MECHANISMS FOR TEMPORAL NUMEROSITY
IN AUDITION

MECHANISMS FOR TEMPORAL NUMEROSITY IN AUDITION

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SCOPE AND CONTENTS:

This research investigates human perception of brief auditory events presented sequentially. Following an analysis of current, relevant theories, two experiments are described. The results of the first experiment show that two trains consisting of n and n+1 pulses become more difficult to discriminate from each other as (i) the time between the pulses decreases (ii) the number of pulses in the trains increase and (iii) the size of the set of stimulus trains increases. The results of Experiment 2 indicate that discrimination between a pair of "empty" intervals differing only by a constant duration depends on the time between the pulses marking the beginning and end of the intervals and not on the duration of the markers. Discrimination decreases as the durations of the pair of intervals increases.

No current theory accounts for the observed relationships although the neural counting theory proposed by Creelman (1962) is partially successful.

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CHAPTER 1

INTRODUCTION

The perception of number of sequentially presented sensory events has been investigated most thoroughly to date by C.T. White and his associates. The focus of interest of a series of studies begun in the 1950s (e.g. Cheatham and White, 1952; Cheatham and White, 1954; Forsyth and Chapanis, 1958; White, Cheatham and Armington, 1953; White and Cheatham, 1959; White, 1963) was the report of number of short duration events as a function of both number of events presented in a given modality ("the temporal numerosity function") and rate of presentation of events.

Auditory perception of number was investigated by Cheatham and White (1954). The stimuli used in the study were trains of square-wave pulses. Each pulse was 11 msec. in duration and 1000 Hz in frequency. Rates of presentation of pulses within a train were 10, 15 and 30 pulses per sec. The numbers of pulses comprising trains presented at the three rates respectively were: 1 to 8, 1 to 11 and 1 to 17. The results indicated that:

(i) for any rate, mean number reported increased as a function of number presented

(ii) for rates of 10, 15 and 30 pulses per sec. the mean number reported was less than number presented for trains of more than 4, 3 and 2 pulses respectively

(iii) for any presented number greater than 2, number reported decreased as rate of presentation increased.

(iv) the slopes of the functions relating mean number reported to total train duration were 9 pulses (reported) per sec. for objective rates of 10 and 15 per sec. and 11 per sec. for the 30 per sec. rate of presentation. The ranges of durations for the three objective rates were 11-711 msec., 11-678 msec. and 11-544 msec.

The data were re-analyzed in terms of *modal* number reported by White (1963). The skewness of the distribution of responses for numbers greater than five suggested to White that the mean was not an appropriate measure of central tendency. A graph relating number reported to total train duration for an objective rate of 30 pulses per sec. indicated a relative decrease in the slope of the function for trains whose total durations exceeded 300 msec. The slope, interpreted as the "perceived" rate of occurrence of events, was 12-13 pulses per sec. for train durations ranging from 11 to 300 msec. and was 6-7 pulses per sec. for train durations ranging from 300 to 544 msec. White noted that the slopes of the functions relating modal number reported to total duration of trains greater than 300 msec. was the same whether the trains were comprised of visual, auditory or tactile events. This observation led to the conclusion that a central temporal process was responsible for the rate of change in number reported.

The validity of White's interpretation of slope as the "perceived" rate of occurrence of events would appear to rest (i) on the assumption that Ss' reports were counts of discrete events "perceived" and (ii) on the independence of reported numbers and the size of the set of stimulus patterns. If the number reported in response to a particular train were dependent upon the characteristics of the set of trains presented during a session, then White's interpretation could not be accepted. An examination of the experimental procedure in the temporal numerosity studies does in fact suggest that an alternative interpretation of results can be offered. In all of White's studies a variation of the method of constant stimuli was used. A block of trials consisted of ten presentations in random order of each member of a set of trains. Rate was fixed for the block. The number of events comprising trains in the block were pre-determined for a particular rate. After each trial Ss reported their estimate of the number presented. This design would allow the subject to make consistent judgments of the stimulus in terms of either (i) number of events "perceived" or (ii) total duration (i.e. onset of first pulse to offset of last pulse.). Judgments based on total duration would depend on the availability of a rule for translating total duration into reported number.

The present research is a further investigation of the human observer's perception of short duration events presented in sequence. For the range of total durations studied by White, number reported appears to depend on the rate of occurrence of pulses, when number presented is held constant. The main question of this thesis is whether number is a count of discrete central events or whether reported number is based on discriminable differences in some continuous dimension.

I. Statement of Approach

The results of the present experiments will be examined within the frameworks of several current theories. The theories chosen for study describe, in particular, central mechanisms for coding the duration of a brief visual or an auditory stimulus and the successiveness of a pair of stimuli. The theories fall into three major classes. The first class describes the coding of brief sensory stimuli in terms of passage of time in the central nervous system that is independent of these stimuli. The stimulus serves as a signal to count internal neural events that mark time. The second class of theories suggest that the central nervous system codes only the presence of the stimulus. It is assumed that some finite amount of time is required for this registration of the occurrence of an event. If a second brief sensory event occurs within this time period, the processing of the first event will not be completed. The third major class of theories describe the conversion of stimulus energy into excitation in the central nervous system. Both the amount of this excitation and its distribution in time provide the information that allows the observer to recognize the stimulus.

The theories that will be considered are:

(i) the Central Unit of Duration or "Moment" Theory (Stroud, 1955; Kristofferson, 1965)
(ii) Information Processing Theory (Haber, 1968)

(iii) the Theory of Signal Detection (Swets, Tanner and Birdsall, 1961)
(iv) Duration Discrimination
(v) Discrimination of a Change in Excitation

Since no one of these theories was developed to explain either the coding of a train of brief pulses or the conversion from a central code to a numerical response, one of the goals of the thesis will be to extend them to allow for the analysis of temporal numerosity data.

II. The Central Unit of Duration

The central temporal process, to which White ascribes limitations in number perceived, has been variously described by McReynolds (1953), Stroud (1955), Walter (1950) and Wiener (1948). The focus of Stroud's temporal process is the coding of stimulus information in the central nervous system. According to the theory "physical time t is represented in the experience of man as psychological time T." Psychological time is assumed to have various characteristics:

(1) T is not a continuous variable.

(2) The transformation or coding of all physical information (e.g. luminosity) into experience (the psychological dimension, brightness) is dependent on T. The coding is accomplished either by means of an integrative scanning process or by sampling at successive points in real time. The unit of psychological time T is defined as the period of the scan or as the time between successive samples.

(3) Changes in a dimension of the physical stimulus that occur within the period of the scan will not be represented in experience. Changes occurring between successive scans or samples will be represented in experience.

(4) Data suggested to Stroud that the period of the scan or time between samples ranged between

> 50 and 200 msec. For most data a constant value of 100 msec. for the unit of psychological time or "moment" allowed an adequate summary of psychophysical functions.

These assumptions imply that physical events which differ from one another in t but not in T will be experienced to occur simultaneously. The maximum "perceived" rate of occurrence of events will be one per moment. For auditory stimuli, Stroud suggests that pressure variations in real time may be transformed or coded into several functions of T such as pitch, loudness or quality.

White's temporal numerosity experiments were designed to verify the prediction that there should be a definite limit to the perceived rate of stimulation. As noted above this prediction was stated explicitly by Stroud. It implies that given the limiting rate, one can calculate the duration of the hypothetical moment or central duration unit. If the central processing mechanism described by Stroud does in fact account for the reports of number perceived and if White's subjects are reporting number perceived, then White's data indicate that for train durations less than 300 msec. the perceived rate of stimulation is 12-13 pulses per second. The moment must be approximately 84 msec. For train durations equal to or greater than 300 msec. the central unit must be approximately 167 msec. That point in time about 300 msec. after the onset of stimulation was conceived of by White (1963) as a point of transition between two central processes.

In suggesting that a central unit of duration (more specifically Stroud's moment) may account for the shape of the function relating reported number and train duration, one must make several implicit assumptions about the translation of input into the reported count. Assumptions suggested in White's papers but not clearly stated are:

(1) the subject's report is the outcome of integrative scanning during successive moments of time
(2) several stimulus pulses occurring within one moment will be coded as one pulse
(3) the subject's response will depend on moments counted rather than on the stimulus input
(4) to be counted, a moment must contain at least one stimulus event
(5) the subject's report of number perceived will be equal to the number of moments counted
(6) the subject's memory of the number of moments counted will not change as a function of the total duration of the train.

It is unclear from White's (1963) discussion whether the operation of the scanning mechanism is independent of the train of pulses or whether it is triggered into operation by the onset of the first pulse in the train. White and Eason (1966) present evidence supporting the argument that the hypothetical scanning mechanism is initiated by the stimulus. In this later paper, the evoked cortical response is suggested as the mediator between number presented and number reported. Like the temporal numerosity function described by White (1963), the evoked cortical response (ECR, measured from the occipital cortex) to a single flash of light consists of two main parts: (i) a complex transient response ending about 250 msec. after the stimulus onset and (ii) a rhythmic afterdischarge which appears as the transient ends. Components of the rhythmic aftereffect are separated by approximately 100 msec. This evoked pattern to a single flash does not appear to be different when the stimulus is a train of 25 flashes presented at the rate of 50 flashes per second. However, it is influenced by the nature of the stimulus conditions at the time of onset of the train. According to White and Eason's analysis, during the first segment of the ECR, "there seems to be a one-to-one relationship between the components of the response pattern and the number of flashes which can be perceived. During the second segment there seems to be a two-to-one relationship between the cyclic brain processes and the perceived rate." Number perceived is limited by the total duration of the stimulus train.

Kristofferson (1965) argues that the unit of duration is independent of the stimulus. He postulates a central time base composed of adjacent quanta of time. The quanta are separated by time points generated by a clock in the brain. The operation of the clock is independent of events external to the observer. Kristofferson's theory predicts that discrimination of the successiveness of a pair of events in different modalities will depend on the occurrence of a time point between these events. More specifically, the probability that the interval between the events will cover a time point will increase linearly from zero to 1.00 as this interval increases from zero msec. to the duration of one quantum. Experimental data in both successiveness discrimination and reaction time studies (Kristofferson, 1967) suggest that the quantum is approximately 50 msec. in duration.

Kristofferson's theory at its present stage of development does not make predictions about the discrimination of successive events within a single modality. However, the addition of several explicit assumptions to the theory allows it extension to temporal numerosity. Let us assume that:

(1) onset of a stimulus results in excitation in a display area of the auditory system

(2) a memory counter associated with the display area can take on one of two values (d=0, 1)

(3) presentation of excitation automatically sets the memory counter to 1 if it is in state 0

(4) offset of a stimulus pulse results in a decrease to zero of excitation in the display area

(5) at each time point generated by the clock, the display area is sampled. Absence of excitation in the display area during sampling resets the memory counter to 0.

(6) offset of the last pulse in the sequence is signalled by a criterion number of successive samplings of an empty display area

(7) at train termination a count is generated. The count is equal to the number of 1 states in the memory counter.

This new model predicts that stimulus events in the same modality must be separated by at least one quantum of time (i.e. must fall in adjacent quanta) to be counted as distinct events.

In a train of n events the count generated will depend on:

(1) the number (n) of pulses in the train

(2) the periodicity of the hypothetical time base (q)

(3) the duration of the pulses (p) comprising the train

(4) the interpulse interval (ipi).

Since the generation of sampling points is independent of the sequence of stimulus pulses, onset of the first event in a train is equally likely to arrive at any point during the current quantum. Thus, several outcomes or counts with different probabilities of occurrence may be generated for a given train.

This theory assumes that the response will be the same as the coded input. However, it has been shown (Swets, 1961) that the subject's response in many psychophysical experiments may be influenced by such non-perceptual factors as: the probability of a particular stimulus within the set of stimuli to be presented, expectations of the subject and the values associated with certain stimulus response contingencies. Thus, if a subject is asked to discriminate between two trains of different n his performance may reflect both

(1) the degree of overlap of the predicted distributions of counts

(2) response bias: the count generated will with some probability be associated with each member of the set of admissible response alternatives.

White, in his studies of temporal numerosity, has used the free response paradigm exclusively. Thus, the set of admissible response alternatives available to S is infinite. Frequency of numerical response data presented in the vision, audition and touch articles of White and his associates show that number reported never exceeded number presented. But subjects did tend to distribute their responses in at least two or three categories. The data imply that if the count is completely determined by the stimulus, the subject must be holding simultaneously several biases for report. In order to limit the bias structure, the response set in the present experiments will be restricted to those numbers of events actually presented to the subject. The stimulus set will be restricted to at most three trains each comprised of fewer than five stimulus pulses in order to minimize the effects of memory factors on numerosity judgements.

A tree diagram predicting the discriminability of any pair of trains containing n and n+1 pulses and having the same values of p and ipi is shown in Figure 1. The model presented contains both a sensory process and a response process. The sensory process consists of the translation or coding of the stimulus pattern S_n, where n represents the number of pulses in the train, into a count K_c. The subscript c is the number of 1 states in the hypothetical memory counter. More simply,

$$c = 1 + t \tag{1}$$

where t is equal to the number of interpulse intervals covering time points. The probability that a given train of pulses S_n will be translated into a particular count K_c is represented by $\alpha_{n,c}$. This probability depends on the stimulus parameters p, ipi and n and on the theoretical parameter q. The probability distribution of counts for a particular train takes into account the asynchrony between the onset of the

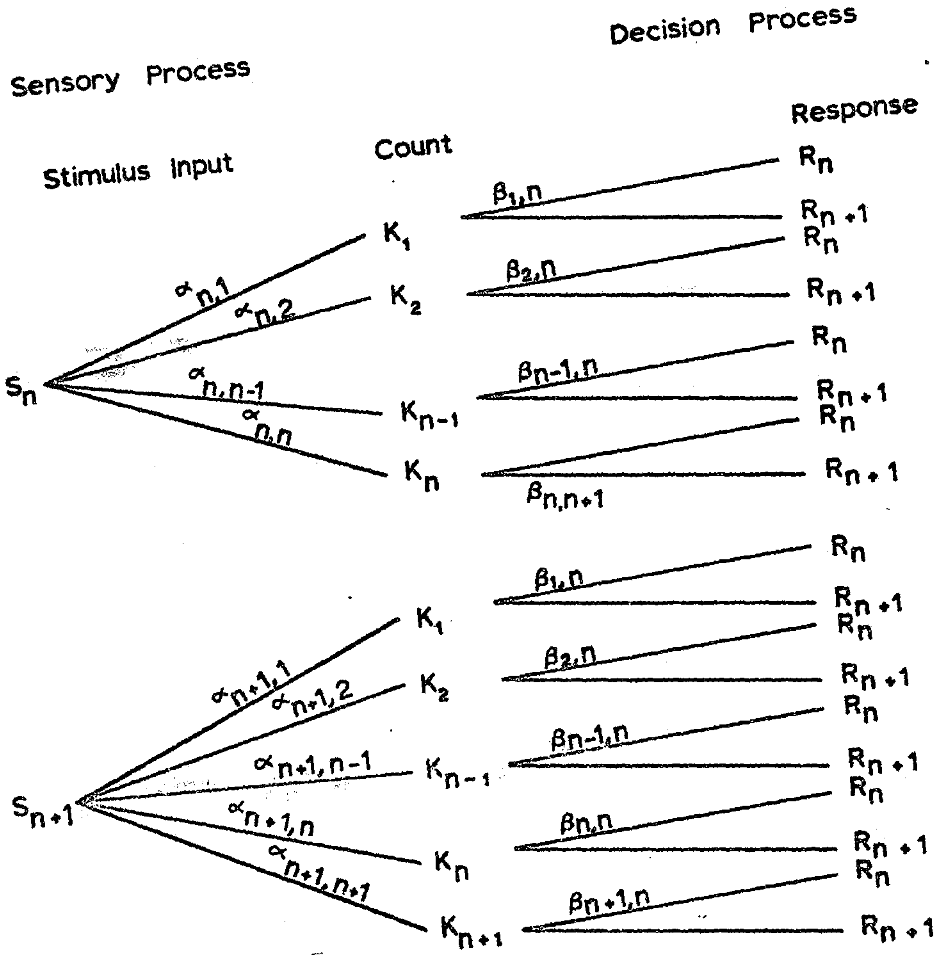


Figure 1. Schematic representation of the quantal model.

pulse train and the current quantum. The sensory process predicts that counts generated will always be equal to or less than n. The counts produced by a particular train S_n need not be consecutive integers. A numerical example of the calculation of $\alpha_{n,c}$ is presented in Appendix A.

The decision process consists of the translation of the count K_c into the response alternatives defined by the task. The probability that K_c will result in a particular response R_j (j=n, n+1) is represented by $B_{c,j}$. In the two response alternative model presented

$$B_{c,n+1} = 1 - B_{c,n} \tag{2}$$

Each value of c will be associated with different pairs of biases for report. That is, the bias for report will depend on the particular sensory state. For a block of trials the values of these biases will be determined by the presentation probabilities of the trains presented and by the distribution of counts generated by these trains.

If S̲ is required to discriminate between equally likely trains of n and n+1 pulses, the probability of a correct response Pr(C) is given by the equation

$$\Pr(C) = \sum_{j=n}^{n+1} \Pr(R_j | S_j)\Pr(S_j) \tag{3}$$

where $\Pr(S_j)$ is the marginal probability of presenting j events and $\Pr(R_j|S_j)$ is the conditional probability of responding j given a train containing j pulses. The expressions for $\Pr(R_n|S_n)$ and $\Pr(R_{n+1}|S_{n+1})$ may be determined from the tree diagram:

$$\Pr(R_n|S_n) = \alpha_{n,1}\,\beta_{1,n} + \alpha_{n,2}\,\beta_{2,n} + \cdots\ \alpha_{n,n-1}\,\beta_{n-1,n} + \alpha_{n,n}\,\beta_{n,n}$$

$$= \sum_{c=1}^{n} \alpha_{n,c}\,\beta_{c,n} \tag{4}$$

$$\Pr(R_{n+1}|S_{n+1}) = \alpha_{n+1,1}\,\beta_{1,n+1} + \alpha_{n+1,2}\,\beta_{2,n+2} + \cdots + \alpha_{n+1,n-1}\,\beta_{n-1,n+1} + \alpha_{n+1,n}\,\beta_{n,n+1} + \alpha_{n+1,n+1}\,\beta_{n+1,n+1}$$

$$= \sum_{c=1}^{n+1} \alpha_{n+1,c}\,\beta_{c,n+1} \tag{5}$$

III. Information Processing Theory

An alternative to the quantum model described above is the theory of discrete item processing outlined by Haber (1968). According to Haber, visual displays of items (e.g. letters or numbers) are initially encoded into a short term visual storage (STVS). STVS is defined as an "unlabelled neural representation of the retinal patterns or some receptive field neural encoding of that pattern". Processing of the items (information) in the display consists of the serial labelling and transfer of each from STVS into a short term memory. The time to process an item (PT) may depend on the requirements of a particular task. It is assumed that the serial processing begins with the onset of the display and may be terminated before completion either by decay of the STVS or by the presentation of an interfering event such as visual noise (a random pattern of lines, curves and nonsense forms). In the latter case processing of the STVS stops and processing of the visual noise begins. Items in the STVS that have been processed already will be preserved.

In applying the "process-stopping" model described to serially presented items Haber and Nathanson (1969) assume that when these items occupy the same retinal location, each successive item may function as visual noise for the one preceding. Thus, "if it takes X msec. to read out a letter (i.e. PT equals X msec.), then any rate (of presentation of items) yielding a processing time per letter (i.e. a time between successive onsets of items) greater than X should yield perfect performance. Processing times less than X should produce less accuracy--sometimes the letter will be read out and sometimes not--with a higher probability attached to longer processing times." The latter statement implies that information available from partially processed items may lead with some probability to correct identification of the items.

In the Haber and Nathanson study subjects were presented sequences of 4,5,6,7 and 8 letters against a dark background. For each of these lengths of sequence all combinations of five _on_ times (10, 25, 50, 100 and 150 msec.) for letters and five _off_ times (10, 25, 50, 100 and 150 msec.) were investigated. Processing time available was operationally defined as the time between onset of successive items in the sequence or as the sum of the _on_ and _off_ times. At the end of each sequence _S_ was required to report the letters seen.

Subjects' reports were scored for the number of letters correctly reported in their proper position. Thus, _Ss_ were required to recall two items of information: the letters presented on a given trial and the position of each letter in the series presented. The results showed that available processing time or time between lines was a better predictor of performance than was either the on or the off time. For each word length, the more time available for processing, the higher the probability of a correct response. Probability of a correct response was interpreted as an indicant of number of letters processed in the hypothetical system.

For any condition (word length x on time x off time) the time needed by the system to process each letter correctly (PT) was calculated from the formula:

$$PT = \frac{\text{on time} + \text{off time}}{Pr(RO)} \tag{6}$$

where Pr(RO) refers to the probability that any given letter was correctly reported.

$$Pr(RO) = \frac{\text{number of letters correctly reported}}{\text{number of letters in the sequence}} \tag{7}$$

For any given word length PT remained fairly constant and independent of both the on and off times. However, as the number of items in the sequence increased from 4 to 8 the calculated PT increased from 65 to 110 msec. The latter finding was attributed to the absence of a noise mask following the last item in the sequence. It was also suggested that the increase might be due to a limitation of memory span rather than processing.

Support for the process-stopping model in the Haber and Nathanson paper suggests that the theory may be an important alternative to the moment model for explaining the perception of number. A simple model extending the theory to temporal numerosity may be derived. Like the quantal model the information processing model assumes both a sensory process and a response process. Again, the sensory process consists of the coding of a number of stimulus pulses S_n into a count K_c. However, c may assume only one value. This value will depend on the time (ipi + p) msec. between the onsets of successive pulses. If (ipi + p) is greater than or equal to PT, a one will be stored in memory for each pulse in the sequence. The value of c will be the number of 1 s stored. If (ipi + p) is less than PT then c should equal zero. However, for any value of (ipi + p) absence of a noise mask at the end of the sequence will allow the complete processing of the final pulse in the sequence presented.

The information processing model assumes as did traditional moment theory (c.f. White) that a sensory process alone will determine the subject's judgment of number. Non-sensory factors are ignored. Thus, if none of the admissible responses is equal to c, we will assume that the probabilities of reporting each of these responses given K_c are equal. If one of the responses is equal to the count, then the probability of that response given K_c is 1.00.

If S̲ is required to discriminate two equally likely trains of n and n+1 events, with the same values of p and ipi, then

(i) for ipi $\geqslant$ PT

$$Pr(C) = Pr(S_n) \quad Pr(R_n|S_n) + Pr(S_{n+1}) \quad Pr(R_{n+1}|S_{n+1})$$

$$= .50 \quad (1.00) + .50 \quad (1.00)$$

$$= 1.00 \qquad (8)$$

(ii) for ipi<PT (and in the absence of a noise mask)

$$Pr(C) = Pr(S_n) \quad Pr(R_n|S_n) + Pr(S_{n+1}) \quad Pr(R_{n+1}|S_{n+1})$$

$$= .50 \quad (.50) + .50 \quad (.50)$$

$$= .50 \qquad (9)$$

A schematic representation of this model is shown in Figure 2.

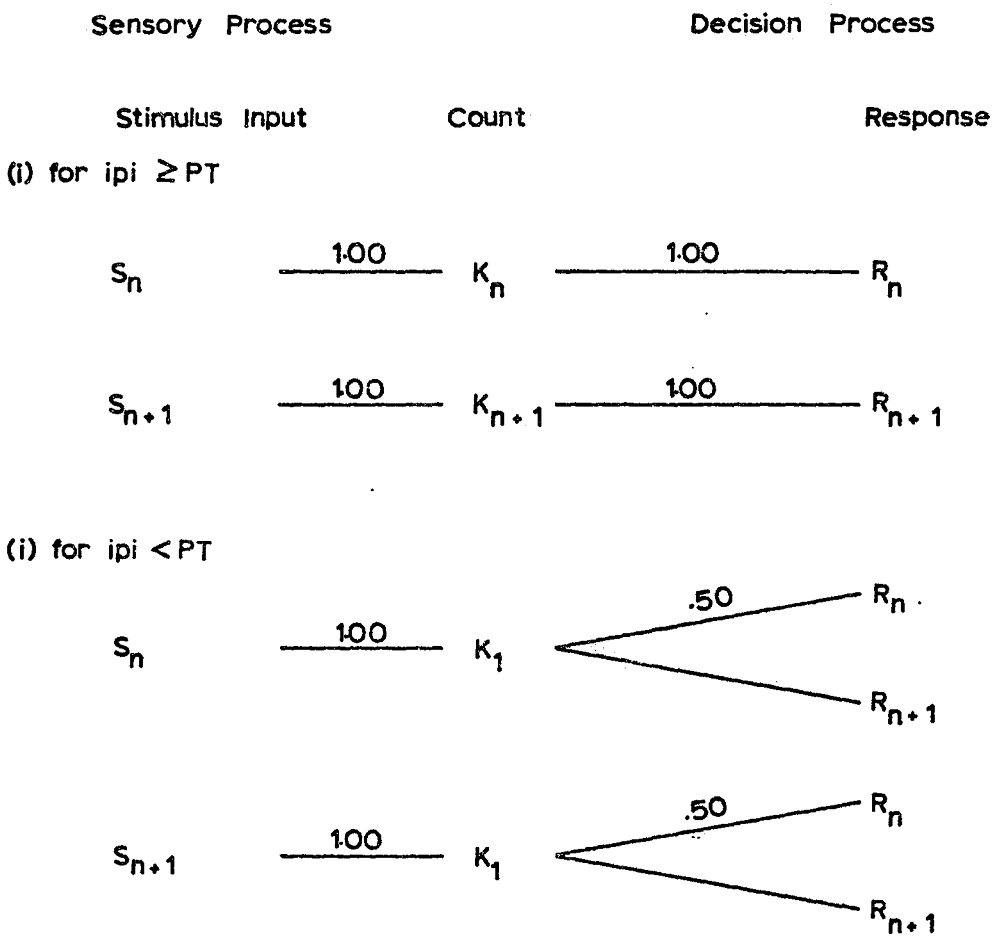


Figure 2. Schematic representation of the information processing model.

IV. The Theory of Signal Detection (TSD)

Chistovich and Ivanova (1950) and von Bekesy (1960) have suggested that if the gaps occurring between a pair of auditory pulses are too small to allow detection of two distinct events, the subject may report one "notched" stimulus or a "rough" event. It is conceivable that the amount and character of the roughness reported will depend not only on the size of the interpulse interval but in addition on the number of pulses in the train presented and on the duration of pulses. More generally, roughness may be defined in terms of the distribution of energy over time.

Suppose that Ss in the temporal numerosity experiment are discriminating distribution of energy in time rather than number. If such distributions produced by any pair of stimulus trains give rise to overlapping distributions of excitation values in the central nervous system, then S's response on any trial would depend on his decision rule for dividing the continuum of excitation values into two sets--one for each possible stimulus train.

According to the Theory of Signal Detection (Swets, Tanner and Birdsall, 1961) the observer is viewed as a tester of statistical hypotheses. On each experimental trial he makes a decision based on sensory excitation. Sensory excitation is a vector of N dimensions. In the absence of a signal excitation generated by noise in the sensory system will be present. It is assumed that any particular value of excitation may arise with given probabilities from either signal plus noise or from noise alone.

In the TSD model the excitation continuum is identified with likelihood ratio: the probability density that the sensory datum on any trial is contingent on signal plus noise relative to the probability density that the excitation has arisen from noise alone. More specifically, excitation is identified with that transformation of likelihood ratio that results in Gaussian density functions on the excitation continuum.

The distance between the means of the normal density functions divided by the standard deviation of the noise distribution is the average discriminability (d') of signal plus noise and noise alone. For any trial it is assumed that the observer matches the likelihood ratio of the observation made on that trial against some criterion value which allows him to accept or reject the hypothesis that a signal has been presented. While d' depends on sensory events, the criterion is influenced only by nonsensory factors.

The following assumptions will be added to the TSD model described above. They suggest a sensory process for generating Gaussian distributions of excitation on the decision axis given trains of n and n+1 pulses.

(1) In the absence of a signal the auditory display area contains excitation contingent on noise.

Values of the noise excitation are normally distributed with mean $E(S_o)$ equal to zero and variance Var (S_o) equal to 1.00.

(2) On any trial the presentation of a given stimulus train S_n adds a constant amount of sensory excitation to the display area.

(3) The amount of sensory excitation generated by the pulse train will depend on the interaction between successive pulses. This interaction depends in turn on the values of n, p and ipi.

(4) Changes in the relationship between n, ipi, and p will produce a unidimensional change in sensory excitation.

(5) Since the excitation of the signal train on any trial is added to noise and the noise has some variance, the signal plus noise excitation will be normally distributed with mean $E(S_n)$ equal to X_n and variance Var (S_n) equal to Var (S_o).

In accordance with TSD the discriminability of a pair of trains S_n and S_{n+1} may be expressed as the distance between the means of their hypothetical distributions of excitation on the decision axis

$$d' = \frac{E(S_{n+1}) - E(S_n)}{(\text{Var } S_o)^{1/2}} \tag{10}$$

Since the exact nature of the cumulative effects of n, p and ipi on stimulus excitation are unknown, no predictions will be made within the context of TSD about the effect of changes in these variables on discrimination of pulse trains. The measure d' will be used as a descriptive statistic and the consistency of the theory will be examined.

V. Duration Discrimination

It was suggested earlier that performance in the temporal numerosity experiment might be based on the subject's estimation of the total duration of the stimulus train. If this were the sole basis of performance, then S would have to remain insensitive to such stimulus dimensions as (a) the relation between pulse duration and interpulse interval and (b) the number of events presented. Constant train durations should yield the same response. These predictions appear to be supported by White's (1963) data.

How might the response change as a function of changes in total duration? Suppose the subject was asked to compare two trains of durations D and (D + ΔD) msec. respectively. If the duration of the shorter of the two trains remained constant, linear increments in ΔD for the second train would result in linear increments in the ratio $\frac{\Delta D}{D}$. However, if ΔD remained constant and D was varied, then the ratio ΔD/D would decrease exponentially as a function of linear increases in the duration of the shorter train. Data reported in the literature both supports and brings evidence to bear against the possibility that S is sensitive to changes in the ratio ΔD/D in duration discrimination tasks. The experiments are primarily concerned with the just noticeable difference (or differential threshold) for duration and the stimuli used are continuous events.

Henry (1948) asked subjects to report whether a series of durations presented was constant or whether some duration D alternated with D plus some particular increment ΔD. The durations used were comprised of segments of a sine wave of approximately 500 Hz and were 50 db above treshold. Series containing the homogeneous and alternating stimuli were presented in random order. The results indicated that the value of $\frac{\Delta D}{D}$ resulting in 75% correct judgments (the Weber ration or differential threshold) decreased linearly from .281 to .143 as D increased from 32 to 480 msec. More explicitly, contrary to Weber's Law, ΔD was not a constant proportion of D although it did appear to increase with increases in the stimulus duration judged.

Small and Campbell (1962) used the method of constant stimulus differences to determine the DL (difference threshold) for equally loud standard durations of .4, 4, 40 and 400 msec. Variable stimulus durations ranging from 0.25 to 2.5 times the duration of the standard were used. Measurements of the differential threshold, defined as one-half the difference between the variable stimulus value yielding 25% longer judgments and the value yielding 75% longer judgments supported the findings of Henry that $\frac{\Delta D}{D}$ decreased as the standard duration increased. Differences in the Weber ratio as a function of frequency (250 Hz, 5000 Hz and noise) appeared at the shortest durations used. These differences were attributed to changes in the acoustic waveform at stimulus termination for the 250 Hz tone and to decreases in the loudness of the variable stimulus correlated with decreases in its duration.

A more recent study by Creelman (1962) investigated the effects of both the duration and signal voltage of an auditory stimulus, mixed with continuous background noise, on duration discrimination. A two-alternative temporal forced-choice procedure was used. On each trial subjects were presented two stimuli of different durations separated by an inter-stimulus interval of .8 sec. For any block of 100 trials the shorter, base duration (D msec.) was equally likely to occur in the first or second position. S was required to report the ordinal position of the longer stimulus $(D + \Delta D)$ msec. In each of four experiments reported, each value of the independent variable was presented for 200 or 300 trials. A second value was then chosen and the procedure repeated. When all values had been presented the experiment was replicated with a new random order of stimulus values. Replications continued until approximately 1000 observations had been obtained for each value.

Experiment 1 studied the effects on duration discrimination of signal voltage. With the stimulus durations for any trial fixed at 100 and 130 msec., d'_{12} (a measure of discrimination derived from TSD) increased as a negatively accelerated function of signal voltage for values ranging from .01 to .05 volts.

The second experiment investigated the observers' ability to discriminate a constant increment of 10 msec. from base durations ranging from 20 msec. to 320 msec. Signal voltage was held constant at .084 v. The results of four observers indicated that d'_{12} decreased with increases in D.

Experiment 3 studied discrimination as a function of the increment in duration ΔD. Base duration was fixed at 160 msec. Values of ΔD were 10, 20, 40, 80 and approximately 160 msec. Discriminability increased as a function of increases in ΔD. There appeared to be no interaction between this effect and the effect of a change in signal voltage from .010 v. to .042 v.

But Experiment 4, designed to measure discrimination as a function of base duration and signal voltage, did show an interaction of effects of the two independent variables. The effect of increasing signal voltage from .010 to .042 v. resulted in a steeper decrease in discriminability as base duration increased from 40 to approximately 600 msec.

The final experiment in the series investigated changes in discrimination for constant values of both $\frac{\Delta D}{D}$ and constant energy in the increment ΔD. D varied from 40 to 1600 msec. For this range of base durations two levels of signal to noise ratio ($2E/N_o = 100, 400$) were used. The ratio $\frac{\Delta D}{D}$ was held constant at 1/8. For increments in ΔD ranging from 5 to 200 msec. the product of the square of increment voltage and increment duration ($V_s^2 \times \Delta D$) remained constant. (Thus, when ΔD was doubled V_s decreased by a factor of $\sqrt{2}$). The results indicated that for both signal to noise ratios performance remained fairly constant giving some support to Weber's Law.

The data described above support our earlier statements concerning the relationship between subjects' ability to discriminate differences in duration and the ratio of ΔD, an increment in duration, to D some base duration. These studies indicate the amount of change in specific performance measures that one might expect given a wide range of values of both D and ΔD. Discrimination in temporal numerosity studies exceeding the limits defined by the duration discrimination studies would lead to the rejection of the hypothesis that total duration is the only cue to number.

VI. Discrimination of Energy

The results of the studies reported both by Small and Campbell and by Creelman and discussed above suggest, that for auditory signals of short duration, subjects' discrimination of differences in duration may be based on cues other than or in addition to changes in duration. In the Creelman study, when signal energy is not held constant, the rate of change in d' as a function of base duration depends on signal voltage. On the other hand, sensitivity remains constant for a constant $\frac{\Delta D}{D}$, when differences in energy of pairs of stimuli presented is held constant.

Evidence for a trading relation between time and intensity

$$I \times T = C \tag{11}$$

for the detection of auditory signals has been reviewed by Green, Birdsall and Tanner (1957). Whereas previous research was aimed at determining the intensity and duration of signal or pattern of input necessary to produce a constant detectability (e.g. absolute threshold) Green et al were interested in those changes in signal duration and intensity resulting in a change in detectability.

The paradigm of the experiments was the four-alternative forced choice test. A measure of discrimination, the index d', was derived for the paradigm from the Theory of

Signal Detection. The signal was a tone of 1000 Hz presented for an integral number of cycles from a positive-going zero crossing of the sine wave. On all trials only one of four adjacent time periods defined for S contained the signal presented in noise. The other intervals contained noise only. The length of the time periods depended on the duration of the signal. The observer's task was to choose that interval containing the signal.

In the first of three experiments the power of the signal and duration were varied independently. Values of E/N_o ranged from 10 to 40. (E represents the signal energy, i.e. acoustic power multiplied by duration, and N_o refers to the noise spectrum level). The results indicated that for a constant signal duration, d' increased as a linear function of acoustic power.

Pairs of values for signal power and duration were selected for Experiment 2 such that the energy of the signal E/N_o remained constant. Thus, as signal duration increased from 5 to 1000 msec., signal power was reduced. Data for four observers showed that detectability increased as a linear function of duration to approximately 20 msec. and then remained constant for durations ranging from 20 to approximately 100 msec. Further increases in duration were accompanied by linear decreases in d'.

In Experiment 3 signal duration was varied for different values of signal power. For values of signal power

(P/N_o) equal to 867.4, 243.4, 87.6 and 33.7 the ranges of duration was approximately 5 to 25 msec., 20 to 100 msec., 70 to 275 msec. and 250 to 3000 msec. respectively. For each value of signal power, d' increased with increases in duration over the entire range studied.

The experiments reported both by Creelman and by Green et al imply that, in detecting and discriminating signals varying over a wide range of durations, Ss are not sensitive solely to duration or to amplitude. Rather, Ss respond to some quantity which represents an integration of signal duration and amplitude. We shall label this quantity excitation in the auditory system. According to Green et al the nature of the integrative process changes as a function of duration. Linear integration of acoustic power over time can only be expected for signals ranging from approximately 20 to 100 msec. For signals shorter than 20 msec., the authors assume some loss of physical energy present in the stimulus. For longer signals they assume less than perfect integration. Over any of the three ranges of durations, we might expect in addition that the shape of the integration or excitation function would depend on the signal to noise ratio.

Evidence for cumulative effects of power for signal durations ranging from about 20 msec. to 3 sec. suggests the necessity of studying the integrative process in pulse trains and of evaluating this integration as a cue to number presented.

VII. Discrimination of a Change in Excitation

The preceding discussion has brought the theoretical mechanism underlying discrimination of number from coding in terms of total duration to coding in terms of stimulus excitation. It has been suggested that for signals of short duration this excitation is the product of duration and amplitude. The studies described above were concerned with the detectability and discriminability of single stimuli. One implication of their results for discriminability of pulses presented in sequence might be that discrimination of number is a function of *S*'s ability to detect a change in excitation in the intervals between successive pulses. The change in the level of excitation during the intervals would presumably depend on the duration of the interval, and on the rate of decay during this interval of excitation produced by a previous pulse.

Miller and Taylor (1948) reported that the sensation level at which interrupted noise could not be distinguished from a continuous noise (critical flutter frequency) increased (i) as the rate of presentation of noise pulses increased and (ii) for any rate as the on to off ratio increased from .5 to .75 to .90. In a more recent study, Gesheider (1967) studied *Ss* ability to resolve auditory and cutaneous stimulus doublets. The auditory stimuli on any trial were two one msec. square-wave electrical pulses delivered binaurally from vibrators suspended one inch from the subject's ears. The results indicated that as sensation level of the pulses increased from 10 to 40 db SL, (measurements relative to absolute threshold) Δt (the threshold for reporting two pulses rather than one) decreased from approximately 4 to 2 msec. Subsequent studies showed that Δt depended on the relative amplitude of the pair of events. With the intensity of the first stimulus held constant at 20 db SL, variation of the intensity of the second pulse from 10 to 35 db SL resulted in a decrease in Δt from 6 msec. to approximately 2 msec. When the second stimulus was held constant at 20 db, Δt decreased from about 3.2 to 2.9 msec. as intensity of the first pulse increased from 10 db to 15 db and then increased with increases in intensity from 15 to 35 db. The results of the studies were interpreted in terms of the suppressive effects of a first event on the neural response of the second.

The phenomenon reported by Gescheider may be subsumed under the more general heading of auditory masking. Demonstrations of both forward and backward masking have been reported by both Russian and American researchers (e.g. Chistovich and Ivanova, 1958; Raab, 1963; and Samoilova, 1959). However, in most of the experiments that appear in the literature, masking is measured as a function of the time interval between the two events. With the exception of some recent work by Green (in press) little attention is given to the characteristics of the events themselves. The magnitude of masking is usually defined in terms of the shift in the absolute threshold of the masked tone.

Raab (1961), for example, presented trials consisting of a masking pulse alone and a masking pulse plus "probe stimulus". The two types of events were spaced 0.8 sec. apart. Ss were required to report which mask was accompanied by the probe. For given values of Δt (the time between probe and mask) the intensity of the probe was decreased from at least 10 db above hearing threshold to that intensity yielding fewer than six out of seven correct judgements. The probe threshold under these conditions was defined as midway between the first intensity of the probe yielding six out of seven correct detections of the probe and that intensity where S failed to reach this performance criterion. Probe thresholds plotted were means of at least three such determinations. Sensation levels of the mask studied were 85 and 70 db. Durations of both the mask and probe were .2 msec. The results showed that both forward and backward masking (defined by positive and negative values respectively of Δt) did occur. The forward effect however appeared to be greater and longer lasting than the backward. For both positive and negative values of Δt the greater the amplitude of the masking stimulus the greater the intensity of the probe necessary for threshold performance. For the weaker mask, forward masking as defined by probe threshold decreased to the level of the unmasked threshold as Δt increased from 0 to 50 msec. For the stronger mask, the probe threshold reached the unmasked threshold at Δt equal to approximately 100 msec.

Plomp (1964) studied in somewhat greater detail the shape of the decay function implied by forward masking. On each trial Ss were presented two pairs of square-wave noise pulses. The duration of the first pulse in each pair (200 msec.) was long enough to ensure that the growth of excitation had reached asymptote at pulse offset. The second pulse in the pair ended 200 msec. after the offset of the first pulse. However, for one pair onset of the second pulse was contiguous with offset of the first pulse and in the second pair, some time Δt intervened between the two events. On each trial the time gap occurred at random in one of the two pairs. Ss were required to choose the pair containing the gap. The minimum time gap that could be heard was measured as a funtion of the sensation level of the second pulse. For levels of the first pulse at 65, 45, and 25 db SL, level of the second pulse was varied between the sensation level of the first pulse and 15 db. For pairs of values of the first and second pulse the threshold value of Δt (i.e. that value of Δt detectable on 75 out of 100 trials) was determined. The results showed that threshold Δt decreased linearly with linear increases in the sensation level of the second pulse. For constant values of pulse two the log of threshold Δt decreased with decreases in the level of pulse one.

Plomp's primary interest was in the rate of decay of auditory sensation produced by the first of two pulses. He assumed that an interruption or time gap Δt would only be perceived when the difference in sensation level (Δs) between the residual value of the first pulse after Δt msec. and the level of the second pulse exceeded a critical amount. For any value of pulse two Δs would increase as the time gap increased. For each threshold Δt measured in the experiment described above its associated value of pulse two was interpreted as a good approximation of the residual value of pulse one at Δt msec. after the offset of pulse one. Extrapolation of the function relating log Δt and sensation level of the first pulse indicated the time needed for the residual of pulse one to reach absolute threshold. Extrapolation of the three functions plotted for the three levels of pulse one showed that the first pulse decayed to absolute threshold at approximately 225 msec. after its offset. The result was independent of the SL of pulse one.

The work reported by Miller and Taylor, Gescheider and Plomp support the notion that number of pulses reported may within certain limits be determined by the number of detectable changes in excitation that occur during the period of stimulation. This hypothesis is related to the theory described earlier that the observer looks at the cumulative effects of stimulation. It was suggested that these cumulative effects could depend on the relationship between n, p and ipi. The studies reviewed above point to pulse amplitude as another relevant variable. To study changes in excitation during the interpulse interval as a function of pulse duration and amplitude, it would be necessary to control for the effects of n, number of successive ipis and changes in the total duration of the stimulus train. We could eliminate the effects of these latter variables by restricting the subject's task to recognition of a single interpulse interval, while holding fixed differences in the total durations of stimulus trains judged.

The following models make predictions about the subject's ability to discriminate a gap given assumptions about (a) the sensory effects of the stimulus pulse parameters and (b) the observer's decision strategy.

a. Models for Gap Discrimination

Suppose the observer is asked to compare the two patterns of events presented in Figure 3. Both are composed of a train of two auditory signals. The first square-wave pulse in each sequence has the same duration d_1 msec. and the same amplitude a_1 volts. Similarly, the second pulse in each sequence has the same duration d_2 msec. and amplitude a_2 volts. In the first pattern the pulses are separated by ipi

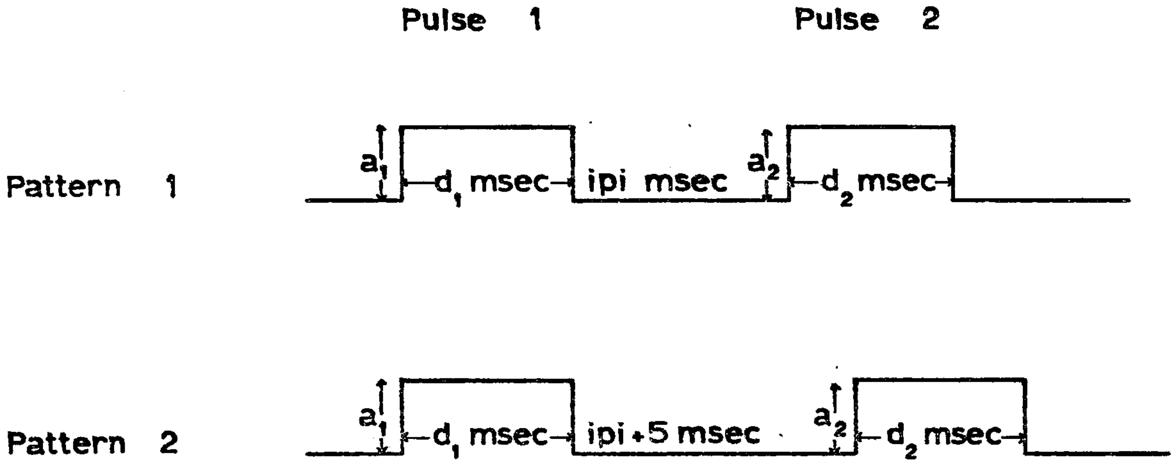


Figure 3. Stimulus patterns for study of the recognition of a single interpulse interval.

msec. and in the second by (ipi + 5) msec. S must distinguish between pattern 1 and pattern 2.

Model I: Exponential Growth, Exponential Decay

Several variations of the Exponential Growth, Exponential Decay (EGED) model will be considered. Each consists of the same assumptions for the coding of signals. They differ in terms of the cognitive or decision strategies assumed to operate.

Sensory Processing Assumptions

1. In the absence of a signal a display area in the brain set to receive auditory signals will contain excitation contingent on noise in the nervous system. The amount of such excitation may take on a number of values. The N dimensional vector representation of these values is assumed to be normally distributed (Swets, Tanner, and Birdsall, 1961). To simplify computation the expected value of the noise excitation, E(X), is assumed equal to zero, and the variance, VAR(X), is assumed equal to 1.00.

2. According to Plomp (1961), the onset of a sinusoidal signal of amplitude a_1 volts and duration d_1 msec. results in the exponential growth of excitation in the auditory display area. Let the excitation produced by a pattern V (for v=1,2) in Figure 3 be represented by X_v. The excitation

contingent on a particular pulse p (for p=1,2) in this pattern will be denoted $X_{v,p}$.

3. For any value of d_1 the sensory effect is the same on repeated presentations. However, this sensory effect is displayed against a variable noise background. Therefore, as d_1 approaches zero msec. the expected value of the excitation produced by the first pulse in either pattern $E(X_{v,1})$ will approach zero. As d_1 increases linearly, $E(X_{v,1})$ will grow exponentially to an asymptotic value αa_1. The value of αa_1 is a constant proportion of the amplitude of the signal. Thus, at the moment of pulse offset

$$E(X_{v,1}) = \alpha a_1 (1-e^{-cd_1}) \tag{12}$$

where c represents the rate of growth of excitation during pulse presentation.

4. At pulse offset the excitation in the display area decays exponentially to zero. For pattern 1 in Figure 3 at the end of ipi msec.,

$$E(X_{1,1}) = [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi)} \tag{13}$$

where b is the rate of decay of excitation.

The expected value of the excitation of pulse 1 for the second pattern in Figure 3, $E(X_{2,1})$, at the moment of onset of pulse two may be described by the equation

$$E(X_{2,1}) = [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi+5)} \qquad (14)$$

Decision Process

Variation 1: First Event Strategy

The onset of the second pulse acts as a signal to sample the display area and to record in short term memory the amount of excitation present. The act of sampling results in the loss of information contained in the excitation produced by the second pulse. Thus, S's sensitivity (d') to the difference between the two patterns must depend on the differences in the residual values of the first pulse in the patterns at the time of sampling. This difference is the distance between the means $E(X_{1,1})$ and $E(X_{2,1})$ of the distributions of excitation.

$$\begin{aligned} d' &= E(X_{1,1}) - E(X_{2,1}) \\ &= [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi)} - [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi+5)} \\ &= [\alpha a_1(1-e^{-cd_1})]\ [e^{-b(ipi)}(1-e^{-5b})] \qquad (15) \end{aligned}$$

Equation 15 predicts that the measure of S's sensitivity to the difference between the two patterns d' will

(a) decrease exponentially with a limit of zero as ipi increases for fixed α, a_1, cd, and b

(b) increase exponentially as the duration of the first pulse in the patterns increases for fixed α, a_1, b, c and ipi

(c) increase linearly as the amplitude of the first pulse in the stimulus pairs increases for fixed α, a_1, b, c and ipi

(d) not be affected by changes in the duration and amplitude of the second pulse with the other parameters fixed.

Variation 2: Excitation Summation Strategy

The second variation assumes that <u>S</u> bases his decision on a comparison of the summated excitation produced by each train of events. Excitation in the display area is sampled at the offset of the second pulse and will equal the expected value of the residual excitation of pulse one at the onset of pulse 2, $E(X_{v,1})$, plus the expected value of the excitation produced by the second pulse, $E(X_{v,2})$. Thus, the expected values of the excitation for the two patterns $E(X_1)$ and $E(X_2)$ are expressed by the equations:

$$E(X_1) = E(X_{1,1}) + E(X_{1,2}) = [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi)} + \alpha a_2(1-e^{-cd_2}) \qquad (16)$$

$$E(X_2) = E(X_{2,1}) + E(X_{2,2})$$

$$= [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi+5)} + \alpha a_2(1-e^{-cd_2}) \quad (17)$$

The observer's sensitivity to the difference between the patterns is summarized by the equation:

$$d' = E(X_1) - E(X_2)$$

$$= [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi)} + \alpha a_2(1-e^{-cd_2})$$

$$- [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi+5)} - \alpha a_2(1-e^{-cd_2})$$

$$= [\alpha a_1(1-e^{-cd_1})] \; [e^{-b(ipi)}(1-e^{-5b})] \quad (18)$$

Since equations 18 and 15 are identical, the predictions of variation 2 will be the same as those of variation 1.

Variation 3: Increment Strategy

The third variation assumes that excitation contingent on the first pulse is stored in one display location of the brain. Onset of the second pulse results in (a) the sampling of the first display area and short term storage of the amount of excitation present and (b) accumulation of excitation contingent on the second pulse in a second display area. At the offset of pulse two the second display area is sampled. The difference in the expected values of the excitation in the two display areas, $E(X_v)$, is computed for each pattern as follows:

$$\Delta E(X_1) = E(X_{1,2}) - E(X_{1,1})$$

$$= \alpha a_2(1-e^{-cd_2})-[\alpha a_1(1-e^{-cd_1})]e^{-b(ipi)} \quad (19)$$

where Δ refers to the increment in the excitation of the second pulse in the pattern as compared with the first.

$$\Delta E(X_2) = E(X_{2,2}) - E(X_{2,1})$$

$$= \alpha a_2(1-e^{-cd_2}) - [\alpha a_1(1-e^{-cd_1})]e^{-b(ipi+5)} \quad (20)$$

S's sensitivity to the difference between the two patterns will depend on the difference between the expected values of the differences for the two patterns.

$$d' = \Delta E(X_2) - \Delta E(X_1)$$

$$= [\alpha a_1(1-e^{-cd_1})] [e^{-b(ipi)} (1-e^{-5b})] \quad (21)$$

Since equation 21 is the same as equation 15 the predicted relationships between d' and stimulus variables are the same as those for variations 1 and 2.

Model II: Exponential Growth, Linear Decay

Three variations of the Exponential Growth, Linear Decay (EGLD) model will be described. Model II differs from Model I in the sensory processing assumptions. The decision strategies will be the same.

Sensory Processing Assumptions

Assumptions 1, 2 and 3 of Model II are the same as those described for Model I.

4. At pulse offset excitation in the display area decays linearly. After any interpulse interval ipi the amount of decay is a constant percentage ϕ of the excitation present at the start of the interval. Thus, at the moment of onset of pulse 2, the expected values of the excitation for the first pulse in each pattern will be described by the equations:

$$E(X_{1,1}) = [\alpha a_1(1-e^{-cd_1})] - \phi(ipi)[\alpha a_1(1-e^{-cd_1})] \quad (22)$$

$$E(X_{2,1}) = [\alpha a_1(1-e^{-cd_1})] - \phi(ipi+5)[\alpha a_1(1-e^{-cd_1})] \quad (23)$$

Decision Process

Variation 1: First Event Strategy

(for a description see pages 45, 46)

$$d' = E(X_{1,1}) - E(X_{2,1})$$

$$= 5\phi\ [\alpha a_1(1-e^{-cd_1})] \qquad (24)$$

The model predicts that S's sensitivity to the difference between the two patterns will

(a) not depend on ipi for fixed ϕ, α, a_1, c, and d_1

(b) increase exponentially as d_1 increases for fixed ϕ, α, a_1 and c

(c) increase linearly with increases in a_1 for fixed ϕ, α, c and d_1

(d) not depend on changes in the amplitude and duration of the second pulse in each sequence.

Variation 2: Excitation Summation Strategy

(for a description see pages 46, 47)

$$d' = E(X_1) - E(X_2) = \{E(X_{1,1}) + E(X_{1,2})\}$$

$$- \{E(X_{2,1}) + E(X_{2,2})\}$$

$$= \{\alpha a_1(1-e^{-cd_1}) - \phi ipi[\alpha a_1(1-e^{-cd_1})]$$

$$+ \alpha a_2(1-e^{-cd_2})\}$$

$$- \{\alpha a_1(1-e^{-cd_1}) - \phi(ipi+5)[\alpha a_1(1-e^{-cd_1})]$$

$$+ \alpha a_2(1-e^{-cd_2})\}$$

$$= 5\phi\ [\alpha a_1(1-e^{-cd_1})] \qquad (25)$$

The predicted relationships between d' and the stimulus variables will be the same as those for variation 1, since equation 25 is the same as equation 24.

Variation 3: Increment Strategy

(for a description see pages 47, 48)

$$\Delta E(X_1) = \{[\alpha a_1(1-e^{-cd_2})]\} - \{[\alpha a_1(1-e^{-cd_1})]$$

$$- \phi ipi[\alpha a_1(1-e^{-cd_1})]\} \qquad (26)$$

$$\Delta E(X_2) = \{[\alpha a_2(1-e^{-cd_2})]\} - \{[\alpha a_1(1-e^{-cd_1})]$$

$$- \phi(ipi+5)[\alpha a_1(1-e^{-cd_1})]\} \qquad (27)$$

$$d' = \Delta E(X_2) - \Delta E(X_1) = 5\phi[\alpha a_1(1-e^{-cd_1})] \qquad (28)$$

Since equations 28 and 24 are the same, the predicted relationships between d' and the stimulus variables will be the same as those for variations 1 and 2.

In summary, for the experimental conditions outlined, sensory models I and II combined with decision strategies 1, 2 and 3 predict the same general result since the second pulse is the same for the two patterns. _S_'s ability to discriminate between the patterns will not depend upon the parameters describing pulse 2 but will be dependent on the amplitude and duration of pulse 1. A change in performance contingent on variation of either the duration or the amplitude of pulse 2 would clearly invalidate the three variations described for each of the two major models.

VIII. Plan of Experiments

The first experiment was designed to investigate the observer's ability to distinguish a train consisting of n pulses from trains of n+1 and n-1 pulses respectively. For a constant n, the time between pulses ipi and the number (m) of different trains in the stimulus set were varied.

The second experiment studied the observer's ability to discriminate a pair of interpulse intervals of ipi and (ipi+5) msec. as a function of ipi and of the duration of the first and second pulses.

CHAPTER II

METHOD

A. EXPERIMENT 1

Subjects:

Four male students ranging in age from 17 to 28 years were paid to serve as subjects. Two had previously participated in visual detection studies and two were experimentally naive.

Apparatus:

During each testing session the subject was seated in a sound proof cubicle, isolated from the experimenter's control room. Instructions and information were presented to S by means of an inter-communication system between the two rooms.

The temporal sequence of events on each trial was controlled by a small computer (Model PDP-8/S, Digital Equipment Corporation, Maynard, Massachusetts) interfaced to an electronic switch (Model 829E, Grason-Stadler, West Concord, Massachusetts). The electronic switch gated a sine wave at zero crossings in its cycle for presentation of square wave pulses. The sine wave, delivered to the switch by an audio-oscillator (Model 201C, Hewlett-Packard, Toronto, Ontario), was 2000 Hz(±10Hz) and 60 decibels SPL (±2db, referred to .0002 μ bar) measured at the earphone.

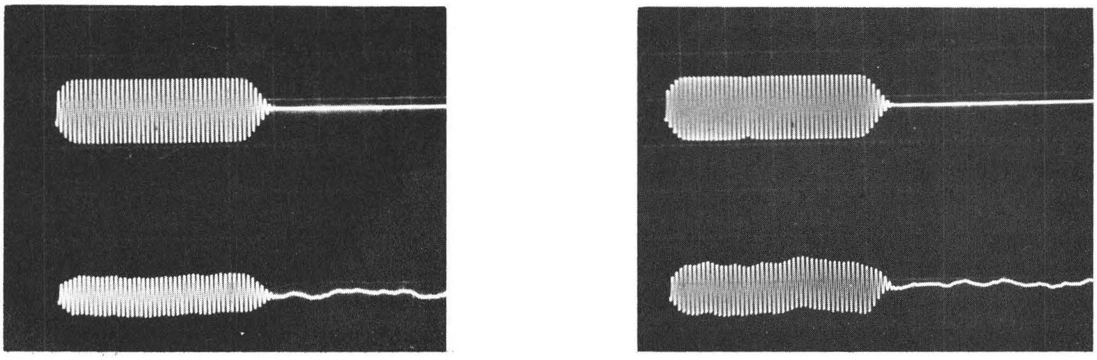
The duration of each auditory pulse was programmed for 10 msec. The rise-decay time of the pulse was controlled by the electronic switch and was set for 2.5 msec. Thus, the pulse presented to the observer was 12.5 msec. in duration. When the computer signalled pulse onset, the amplitude of the signal increased approximately linearly for 2.5 msec. and remained fixed for 7.5 msec. at the amplitude specified. At the end of this duration of 10 msec. the computer signalled pulse offset and the amplitude of the pulse decayed approximately linearly to zero in 2.5 msec.

Pulses were presented binaurally. The response of the earphone to a train of two pulses separated by 2,3, 4 and 5 msec. is shown in Figure 4.

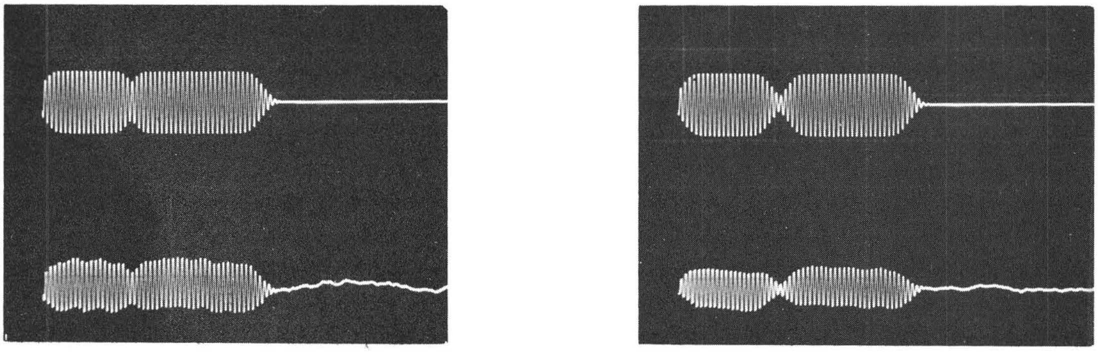
Procedure:

Each subject was presented three experimental tasks. In Task 2-13 S̲ was informed that 1, 2 or 3 pulses might occur on each of 150 trials for a given block. In Task 2-1 S̲ knew that 1 or 2 pulses would be equally likely to occur on each of 100 trials and in Task 2-3, 2 or 3 pulses. The stimulus alternatives for the tasks are described in detail in Table 1.

The experimental trial is described in Figure 5. For all tasks S̲ was required to rate his confidence on a five-



ipi = 2msec. ipi = 3msec.



ipi = 4msec. ipi = 5msec.

Figure 4. The response of the earphone to a pair of stimulus pulses presented in Experiment 1. The upper trace in each photograph is the output from the electronic switch, the lower trace is the output of the earphone. (Scale = 5 msec/cm)

TABLE 1 Stimulus Patterns for Tasks in Experiment 1

Task	Number of Pulses	Interpulse Interval (msec.)
2-13	1	-
	2	ipi
	3	ipi
2-1	1	-
	2	ipi
2-3	2	ipi
	3	ipi

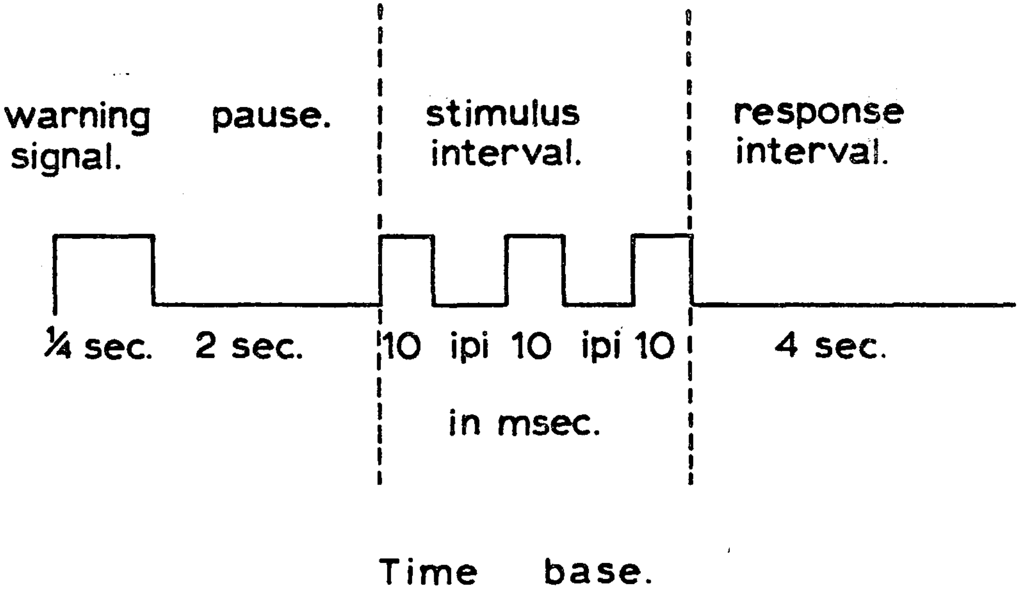


Figure 5. Temporal sequence of events on each trial of Experiment 1.

point scale that two pulses had been presented. Depression of the first microswitch in the array of response keys signified S's choice of Category 1 ("very sure that 2 pulses have occurred".) Depression of the fifth microswitch signified S's choice of Category 5 ("very sure that 2 pulses have not occurred".)

Each S was presented a given task for ten blocks of trials before attempting the next in the series of three. For each task the random presentation of stimulus alternatives was the same for each block and subject. On any day of testing three or four blocks were given. Tasks were completed in the order 2-1, 2-3, and 2-13.

Values of the interpulse interval (ipi) were programmed for 2, 3, 4 and 5 msec. (The results of extensive pre-testing had shown that given an ipi of 10 msec., Ss were able to discriminate 1 from 2 pulses on at least 95% of the trials presented.) The value of ipi remained fixed for the three tasks and was changed only after presentation of the thirty blocks of trials required for completion of the set of tasks. Order of presentation of ipis for all Ss was: 5, 4, 3 and 2 msec.

The instructions presented to S at the start of each day's run were:

> "This is an experiment in auditory perception. On each of 100(150) trials, I will present a sequence of 1 or 2 (2 or 3 OR 1, 2 or 3) very brief auditory pulses. You will be required to rate your confidence

> on a 5-point scale that 2 pulses have occurred. At the completion of the sequence press Key 1 if you are very sure that there were 2 pulses, Key 5 if you are very sure that there were not 2 pulses (i.e. that there was 1 pulse OR 3 pulses OR 1 or 3 pulses). You have about 4 seconds in which to make your response. You must respond on every trial.
>
> A short warning tone followed by a pause will indicate the start of each trial."

Prior to each day's run, Ss were told the number of correct responses made during the previous day's blocks of trials.

B. EXPERIMENT 2

Subjects:

Four male students ranging in age from 22 to 28 years were paid to serve as subjects. Three had previously participated in auditory and visual signal detection experiments. The fourth observer was experimentally naive.

Apparatus:

The apparatus used was described for Experiment I (page 54). There were several modifications. Auditory pulses were segments of a sine wave of approximately 2000 Hz (±10Hz) and .3 volts (measured at the source or 57 db SPL measured to the earphone). The pulse was of variable duration and had a rise-decay time of 1 msec. Pulses were presented binaurally. The response of the earphone to a train of two stimulus pulses is shown in Figure 6.

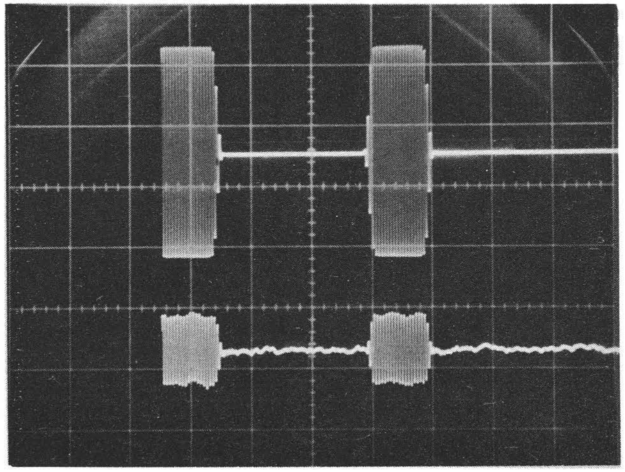
A panel of four microswitches interfaced to the computer was provided for a communication of _S_'s responses.

Procedure:

Each subject attempted two experimental problems in sequence.

Problem 1

Subjects were presented a block of 100 trials



ipi = 25 msec.

Figure 6. The response of the earphone to a pair of stimulus pulses presented in Experiment 2. The upper trace is the output from the electronic switch, the lower trace is the output of the earphone. (Scale = 10 msec/cm)

consisting of two equally likely, randomly presented sequences of events. The first pattern consisted of two 10 msec. pulses (as programmed by the computer) separated by an interpulses interval of ipi msec. and the second pattern consisted of two 10 msec. pulses separated by (ipi+5) msec. The value of ipi ranged from 10 to 60 msec. in steps of 5 msec. Additional values of ipi presented to any S were determined by the results obtained for the initial range chosen. The order of presentation of ipi was randomly determined for each subject.

During each day's run S was presented three or four blocks of trials. A given value of ipi was presented until S's performance reached its upper limit. The data was disgarded and eight more blocks of trials were presented. The next value of ipi was then introduced. At the start of each new condition the subject listened to a random block of ten trials, five for each of the two patterns to be presented on the following blocks of trials.

The sequence of events defining an experimental trial is described in Figure 5. On each trial S was required to rate on a four point scale his confidence that the shorter of the two intervals was presented. Depression of the first microswitch in the array of response keys signified S's choice of Category 1 ("very sure that the short interval was presented"). Depression of the fourth microswitch signified S's choice of Category 4 ("very sure that the short interval has not occurred"). At the end of the four second response period a buzzer sounded if the short interval had in fact been presented.

Instructions to the subject presented prior to the first block of trials in each new condition were:

> "This is an experiment in auditory perception On each of 100 trials you will be presented with a sequence of two very short auditory pulses. The pulses on any trial will be separated by one of two possible interpulse intervals, which we shall designate as a short and a long interval. You will be required to rate your confidence on a 4-point scale that the short interpulse interval has been presented.
>
> A trial will begin with a warning signal consisting of a flash of light. After a pause of 2 sec. the auditory pulses will be presented. You will then have 4 sec. to respond. If you are sure that the short interval was presented depress Key 1. If you think the short interval occurred use Key 2. If you think it was not presented depress Key 3 and if you are sure it was not, Key 4. Try to use the keys equally often.
>
> A buzzer following your response will indicate that the short interval was in fact presented.
>
> We will begin today's session with a practice block of 10 trials. On five trials the shorter interval will occur and on five, the longer interval. The order of the two patterns will be: short, short, short, long, long, short, long, long, short, long. Listen but do not respond."

Prior to each day's run S was told the number of correct responses made in each of the previous day's blocks of trials.

Problem 2

The subject's task and the procedure for problem 2 were the same as those for problem 1. However, the intervals ipi and ipi+5 in the two patterns presented remained fixed at 15 and 20 msec. respectively for one subject and at 25 and 30 msec. for three subjects.

In this study the durations of the first and second pulses (d_1 and d_2) in the two patterns were varied. The values of d_1 and d_2 were always the same for both patterns and ranged from 4 to 16 msec. in steps of 4 msec. Changes in pulse 2 preceeded changes in pulse 1 for all subjects. The duration of the constant pulse was fixed at 10 msec. The order of presentation of the durations of the variable pulse was counterbalanced across subjects.

CHAPTER III

RESULTS AND THEORETICAL ANALYSIS OF EXPERIMENT 1

A. Raw Data

The estimated probability of a correct response $\hat{Pr}(C)$ is presented as a function of interpulse interval (ipi) for each subject and task in Figure 7. For this preliminary analysis of results the response matrix, comprised of five cells (one for each response key), has been collapsed into two cells. Key A and Key B responses have been interpreted as report of two pulses, R_2. Keys C, D and E responses have been grouped together as $R_{\bar{2}}$ and interpreted as report of $\bar{2}$ (i.e. reports of numbers other than two, depending on the experimental task). The response probabilities associated with each response key are presented for each combination of subject task and interpulse interval in Appendix A, Tables 1, 2 and 3. The probabilities summed over Keys A and B and Keys C, D and E respectively are shown in Appendix A, Table 4.

The $\hat{Pr}(C)$ for Task 2-1 was estimated by the formula:

$$\hat{Pr}(C) = \hat{Pr}(S_2) \cdot \hat{Pr}(R_2/S_2) + \hat{Pr}(S_1) \cdot \hat{Pr}(R_{\bar{2}}/S_1) \qquad (29)$$

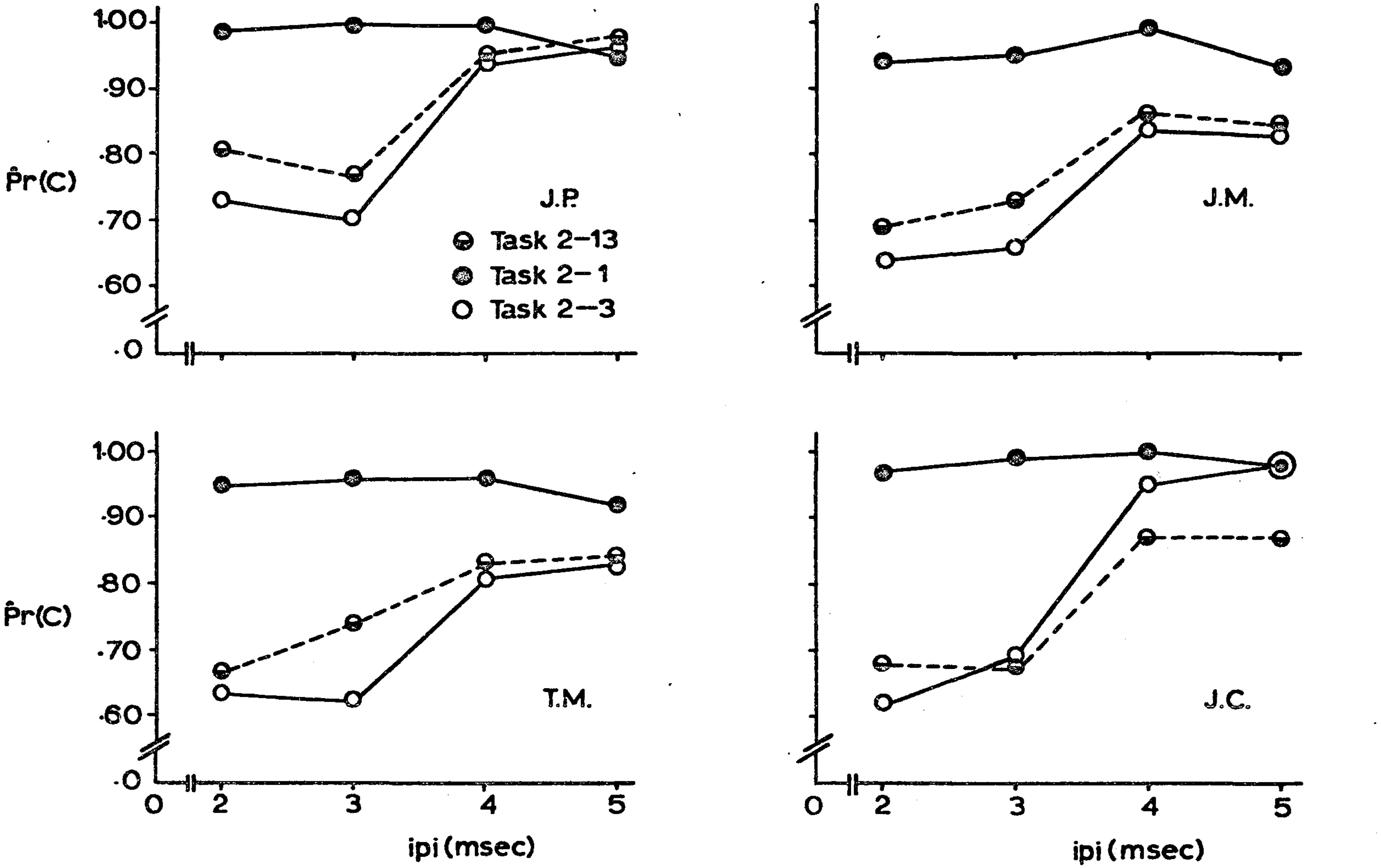


Figure 7. P̂r(C) as a function of task and ipi for four Ss.

where $\hat{Pr}(R_2/S_2)$ represents the conditional probability of pressing Key A or Key B given a train of 2 pulses, and $\hat{Pr}(R_{\bar{2}}/S_1)$ is the conditioned probability of pressing Keys C, D, or E given 1 pulse. For Task 2-3

$$\begin{aligned}\hat{Pr}(C) &= \hat{Pr}(S_2) \cdot \hat{Pr}(R_2/S_2) \\ &\quad + \hat{Pr}(S_3) \cdot \hat{Pr}(R_{\bar{2}}/S_3) \qquad (30)\end{aligned}$$

and for Task 2-13

$$\begin{aligned}\hat{Pr}(C) &= \hat{Pr}(S_2) \cdot \hat{Pr}(R_2/S_2) \\ &\quad + \hat{Pr}(S_1) \cdot \hat{Pr}(R_{\bar{2}}/S_1) \\ &\quad + \hat{Pr}(S_3) \cdot \hat{Pr}(R_{\bar{2}}/S_3) \qquad (31)\end{aligned}$$

The data points plotted in Figure 7 are based on the conditional response probabilities in the data matrices obtained for each of 10 blocks of trials. Each data point for Tasks 2-1 and 2-3 is based on 1000 trials. Each data point for Task 2-13 is based on 1500 trials. The standard deviations of the proportions estimated are less than .016. $\hat{Pr}(C)$ shows an increase for Tasks 2-3 and 2-13 as the ipi increases from 3 to 4 msec. For both Tasks 2-3 and 2-13 and for each subject we tested the hypothesis that the frequencies of a correct response for each of the values of ipi was a constant that was equal to the mean value of these frequencies. A goodness of fit test (Hayes, 1963) showed that this hypothesis could be rejected at the .001 level. The values of chi-square for this test were greater than 16.266 with 3df. In Task 2-1 a slight systematic increment in $\widehat{\Pr}(C)$ as ipi increases from 2 to 4 msec. is apparent in the data of two subjects (JM and JC). For no subject can we reject the hypothesis at the .05 level that the frequency of a correct response for each value of ipi is a constant that is equal to their mean. The values of chi-square for this test were less than 7.815 with 3df.

The $\widehat{\Pr}(C)$ for each block of trials is shown for all combinations of subject, task and ipi in Appendix A, Figure 1. In this analysis, it can be seen that $\widehat{\Pr}(C)$ increases systematically over Blocks 1 to 4 for the first ipi (5 msec.) presented in each of the three experimental tasks.

The $\widehat{\Pr}(C)$ for trials on which 2 pulses were presented is shown as a function of ipi and task in Figure 8. The data in Figure 8 indicate that

(i) in Tasks 2-3 and 2-13 $\widehat{\Pr}(C)$ increases as ipi increases from 3 to 4 msec. A slight comparable change is evident in Task 2-1 for two *Ss* (TM and JM).

(ii) in 15 out of 16 instances (4 *Ss* x 4 ipi s)

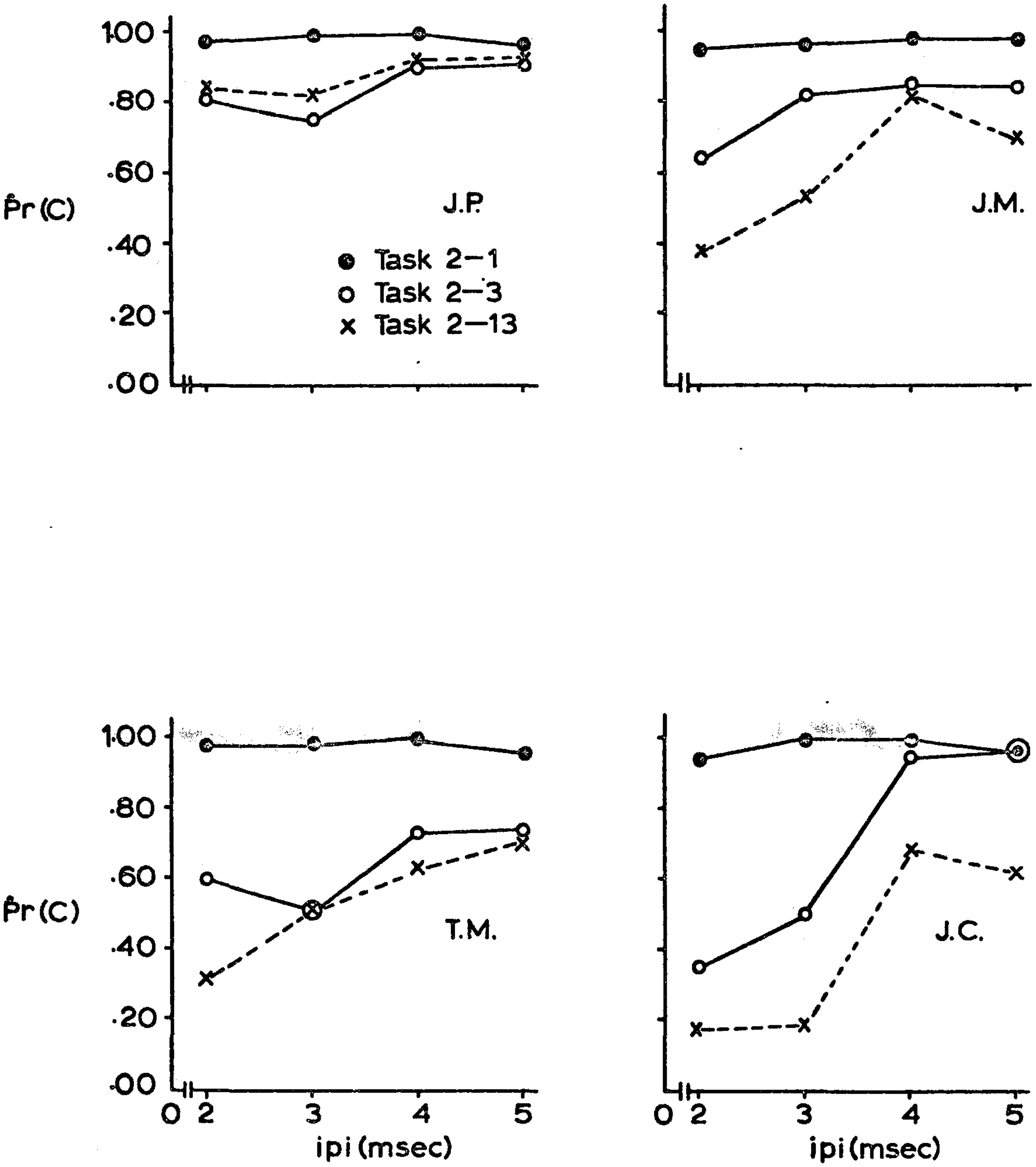


Figure 8. Probability of correctly identifying a train of two pulses.

$\hat{P}r(C)$ is relatively greater in Task 2-1 than in Task 2-3

(iii) in 11 out of 16 instances $\hat{P}r(C)$ for Task 2-13 is less than $\hat{P}r(C)$ for either Tasks 2-1 or 2-3

In summary, the probability of correctly reporting the presentation of 2 pulses depends upon (a) the time between successive pulses in the train, (b) the nature of the alternative train which is being presented during the same series of trials, i.e. whether it consists of one or three pulses and (c) the number of different alternative trains which occur within the series, i.e. whether one or two.

B. Theoretical Analysis

I. The Quantal Model

(a) Collapsed Response Matrix

A specific variation of the general form of the quantal model presented schematically in Figure 1 for n and n+1 pulses is shown for stimulus trains of 1, 2 and 3 pulses in Figure 9. An R_2 report is the sum of Key A and Key B responses. An $R_{\bar{2}}$ is the sum of Keys C, D and E responses.

In Task 2-1, $R_{\bar{2}} = R_1$ and

$$\begin{aligned} Pr(C) &= Pr(S_2) \cdot Pr(R_2/S_2) + Pr(S_1)Pr(R_1/S_1) \\ &= .50\ [\alpha_{21}\beta_{12} + \alpha_{22}(1-\beta_{2\bar{2}})] \\ &\quad + .50\ [\alpha_{11}(1-\beta_{12})] \end{aligned} \tag{32}$$

For Task 2-3, $R_{\bar{2}} = R_3$ and

$$\begin{aligned} Pr(C) = {} & .50[\alpha_{21}\beta_{12} + \alpha_{22}(1-\beta_{2\bar{2}})] \\ & + .50\ [\alpha_{31}(1-\beta_{12}) + \alpha_{32}\beta_{2\bar{2}} \\ & + \alpha_{33}(1-\beta_{32})] \end{aligned} \tag{33}$$

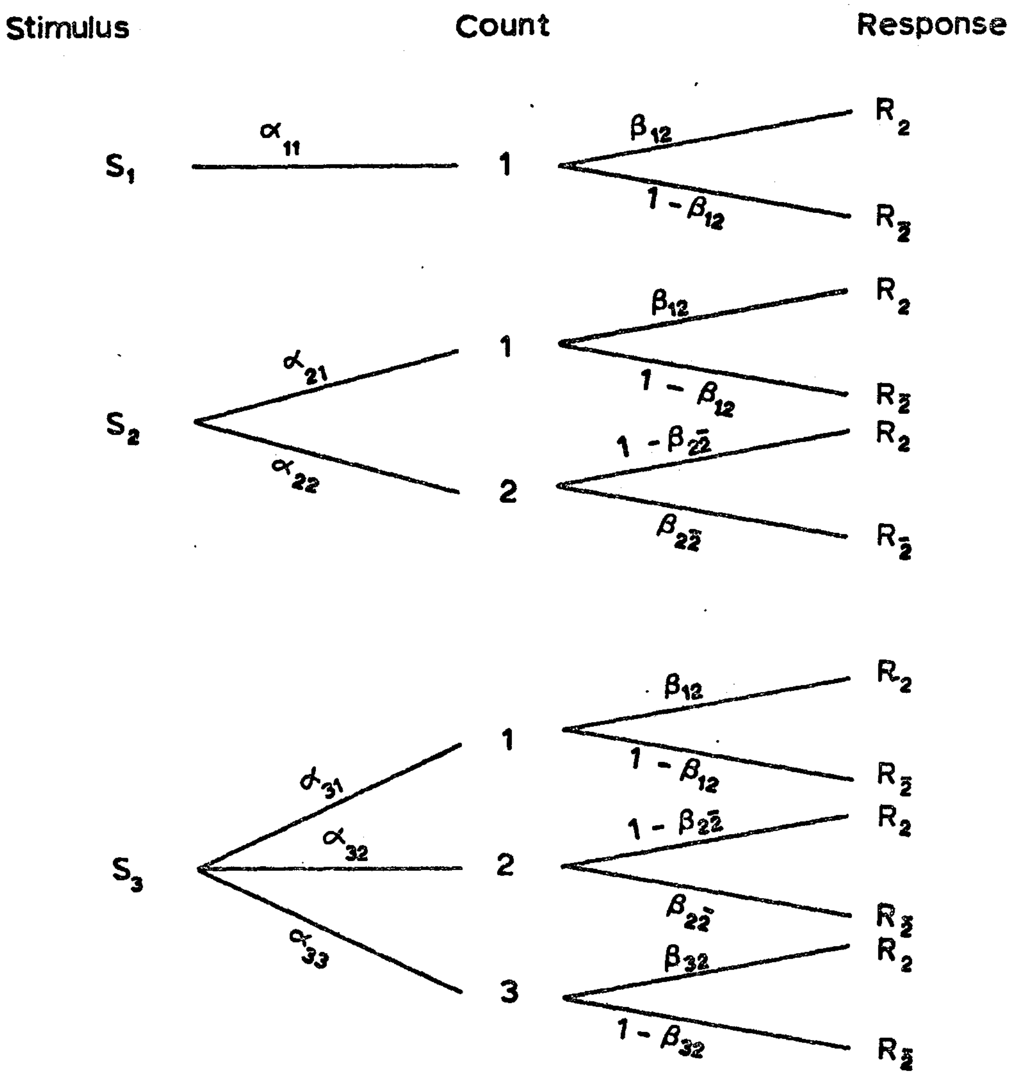


Figure 9. Schematic representation of the quantal model for stimulus trains of 1, 2 and 3 pulses.

and for Task 2-13, $R_{\bar{2}} = R_1 + R_3$ and

$$\begin{aligned} Pr(C) = {} & .333[\alpha_{11}\ (1-\beta_{12})] \\ & + .333[\alpha_{21}\beta_{12}+\alpha_{22}(1-\beta_{2\bar{2}})] \\ & + .333[\alpha_{31}(1-\beta_{12}) + \alpha_{32}\beta_{2\bar{2}} \\ & + \alpha_{33}(1-\beta_{32})] \end{aligned} \tag{34}$$

Substitution in equations 32, 33 and 34 of (i) values of the sensitivity parameters α calculated for particular values of ipi, n, p and an assumed value of q (see Appendix A) and (ii) all possible combinations of values of the biases β ranging from zero to 1.00 results in a range of values of Pr(C) for each experimental condition. In Figure 10 the maximum and minimum values of Pr(C) are plotted for the three tasks as a function of ipi and three assumed values of q in Figure 10. This graph shows that

(i) when q equals 10 and 50 msec., Pr(C) max. increases and Pr(C) min. decreases as ipi increases from 2 to 5 msec. in Tasks 2-1 and 2-3. Both Pr(C) max. and Pr(C) min. are fixed for all values of ipi in Task 2-13.

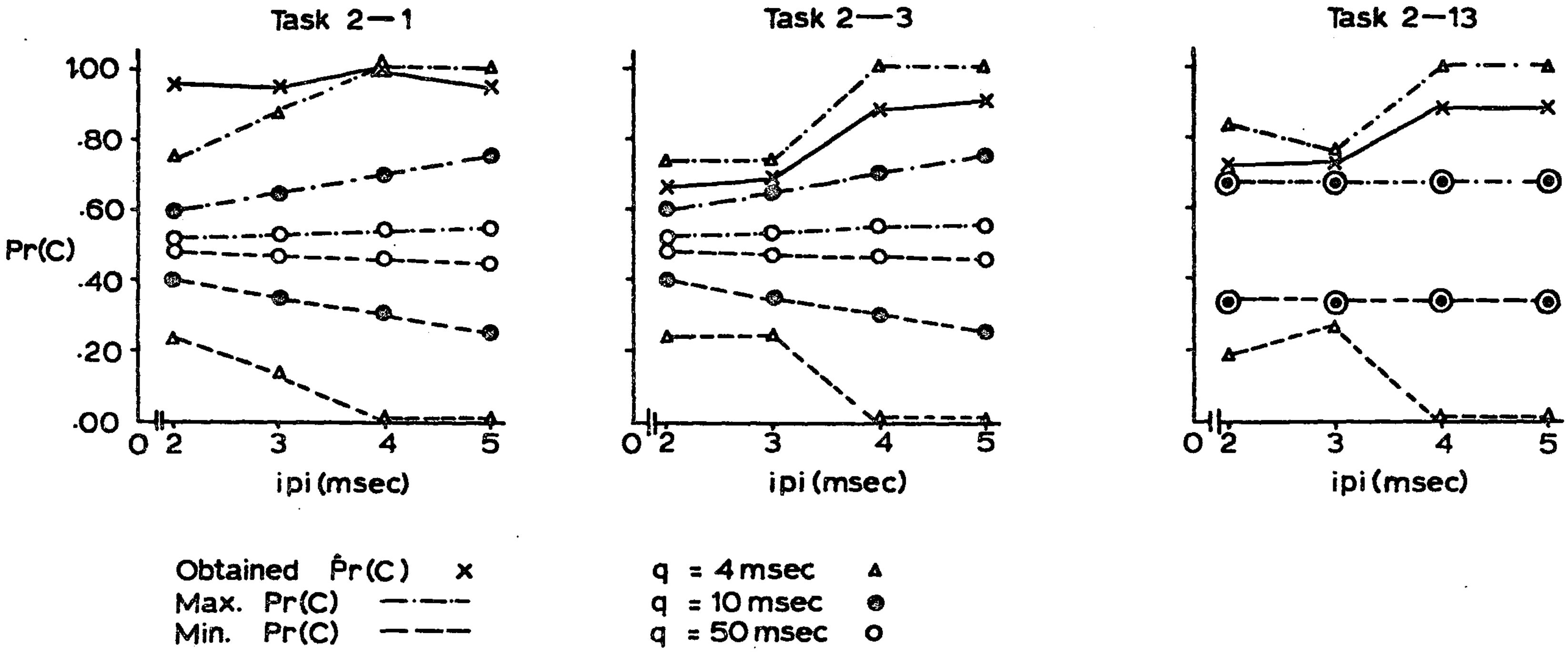


Figure 10. Comparison of the obtained P̂r(C) for four S's with the range of Pr(C) predicted by the quantal model.

(ii) when q equals 4 msec., the Pr(C) max. increases the Pr(C) min. decreases as ipi increases from 3 to 4 msec. in Tasks 2-3 and 2-13.

As shown in Figure 10, the $\hat{Pr}(C)$ obtained for four *S*s in Task 2-1 will lie outside the performance boundaries defined by the max. and the min. Pr(C) at each ipi when q is 4 msec. The $\hat{Pr}(C)$ obtained for four *S*s in Tasks 2-3 and 2-13 will lie within the predicted performance boundaries when q lies between 4 and 10 msec.

The observed $\hat{Pr}(C)$ for trials on which 2 pulses were presented is shown in Table 2. The data for the group of *S*s like the data presented for individual *S*s, indicates that correct identification of 2 pulses depends on the ipi, the size of the stimulus set and the composition of the stimulus set. The quantal model accounts for the effect of ipi in terms of the theoretical distribution of counts generated by the train. As the ipi decreases, the probability of a count of 2 decreases and the probability of a count of 1 increases. Thus, if the biases (β) for report are fixed, the probability of reporting 2 given 2 pulses should decrease.

The counts K_c generated by a train of 2 pulses are independent of the counts generated by other trains in the set of stimulus alternatives. Thus, within the quantal framework changes in performance as a function of the size of the set of alternatives or the composition of the set can

TABLE 2 The Probability of Reporting "2" on "2" Pulse Trials in Three Experimental Tasks

Task	ipi (msec.)			
	2	3	4	5
2-1	.961	.982	.993	.967
2-3	.599	.646	.856	.865
2-13	.428	.506	.761	.723

only occur if the biases for report have changed. For each of the three experimental tasks it should be possible to solve for the values of the biases (β)

either (i) by substitution for a fixed value of ipi and task of values of the sensitivity parameter α in the equation for Pr(C)

or (ii) by solving, for a given task and two values of ipi, a pair of simultaneous equations for Pr(C) that contain the unknowns α and β.

The data obtained and the experimental design will not allow us to use either of these methods to determine β. First, as discussed above, the data can only suggest a range of values for q. Hence α is unknown. Secondly, since the different values of ipi presented for each task were not used during the same blocks of trials, we cannot assume that the values of the bias parameters for these values of ipi will be the same. In short, we cannot test the prediction of the quantal-two alternative response model that the size and composition of the stimulus set affects bias rather than sensitivity.

(b) Rating Data

The quantal theory is a state model based on a normative process. That is, any stimulus train will generate any one of a set of states, each with a fixed probability of occurrence. These states are discrete counts. The probability of occurrence of any particular count depends on the stimulus parameters n, ipi, p, the theoretical parameter q and the phase relation between the theoretical time base and the onset of the stimulus train. The obvious fact that an observer can rate his confidence that a particular event has occurred suggests either (i) that he can differentiate among successive occurrences of the same sensory state or (ii) that he is responding probabilitistically for any given state.

Krantz (1969, p.312) suggests that "*S* can easily find a variety of 'irrelevant' factors to serve as a basis for varying his confidence responses. These 'irrelevant' factors (e.g. his momentary state of attention or alertness, etc.) may correspond to a multiplicity of internal states...that in the detection situation are not correlated with presence or absence of a signal". Krantz argues further that if the observer cannot "classify his internal states sharply into sensory and nonsensory components...it seems very likely that the response distribution in D, *the detect state*,[1.] will overlap the distribution in $\bar{D}$, *the nondetect state*." In other words, one cannot assert *a priori* that the observer will use a subset of confidence categories exclusively for a particular sensory state.

Extending Krantz's arguments to the recognition paradigm, let us replace the two response alternatives, R_2 and $R_{\bar{2}}$ in the quantal model with the response alternatives R_A, R_B, R_C, R_D, and R_E for the five confidence categories actually used by S. A tree diagram describing the quantal-confidence rating model for trains of 1, 2 and 3 pulses is shown in Figure 11. For the modified response process the probability of choosing any category g (for g equal to A, B, C, D or E) given states or counts of 1, 2 and 3 are β_{1g}, β_{2g}, and β_{3g} respectively.

This confidence rating model implies that the observer establishes a number of criteria for report. These criteria are determined by nonsensory factors. Any criterion represents a division between two exclusive subsets of confidence categories. Thus, if the observer has available and uses five such categories, he must simultaneously maintain four criteria. The most stringent criterion for responding 2 pulses (greatest certainty that 2 pulses were presented) divides Category A from Categories B, C, D and E. The most lax criterion separates Categories A, B, C and D from category E. By considering each criterion from stringent to lax in turn for a pair of stimuli consisting of 1 and 2 or 2 and 3 pulses and plotting $\hat{Pr}(R_2/S_2)$, the probability of a hit, P(H), as a function of $\hat{Pr}(R_2/S_{\bar{2}})$, the probability of a false alarm,

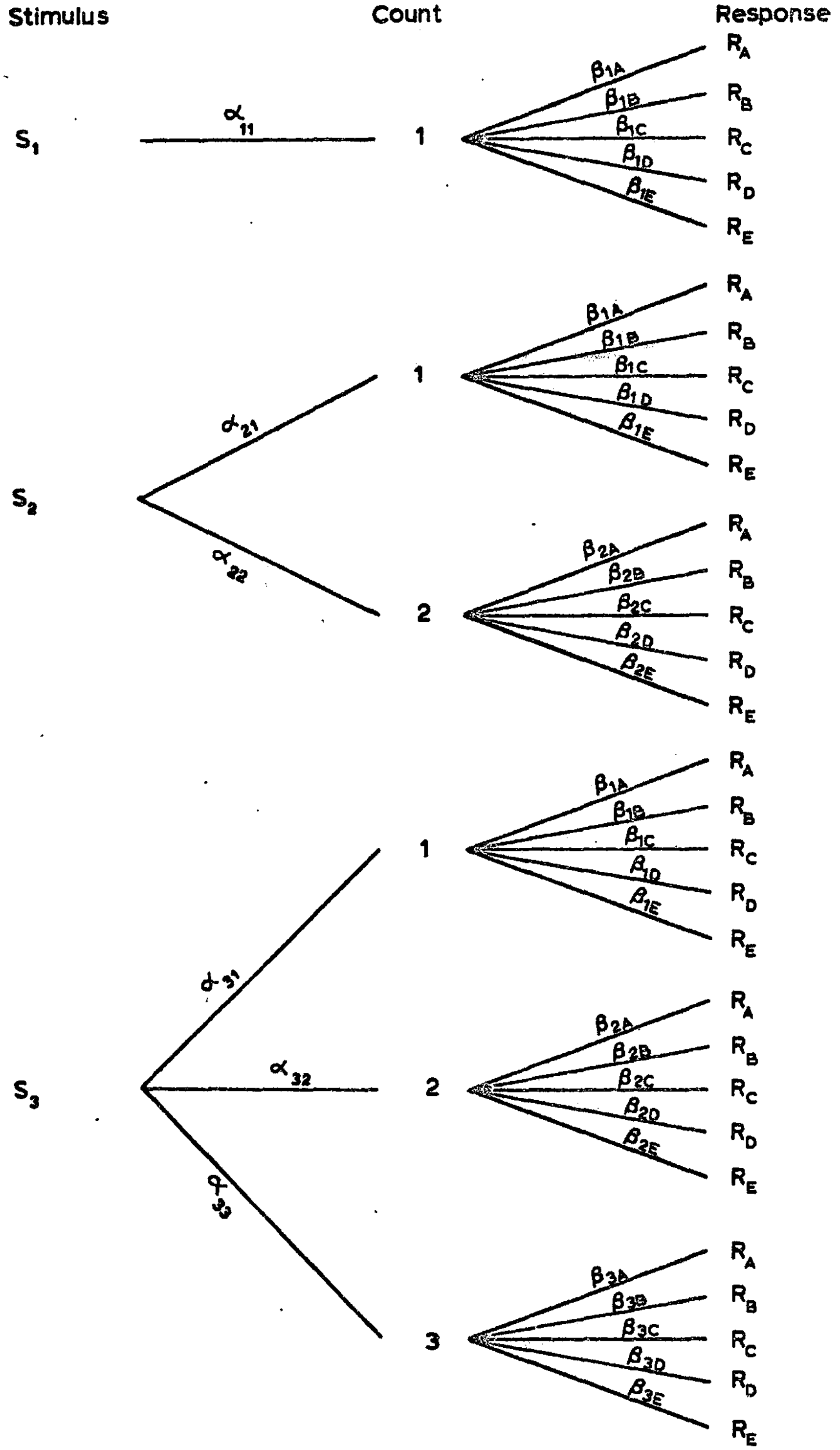


Figure 11. Schematic representation of the quantal-confidence rating model for stimulus trains of 1, 2 and 3 pulses.

P(FA), we can trace out the empirical receiver operating characteristic (ROC) for every combination of subject, pair of trains and ipi.

The theoretical ROC for any condition in Experiment 1 consists of four points. Beginning with the most stringent and progressing to the most lax criterion, the coordinates of the points in Task 2-1 are:

Point 1

$$Pr(R_A/S_1) \; ; \; Pr(R_A/S_2)$$

where $Pr(R_A/S_1) = \alpha_{11}\beta_{1A} = \beta_{1A}$

and $Pr(R_A/S_2) = \alpha_{21}\beta_{1A} + \alpha_{22}\beta_{2A}$

Point 2

$$Pr(R_A + R_B/S_1) \; ; \; Pr(R_A + R_B/S_2)$$

where $Pr(R_A + R_B/S_1) = \alpha_{11}\beta_{1A} + \alpha_{11}\beta_{1B}$

$$= \beta_{1A} + \beta_{1B}$$

and $\Pr(R_A + R_B/S_2) = \alpha_{21}\beta_{1A} + \alpha_{21}\beta_{1B} + \alpha_{22}\beta_{2A} + \alpha_{22}\beta_{2B}$

$= \alpha_{21} (\beta_{1A} + \beta_{1B}) + \alpha_{22}(\beta_{2A} + \beta_{2B})$

Point 3

$\Pr(R_A + R_B + R_C / S_1) ; \Pr(R_A + R_B + R_C / S_2)$

where $\Pr(R_A + R_B + R_C / S_1) = \beta_{1A} + \beta_{1B} + \beta_{1C}$

and $\Pr(R_A + R_B + R_C / S_2) = \alpha_{21}(\beta_{1A} + \beta_{1B} + \beta_{1C})$

$+ \alpha_{22}(\beta_{2A} + \beta_{2B} + \beta_{2C})$

Point 4

$\Pr(R_A + R_B + R_C + R_D / S_1) ; \Pr(R_A + R_B + R_C + R_D / S_2)$

where $\Pr(R_A + R_B + R_C + R_D / S_1) = \beta_{1A} + \beta_{1B} + \beta_{1C} + \beta_{1D}$

and $\Pr(R_A + R_B + R_C + R_D / S_2 = \alpha_{21}(\beta_{1A} + \beta_{1B} + \beta_{1C} + \beta_{1D})$

$+ \alpha_{22}(\beta_{2A} + \beta_{2B} + \beta_{2C} + \beta_{2D})$

The general expression for the coordinates of the kth point is:

$$\left[\sum_{g=A}^{k} \beta_{1g}\right] ; \left[\alpha_{21} \sum_{g=A}^{k} \beta_{1g} + \alpha_{22} \sum_{g=A}^{k} \beta_{2g}\right] \tag{35}$$

The general expression for the coordinates of the kth point in Task 2-3 is:

$$\left[\alpha_{31} \sum_{g=A}^{k} \beta_{1g} + \alpha_{32} \sum_{g=A}^{k} \beta_{2g} + \alpha_{33} \sum_{g=A}^{k} \beta_{3g}\right] ; \left[\alpha_{21} \sum_{g=A}^{k} \beta_{1g} + \alpha_{22} \sum_{g=A}^{k} \beta_{2g}\right] \tag{36}$$

As Krantz (1969) points out, successive points on the theoretical ROC curve lie on the same straight line if and only if the ratios of corresponding biases for any pair of states (e.g. β_{2g}/β_{1g}) are the same value for all values of g. The theoretical ROC curve will consist of two straight line segments if the ratio of biases for any pair of states takes on exactly two values, one value for $g \leqslant g_o$, another for $g > g_o$.

In either Task 2-1 or 2-3 the position of the theoretical ROC curve relative to the positive diagonal of the ROC space, and the slopes of the line segments comprising the curve depend on the distributions of states or counts and on the bias structure. The quantal confidence-rating model predicts that the ROC for trains of n and n+1 pulses will move closer to the positive diagonal either (i) as the distributions of counts generated by the pair of trains become more "similar" or (ii) as the ratio of corresponding biases for any pair of counts decreases. For trains of n and n+1 pulses if ipi is less than q, and n is equal to 1, 2 or 3, the distributions of counts become more alike both as the time between pulses (ipi) decreases and as n increases. An increase in the size of the stimulus set would not affect the distribution of counts for any particular stimulus train. Thus, the effect of size of set could only be accounted for within the model by a change in response bias.

Empirical ROC curves based on the pooled data of four Ss are presented for each ipi in Figure 12 for discrimination of 1 from 2 and 2 from 3 pulses in the three experimental tasks. Since each ROC is comprised of only four points, the shape of the function connecting the points is not clearly defined. As predicted by the quantal confidence rating (Q-CR) model, as the ipi decreases the ROC for any pair of stimuli approaches the positive diagonal. For a fixed ipi the ROC for discrimination of 2 from 3 pulses in Task 2-3 is closer to the positive diagonal than the ROC for discrimination of 1 from 2 pulses in Task 2-1.

Comparisons of ROC curves obtained in Tasks 2-13 for discrimination of 1 from 2 and 2 from 3 pulses with ROC curves from Tasks 2-1 and 2-3 indicates that for a fixed ipi

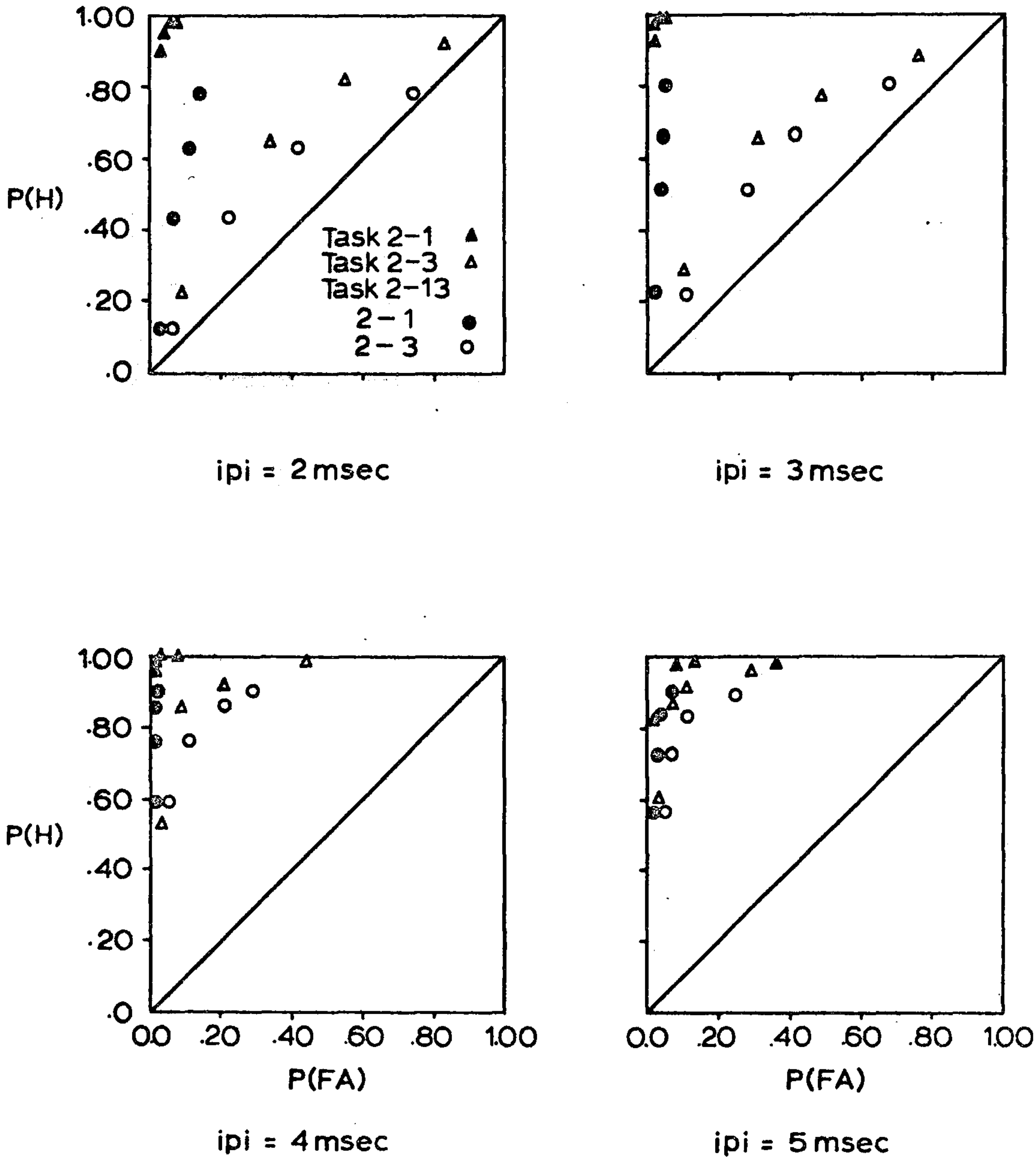


Figure 12. Empirical ROC curves based on the data of four Ss for each experimental condition.

discrimination of a pair of trains becomes more difficult (i.e. the ROC approaches the positive diagonal) as the number of possible stimuli increases from 2 to 3. This effect appears to be due to the greater likelihood of responding incorrectly to trains of 2 and 3 pulses. In both Tasks 2-1 and 2-13 the probability of correctly responding R_E given 1 pulse is close to 1.00.

Although the Q-CR model describes the results of Experiment 1, it provides a poor explanation of the data. The ROC predicted by the model does not allow us to distinguish unequivocally between the effects of a change in the distribution of counts (or sensitivity) and response bias.

II. The Information Processing Model

The information processing model for numerosity judgments predicted that for any pair of consecutive numbers, Pr(C) would decrease as a step function from 1.00 to .50 when the processing time available (p+ipi) was less than the time required to process a single pulse. The $\hat{Pr}$(C) obtained for four S̲s was presented in Figure 10 for the three experimental tasks.

The data for Task 2-1 show that $\hat{Pr}$(C) is approximately 1.00 when values of ipi range from 2 to 5 msec. These values of ipi added to a pulse duration of 10 msec. give processing times ranging from 12 to 15 msec. Thus, this result implies that the theoretical processing time (PT) must be less than 12 msec.

In Task 2-3 the $\widehat{Pr}(C)$ is approximately .90 for ipi equal to or greater than 4 msec. and .67 for ipi less than 4 msec. Although the change in performance at ipi equal to 4 msec. suggests that PT is 14 msec. the obtained values of $\widehat{Pr}(C)$ are inconsistent with those predicted by the model.

In Task 2-13 as in Task 2-3 the obtained $\widehat{Pr}(C)$ increases as ipi increases from 3 to 4 msec. This trend in the data implies that PT is 14 msec. However, again the increase in $\widehat{Pr}(C)$ from .73 to .88 is not consistent with the change from .50 to 1.00 predicted by the information processing model.

According to the model, context (for this experiment the number of stimulus alternatives) should not affect the judgment of number. Thus, the value of $\widehat{Pr}(C)$ obtained for each ipi should be the same whether we compare the data for the three tasks or the data for presentation of a train of n pulses in the three tasks. The data presented in Figure 10 show that for each value of ipi $\widehat{Pr}(C)$ in Task 2-1 exceeds $\widehat{Pr}(C)$ obtained for Tasks 2-3 and 2-13. In Table 2 it was shown that the $\widehat{Pr}(C)$ for trials on which 2 pulses were presented depends on both the composition and the size of the stimulus set.

In summary, the information processing model provides us with a poor description of performance. It does not predict the values of $\hat{Pr}(C)$ obtained in the 2-alternative tasks and it does not account for the effect on recognition of a particular stimulus of increasing the size of the set of stimulus alternatives.

III. Duration Discrimination

The duration discrimination hypothesis suggested that performance in the temporal numerosity experiment might vary because of changes in the ratio $\Delta D/D$. To test this hypothesis, we examined the functional relationship between the $\hat{Pr}(C)$ for four *S*s and $\Delta D/D$ for all values of the ipi in Tasks 2-1 and 2-3. Base duration D was set equal to the total duration of the shorter of the two trains presented for discrimination. The difference in total duration between the two trains was ΔD.

In Task 2-1 the duration of the train consisting of 1 pulse was 10 msec. The value of ΔD, increased from 12 to 15 msec. Thus, as the ipi increased from 2 to 5 msec., the ration $\Delta D/D$ increased linearly from 1.2 to 1.5. The data of Henry (1948) indicate that when D is 32 msec. the Weber ratio, that value of $\Delta D/D$ resulting in correct recognition on .75 of the trials, is .281. Extrapotation of Henry's data shows that if D were equal to 10 msec. the Weber ratio would have to be greater than .281. In the present study, it was observed that for D equal to 10 msec. $\hat{Pr}(C)$ remained close to 1.00 as $\Delta D/D$ increased from 1.2 to 1.5.

This level of performance would be expected on the basis of Henry's data.

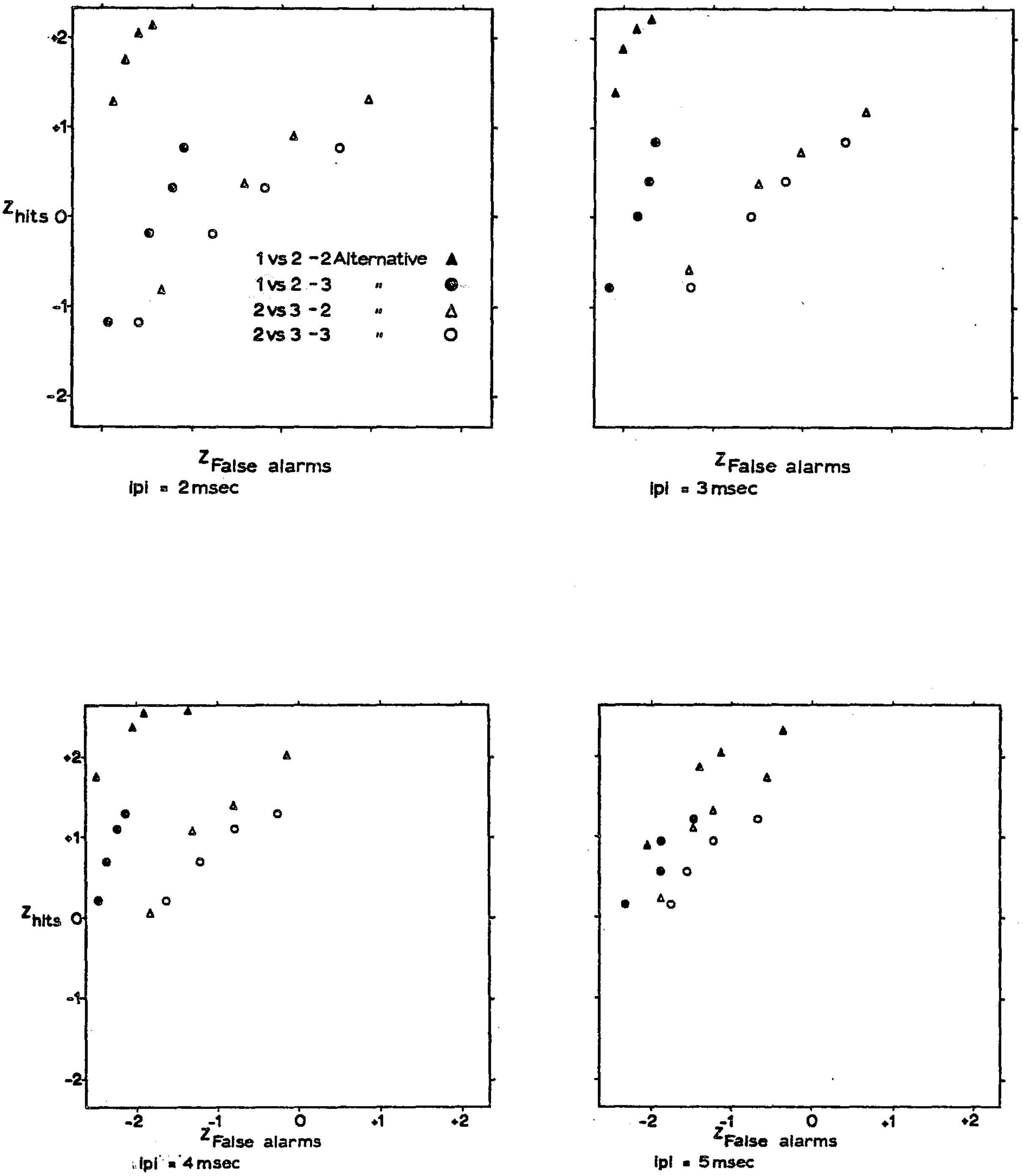
In Task 2-3 the duration of the train of 2 pulses increased from 22 to 25 msec. as the ipi increased from 2 to 5 msec. Thus, for ipi equal to 2, 3, 4 and 5 msec. $\Delta D/D$ was .545, .565, .583, and .600 respectively. From Henry's data one would expect that for all values of the ipi in Task 2-3 $\Delta D/D$ would be great enough for discrimination on at least .75 of the trials. However, we observe that as the ipi increases from 2 to 5 msec. the values of the obtained $\hat{Pr}(C)$ are .66, .67, .88 and .90. An obtained $\hat{Pr}(C)$ less than .75 suggests that Ss are not using all of the information available in the ratio $\Delta D/D$.

Photographs of pairs of stimulus pulses separated by 2, 3, 4 and 5 msec. were shown in Figure 4. These photographs indicated that a gap did not occur between a pair of pulses until the ipi was 4 msec. When the ipi was less than 4 msec., the gap was obscured by the rise-decay time of 2.5 msec. for each pulse. We have noted above that as the ipi increases from 3 to 4 msec. the $\hat{Pr}(C)$ observed for Task 2-3, shows a dramatic change from .67 to .88. The change in $\Delta D/D$ for this increase in ipi is only .018. The data support the hypothesis that in Task 2-3 Ss are attending to a change in stimulation during the presentation of the train and not to a comparison of the total durations of the trains presented.

IV. Theory of Signal Detection (TSD)

Two assumptions of the Theory of Signal Detection presented above (see page 26) were that a pair of trains of n and n+1 pulses generated Gaussian distributions of excitation of equal variance. We can test these assumptions by examining the ROC curves. In TSD sensitivity is independent of response bias. As the measure of sensitivity d' decreases to zero the ROC moves toward the positive diagonal of the ROC space. The measure d' decreases both as (i) the distance between the means of the signal and noise distributions decrease and (ii) as the variance of the noise distribution increases. For a fixed d' the position of the criterion on the decision or likelihood ratio axis corresponds to a position on the ROC curve.

In the present study the subject was required to maintain a number of criteria simultaneously. As described above (see page 80) the empirical ROC curve is generated by considering each criterion from stringent to lax in turn. The ROC curves based on the data of four Ss for pairs of stimuli within each experimental task was presented in Figure 12. A common way of depicting the ROC curve entails transformation of the coordinates P(H) and P(FA) of each point on the curve into standard normal deviates(Z_{Hits} and $Z_{\text{false alarms}}$). Transformations of ROC curves for discrimination of stimulus trains of 1 and 2 pulses and 2 and 3 pulses in the three tasks are shown in Figure 13. If the variances



ipl = 2 msec

ipl = 3 msec

ipl = 4 msec

ipl = 5 msec

Figure 13. Transformed empirical ROC curves based on the data of four S's for each experimental condition.

of the underlying distributions for any pair of trains are Gaussian, the ROC points plotted in standard normal deviates should lie on the same straight line. If the distributions are of equal variance this line should have a slope of 1.00 (Green and Swets, 1966). Slopes greater than 1.00 indicate that the variance of the signal distribution (for this experiment the distribution generated by the "2"-pulse train) is less than the variance of the noise distribution (generated by trains of 1 or 3 pulses). Slopes less than 1.00 imply that the variance of the signal distribution is greater than the variance of the noise distribution.

An analysis for d' and slope of ROC curves generated by rating data has been published by Ogilvie and Creelman (1968). Their measure of sensitivity d* is the maximum likelihood estimate of the distance between the means of logistic distributions. A conversion formula

$$d'_e = d^* \times .61$$

allows us to estimate the distance between the means of the normal distributions for the midline criterion (i.e. that position of the criterion for which the probability of responding "signal" given a sample of excitation from the noise distribution is equal to the probability of responding "noise" given a sample of excitation from the signal distribution). When the variances of the underlying Gaussian distributions are unequal the measure d'_e weights the standard deviation units of the two distributions. That is,

$$d'_e = \frac{2r}{r+1} Z_N - \frac{2}{r+1} Z_{SN}$$

where r is the ratio of the standard deviation of the noise distribution to the standard deviation of the signal distribution. The symbol Z_N is the transformation to standard normal deviates of the probability of a false alarm (i.e. the probability of a reporting a signal given noise) and Z_{SN} is the transformation to standard normal deviates of the probability of a hit (i.e. the probability of reporting a signal given a signal).

The measures of sensitivity (d'_e) are presented for combinations of subject, task, ipi and pairs of trains of 1 and 2 and 2 and 3 pulses in Figure 14. The rating data could not be analyzed in 10 out of 64 instances either because the high level of performance or because the S had not used all the rating categories. Inspection of the data available indicates that d'_e

(i) increases for any pair of stimuli as the ipi increases from 3 to 4 msec.

(ii) decreases in the 2-alternative tasks as the values of the pair of consecutive numbers (n and n+1) in the trains presented increase

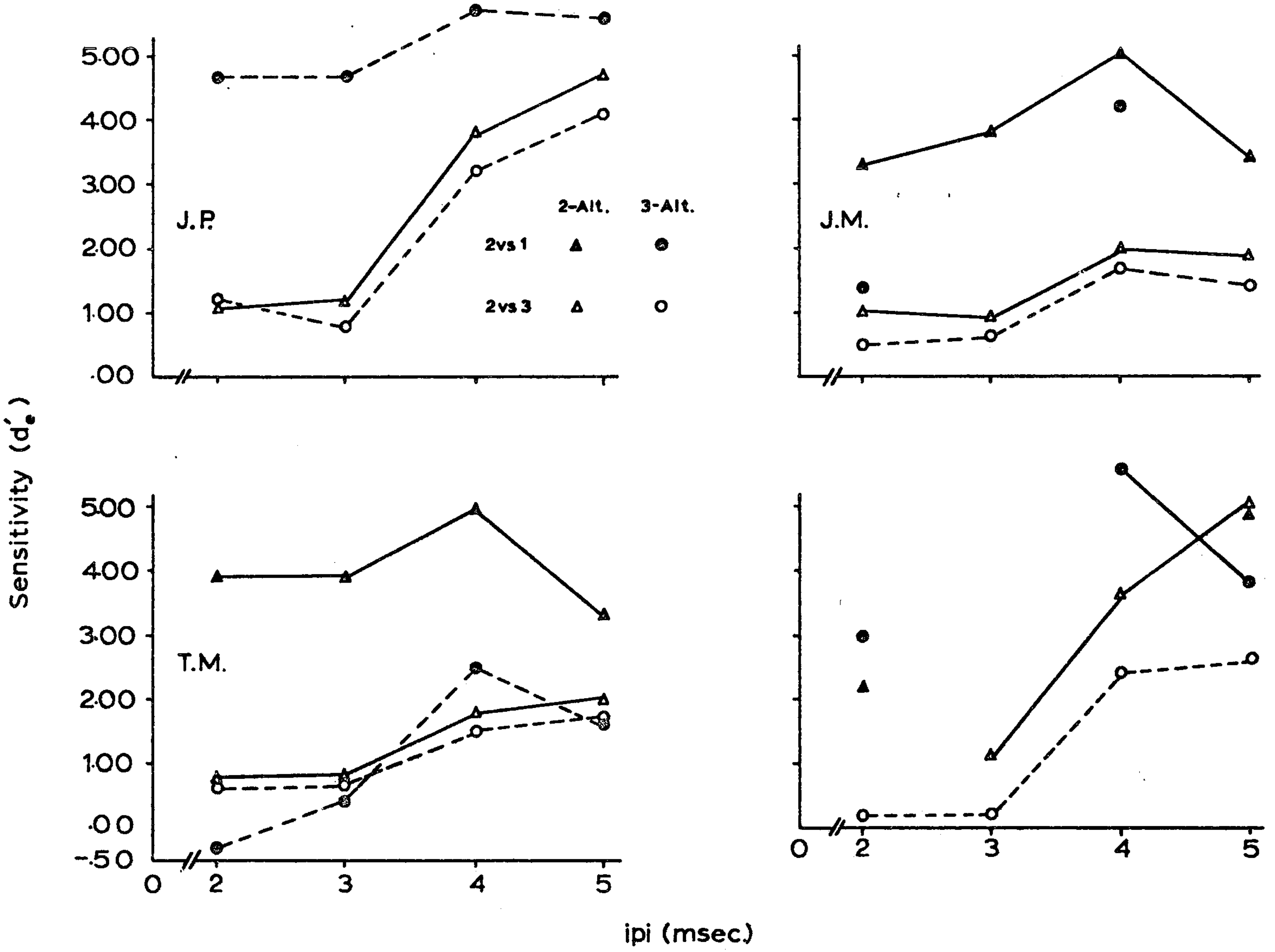


Figure 14. Discriminability of pairs of stimuli in the three experimental tasks.

(iii) decreases for trains of 2 and 3 pulses as the number of alternative stimuli increases.

It should be noted that for large values of d* the estimate of d'_e plotted in Figure 14 is too high. For d'_e greater than 2.5 the error is approximately 10%. The standard error of the estimate of d'_e for each combination of subject, task and ipi is presented in Appendix A, Table 5.

A chi-square test for linearity of the four points comprising each empirical ROC curve (Ogilvie and Creelman, 1968) indicated that the chi-square statistic was non-significant ($p > .05$, 2df) in 43 out of the 54 instances available for examination. The slopes of the lines are presented for each experimental condition in Table 3. In Task 2-1 the slopes of 7 out of 10 significantly linear functions did not deviate significantly from 1.00 ($p > .05$). In Tasks 2-3 the slopes of 9 out of 13 lines were not significantly different from 1.00. Discrimination of 1 and 2 pulses in Task 2-13 resulted in linear ROC curves in 10 out of 13 cases. Of these 10, only 4 could be described by a slope of 1.00. Discrimination of 2 and 3 pulses at each ipi in Task 2-13 yielded 10 out of 15 significantly linear fits. Of these 10, 5 had slopes of 1.00.

A cumulative probability distribution for the values of chi-square obtained for the 54 lines is presented in Appendix A, Figure 5. A comparison of the obtained distribution with the theoretical cumulative probability distribution for

TABLE 3 Slopes of Straight Line Fits to ROC Curves

		1 vs 2		2 vs 3	
Subject	ipi (msec.)	Task 2-1	Task 2-13	Task 2-3	Task 2-13
JP	2	-	1.374[1]	1.627[1]	1.205*
	3	-	3.080	1.209[1]	1.035*+
	4	-	1.949[1]	2.267	1.221[1]
	5	.671	1.966	1.403[1]	0.635
JM	2	1.747	1.868*	.986*+	.782
	3	0.715[1]	-	1.037[1]	.970[1]
	4	1.367[1]	2.473+	1.145[1]	.869*+
	5	1.299*+	-	1.419	.703
TM	2	1.000	.238*	2.206[1]	1.142[1]
	3	1.587[1]	.110	1.534[1]	1.000
	4	0.714[1]	.615[1]	1.202*+	1.110[1]
	5	0.596	.443	.677	1.062*+
JC	2	1.000	2.497*	-	-
	3	-	-	.789	.854*
	4	-	4.609	1.339[1]	1.067
	5	.689[1]	1.638[1]	1.991[1]	.765[1]

* significant deviation of ROC points about best fitting line

[1] slope of 1.00 doesn't differ significantly from best slope

\+ ROC points may be fit by a line of slope 1.00 but the deviation of the points about this line is significant

2df shows that the two distributions are similar. If we exclude the six most extreme values of chi-square obtained (i.e. $X^2 > 14.0$) the expected value empirical distribution of chi-square is 2.15.

The analysis presented in Table 3 suggests that the assumptions of normality and of equal variance of the theoretical distributions are valid for discrimination of trains of 1 and 2 pulses and 2 and 3 pulses in Tasks 2-1 and 2-3 respectively. For Tasks 2-13 the equal variance assumption appears to be a poor one. The data imply that presentation of trains of 1, 2 or 3 pulses within the 3-alternative task results in a change in the variance of at least one of the underlying distributions.

Durlach and Braida (1969, p. 374) have suggested that for auditory stimuli "the transformation from the stimuli to the decision variable is composed of a transformation from the acoustic waveforms to *sensations*, followed by a transformation from the sensations to the decision variable" and that "two types of noise...contribute to the random behavior of the decision variable: *sensation noise and memory noise*." The memory noise depends on the experimental paradigm. For the one interval paradigm (consisting of presentation of a single stimulus on each experimental trial), the memory noise results from the subject's "attempt to compare the sensation with the general context of sounds in the experiment." He tries to remember a verbal representation of this comparison. It is assumed that the width of the context and hence the memory noise will increase as the range of the set of stimuli increases. This increase in memory noise will result in an equal increase in the variance of each of the theoretical distributions for the set of stimuli on the decision axis. Thus, for any pair of stimuli an increase in the set should produce a decrease in the distance between the means of the distributions, measured in the units of one of these distributions. Context then will affect sensitivity.

The results for discrimination of trains of two and three pulses presented in Figure 14 show a decrease in sensitivity as a function of an increase in set size, as predicted by the Durlach and Braida model. However, the slopes of the ROC curves for discrimination of trains of two and three pulses in Task 2-13 suggests that if the decrease in sensitivity is attributable to an increase in the variance of the underlying distributions, these distributions have not changed by equal amounts.

One of the assumptions of the TSD model presented above (see page 26) was that "changes in the relationship between n, p and ipi would produce unidimensional changes in sensory excitation. To test the validity of this assumption for changes in n we can examine in Task 2-13 the relationship among distances between the means of the theoretical distributions for trains of 1, 2 and 3 pulses. If the excitation continuum is unidimensional then knowledge of the distances between the means of any two pairs of distributions should allow us to predict the distance for the third pair.

Since the underlying distributions for trains presented in Task 2-13 were not equal in variance our measure of the distance between any pair of means, d'_e, represents a weighting of the standardized units of the distributions studied. Thus, in order to compare distances we must convert measures of d'_e to the units of one distribution. The standard deviation for the distribution contingent on presentation of a train of two pulses was chosen. According to Green and Swets (1966) the relationship between d'_e and $d'_{\Delta m}$, the distance between the means measured in the units of the standard deviation of the noise distribution (i.e. the distribution produced by presentation of 1 or 3 pulses in the present study) is expressed as

$$d'_{\Delta m} = \frac{d'_e}{2}\left(\frac{1}{r}\right) + \frac{d'_e}{2} \tag{37}$$

where r is equal to the ratio of the noise to the signal standard deviation. This distance expressed in the units of the standard deviation of the signal or "2" distribution ($d'_{\Delta s}$) may be calculated from the formula

$$d'_{\Delta s} = d'_{\Delta m} \times r \tag{38}$$

In the case of comparison of trains of 1 and 3 pulses d'_e was expressed in the units of the distribution for 1 pulse and the ratio of the standard deviations of the distributions for one and two pulses (σ_1/σ_3) was used to convert $d'_{\Delta m}$ to $d'_{\Delta s}$. This method of calculating $d'_{\Delta s}$ for distributions of 1 and 3 pulses is possible only if

$$\frac{\sigma_1}{\sigma_3} = \frac{\sigma_1}{\sigma_2} \div \frac{\sigma_3}{\sigma_2} \qquad (39)$$

The truth of equation 39 rests on the validity of the unidimensional assumption.

The discrepancy between the ratio $\frac{\sigma_1}{\sigma_3}$ predicted from equation 39 and the same ratio obtained from the data are shown for each value of ipi in Table 4. Except for one instance the discrepancies are always in the same direction. In 7 out of 12 instances the discrepancy is less than .1 and in 10 out of 12 instances it is less than .50.

Examination in Table 5 of the relationship among the distances between the means of distribution for trains 1, 2 and 3 pulses show that in 13 out of 14 instances the data can be described by the expression

$$d'_{12} = d'_{13} + d'_{32} \qquad (40)$$

where d'_{12} represents the distance between the means of the distributions for 1 and 2 pulses in the units of the standard deviation of the "2" distribution. The absolute magnitude of

TABLE 4 Predicted and Observed Ratios of Standard Deviations For Distributions of 1, and 3 Pulses

Subject	ipi (msec.)	Predicted $\frac{\sigma_1}{\sigma_3}$	Observed $\frac{\sigma_1}{\sigma_3}$	Discrepancy (Pred.-Obser.)
JP	2	1.140	2.201	-1.061
	3	2.976	3.449	- .473
	4	1.596	1.828	- .232
	5	3.096	3.183	- .087
JM	2	2.389	2.678	- .289
	3	-	-	-
	4	2.846	3.411	- .565
	5	-	-	-
TM	2	.208	.206	+ .002
	3	.110	.198	- .088
	4	.554	.578	- .024
	5	.417	.461	- .044
JC	2	2.830	-	-
	3	-	-	-
	4	4.320	4.344	- .024
	5	2.140	2.216	- .076

TABLE 5 Predicted and Observed d'_{12}

Subj.	ipi (msec)	Observed d'_{13}	Observed d'_{32}	Predicted d'_{12} (Obs.d'_{13}+Obs.d'_{32})	Observed d'_{12}	Discrepancy (Pred.d'_{12}-Obs.d'_{12})
JP	2	4.364	1.279	5.643	5.591	.052
	3	8.975	.834	9.809	9.650	.159
	4	5.138	3.553	8.691	8.433	.258
	5	5.064	3.393	8.457	8.291	.166
JM	2	1.527	.428	1.955	1.951	.004
	3	-	-	-	-	-
	4	5.802	1.608	7.410	7.258	.152
	5	-	-	-	-	-
TM	2	- .803	.621	- .182	- .173	- .009
	3	- .148	.680	.432	.244	.188
	4	.434	1.614	2.048	1.994	.054
	5	- .506	1.784	1.298	1.172	.126
JC	2	-	-	-	-	-
	3	-	-	-	-	-
	4	13.219	1.372	14.591	15.648	-1.057
	5	2.709	2.303	5.012	4.999	.013

the discrepancy between observed values of d'_{12} and the values obtained from the summation of the obtained d'_{13} and d'_{32} is less than .20 in 10 out of 12 cases examined for four Ss. Except for 2 instances the sign of the discrepancies is the same. Thus, within the limits of the observed discrepancies, the assumption of unidimensional scaling of excitation for changes in n appears to describe the data fairly well, although a small, systematic departure appears to exist.

CHAPTER IV

RESULTS AND THEORETICAL ANALYSIS OF EXPERIMENT 2

A. Raw Data

The probability of a correct response in Experiment 2 was calculated from the equation

$$\hat{Pr}(C) = .50\ [\hat{Pr}(R_s/S_s)] + .50\ [\hat{Pr}(R_l|S_l)] \qquad (41)$$

where $\hat{Pr}(R_s/S_s)$ is the conditional probability of responding "short" given the stimulus pattern containing the short interval (ipi msec.) and $\hat{Pr}(R_l|S_l)$ is the conditional probability of responding "long" when presented the pattern containing the long interval (ipi + 5 msec.) For this analysis of results the data matrix for each of the two stimulus patterns in any condition was reduced to two cells. Key A and Key B responses were summed and interpreted as report of "short". Key C and Key D responses were grouped and interpreted as report of "long". The probabilities associated with each of the four keys given short and long intervals respectively are presented for combinations of subject, and experimental condition in Appendix B, Tables 1, 2 and 3.

The $\hat{Pr}(C)$ for problem 1 is presented as a function of ipi in Figure 15 for each of four Ss. Each data point is

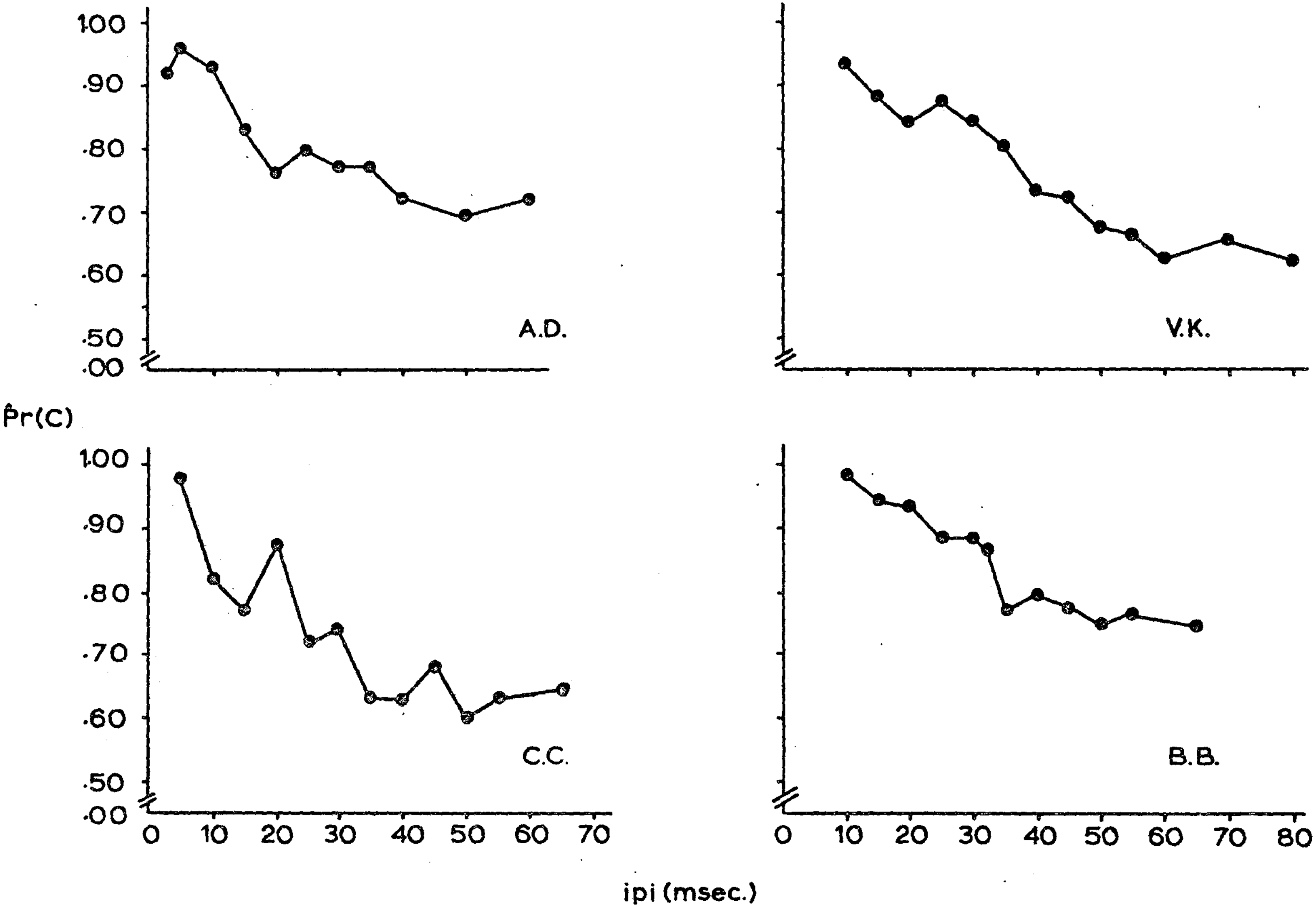


Figure 15. Probability of correct response as a function of ipi.

based on 800 trials. The standard deviation of each estimate of proportion is less than .018. It is apparent from these graphs that $\hat{Pr}(C)$ decreases as ipi increases. For the range of ipi studied the minimum value of $\hat{Pr}(C)$ is reached at approximately 50 msec. An increase of 10 msec. for subject A.D. and of 15 msec. for subjects C.C. and B.B. beyond an ipi of 50 msec. and an increase of 20 msec. beyond an ipi of 60 msec. for subject V.K. does not produce a systematic decrease in $\hat{Pr}(C)$.

The obtained $\hat{Pr}(C)$ for problem 2 is presented as a function of the durations of pulses 1 and 2 respectively in Figure 16. The data points presented for each *S* are based on 800 trials each. The standard deviation of each estimate of proportion is less than .018. Comparison of the data obtained for the four *Ss* suggests that $\hat{Pr}(C)$ does not change systematically as the durations of either pulse 1 or pulse 2 increase from 4 to 16 msec., while the other pulse remains fixed at 10 msec. Goodness of fit tests (Hayes, 1963) indicate that

(i) for each of four *Ss* we cannot reject the hypothesis at the .05 level that the frequencies of a correct response for each of the four durations of pulse 1 are equal to a constant that is the mean of these frequencies. The values of chi-square obtained for each *S* were less then

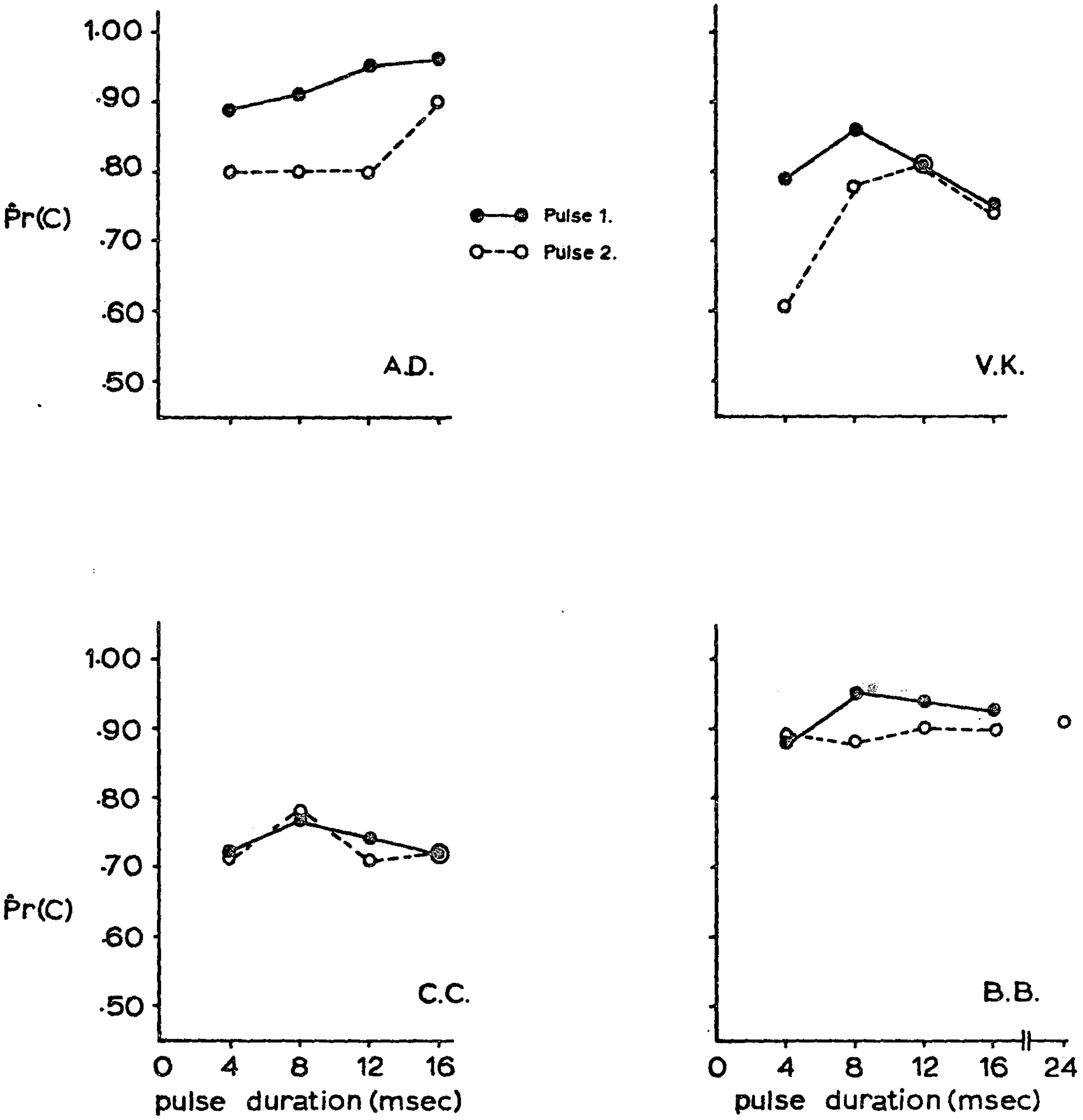


Figure 16. Probability of a correct response as a function of the duration of pulse 1 with pulse 2 constant and of pulse 2 with pulse 1 constant.

7.814 with 3 df.

(ii) for each of three Ss we cannot reject the hypothesis at the .05 level that the frequencies of a correct response for each of the four durations of pulse 2 are equal to a constant that is the mean of these frequencies. The values of chi-square for two Ss were less than 7.815 with 3 df. For the third S the value of chi-square was less than 9.488 with 4 df.

The number of practice blocks that preceded the eight blocks of relatively stable performance represented in Figures 15 and 16 for each condition is presented for each subject in Appendix B, Tables 4 and 5. The data in Tables 4 and 5 indicate that the amount of practice needed depends both on the subject and on the experimental condition. The number of practice blocks which were required when changing to a new experimental condition did not decrease as a function of the number of conditions which had been experienced previously.

B. Theoretical Analysis

The Exponential Growth, Exponential Decay and Exponential Growth, Linear Decay models assumed that the stimulus patterns in Experiment 2 generated Gaussian distributions of excitation. A measure of sensitivity (the distance between the means of the theoretical distributions) is presented for each condition in problem 1 in Figure 17. The short pattern was arbitrarily defined as the signal and the long pattern as the noise. The graph shows that

(i) for three Ss (A.D., V.K. and B.B.) d'_e appears to decrease as ipi increases

(ii) for all Ss d'_e does not reach zero for the range of values of ipi investigated

The standard error of the estimate of d'_e for each condition is presented in Appendix B, Table 6. It should be noted that the error in estimating d'_e using the Ogilvie and Creelman method increases as d'_e increases beyond 2.50.

A measure of discrimination (d'_e) for patterns in each condition of problem 2 is shown in Figure 18. It is apparent from a comparison of the data for the four Ss that d'_e does not change systematically for changes in the duration of either pulse 1 or pulse 2. The standard error of the estimate of d'_e for each condition is presented in Appendix B, Table 7.

The slopes of the ROC curves based on the rating responses and plotted in terms of standard normal deviates are presented for problems 1 and 2 in Tables 6 and 7

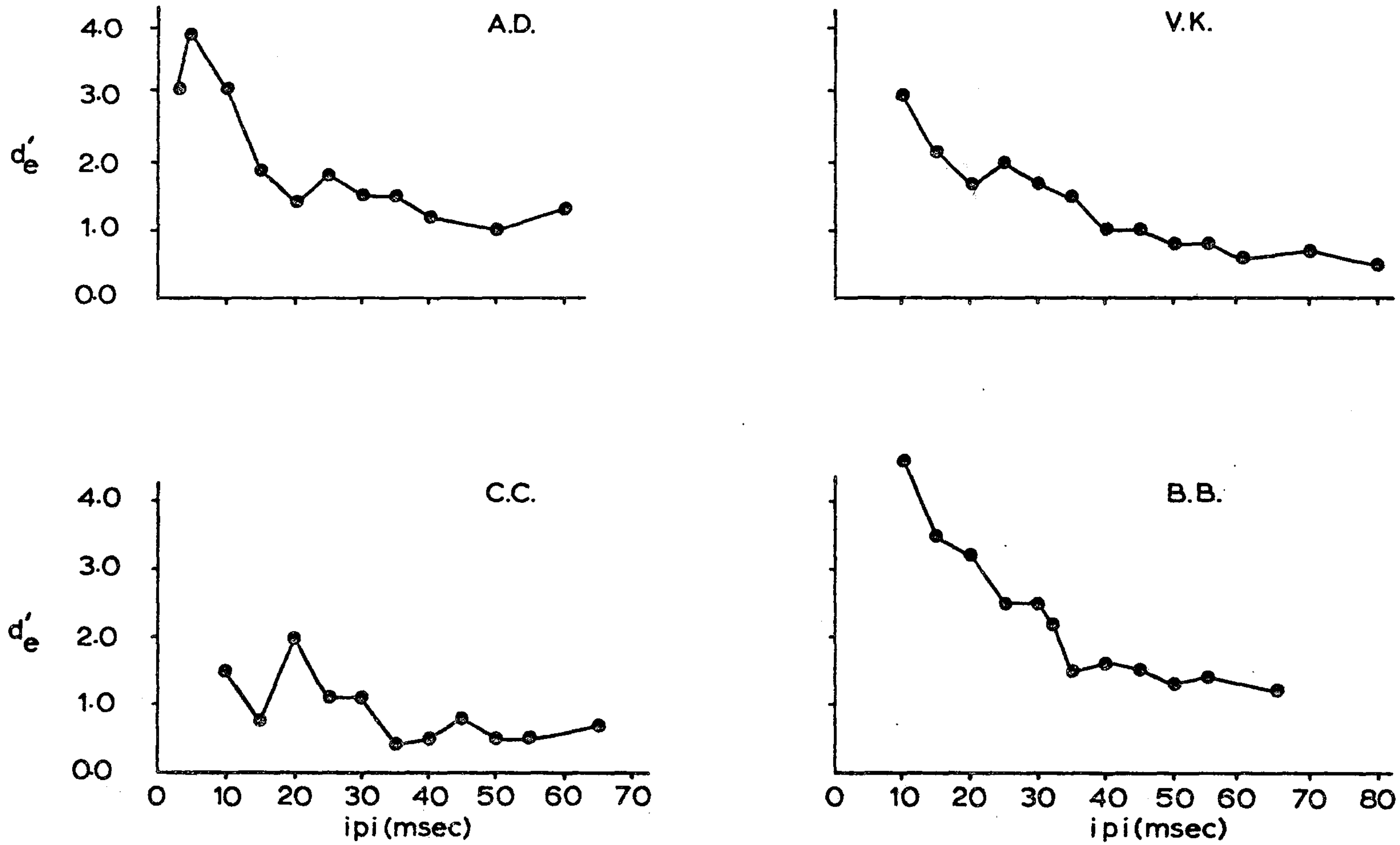


Figure 17. Discriminability of a pair of intervals ipi and (ipi+5)msec. as a function of ipi.

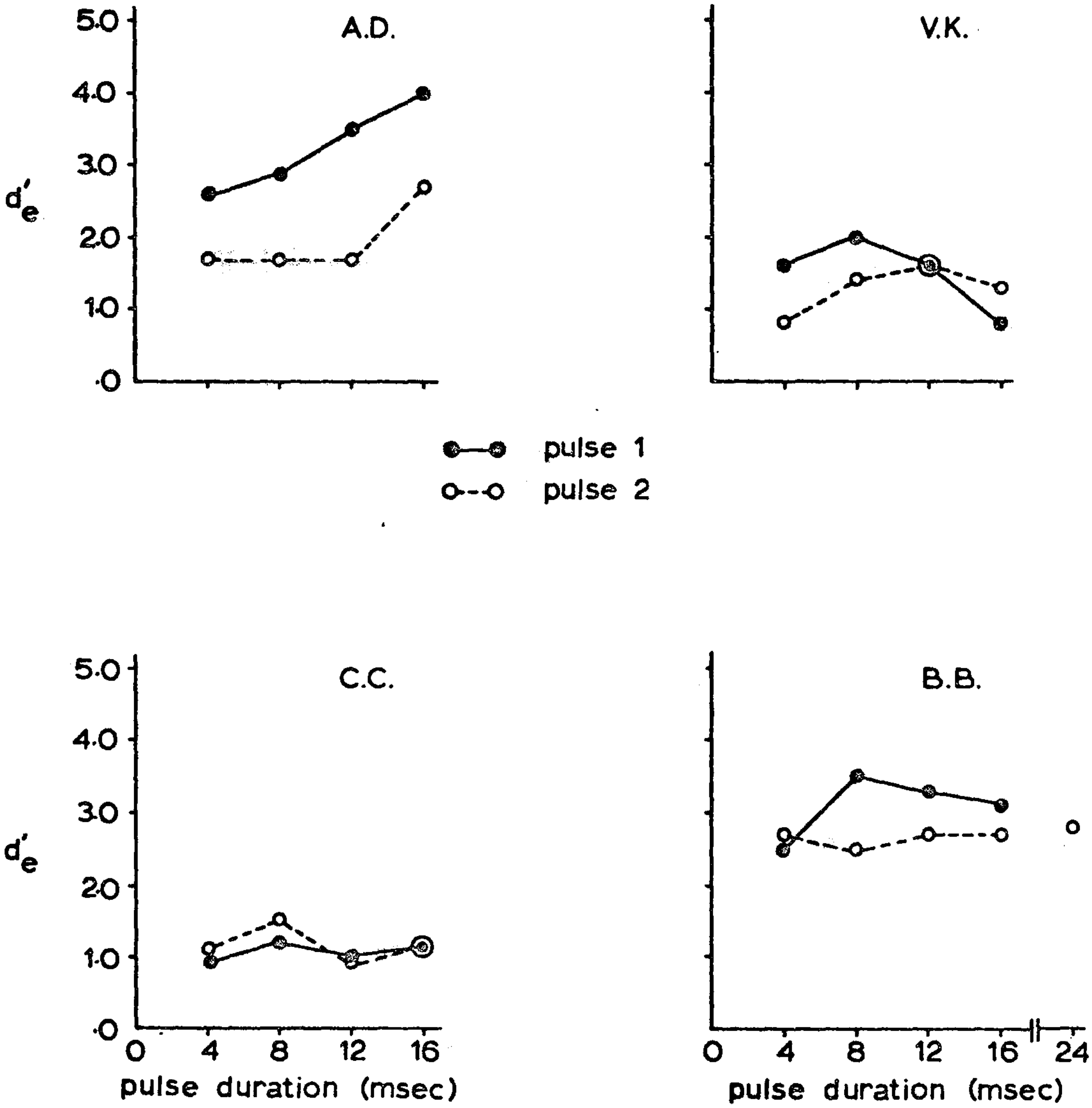


Figure 18. Discriminability of a pair of intervals as a function of pulse duration.

TABLE 6 Slopes of Straight Line Fits to ROC Curves Obtained in Experiment 2, Problem 1

ipi (msec.)	Subjects			
	A.D.	V.K.	C.C.	B.B.
3	1.431[1]	-	-	-
5	1.660[1]	-	-	-
10	1.167[1]	.689[*]	2.019[*]	2.036[*]
15	-	1.529[*]	2.640[*]	1.371[1]
20	1.406	0.937[*+]	1.195[*1]	1.038[1]
25	1.148[1]	1.446[*]	1.041[*1]	1.123[1]
30	1.174[1]	1.211[*+]	1.310[*]	1.026[1]
32	-	-	-	1.053[1]
35	1.134[1]	1.094[*+]	1.274[*+]	1.081[1]
40	1.096[*1]	1.460[*]	1.290[*]	1.015[1]
45	-	1.013[*+]	1.142[*+]	0.910[1]
50	1.068[1]	1.312	1.116[1]	1.024[1]
55	-	1.044[*1]	1.283[*]	1.015[1]
60	.973[*1]	1.174[1]	-	.964[1]
65		-	1.269	
70		1.045[*1]		
75		-		
80		1.319		

[*] sig. deviation of ROC points about best fitting line

[1] line of slope 1.00 doesn't differ significantly from best line

[+] ROC points may be fit by a line of slope 1.00 but the deviation of the points about this line is significant.

TABLE 7 Slopes of Straight Line Fits to ROC Curves Obtained in Experiment 2, Problem 2

Pulse	Pulse Duration (msec.)	Subjects A.D.	V.K.	C.C.	B.B.
1	4	1.080[1]	1.337[*]	2.495[*]	1.299[1]
	8	.883[*1]	1.769[*]	2.308[*]	1.605[+]
	12	.829[1]	1.704[*]	1.667[*]	.783[*+]
	16	1.180[1]	1.509	1.318[*]	.851[1]
2	4	.939[1]	1.094[*1]	1.265	.925[1]
	8	1.232[1]	1.620[*]	2.144[*]	1.013[1]
	12	1.041[1]	1.489[*]	1.894[*]	1.425
	16	.988[1]	1.357[*]	1.458[*]	1.189[1]
	24	-	-	-	1.175[1]

* sig. deviation of ROC points about best fitting line

1 slope of 1.00 doesn't differ significantly from best slope

\+ ROC points may be fit by a line of slope 1.00 but the deviation of the points about this line is significant.

respectively. For subjects A.D. and B.B. the Gaussian assumption appears to provide a good model for the data. For subject A.D. a chi-square test for linearity of ROC points shows a significant deviation ($p<.05$, 1 df) from a straight line fit in only 2 out of 9 conditions in problem 1 and in 1 out of 8 conditions in problem 2. For subject B.B. the chi-square statistic is significant in only 1 out of 9 cases for problem 2. A test for the slopes of the lines shows that for subject A.D. the slope of the line is not significantly different from 1.00 ($p > .05$) for 6 out of the 7 acceptably linear fits in problem 1 and for 7 out of the 7 linear fits in problem 2. For subject B.B. the slope is not significantly different from 1.00 for 11 out of the 12 acceptable lines in problem 1 and for 6 of the 7 lines in problem 2. The data for subjects V.K. and C.C. indicate that the Gaussian assumption cannot be accepted. For both Ss the chi-square statistic is significant for all but 1 or 2 conditions in each of problems 1 and 2.

The three variations of the Exponential Growth, Linear Decay model predicted that sensitivity to the difference between patterns 1 and 2 would be independent of ipi. When the assumption of underlying Gaussian distributions of equal variance holds (for subjects A.D. and B.B.) this prediction is not supported. According to these three variations of the model, sensitivity should increase exponentially as the duration of the first pulse (d_1) increases linearly and should be independent of changes in the duration of pulse 2 (d_2). Only the second of these two predictions is supported by the data.

The three variations of the Exponential Growth, Exponential Decay model predicted that sensitivity would decrease exponentially to zero as a function of linear increases in ipi. The data of subjects A.D. and B.B. for problem 1 may have these properties. However, it is not clear that the obtained functions can be described mathematically as negative exponentials or that they will reach zero for some value of ipi greater than 60 msec. Variations 1, 2 and 3 predicted the observed insensitivity to changes in d_2. Every one of these models predicted that sensitivity would be affected by changes in d_1 and it was not.

CHAPTER V

CONCLUSIONS AND DISCUSSION

This research began as an attempt to investigate further White's (1963) conclusion that the functional relationship between the number reported and the number of pulses presented in a train of brief, auditory pulses entirely depends on a central process. The temporal characteristics of this process, in particular, were said to limit the perceived rate of occurrence of events. White and Eason (1966) stated more specifically that for visual numerosity functions the number reported is related to the number of successive components in the evoked cortical response pattern that occur during the total duration of the stimulus train. Other central mechanisms for both number and duration of visual and auditory stimuli had been suggested by the theories and experimental data of such researchers as Kristofferson, Haber, Creelman, Green and Plomp. In view of the diversity of possible theoretical explanations of the temporal numerosity function, the focus of interest of the present work broadened to that of comparing the success of several theoretical explanations of the observer's counting behavior.

A. Empirical Findings

Experiment 1 was a study of the discrimination of the number of pulses presented in a train and of the effect of context on the discrimination of number. The stimulus variables that were manipulated were

(i) the number of pulses in the trains presented for discrimination

(ii) the time between successive pulses in these trains

(iii) the number of different trains in the stimulus set.

The subjects were told the composition of the stimulus set and were asked to limit their responses to a given set of confidence categories.

The results of Experiment 1 showed that for each of four Ss the discrimination between a pair of trains containing consecutive numbers (n and n+1) of pulses improved as these numbers decreased. The probability of correctly discriminating between trains of 1 and 2 pulses was close to 1.00 when the interval between the pulses in the 2-pulse train ranged from 2 to 5 msec. For trains of 2 and 3 pulses the data indicated that the time between pulses would have to be greater than 5 msec. for the same level of performance to be reached.

The data of Task 2-13 showed the effect of context on the correct identification of a particular train. As the number of trains in the set of stimuli increased from two to three, the probability of correctly identifying a train of 2 pulses decreased.

Experiment 2 represented an effort to increase our understanding of the nature of the interaction between the parameters of the stimulus train. We began by studying discrimination between two "empty" time intervals, each bounded by a pair of short auditory pulses and differing in duration by a constant 5 msec. as a function of

(i) the duration of the shorter interval
(ii) the durations of the pulses marking the beginning and end of these intervals.

Examination of the probability of a correct response for each condition in Experiment 2 indicated that

(i) when the durations of pulses 1 and 2 were fixed at 10 msec., performance decreased as the shorter interval increased from 10 to approximately 50 msec.
(ii) when the pair of interpulse intervals were fixed, performance did not vary as a function of changes from 4 to 16 msec. in the duration of either pulse 1 or pulse 2 with the other pulse fixed at 10 msec.

These data suggest that for the conditions of Experiment 2 performance depends only on the interval between the offset of pulse 1 and the onset of pulse 2.

B. Theoretical Interpretations

I. Experiment 1

The theoretical analyses of the results of Experiment 1, and in particular of Tasks 2-1 and 2-3, imply that the discrimination of a pair of trains does not depend solely on the difference in the total durations of these trains. The results do suggest that discrimination depends on the processing of individual pulses in the trains presented. It was noted that the probability in correctly discriminating 2 from 3 pulses in Task 2-3 increased sharply when ipi was large enough to allow for a decrease in pulse amplitude between stimulus pulses. It is clear from the effect on discrimination of the time between pulses that the effect of a particular pulse in the nervous system is related to the proximity of that pulse to its neighbours. However, the results of the three tasks are not consistent with the predictions of our model extending Haber's (1968) theory of information processing to temporal numerosity. In particular, this theory predicts that the processing of a pulse will depend only on the time between the onsets of successive pulses and that the discrimination of a pair of trains of n and n+1 pulses will be independent of the value of n, and of the size of the stimulus set.

(a) The Quantal Theory

The quantal theory assumed that the perception of a discontinuity or interpulse interval in a pulse train would depend on the occurrence of time points in a central time base. More specifically an interpulse interval would be detected only when a time point occurred between successive pulses. Pulses occurring within a quantum or frame of time would not be distinguished as separate.

According to the quantal model, the overlap between the distributions of central counts for pairs of trains used in the present study should increase

(i) as the interpulse interval decreases

(ii) as the numbers n and n+1 in a pair of trains increase.

In both of these cases, if the biases for report remain fixed, the probability of a correct response should decrease. The observed changes in $\hat{Pr}(C)$ support these predictions.

The effect of context shown in comparing the data of Tasks 2-13 with the data of Tasks 2-1 and 2-3 rules out a counting model of the type described by White (1963). If the correct identification of a train of 2 pulses becomes more difficult when S is expecting both trains of 1 and 3 pulses, then there cannot be a one to one relationship between the sensory count and the number reported. The quantal model accounts for the effect of context in terms of response bias. However, we were not able to test for a change in response bias, as we varied the size of the stimulus set.

Within the framework of the quantal two-alternative response model the data and the experimental design did not allow us to determine the bias parameters for a particular train in each of three experimental tasks. The quantal confidence-rating model predicts the effect of changes in both sensory parameters and response bias in terms of the position of the ROC curve. If either of these sets of parameters produces greater overlap in the distributions of rating responses for a pair of trains (indicating less discrimination) the ROC curve will be closer to the positive diagonal of the ROC space. Although the obtained ROC curve for trains of 1 and 2 _and_ 2 and 3 pulses in fact indicated poorer discrimination in the 3-alternative as compared with the 2-alternative task, the effect cannot be unequivocally attributed to a change in response bias.

The data obtained for the three tasks in Experiment 1 cannot be interpreted as evidence for a particular value of q. For the relatively small values of ipi and n used in these tasks the value of q would have to be equal to or less than 10 msec., for the model to describe the data obtained. This value for the unit of duration is considerably less than the value of 100 msec. suggested by White's (1963) data and the value of 50 msec. suggested by Kristofferson's (1967) experiments. The relatively high level of performance in these tasks suggests that subjects are using a more efficient basis for estimating number than the discrete values of the count suggested by the quantal theory.

(b) The Theory of Signal Detection

Our model that extends the Theory of Signal Detection to allow for a description of counting behavior suggests that the effective stimulus for the observer is the amount of sensory excitation accumulated during the presentation of a train of pulses. This accumulation is a continuous random variable that depends presumably on the build-up and decay in the nervous system of the excitation produced by each pulse. On any trial the observer's judgment will depend on the probability that his sample of accumulated excitation resulted from the presentation of one stimulus train relative to the probability that this summed excitation was produced by a second train.

An analysis of the data for Tasks 2-1 and 2-3 within the TSD framework supports the basic assumption of this model that trains of pulses give rise to Gaussian distributions of excitation. Further, these distributions are equal in variance. For the 2-alternative tasks

(i) as the time between pulses in a pair of trains increases, the distance between the means of the pair of theoretical distributions generated by these trains increases

(ii) as the number of pulses (n and n+1) presented increases, the distance between the means of the pair of theoretical distributions will decrease.

The TSD analysis of the data for Task 2-13 shows that although the assumption of a Gaussian distribution of excitation for each train of pulses may be acceptable, the equal variance assumption can be rejected. As discussed above (page 99) Durlach and Braida (1969) have suggested that as the range of a set of stimulus intensities increases, the variances of the underlying Gaussian distributions for each stimulus in the set will increase. As the variance increases, the distance between the means of any two of these distributions, measured in the standard deviation units of one of the distributions, will decrease. If context in the present experiment is equivalent to an increase in the range, then for a fixed interpulse interval the measure d' should be less for a pair of trains presented in the 3-alternative as compared with the 2-alternative situation. A comparison of the data for discrimination between trains of 2 and 3 pulses in Tasks 2-3 and 2-13 supports this prediction.

Examining within the TSD framework the relationships among distances between pairs of means, we find that *Ss* appear to order their impressions of the stimulus along a single sensory dimension. Within certain limits knowledge of any two distances allows us to predict the third. For three *Ss* (J.P., J.M., and J.C.) the order of magnitude of these distances indicates that *Ss* are rating their impressions in terms of the relative degree of "twoness" in the stimulus train. Presentation of a train of 3 pulses is more like the presentation of a train of 2 pulses than is 1 pulse. The distance between the means of the theoretical distributions for 1 and 3 pulses is less than the distance between the means of the distributions for 1 and 2 pulses.

For subject T.M. the relative positions of the means of the theoretical distributions for trains of 1, 2 and 3 pulses appears to change as the time between pulses increases. For interpulse intervals of 3, 4 and 5 msec. this S is most likely to respond "2" when 2 pulses are presented. However, when the ipi is equal to 2 msec., a report of 2 is most likely when 1 pulse is presented.

II. Experiment 2

The second experiment was an attempt to isolate the process involved in producing the cumulative sensory effect discussed in the TSD interpretation of the results of Experiment 1.

(a) The Spectral Distribution of Energy of the Acoustic Wave-form for Each Train as a Cue for Discrimination of Empty Intervals

It could be argued that, when comparing trains containing only two pulses, the observer could capitalize on differences in the spectral distributions of energies of the patterns presented in each condition. The spectral distribution of energy for an acoustic waveform describes the amount of energy concentrated at frequencies other than the central or carrier frequency of the pulse (Licklider, 1951). This spread of energy is due to a change in the amplitude or frequency of the pulse. In a train of pulses 100% amplitude modulation occurs at the offset of each pulse. However, the resultant spread of energy will depend on the duration of the pulse and on the interpulse interval. More specifically, as discussed by Garner (1947 a) the spectral distribution of energy for repeated tones is determined by the on-off ratio of the tones. The central component of this frequency spectrum (the component with the greatest energy) is determined by the carrier or modulated frequency (i.e. the frequency of the tone) and the relative position of sideband components in the spectrum is determined by the frequency of repetition of pulses. For a series of pulsed sine-waves the sideband components are spaced symmetrically on either side of the central component. Inertia of the diaphragm of the earphone in following the abrupt changes in amplitude at the pulse onset and offset will introduce changes in the frequency spectrum of the stimulus pulse.

Thus the frequency spectrum for a pattern of two 10 msec pulses separated by ipi msec. could differ from the spectrum of a pattern of 10 msec. pulses separated by ipi + 5 msec. The difference would depend on ipi. Within certain limits the problems of spread of energy and inertia have been circumvented by the relatively slow rise and decay times of pulses. A photograph comparing the response of the earphone with the stimulus pulse (see Figure 4) shows that the waveform changes very little as a result of its passage across the diaphragm.

(b) The Exponential Growth, Exponential Decay and Exponential Growth, Linear Decay Models

An analysis of the empirical ROC curves for each condition in problems 1 and 2 of Experiment 2 showed that for two Ss (A.D. and B.B.) the basic assumptions of our

TSD models were supported. That is, for no condition could we reject statistically the assumption that the patterns generated Gaussian distributions of equal variance. However, the data for Experiment 2 supported neither the predictions of the Exponential Growth, Exponential Decay nor the Exponential Growth, Linear Decay models. The EGED model predicted that sensitivity to the difference between the pair of patterns presented in each condition would decrease exponentially to zero as the interpulse interval (ipi) increased. The EGLD model predicted that performance would be independent of ipi. All variations of both models led us to expect that the observer would be sensitive to a change in the duration of the first pulse.

Although we can reject the EGLD model, the data are not sufficient to allow us to reject the EGED model. First, we have not investigated the effect on sensitivity of increasing ipi beyond 60 msec. for all subjects. It may be that the measure of sensitivity will eventually decrease to zero as ipi increases. Secondly, we could expect the observed insensitivity to changes in the duration of pulse 1 if the excitation produced by this pulse reaches its maximum sensation level at a pulse duration less than the smallest value of d_1 used in the experiment.

If we can extend the conclusions concerning the effect of duration on the absolute hearing threshold for tones to detectability of supra-threshold tones the latter argument is not supported by findings reported in the literature. Plomp and Bouman (1959) describe the results of a study by Hughes (1946) concerning the course of the absolute hearing threshold and the duration of a single pulse. The durations studied ranged between 63 and 739 msec. The frequencies studied were 250, 500, 1000 and 2000 Hz. The results indicated that the threshold intensity was inversely proportional to the duration. For the range of durations studied the results were interpreted as evidence for a complete integration of the energy in the pulse for threshold detectability. Garner (1947 b) measured the absolute hearing threshold for a pulse ranging in duration from 1 to 100 msec. The frequencies studied were 250, 1000 and 4000 Hz. His results like those of Hughes suggested a linear integration of energy in the pulse. Garner points out that as his tone became shorter the spectral distribution of energy increased. He argues that the integration of energy for threshold detectability would be restricted to energy contained in a restricted band of frequencies.

(c) Duration Discrimination

The data from Experiment 2 suggest that for the conditions of the present study the observer's judgment depends only on the durations of the interpulse intervals in the patterns presented for comparison. Creelman (1962) suggests that the observer measures the duration of a continuous auditory signal by "counting pulses" that occur during the duration to be judged. These pulses are the firings of hypothetical independent elements in the nervous system. It is assumed that each element has a fixed probability of firing at any moment. The intervals between the times of firing of the pool of elements will be randomly distributed.

The probability distribution for the number of firings or "counts" that occur during an interval T is described by the Poisson distribution (Feller, 1957). The mean number of counts and the variance of the distribution of counts produced will be λT, where λ reflects the probability that a given element in the pulse source will be active at a given time, and T is the duration of the stimulus. For a large λT, the Poisson distribution of counts will be closely approximated by the normal distribution.

In applying Creelman's model to "empty" duations of time we must assume that

(i) the activity of the hypothetical pulse source does not depend on continuous sensory stimulation

(ii) the pulse counter begins counting precisely at the offset of the first pulse or marker and stops counting precisely at the onset of the second stimulus pulse.

For any pair of interpulse intervals equal to the values of ipi and (ipi + 5) msec. the model states the following:

(i) The underlying distributions of counts will be approximately Gaussian.

(ii) The mean (and the variance) of these underlying distributions will be equal to (λ ipi) and λ(ipi + 5) msec. respectively. Thus, the absolute difference between the mean numbers of counts will be independent of ipi and always equal to 5λ. The distribution of counts produced by the longer of the two "empty" intervals will have the greater variance.

(iii) The distance between the means of the theoretical distributions measured in the standard deviation units of the distribution for the shorter interval will be expressed by the equation

$$d' = \frac{\lambda(\text{ipi} + 5) - \lambda(\text{ipi})}{\sqrt{\lambda \text{ipi}}}$$

$$= \frac{\sqrt{\lambda}\ 5}{\sqrt{\text{ipi}}} \qquad (42)$$

Thus, sensitivity to the difference between the two intervals ipi and (ipi + 5) msec. as measured by d', should decrease as a negative power function of ipi.

(iv) When the criterion for report is fixed relative to the means of the distributions on the dimension of theoretical counts, as the value of ipi increases linearly, the probability of reporting "long" given the relatively longer pattern, $\hat{Pr}(R_1/S_1)$, will decrease and the probability of reporting "long" given the short pattern, $\hat{Pr}(R_1/S_s)$, will increase.

The data for Experiment 2, problem 1 indicate that the assumption of underlying Gaussian distributions for the patterns presented in each condition is acceptable for two of the four Ss (A.D. and B.B.). The slope of the ROC functions presented in Table 6 for each condition are equal to the ratio of the standard deviation of the noise distribution (i.e. the theoretical distribution produced by the longer interval) to the standard deviation of the signal distribution (for the short interval). The Ogilvie and Creelman analysis of the ROC functions for each condition showed that for subjects A.D. and B.B. the ROC points were fit satisfactorily by lines of slope 1.00. However, the data presented in Table 6 shows that the obtained slopes of the lines are consistently greater than 1.00. That is, the standard deviation of the noise distribution is consistently greater than the standard deviation of the signal distribution, although not significantly so for single cases. Further, as shown in Figure 19, this ratio decreases as the short interval increases. Since the Ogilvie and Creelman analysis provides poor estimates of sensitivity and slope of ROC functions for high levels of performance, the ratios obtained for small values of ipi are in error. We have found that the error in estimating d'_e decreases as d'_e decreases and is negligible when d'_e is approximately 2.50. The measure d'_e is greater than 2.50 when ipi is less than 15 msec. for subject A.D. and when ipi is less than 25 msec. for subject B.B.

The measure $d'_{\Delta s}$ of the distance between the means of the theoretical distributions expressed in the standard deviation units of the distribution for the short interval is shown for subjects A.D. and B.B. for each condition of Experiment 2, problem 1 in Figure 20. This measure was calculated by substituting the values of d'_e and the slope obtained for the rating data in each condition in equation 38. The error in estimating d'_e and slope and thus $d'_{\Delta s}$ makes it difficult to specify the exact form of the function relating $d'_{\Delta s}$ and ipi. It appears that for the range of values of ipi studied, for subject A.D. the measure $d'_{\Delta s}$ has reached its lowest value on the function when ipi is equal to approximately 50 msec. For subject B.B. it is not clear whether $d'_{\Delta s}$ has reached its minimum value when ipi is equal

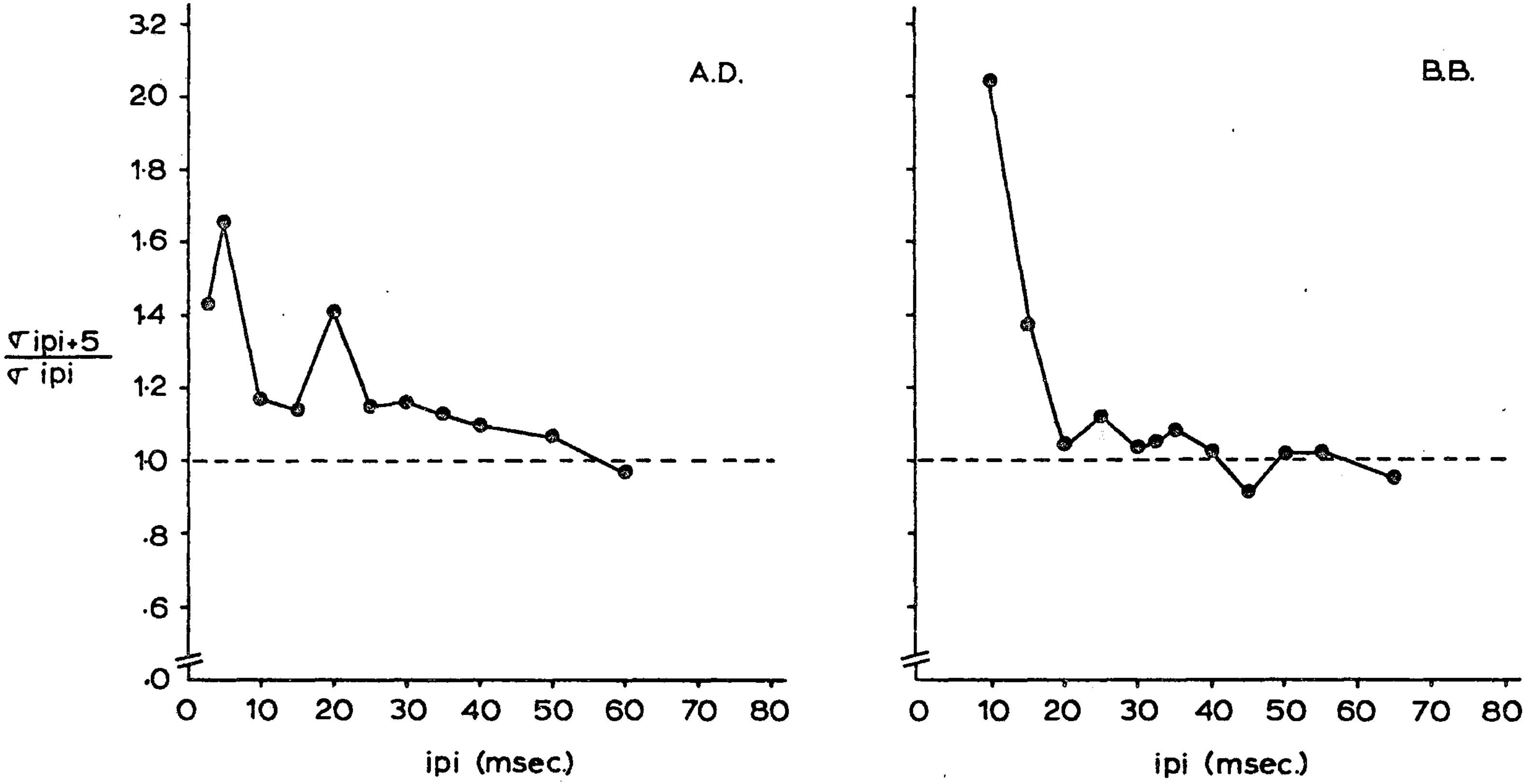


Figure 19. Ratio of standard deviations of distributions for short and long intervals as a function of ipi.

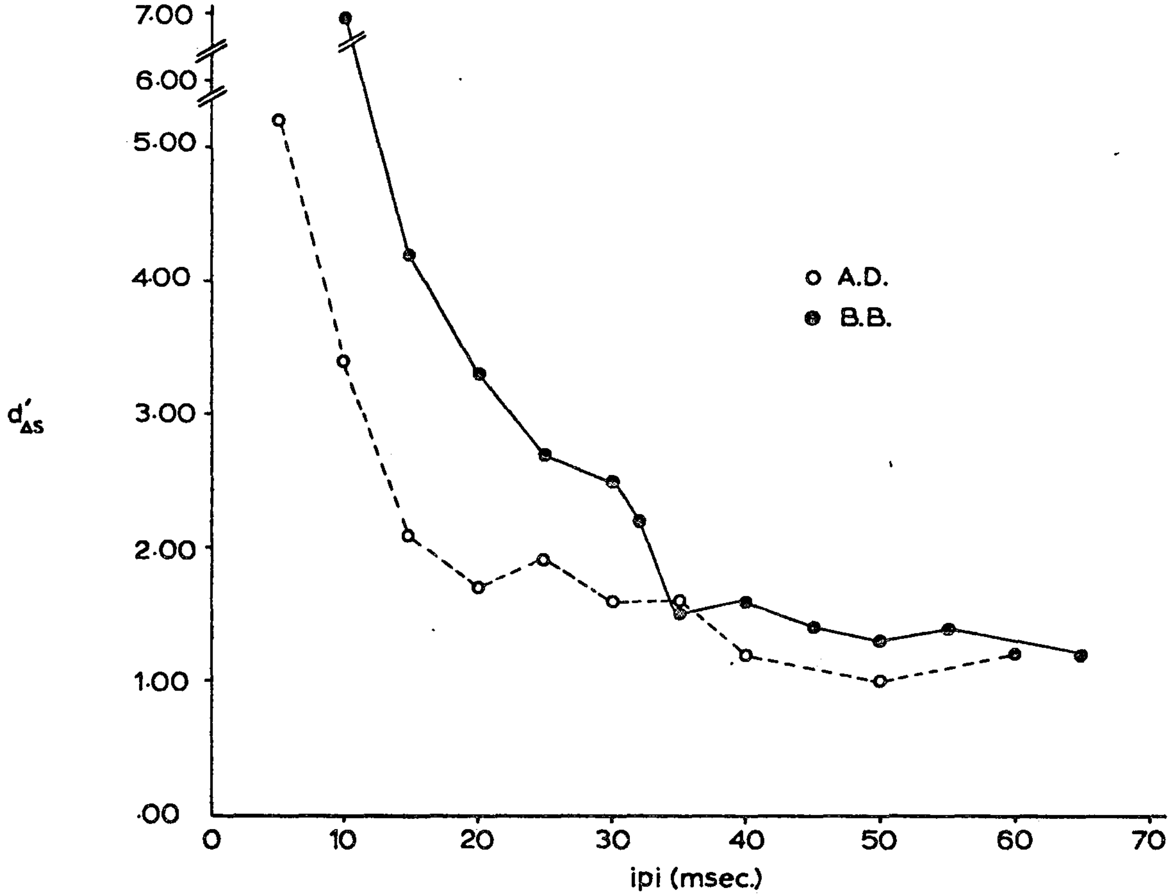


Figure 20. The distance between the means of theoretical distributions for the short and long intervals in the standard deviation units of the distribution for the short interval.

to 65 msec.

The probabilities $\hat{Pr}(R_1/S_1)$ and $\hat{Pr}(R_1/S_s)$ are presented as a function of ipi in Figure 21. Each data point is based on 400 trials. It is apparent from the figure that $\hat{Pr}(R_1/S_1)$ decreases and $\hat{Pr}(R_1/S_s)$ increases as ipi increases. For both subjects A.D. and B.B. both probabilities appear to be stable for values of ipi equal to and greater than approximately 50 msec.

The values of the theoretical parameter λ in the Creelman model are plotted in Figure 22. These values were obtained by substituting the values of $d'_{\Delta s}$ and ipi for each condition in Equation 42. According to the theory the parameter λ should be constant for any subject throughout the range of values of ipi. The graph indicates that when ipi ranges from 15 to 60 msec. the value of λ for subject A.D. fluctuates about a mean value of 2.95. For subject B.B. as ipi increases from 15 to 32 msec. λ decreases from 10.2 to 6.2. When ipi is greater than 32 msec. for this subject λ fluctuates about a mean value of 3.79. The decrease in λ for values of ipi between 15 and 25 msec. may reflect poor estimates of $d'_{\Delta s}$ in this range.

In summary, within the limits of the observed fluctuation in λ, the data of two Ss in Experiment 2 provide some support for the predictions of Creelman's counting theory when ipi is less than 50 msec. There are some indications in the data contrary to Creelman's predictions

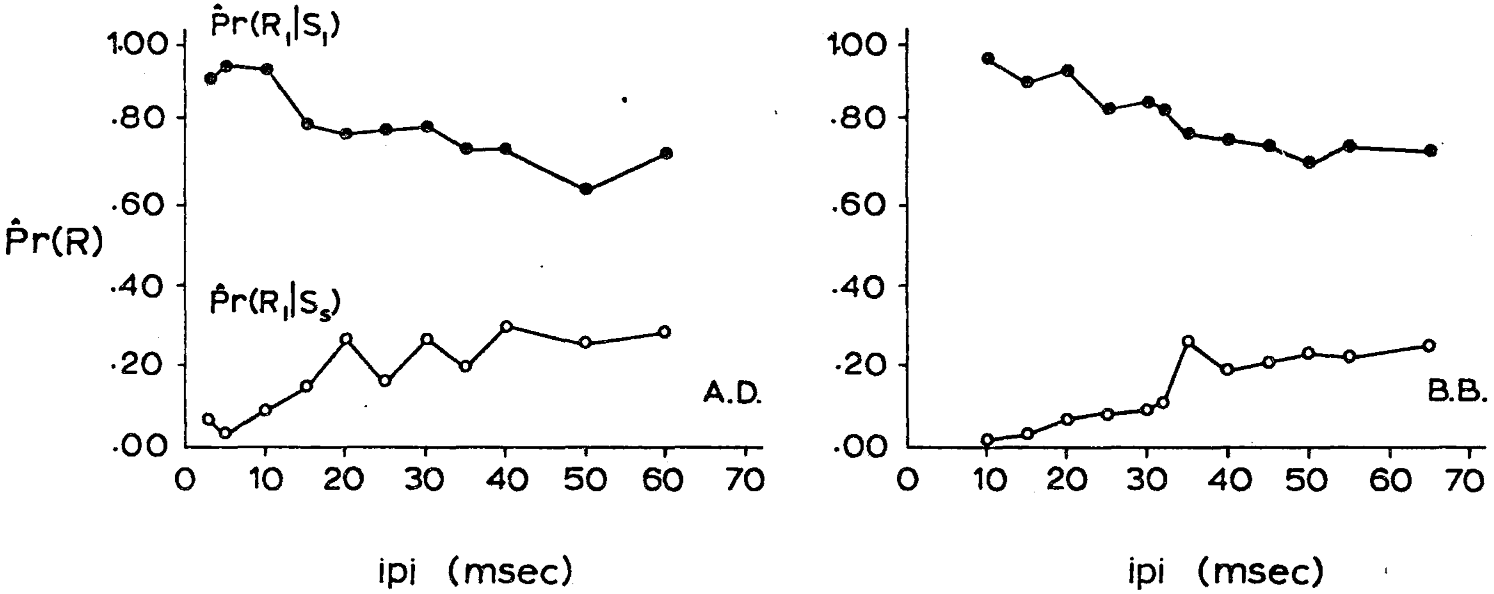


Figure 21. Probability of reporting "long" for short and long intervals as a function of ipi.

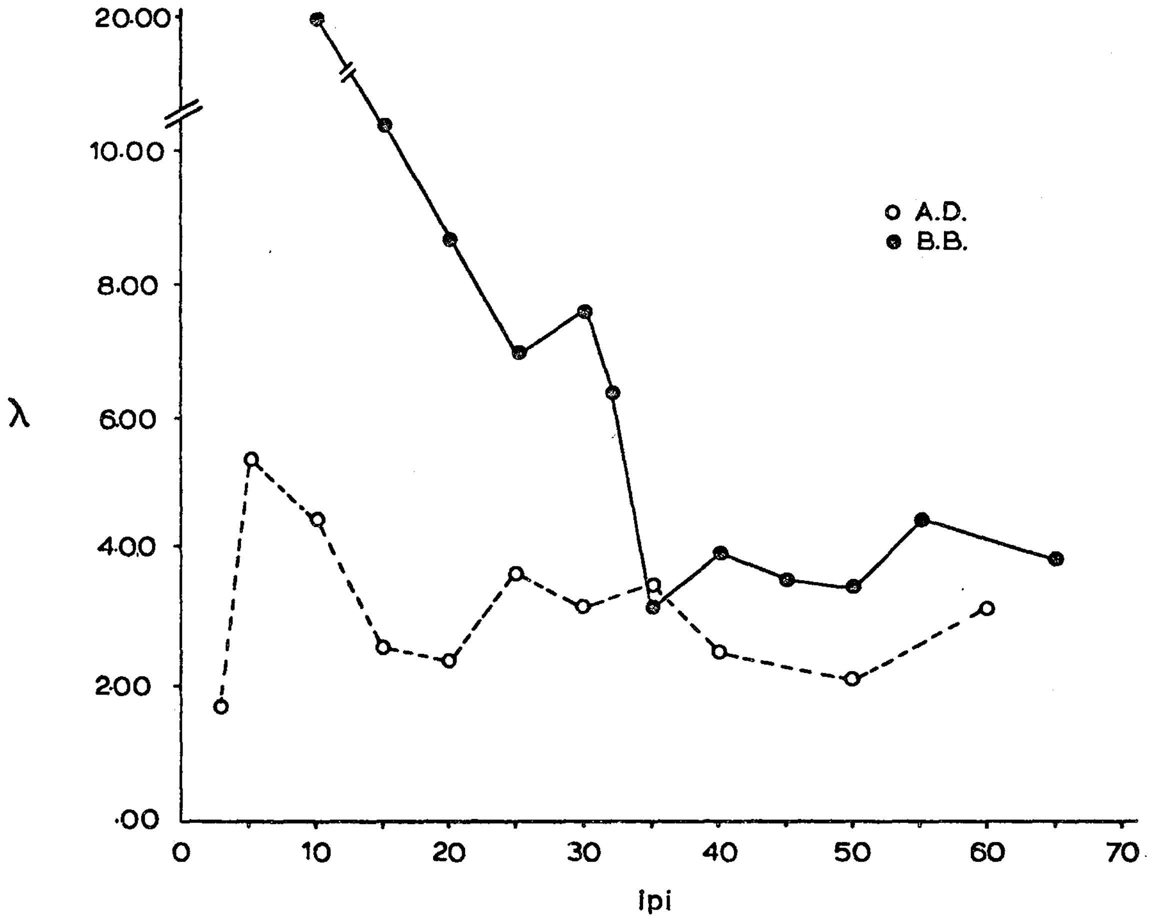


Figure 22

Estimates of the parameter λ of Creelman's counting model for each experimental condition.

that for values of ipi greater than approximately 50 msec. the variances of the theoretical distributions of counts are equal. More specifically, for values of ipi in the neighbourhood of 50 msec.

(i) the ratio of standard deviations of the theoretical distributions of counts for a pair of intervals fluctuates about 1.00

(ii) the rate of change in $d'_{\Delta s}$ appears to be close to zero

(iii) the probabilities $\hat{Pr}(R_1/S_1)$ and $\hat{Pr}(R_1/S_s)$ are fairly stable.

The implication of the data for Experiment 2 that sensitivity to an increment of 5 msec. remains constant for values of ipi greater than 50 msec. has been corroborated by Kristofferson (unpubl.) for empty intervals ranging from 50 to 100 msec. Also, Allan, Kristofferson and Wiens (1970) have shown that theoretical Gaussian distributions generated by continuous visual stimuli ranging in duration from 50 to 150 msec. are equal in variance. Suppose we assume on the basis of these results that performance does remain constant in Experiment 2, problem 1 of the present study when ipi is greater than 50 msec. If we fit the change in the observed $\hat{Pr}(C)$ for the range of values of ipi used with two straight line segments:

(i) the best fitting line for values of

$\hat{Pr}(C)$ obtained when ipi is equal to or less than 50 msec.

and (ii) the best line of zero slope for values of $\hat{Pr}(C)$ obtained when ipi is greater than 50 msec.

we can account for 87, 96, 82 and 94% of the variance in the data of subjects A.D., V.K., C.C. and B.B. respectively. This index was calculated from the formula

$$\frac{x-y}{x} \times 100$$

where x is the sum of the squared deviations of $\hat{Pr}(C)$ obtained for the whole range of value of ipi about the average of these values of $\hat{Pr}(C)$ and y is the sum of

(i) the squared deviations of the obtained values of $\hat{Pr}(C)$ for ipi less than and equal to 50 msec., about the best fitting straight line through these points

and (ii) the squared deviations of $\hat{Pr}(C)$ for ipi greater than 50 msec. about the mean of these values of $\hat{Pr}(C)$.

The values of ipi at which the two line segments intersect are approximately 45, 59, 43 and 48 msec. for subjects A.D., V.K., C.C. and B.B. respectively.

These findings imply that Creelman's theory is limited to "empty" durations that are less than or equal to 50 msec. In terms of the theory the mean and variance of the counts for an interval have an upper limit of λ 50. The duration of approximately 50 msec. at which the upper limit in the mean and the variance may be reached for Creelman's model has been reported as the duration of the "moment" or psychological unit of duration for successiveness discrimination (Kristofferson, 1967). If we assume

(i) that "the onset of stimulation triggers some central process...which interacts with afferent neural activity..." (White and Eason, 1966, p.8)

(ii) that this central process functions as a scanning mechanism in the brain (for a review of theories of excitability cycles and cortical scanning see Harter, 1967)

(iii) that the duration of a scan is 50 msec.

then the number of pulses accumulated during a stimulus duration could be limited by the duration of this scan. Thus, when the scan terminates, the hypothetical pulse counter would be reset to zero. It has been suggested by several researchers (e.g. Pitts and McCulloch, 1947; Stroud, 1949) that neural events that occur during such a scan may be averaged although they are not distinguishable from one another.

To apply Creelman's model to the more general problem of temporal numerosity, one might assume a summation of theoretical counts for the interpulse intervals in the train. Thus, for a constant ipi, as n, the number of stimulus pulses in the train, increases the mean and variance of the number of counts should increase. For a constant number of pulses the mean and variance of the theoretical count should increase as ipi increases.

For Tasks 2-1 and 2-3 of Experiment 1 we have shown that the Gaussian model provides a good fit to the ROC curves. Examination of the slopes of the best fitting lines to the ROC points for all values of ipi presented in Table 3 indicates that in Tasks 2-3 the standard deviation of the theoretical distribution for a train of 3 pulses is greater than the standard deviation of the distribution for 2 pulses in 12 out of 15 instances for four observers. For Task 2-1 the estimates of d'_e may be poor and the ratios not valid. Unfortunately, the range of values of ipi for Task 2-3 is not great enough to allow an examination of the change in variance as a function of ipi.

The total duration of stimulus trains presented in Experiment 1 ranged from 20 to 40 msec. Total durations of trains of 2 pulses in Experiment 2 exceeded 65 msec. for only one subject. Data presented in Experiment 2 suggest that the theoretical counting mechanism which appears to describe the data for two Ss for durations less than 50 msec. would provide a poor explanation for longer trains of events. It may be that for these longer durations the observer would turn to a count of moments or brain scans spanned by the train or to a count of the number of components in the evoked cortical response, as suggested by White and Eason (1966).

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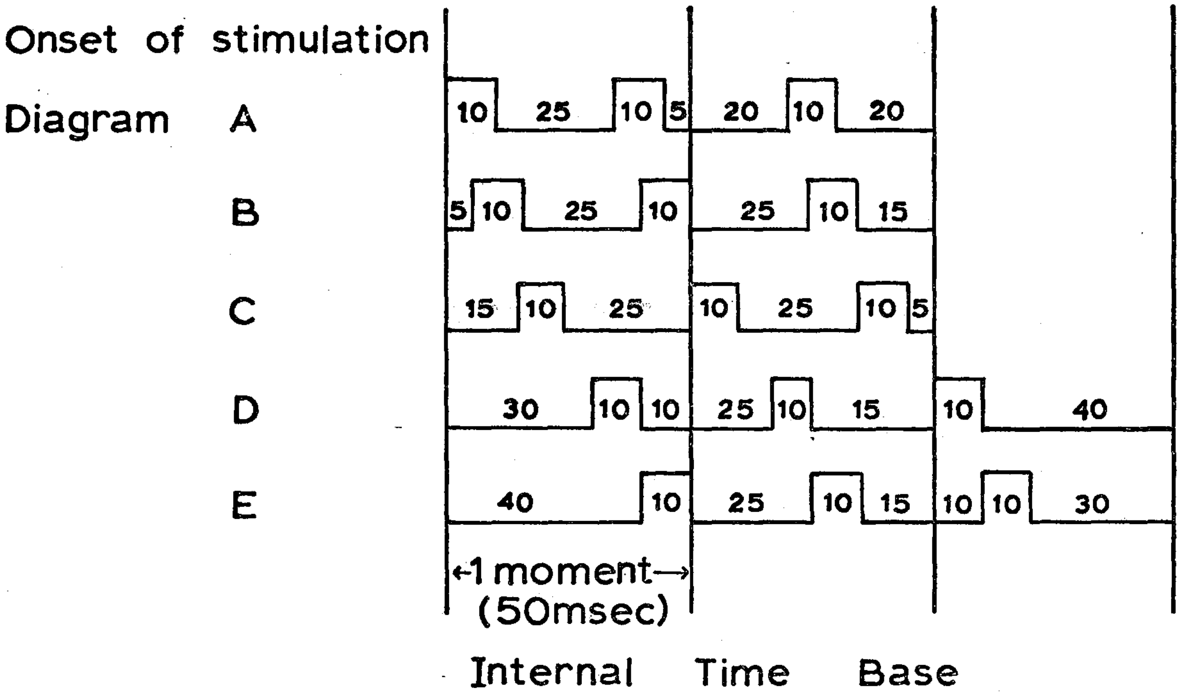
NOTES:

1. Italicized words added

APPENDIX A

Sample Calculation of $\alpha_{n,c}$



Suppose that

$q = 50$ msec.

$n = 3$

$p = 10$ msec.

$ipi = 25$ msec.

We assume that the onset of the first event in a train is equally likely to occur at any point during a current or ongoing quantum. As shown in Diagram A if the onset of stimulation of the train of three pulses is synchronous with a time point, then the second interpulse interval in the stimulus sequence will cover the next internal time point or "tick of the clock". This interval will continue to cover a time point as the lag between the first time point and the onset of the first pulse in the train increases from 5 to 14 msec. in Diagram B, no interval will cover a time point and a count of "1" will be generated. In Diagram C as the lag increases from 15 to 29 msec. the first interpulse interval will cover a time point and again a count of "1" will be generated. In Diagram D we note that for lags of 30 to 39 msec. both interpulse intervals will cover time points and a count of "3" will be generated. Finally in Diagram E if the onset of stimulation occurs during the last 10 msec. of an ongoing quantum the second interpulse interval will contain a time point, and the sensory count will be "2".

In summary, no intervals will cover a time point (and thus a count of "1" will be generated) if the onset of stimulation occurs synchronously with 10 of the possible 50 msec. of the ongoing quantum. Thus the probability of a count of "1", $\alpha_{3,1}$, will be 10/50 or .20. The probability that either the first or the second interpulse interval (but not both) will cover a time point, $\alpha_{3,2}$ will be 30/50 or .60. The probability that both intervals will cover time points, is 10/50 or .20.

TABLE 1 Conditional Probabilities of a Response for Trains of One and Two Pulse in Task 2-1

		2 Pulses Response Category						1 Pulse Response Category					
Subject	ipi (msec.)	A	B	C	D	E	Σ	A	B	C	D	E	Σ
J.P.	2	.880	.094	.014	.000	.012	1.000	.004	.000	.000	.016	.980	1.000
	3	.932	.058	.008	.002	.000	1.000	.000	.000	.000	.024	.976	1.000
	4	.938	.054	.006	.000	.002	1.000	.002	.000	.004	.062	.932	1.000
	5	.720	.238	.020	.016	.004	.998	.002	.058	.086	.524	.329	.999
J.M.	2	.787	.158	.034	.010	.010	.999	.027	.040	.040	.054	.839	1.000
	3	.784	.122	.044	.008	.042	1.000	.004	.008	.016	.014	.958	1.000
	4	.942	.040	.010	.002	.006	1.000	.008	.004	.010	.022	.956	1.000
	5	.938	.042	.012	.000	.008	1.000	.056	.062	.032	.154	.695	.999
T.M.	2	.982	.000	.000	.000	.018	1.000	.084	.000	.002	.006	.908	1.000
	3	.978	.006	.000	.004	.012	1.000	.060	.004	.000	.024	.912	1.000
	4	.968	.028	.000	.000	.004	1.000	.008	.066	.006	.136	.784	1.000
	5	.770	.189	.014	.008	.019	1.000	.006	.107	.084	.165	.637	.999
J.C.	2	.942	.000	.034	.000	.024	1.000	.010	.000	.014	.000	.976	1.000
	3	.993	.002	.000	.000	.004	.999	.010	.000	.020	.000	.970	1.000
	4	1.000	.000	.000	.000	.000	1.000	.000	.000	.002	.000	.998	1.000
	5	.842	.128	.016	.006	.008	1.000	.002	.006	.014	.074	.904	1.000

TABLE 2 Conditional Probabilities of a Response for Trains of Two and Three Pulses in Task 2-3

		2 Pulses Response Categories						3 Pulses Response Categories					
Subject	ipi (msec)	A	B	C	D	E	Σ	A	B	C	D	E	Σ
J.P.	2	.002	.804	.070	.124	.000	1.000	.000	.348	.078	.564	.010	1.000
	3	.032	.722	.026	.208	.012	1.000	.006	.342	.018	.482	.152	1.000
	4	.414	.486	.092	.008	.000	1.000	.010	.018	.062	.256	.654	1.000
	5	.321	.593	.080	.006	.000	1.000	.000	.008	.040	.261	.691	1.000
J.M.	2	.203	.639	.123	.032	.000	.997	.034	.532	.213	.216	.004	.999
	3	.368	.455	.080	.078	.018	.999	.128	.379	.148	.286	.057	.998
	4	.403	.447	.100	.042	.008	1.000	.032	.146	.207	.362	.254	1.001
	5	.601	.243	.076	.066	.014	1.000	.082	.104	.104	.257	.454	1.001
T.M.	2	.592	.010	.000	.032	.366	1.000	.329	.002	.000	.016	.653	1.000
	3	.495	.012	.000	.032	.461	1.000	.246	.012	.002	.008	.732	1.000
	4	.344	.382	.020	.180	.074	1.000	.048	.066	.002	.268	.616	1.000
	5	.528	.208	.008	.124	.132	1.000	.022	.052	.008	.148	.770	1.000
J.C.	2	.056	.292	.482	.170	.000	1.000	.006	.102	.534	.358	.000	1.000
	3	.220	.280	.401	.094	.004	.999	.024	.090	.570	.304	.010	.998
	4	.938	.008	.048	.004	.002	1.000	.042	.008	.180	.034	.736	1.000
	5	.946	.020	.030	.002	.002	1.000	.010	.000	.026	.016	.948	1.000

TABLE 3 Conditional Probabilities of a Response for Trains of 1, 2 and 3 Pulses in Task 2-13

		2 Pulses						1 Pulse					
		Response Categories						Response Categories					
Subject	ipi (msec.)	A	B	C	D	E	Σ	A	B	C	D	E	Σ
J.P.	2	.126	.714	.070	.074	.016	1.000	.000	.004	.004	.016	.976	1.000
	3	.150	.656	.036	.094	.064	1.000	.004	.004	.000	.006	.986	1.000
	4	.773	.144	.044	.026	.012	.999	.002	.000	.002	.004	.992	1.000
	5	.658	.262	.058	.010	.012	1.000	.002	.002	.000	.006	.990	1.000
J.M.	2	.064	.320	.144	.327	.145	1.000	.016	.120	.086	.064	.712	.998
	3	.189	.336	.108	.174	.187	.994	.026	.044	.020	.022	.888	1.000
	4	.416	.403	.100	.040	.040	.999	.006	.010	.008	.004	.972	1.000
	5	.445	.200	.108	.098	.148	.999	.000	.008	.010	.004	.978	1.000
T.M.	2	.304	.008	.002	.014	.672	1.000	.090	.036	.002	.010	.862	1.000
	3	.508	.002	.000	.004	.486	1.000	.034	.004	.000	.004	.958	1.000
	4	.563	.070	.000	.058	.308	.999	.008	.002	.000	.008	.982	1.000
	5	.568	.136	.000	.096	.200	1.000	.024	.058	.004	.144	.770	1.000
J.C.	2	.002	.172	.598	.186	.042	1.000	.000	.008	.060	.024	.908	1.000
	3	.042	.142	.472	.286	.058	1.000	.000	.006	.028	.010	.955	.999
	4	.615	.060	.262	.032	.030	.999	.004	.000	.004	.000	.992	1.000
	5	.588	.036	.254	.038	.083	.999	.008	.000	.016	.004	.972	1.000

TABLE 3 CONT'D

3 Pulses

Subject	ipi (msec.)	A	B	C	D	E	Σ
		Response Categories					
J.P.	2	.048	.344	.104	.470	.034	1.000
	3	.062	.414	.046	.336	.142	1.000
	4	.026	.026	.044	.190	.714	1.000
	5	.002	.006	.038	.188	.766	1.000
J.M.	2	.014	.170	.123	.494	.198	.999
	3	.080	.175	.096	.265	.381	.997
	4	.036	.182	.157	.363	.262	1.000
	5	.048	.062	.092	.227	.570	.999
T.M.	2	.166	.008	.000	.006	.820	1.000
	3	.250	.000	.000	.008	.742	1.000
	4	.106	.018	.000	.038	.838	1.000
	5	.082	.026	.000	.088	.804	1.000
J.C.	2	.002	.124	.562	.304	.008	1.000
	3	.028	.107	.426	.421	.018	1.000
	4	.034	.012	.185	.142	.626	.999
	5	.010	.002	.082	.042	.863	.999

TABLE 4 Estimated Conditional Probabilities of Responding 2 and $\bar{2}$ for Combinations of Subject, Task, and ipi in Experiment 1

Subject	Task	ipi (msec.)		$\hat{Pr}(R_A+R_B/S)$	$\hat{Pr}(R_C+R_D+R_E/S)$	Σ
JP	2-1	2	S_1	.004	.996	1.000
			S_2	.974	.026	1.000
	2-3		S_2	.806	.194	1.000
			S_3	.348	.652	1.000
	2-13		S_1	.004	.996	1.000
			S_2	.840	.160	1.000
			S_3	.392	.608	1.000
	2-1	3	S_1	.000	1.000	1.000
			S_2	.990	.010	1.000
	2-3		S_2	.754	.246	1.000
			S_3	.348	.652	1.000
	2-13		S_1	.008	.992	1.000
			S_2	.806	.194	1.000
			S_3	.476	.524	1.000
	2-1	4	S_1	.002	.998	1.000
			S_2	.992	.008	1.000
	2-3		S_2	.900	.100	1.000
			S_3	.028	.972	1.000
	2-13		S_1	.002	.998	1.000
			S_2	.917	.082	1.000
			S_3	.052	.948	1.000

TABLE 4 CONT'D

JP	2-1	5	S_1	.060	.939	.999
			S_2	.958	.040	.998
	2-3		S_2	.914	.086	1.000
			S_3	.008	.992	1.000
	2-13		S_1	.004	.996	1.000
			S_2	.920	.080	1.000
			S_3	.008	.992	1.000

TABLE 4 CONT'D

Subject	Task	ipi (msec.)		$\hat{Pr}(R_A+R_B/S)$	$\hat{Pr}(R_C+R_D+R_E/S)$	Σ
JM	2-1	2	S_1	.067	.933	1.000
			S_2	.945	.054	.999
	2-3		S_2	.842	.155	.997
			S_3	.566	.433	.999
	2-13		S_1	.136	.862	.998
			S_2	.384	.616	1.000
			S_3	.184	.815	.999
	2-1	3	S_1	.012	.988	1.000
			S_2	.906	.094	1.000
	2-3		S_2	.823	.176	.999
			S_3	.507	.491	.998
	2-13		S_1	.070	.930	1.000
			S_2	.525	.469	.994
			S_3	.255	.742	.997
	2-1	4	S_1	.012	.988	1.000
			S_2	.982	.018	1.000
	2-3		S_2	.850	.150	1.000
			S_3	.178	.823	1.001
	2-13		S_1	.016	.984	1.000
			S_2	.819	.180	.999
			S_3	.218	.782	1.000

TABLE 4 CONT'D

JM	2-1	5	S_1	.118	.881	.999
			S_2	.980	.020	1.000
	2-3		S_2	.844	.156	1.000
			S_3	.186	.815	1.001
	2-13		S_1	.008	.992	1.000
			S_2	.645	.354	.999
			S_3	.110	.889	.999

TABLE 4 CONT'D

Subject	Task	ipi(msec.)		$\hat{P}r(R_A+R_B/S)$	$\hat{P}r(R_C+R_D+R_E/S)$	Σ
TM	2-1	2	S_1	.084	.916	1.000
			S_2	.982	.018	1.000
	2-3		S_2	.602	.398	1.000
			S_3	.331	.669	1.000
	2-13		S_1	.126	.874	1.000
			S_2	.312	.688	1.000
			S_3	.174	.826	1.000
	2-1	3	S_1	.064	.936	1.000
			S_2	.984	.016	1.000
	2-3		S_2	.507	.493	1.000
			S_3	.258	.742	1.000
	2-13		S_1	.038	.962	1.000
			S_2	.510	.490	1.000
			S_3	.250	.750	1.000
	2-1	4	S_1	.074	.926	1.000
			S_2	.996	.004	1.000
	2-3		S_2	.726	.274	1.000
			S_3	.114	.886	1.000
	2-13		S_1	.010	.990	1.000
			S_2	.633	.366	.999
			S_3	.124	.876	1.000

TABLE 4 CONT'D

TM	2-1	5	S_1	.113	.886	.999
			S_2	.959	.041	1.000
	2-3		S_2	.736	.264	1.000
			S_3	.074	.926	1.000
	2-13		S_1	.082	.918	1.000
			S_2	.704	.296	1.000
			S_3	.108	.892	1.000

TABLE 4 CONT'D

Subject	Task	ipi(msec.)		$\hat{P}r(R_A+R_B/S)$	$\hat{P}r(R_C+R_D+R_E/S)$	Σ
JC	2-1	2	S_1	.010	.990	1.000
			S_2	.942	.058	1.000
	2-3		S_2	.348	.652	1.000
			S_3	.108	.892	1.000
	2-13		S_1	.008	.992	1.000
			S_2	.174	.826	1.000
			S_3	.126	.874	1.000
	2-1	3	S_1	.010	.990	1.000
			S_2	.995	.004	.999
	2-3		S_2	.500	.499	.999
			S_3	.114	.884	.998
	2-13		S_1	.006	.993	.999
			S_2	.184	.816	1.000
			S_3	.135	.865	1.000
	2-1	4	S_1	.000	.998	.998
			S_2	1.000	.000	1.000
	2-3		S_2	.946	.054	1.000
			S_3	.050	.950	1.000
	2-13		S_1	.004	.996	1.000
			S_2	.675	.324	.999
			S_3	.046	.953	.999

TABLE 4 CONT'D

JC	2-1	5	S_1	.008	.992	1.000
			S_2	.970	.030	1.000
	2-3		S_2	.966	.034	1.000
			S_3	.010	.990	1.000
	2-13		S_1	.008	.992	1.000
			S_2	.624	.375	.999
			S_3	.012	.987	.999

TABLE 5 Standard Error of the Maximum Likelihood Estimate of d'_e (Ogilvie and Creelman, 1968) for each Condition in Experiment 1

Task	Pair	ipi (msec)	Subject			
			J.P.	J.M.	T.M.	J.C.
2-1	1 vs 2	2	-	.248	.372	.635
		3	-	.318	.455	-
		4	-	.460	.482	-
		5	.298	.268	.267	.435
2-3	2 vs 3	2	.181	.142	.144	-
		3	.138	.125	.217	.136
		4	.318	.157	.150	.281
		5	.446	.153	.162	.495
2-13	1 vs 2	2	.433	.143	.333	.242
		3	.642	-	.460	-
		4	.738	.414	.753	.885
		5	.664	-	.163	.416
2-13	2 vs 3	2	.140	.118	.557	-
		3	.131	.117	.136	.119
		4	.230	.146	.196	.179
		5	.342	.137	.159	.197
2-13	1 vs 3	2	.436	.151	.458	-
		3	.712	.241	.846	-
		4	1.668	.512	1.305	1.443
		5	1.374	-	.261	.916

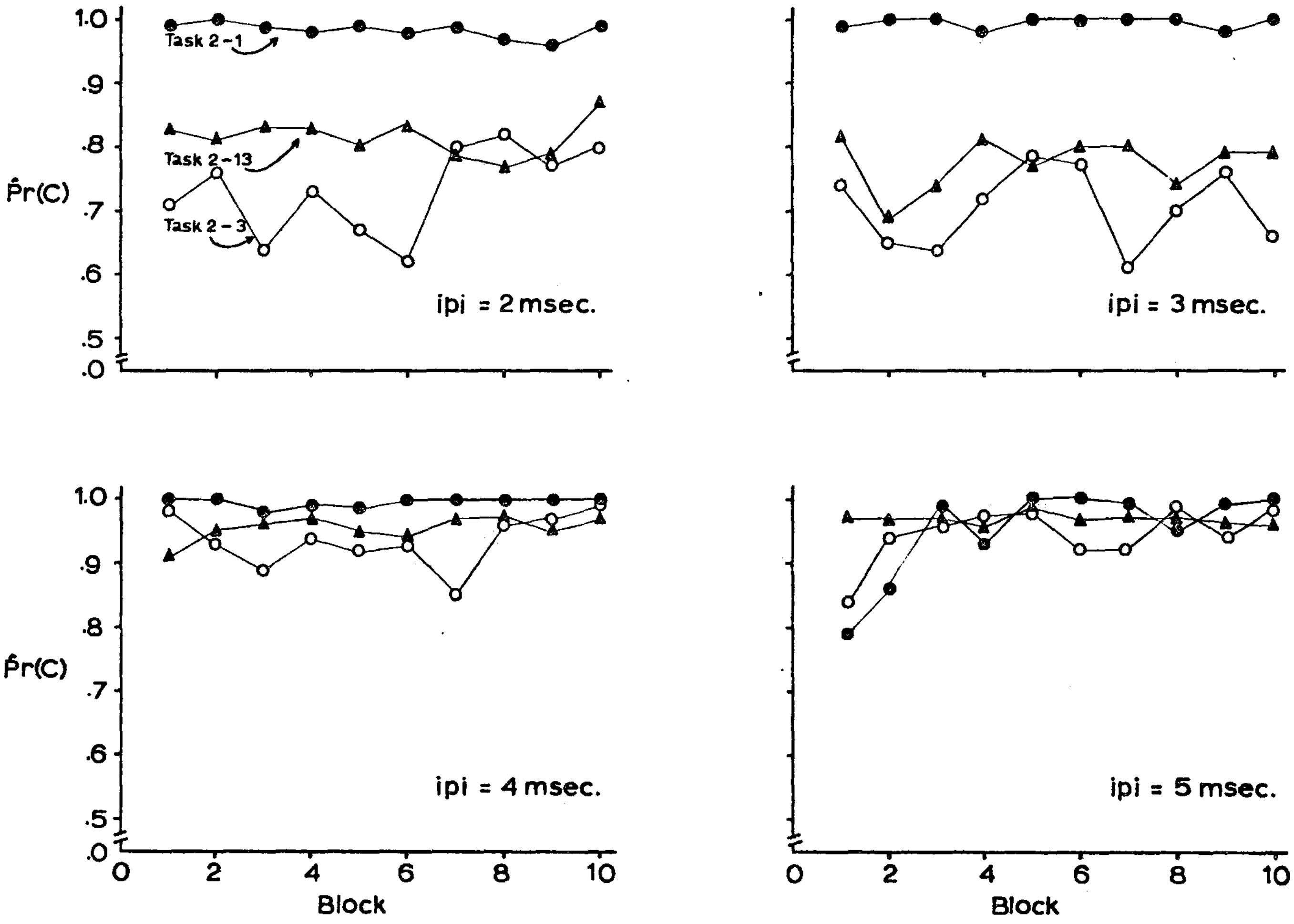


Figure 1. Probability of a correct response for 10 blocks of trials in Experiment 1. (J.P.)

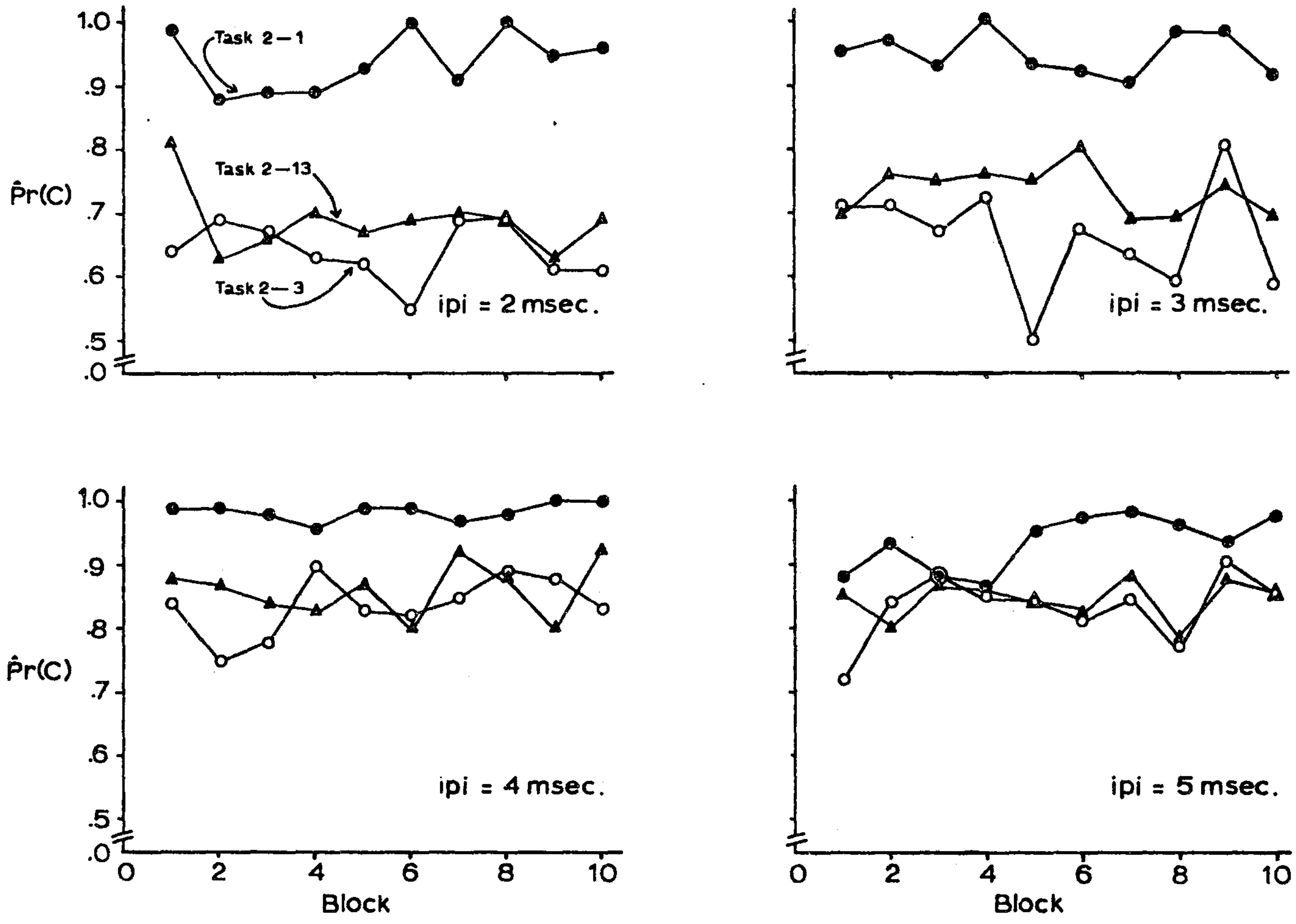


Figure 2. Probability of a correct response for 10 blocks of trials in Experiment 1. (J.M.)

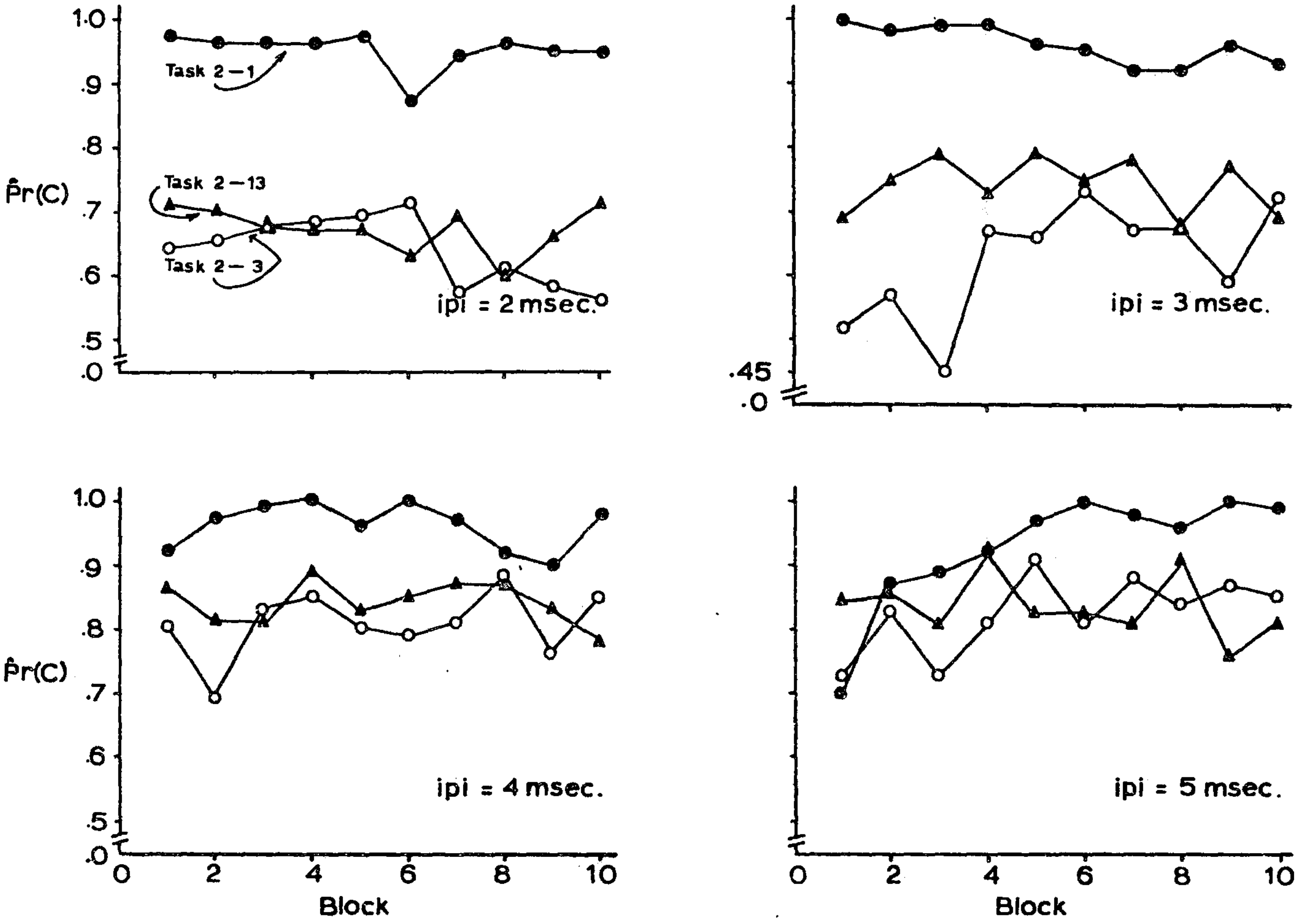


Figure 3. Probability of a correct response for 10 blocks of trials in Experiment 1. (T.M.)

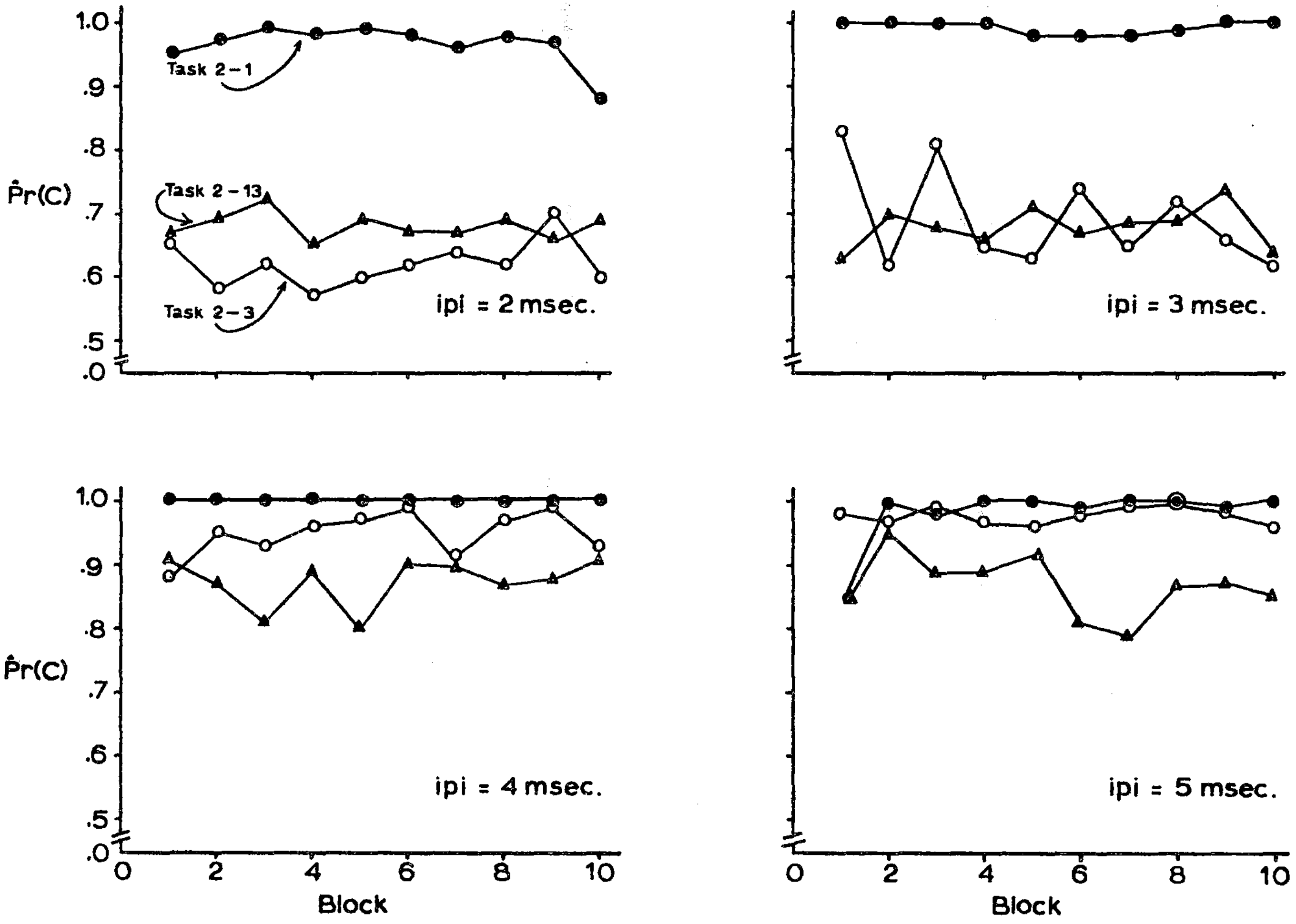


Figure 4. Probability of a correct response for 10 blocks of trials in Experiment 1.(J.C.)

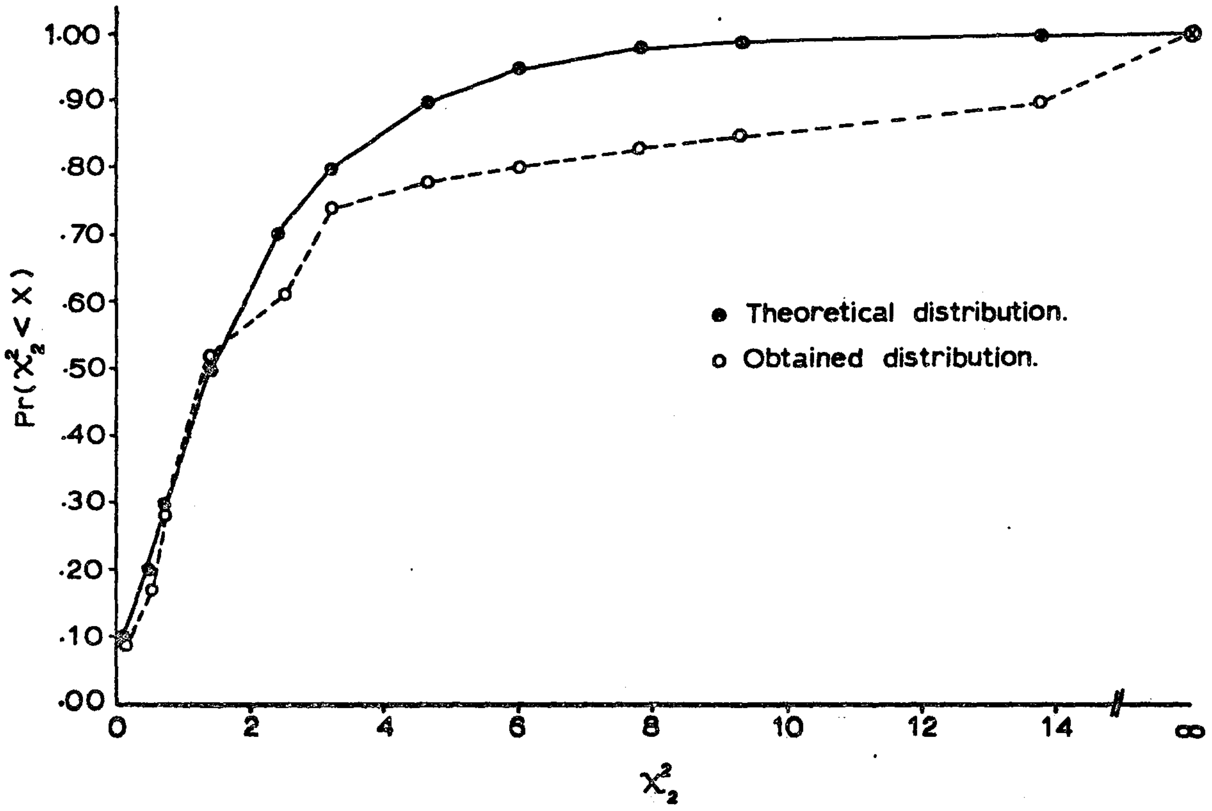


Figure 5. A comparison of the theoretical cumulative chi-square distribution for 2 df with the cumulative distribution for chi-square values obtained in Experiment 1.

APPENDIX B

TABLE 1 Estimated Probability of a Repsonse for Short and Long Interval Trials in Experiment 2, Problem 1

		Interval ipi					Interval ipi + 5				
		Response Category					Response Category				
Subject	ipi (msec)	A	B	C	D	Σ	A	B	C	D	Σ
A.D.	3	.413	.517	.070	.000	1.000	.008	.073	.427	.490	.998
	5	.507	.463	.030	.000	1.000	.005	.043	.283	.669	1.000
	10	.352	.561	.084	.003	1.000	.005	.058	.530	.470	1.000
	15	.365	.484	.144	.007	1.000	.025	.173	.521	.281	1.000
	20	.049	.677	.273	.000	.999	.013	.203	.708	.076	1.000
	25	.057	.778	.161	.003	.999	.005	.208	.699	.089	1.001
	30	.030	.706	.262	.003	1.001	.003	.199	.705	.093	1.000
	35	.294	.507	.167	.032	1.000	.043	.221	.397	.340	1.001
	40	.030	.670	.295	.005	1.000	.000	.264	.634	.102	1.000
	45										
	50	.110	.626	.244	.020	1.000	.023	.339	.500	.139	1.001
	55										
	60	.221	.494	.263	.022	1.000	.020	.247	.524	.209	1.000
	65										

Cont'd

TABLE 1 CONT'D

V.K.	5										
	10	.562	.340	.075	.020	.997	.010	.038	.527	.425	1.000
	15	.677	.249	.037	.037	1.000	.091	.068	.219	.622	1.000
	20	.451	.380	.104	.064	.999	.068	.093	.454	.386	1.001
	25	.433	.480	.052	.035	1.000	.069	.114	.343	.475	1.001
	30	.471	.393	.070	.067	1.001	.089	.101	.314	.496	1.000
	35	.220	.555	.166	.059	1.000	.061	.111	.523	.306	1.001
	40	.305	.499	.087	.109	1.000	.134	.204	.201	.461	1.000
	45	.417	.366	.127	.091	1.001	.134	.207	.352	.306	.999
	50	.415	.371	.137	.077	1.000	.211	.233	.266	.289	.999
	55	.332	.449	.111	.107	.999	.141	.316	.267	.276	1.000
	60	.377	.397	.161	.065	1.000	.214	.323	.269	.194	1.000
	65										
	70	.307	.483	.126	.084	1.000	.141	.356	.300	.202	.999
	75										
	80	.310	.477	.107	.107	1.001	.219	.337	.171	.273	1.000

TABLE 1 CONT'D

C.C.	5	.853	.145	.000	.003	1.001	.023	.008	.103	.867	1.001
	10	.792	.119	.040	.050	1.001	.245	.018	.189	.548	1.000
	15	.740	.201	.042	.017	1.000	.344	.063	.359	.235	1.001
	20	.640	.236	.070	.055	1.001	.091	.050	.279	.580	1.000
	25	.470	.277	.163	.089	.999	.151	.154	.356	.338	.999
	30	.620	.198	.132	.050	1.000	.247	.096	.378	.280	1.001
	35	.604	.152	.157	.088	1.001	.434	.068	.297	.201	1.000
	40	.487	.213	.202	.099	1.001	.345	.088	.339	.228	1.000
	45	.525	.250	.154	.072	1.001	.261	.147	.375	.217	1.000
	50	.148	.612	.210	.030	1.000	.089	.475	.351	.086	1.001
	55	.392	.328	.193	.087	1.000	.254	.201	.308	.237	1.000
	60										
	65	.212	.561	.195	.033	1.001	.113	.386	.334	.168	1.001

TABLE 1 CONT'D

B.B.	5										
	10	.675	.310	.015	.000	1.000	.005	.023	.303	.670	1.001
	15	.705	.268	.028	.000	1.001	.010	.083	.255	.652	1.000
	20	.623	.308	.062	.008	1.001	.008	.055	.280	.655	.998
	25	.635	.291	.057	.018	1.001	.033	.129	.270	.569	1.001
	30	.580	.328	.075	.018	1.001	.025	.116	.354	.505	1.000
	32	.520	.369	.079	.032	1.000	.038	.126	.326	.510	1.000
	35	.474	.271	.177	.078	1.000	.083	.134	.275	.509	1.001
	40	.473	.341	.137	.050	1.001	.071	.164	.381	.385	1.001
	45	.519	.270	.155	.057	1.001	.081	.167	.392	.361	1.001
	50	.438	.337	.174	.052	1.001	.086	.204	.380	.330	1.000
	55	.473	.307	.158	.062	1.000	.086	.167	.354	.394	1.001
	60										
	65	.427	.325	.144	.104	1.000	.091	.174	.293	.443	1.001

TABLE 2 Estimated Probability of a Response for Short and Long Intervals as a Function of Duration of Pulse 1 - Experiment 2

Subject	Pulse 1 Duration (msec.)	Interval ipi Response Category					Interval ipi + 5 Response Category				
		A	B	C	D	Σ	A	B	C	D	Σ
A.D.	4	.409	.520	.067	.005	1.001	.008	.152	.384	.457	1.001
	8	.432	.499	.067	.003	1.001	.000	.119	.452	.429	1.000
	12	.492	.456	.047	.005	1.000	.003	.053	.434	.510	1.000
	16	.616	.354	.027	.003	1.000	.005	.038	.359	.598	1.000
V.K.	4	.327	.594	.057	.022	1.000	.061	.283	.366	.291	1.001
	8	.542	.411	.020	.027	1.000	.099	.135	.306	.460	1.000
	12	.384	.546	.052	.017	.999	.099	.215	.413	.274	1.001
	16	.301	.603	.072	.025	1.001	.091	.315	.286	.308	1.000
C.C.	4	.693	.268	.030	.010	1.001	.347	.184	.275	.194	1.000
	8	.693	.270	.023	.015	1.001	.268	.159	.331	.242	1.000
	12	.543	.365	.070	.022	1.000	.247	.189	.423	.141	1.000
	16	.556	.299	.110	.035	1.000	.206	.201	.362	.231	1.000
B.B.	4	.506	.405	.079	.010	1.000	.025	.124	.301	.550	1.000
	8	.535	.425	.037	.003	1.000	.013	.050	.334	.603	1.000
	12	.570	.379	.032	.020	1.001	.010	.051	.371	.568	1.000
	16	.537	.406	.045	.012	1.000	.008	.078	.422	.492	1.000

TABLE 3 Probability of a Response for Short and Long Intervals as a Function of Duration of Pulse 2 - Experiment 2

Subject	Pulse 1 Duration (msec.)	Interval ipi Response Category					Interval (ipi+5) Response Category				
		A	B	C	D	Σ	A	B	C	D	Σ
A.D.	4	.215	.609	.161	.015	1.000	.013	.221	.581	.186	1.001
	8	.274	.571	.145	.010	1.000	.031	.218	.547	.284	1.000
	12	.283	.539	.161	.017	1.000	.023	.197	.509	.272	1.001
	16	.420	.488	.079	.013	1.000	.010	.099	.432	.459	1.000
V.K.	4	.184	.512	.221	.082	.999	.086	.281	.402	.231	1.000
	8	.427	.487	.059	.027	1.000	.131	.221	.401	.248	1.001
	12	.468	.451	.052	.030	1.001	.116	.187	.404	.298	1.000
	16	.278	.640	.059	.022	.999	.086	.360	.391	.164	1.001
C.C.	4	.225	.643	.119	.012	.999	.079	.372	.451	.099	1.001
	8	.469	.482	.045	.005	1.001	.148	.239	.435	.178	1.000
	12	.508	.413	.069	.010	1.000	.257	.240	.409	.093	.999
	16	.168	.691	.128	.013	1.000	.084	.327	.509	.081	1.001
B.B.	4	.617	.316	.050	.018	1.001	.018	.128	.314	.540	1.000
	8	.460	.445	.080	.015	1.000	.015	.133	.379	.473	1.000
	12	.491	.426	.077	.005	.999	.023	.095	.366	.516	1.000
	16	.561	.359	.067	.012	.999	.023	.088	.331	.558	1.000
	24	.516	.398	.082	.005	1.001	.010	.086	.402	.502	1.000

TABLE 4 Number of Practice Blocks Required for Asymptotic Performance in Experiment 2, Problem 1

	Subject							
	A.D.		V.K.		C.C.		B.B.	
ipi (msec.)	Order of Presentation	No. of Prac. Blks.	Order of Presentation	No. of Prac. Blks.	Order of Presentation	No. of Prac. Blks.	Order of Presentation	No. of Prac. Blks.
3	1	2	-	-	-	-	-	-
5	2	4	-	-	8	6	-	-
10	4	9	1	14	3	1	12	16
15	3	5	7	6	9	3	1	8
20	5	2	2	12	2	1	4	6
25	8	9	5	9	1	9	2	1
30	6	1	3	26	6	4	5	5
32	-	-	-	-	-	-	11	6
35	9	2	6	20	4	1	3	1
40	7	4	4	14	5	2	8	2
45	-	-	10	11	7	2	6	2
50	10	5	8	10	11	2	7	2
55	-	-	12	3	10	2	9	2
60	11	8	9	13	-	-	--	--
65	-	-	-	--	12	8	10	1
70			11	8				
75			-	-				
80			13	5				

TABLE 5 Number of Practice Blocks Required for Asymptotic Performance in Experiment 2, Problem 2

		Subject							
		A.D.		V.K.		C.C.		B.B.	
pulse	pulse duration (msec.)	Order of Presentation	No. of Prac. Blks.	Order of Presentation	No. of Prac. Blks.	Order of Presentation	No. of Prac. Blks.	Order of Presentation	No. of Prac. Blks.
1	4	1	8	3	3	4	4	2	7
	8	2	5	1	5	3	7	4	1
	12	3	6	4	5	2	1	1	1
	16	4	4	2	15	1	1	3	12
2	4	3	6	1	3	2	6	4	10
	8	2	3	3	2	4	7	1	6
	12	1	5	4	7	3	6	2	3
	16	4	4	2	6	1	6	3	7
	24	-	-	-	-	-	-	5	6

TABLE 6 Standard Error of the Maximum Likelihood Estimate of d'_e (Ogilvie and Creelman, 1968) For Each Condition in Experiment 2, Problem 1

ipi (msec.)	Subject A.D.	V.K.	C.C.	B.B.
3	.267	–	–	–
5	.363	–	–	–
10	.270	.256	.183	.486
15	.179	.190	.176	.313
20	.171	.164	.176	.269
25	.180	.183	.140	.209
30	.169	.164	.147	.207
32	–	–	–	.187
35	.157	.159	.147	.157
40	.157	.143	.135	.157
45	–	.140	.138	.153
50	.148	.136	.140	.148
55	–	.136	.133	.150
60	.152	.133	–	–
65	–	–	.139	.145
70	–	.136		
75		–		
80		.134		

TABLE 7 Standard Error of the Maximum Likelihood Estimate of d'_e (Ogilvie and Creelman, 1968) for each Condition in Experiment 2, Problem 2

Pulse	Pulse Duration (msec.)	Subject			
		A.D.	V.K.	C.C.	B.B.
1	4	.229	.174	.166	.209
	8	.255	.186	.168	.307
	12	.305	.172	.149	.280
	16	.363	.160	.145	.267
2	4	.171	.137	.153	.226
	8	.170	.159	.170	.209
	12	.169	.164	.149	.225
	16	.225	.163	.154	.224
	24	-	-	-	.235