A game theoretic framework for strategic production planning

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Abstract

A game theoretic framework for strategic refinery production planning is presented in which strategic planning problems are formulated as non-cooperative potential games whose solutions represent Nash equilibria. The potential game model takes the form of a nonconvex nonlinear program (NLP) and we examine an additional scenario extending this to a nonconvex mixed integer nonlinear program (MINLP). Tactical planning decisions are linked to strategic decision processes through a potential game structure derived from a Cournot oligopoly-type game in which multiple crude oil refineries supply several markets. The resulting production planning decisions are rational in a game theoretic sense and are robust to deviations in competitor strategies. These solutions are interpreted as mutual best responses yielding

maximum profit in the competitive planning game. Two scenarios are presented which illustrate the utility of the game theoretic framework in the analysis of production planning problems in competitive scenarios.

1. Introduction

Strategic production planning plays a vital role in modern organizations as a tool for strategic and tactical decision making at an organization-wide level [1]. In a comprehensive review of refinery supply chain planning models Sahebi, Nickel, and Ashayeri identify crude oil supply chain planning optimization as an imperative source of competitive advantage in the refining business [2]. Few papers exist in which refinery production planning has been examined in a competitive context where the presence of separate refiners competing for limited market share is taken into account at the strategic or tactical planning levels. Game theory provides the tools to investigate competitive interactions and has seen wide use in process systems engineering in areas where the interactions between competing entities are of fundamental interest. Of note is the area of electricity market modelling in deregulated power markets, where the ability of interested power suppliers to “game” established auction and distribution systems is well known. Bajpai and Singh review game theoretic methodologies used in modelling strategic decision making processes in electrical markets [3]. Also of note is the area of distributed model predictive control (MPC) in which the control actions of separate but interacting controllers are managed using game theoretic principles. Scattolini reviews game theoretic and other distributed MPC architectures [4].

Game theoretic principles have seen use in engineering supply chain planning literature to solve cooperative and competitive problems. Gjerdrum, Shah, and Papageorgiou have implemented Nash bargaining objective functions to determine fair profit allocation among members of multienterprise supply chains [5][6]. Pierru used Aumann-Shapley cost sharing to allocate carbon dioxide emissions to various products in an oil refinery [7]. Bard, Plumer, and Sourie used a bilevel formulation to investigate interactions between governments and biofuels producers as a Stackelberg game where the government leads by enacting policy [8]. Bai, Ouyang, and Pang have used a bilevel formulation to solve a competitive biofuel refinery location and planning problem as a Stackelberg game wherein the biofuel refiner takes the role of the leader and farmers follow by adjusting their land use [9]. Yue and You used KKT conditions to reduce the bilevel program describing a Stackelberg game into a single nonconvex MINLP whose global optimum is a Stackelberg equilibrium [10]. Zamarripa *et al* have developed a framework for solving cooperative and competitive supply chain problems through enumeration of the payoff matrix in multi-objective scenarios, yielding Nash equilibria in almost all cases [11][12][13].

With the exception to the works of Zamarripa *et al*, the applications of game theoretic principles in engineering supply chain literature do not yield Nash equilibrium planning results, and rely instead on other game theoretic constructs. In particular the use of a Stackelberg game allows the planning decisions of a leader to be optimized such that the followers are constrained to Nash equilibrium strategies. The Stackelberg framework is not appropriate if no single competitor can be identified as a leader or does not have the capacity to implement a strategy before competitors can react [14][15]. The framework proposed by Zamarripa *et al* yields Nash equilibria in most cases, but does not under certain conditions, as they observed in [13]. Since their method is based on enumeration of a finite strategy matrix, and the framework examines only pure strategy solutions (as opposed to mixed strategies) a Nash equilibrium is not guaranteed to exist in all cases [16]. There is thus a gap in engineering supply chain literature where supply chain planning problems in competitive scenarios cannot be effectively solved to Nash equilibrium strategies. We address this problem with a game theoretic framework for strategic and tactical production planning which generates production plans representative of Nash equilibria between competing producers and we illustrate the properties of this framework using a set of competing oil refiners. Our framework treats production planning problems as continuous games (also referred to as infinite games) which guarantees that at least one Nash equilibrium will exist [17][18][19]. Problems are formulated as potential games, and Nash equilibrium solutions are identified as the global maxima of a potential function objective [20]. This potential game framework circumvents many of the problems which arise in the application of game theoretic models to production planning as the planning and game theoretic aspects of the problem are defined by a single objective function which can be solved using conventional NLP and MINLP solvers. The contributions and novel elements of this work are:

- A framework under which strategic production planning problems can be solved in a game theoretic context using a potential game formulation yielding solutions forming Nash equilibria;
- A modification to the Cournot oligopoly model which uses a defined demand level as a modifier of price behaviour;
- Two case studies which illustrate the utility of the game theoretic framework in relevant planning scenarios which exemplify its potential applications to strategic and tactical production planning.

The paper is presented as follows: Background material on game theory and potential games is provided in brief in section 2. Problem statements are outlined in section 3. Equations and model formulation, including the formulation of each of the scenarios and their variants, are presented in section 4, along with the formulation of the demand-based Cournot oligopoly. Results, interpretations, and discussion are presented in section 5, and conclusions are drawn in section 6.

2. Background

2.1. Nash equilibrium

The concept of the Nash equilibrium as a solution to a noncooperative game has been studied extensively and has different interpretations in various types of game theoretic problems [16][21][22]. We present elements of Nash equilibrium theory pertinent to the development of our potential game framework. Denoting the game as G and the strategy sets of each of N players as S_n with strategies $s_n \in S_n$ then a Nash equilibrium of G is defined as a set of strategies $G\{s_1^*, \dots, s_N^*\}$ where s_n^* represents player n's equilibrium strategy. Each player has an objective function $J_n\{s_n, s_{-n}\}$; a Nash equilibrium strategy has the property in Eq. (1).

$$J_n\{s_n, s_{-n}^*\} \leq J_n\{s_n^*, s_{-n}^*\} \quad \forall n \in N, s_n \in S_n \tag{1}$$

By using a non-strict inequality this definition allows multiple equilibria to exist with the same value. Such a case is referred to as a weak Nash equilibrium. Where an equilibrium satisfies the definition to strict inequality, the resulting Nash equilibrium is termed strict [23]. This result has several implications on the meaning of equilibrium. There is no alternate strategy available to any single player which yields a unilateral increase in their objective; equilibrium strategies are a set of mutual best responses among the set of players. The mathematical definition of the Nash equilibrium corresponds to an assumption that every participant in the game has full knowledge of the game and the strategy sets of all other players in order to formulate their own strategies. The Nash equilibrium may also be interpreted as a maximizer of the set of player objectives in Eq. (2).

$$s_n^* = argmax\{J_n(s_n, s_{-n}^*)\} \quad \forall n \in N, s_n \in S_n \tag{2}$$

Each player's objective is maximized with regard to the best responses of all other players, which are usually not the global maximizers of $J_n(s_n, s_{-n})$ with respect to both strategy sets S_n and S_{-n}. Where the players' objectives are continuous and differentiable functions of strategy variables $s_n \in S_n$ the Nash equilibrium is defined by solving the set of equations in Eq. (3) [22].

$$\frac{\partial J_n(s_n, s_{-n})}{\partial s_n} = 0 \quad \forall s_n \in S_n, n \in N \tag{3}$$

Multiple Nash equilibria may exist in a continuous game. Calculation of all Nash equilibria which exist in a game is an NP-hard problem, although heuristics exist which allow additional equilibria to be characterized [24][25].

Games can be defined such that participants' strategy spaces are not independent. Such games are referred to as generalized Nash equilibrium problems (GNEP) [26][27][28]. In a GNEP player strategies are defined in terms of a strategy set $s_n \in S_n(s_{-n})$ which is dependent on competing players' chosen strategies. Constraints on player strategies make analytical solutions more difficult to obtain [28]. The solution to a GNEP is referred to as a generalized Nash equilibrium, and shares many of the same properties of a Nash equilibrium. Generalized Nash equilibrium strategies have the definition in Eq. (4).

$$s_n^* = argmax\{J_n(s_n, s_{-n}^*)\} \quad \forall n \in N, s_n \in S_n(s_{-n}) \tag{4}$$

The generalized Nash equilibrium is defined by the KKT conditions corresponding to players' problems, and multiple generalized Nash equilibria may be defined this way. Selection of one equilibrium as a solution from among all possible generalized Nash equilibria is facilitated by the concept of the normalized Nash equilibrium. Normalization is accomplished by imposing a set of relative weightings on the dual variables, which for convex games guarantees that a unique normalized Nash equilibrium exist for each unique set of weightings [29][30]. This definition provides a selection mechanism for Nash equilibrium solutions in constrained problems.

2.2. Potential games and the potential function

For a subclass of games called potential games, the system of equations defining the Nash equilibria can be used to formulate a potential function whose maxima correspond to the Nash equilibria of the game. Early work demonstrating existence of the potential function was formalized by Bergstrom and Varian in 1985 [31], and Slade in 1989 [32] and 1994 [33]. The class of potential games and the associated nomenclature were characterized by Monderer and Shapley in 1996 [20]. With game equilibria defined as objective function maxima, potential games can be solved using optimization tools. The equilibria defined by potential games may be strict, weak, or of the generalized type [34][35].

A potential function can be derived from the individual objectives $J_n(s_n, s_{-n})$ of the players in a game. All objective functions must be of the form in Eq. (5).

$$J_n(s_n, s_{-n}) = \Psi(s_n, s_{-n}) + \Omega_n(s_n) + \Theta_n(s_{-n}) \quad \forall n \in N \tag{5}$$

In this form each player's objective consists of three parts: Ψ is a term common to all players and a function of all players' strategy variables; Ω_n is a term unique to each player and is a function exclusively of that player's strategy variables; and Θ_n is a term unique to each player which contains only the variables associated with the other players. The potential function is formulated as in Eq. (6).

$$Z(s_n, s_{-n}) = \Psi(s_n, s_{-n}) + \sum_n \Omega_n(s_n) \tag{6}$$

The potential function yields the same definition of the Nash equilibrium as defined in section 2.1: its derivative with respect to any individual player's strategy variable yields the derivative of that player's objective function, as in Eq. (7).

$$\frac{\partial Z(s_n, s_{-n})}{\partial s_n} = \frac{\partial}{\partial s_n}\left(\Psi(s_n, s_{-n}) + \Omega_n(s_n)\right) = \frac{\partial J_n(s_n, s_{-n})}{\partial s_n} \tag{7}$$

In this interpretation it is apparent that the maxima of the potential function must be solutions to the set of partial differential equations obtained by equating each player's derivative to zero, and are therefore Nash equilibria by definition. These concepts extend to constrained games and the generalized Nash equilibrium; the maxima of the potential function subject to strategy space constraints are generalized Nash equilibria [29].

2.3. Cournot oligopoly

The Cournot oligopoly is a classic economic model used to examine market competition, which we use to structure game theoretic interactions between competitors. The Cournot oligopoly has seen widespread use in economics and defines a game in which a set N of producers of a single homogeneous good each must decide how much of that good to sell to a market [21][36]. The realized price for the good is a function of the collective amount the players deliver to the market. This scenario results in a game in which each player's only strategic decision is a production volume. We focus on a Cournot game with particular assumptions: that the game is static, meaning all decision making occurs instantaneously and simultaneously, and also a state of complete information, meaning that players are always aware of their competitors' decisions [21].

Each market participant attempts to maximize its profits according to a function $J_n = Pr \cdot q_n - c_n$ where Pr represents market price, q_n is the amount supplied by producer n, each of whom has a production cost c_n. The solution to this game theoretic model is a Nash equilibrium in terms of the quantities of product q_n that each of the N producers supply. The market price Pr is a function of q_n and the most common interpretation is that in Eq. (8).

$$Pr = A - \sum_n q_n \tag{8}$$

A is a parameter indicative of the marginal value of the first unit sold on the market. This equation interpretation is presented in similar form in [21][32][36]. Player objective functions can be rewritten as functions of total production of the form in Eq. (9).

$$J_n = \left(A - \sum_{n'} q_{n'}\right) q_n - c_n \tag{9}$$

The Nash equilibrium in terms of q_n for this oligopoly problem is defined by the solution to the set of best response equations in Eq. (10).

$$\frac{\partial J_n}{\partial q_n} = 0 \quad \forall n \in N \tag{10}$$

The Cournot oligopoly presented here is a potential game [33]. The corresponding potential function has the form in Eq. (11).

$$Z = \sum_n (A \cdot q_n - q_n^2 - c_n) - \sum_{\substack{n,n' \\ n<n'}} (q_n q_{n'}) \tag{11}$$

The derivatives of this potential function with respect to each q_n yield the partial derivatives of each player's objective J_n with respect to q_n. The maxima of this function are Nash equilibria.

3. Problem statement

We examine strategic refinery production planning in a game theoretic framework to investigate the effects of competition on strategic planning decisions. In this framework individual refineries are owned and operated by single, competing refiners such that each refinery is considered to be an individual competitor in a game theoretic sense. Each refiner produces the same set of petroleum products as the others and has access to the same crude oil stocks. Refineries are identical in configuration, but vary in capacity.

Refiners are faced with a production planning problem in which multiple target markets exist and each market is characterized by its own nominal demand levels, corresponding nominal prices, and status as either a local or a global market. Local markets are those in which refiners are physically located, while global markets do not contain refineries. Refiners are collectively obligated to satisfy product supply constraints in their local market, and refiners outside that market cannot export product there for sale. We interpret this arrangement as a form of price protection between the local market population and the refiners operating there; refiners may import product for local market sale to make up production slack. Since global markets do not contain refiners, they are reliant upon imports. Global markets are connected to local markets by pipelines, and any refiner with access to a pipeline may export product to a global market without limit.

The refinery market is formulated with Cournot oligopoly pricing. Product prices in each market are variable functions of the collective market supply of that product; refiners do not control prices, but do influence them with their production decisions. Pricing is based on the concept of inverse demand; prices decrease in response to a market supply in excess of demand, and increase when supply falls short of demand. This pricing structure assumes that prices adjust to a point where all supplied product is sold and the market clears. Each refiner has the objective of maximizing its profit independently of the others. A refiner's individual problem is thus to:

- Determine the amounts of each product which should be sold in its local market in order to satisfy local supply constraints in concert with its competitors, and whether any product should be imported, in order to maximize its own profit (a strategic decision).
- Determine the amounts of each product which should be sold to global markets accounting for all competitors with market access in order to maximize its profit (a strategic decision).
- Determine how much of each crude oil stock to purchase and how to process the purchased stocks into the desired products in the most cost efficient manner (production planning decisions).

Each refiners' decision variables are:

- Crude stock purchase volumes.
- Blend volumes and unit operating modes.
- Product volumes and shipping destinations, including imports.

This game theoretic production planning problem is formulated as a potential game taking the form of an NLP. Local and global maxima of the potential function objective are defined as Nash equilibrium strategies in terms of refiner production decisions, and may be generalized Nash equilibria [29][33]. Due to the local market supply constraint forcing refiners to satisfy production within specified limits, the solutions obtained from this model are characterized as generalized Nash equilibria when the constraint is enforced. Figure 1 illustrates an arrangement of refiners, consumers, and markets with three local markets and two global markets; one refiner has a local market monopoly because no other refiners have access to its point of sale, the rest compete in local oligopolies, and all compete in either one or two global market oligopolies.

Two scenarios are presented under this framework in which multiple refiners compete under different conditions. Each scenario explores different aspects of production planning problems in a competitive context. The scenarios are described in the following sections.

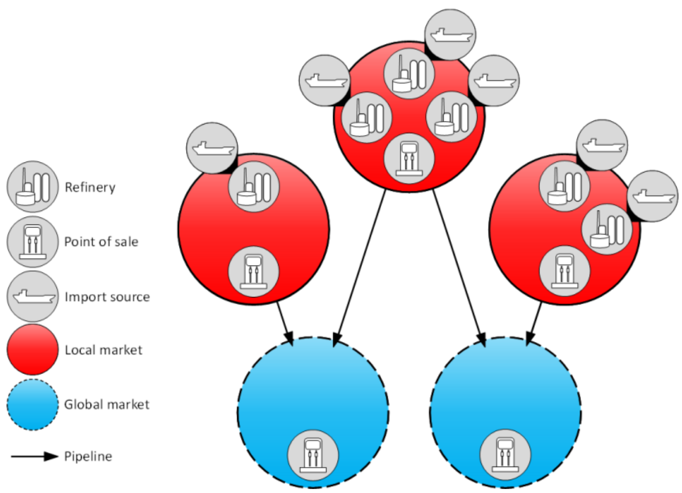

Figure 1. Sample arrangement of local and global markets.

3.1. Scenario 1 (S1) - Competition for market share

This scenario examines refiners competing in the petroleum market and forms a point of comparison with other production planning approaches. Each refiner is capable of producing the same set of six products. Unit capacity constraints in the production planning model limit the ability of any individual refiner to process more than a certain total throughput regardless of market driving forces. It will be shown by removing unit capacity constraints and allowing refiners to produce potentially infinite volumes that there exist Nash equilibria as the global maxima of the unconstrained potential function and that both types of solution share similar properties. The production planning problem is also solved under a fixed price profit maximization objective for comparison using the sum of refiner profits as the objective function. With fixed prices the refiners are not in competition. It will be shown that results obtained using a fixed price approach differ from game theoretic results, and that the strategic plans derived are not competitively rational.

3.2. Scenario 2 (S2) - Elimination of inefficient competitors

This scenario examines refiners in competition where the market structure may change. A subset of refineries are considered to be more efficient and competitive than the remainder and are labeled low-cost refineries, denoted by the subset $LCN \subseteq N$. The remaining refineries are, by virtue of their size, age, or technological obsoleteness, rendered less competitive than the low-cost refineries and are termed high-cost refineries, denoted as part of the subset $HCN \subseteq N$. All refiners are in competition regardless of low or high-cost status, and the low cost refiners need to decide whether to shut down high-cost refineries and obtain additional market share for themselves. Examples of such industry structure exist in western Canada where several small refiners compete with large capacity refiners [37].

Cournot limit theorem states that all else being equal a market with fewer competitors maintains higher prices [38]. Based on this theorem, any option to reduce the number of competitors is a positive decision for the remaining refiners. We differentiate our scenario from this theorem by assuming that high-cost refineries are older, obsolete, and require higher market prices than their low-cost competitors to remain profitable. As long as the high-cost refineries remain active, all refiners gain the benefit of the higher local market prices; if the high-cost refineries shut down, prices drop to reflect the competitive margins of the low-cost refineries. Low-cost refiners have the option either to drive a high-cost refiner out of business by aggressively supplying the local market, or allowing the high-cost refiner to continue to operate;

this decision occurs by consensus among the low-cost refiners in order to avoid cartel game mechanics [36]. The consensus decision is modelled as a binary variable such that this scenario is a nonconvex MINLP. The question in this game is under what market conditions a high-cost refiner is allowed to remain in operation. It will be shown that an inclusion region can be characterized based on market demand levels and the price increase associated with the high-cost refiner.

4. Models and Formulation

4.1. Production planning model

The refinery production planning model consists of the set of equations which describe how crude oil is transformed into intermediates and products. We use a simplified linear yield-based model similar to that used by Castillo Castillo and Mahalec [39]. A schematic of the refinery is shown in Figure 2 illustrating the pathways that crude oil, production intermediates, and products take through the process units. Each refinery consists of a crude distillation unit (CDU) two hydrotreaters (HT1 and HT2) a hydrocracker (HC) a fluid catalytic cracker (FCC) and a catalytic reformer (CR). Blending of intermediates into products occurs in a gas blender (GB) and a diesel blender (DB). The eight intermediates of interest are straight run light naphtha (srln) hydrocracker light naphtha (hcln) catalytic cracker light naphtha (fccln) heavy naphtha (fcchn) light cycle oil (fcclco) straight run distillate (srds) hydrocracker distillate (hcds) and reformate (rft). The six products are regular, mid-grade, and premium gasoline (reg, mid, and pre) and diesel grades 1, 2, and 4 (de1, de2, and de4).

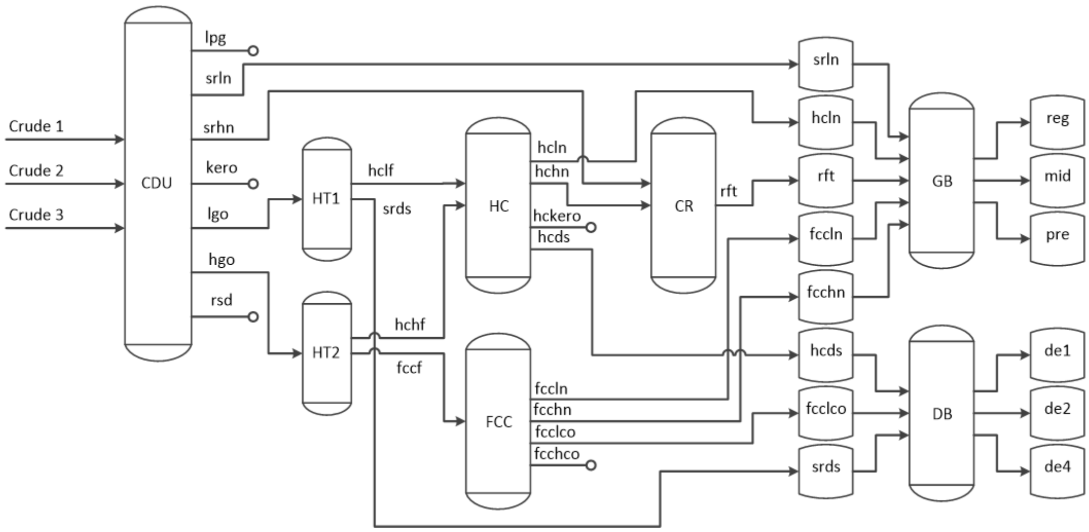

Figure 2. Refinery model schematic.

The plant processes crude oil following the layout in Figure 2. For each crude oil which enters the CDU there is a yield corresponding to the type of crude processed and operating mode (either max diesel mode or max naphtha mode) which dictates the amounts of outputs produced. The refinery model considers primarily those streams involved in the production of gasoline and diesel products. The streams denoting the CDU output of light product gasses (lpg) kerosene (kero) and residuals (rsd) are assumed to be sold at fixed price in order to keep the model relatively small. This assumption impacts neither the qualitative pattern of the results nor the conclusions. Similarly, the HC kerosene stream (hckero) and FCC heavy cycle oil (fcchco) are calculated but not included in profit calculations. All other streams in Figure 2 indicate movements of material through the refinery linking crude oil to gasoline and diesel products.

Refinery efficiency cost reflects the cost a refiner faces due to operating away from its efficient operating throughput. It is meant to represent complex unit and process operating costs incurred from nonstandard plant operation. The efficiency cost curve is modelled as a quadratic function with vertex coordinates $\left(EC_H(n), EC_K(n), EC_P(n)\right)$ for each refinery n, where $EC_H(n)$

indicates the design throughput with the efficient cost $EC_K(n)$. $EC_P(n)$ represents parabolic focal length and determines how efficiency costs increase with deviation from $EC_H(n)$. A quadratic efficiency function is calculated with the parameters in Eqs. (12), (13), and (14).

$$EC_A(n) = \frac{1}{4\big(EC_P(n)\big)} \tag{12}$$

$$EC_B(n) = -\frac{EC_H(n)}{2\big(EC_P(n)\big)} \tag{13}$$

$$EC_C(n) = \frac{\big(EC_H(n)\big)^2}{4\big(EC_P(n)\big)} + EC_K(n) \tag{14}$$

The total efficiency cost experienced by a refiner is the quadratic efficiency cost multiplied by the total output from the refinery cumulatively over all products and planning periods, and is cubic overall. Total efficiency cost is defined by Eq. (15), where the variable $Prc(t,p,n)$ indicates the amount of a product produced in a given time period by a refiner.

$$\begin{aligned} TEC(n) = {} & EC_A(n)\left[\sum_{t,p} Prc(t,p,n)\right]^3 + EC_B(n)\left[\sum_{t,p} Prc(t,p,n)\right]^2 \\ & + EC_C(n)\sum_{t,p} Prc(t,p,n)\,, \forall n \in N \end{aligned} \tag{15}$$

The production planning model equations are included as supplementary material. The refinery production planning model consists of Eqs. (A 1) to (A 41). Model equation variants specific to a scenario are outlined in sections 4.4 and 4.5.

4.2. A demand-based Cournot oligopoly

We present a modified Cournot oligopoly designed for use with this work which assumes that if the total market production level of a product is equal to a nominal market demand level, denoted $D(p,w)$, then the market price of that product will take a value $B(p,w)$. As in the classic Cournot model, price varies linearly with total market supply, where individual producer amounts are denoted $Tpd(p,n,w)$ with a product index p. The marginal value of the first unit of a product to enter the market is defined as $A(p,w)+B(p,w)$, thus the price of a product p varies according to Eq. (16).

$$Pr(p,w) = A(p,w) + B(p,w) - \frac{A(p,w)}{D(p,w)} \sum_{n} Tpd(p,n,w) \quad \forall p \in P, w \in W \tag{16}$$

The competitor profit function is defined in Eq. (17) using the definition of market price in Eq. (16) and cost total $TotCost(n)$. This demand-based Cournot oligopoly problem is a potential game and forms the basis of the game theoretic refinery planning framework.

$$J(n) = \sum_{p,w} \big(\mathrm{Pr}(p,w)\, Tpd(p,n,w)\big) - TotCost(n) \quad \forall n \in N \tag{17}$$

4.3. Potential function formulation

The potential function corresponding to the set of objectives defined in Eq. (17) consists of the common part Ψ and the unique parts $\Omega(n)$ of the objectives $J(n)$, and is defined in Eq. (18) with definitions for Ψ and $\Omega(n)$ in Eqs. (19) and (20).

$$\max Z$$

$$Z = \Psi + \sum_{n} \Omega(n) \tag{18}$$

$$\Psi = \sum_{p,w} \left[-\frac{A(p,w)}{D(p,w)} \sum_{\substack{n,n' \\ n<n'}} Tpd(p,n,w)Tpd(p,n',w) \right] \tag{19}$$

$$\Omega(n) = \sum_{p,w} \left[\left(A(p,w) + B(p,w) - \frac{A(p,w)}{D(p,w)} Tpd(p,n,w) \right) Tpd(p,n,w) \right] - TotCost(n) \quad \forall n \in N \tag{20}$$

This form of the potential function serves as the model objective and its maxima are Nash equilibria of strict, weak, or generalized types depending on the included constraints. The total product leaving a refinery $Tpd(p,n,w)$ is defined in Eq. (21) as the sum of the product the refinery produces and the amounts which it imports. These variables link the refinery planning model to the potential function.

$$Tpd(p,n,w) = \sum_{t} Dlv(t,p,n',w) + Imp(p,n,w) \tag{21}$$

4.4. Fixed-price analysis

Current practices generally use fixed prices in refinery planning models; we compare the outcomes of such analyses with game theoretic results. Our scenarios are examined under a fixed-price profit maximization framework using the objective in Eq. (22). This objective is the total profit of all refiners. Revenues are calculated based on fixed market prices $F(p,w)$ and are linear calculations; the only nonlinearity in this variant is the efficiency cost calculation.

$$\max Z \tag{22}$$

$$Z = \sum_{n} \left(\sum_{p} \big(F(p,w) Tpd(p,n,w) \big) - TotCost(n) \right)$$

The local market supply constraints defined by Eqs. (A 40) and (A 41) enforce refiner coordination. With exception to these constraints, refiners are independent of one another in terms of their decision making; their profits are not interdependent under this objective.

4.5. Model alterations for Scenario 2

The game theoretic model is modified such that high-cost refiners are linked to a binary variable that is incorporated into the high-cost refiner model equations in order to allow all flow rates, inventories, and outputs to be set to zero, effectively shutting down those refiners. The continued participation of high-cost refiners is dependent on a binary variable $Y_{HCN}(w)$. High-cost refiners are also limited to a decreased production level $HCNset$ using Eq. (23), and are prevented from making import purchases in this scenario. These changes define the characteristics of the high-cost refiner, along with its parameter values.

$$\sum_{t,p} Prc(t,p,n) = Y_{HCN}(w) HCNset \quad \forall\ (n,w) \in WHCN(n,w) \tag{23}$$

The participation binary is also used to relax variable bound constraints in Eqs. (A 3), (A 4), (A 10), (A 12), (A 13), (A 19), (A 20), (A 27), and (A 29)-(A 32); each use the binary variable to reduce a constraint value to zero if the value of $Y_{HCN}(w)$ is zero in order to deactivate the high-cost refiner model. The potential function term $\Omega(n)$ is altered to include the price increase $AHC(p,w)$ corresponding to the presence or absence of the high-cost refiner defined in Eq. (24).

$$\Omega(n) = \sum_{\substack{p,w \\ w \in WN(n,w)}} \left[\left(A(p,w) + AHC(p,w) \sum_{\substack{w' \\ w' \in WLN}} Y_{HCN}(w') + B(p,w) - \frac{A(p,w)}{D(p,w)} Tpd(p,n,w) \right) Tpd(p,n,w) \right] - TotCost(n) \quad \forall n \in N \tag{24}$$

This version of $\Omega(n)$ contains a bilinear term of the form $Y_{HCN}(w)Tpd(p,n,w)$ which has an exact linearization obtained by introducing two variables and the constraints in Eqs. (25), (26), and (27). This linearization technique reduces the number of model nonlinearities and is described in more detail by You and Grossmann [40]. The presented formulation allows multiple high-cost refiners to exist in a single local market, and dictates their activity on an all-or-none basis. The upper bound $\overline{Tpd}(p,n,w)$ represents the total combined processing capacity of a refiner plus its product import limit.

$$TP(p,n,w,w') + TP1(p,n,w,w') = Tpd(p,n,w) \quad \forall p \in P, n \in N, (w,w') \in W \tag{25}$$

$$TP(p,n,w,w') \leq Y_{HCN}(w')\overline{Tpd}(p,n,w) \quad \forall p \in P, n \in N, (w,w') \in W \tag{26}$$

$$TP1(p,n,w) \leq \left(1 - Y_{HCN}(w')\right)\overline{Tpd}(p,n,w) \quad \forall p \in P, n \in N, (w,w') \in W \tag{27}$$

With this linearization Eq. (24) can be rewritten as in Eq. (28), which is the form of the equation implemented in the elimination scenario and is denoted $\Omega_K(n)$ in order to differentiate it from the version used in other scenarios.

$$\Omega_{\mathrm{K}}(n) = \sum_{\substack{p,w \\ w \in WN(n,w)}} \Bigg[\big(A(p,w) + B(p,w)\big) Tpd(p,n,w) + \left(AHC(p,w) \sum_{\substack{w' \\ w' \in WLN}} TP(p,n,w,w') \right) - \frac{A(p,w)}{D(p,w)} \big(Tpd(p,n,w)\big)^2 \Bigg] - TotCost(n) \quad \forall n \in N \tag{28}$$

The elimination scenario is intended to determine whether low-cost refiners in a local market are better off with or without high-cost refiners. High-cost refiners' production decisions cannot be part of the potential function; game potentials with opposing values of $Y_{HCN}(w)$ compare the accrued value of the entire set of refiners including the local market price increase against that of the just the low-cost refiners without the increase. In order to model the question appropriately the potential function in this scenario must capture only the interests of the low-cost refiners, with the high-cost refiner's production decisions fixed to rational values. A two-stage solution process is used to solve this problem. In the first stage all refiners are active under a full potential function generating the optimal game theoretic production decisions. This first stage amounts to solving a Scenario 1 problem where the high-cost refiner is unable to import product and generates a price increase. In the second stage of the solution process the potential function is generated with $\Omega_{\mathrm{K}}(n)$ for the low-cost refiners which captures their profits and the price increase coming from the high-cost refiner. The variable Ψ is generated over all refiners and captures the decrease in price caused by their collective production decisions, including the fixed first stage values assigned to the high-cost refiner. The second stage thus represents the interests of only the low-cost refiners. Any product profile could be assigned to the high-cost refiner in order to solve the Scenario 2 problem; generating the high-cost refiner's profile using the first stage ensures that the decision is rational with respect to game theoretic analysis. The potential function used in the second stage is formulated as in Eq. (29).

$$\max Z$$

$$Z = \Psi + \sum_{n \in LCN} \Omega_{\mathrm{K}}(n) \tag{29}$$

The fixed price approach to the elimination problem is formulated using the objective function in Eq. (30).

$$\max Z$$

$$Z = \sum_{\substack{n \\ n \in LCN}} \left(\sum_{p} \left(\left(F(p,w) + AHC(p,w) \sum_{\substack{w' \\ w' \in WLN}} Y_{HCN}(w') \right) Tpd(p,n,w) \right) - TotCost(n) \right) \tag{30}$$

5. Results and Discussion

5.1. Scenario basis and data

The scenarios presented in this work are based as much as possible on the Canadian national fuel market using data from 2014. Each scenario is based on the same example involving three refineries acting in a local market $LM1$ with access to an global market $EM1$.

Product demand is scaled down to an appropriate level corresponding to the total combined production capacity existing among the three refineries. All numerical and structural problem data are included as supplementary material.

Market demands for the six products in the scenarios are calculated based on historical Canadian national consumption using data from Statistics Canada for gasoline and diesel products. The published net consumer sales of gasoline and diesel provide the baseline for demand, but the reported net gasoline and diesel sales are not listed by grade [41]. The fraction of demand associated with each product grade is calculated using the Canadian gasoline and diesel totals assuming that the sales by grade can be approximated using consumer sales data for the relevant gasoline and diesel product grades in the USA in 2014 made available by the EIA [42][43]. These values are scaled to 18% and 21% of the real total in order to create local and global market demand totals with values scaling to the same order of magnitude as the combined production capacity of the three refineries.

The product pricing structure is based on weekly national average price data from 2014 published by Natural Resources Canada. Data are available for regular, mid-grade, and premium gasoline [44][45][46]. Data is also available for diesel fuel, but due to changes in the sale of diesel fuels the majority of diesel fuel sold for commercial purposes consists of a single grade [47][48]. We take the average price of each corresponding fuel in 2014 as $B(p, w)$ in both markets. In order to account for the different grades of diesel, the average diesel price obtained from Natural Resources Canada data is assigned to fuel oil type 2 and the price for fuel oil types 1 and 4 are calculated as one standard deviation above and below that price based on the available data. The marginal value of the first unit of each product on the market is calculated using a value of $A(p, w)$ equal to three standard deviations of price. Hence a market supply of a product twice the magnitude of $D(p, w)$ results in prices three standard deviations below average. The values of $AHC(p, w)$ are taken to be 5% of $A(p, w)$ in the local market and zero elsewhere; $CI(p, w)$ is 120% of $B(p, w)$, and $F(p, w)$ is calculated as the sum of $A(p, w)$ and $B(p, w)$ meaning that the fixed price in the corresponding examples is the price of the first unit sold in the equivalent game theoretic scenario.

The primary operating cost burden to the refiner is the purchase price of crude oil. In this work crude oil prices are considered to be fixed. Three crude oil stocks are available on the market: a light sweet, a medium, and a heavy sour variety. The prices for these crude oils are chosen from representative average monthly prices of benchmark crude oils reported by Natural Resources Canada, using Canadian Light Chicago, Canadian Light Sweet, Canadian Heavy Chicago, and Canadian Heavy Hardisty prices to generate three representative crude stock prices at 610.20, 577.30, and 535.04 dollars per cubic meter [49]. Yield values in the production planning model are calculated based on assays for three crude oil stocks produced by ExxonMobil: Hibernia, a light blend, Terra Nova, a medium crude, and Cold Lake, a heavy sour crude [50][51][52]. Yields are computed for max naphtha and max diesel CDU operating modes using data from Fu, Sanchez, and Mahalec [53].

The two scenarios presented in this work are referred to using the notation S1 and S2 for convenience. Associated with each of these scenarios are two additional variations. The first is a version of the game theoretic scenario excluding upper capacity limits on refinery units such that refiners are capable of processing unlimited amounts of material. This variant is intended to illustrate the relationship between unit capacity constraints Nash equilibria, and is denoted by adding –G to the scenario name. The second variant solves the scenario problem under fixed prices for contrast with the game theoretic planning results. The fixed price variants are referred to by appending each model name with –F. Equation listings for each scenario are given in Table 1. All set and parameter data is included as supplementary material.

Table 1. Scenario nomenclature and corresponding equations.

Scenario	Equation list
S1	(15), (18) to (21), and (A 1) to (A 41)
S1-G	(15), (18) to (21), (A 1) to (A 41), excluding: (A 4), (A 13), (A 20), and (A 30)
S1-F	(15), (22), and (A 1) to (A 41)
S2	(15), (21), (23), (25) to (29), and (A 1) to (A 41)
S2-G	(15), (21), (23), (25) to (29), and (A 1) to (A 41), excluding: (A 4), (A 13), (A 20), and (A 30)
S2-F	(15), (23), (30), and (A 1) to (A 41)

5.2. Scenario 1 results

Results are characterized entirely by refiner production decisions interpreted through the potential function as profits. Market prices and individual refiner profits are implicitly defined in the potential function and do not appear directly in the model. The production volumes resulting in S1 are presented in Figure 3. Local and import production volumes satisfy local market demand while production excesses of gasoline products are exported in order to take advantage of higher global market prices. Refiners have similar optimal production plans which scale according to refinery capacity. In the case that all three refiners are identical in size, their production volumes will be symmetric. Import volumes scale inversely with refinery size; smaller refiners import more than larger ones.

Refiners do not have the capacity to satisfy local market diesel demand to the minimum constraint level and must import to do so. The total amount of each diesel product that each refiner produces is identical. Each refiner produces diesel products up to capacity and imports the balance of its share. Larger refiners are able to produce more diesel in-house and import less.

The cumulative amount produced by the refiners satisfies precisely the lower supply limit. Similarly refiners produce identical amounts of diesel 1 without imports; the cumulative volume satisfies the minimum supply. This pattern is not observed for gasoline products, in which case refiners produce amounts varying with their capacity for both local and global markets. This result suggests that game theoretic planning drives refiners to compete on marginally more profitable products and to allocate production equally for less profitable products. We note that the observed results are strongly dependent on refinery production capacities; e.g. refineries configured for diesel would have a reversed pattern of production volumes.

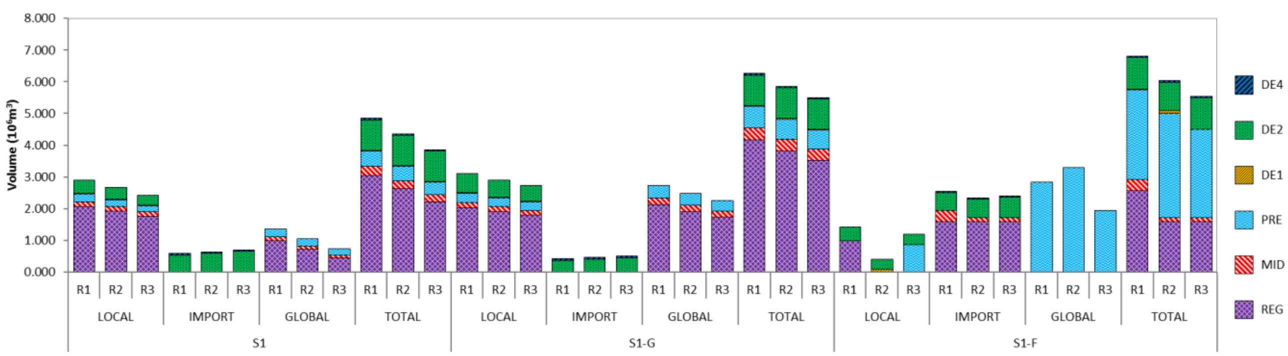

Figure 3. S1 production volume breakdown and totals by refiner and scenario variant.

The results of S1-G are presented in Figure 3 for comparison with S1. The total amount of diesel product is unchanged from S1 (minimum local supply is satisfied) but the amounts of gasoline product produced are increased. No product is supplied in the local market at an amount large enough to reach the upper constraint level; the solution in the local market is defined by Nash equilibrium prices, not local supply constraints. Import amounts in S1-G are unchanged from S1. This outcome suggests that with unlimited production capacity it is not profitable to produce any more diesel products than those required in the local market, and that the most cost effective means of obtaining those products is to produce a fraction and import the balance, where the decision is driven by prices, not capacity constraints.

Refiners export more gasoline product to the global market in S1-G than S1. Despite unconstrained capacities the refiners halt production at a price point representing a Nash equilibrium, contrasting with the different equilibrium obtained in S1 defined by capacity constraints. Refiners continue to plan production according to their sizes due to efficiency costs.

The production volumes obtained from the solution of S1-F are shown in Figure 3. Total local market supply satisfies the upper and lower demand constraints, but production volumes associated with each refiner do not follow a competitively rational pattern. In S1-F prices are fixed and the objective is total refiner profit. The allocation of production is that which maximizes total market profit regardless of individual profits. This result is unobtainable barring a monopoly; an individual refiner will not yield profits because a competitor has a better marginal gain. This fixed price approach does not generate rational behaviour. Refiners only export premium gasoline as there is no consideration of market demand levels. We present this result to illustrate the driving forces at play in fixed priced models.

The prices and profits associated with these three cases and for all scenarios are collected in Table 2 and Table 3. In S1-G refiners produce more than in S1; refiners R1 and R2 lose profits in S1-G while refiner three gains. This illustrates the rationality of Nash equilibria: refiners will not be worse off in terms of their own profits if any others behave differently; they can make gains if competitors deviate from equilibrium strategies. In this case R1 lost profits in S1-G relative to S1, but could stand to gain if R2 or R3 made a non-equilibrium plan. The profits reported in S1-F by comparison are much higher, and are unrealistic in a game-theoretic sense.

Table 2. Profit values by scenario (10^6 CAD).

	R1	R2	R3
S1	97.82	52.81	4.24
S1-G	71.61	50.73	30.18
S1-F	1102.98	971.85	899.27
S2	47.96	0.72	121.11
S2-G	132.81	95.60	0.00
S2-F	1112.83	1014.83	646.16

The prices in scenarios S1 and S1-G illustrate the Cournot property: as refiners supply more of a product, its price drops. In S1-F the prices reported are the highest possible due to the assumption of fixed prices (under FIXED heading). The equivalent game theoretic prices (under EQUIV. heading) corresponding to the market supplies in S1-F are correspondingly lower and, in the case of the price of premium gasoline in the global market, substantially lower than those observed in the game theoretic version where consideration of market demand and price limits the total volume of premium gasoline supplied to a rational level.

Table 3. Scenario prices (CAD/m^3). S1-F and S2-F show fixed price values and the equivalent game theoretic Cournot prices corresponding to the production levels in those scenarios.

MARKET	PRODUCT	S1	S1-G	S1-F		S2	S2-G	S2-F	
				FIXED	EQUIV.			FIXED	EQUIV.
LOCAL	REG	912.18	912.18	1130.49	909.72	916.22	912.18	1134.53	909.72
	MID	977.00	958.01	1183.36	911.64	980.82	977.00	1187.18	911.64
	PRE	1028.12	971.91	1238.94	959.67	1032.02	1021.10	1242.84	959.67
	DE1	1089.39	1089.39	1238.58	1042.34	1092.15	1089.39	1241.34	1042.34
	DE2	1034.13	1034.13	1183.33	1036.14	1036.90	1034.13	1186.09	1036.14
	DE4	978.88	978.88	1128.07	980.89	981.64	978.88	1130.83	980.89
GLOBAL	REG	1003.27	924.58	1049.64	1049.64	1008.15	958.00	1049.64	1043.54
	MID	1017.59	938.91	1106.93	1106.93	1022.47	977.10	1106.93	1086.84
	PRE	1031.07	952.39	1160.86	-367.68	1035.96	995.07	1160.86	70.25
	DE1	1183.33	1183.33	1183.33	1183.33	1183.33	1183.33	1183.33	1183.33

	DE2	1128.07	1128.07	1128.07	1128.07	1128.07	1128.07	1128.07	1128.07
	DE4	1072.81	1072.81	1072.81	1072.81	1072.81	1072.81	1072.81	1072.81

5.3. Scenario 2 results

The purpose of this scenario is to examine conditions under which strategic planning can incorporate large scale decisions affecting the structure of the market. Results are presented for S2 in Figure 4 in which the high-cost refiner R3 continues to participate in the market, and are similar to the production plans in S1, but the high-cost refiner cannot import in this scenario and has limited production. The high-cost refiner distributes its market share in the same way as the low-cost refiners; it produces less than in S1, since its share is constrained.

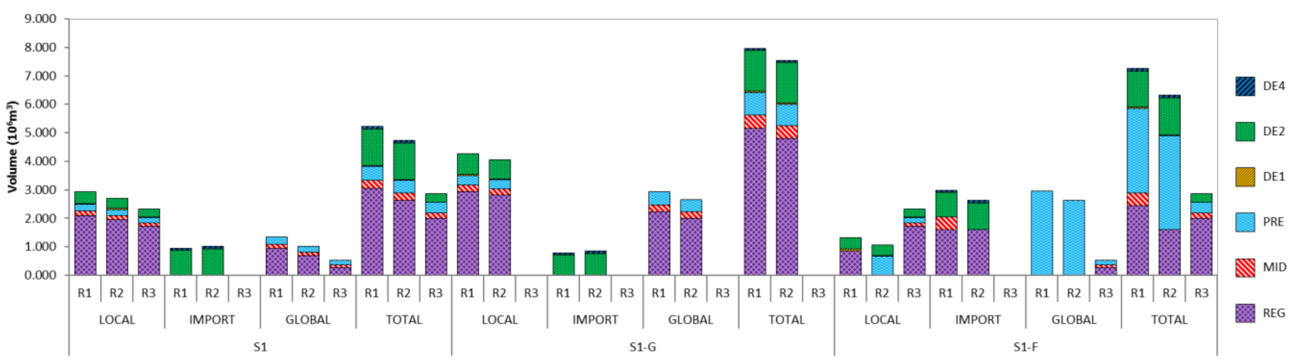

Figure 4. S2 production volume breakdown and totals by refiner and scenario variant.

An unconstrained version of this problem is presented as S2-G in which low-cost refiners have unconstrained capacity while the high-cost refiner (if active) is limited to producing the profile determined from the first stage calculation. In S2-G the high-cost refiner is shut down, as can be seen in Figure 4. In this scenario the low-cost refiners benefit more from the increased local market share obtained by shutting down the high-cost refiner than by having higher local market prices. The optimal decision in S2-G differs from that in S2; with unconstrained capacity the low-cost refiners are better off without the high-cost refiner, whereas in S2 the high-cost refiner’s contributions in the local market allow the low-cost refiners greater access to global markets. There are thus multiple factors, both refinery-specific and market-based, influencing the participation of the high-cost refiner.

A fixed price case is presented as S2-F. As in S2 the high-cost player remains active, but the production decisions made by the low-cost refiners are irrational, following the same patterns as S1-F, even though the high-cost refiner’s allocation is rational from a game theoretic standpoint.

In order to investigate the influence of local and global market demands on the high-cost refiner’s participation in scenario S2 and S2-F these two cases are solved over a grid of demand values and for six different values of the high-cost refiner price increase $AHC(p,w)$. Demand scaling factors for all products are taken at 19 even intervals ranging from 6% to 25.3% of the demand values $D(p,w)$ included as supplementary data. The test values for the price increase $AHC(p,w)$ are taken at 10% intervals of the values of $A(p,w)$ ranging from 5% up to 55%.

Objective values are calculated for the six values of $AHC(p,w)$ and are used to define contours characterizing the inclusion region boundary. These boundary lines are visualized in Figure 5 as solid lines labeled with the associated price increase percentage. As the high-cost refiner’s presence brings larger price increases, the minimum local and global demands at which it will be shut down decrease. This result is intuitive; as the benefit accrued by low-cost refiners increases, they become more tolerant of the high-cost refiner in smaller markets.

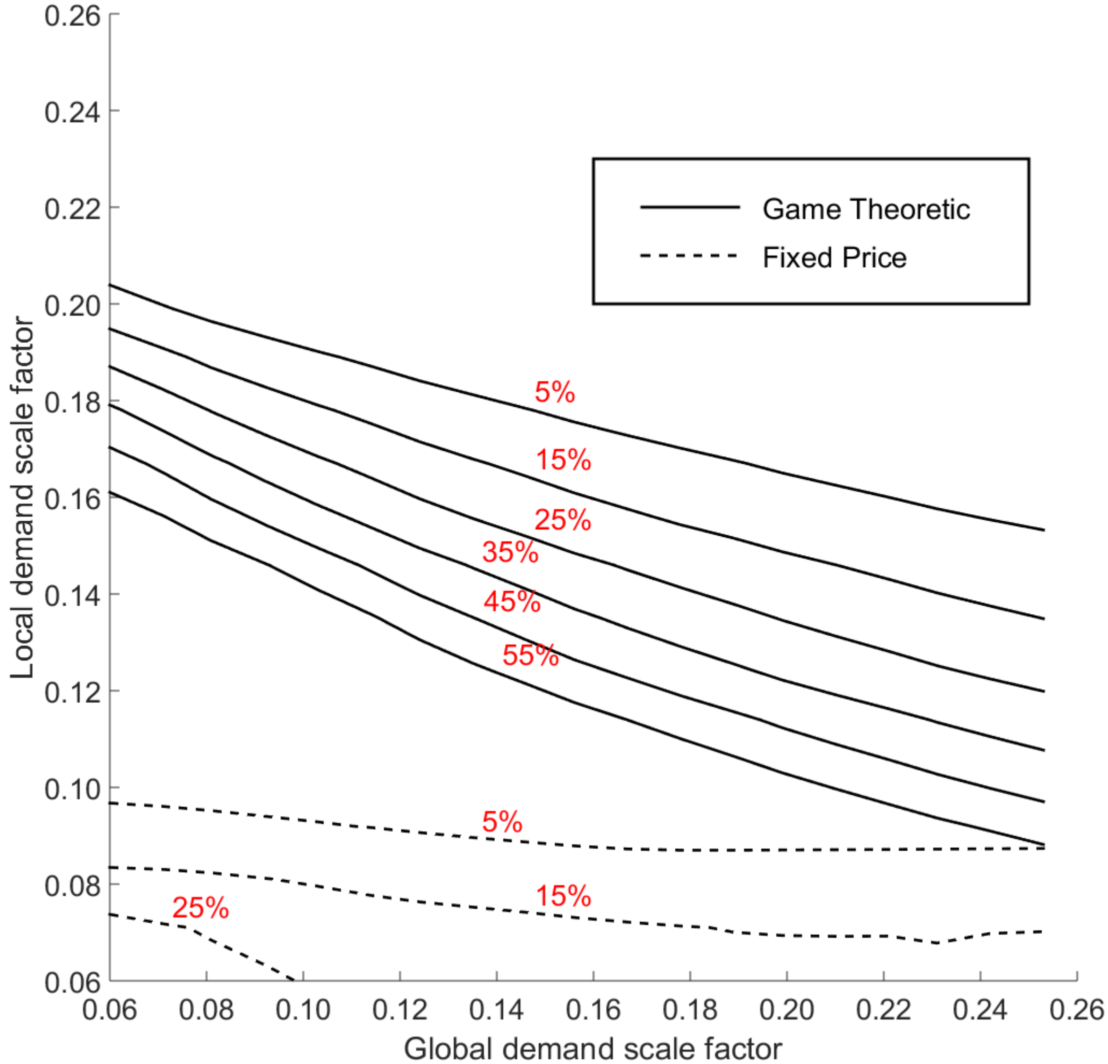

Figure 5. Inclusion region boundary characterization for S2; boundaries define paired local and global demand levels (as scaled values) below which low-cost refiners shut down the high-cost refiner with the indicated price increase.

Results are shown for the inclusion region associated with S2-F as dashed lines. Boundary lines are shown for values of $AHC(p,w)$ equal to 5%, 15%, and 25% of $A(p,w)$ for local market prices; no such boundary lines are found for values of 35% or greater, in which case the high cost refiner is allowed to remain active for all demand levels tested. Under fixed price analysis, the low-cost refiners will mistakenly allow the high-cost refiner to operate when its impacts will reduce their profits, instead of increasing them as they would predict.

5.4. Existence of multiple equilibria

Multiple Nash equilibria may exist in continuous games; our potential game formulation is nonconvex and we identify equilibria of the generalized type as globally optimal solutions. The existence of equilibria with equal objective value is of interest to determine whether normalization is required. To ascertain whether equal-valued equilibria exist in this problem, we constrain the potential function to the obtained optimal value Z^* and modify the objective function to drive refiners' individual decision variables to different values corresponding to the same optimum. Changes in $Dlv(t,p,n,w)$ or $Imp(p,n,w)$ yielding Z^* constitute equal valued alternative equilibria. We were unable to find alternative Nash equilibria using this approach, suggesting that there is a single globally optimal Nash equilibrium under the implemented formulation and data set. Since the equilibrium is unique, we are not concerned with normalization to characterize a best equilibrium solution [30].

5.5. Model solution statistics

All results are generated on a Dell Optiplex 9010 computer with Intel Core-i7-3770 CPU and a 3.40 GHz processor running the Windows 10 64-bit operating system. Models are solved using GAMS 24.7.1 with ANTIGONE 1.1 [54] warm starting with CONOPT 3.17A or DICOPT. Solution data for each scenario are given in Table 4 including preprocessing results.

Solution results consist of the objective function value and optimality gap data reported by the solver, as well as the model statistics generated by GAMS. The model status indicates the optimality of the solution achieved; scenarios with a model status of 1 are solved to global optimality; a model status of 2 indicates local optimality. All scenarios presented are solved to global optimality with a relative gap of 1×10^{-9}. The processed model sizes generated by ANTIGONE are also reported; these indicate the types of equations, variables, and nonlinear terms detected by the solver and provide additional model information. The relative gap and CPU time reported by ANTIGONE are also included. The CPU times are essentially the same as those reported by GAMS, but exclude model generation time.

Table 4. Solution data.

	S1	S1-G	S1-F	S2	S2-G	S2-F
Model statistics (GAMS)						
Single equations	1833	1713	1753	2121	2009	1825
Single variables	1245	1245	1165	1376	1376	1140
Nonlinear entities	111	111	3	55	55	20
Solve summary (ANTIGONE)						
Objective value	871.7855	1182.4504	2974.0996	354.0593	864.1746	2127.6622
Model status	1	1	1	1	1	1
Resource usage	0.364	1.534	0.151	0.583	0.885	0.541
After pre-processing						
Variables	531	531	513	498	498	438
Continuous	531	531	513	497	497	437
Binary				1	1	1
Equations	1153	1153	1063	1172	1162	1004
Linear	1077	1077	1060	1132	1122	1000
Convex nonlinear						
Nonconvex nonlinear	76	76	3	40	40	4
Nonlinear terms	348	348	6	194	194	80
Bilinear/quadratic	345	345	3	192	192	78
Sigmoidal	3	3	3	2	2	2
Solve statistics						
Relative gap	1.00E-09	1.00E-09	1.00E-09	1.00E-09	1.00E-09	1.00E-09
Total time (CPU s)	0.35	1.43	0.15	0.56	0.81	0.44

6. Conclusions

We have presented a game theoretic strategic production planning framework based on a modified Cournot oligopoly formulated as a potential game which we use to solve strategic refinery production planning problems to Nash equilibrium solutions. Two scenarios have been presented illustrating competitive behaviour in production planning problems. The first scenario illustrates competitive behaviour in the game theoretic sense and contrasts those results with equivalent fixed price planning results. The second scenario extends the framework to include a decision to shut down a competitor by claiming its market share. This scenario illustrates how competitive behaviour manifests in problems involving market restructuring and that the inclusion of planning decisions influences the outcome. In both scenarios, the results of production planning in a game theoretic framework were contrasted with those obtained by solving the same problem under the assumption of fixed prices. The proposed potential game framework demonstrates that refinery production planning benefits from game theoretic analysis. Making production planning decisions in a competitive context is a non-obvious problem, particularly so when market restructuring decisions are involved, and using fixed price methods does not yield rational solutions in either case. The importance of rational planning arises from the reality that most industries operate under competition. Game theoretic methods are able to generate rational planning outcomes to these difficult problems.

7. Notation

7.1. Sets

BL	(bl) set of blenders
I	(i) set of process streams
IC	(i) set of crude oils streams entering refinery
M	(m) set of unit operating modes
N	(n) set of refineries
P	(p) set of products
Q	(q) set of quality properties
W	(w) set of markets

T	(t) set of time periods in which planning takes place
TK	(tk) set of tanks for intermediates
U	(u) set of process units
BL_{BLEND}	(i) all streams entering a blender
BL_{IN}	(bl, i) streams entering blender bl
BL_{OUT}	(bl, p) streams leaving a blender
$BL_{OUT,VOL}$	(bl, p, q) product p with volume-based properties q leaves blender bl
$BL_{OUT,WT}$	(bl, p, q) product p with weight-based properties q leaves blender bl
$BL_{OUT,NL}$	(bl, p, q) product p with nonlinear properties q leaves blender bl
$BLIP$	(bl, i, p) product of BL_{IN} and BL_{OUT}
DWN	(p', p) product p' may be mixed with product p for delivery to market
LCN	(n) refineries classified as low cost
HCN	(n) refineries classified as high cost
$NPrest$	(n, p) refineries with production limits on product p
P_g	(p) gasoline products
P_d	(p) diesel products
Q_g	(q) gasoline properties
Q_d	(q) diesel properties
Q_{VOL}	(q) volume-based quality properties
Q_{WT}	(q) weight-based quality properties
W_E	(w) global markets
W_L	(w) local markets

WLN (n, w) refinery n is located in local market w

WLE (w, w') refiners in local market w can sell to global market w'

WN (n, w) refiner n can sell to market w

$WLCN$ (n, w) low cost refiner n is located in market w

$WHCN$ (n, w) high cost refiner n is located in market w

TK_{IN} (tk, i) streams entering intermediate tank tk

TK_{OUT} (tk, i) streams leaving intermediate tank tk

U_{IN} (u, i) streams i entering unit u

U_{OUT} (u, i) streams i leaving unit u

U_C (u) subset of U for certain constraints

UM (u, m) units u which can operate in a mode m

UM_C (u, m) subset of UM for certain constraints

$UM_{OUT,C}$ (i, u, m) product of U_{OUT} and UM_C

7.2. Parameters

$A(p, w)$ Marginal value of first unit of product p in market w

$AHC(p, w)$ Additional marginal value associated with product p if high cost refineries are active in market w

$B(p, w)$ Marginal value of product p when market w supply is $D(p, w)$

$CI(p, w)$ Import cost of product p to local market w from elsewhere

$D(p, w)$ Expected market demand for product p in market w

$\underline{D}(p, w)$ Minimum demand for product p in local market w

$\overline{D}(p, w)$ Maximum demand for product p in local market w

$F(p,w)$	Fixed sale price for product p in market w
$Pr(p,w)$	Price of product p in market w
$HCNset$	High-cost refinery production level, as a fraction of market demand
$EC_H(n)$	Efficiency cost parameter H for n
$EC_K(n)$	Efficiency cost parameter K for n
$EC_P(n)$	Efficiency cost parameter P for n
$EC_A(n)$	Efficiency cost parameter A for n
$EC_B(n)$	Efficiency cost parameter B for n
$EC_C(n)$	Efficiency cost parameter C for n
Cap	Percentage rated capacity
$Cost(i)$	Cost of crude oil stream $i \in IC$
$MaxProd(u)$	Maximum production rate on unit u
$MinProd(u)$	Minimum production rate on unit u
$\overline{V}(tk)$	Maximum holding in intermediate tank tk
$\underline{V}(tk)$	Minimum holding in intermediate tank tk
$V_{ini}(tk)$	Initial holding in intermediate tank tk
$\overline{VP}(p)$	Maximum holding in product tank p
$\underline{VP}(p)$	Minimum holding in product tank p
$VP_{ini}(p)$	Initial holding in product tank p
$BlendMax(bl)$	Maximum blending rate for blender bl
$BlendMin(bl)$	Minimum blending rate for blender bl
$BLcost(bl)$	Cost of operating blender bl

$\tau(t)$	Duration of time period t
$OpCost(u,m)$	Operating cost of unit u in mode m
$ProdRest(n,p)$	Restriction on refinery n production level of product p
$qq(i,q)$	Quality property q of stream i
$\overline{Q}(q,p)$	Maximum quality specification of property q for product p
$\underline{Q}(q,p)$	Minimum quality specification of property q for product p
$\overline{R}(i,p)$	Maximum specification of stream i for product p
$\underline{R}(i,p)$	Minimum specification of stream i for product p
TC	Time scaling cost factor
$\overline{Tpd}(p,n,w)$	Total product variable upper bound
$Y(i,m,u)$	Yield of stream i from unit u operating in mode m
$X(i,m,i')$	CDU yield of stream i from feed of crude oil $i' \in IC$ operating in mode m

7.3. Continuous variables

Z	Objective function value
Ψ	Potential function term
Ω_n	Potential function term
$TEC(n)$	Total efficiency cost for n
$FA(t,u,n)$	Inlet feed to unit u in period t for refinery n
$FV(t,i,n)$	Volumetric flow of stream i in period t for refinery n
$FVM(m,t,u,n)$	Inlet feed to unit u in period t in mode m for refinery n
$FVM_{IN}(m,t,i,n)$	Inlet feed of stream i in period t in mode m

$FVM_{OUT}(i,m,t,u,n)$	Volumetric flow of stream i leaving unit u in mode m in period t
$VP(t,p,n)$	Product tank inventory of p in period t
$V(tk,t,n)$	Intermediate tank inventory tk in period t
$VB(i,p,t,n)$	Volume of intermediate i used to produce product p in period t
$VBlend(t,p,n)$	Volume of product p blended in period t
$VBlendT(t,bl,n)$	Total volume blended by blender bl in period t
$Prc(t,p,n)$	Volume of product p produced in period t by refiner n
$Dlv(t,p,n,w)$	Volume of product p for delivery to market w produced in period t
$Imp(p,n,w)$	Volume of product p imported by refiner n in local market w
$CrudeOilCost(n)$	Total cost of all crude oil purchased by refinery n
$UnitOpCost(n)$	Total unit operating cost in refinery n
$BlendOpCost(n)$	Total blending cost in refinery n
$ImpCost(n)$	Cost of imports for refiner n
$TimeCost(n)$	Cost of production timing for refinery n
$TotCost(n)$	Total cost for refiner n excluding upgrades
$Tpd(p,n,w)$	Total product p leaving refinery n for sale to market w
$TP(p,n,w)$	Linearization variable for $TotProd(p,n,w)$
$TP1(p,n,w)$	Linearization variable for $TotProd(p,n,w)$

7.4. Binary variables

$Y_{HCN}(w)$	Decision variable dictating whether high-cost refiners remains in a market w

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1. Set elements

Set	Indices	Elements
BL	(bl)	GB, DB
I	(i)	crude1, crude2, crude3, lpg, srln, srhn, kero, lgo, hgo, rsd, rft, srds, hclf, hchf, hcln, hckero, hcds, hchn, fccf, fccln, fcchn, fcclco, fcchco, srln_tk, rft_tk, hcln_tk, fccln_tk, fcchn_tk, srds_tk, hcds_tk, fcclco_tk
IC	(i)	crude1, crude2, crude3
M	(m)	1, 2
N	(n)	R1, R2, R3
P	(p)	REG, MID, PRE, DE1, DE2, DE4
Q	(q)	RON, MON, ARO, FLS, CNU, SUL, SG, RVP
W	(w)	LM1, EM1
T	(t)	1, 2
TK	(tk)	tk1, tk2. Tk3, tk4, tk5, tk6, tk7, tk8
U	(u)	CDU, CR, HC, FCC, HT1, HT2
UPG	(upg)	uHCproc, uHTproc, uCDUcap, uCRcap, uHCcap, uFCCcap, uGBcap, uDBcap
BL_{BLEND}	(i)	(GB).(srln_tk, rft_tk, hcln_tk, fccln_tk, fcchn_tk), (DB).(srds_tk, hcds_tk, fcclco_tk)
BL_{IN}	(bl,i)	srln_tk, rft_tk, hcln_tk, fccln_tk, fcchn_tk, srds_tk, hcds_tk, fcclco_tk
BL_{OUT}	(bl,p)	(GB).(REG, MID, PRE), (DB).(DE1, DE2, DE4)
$BL_{OUT,VOL}$	(bl,p,q)	(GB).(REG, MID, PRE).(RON, MON, ARO, SG), (DB).(DE1, DE2, DE4).(FLS, CNU, SG)
$BL_{OUT,WT}$	(bl,p,q)	(GB).(REG, MID, PRE).(RVP)
$BL_{OUT,NL}$	(bl,p,q)	(DB).(DE1, DE2, DE4).(SUL)
$BLIP$	(bl,i,p)	$BL_{IN} \cdot BL_{OUT}$
DWN	(p',p)	(REG).(REG), (MID).(REG, MID), (PRE).(REG, MID, PRE), (DE1).(DE1), (DE2).(DE1, DE2), (DE4).(DE1, DE2, DE4)
LCN	(n)	R1, R2
HCN	(n)	R3
$NPrest$	(n,p)	(R1, R2, R3).(REG)
P_g	(p)	REG, MID, PRE
P_d	(p)	DE1, DE2, DE4
Q_g	(q)	RON, MON, ARO, SG, RVP
Q_d	(q)	FLS, CNU, SUL, SG
Q_{VOL}	(q)	RON, MON, ARO, FLS, CNU, SG, RVP
Q_{WT}	(q)	SUL
W_E	(w)	EM1
W_L	(w)	LM1
WLN	(n,w)	(R1, R2, R3).(LM1)
WLE	(w,w')	(LM1).(EM1)
WN	(n,w)	(R1, R2, R3).(LM1, EM1)

$WLCN$	(n, w)	(R1, R2).(LM1)
$WHCN$	(n, w)	(R3).(LM1)
TK_{IN}	(tk, i)	(tk1).(srln), (tk2).(rft), (tk3).(hcln), (tk4).(fccln), (tk5).(fcchn), (tk6).(srds), (tk7).(hcds), (tk8).(fcclco)
TK_{OUT}	(tk, i)	(tk1).(srln_tk), (tk2).(rft_tk), (tk3).(hcln_tk), (tk4).(fccln_tk), (tk5).(fcchn_tk), (tk6).(srds_tk), (tk7).(hcds_tk), (tk8).(fcclco_tk)
U_{IN}	(u, i)	(CDU).(crude1, crude2, crude3), (CR).(srhn, hchn), (HC).(hclf, hchf), (FCC).(fccf), (HT1).(lgo), (HT2).(hgo)
U_{OUT}	(u, i)	(CDU).(lpg, srln, srhn, kero, lgo, hgo, rsd), (CR).(rft), (HC).(hcln, hchn, hckero, hcds), (FCC).(fccln, fcchn, fcclco, fccchco), (HT1).(srds, hclf), (HT2).(hchf, fccf)
U_C	(u)	CDU, CR, HC, FCC
UM	(u, m)	(CDU, CR, HC, FCC).(1, 2),(HT1, HT2).(1)
UM_C	(u, m)	(CR, HC, FCC).(1, 2), (HT1).(1)
$UM_{OUT,C}$	(i, u, m)	$U_{OUT} \cdot UM_C$
$ProcUp$	(upg)	uHCproc, uHTproc
$CapUp$	(upg)	uCDUcap, uCRcap, uHCcap, uFCCcap, uGBcap, uDBcap
SUD	(i, upg)	(crude2).(uHTproc), (crude3).(uHCproc, uHTproc)
UUD	(u, upg)	(CDU.uCDUcap), (CR.uCRcap), (HC.uHCcap), (FCC.uFCCcap)
BUD	(bl, upg)	(GB).(uGBcap), (DB).(uDBcap)

2. Parameter values

Table 1. $A(p, w)$ (CAD/m3)

	REG	MID	PRE	DE1	DE2	DE4
LM1	245.30	226.43	232.72	163.54	163.54	163.54
EM1	163.54	150.96	157.24	113.22	113.22	113.22

Table 2. $AHC(p, w)$ (CAD/m3)

	REG	MID	PRE	DE1	DE2	DE4
LM1	4.04	3.82	3.90	2.76	2.76	2.76

Table 3. $B(p, w)$ (CAD/m3)

	REG	MID	PRE	DE1	DE2	DE4
LM1	886.86	956.05	1006.37	1075.56	1018.95	962.34
EM1	886.86	956.05	1006.37	1075.56	1018.95	962.34

Table 4. $CI(p)$ (CAD/m3)

	REG	MID	PRE	DE1	DE2	DE4
	1065.51	1144.89	1205.63	1287.38	1221.07	1154.76

Table 5. $\boldsymbol{D(p,w)}$ **(10^6 m^3/year)**

	REG	MID	PRE	DE1	DE2	DE4
LM1	6.38	0.48	0.71	0.08	3.19	0.17
EM1	7.45	0.56	0.83	0.10	3.73	0.20

Table 6. $\boldsymbol{\underline{D}(p,w)}$ **(10^6 m^3/year)**

	REG	MID	PRE	DE1	DE2	DE4
LM1	5.75	0.43	0.64	0.07	2.87	0.16

Table 7. $\boldsymbol{\overline{D}(p,w)}$ **(10^6 m^3/year)**

	REG	MID	PRE	DE1	DE2	DE4
LM1	7.66	0.57	0.86	0.10	3.83	0.21

Table 8. $\boldsymbol{F(p,w)}$ **(CAD/m3)**

	REG	MID	PRE	DE1	DE2	DE4
LM1	1132.16	1182.48	1239.09	1239.09	1182.48	1125.87
EM1	1050.40	1107.00	1163.61	1182.48	1125.87	1075.56

Table 9. $\boldsymbol{HCNset}$ **(10^6 m^3/year)**

R3	2.86

Table 10. Efficiency cost curve parameters

	R1	R2	R3
$EC_H(n)$ (10^6 m^3)	5.56	4.79	4.12
$EC_K(n)$ (CAD/m^3)	6.04	5.98	6.16
$EC_P(n)$ ((m^3)^3/CAD)	2.009×10^{11}	2.010×10^{11}	2.011×10^{11}
$EC_A(n)$ (CAD/(m^3)^3)	1.24×10^{-12}	1.24×10^{-12}	1.24×10^{-12}
$EC_B(n)$ (CAD/(m^3)^2)	-1.38×10^{-5}	-1.19×10^{-5}	-1.02×10^{-5}
$EC_C(n)$ (CAD/ m^3)	44.45	34.49	27.27

Table 11. $\boldsymbol{Cap}$

	0.65

Table 12. $\boldsymbol{Cost(i)}$ **(CAD/m^3)**

crude 1	610.20
crude 2	577.30
crude 3	535.04

Table 13. $\boldsymbol{MaxProd(u)}$ **(10^3 m^3/day)**

	R1	R2	R3
CDU	18.28	15.90	13.51
CR	5.30	5.30	5.30
HC	10.60	10.60	10.60
FCC	10.60	10.60	10.60
GB	10.60	10.60	10.60
DB	9.54	9.54	9.54

Table 14. $\boldsymbol{MinProd(u)}$ **(10^3 m^3/day)**

	R1	R2	R3
CDU	9.54	7.95	7.15
CR	1.06	1.06	1.06
HC	0.53	0.53	0.53
FCC	0.53	0.53	0.53

Table 15. Intermediate tank capacity data (10^3 m^3)

	$\overline{V}(tk)$	$\underline{V}(tk)$	$V_{ini}(tk)$
tk1	47.70	0	0
tk2	47.70	0	0
tk3	47.70	0	0
tk4	47.70	0	0
tk5	47.70	0	0
tk6	47.70	0	0
tk7	47.70	0	0
tk8	47.70	0	0

Table 16. Product tank capacity data (10^3 m^3)

	$\overline{VP}(p)$	$\underline{VP}(p)$	$VP_{ini}(p)$
REG	159	1.59	1.59
MID	159	1.59	1.59
PRE	159	1.59	1.59
DE1	159	1.59	1.59
DE2	159	1.59	1.59
DE4	159	1.59	1.59

Table 17. Blender capacity data (10^3 m^3/month)

	$BlendMax(bl)$	$BlendMin(bl)$
GB	318	4.70
DB	286	4.70

Table 18. $\boldsymbol{BLcost(bl)}$ **(CAD/m³)**

GB	6.29×10^{-2}
DB	6.29×10^{-2}

Table 19. $\boldsymbol{\tau(t)}$ **(months)**

1	6
2	6

Table 20. $\boldsymbol{OpCost(u,m)}$ **(CAD/m³)**

	1	2
CDU	1.95	1.41
CR	2.61	5.43
HC	3.37	2.62
FCC	2.12	2.07
GB	0.21	0.21
DB	2.20	2.20

Table 21. $\boldsymbol{ProdRest(n,p)}$ **(10^3 m³/year)**

R1,REG	318
R2,REG	318
R3,REG	318

Table 22. $\boldsymbol{qq(i,q)}$

	RON	MON	ARO	FLS	CNU	SUL	SG	RVP
srln	69.4	64.2	0	0	0	0	0.694	2.378
rft	103	90.8	74.9	0	0	0	0.818	2.378
hcln	93.2	81.6	18	0	0	0	0.751	12.335
hcds	0	0	0	56	50	0.008	0.832	0
fccln	87.7	75.8	25	0	0	0	0.713	13.876
fcchn	82.3	73.5	20	0	0	0	0.764	19.904
fcclco	0	0	0	53	50	0.009	0.802	0
srds	0	0	0	46	40	0.008	0.852	0

Table 23. $\boldsymbol{\overline{Q}(q,p)}$

	REG	MID	PRE	DE1	DE2	DE4
RON	200	200	200	200	200	200
MON	200	200	200	200	200	200
ARO	60	50	45	200	200	200
FLS	200	200	200	200	200	200

CNU	200	200	200	200	200	200
SUL	0.01	0.01	0.01	0.01	0.01	0.05
SG	0.81	0.81	0.81	0.85	0.87	0.9
RVP	15.6	15.6	15.6			

Table 24. $\underline{\boldsymbol{Q}}(\boldsymbol{q},\boldsymbol{p})$

	REG	MID	PRE	DE1	DE2	DE4
RON	88	91	94	0	0	0
MON	75	78	81	0	0	0
ARO	0	0	0	0	0	0
FLS	0	0	0	40	45	55
CNU	0	0	0	40	40	30
SUL	0	0	0	0	0	0
SG	0.7	0.7	0.7	0.81	0.81	0.81
RVP	0	0	0	0	0	0

Table 25. $\overline{\boldsymbol{R}}(\boldsymbol{i},\boldsymbol{p})$

	REG	MID	PRE	DE1	DE2	DE4
srln	1	1	1	0	0	0
rft	1	1	1	0	0	0
hcln	1	1	1	0	0	0
hcds	1	1	1	0	0	0
fccln	1	1	1	0	0	0
fcchn	0	0	0	1	1	1
fcclco	0	0	0	1	1	1
srds	0	0	0	1	1	1

Table 26. $\underline{\boldsymbol{R}}(\boldsymbol{i},\boldsymbol{p})$

	REG	MID	PRE	DE1	DE2	DE4
srln	0	0	0	0	0	0
rft	0	0	0	0	0	0
hcln	0	0	0	0	0	0
hcds	0	0	0	0	0	0
fccln	0	0	0	0	0	0
fcchn	0	0	0	0	0	0
fcclco	0	0	0	0	0	0
srds	0	0	0	0	0	0

Table 27. $\boldsymbol{TC}$ **(CAD/m^3)**

	0.314

Table 28. $\boldsymbol{UPGcost(upg)}$ **(10^6 CAD)**

uHCproc	7.00
uHTproc	3.63
uCDUcap	8.40
uCRcap	2.475
uHCcap	6.00
uFCCcap	10.50
uGBcap	1.00
uDBcap	1.00

Table 29. $\boldsymbol{UPGcost(upg)}$ **(10^3 m^3/day)**

uCDUcap	9.27
uCRcap	2.65
uHCcap	5.30
uFCCcap	5.30
uGBcap	5.30
uDBcap	4.77

Table 30. $\boldsymbol{Y(i,m,u)}$ **(yield fraction)**

	CR	HC	FCC	HT1
rft.1	0.8			
rft.2	0.9			
hcln.1		0.5		
hchn.1		0.3		
hckero.1		0.1		
hcds.1		0.1		
hcln.2		0.3		
hchn.2		0.2		
hckero.2		0.2		
hcds.2		0.3		
fccln.1			0.5	
fcchn.1			0.3	
fcclco.1			0.1	
fcchco.1			0.1	
fccln.2			0.3	
fcchn.2			0.2	
fcclco.2			0.2	
fcchco.2			0.3	
srds.1				0.072
hclf.1				0.928

Table 31. $\boldsymbol{X(j,m,i)}$ **(%)**

		Crude 1	Crude 2	Crude 3
1	lpg	2.18	1.45	0.86
	srln	6.37	5.91	12.21
	srhn	17.14	16.19	8.00
	kero	15.83	15.21	5.87
	lgo	13.25	13.60	6.73
	hgo	29.87	30.60	29.99
	rsd	16.57	17.05	36.33
2	lpg	1.97	1.23	0.76
	srln	5.76	5.30	10.79
	srhn	15.50	14.51	7.07
	kero	12.12	11.49	2.16
	lgo	25.16	25.51	18.64
	hgo	26.17	26.91	27.39
	rsd	14.52	14.99	33.18

3. Refinery Production Planning Model

The purpose of the production planning model equations is to determine the volumes of products that the refinery should produce in order to satisfy the model objective and what crude oil stocks, intermediate products, and blending strategies must be used in order to satisfy quality constraints associated with each product. Flow of material between process units is defined based on inclusion of set elements in equation definitions. The total volumetric inlet flow to each unit in the refinery is defined by Eq. (A 1). Inlet flow is broken down by mode for those units which have multiple operating modes in Eq. (A 2). The minimum and maximum total inlet flows into a unit are defined by Eqs. (A 3) and (A 4).

$$FA(t,u,n) = \sum_{\substack{m \\ m \in UM}} FVM(m,t,u,n) \quad \forall t \in T, u \in U, n \in N \qquad \textbf{(A 1)}$$

$$FVM(m,t,u,n) = \sum_{\substack{i \\ i \in U_{IN}}} FVM_{IN}(m,t,i,n) \quad \forall t \in T, n \in N, (u,m) \in UM \qquad \textbf{(A 2)}$$

$$FA(t,u,n) \geq MinProd(u)\tau(t) \quad \forall t \in T, u \in U_C, n \in N \qquad \textbf{(A 3)}$$

$$FA(t,u,n) \leq MaxProd(u)\tau(t) \quad \forall t \in T, u \in U_C, n \in N \tag{A 4}$$

Volumetric flow rates of streams exiting a unit are defined using a similar set of equations. Streams entering a unit have a corresponding stream or streams leaving that unit which are defined by specific yield values. Yield relationships are governed by Eq. (A 5). The total volume leaving each unit is defined by Eq. (A 6).

$$\begin{gathered} FVM_{OUT}(i,m,t,u,n) = Y(i,m,u)FVM(m,t,u,n) \\ \forall t \in T, n \in N, (i,u,m) \in UM_{OUT,C} \end{gathered} \tag{A 5}$$

$$FA(t,u,n) = \sum_{\substack{i \\ i \in U_{OUT}}} FV(t,i,n) \quad \forall t \in T, u \in U_C, n \in N \tag{A 6}$$

Unit outlet and inlet volumetric flow rates are calculated on a stream basis using Eqs. (A 7) and (A 8).

$$FV(t,i,n) = \sum_{\substack{m \\ m \in UM}} FVM_{OUT}(i,m,t,u,n) \quad \forall t \in T, n \in N, (i,u) \in U_{OUT} \tag{A 7}$$

$$FV(t,i,n) = \sum_{\substack{m \\ m \in UM}} FVM_{IN}(m,t,i,n) \quad \forall t \in T, n \in N, (i,u) \in U_{IN} \tag{A 8}$$

The CDU uses Eq. (A 9) to compute intermediate yields based on the crude streams entering the unit.

$$\begin{gathered} FVM_{OUT}(i,m,t,u,n) = \sum_{\substack{i' \\ i' \in U_{IN}}} X(i,m,i')FVM_{IN}(m,t,i',n) \\ \forall m \in M, i \in I, t \in T, n \in N, u = CDU \end{gathered} \tag{A 9}$$

The holdings of refining intermediates are defined by balance equations around the intermediate tanks and the initial tank content in Eqs. (A 10) and (A 11), and by constraints which maintain the tank level between its maximum and minimum values in Eqs. (A 12) and (A 13).

$$\sum_{\substack{i \\ i\in TK_{IN}}} FV(t,i,n) - \sum_{\substack{i \\ i\in TK_{OUT}}} FV(t,i,n) + V_{ini}(tk,n) - V(tk,t,n) = 0 \quad \forall tk \in TK, t = 1, n \in N \tag{A 10}$$

$$\sum_{\substack{i \\ i\in TK_{IN}}} FV(t,i,n) - \sum_{\substack{i \\ i\in TK_{OUT}}} FV(t,i,n) + V(tk,t-1,n) - V(tk,t,n) = 0 \quad \forall tk \in TK, t > 1, n \in N \tag{A 11}$$

$$V(tk,t,n) \geq \underline{V}(tk) \quad \forall tk \in TK, t \in T, n \in N \tag{A 12}$$

$$V(tk,t,n) \leq \overline{V}(tk) \quad \forall tk \in TK, t \in T, n \in N \tag{A 13}$$

The process of blending refining intermediates into products is governed by a number of equations and constraints which dictate blend volumes and quality specifications. The volume of a stream to be blended into a particular product is defined by Eq. (A 14). The blended volume of a product is defined by Eq. (A 15).

$$\sum_{\substack{p \\ p\in BL_{OUT}}} VB(i,p,t,n) = FV(t,i,n) \quad \forall t \in T, n \in N, (bl,i) \in BL_{IN} \tag{A 14}$$

$$\sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n) = VBlend(t,p,n) \quad \forall t \in T, n \in N, (bl,p) \in BL_{OUT} \tag{A 15}$$

The minimum and maximum fractions of an intermediate allowed in the blending of a product are defined by Eqs. (A 16) and (A 17).

$$VB(i,p,t,n) \geq RMin(bl)VBlend(t,p,n) \quad \forall t \in T, n \in N, (bl,i,p) \in BLIP \tag{A 16}$$

$$VB(i,p,t,n) \leq RMax(bl)VBlend(t,p,n) \quad \forall t \in T, n \in N, (bl,i,p) \in BLIP \tag{A 17}$$

The total volume processed in a blender is defined by Eq. (A 18). This volume must be within the lower and upper capacity values for each blender, reflected through the constraints in Eqs. (A 19) and (A 20).

$$\sum_{\substack{i,p \\ (i,p)\in BLIP}} VB(i,p,t,n) = VBlendT(t,bl,n) \quad \forall bl \in BL, t \in T, n \in N \tag{A 18}$$

$$VBlendT(t,bl,n) \geq BlendMin(bl)\tau(t) \quad \forall bl \in BL, t \in T, n \in N \tag{A 19}$$

$$VBlendT(t,bl,n) \leq BlendMax(bl)\tau(t) \quad \forall bl \in BLt \in T, n \in N \tag{A 20}$$

Quality properties are divided into three groups: properties based on volume, based on weight, and based on nonlinear relationships. The upper and lower bounds for each property are defined by Eqs. (A 21) to (A 26).

$$\sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q) \geq \underline{Q}(q,p)VBlend(t,p,n) \quad \forall t \in T, n \in N, (bl,p,q) \in BL_{OUT,VOL} \tag{A 21}$$

$$\sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q) \leq \overline{Q}(q,p)VBlend(t,p,n) \quad \forall t \in T, n \in N, (bl,p,q) \in BL_{OUT,VOL} \tag{A 22}$$

$$\sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q)qq(i,q') \geq \underline{Q}(q,p) \sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q') \quad \forall t \in T, n \in N, q' = SG, (bl,p,q) \in BL_{OUT,WT} \tag{A 23}$$

$$\sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q)qq(i,q') \leq \overline{Q}(q,p) \sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q') \quad \forall t \in T, n \in N, q' = SG, (bl,p,q) \in BL_{OUT,WT} \tag{A 24}$$

$$\sum_{\substack{i \\ i\in BL_{IN}}} VB(i,p,t,n)qq(i,q)^{1.25} \geq \underline{Q}(q,p)^{1.25}VBlend(t,p,n) \quad \forall t \in T, n \in N, (bl,p,q) \in BL_{OUT,NL} \tag{A 25}$$

$$\sum_{\substack{i \\ i \in BL_{IN}}} VB(i,p,t,n) qq(i,q)^{1.25} \leq \overline{Q}(q,p)^{1.25} VBlend(t,p,n) \qquad \forall t \in T, n \in N, (bl,p,q) \in BL_{OUT,NL} \tag{A 26}$$

The products produced by blending are either stored in product tanks or delivered to a market for sale. The product tank balances for the initial tank condition and for subsequent time periods take the form of Eqs. (A 27) and(A 28). The maximum and minimum product tank levels are defined by Eqs. (A 29) and (A 30).

$$VBlend(t,p,n) + VP_{ini}(p) - VP(t,p,n) - Prc(t,p,n) = 0 \qquad \forall t = 1, p \in P, n \in N \tag{A 27}$$

$$VBlend(t,p,n) + VP(t-1,p,n) - VP(t,p,n) - Prc(t,p,n) = 0 \qquad \forall t > 1, p \in P, n \in N \tag{A 28}$$

$$VP(t,p,n) \geq \underline{VP}(p) \quad \forall t \in T, p \in P, n \in N \tag{A 29}$$

$$VP(t,p,n) \leq \overline{VP}(p) \quad \forall t \in T, p \in P, n \in N \tag{A 30}$$

At the end of the planning horizon all tank levels should return to their minimum levels. Equations (A 31) and (A 32) enforce this constraint for the sets of intermediate and product tanks.

$$V(tk,t,n) = \underline{V}(tk) \quad \forall tk \in TK, t = T, n \in N \tag{A 31}$$

$$VP(t,p,n) = \underline{VP}(p) \quad \forall t = T, p \in P, n \in N \tag{A 32}$$

The total amount of each product produced by a refiner is delivered to a market for sale. Eq. (A 33) defines the balance between the products produced and those delivered to a market.

$$Prc(t,p,n) - \sum_{w} Dlv(t,p,n,w) = 0 \quad \forall t \in T, p \in P, n \in N \tag{A 33}$$

The costs of crude oil, unit operation, and blender operation are defined by Eqs. (A 34), (A 35), and (A 36), respectively.

$$CrudeOilCost(n) = \sum_{t} \left[(1 + 0.01t) \sum_{\substack{i \\ i \in IC}} Cost(i) FV(t,i,n) \right] \quad \forall n \in N \qquad \textbf{(A 34)}$$

$$UnitOpCost(n) = \sum_{m,n,u} OpCost(u,m) FVM(m,t,u,n) \quad \forall n \in N \qquad \textbf{(A 35)}$$

$$BlendOpCost(n) = \sum_{bl,n} BLcost(bl)\, VBlendT(t,bl,n) \quad \forall n \in N \qquad \textbf{(A 36)}$$

Refiners are able to import products from another seller located elsewhere whose prices are fixed at values of $CI(p,w)$ for refiners in local markets. Buyers in local and global markets do not have access to this purchasing channel; refiners may purchase imports at a price $CI(p,w)$ and sell them in their local market at the market price $\Pr(p,w)$. Imports cannot be sold in global markets and are limited to an amount of 1.589×10^6 m^3 per year of each product by each refiner as a reasonable upper limit. The cost of imports incurred by a refiner is defined by Eq. (A 37).

$$ImpCost(n) = \sum_{p,w} CI(p,w)\, Imp(p,n,w) \quad \forall n \in N \qquad \textbf{(A 37)}$$

Refiners also incur time-based costs which are calculated based on the total amount produced in a given time period and which decrease in each subsequent time period in the planning horizon. Eq. (A 38) defines this cost value which serves, all else being equal, to make production near the end of the planning horizon more efficient.

$$TimeCost(n) = TC \sum_{t,p} (1 - 0.01 \cdot t) Produce(t,p,n) \quad \forall n \in N \qquad \textbf{(A 38)}$$

For convenience of equation writing we define the variable $TotCost(n)$ as in Eq. (A 39).

$$TotCost(n) = CrudeOilCost(n) + UnitOpCost(n) + BlendOpCost(n) + TEC(n) + ImpCost(n) + TimeCost(n) \quad \forall n \in N \qquad \textbf{(A 39)}$$

Deliveries to global markets are unrestricted and are driven purely by competition, but deliveries to local markets by the refiners situated in those markets face contracts stipulating that neither too low a supply of any one product, nor more than the market can absorb, be collectively produced. In local markets the collective supply from local refiners is constrained to fall within upper and lower bounds. Since the model is formulated as a deterministic static game refiners are capable of making competitive plays guaranteed to satisfy these constraints, which take the form in Eqs. (A 40) and(A 41).

$$\sum_{t,n} Dlv(t,p,n,w) \geq \underline{D}(p,w) \quad \forall p \in P, w \in W_L \qquad \textbf{(A 40)}$$

$$\sum_{t,n} Dlv(t,p,n,w) \leq \overline{D}(p,w) \quad \forall p \in P, w \in W_L \qquad \textbf{(A 41)}$$

4. Lists of Figures and Tables

4.1. List of Tables

4.2. List of Figures