transfer	
Waste	
Chains -- Floor	M
Green	M
Cut-Off Saws -- Chain	M
Drag	M
Debarking Drums	H
Feeds -- Edger	M
Gang	H
Trimmer	M

Application	L.C.
LUMBER INDUSTRY, Continued	
Log Deck	H
Log Hauls -- Incline	
-- Well Type	H
Log Turning Devices	H
Planer Feed	M
Planer Tilting Hoists	M
Rolls -- Live-Off Brg.	
-- Roll Cases	H
Sorting Table	M
Tipple Hoist	M
Transfers -- Chain	M
Transfers -- Craneway	M
Tray Drives	M
Veneer Lathe Drives	
MACHINE TOOLS	
Bending Roll	M
Notching Press -- Belt Driven	
Plate Planer	H
Punch Press -- Gear Driven	H
Tapping Machines	H
Other Machine Tools	
Main Drives	M
Auxiliary Drives	U
METAL MILLS	
Bridle Roll Drives	H
Draw Bench -- Carriage	H
Draw Bench -- Main Drive	H
Forming Machines	H
Pinch Dryer & Scrubber Rolls, Reversing	
Slitters	M
Table Conveyors	
Non-Reversing	M
Reversing	M
Winding Reels -- Strip	M
Wire Drawing & Flattening Machine	M
Wire Winding Machine	M
MILLS, ROTARY TYPE	
Ball	H
Cement Kilns	
Dryers & Coolers	M
Kilns	M
Pebble	H
Rod	H
Tumbling Barrels	H
MIXERS	
Concrete Mixers, Continuous	M
Concrete Mixers, Intermittent	M
Constant Density	U
Variable Density	M
OIL INDUSTRY	
Chillers	M
Oil Well Pumping	M
Paraffin Filter Press	M
Rotary Kilns	M
PAPER MILLS	
Agitators (Mixers)	M
Barker Auxiliaries, Hydraulic	M
Barker, Mechanical	M
Barking Drum	H
Beater & Pulper	M
Bleacher	U
Calenders	M
Calenders -- Super	M
Converting Machines, except Cutters, Platers	M
Conveyors	U
Conveyors, Log	H
Couch	M
Cutters, Platers	H
Cylinders	M
Dryers	M
Felt Stretcher	M
Felt Whipper	H
Jordans	U
Presses	U
Pulp Machines, Reel	M
Stock Chests	M
Suction Roll	U
Washers and Thickeners	M
Winders	U
PRINTING PRESSES	U
PULLERS	
Barge Haul	H
PUMPS	
Centrifugal	U
Proportioning	M
Reciprocating	
Single Acting	
3 or more Cylinders	M
Double Acting	
2 or more Cylinders	M
Single Acting	
1 or 2 Cylinders	
Double Acting	
Single Cylinder	
Rotary -- Gear Type	U
-- Lobe, Vane	U
RUBBER INDUSTRY	
Mixer	H

Application	L.C.
RUBBER INDUSTRY, Continued	
Rubber Calender	M
Rubber Mill (2 or more)	M
Sheeter	M
Tire Building Machines	
Tire & Tube Press Openers	
Tubers and Strainers	M
SEWAGE DISPOSAL EQUIPMENT	
Bar Screens	U
Chemical Feeders	U
Collectors, Circuline or Straightline	U
Dewatering Screens	M
Grit Collectors	U
Scum Breakers	M
Slow or Rapid Mixers	M
Sludge Collectors	U
Thickeners	M
Vacuum Filters	M
SCREENS	
Air Washing	U
Rotary -- Stone or Gravel	M
Traveling Water Intake	U
SLAB PUSHERS	M
STEERING GEAR	M
STOKERS	U
TEXTILE INDUSTRY	
Batchers	M
Calenders	M
Card Machines	M
Cloth Finishing Machines, (Washers, Pads, Tenters) (Dryers, Calenders, etc.)	M
Dry Cans	M
Dryers	M
Dyeing Machinery	M
Knitting Machines (looms, etc.)	
Looms]	M
Mangles	M
Nappers	M
Pads	M
Range Drives	
Slashers	M
Soapers	M
Spinners	M
Tenter Frames	M
Washers	M
Winders (Other than Batchers)	M
Yarn Prepartory Machines (Cards, Spinners, Slashers etc.)	M
WINDLASS	M

TABLE 1.4 General Classification of Power Sources.[5]

The following chart lists and classifies the character of general classes of prime movers for gear speed reducers. These should only be taken as minimum suggestions in classifying a power source.

Nomenclature :

U = uniform

L = light shock

M = medium shock

H = heavy shock

Prime Mover	Duration of Service		
	Less than 3 hrs./day	Less than 10 hrs./day	24 hrs./day
Electric Motors	U	U	L
Multi-Cylinder Internal Combustion Engine	U	L	M
Single Cylinder Internal Combustion Engine	L	M	H

NOTE: In general NCUT = 2 will represent the most realistic manufacturing method. The radius of curvature of the rounded corner of the hob teeth simulated in the program is a function of the clearance.

NLOAD	=	0	for computer determined load location on gear tooth
	=	1	for tooth tip-loading
	=	2	for highest point of single tooth contact loading

NOTE: For general purposes the design should be carried out assuming tip-loading (i.e. NLOAD = 1), since the theory used to determine if the actual loading is tip loading or highest point of single tooth contact loading, cannot be guaranteed in all cases. At the same time the NLOAD = 0 usage takes an excessive amount of computer time. The NLOAD = 0 and NLOAD = 2 facilities should only be used to analyze a given design. If they are implemented in an optimization search, the starting values for the search should be feasible results from a tip-loading solution.

NQUAL AGMA quality number

NOTE: Table 1.5 suggests quality numbers for various types of machinery.

OPTIMIZATION OPTIONS (integer numbers)

ISTRT	=	0	computer evaluated starting values employed as initial optimization values
	=	1	user suggested starting values employed as initial optimization values

STRT(I) Array containing n user suggested starting values where n specified the number of independent variables.

NOTE: Because of the options in defining which quantities are to be design variables, it is not possible for the user to relate design variables to elements of STRT(I) until one run has been made. Thus, for the first run, the user must accept the internal starting values. The resultant output, then relates design variables to elements of X(I), which corresponds to STRT(I).

Table 1.5 Applications and Suggested Quality Numbers

This table indicates a tabulation of many industrial and end use applications for spur gearing with typical AGMA Quality Numbers for many applications. When selecting a Quality Number for an industry or an application which is not shown use a similar industry or application as a guide. The AGMA Quality Number shown opposite each item of equipment identifies the quality of gearing generally used. Ther may be certain designs or operating conditions that would justify specifying gears to lower or higher Quality Number. In the interest of economy, use the lowe Quality Number shown, unless some of the conditions of the equipment or its operation indicate the use of the higher Quality Number.

Application	*Quality Numbers	Application	*Quality Numbers
AEROSPACE		BAILING MACHINE	5– 7
Actuators	7–11		
Control Gearing	10–12	BOTTLING INDUSTRY	
Engine Accessories	10–13	Capping	6– 7
Engine Power	10–13	Filling	6– 7
Engine Starting	10–13	Labeling	6– 7
Loading Hoist	7–11	Washer, Sterilizer	6– 7
Propeller Feathering	10–13		
Small Engines	12–13	BREWING INDUSTRY	
		Agitator	6– 8
AGRICULTURE		Barrel Washer	6– 8
Baler	3– 7	Cookers	6– 8
Beet Harvester	5– 7	Filling Machines	6– 8
Combine	5– 7	Mash Tubs	6– 8
Corn Picker	5– 7	Pasteurizer	6– 8
Cotton Picker	5– 7	Racking Machine	6– 8
Farm Elevator	3– 7		
Field Harvester	5– 7	BRICK-MAKING MACHINERY	5– 7
Peanut Harvester	3– 7		
Potato Digger	5– 7	BRIDGE MACHINERY	5– 7
AIR COMPRESSOR	10–11	BRIQUETTE MACHINES	5– 7
AUTOMOTIVE INDUSTRY	10–11		

*Quality Numbers are inclusive, from lowest to highest numbers shown.

Application	*Quality Numbers
CEMENT INDUSTRY	
(Quarry Operation)	
Conveyor	5– 6
Crusher	5– 6
Diesel-Electric Locomotive	8– 9
Electric Dragline (cast gear)	3
(cut gear)	6– 8
Electric Locomotive	6– 8
Electric Shovel (cast gear)	3
(cut gear)	6– 8
Elevator	5– 6
Locomotive Crane (cast gear)	3
(cut gear)	5– 6
(Plant Operation)	
Air Separator	5– 6
Ball Mill	5– 7
Compeb Mill	5– 6
Conveyor Mill	5– 6
Cooler	5– 6
Elevator	5– 6
Feeder	5– 6
Filter	5– 6
Kiln	5– 6
Kiln Slurry Agitator	5– 6
Overhead Crane	5– 6
Pug, Rod, and Tube Mills	5– 6
Pulverizer	5– 6
Raw and Finish Mill	5– 6
Rotary Dryer	5– 6
Slurry Agitator	5– 6
CHEWING GUM INDUSTRY	
Chicle Grinder	6– 8
Coater	6– 8
Mixer-Kneader	6– 8
Molder-Roller	6– 8
Wrapper	6– 8
CHOCOLATE INDUSTRY	
Glazer, Finisher	6– 8
Mixer, Mill	6– 8
Molder	6– 8
Presser, Refiner	6– 8
Tampering	6– 8
Wrapper	6– 8
CLAY WORKING MACHINERY	5– 7
COMMERCIAL METERS	
Gas	7– 9
Liquid, Water, Milk	7– 9
Parking	7– 9
COMPUTING AND ACCOUNTING MACHINES	
Accounting – Billing	9–10
Adding Machine – Caluclator	7– 9
Addressograph	7
Bookkeeping	9–10
Cash Register	7
Comptometer	6– 8
Computing	10–11
Data Processing	7– 9
Dictating Machine	9
Typewriter	8

Application	*Quality Numbers
CONSTRUCTION EQUIPMENT	
Backhoe	6– 8
Cranes, Open Gearing	3– 6
Enclosed Gearing	6– 8
Ditch Digger	3– 8
Transmission	6– 8
Drag Line	5– 8
Dumpster	6– 8
Paver, Loader	3
Transmission	8
Mixer	3– 5
Swing Gear	3– 5
Mixing Bucket	3
Shaker	8
Shovels, Open Gearing	3– 6
Enclosed Gearing	6– 8
Stationary Mixer, Transmission	8
Drum Gears	3– 5
Stone Crusher, Transmission	8
Conveyor	6
Truck Mixer, Transfer Case	9
Drum Gears	3– 5
CRANES	
Boom Hoist	5– 6
Gantry	5– 6
Load Hoist	5– 7
Overhead	5– 6
Ship	5– 7
CRUSHERS	
Ice, Feed	6– 8
Portable and Stationary	6– 8
Rock, Ore, Coal	6– 8
DAIRY INDUSTRY	6– 7
Bottle Washer	7– 9
Homogenizer	7– 9
Separator	
DAMS AND LOCKS	5– 7
Tainter Gates	
DISH WASHER	5– 7
Commercial	
DISTILLERY INDUSTRY	5– 7
Agitator	5– 7
Bottle Filler	6– 7
Conveyor, Elevator	6– 8
Grain Pulverizer	5– 7
Mash Tub	5– 7
Mixer	5– 7
Yeast Tub	
ELECTRIC FURNACE	5– 7
Tilting Gears	
ELECTRONIC INSTRUMENT CONTROL AND GUIDANCE SYSTEMS	
Accelerometer	10–12
Airborne Temperature Recorder	12–13
Aircraft Instrument	12
Altimeter-Stabilizer	9–11

*Quality Numbers are inclusive, from lowest to highest numbers shown.

Application	*Quality Numbers
Analog Computer	10–12
Antenna Assembly	7– 9
Anti-Aircraft Detector	12
Automatic Pilot	9–11
Digital Computer	10–12
Gun-Data Computer	12–13
Gyro Caging Mechanism	10–12
Gyroscope-Computer	12–13
Pressure Transducer	12–13
Radar, Sonar, Tuner	10–12
Recorder, Telemeter	10–12
Servo System Component	9–11
Sound Detector	9
Transmitter, Receiver	10–12
ENGINES	
Diesel, Semi-Diesel and Internal Combustion	
Engine Accessories	10–12
Supercharger	10–12
Timing Gearings	10–12
Transmission	8–10
FARM EQUIPMENT	
Milking Machine	6– 8
Separator	8–10
Sweeper	4– 6
FLOUR MILL INDUSTRY	
Bleacher	7– 8
Grain Cleaner	7– 8
Grinder	7– 8
Hulling	7– 8
Milling, scouring	7– 8
Polisher	7– 8
Separator	7– 8
FOUNDRY INDUSTRY	
Conveyor	5– 6
Elevator	5– 6
Ladle	5– 6
Molding Machine	5– 6
Overhead Cranes	5– 6
Sand Mixer	5– 6
Sand Slinger	5– 6
Tumbling Mill	5– 6
HOME APPLIANCES	
Blender	6– 8
Mixer	7– 9
Timer	8–10
Washing Machine	8–10
MACHINE TOOL INDUSTRY	
Hand Motion (other than Indexing and Positioning)	6– 9
Feed Drives	8 and up
Speed Drives	8 and up
Multiple Spindel Drives	8 and up
Power Drives, 0-800 FPM	6– 8
800-2000 FPM	8–10
2000-4000 FPM	10–12
Over 4000 FPM	12 and up
Indexing and Positioning – Approximate Positioning	6–10
Accurate Indexing and Positioning	12 and up
MARINE INDUSTRY	
Anchor Hoist	6– 8
Cargo Hoist	7– 8
Conveyor	5– 7
Davit Gearing	5– 7
Elevator	6– 7
Small Propulsion	
Steering Gear	10–12
Winch	8
	5– 8
METAL WORKING	
Bending Roll	5– 7
Draw Bench	6– 8
Forge Press	5– 7
Punch Press	5– 7
Roll Lathe	5– 7
MINING AND PREPARATION	
Agitator	
Breaker	5– 6
Car Dump	5– 6
Car Spotter	5– 7
Centrifugal Drier	7– 8
Clarifier	7– 8
Classifier	7– 8
Coal Digger	6–10
Concentrator	5– 6
Continuous Miner	6– 7
Cutting Machine	6–10
Conveyor	5– 7
Drag Line, Open Gearing	3– 6
Enclosed Gearing	6– 8
Drills	5– 6
Drier	5– 6
Electric Locomotive	6– 8
Elevator	5– 6
Feeder	6– 8
Flotation	5– 6
Grizzly	5– 6
Hoists, Skips	7– 8
Loader (Underground)	5– 8
Rock Drill	5– 6
Rotary Car Dump	6– 8
Screen (Rotary)	7– 8
Screen (Shaking)	7– 8
Separator	5– 6
Sedimentation	5– 6
Shaker	6– 8
Shovel	3– 8
Triple Gearing	5– 7
Washer	6– 8
PAPER AND PULP	
Bag Machines	6– 8
Bleacher, Decker	
Box Machines	6– 8
Building Paper	6– 8
Calender	6– 8
Chipper	6– 8
Coating	6– 8
Digester	
Envelope Machines	6– 8
Food Container	6– 8

* Quality Numbers are inclusive, from lowest to highest numbers shown.

Application	*Quality Numbers
Glazing	6– 8
Grinder	
Log Conveyor – Elevator	5– 7
Mixer, Agitator	6– 8
Paper, Machine	
Auxiliary	8– 9
Main Drive	10–12
Press, Couch, Drier Rolls	6– 8
Save-all	
Slitting	10–12
Steam Drum	6– 8
Varnishing	6– 8
Wall Paper Machines	6– 8
PAVING INDUSTRY	
Aggregate Drier	5– 7
Aggregate Spreader	5– 7
Asphalt Mixer	5– 7
Ashpalt Spreader	5– 7
Concrete Batch Mixer	5– 7
PHOTOGRAPHIC EQUIPMENT	
Aerial	10–12
Commercial	8–10
PRINTING INDUSTRY	
Press, Book	9–11
Flat	9–11
Magazine	9–11
Newspaper	9–11
Roll Reels	6– 7
Rotary	9–11
Book Binding	
PUMP INDUSTRY	
Liquid	10–12
Rotary	6– 8
Slush-duplex-triplex	6– 8
Vacuum	6– 8
QUARRY INDUSTRY	
Conveyor-Elevator	6– 7
Crusher	5– 7
Rotary Screen	7– 8
Shovel-Electric-Diesel	
RADAR AND MISSILE	
Antenna Elevating	8–10
Data Gear	10–12
Launch Pad Azimuth	8
Ring Gear	9–12
Rotating Drive	10–12
RAILROADS	
Construction Hoist	5– 7
Wrecking Crane	6– 8
RUBBER AND PLASTICS	
Boot and Shoe Machines	6– 8
Drier, Press	6– 8
Extruder, Strainer	6– 8
Mixer, Tuber	6– 8
Refiner, Calender	5– 7

Application	*Quality Numbers
Rubber Mill, Scrap Cutter	5– 7
Tire Building	6– 8
Tire Chopper	5– 7
Washer, Banbury Mixer	5– 7
SMALL POWER TOOLS	
Bench Grinder	6– 8
Drills-Saws	7– 9
Hair Clipper	7– 9
Hedge Clipper	7– 9
Sander, Polisher	8–10
Sprayer	6– 8
SPACE NAVIGATION	
Sextant and Star Tracker	13 and up
STEEL INDUSTRY	
Auxiliary and Miscellaneous Drives	
Annealing Furnance	5– 6
Bending Roll	5– 6
Blooming-Mill Manipulator	5– 6
Blooming-Mill Rack and Pinion	5– 6
Blooming-Mill Side Guard	5– 6
Car Haul	5– 6
Coil Conveyor	5– 6
Edger Drives	5– 6
Electrolytic Line	6– 7
Flange-Machine Ingot Buggy	5– 6
Leveler	6– 7
Magazine Pusher	6– 7
Mill Shear Drives	6– 7
Mill Table Drives (under 800 ft/min)	5– 6
Mill Table Drives (over 800-1800 ft/min)	6– 7
Mill Table Drives (over 1800 ft/min)	8
Nail and Spike Machine	5– 6
Piler	5– 6
Plate Mill Rack and Pinion	5– 6
Plate Mill Side Guards	5– 6
Plate Turnover	5– 6
Preheat Furnace Pusher	5– 6
Processor	6– 7
Pusher Rack and Pinion	5– 6
Rotary Furnace	5– 6
Shear Depress Table	5– 6
Slab Squeezer	5– 6
Slab-Squeezer Rack and Pinion	5– 6
Slitter, Side Trimmer	6– 7
Tension Reel	6– 7
Tilt Table, Upcoiler	5– 6
Transfer Car	5– 6
Wire Drawing Machine	6– 7
Blast Furnace, Coke Plant	
Open-Hearth and Soaking Pits	
Miscellaneous Drives	
Bessemer Tilt-Car Dump	5– 6
Coke Pusher, Distributor	5– 6
Conveyor, Door Lift	5– 6
Electric-Furnace Tilt	5– 6
Hot Metal Car Tilt	5– 6
Hot Metal Charger	5– 6
Jib Hoist, Dolomite Machine	5– 6
Larry Car,	5– 6
Mixing Bin, Mixer Tilt	5– 6

*Quality Numbers are inclusive, from lowest to highest numbers shown.

Application	*Quality Numbers
Ore Crusher, Pig Machine	5– 6
Pulverizer, Quench Car	5– 6
Shaker, Stinter Conveyor	5– 6
Stinter Machine Skip Hoist	5– 6
Slag Crusher, Slag Shovel	5– 6
Primary and Secondary Rolling Mill Drives	
Blooming and Plate Mill	5– 6
Heavy Duty Hot Mill Drives	5– 6
Slabbing and Strip Mill	5– 6
Hot Mill Drives	
Sendzimer-Stekel	7– 8
Tandem-Temper-Skin	6– 7
Cold Mill Drives	
Bar, Merchant, Rail, Rod	5– 6
Structural, Tube	5– 6
Mill Gearing	
Billet Mills	
Free Roughing	5– 6
Tandem Roughing	5– 6
Finishing	5– 6
Cold Mills	
Reversing	7– 8
Tandem	7– 8
Temper	7– 8
Foil	7– 8
Hot Mills	
Blooming and Slabbing Mills	5– 6
Continuous Hot Strip Mills	5– 6
Free Reversing Roughing	5– 6
Tandem Roughing (including Scalebreaker)	5– 6
Finishing	6– 7
Merchant Mills	
Roughing	6– 7
Intermediate	7– 8
Finishing	7– 9
Plate Mills	
Reversing Roughing	5– 6
Unidirectional Roughing	5– 6
Unidirectional Finishing	5– 6
Rod Mills	
Roughing	6– 7
Intermediate	7– 8
Finishing	10–12
High Speed	12–14
Skelp Mills	
Roughing	6– 7
Intermediate	7– 8
Finishing	7– 9
Structural and Rail Mills	
Heavy	
Reversing Rougher	5– 6
Finishing	5– 6

Application	*Quality Numbers
Light	
Roughing	5– 6
Finishing	5– 6
Overhead Cranes	
Billet Charger, Cold Mill	5– 6
Bucket Handling	5– 6
Car Repair Shop	5– 6
Cast House, Coil Storage	5– 6
Charging Machine	5– 6
Cinder Yard, Hot Top	5– 6
Coal and Ore Bridges	5– 6
Electric Furnace Charger	5– 6
Hot Metal, Ladle	5– 6
Hot Mill, Ladle House	5– 6
Jib Crane, Motor Room	5– 6
Mold Yard, Rod Mill	5– 6
Ore Unloader, Stripper	5– 6
Overhead Hoist	5– 6
Pickler Building	5– 6
Pig Machine, Sand House	5– 6
Portable Hoist	5– 6
Scale Pit, Shipping	5– 6
Scrap Balers and Shears	5– 6
Scrap Preparation	5– 6
Service Shops	5– 6
Skull Cracker	5– 6
Slab Handling	5– 6
Precision Gear Drives	
Diesel Electric Gearing	8– 9
Flying Shear	9–10
Shear Timing Gears	9–10
High Speed Reels	8– 9
Locomotive Timing Gears	9–10
Pump Gears	8– 9
Tube Reduction Gearing	8– 9
Turbine	9–10
MISCELLANEOUS	
Clocks	6
Counters	7– 9
Fishing Reel	6
Gauges	8–10
IBM Card Puncher, Sorter	8
Metering Pumps	7– 8
Motion Picture Equipment	8
Popcorn Machine, Commercial	6– 7
Pumps	5– 7
Sewing Machine	8
Slicer	7– 8
Vending Machines	6– 7

*Quality Numbers are inclusive, from lowest to highest numbers shown.

NOF1	=	1	minimum volume optimized
	=	0	minimum volume not optimized
NOF2	=	1	maximum contact ratio optimized
	=	0	maximum contact ratio not optimized
NOF3	=	1	minimum centre distance optimized
	=	0	minimum centre distance not optimized
NOF4	=	1	minimum face width optimized
	=	0	minimum face width not optimized

NOTE: For further information on the optimization criteria see Section 1.3 or reference [1].

1.5 HOW TO SET UP CALLING PROGRAM

The calling program must have exactly the form of the following example with decimal points added where indicated. Labelled COMMON blocks listed below are placed in the calling program to transfer data to and from the package. Blocks BLKO to BLK4A inclusive are sufficient to input all required data. If, however, output results from the package are used for other analysis (e.g. plotting routines, etc.), blocks BLKO to BLK15 inclusive are required. Following the list of input data, the CALL SPUR statement calls the package. The CALL TOCKS and CALL TOCKP statements are CDC 6400 subroutines to print out the amount of execution time between the CALL statements. An equivalent subroutine to print executive time, if desired, as well as other control cards will be required in accordance with the computer used.

```
COMMON/BLKO /IDATA,IPRINT,IWRITE,NTYPE
COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,U1,U2
COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
COMMON/BLK3 /ADD1,ADDK2,DEDK1,DEDK2,ADD1,ADD2 DED1,DED2
COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
COMMON/BLK3B/BLL,BLU,BLR,CDR
COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
COMMON/BLK5 /BP,CP,DP,PAR,PLV,RATIO,RPM1,RPM2,SHAFT1,SHAFT2,TEETH2
COMMON/BLK6 /PR1,PR2,RB1,RB2,RI1,RI2,RM1,RM2,RO1,RO2,RU1,RU2
COMMON/BLK7 /ADDL1,ADDL2,CCC1,CCC2,CRATIO,EFF
COMMON/BLK7A/HUBL1,HUBL2,HUBR1,HUBR2,RIM1,RIM2,WEB1,WEB2,VOL1,VOL2
COMMON/BLK7B/ANGC1,ANGC2,ANGL1,ANGL2,RL1,RL2,RLL1,RLL2,RLM1,RLM2
COMMON/BLK7C/XKEY1,XKEY2,VOLMIN,VOLMAX,XLA,XLR,TO1,TO2,TP1,TP2
COMMON/BLK8 /CE,CF,CH,CJ,CL1,CL2,CM,CO,CR,CS,CT,CV
COMMON/BLK8A/QJ1,QJ2,QL1,QL2,QM,QO,QR,QS,QT,QV
COMMON/BLK9 /COD,CODL1,CODL2,QOD,QODL1,QODL2
COMMON/BLK10 /SB1,SB2,SBM1,SBM2,SS1,SS2,SSM1,SSM2
COMMON/BLK10A/PAB1,PAB2,PAW1,PAW2,TORQ1,TORQ2,WA,WR,WT,WN
COMMON/BLK11 /J,K,N,NN,NCD,NFW,NTOOTH,NDRIVE,NNLOAD,NOPT,NOFN,PI
COMMON/BLK11A/NVAR(8),NSTD(8),NOF(4)
COMMON.BLK13 /BBA1,BBA2,BBX1,BBX2,BBY1,BBY2,RT1,RT2
COMMON/BLK14 /TOLR1,TOLR2,TOLP1,TOLP2,PTOL1,PTOL2,TOLL1,TOLL2
COMMON/BLK14A/TTCT1,TTCT2,TCT1,TCT2,TPTL1,TPTL2,TPTU1,TPTU2
COMMON/BLK14B/TPTE1,TPTE2,TPTV1,TPTV2,CDTOLL,CDTOLU,ERR
COMMON/BLK15 /BLMIN,BLMINT,BLMAX,BLMAXT,BLMAXU,DELBL,BL1,BL2
```

The following example, Table 1.6 has all data and results printed for demonstration purposes.

1.6 OUTPUT INFORMATION

The input data is reprinted in the output for reference according to the printout options, along with the variable list, initial values for the independent variables, and input data from the optimization technique. The intermediate results printed present the true optimization function; the artificial optimization function, a modified optimization function to include constraints; and the design variables after every cycle requested. On completion of the optimization procedure, the optimum value of the criterion characteristic is printed along with the corresponding design variables and the inequality

TABLE 1.6 SAMPLE CALLING PROGRAM

```
      PROGRAM MAIN(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON/BLK0 /IDATA,IPRINT,IWRITE,NTYPE
      COMMON/BLK1 /BHN1,BHN2,E1,E2,RHO1,RHO2,SAC1,SAC2,SAF1,SAF2,U1,U2
      COMMON/BLK2 /HP,RPMI,RPMO,SHAFTI,SHAFTO,CD,FW,PAD,TEETH1
      COMMON/BLK3 /ADDK1,ADDK2,DEDK1,DEDK2,ADD1,ADD2,DED1,DED2
      COMMON/BLK3A/CDMAX,CDMIN,FWMAX,FWMIN,PADMAX,PADMIN
      COMMON/BLK3B/BLL,BLU,BLR,CDR
      COMMON/BLK4 /CYCLE,DRIVEN,DRIVER,NCUT1,NCUT2,NLOAD,NQUAL,RELI,TEMP
      COMMON/BLK4A/ISTRT,STRT( 8),NOF1,NOF2,NOF3,NOF4
      DATA IDATA,IPRINT,IWRITE/ 1, 1, 1/
      DATA E1,E2,RHO1,RHO2,U1,U2/2*3.0E+07,2*0.283,2*0.33/
      DATA SAC1,SAC2,SAF1,SAF2,BHN1,BHN2/1.28E+05,1.15E+05,3.6E+04,
     13.24E+04,300.0,255.0/
      NTYPE=1
      HP=50.0
      RPMI=1750.0
      RPMO=350.0
      SHAFTI=2.0
      SHAFTO=2.0
      ADDK1=3HSTD
      ADDK2=3HSTD
      DEDK1=3HSTD
      DEDK2=3HSTD
      FW=3HVAR
      CD=3HVAR
      PAD=20.0
      TEETH1=3HVAR
      CDMAX=30.0
      CDMIN=0.0
      FWMAX=6.0
      FWMIN=0.0
      PADMAX=30.0
      PADMIN=14.5
      CYCLE=1.0E+10
      DRIVEN=3.0
      DRIVER=1.0
      NLOAD=1
      NCUT1=2
      NCUT2=2
      NQUAL=8
      RELI=0.995
      TEMP=100.0
      BLL=1.0
      BLU=1.5
      BLR=0.5
      CDR=1.0
      ISTRT=0
      NOF1=1
      NOF2=1
      NOF3=1
      NOF4=1
      CALL TOCKS
      CALL SPUR
      CALL TOCKP
      STOP
      END
```

TABLE 1.7 PRINTED OUTPUT OF EXAMPLE

SPUR GEAR DESIGN...INPUT DATA

POWER REQUIREMENTS

HORSEPOWER TRANSMITTED.	HP	=	5.00000000E+01
INPUT SPEED (RPM).	RPMI	=	1.75000000E+03
OUTPUT SPEED (RPM).	RPMO	=	3.50000000E+02

GEAR MATERIAL PROPERTIES

			PINION	GEAR
MAXIMUM ALLOWABLE FATIGUE STRESS (PSI).	SAF	=	3.60000000E+04	3.24000000E+04
MAXIMUM ALLOWABLE COMPRESSIVE STRESS (PSI).	SAC	=	1.28000000E+05	1.15000000E+05
MODULUS OF ELASTICITY (PSI)	E	=	3.00000000E+07	3.00000000E+07
POISSONS RATIO.	U	=	3.30000000E-01	3.30000000E-01
DENSITY (LBS/CU. IN.)	RHO	=	2.83000000E-01	2.83000000E-01
BRINELL HARDNESS.	BHN	=	3.00000000E+02	2.55000000E+02

INPUT LAYOUT

			PINION	GEAR
PRESSURE ANGLE (DEGREES).	PAD	=	2.00000000E+01	
NUMBER OF TEETH	TEETH	=	VARIABLE	
CENTRE DISTANCE (INCHES).	CD	=	VARIABLE	
FACE WIDTH (INCHES)	FW	=	VARIABLE	
ADDENDUM (INCHES)	ADD	=	1.0/DP	1.0/DP
DEDENDUM (INCHES)	DED	=	1.250/DP	1.250/DP

INPUT REQUIREMENTS

```
****************************************************************************************
MODE OF LOADING . . . . . . . . . . . . DRIVEN  =  1.0 LOAD ON DRIVEN MACHINE - UNIFORM
                                                =  2.0 LOAD ON DRIVEN MACHINE - MODERATE
                                                =  3.0 LOAD ON DRIVEN MACHINE - HEAVY
MODE OF POWER SOURCE. . . . . . . . . . DRIVER  =  1.0 POWER SOURCE - UNIFORM
                                                =  2.0 POWER SOURCE - LIGHT SHOCK
                                                =  3.0 POWER SOURCE - MEDIUM SHOCK

CUTTING TOOL TYPE . . . . . . . . . . .   NCUT  =  1 IF GEAR CUT BY RACK WITH SHARP
                                                     CORNERED TEETH
                                                =  2 IF GEAR CUT BY RACK WITH ROUNDED
                                                     CORNERS

MODE OF LOAD LOCATION ON TOOTH. . . . .  NLOAD  =  0 FOR PROGRAM DETERMINED
                                                =  1 FOR TOOTH TIP LOADING
                                                =  2 FOR POINT OF HIGHEST SINGLE TOOTH
                                                     CONTACT LOADING
****************************************************************************************

REQUIRED NUMBER OF LOADING CYCLES . . .  CYCLE  =              1.00000000E+10

REQUIRED RELIABILITY. . . . . . . . . .   RELI  =              9.95000000E-01

GEAR BLANK TEMPERATURE (DEGREES F). . .   TEMP  =              1.00000000E+02

ACTUAL MODE OF LOADING. . . . . . . . . DRIVEN  =              3.00000000E+00

ACTUAL MODE OF POWER SOURCE . . . . . . DRIVER  =              1.00000000E+00

RATIO OF PINION TOOTH THINNING TO
TOTAL TOOTH THINNING. . . . . . . . . .    BLR  =              5.00000000E-01

MINIMUM BACKLASH MODIFICATION FACTOR. .    BLL  =              1.00000000E+00

MAXIMUM BACKLASH MODIFICATION FACTOR. .    BLU  =              1.50000000E+00

CENTRE DISTANCE TOLERANCE
MODIFICATION FACTOR . . . . . . . . . .    CDR  =              1.00000000E+00

LOAD LOCATION MODE. . . . . . . . . . .  NLOAD  =              1

CUTTING TOOL TYPE A) PINION . . . . . .  NCUT1  =              2

                  B) GEAR   . . . . . .  NCUT2  =              2

A.G.M.A. QUALITY NUMBER . . . . . . . .  NQUAL  =              8
```

```
USER SIZE LIMITATIONS
---------------------

MAXIMUM CENTRE DISTANCE (INCHES). . . .  CDMAX   =        3.00000000E+01
MINIMUM CENTRE DISTANCE (INCHES). . . .  CDMIN   =        0.
MAXIMUM FACE WIDTH (INCHES) . . . . . .  FWMAX   =        6.00000000E+00
MINIMUM FACE WIDTH (INCHES) . . . . . .  FWMIN   =        0.
MAXIMUM PRESSURE ANGLE (DEGREES). . . . PADMAX   =        3.00000000E+01
MINIMUM PRESSURE ANGLE (DEGREES). . . . PADMIN   =        1.45000000E+01
INPUT SHAFT DIAMETER. . . . . . . . . . SHAFTI   =        2.00000000E+00
OUTPUT SHAFT DIAMETER . . . . . . . . . SHAFTO   =        2.00000000E+00


VARIABLE LIST AND INITIAL VALUES
--------------------------------

NUMBER OF PINION TEETH. . . . . . . . . TEETH1   =        X( 1)
CENTRE DISTANCE (INCHES). . . . . . . .     CD   =        X( 2)
FACE WIDTH (INCHES) . . . . . . . . . .     FW   =        X( 3)

COMPUTED STARTING VALUES. . . . . . .XSTRT( 1)   =        1.80000000E+01
                                     XSTRT( 2)   =        1.46114494E+01
                                     XSTRT( 3)   =        2.02155558E+00

MODIFIED STARTING VALUES. . . . . . .XSTRT( 1)   =        1.80000000E+01
                                     XSTRT( 2)   =        1.46114494E+01
                                     XSTRT( 3)   =        2.02155558E+00
```

OPTIMIZATION USING DIRECT SEARCH METHOD...SEEK1

DATA INPUT

NUMBER OF INDEPENDENT VARIABLES	N	=	3
NUMBER OF INEQUALITY (.GE.0.0) CONSTRAINTS. . . .	NCONS	=	25
NUMBER OF EQUALITY CONSTRAINTS.	NEQUS	=	0
INPUT DATA PRINTED OUT FOR (IDATA.NE.0)	IDATA	=	1
INTERMEDIATE OUTPUT EVERY IPRINT ITERATIONS . . .	IPRINT	=	1
NUMBER OF DIRECTED RANDOM SEARCHES PERMITTED. . .	NSHOT	=	2
NUMBER OF TEST POINTS IN DIRECTED RANDOM SEARCH .	NTEST	=	100
MAXIMUM NUMBER OF ITERATIONS.	MAXM	=	300
FRACTION OF RANGE USED AS STEP SIZE	F	=	1.00000000E-02
FRACTION OF RANGE USED FOR CONVERGENCE CRITERION.	G	=	1.00000000E-02

ESTIMATED UPPER BOUND ON RANGE OF X(I). . . . . . RMAX(I) =

1.00000000E+02 1.50000000E+02 3.00000000E+01

ESTIMATED LOWER BOUND ON RANGE OF X(I). . . . . . RMIN(I) =

0. 0. 0.

STARTING VALUES OF X(I) . . . . .XSTRT(I) =

1.80000000E+01 1.46114494E+01 2.02155558E+00

INTERMEDIATE RESULTS

	U	UART	INDEPENDENT VARIABLES X(I)		
1	6.30710787E+00	4.30700762E+25	1.90000000E+01	1.61114494E+01	2.32155558E+00
2	7.24773908E+00	7.24773908E+00	2.10000000E+01	1.91114494E+01	2.92155558E+00
3	7.14892274E+00	7.14892274E+00	2.10000000E+01	1.91114494E+01	2.77155558E+00
4	7.09820465E+00	7.09820465E+00	2.12500000E+01	1.91114494E+01	2.69655558E+00
5	7.09602283E+00	7.09602283E+00	2.13750000E+01	1.91114494E+01	2.69655558E+00
6	7.09494108E+00	7.09494108E+00	2.14375000E+01	1.91114494E+01	2.69655558E+00
7	7.08875645E+00	7.08875645E+00	2.14687500E+01	1.91114494E+01	2.68718058E+00
8	7.08721520E+00	7.08721520E+00	2.14765625E+01	1.91114494E+01	2.68483683E+00
9	7.08581015E+00	7.08581015E+00	2.14765625E+01	1.91114494E+01	2.68249308E+00

```
    OPTIMUM SOLUTION FOUND

MINIMUM   U =   7.08581015E+00

       X( 1) =   2.14765625E+01
       X( 2) =   1.91114494E+01
       X( 3) =   2.68249308E+00

INEQUALITY CONSTRAINTS

    PHI( 1) =   1.32468307E+04
    PHI( 2) =   1.26602796E+04
    PHI( 3) =   1.15090539E+04
    PHI( 4) =   2.75550552E+00
    PHI( 5) =   6.35059287E-02
    PHI( 6) =   1.55135584E+00
    PHI( 7) =   3.70734600E+00
    PHI( 8) =   1.08095413E-01
    PHI( 9) =   1.57825383E-01
    PHI(10) =   2.62067335E-01
    PHI(11) =   7.41562243E-02
    PHI(12) =   7.41562243E-02
    PHI(13) =   1.0[illegible]885506E+01
    PHI(14) =   1.91114494E+01
    PHI(15) =   3.31750692E+00
    PHI(16) =   2.68249308E+00
    PHI(17) =   1.00000000E+01
    PHI(18) =   5.50000000E+00
    PHI(19) =   1.20265394E-06
    PHI(20) =   6.99381424E-04
    PHI(21) =   6.47564421E-04
    PHI(22) =   1.24529045E-01
    PHI(23) =   1.54819637E-01
    PHI(24) =   2.49182683E-01
    PHI(25) =   2.67633797E-01
```

```
RESULTS AT LAST ITERATION

        U =  7.07701722E+00

   X(  1) =  2.20000000E+01
   X(  2) =  1.91114494E+01
   X(  3) =  2.68249308E+00

INEQUALITY CONSTRAINTS

 PHI(  1) =  1.28767941E+04
 PHI(  2) =  1.22811192E+04
 PHI(  3) =  1.18372455E+04
 PHI(  4) =  3.30947128E+02
 PHI(  5) =  7.07480700E-02
 PHI(  6) =  1.55863199E+00
 PHI(  7) =  3.71440349E+00
 PHI(  8) =  1.15152895E-01
 PHI(  9) =  1.55815558E-01
 PHI( 10) =  2.56860375E-01
 PHI( 11) =  7.23918539E-02
 PHI( 12) =  7.23918539E-02
 PHI( 13) =  1.08885506E+01
 PHI( 14) =  1.91114494E+01
 PHI( 15) =  3.31750692E+00
 PHI( 16) =  2.68249308E+00
 PHI( 17) =  1.00000000E+01
 PHI( 18) =  5.50000000E+00
 PHI( 19) = -1.35912111E-04
 PHI( 20) =  8.47275745E-06
 PHI( 21) =  6.56313380E-04
 PHI( 22) =  1.22291374E-01
 PHI( 23) =  1.51254708E-01
 PHI( 24) =  2.43113103E-01
 PHI( 25) =  2.61595165E-01
```

SPUR GEAR DESIGN...RESULTS OF LAST ITERATION

VIOLATED CONSTRAINT EVALUATION

FOR MORE COMPLETE EXPLANATION...SEE MANUAL

PHI(19) BACKLASH RANGE EXCEEDED

TRY A) INCREASING BACKLASH RANGE
     B) INCREASING A.G.M.A. QUALITY NUMBER

GEAR LAYOUT

			PINION	GEAR
PRESSURE ANGLE (DEGREES).	PAD	=	2.00000000E+01	
CENTRE DISTANCE (INCHES).	CD	=	1.91114494E+01	
FACE WIDTH (INCHES)	FW	=	2.68249308E+00	
NUMBER OF TEETH	TEETH	=	2.20000000E+01	1.10000000E+02
ADDENDUM (INCHES)	ADD	=	2.89567416E-01	2.89567416E-01
DEDENDUM (INCHES)	DED	=	3.61959269E-01	3.61959269E-01
CLEARANCE (INCHES).	CCC	=	7.23918539E-02	7.23918539E-02
PITCH CIRCLE RADIUS (INCHES).	PR	=	3.18524157E+00	1.59262079E+01
BASE CIRCLE RADIUS (INCHES)	RB	=	2.99314800E+00	1.49657400E+01
ADDENDUM CIRCLE RADIUS (INCHES)	RO	=	3.47480899E+00	1.62157753E+01
DEDENDUM CIRCLE RADIUS (INCHES)	RI	=	2.82328230E+00	1.55642486E+01
MAXIMUM ADDENDUM CIRCLE RADIUS BEFORE INTERFERENCE OCCURS (INCHES) . .	RM	=	7.18921247E+00	1.63309282E+01
UNDERCUT LIMIT RADIUS (INCHES).	RU	=	2.74024723E+00	1.39908036E+01
MINIMUM CIRCULAR TOOTH THICKNESS AT PITCH CIRCLE (INCHES).	TP	=	4.45907230E-01	4.44068445E-01
TOOTH THICKNESS AT ADDENDUM CIRCLE (INCHES)	TO	=	1.94683227E-01	2.23646562E-01
BASE PITCH (INCHES)	BP	=	8.54841070E-01	
CIRCULAR PITCH (INCHES)	CP	=	9.09702865E-01	
DIAMETRAL PITCH (TEETH/INCH).	DP	=	3.45342724E+00	
EFFICIENCY.	EFF	=	9.93585844E-01	
GEAR RATIO (GEAR TEETH /PINION TEETH) .	RATIO	=	5.00000000E+00	
CONTACT RATIO	CRATIO	=	1.72169330E+00	

LOADING ANALYSIS

			PINION	GEAR
HORSEPOWER TRANSMITTED.	HP	=	5.00000000E+01	
MAXIMUM ALLOWABLE POWER...BENDING (HP).	PAB	=	8.39186957E+01	8.74513047E+01
MAXIMUM ALLOWABLE POWER...WEAR (HP) . .	PAW	=	6.23480653E+01	5.03267312E+01
NORMAL LOAD (LBS)	WN	=	6.01615590E+02	
TANGENTIAL LOAD (LBS)	WT	=	5.65333730E+02	
RADIAL LOAD (LBS)	WR	=	2.05764050E+02	
AXIAL LOAD (LBS).	WA	=	0.	
PITCH LINE VELOCITY (FPM)	PLV	=	2.91863003E+03	
SHAFT SPEED (RPM)	RPM	=	1.75000000E+03	3.50000000E+02
TORQUE (FT-LBS)	TORQ	=	1.50060375E+02	7.50301875E+02
LOAD ANGLE (RADIANS).	ANGL	=	5.04792971E-01	3.88322716E-01
RADIUS TO LOAD ON TOOTH CENTRELINE (INCHES)	RL	=	3.41966731E+00	1.61696437E+01

LOADING ANALYSIS FOR TIP LOADING

MATERIAL PROPERTIES AND STRESS ANALYSIS

			PINION	GEAR
MODULUS OF ELASTICITY (PSI)	E	=	3.00000000E+07	3.00000000E+07
POISSONS RATIO.	U	=	3.30000000E-01	3.30000000E-01
DENSITY (LBS/CU. IN.)	RHO	=	2.83000000E-01	2.83000000E-01
BRINELL HARDNESS.	BHN	=	3.00000000E+02	2.55000000E+02
MAXIMUM ALLOWABLE FATIGUE STRESS (PSI).	SAF	=	3.60000000E+04	3.24000000E+04
MAXIMUM ALLOWABLE BENDING STRESS (PSI).	SBM	=	3.18635955E+04	2.86772360E+04
ACTUAL BENDING STRESS (PSI)	SB	=	1.89848014E+04	1.63961167E+04
MAXIMUM ALLOWABLE COMPRESSIVE STRESS (PSI).	SAC	=	1.28000000E+05	1.15000000E+05
MAXIMUM ALLOWABLE WEAR STRESS (PSI) . .	SSM	=	1.13292784E+05	1.01786486E+05
ACTUAL WEAR STRESS (PSI).	SS	=	1.01455539E+05	1.01455539E+05

MODIFICATION FACTORS

C-FACTORS EMPLOYED IN WEAR STRESS ANALYSIS
Q-FACTORS EMPLOYED IN BENDING STRESS ANALYSIS

BENDING ANALYSIS GEOMETRY FACTOR FOR THE PINION.	QJ1	=	2.57264737E-01	
BENDING ANALYSIS GEOMETRY FACTOR FOR THE GEAR.	QJ2	=	2.97882726E-01	
WEAR ANALYSIS GEOMETRY FACTOR	CJ	=	1.15565196E-01	
ELASTIC COEFFICIENT FACTOR.	CE	=	2.31476801E+03	
SURFACE CONDITION FACTOR.	CF	=	1.00000000E+00	
LOAD DISTRIBUTION CORRECTION FACTOR . .	CM,QM	=	1.84317547E+00	1.84317547E+00
OVERLOAD CORRECTION FACTOR.	CO,QO	=	1.75000000E+00	1.75000000E+00
SIZE CORRECTION FACTOR.	CS,QS	=	1.00000000E+00	1.00000000E+00
VELOCITY CORRECTION FACTOR.	CV,QV	=	4.80656708E-01	4.80656708E-01
OVERALL DERATING FACTOR	COD,QOD	=	6.71072935E+00	6.71072935E+00
HARDNESS RATIO FACTOR	CH	=	1.00000000E+00	
LIFE CORRECTION FACTOR FOR THE PINION.	CL1,QL1	=	1.00000000E+00	1.00000000E+00
LIFE CORRECTION FACTOR FOR THE GEAR.	CL2,QL2	=	1.00000000E+00	1.00000000E+00
RELIABILITY CORRECTION FACTOR	CR,QR	=	1.12981600E+00	1.12981600E+00
TEMPERATURE CORRECTION FACTOR	CT,QT	=	1.00000000E+00	1.00000000E+00
OVERALL LIFE DERATING FACTOR FOR THE PINION.	CODL1,QODL1	=	8.85099875E-01	8.85099875E-01
OVERALL LIFE DERATING FACTOR FOR THE GEAR.	CODL2,QODL2	=	8.85099875E-01	8.85099875E-01

BACKLASH ANALYSIS

DESIRED MINIMUM BACKLASH AT STANDARD CENTRE DISTANCE (INCHES).	BLMIN	=		8.16303270E-03
DESIRED MAXIMUM BACKLASH AT STANDARD CENTRE DISTANCE (INCHES).	BLMAX	=		1.95912785E-02
ACTUAL MINIMUM BACKLASH AT STANDARD CENTRE DISTANCE (INCHES).	BLMINT	=		8.16303270E-03
ACTUAL MAXIMUM BACKLASH AT STANDARD CENTRE DISTANCE (INCHES).	BLMAXT	=		1.02489637E-02
MAXIMUM BACKLASH AT CENTRE DISTANCE TOLERANCE LIMIT (INCHES).	BLMAXU	=		2.43784740E-02
MAXIMUM TOOTH THINNING FOR BACKLASH INCLUDING MACHINING TOLERANCE (INCHES).	BL	=	8.94420254E-03	1.07829880E-02

TOLERANCES

			PINION	GEAR
RUNOUT TOLERANCE (INCHES)	TOLR	=	4.94661224E-03	7.25542286E-03
PITCH TOLERANCE (INCHES).	TOLP	=	1.10397627E-03	1.46783565E-03
PROFILE TOLERANCE (INCHES).	PTOL	=	1.60779290E-03	2.13689656E-03
LEAD TOLERANCE (INCHES)	TOLL	=	1.00522712E-03	1.00522712E-03
TOOTH TO TOOTH COMPOSITE TOLERANCE (INCHES).	TTCT	=	1.92198876E-03	1.85677780E-03
TOTAL COMPOSITE TOLERANCE (INCHES). . .	TCT	=	6.68006026E-03	9.20607108E-03
TOOTH THICKNESS TOLERANCE (INCHES). . .	TPTU	=	+ 1.39062064E-03	+ 6.95310320E-04
	TPTL	=	- 0.	- 0.
TOOTH THICKNESS VARIATION FROM TOOTH ELEMENT ERRORS (INCHES)	TPTV	=	1.39909340E-03	1.35162370E-03
TOOTH THICKNESS VARIATION FROM TOOTH ELEMENT ERRORS AND RUNOUT (INCHES). . .	TPTE	=	4.86268619E-03	6.70147170E-03
CENTRE DISTANCE TOLERANCE (INCHES). . .	CDTOLU	=	+ 6.37048314E-03	
	CDTOLL	=	- 0.	
ERROR IN ACTION	ERR	=	4.47415665E-03	

GEAR BLANK DIMENSIONS			PINION	GEAR
HUB LENGTH (INCHES)	HUBL	=	2.68249308E+00	2.68249308E+00
OUTER HUB RADIUS (INCHES)	HUBR	=	1.75000000E+00	1.75000000E+00
INNER RIM RADIUS (INCHES)	RIM	=	2.17175562E+00	1.49127219E+01
WEB THICKNESS (INCHES).	WEB	=	1.34124654E+00	1.34124654E+00
GEAR BLANK VOLUME (CUBIC INCHES). . . .	VOL	=	9.47842619E+01	1.29180388E+03

SPUR GEAR DESIGN...COMPLETE

15.567 SECONDS

constraints for those users familiar with optimization theory. The printout then continues with the violated constraint evaluation, if necessary, and the independent - dependent design variable list. The output results of the example problem are printed in Table 1.7.

REFERENCES

(1) Stratton, J.D., "Optimum Computer Design of External Spur Gears", Masters Thesis 1972, McMaster University.

(2) AGMA 210.02, "AGMA Standard for Surface Durability (Pitting) of Spur Gear Teeth", January 1965.

(3) AGMA 220.02, "AGMA Standard for Rating the Strength of Spur Gear Teeth", August 1966.

(4) AGMA 150.03, "Application Classification for Spur, Helical, Herringbone and Bevel Gear Gearmotors", April 1968.

(5) AGMA 151.02, "Application Classification for Helical Herringbone and Spiral Bevel Gear Reduces", December 1963.

NOTE: All other AGMA Publications employed in the design package have been noted in reference (1).