TRANSITION FROM HIGH SCHOOL TO UNIVERSITY CALCULUS.

A STUDY ON THE TRANSITION OF STUDENTS FROM HIGH SCHOOL TO UNIVERSITY CALCULUS AT MCMASTER UNIVERSITY.

By:

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance of a thesis entitled "A STUDY ON THE TRANSITION OF STUDENTS FROM HIGH SCHOOL TO UNIVERSITY CALCULUS AT MCMASTER UNIVERSITY" by Marcella Fioroni in partial fulfillment of the requirements for the degree of **Master of Science**.

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Abstract

For many students, the transition from high school to university is difficult. This thesis uses data from a Mathematics Background Questionnaire ("the survey"), completed by first year mathematics students at McMaster University, to answer several questions related to transition in first year Calculus. In addressing these questions, we establish that transition from high school to university mathematics is, in fact, a problem here at McMaster and claim that the Mathematics Background Questionnaire can be used as a tool to help overcome some of these transitional barriers.

The purpose of this study is twofold. Firstly, the study examines students' responses to the survey, and seeks a relationship between these responses and overall performance in the course MATH 1A3. A second purpose for this study is to monitor student performance over the years 2002, 2003 and 2005, based on background information and the results of a particular survey question. This is interesting and valuable information since this period includes the students before, during and after the double cohort group.

It was found that students before and during the double cohort year (2002 and 2003) performed better both on the survey and in MATH 1A3 than the students after the double cohort year (2005). We discuss possible reasons for this; for example, fewer high school mathematics courses taken by the students in 2005 and problems with implementation of the new high school mathematics curriculum in 2000.

Our most important result is that our survey data revealed the following relationship: high survey scores are indicative of high overall final grades in MATH 1A3; low survey scores do not predict overall final grades. Thus, based on our ability to use our questionnaire to make predictions about a student's performance, we can take steps towards helping students make their transition to university calculus more successful. At this point, it is important to focus on incorporating survey feedback to students so that they have a better idea of what to expect in MATH 1A3 and how to proceed in the course.

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To my patient and caring family and friends – thank you for understanding me and not understanding math.

To family, friends and Harper.

Contents

Introduction

For many students, the transition from high school to university is difficult. This is not a new discovery – transition has been acknowledged and written about in recent years. Still, it is a problem that persists. Specifically, it is a problem that persists here, at McMaster University in the Mathematics and Statistics Department (and there is evidence of this problem in other departments at McMaster). Students entering first year Calculus at McMaster are faced with the challenges of both adjusting to the university environment and adjusting to university mathematics. This has become increasingly more difficult for graduates of Ontario high schools, since the Ontario high school curriculum was revised in 2000.

To date, there is no literature about transition from high school to university mathematics at McMaster University. This thesis uses data from a Mathematics Background Questionnaire, completed by first year mathematics students at McMaster University, to answer several questions related to transition in first year Calculus. In addressing these questions, we establish that transition from high school to university mathematics is, in fact, a problem here at McMaster and claim that the Mathematics Background Questionnaire can be used as a tool to help overcome some of these transitional barriers. In doing so, we hope to stimulate interest in the topics of transition and Mathematics Education at McMaster University.

Statement of the Problem

Each year, over 1500 students take MATH 1A3, called Calculus for Science, at McMaster University. The average for students completing this course in 2005 was 67.40% (C^+). In 2005, 200 students failed and 150 students dropped the course (out of 1628 students enrolled originally).

Calculus for Science is a mandatory course for most Science programs at McMaster. As a result, MATH 1A3 draws students from a range of different faculties attracting students with various amounts of mathematical background. For some students, MATH 1A3 will be the only mathematics course they take in university. For others, there will be several mathematics courses that follow.

Most students entering MATH 1A3 attended high school in Ontario - some students for all years of high school and some students just for their final year (this is true for many international students who come to Canada to complete their last year of high school before attending university). These, then, are students who have been educated in Ontario according to a curriculum that has undergone major reconstruction in recent years.

So the students that enter MATH 1A3, as a whole, are faced with many adversities. Some are typical of the transition from high school to university: change in environment, class size, ranging background mathematics education, changing expectations of instructors, etc. But in addition to this, most are also challenged with the process of adjusting to the recent curriculum changes in Ontario. In Ontario, starting September 2000, the number of years spent in high school was reduced from 5 years to 4 years which affected all high school students. Students taking mathematics courses, however, were subject to another change major change in the curriculum: the 2000 high school mathematics curriculum reflects a different ideology focusing more on teaching for mathematical thinking, communication and application (this will be described in more detail in later sections).

Although the overall average for Calculus for Science in the past few years was a respectable 70.91% (averaged over 2002, 2003 and 2005), not all students are successful. Throughout the semester, students show inadequacies in a range of areas – both procedural and conceptual.

In this paper, the problem that we are addressing is that of transition of students at McMaster University from high school mathematics to MATH 1A3. This problem is highly relevant – once a student is accepted at McMaster University, it becomes *the university's* responsibility to support students and give them the help they need to succeed in MATH 1A3 (and all courses, for that matter). We have one semester to effectively teach the course material to all students, regardless of background, program, etc., to the best of our ability. Researching this problem will help both students and instructors with the transitional difficulties in teaching and learning MATH 1A3.

Purpose of the Study

The purpose of this study is twofold. Firstly, the study examines students' responses to a Mathematics Background Questionnaire ("the survey"), and seeks a relationship between these responses and overall performance in the course. If strong patterns are found, we hope to use this information to loosely predict student performance in the course before the term is underway. Using this prediction, we can help weaker students by directing them to university resources (for example, the Math Help Center, TA and instructor office hours, etc.) or suggest some options with them (for example, taking the MATH 1K3 Calculus bridging course, private tutoring, etc.). A second purpose for this study is to monitor student performance over a span of 4 years, based on background information and the results of a particular survey question. This is interesting and valuable information since this period includes the students before, during and after the double cohort group.

With these two key purposes in mind, the overall objective of this paper is to provide literature specific to McMaster University that will inform educators, students, parents, administration and any interested party on transitional issues here, and stimulate interest in smoothing the transition from high school to university mathematics at McMaster University.

Research Questions

In our paper, we aim to address the following questions:

- Does a student's background information help predict what to expect from the student in MATH 1A3? For example, does the quantity and/or type of mathematics courses taken in high school have an effect on students' performance in MATH 1A3?

- How do students before, during and after the double cohort perform on the Mathematics Background Questionnaire? How do students before, during and after the double cohort perform in MATH 1A3?

- How do students before, during and after the double cohort perform on communications and applications oriented questions? What are some typical responses by students to these questions?

- Is there a significant correlation between performance on the Mathematics Background Questionnaire and a student's final grade in MAT 1A3?

Theoretical Perspective

When studying the cognitive processes of learning, there has been plenty of research done from a theoretical viewpoint that can be used to explain findings. For example, researchers often reference Piaget's work in developmental psychology. In this thesis, though, we are not studying students' learning processes in first year Calculus (however, we do address issues such as procedural and conceptual learning in the Discussion section of this paper). Instead, we are focusing on the process of transition from high school to university mathematics and the problems associated with this process.

To date, there is no theory in existence that models transition from high school to university. Presently, there exists literature on the topic of transition – see the Literature Review – but no work has been published giving a theoretical model for transition from high school to university (there is no "Piaget Model of Transition" so to speak). As a result, this thesis does not have a theoretical framework to work from and cannot be analyzed in a theoretical context. What this paper does, though, is contribute to the field of research on transition on a practical level, adding to literature of the type discussed in the Literature Review.

Definition of Terms

In 2000, Ontario introduced a new high school curriculum. This new curriculum reduced the number of courses required to graduate secondary school. More specifically, OAC courses (Ontario Academic Credit courses, taken after grade 12) were removed entirely, reducing a student's high school term from 5 years to 4 years typically. The implications of this change were felt in 2003, when secondary schools were graduating students taught using the old curriculum and the new curriculum at the same time. That is, in 2003, Ontario students having OAC courses were leaving high school with students having grade 12 as their highest level of secondary education. This group of students who graduated from high school in 2003, some students with OAC education and some with grade 12, are referred to as the **double cohort** group.

When describing student knowledge, we speak of it as **procedural** and **conceptual**. **Procedural knowledge** refers to a student's knowledge of mathematical procedures and the ability to identify and correctly execute algorithms for solving a given problem. In particular, students with procedural knowledge have "(a) knowledge of the format and syntax of the symbol representation system and (b) knowledge of rules and algorithms, some of which are symbolic, that can be used to complete mathematical tasks". Students may exhibit procedural knowledge without knowing exactly what they are doing or why they are doing it. More importantly, they may not know how to interpret the mathematical task at hand or where it fits into the larger mathematical scheme.

Conceptual knowledge, on the other hand, refers to understanding of the mathematical ideas behind the problem being solved. This is both in the context of the problem itself (for example, understanding why setting a derivative to zero may yield an extreme value) and in the context of the larger mathematical concept (for example, using extreme values to curve sketch). Furthermore, understanding of relationships between different mathematical concepts and representations underlying the mathematics exemplifies conceptual knowledge.

It is important to note that both procedural and conceptual knowledge are required to fully grasp mathematical problems and solve them correctly on a consistent basis. Also, one should realize that procedural and conceptual knowledge are not separate from each other. That is, conceptual knowledge fosters procedural learning since after a problem is understood by a student, it is solved using mathematical skills. Similarly, procedural knowledge fosters conceptual knowledge since after a solution is computed, the student will evaluate and interpret the result and be lead to find meaning in the answer.

About 20 years ago, a new school of thought was developed concerning the way in which mathematics should be taught. It suggested a shift in the main focus of mathematics teaching from performing computations and rote practice (procedural knowledge) to the understanding of mathematical ideas and experiences (conceptual knowledge). This new ideology, named the **mathematics reform**, swept North America and was accepted by some and criticized by others. In Ontario, the changes to the high school mathematics curriculum in 2000 catered to the goals of the mathematics reform, some of which will be mentioned in this paper.

Delimitations and Limitations of the Study

Delimitations:

This study has confined itself to consider only one question on the Mathematics Background Questionnaire. The question we chose as our focus is described in detail in the Methodology section of this paper. Also, in obtaining participants for this study, we were restricted to the number of students who were interested in volunteering to complete the questionnaire. This amounted to roughly 200-300 first year students at McMaster

University enrolled in Calculus for Science. In this process, our study sample was confined to between 197-317 participants for each year.

Limitations:

Since we choose to focus on one specific question for our short study, we have sacrificed breadth for depth. Also, we are limited by the logistics of the project. Namely, the students at McMaster University were the only population sampled, which makes it hard to generalize any conclusions to an Ontario-wide or international level. Furthermore, this study only concerns itself with the material directly related to MATH 1A3 (Calculus for Science) and no other first year mathematics course. Of course, we are also limited by time, since this questionnaire must be completed in one lecture hour (50 minutes). Ideally, we would like to conduct personal interviews with students, but, due to the time required for interviewing, this cannot be made a reality.

Significance of the Study

This study has immediate significance for the students and instructors at McMaster University, particularly for those involved in MATH 1A3 in some way. The results presented in this paper are based entirely on information gathered from students in MATH 1A3, and the discussion herein provides valuable feedback to them. It also provides information about previous MATH 1A3 classes and the changes that have been made in the past to improve the course. Thus, this study will inform past, present and future students and instructors of the course.

The Mathematics and Statistics Department will also find this paper useful for purposes of feedback and improvement within the Department. In particular, those who are involved in making decisions about things such as admission requirements, bridging courses, the McMaster Math Help Centre, etc. will find this paper informative. This paper presents more specific and detailed feedback to be considered, rather than just final grades and instructor evaluations. Since this study includes survey samples before, during and after the Ontario high school mathematics curriculum changes made in 2000, our results will be interesting for the Department to consider in light of the new high school curriculum changes being implemented in 2007.

Several institutions have shown interest in our data and data analysis even before this paper was underway. In particular, the Ontario Mathematics Education Forum – a forum comprised of university instructors, high school teachers, school boards, Ministry of Education, and general public – expressed curiosity and support of our research after a presentation of our preliminary results (see appendix for attached power point presentation). To our knowledge, no other university has such data and cannot provide as detailed information about transition at an Ontario university. Again, considering the forthcoming changes in the mathematics curriculum, this paper will serve to inform members of the Forum.

Our study is unique in that it examines transition at an Ontario university. This paper adds to literature on transition at other universities worldwide so that transition may be studied on an international scale. Furthermore, our research involves a large amount of raw data (hundreds of participants each year) which is much different than the typical case study analysis (fewer participants, more detailed data). This is significant to our data analysis because having such a large sample population makes our statistical findings robust.

Review of the Literature

Informally, there has been interest in the field of Mathematics Education at the tertiary (university and college) level for about 30 years (Artigue, 2001). However, it was only recently that Mathematics Education has become something of a hot topic in both mathematics and education faculties. This is happening on an international scale. As the topic began proving itself relevant in the educational system, research quickly moved forward. Although most literature on topics in mathematics education at the tertiary level has not yet been developed from a theoretical viewpoint, there is an abundance of articles written by mathematics educators and advocates of improved mathematics education. A large portion of these articles are survey studies and dissemination of university-specific anecdotes revealing what worked and what did not in their respective mathematics or education faculties. These writings have been the primary source for informing those who are interested in researching mathematics education. In this section of our paper, we briefly review literature that has influenced our research and observations.

Some articles that have informed this paper are those on the larger topic of the role of research in mathematics education. In her paper entitled "What Can We Learn from Research at the University Level?" (2001), Michele Artigue claims that although educational research is highly diverse and often research results do not easily produce practical results, she "is convinced that existing research can greatly help us today, if we make its results accessible to a large audience and make the necessary efforts to better link research and practice" (Artigue, 2001, pg. 207). Her research focuses on university material and students' learning processes at this level. She believes that "research helps our understanding of the complexity of the necessary cognitive constructions and, at the same time, shows the insensitivity of the educational system to this complexity" (Artigue, 2001, pg. 216). This author points out that though research in education cannot give general solutions to the problem of teaching and learning at the university level, research has in the past promoted changes in universities that have had some success. This can be said about McMaster University – informal research has initiated developments such as the Math Help Centre, training for mathematics TA's and adjustments in the curriculum.

In the paper "Tertiary Mathematics Education Research and its Future" (2001), Annie Selden and John Selden express beliefs similar to Artigue in that research in mathematics education is useful for purposes of informing a larger audience (Selden and Selden, 2001). In particular, this information is valuable for teaching and curriculum development (Selden and Selden, 2001, pg. 247). The authors emphasize that mathematics education at the university or college level is largely unstudied; literature on the topic is limited, there are few programs that offer and/or support research in the field and there is the additional obstacle that many who wish to study mathematics education do not have the required mathematical background to do so (Selden and Selden, 2001). More important is the "low status" that some mathematicians place on studying mathematics education (Selden and Selden, 2001, pg. 247). As a result, research in mathematics education is having trouble finding its "academic home" (Selden and Selden, 2001, pg. 247). In fact, our paper is being written as part of a Masters Program at McMaster University (Mathematics and Statistics Department) under special permission from the Department. In Canada, there is very little funding available for research in mathematics education and currently there is no program available to those who wish to pursue this research at McMaster University. In the paper by Selden and Selden (2001), it is suggested that graduate students who are currently studying mathematics – and are "tomorrow's university mathematics teachers" – should seriously consider the benefits of research in mathematics education. This would be easier if more programs, seminars, mini-courses or courses were available to them.

Alan Schoenfield distinguishes between mathematics research and mathematics education research in his paper "Purposes and Methods of Research in Mathematics Education". The most significant difference is that in mathematics research, there are proofs to support theories whereas in mathematics education, theories are works in progress (Schoenfield, 2000, pg. 643). It is possible that this is the underlying reason some mathematicians deem research in mathematics education unfit for them. Schoenfield feels that many mathematicians view research in mathematics education as a quest to answer only one question – "tell me what works in the classroom". Of course, this question is not immediately answerable; the purpose of research in mathematics education is to explore this and related questions (Schoenfield, 2000). Schoenfield says "an understanding of the differences [between mathematics and mathematics education] is essential if one is to appreciate (or better yet, contribute to) work in the field. Findings are rarely definitive; they are usually suggestive". Since mathematics education is still a young field, it is important to define what this research actually is in order for it to establish a reputation for itself. Only then will programs and funding stabilize, along with research work and practical results.

One area of mathematics education that has received particular attention is the topic of transition from secondary (high school) to tertiary (university or college) education (this is the main topic of our paper). These papers focus mainly on first year university students and the obstacles faced by the students and institutions during this transitional time. The paper by De Guzman, Hodgson, Robert and Villani (1998) discusses the difficulties in transition for students entering first year who are either specializing in mathematics or taking mathematics as part of their degree (i.e., engineering or teaching). The authors used a questionnaire to survey students in Canada, France and Spain and also informally surveyed some first year university mathematics teachers. In doing so, the authors proceed to list apparent difficulties in the transition from secondary to tertiary mathematics (epistemological and cognitive, sociological and cultural, didactical) and some suggested methods to help reduce these transitional difficulties (for example, providing orientation for first year students). The same issue is addressed in the paper written by Gruenwald, Klymchuk, Jovanoski (preprint, 2004), but these authors choose to focus their survey on university lecturers. More specifically, they surveyed 63 university lecturers from 24 countries asking questions about transitional difficulties at their respective universities and what their department has done (or could do) to smooth this transition. Leigh Wood (2001) studies what she calls the interface between secondary and tertiary mathematics. This paper raises many questions that educators should consider when designing courses or curricula and describes several options for support services for students (Wood, 2001). Wood reminds us that numbers of students taking mathematics is on the increase and "mathematics needs to adapt to these changing circumstances and make all students who want to study at the tertiary level welcome" (Wood, 2001).

All three papers present discussions and conclusions as case studies from various universities. There are a number of papers that have been written in similar spirit to these. Our paper is guided by and related to these writings in that our research involves studying transition, but it is unique in that our data was collected from students at McMaster University.

In 2000, the Ontario Ministry of Education revised the high school mathematics curriculum for the first time in 15 years. This was part of a massive mathematics reform that swept North America. Hence, Ontario curriculum guideline documents from 1985 and 2000 have become important pieces of literature for our study. Data collected for this paper is critiqued in light of the guidelines provided by the Ontario Ministry of Education to high school educators before and after 2000.

Most closely related to our paper are several articles written by Miroslav Lovric and Ann Kajander. Their work focuses on the topic of transition from high school to university calculus at McMaster University and Lakehead University. The authors use the same Mathematics Background Questionnaire that is used in this paper (in fact, we share the same data for the McMaster University questionnaires). Lovric and Kajander have used this data to improve their teaching of first year calculus courses. Their papers report the process of analyzing questionnaire results, correlating these results with final marks and their attempts to adjust to help smooth the transition. Of course, the content of these papers overlaps with this paper to some degree; however this paper is more extensive and aims to set the ground work for future research at McMaster University.

Methodology

A Mathematics Background Questionnaire was developed by Dr. Miroslav Lovric, whose motivation for creating the survey was to produce a tool to gauge the level of mathematics knowledge of students in his highly populated first year Calculus for Science course at McMaster University. The enrollment for first year Calculus at McMaster sits around 1500-1600 students, ranging from local to international students, with varying degrees of mathematical background and experience.

The survey is composed of two sections. The first page asks students to provide demographic information as well as to comment on their high school experiences and expectations in the Calculus for Science course. More specifically, students are asked to list any high school(s) they attended and the language spoken in their parental homes. This is followed by the questions "Describe your experiences with high school mathematics courses that you took" and "What are your expectations about the Calculus course that you are taking now?" The remainder of the survey consists of math problems that are designed to test a very specific set of knowledge and skills. In particular, the survey addresses:

- Basic technical and computational skills (fractions, equations)
- Basic notions for functions (range, composition)
- Familiarity with transcendental functions (exponential, logarithm, trigonometric functions)
- Written communication of mathematics ideas ("explain" type questions)
- Proficiency in multi-step problems
- Drawing and interpreting graphs of functions
- Applied problems (involving computation and interpretation)

The survey assesses these areas in the form of 12 mathematics problems. Dr. Lovric consulted high school mathematics teachers and used his own classroom experience to select problems to include in the survey. Two different versions of the survey were created, called Survey A and Survey B. Most of the questions are different on the different surveys, but some appear on both. The survey was altered slightly over the years, but still reflects the original purpose completely.

The primary purpose of the survey was to quickly provide an overall sense for the students' strengths and weaknesses in the early days of the semester. In this way, Dr. Lovric was able to tailor the early stages of the course according to the general performance on the survey problems. For example, past survey responses have revealed that the topic of functions must be taught from a very elementary level. Students were fairly comfortable with the concept of domain, but questions involving the concept of range were answered poorly (in particular, the range of $y = |x|$ was not done well). Surveys also conveyed an incomplete understanding of Trigonometry, prompting a heavier emphasis on Trigonometry in the MATH 1A3 course. Responses illustrated students had a good understanding of basic functions (linear and quadratic functions) and would not require extra time on this topic. Overall, the surveys proved to be a useful tool for collecting information about student knowledge quickly and early enough in the term for Dr. Lovric to act on any problem areas.

Prior to the creation of the survey, it was foreseen that the data collected by students in MATH 1A3 would contain valuable information for Mathematics Education researchers. With the potential for research value in mind, an important statement was included on the survey stating to students that their participation in the survey is optional and that their responses would never be reported on an individual basis. When applying for ethics approval, this statement allowed us to use the surveys for research purposes.

Due to the large class size of MATH 1A3 at McMaster University, each year the students are divided into several sections (in 2006, there are 5 sections). It is always the case that one section has an extra lecture in the first week. As to not disturb the lecture schedule, the section with the extra lecture is chosen for participation in the survey.

The survey is intentionally unannounced to the students to ensure they will not study for it. It is desirable to test them without having studied in order to capture their knowledge strictly out of high school study, and not what they may have studied on their own in preparation for the survey. A representative explains the survey and its benefits to the class, clearly stating that the survey is voluntary and that responses will not be reported individually. The students are also advised to try to answer the survey on their own and to avoid consulting their neighbours. It is explained that if the surveys are completed using a group effort, it will not be a true representation of individual knowledge. Once this has been explained and the papers distributed (this takes about 10 minutes) the remainder of the class period is used by students to complete the survey.

Roughly 200-300 completed surveys are collected and marked. The marking scheme can be found later in this section. All results were collected and entered into an excel file, including demographic details. The results were then merged (by a third party) with students' final grade in MATH 1A3. Any identifying information was then removed and the resulting file was used to examine the data.

Dr. Lovric began using the survey in 2000, and continued to do so every year thereafter to this date (except for 2004 when there was no extra lecture in the first week of class). The data collected prior to 2006 required ethics approval in order to be used for research. McMaster Ethics Review Board approved our application on April 5th, 2006, with Research Ethics Approval Certificate number 2006-118. This certificate allows us to use data collected up to and including 2005. For this data set, Dr. Miroslav Lovric is the supervisor and Marcella Fioroni is the student investigator.

After 2005, the technique for administering the tests changed slightly to abide by ethics clearance procedures. This was done to ensure we have ethics approval for research on data as it is collected (not retroactively, as it was up to and including 2005). The procedure is now a two-step process where a representative must visit the classroom twice. The first visit is to distribute the surveys to be completed by the students as before. During the second visit, the representative hands out an information sheet to the students (for students to keep), explaining the research value contained in the surveys. They are advised that their responses will never be reported individually and that they have the option of pulling their survey out of the research study at any time. Students are given the opportunity to ask questions about the research. When this is done, a permission form is given to the students. They have the option to grant permission for their survey to be used for research purposes, or they can simply leave the form blank if they do not wish to participate. Forms are collected and later matched with the completed forms. Only these surveys are marked and used for research. The second visit takes roughly 15 minutes of class time. This process received clearance by McMaster Ethics Review Board in August, 2006 with Research Ethics Approval Certificate number 2006-013. Starting 2006, the title for the project was broadened with the student investigator left unlisted, allowing more flexibility for future work.

We discussed implementing this permission process as an on-line option; however this may result in fewer participants if we put the onus on the students to find the form. This could be a problem if a large number of students do not grant permission since we hope to maintain a sample size of roughly 200 students per year. It proved to be successful with the second visit to the classroom and providing students with quick access to the forms.

In this paper, we chose to focus on one particular survey question. The question provides a graph of a position function s(t) and, in reference to s(t), asks "Describe the velocity v(t) as increasing or decreasing. Explain how you know.". The question was chosen because it is conceptual in nature and is applications based. It was also chosen because curriculum changes made in 2000 are reflective of the mathematics reform ideology – students need to be engaged in mathematical thinking and work on understanding and communicating mathematics. Students are expected to be able to describe the velocity using tools from mathematics, such as slope of tangent, concavity, distance traveled over equal time intervals, etc.

The focus question was marked several different ways before a suitable scheme was found. At first, a 0/1/2 scheme was used. Typically, this meant 0 for an incorrect answer, 1 for the correct response with incorrect explanation or no explanation at all, 2 marks for correct response and correct explanation. Due to the varying degree of "correctness" for a response awarded 2 points in the 0/1/2 scheme, the scheme was replaced by a 0/1/2/3/4 scheme. This scheme also proved to be insufficient, mostly due to the quality of the responses that received 4 points. Finally a 0/1/2/3/4/5 scheme was developed in order to best capture varying degrees of correctness (see detailed scheme below) and used to grade student responses over the 3 years. A second scheme (borrowed from Varsavsky, 2003) was used in order to capture a more qualitative analysis of student responses. Expected responses as well as responses that appeared commonly throughout the survey were given letter codes. Each student response was matched to as many codes necessary to properly represent their answer. See details for the letter coding scheme below. The results of both the numerical and the letter coding schemes were collected in tables, allowing for analysis across class years.

Numeric Grading Scheme:

Score	Description
0	Incorrect answer (decreasing). If it was accompanied by an explanation, the explanation received a letter code.
1	Correct answer (increasing) without an accompanying explanation.
2	Correct answer with some attempt at an explanation. In this case, the explanation is either mathematically incorrect, not relevant (eg. *Increasing, since the slope is positive*) or nonsensical (eg. *Increasing by the way the slope is*).
3	Correct answer with accompanying explanation. In this case, the explanation is either on the right track, but not precise enough (eg. *Velocity is increasing because as the independent variable (time) increases, the dependent variable (distance) also increases, and it increases in the form of a curve going up to the*

	right) or partly correct and partly incorrect (eg. *The velocity is increasing b/c velocity is the slope. The slope is getting "steeper" and "steeper" therefore the velocity is increasing. They're going a greater distance in a shorter amount of time*).
4	Correct answer with accompanying explanation. In this case, nothing about their statement is incorrect, but the response is either not completely precise or there is some ambiguity in the terminology (eg. *The velocity is increasing, because the graph becomes steeper as time progresses*).
5	Correct answer with accompanying explanation. The explanation is precise and at most would have a minor detail missing. (eg. *Increasing since the slope of the tangents are increasing over time* and *Increasing because the tangents to the curve are increasing* will both receive 5)

Table 1: The numeric grading scheme used to mark the velocity question on the survey. Some examples are included.

Letter Coding Scheme:

Letter Code	Description
P	Uses the shape of the curve to explain their answer. If stated properly, this could be adequate (i.e., *the velocity is increasing because the curve is concave up*).
M	Involves the (inadequate) statement that more distance is covered in less time.
R	Involves the adequate statement that more distance is being covered over set (equal) time intervals.
G	Uses the assumption that the graph is a specific curve such as a parabola, exponential or power curve (inadequate, but on the right track).
B	Mentions that the slope/curve/graph/slope of curve/slope of graph is increasing. If they mention the slope of the tangent is increasing, they get full credit.
S	Says the slope is positive (inadequate).
D	Uses or mentions derivatives in some way; basically, understands the idea that a derivative is involved.
I	Says the velocity is increasing because as time increases, position/distance/displacement increases.
V	Says the velocity is increasing because the velocity/speed is increasing (this response was surprisingly not rare).
T	Mentions the term ‘tangent’ somewhere in their response.

Table 2: The letter coding scheme used to code the velocity question on the survey.

After the surveys were marked, they were entered into spreadsheets for analysis. Every question was entered, except for two appearing on the first page (“Describe your experiences with high school mathematics courses that you took” and “What are your expectations about the Calculus course that you are taking now?”). These questions are will be entered at a later date when survey data is analyzed in its entirety.

Research Design

In this section, the Mathematics Background Questionnaire is described in full detail. Each survey question is stated, along with a quick rationale of its purpose on the survey. Dr. Lovric constructed the survey based on recommendations from high school mathematics teachers and his own experiences teaching first year Calculus at McMaster University.

The survey format was chosen in order to collect a large amount of data in a short amount of time. Calculators were not permitted (some conceptual questions involving reasoning with decimals and fractions would be reduced to trivial questions if calculators are used).

Over the years, the order of questions has been shuffled to test the likelihood of a question being skipped because of its place on the survey. For example, it is possible that the last question on a survey is skipped by students simply because it is the last question, not because it was beyond the student's capability. The questions described below were all on the survey, but may have appeared in different order.

Cover Page of Survey (Surveys A and B):

Two versions of the survey were created called Survey A and Survey B. The primary difference in the surveys is the heavy weighting of Survey B towards "thinking" questions. All questions involve reasoning rather than routine operations to find solutions. The purpose of this shift in Survey B is to monitor the changes in curriculum towards producing students with "skills of reasoning, problem solving, and communication; and, most importantly, with the ability and the incentive to continue learning on their own" (Ministry of Education, 2000). Survey A contains questions of both procedural and conceptual nature, but does not focus as dramatically on reasoning and problem solving as Survey B.

For the purposes of this paper, we only consider Survey A in our analysis and not Survey B. Survey B will be studied in detail at a later date. As such, our description that follows for Survey A contains more detail than Survey B with regards to curriculum guidelines that each survey question addresses. These curriculum guidelines are used in our data analysis.

Both surveys share the same first page, which asks students the following:

- Gender
- Age
- High school(s) attended (stating school name, location, and how long it was attended)
- High school math marks
- High school courses taken
- Language spoken at home
- Description of experiences with high school mathematics
- Expectations of Calculus course being taken presently (at McMaster)

These questions are asked in order to collect the demographic data of the students. Age and high school math courses taken were of particular interest to obtain information about students before, during and after the double cohort group. Language spoken at home reflects the cultural diversity of the students in the Calculus for Science course.

The questions about experience and expectations provide a purely qualitative analysis of participants for us to consider. Students' thoughts on their high school mathematics experience give an overall impression of whether incoming students feel positively or negatively towards learning mathematics. The question asking expectations of the course communicates students' fears and anxieties about the future year, as well as their ideas of what university calculus is.

SURVEY A:

1. Indicate whether each of the following formulas is correct or not. Circle your choice. You do not have to justify your answer.
(a) $x^2 + y^2 = (x - y)(x + y)$
(b) $\left(e^x\right)^2 = e^{x^2}$
(c) $\ln(2x) = 2\ln(x)$

This question was chosen because it:
- tests knowledge of basic rules on exponentials, logarithms and factoring/expansion;
- is assumed knowledge for MATH 1A3.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:
- requires "facility in the algebraic manipulation of polynomials" and the ability to "solve exponential and logarithmic equations" as per the Ontario curriculum for grade 12 in 2000 (Ministry of Education, 2000);
- requires "solving problems involving exponential and logarithmic functions" as per the Ontario curriculum for grade 12 in 1985 (Ministry of Education, 1985).

2. The revenue of a company is modeled by $R(x) = x(50 - x)$, where x is the price per item, $0 \le x \le 50$.
(a) Determine the rate of change of the revenue with respect to the price when the price is 10 dollars and when the price is 15 dollars.
(b) Explain what the values of the rate of change above mean to the company.

This question was chosen because it:
- is a multi-step question, but does not make the steps explicit for the student (the student is expected to do this on his/her own);
- tests understanding of modeling a revenue function using a parabola;

- requires conceptual knowledge and understanding of rates of change/derivatives, as well as procedural skills, and the student's ability to explain the meaning of the solution.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- tests whether the student can "determine and interpret rates of change of functions drawn from the natural and social sciences" (Ministry of Education, 2000);
- tests ability to explain what the rate of change means in the context of the question (i.e., the student should be able to "compare the key features of a mathematical model with the features of the application it represents") (Ministry of Education, 2000);
- tests if student can "communicate findings clearly and concisely, using an effective integration of essay and mathematical forms" (Ministry of Education, 2000);
- tests whether the student can "[solve] problems involving rates of change in the natural and social sciences" (note there is no mention of "interpreting" here) (Ministry of Education, 1985).

3. Indicate whether each of the following statements is correct or not (circle your choice). Explain your answer.

(a) $f(x) = (x + a)(x + b)$, then the graph of $f(x)$ cuts the x-axis at both a and b.

CORRECT   NOT CORRECT

(b) If $a > b$, then $\frac{1}{a} < \frac{1}{b}$ for all real numbers $a, b \neq 0$.

CORRECT   NOT CORRECT

This question was chosen because it:

- tests ability to reason with quadratics in factored form and with the function $1/x$;
- tests for problem solving skills since these questions have no set algorithm for a solution and are not necessarily solved by inspection;
- requires knowledge of roots (or solutions) of a quadratic, familiarity with fractions and how one must consider different cases to a problem before attempting to solve it.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- tests "facility in the algebraic manipulation of polynomials", more specifically, ability to "determine an equation to represent a given graph of a polynomial function, using methods appropriate to the situation" (Ministry of Education, 2000);
- tests ability to "determine, through investigation, the properties of $1/x$" as part of the grade 11 curriculum for 2000 (Ministry of Education, 2000);
- tests ability to "[identify] the intercepts of polynomial and rational functions" (Ministry of Education, 1985);

- as part of the grade 11 curriculum, tests ability to "[investigate] examples of functions" (Ministry of Education, 1985).

4. A ball is thrown from a building into the air and falls on the ground below. The height of the ball t seconds after being thrown is $y = -5t^2 + 30t + 35$ metres.
(a) Determine the maximum height of the ball above the ground.
(b) After how many seconds does the ball hit the ground?

This question was chosen because it:
- requires understanding of trajectory of a thrown ball modeled by a parabola;
- can be solved using different methods (i.e., completing the square, taking derivatives, quadratic equation);
- requires conceptual knowledge to properly set up the algebra and procedural knowledge to solve the computation.

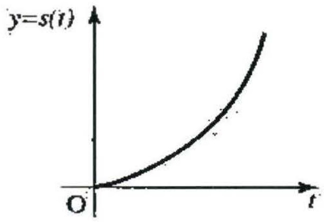
Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:
- may require determination of "the characteristics of the graphs of polynomial functions" and "facility in the algebraic manipulation of polynomials" if the student chooses to solve by completing the square or by quadratic equation (Ministry of Education, 2000);
- may require an understanding of "the key features of the graph of [a] function (i.e., intervals of increase and decrease, critical points …), using the techniques of differential calculus" if the student chooses to solve using differentiation (Ministry of Education, 2000).
- may require the ability to "[define] a critical value of a function as number for which the derivative is zero or does not exist" (Ministry of Education, 1985).

5. What is the range of the function $h(x) = |x|$?

This question was chosen because it:
- tests knowledge of range and techniques for determining range;
- requires familiarity with the absolute value function;
- can be solved using different methods (i.e., state the range of the given function $y = |x|$ from memory; graph the function and deduce the range from the graph).

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:
- there are no objectives relating directly to absolute value in the curriculum guideline for 2000 other than students must "describe intervals and distances using absolute-value notation (Ministry of Education, 2000);
- requires "identifying the relationship between the graph of $y = f(x)$ and [...] $y = |f(x)|$ " and the use of the "concept of absolute value" (Ministry of Education, 1985).

6. A position function $y = s(t)$ is given below.



(a) Describe the velocity $v(t)$ as increasing or decreasing. Explain how you know.
(b) Is the acceleration $a(t)$ positive or negative?. Explain how you know.

This question was chosen because it:

- requires understanding of how position, velocity and acceleration are related;
- tests if student can "demonstrate an understanding of the graphical definition of the derivative of a function" and/or "demonstrate an understanding of the relationship between the derivative of a function and the key features of its graph".
- tests ability to communicate their answer;
- requires conceptual knowledge.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- tests if student can "identify the nature of the rate of change of a given function, and the rate of change of the rate of change, as they relate to the key features of the graph of that function (Ministry of Education, 2000);
- tests if student can "solve problems of rates of change drawn from a variety of applications (including distance, velocity, and acceleration) (Ministry of Education, 2000);
- tests if student can "communicate findings clearly and concisely, using an effective integration of essay and mathematical forms" (Ministry of Education, 2000);
- there is no clear objective for this in the 1985 curriculum guideline, though "solving problems involving rates of change in the natural and social sciences" or "solving problems involving displacement, velocity, acceleration, and time using differential equations" should have helped students answer this question (Ministry of Education, 1985).

7. Solve the equation $\dfrac{x^2+5x+6}{x^2+7x+10} = 2.$

This question was chosen because it:

- requires algebraic skills;
- tests conceptual knowledge (i.e., domain of a function before and after algebraic manipulation);
- can be solved using different methods (i.e., factor and cancel; cross multiply).

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- requires "facility in the algebraic manipulation of polynomials" (Ministry of Education, 2000);
- there is no clear objective mentioning domain of rational functions, however this is implied in the curve sketching section (Ministry of Education, 2000);
- uses the concept in the grade 12 curriculum of considering the domain when "dividing a polynomial by a binomial" (this question is an extension of this concept) (Ministry of Education, 2000).

8. Solve the equation $4^x = 16^{2x-2}$.

This question was chosen because it:

- requires knowledge of exponential functions and possibly logarithms and their laws (if logarithms are used to solve the problem);
- tests ability to solve for variables in exponents;
- can be solved using different methods (i.e., finding a common base, then using the equality to solve; taking the logarithm of both sides).

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- requires the student to "solve exponential and logarithmic equations, using the laws of logarithms" (Ministry of Education, 2000);
- tests if student can "[solve] problems involving exponential and logarithmic functions" (grade 12 curriculum) (Ministry of Education, 1985).

9. Compute the composition $(g \circ f)(x)$ or $g(f(x))$ of the functions $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x} + 1$.

This question was chosen because it:

- requires proficiency with composition of functions in a general form (in terms of x, not evaluated at a point);
- can be used to teach principles of the chain rule;
- tests understanding of different notations for composition of functions.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- tests if student can "determine the composition of two functions expressed in function notation" (Ministry of Education, 2000);
- requires student to "[define] f(g(x)); [determine] the composite function" (grade 12 curriculum) (Ministry of Education, 1985).

10. Sketch the graph of the function $y = (x-1)^2 + 2$.

This question was chosen because it:

- is a multi-step question, but does not make the steps explicit for the student (the student is expected to do this on his/her own);
- involves translations of the well-known parabola function;
- can be solved using different methods (i.e., translations; table of values) though in this case, one method is preferred (the translation method);
- checks whether students will use the conceptual approach of translating a parabola or reduce the problem and take a procedural approach by using a table of values.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- tests if the student can "represent transformations (e.g., translations, reflections, stretches) of the functions defined by $f(x) = x$, $f(x) = x^2$..., using function notation" (grade 11 curriculum) (Ministry of Education, 2000);
- requires the use of "techniques of graphing using transformations" (grade 12 curriculum) (Ministry of Education, 1985).

11. Solve the equation $\log x + \log(x+7) = \log 4 + \log 2$.

This question was chosen because it:

- involves the use of exponential laws and/or laws of logarithms;
- requires knowledge of the domain of the logarithm function;
- can be solved using different methods (i.e., exponentials; laws of logarithms)

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- requires the student to "solve exponential and logarithmic equations, using the laws of logarithms" (Ministry of Education, 2000);
- requires the student to "determine the real...roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic equation, completing the square)" (grade 11 curriculum) (Ministry of Education, 2000);
- tests if student can "[solve] problems involving exponential and logarithmic functions" (grade 12 curriculum) (Ministry of Education, 1985);
- tests if student can "[solve] quadratic equations by factoring and by equation" (grade 12 curriculum) (Ministry of Education, 1985).

12. Find an equation of a line perpendicular to the line $2x + y - 4 = 0$ that goes through the point $(1, -2)$.

This question was chosen because it:

- is a multi-step question, but does not make the steps explicit for the student (the student is expected to do this on his/her own);
- involves the skill of solving for the equation of a line;
- tests if students know the relationship between perpendicular lines.

Relating directly to grade 12 (2000) and OAC (1985) and curriculum guidelines, this question:

- there is no direct objective for this in the upper year high school curricula for 1985 or 2000. Guidelines associated with solving equations of lines appear in the grade 10 curriculum guidelines in 1985 and do not appear in the curriculum guidelines for grades 11 and 12 in 2000. This is taught in lower high school years.

SURVEY B:

1. Multiple choice. Circle the correct answer
What 40% means?
(a) one-quarter (b) one in four (c) every 40^{th} person (d) 4 out of 10

This question was chosen because it:

- tests student's knowledge of the concept of percentage as part of a whole;
- asks student to distinguish between a correct answer and several common misconceptions.

2. Arrange the following numbers from the smallest to the largest:
$\frac{100}{0.01}$, $100 \cdot 0.01$, $\frac{100}{0.02}$, $\frac{200}{0.01}$, and $200 \cdot 0.01$.

This question was chosen because it:

- tests understanding of division and multiplication of decimals (i.e., dividing by 0.01 results in something larger than the dividend);
- tests understanding of the concept of changing the magnitude of the numerator and/or denominator in a fraction.

3. Determine which of the two fractions $\frac{7}{8}$ or $\frac{78}{87}$ is larger. Explain your reasoning.

This question was chosen because it:

- tests understanding of same concepts as in (2);
- requires ability of the student to reason and explain his/her result.

4. Find $\frac{1\frac{3}{4}}{\frac{1}{2}}$. Write down a mathematical question to which $\frac{1\frac{3}{4}}{\frac{1}{2}}$ is the answer.

This question was chosen because it:

- involves the skill of dividing one fraction by another (one fraction is mixed);

- tests ability to create a relevant and accurate context for the mathematical problem given;

5. In how many different ways can you pay 50 cents in change (i.e., using pennies, nickels, dimes and quarters)?

This question was chosen because it:
- requires some version of an exhaustive search to answer;
- tests ability to form an organized and logical method to answer the question (i.e., some sort of tree model);
- tests understanding of permutations;
- requires persistence – it is a problem that can not be solved immediately.

6. You are driving on a road where the speed limit is 70 miles per hour. The speedometer in your car reads 120 kilometers per hour. Are you driving above or below the speed limit? Explain your answer. (Recall that 1 mile is approximately 1.6 kilometers.)

This question was chosen because it:
- tests proficiency in working with ratios (i.e., setting them up, using them properly);
- requires conceptual understanding in order to transfer the word problem into a mathematical problem.

7. (a) What is the sum of the angles in a triangle?
(b) What is the sum of the angles in a hexagon? (Hint: divide the hexagon into triangles.)

This question was chosen because it:
- tests general knowledge about triangles (i.e., angles add up to 180^{o});
- requires students to reduce a problem into a smaller problem that is easily answered (i.e., a hexagon is divided into triangles).

8. If the radius of a sphere doubles, how does its volume change? Explain your answer.

This question was chosen because it:
- requires knowledge of a sphere (i.e., equation for volume);
- tests understanding of how parameters are related in an equation (i.e., relationship between radius and volume);
- tests concept of comparing new quantities with original quantities;
- requires the ability to explain what is changing, why it is changing and how it is changing.

9. Consider the following definition: "A function is called one-to-one if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$."

(a) Explain in words (without using any symbols) when is a given function one-to-one.
(b) Explain in words (without using any symbols) when is a given function NOT one-to-one.
(c) Give an example of a function (sketch its graph, or give a formula) that is not one-to-one. Explain why it is not one-to-one.
(d) Is the function $f(x) = \sin x$ one-to-one? Explain.
(e) Can the statement "if $f(x_1) \neq f(x_2)$ then $x_1 \neq x_2$" be also used to define a one-to-one function? Explain why or why not.

This question was chosen because it:

- requires the ability to learn a new concept and apply it immediately (i.e., one to one functions);
- asks for explanations of why functions fit or do not fit the definition given;
- asks for examples of functions that do not obey the definition given (i.e., are not one to one);
- tests if the student can realize that the converse is not true and explain why it is not true.

10. Let $f(x) = x^2 + x - 2$. Find $\dfrac{f(x+h) - f(x)}{h}$ and simplify.

This question was chosen because it:

- tests the procedural skill of evaluating $f(x+h)$ for a given function;
- requires the ability to simplify the quantity involving the variables x and h.

11. (a) Write down the precise statement of Pythagora's Theorem. (Drawing a picture will not suffice).
(b) State Pythagora's Theorem in words, without using any formulas (say, as you would if you have to explain it to a friend over the telephone).
(c) Explain why $\sin^2 x + \cos^2 x = 1$.

This question was chosen because it:

- requires precise knowledge of Pythagora's Theorem (a theorem used extensively in high school);
- requires an understanding of the theorem to be able to explain it in words, without using mathematics;
- tests understanding of the relationship between trigonometry and Pythagora's Theorem.

<u>Sample Size</u>

<u>Participants and Determining the Sample Size:</u>

The enrollment in first year Calculus at McMaster University is approximately 1500-1600 students. The students are randomly divided into lecture sections of about 300-400 students per section. Each year, one section usually has one extra lecture in the first week of class (2 lectures instead of just 1) and it is this lecture section that is chosen for participation in the survey. Since the section was created randomly (but with regard to student availability), the sample contains a diverse group of students and is a good representative sample of the population.

Of the students in the selected section, there may be students who do not wish to participate in the survey. In this way, the students collectively determine the final number of completed surveys and therefore set the *initial* sample size.

For the purposes of this paper, our initial sample sizes for years 2002, 2003 and 2005 are 317, 226 and 197 students respectively. Note that since our study focuses on a question from Survey A, these initial samples are for Survey A only, not Survey B (the numbers for Survey B are not yet determined).

The initial sample sizes discussed above are further reduced by students who chose to leave questions blank. Blank responses were not marked and not included in the sample population. Thus, *final* sample size varies depending on the question being examined. For example, on the topic of age of participants, our sample sizes for 2002, 2003 and 2005 are 315, 225 and 192 students respectively (since there were 2, 1 and 5 blanks respectively). We based all subsequent calculations on the final sample size associated with a given question.

We have taken considerable care when analyzing data to specifically mention how many participants were involved in the analysis. The number of participants is indicated in parentheses beside the appropriate data.

When exploring correlations between survey performance and final grades in MAT 1A3, it should be noted that students who did not complete the course were not considered. A course was deemed incomplete if the student did not write the final exam. These students' surveys were, however, included in collecting demographic data and also performance on the survey itself.

Some Demographic Data of the Participants:

Survey results show that students range in age, high school experience and ethnic background. Analyzing the demographic data of the population, we found the following:

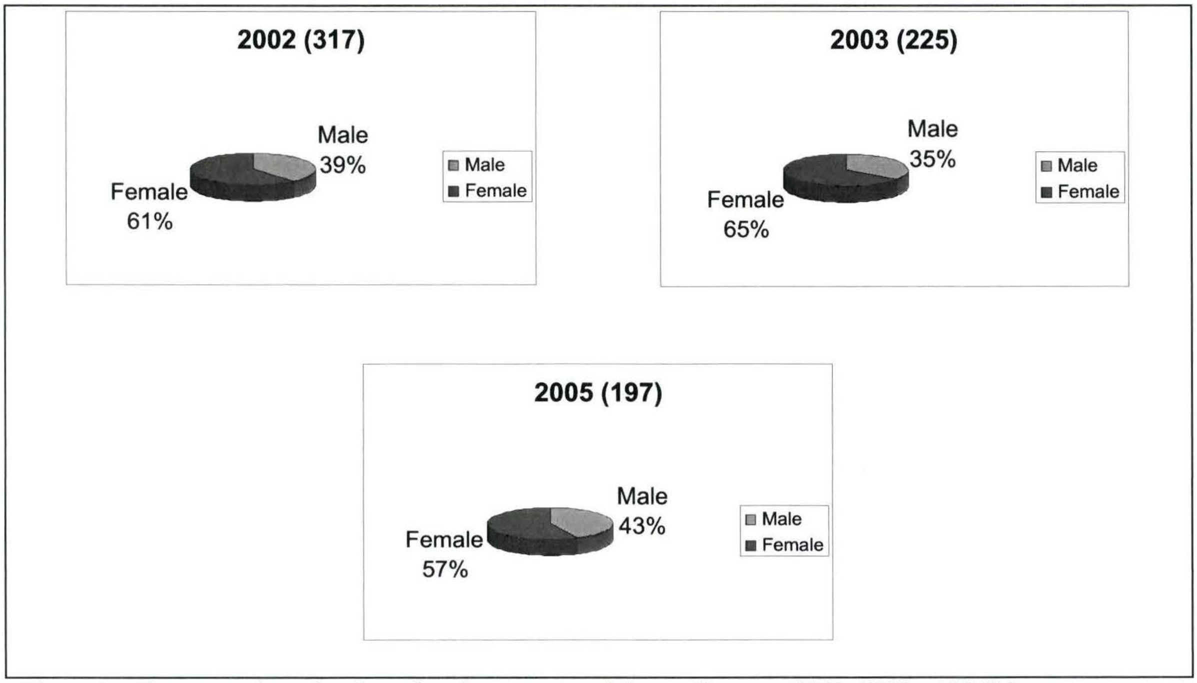


Figure 1: The proportion of male to female survey participants for years 2002, 2003 and 2005.

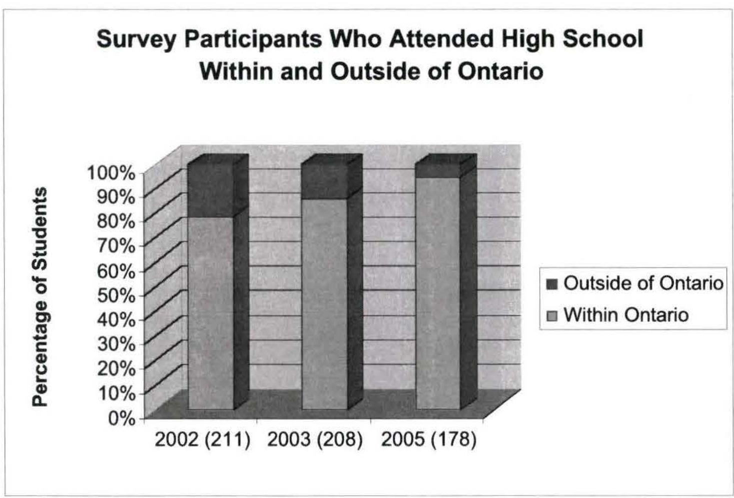


Figure 2: Percentage of survey participants who attended high school within Ontario and outside Ontario in 2002, 2003 and 2005.

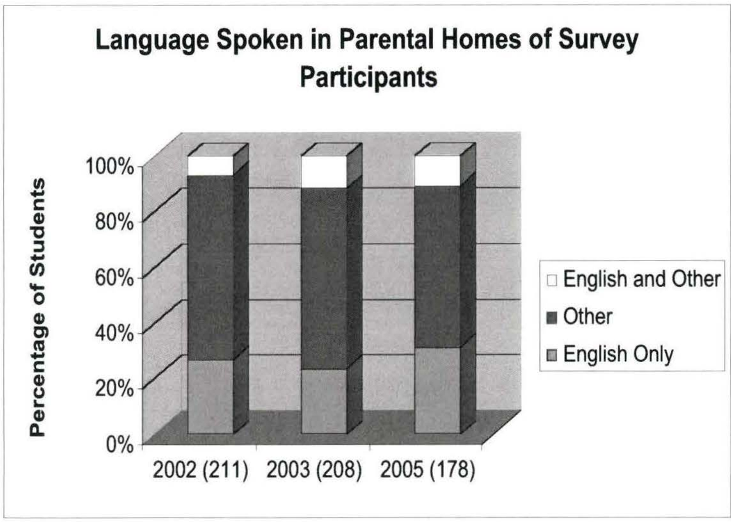


Figure 3: Percentage of survey participants who speak English (only), a language other than English (only) or both English and another language in their parental homes in 2002, 2003 and 2005.

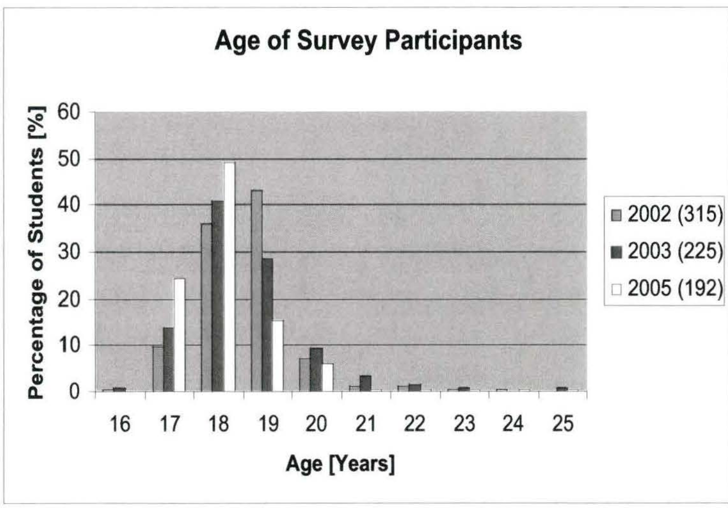


Figure 4: The age of survey participants in 2002, 2003 and 2005.

These figures can also be found in the appendix, as part of a power point presentation delivered at the Ontario Mathematics Education Forum meeting held at Field's Institute on October 21st, 2006. The entire presentation is appended to this paper.

The figures reveal some interesting data about the students in MAT 1A3. The majority of students in MAT 1A3 who participated in our study are female (the percentage ranging from 57%-65%). Although most students attended high school within Ontario, the majority of students speak a language other than English in their homes, while most others speak English and at least one other language. There were relatively few students who speak only English in their parental homes, indicating high ethnic diversity in MAT 1A3. Both the gender and the language statistics coincide with McMaster University's reputation for having overall a greater proportion of female students and high diversity. This implies further that our sample size is representative of the students in MATH 1A3.

It is shown in Figure 1 that the age of students in MATH 1A3 decreased from 2002 to 2005. Where in 2002, the majority of students in MATH 1A3 were 19 years, with 18 years being the next largest age category, in 2003 the majority were 18 and 19, then 18 and 17 in 2005. This speaks to the change in age of students in university before, during and after the double cohort group. Universities are now admitting younger students than before the curriculum changes in 2000.

Variables in the Study

In this study we correlate quantity and type of background mathematics knowledge determined by high school courses taken (independent variable) with students' final grades in MATH 1A3 (dependent variable). These variables form a set that addresses the question "Does the quantity and/or type of mathematics courses taken in high school have an affect on students' performance in MATH 1A3?".

We also examine student responses to one particular survey question quantified by a numeric and letter coded grading scheme. We are then able to comment on overall student performance on the survey problem while addressing the question "How do students before, during and after the double cohort perform on communications oriented questions?". In this case, the independent variable is the year being considered and the dependent variable is student performance for that year.

There are other variables examined that are not formalized as being independent or dependent. For example, when we examine student responses of the questions, we discuss qualitatively some common threads without basing them on particular variables. These are observations that add value to the discussion, but are not quantified using statistics.

Data Analysis

In this section, we objectively present the survey data collected that is pertinent to our research questions. We discuss the relevance of the data analyzed in the Discussion section that follows.

Mathematical Background (High school courses taken by participants):

In the first part of the survey, students are asked to list the mathematics courses they took in their final year of high school. From this information, we were able to deduce how many mathematics courses and which mathematics courses each student took. High school Calculus is a prerequisite for MATH 1A3 at McMaster University; a second high school mathematics course (if taken) can be chosen from Algebra and Geometry or Finite Mathematics (changed to Geometry and Discrete Mathematics and Finite Mathematics, respectively, in 2000). The following figure and table relate the number and type of mathematics courses taken in high school with a student's final grade in MATH 1A3:

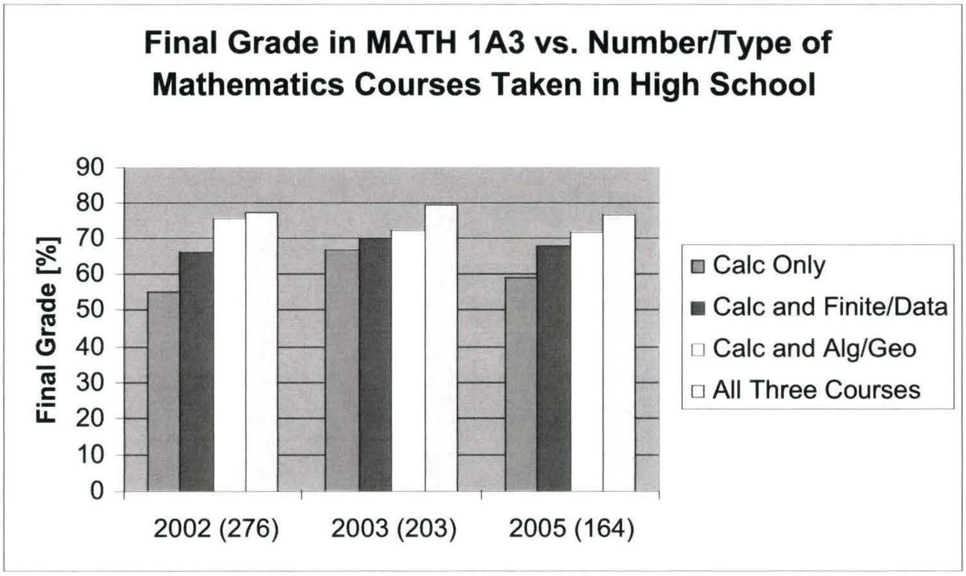


Figure 5: Final grades of survey participants vs. number and type of mathematics courses taken in high school (for years 2002, 2003 and 2005).

	FINAL GRADE IN MATH 1A3 [%]				Overall
YEAR (# Participants)	Calc. Only	Calc. and Finite/Data	Calc. and Alg./Geo.	All Three	**Average for MATH 1A3**
2002 (276)	55.37	66.17	75.37	77.47	**71.67**
2003 (203)	66.66	70.09	72.35	79.59	**73.65**
2005 (164)	59.32	68.04	71.76	76.73	**67.40**
AVERAGE	**60.04**	**68.06**	**73.48**	**78.04**	

Table 3: Final grades of survey participants vs. number and type of mathematics courses taken in high school (for years 2002, 2003 and 2005). The overall average final grade for MATH 1A3 for the appropriate years is also provided for comparison.

Figure 5 and Table 3 illustrate the relationship between number (1, 2 or 3) and type (calculus, algebra and geometry/geometry and discrete mathematics and finite mathematics/data management) of mathematics courses taken in high school with students' final grades in MATH 1A3. It is obvious from the figure that the students having taken more mathematics courses in high school attain higher grades in MATH 1A3. Note that students having taken 2 mathematics courses in high school are divided into 2 categories, namely: (i) those whose 2 courses were Calculus and Algebra and Geometry (or Advanced Functions and Introductory Calculus and Geometry and Discrete Mathematics), and (ii) those whose 2 courses were Calculus and Finite Mathematics (or Advanced Functions and Introductory Calculus and Mathematics of Data Management). Students who fell in category (i) attained a higher final grade (73.48%) in MATH 1A3 than the students in category (ii) (68.06%).

Furthermore, the 289 students surveyed in 2002 took an average of 2.34 high school mathematics courses per person. The final average for the course in 2002 (including all students, not just those surveyed) was 71.67%. In 2003, the survey data indicates students took an average of 2.19 high school courses per person (with a course average of

73.65%) and in 2005, an average of 1.83 high school courses per student (with a course average of 67.40%).

The Double Cohort Group:

In this section, we present results of the 2003 double cohort group. We were able to use survey data to determine whether students studied high school mathematics according to the old curriculum, new curriculum or both old and new curricula (some students took a mixture of grade 12 "new curriculum" and OAC "old curriculum" courses). The population of students who participated in our survey in 2003 had the following composition:

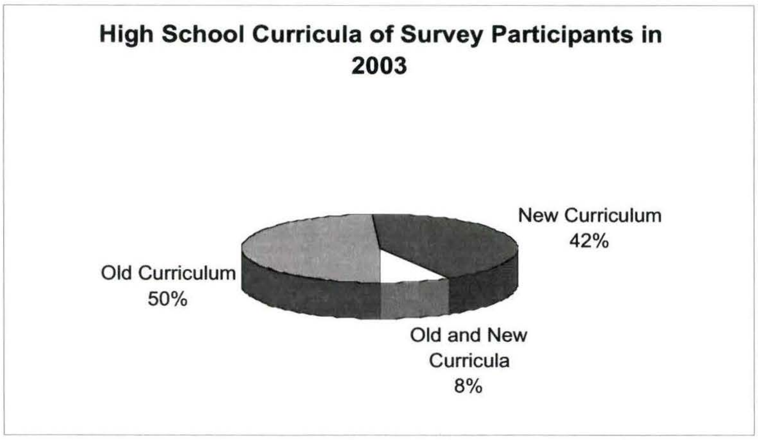


Figure 6: Proportion of 2003 survey participants who were educated according to the new curriculum, old curriculum or both old and new curricula.

Half of the students were educated using the curriculum guidelines before 2000, meaning they completed their final year of high school taking OAC courses. 42% of students were educated using the curriculum guidelines after 2000, meaning grade 12 was their final set of courses. Finally, only 8% of students were educated using a mixture of both curricula (having some OAC courses and some grade 12 courses as their final high school courses).

The next figure shows the data regarding the overall survey performance of students before, during and after the double cohort year. The high school curriculum is indicated by the numbers 1, 2 and 3 which code for old curriculum, new curriculum and both old and new curricula respectively. It is important to note that in 2005, there were only 2 students from the old curriculum (code 1) and only 4 students from both curricula (code 3). These numbers have been considered negligible and are omitted for this analysis. Survey performance of these students was the following:

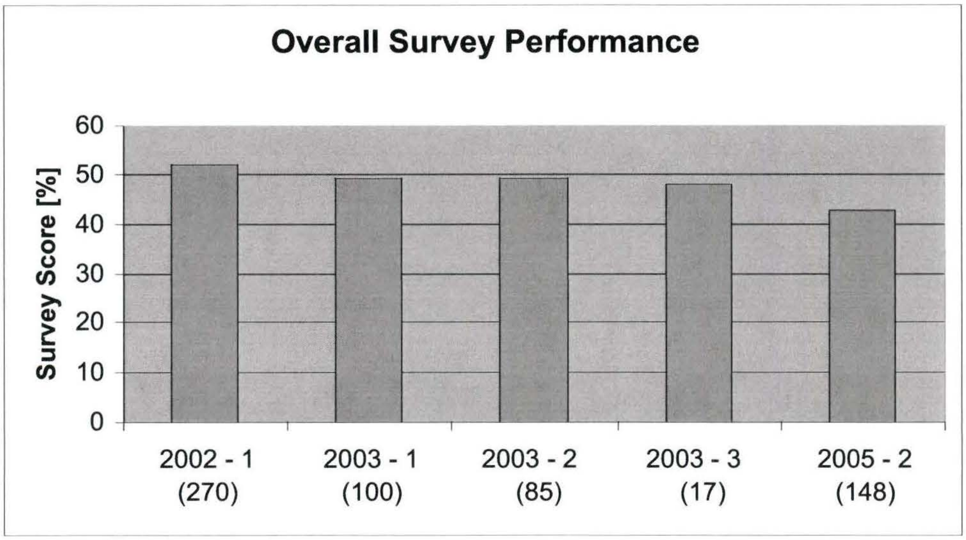


Figure 7: Overall survey performance for years 2002, 2003 and 2005. The codes 1, 2 and 3 indicate high school curriculum of the student (old curriculum, new curriculum or both old and new curricula, respectively).

From Figure 7, we conclude that students who studied under the old curriculum in 2002 had the highest overall performance on the survey with 51.97%. In 2003, the group as a whole performed slightly worse than 2002 with 48.94% (old curriculum), 49.23% (new curriculum) and 48.00% (both curricula) overall survey scores. In 2005, there was a more severe drop in performance – this group of students entirely from the new curriculum scored 42.83%.

We turn next to the double cohort's performance in MATH 1A3. The overall average in MATH 1A3 in 2003 was 73.65%. The following figure shows how the top scores and low scores in MATH 1A3 were divided among the survey participants in 2003:

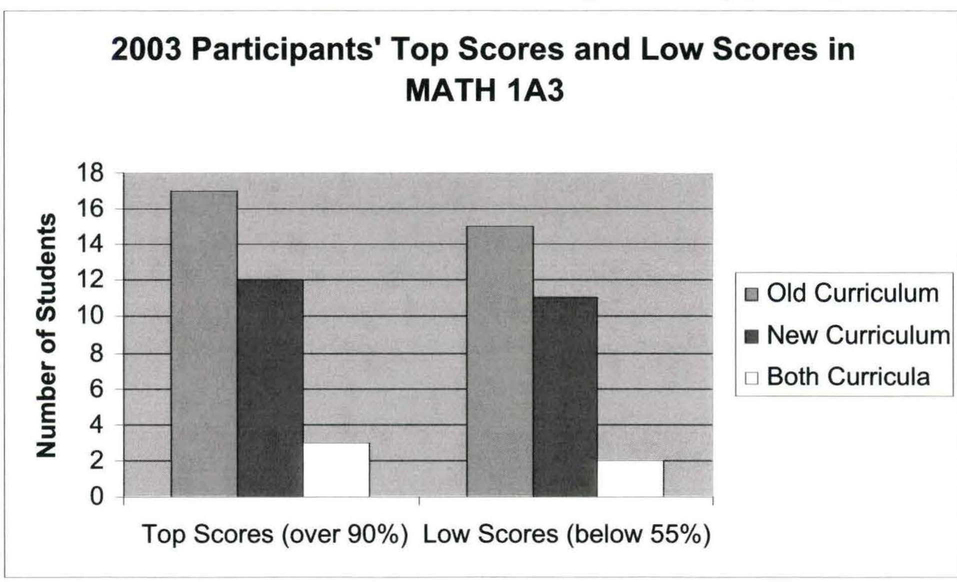


Figure 8: Top scores and low scores in MATH 1A3 of survey participants in 2003, separated by curriculum background (either old curriculum, new curriculum or both old and new curricula).

This figure shows that of the students attaining a mark of 90% or greater in MATH 1A3, most students were educated under the old curriculum, with students educated under the new curriculum next in quantity and the students educated under both old and new curricula least in quantity. The same trend is noticed for students with marks below 55%. It is clear from this graph that the trend is a result of the composition of the population (see Figure 6). That is, since more students that took MATH 1A3 were educated under the old curriculum, these students represent more of the population in the class, and hence more of the population attaining a grade above 90% or below 55%.

A more appropriate approach considers the percentage of survey participants who achieved high and low marks (rather than quantity). Note that 2003 survey participants who were educated under both the old and new curricula comprise a small portion of the population. The data pertaining to this group as a percentage is misleading on the graph below, but it is included for completeness nonetheless.

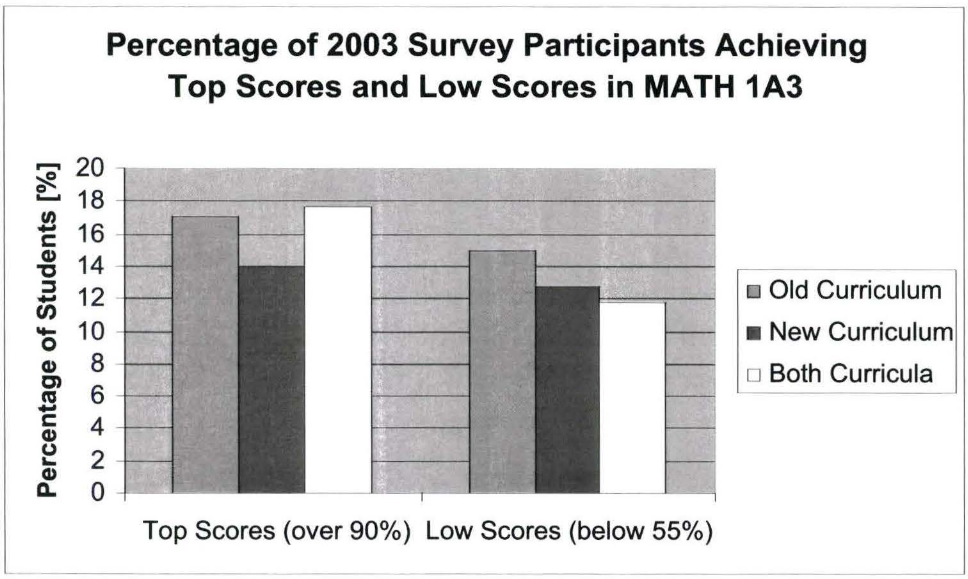


Figure 9: Top scores and low scores in MATH 1A3 (percentage) of survey participants in 2003, separated by curriculum background (either old curriculum, new curriculum or both old and new curricula).

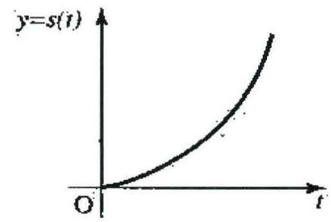
In Figure 9 it is shown that in 2003 both cohorts of students (old curriculum and new curriculum) were capable of achieving a final grade greater than 90%, and did so with roughly the same probability. 17.00% of the OAC survey participants achieved higher than 90% as did 13.95% of the grade 12 participants. Similarly, the likelihood that an OAC student's final grade in MATH 1A3 was below 55% was roughly equal to that of a grade 12 student (15.00% and 12.97% respectively).

It should also be noted that in 2003, the overall average for survey participants educated under the old curriculum was 74.24% and the overall average for participants educated under the new curriculum was 73.54% (recall the overall average for MATH 1A3 in 2003 was 73.65%).

A Closer Look at the Velocity Question:

We have chosen to study the survey question on velocity in more detail in this paper. The question is described below:

6. A position function $y = s(t)$ is given below.



(a) Describe the velocity $v(t)$ as increasing or decreasing. Explain how you know.

As was mentioned previously, this question was chosen because it is reflective of changes made to the Ontario curriculum in 2000 as part of the Mathematics Reform. Note that part (b) was excluded from the analysis, which asks students if the acceleration is negative or positive. It was omitted because many students responded correctly that the acceleration is positive, but would base this on their response on part (a) that may not have been answered properly. For example, a student may use incorrect arguments to conclude that the velocity is increasing, and then respond "acceleration is positive because velocity is increasing" which is a correct response. For this reason, we have postponed the analysis of part (b).

Also, as part of the survey question shuffling described in Research Design, the velocity question appeared as number 6 on some surveys and as number 12 on other surveys in order to alleviate the problem of skipping (students had a tendency to skip the question when it was placed as number 12, the last survey question). In all survey years, both orderings were distributed to the students, some with the velocity question as number 6 and some with the velocity question as number 12. The following figure shows the percentage of blank responses for the velocity question over the years 2002, 2003, and 2005:

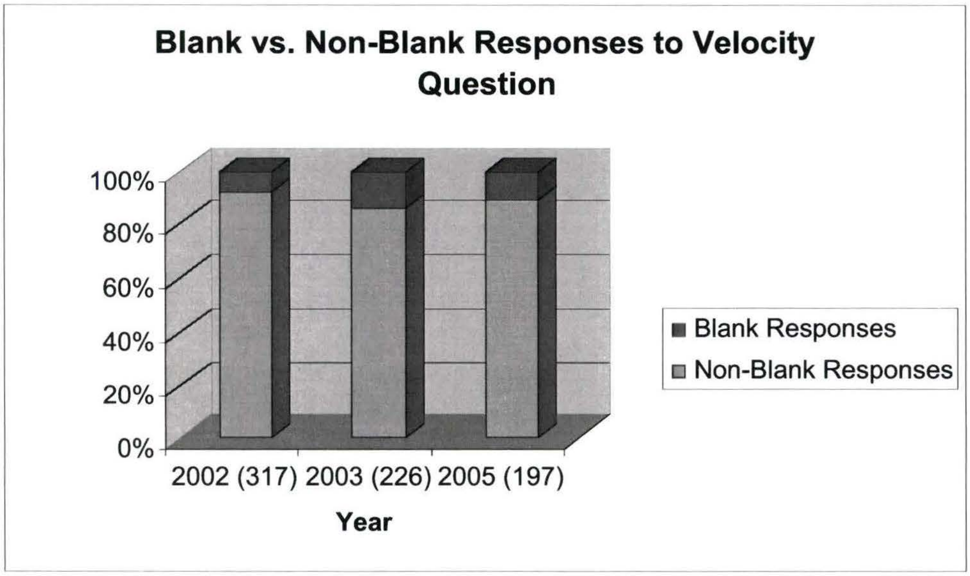


Figure 10: Percentage of blank and non-blank responses to the velocity question for years 2002, 2003 and 2005.

The number of blank responses falls between 7.9-13.7% for the three years. Since this range is narrow, we have successfully ensured that blank responses will not significantly affect our relative sample size and the data can be analyzed in a straightforward manner.

The velocity question was marked according to the numeric and letter coding scheme described in the Methodology section. It was first marked using the numeric scheme, receiving a number grade between 0 and 5. For reference, the numeric scheme is repeated below:

Numeric Scheme:
0 - Incorrect answer (decreasing). If it was accompanied by an explanation, the explanation received a letter code.
1 - Correct answer (increasing) without an accompanying explanation.
2 - Correct answer with some attempt at an explanation. In this case, the explanation is mathematically incorrect, not relevant or nonsensical.
3 - Correct answer with accompanying explanation. In this case, the explanation is either on the right track, but not precise enough or partly correct and partly incorrect.
4 - Correct answer with accompanying explanation. In this case, nothing about their statement is incorrect, but the response is either not completely precise or there is some ambiguity in the terminology.
5 - Correct answer with accompanying explanation. The explanation is precise and at most would have a minor detail missing.

The following figure reports the results of the numeric grading of the velocity question for 2002, 2003 and 2005:

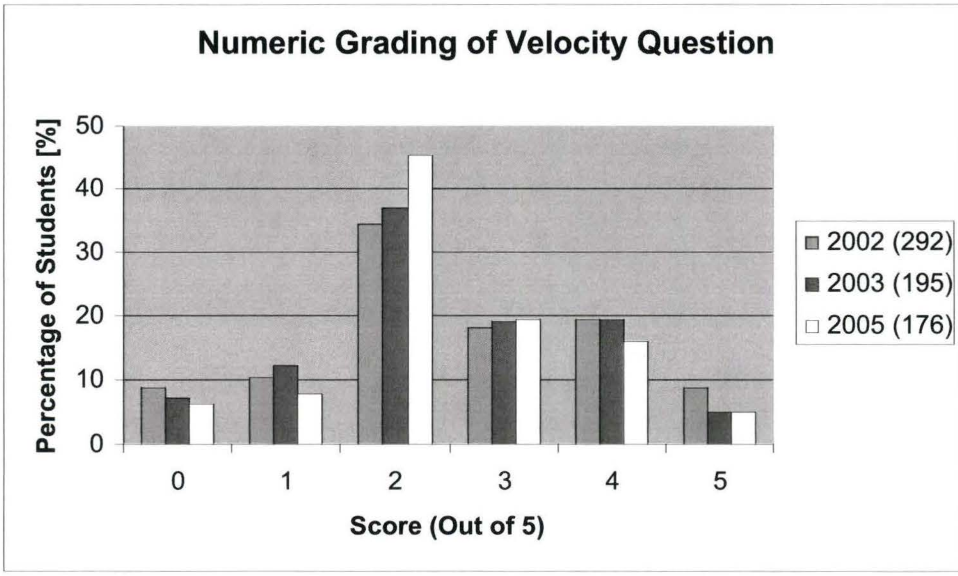


Figure 11: Results of the numeric grading of the velocity question based on the numeric scheme described above.

The numeric scheme was carefully devised to distinguish quality of students' answers. Figure 11 is useful in that it shows the percentage of students scoring 0 to 5 on the velocity question and, as is shown, very few students attained a perfect score. However, this figure does not tell us much about the differences in performance for years 2002, 2003 and 2005. If some of the categories are combined, though, we are able to visualize the trends more easily. For this purpose, we have clustered the scores 1 and 2 together and 4 and 5 together. The clustering of 1 and 2 is reasonable since both of these scores represent responses that correctly identify velocity as increasing, but do not say anything beyond that (i.e., says nothing at all (a mark of 1) or gives irrelevant justification (a mark of 2)). Similarly, the clustering of 4 and 5 is reasonable since both of these marks are given for correct answers, but the response receiving a 4 is not precise enough to receive a 5. The next figure reports the results after clustering:

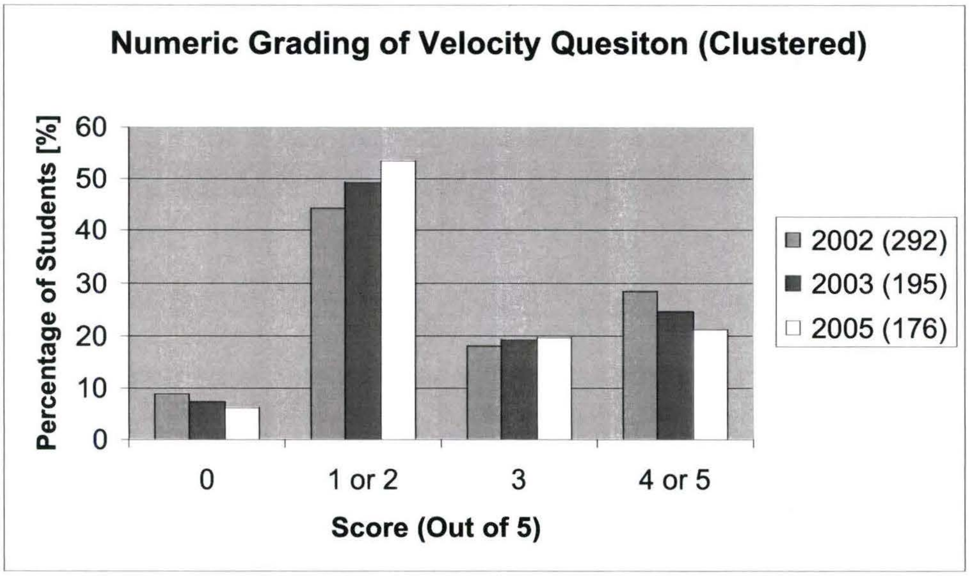


Figure 12: Results of the numeric grading of the velocity question based on clustering the numeric scheme described above.

In this figure, we are able to see trends in the data based on clustered scores for 2002, 2003 and 2005. If we examine scores category by category, we notice that student scores in the "4 or 5" category decreased by 7.4% from 2002 to 2005. Student scores in the "1 or 2" category increased by 8.9% from 2002 to 2005. Scores in the "3" category stayed roughly the same for the three years, with just a slight increase of 1.1% over the three years. From 2002 to 2005, the score on this question appears to be shifting from the "4 or 5" category towards the "1 or 2" category. However, scores in the "0" category decreased by 2.7% which is encouraging since this is the response that is entirely incorrect (i.e., velocity is decreasing).

The question was also coded using the letter coding scheme mentioned above. It is reproduced here for reference:

Letter Coding Scheme:

P - Uses the shape of the curve to explain their answer. If stated properly, this could be adequate.

M - Involves the (inadequate) statement that more distance is covered in less time.

R - Involves the adequate statement that more distance is being covered over set (equal) time intervals.

G - Uses the assumption that the graph is a specific curve such as a parabola, exponential or power curve. (inadequate, but on the right track)

B - Mentions that the slope/curve/graph/slope of curve/slope of graph is increasing. If they mention the slope of the tangent is increasing, they get full credit.

S - Says the slope is positive (inadequate)

D - Uses or mentions derivatives in some way; basically, understands the idea that a derivative is involved

I - Says the velocity is increasing because as time increases, position/distance/ displacement increases.

V - Says the velocity is increasing because the velocity/speed is increasing (this response was surprisingly not rare)
T - Mentions the term 'tangent' somewhere in their response

The following figure reports the codes used by survey participants in their velocity question responses:

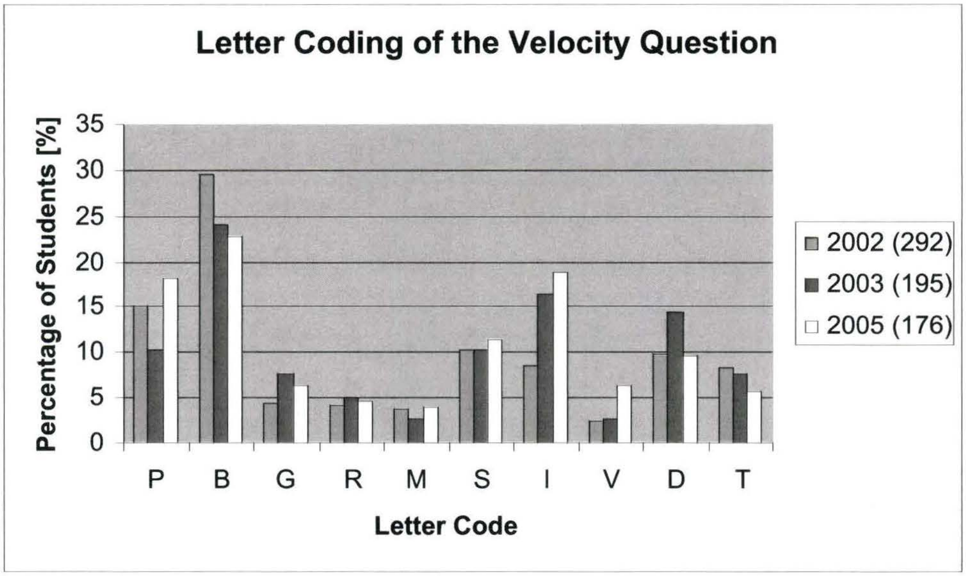


Figure 13: Results of the letter coding of the velocity question based on the numeric scheme described above.

As was the case with Figure 11, Figure 13 shows the percentage of students who used a particular code in their response to the velocity question. Again, this figure makes it difficult to observe trends in the data over the years 2002, 2003 and 2005. For this purpose, we have divided the letter codes into three categories: (i) indicators of good understanding, (ii) indicators of poor understanding and (iii) not an indicator. In particular, we have decided the codes T, D, G, R and B are indicators of good understanding whereas the codes I, V and M are indicators of poor understanding. The codes S and P are not indicators (i.e., responses could be either correct or incorrect and still contain one of these codes). The following figure presents the data from Figure 13, but categorized as indicators of good and poor understanding (category (iii) above is omitted):

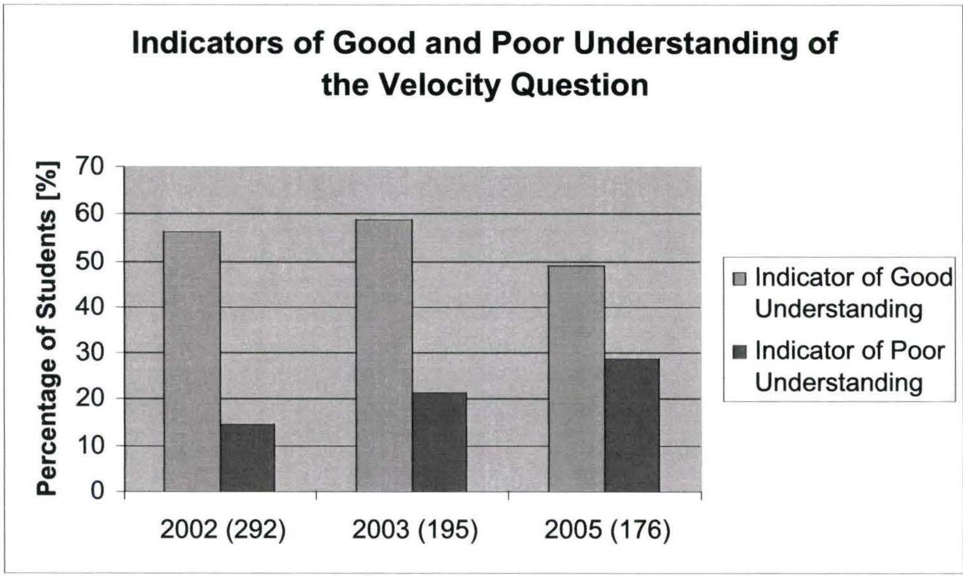


Figure 14: Results of the letter coding of the velocity question presented as indicators of good and poor understanding. Indicators of good understanding are the codes T, D, G, R and B. Indicators of poor understanding are the codes I, V and M. The codes S and P are not indicators.

Figure 14 is more indicative of the overall picture of the participants' understanding of the velocity question. Though there is a glitch in the pattern in 2003, we can see that from 2002 to 2005, the responses indicating good understanding has dropped by 7.3%. From 2002 to 2005, there is a consistent increase in the percentage of students who indicate poor understanding in their responses. These indicators have increased by 14.3% from 2002, almost doubling in frequency.

Next, we perform an analysis of the correlation between participants' scores on the velocity question and performance in MATH 1A3. The following table shows the correlations for years 2002, 2003 and 2005:

Year	Correlation
2002 (292)	0.244341
2003 (195)	0.259604
2005 (176)	0.288895

Table 4: Correlation of performance on the velocity question with overall performance in MATH 1A3 for years 2002, 2003, 2005.

From the results listed in Table 4, we conclude that students' performance on the velocity question is not a good indicator of overall performance in MATH 1A3.

Overall Survey Performance:

The Mathematics Background Questionnaire was originally designed to assess the level and range of student knowledge of Calculus upon entering MATH 1A3 at McMaster University. In this thesis, we are interested in using the survey for a second purpose: to see whether performance on the survey can predict performance in MATH 1A3. It is important to note here that the survey questions were exactly the same on the questionnaires for years 2002, 2003 and 2005. This control allows us to compare the survey data across these three years. The following figure and table present our results of this analysis:

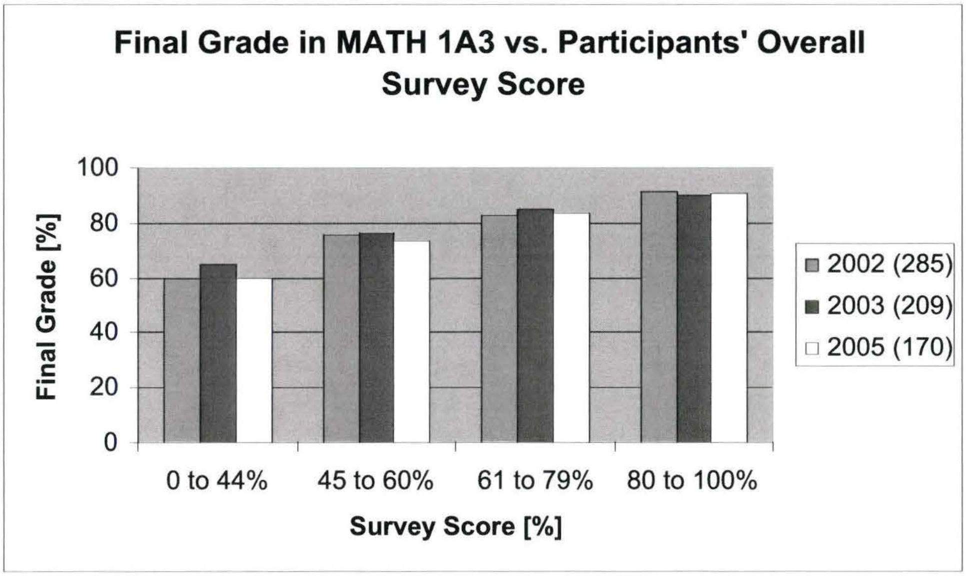


Figure 15: Final grades of survey participants vs. performance on the Mathematics Background Questionnaire, "the survey" (for years 2002, 2003 and 2005).

	FINAL GRADE IN MATH 1A3 [%]				Overall Average for MATH 1A3
YEAR (# Participants)	Survey Score 0 to 11	Survey Score 12 to 15	Survey Score 16 to 19	Survey Score 20 to 25	
2002 (285)	59.73	75.67	82.42	91.62	**71.67**
2003 (209)	64.53	76.52	85.11	90.14	**73.65**
2005 (170)	59.96	73.23	83.22	90.32	**67.40**
AVERAGE	**61.41**	**75.14**	**83.58**	**90.69**	

Table 5: Final grades of survey participants vs. performance on the Mathematics Background Questionnaire, "the survey" (for years 2002, 2003 and 2005). The overall average final grade for MATH 1A3 for the appropriate years is also provided for comparison.

Figure 15 and Table 5 clearly indicate a relationship between average survey performance and average overall performance in MATH 1A3. In all three years, students who scored a failing grade on the survey attained (on average) an overall course average falling in the range of 59.73-64.53%. As we move across categories it is shown that, on average, as survey performance increases, so does overall performance in MATH 1A3. It is interesting to note that students who scored 80% and above on the survey attained an overall final grade of over 90% on average.

The following figure shows the converse relationship. We categorized participants based on final grade in MATH 1A3 and analyzed survey scores according to these categories:

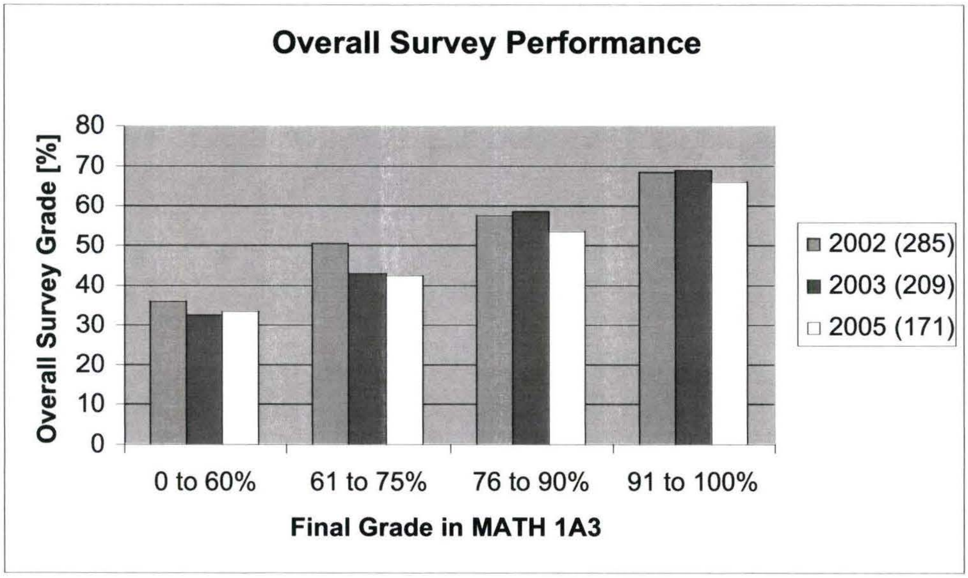


Figure 16: In this graph, students are put in categories based on final grade in MATH 1A3. We then average survey results for each category.

This graph shows that if we look at students' final marks, we can generalize the students' performance on the survey. This relationship is the same as the one found in Figure 15 above. Not only do survey scores predict overall averages (within a range), overall averages predict survey scores (within a range).

In the above analysis, we speak of survey scores and final grades as averages. Here, we present the data using individual data points. Exact survey scores are plotted with exact final grades in MATH 1A3 in the following scatter plots:

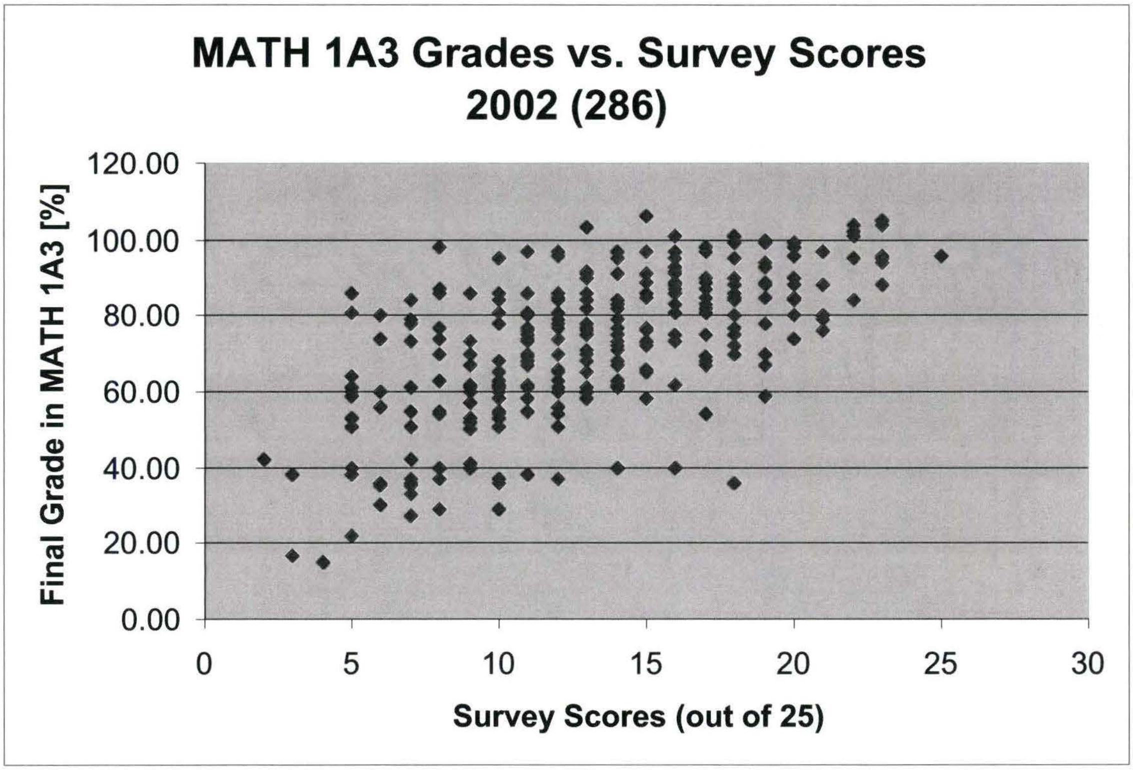


Figure 17: Individual data points of 2002 survey participants showing final grade in MATH 1A3 vs. survey scores.

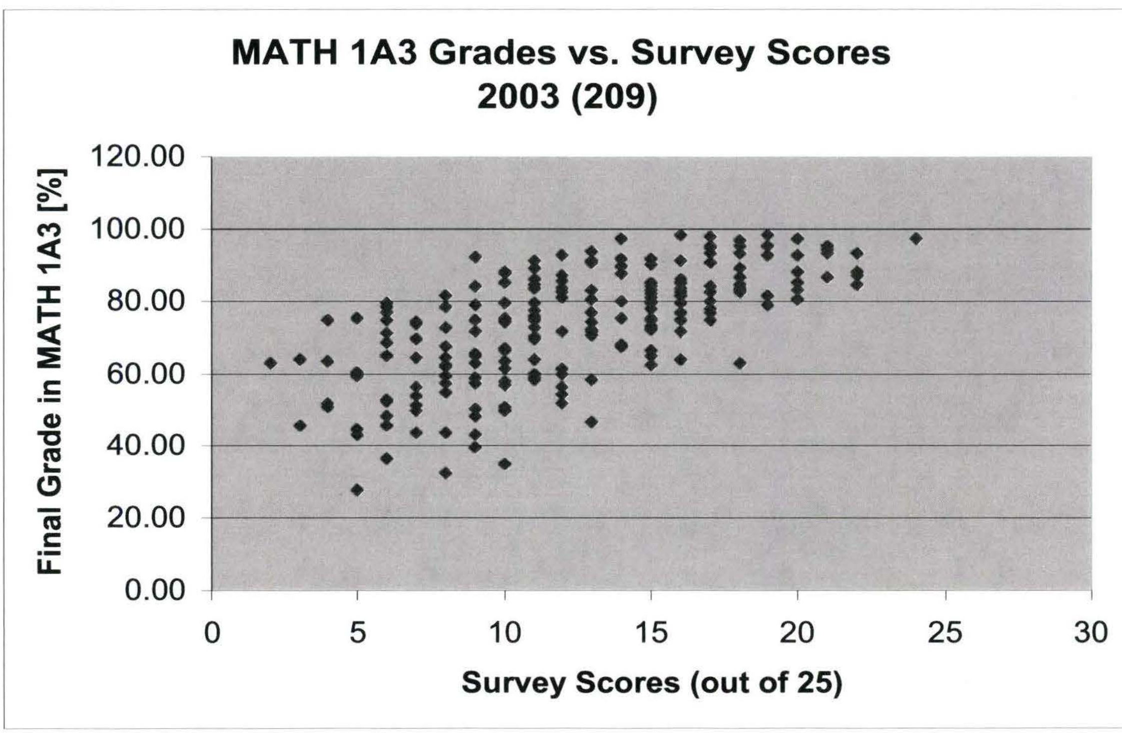


Figure 18: Individual data points of 2003 survey participants showing final grade in MATH 1A3 vs. survey scores.

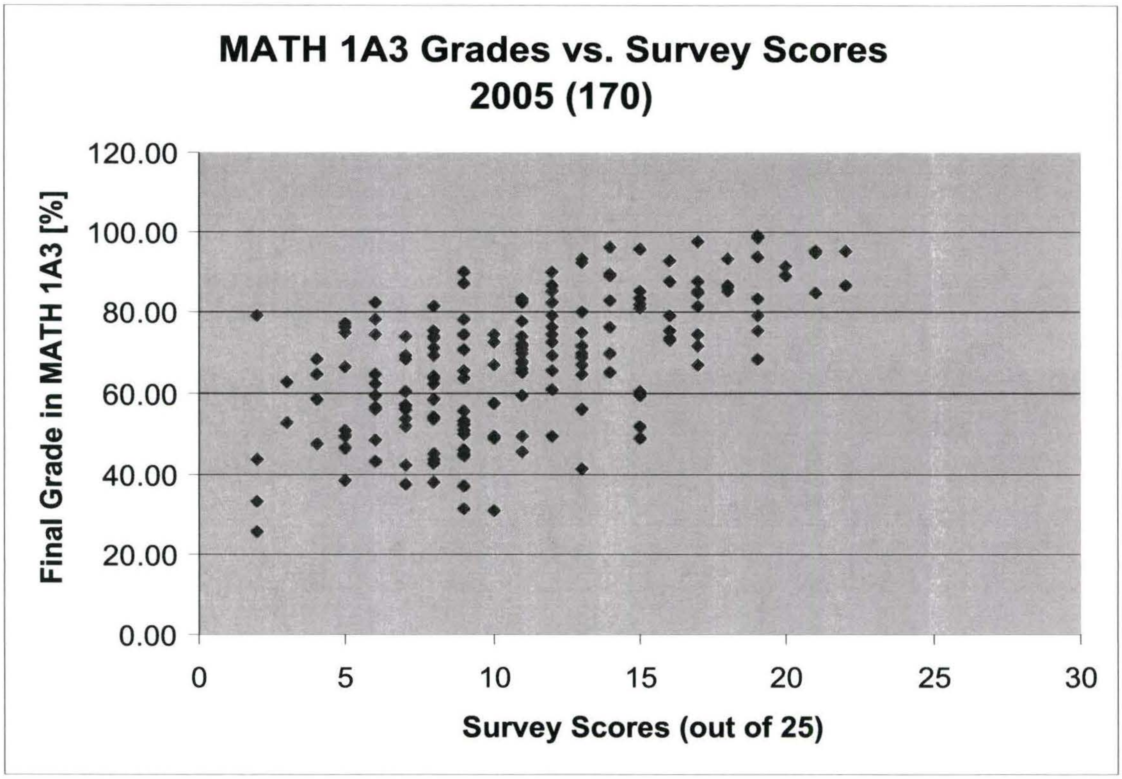


Figure 19: Individual data points of 2005 survey participants showing final grade in MATH 1A3 vs. survey scores.

The scatter plots in Figures 17, 18 and 19 show a general upper triangular shape. That is, if we were to draw a diagonal line from the point (0, 0), we would be able to draw it such that all data points would lie above this diagonal line. This implies that we can make the following generalization about this survey data: high survey scores are indicative of high overall final grades; low survey scores do not predict overall final grades. Students who have low survey scores will not necessarily have low final grades – in fact, many participants scoring low marks on the survey still end up with very high grades in MATH 1A3. However, we can also make the following generalization: of the students who do poorly in MATH 1A3 (say, less than 60%), none of these students had outstanding survey scores. Of the students with a grade of 60% or lower, the highest survey scores in 2002, 2003 and 2005 were 36%, 44% and 60% respectively.

The following table lists the correlation of the individual data points:

Year	Correlation
2002 (286)	0.629
2003 (209)	0.662
2005 (170)	0.610

Table 6: Correlation of survey performance with overall performance in MATH 1A3 for years 2002, 2003, 2005.

Since a correlation value between 0.5 and 0.8 indicates moderate correlation of data, these values confirm the above results – survey performance is a good indicator of overall performance in MATH 1A3.

Discussion

In this section, we interpret and attempt to explain the results of the previous section. We also discuss the relevance of the findings to teaching mathematics at McMaster University. In this section, we first discuss the survey and data analysis locally (interesting background statistics, specific student responses, basically any information the survey reports) and then globally (overall survey performance, what this predicts and the value of this survey beyond intrinsic results).

The first step in addressing the issue of transition from high school to university mathematics (aside from realizing it *is* an issue) is to understand and accept some of the problems that are unavoidable in a university as large as McMaster. Specifically, it is impossible to precisely predict the features of incoming students' mathematical knowledge and skills since students come from various backgrounds. Admissions requirements change, students change and, as we well know in Ontario, high school curricula change. Moreover, the nature of mathematics learning is complex. Mathematics requires mastery of a concept before moving to another concept; one cannot successfully divide without first understanding multiplication and one cannot integrate without first understanding differentiation. A student who is not successful in overcoming transitional obstacles early in the MATH 1A3 term may not be able to recover and persevere in the course.

We have analyzed the survey with this and our research questions in mind. The first question we address is the following: *Does a student's background information help predict what to expect from the student in MATH 1A3? For example, does the quantity and/or type of mathematics courses taken in high school have an affect on students' performance in MATH 1A3?* From the results above (see Figure 5) we have discovered that the number and type of background courses in high school has a strong relationship with final grade in MATH 1A3. There are many possible reasons for this relationship, the most obvious being more exposure and more time devoted to a topic usually means a student will have a better understanding of the material. Even though the different high school courses cover different material, engaging a student in any mathematical thinking will improve mathematical understanding as a whole. This is supported by our survey results.

It is interesting to note that in Figure 5 students who took the OAC Calculus and Algebra and Geometry combination (or the grade 12 counterparts) performed better in MATH 1A3 than the students who took the OAC Calculus and Finite Mathematics combination (or the grade 12 counterparts). This is likely due to the higher level of abstraction and mathematical thinking in the Algebra course as opposed to the more routine, algorithmic work that is required in the Finite course.

There are two reasons this information about high school background and performance in MATH 1A3 is significant to McMaster University. Firstly, it is important to consider these results when deciding on admission requirements for Science programs since students admitted to these programs must take MATH 1A3. Since a greater number of high school mathematics courses correlates with better performance in MATH 1A3, perhaps admission requirements should change from just one mathematics credit (Advanced Functions and Introductory Calculus) to two credits (this course plus one of Geometry and Discrete Mathematics or Mathematics of Data Management). If more students enter MATH 1A3 with this additional senior mathematics credit, our results predict that more students will experience success in the course. This additional course requirement may seem unreasonable at first, however, most students surveyed already take more than one grade 12 mathematics course in high school. The resistance to such a change would not be overwhelming, though it is true that including this new requirement might deter some students from applying to McMaster University. We also need to consider how important mathematics is to Science students. Perhaps an undergraduate Biology student would not have the need for an extensive mathematical background and can get by without a deep understanding of MATH 1A3 material (recall that students with one high school mathematics course scored an average of 60.04% in MATH 1A3). But what happens when this student applies to graduate school? Will this student be sufficiently prepared in mathematics to be able to do the research he or she is interested in?

Secondly, the relationship between number and type of high school mathematics courses and performance in MATH 1A3 is significant in light of the recent changes to the Ontario mathematics curriculum. The Ontario high school mathematics curriculum changed in 2000 and the effects of these changes were felt in universities in 2003. Removing the OAC year in the curriculum has resulted in students entering university at a younger age than before 2003 (see Figure 4). Moreover, students enrolled in MATH 1A3 now spend only 4 years on average in high school and are able to graduate with as little as 4 mathematics courses (and at most 6). Previously, MATH 1A3 students spent 5 years in high school on average, and would graduate with a minimum of 5 mathematics courses (and at most 7). In 2002, 2003 and 2005, we have seen a decline in the average number of high school mathematics courses taken per student from 2.34 courses (in 2002, before the double cohort) to 1.83 courses (in 2005, after the double cohort). As previously mentioned, survey results show that students with fewer final year mathematics courses perform poorly in MATH 1A3 in comparison to students with more mathematics courses.

It is appropriate here to mention that, as this paper is being written, the Ontario Ministry of Education is preparing for a new high school mathematics curriculum being implemented in 2007 (this information comes to us via the Ontario Mathematics Education Forum meetings we attend on a monthly basis – members of the Ministry of Education gave a presentation on the new 2007 curriculum in September 2006). Although the final details are still being discussed, the major change in this new curriculum is the separation of the Advanced Functions and Introductory Calculus into two courses: one containing material on advanced functions and one containing material on introductory calculus. The Advanced Functions course will be a prerequisite to the

Calculus course. In other words, students will have to complete four mathematics courses, including Advanced Functions, before taking Vectors and Calculus, their fifth mathematics course. The analysis and discussion in our paper support this new high school arrangement since it provides evidence that students who take more mathematics courses in high school should have a more successful experience in university.

The combination of younger and less mathematically experienced incoming students exacerbates the problem of transition. This brings us to the next question: *How do students before, during and after the double cohort perform on the Mathematics Background Questionnaire? How do students before, during and after the double cohort perform in MATH 1A3?* After the above discussion regarding background mathematics courses, it is not surprising that students in 2003, before the double cohort (with 5 – 7 high school mathematics courses), attained the highest overall survey average of the 3 years. The students in 2005, after the double cohort (with 4 – 6 high school mathematics courses), had the lowest overall survey average of the 3 years. Interestingly, in 2003, during the double cohort year, survey participants from the old curriculum and new curriculum scored roughly the same overall survey averages (both groups scored lower than 2002, but higher than 2005). The fact that both groups in the double cohort performed similarly on the survey follows from a difficulty in high schools in distinguishing between grade 12 and OAC teaching while both groups were in their final year. That is, the new grade 12 material is very similar to OAC material, but the teaching and assessment of it was supposed to change according to the 2000 curriculum guidelines. While both cohorts were in their final year, though, teachers may not have implemented new teaching and assessment methods entirely. In fact, in some cases, both grade 12 and OAC students were put into the same classroom and taught exactly the same material.

The fact that survey data shows a trend towards lower performance over the years is cause for concern since the survey questions (which reflected the 2000 curriculum objectives) were exactly the same for all three groups. More importantly, final grades in MATH 1A3 are also discouraging. MATH 1A3 has had to adjust to accommodate the new curriculum students, mostly by removing part of the course content and by teaching material that was previously considered review. Also, the Mathematics and Statistics Department has developed several resources to help students succeed. A document called the Mathematics Review Manual was created and is available for students to brush up on mathematical concepts relevant to the course before entering MATH 1A3. MATH 1A3 teaching assistants are required to take the course "Teaching Mathematics" in which they discuss teaching mathematical concepts relevant to MATH 1A3. No other university requires this of their teaching assistants. In the mathematics building, a drop-in centre was created called the Mathematics Help Centre, open 34 hours a week, that offers free tutoring by TA's for students taking MATH 1A3.

Still, the overall course averages in 2002, 2003 and 2005 were 71.67%, 73.65% and 67.40% respectively. It should be noted here that there is a general consensus among educators that 2003 was a year of exceptions for first year university mathematics. Grades were hard to predict and many students who felt the pressure of curricular changes went the extra mile to ensure their success. So even though many feared a disastrous year, students attained an impressive overall average of 73.65% (for our survey participants, old curriculum students scored an average of 74.24% and new curriculum students scored an average of 73.54%). In 2005, however, students were not driven by these curricular pressures. Despite the adjustments made to MATH 1A3 and the resources available to students, the overall average for the 2005 class was more than 6% lower than the 2002 class. The difference of 6% could be as significant a drop as two letter grades according to McMaster University's grading scheme.

It appears as though removing the OAC year has put the 2005 group at a disadvantage. With fewer mathematics courses and less time spent in high school, students are not able to develop the same level of maturity mathematically and otherwise. Naturally, this has implications for other mathematics courses these students might be taking in university. Moreover, with less repetition comes a less robust understanding and this can contribute to more significant summer losses for students.

Of course, age and mathematical maturity are not the only possible contributors to the poor survey performance and low average in 2005. We also need to recognize that the 2000 curriculum is vastly different than the 1985 edition and that high school teachers need to successfully adjust to these changes before their students can. The new curriculum reflects the ideas of the Mathematics Reform, that is, it advocates mathematical communication, self-discovery, mathematical thinking and conceptual understanding. It expects students to "demonstrate an understanding of", "describe the nature of", "compare", "describe the significance" and "pose problems and formulate hypothesis", among others (Ministry of Education, 2000). The 1985 curriculum, such expectations do not appear, rather, students are asked to "determine", "solve", "define" and "investigate", among others (Ministry of Education, 1985). With such a major shift in curriculum, it is expected that there will be a lag in implementation of the new curriculum as teachers redesign their classroom materials and become comfortable teaching in a new way.

Furthermore, the students entering university after 2003 are simply receiving a different education and this could be producing the results we observe. Assuming the 2000 curriculum is being taught properly – that is, assuming teachers have adjusted to teaching the new curriculum – it is possible that the underlying issue is that of the content of the new curriculum. Perhaps the changes made in 2000 are not inducing the results that were expected. If this is the case, it could mean major reconstruction and reconsideration of the psychology used in creating the 2000 mathematics curriculum.

The psychology of the 2000 mathematics curriculum leads into the discussion of our next research question, which was based on the following statement:

"The importance of communication in mathematics is a highlight of the secondary school curriculum. In all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings." (Ministry of Education, 2000)

This theme is meant to reflect the ideals of the mathematics reform. Our expectation is that students educated using these new curricular guidelines should be better equipped to answer questions that require communication of mathematical ideas. This was the motivation to ask the question: *How do students before, during and after the double cohort perform on communications oriented questions? What are some typical responses by students to these questions?* We have chosen to study the velocity question on the survey because students are asked to answer the question and then explain their answer. Moreover, it is qualitative in nature and requires conceptual knowledge which is also a feature of the mathematics reform.

In short, students graduating from the new curriculum are not performing as expected. In fact, they are performing more poorly than the graduates before and during the double cohort. Not only did the scores for the 2005 group fall below that of the 2002 and 2003 groups, the indicators of good understanding decreased and, interestingly, the indicators of poor understanding increased. It seems as though the implementation of the new curriculum actually had the opposite effect than the intended objective of improved conceptual understanding and mathematical communication skills. In the discussion above, we have already touched on some possible reasons for this outcome: fewer high school mathematics courses, adjustment to the curriculum change, etc. Instead of reiterating the discussion, we would like to present some interesting, but typical, student responses to this question and what they tell us about students' mathematical thinking. For each response, we have included the survey year (2002, 2003 or 2005) and the student's mark in brackets (0 to 5):

The Geometric Approach:

2003 (3): the velocity is increasing because $v(t)$ is the derivative of $s(t)$. If $s(t)$ is x^2 then $v(t)=2x$ which is positive.

2003 (3): $x>0$, $y=ax^2 + bx + c$ (curve as given above), $y'=ax+b$, $y'=ax$, $a>0$, so v is increasing.

2002 (4): Use pretend equation $s(t)=x^2$, therefore $v(t)=2x$. Therefore, velocity is increasing because the slope is increasing and slope indicates velocity.

2005 (3): Increasing, this graph is increasing at an exponential rate therefore $y=x^2$ so $v(t)=y'$, $y'=2x$ therefore when x increases, y' increases.

In the above responses, the students correctly answered that the velocity is increasing. What is interesting about the responses is that the students assumed that the given position curve was a quadratic function. This information was not given in the question (in fact, this curve could just as easily be assumed to be an exponential function). Furthermore, the assumption changed the nature of the question – by assuming a quadratic function, the student was able to associate an equation to the curve, and use differentiation to find an equation for velocity. In doing so, the question is changed from a conceptual question to a procedural one. Instead of speaking generally about the shape of the curve and how the changing tangents illustrate increasing velocity, the student limited their response to cover only quadratic functions. This evidence supports the claim that one of the common problems in transition from high school to university mathematics is that students tend to rely on surface learning as opposed to deep learning. In this case, reducing a problem to a set of equations that yield a positive second derivative, therefore increasing velocity, requires computational skills and knowledge of rules (i.e., positive derivative implies increasing original quantity). There is no association with this increase in velocity with the shape of the position function other than the function that has been superficially assumed by the student. On the other hand, what is encouraging about these responses is that it is true that every parabola that opens upwards has increasing velocity.

Using Elimination:

2002 (3): The velocity is increasing because the line is not straight.

2002 (3): Velocity is increasing because the line is not perfectly straight thus not being constant. The line on the graph is slightly curved.

2003 (3): V(t) increases when t increase v=s/t if v(t)=0 the graph of y=s(t) should be a straight line. However, the graph is not a straight line, so v(t) is not equal to 0 and because of y=s(t) always increases, so v(t) must be increasing.

2005 (3): It is known that the function is increasing because the graph curves upwards. If the position graph had a constant slope then is would have a constant velocity however it seems to have an exponential curve therefore velocity is increasing.

In these responses, students answered the question based on what they knew the curve *was not*. That is, they knew that for velocity to be constant, the position function would have to be the straight diagonal line $y = x$. Since the given graph was *not* the function $y = x$, the velocity cannot be constant and, by the process of elimination, the student concludes that the velocity is increasing. The fact that these students conclude that the velocity is increasing (as opposed to decreasing) has value because some students might still come to the conclusion that the function is decreasing (see Common Misconceptions). Also, this response does not rely on procedural knowledge. The student is using the concept of what it means to have constant velocity (as opposed to increasing or decreasing) and then works backwards from there. Based on the shape of the given function, as compared to the shape of a function with constant or decreasing velocity, the student can deduce that the velocity is increasing. Though these responses are not very precise – the student doesn't explain *why* the velocity is increasing, rather, simply that it is increasing because it is not constant or decreasing – these responses indicate a more conceptual approach than assuming the graph to be a quadratic, as in the above section.

Common Misconceptions:

Misconception 1:

2002 (2): The velocity function v(t) is the derivative of s(t) is a positive slope therefore the velocity is increasing.

2003 (2): The velocity is increasing since it is a positive slope.

In these responses, the students correctly answer that the velocity is increasing, but the students base their responses on the fact that the derivative of the function is positive. Of course, this is insufficient because it is possible to have a position function having a positive derivative and decreasing velocity (for example, if $s(t) = \sqrt{t}$ was the given position function). So the responses here are deemed irrelevant, since a positive derivative does not imply increasing velocity. In this case, students are using a procedural approach to answer the question. This response is actually based on the First Derivative Test that is taught in high school in some form (probably in the curve sketching unit). That is, if the first derivative of a function is positive, the function is increasing. Of course, in this survey question, we are not asking if the function is increasing, but whether or not the *derivative* of the function is increasing. It appears as though students were just applying a rule that was memorized, but doing so incorrectly.

Misconception 2:

2002 (0): The velocity is decreasing b/c the graph is going up sharply and beginning to even out.

2003 (0): Velocity is decreasing because the slope of the graph is increasing at a slower rate towards the end.

2003 (0): Velocity is decreasing. The tangent value is getting smaller.

It should be mentioned that this misconception was not as common as others – but it did appear enough to be mentioned here. In these responses, there is a solid understanding that how the tangent to the graph is changing needs to be considered when answering this question. However, the students seem to have confused exactly what it is about the slope that determines whether the velocity is increasing or decreasing. Instead of examining the direction in which the slope's magnitude is changing (i.e., that the slopes of the tangents are getting bigger), the students here consider how the changes in the slope are changing (i.e., between 1s and 2s the slope increases more than it does from 2s to 3s, meaning the change in slope is decreasing). These students only have a partial understanding of how position and velocity are related and simply attempted the answer by guessing the relationship. Although this approach is conceptual rather than procedural, it yields the incorrect answer.

Misconception 3:

2003 (3): Increasing. Covers more distance over in less time. Position is changing quicker and quicker.

2005 (3): The velocity is increasing as shown by the curve of the graph. A greater distance is being covered in a less amount of time.

This misconception was very common among responses receiving a mark of 3. There are two main problems with this response. The first problem has to do with mathematical convention in rates of change. In particular, when we talk about rates of change, we usually talk about a change in one quantity, x, over a change in time, t, giving the function $\frac{\Delta x}{\Delta t}$. It is convention to define change in time, Δt, as a quantity that does not fluctuate throughout the question. That is, if we say $\Delta t = 1s$, it remains 1s for the entirety of the analysis and Δx is studied based on this time interval. In the above responses, the students do not follow this convention and speak of the distance being changed over "less time". Although we can make sense of it, this does not follow the theory used when discussing rates of change. Secondly, for this position function, even though it is true that more distance can be covered in less time, it is also true that an equal amount of distance can be covered in less time. That is, if we choose our first time interval to be 1s with a distance traveled of 1m, for instance, it is possible to find a second time interval in which another 1m is traveled. So, we can also say that the same distance is being traveled in less time and these responses are not necessarily true for all "lesser" time intervals. Though students show the use of mathematical reasoning in this question, the conceptual understanding is not clear and the answer is not precise or fully valid. Note that a better answer is the following:

2002 (5): The velocity is increasing as the distance for the same time interval increases as time passes.

Communication:

Trouble with Terminology:

2005 (2): The velocity increases because as the time increases, so does the speed. If you derive this graph, you will find that it is above the x-axis and therefore will increase in velocity.

2005 (2): Increasing because of a positive ?? line. The value b will always be > a.

These responses show that students have problems using mathematical terminology in their answers, even with the guideline in the 2000 mathematics curriculum stating that students should be able "communicate findings clearly and concisely, using an effective integration of essay and mathematical forms" (Ministry of Education, 2000). In the first response above, the student confuses the word "derive" with "differentiate". (The author knows from personal teaching experience that this mistake is common). Though these two words sound similar, they have extremely different mathematical meanings. In the second response, the student has forgotten the word "tangent". This is alarming since students spend time in high school learning how to calculate the equations of tangent lines. It seems as though this student got by without having to incorporate the word "tangent" into their previous work.

Good Communication, Wrong Idea:

2002 (2): Let x1<x2 and they are any number in the domain. We can see y2>y1 from the graph. So, the velocity is increasing.

2003 (3): v=d/t, this is because velocity is the slope or derivative of the graph. Also, the curve is increasing to the right, thus as the time is increasing, the velocity is increasing. This is because the v and d are directly proportional. As the velocity increases, time decreases.

2005 (3): It is increasing because as time is increasing in the t axis, which is the dependent variable, the value of y axis also increases which is the independent variable. So they are increasing across the graph as the numbers increase i.e., they are increasing functions.

2003 (0): 1. The curve is increasing rapidly. 2. If you take the derivative of the curve you will get a straight line with a positive slope.

In the first example above, the student has shown the ability to effectively "integrate" mathematical ideas into a sentence structure (Ministry of Education, 2000). However, though the inequalities are true, it does not follow that the velocity is increasing (here, the argument given is enough to say that s(t) is increasing, but not its derivative). In statements two and three, we see examples of very well written responses. The students' thoughts are clear and they use complete sentences with appropriate terminology. Even though the quality of sentence is good, the content does not deserve full credit. Neither of the statements say anything that support the fact that velocity is increasing. In fact, one statement even claims that "as the velocity increases, time decreases" which doesn't make sense. Finally, in the last response, we see another well written response that has even been broken into two points for clarity. Again, although both the statements are true, neither of them even answer the question. It is not indicated anywhere in the response whether the student thinks the velocity is increasing or decreasing. This response is incomplete.

It should be noted here that often well written, complete and clear responses can be deceiving and very difficult to mark. When a student presents organized thoughts, albeit incorrect, there is a tendency to want to give more credit for their response. After all, communication is a "highlight" of the new curriculum (Ministry of Education, 2000). However, should credit be given for well written answers that are void of correct mathematical content? In this thesis, we have marked solely on correctness. What is the reason that students performed badly on this question, according to marking for correctness as opposed to communication? Why did the survey participants in 2005 perform the worst out of the three years when marking this way? What does the new curriculum actually foster?

Vague Responses:

2002 (3): The velocity is increasing cause the y' is increasing.

2002 (3): The velocity is increasing because the value of graph is increasing more steeply.

2003 (3): The velocity is increasing because the line becomes more vertical as time goes on.

2005 (2): The velocity is increasing because the graph is going up to the right, and it started at zero, if it started higher on the graph, then fell down to the right, then it would be decreasing.

In the responses above, all students correctly answered the question and show evidence of understanding the reasons for their answer, but they are too vague to make this understanding clear. For example, in the first response, the student knows that y' is increasing, but does not explain how they know this. In fact, all the above statements lack the explanation of their claims and, even though it is clear they are on the right track, the answers are too vague to deserve full credit.

Although the students' scores were low on the velocity question and performance has been worsening over the years (see Figure 12), it is encouraging to know that performance on this question has a poor correlation with final marks in MATH 1A3 (see Table 4). This means that students who score poorly on the velocity do not necessarily perform badly in MATH 1A3. This is likely due to the misalignment of curriculum guidelines and teaching methods when the curriculum first changed in 2000. Though the velocity question is not a good predictor of course performance, it is still useful to examine students' responses, as we did above, to note problems with communication of mathematical ideas and conceptual understanding of graphs and their derivatives.

In addition to correlating the scores for the velocity question with final marks in MATH 1A3, we also studied the correlation between overall survey performance with final marks in MATH 1A3. This brings us to the last research question we discuss in this paper: *Is there a significant correlation between performance on the Mathematics Background Questionnaire and a student's final grade in MATH 1A3?* This, in fact, is the most important question to be considered. The previous questions have all examined the data on the survey closely, extracting statistics used to evaluate and critique the school system and how it is contributing to transitional difficulties for students and instructors. However, this final question is asked with the intent of using the survey as a tool to help prevent some of the first year transitional obstacles (that are in our power to avoid) and to report progress made up to now. Continuing to use this survey in future years, combined with the data already collected, would create a robust assessment of the student development in MATH 1A3 at McMaster University. This is important not only for McMaster University, but also for all universities in Ontario, since McMaster is the only university in Ontario to have such data.

As is shown in Figures 17, 18 and 19 and Table 6, performance on the Mathematics Background Survey has a good correlation with performance in MATH 1A3. In particular, students who do well on the survey do well in MATH 1A3. Since the questions on the survey were carefully selected, it makes sense that these quantities would correlate well. More importantly, students who do not score well on the survey may or may not perform poorly in MATH 1A3. Evidence shows that some of these students perform badly overall, but some recover and are able to be successful in the course.

We would like to stress the importance of this discovery. Knowing that the Mathematics Background Questionnaire is useful in predicting student performance in MATH 1A3 helps us in our battle to eliminate the transitional gap for students entering McMaster University. If we were to mark surveys immediately and provide feedback (i.e., survey scores) to students early in the term, we could inform them of their predicted performance in the course based on our research. In particular, based on the statistics presented in this paper, we can advise students who did not do well on the survey to seek extra help and remind them of the resources that McMaster University provides (Mathematics Review Manual, tutorials, the Math Help Centre). It is important to inform students that even though the onus is on the university to help students succeed, the only way we will see results is if students take the responsibility to do the work that is expected of them.

Conclusions and Future Work

In this paper, we presented data collected from a Mathematics Background Questionnaire that is completed by students in MATH 1A3 in their first week of lecture. It was revealed that, in addition to guiding lectures early in the semester, the surveys are useful for examining the differences in student performance in MATH 1A3 from year to year and also for studying the relationship between students' mathematics backgrounds and performance in MATH 1A3. Most importantly, our analysis has shown that the Mathematics Background Questionnaire can be used to loosely predict student performance in MATH 1A3. In particular, high survey scores are indicative of high overall final grades; low survey scores do not predict overall final grades and these students may be successful or unsuccessful in MATH 1A3.

Our analysis of survey performance before, during and after the double cohort year in 2003 indicates that students are having a hard time adjusting to the Ontario mathematics curriculum implemented in 2000. Average performance in 2005 on both the survey and overall grade in MATH 1A3 was low compared to 2002 and 2003, implying students studying according to the new curriculum are doing worse than students in previous years.

It is true that students are younger and more mathematically inexperienced than before, but these are not the only factors contributing to lower performance. Although the 2000 curriculum incorporates ideas of the mathematics reform, it is possible that the good intentions of the new curriculum are not creating the results they predicted. Does this new ideology cause more harm than good? Are teachers implementing these changes the way the Ministry of Education had hoped? Are we missing the point? More importantly, are the students missing the point? At this point in our research, we only have data up to 2005. It will be interesting to examine the course average in 2006 to see whether a downward trend is developing or if this group will have success in MATH 1A3. It will also be interesting to see how the changes in the Ontario mathematics curriculum in 2007 will affect student performance in MATH 1A3 in the future.

It appears as though transition is still difficult for students entering university. The good news is that, based on our ability to use our questionnaire to make predictions about a student's performance, we can take steps towards helping students make their transition to university calculus more successful. At this point, it is important to focus on incorporating survey feedback to students so that they have a better idea of what to expect in MATH 1A3 and how to proceed in the course.

The next natural addition to this project is to update it with 2006 survey data and course performance and observe any trends over 2002, 2003, 2005 and 2006. This should be completed in early 2007 once the MATH 1A3 2006 final grades have been finalized. In addition to this, we would like to study all survey questions in detail, including the narrative questions on the front page of the survey. For the velocity question, we would like to incorporate an indicator for communication and remark and analyze surveys based on this indicator. Survey B also needs to be analyzed. We have also discussed adding questions to the survey. For example, what are students' expected grades in MATH 1A3? How many other mathematics courses are they taking in university? Did students use graphing calculators in high school? Did this use in high school help or harm them in university?

Also, we would like to conduct interviews with a small group of students taking MATH 1A3. It would be interesting to monitor their studying techniques and progress throughout the term. In particular, it would be useful to monitor students who perform poorly on the survey, but recover and do well in MATH 1A3 to see what they did in order to overcome transitional difficulties. This would allow us to provide MATH 1A3 students with feedback via survey scores *and* supply information on how past students with low survey scores were able to persevere in the course.

APPENDIX A: MATHEMATICS BACKGROUND QUESTIONNAIRE

Name (please print): ________________________________

Student No.: ______________ Faculty: ____________________

MATHEMATICS BACKGROUND QUESTIONNAIRE 2003

Note: The information you are sharing will be used to gain information on your high school mathematics background. It will help your mathematics instructor better plan and design the calculus course. Your responses will be kept confidential and will never be reported individually. **Thank you for taking the time to complete this questionnaire.**

• Your gender: __________________ Age (years and months): _________________

• How many years did you go to high school? What high school(s) did you attend (for each school, list: name, location (city and country), and for how long you attended it)

• Your high school math marks. Indicate the marks that you got for the courses below. If you do not remember the mark, write 'yes' instead.

OAC Calculus: _____ OAC Finite Math: _____ OAC Algebra and Geometry: _____
Advanced Functions and Introductory Calculus U: _______

Geometry and Discrete Math U: _____ Mathematics of Data Management U: _____

• What language do you speak most often in your parental home(s)?

Describe your experiences with high school mathematics courses that you took.

What are your expectations about the Calculus course that you are taking now?

Try to solve as many problems as you can.
Most probably you will not be able to do all of them.

1. Indicate whether each of the following formulas is correct or not. Circle your choice. You do not have to justify your answer.

(a) $x^2 + y^2 = (x-y)(x+y)$ CORRECT NOT CORRECT

(b) $(e^x)^2 = e^{x^2}$ CORRECT NOT CORRECT

(c) $\ln(2x) = 2\ln x$ CORRECT NOT CORRECT

2. The revenue of a company is modeled by $R(x) = x(50-x)$, where x is the price per item, $0 \le x \le 50$.

(a) Determine the rate of change of the revenue with respect to the price when the price is10 dollars and when the price is 15 dollars.

(b) Explain what the values of the rate of change above mean to the company.

3. Indicate whether each of the following statements is correct or not (circle your choice). Explain your answer.

(a) If $f(x) = (x + a)(x + b)$, then the graph of $f(x)$ cuts the x-axis at both a and b.

CORRECT NOT CORRECT

(b) If $a > b$, then $\frac{1}{a} < \frac{1}{b}$ for all real numbers $a, b \neq 0$.

CORRECT NOT CORRECT

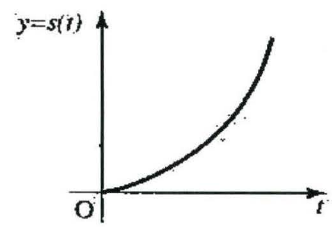
4. A ball is thrown from a building into the air and falls on the ground below. The height of the ball t seconds after being thrown is $y = -5t^2 + 30t + 35$ metres.

(a) Determine the maximum height of the ball above ground.

(b) After how many seconds does the ball hit the ground?

5. What is the range of the function $h(x) = |x|$?

6. A position function $y = s(t)$ is given below.



(a) Describe the velocity v(t) as increasing or decreasing. Explain how you know.

(b) Is the acceleration a(t) positive or negative? Explain how you know.

7. Solve the equation $\dfrac{x^2+5x+6}{x^2+7x+10} = 2.$

8. Solve the equation $4^x = 16^{2x-2}.$

9. Compute the composition $(g \circ f)(x)$ or $g(f(x))$ of the functions $f(x) = x^2 + 1$ and $g(x) = \dfrac{1}{x} + 1.$

10. Sketch the graph of the function $y = (x-1)^2 + 2$.

11. Solve the equation $\log x + \log(x+7) = \log 4 + \log 2$.

12. Find an equation of a line perpendicular to the line $2x + y - 4 = 0$ that goes through the point (1,-2).

APPENDIX B: LETTER OF CONSENT

McMaster University

Inspiring Innovation and Discovery

1 September 2006

Letter of Consent

A Study of Correlation Between Background Preparation and Success in Undergraduate Mathematics Courses at McMaster University

Principal Investigator: Dr. Miroslav Lovric, Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada; tel. (905) 525-9140 ext. 27362

Why are we doing this study? This survey will provide us an insight into the knowledge of our incoming students, and will suggest ways of changing our math courses (beyond Math 1A3) to help students learn mathematics better. Moreover, it will help us answer important questions, such as: 'What are incoming students' high school experiences with mathematics and how those influence their university mathematics performance?' or 'How do grade 12 curriculum changes reflect in students' performance in university mathematics courses?'

What will happen during the study? In early September, you filled in the survey that helped us modify the course you are taking now. We would like to use the same survey to conduct further research, and we need your permission to use your survey. You will not be asked to further participate in this research in any way.

Will anything bad happen during the study? You might have felt uncomfortable answering math questions in the survey, and we have already addressed this issue.

What good things could happen if I participate? We hope to learn a lot about how reforms in high school mathematics prepare students (in terms of skills and knowledge) for university courses in mathematics. Understanding our students' background and learning needs will help us modify appropriately our undergraduate courses and create new ones. Other universities will learn about our research as well.

Who will know what I said or did in the study? The information you provided in the survey will be kept confidential to the full extent of the law and I (Dr. Lovric) will treat all information provided to me as subject to researcher-participant privilege. The results will never be reported individually. As a matter of fact, once your survey results are correlated with Math 1A3 marks, all identifying information will be removed. Researchers (myself and graduate students) will be studying the data that no longer contains information that could identify you in any way. The surveys will be locked in a cabinet, and once the research is completed, will be destroyed using the procedures for destruction of confidential material.

What if I change my mind about participating in the study? It is your choice to be part of this study. If you decide to participate, you can decide to stop at any time, even after signing this consent form. If you decide to stop participating, there will be no consequences whatsoever to you.

Study Debriefing. Preliminary research results, once completed, will be linked to the Math 1A3 web page.

Information about Participating as a Study Subject. If you have questions or require more information about the study itself, please contact Dr Lovric (HH 411, ext. 27362, lovric@mcmaster.ca)

This study has been reviewed and approved by the McMaster Research Ethics Board. If you have concerns or questions about your rights as a participant or about the way the study is conducted, you may contact:

McMaster Research Ethics Board Secretariat
Telephone: (905) 525-9140 ext. 23142
c/o Office of Research Services
E-mail: ethicsoffice@mcmaster.ca

CONSENT

I have read the information presented in the information letter about the study being conducted by Dr Miroslav Lovric, of McMaster University. I have had the opportunity to ask questions about my involvement in this study, and to receive any additional details I wanted to know about the study. I understand that I may withdraw from the study at any time, if I choose to do so, and I agree to participate in this study. I have been given a copy of this form.

Name of Participant (please print): ________________________________

ID number: ________________________________

Signature: ________________________________

APPENDIX C: POWER POINT PRESENTATION, OCTOBER 21ST, 2006

Mathematics Background Questionnaire

Some Preliminary Survey Results for Introductory Calculus at McMaster University

Introduction and Background

- This is a project by graduate student Marcella Fioroni , a M.Sc. candidate at McMaster University (supervisor Dr. Miroslav Lovric)
- The Mathematics Background Questionnaire ("the survey ") was created and administered by Dr. Miroslav Lovric , originally as a diagnostic tool.
- Results of the survey were found to have information that is useful for research in Mathematics Education

Demographic Information

- Each year we obtained a sample size of roughly 200 -300 participants from a class of approximately 1500 students
  (a representative sample size)

- In the first part of the survey, students were asked for demographic information

A

Name (please print): ____________________

Student No.: ____________ Faculty: ____________

MATHEMATICS BACKGROUND QUESTIONNAIRE 2006

The information you are sharing will be used to gain information on your high school mathematics background. It will help your mathematics instructor better plan and design the calculus course. Your responses will be kept confidential and will never be reported individually. **Thank you for taking the time to complete this questionnaire.**

- Your gender: ____________ Age (years and months): ____________
- How many years did you go to high school? What high school(s) did you attend (for each school, list: name, location (city and country), and for how long you attended it)

- Your high school math marks. Indicate the marks that you got for the courses below. If you do not remember the mark, write 'yes' instead.

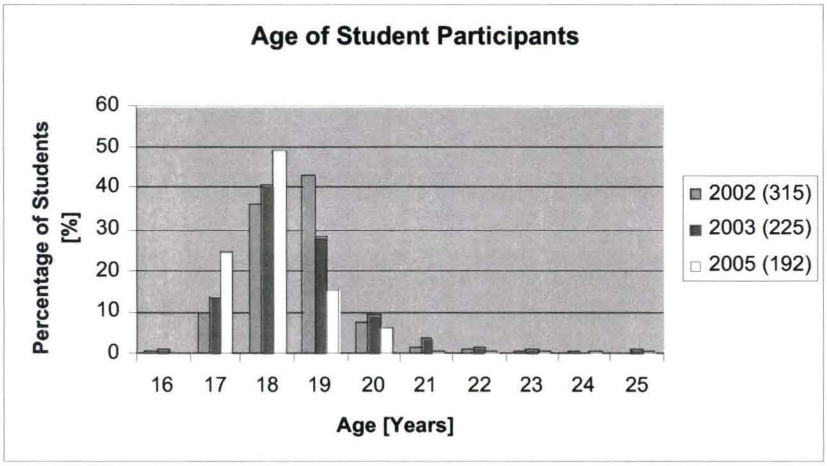
Advanced Functions and Introductory Calculus U: ____________

Geometry and Discrete Math U: ____________ Mathematics of Data Management U: ____________

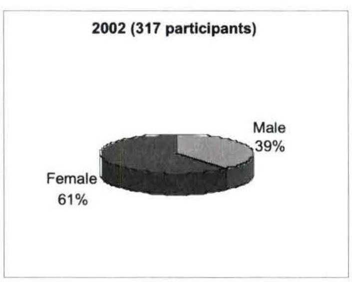
If you took courses not listed above in your final year in high school, give their name(s) and marks you got:

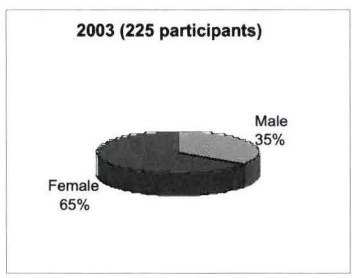
- What language do you speak most often in your parental home(s)?

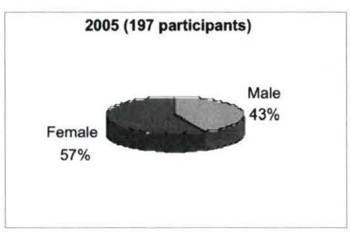
____

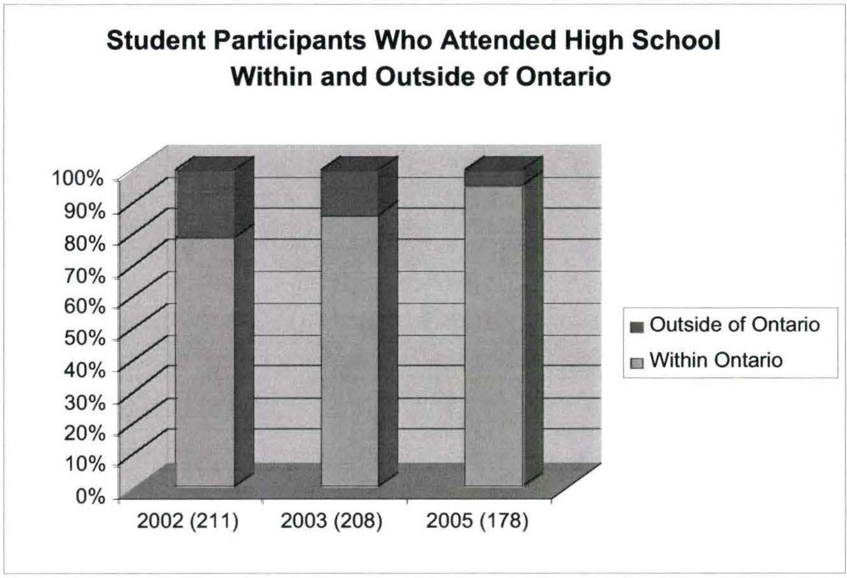


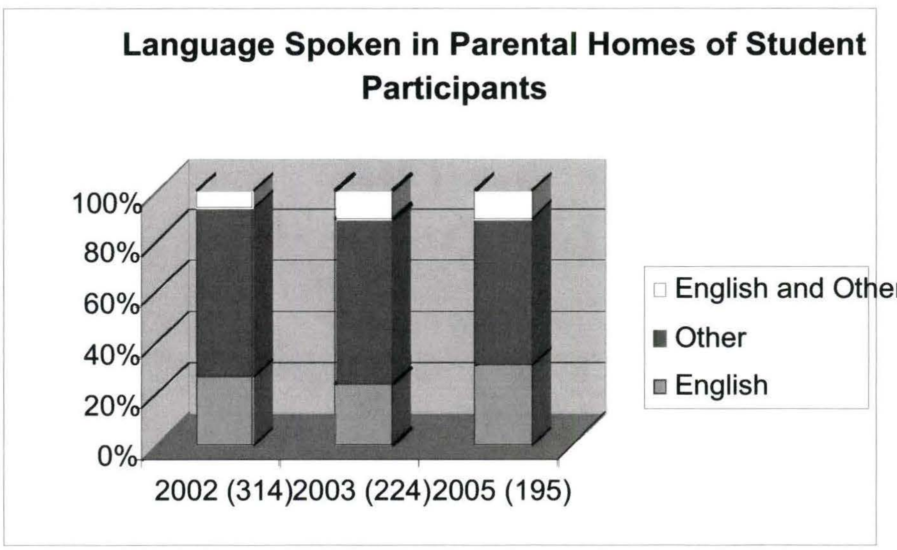
Gender of Student Participants








Student Participants Who Attended High School Within and Outside of Ontario
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
2002 (211)
2003 (208)
2005 (178)
Outside of Ontario
Within Ontario


Language Spoken in Parental Homes of Student Participants
100%
80%
60%
40%
20%
0%
2002 (314)
2003 (224)
2005 (195)
English and Other
Other
English

Purpose

The purpose of this project is two -fold. We will:

- analyze survey results in light of the recent High School curriculum changes in grade 12 mathematics.
- correlate survey results with students ' final grade in the course.

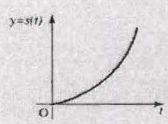
Correlation

- Students taking only 1 math course in high school on average scored 53.16% in first year Calculus
- Students taking 2 math courses in high school on average scored 66.26% in first year Calculus
  - Calc/Alg combo: 69.49%
  - Calc/Finite or Calc/Data combo: 63.02%
- Students taking all 3 high school math courses on average scored 74.66% in first year Calculus

Focus

- The survey contains 12 math problems.
- For this project, we focus on one particular survey question.

6. A position function $y = s(t)$ is given below.



(a) Describe the velocity $v(t)$ as increasing or decreasing. Explain how you know.

(b) Is the acceleration $a(t)$ positive or negative? Explain how you know.

Focus

This question was chosen because:

- it is conceptual (not procedural) in nature.
- it should reflect changes made in the High School Mathematics curriculum in 2000 (it is an applications problem and requires communication of a mathematical idea).

Grading Scheme

- Questions were graded both numerically and by letter code.
- Numeric scores ranged from 0 to 5. Letter codes were assigned from P, G, B, R, M, S, I, V, D, T.

Sample Answers

"V(t) is increasing seeing as the slope of the tangent is increasing " (2003, 5BT)

"Increasing because the slope is increasing " (2005, 4B)

"Use pretend equation s(t)=x^2, therefore v(t)=2x. Therefore velocity is increasing because the slope is increasing and slope indicates velocity" (2002, 4BG)

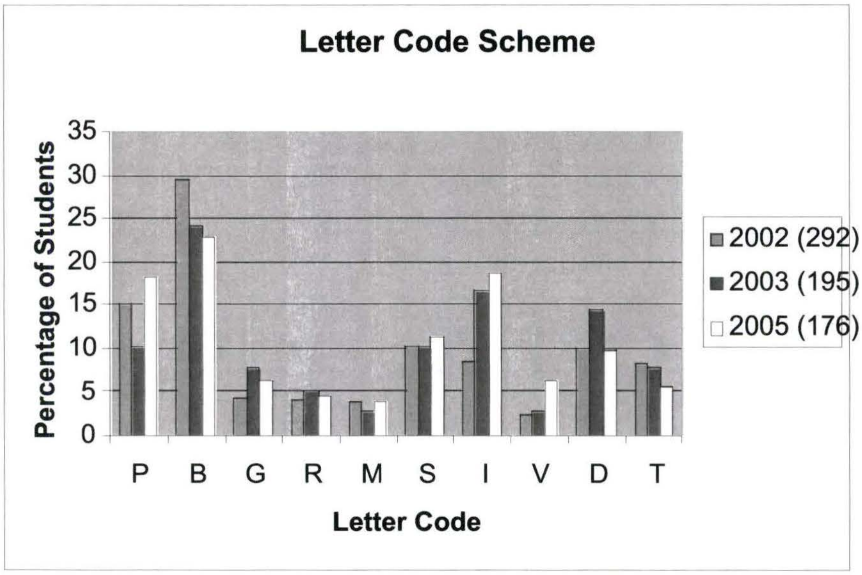
Sample Answers

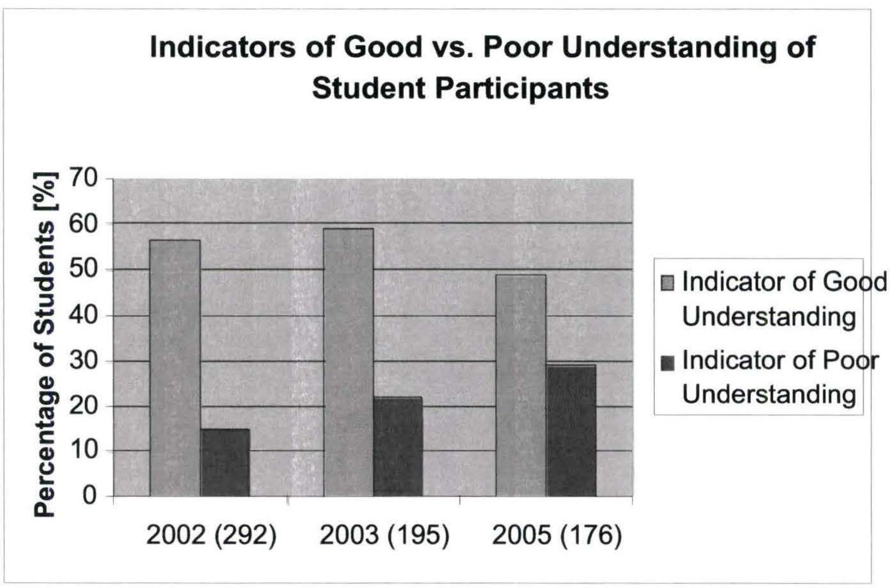
"Take first derivative it is increasing " (2003, 3D)

"Let x1<x2 and they are any number in the domain. We can see y2>y1 from the graph. So, the velocity is increasing. " (2003, 2I)

"Velocity is increasing because the slope of the tangent line is positive " (2003, 2ST)

"The velocity is increasing because as time increases, the velocity increases " (2005, 2V)

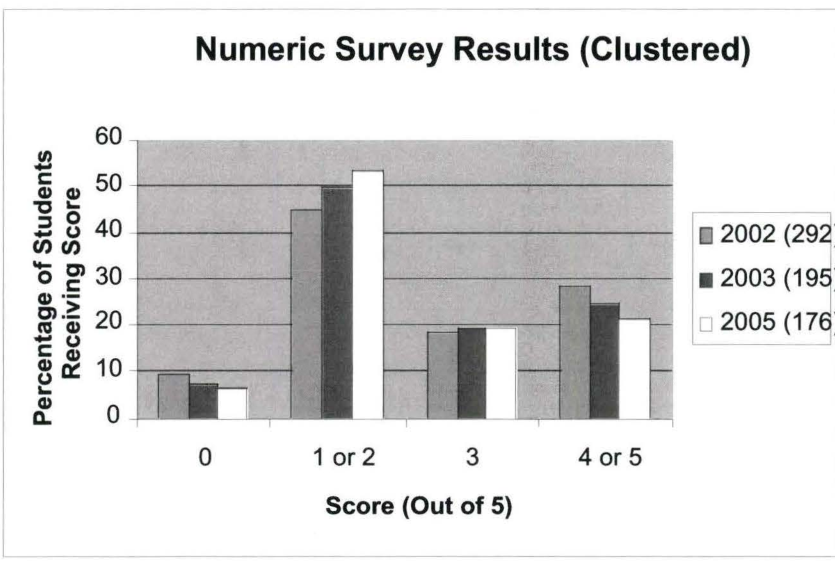

Letter Code Scheme
Percentage of Students
35
30
25
20
15
10
5
0
P
B
G
R
M
S
I
V
D
T
Letter Code
2002 (292)
2003 (195)
2005 (176)


Indicators of Good vs. Poor Understanding of Student Participants
Percentage of Students [%]
70
60
50
40
30
20
10
0
2002 (292)
2003 (195)
2005 (176)
Indicator of Good Understanding
Indicator of Poor Understanding

Interesting Results

- From 2002 to 2005, student responses containing indicators of good understanding dropped from 56.2% to 48.9%

- From 2002 to 2005, student responses containing indicators of poor understanding increased from 14.7% to 29.0%



Interesting Results

- Student scores in the "4 or 5" category decreased by 7.4% from 2002 to 2005
- Student scores in the "1 or 2" category increased by 8.9% from 2002 to 2005
- Scores in the "3" category stayed roughly the same
- Scores in the "0" category decreased by 2.7%

Why is the transition getting harder?

- Mathematical maturity of students (procedural vs. conceptual learning)
- Summer losses
- Confidence and care in answers
- Different education
- University courses need more adjustment

Can we make the transition smoother?

- Math Background surveys as diagnostic tools; adjust courses
- Collaboration between High School and University teachers
- Changes in curriculum (both in High School and University)

Thank you

Future Work

"The importance of communication in mathematics is a highlight of the secondary school curriculum. In all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings "

- We would like to incorporate an indicator for communication in our study

Letter Coding Scheme

- P – uses the shape of the curve to explain their answer. If stated properly, this could be adequate (i.e., the velocity is increasing because the curve is concave up) or not (eg, the curve goes up and to the right)
- M – involves the (inadequate) statement that more distance is covered in less time.
- R – involves the adequate statement that more distance is being covered over set (equal) time intervals
- G – uses the assumption that the graph is a specific curve such as a parabola, exponential or power curve. (not completely adequate, but on the right track)
- B – mentions that the slope/curve/graph/slope of curve/slope of graph is increasing. If they mention the slope of the tangent is increasing, they get full credit.
- S – says the slope is positive (inadequate)

Letter Coding Scheme Cont'd

- D – uses or mentions derivatives in some way; basically, understands the idea that a derivative is involved
- I – says the velocity is increasing because as time increases, position/distance/displacement increases.
- V – says the velocity is increasing because the velocity/speed is increasing (this response was surprisingly not rare)
- T – mentions the term 'tangent' somewhere in their response

Indicators:

- Indicators of good understanding: D, T, B, R, G
- Indicators of poor understanding: I, V, M

Numerical Coding Scheme

0 – incorrect answer (decreasing). If it was accompanied by an explanation, the explanation received a letter code.

1 – correct answer (increasing) without an accompanying explanation

2 – correct answer with some attempt at an explanation. In this case, the explanation is either mathematically incorrect, not relevant (eg. Increasing, since the slope is positive) or nonsensical.

3 – correct answer with accompanying explanation. In this case, the explanation is either on the right track, but not precise enough or partly correct and partly incorrect.

4 – correct answer with accompanying explanation. In this case, nothing about their statement is incorrect, but the response is either not completely precise or there is some ambiguity in the terminology.

5 – correct answer with accompanying explanation. The explanation is precise and at most would have a minor detail missing.

APPENDIX D: MCMASTER UNIVERSITY GRADING SYSTEM

Grade	Equivalent Grade Point	Equivalent Percentages
A+	12	90-100
A	11	85-89
A-	10	80-84
B+	9	77-79
B	8	73-76
B-	7	70-72
C+	6	67-69
C	5	63-66
C-	4	60-62
D+	3	57-59
D	2	53-56
D-	1	50-52
F	0	0-49 -- Failure

REFERENCES

Adler, J. and Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education, *Journal for Research in Mathematics Education* **37**(4), 270-296.

Alexander, K.L., Entwisle, D.R. and Olson, L.S. (2001). Schools, achievement and inequality: A seasonal perspective, *Educational Evaluation and Policy Analysis* **23**(2), 171-191.

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning, *Journal of Mathematics Teacher Education* **9**, 33-52.

Artigue, M. (2001) What can we learn from educational research at the university level? In D. Holton (Ed.), *The Teaching and Learning of Mathematics at the University Level: An ICMI Study* (pp. 207-220). The Netherlands: Kluwer Academic Publishers.

Cai, J., Magone, M. E., Wang, N. & Lane, S. (1996). Describing student performance qualitatively. *Mathematics Teaching in the Middle School* **1**(10), 828-835.

Crawford, K., Gordon, S., Nicholas, J. and Prosser, M. (1998). Qualitatively different experiences of learning mathematics at university, *Learning and Instruction* **8**(5), 455-468.

De Guzman, M., Hodgson, B.R., Robert, A. and Villani, V. (1998) Difficulties in the passage from secondary to tertiary education, *Proceedings of the International Congress of Mathematicians* **3**, 747-762.

Devlin, M. (2002). An improved questionnaire for gathering student perceptions of teaching and learning, *Higher Education* **3**, 289-301.

Doerr, H.M. (2006). Examining the tasks of teaching when using students' mathematical thinking, *Educational Studies in Mathematics* **62**, 3-24.

Gruenwald, N., Klymchuk, S. and Jovanski, Z. (2004) Reducing the gap between the school and university mathematics: University lecturers' perspective. *The New Zealand Mathematics Magazine* **41**(3), 12-24.

Handa, Y. (2003). A phenomenological exploration of mathematical engagement: Approaching an old metaphor anew, *For the Learning of Mathematics* **23**(1), 22-28.

Kajander, A. and Lovric, M. (2004b). Transition from secondary to post-secondary mathematics: Changing features of students' mathematical knowledge and skills and their influence on students' success. *Proceedings of the 26th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Toronto: University of Toronto.

Kajander, A. and Lovric, M. (2005). Transition from secondary to tertiary mathematics, McMaster University experience, International Journal of Mathematics, Science and Technology, **36**(2-3), 149-160.

Kajander, A. and Lovric, M. (2005b) Mathematics Background Survey – An insight into students' preparation for university mathematics courses. *Proceedings of the First Africa Regional Congress of the International Commission on Mathematical Instruction. Johannesburg*, South Africa: University of the Witwatersrand.

Kajander, A. and Lovric, M. (2004) The double cohort: Preliminary research on the transition from secondary to university level mathematics, *The Ontario*

Mathematics Gazette **42**(4), 33-36.
MacGillivray, H. and Cuthbert, R. (2003) Investigating weaknesses in the underpinning mathematical confidence of first year engineering students. *Proceedings of the Australasian Engineering Education Conference*, 358-368.
Ministry of Education (1985). Curriculum Guideline – Mathematics, intermediate and senior divisions. Toronto: Queen's Printer for Ontario.
Ministry of Education (2000). The Ontario Curriculum grades 11 and 12 - Mathematics. Toronto: Queen's Printer for Ontario.
Novotna, J. (2000) Students' levels of understanding of word problems. *Proceedings of the ICME-9 Conference*, Tokyo, Japan 2000.
Seldon, A. and Seldon, J. (2001). Tertiary mathematics education research and its future. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at the University Level: An ICMI Study* (pp. 237-254). The Netherlands: Kluwer Academic Publishers.
Schoenfeld, A.H. (2000). Purposes and methods of research in mathematics education, *Notices of the American Mathematical Society* **47**(6), 641-649.
Varsavsky, C. (2003, November). *What students say they learned – A case study in first year calculus*. Paper presented at Remarkable Delta '03 conference, Queenstown, New Zealand. Abstract Retrieved September 1, 2006 from the World Wide Web: http://www.maths.otago.ac.nz/delta03/.
Wood, L. (2001) The secondary-tertiary interface. In D. Holton (Ed.), *The Teaching and Learning of Mathematics at the University Level: An ICMI Study* (pp. 87-98). The Netherlands: Kluwer Academic Publishers.