# POLYHEDRA COMPUTATION UNDER SYMMETRY

# POLYHEDRA COMPUTATION UNDER SYMMETRY

By

GABRIEL INDIK, B.Sc.

### A Thesis

Submitted to the School of Graduate Studies in Partial Fulfilment of the Requirements

for the Degree

Master of Science

McMaster University

© Copyright by Gabriel Indik, May 2006

MASTER OF SCIENCE (2006) (Computing and Software)

McMaster University Hamilton, Ontario

TITLE:

Polyhedra Computation

**Under Symmetry** 

AUTHOR:

Gabriel Indik, B.Sc.

SUPERVISOR:

Dr. Antoine Deza

NUMBER OF PAGES: xiv, 86.

### Abstract

The last 15 years have seen significant progress in the development of general purpose algorithms and software for polyhedral computation. Many polytopes of practical interest have enormous output complexity and are often highly degenerate, posing severe difficulties for known general purpose algo-They are, however, highly structured and attention has turned to exploiting this structure, particularly symmetry. We focus on polytopes arising from combinatorial optimization problems. In particular, we study the metric polytope associated to the well-known maxcut and multicommodity flow problems, as well as to finite metric spaces. To tackle the huge size of the problem – hundreds of trillions of vertices – a parallel orbitwise enumeration algorithm was implemented and run on Shared Hierarchical Academic Research Computing Network (SHARCNET) clusters. Exploiting the high degree of symmetry, we provide for the first time a description of the highly degenerate metric polytope in dimension 36. The description consists of 1 056 368 orbits and is conjectured to be complete. While the validity of the dominating set conjecture [Laurent-Poljak 1992 is proven for the overwhelming majority of the known vertices of the metric polytope, we disprove the conjecture by exhibiting counterexamples in dimensions 36 and 45.

### Acknowledgments

The thesis was written under the guidance and with the help of my supervisor, Dr. Antoine Deza. His valuable advice and extended knowledge of the area helped me to do my best while working on the thesis. My special thanks are to the members of the examination committee: Dr. Franya Franck (Chair), Dr. Tamás Terlaky and Dr. Antoine Deza.

I would like to thank to the Shared Hierarchical Academic Research Computing Network (SHARCNET) for their generous allocation of CPU time on their shared memory and distributed clusters. Also, I appreciate the great aid and support from all the members of the Advanced Optimization Laboratory, my special thanks to Imre Pólik for his generous assistance through out the development of the project.

Finally, I am indebted to thank my family and friends for their patience, understanding and continuous support.

## Contents

A	Abstract								
A	Acknowledgments								
Li	List of Figures								
Li	st of	Tables	xi						
1	roduction	1							
	1.1	Polytopes	3						
	1.2	Thesis Outline	5						
2	Polyhedra Computation								
	2.1	The vertex Enumeration Problem	7						
	2.2	Pivoting Algorithms	9						
	2.3	Incremental Algorithms	9						
3	Pol	yhedra Computation Under Symmetry	11						
	3.1	Generating Vertices with Symmetries	13						
		3.1.1 Combinatorial Polytopes	13						
		3.1.2 Orbitwise Enumeration Algorithm	16						

	3.2	2 Generating Vertices of the Metric Polytope						
		3.2.1	Results and Conjectures on the Skeleton of the Metric					
			Polytope	20				
		3.2.2	Heuristic: Skipping High Degeneracy	21				
4	Soft	tware 1	Implementation: Parameterization, Design and Par-					
	alle	lizatio	n	23				
	4.1	Paran	neterization	23				
	4.2	Design	1	27				
		4.2.1	Orbit Database	28				
		4.2.2	Orbitwise Adjacency Calculator	28				
		4.2.3	Enumeration Controller	28				
	4.3	Parall	elization	28				
	4.4	Comp	utation	30				
5	Cou	ınterez	camples to the Dominating Set Conjecture	33				
	5.1	5.1 Counterexamples to the Dominating Set Conjecture						
	5.2	5.2 Related Questions						
		5.2.1	The Diameter of the Metric Polytope	34				
		5.2.2	The No-cut Set Conjecture	35				
	5.3	Count	erexample Generation and Verification	35				
		5.3.1	Counterexample Generation	36				
		5.3.2	Counterexample Verification	36				
		5.3.3	Counterexample Incidence and Adjacency Lists	37				
			Incidence List	37				
			Adjacency List	38				

	5.3.4	Computationally Challenging Orbits	40			
	5.3.5	Counterexample for the Metric Polytope on 10 Nodes $$ .	41			
6	Conclusion	ns	43			
$\mathbf{A}$	Metric Po	lytope met <sub>9</sub> Orbitwise Description	45			
В	Domination tope met <sub>9</sub>	ng Set Conjecture Counterexamples for Metric Poly-	49			
C	Software I	Publication	81			
Bi	Bibliography					

## List of Figures

1.1	Platonic solids	٠	٠		•			4
4.1	Orbit distribution pattern in metric polytope							24
4.2	$Computation\ window\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .$					*		25
4.3	Lowest incidence initial orbit set computation $\ .$				•			26
4.4	Targeting the next cluster of tractable orbits $ . $ .							27
4.5	Orbit computation distribution			•				27
4.6	Orbitwise enumeration main modules							29
4.7	Process parallelization							30
4.8	Incremental computation of the metric polytope		·					32
C.1	MPI-OWE Home Page							81

## List of Tables

4.1	Clusters used in the computation	31
A.1	Orbits of met <sub>9</sub> by incidence with minimum and maximum adja-	
	cency	46
A.2	Orbits of met <sub>9</sub> by incidence with minimum and maximum de-	
	nominator	47
A.3	Orbits of met <sub>9</sub> by denominator with minimum and maximum	
	incidence and adjacency	48
B.1	Counterexamples for $met_9$	50

M.Sc. Thesis - Gabriel Indik

McMaster - Computing and Software

## Chapter 1

### Introduction

Convex polytopes are the d-dimensional analogues of 2-dimensional convex polygons and 3-dimensional convex polyhedra. To a large extent the geometry of polytopes is just that of  $\mathbb{R}^d$  itself. These geometric objects of relevant importance in various areas of mathematics and other disciplines have been studied since antiquity (e.g., the platonic solids). Interest in the theory of convex polytopes grew tremendously in the second half of the 20th century due to its relation with linear programming (i.e., optimizing a linear function over the solutions of a system of linear inequalities). Dantzig's Simplex Algorithm, developed in the late 40's, showed that geometric and combinatorial knowledge of convex polytopes is key for finding and analyzing solution procedures for linear programming problems.

A convex polytope can be defined as the bounded intersection of a finite set  $\mathcal{H}$  of halfspaces. The well known theorem of Minkowski-Weyl states that polytopes can also be defined as the convex hull of its set characterization of polytopes give rise to two closely related computational problems: how to compute vertex enumeration problem, and how to compute  $\mathcal{H}$  from as the facet

enumeration problem. These two problems are essentially equivalent under the point/hyperplane duality.

The vertex/facet enumeration of combinatorial polytopes, i.e. polytopes arising from combinatorial optimization problems, is often trivial for the very first cases and then suddenly the so-called combinatorial explosion occurs even for small instances. While these polytopes turn out to be quickly intractable for enumeration algorithms designed for general polytopes, algorithms using their rich combinatorial features can exhibit surprisingly strong performances. In the last five years different research groups have proposed new enumeration techniques for combinatorial polytopes, in particular the metric polytope, that exploit their large symmetry groups making it possible tackle problems that until now have been intractable.

In this thesis we analyze in detail the vertex enumeration problem applied to the metric polytope. We begin by studying the two main classes of enumeration algorithms designed for general polytopes, namely incremental and pivoting based algorithms. We show how combinatorial polytopes quickly turn intractable for these methods. We then analyze how the rich combinatorial structure of the metric polytope can be exploited to perform enumerations that would otherwise be impossible. Specifically by partitioning its vertices into symmetry groups called orbits and performing an orbitwise enumeration based on adjacency decomposition. A parallel implementation of an orbitwise enumeration algorithm is then developed and experimental computations are carried out on SHARCNET clusters. We perform first-time computation of metric polytope in dimensions 36 and 24 and use its results to determine new combinatorial properties in order to reach tighter relaxations, thereby gener-

ating closer approximations. In particular, we exhibit a set of vertices that are counter examples for the dominant set conjecture of Laurent and Poljak obtained from the computation of these two polytopes. We also exhibit a set of vertices that reveal the limitations of certain heuristics used for computing the structure of polyhedra.

### 1.1 Polytopes

We recall some definitions and elementary properties concerning polyhedra. A complete presentation can be found in BAYER AND LEE [4] BRØNDSTED [5], GRÜNBAUM [23], McMullen and Shephard [27], and Ziegler [32]. A convex polytope is an intersection of a finite number of closed half spaces in  $\mathbb{R}^d$ . Since we do not consider non-convex polyhedra, we often omit the term convex. Let P be a d-dimensional polytope, a linear inequality  $c \cdot x \leq c_0$  is valid for P if it is satisfied for all points  $x \in P$ . A face F of P is any set of the form:

$$F = P \cap \{x \in \mathbb{R}^d : c \cdot x = c_0\}$$

where  $c \cdot x \leq c_0$  is a valid inequality for P. The dimension of a face is the dimension of its affine hull. A proper face of P is a face F such as  $F \neq \emptyset$ . The faces of dimension 0, 1, d-2 and d-1 are respectively called the vertices, edges, ridges) and facets of the polytope. One of the earliest results in the field is the generalization by Schlafli in 1852 of Euler's relation stating that the alternating sum of the number of i-faces (including the improper faces  $\emptyset$  and P itself) equals zero. For the case d=3, it was discovered by Euler in 1752. The face lattice of a convex polytope is the set of all its faces partially ordered by inclusion. Two polytopes are combinatorially equivalent, respectively dual, if there is a bijection between their faces with preserves, respectively reverses,

the inclusion relations.



Figure 1.1: Platonic solids

A d-dimensional polytope with exactly d+1 vertices is called a *simplex*. A polytope such that each vertex belongs to exactly d edges is simple, and a polytope such that each facet contains exactly d vertices is simplicial. A d-dimensional polytope is called k-simplicial if each k-face is a simplex. The dual of a k-simplicial polytope is called k-simple. Figure 1.1 illustrates different types of polytopes in dimension 3, namely the five platonic solids: tetrahedron, cube, octahedron, dodecahedron and isocahedron. We recall some definitions and elementary properties concerning the graph of a polytope. A complete presentation can be found in BRØNDSTED [5] and ZIEGLER [32]. The main reference for the general graph theory is Brouwer, Cohen and Neumaier [6]. The vertices and edges of a d-dimensional polytope P clearly form an undirected graph G(P) called the skeleton of P. The diameter  $\delta(P)$  of a polytope P is the diameter of its skeleton, that is, the smallest number such that any two vertices of P can be connected by a path with at most edges. In this thesis we will consider only non-redundant representation of bounded convex polyhedra. An representation is redundant is removing an hyperplane does not change the polyhedra.

### 1.2 Thesis Outline

The current Chapter 1 is an introduction to convex polyhedra and some motivation to study the structure of these objects.

Chapter 2 deals the vertex enumeration problem for combinatorial polyhedra and its complexity. We also discuss the strengths and limitations of the two main classes of algorithms for vertex enumeration: pivoting and incremental algorithms.

Chapter 3 introduces the adjacency decomposition method with exploits the large symmetry group of most combinatorial polytopes and enable us to to tackle problems that would be intractable by pivoting or incremental algorithms.

Chapter 4 is devoted to the technical design and development of a software that implements the adjacency decomposition method for vertex enumeration. We also describe how the software was used on large scale SHARCNET parallel clusters, leading to the first time computation of a particulary challenging polytope: the 36-dimensional metric polytope met<sub>9</sub> on 9 nodes.

In Chapter 5 we introduce a theoretical result yielded from the computation met<sub>9</sub>: we provide counterexamples in dimension 36 and 45 for the dominating set conjecture of MONIQUE LAURENT AND SVATOPLUK POLJAK open since 1992.

Finally, Chapter 6 contains concluding remarks and suggestions for future work.

Appendix A contains some of the most important numerical results obtained from the computation of the 36-dimensional metric polytope met<sub>9</sub>.

Appendix B contains the set of orbits of counterexamples for the domi-

M.Sc. Thesis - Gabriel Indik

nating set conjecture.

Appendix C contains information about the online publication of the software developed in this thesis as well as sample input and output files for computing the metric polytope  $\text{met}_n$  for  $n=3,\ldots,9$ .

## Chapter 2

## **Polyhedra Computation**

### 2.1 The vertex Enumeration Problem

A d-dimensional convex polyhedron is the intersection of a finite number m of non-redundant halfs-paces  $\mathcal{H} = \{H_1, H_2, \dots, H_{|\mathcal{H}|}\}$  of  $\mathbb{R}^d$ . A bounded polytope P can also be expressed as the convex hull of its vertices  $\mathcal{V} = \{v_1, v_2, \dots, v_{|\mathcal{V}|}\}$ . These descriptions of P will be referred to as the halfspace and vertex descriptions, respectively. In this thesis we consider only non-redundant and bounded convex polyhedra. The size of a polytope, denoted  $\operatorname{size}(P) = (|\mathcal{H}| + |\mathcal{V}|)d$ , is the space required to store both descriptions of a polytope. There are three closely related computational problems concerning the two descriptions of a polytope:

- The vertex enumeration problem asks to compute  $\mathcal V$  from  $\mathcal H$ .
- The facet enumeration problem asks to compute  $\mathcal{H}$  from  $\mathcal{V}$ .
- The polytope verification problem asks to decide whether a given vertex description  $\mathcal{V}$  and halfspace description  $\mathcal{H}$  determine the same polytope.

The first two problems are equivalent under point/hyperplane duality. Either of the first two can be trivially used to solve the third. To solve the vertex

enumeration using polytope verification, verify that all vertices have been found for each facet of P. If not, recurse on that facet. In at most  $d |\mathcal{H}|$  polytope verification calls, the algorithm finds a new vertex. In this thesis we focus on the vertex//facet enumeration problems which are essentially equivalent under point/hyperplane duality. We therefore restricted the discussion to the vertex enumeration problem. When measuring the efficiency of a vertex enumeration algorithm an important factor is the large gap between the upper and lower bound of the output size, i.e., the number of vertices. McMullen [26] upper bound theorem give that the number of vertices of a d-dimensional polytope P defined as the intersection of  $|\mathcal{H}|$  halfspaces can be as large as:

$$V_{max} = \binom{|\mathcal{H}| - \lceil \frac{d}{2} \rceil}{\lfloor \frac{d}{2} \rfloor} + \binom{|\mathcal{H}| - 1 - \lceil \frac{d-1}{2} \rceil}{\lfloor \frac{d-1}{2} \rfloor}$$

BARNETTE [3] lower bound theorem (for simple polytope) gives that the number of vertices could be as few as:

$$V_{min} = (|\mathcal{H}| - d)(d - 1) + 2$$

This large possible gap suggests that the performance of vertex enumeration algorithms be measured not only in terms of input size  $d|\mathcal{H}|$  but also in terms of output size  $d|\mathcal{V}|$ .

The main vertex enumeration algorithms can be viewed as not just generating all vertices of a polytope P but actually the skeleton of P, i.e. the graph formed by its vertices and edges. There are essentially two main classes of algorithms for producing these graphs: graph traversal algorithms and incremental algorithms.

### 2.2 Pivoting Algorithms

Pivoting algorithms, also called graph traversal algorithms, first find one vertex of the skeleton and then attempt to identify all vertices and edges of the graph by traversing it in some fashion. In the case of vertex enumeration each vertex v of a d-polytope P can be identified by a basis, i.e. d facets that contain v and whose spanning hyperplanes are affinely independent. Two simple vertices of P are connected by an edge of P if they have bases that differ in exactly one member. Going from a vertex to an adjacent one during the graph traversal amounts to changing this one member of the basis. This operation is known as pivoting in the simplex algorithm for linear programming. For this reason graph traversal algorithms for vertex enumeration are also known as pivoting algorithms. In the context of facet enumeration going from one facet to a neighboring can be viewed as rotating a supporting hyperplane about the common ridge. Representatives of this class of graph traversal algorithms are the gift wrapping algorithm of Chand and Kapur [7], Seidel [31] algorithm and the reverse search algorithm of Avis and Fukuda [2]

### 2.3 Incremental Algorithms

Incremental algorithms for the vertex enumeration problem compute the vertex description by intersecting the defining halfspaces sequentially. An initial simplex is constructed from a subset of d+1 affinely independent halfspaces and its vertices and skeleton are computed. Additional halfspaces are introduced sequentially and the vertex description and skeleton are updated at each stage. Essentially such an update amounts to identifying and removing all vertices

that are not contained in the new halfspace, introducing new vertices for all intersections between edges and the bounding hyperplane of the new halfspace, and generating the new edges between these new vertices. The first explicit description of such an algorithm, now widely known as the double description method, appeared in the pioneering paper of MOTZKIN [?] in 1953. We should mention that so-called Fourier-Motzkin elimination can be viewed just as a dual formulation of the double description method and thus falls in the class of incremental algorithms. All incremental algorithms can use different insertion orders. One can distinguish between static insertion orders, which are determined from the inputs before the actual incremental algorithm is started, and dynamic insertion orders, where the next halfspace to be considered is a function of the current polytope and the remaining halfspaces. Typical static orderings are minindex (process in order as the input happens to be), lexicographic (process halfspaces in lexicographic order of their suitably normalized coefficient vectors), and random. Typical dynamic orderings are maxcut and mincut (as next halfspace choose the one whose complement constrains the most or least vertices of the current polytope). The same algorithm applied to the same input can have vastly different running times when different insertion orders are used. Thus choosing a good insertion order is crucial.

## Chapter 3

# Polyhedra Computation Under Symmetry

A full d-dimensional bounded polytope P can be defined either by the set  $\mathcal{H}$  of its facet-inducing linear inequalities or as the convex hull of its vertex set  $\mathcal{V}$ . While these two representations are theoretically equivalent, converting one into the other is a difficult computational problem. The computation of  $\mathcal{H}$  from  $\mathcal{V}$  is the facet enumeration problem and the computation of  $\mathcal{V}$  from  $\mathcal{H}$  is the vertex enumeration problem. These two problems being equivalent by the vertex/facet duality, we mainly consider the vertex enumeration problem.

The two main enumerations are: the pivoting methods and the incremental methods. The basic pivoting method first finds one vertex of P and then identifies all vertices (and edges) by moving from one vertex to an adjacent one. In this method, each vertex v is described by a basis- i.e. d affinely independent inequalities containing v. Moving from one vertex to an adjacent one amounts to change one member of the basis in some proper way. The basic insertion method first selects d+1 affinely independent inequalities and computes the vertices and edges of the associated d-simplex. Then at each step k one of

the remaining inequalities  $H_k$  is inserted and the vertex and edge description is updated by removing the vertices cut off by the newly inserted inequality  $H_k$  and adding new vertices (and edges) created by the intersections of edges of the intermediate polytope  $P_k$  with the newly inserted inequality  $H_k$ . For a detailed presentation of main existing algorithms we refer to AVIS, BREMMER AND SEIDEL [1] and references therein.

Even though these algorithms often perform quite well for many cases in particular for low dimensional and simple polytopes - and despite the fact that the vertex enumeration problem has been extensively studied by many authors - see for instance [2, 7, 21, 28, 30] - there is no satisfying algorithm for generating the vertices of a general polytope given by its facets. In [1] AVIS, Bremmer and Seidel underlined the main weakness of the pivoting and insertion methods: "...inability to deal with degeneracies or inability to control the sizes of intermediate results". For each main class of algorithms, they exhibited a family of polytopes for which the performances can be very poor. Degeneracy occurs when the incidence  $Icd_v$  of a vertex v - i.e. the number of facets containing v - is strictly higher that the dimension d. A highly degenerate vertex might create a very large number of bases and a naive pivoting algorithm would visit each of them. One can deal with degeneracy either by the perturbation technique - which makes the polytope simple but might create a large number of additional vertices, see [1] for details. As for insertion algorithms, they are very sensitive to the order in which the inequalities are inserted. Computing the entire face lattice of P is a computationally extremely expensive approach.

In this thesis we focus on combinatorial polytopes, i.e. polytopes arising

from combinatorial optimization problems. On one hand, the so-called *combinatorial explosion* occurs even for small instances and these polytopes, often trivial for the very first cases, become quickly intractable by enumeration algorithm designed for solving general polytope. On the other hand, tailor-made algorithms using their rich combinatorial features can exhibit surprisingly strong performances. For example, Christof and Reinelt [8] computed large instances of the *traveling salesman polytope*, the *linear ordering polytope* and the *cut polytope* using their large symmetry group. In a similar approach, in addition to its large symmetry group, we used a partial knowledge of its combinatorial structure to orbitwise enumerated the vertices of another combinatorial polytope: the *metric polytope* met<sub>n</sub>.

### 3.1 Generating Vertices with Symmetries

### 3.1.1 Combinatorial Polytopes

We first present some polyhedra with application in combinatorial optimization. In particular we consider polyhedra associated to problems which are symmetric. We recall that the symmetry group Is(P) of a polytope P is the group of isometries preserving P. Typical examples of polytopes with large symmetry group are polytopes associated to problems arising from the complete directed graph  $D_n$  or the undirected graph  $K_n$  on n nodes. Some well know polyhedra are: the traveling salesman polytope  $tsp_n$  which is the convex hull of all the incidence vectors of all Hamiltonian cycles of  $K_n$  and the linear ordering polytope  $lo_n$  which is the convex hull of the incidence vectors of all acyclic tournaments of  $D_n$ . The isometries preserving  $tsp_n$  are induced by the n! permutations on  $V_n = \{1, 2, ..., n\}$ , that is,  $Is(tsp_n) \simeq Sym(n)$ . In this thesis we consider polytope with even larger symmetry group: the cut and metric polyhedra. The  $\binom{n}{2}$ -dimensional cut polytope cut<sub>n</sub> is usually introduced as the convex hull of the incidence vectors of all the cuts of  $K_n$ . More precisely, given a subset S of  $V_n$ , the cut determined by S consists of the pairs (i,j) of elements of  $V_n$  such that exactly one of i,j is in S. By  $\delta(S)$  we denote both the cut and its incidence vector in  $\mathbb{R}^{\binom{n}{2}}$ , that is,  $\delta(S)_{ij} = 1$  if  $|\{i,j\} \cap S| = 1$  and  $\delta(S)_{ij} = 0$  otherwise for  $1 \leq i < j \leq n$ . By abuse of language, we use the term cut for both the cut itself and its incidence vector, so  $\delta(S)_{ij}$  are considered as coordinates of a point in  $\mathbb{R}^{\binom{n}{2}}$ . The cut polytope cut<sub>n</sub>, which is also called the complete bipartite subgraphs polytope, is the convex hull of all  $2^{n-1}$  cuts, and the cut cone Cut<sub>n</sub> is the conic hull of all  $2^{n-1} - 1$  nonzero cuts.

The cut polytope and one of its relaxation - the metric polytope - can also be defined in terms of finite metric space in the following way. For all 3-sets  $\{i,j,k\}\subset\{1,\ldots,n\}$ , we consider the following inequalities:

$$x_{ij} - x_{ik} - x_{jk} \le 0 (2.1)$$

$$x_{ij} + x_{ik} + x_{jk} \le 2 (2.2)$$

The inequalities (2.1) induce the  $3\binom{n}{3}$  facets which define the *metric cone*. Then, bounding the latter by the inequalities (2.2) we obtain the *metric polytope*  $\text{met}_n$ . The  $3\binom{n}{3}$  (resp.  $\binom{n}{3}$ ) facets defined by (2.1) (resp. (2.2)) can be seen as triangle (resp. perimeter) inequalities for distance  $x_{ij}$  on  $\{1, 2, ..., n\}$ . While the cut cone is the conic hull of all, up to a constant multiple,  $\{0, 1\}$ -valued extreme rays of the metric cone, the cut polytope  $\text{cut}_n$  is the convex hull of all  $\{0, 1\}$ -valued vertices of the metric polytope. The link with finite metric spaces

is the following: there is an evident 1-1 correspondence between the elements of the metric cone and all the semi-metrics on n points, and the elements of the cut cone correspond precisely to the semi-metrics on n points that are isometrically embeddable into some  $l_1^s$ .

One of the motivations for the study of these polyhedra comes from their applications in combinatorial optimization, the most important being the maxcut problem. Given a graph  $G = (V_n, E)$  and nonnegative weights  $w_e$ ,  $e \in E$ , assigned to its edges, the max-cut problem consists in finding a cut  $\delta(S)$  whose weight  $\sum_{e \in \delta(S)} w_e$  is as large as possible. It is a well-known NP-complete problem. By setting  $w_e = 0$  if e is not an edge of G, we can consider without loss of generality the complete graph on  $V_n$ . Then the max-cut problem can be stated as a linear programming problem over the cut polytope cut<sub>n</sub> as follows:

$$\max \qquad w^T \cdot x$$
 subject to 
$$x \in \text{cut}_n \ .$$

Since the metric polytope is a relaxation of the cut polytope, optimizing  $w^T \cdot x$  over cut<sub>n</sub> instead of met<sub>n</sub> provides an upper bound for the max-cut problem.

The polytope  $\operatorname{cut}_n$  is a  $\binom{n}{2}$  dimensional 0-1 polyhedron with  $2^{n-1}$  vertices and  $\operatorname{met}_n$  is a polytope of same dimension with  $4\binom{n}{3}$  facets inscribed in the cube  $[0,1]^{\binom{n}{2}}$ . We have  $\operatorname{cut}_n \subseteq \operatorname{met}_n$  with equality only for  $n \leq 4$ . Any facet of the metric polytope contains a facet of the cut polytope and the vertices of the cut polytope are vertices of the metric polytope, in fact the cuts are precisely the integral vertices of the metric polytope. Actually the metric polytope  $\operatorname{met}_n$  wraps the cut polytope  $\operatorname{cut}_n$  very tightly since, in addition to the vertices, all edges and 2-faces of  $\operatorname{cut}_n$  are also faces of  $\operatorname{met}_n$ , for 3-faces it is false for  $n \geq 4$ , see [10]. Any two cuts are adjacent both on  $\operatorname{cut}_n$  and on  $\operatorname{met}_n$ . While

the diameter of the dual of the metric polytope  $\operatorname{met}_n^*$  is 2, the diameters of the dual cut polytope  $\operatorname{cut}_n^*$  and  $\operatorname{met}_n$  are respectively conjectured to be 4 and 3, see [9, 25]. We recall that the skeleton of a polytope is the graph formed by its vertices and edges. For a detailed study of those polytopes and their applications in combinatorial optimization we refer to Deza and Laurent [17] and Poljak and Tuza [29].

### 3.1.2 Orbitwise Enumeration Algorithm

As stressed earlier, one important feature of the metric and cut polytopes is their very large symmetry group. More precisely, for  $n \geq 5$ , we have  $Is(\text{met}_n) = Is(\text{cut}_n)$  and both are induced by permutations on  $V_n = \{1, \ldots, n\}$  and switching reflections by a cut and, for  $n \geq 5$ , we have  $|Is(\text{met}_n)| = 2^{n-1}n!$ , see [15]. Given a cut  $\delta(S)$ , the switching reflection  $r_{\delta(S)}$  is defined by  $y = r_{\delta(S)}(x)$  where  $y_{ij} = 1 - x_{ij}$  if  $(i, j) \in \delta(S)$  and  $y_{ij} = x_{ij}$  otherwise. As these symmetries preserve the adjacency relations and the linear independency, all faces of met<sub>n</sub> are partitioned into orbits of faces equivalent under permutations and switchings.

**Proposition 3.1.1** The vertices of the metric polytope  $\operatorname{met}_n$  are partitioned into orbits of its symmetry group. Let  $v_i$  be a vertex of  $\operatorname{met}_n$ ,  $\operatorname{Icd}_{v_i}$  its incidence,  $\operatorname{Adj}_{v_i}$  its adjacency and  $O_i$  the orbit generated by the action of  $\operatorname{Is}(\operatorname{met}_n)$  on  $v_i$ . For any vertex  $v \in O_i$ , we have  $\operatorname{Icd}_v = \operatorname{Icd}_{v_i}$  and  $\operatorname{Adj}_v = \operatorname{Adj}_{v_i}$ .

In other words, the neighborhood, that is, the set of vertices adjacent to a given vertex, is equivalent up to permutations and switchings for all vertices belonging to the same orbit. Proposition 3.1.1 leads to the following orbitwise enumeration algorithm. The main two subroutines are the enumeration

of the neighborhood N of a vertex v and the computation of the canonical representative  $\tilde{v}$  of the orbit O containing v. Starting from an initial vertex  $v_1$  the algorithm computes the canonical representative  $\tilde{v}_1$  of the orbit  $O_1$  generated by  $v_1$ , computes its neighborhood  $N_1$ , identifies new orbits contained in  $N_1$ , updates the list L of canonical representatives and then picks up the next canonical representative in L which neighborhood is not yet computed. The algorithm terminates when there is no more such canonical representative in L and outputs L. Since the skeleton of a polytope is connected, this algorithm finds all orbits.

### Orbitwise Enumeration Algorithm

### begin

Find an initial vertex  $v_1$ ;

Compute the canonical representative  $\tilde{v}_1$  of the orbit  $O_1$  generated by  $v_1$ ;

Mark  $\tilde{v}_1$  with a " $\circ$ ";  $\setminus *$  neighborhood not yet computed  $* \setminus$ 

Initialized the list L of canonical representatives with  $\tilde{v}_1$ ;

while L contains a vertex  $\tilde{v}_i$  marked " $\circ$ " do

#### begin

Compute the neighborhood  $N_i$  of  $\tilde{v}_i$ ;

For each vertex  $v_k$  adjacent to  $\tilde{v}_i$ ;

Compute the canonical representative  $\tilde{v}_k$  of the orbit  $O_k$  generated by  $v_k$ ;

```
If \tilde{v}_k \in L, disregard it;

If \tilde{v}_k \notin L, mark it with a "\circ" and add it to L;

Mark \tilde{v}_i with a "\bullet"; \* neighborhood computed *\

end

Output L;
```

#### end

**Lemma 3.1.2** Let I be the number of orbits,  $Icd_i$  and  $Adj_i$  the incidence and the adjacency of the orbit  $O_i$  for  $i=1,\ldots,I$ . The neighborhood enumeration subroutine is called exactly I times and each neighborhood is generated by  $Icd_i$  facets. The canonical representative computation subroutine is called exactly  $\sum_{i=1...I} Adj_i$  times.

#### Remark 3.1.3

- (i) The orbitwise enumeration algorithm performs I classic vertex enumerations for smaller sub-polytopes (one for each orbit of neighborhoods) instead of performing one large classic vertex enumeration (the whole polytope).
- (ii) The computation is independent of the choice of the initial vertex  $v_1$ .
- (iii) In case of very high degeneracy, the subroutine computing the canonical representative has to be called a large number of times and some of the neighborhoods might represent a large fraction of the whole polytope.

For i = 1, ..., I, the algorithm get the orbitwise incidence  $Icd_i$  (by simply checking which inequality is satisfied with equality) as input for the neighborhood enumeration subroutine and get the adjacency  $Adj_i$  as output of this

subroutine. By simply counting the number of times a vertex equivalent to the canonical representative  $\tilde{v}_k$  is found in  $N_i$ , we get the *orbitwise adjacency table*, that is, the  $I \times I$  matrix Adj with  $\mathrm{Adj}_{i,j}$  the number of vertices of the orbit  $O_j$  adjacent to  $v \in O_i$ . Assuming we know the size of one orbit, from the matrix Adj we can get the size of the other orbit using the following easy relation:  $\mathrm{Adj}_{i,j} \times |O_i| = \mathrm{Adj}_{j,i} \times |O_j|$ .

Remark 3.1.4 The output, that is, the list of canonical representatives  $\tilde{v}_i$  for  $i=1,\ldots,I$ , is minimally compact. Additionally to the vertex enumeration, the algorithm computes the orbits invariants  $Adj_i$ ,  $Icd_i$  and  $|O_i|$ . The orbitwise adjacency table Adj reveals the skeleton. The total number of vertices is simply  $\sum_{i=1,\ldots,I} |O_i|$  and the full list of vertices can be generated by the action of the symmetry group on each representative  $\tilde{v}_i$ .

## 3.2 Generating Vertices of the Metric Polytope

The adjacency check and the heuristics presented in this section are valid for other combinatorial polytopes but - without theoretical improvement - we restrict ourself to the metric polytope case. Insertion algorithms usually handle better high degeneracy that pivoting algorithms. The metric polytope  $\text{met}_n$  being extremely degenerate (the cut incidence  $\text{Icd}_{\delta(S)} = 3\binom{n}{3}$  is much larger than the dimension  $d = \binom{n}{2}$ ), we choose an insertion algorithm for the neighborhood enumeration subroutine: the *cddlib* implementation of the double description method [20]. In the following, we always assume that the neighborhood enumeration subroutine is performed by an insertion algorithm. Item (*iii*) of Remark 3.1.3 indicates that even the neighborhoods of highly degenerate poly-

topes might be beyond problems currently solvable by insertion algorithm. In Section 3.2.2, we present heuristics addressing this issue.

## 3.2.1 Results and Conjectures on the Skeleton of the Metric Polytope

We recall some results on the vertices of the metric polytope and a conjecture which can be seen as complementary to the LAURENT-POLJAK conjecture. The cuts are the only integral vertices of  $\text{met}_n$ . All other vertices with are not fully fractional are so-called trivial-extensions of a vertex of  $\text{met}_{n-1}$ . In other words, the new vertices are the fully fractional ones. The  $(\frac{1}{3}, \frac{2}{3})$ -valued fully fractional vertices are well studied and include the anticut orbit formed by the  $2^{n-1}$  anticuts  $\bar{\delta}(S) = \frac{2}{3}(1,\ldots,1) - \frac{1}{3}\delta(S)$ . Metric polytopes  $\text{met}_3$  and  $\text{met}_4$  have, respectively, 4 and 8 vertices made of cuts as  $\text{met}_3 = \text{cut}_3$  and  $\text{met}_4 = \text{cut}_4$ . The 32 vertices of  $\text{met}_5$  form 2 orbits: 16 cuts and 16 anticuts. The metric polytope  $\text{met}_6$  has 544 vertices (3 orbits), see [25], and  $\text{met}_7$  has 275 840 vertices (13 orbits), see [11]. The metric polytope  $\text{met}_8$  has 1 550 825 600 vertices partitioned into 533 orbits.

**Property 3.2.1** [25] Let v be a vertex of  $met_n$ , we have:  $Icd_v \leq Icd_{\delta(S)} = 3\binom{n}{3}$  with equality only for  $v = \delta(S)$  and, if v is fully fractional,  $Icd_v \leq Icd_{\bar{\delta}(S)} = \binom{n}{3}$  with equality only for  $v = \bar{\delta}(S)$ .

Conjecture 3.2.2 [25] Any vertex of the metric polytope is adjacent to a cut.

Conjecture 3.2.3 [13] For  $n \geq 6$ , the restriction of the skeleton of the metric polytope  $met_n$  to the non-cut vertices is connected.

While Conjecture 3.2.2 underlines the extreme connectivity of the cuts, Conjecture 3.2.3 means that the skeleton remains connected without the cuts. Recall that the cuts form a clique in both the cut and metric polytopes. Therefore, if Conjecture 3.2.2 holds, the cuts would be a dominant clique in the skeleton of  $\text{met}_n$  implying that its diameter would satisfy  $\delta(\text{met}_n) \leq 3$ . Conjecture 3.2.2 holds for  $n \leq 8$ . The computational applications of Conjecture 3.2.3 - which holds for  $n \leq 8$  and is strongly believed to be true - are developed in Section 3.2.2.

### 3.2.2 Heuristic: Skipping High Degeneracy

If Conjecture 3.2.3 holds, all orbits can be found by the following metric cone skipping heuristic: disregard  $\tilde{v}_k$  if  $\tilde{v}_k = \delta(S)$ . In other words, disregard the neighborhood of the cuts, that is, essentially the metric cone  $\mathrm{Met}_n$ . This neighborhood being, far from other, the largest as  $\mathrm{Adj}_{\delta(S)} \gg \mathrm{Adj}_{v \neq \delta(S)}$ , the heuristic removes the hardest neighborhood enumeration. Note that cuts are very easy to recognize as  $\delta(S)$  is uniquely characterised by its incidence:  $\mathrm{Icd}_{\delta(S)} = 3\binom{n}{3}$ . For the metric on 8 nodes, while  $\mathrm{Adj}_{\delta(S)} = 119\ 269\ 588$  - i.e. 7.7% of the total number of vertices of  $\mathrm{met}_8$  - the enumeration of the other 532 neighborhoods generates (with multiplicity)  $\sum_{i=1...I}^{v \neq \delta(S)} \mathrm{Adj}_i = 780\ 711$  vertices - i.e. less than 0.05% of the total number of vertices. One can easily get the neighborhood of the cut from the orbitwise adjacency table  $\mathrm{Adj}$ . Taking the column and the row corresponding to the cuts, we have:  $\mathrm{Adj}_{\delta(S),i} \times 2^{n-1} = \mathrm{Adj}_{i,\delta(S)} \times |O_i|$  where  $\mathrm{Adj}_{i,\delta(S)}$  is the number of cuts adjacent to  $v_i \in O_i$ .

**Proposition 3.2.4** If true, Conjecture 3.2.3 would be a certificate that the "metric cone skipping heuristic" gives a complete description of the metric poly-

tope by generating only a very small fraction of the vertices.

One can further decrease the computation time by skipping not only the orbit with the highest incidence (the cuts) but all orbits with arbitrarily high incidence. In this case, a certificate for a complete description is that the restriction of the skeleton of  $met_n$  to the low incidence vertices is connected. This heuristic is particularly suitable for partial enumeration purpose and the choice of the initial vertex become an important factor.

## Chapter 4

## Software Implementation: Parameterization, Design and Parallelization

### 4.1 Parameterization

Dealing with combinatorial polytope is a major computational challenge due to the usually enormous size and complexity resulting from the high dimensions and degeneracies. In order to perform large scale computations, a particular attention has to be given to the analysis of the combinatorial and geometric properties of the input. The first task is to carefully investigate the behavior, including the adjacency and incidence relationships, for known metric polytopes  $met_n$  for  $n=3,\ldots,8$ . Identify potentially easy/hard to compute neighborhoods is a key factor for a successful implementation of the adjacency decomposition method. As stressed earlier, the computation of the neighborhood of highly degenerate vertices with large adjacency can take a considerable amount of resources both in memory and CPU time. Given the usually high correlation between high incidence and high adjacency (and small size orbit), the orbit-

wise enumeration algorithm developed to compute met<sub>8</sub> used the *skipping high degeneracy* heuristic. The description of met<sub>8</sub> was later proven complete using additional computation of the upper face-lattice. The skipping high degeneracy heuristic consist in simply filtering out potential intractable vertices by checking if their incidence is higher than a fixed threshold value. Checking the incidence is a straightforward computation. This approach was successful for the computation of met<sub>8</sub> as all its 1 550 825 600 can be found using this heuristic. Note that while the number of vertices of met<sub>8</sub> is huge, they are partitioned in 533 orbits which is a relatively small figure. The gap with met<sub>9</sub> is large as this 36-dimensional polytope has an estimate of a hundred billion vertices, partitioned about a million orbits. The heuristic was therefore refined. Fig.4.1 gives an overview of the distribution of orbits by incidence and adjacency for met<sub>8</sub>. As it can be observed, the majority of the orbits are of low

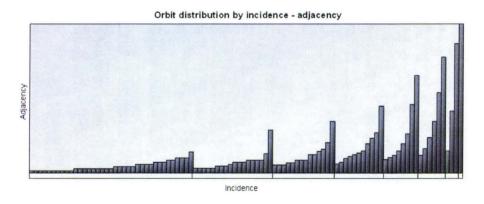


Figure 4.1: Orbit distribution pattern in metric polytope

incidence, and the adjacency distribution through out these orbits is also low but with a small peak: A small subset of low incidence orbits have relatively high adjacency. The higher the incidence, the smaller the group of orbits, and the taller the peak. This pattern is then repeated through all the orbits until reaching the orbits of very large incidence which constitute the smallest group but have the largest adjacency.

Due to the large number of orbits met<sub>9</sub>, the skipping highest degeneracy heuristic was refined to take in account the adjacency and computation-time in addition to incidence. The intractability is a result mostly of the adjacency witch is only estimated by the incidence. As illustrated by the incidence-adjacency distribution graph, some orbits with high incidence have with relatively moderate adjacency and could be computed. Incidence and adjacency upper and lower bounds were therefore added into the design of the software as parameter These bounds create a *computation window* as shown in Fig.4.2. The computation window allow us to introduce a new termination criteria: the

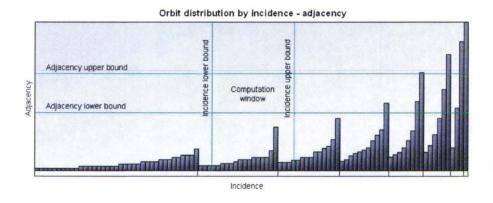


Figure 4.2: Computation window

enumeration will be considered complete when the set of orbits that are found within the computation window have all been computed. At the initial state of the vertex enumeration process, the computation window can be set to include only the set of orbits with the lowest known incidence (which have the low adjacency). This will ensure that the computation can be completed since all such orbits to be computed are tractable. This is shown in Fig.4.3. In this

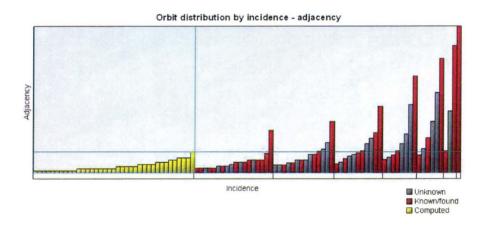


Figure 4.3: Lowest incidence initial orbit set computation

figure we can see how the computation of a small set of low incidence orbits can reveal a large amount of information about the rest of the structure of the polytope. The key idea is to turn the enumeration incremental: Once a set of orbits has been computed, resume the computation from that point, so no orbit has to be computed twice. The information obtained within each computation can be used to fine tune the next one, given that at each stage new orbits will be discovered, toward a complete description of the polytope. Once the first computation has been completed, the computation window can be set to include the next cluster of tractable orbits, as shown in Fig.4.4. Every time a new computation window is set, additional tractable orbits are computed, inducing the discovery of additional orbits. Intractable orbits can be detected in time and properly skipped. This process is then repeated till all tractable orbits have been computed as shown in Fig.4.5.

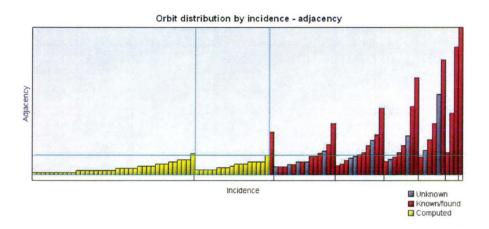


Figure 4.4: Targeting the next cluster of tractable orbits

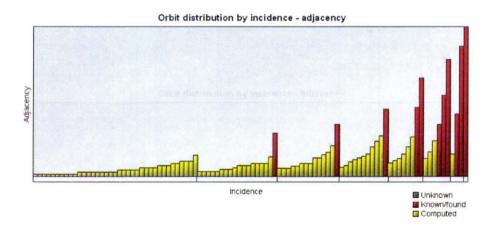


Figure 4.5: Orbit computation distribution

# 4.2 Design

An object oriented design approach was adopted for the implementation of the orbitwise enumeration algorithm. For this, a set of entities (objects) were defined, each one with its set of responsibilities. These objects are described next.

#### 4.2.1 Orbit Database

This object's responsibility is to store in memory the set of orbits under computation, as well as saving and retrieving them from a secondary storage device, such as a hard disk drive. Also, this object will be responsible for keeping track of the status of each orbit, whether it has been computed or not, found or unknown.

#### 4.2.2 Orbitwise Adjacency Calculator

This object's responsibility is to compute orbits. This is, given the coordinate of a vertex, obtain the set of adjacent vertices and return the canonical representative of the orbits they belong to. This is performed by utilizing TCDDLib, a n enhanced version of the Double Description Method Library which includes tools for incidence and adjacency control as well as time-out mechanisms.

#### 4.2.3 Enumeration Controller

This object's responsibility is to coordinate the other objects in the process. This includes initializing the Orbit database, extracting the orbits to be computed from it and feeding them into the Orbitwise adjacency calculator. This object is also responsible for program initialization and termination.

The interaction between these three main objects is detailed in Fig.4.6.

#### 4.3 Parallelization

It is important to observe that throughout the enumeration, the computation of an orbit requires only the coordinates of one of its vertices. This provides the

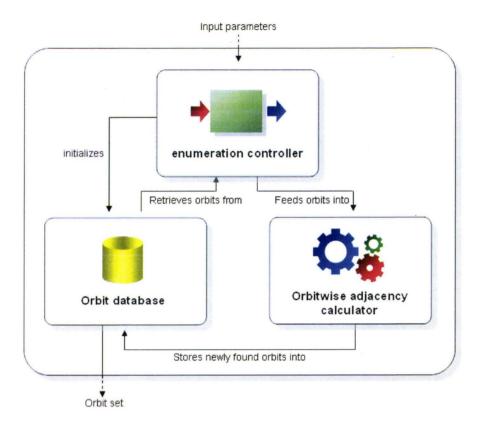


Figure 4.6: Orbitwise enumeration main modules

ideal setting to parallelize the system given that multiple orbits could be computed concurrently as there is no need to share data in between their computations. For this, we first separate the main process introduced in the preceding section into sub processes: a master process that will contain the enumeration controller and orbit database objects, and a variable number of slave processes, each containing an instance of the orbitwise adjacency calculator object. This is shown in Fig.4.7.

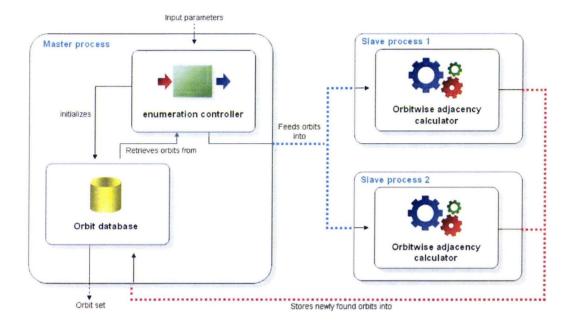


Figure 4.7: Process parallelization

## 4.4 Computation

The computation of the 36 dimensional metric polytope met<sub>9</sub> was performed on SHARCNET clusters. Established in 2001, SHARCNET is consortium of colleges and universities in a *cluster of clusters* of high performance computers, linked by advanced fibre optics. It is aimed to:

- Accelerate the production of research results to meet the needs of researchers and industry partners in today's competitive fast-paced business environments
- Unite students, researchers and companies by providing cutting edge computational expertise and hardware

Name	silky	Name	wobbe
Make	Silicon Graphics	Make	Hewkett Packard
Class	Shared memory	Class	Distributed cluster
Architecture	Intel Itanium 2	Architecture	AMD Opteron
CPUs	128	CPUs	208
OS	Linux 2.4.21	OS	Fedora Core 3
Location	UWO	Location	McMaster University

Table 4.1: Clusters used in the computation

• Link academic researchers and corporate partners in a search for new business opportunities

SHARCNET was founded by McMaster University, University of Guelph, University of Windsor, Wilfrid Laurier University, The University of Western Ontario, Fanshawe College and Sheridan College. In 2003 it was expanded with the University of Waterloo, Brock University, York University, The University of Western Ontario and the Institute of Technology. Finally, it gather some more new members in 2005, with the inclusion of Trent University, Laurentian University and Lakehead University.

The details of the two SHARCNET clusters used for this project are summarized in Table 4.1.

The computation was performed through 33 incremental steps, each including a greater variety of orbits by adjusting the computation window based on previous computations. The overall computation time was two months. The progress of the computation is presented in Fig.4.8.

Conjecture 4.4.1 The current description of metric polytope met<sub>9</sub> is complete, i.e. the vertices of met<sub>9</sub> are partitioned into the 1 056 368 orbits.

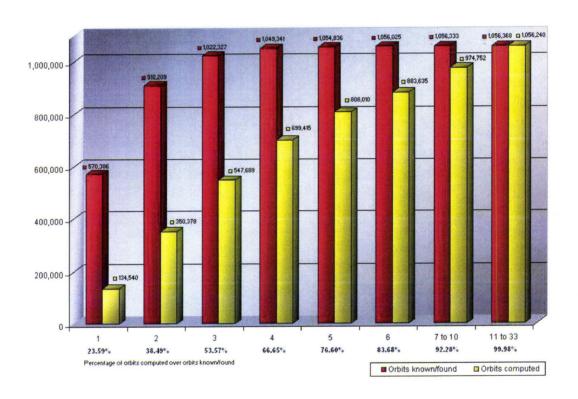


Figure 4.8: Incremental computation of the metric polytope

# Chapter 5

# Counterexamples to the Dominating Set Conjecture

# 5.1 Counterexamples to the Dominating Set Conjecture

LAURENT AND POLJAK [25] conjectured that every fractional vertex of the metric polytope met<sub>n</sub> is adjacent to some integral vertex, i.e., to a cut. Since we have met<sub>3</sub> = cut<sub>3</sub> and met<sub>4</sub> = cut<sub>4</sub>, the conjecture, which is commonly called the dominating set conjecture, is obviously true for the 4 vertices of met<sub>3</sub> and for the 8 vertices of met<sub>4</sub>. The conjecture holds for the 32 vertices of met<sub>5</sub> and the 544 vertices of met<sub>6</sub> as well as for several classes of vertices of met<sub>7</sub>, see [24]. The conjecture was further substantiated by the computation of met<sub>7</sub> and met<sub>8</sub>. The 275 840 vertices of met<sub>7</sub> and the 1 550 825 600 vertices of met<sub>8</sub> are adjacent to a cut, see [11, 13].

While the overwhelming majority of the known vertices of met<sub>9</sub> satisfy the dominating set conjecture, we found few counterexamples. In particular, we exhibit the following fractional vertex not adjacent to any integral vertex. **Proposition 5.1.1** The neighbors of the fractional vertex  $\frac{1}{9}(2, 2, 3, 3, 4, 4, 5, 5, 4, 3, 5, 6, 6, 3, 3, 5, 5, 2, 4, 3, 5, 6, 3, 3, 6, 6, 5, 3, 2, 6, 6, 3, 3, 5, 3, 4) of the metric polytope met<sub>9</sub> are all fractional.$ 

The vertex given in Proposition 5.1.1, as well as other vertices not adjacent to any cut, were found by an extensive computer search of the vertices of the 36-dimensional metric polytope met<sub>9</sub>. Note that while finding a vertex providing a counterexample to the dominating set conjecture is computationally challenging, to verify that a given vertex is indeed not adjacent to a cut is easy if the vertex is quasi-simple, i.e., if the incidence of the given vertex is equal to the dimension plus one. For example, one can easily check, see Section 5.3, that the vertex given in Proposition 5.1.1 satisfies with equalities 37 of the 336 inequalities defining met<sub>9</sub> and is adjacent to 37 vertices which are all fractional.

### 5.2 Related Questions

#### 5.2.1 The Diameter of the Metric Polytope

Since any pair of cuts forms an edge of  $\operatorname{met}_n$ , the dominating set conjecture would imply that the diameter  $\delta(\operatorname{met}_n)$  of the metric polytope satisfies  $\delta(\operatorname{met}_n) \leq 3$ . We recall that the diameter of a polytope P is the smallest number  $\delta(P)$  such that any two vertices of P can be connected by a path with at  $\operatorname{most} \delta(P)$  edges. We have  $\delta(\operatorname{met}_3) = \delta(\operatorname{met}_4) = 1$ ,  $\delta(\operatorname{met}_5) = \delta(\operatorname{met}_6) = 2$  and  $\delta(\operatorname{met}_7) = \delta(\operatorname{met}_8) = 3$ . While the diameter of the restriction of  $\operatorname{met}_9$  to its known vertices appears to be less than 3, it is not clear that the diameter of  $\operatorname{met}_n$  is bounded by a constant.

#### 5.2.2 The No-cut Set Conjecture

Conjecture 3.2.3 can be seen as complementary to the dominating set conjecture both graphically and computationally: For any pair of vertices, while the dominating set conjecture implies that there is a path made of cuts joining them, i.e., the cut vertices form a dominating set, Conjecture 3.2.3 means that there is a path made of non-cut vertices joining them, i.e., the cut vertices do not form a cut-set. On the other hand, while the dominating set conjecture means that the enumeration of the extreme rays of the metric cone Met<sub>n</sub> is enough to obtain the vertices of the metric polytope  $met_n$ , Conjecture 3.2.3 means that we can obtain the vertices of  $met_n$  without enumerating the extreme rays of  $met_n$ .

# 5.3 Counterexample Generation and Verification

One important feature of the metric and cut polyhedra is their very large symmetry group. We recall that the symmetry group Is(P) of a polyhedron P is the group of isometries preserving P and that an isometry is a linear transformation preserving the Euclidean distance. For  $n \geq 5$ , the symmetry groups of the polytopes  $\text{met}_n$  and  $\text{cut}_n$  are isomorphic and induced by permutations on  $V_n$  and switching reflections by a cut, see [15], and the symmetry groups of the cones  $\text{Met}_n$  and  $\text{Cut}_n$  are isomorphic to Sym(n). Given a cut  $\delta(S)$ , the switching reflection  $r_{\delta(S)}$  is defined by  $y = r_{\delta(S)}(x)$  where  $y_{ij} = 1 - x_{ij}$  if  $(i, j) \in \delta(S)$  and  $y_{ij} = x_{ij}$  otherwise.

#### 5.3.1 Counterexample Generation

The vertices of  $met_n$  are partitioned into orbits under the action of the symmetry group  $Is(met_n)$ . Using a parallel implementation of an orbitwise enumeration algorithm, 1 056 368 orbits of vertices of met<sub>9</sub> were computed on Shared Hierarchical Academic Research Computing Network (SHARCNET) clusters. Among these 1056368 orbits, 1221 provide counterexamples to the dominating set conjecture, including 483 made of quasi-simplicial vertices. As the dimension of met<sub>9</sub> is 36, these quasi-simplicial counterexamples satisfy with equality exactly 37 inequalities. The remaining 738 orbits providing counterexamples to the dominating set conjecture are made of vertices satisfying with equality at least 38 inequalities. Some of these counterexamples have a relatively large incidence and adjacency: For example, the vertex  $\frac{1}{9}(1, 2, 3, 3, 4, 4, 4, 6, 3, 4,$  $4,\ 3\ 3,\ 5,\ 7,\ 5,\ 5,\ 6,\ 6,\ 2,\ 4,\ 6,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 3,\ 6,\ 6,\ 4,\ 6,\ 4,\ 2)\ satisfies$ with equality 44 inequalities and has 84 fractional adjacent vertices, and the  $vertex \ \frac{1}{12}(3,\ 3,\ 3,\ 6,\ 7,\ 7,\ 7,\ 7,\ 6,\ 6,\ 3,\ 4,\ 4,\ 4,\ 4,\ 6,\ 3,\ 4,\ 4,\ 4,\ 4,\ 4,\ 9,\ 4,\ 4,\ 8,\ 8,$ 7, 7, 7, 8, 4, 4, 4, 8) satisfies with equality 43 inequalities and has 202 fractional adjacent vertices. See Appendix B for a complete list of the known counterexamples.

### 5.3.2 Counterexample Verification

For a given quasi-simple vertex, one can easily verify that all the adjacent vertices are fractional by performing 3 simple computations which we illustrate using the vertex given in Proposition 5.1.1.

(i) Check which of the 336 inequalities of met<sub>9</sub> are satisfied with equality by the given vertex. For the vertex given in Proposition 5.1.1, we obtain the 37 inequalities given in Section 5.3.3.

- (ii) Compute the pointed cone formed by the active inequalities identified in(i). For the vertex given in Proposition 5.1.1, we obtain a quasi-simplicial cone with 37 extreme rays.
- (iii) From the given vertex, follow each extreme ray of the cone computed in (ii) until at least one of the inequalities defining met<sub>9</sub> is violated. The obtained points are the adjacent vertices. For the vertex given in Proposition 5.1.1, we obtain the 37 adjacent vertices given in Section 5.3.3. As these 37 adjacent vertices are all fractional, the vertex given in Proposition 5.1.1 is indeed a counterexample.

Note that, while the computation (ii) can be extremely expensive for a highly degenerate vertex in high dimension, it can be done efficiently if the given vertex is quasi-simple. It takes less than a second of CPU time for the vertex given in Proposition 5.1.1 using enumeration packages such as lrs or cdd. Computations (i) and (iii) are straightforward and take less than a second of CPU time.

# 5.3.3 Counterexample Incidence and Adjacency Lists Incidence List

The vertex given in Proposition 5.1.1 satisfies with equalities the following 37 inequalities of met<sub>9</sub>:  $\Delta_{6,7,\bar{9}}$ ,  $\Delta_{5,\bar{8},9}$ ,  $\Delta_{5,\bar{7},9}$ ,  $\Delta_{\bar{5},7,8}$ ,  $\Delta_{5,6,\bar{8}}$ ,  $\Delta_{4,\bar{7},9}$ ,  $\Delta_{4,\bar{6},9}$ ,  $\Delta_{4,\bar{6},8}$ ,  $\Delta_{\bar{4},6,7}$ ,  $\Delta_{4,5,9}$ ,  $\Delta_{4,5,\bar{7}}$ ,  $\Delta_{3,\bar{6},9}$ ,  $\Delta_{\bar{3},6,7}$ ,  $\Delta_{3,5,\bar{8}}$ ,  $\Delta_{3,4,\bar{6}}$ ,  $\Delta_{2,7,\bar{9}}$ ,  $\Delta_{2,6,\bar{9}}$ ,  $\Delta_{2,6,\bar{8}}$ ,  $\Delta_{2,6,7}$ ,  $\Delta_{2,5,\bar{8}}$ ,  $\Delta_{\bar{2},4,9}$ ,  $\Delta_{\bar{2},4,8}$ ,  $\Delta_{2,\bar{4},7}$ ,  $\Delta_{2,\bar{4},6}$ ,  $\Delta_{2,\bar{3},6}$ ,  $\Delta_{1,\bar{5},8}$ ,  $\Delta_{\bar{1},4,5}$ ,  $\Delta_{1,\bar{3},8}$ ,  $\Delta_{1,\bar{3},6}$ ,  $\Delta_{\bar{1},3,5}$ ,  $\Delta_{\bar{1},3,4}$ ,  $\Delta_{1,\bar{2},9}$ ,  $\Delta_{1,\bar{2},8}$ ,  $\Delta_{\bar{1},2,7}$ ,  $\Delta_{\bar{1},2,6}$ ,  $\Delta_{\bar{1},2,5}$ ,  $\Delta_{\bar{1},2,3}$  where the triangle inequality (2.1) and the perimeter inequality (2.2) are respectively denoted by  $\Delta_{i,j,\bar{k}}$  and

 $\Delta_{i,j,k}$ .

#### **Adjacency List**

The vertex given in Proposition 5.1.1 is adjacent to the following 37 fractional vertices of met<sub>9</sub>:

```
\frac{1}{3}(1,1,1,1,1,1,2,2,1,1,2,2,2,1,1,2,2,1,1,1,2,2,1,1,2,2,1,1,2,2,1,1,2,2,1,1,2,1,1)
\frac{1}{3}(1,1,1,1,1,1,2,2,2,1,2,2,2,1,1,1,2,0,2,1,1,2,1,1,2,2,2,1,1,2,2,1,1,2,1,1)
\frac{1}{3}(1,1,1,1,1,1,2,2,2,1,2,2,2,1,1,2,2,1,1,1,2,2,1,1,2,2,2,1,1,2,2,1,1,2,2,1,1,2,1,1)
\frac{1}{2}(1,1,1,1,2,1,2,2,1,1,2,2,2,1,1,2,2,1,1,1,2,2,1,1,2,2,2,1,1,2,2,1,1,2,1,1)
\frac{1}{2}(1,1,1,1,2,1,2,2,2,1,2,2,2,1,1,2,2,1,1,1,2,2,1,1,2,2,2,1,1,2,2,1,1,2,1,1)
\frac{1}{3}(1,1,1,1,1,2,2,2,2,2,2,2,3,1,1,2,2,0,3,1,1,2,2,1,3,3,2,1,1,3,3,1,1,2,2,2)
\frac{1}{6}(1,1,2,2,3,3,3,3,2,2,3,4,4,2,2,3,3,2,2,2,4,4,2,2,4,4,3,2,1,4,4,2,2,3,2,3)
\frac{1}{6}(1,1,3,1,3,3,2,3,2,2,2,4,4,1,2,4,2,2,2,1,4,4,2,2,3,4,4,2,1,4,4,3,2,3,2,3)
\frac{1}{6}(1,1,3,2,3,3,3,3,2,2,3,4,4,2,2,4,3,2,2,2,4,4,2,2,4,4,3,2,1,4,4,2,2,3,2,3)
\frac{1}{6}(1,1,3,2,3,3,3,3,2,2,3,4,4,2,2,4,3,2,2,2,4,5,2,2,4,4,3,3,1,3,4,2,2,4,2,2)
\frac{1}{6}(2,1,2,2,2,2,4,4,3,2,4,4,4,2,2,3,3,1,3,3,3,4,2,2,4,4,4,2,2,4,4,2,2,4,2,2)
```

 $\frac{1}{7}(1,1,3,2,3,3,3,3,2,2,3,4,4,2,2,4,3,2,2,2,4,5,2,2,4,4,3,3,1,5,4,2,2,4,2,4)$  $\frac{1}{7}(1,1,3,2,3,4,3,3,2,2,3,4,5,2,2,4,3,2,3,2,4,5,2,3,4,4,3,2,1,5,5,2,2,3,3,4)$  $\frac{1}{7}(1,1,3,2,3,4,3,4,2,2,3,4,5,2,3,4,3,2,3,2,5,5,2,3,4,5,3,2,1,4,5,2,3,3,2,3)$  $\frac{1}{7}(2,2,2,3,3,2,5,4,4,2,5,5,4,3,2,4,5,1,4,3,4,5,3,2,5,4,4,3,2,5,5,2,3,5,2,3)$  $\frac{1}{7}(2,2,2,3,3,3,5,5,4,2,5,5,5,5,3,3,4,5,1,3,3,3,5,3,3,5,5,4,2,2,4,4,2,2,4,2,2)$  $\frac{1}{6}(1,2,4,2,5,5,4,4,3,3,3,6,6,3,3,6,4,3,3,2,6,6,3,3,6,6,5,3,2,6,6,3,3,5,3,4)$  $\frac{1}{6}(2,2,3,3,4,4,5,3,4,3,5,6,6,3,3,5,5,2,4,3,5,6,3,3,6,6,5,3,2,6,6,3,3,5,3,4)$  $\frac{1}{6}(2,2,3,3,4,4,5,5,4,3,5,6,6,3,3,5,5,2,4,3,3,6,3,3,6,6,5,3,2,6,6,3,3,5,3,4)$  $\frac{1}{6}(2,2,3,3,4,4,5,5,4,3,5,6,6,3,3,5,5,2,4,3,5,6,3,3,6,6,5,3,2,6,6,3,3,5,3,6)$  $\frac{1}{6}(2,2,3,3,4,4,5,5,4,3,5,6,6,3,3,5,5,2,4,3,5,6,3,3,6,6,3,3,2,6,6,3,3,5,3,4)$  $\frac{1}{6}(2,2,3,3,4,4,5,5,4,3,5,6,6,3,3,5,5,2,4,3,5,6,3,3,6,6,5,3,2,6,6,3,3,3,3,4)$  $\frac{1}{6}(2,2,3,3,4,4,5,5,4,3,5,6,6,3,3,5,5,2,6,3,5,6,3,3,6,6,5,3,2,6,6,3,3,5,3,4)$  $\frac{1}{6}(2,2,3,3,4,6,5,5,4,3,5,6,6,3,3,5,5,2,4,3,5,6,3,3,6,6,5,3,2,6,6,3,3,5,3,4)$  $\frac{1}{6}(3,2,2,4,3,3,6,6,5,3,5,6,6,3,3,4,6,1,5,4,4,6,3,3,6,6,5,3,2,6,6,3,3,5,3,4)$  $\frac{1}{10}(2, 2, 4, 3, 5, 4, 5, 5, 4, 4, 5, 7, 6, 3, 3, 6, 5, 3, 4, 3, 7, 7, 3, 4, 7, 7, 6, 3, 2, 6, 7, 4, 4, 5, 3, 4)$  $\frac{1}{10}(2,2,4,3,5,5,5,6,4,4,5,7,7,3,4,6,5,3,3,3,6,7,3,3,7,6,6,4,2,7,6,4,3,6,3,5)$  $\frac{1}{10}(3,3,2,4,4,3,7,7,6,3,7,7,6,4,4,5,7,1,6,4,4,6,4,3,7,7,6,3,3,7,7,3,3,6,4,4)$  $\frac{1}{12}(3,3,3,5,5,5,5,7,7,6,4,8,8,8,4,4,6,8,2,6,4,6,8,4,4,8,8,8,4,4,8,8,4,4,8,4,4)$  $\frac{1}{12}(3,3,3,5,5,5,5,8,7,6,4,8,8,8,5,4,6,8,2,6,5,6,8,4,4,7,8,6,4,3,8,8,3,4,7,4,5)$  $\frac{1}{12}(3,3,3,5,5,5,5,8,7,6,4,8,8,8,5,4,6,8,2,6,5,6,8,4,4,9,8,8,4,3,8,8,5,4,7,4,5)$ 

#### 5.3.4 Computationally Challenging Orbits

Some vertices are extremely hard to find by the adjacency decomposition method. For example, for met<sub>9</sub> the hardest to find vertex is  $v^* = \frac{1}{7}(1, 2, 2, 3, 3, 3, 3, 4, 3, 3, 2, 2, 4, 4, 3, 4, 3, 3, 1, 5, 4, 3, 5, 3, 5, 2, 4, 4, 2, 1, 2, 2, 3, 4, 5, 3). The vertex <math>v^*$  is moderately degenerate as its incidence is 42 (i.e dimension plus 6) but its adjacency is minimal as  $\operatorname{Adj}_{v^*} = \operatorname{Icd}_{v^*} = 42$ . All the 42 neighbors of  $v^*$  are highly degenerate (their incidence ranges from 57 to 252) and have a very large adjacency: the smallest being 48 969. In other words, this relatively small (incidence and adjacency wise) vertex  $v^*$  is surrounded by very large vertices. This phenomenon reveals the limitations of the skipping highest degeneracy heuristic: a vertex surrounded by a wall of highly degenerate vertices may not be find. While this phenomenon is not surprising as polytopes of large dimension are known to be odd, the vertex  $v^*$  is, to the best of our knowledge, the first known example.

The following vertices corresponds to some extremal (maximal) values found among the 1 056 368 orbits of met<sub>9</sub>:

- Incidence (Icd<sub>max</sub> = 88):  $\frac{1}{6}(0,0,1,2,2,2,2,4,0,1,2,2,2,2,4,1,2,2,2,2,4,1,3,3,3,3,2,2,4,2,4,4,2,4,2,2)$  $\frac{1}{7}(0,0,1,2,2,4,4,4,0,1,2,2,4,4,4,1,2,2,4,4,4,3,3,3,5,5,4,2,2,2,2,2,2,2,2,2,4)$
- Denominator (maximum denominator 43):

 $\frac{1}{43}(10, 10, 10, 16, 19, 20, 20, 26, 14, 16, 26, 29, 16, 30, 16, 20, 14, 29, 30, 16, 16, 26, 13, 10, 14, 16, 15, 16, 12, 10, 13, 27, 13, 14, 26, 14)$   $\frac{1}{43}(10, 10, 10, 16, 19, 20, 24, 26, 14, 16, 26, 29, 16, 30, 16, 20, 14, 29, 30, 16, 16, 26, 13, 10, 14, 16, 15, 16, 12, 10, 13, 27, 13, 14, 26, 14)$ 

# 5.3.5 Counterexample for the Metric Polytope on 10 Nodes

The computation of the 45 dimensional metric polytope met<sub>10</sub> was started and more than a quarter of a million of orbits were found. These orbits include 992 counterexamples to the dominating set conjecture in dimension 45. Preliminary investigations indicate that a significant fraction of the vertices of met<sub>10</sub> provide counterexamples to the dominating set conjecture.

The following two vertices with incidence and adjacency equal to 46 are counterexamples to the dominating set conjecture for  $met_{10}$ :

```
\begin{array}{l} \frac{1}{10}(2,2,3,3,4,4,5,6,6,3,2,5,6,6,4,4,5,5,4,3,6,7,6,4,6,7,4,3,6,3,7,4,3,6,3,3,4,3,4,5,2,2,7,3,4) \end{array}
```

 $<sup>\</sup>frac{1}{75}(17, 17, 18, 31, 31, 36, 48, 48, 49, 34, 29, 20, 48, 19, 31, 31, 38, 35, 48, 48, 53, 31, 53, 44, 49, 19, 48, 36, 30, 31, 30, 39, 17, 35, 18, 29, 17, 17, 48, 22, 44, 53, 22, 31, 53)$ 

M.Sc. Thesis - Gabriel Indik

McMaster - Computing and Software

# Chapter 6

# Conclusions

This thesis deals with the vertex enumeration problem for convex polytope in general dimension. We focus on polytopes arising from combinatorial optimization problems. In particular, we study the metric polytope associated to the well-known maxcut and multicommodity flow problems, as well as to finite metric spaces. To tackle the huge size of the problem – hundreds of trillions of vertices – a parallel orbitwise enumeration algorithm was implemented and run on Shared Hierarchical Academic Research Computing Network (SHARCNET) clusters. Considerable effort were put into the implementation in order to develop an efficient portable and scalable ANSI C software based on the MPI (Message Passing Interface) specification which was run on parallel clusters, both distributed and shared memory architectures. Exploiting the high degree of symmetry, we provide for the first time a description of the highly degenerate metric polytope in dimension 36. The description consists of 1 056 368 orbits and is conjectured to be complete. While the validity of the dominating set conjecture [Laurent-Poljak 1992] is proven for the overwhelming majority of the known vertices of the metric polytope, we disprove the conjecture by

M.Sc. Thesis - Gabriel Indik

McMaster - Computing and Software

exhibiting counterexamples in dimensions 36 and 45.

# Appendix A

Metric Polytope met<sub>9</sub> Orbitwise Description

				1				
inc	min adj	max adj	count		inc	min adj	max adj	count
37	37	45	148192		62	409	160000	175
38	38	233	207111		63	617	196626	127
39	39	458	193507		64	1315	158163	81
40	40	862	150846		65	3024	195111	89
41	41	1763	108418		66	5160	191206	63
42	42	3558	75583		67	4987	131633	54
43	46	13163	53203		68	8786	126734	36
44	48	15676	37903		69	10127	152956	24
45	51	26913	26065		70	15770	162887	38
46	54	75046	17473		71	17652	166487	19
47	57	125066	11487		72	32176	134949	8
48	60	193529	7655		73	20088	136570	10
49	80	192128	5035		74	23780	130824	11
50	72	189986	3554		75	48100	119709	5
51	101	174988	2191		76	8703	148374	3
52	104	174403	1699		77	5935	77523	4
53	166	116747	1193		78	8694	189323	5
54	207	197339	966		79	112206	188726	4
55	251	156126	856		80	20631	107055	5
56	193	145496	737		81	29290	175032	3
57	296	175729	558		82	54711	176274	2
58	273	127651	440		84	28713	122478	2
59	382	164277	338		86	95055	137259	2
60	314	137709	273		88	147990	151206	2
61	1057	159418	185					
0.1	2001	100110		J				

Table A.1: Orbits of met<sub>9</sub> by incidence with minimum and maximum adjacency

inc	min den	max den	count	inc	min den	max den	count
37	3	43	148192	71	3	7	20
38	3	38	207111	72	5	7	9
39	3	40	193507	73	3	7	10
40	3	35	150846	74	3	6	16
41	3	38	108418	75	3	7	6
42	3	31	75583	76	3	6	5
43	3	31	53203	77	3	6	6
44	3	27	37903	78	3	7	5
45	3	29	26065	79	4	7	8
46	3	25	17473	80	3	6	16
47	3	21	11487	81	3	7	3
48	3	23	7656	82	3	7	3
49	3	19	5038	83	3	5	2
50	3	17	3557	84	3	5	4
51	3	17	2196	85	4	5	3
52	3	15	1704	86	5	7	2
53	3	15	1197	88	6	7	2
54	3	15	970	89	3	4	3
55	3	15	860	90	4	4	2
56	3	15	740	91	3	5	2
57	3	14	561	93	4	4	1
58	3	15	442	97	4	4	2
59	3	14	339	98	3	3	1
60	3	12	275	104	3	3	1
61	3	12	187	105	3	3	1
62	3	12	175	111	3	3	1
63	3	11	131	112	3	3	1
64	3	12	85	118	3	3	1
65	3	11	90	122	3	3	1
66	3	11	63	124	3	3	1
67	3	11	57	129	3	3	1
68	3	9	39	144	3	3	1
69	3	9	25	252	1	1	1
70	3	9	52				

Table A.2: Orbits of met<sub>9</sub> by incidence with minimum and maximum denominator

den         min icd         max icd         min adj         max adj         count           3         37         84         45         197339         854           4         41         70         48         98739         485           5         40         86         42         196626         891           6         37         88         37         195111         22511           7         37         88         37         195111         22511           8         37         70         37         188726         16104           8         37         70         37         32612         111245           10         37         65         37         20260         108904           11         37         66         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         58         37         12942         123678           14         37         58         37         1394         88949						
4         41         70         48         98739         485           5         40         86         42         196626         891           6         37         88         37         195111         22511           7         37         88         37         195111         22511           7         37         88         37         195111         22511           7         37         3493         45482         9           9         37         70         37         31493         45482           9         37         70         37         32612         111245           10         37         65         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>						
5         40         86         42         196626         891           6         37         88         37         195111         22511           7         37         88         37         188726         16104           8         37         70         37         31493         45482           9         37         70         37         32612         111245           10         37         65         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         57         37         2842         123678           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         49         37         483         35960           19         37         49         37         408         27957           20 </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>						
6         37         88         37         195111         22511           7         37         88         37         188726         16104           8         37         70         37         31493         45482           9         37         70         37         32612         111245           10         37         65         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         59         37         1720         74339           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         488         35960           19         37         46         37         381         15243           21 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>						
7         37         88         37         188726         16104           8         37         70         37         31493         45482           9         37         70         37         32612         111245           10         37         65         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         57         37         2842         123678           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>						
8     37     70     37     31493     45482       9     37     70     37     32612     111245       10     37     65     37     20260     108904       11     37     67     37     7640     99223       12     37     64     37     3689     143409       13     37     57     37     2842     123678       14     37     59     37     1720     74339       15     37     58     37     1394     88949       16     37     50     37     685     59258       17     37     51     37     528     39031       18     37     49     37     483     35960       19     37     49     37     408     27957       20     37     46     37     381     15243       21     37     46     37     381     15243       21     37     46     37     381     15243       21     37     46     37     190     7586       23     37     48     37     100     6389       24     37     46     37 <td< td=""><td></td><td></td><td></td><td></td><td></td><td>Market Control of the Control of the</td></td<>						Market Control of the
9         37         70         37         32612         111245           10         37         65         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         57         37         2842         123678           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         47         37         467         15442           22         37         46         37         190         7586           23						
10         37         65         37         20260         108904           11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         57         37         2842         123678           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         47         37         467         15442           22         37         46         37         190         7586           23         37         48         37         210         6389           24						
11         37         67         37         7640         99223           12         37         64         37         3689         143409           13         37         57         37         2842         123678           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         47         37         467         15442           22         37         46         37         190         7586           23         37         48         37         210         6389           24         37         46         37         118         3020           26         <					1,100,000,000,000,000,000	
12       37       64       37       3689       143409         13       37       57       37       2842       123678         14       37       59       37       1720       74339         15       37       58       37       1394       88949         16       37       50       37       685       59258         17       37       51       37       528       39031         18       37       49       37       483       35960         19       37       49       37       483       35960         19       37       49       37       408       27957         20       37       46       37       381       15243         21       37       46       37       190       7586         23       37       48       37       210       6389         24       37       46       37       118       3020         26       37       42       37       96       1819         27       37       45       37       122       1948         28       37       42						
13         37         57         37         2842         123678           14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         46         37         190         7586           23         37         46         37         190         7586           23         37         48         37         210         6389           24         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37 </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>						
14         37         59         37         1720         74339           15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         46         37         190         7586           23         37         46         37         190         7586           23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         101         620           29         37         45         37         72         509           30         37						
15         37         58         37         1394         88949           16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         46         37         190         7586           23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37	13	37				
16         37         50         37         685         59258           17         37         51         37         528         39031           18         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         47         37         467         15442           22         37         46         37         190         7586           23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         101         620           29         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37	14	37				33 (0-225, 35, 35-2
17         37         51         37         483         35960           19         37         49         37         483         35960           19         37         49         37         408         27957           20         37         46         37         381         15243           21         37         46         37         190         7586           23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         75         413           31         37         43         37         75         413           31         37         4						88949
18     37     49     37     483     35960       19     37     49     37     408     27957       20     37     46     37     381     15243       21     37     47     37     467     15442       22     37     46     37     190     7586       23     37     48     37     210     6389       24     37     44     37     120     4236       25     37     46     37     118     3020       26     37     42     37     96     1819       27     37     45     37     122     1948       28     37     42     37     101     620       29     37     45     37     72     509       30     37     43     37     75     413       31     37     43     37     75     413       31     37     43     37     49     158       34     37     40     37     46     48       35     37     40     37     46     48       35     37     40     37     46     48	16	37	50	37	685	59258
19     37     49     37     408     27957       20     37     46     37     381     15243       21     37     47     37     467     15442       22     37     46     37     190     7586       23     37     48     37     210     6389       24     37     44     37     120     4236       25     37     46     37     118     3020       26     37     42     37     96     1819       27     37     45     37     122     1948       28     37     42     37     101     620       29     37     45     37     72     509       30     37     43     37     75     413       31     37     43     37     75     413       31     37     43     37     49     158       34     37     40     37     46     48       35     37     40     37     46     48       35     37     39     37     40     28       37     37     37     40     22       38	17	37	51	37	528	39031
20         37         46         37         381         15243           21         37         47         37         467         15442           22         37         46         37         190         7586           23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         75         413           31         37         43         37         49         158           34         37         40         37         46         48           35         37         40 <td>18</td> <td>37</td> <td>49</td> <td>37</td> <td>483</td> <td>35960</td>	18	37	49	37	483	35960
21         37         47         37         467         15442           22         37         46         37         190         7586           23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         75         413           31         37         43         37         49         158           34         37         40         37         49         158           34         37         40         37         46         48           35         37         39	19	37	49	37	408	27957
22     37     46     37     190     7586       23     37     48     37     210     6389       24     37     44     37     120     4236       25     37     46     37     118     3020       26     37     42     37     96     1819       27     37     45     37     122     1948       28     37     42     37     101     620       29     37     45     37     72     509       30     37     43     37     75     413       31     37     43     37     64     229       32     37     41     37     100     165       33     37     40     37     46     48       35     37     40     37     46     48       35     37     39     37     40     28       37     37     39     37     40     22       38     38     41     38     45     2       39     37     37     37     37     17       40     39     39     40     40     1       41	20	37	46	37	381	15243
23         37         48         37         210         6389           24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         46         48           35         37         40         37         46         48           35         37         39         37         40         28           37         37         37         40         22           38         38         41         38	21	37	47	37	467	15442
24         37         44         37         120         4236           25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         39         37         40         28           37         37         39         37         40         28           37         37         37         37         17           40         39         39         40 <t< td=""><td>22</td><td>37</td><td>46</td><td>37</td><td>190</td><td>7586</td></t<>	22	37	46	37	190	7586
25         37         46         37         118         3020           26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37	23	37	48	37	210	6389
26         37         42         37         96         1819           27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         37         17           40         39         39         40 <td>24</td> <td>37</td> <td>44</td> <td>37</td> <td>120</td> <td>4236</td>	24	37	44	37	120	4236
27         37         45         37         122         1948           28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37	25	37	46	37	118	3020
28         37         42         37         101         620           29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37	26	37	42	37	96	1819
29         37         45         37         72         509           30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37         4	27	37	45	37	122	1948
30         37         43         37         75         413           31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37         4	28	37	42	37	101	620
31         37         43         37         64         229           32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37         4	29	37	45	37	72	509
32         37         41         37         100         165           33         37         40         37         49         158           34         37         40         37         46         48           35         37         40         37         46         57           36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37         4	30	37	43	37	75	413
33     37     40     37     49     158       34     37     40     37     46     48       35     37     40     37     46     57       36     37     39     37     40     28       37     37     39     37     40     22       38     38     41     38     45     2       39     37     37     37     17       40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4	31	37	43	37	64	229
34     37     40     37     46     48       35     37     40     37     46     57       36     37     39     37     40     28       37     37     39     37     40     22       38     38     41     38     45     2       39     37     37     37     17       40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4	32	37	41	37	100	165
35     37     40     37     46     57       36     37     39     37     40     28       37     37     39     37     40     22       38     38     41     38     45     2       39     37     37     37     17       40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4	33	37	40	37	49	158
36         37         39         37         40         28           37         37         39         37         40         22           38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37         4	34	37	40	37	46	48
37     37     39     37     40     22       38     38     41     38     45     2       39     37     37     37     37     17       40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4	35	37	40	37	46	57
38     38     41     38     45     2       39     37     37     37     17       40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4	36	37	39	37	40	28
38         38         41         38         45         2           39         37         37         37         17           40         39         39         40         40         1           41         37         37         37         37         2           42         37         37         37         37         4	37	37	39	37	40	22
39     37     37     37     17       40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4						
40     39     39     40     40     1       41     37     37     37     37     2       42     37     37     37     37     4						
41     37     37     37     37     2       42     37     37     37     37     4		40.00				
42 37 37 37 37 4						
					1,000	
	43	37	37	37		

Table A.3: Orbits of  $met_9$  by denominator with minimum and maximum incidence and adjacency

# Appendix B

Dominating Set Conjecture Counterexamples for Metric Polytope met9

Incidence	Adjacency	Count
37	37	483
38	38	18
38	39	368
38	46	1
38	62	6
39	42	85
39	48	104
39	55	44
39	75	3
39	88	1
40	42	12
40	44	18
40	45	2
40	48	1
40	68	1
40	75	31
40	83	6
40	100	1
40	120	1
41	49	3
41	51	5
41	58	2
41	60	3
41	80	2
41	87	2
41	108	1
41	112	1
41	116	2
41	136	4
41	142	1
42	60	1
42	158	2
43	51	1
43	74	3
43	202	1
44	84	1

Table B.1: Counterexamples for met<sub>9</sub>

```
\frac{1}{15}(3,4,5,5,5,6,7,10,7,8,8,8,5,4,9,5,5,7,10,11,6,10,10,9,6,5,10,9,6,5,9,4,5,9,4,5)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 4, 10, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 6, 5, 10, 5, 9)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 15, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{18}(3, 5, 6, 6, 8, 8, 10, 11, 8, 7, 7, 5, 11, 7, 12, 11, 11, 13, 7, 5, 6, 6, 6, 12, 6, 5, 12, 6, 12, 5, 6, 12, 7, 6, 11, 7)
\frac{1}{15}(3, 4, 5, 5, 8, 8, 8, 8, 7, 8, 8, 5, 5, 5, 11, 9, 9, 4, 10, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 6)
\frac{1}{27} (6, 6, 9, 9, 13, 13, 18, 18, 12, 7, 15, 7, 9, 16, 16, 9, 11, 19, 19, 12, 12, 18, 10, 10, 9, 9, 8, 8, 9, 9, 16, 9, 9, 7, 17, 18)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 4, 7, 4, 8, 6, 5, 8, 9, 8, 4, 7, 4, 3, 4, 4, 3, 4, 5, 3, 7, 8, 4, 3, 7, 4)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 8, 9, 4, 6, 7, 8, 4, 5, 8, 7, 4, 4, 3, 4, 7, 3, 4, 3, 4, 5, 8, 7, 4, 7)
\frac{1}{12}(3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 5, 4, 8, 6, 7, 4, 9, 8, 4, 7, 4, 3, 4, 4, 5, 4, 3, 3, 5, 4, 8, 7, 5, 4)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 5, 7, 7, 7, 9, 4, 5, 4, 4, 8, 3, 4, 8, 4, 5, 8, 4, 7, 8, 7, 4, 3)
\frac{1}{25}(4,6,7,9,13,14,15,16,10,9,9,17,10,17,16,9,7,7,16,9,14,16,8,7,8,9,10,9,16,7,9,16,17,7,16,17)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 11, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{15}(3,4,5,5,6,8,8,8,7,8,8,5,5,5,11,9,9,10,4,10,4,10,5,5,5,5,5,5,5,5,5,10,10,6,10,8,6)
\frac{1}{15}(3, 4, 5, 5, 8, 9, 9, 9, 7, 6, 8, 11, 6, 6, 10, 9, 9, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 4, 5, 5, 9, 10, 9, 9)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 7, 4, 8, 4, 5, 8, 4, 8, 3, 4, 4, 7, 8, 5, 4, 3)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 8, 8, 9, 4, 6, 7, 4, 4, 5, 8, 7, 4, 6, 3, 4, 3, 5, 4, 3, 8, 7, 4, 3, 4, 7)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 4, 8, 4, 7, 8, 4, 8, 3, 4, 4, 5, 8, 7, 4, 3)
\frac{1}{12}(3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 7, 7, 7, 3, 4, 7, 5, 7, 7, 9, 4, 4, 4, 8, 8, 7, 8, 4, 4, 5, 4, 4, 3, 8, 7, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 4, 7, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 4, 4, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 8, 3, 5)
\frac{1}{11} (2, 3, 4, 4, 5, 6, 6, 7, 5, 4, 4, 7, 4, 8, 7, 5, 7, 6, 3, 3, 4, 4, 5, 8, 4, 3, 3, 4, 4, 3, 3, 7, 6, 4, 7, 7)
\frac{1}{15}(3,4,5,5,8,8,8,8,7,6,8,5,5,5,11,9,9,4,10,10,4,10,5,5,5,5,5,5,5,5,10,10,8,10,6,6)
\frac{1}{15}(3,4,5,5,8,8,8,8,7,6,6,5,5,5,11,9,9,4,10,10,4,10,5,5,5,5,5,5,5,5,10,10,6,10,6,6)
\frac{1}{12}(3,3,3,4,6,7,7,7,4,6,4,5,4,8,8,6,5,9,8,4,4,7,3,4,4,4,4,3,4,4,7,5,5,4,4,8)
\frac{1}{32}(6,8,11,11,16,17,19,22,14,11,11,10,23,13,20,11,11,22,9,11,14,22,11,12,22,11,11,12,22,11,13,11,10,10,23,21)
\frac{1}{19} (4, 5, 6, 7, 10, 11, 11, 11, 9, 10, 11, 14, 7, 7, 13, 11, 12, 5, 6, 12, 6, 7, 6, 11, 5, 5, 13, 6, 12, 6, 7, 7, 11, 6, 12, 6)
\frac{1}{15}(3, 4, 5, 8, 8, 8, 8, 9, 7, 8, 5, 5, 5, 11, 10, 9, 4, 6, 6, 4, 5, 5, 5, 5, 5, 4, 10, 10, 6, 5, 10, 6, 5, 10, 5, 9)
\frac{1}{11} (2, 3, 4, 4, 6, 6, 7, 7, 5, 4, 4, 4, 8, 7, 7, 5, 7, 3, 3, 4, 6, 4, 8, 4, 3, 5, 4, 4, 3, 3, 4, 7, 3, 7, 7, 4)
\frac{1}{12}(3, 3, 3, 4, 5, 6, 7, 7, 4, 4, 7, 4, 5, 4, 8, 6, 5, 8, 9, 8, 4, 7, 4, 3, 6, 4, 3, 4, 5, 3, 7, 8, 4, 3, 7, 4)
\frac{1}{12} (2, 3, 4, 6, 6, 6, 8, 8, 5, 4, 4, 4, 8, 6, 6, 7, 3, 7, 3, 7, 7, 8, 8, 4, 4, 4, 8, 6, 4, 4, 4, 4, 4, 4, 4, 8)
\frac{1}{15}(3, 4, 5, 5, 8, 9, 9, 9, 7, 6, 8, 11, 6, 6, 10, 9, 9, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 4, 5, 5, 9, 10, 6, 9)
\frac{1}{15} (3, 4, 5, 7, 8, 8, 8, 9, 7, 6, 10, 5, 5, 11, 10, 9, 5, 4, 6, 4, 5, 10, 5, 5, 5, 4, 5, 5, 5, 10, 10, 6, 5, 6, 5, 9)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 6, 7, 6, 5, 7, 7, 3, 9, 4, 7, 7, 7, 5, 3, 4, 4, 8, 4, 4, 7, 4, 8, 8, 3, 4, 4, 7, 8, 7, 5)
\frac{1}{21}(4,6,6,7,8,11,12,13,8,10,7,6,15,14,9,12,13,14,7,6,7,7,14,5,14,7,7,8,7,14,9,14,7,13,6,7)
\frac{1}{23}(6, 8, 11, 11, 14, 17, 19, 22, 14, 11, 11, 10, 23, 13, 20, 11, 11, 22, 9, 11, 14, 22, 11, 12, 22, 11, 11, 12, 22, 11, 13, 11, 10, 10, 23, 21)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{21}(4, 6, 6, 8, 10, 11, 13, 13, 8, 10, 6, 14, 15, 9, 13, 12, 14, 6, 7, 13, 7, 14, 14, 5, 7, 7, 14, 9, 7, 7, 13, 7, 7, 6, 12, 14)
\frac{1}{22}(4,6,7,8,12,13,14,14,10,11,8,16,9,10,14,7,14,6,7,8,8,13,7,14,15,7,8,9,6,6,7,8,14,15,7,8)
\frac{1}{15}(3,4,5,5,6,7,10,10,7,8,8,5,4,7,9,5,7,10,11,6,6,10,5,6,5,5,5,4,5,5,9,6,4,9,5,10)
\frac{1}{12} (2, 3, 4, 6, 6, 8, 8, 8, 5, 4, 4, 8, 6, 8, 8, 7, 3, 3, 5, 7, 7, 8, 4, 4, 4, 4, 6, 4, 4, 4, 8, 4, 4, 4, 8)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 4, 8, 4, 5, 8, 4, 8, 3, 4, 4, 5, 8, 7, 4, 3)
```

```
\frac{1}{9} (2, 2, 3, 3, 4, 4, 4, 5, 4, 5, 5, 2, 4, 6, 3, 5, 5, 6, 6, 4, 3, 6, 3, 3, 3, 6, 3, 3, 6, 6, 4, 3, 2, 3, 3)
\frac{1}{18}(4,4,6,6,9,10,10,10,8,6,10,5,6,6,6,6,10,13,6,12,12,12,7,12,6,6,5,4,6,10,7,11,11,6,6,12)
\frac{1}{24}(3, 5, 7, 9, 9, 12, 13, 16, 8, 8, 8, 8, 9, 16, 15, 12, 8, 14, 7, 8, 15, 16, 16, 17, 8, 9, 16, 15, 8, 7, 7, 8, 9, 15, 8, 7)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 4, 5, 4, 5, 4, 8, 6, 5, 8, 9, 8, 4, 7, 4, 3, 4, 4, 3, 4, 5, 3, 7, 8, 4, 3, 7, 4)
\frac{1}{15} (3, 4, 5, 5, 8, 9, 9, 9, 7, 6, 6, 11, 6, 6, 10, 9, 9, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 4, 5, 5, 9, 10, 6, 9)
\frac{1}{27}(5, 7, 8, 11, 14, 14, 14, 16, 12, 11, 8, 9, 9, 19, 11, 15, 18, 9, 9, 7, 9, 19, 18, 18, 8, 16, 9, 9, 11, 9, 18, 10, 18, 10, 18, 8)
\frac{1}{33}(6, 8, 11, 11, 16, 17, 19, 22, 14, 11, 11, 10, 23, 13, 20, 11, 11, 22, 9, 11, 14, 22, 11, 12, 22, 11, 11, 12, 22, 11, 13, 11, 10, 10, 23, 13)
\frac{1}{19}(4, 4, 7, 7, 8, 9, 13, 13, 8, 7, 11, 6, 13, 13, 13, 11, 7, 12, 5, 9, 11, 8, 13, 6, 6, 6, 5, 12, 6, 6, 7, 7, 7, 6, 6, 12)
\frac{1}{15} (3, 4, 5, 5, 8, 9, 9, 9, 7, 6, 6, 11, 6, 6, 10, 9, 9, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 4, 5, 5, 9, 10, 6, 6)
\frac{1}{22}(4,6,7,8,12,13,14,14,10,9,8,16,9,10,14,7,14,6,7,8,8,13,7,14,15,7,8,13,6,6,7,8,14,15,7,8)
\frac{1}{24}(3, 5, 7, 9, 9, 12, 13, 16, 8, 8, 8, 8, 9, 16, 13, 10, 8, 14, 7, 8, 15, 16, 16, 17, 8, 9, 16, 15, 8, 7, 7, 8, 9, 15, 8, 7)
\frac{1}{15} (3, 4, 5, 5, 6, 6, 6, 9, 5, 8, 8, 5, 5, 5, 10, 9, 9, 10, 10, 10, 5, 8, 3, 9, 9, 4, 11, 5, 5, 4, 6, 6, 7, 10, 5, 5)
\frac{1}{18}(3,5,5,7,7,7,8,12,6,8,6,6,10,5,11,6,6,12,6,11,7,12,6,12,13,7,10,12,11,5,6,11,5,5,7,6)
\frac{1}{16} (3, 4, 6, 6, 6, 8, 11, 11, 7, 5, 5, 9, 11, 10, 10, 4, 10, 6, 4, 9, 9, 10, 10, 6, 5, 5, 4, 6, 5, 9, 10, 5, 5, 11, 5, 10)
\frac{1}{21} (4, 6, 6, 7, 8, 10, 11, 13, 8, 10, 7, 6, 14, 15, 9, 12, 13, 14, 6, 7, 7, 7, 14, 14, 5, 7, 7, 7, 8, 14, 14, 9, 7, 13, 7, 6)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 5, 8, 8, 5, 5, 5, 10, 9, 9, 10, 10, 10, 5, 8, 3, 9, 9, 4, 11, 5, 5, 4, 6, 6, 7, 10, 5, 5)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 14, 10)
\frac{1}{15}(3, 4, 5, 6, 8, 8, 8, 9, 7, 8, 5, 5, 5, 11, 10, 9, 10, 4, 6, 4, 5, 5, 5, 5, 5, 4, 10, 10, 6, 5, 10, 6, 5, 6, 5, 9)
\frac{1}{18}(4,4,6,6,8,9,10,10,8,6,10,6,5,6,6,6,10,12,13,6,12,12,6,7,12,6,10,5,4,6,11,6,12,7,11,6)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 15, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 16, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{19}(4,5,7,7,10,11,12,12,9,7,11,14,7,8,12,12,12,5,6,7,7,6,7,8,5,5,13,6,7,5,7,6,12,13,11,6)
\frac{1}{15}(3, 4, 4, 5, 7, 7, 8, 9, 5, 7, 8, 4, 10, 5, 10, 6, 5, 5, 5, 10, 5, 9, 11, 5, 4, 5, 10, 6, 5, 4, 6, 9, 6, 5, 10, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 5, 6, 7, 6, 5, 5, 7, 8, 9, 4, 7, 7, 7, 7, 3, 4, 8, 4, 8, 4, 4, 4, 8, 4, 4, 4, 8, 3, 4, 4, 5)
\frac{1}{18}(4,4,6,9,10,10,10,10,8,8,13,6,6,12,12,10,5,6,6,6,6,5,4,12,6,10,7,7,11,11,12,6,6,6,6,12)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 5, 7, 7, 3, 4, 7, 7, 5, 7, 9, 4, 8, 4, 4, 8, 4, 4, 4, 8, 4, 8, 4, 8, 4, 3, 5)
\frac{1}{18}(4,4,6,6,8,9,10,10,8,6,10,12,13,6,12,10,10,6,5,6,6,12,6,7,12,6,10,5,4,6,11,6,12,7,11,6)
\frac{1}{10}(4,5,7,7,10,11,12,12,9,7,11,14,7,8,12,12,12,5,6,7,7,6,7,8,5,5,13,6,7,5,7,6,12,13,11,10)
\frac{1}{15}(3,4,5,5,6,8,8,8,7,6,6,5,5,5,11,9,9,10,4,10,4,10,5,5,5,5,5,5,5,5,10,10,6,10,6,6)
\frac{1}{25}(4,6,7,9,13,14,15,16,10,9,13,17,10,17,16,9,7,7,16,9,14,16,8,7,8,9,10,9,16,7,9,16,17,7,16,17)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 11, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 8, 8, 9, 4, 6, 7, 4, 4, 7, 8, 7, 4, 6, 3, 4, 3, 5, 4, 3, 8, 7, 4, 3, 4, 7)
\frac{1}{18}(4,4,6,6,9,10,10,10,8,6,10,13,6,12,12,10,10,5,6,6,6,12,7,12,6,6,5,4,6,10,7,11,11,6,6,12)
\frac{1}{30}(5, 8, 10, 12, 16, 16, 20, 20, 13, 11, 9, 11, 21, 17, 19, 18, 20, 8, 8, 12, 12, 20, 20, 10, 20, 10, 12, 12, 8, 10, 10, 20, 10, 10, 20, 10)
\frac{1}{27} (6, 6, 9, 9, 9, 13, 15, 18, 12, 9, 9, 11, 19, 19, 12, 11, 11, 15, 7, 9, 16, 18, 18, 10, 9, 18, 18, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{12}(3, 3, 3, 4, 6, 7, 7, 7, 4, 6, 4, 4, 4, 8, 8, 6, 5, 3, 8, 4, 4, 7, 9, 4, 4, 4, 8, 3, 4, 4, 5, 5, 5, 4, 4, 8)
\frac{1}{18}(4,4,6,6,8,9,10,10,8,6,10,12,13,6,12,10,10,6,5,6,6,12,6,7,12,6,6,5,4,10,11,6,12,7,11,6)
\frac{1}{20}(4, 6, 8, 12, 12, 13, 13, 14, 10, 8, 8, 16, 9, 9, 14, 14, 8, 6, 7, 7, 8, 12, 8, 7, 7, 6, 8, 15, 15, 6, 7, 7, 14, 14, 9, 13)
\frac{1}{15} (3, 4, 5, 7, 8, 9, 9, 9, 7, 6, 10, 11, 6, 6, 6, 9, 5, 4, 5, 5, 5, 4, 5, 4, 6, 6, 9, 8, 4, 6, 5, 5, 5, 10, 10, 10)
\frac{1}{15} (3, 4, 5, 7, 8, 8, 8, 9, 7, 6, 10, 5, 5, 11, 10, 9, 5, 4, 6, 4, 5, 6, 5, 5, 5, 4, 5, 5, 5, 10, 10, 8, 5, 10, 5, 9)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 8, 5, 10, 10, 6, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 8, 5, 6, 5, 9)
\frac{1}{12} (3, 3, 3, 4, 6, 7, 7, 7, 4, 6, 4, 8, 4, 8, 8, 6, 5, 9, 8, 4, 4, 7, 3, 4, 4, 4, 4, 3, 4, 4, 7, 5, 5, 4, 4, 8)
```

```
\frac{1}{30}(6, 8, 8, 12, 15, 16, 16, 18, 12, 14, 8, 21, 10, 10, 12, 16, 20, 9, 20, 20, 10, 20, 7, 10, 10, 10, 13, 10, 10, 10, 11, 11, 9, 20, 20, 20)
\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,10,5,4,12,6,11,6,6,12,9,7,11,12,6,6)
\frac{1}{14}(2,4,5,5,7,7,8,8,6,5,7,5,9,6,10,9,9,5,5,4,4,10,6,4,5,5,4,6,5,5,4,9,5,5,9,4)
\frac{1}{12}(3, 3, 3, 4, 6, 7, 7, 7, 4, 5, 4, 4, 4, 8, 8, 6, 5, 3, 8, 4, 4, 7, 9, 4, 4, 4, 8, 3, 4, 4, 5, 5, 7, 4, 4, 8)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 4, 8, 4, 5, 8, 4, 8, 3, 4, 4, 7, 8, 7, 4, 3)
\frac{1}{22}(4,6,8,12,12,13,13,14,10,8,8,16,9,9,14,14,8,6,7,7,8,12,8,7,7,6,8,15,15,6,7,7,14,14,9,9)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 4, 5, 4, 8, 6, 5, 8, 9, 8, 4, 7, 4, 3, 4, 4, 3, 4, 5, 3, 7, 8, 4, 3, 7, 4)
\frac{1}{21}(4,6,6,8,10,11,13,13,8,10,6,14,15,9,9,12,14,6,7,7,13,14,14,5,7,7,14,9,7,7,13,7,7,8,6,14)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 4, 5, 8, 6, 5, 8, 8, 9, 4, 7, 4, 6, 3, 4, 3, 5, 4, 3, 8, 7, 4, 3, 4, 7)
\frac{1}{27}(6, 6, 9, 9, 13, 13, 18, 18, 12, 7, 15, 7, 9, 16, 16, 9, 11, 19, 19, 12, 12, 18, 10, 10, 9, 9, 8, 8, 9, 9, 16, 9, 9, 7, 11, 18)
\frac{1}{11}(2, 3, 4, 4, 5, 6, 6, 7, 5, 4, 4, 7, 4, 8, 7, 5, 7, 6, 3, 3, 4, 6, 5, 8, 4, 3, 3, 4, 4, 3, 3, 7, 4, 4, 7, 7)
\frac{1}{12}(3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 4, 4, 8, 8, 7, 8, 4, 4, 7, 4, 4, 7, 8, 3, 5)
\frac{1}{12}(3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 5, 4, 8, 6, 7, 4, 9, 8, 4, 7, 4, 3, 4, 4, 5, 4, 3, 3, 7, 4, 8, 7, 5, 4)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 5, 4, 8, 6, 7, 4, 9, 8, 4, 7, 4, 3, 4, 4, 7, 4, 3, 3, 5, 4, 8, 7, 5, 4)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 8, 5, 10, 10, 6, 5, 9, 4, 10, 4, 5, 5, 5, 5, 4, 10, 8, 5, 6, 5, 9)
\frac{1}{15}(3,4,5,5,6,6,6,9,5,6,6,5,5,9,10,9,9,10,10,10,5,8,5,5,11,4,9,9,3,4,10,6,5,6,5,7)
\frac{1}{18}(3, 5, 6, 6, 6, 10, 10, 11, 8, 7, 7, 9, 7, 13, 12, 7, 11, 11, 5, 5, 6, 12, 6, 7, 6, 12, 6, 5, 6, 12, 5, 6, 7, 11)
\frac{1}{15} (3, 4, 5, 5, 7, 8, 10, 10, 7, 8, 8, 4, 5, 9, 9, 5, 9, 11, 10, 6, 6, 10, 6, 5, 5, 5, 4, 5, 5, 5, 9, 5, 5, 4, 6, 10)
\frac{1}{15}(3, 4, 5, 5, 6, 8, 8, 8, 7, 8, 8, 5, 5, 5, 11, 9, 9, 10, 4, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 6)
\frac{1}{27}(5, 7, 8, 10, 11, 14, 14, 16, 12, 11, 9, 8, 9, 19, 11, 15, 9, 18, 9, 7, 9, 18, 19, 18, 8, 16, 9, 18, 10, 18, 9, 11, 9, 10, 18, 8)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 8, 9, 4, 6, 7, 8, 4, 5, 8, 7, 4, 4, 3, 4, 5, 3, 4, 3, 4, 5, 8, 7, 4, 7)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 7, 7, 7, 3, 4, 7, 5, 7, 7, 9, 4, 4, 4, 8, 8, 4, 8, 4, 4, 8, 4, 4, 3, 8, 4, 5)
\frac{1}{19}(4,5,7,7,10,11,12,12,9,7,11,14,7,8,12,12,12,5,6,7,7,10,7,8,5,5,13,6,7,5,7,6,12,13,7,6)
\frac{1}{14}(2, 4, 5, 5, 7, 7, 8, 8, 6, 5, 7, 5, 9, 6, 10, 9, 9, 5, 5, 4, 4, 10, 8, 4, 5, 5, 4, 6, 5, 5, 4, 9, 5, 5, 9, 4)
\frac{1}{22}(4,6,7,8,12,13,14,14,10,9,8,16,9,10,14,7,14,6,7,8,8,9,7,14,15,7,8,9,6,6,7,8,14,15,7,8)
\frac{1}{21}(4, 6, 6, 8, 11, 12, 13, 13, 8, 10, 6, 15, 14, 9, 13, 12, 14, 7, 6, 13, 7, 14, 5, 14, 7, 7, 9, 14, 7, 7, 13, 6, 12, 7, 7, 14)
\frac{1}{15}(3, 4, 5, 5, 6, 6, 8, 9, 5, 8, 8, 5, 5, 5, 10, 9, 9, 10, 10, 10, 5, 8, 3, 9, 9, 4, 11, 5, 5, 4, 6, 6, 7, 10, 5, 5)
\frac{1}{24} (5, 6, 8, 8, 8, 10, 12, 16, 11, 7, 9, 13, 7, 17, 15, 6, 10, 10, 16, 6, 10, 16, 16, 10, 10, 8, 16, 16, 8, 8, 6, 16, 8, 10, 8, 16)
\frac{1}{18}(4,4,6,8,8,9,10,10,8,8,12,12,13,6,6,10,6,6,5,6,6,6,10,5,4,12,12,11,6,6,11,6,6,9,7,12)
\tfrac{1}{0}(2,2,3,3,4,4,4,5,4,3,3,4,6,6,3,5,5,6,2,4,3,6,3,3,5,6,3,3,5,6,4,2,3,6,3,3)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 15, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{11} (2, 3, 4, 5, 6, 6, 6, 7, 5, 4, 7, 4, 4, 8, 7, 7, 6, 3, 5, 3, 4, 3, 4, 4, 4, 3, 3, 5, 7, 4, 8, 4, 7, 4, 3, 7)
\frac{1}{30}(6, 8, 8, 12, 12, 12, 15, 18, 12, 14, 8, 10, 10, 21, 12, 16, 20, 20, 20, 9, 10, 20, 10, 10, 7, 10, 10, 10, 13, 10, 20, 11, 20, 11, 20, 9)
\frac{1}{33}(6, 8, 11, 11, 14, 17, 19, 22, 14, 11, 11, 10, 23, 13, 20, 11, 11, 22, 9, 11, 14, 22, 11, 12, 22, 11, 11, 12, 22, 11, 13, 11, 10, 10, 23, 13)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 10, 10, 9, 18, 18, 16, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{9} (2, 2, 3, 3, 4, 4, 5, 5, 4, 3, 5, 6, 6, 3, 3, 5, 5, 2, 4, 3, 5, 6, 3, 3, 6, 6, 5, 3, 2, 6, 6, 3, 3, 5, 3, 4)
\frac{1}{15}(3, 4, 4, 5, 7, 7, 8, 9, 5, 7, 6, 4, 10, 5, 10, 6, 5, 5, 5, 10, 5, 9, 11, 9, 4, 5, 10, 6, 5, 4, 6, 7, 6, 5, 10, 5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 11, 11, 9, 7, 7, 9, 14, 7, 15, 8, 8, 12, 13, 6, 6, 14, 14, 7, 14, 8, 14, 7, 14, 8, 13, 6, 6, 7, 7, 12)
\frac{1}{18}(4,4,6,6,9,10,10,12,8,6,8,13,6,6,12,10,10,5,6,6,8,12,7,12,12,6,5,6,6,6,11,11,11,12,6,6)
\frac{1}{15} (3, 4, 4, 5, 7, 7, 8, 9, 5, 7, 8, 4, 10, 5, 10, 6, 5, 5, 5, 10, 5, 9, 11, 5, 4, 5, 10, 6, 5, 4, 6, 7, 6, 5, 10, 5)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 8, 7, 6, 6, 5, 5, 5, 11, 9, 9, 10, 4, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 8, 6)
```

```
\frac{1}{27} (6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 11, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{15} (3, 4, 4, 5, 8, 8, 9, 9, 5, 7, 8, 5, 11, 10, 10, 6, 5, 10, 10, 5, 5, 9, 4, 4, 5, 5, 5, 5, 4, 6, 6, 5, 5, 9, 5, 10)
\frac{1}{21} (4, 6, 6, 8, 10, 11, 13, 13, 8, 10, 6, 14, 15, 9, 13, 12, 14, 6, 7, 13, 7, 14, 14, 5, 7, 7, 14, 9, 7, 7, 13, 7, 7, 6, 8, 14)
\frac{1}{18}(3, 5, 6, 6, 6, 8, 10, 11, 8, 7, 7, 9, 5, 7, 12, 11, 11, 7, 13, 5, 6, 6, 6, 12, 12, 5, 12, 6, 6, 5, 6, 6, 7, 12, 7, 7)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 11, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{15}(3, 4, 4, 5, 7, 7, 8, 9, 5, 7, 8, 4, 10, 5, 10, 8, 5, 5, 5, 10, 5, 9, 11, 5, 4, 5, 10, 10, 5, 4, 6, 9, 6, 5, 10, 5)
\frac{1}{12}(3,3,4,4,4,4,6,7,6,5,5,7,7,3,4,7,7,7,9,4,8,4,4,8,7,4,4,8,7,8,4,3,4,5,5)
\frac{1}{15} (3, 4, 5, 5, 5, 6, 7, 10, 7, 8, 8, 8, 5, 4, 9, 5, 5, 9, 10, 11, 6, 10, 10, 9, 6, 5, 10, 9, 6, 5, 5, 4, 5, 9, 4, 5)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 13, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 14, 14)
\frac{1}{16}(4, 4, 6, 6, 6, 8, 11, 11, 8, 4, 6, 10, 12, 9, 9, 10, 10, 6, 4, 7, 7, 10, 6, 8, 5, 5, 4, 6, 5, 5, 10, 5, 5, 5, 5, 10)
\frac{1}{18}(3, 5, 6, 6, 10, 10, 10, 11, 8, 7, 9, 7, 7, 13, 12, 11, 11, 5, 7, 5, 6, 6, 12, 12, 6, 5, 6, 6, 12, 5, 12, 6, 7, 6, 7, 11)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 7, 4, 8, 4, 7, 8, 4, 8, 3, 4, 4, 7, 8, 5, 4, 3)
\frac{1}{16}(3, 4, 6, 6, 6, 8, 11, 11, 7, 5, 5, 9, 11, 8, 10, 4, 10, 6, 4, 9, 9, 10, 10, 6, 5, 5, 4, 6, 9, 5, 10, 5, 5, 5, 11, 10)
\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,6,10,5,4,12,6,11,6,6,12,7,7,11,12,6,6)
\frac{1}{11} (2, 3, 4, 4, 5, 5, 6, 6, 5, 4, 4, 7, 7, 4, 8, 7, 7, 6, 6, 3, 3, 8, 3, 5, 4, 4, 5, 3, 4, 4, 6, 3, 7, 3, 7, 4)
\frac{1}{18}(3, 5, 6, 6, 6, 8, 10, 11, 8, 7, 9, 9, 5, 7, 12, 11, 11, 11, 13, 5, 6, 6, 6, 12, 12, 5, 12, 6, 6, 11, 6, 6, 11, 12, 7, 11)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 4, 4, 8, 8, 5, 8, 4, 4, 7, 4, 4, 7, 8, 3, 5)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 13, 16, 16, 6, 8, 8, 10, 12, 10, 16, 16, 6, 16, 8, 8, 6, 8, 8, 16, 16, 14, 14)
\frac{1}{18}(3, 5, 6, 6, 6, 8, 10, 11, 8, 7, 9, 9, 5, 7, 12, 11, 7, 11, 13, 5, 6, 6, 6, 12, 12, 5, 12, 6, 6, 7, 6, 6, 5, 12, 7, 7)
\frac{1}{15} (3, 4, 5, 8, 8, 8, 8, 9, 7, 6, 5, 5, 5, 11, 10, 9, 4, 6, 6, 4, 5, 5, 5, 5, 5, 4, 10, 10, 8, 5, 10, 6, 5, 6, 5, 9)
\frac{1}{20} (4, 6, 7, 8, 12, 13, 14, 14, 10, 9, 8, 16, 9, 10, 14, 7, 14, 6, 7, 8, 8, 13, 7, 14, 15, 7, 8, 9, 6, 6, 7, 8, 14, 15, 7, 8)
\frac{1}{21}(4,5,7,7,7,10,11,11,9,7,7,11,14,7,15,12,12,12,13,6,6,14,14,7,14,8,14,7,14,8,13,6,6,7,7,12)
\frac{1}{15} (3, 4, 5, 8, 8, 8, 8, 9, 7, 8, 5, 5, 5, 11, 10, 9, 4, 6, 10, 4, 5, 5, 5, 5, 5, 4, 10, 10, 6, 5, 10, 6, 5, 6, 5, 9)
\frac{1}{15} (3, 4, 5, 5, 8, 8, 8, 8, 7, 6, 6, 5, 5, 5, 11, 9, 9, 4, 10, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 8, 10, 6, 6)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 5, 6, 8, 9, 5, 5, 10, 9, 9, 10, 10, 10, 5, 8, 3, 9, 9, 4, 11, 5, 5, 4, 6, 6, 7, 10, 5, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 4, 4, 8, 8, 5, 8, 4, 4, 7, 4, 4, 7, 8, 3, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 6, 7, 6, 5, 5, 7, 3, 9, 4, 7, 7, 5, 5, 3, 4, 8, 4, 4, 4, 3, 4, 4, 4, 7, 8, 8, 3, 8, 7, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 5, 7, 9, 4, 7, 7, 7, 5, 3, 4, 4, 4, 8, 8, 7, 8, 4, 4, 3, 4, 4, 5, 8, 3, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 5, 6, 8, 6, 4, 5, 7, 8, 9, 5, 7, 7, 7, 4, 3, 7, 8, 8, 4, 5, 4, 8, 4, 4, 4, 4, 4, 4, 7, 3, 4)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 15, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{15} (3, 4, 5, 5, 6, 6, 8, 8, 7, 8, 8, 5, 5, 5, 11, 9, 9, 10, 10, 4, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 8)
\frac{1}{15}(3,4,5,5,6,6,8,8,7,6,6,5,5,5,11,9,9,10,10,4,4,10,5,5,5,5,5,5,5,5,10,10,6,10,6,8)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 15, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{18}(4,4,6,6,8,8,9,10,8,6,10,6,6,5,6,6,10,12,12,13,6,12,6,6,7,12,6,10,5,4,12,11,6,11,6,7)
\frac{1}{30}(6, 8, 8, 12, 12, 15, 16, 18, 12, 14, 8, 10, 21, 10, 12, 16, 20, 20, 9, 20, 10, 20, 10, 7, 10, 10, 10, 13, 10, 10, 11, 20, 20, 11, 9, 20)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 9, 9, 5, 7, 8, 5, 11, 10, 10, 6, 5, 10, 10, 5, 5, 9, 4, 4, 5, 5, 5, 5, 4, 6, 8, 5, 5, 9, 5, 10)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 9, 4, 8, 6, 7, 4, 5, 8, 4, 7, 4, 3, 4, 6, 3, 4, 3, 5, 7, 4, 8, 7, 3, 4)
\frac{1}{15}(3, 4, 4, 5, 6, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 6, 4, 4, 5, 5, 5, 5, 4, 10, 10, 5, 6, 5, 9)
\frac{1}{15} (3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 6, 10, 5, 11, 10, 6, 5, 5, 10, 10, 5, 9, 5, 4, 4, 5, 6, 5, 5, 4, 5, 5, 10, 6, 5, 9)
\frac{1}{22}(4,6,7,8,12,13,14,14,10,11,8,16,9,10,14,7,14,6,7,8,8,13,7,14,15,7,8,13,6,6,7,8,14,15,7,8)
\frac{1}{12} (2, 3, 4, 4, 6, 6, 6, 6, 5, 6, 6, 4, 4, 8, 8, 7, 7, 3, 5, 3, 7, 8, 4, 8, 4, 4, 4, 8, 4, 4, 8, 6, 4, 4, 4, 8)
```

```
\frac{1}{15}(3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 6, 4, 5, 5, 10, 6, 5, 5, 10, 10, 5, 9, 11, 4, 6, 5, 10, 9, 9, 4, 7, 5, 6, 10, 5, 5)
\frac{1}{15}(3, 4, 5, 5, 8, 9, 9, 9, 7, 6, 6, 11, 6, 6, 10, 9, 9, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 4, 5, 5, 9, 10, 9, 9)
\frac{1}{15}(3, 4, 5, 7, 8, 9, 9, 9, 7, 6, 10, 11, 6, 6, 6, 9, 5, 4, 5, 5, 5, 4, 5, 4, 6, 8, 9, 6, 6, 4, 5, 5, 5, 10, 10, 10)
\frac{1}{15}(3, 4, 5, 5, 5, 6, 7, 10, 7, 8, 8, 8, 5, 4, 9, 5, 5, 9, 10, 11, 6, 10, 10, 5, 6, 5, 10, 9, 6, 5, 9, 4, 5, 9, 4, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 5, 7, 9, 4, 7, 7, 7, 5, 3, 4, 4, 4, 8, 8, 7, 8, 4, 4, 3, 4, 4, 7, 8, 3, 5)
\frac{1}{24}(5, 6, 8, 8, 8, 10, 12, 16, 11, 7, 9, 13, 7, 17, 15, 6, 10, 10, 16, 6, 10, 16, 16, 10, 12, 8, 16, 16, 8, 8, 6, 16, 8, 10, 8, 16)
\frac{1}{21}(4,5,7,7,7,10,11,11,9,7,7,11,14,7,15,8,12,12,13,6,6,14,14,7,14,8,14,7,14,8,13,6,6,7,7,12)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 5, 7, 7, 3, 4, 7, 7, 5, 7, 9, 4, 8, 4, 4, 8, 7, 4, 4, 8, 7, 8, 4, 5, 4, 3, 5)
\frac{1}{22}(4,6,8,8,12,13,13,14,10,8,8,16,9,9,14,8,14,6,7,7,8,12,8,15,15,6,8,7,7,6,7,7,14,14,13,13)
\frac{1}{15} (3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 8, 4, 5, 5, 10, 6, 5, 9, 10, 10, 5, 9, 11, 4, 6, 5, 10, 5, 5, 4, 7, 5, 6, 10, 5, 5)
\frac{1}{21}(4,5,7,7,7,10,11,11,9,7,7,9,14,7,15,12,12,12,13,6,6,14,14,7,14,8,14,7,14,8,13,6,6,7,7,12)
\frac{1}{11} (2, 3, 4, 4, 4, 6, 7, 7, 5, 4, 4, 4, 8, 5, 7, 7, 7, 7, 3, 4, 4, 4, 6, 6, 3, 3, 8, 4, 7, 3, 6, 3, 5, 3, 7, 4)
\frac{1}{22}(4, 5, 7, 8, 11, 13, 13, 15, 9, 7, 8, 15, 9, 15, 15, 8, 13, 6, 14, 8, 10, 15, 14, 6, 8, 8, 7, 13, 7, 7, 8, 14, 6, 6, 14, 14)
\frac{1}{12}(3,3,3,4,6,7,7,7,4,6,4,5,4,8,8,6,5,9,8,4,4,7,3,4,4,4,4,3,4,4,7,5,7,4,4,8)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 7, 4, 8, 4, 5, 8, 4, 8, 3, 4, 4, 5, 8, 5, 4, 3)
\frac{1}{15} (3, 4, 5, 5, 5, 6, 7, 10, 7, 8, 8, 8, 5, 4, 9, 5, 5, 9, 10, 11, 6, 10, 10, 9, 6, 5, 10, 9, 6, 5, 9, 4, 5, 9, 4, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 6, 7, 6, 5, 7, 7, 3, 9, 4, 7, 7, 7, 5, 3, 4, 4, 8, 4, 4, 7, 4, 8, 8, 3, 4, 4, 7, 8, 5, 5)
\frac{1}{10} (4, 4, 7, 7, 8, 9, 13, 13, 8, 7, 11, 6, 13, 13, 13, 11, 7, 12, 5, 9, 9, 8, 13, 6, 6, 6, 5, 12, 6, 6, 7, 7, 7, 6, 6, 12)
\frac{1}{10}(4,4,6,6,7,9,11,12,8,6,6,7,13,13,8,8,10,11,5,7,12,12,13,13,7,6,13,13,11,6,6,6,7,12,7,5)
\frac{1}{10}(4, 5, 7, 7, 10, 11, 12, 12, 9, 7, 11, 14, 7, 8, 12, 12, 12, 5, 6, 7, 7, 10, 7, 8, 5, 5, 13, 6, 7, 5, 7, 6, 12, 13, 11, 10)
\frac{1}{27} (6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 15, 15, 7, 9, 16, 15, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{18}(3, 5, 6, 6, 6, 8, 10, 11, 8, 7, 9, 9, 5, 7, 12, 11, 11, 11, 13, 5, 6, 6, 6, 12, 12, 5, 12, 6, 6, 11, 6, 6, 11, 12, 7, 7)
\frac{1}{25}(4,6,7,9,13,14,15,16,10,9,13,17,10,17,16,9,7,7,16,9,14,16,8,7,8,9,14,9,16,7,9,16,17,7,16,17)
\frac{1}{22}(4, 5, 7, 8, 11, 13, 13, 15, 9, 7, 8, 15, 9, 15, 15, 8, 13, 6, 14, 8, 12, 15, 14, 6, 8, 8, 7, 13, 7, 7, 8, 14, 6, 6, 14, 14)
\frac{1}{15}(3, 4, 4, 5, 7, 7, 8, 9, 5, 7, 6, 4, 10, 5, 10, 6, 5, 5, 5, 10, 5, 9, 11, 9, 4, 5, 10, 6, 5, 4, 6, 9, 6, 5, 10, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 5, 7, 7, 7, 9, 4, 5, 4, 4, 8, 3, 4, 8, 4, 5, 8, 4, 5, 8, 7, 4, 3)
\frac{1}{21}(4,6,6,8,11,12,13,13,8,10,6,15,14,9,13,12,14,7,6,13,7,14,5,14,7,7,9,14,7,7,13,6,8,7,7,14)
\frac{1}{12} (2, 3, 4, 4, 6, 6, 6, 6, 5, 6, 6, 4, 4, 8, 8, 7, 7, 3, 5, 3, 5, 8, 4, 8, 4, 4, 4, 8, 4, 4, 8, 6, 4, 4, 4, 8)
\frac{1}{12} (3, 3, 3, 4, 6, 7, 7, 7, 4, 6, 4, 5, 4, 8, 8, 6, 5, 9, 8, 4, 4, 7, 3, 4, 4, 4, 4, 3, 4, 4, 7, 7, 7, 4, 4, 8)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 12, 10, 16, 16, 6, 16, 8, 8, 6, 8, 8, 16, 16, 10, 10)
\frac{1}{10}(4,5,7,7,10,11,12,12,9,7,11,14,7,8,12,12,12,5,6,7,7,6,7,12,5,5,13,6,7,5,7,6,12,13,7,10)
\frac{1}{15}(3,4,5,5,6,6,8,8,7,6,8,5,5,5,11,9,9,10,10,4,4,10,5,5,5,5,5,5,5,5,10,10,6,10,6,6)
\frac{1}{15}(3, 4, 5, 7, 8, 8, 8, 9, 7, 6, 10, 5, 5, 11, 10, 9, 5, 4, 6, 4, 5, 6, 5, 5, 5, 4, 5, 5, 5, 10, 10, 6, 5, 10, 5, 9)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 16, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{21}(4,6,6,8,11,11,12,13,8,10,6,7,15,14,9,12,14,13,7,6,7,14,7,5,14,7,7,9,14,7,8,7,14,13,6,7)
\frac{1}{18}(4,4,6,9,10,10,10,10,8,8,13,6,6,12,12,10,5,6,6,6,6,5,4,12,6,10,9,7,11,11,12,6,6,6,6,12)
\frac{1}{18}(4,4,6,8,8,9,10,10,8,8,12,12,13,6,6,10,6,6,5,6,6,10,5,4,12,12,11,6,6,11,6,6,7,7,12)
\frac{1}{15}(3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 8, 4, 5, 5, 10, 6, 5, 5, 10, 10, 5, 9, 11, 4, 6, 5, 10, 5, 5, 4, 7, 9, 6, 10, 5, 5)
\frac{1}{15}(3,4,5,6,6,7,9,10,7,8,5,9,4,6,9,5,10,10,11,5,6,5,5,6,10,5,10,9,5,4,5,5,6,10,5,5)
\frac{1}{15}(3, 4, 5, 5, 8, 8, 8, 8, 7, 8, 8, 5, 5, 5, 11, 9, 9, 4, 10, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 8, 10, 6, 6)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 5, 7, 7, 7, 9, 4, 5, 4, 4, 8, 3, 4, 8, 4, 7, 8, 4, 7, 8, 7, 4, 3)
```

```
\frac{1}{24}(5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 12, 10, 16, 16, 6, 16, 8, 8, 6, 8, 8, 16, 16, 10, 14)
\frac{1}{10}(4,5,6,7,10,11,11,11,9,10,11,14,7,7,13,11,12,5,6,12,6,7,6,7,5,5,13,6,12,6,7,7,11,6,12,6)
\frac{1}{12}(3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 7, 7, 7, 3, 4, 7, 7, 7, 9, 4, 4, 4, 8, 8, 7, 8, 4, 4, 3, 4, 4, 5, 8, 7, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 5, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 8, 4, 4, 5, 8, 4, 4, 5, 8, 8, 3, 4, 7, 4, 3)
\frac{1}{18}(4,4,6,6,8,8,9,10,8,6,10,12,12,13,6,10,10,6,6,5,6,12,6,6,7,12,6,10,5,4,12,11,6,11,6,7)
\frac{1}{27} (6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 15, 15, 7, 9, 16, 9, 9, 11, 19, 12, 18, 18, 14, 16, 9, 18, 10, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{21} (4, 6, 6, 8, 11, 12, 13, 13, 8, 10, 6, 15, 14, 9, 9, 12, 14, 7, 6, 7, 13, 14, 5, 14, 7, 7, 9, 14, 7, 7, 13, 8, 6, 7, 7, 14)
\frac{1}{11} (2, 3, 4, 4, 4, 6, 7, 7, 5, 4, 4, 4, 8, 5, 7, 7, 7, 7, 3, 4, 4, 6, 6, 6, 3, 3, 8, 4, 7, 3, 6, 3, 5, 3, 7, 4)
\frac{1}{11} (2, 3, 4, 4, 5, 6, 6, 7, 5, 4, 4, 7, 4, 8, 7, 5, 7, 6, 3, 3, 4, 4, 5, 8, 4, 3, 3, 4, 4, 3, 3, 7, 4, 4, 7, 7)
\frac{1}{15} (3,4,5,6,8,8,8,9,7,8,5,5,5,11,10,9,10,4,6,4,5,5,5,5,4,10,10,6,5,10,8,5,6,5,9)
\frac{1}{19}(4,4,6,6,7,9,11,12,8,6,6,7,13,13,8,8,8,11,5,7,12,12,13,13,7,6,13,13,11,6,6,6,7,12,7,5)
\frac{1}{15} (3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 8, 5, 4, 5, 10, 6, 5, 10, 9, 10, 5, 9, 6, 11, 4, 5, 5, 10, 5, 4, 5, 10, 5, 7, 6, 5)
\frac{1}{18}(3, 5, 5, 7, 7, 8, 9, 12, 6, 8, 6, 10, 5, 6, 11, 6, 6, 6, 7, 12, 7, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 7, 11, 6, 5)
\frac{1}{12} (2, 3, 4, 6, 6, 6, 8, 8, 5, 4, 4, 4, 8, 6, 6, 7, 3, 5, 3, 7, 7, 8, 8, 4, 4, 4, 8, 6, 4, 4, 4, 4, 4, 4, 4, 8)
\frac{1}{24} (5, 6, 8, 8, 8, 10, 12, 16, 11, 9, 13, 13, 7, 17, 15, 14, 6, 10, 16, 6, 10, 16, 16, 16, 8, 8, 16, 10, 12, 8, 6, 16, 8, 10, 8, 16)
\frac{1}{24}(3, 5, 7, 9, 9, 12, 13, 16, 8, 8, 8, 8, 9, 16, 13, 12, 8, 14, 7, 8, 15, 16, 16, 17, 8, 9, 16, 15, 8, 7, 7, 8, 9, 15, 8, 7)
\frac{1}{15} (3, 4, 5, 5, 5, 7, 8, 10, 7, 8, 8, 8, 4, 5, 9, 5, 7, 9, 11, 10, 6, 10, 10, 6, 5, 5, 10, 4, 5, 5, 6, 5, 5, 9, 5, 4)
\frac{1}{18}(4,5,5,7,7,9,11,11,7,9,5,11,13,7,7,10,12,6,6,12,12,12,8,4,6,6,6,8,6,6,12,6,6,6,6,12)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 7, 6, 8, 5, 5, 11, 6, 9, 9, 10, 10, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 10, 6, 5, 6, 5, 5)
\frac{1}{15}(3, 4, 5, 5, 8, 9, 9, 9, 7, 6, 8, 11, 6, 6, 10, 9, 9, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 4, 5, 5, 9, 10, 6, 6)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 5, 7, 9, 4, 5, 7, 7, 5, 3, 4, 4, 4, 8, 8, 7, 8, 4, 4, 3, 4, 4, 7, 8, 3, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 4, 8, 4, 7, 8, 4, 8, 3, 4, 4, 7, 8, 7, 4, 3)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 7, 7, 7, 7, 9, 4, 7, 4, 4, 8, 3, 4, 8, 4, 5, 8, 4, 5, 8, 5, 4, 3)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 14, 14)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 4, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 4, 4, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 8, 3, 5)
\frac{1}{15}(3,4,5,5,6,8,8,9,7,8,8,5,5,11,6,9,9,10,10,4,5,10,5,5,5,10,5,5,5,10,10,6,5,6,5,5)
\frac{1}{15}(3,4,5,5,6,6,8,9,7,6,6,5,5,11,6,9,9,10,10,4,5,10,5,5,5,10,5,5,5,10,10,6,5,6,5,5)
\frac{1}{12}(3, 3, 3, 4, 6, 7, 7, 7, 4, 5, 4, 4, 4, 8, 8, 6, 5, 3, 8, 4, 4, 7, 9, 4, 4, 4, 8, 3, 4, 4, 5, 5, 5, 4, 4, 8)
\frac{1}{18}(3, 5, 6, 6, 6, 8, 10, 11, 8, 7, 7, 9, 5, 7, 12, 11, 11, 7, 13, 5, 6, 6, 6, 12, 12, 5, 12, 6, 6, 5, 6, 6, 11, 12, 7, 7)
\frac{1}{15}(3, 4, 4, 5, 6, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 6, 4, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{18}(4,4,6,6,8,8,9,10,8,6,10,6,6,5,6,6,10,12,12,13,6,12,6,6,7,12,6,10,5,4,12,11,6,11,6,9)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 5, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 8, 4, 4, 4, 8, 4, 4, 4, 8, 8, 3, 4, 7, 4, 4)
\frac{1}{15}(3, 4, 5, 6, 8, 9, 9, 9, 7, 6, 9, 11, 6, 6, 10, 9, 10, 4, 5, 5, 5, 5, 5, 6, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 10, 10)
\frac{1}{15}(3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 8, 4, 5, 5, 10, 8, 5, 9, 10, 10, 5, 9, 11, 4, 6, 5, 10, 5, 5, 4, 7, 5, 6, 10, 5, 5)
\frac{1}{27}(5, 7, 8, 10, 10, 11, 14, 16, 12, 11, 9, 9, 8, 19, 11, 15, 9, 9, 18, 7, 9, 18, 18, 19, 8, 10, 18, 9, 10, 18, 9, 10, 18, 11, 9, 8)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 4, 10, 4, 5, 5, 5, 5, 4, 10, 8, 5, 6, 5, 9)
\frac{1}{17}(2, 5, 5, 5, 6, 7, 7, 11, 5, 7, 7, 6, 5, 9, 11, 10, 10, 11, 10, 4, 6, 10, 11, 6, 12, 6, 11, 12, 6, 6, 11, 11, 5, 6, 6, 6)
\frac{1}{15}(3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 6, 4, 5, 5, 10, 6, 5, 5, 10, 10, 5, 9, 11, 4, 6, 5, 10, 9, 9, 4, 9, 5, 6, 10, 5, 5)
\frac{1}{15} (3, 4, 5, 8, 8, 8, 9, 9, 7, 6, 5, 5, 11, 10, 10, 9, 4, 6, 4, 5, 5, 5, 5, 5, 4, 6, 10, 8, 5, 5, 10, 5, 5, 9, 5, 10)
\frac{1}{15}(3, 4, 5, 5, 6, 6, 6, 9, 5, 6, 8, 5, 5, 9, 10, 9, 9, 10, 10, 10, 5, 8, 9, 9, 3, 4, 5, 5, 11, 4, 10, 6, 5, 6, 5, 7)
\frac{1}{22}(4, 6, 7, 8, 12, 13, 14, 14, 10, 11, 8, 16, 9, 10, 14, 7, 14, 6, 7, 8, 8, 9, 7, 14, 15, 7, 8, 13, 6, 6, 7, 8, 14, 15, 7, 8)
```

```
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 4, 5, 5, 7, 9, 4, 7, 7, 7, 7, 3, 4, 4, 4, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 8, 3, 5)
\frac{1}{15}(3, 4, 5, 8, 8, 8, 8, 9, 7, 6, 5, 5, 5, 11, 10, 9, 4, 6, 6, 4, 5, 5, 5, 5, 5, 4, 10, 10, 6, 5, 10, 6, 5, 6, 5, 9)
\frac{1}{24} (5, 6, 8, 10, 12, 14, 14, 16, 11, 9, 7, 17, 9, 9, 15, 14, 16, 6, 8, 8, 10, 16, 8, 16, 16, 8, 10, 8, 8, 8, 8, 8, 8, 16, 16, 8, 8)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 6, 7, 6, 5, 5, 7, 3, 9, 4, 7, 7, 5, 5, 3, 4, 8, 4, 4, 4, 3, 4, 4, 4, 5, 8, 8, 3, 8, 7, 5)
\frac{1}{27}(5, 7, 8, 10, 11, 14, 14, 16, 12, 11, 9, 8, 9, 19, 11, 15, 9, 18, 9, 7, 9, 18, 19, 18, 8, 10, 9, 18, 10, 18, 9, 11, 9, 10, 18, 8)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 8, 9, 4, 6, 7, 8, 4, 5, 8, 7, 4, 4, 3, 4, 7, 3, 4, 3, 4, 7, 8, 7, 4, 7)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 7, 7, 7, 7, 3, 4, 7, 7, 7, 7, 9, 4, 4, 4, 8, 4, 3, 8, 4, 8, 7, 4, 8, 7, 4, 5, 5)
\frac{1}{22}(4, 6, 8, 12, 12, 13, 13, 14, 10, 8, 8, 16, 9, 9, 14, 14, 8, 6, 7, 7, 8, 12, 8, 7, 7, 6, 8, 15, 15, 6, 7, 7, 14, 14, 13, 13)
\frac{1}{19} (4, 4, 7, 7, 8, 9, 13, 13, 8, 7, 11, 6, 13, 13, 13, 11, 7, 12, 5, 9, 9, 12, 13, 6, 6, 6, 5, 12, 6, 6, 7, 7, 11, 6, 6, 12)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 6, 5, 10, 10, 10, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{18}(3, 5, 6, 6, 10, 10, 10, 11, 8, 7, 7, 7, 7, 13, 12, 11, 11, 5, 7, 5, 6, 6, 6, 6, 12, 5, 12, 12, 6, 5, 12, 6, 7, 6, 7, 11)
\frac{1}{15}(3, 4, 5, 5, 6, 8, 8, 8, 7, 6, 8, 5, 5, 5, 11, 9, 9, 10, 4, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 8, 6)
\frac{1}{12} (3, 3, 3, 4, 6, 7, 7, 7, 4, 6, 4, 4, 4, 8, 8, 6, 5, 3, 8, 4, 4, 7, 9, 4, 4, 4, 8, 3, 4, 4, 5, 5, 7, 4, 4, 8)
\frac{1}{18}(4,4,6,6,8,9,10,10,8,6,10,6,5,6,6,6,10,12,13,6,12,12,6,7,12,6,6,5,4,10,11,6,12,7,11,6)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 8, 8, 9, 4, 6, 7, 4, 4, 7, 8, 7, 4, 4, 3, 4, 3, 5, 4, 3, 8, 7, 4, 3, 4, 7)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 5, 7, 7, 7, 9, 4, 5, 4, 4, 8, 3, 4, 8, 4, 4, 8, 4, 4, 8, 7, 4, 4)
\frac{1}{12}(3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 7, 7, 7, 9, 4, 7, 7, 7, 3, 4, 4, 4, 8, 4, 5, 8, 4, 8, 7, 4, 8, 7, 4, 3, 5)
\frac{1}{15}(3, 4, 5, 8, 9, 9, 9, 9, 7, 6, 11, 6, 6, 6, 10, 9, 4, 5, 5, 5, 5, 5, 4, 9, 9, 4, 5, 5, 5, 9, 10, 10, 5, 10, 5, 5)
\frac{1}{18}(3, 5, 6, 6, 8, 8, 10, 11, 8, 7, 9, 5, 11, 7, 12, 11, 11, 13, 11, 5, 6, 6, 12, 6, 12, 5, 6, 12, 6, 5, 6, 12, 7, 6, 7, 7)
\frac{1}{15} (3, 4, 5, 5, 6, 6, 8, 9, 5, 6, 8, 5, 9, 5, 10, 9, 9, 10, 10, 10, 5, 8, 9, 3, 9, 4, 5, 11, 5, 4, 6, 10, 5, 6, 7, 5)
\frac{1}{15}(3, 4, 5, 6, 8, 9, 9, 9, 7, 6, 9, 11, 6, 6, 10, 9, 6, 4, 5, 5, 5, 5, 5, 10, 10, 4, 10, 5, 5, 9, 5, 5, 9, 10, 6, 6)
\frac{1}{18}(3, 5, 6, 6, 8, 8, 10, 11, 8, 7, 7, 5, 11, 7, 12, 11, 11, 13, 7, 5, 6, 6, 6, 12, 6, 5, 12, 6, 12, 5, 6, 12, 7, 6, 7, 7)
\frac{1}{24}(5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 13, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 10, 14)
\frac{1}{18}(4,5,5,7,9,11,11,11,7,9,5,13,7,7,11,10,12,6,12,12,6,12,4,6,6,8,8,6,6,6,6,6,12,12,6,6)
\frac{1}{18}(3, 5, 6, 6, 6, 10, 10, 11, 8, 7, 9, 9, 7, 13, 12, 11, 7, 11, 5, 5, 6, 12, 6, 12, 6, 5, 6, 12, 6, 11, 6, 12, 5, 6, 7, 11)
\frac{1}{14}(2, 4, 5, 5, 7, 7, 8, 8, 6, 5, 7, 5, 9, 6, 10, 9, 9, 5, 5, 4, 4, 10, 6, 4, 5, 5, 4, 8, 5, 5, 4, 9, 5, 5, 9, 4)
\frac{1}{15}(3, 4, 5, 5, 6, 8, 8, 8, 7, 6, 8, 5, 5, 5, 11, 9, 9, 10, 4, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 6)
\frac{1}{15} (3, 4, 5, 5, 6, 6, 8, 9, 5, 6, 6, 5, 9, 5, 10, 9, 9, 10, 10, 10, 5, 8, 5, 11, 5, 4, 9, 3, 9, 4, 6, 10, 5, 6, 7, 5)
\frac{1}{18}(4,4,6,6,9,10,10,10,8,6,10,13,6,12,12,10,10,5,6,6,6,6,12,7,12,6,6,5,4,6,10,9,11,11,6,6,12)
\frac{1}{24}(5, 6, 8, 8, 8, 12, 12, 16, 11, 7, 9, 13, 7, 17, 15, 6, 10, 10, 16, 6, 10, 16, 16, 10, 10, 8, 16, 16, 8, 8, 6, 16, 8, 10, 8, 16)
\frac{1}{12}(3,3,3,4,5,6,7,7,4,6,5,8,9,4,8,6,7,4,7,8,4,7,4,3,4,6,3,4,3,5,7,4,8,7,3,4)
\frac{1}{25}(4,6,7,9,13,14,15,16,10,9,9,17,10,17,16,9,7,7,16,9,14,16,8,7,8,9,14,9,16,7,9,16,17,7,16,17)
\frac{1}{15} (3, 4, 5, 5, 7, 8, 10, 10, 7, 8, 8, 4, 5, 7, 9, 5, 9, 11, 10, 6, 6, 10, 6, 5, 5, 5, 4, 5, 5, 5, 9, 5, 5, 6, 4, 10)
\frac{1}{15}(3,4,5,5,6,6,8,8,7,8,8,5,5,5,11,9,9,10,10,4,4,10,5,5,5,5,5,5,5,5,5,10,10,6,10,6,6)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 8, 5, 10, 10, 10, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 5, 7, 9, 4, 7, 7, 7, 7, 3, 4, 7, 8, 4, 4, 5, 8, 4, 4, 7, 8, 8, 3, 4, 5, 4, 3)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 8, 9, 4, 6, 7, 8, 4, 5, 8, 7, 4, 4, 3, 4, 5, 3, 4, 3, 4, 7, 8, 7, 4, 7)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 8, 5, 10, 10, 6, 5, 9, 4, 10, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 7, 7, 7, 7, 9, 4, 7, 4, 4, 8, 3, 4, 8, 4, 7, 8, 4, 7, 8, 5, 4, 3)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 7, 7, 7, 3, 4, 7, 7, 7, 7, 9, 4, 7, 4, 4, 8, 3, 4, 8, 4, 5, 8, 4, 7, 8, 5, 4, 3)
\frac{1}{18}(3, 5, 6, 6, 6, 10, 10, 11, 8, 7, 9, 9, 7, 13, 12, 11, 7, 11, 5, 5, 6, 12, 6, 12, 6, 5, 6, 12, 6, 7, 6, 12, 5, 6, 7, 11)
```

```
\frac{1}{6}(2, 2, 3, 3, 4, 4, 5, 5, 4, 3, 5, 6, 6, 3, 3, 5, 5, 2, 4, 3, 5, 6, 3, 3, 4, 6, 5, 3, 2, 6, 6, 3, 3, 3, 3, 4)
\frac{1}{10} (4, 4, 6, 6, 8, 9, 10, 10, 8, 6, 10, 6, 5, 6, 6, 6, 10, 12, 13, 6, 12, 12, 6, 7, 12, 6, 10, 5, 4, 6, 11, 6, 12, 9, 11, 6)
\frac{1}{15} (3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 8, 5, 4, 5, 10, 6, 5, 10, 5, 10, 5, 9, 6, 11, 4, 5, 5, 10, 9, 4, 5, 10, 5, 7, 6, 5)
\frac{1}{15} (3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 6, 5, 4, 5, 10, 6, 5, 10, 5, 10, 5, 9, 6, 11, 4, 5, 9, 10, 9, 4, 5, 10, 5, 7, 6, 5)
\frac{1}{15}(3, 4, 6, 6, 8, 9, 9, 10, 7, 5, 9, 11, 6, 6, 9, 10, 6, 4, 5, 5, 6, 6, 6, 5, 9, 4, 10, 5, 5, 10, 5, 5, 10, 10, 5, 5)
\frac{1}{18}(3, 5, 6, 6, 8, 8, 10, 11, 8, 7, 9, 5, 11, 7, 12, 11, 11, 13, 7, 5, 6, 6, 12, 6, 12, 5, 6, 12, 6, 5, 6, 12, 7, 6, 11, 7)
\frac{1}{18} (4, 4, 6, 6, 9, 10, 10, 10, 8, 6, 10, 5, 6, 6, 6, 6, 10, 13, 6, 12, 12, 12, 7, 12, 6, 6, 5, 4, 6, 10, 9, 11, 11, 6, 6, 12)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 5, 7, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 4, 4, 8, 4, 8, 4, 8, 4, 4, 8, 4, 4, 3, 5)
\frac{1}{22}(4,6,8,8,12,13,13,14,10,8,8,16,9,9,14,8,14,6,7,7,8,12,8,15,15,6,8,7,7,6,7,7,14,14,9,13)
\frac{1}{6}(2, 2, 3, 3, 3, 4, 4, 5, 4, 3, 3, 5, 6, 6, 3, 5, 5, 3, 2, 4, 5, 6, 2, 3, 5, 4, 6, 3, 3, 6, 5, 3, 6, 6, 3, 3)
\frac{1}{27} (6, 6, 9, 9, 9, 13, 15, 18, 12, 9, 9, 11, 19, 19, 12, 11, 15, 15, 7, 9, 16, 18, 18, 10, 9, 18, 18, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{15} (3, 4, 5, 5, 8, 8, 8, 8, 7, 6, 8, 5, 5, 5, 11, 9, 9, 4, 10, 10, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 6)
\frac{1}{16} (3, 4, 5, 8, 8, 8, 9, 9, 7, 6, 5, 5, 11, 10, 10, 9, 4, 6, 4, 5, 5, 5, 5, 5, 4, 6, 10, 6, 5, 5, 10, 5, 5, 9, 5, 10)
\frac{1}{15} (3, 4, 5, 7, 8, 9, 9, 9, 7, 6, 10, 11, 6, 6, 6, 9, 5, 4, 5, 5, 5, 4, 5, 4, 6, 6, 9, 6, 4, 6, 5, 5, 5, 10, 10, 10)
\frac{1}{24}(5,6,10,10,12,14,14,16,11,7,15,17,9,9,13,16,16,6,8,8,10,12,10,16,16,6,16,8,8,6,8,8,16,16,10,10)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 9, 9, 5, 7, 8, 5, 11, 10, 10, 6, 5, 10, 6, 5, 5, 9, 4, 4, 5, 5, 5, 5, 4, 10, 6, 5, 5, 9, 5, 10)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 10, 14)
\frac{1}{15} (3, 4, 5, 8, 9, 9, 9, 9, 7, 6, 11, 6, 6, 6, 10, 9, 4, 5, 5, 5, 5, 5, 4, 6, 9, 4, 5, 5, 5, 9, 10, 10, 5, 10, 5, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 4, 5, 5, 7, 9, 4, 5, 7, 7, 7, 3, 4, 4, 4, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 8, 3, 5)
\frac{1}{27} (6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 16, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{15} (3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 8, 5, 4, 5, 10, 6, 5, 10, 5, 10, 5, 9, 6, 11, 4, 5, 5, 10, 9, 4, 5, 10, 5, 9, 6, 5)
\frac{1}{15} (3, 4, 5, 5, 5, 6, 7, 10, 7, 8, 8, 8, 5, 4, 9, 5, 5, 7, 10, 11, 6, 10, 10, 5, 6, 5, 10, 9, 6, 5, 9, 4, 5, 9, 4, 5)
\frac{1}{15} (3, 4, 5, 8, 8, 8, 8, 9, 7, 8, 5, 5, 5, 11, 10, 9, 4, 6, 10, 4, 5, 5, 5, 5, 5, 4, 10, 10, 8, 5, 10, 6, 5, 6, 5, 9)
\frac{1}{15} (3, 4, 5, 5, 5, 6, 7, 10, 7, 8, 8, 8, 5, 4, 9, 5, 5, 7, 10, 11, 6, 10, 10, 9, 6, 5, 10, 9, 6, 5, 5, 4, 5, 9, 4, 5)
\frac{1}{10}(4,4,7,7,8,9,13,13,8,7,11,6,13,13,13,11,7,12,5,11,11,8,13,6,6,6,5,12,6,6,7,7,7,6,6,12)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 13, 16, 16, 6, 8, 8, 10, 12, 10, 16, 16, 6, 16, 8, 8, 6, 8, 8, 16, 16, 10, 14)
\frac{1}{21}(4, 6, 6, 8, 10, 11, 13, 13, 8, 10, 6, 14, 15, 9, 9, 12, 14, 6, 7, 7, 13, 14, 14, 5, 7, 7, 14, 9, 7, 7, 13, 7, 7, 12, 6, 14)
\frac{1}{18}(3,5,5,7,7,8,9,12,8,8,6,10,5,6,11,6,6,6,11,12,7,12,12,13,6,7,12,11,6,5,5,6,7,11,6,5)
\frac{1}{22}(4, 5, 7, 8, 11, 13, 13, 15, 9, 7, 8, 15, 9, 15, 15, 8, 13, 6, 14, 8, 12, 15, 14, 6, 8, 8, 7, 9, 7, 7, 8, 14, 6, 6, 14, 14)
\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,6,6,5,4,12,10,11,6,6,12,7,7,11,12,6,6)
\frac{1}{11} (2, 3, 4, 4, 5, 5, 6, 6, 5, 4, 4, 7, 7, 4, 8, 7, 7, 6, 6, 3, 3, 8, 3, 5, 4, 4, 7, 3, 4, 4, 6, 3, 7, 3, 7, 4)
\frac{1}{15} (3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 6, 5, 4, 5, 10, 6, 5, 10, 5, 10, 5, 9, 6, 11, 4, 5, 9, 10, 5, 4, 9, 10, 5, 7, 6, 5)
\frac{1}{18} (4, 4, 6, 6, 8, 9, 10, 10, 8, 6, 10, 6, 5, 6, 6, 6, 10, 12, 13, 6, 12, 12, 6, 7, 12, 6, 6, 5, 4, 10, 11, 6, 12, 9, 11, 6)
\frac{1}{27}(5, 7, 8, 10, 10, 11, 14, 16, 12, 11, 9, 9, 8, 19, 11, 15, 9, 9, 18, 7, 9, 18, 18, 19, 8, 16, 18, 9, 10, 18, 9, 10, 18, 11, 9, 8)
\frac{1}{27}(5,7,8,11,14,14,14,16,12,11,8,9,9,19,11,15,18,9,9,7,9,19,18,18,8,10,9,9,11,9,18,10,18,10,18,8)
\frac{1}{24}(3, 5, 7, 9, 9, 12, 13, 16, 8, 8, 8, 8, 9, 16, 15, 10, 8, 14, 7, 8, 15, 16, 16, 17, 8, 9, 16, 15, 8, 7, 7, 8, 9, 15, 8, 7)
\frac{1}{15} (3, 4, 5, 7, 8, 9, 9, 9, 7, 6, 10, 11, 6, 6, 6, 9, 5, 4, 5, 5, 5, 4, 5, 4, 6, 8, 9, 8, 6, 4, 5, 5, 5, 10, 10, 10)
\frac{1}{24} (5, 6, 8, 8, 8, 12, 12, 16, 11, 9, 13, 13, 7, 17, 15, 14, 6, 10, 16, 6, 10, 16, 16, 16, 8, 8, 16, 10, 12, 8, 6, 16, 8, 10, 8, 16)
\frac{1}{20}(5, 8, 10, 12, 16, 16, 20, 20, 13, 11, 9, 11, 21, 15, 19, 18, 20, 8, 8, 12, 12, 20, 20, 10, 20, 10, 12, 12, 8, 10, 10, 20, 10, 10, 20, 10)
\frac{1}{15}(3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 8, 5, 4, 5, 10, 6, 5, 10, 5, 10, 5, 9, 6, 11, 4, 5, 5, 10, 5, 4, 9, 10, 5, 7, 6, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 4, 7, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 4, 4, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 8, 3, 5)
```

```
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 15, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 10, 16, 9, 18, 10, 16, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 7, 7, 7, 7, 3, 4, 7, 4, 8, 4, 7, 8, 4, 8, 3, 4, 4, 5, 8, 5, 4, 3)
\frac{1}{15} (3, 4, 4, 5, 8, 8, 9, 9, 5, 7, 8, 5, 11, 10, 10, 8, 5, 10, 6, 5, 5, 9, 4, 4, 5, 5, 5, 5, 4, 10, 6, 5, 5, 9, 5, 10)
\frac{1}{15}(3, 4, 4, 5, 7, 7, 8, 9, 5, 7, 8, 4, 10, 5, 10, 8, 5, 5, 5, 10, 5, 9, 11, 5, 4, 5, 10, 10, 5, 4, 6, 7, 6, 5, 10, 5)
\frac{1}{22}(4, 6, 8, 8, 12, 13, 13, 14, 10, 8, 8, 16, 9, 9, 14, 8, 14, 6, 7, 7, 8, 12, 8, 15, 15, 6, 8, 7, 7, 6, 7, 7, 14, 14, 9, 9)
\frac{1}{15}(3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 8, 5, 10, 10, 6, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 6, 5, 10, 5, 9)
\frac{1}{9}(2, 2, 3, 3, 4, 4, 4, 5, 4, 5, 5, 2, 4, 4, 3, 5, 5, 6, 6, 6, 3, 6, 3, 3, 3, 4, 5, 3, 3, 2, 6, 6, 3, 6, 3, 3)
\frac{1}{15}(3, 4, 5, 5, 6, 7, 10, 10, 7, 8, 8, 5, 4, 7, 9, 5, 9, 10, 11, 6, 6, 10, 5, 6, 5, 5, 5, 4, 5, 5, 9, 6, 4, 9, 5, 10)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 5, 6, 6, 9, 5, 5, 10, 9, 9, 10, 10, 10, 5, 8, 3, 9, 9, 4, 11, 5, 5, 4, 6, 6, 7, 10, 5, 5)
\frac{1}{21} (4, 6, 6, 8, 10, 11, 11, 13, 8, 10, 6, 14, 7, 15, 9, 12, 14, 6, 13, 7, 7, 14, 14, 7, 5, 7, 14, 7, 9, 7, 7, 13, 7, 8, 14, 6)
\frac{1}{27}(6, 6, 9, 9, 13, 15, 18, 18, 12, 7, 15, 7, 9, 16, 16, 9, 11, 19, 19, 12, 12, 18, 10, 10, 9, 9, 8, 8, 9, 9, 16, 9, 9, 7, 11, 18)
\frac{1}{24}(5, 6, 8, 8, 8, 12, 12, 16, 11, 7, 9, 13, 7, 17, 15, 6, 10, 10, 16, 6, 10, 16, 16, 10, 12, 8, 16, 16, 8, 8, 6, 16, 8, 10, 8, 16)
\frac{1}{15} (3, 4, 4, 5, 6, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 8, 5, 10, 10, 6, 5, 9, 6, 4, 4, 5, 5, 5, 5, 4, 10, 10, 5, 6, 5, 9)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 13, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 10, 10)
\frac{1}{15} (3, 4, 5, 5, 6, 6, 8, 9, 7, 8, 8, 5, 5, 11, 6, 9, 9, 10, 10, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 10, 6, 5, 6, 5, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 4, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 4, 4, 8, 8, 8, 8, 4, 4, 4, 4, 4, 4, 8, 3, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 4, 4, 8, 8, 7, 8, 4, 4, 7, 4, 4, 7, 8, 3, 5)
\frac{1}{18}(4,4,6,6,8,9,10,10,8,6,10,12,13,6,12,10,10,6,5,6,6,12,6,7,12,6,6,5,4,10,11,6,12,9,11,6)
\frac{1}{12} (3, 3, 4, 4, 4, 5, 6, 8, 6, 4, 5, 5, 8, 9, 5, 7, 7, 7, 4, 3, 7, 8, 8, 4, 5, 4, 8, 4, 4, 4, 4, 4, 4, 7, 3, 4)
\frac{1}{15}(3, 4, 5, 7, 8, 8, 8, 9, 7, 6, 10, 5, 5, 11, 10, 9, 5, 4, 6, 4, 5, 10, 5, 5, 5, 4, 5, 5, 5, 10, 10, 8, 5, 6, 5, 9)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 11, 11, 9, 7, 7, 9, 14, 7, 15, 8, 12, 12, 13, 6, 6, 14, 14, 7, 14, 8, 14, 7, 14, 8, 13, 6, 6, 7, 7, 12)
\frac{1}{15}(3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 6, 4, 5, 5, 10, 8, 5, 5, 10, 10, 5, 9, 11, 4, 6, 5, 10, 5, 5, 4, 7, 9, 6, 10, 5, 5)
\frac{1}{21} (4, 6, 6, 8, 11, 12, 13, 13, 8, 10, 6, 15, 14, 9, 9, 12, 14, 7, 6, 7, 13, 14, 5, 14, 7, 7, 9, 14, 7, 7, 13, 12, 6, 7, 7, 14)
\frac{1}{15}(3, 4, 4, 5, 6, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 6, 5, 10, 10, 10, 5, 9, 6, 4, 4, 5, 9, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{19}(4,4,6,6,7,9,11,12,8,6,6,7,13,13,8,8,10,11,5,7,12,12,13,13,11,6,13,13,7,6,6,6,7,12,7,5)
\frac{1}{15} (3, 4, 5, 8, 9, 9, 9, 9, 7, 6, 11, 6, 6, 6, 10, 9, 4, 5, 5, 5, 5, 5, 4, 6, 6, 4, 5, 5, 5, 9, 10, 10, 5, 10, 5, 5)
\frac{1}{15}(3,4,5,6,6,7,9,9,7,6,9,9,4,6,10,9,6,10,11,5,5,5,5,10,10,4,10,5,5,5,5,5,5,10,6,6)
\frac{1}{18}(3, 5, 6, 6, 8, 8, 10, 11, 8, 7, 7, 5, 11, 7, 12, 11, 11, 13, 11, 5, 6, 6, 6, 12, 6, 5, 12, 6, 12, 5, 6, 12, 7, 6, 7, 7)
\frac{1}{14} (2, 4, 5, 5, 7, 7, 8, 8, 6, 5, 7, 5, 9, 6, 10, 9, 9, 5, 5, 4, 4, 10, 8, 4, 5, 5, 4, 8, 5, 5, 4, 9, 5, 5, 9, 4)
\frac{1}{15}(3, 4, 5, 5, 6, 6, 8, 8, 7, 6, 6, 5, 5, 5, 11, 9, 9, 10, 10, 4, 4, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 6)
\frac{1}{15} (3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 5, 6, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 6, 4, 4, 5, 5, 5, 5, 4, 10, 6, 5, 8, 5, 9)
\frac{1}{15}(3, 4, 5, 6, 6, 6, 7, 9, 7, 6, 9, 9, 9, 4, 10, 9, 6, 6, 10, 11, 5, 5, 5, 5, 10, 4, 10, 10, 5, 5, 10, 5, 5, 5, 5, 6)
\frac{1}{27}(6, 6, 9, 9, 13, 15, 18, 18, 12, 7, 15, 7, 9, 16, 16, 9, 11, 19, 19, 12, 12, 18, 10, 10, 9, 9, 8, 8, 9, 9, 16, 9, 9, 7, 17, 18)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 7, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 4, 8, 4, 4, 8, 4, 8, 3, 4, 4, 4, 8, 7, 4, 4)
\frac{1}{18}(4,4,6,6,8,9,10,10,8,6,10,12,13,6,12,10,10,6,5,6,6,12,6,7,12,6,10,5,4,6,11,6,12,9,11,6)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 5, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 8, 4, 4, 7, 8, 4, 4, 7, 8, 8, 3, 4, 7, 4, 3)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 14, 10, 9, 18, 18, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{18}(3, 5, 6, 6, 6, 8, 10, 11, 8, 7, 9, 9, 5, 7, 12, 11, 7, 11, 13, 5, 6, 6, 6, 12, 12, 5, 12, 6, 6, 11, 6, 6, 5, 12, 7, 7)
\frac{1}{22}(4,6,7,8,12,13,14,14,10,11,8,16,9,10,14,7,14,6,7,8,8,9,7,14,15,7,8,9,6,6,7,8,14,15,7,8)
\frac{1}{15}(3, 4, 5, 5, 7, 8, 10, 10, 7, 8, 8, 4, 5, 7, 9, 5, 7, 11, 10, 6, 6, 10, 6, 5, 5, 5, 4, 5, 5, 5, 9, 5, 5, 6, 4, 10)
```

```
\frac{1}{12}(2, 3, 4, 4, 6, 6, 6, 6, 6, 6, 6, 6, 4, 8, 8, 8, 7, 7, 3, 3, 5, 5, 8, 4, 4, 4, 4, 4, 4, 4, 4, 6, 4, 4, 8, 8, 8)
\frac{1}{15} (3, 4, 5, 5, 6, 6, 8, 9, 7, 6, 8, 5, 5, 11, 6, 9, 9, 10, 10, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 10, 6, 5, 6, 5, 5)
\frac{1}{15} (3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 8, 5, 10, 10, 6, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{18}(3, 5, 6, 6, 6, 10, 10, 11, 8, 7, 7, 9, 7, 13, 12, 7, 11, 11, 5, 5, 6, 12, 6, 12, 6, 11, 6, 12, 6, 5, 6, 12, 5, 6, 7, 11)
\frac{1}{24} (5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 12, 10, 16, 16, 6, 16, 8, 8, 6, 8, 8, 16, 16, 14, 14)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 4, 5, 8, 6, 5, 8, 8, 9, 4, 7, 4, 4, 3, 4, 3, 5, 4, 3, 8, 7, 4, 3, 4, 7)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 5, 7, 9, 4, 5, 7, 7, 7, 3, 4, 5, 8, 4, 4, 5, 8, 4, 4, 7, 8, 8, 3, 4, 7, 4, 3)
\frac{1}{16} (4, 4, 5, 6, 8, 9, 9, 10, 8, 9, 10, 4, 5, 11, 6, 9, 10, 12, 5, 5, 10, 5, 5, 6, 4, 5, 10, 5, 5, 10, 9, 7, 10, 10, 5, 5)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 15, 18, 12, 9, 9, 11, 19, 19, 12, 15, 15, 15, 7, 9, 16, 18, 18, 10, 9, 18, 18, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{18}(3, 5, 6, 6, 8, 8, 10, 11, 8, 7, 9, 5, 11, 7, 12, 11, 11, 13, 7, 5, 6, 6, 12, 6, 12, 5, 6, 12, 6, 5, 6, 12, 7, 6, 7, 7)
\frac{1}{15}(3,4,5,5,6,6,8,8,7,6,8,5,5,5,11,9,9,10,10,4,4,10,5,5,5,5,5,5,5,5,5,10,10,6,10,6,8)
\frac{1}{12}(3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 5, 4, 8, 6, 7, 4, 9, 8, 4, 7, 4, 3, 4, 4, 7, 4, 3, 3, 7, 4, 8, 7, 5, 4)
\frac{1}{15} (3, 4, 4, 5, 6, 8, 8, 9, 5, 7, 8, 5, 5, 11, 10, 6, 5, 10, 10, 10, 5, 9, 6, 4, 4, 5, 5, 5, 5, 4, 10, 6, 5, 6, 5, 9)
\frac{1}{24}(5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 11, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 10, 10)
\frac{1}{15}(3, 4, 5, 5, 8, 8, 8, 9, 7, 8, 8, 5, 5, 11, 6, 9, 9, 10, 10, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 10, 6, 5, 6, 5, 5)
\frac{1}{27} (6, 6, 9, 9, 9, 13, 15, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 10, 10, 9, 18, 18, 16, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 11, 11, 9, 7, 7, 11, 14, 7, 15, 8, 8, 12, 13, 6, 6, 14, 14, 7, 14, 8, 14, 7, 14, 8, 13, 6, 6, 7, 7, 12)
\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,6,5,4,12,10,11,6,6,12,9,7,11,12,6,6)
\frac{1}{16}(4,4,5,6,8,9,9,10,8,9,10,4,5,11,6,9,10,12,5,5,10,5,5,6,4,5,10,5,5,10,7,7,10,10,5,5)
\frac{1}{15}(3, 4, 5, 8, 8, 8, 9, 9, 7, 6, 5, 5, 11, 10, 10, 9, 4, 6, 4, 5, 5, 5, 5, 5, 4, 10, 10, 6, 5, 5, 6, 5, 5, 9, 5, 10)
\frac{1}{15} (3, 4, 4, 5, 8, 8, 8, 9, 5, 7, 6, 5, 5, 11, 10, 6, 5, 10, 10, 6, 5, 9, 4, 6, 4, 5, 5, 5, 5, 4, 10, 8, 5, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 6, 5, 4, 5, 10, 6, 5, 10, 5, 10, 5, 9, 6, 11, 4, 5, 9, 10, 9, 4, 5, 10, 5, 9, 6, 5)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 8, 8, 9, 4, 6, 7, 4, 4, 5, 8, 7, 4, 4, 3, 4, 3, 5, 4, 3, 8, 7, 4, 3, 4, 7)
\frac{1}{10}(4,5,7,7,10,11,12,12,9,7,11,14,7,8,12,12,12,5,6,7,7,6,7,8,5,5,13,6,7,5,7,6,12,13,7,6)
\frac{1}{18}(3,5,5,7,7,7,8,12,6,8,6,6,10,5,11,6,6,12,6,7,11,12,6,12,13,7,6,12,11,5,6,11,5,5,7,6)
\frac{1}{22}(4,6,7,8,12,13,14,14,10,9,8,16,9,10,14,7,14,6,7,8,8,9,7,14,15,7,8,13,6,6,7,8,14,15,7,8)
\frac{1}{15} (3, 4, 5, 8, 8, 8, 9, 9, 7, 6, 5, 5, 11, 10, 10, 9, 4, 6, 4, 5, 5, 5, 5, 5, 4, 10, 10, 8, 5, 5, 6, 5, 5, 9, 5, 10)
\frac{1}{22}(4,5,7,8,11,13,13,15,9,7,8,15,9,15,15,8,13,6,14,8,10,15,14,6,8,8,7,9,7,7,8,14,6,6,14,14)
\frac{1}{15} (3, 4, 4, 5, 6, 7, 8, 9, 5, 7, 8, 5, 4, 5, 10, 8, 5, 10, 9, 10, 5, 9, 6, 11, 4, 5, 5, 10, 5, 4, 5, 10, 5, 7, 6, 5)
\frac{1}{15} (3, 4, 4, 5, 7, 8, 8, 9, 5, 7, 6, 10, 5, 11, 10, 6, 5, 5, 10, 10, 5, 9, 5, 4, 4, 5, 6, 5, 5, 4, 5, 5, 10, 8, 5, 9)
\frac{1}{27}(6, 6, 9, 9, 9, 13, 13, 18, 12, 7, 11, 15, 7, 9, 16, 9, 9, 11, 19, 19, 12, 18, 18, 14, 10, 9, 18, 18, 10, 9, 8, 8, 9, 16, 9, 7)
\frac{1}{18}(3,5,5,7,7,8,9,12,8,8,6,10,5,6,11,6,6,6,7,12,7,12,12,13,6,7,12,11,6,5,5,6,7,11,6,5)
\frac{1}{18}(4,4,6,6,9,10,10,12,8,6,10,13,6,6,12,10,10,5,6,6,8,12,7,12,12,6,5,6,6,6,11,11,11,12,6,6)
\frac{1}{18}(4,4,6,6,8,8,9,10,8,6,10,12,12,13,6,10,10,6,6,5,6,12,6,6,7,12,6,10,5,4,12,11,6,11,6,9)
\frac{1}{27}(6,6,9,9,9,13,13,18,12,7,11,15,7,9,16,9,9,11,19,19,12,18,18,14,16,9,18,10,10,9,8,8,9,16,9,7)
\frac{1}{24}(5, 6, 10, 10, 12, 14, 14, 16, 11, 7, 15, 17, 9, 9, 13, 16, 16, 6, 8, 8, 10, 8, 10, 16, 16, 6, 16, 8, 8, 6, 8, 14, 16, 16, 14, 10)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 7, 8, 6, 5, 5, 7, 9, 4, 7, 7, 7, 7, 3, 4, 7, 8, 4, 4, 5, 8, 4, 4, 5, 8, 8, 3, 4, 5, 4, 3)
\frac{1}{12}(3,3,4,4,4,6,7,8,6,5,5,7,9,4,7,7,7,7,3,4,7,8,4,4,7,8,4,4,7,8,8,3,4,5,4,3)
\frac{1}{12}(3,3,4,4,4,4,6,7,6,3,5,5,7,9,4,5,7,7,5,3,4,4,4,8,8,7,8,4,4,3,4,4,5,8,3,5)
\frac{1}{17}(2, 5, 5, 5, 6, 7, 7, 11, 5, 7, 7, 6, 5, 9, 11, 8, 10, 11, 10, 4, 6, 10, 11, 6, 12, 6, 11, 12, 6, 6, 11, 11, 5, 6, 6, 6)
\frac{1}{11} (2, 3, 4, 4, 5, 6, 6, 7, 5, 4, 4, 7, 4, 8, 7, 5, 7, 6, 3, 3, 4, 6, 5, 8, 4, 3, 3, 4, 4, 3, 3, 7, 6, 4, 7, 7)
```

```
\frac{1}{15} (3, 4, 5, 5, 7, 8, 10, 10, 7, 8, 8, 4, 5, 9, 9, 5, 7, 11, 10, 6, 6, 10, 6, 5, 5, 5, 4, 5, 5, 5, 9, 5, 5, 4, 6, 10)
\frac{1}{12} (3, 3, 3, 4, 6, 7, 7, 7, 4, 5, 4, 8, 4, 8, 8, 6, 5, 9, 8, 4, 4, 7, 3, 4, 4, 4, 4, 3, 4, 4, 7, 5, 5, 4, 4, 8)
\frac{1}{10} (4, 4, 6, 8, 8, 9, 10, 10, 8, 10, 6, 12, 13, 6, 12, 10, 12, 10, 5, 6, 6, 12, 10, 5, 4, 6, 6, 7, 12, 6, 5, 6, 12, 7, 11, 6)
\frac{1}{12} (2, 3, 4, 4, 4, 5, 6, 8, 5, 4, 4, 6, 3, 8, 8, 7, 7, 5, 8, 3, 5, 8, 8, 5, 4, 4, 8, 5, 8, 4, 3, 8, 4, 5, 7, 4)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,8,12,7,11,6,6,5,5,6,6,10,5,7,5,7,12)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 12, 9, 14, 7, 7)
\frac{1}{18}(3,5,5,7,9,9,10,11,8,8,6,6,6,13,12,10,6,6,6,5,6,12,4,8,5,6,12,12,7,6,12,7,6,11,6,11)
\frac{1}{15}(3, 4, 5, 5, 6, 8, 8, 9, 7, 6, 6, 9, 5, 11, 6, 9, 9, 8, 4, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 4, 10, 5, 6, 9, 5)
\frac{1}{21} (4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 9, 4, 4, 5, 9, 10, 4, 5, 5, 5, 10, 6, 5, 9)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,10,13,13,8,6,10,6,5,5,8,12,12,9,11,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{12} (3, 3, 3, 3, 5, 6, 7, 7, 4, 4, 6, 4, 5, 4, 8, 6, 6, 4, 3, 4, 4, 6, 8, 9, 8, 4, 8, 3, 4, 4, 7, 8, 4, 7, 5, 4)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,10,5,5,8,8,10,8,13,13,8,8,12,7,11,6,6,5,5,6,6,10,5,7,5,11,12)
\frac{1}{22} (4, 6, 8, 12, 12, 13, 13, 14, 10, 8, 8, 16, 9, 9, 14, 14, 8, 6, 7, 7, 8, 8, 8, 7, 7, 6, 8, 15, 15, 6, 7, 7, 14, 14, 13, 13)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{15}(3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 5, 4, 4, 9, 5, 10, 6, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,10,12,15,15,9,14,14,7,13,7,14,7,5,7,7,13,7,12,6,6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 5, 9, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4, 4, 4, 6, 8, 9, 10, 10, 6, 6, 10, 6, 7, 6, 12, 8, 10, 6, 5, 6, 6, 10, 12, 13, 12, 6, 6, 5, 6, 4, 11, 12, 6, 11, 9, 6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{18} (4, 4, 6, 6, 9, 9, 12, 12, 8, 6, 8, 13, 13, 8, 8, 6, 10, 5, 5, 8, 8, 12, 7, 11, 6, 6, 5, 5, 6, 6, 10, 5, 7, 5, 11, 12)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 8, 12, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 9, 7, 12, 6, 6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 8, 9, 14, 7, 7)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 9, 10, 9, 4, 4, 5, 5, 10, 6, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,6,10,6,11,6,12,8,10,6,5,6,6,10,12,13,12,6,6,5,6,4,11,12,6,11,9,6)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 5, 4, 10, 4, 9, 5, 5, 9, 10, 10, 8, 5, 6, 5, 5)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 9, 4, 6, 4, 5, 9, 10, 6, 4, 5, 5, 5, 5, 10, 10, 6, 5, 10, 5, 9)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,11,11,6,6,12,8,8,10,15,15,9,14,14,7,13,7,14,7,9,7,7,5,7,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,12,6,12,9,9,10,6,12,6,10,13,7,5,5,13,7,5,5,8,10,8,6,12,6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 6, 4, 9, 5, 5, 9, 10, 10, 8, 5, 6, 5, 9)
\frac{1}{12} (3, 3, 4, 4, 4, 6, 6, 7, 6, 5, 7, 7, 3, 5, 4, 7, 5, 7, 9, 3, 4, 4, 4, 8, 8, 3, 8, 4, 4, 3, 4, 4, 7, 8, 5, 5)
\frac{1}{15} (3, 4, 5, 6, 6, 8, 9, 9, 5, 8, 5, 9, 5, 6, 10, 9, 10, 6, 10, 5, 5, 5, 7, 5, 4, 4, 6, 10, 9, 5, 6, 3, 11, 9, 5, 8)
\frac{1}{21}(4, 5, 7, 9, 11, 11, 12, 12, 9, 9, 13, 15, 15, 8, 8, 12, 14, 6, 6, 7, 7, 12, 6, 6, 5, 9, 8, 14, 7, 7, 12, 11, 7, 7, 7, 14)
\frac{1}{15}(3, 3, 5, 5, 5, 5, 7, 7, 6, 4, 6, 6, 8, 4, 10, 8, 4, 8, 6, 10, 4, 10, 10, 10, 10, 8, 10, 10, 10, 10, 6, 10, 10, 4, 4, 10, 6)
\frac{1}{22}(4, 5, 6, 8, 12, 12, 13, 14, 9, 10, 12, 8, 16, 9, 14, 7, 7, 15, 7, 14, 9, 14, 8, 6, 7, 8, 8, 8, 7, 6, 8, 15, 6, 7, 14, 9)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,10,6,6,10,13,13,8,12,12,7,7,6,12,7,11,6,5,11,6,10,5,5)
\frac{1}{18}(3, 5, 5, 9, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 4, 8, 12, 5, 6, 6, 6, 6, 6, 6, 6, 12, 12, 9, 6, 12, 7, 6, 7, 6, 11)
\frac{1}{18}(3,5,5,7,10,11,11,11,8,8,6,13,8,8,12,10,6,5,6,6,12,12,5,6,10,6,7,12,12,6,11,5,11,12,6,6)
\frac{1}{15}(3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 9, 3, 8, 8, 10)
```

```
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 7, 6, 8, 9, 5, 11, 6, 9, 9, 6, 4, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 4, 10, 5, 6, 9, 5)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,8,12,5,5,6,6,7,7,6,6,10,5,7,5,7,12)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,10,6,10,10,13,13,8,12,12,7,11,6,12,7,7,6,5,5,6,10,5,5)
\frac{1}{18}(4,4,4,8,8,9,10,12,6,8,6,6,5,12,10,8,12,12,5,6,8,6,12,13,12,8,12,7,6,12,7,6,4,11,5,6)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 12, 12, 12, 15, 15, 9, 14, 14, 5, 7, 7, 14, 9, 7, 7, 13, 7, 7, 12, 6, 6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,10,10,8,13,13,8,12,12,7,11,6,12,7,11,6,5,5,6,10,11,5)
\frac{1}{18}(3, 5, 5, 7, 9, 9, 11, 13, 8, 8, 6, 6, 12, 12, 12, 10, 12, 4, 6, 6, 10, 12, 6, 4, 6, 8, 12, 8, 6, 6, 10, 6, 6, 8, 4, 12)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 9, 4, 4, 9, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{18} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 8, 8, 10, 12, 6, 6)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 10, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,6,5,4,8,6,11,6,6,12,9,7,11,12,6,6)
\frac{1}{18}(3, 5, 5, 7, 7, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 6, 6, 6, 5, 6, 12, 12, 4, 5, 6, 12, 12, 11, 6, 12, 11, 6, 9, 6, 11)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,10,12,6,6)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 9, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{22}(4, 6, 8, 12, 12, 13, 13, 14, 10, 8, 8, 16, 9, 9, 14, 14, 8, 6, 7, 7, 8, 8, 8, 7, 7, 6, 8, 15, 15, 6, 7, 7, 14, 14, 9, 9)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 8, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 10, 9, 14, 7, 7)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,11,11,8,10,6,6,6,12)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 7, 4, 4, 3, 8, 6, 7, 8, 8, 3, 4, 7, 4, 4, 9, 4, 3, 7, 8, 3, 8, 5, 4, 5, 4, 7)
\frac{1}{18}(4,4,6,8,8,9,10,10,8,10,6,12,13,6,12,10,12,10,5,6,6,12,10,5,4,6,6,7,12,6,5,6,12,9,11,6)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,8,8,6,7,6,12,8,10,12,13,12,6,10,6,5,6,6,6,5,6,4,11,12,6,11,7,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,10,6,12,9,9,12,4,12,6,10,7,13,5,5,7,13,5,5,8,12,6,8,10,6)
\frac{1}{10}(4, 4, 7, 7, 8, 9, 13, 13, 8, 7, 11, 6, 13, 13, 13, 11, 7, 12, 5, 9, 9, 12, 13, 6, 6, 6, 5, 12, 6, 6, 7, 7, 7, 6, 6, 12)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,11,8,10,6,6,6,12)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,6,8,6,7,6,12,8,10,6,5,6,6,10,12,13,12,6,6,5,6,4,11,12,6,11,9,6)
\frac{1}{12} (3, 3, 3, 3, 5, 6, 7, 7, 4, 4, 6, 4, 7, 4, 8, 6, 6, 4, 3, 4, 4, 6, 8, 9, 8, 4, 8, 3, 4, 4, 7, 8, 4, 7, 5, 4)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{10} (4, 4, 6, 6, 9, 9, 12, 12, 8, 8, 10, 13, 13, 8, 8, 10, 10, 5, 5, 8, 8, 12, 5, 5, 6, 6, 7, 11, 6, 6, 10, 5, 7, 5, 11, 12)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{10}(4,4,6,6,7,9,11,12,8,6,6,7,13,13,8,8,8,11,5,7,12,12,13,13,7,6,13,13,7,6,6,6,7,12,7,5)
\frac{1}{12} (3, 3, 3, 5, 5, 5, 6, 7, 6, 6, 4, 4, 4, 3, 4, 6, 4, 4, 8, 9, 4, 8, 8, 4, 3, 4, 8, 4, 7, 8, 4, 7, 8, 7, 4, 7)
\frac{1}{15}(3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 7, 3, 6, 6, 10)
\frac{1}{18}(4,4,6,9,10,10,10,10,8,8,13,6,6,12,12,10,5,6,6,6,5,4,8,6,6,7,7,11,11,12,6,6,6,6,12)
\frac{1}{15}(3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 7, 4, 4, 10, 5, 7, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 3, 8)
\frac{1}{18}(3, 5, 5, 7, 9, 9, 10, 11, 8, 8, 6, 6, 12, 13, 12, 10, 6, 6, 12, 5, 6, 12, 4, 6, 5, 6, 12, 6, 11, 6, 6, 7, 6, 7, 12, 11)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 9, 4, 4, 9, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{22}(4, 5, 6, 8, 12, 12, 13, 14, 9, 10, 12, 8, 16, 9, 14, 7, 7, 15, 7, 14, 9, 14, 8, 6, 7, 8, 8, 8, 7, 6, 8, 15, 6, 7, 14, 13)
\frac{1}{15}(3,3,5,5,5,7,7,10,6,4,6,8,4,10,9,8,8,6,10,4,9,10,10,6,10,5,10,10,4,5,4,10,5,6,9,5)
\frac{1}{18}(4,4,4,8,8,9,10,12,6,8,6,6,5,12,10,8,12,12,5,6,8,6,12,13,12,8,12,7,6,12,9,6,4,11,5,6)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,10,5,5,8,8,10,8,13,13,8,8,12,7,11,6,6,5,5,6,6,10,5,11,5,7,12)
```

```
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 10, 4, 5, 9, 4, 4, 5, 5, 5, 10, 8, 5, 9)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 12, 8, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 12, 9, 14, 7, 7)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 8, 13, 13, 8, 6, 10, 10, 5, 5, 10, 12, 12, 7, 11, 6, 12, 7, 11, 6, 5, 11, 6, 10, 5, 5)
\frac{1}{15}(3,4,4,8,8,8,9,9,7,7,5,5,11,10,10,8,4,6,4,5,9,10,6,4,5,5,10,8,5,5,10,5,5,9,5,10)
\frac{1}{18} (3, 5, 5, 7, 9, 9, 11, 13, 8, 8, 6, 6, 8, 12, 12, 10, 12, 4, 6, 6, 10, 12, 6, 4, 6, 8, 12, 8, 6, 6, 10, 6, 6, 10, 4, 12)
\frac{1}{18} (4, 4, 6, 9, 10, 10, 10, 10, 8, 8, 13, 6, 6, 12, 12, 10, 5, 6, 6, 6, 6, 5, 4, 8, 6, 6, 9, 7, 11, 11, 12, 6, 6, 6, 6, 12)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,10,10,6,6,12)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,8,12,5,5,6,6,7,7,6,6,10,5,7,5,11,12)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,8,10,10,13,13,8,10,10,10,5,5,8,12,12,5,5,6,12,7,7,6,7,7,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,10,12,6,6)
\frac{1}{21}(3, 5, 7, 9, 11, 11, 14, 14, 8, 8, 6, 8, 14, 11, 13, 12, 14, 6, 6, 13, 13, 14, 14, 6, 7, 7, 14, 8, 7, 7, 8, 7, 7, 7, 74)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,10,6,12,9,9,12,4,12,6,10,7,13,5,5,7,13,5,5,8,12,6,8,10,6)
\frac{1}{22}(4, 5, 6, 8, 12, 12, 13, 14, 9, 10, 8, 8, 16, 9, 14, 7, 7, 15, 7, 14, 13, 14, 8, 6, 7, 8, 8, 8, 7, 6, 8, 15, 6, 7, 14, 9)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{15} (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 9, 4, 4, 10, 9, 5, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 9, 8)
\frac{1}{18}(3, 3, 5, 7, 7, 8, 9, 12, 6, 6, 6, 6, 7, 12, 9, 8, 6, 10, 5, 6, 11, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 7, 11, 6, 5)
\frac{1}{21}(4, 5, 7, 9, 11, 11, 12, 12, 9, 9, 13, 15, 15, 8, 8, 12, 14, 6, 6, 7, 7, 8, 6, 6, 5, 9, 8, 14, 7, 7, 12, 11, 13, 7, 7, 14)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 7, 6, 6, 9, 5, 11, 6, 9, 9, 6, 4, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 4, 10, 5, 6, 9, 5)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,8,8,6,7,6,12,8,10,12,13,12,6,10,6,5,6,6,6,5,6,4,11,12,6,11,9,6)
\frac{1}{18}(3, 5, 5, 9, 9, 9, 10, 11, 8, 8, 6, 6, 12, 13, 12, 10, 4, 12, 6, 5, 6, 6, 6, 12, 5, 6, 12, 6, 9, 6, 6, 11, 6, 7, 12, 11)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,11,7,8,10,6,6,6,12)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 4, 3, 8, 6, 7, 8, 8, 3, 4, 7, 4, 4, 9, 4, 3, 5, 8, 3, 8, 5, 4, 5, 4, 7)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 11, 5, 7, 5, 5, 9, 10, 10, 8, 6, 4, 4, 5, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 7, 3, 10)
\frac{1}{21}(3, 5, 7, 9, 11, 11, 14, 14, 8, 8, 6, 8, 14, 13, 13, 12, 14, 6, 6, 13, 13, 14, 14, 6, 7, 7, 14, 8, 7, 7, 12, 7, 7, 7, 14)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,10,6,12,9,9,12,4,12,6,10,7,13,5,5,11,13,5,5,8,12,6,8,10,6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{18}(3,5,5,7,9,9,10,11,8,8,6,6,6,13,12,10,6,6,6,5,6,12,4,8,5,6,12,12,7,6,12,7,6,7,6,11)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{12}(3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 9, 4, 4, 6, 7, 4, 5, 8, 8, 7, 4, 3, 4, 4, 3, 4, 3, 7, 7, 4, 4, 5, 7, 8)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,8,12,15,15,9,14,14,7,9,7,14,7,13,7,7,5,7,12,6,6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,10,12,15,15,9,14,14,7,9,7,14,7,5,7,7,9,7,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,11,6,8,10,6,12,6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,11,11,6,6,12,8,8,12,15,15,9,14,14,7,13,7,14,7,9,7,7,5,7,12,6,6)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 7, 3, 8, 6, 10)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,4,8,13,13,8,8,10,10,5,5,8,8,12,9,9,6,6,5,5,6,6,10,5,7,5,11,12)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 7, 4, 4, 3, 8, 6, 7, 8, 8, 3, 4, 7, 4, 4, 9, 4, 3, 5, 8, 3, 8, 5, 4, 5, 4, 7)
\frac{1}{15}(3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 5, 4, 4, 10, 5, 9, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 5, 8)
```

```
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 9, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{21}(3, 5, 7, 9, 11, 11, 14, 14, 8, 8, 6, 8, 14, 11, 13, 12, 14, 6, 6, 13, 13, 14, 14, 6, 7, 7, 14, 8, 7, 7, 12, 7, 7, 7, 14)
\frac{1}{15}(3, 3, 5, 5, 5, 5, 7, 7, 6, 4, 6, 6, 8, 4, 10, 8, 4, 8, 6, 10, 4, 10, 10, 10, 10, 8, 10, 10, 10, 10, 8, 10, 10, 4, 4, 10, 6)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(3,5,5,5,9,9,10,11,6,8,8,6,6,13,12,6,8,12,12,11,6,10,6,6,5,6,4,12,5,6,12,9,6,7,6,11)
\frac{1}{21}(4,5,6,6,9,11,11,12,7,10,10,5,7,7,14,7,9,12,14,14,7,12,15,7,7,6,15,5,13,6,10,12,9,14,7,7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 10, 9, 14, 7, 7)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 11, 5, 5, 13, 7, 5, 5, 6, 8, 10, 6, 12, 6)
\frac{1}{21}(3, 5, 7, 9, 11, 11, 14, 14, 8, 8, 6, 8, 14, 11, 11, 12, 14, 6, 6, 13, 13, 14, 14, 6, 7, 7, 14, 8, 7, 7, 12, 7, 7, 7, 14)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 5, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 7, 4, 3, 4, 7, 4, 7, 8, 5)
\frac{1}{18}(3, 5, 5, 7, 7, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 6, 6, 6, 5, 6, 12, 12, 4, 5, 6, 12, 12, 11, 6, 12, 11, 6, 7, 6, 11)
\frac{1}{12} (3, 3, 3, 5, 5, 5, 6, 7, 6, 6, 4, 4, 4, 3, 4, 6, 4, 8, 8, 3, 4, 8, 4, 8, 9, 4, 4, 8, 7, 4, 4, 7, 8, 7, 4, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 8, 8, 7, 5, 8, 3, 5, 5, 8, 5, 4, 4, 4, 3, 8, 4, 4, 5, 5, 5, 4, 4, 8)
\frac{1}{10} (4, 4, 4, 6, 8, 9, 10, 10, 6, 6, 8, 6, 7, 6, 12, 8, 10, 6, 5, 6, 6, 10, 12, 13, 12, 6, 6, 5, 6, 4, 11, 12, 6, 11, 7, 6)
\frac{1}{15}(3, 4, 4, 6, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 6, 4, 4, 5, 5, 10, 4, 4, 5, 9, 10, 6, 5, 5, 6, 5, 5, 9, 5, 10)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 12, 12, 12, 15, 15, 9, 14, 14, 5, 7, 7, 14, 9, 7, 7, 9, 7, 7, 12, 6, 6)
\frac{1}{18}(3, 5, 5, 7, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 6, 6, 6, 5, 6, 12, 4, 8, 5, 6, 12, 12, 7, 6, 12, 9, 6, 7, 6, 11)
\frac{1}{15} (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 5, 4, 4, 10, 5, 9, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 9, 6)
\frac{1}{15} (3, 4, 5, 6, 6, 6, 9, 9, 5, 6, 5, 5, 9, 6, 10, 9, 10, 10, 6, 5, 5, 5, 5, 7, 4, 4, 10, 6, 9, 5, 6, 9, 5, 3, 11, 8)
\frac{1}{21} (4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 8, 8, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{15} (3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 5, 9, 10, 6, 5, 5, 6, 5, 5, 9, 9, 10)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,10,10,6,6,12)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 5, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 7, 4, 3, 4, 7, 4, 5, 8, 5)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,6,8,6,11,6,12,8,10,6,5,6,6,10,12,13,12,6,6,5,6,4,11,12,6,11,9,6)
\frac{1}{15} (3, 4, 5, 6, 8, 8, 9, 9, 5, 6, 9, 5, 5, 6, 10, 9, 6, 10, 10, 5, 5, 7, 5, 5, 4, 4, 6, 6, 3, 11, 10, 9, 5, 9, 5, 8)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,8,12,15,15,9,14,14,7,9,7,14,7,9,7,7,5,7,12,6,6)
\frac{1}{21} (4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 10, 12, 15, 15, 9, 14, 14, 5, 7, 7, 14, 9, 7, 7, 9, 7, 7, 12, 6, 6)
\frac{1}{2}, (4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,11,11,8,8,6,6,6,12)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 9, 4, 4, 6, 7, 4, 5, 8, 8, 7, 4, 3, 4, 4, 3, 4, 3, 5, 7, 4, 4, 5, 5, 8)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,10,8,10,6,12,6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,10,5,5,8,12,12,7,11,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,10,12,15,15,9,14,14,7,13,7,14,7,5,7,7,9,7,12,6,6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,10,10,10,5,5,10,12,12,7,7,6,12,7,7,6,5,11,6,10,5,5)
\frac{1}{10} (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 7, 6, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 7, 13, 5, 11, 7, 8, 10, 6, 6, 6, 12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,11,6,8,8,6,12,6)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,11,13,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,13,13,14)
```

```
\frac{1}{22}(4, 4, 8, 8, 10, 10, 15, 15, 8, 8, 12, 6, 14, 11, 13, 12, 8, 14, 6, 15, 15, 14, 14, 6, 7, 7, 6, 14, 7, 7, 8, 7, 7, 9, 9, 14)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 10, 9, 14, 7, 7)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 7, 4, 4, 9, 10, 7, 4, 4, 5, 5, 10, 6, 5, 5, 3, 10, 8, 5, 9)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,10,8,10,6,12,6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,6,10,8,13,13,8,12,12,7,11,6,12,7,7,6,5,5,6,10,5,5)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 7, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 5, 4, 3, 4, 7, 4, 7, 8, 5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,10,10,6,6,12)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 12, 10, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 13, 7, 12, 6, 6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,7,5,5,10,8,10,6,12,6)
\frac{1}{21}(3, 5, 7, 9, 11, 11, 14, 14, 8, 8, 6, 8, 14, 13, 13, 12, 14, 6, 6, 13, 13, 14, 14, 6, 7, 7, 14, 8, 7, 7, 8, 7, 7, 7, 7, 14)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 11, 6, 8, 8, 6, 12, 6)
\frac{1}{18}(3, 3, 5, 7, 7, 8, 9, 12, 6, 6, 6, 6, 7, 12, 11, 8, 6, 10, 5, 6, 11, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 7, 11, 6, 5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 8, 10, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 13, 7, 12, 6, 6)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 10, 9, 14, 7, 7)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{18}(3, 5, 5, 7, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 6, 6, 6, 5, 6, 12, 4, 8, 5, 6, 12, 12, 7, 6, 12, 9, 6, 11, 6, 11)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,10,8,10,6,12,6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,8,13,13,8,10,10,10,5,5,8,12,12,9,9,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 5, 4, 6, 4, 9, 5, 5, 9, 10, 10, 8, 5, 6, 5, 9)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{18}(4,4,4,8,8,8,9,12,6,8,6,6,12,5,8,8,12,12,6,5,8,6,12,12,13,8,12,6,7,12,6,9,4,11,6,5)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 7, 4, 4, 3, 8, 6, 7, 8, 8, 3, 4, 7, 4, 4, 9, 4, 7, 7, 8, 3, 8, 5, 4, 5, 4, 5)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 10, 4, 5, 5, 5, 5, 10, 10, 6, 5, 6, 5, 9)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 8, 12, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 13, 7, 12, 6, 6)
\frac{1}{22}(4, 5, 6, 8, 12, 12, 13, 14, 9, 10, 8, 8, 16, 9, 14, 7, 7, 15, 7, 14, 13, 14, 8, 6, 7, 8, 8, 8, 7, 6, 8, 15, 6, 7, 14, 13)
\frac{1}{10} (4, 4, 6, 6, 7, 9, 11, 12, 8, 6, 6, 7, 13, 13, 8, 8, 10, 11, 5, 7, 12, 12, 13, 13, 7, 6, 13, 13, 7, 6, 6, 6, 7, 12, 7, 5)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,10,10,10,5,5,10,12,12,7,7,6,12,7,11,6,5,11,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,11,11,8,8,6,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,11,6,8,10,6,12,6)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 5, 7, 5, 7, 3, 4, 4, 8, 4, 8, 5, 4, 8, 4, 3, 4, 8, 3, 4, 5, 5)
\frac{1}{15}(3, 4, 5, 6, 6, 8, 9, 9, 5, 6, 5, 9, 5, 6, 10, 9, 10, 6, 10, 5, 5, 5, 7, 5, 4, 4, 6, 10, 9, 5, 6, 3, 11, 9, 5, 8)
\frac{1}{21}(4, 5, 7, 9, 11, 11, 12, 12, 9, 9, 13, 15, 15, 8, 8, 12, 14, 6, 6, 7, 7, 12, 6, 6, 5, 9, 8, 14, 7, 7, 12, 7, 13, 7, 7, 14)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 8, 10, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 9, 7, 12, 6, 6)
\frac{1}{21}(4,5,6,6,9,11,11,12,7,10,10,5,7,7,14,7,9,8,14,14,7,12,15,7,7,6,15,5,9,6,12,12,9,14,7,7)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,10,10,10,5,5,10,10,10,10,13,13,8,12,12,5,5,6,12,7,7,6,7,7,6,10,5,5)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 9, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 5, 5, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,8,13,13,8,10,6,10,5,5,8,12,12,9,9,6,12,7,11,6,5,5,6,10,5,5)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,11,11,8,10,6,6,6,12)
```

 $\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,11,6,8,8,6,12,6)$  $\frac{1}{12}(2, 3, 4, 4, 4, 6, 6, 6, 5, 6, 6, 6, 4, 8, 8, 7, 7, 7, 3, 3, 5, 4, 4, 4, 6, 2, 8, 8, 4, 4, 8, 4, 4, 4, 4, 8)$  $\frac{1}{15}(3, 4, 4, 4, 8, 8, 9, 9, 5, 7, 7, 5, 11, 10, 10, 6, 8, 10, 10, 5, 5, 8, 4, 4, 5, 9, 10, 4, 5, 9, 6, 5, 5, 9, 5, 10)$  $\frac{1}{21}$  (4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 7, 11, 6, 6, 12, 8, 8, 12, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)  $\frac{1}{21}(4,5,6,6,9,11,11,12,7,10,10,5,7,7,14,7,11,12,14,14,7,12,15,7,7,6,15,5,13,6,12,8,9,14,7,7)$  $\frac{1}{21}$ (4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 12, 9, 14, 7, 7) $\frac{1}{15}(2,4,5,5,5,7,7,10,6,5,5,5,5,9,10,7,7,7,3,3,6,6,6,10,4,5,10,10,10,5,10,10,5,6,5,9)$  $\frac{1}{15}(2,4,5,5,7,7,7,10,6,5,5,5,5,9,10,7,7,3,7,3,6,10,10,6,10,5,10,6,10,5,10,6,5,4,5,9)$  $\frac{1}{18}(4,4,6,8,8,9,10,10,8,10,6,10,5,6,12,10,12,12,13,6,6,6,10,5,4,12,12,11,6,6,5,6,6,7,7,12)$  $\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,6,5,4,8,6,11,6,6,12,7,7,11,12,6,6)$  $\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,11,11,6,6,12,12,8,10,15,15,9,14,14,7,13,7,14,7,9,7,7,5,7,12,6,6)$  $\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,8,12,5,5,6,6,7,11,6,6,10,5,7,5,7,12)$  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,10,10,8,13,13,8,12,12,7,7,6,12,7,11,6,5,5,6,10,5,5)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 10, 8, 10, 6, 12, 6)  $\frac{1}{21}(4,5,6,6,9,9,11,12,7,10,10,5,7,7,14,7,11,8,14,14,7,12,15,7,7,6,15,9,5,6,12,12,9,14,7,7)$  $\frac{1}{10}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 11, 6, 8, 10, 6, 12, 6)  $\frac{1}{10}$  (3, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 10, 12, 12, 11, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 9, 6, 7, 6, 11)  $\frac{1}{15}$  (3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 9, 9, 4, 10, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 6)  $\frac{1}{15}(3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 7, 3, 6, 8, 10)$  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,10,6,12,9,9,12,4,12,6,10,11,13,5,5,7,13,5,5,8,12,6,8,10,6)$  $\frac{1}{18}(4,4,6,8,9,10,10,10,8,8,12,13,6,6,12,10,6,5,6,6,6,6,5,4,8,6,11,6,6,12,7,11,11,12,6,6)$  $\frac{1}{15}(3, 4, 5, 6, 6, 6, 9, 9, 5, 8, 5, 5, 9, 6, 10, 9, 10, 10, 6, 5, 5, 5, 5, 7, 4, 4, 10, 6, 9, 5, 6, 9, 5, 3, 11, 8)$  $\frac{1}{21}(4,5,6,6,9,11,11,12,7,10,10,5,7,7,14,7,11,8,14,14,7,12,15,7,7,6,15,5,9,6,10,8,9,14,7,7)$  $\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 8, 9, 14, 7, 7)$  $\frac{1}{10}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 10, 8, 10, 6, 12, 6)  $\frac{1}{21}$  (4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 12, 9, 14, 7, 7)  $\frac{1}{18}(4,4,4,6,8,9,10,10,6,6,10,6,7,6,12,8,10,6,5,6,6,10,12,13,12,6,6,5,6,4,11,12,6,11,7,6)$  $\frac{1}{21}$  (4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 8, 12, 12, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 5, 7, 7, 13, 7, 12, 6, 6)  $\frac{1}{21}$ (4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 12, 9, 14, 7, 7)  $\frac{1}{21}$ (4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 12, 9, 14, 7, 7) $\frac{1}{21}(3,6,7,7,10,11,14,14,9,6,8,7,14,11,11,13,11,4,5,12,12,14,13,8,7,7,15,6,7,7,9,8,8,7,7,14)$  $\frac{1}{18}(3, 5, 5, 9, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 4, 8, 12, 5, 6, 6, 6, 6, 6, 6, 6, 12, 12, 7, 6, 12, 7, 6, 7, 6, 11)$  $\frac{1}{18}(3, 5, 5, 7, 9, 10, 11, 11, 8, 8, 6, 12, 13, 8, 8, 10, 6, 12, 5, 6, 6, 12, 6, 5, 6, 10, 6, 7, 12, 12, 11, 6, 6, 11, 5, 12)$  $\frac{1}{10}$  (4, 4, 6, 9, 10, 10, 10, 10, 8, 8, 13, 6, 6, 12, 12, 10, 5, 6, 6, 6, 6, 5, 4, 8, 6, 6, 7, 11, 11, 11, 12, 6, 6, 6, 6, 12)  $\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,11,11,6,6,12,12,8,12,15,15,9,14,14,7,13,7,14,7,9,7,7,5,7,12,6,6)$  $\frac{1}{21}$  (4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 12, 9, 14, 7, 7)  $\frac{1}{22}$ (4, 4, 8, 8, 10, 10, 15, 15, 8, 8, 12, 6, 14, 13, 13, 12, 8, 14, 6, 15, 15, 14, 14, 6, 7, 7, 6, 14, 7, 7, 8, 7, 7, 9, 9, 14)  $\frac{1}{21}$  (4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 10, 9, 14, 7, 7)  $\frac{1}{10}$  (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 8, 13, 13, 8, 6, 10, 10, 5, 5, 10, 12, 12, 7, 11, 6, 12, 7, 7, 6, 5, 11, 6, 10, 5, 5)  $\frac{1}{18}$  (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 9, 6, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 7, 13, 5, 11, 7, 8, 8, 6, 6, 6, 12)  $\frac{1}{16}(4,4,4,6,7,7,8,10,6,8,6,5,5,4,10,8,6,11,11,4,6,10,5,5,12,6,5,5,10,4,10,9,5,9,5,6)$ 

```
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 8, 8, 7, 5, 8, 3, 5, 5, 8, 7, 4, 4, 4, 3, 8, 4, 4, 5, 5, 5, 4, 4, 8)
\frac{1}{18} (4, 4, 4, 8, 8, 8, 9, 12, 6, 8, 6, 6, 12, 5, 8, 8, 12, 12, 6, 5, 8, 6, 12, 12, 13, 8, 12, 6, 7, 12, 6, 7, 4, 11, 6, 5)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,8,13,13,8,10,10,10,5,5,8,12,12,9,9,6,12,5,5,6,7,11,6,10,5,5)
\frac{1}{15}(3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 9, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 9, 4, 4, 5, 5, 10, 4, 5, 5, 5, 10, 8, 5, 9)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 8, 8, 10, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 9, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 8, 9, 14, 7, 7)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 5, 4, 4, 9, 5, 10, 6, 5, 5, 9, 10, 8, 5, 5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 8, 8, 10, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 7, 3, 8, 8, 10)
\frac{1}{18} (4, 4, 6, 8, 9, 10, 10, 10, 8, 8, 12, 13, 6, 6, 12, 10, 6, 5, 6, 6, 6, 6, 5, 4, 8, 6, 11, 6, 6, 12, 9, 11, 11, 12, 6, 6)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 10, 10, 5, 5, 8, 10, 8, 10, 13, 13, 8, 12, 12, 7, 11, 6, 12, 5, 5, 6, 7, 7, 6, 10, 5, 5)
\frac{1}{18}(3, 5, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 8, 12, 12, 11, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 7, 6, 7, 6, 11)
\frac{1}{15}(3, 4, 4, 6, 8, 8, 9, 11, 7, 7, 5, 5, 7, 10, 10, 8, 6, 4, 4, 9, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 7, 3, 10)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,13,13,8,6,10,10,5,5,10,12,12,7,11,6,12,7,7,6,5,11,6,10,5,5)
\frac{1}{18}(3, 5, 5, 7, 9, 9, 11, 13, 8, 8, 6, 6, 12, 12, 12, 10, 12, 4, 6, 6, 10, 12, 6, 4, 6, 8, 12, 8, 6, 6, 10, 6, 6, 10, 4, 12)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 7, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 7, 4, 3, 4, 7, 4, 5, 8, 5)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 7, 7, 5, 7, 3, 4, 4, 8, 4, 8, 5, 4, 8, 4, 3, 4, 8, 3, 4, 7, 5)
\frac{1}{15}(3, 4, 5, 6, 8, 8, 9, 9, 5, 8, 9, 5, 5, 6, 10, 9, 6, 10, 10, 5, 5, 7, 5, 5, 4, 4, 6, 6, 3, 11, 10, 9, 5, 9, 5, 8)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 11, 7, 7, 5, 5, 7, 10, 10, 8, 6, 4, 4, 5, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 7, 3, 10)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,8,12,7,11,6,6,5,5,6,6,10,5,11,5,7,12)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,10,10,10,5,5,10,12,12,7,11,6,12,7,11,6,5,11,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,11,6,8,8,6,12,6)
\frac{1}{15} (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 5, 4, 4, 10, 9, 9, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 9, 8)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 6, 8, 6, 10, 4, 7, 10, 10, 6, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 5, 9)
\frac{1}{12} (3, 3, 3, 5, 5, 5, 6, 7, 6, 6, 4, 4, 4, 3, 4, 6, 4, 8, 8, 3, 4, 8, 4, 8, 9, 4, 4, 8, 7, 4, 4, 5, 8, 5, 4, 7)
\frac{1}{15}(3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 9, 4, 4, 9, 5, 10, 6, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 7, 8, 8, 9, 5, 11, 6, 9, 9, 6, 4, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 4, 10, 5, 6, 9, 5)
\frac{1}{18} (3, 5, 5, 7, 9, 9, 11, 13, 8, 8, 6, 6, 8, 12, 12, 10, 12, 4, 6, 6, 10, 12, 6, 4, 6, 8, 12, 10, 6, 6, 10, 6, 6, 8, 4, 12)
\frac{1}{18}(4,4,6,8,8,9,10,10,8,8,12,12,13,6,6,10,6,6,5,6,6,6,6,6,5,4,8,12,11,6,6,11,6,6,9,7,12)
\frac{1}{18} (4, 4, 4, 6, 8, 9, 10, 10, 6, 6, 10, 6, 11, 6, 12, 8, 10, 6, 5, 6, 6, 10, 12, 13, 12, 6, 6, 5, 6, 4, 11, 12, 6, 11, 7, 6)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 8, 8, 6, 10, 4, 7, 10, 10, 8, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,8,13,13,8,6,6,10,5,5,8,12,12,9,11,6,12,7,7,6,5,5,6,10,5,5)
\frac{1}{18}(3, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 10, 12, 12, 7, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 9, 6, 7, 6, 11)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,7,11,6,6,12,8,12,12,15,15,9,14,14,7,9,7,14,7,13,7,7,5,7,12,6,6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,7,11,6,6,12,8,8,10,15,15,9,14,14,7,9,7,14,7,13,7,7,5,7,12,6,6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 10, 9, 14, 7, 7)
```

```
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 12, 12, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 9, 7, 12, 6, 6)
\frac{1}{15} (3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 9, 4, 10, 4, 10, 5, 5, 9, 5, 5, 9, 10, 6, 6)
\frac{1}{10} (3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 4, 9, 4, 8, 8, 8, 3, 4, 8, 8, 5, 8, 4, 5, 8, 4, 7, 7, 4)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,4,8,13,13,8,8,10,10,5,5,8,8,12,9,9,6,6,5,5,6,6,10,5,7,5,7,12)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 9, 4, 4, 5, 9, 10, 4, 5, 5, 5, 10, 6, 5, 9)
\frac{1}{15} (3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 8, 8, 6, 10, 4, 9, 10, 10, 8, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{18}(3, 5, 5, 9, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 4, 8, 12, 5, 6, 6, 6, 6, 5, 6, 12, 12, 7, 6, 12, 11, 6, 7, 6, 11)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,10,13,13,8,10,10,6,5,5,8,12,12,9,9,6,12,5,5,6,7,11,6,10,5,5)
\frac{1}{15} (3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 5, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 6, 6)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 9, 3, 6, 8, 10)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 9, 4, 4, 5, 5, 10, 6, 5, 5, 5, 10, 6, 5, 9)
\frac{1}{12}(2, 3, 4, 4, 4, 5, 6, 8, 5, 4, 4, 6, 3, 8, 8, 7, 7, 5, 8, 3, 5, 8, 8, 5, 8, 4, 8, 7, 4, 4, 3, 8, 4, 5, 5, 4)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,6,5,5,8,12,12,7,11,6,12,5,5,6,7,11,6,10,5,5)
\frac{1}{21}(3, 5, 7, 9, 11, 11, 14, 14, 8, 8, 6, 8, 14, 11, 11, 12, 14, 6, 6, 13, 13, 14, 14, 6, 7, 7, 14, 8, 7, 7, 8, 7, 7, 7, 7, 14)
\frac{1}{18}(3,5,5,7,7,7,10,11,8,8,6,6,6,13,8,10,6,6,12,5,6,12,12,12,5,10,12,6,7,12,6,7,12,7,6,5)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 9, 3, 8, 6, 10)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 5, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 5, 4, 3, 4, 7, 4, 7, 8, 5)
\frac{1}{15} (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 5, 4, 4, 10, 5, 7, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 3, 8)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 6, 8, 6, 10, 4, 9, 10, 10, 6, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,6,8,10,6,12,6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,7,11,6,6,12,8,12,12,15,15,9,14,14,7,13,7,14,7,9,7,7,5,7,12,6,6)
\frac{1}{15}(2, 4, 5, 7, 7, 7, 10, 10, 6, 5, 5, 5, 9, 8, 8, 9, 3, 7, 3, 8, 8, 10, 10, 6, 5, 5, 10, 6, 5, 5, 4, 5, 5, 5, 5, 10)
\frac{1}{12} (2, 3, 4, 4, 4, 6, 6, 6, 5, 4, 4, 6, 4, 8, 8, 7, 7, 7, 3, 3, 5, 8, 4, 8, 4, 6, 4, 8, 4, 6, 4, 6, 2, 4, 4, 8)
\frac{1}{18}(4,4,6,8,8,8,9,10,8,10,6,10,12,5,6,10,12,12,6,13,6,6,10,12,5,4,12,6,11,6,6,5,6,7,12,9)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 10, 9, 14, 7, 7)
\frac{1}{2} (4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 12, 9, 14, 7, 7)
\frac{1}{18}(3, 5, 5, 9, 9, 10, 11, 11, 8, 8, 6, 12, 13, 8, 8, 10, 6, 12, 5, 6, 6, 12, 6, 5, 6, 10, 6, 7, 12, 12, 11, 6, 6, 11, 5, 12)
\frac{1}{18}(3, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 8, 12, 12, 7, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 9, 6, 7, 6, 11)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 6, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 11, 5, 5, 4, 9, 5, 5, 5, 9, 3, 6, 6, 10)
\frac{1}{12}(3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 7, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 7, 4, 3, 4, 7, 4, 7, 8, 5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 8, 8, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 13, 7, 5, 7, 12, 6, 6)
\frac{1}{18}(3, 5, 5, 7, 7, 9, 10, 11, 8, 8, 6, 6, 6, 13, 8, 10, 6, 12, 6, 5, 6, 12, 12, 12, 12, 5, 10, 6, 12, 7, 12, 6, 7, 6, 7, 12, 5)
\frac{1}{15} (3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 9, 9, 10, 8, 5, 5, 6, 5, 5, 9, 9, 10)
\frac{1}{18} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 11, 5, 5, 13, 7, 5, 5, 8, 8, 10, 12, 6, 6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,10,12,15,15,9,14,14,7,9,7,14,7,5,7,7,13,7,12,6,6)
\frac{1}{18}(3,5,5,7,9,9,10,11,8,8,6,6,12,13,12,10,6,6,12,5,6,12,4,6,5,6,12,6,11,6,6,9,6,7,12,11)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,10,12,12,15,15,9,14,14,5,7,7,14,9,7,7,13,7,7,12,6,6)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,11,8,10,6,6,6,12)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,6,10,10,5,5,8,12,12,7,11,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,6,8,6,11,6,12,8,10,6,5,6,6,10,12,13,12,6,6,5,6,4,11,12,6,11,7,6)
```

```
\frac{1}{12}(3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 5, 7, 5, 7, 3, 4, 4, 8, 4, 8, 5, 4, 8, 4, 3, 4, 8, 3, 4, 7, 5)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,6,10,5,5,8,8,12,7,11,6,6,5,5,6,6,10,5,7,5,7,12)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{21}(3, 6, 7, 7, 10, 11, 14, 14, 9, 6, 8, 7, 14, 13, 13, 13, 11, 4, 5, 12, 12, 14, 13, 8, 7, 7, 15, 6, 7, 7, 9, 8, 8, 7, 7, 14)
\frac{1}{15} (3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 9, 4, 10, 4, 10, 5, 5, 9, 5, 5, 5, 10, 8, 6)
\frac{1}{12} (3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 9, 4, 4, 6, 7, 4, 5, 8, 8, 7, 4, 3, 4, 4, 3, 4, 3, 5, 7, 4, 4, 5, 7, 8)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,11,11,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,13,13,14)
\frac{1}{21}(3, 6, 7, 7, 10, 11, 14, 14, 9, 6, 8, 7, 14, 11, 13, 13, 11, 4, 5, 12, 12, 14, 13, 8, 7, 7, 15, 6, 7, 7, 9, 8, 8, 7, 7, 14)
\frac{1}{18} (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 9, 10, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 11, 13, 5, 5, 7, 8, 10, 6, 6, 6, 12)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,8,13,13,8,6,6,10,5,5,8,12,12,9,9,6,12,7,11,6,5,5,6,10,5,5)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 8, 8, 6, 10, 4, 7, 10, 10, 8, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 5, 9)
\frac{1}{15}(3, 3, 5, 5, 5, 5, 7, 7, 6, 4, 6, 6, 8, 4, 10, 8, 4, 8, 6, 10, 4, 10, 10, 10, 6, 10, 10, 10, 10, 6, 10, 10, 4, 4, 10, 6)
\frac{1}{15}(3,4,4,6,8,8,9,11,5,7,5,5,7,10,10,8,6,4,4,5,7,10,4,4,5,7,10,8,5,5,8,5,5,7,3,10)
\frac{1}{15} (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 7, 9, 4, 4, 10, 7, 5, 4, 4, 5, 5, 10, 6, 10, 5, 3, 5, 9, 8)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 8, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 8, 12, 9, 14, 7, 7)
\frac{1}{12} (3, 3, 4, 4, 4, 4, 6, 7, 6, 3, 5, 7, 7, 9, 4, 7, 7, 5, 7, 3, 4, 4, 8, 4, 8, 5, 4, 8, 4, 3, 4, 8, 3, 4, 5, 5)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,6,5,5,8,12,12,7,11,6,12,5,5,6,11,7,6,10,5,5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 7, 11, 6, 6, 12, 8, 12, 10, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,10,12,12,15,15,9,14,14,5,7,7,14,13,7,7,13,7,7,12,6,6)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 8, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,6,10,5,5,8,8,12,7,11,6,6,5,5,6,6,10,5,11,5,7,12)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,10,5,5,8,8,10,8,13,13,8,8,12,7,11,6,6,5,5,6,6,10,5,7,5,7,12)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 6, 4, 5, 5, 5, 9, 10, 10, 6, 5, 6, 5, 9)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 8, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{12}(3, 3, 3, 4, 5, 6, 7, 7, 4, 6, 5, 8, 9, 4, 4, 6, 7, 4, 5, 8, 8, 7, 4, 3, 4, 4, 3, 4, 3, 7, 7, 4, 4, 5, 5, 8)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,8,10,6,7,6,12,8,10,12,13,12,6,10,6,5,6,6,6,5,6,4,11,12,6,11,7,6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,10,10,10,10,13,13,8,12,12,7,7,6,12,7,11,6,5,5,6,10,5,5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 12, 12, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 13, 7, 12, 6, 6)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 8, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 8, 8, 6, 10, 4, 7, 10, 10, 6, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 5, 9)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,11,11,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,9,9,14)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,6,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,10,12,6,6)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,11,13,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,13,9,14)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 6, 8, 6, 10, 4, 7, 10, 10, 8, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 5, 9)
\frac{1}{12}(2, 3, 4, 4, 4, 5, 6, 8, 5, 4, 4, 6, 3, 8, 8, 7, 7, 5, 8, 3, 5, 8, 8, 5, 4, 4, 8, 5, 8, 4, 3, 8, 4, 5, 5, 4)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,7,7,11,6,6,12,8,12,10,15,15,9,14,14,7,13,7,14,7,9,7,7,5,7,12,6,6)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,11,11,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,9,13,14)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,12,12,12,15,15,9,14,14,5,7,7,14,13,7,7,13,7,7,12,6,6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,12,12,15,15,9,14,14,7,13,7,14,7,5,7,7,13,7,12,6,6)
```

```
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 6, 8, 6, 10, 4, 7, 10, 10, 8, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 8, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,11,7,8,8,6,6,6,12)
\frac{1}{15}(3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 6, 6, 8, 10, 10, 4, 5, 5, 10, 10, 4, 9, 9, 10, 6, 5, 5, 6, 5, 5, 5, 5, 10)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 6, 4, 4, 5, 5, 10, 4, 4, 9, 9, 10, 6, 5, 5, 8, 5, 5, 9, 9, 10)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 7, 6, 8, 9, 5, 11, 6, 9, 9, 8, 4, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 4, 10, 5, 6, 9, 5)
\frac{1}{12}(3, 3, 4, 4, 4, 6, 6, 7, 6, 5, 7, 7, 3, 5, 4, 7, 5, 7, 9, 3, 4, 4, 4, 8, 8, 3, 8, 4, 4, 3, 4, 4, 5, 8, 5, 5)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 8, 8, 10, 12, 6, 6)
\frac{1}{18} (4, 4, 6, 6, 9, 9, 12, 12, 8, 4, 8, 13, 13, 8, 10, 10, 10, 5, 5, 8, 8, 12, 9, 9, 6, 6, 5, 5, 6, 6, 10, 5, 7, 5, 7, 12)
\frac{1}{18} (3, 5, 5, 7, 9, 9, 11, 13, 8, 8, 6, 6, 12, 12, 12, 10, 12, 4, 6, 6, 10, 12, 6, 4, 6, 8, 12, 10, 6, 6, 10, 6, 6, 8, 4, 12)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 12, 9, 14, 7, 7)
\frac{1}{10} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 8, 13, 13, 8, 6, 6, 10, 5, 5, 10, 12, 12, 7, 7, 6, 12, 7, 11, 6, 5, 11, 6, 10, 5, 5)
\frac{1}{18}(3, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 8, 12, 12, 7, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 7, 6, 7, 6, 11)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 12, 9, 14, 7, 7)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,8,12,7,11,6,6,5,5,6,6,10,5,7,5,11,12)
\frac{1}{21}(4,5,7,9,11,11,12,12,9,9,13,15,15,8,8,12,14,6,6,7,7,8,6,6,5,13,8,14,7,7,12,7,13,7,7,14)
\frac{1}{12} (3, 3, 3, 5, 5, 5, 6, 7, 6, 6, 4, 4, 6, 3, 4, 6, 4, 4, 8, 9, 4, 8, 8, 6, 3, 4, 8, 4, 5, 8, 4, 5, 8, 3, 4, 7)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 4, 3, 8, 6, 5, 8, 8, 3, 4, 7, 4, 4, 9, 4, 7, 7, 8, 3, 8, 5, 4, 5, 4, 5)
\frac{1}{18}(4,4,6,8,8,8,9,10,8,10,6,10,12,5,6,10,12,12,6,13,6,6,10,12,5,4,12,6,11,6,6,5,6,7,12,7)
\frac{1}{15} (3, 4, 5, 5, 6, 8, 8, 9, 7, 8, 8, 9, 5, 11, 6, 9, 9, 8, 4, 4, 5, 10, 5, 5, 5, 10, 5, 5, 5, 10, 4, 10, 5, 6, 9, 5)
\frac{1}{18}(4,4,6,8,8,9,10,10,8,8,12,12,13,6,6,10,6,6,5,6,6,6,6,5,4,8,12,11,6,6,11,6,6,9,11,12)
\frac{1}{2} (4, 5, 7, 9, 11, 11, 12, 12, 9, 9, 13, 15, 15, 8, 8, 12, 14, 6, 6, 7, 7, 12, 6, 6, 5, 9, 8, 14, 7, 7, 12, 11, 13, 7, 7, 14)
\frac{1}{20} (4, 6, 8, 12, 12, 13, 13, 14, 10, 8, 8, 16, 9, 9, 14, 14, 8, 6, 7, 7, 8, 8, 8, 7, 7, 6, 8, 15, 15, 6, 7, 7, 14, 14, 9, 13)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,11,13,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,9,13,14)
\frac{1}{15}(3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 9, 9, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 6)
\frac{1}{21} (4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 8, 8, 10, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,13,13,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,9,13,14)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,13,13,8,10,10,10,5,5,10,12,12,7,7,6,12,7,7,6,5,11,6,10,5,5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 11, 11, 11, 6, 6, 12, 8, 8, 10, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{12} (3, 3, 3, 5, 5, 5, 6, 7, 6, 6, 4, 4, 4, 3, 4, 6, 4, 8, 8, 3, 4, 8, 4, 8, 9, 4, 4, 8, 7, 4, 4, 5, 8, 7, 4, 5)
\frac{1}{15}(3,4,4,7,8,8,8,9,7,7,10,5,5,11,10,8,5,6,10,4,5,9,6,4,4,5,5,5,5,10,10,10,5,8,5,9)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 12, 8, 10, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 9, 6, 10, 8, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 9, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{18} (4, 4, 6, 8, 8, 9, 10, 10, 8, 8, 12, 12, 13, 6, 6, 10, 6, 6, 5, 6, 6, 6, 6, 5, 4, 8, 12, 11, 6, 6, 11, 6, 6, 7, 7, 12)
\frac{1}{18} (3, 5, 5, 9, 9, 9, 10, 11, 8, 8, 6, 6, 12, 13, 12, 10, 4, 12, 6, 5, 6, 6, 6, 12, 5, 6, 12, 6, 7, 6, 6, 11, 6, 7, 12, 11)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 7, 11, 6, 6, 12, 12, 12, 10, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,10,13,13,8,6,10,6,5,5,8,12,12,9,9,6,12,5,5,6,7,11,6,10,5,5)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{15}(3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 5, 4, 4, 7, 5, 10, 6, 5, 5, 9, 10, 8, 5, 3)
```

```
\frac{1}{21} (4, 5, 7, 9, 11, 11, 12, 12, 9, 9, 13, 15, 15, 8, 8, 12, 14, 6, 6, 7, 7, 8, 6, 6, 5, 9, 8, 14, 7, 7, 12, 11, 7, 7, 7, 14)
\frac{1}{10} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 11, 6, 8, 10, 6, 12, 6)
\frac{1}{18}(3, 5, 5, 7, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 8, 10, 12, 6, 6, 5, 6, 12, 12, 12, 5, 10, 6, 6, 7, 6, 12, 7, 12, 7, 12, 5)
\frac{1}{18}(4,4,4,6,8,9,10,10,6,8,10,6,7,6,12,8,10,12,13,12,6,10,6,5,6,6,6,5,6,4,11,12,6,11,9,6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 9, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 10, 9, 14, 7, 7)
\frac{1}{12} (3, 3, 3, 3, 5, 6, 7, 7, 4, 6, 6, 4, 5, 4, 8, 6, 6, 8, 9, 8, 4, 6, 4, 3, 4, 4, 8, 3, 4, 4, 7, 8, 4, 7, 5, 4)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 10, 13, 13, 8, 10, 10, 10, 5, 5, 10, 12, 12, 7, 7, 6, 12, 7, 11, 6, 5, 11, 6, 10, 5, 5)
\frac{1}{18}(3, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 10, 12, 12, 7, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 7, 6, 7, 6, 11)
\frac{1}{9}(2, 2, 3, 3, 4, 4, 5, 5, 4, 3, 3, 6, 6, 3, 5, 5, 5, 2, 4, 3, 3, 6, 3, 3, 6, 6, 3, 5, 4, 2, 6, 3, 5, 3, 3, 6)
\frac{1}{16} (3, 5, 5, 7, 9, 9, 11, 13, 8, 8, 6, 6, 8, 12, 12, 10, 12, 4, 6, 6, 10, 12, 6, 4, 6, 8, 12, 8, 6, 6, 10, 6, 6, 8, 4, 12)
\frac{1}{18} (4, 4, 6, 8, 8, 9, 10, 10, 8, 8, 12, 12, 13, 6, 6, 10, 6, 6, 5, 6, 6, 6, 6, 5, 4, 8, 12, 11, 6, 6, 11, 6, 6, 7, 11, 12)
\frac{1}{12} (3, 3, 3, 4, 6, 7, 7, 7, 4, 6, 7, 5, 4, 8, 8, 6, 7, 9, 8, 4, 4, 7, 3, 4, 4, 4, 4, 3, 7, 7, 7, 5, 5, 4, 4, 8)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,10,6,12,9,9,12,4,12,6,10,7,13,5,5,11,13,5,5,8,12,6,8,10,6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 8, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 10, 8, 9, 14, 7, 7)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 6, 8, 6, 10, 4, 7, 10, 10, 6, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{15}(3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 6, 8, 6, 10, 4, 9, 10, 10, 8, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{15}(3, 4, 4, 4, 8, 8, 9, 9, 5, 7, 7, 5, 11, 10, 10, 6, 6, 10, 10, 5, 5, 8, 4, 4, 5, 9, 10, 4, 5, 9, 6, 5, 5, 9, 5, 10)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,8,12,5,5,6,6,7,11,6,6,10,5,11,5,7,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,6,8,10,6,12,6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,12,12,15,15,9,14,14,7,9,7,14,7,5,7,7,9,7,12,6,6)
\frac{1}{18} (3, 5, 5, 5, 9, 9, 10, 11, 6, 8, 8, 6, 6, 13, 12, 6, 10, 12, 12, 11, 6, 10, 6, 6, 5, 6, 4, 12, 5, 6, 12, 7, 6, 7, 6, 11)
\frac{1}{18}(4,4,6,8,8,9,10,10,8,10,6,10,5,6,12,10,12,12,13,6,6,6,10,5,4,12,12,11,6,6,5,6,6,9,7,12)
\frac{1}{18}(4,4,6,9,10,10,10,10,8,8,13,6,6,12,12,10,5,6,6,6,6,5,4,8,6,6,9,11,11,11,12,6,6,6,6,12)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 10, 4, 5, 9, 10, 6, 5, 5, 6, 5, 5, 9, 5, 10)
\frac{1}{22}(4,4,8,8,10,10,15,15,8,8,12,6,14,13,13,12,8,14,6,15,15,14,14,6,7,7,6,14,7,7,8,7,7,13,13,14)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 11, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 12, 9, 14, 7, 7)
\frac{1}{15}(3,4,6,6,8,9,9,10,7,5,9,11,6,6,9,10,10,4,5,5,6,4,6,5,5,8,10,5,5,4,5,5,10,10,5,5)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 11, 11, 6, 6, 12, 12, 10, 12, 15, 15, 9, 14, 14, 7, 13, 7, 14, 7, 5, 7, 7, 9, 7, 12, 6, 6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,7,5,5,10,8,10,6,12,6)
\frac{1}{2} (4, 5, 7, 9, 11, 11, 12, 12, 9, 9, 13, 15, 15, 8, 8, 12, 14, 6, 6, 7, 7, 8, 6, 6, 5, 13, 8, 14, 7, 7, 12, 11, 13, 7, 7, 14)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,6,10,8,13,13,8,12,12,7,11,6,12,7,11,6,5,5,6,10,11,5)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 7, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 5, 4, 3, 4, 7, 4, 5, 8, 5)
\frac{1}{15} (3, 3, 5, 5, 5, 7, 7, 10, 6, 4, 6, 8, 4, 10, 9, 8, 8, 6, 10, 4, 7, 10, 10, 6, 10, 5, 10, 10, 4, 5, 4, 10, 5, 6, 9, 5)
\frac{1}{12} (2, 3, 4, 4, 4, 5, 6, 8, 5, 4, 4, 6, 3, 8, 8, 7, 7, 5, 8, 3, 5, 8, 8, 5, 8, 4, 8, 7, 4, 4, 3, 8, 4, 5, 7, 4)
\frac{1}{21}(4,5,7,9,11,11,12,12,9,9,13,15,15,8,8,12,14,6,6,7,7,8,6,6,5,9,8,14,7,7,12,7,13,7,7,14)
\frac{1}{15}(3, 3, 5, 5, 5, 5, 7, 7, 6, 4, 6, 6, 8, 4, 10, 8, 4, 8, 6, 10, 4, 10, 10, 10, 6, 10, 10, 10, 10, 8, 10, 10, 4, 4, 10, 6)
\frac{1}{21}(4,5,7,7,7,10,10,14,9,11,11,11,6,6,12,8,12,12,15,15,9,14,14,7,13,7,14,7,5,7,7,9,7,12,6,6)
\frac{1}{21}(4, 5, 6, 6, 9, 11, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 5, 13, 6, 12, 8, 9, 14, 7, 7)
\frac{1}{21}(4, 5, 6, 6, 9, 9, 11, 12, 7, 10, 10, 5, 7, 7, 14, 7, 9, 12, 14, 14, 7, 12, 15, 7, 7, 6, 15, 13, 5, 6, 12, 10, 9, 14, 7, 7)
```

```
\frac{1}{18} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 10, 8, 10, 6, 12, 6)
\frac{1}{12} (3, 3, 3, 4, 5, 5, 6, 7, 4, 4, 7, 4, 8, 5, 8, 6, 5, 8, 4, 9, 4, 7, 4, 4, 3, 4, 3, 5, 4, 3, 4, 7, 4, 5, 8, 5)
\frac{1}{12}(3, 3, 3, 4, 5, 5, 6, 7, 4, 6, 5, 4, 4, 3, 8, 6, 7, 8, 8, 3, 4, 7, 4, 4, 9, 4, 3, 7, 8, 3, 8, 5, 4, 5, 4, 7)
\frac{1}{18}(3, 5, 5, 9, 9, 9, 10, 11, 8, 8, 6, 6, 6, 13, 12, 10, 4, 8, 12, 5, 6, 6, 6, 6, 5, 6, 12, 12, 9, 6, 12, 11, 6, 7, 6, 11)
\frac{1}{21}(4, 5, 7, 7, 7, 10, 10, 14, 9, 7, 7, 11, 6, 6, 12, 12, 12, 12, 15, 15, 9, 14, 14, 7, 9, 7, 14, 7, 13, 7, 7, 5, 7, 12, 6, 6)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,6,10,5,5,8,10,12,7,11,6,6,5,5,6,6,10,7,5,7,5,12)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 8, 5, 7, 7, 10, 10, 5, 9, 8, 8, 5, 5, 10, 4, 8, 3, 7, 6, 4, 11, 9, 6, 4, 10, 5, 7, 5, 5, 10)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 5, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,10,12,5,5,6,6,7,7,6,6,10,7,5,11,5,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,12,6,12,9,9,10,6,12,6,10,13,7,5,5,13,7,5,5,8,8,8,6,12,6)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 10, 6, 12, 9, 9, 12, 4, 12, 6, 10, 7, 13, 5, 5, 7, 13, 5, 5, 8, 12, 6, 8, 8, 6)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 6, 8, 8, 6, 12, 6)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 7, 8, 5, 5, 7, 5, 10, 10, 9, 8, 8, 6, 9, 11, 4, 8, 10, 5, 5, 4, 6, 9, 3, 4, 5, 5, 10, 10, 5, 7)
\frac{1}{18}(3, 3, 5, 7, 7, 8, 9, 12, 6, 6, 6, 6, 11, 12, 11, 8, 6, 10, 5, 6, 11, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 7, 11, 6, 11)
\frac{1}{15} (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 6, 10, 5, 5, 4, 9, 3, 5, 7, 10)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 6, 4, 4, 5, 5, 10, 4, 4, 5, 7, 10, 6, 5, 5, 8, 5, 5, 9, 3, 10)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,8,10,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,10,6,12,9,9,12,4,12,6,10,7,13,5,5,11,13,5,5,8,12,6,8,8,6)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 7, 8, 5, 7, 7, 5, 10, 10, 9, 8, 8, 10, 5, 5, 4, 8, 6, 3, 9, 4, 6, 11, 9, 4, 5, 5, 10, 10, 7, 5)
\frac{1}{15}(3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 8, 10, 5, 5, 4, 7, 3, 5, 7, 10)
\frac{1}{14} (2, 3, 4, 5, 5, 7, 8, 8, 5, 6, 5, 7, 9, 6, 10, 5, 6, 4, 4, 9, 5, 9, 9, 5, 4, 4, 10, 4, 5, 5, 8, 5, 5, 5, 9, 4)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 5, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{12} (2, 3, 3, 4, 4, 4, 6, 6, 3, 5, 4, 4, 4, 6, 8, 6, 5, 5, 7, 3, 9, 5, 5, 7, 3, 3, 8, 8, 8, 8, 8, 8, 4, 8, 8, 4, 4, 4, 6)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 11, 5, 7, 5, 5, 7, 10, 10, 8, 6, 4, 4, 5, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 9, 3, 10)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 7, 4, 4, 5, 10, 7, 4, 4, 5, 5, 10, 6, 5, 5, 3, 10, 8, 5, 9)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 4, 4, 5, 9, 4, 4, 9, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 9, 9, 4, 6, 6, 12, 9, 9, 10, 12, 12, 6, 10, 13, 9, 5, 5, 13, 9, 5, 5, 4, 8, 8, 12, 6, 6)
\frac{1}{18} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 10, 8, 8, 6, 12, 6)
\frac{1}{15}(3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 5, 9, 10, 8, 5, 5, 6, 5, 5, 9, 9, 10)
\frac{1}{12}(3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 4, 3, 4, 4, 8, 8, 9, 4, 4, 8, 7, 4, 4, 7, 4, 4, 5, 5, 8)
\frac{1}{15} (3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 9, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 10)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 8, 9, 5, 7, 7, 5, 10, 9, 10, 8, 8, 10, 5, 4, 5, 8, 6, 11, 4, 9, 8, 3, 4, 7, 5, 10, 5, 7, 10, 5)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 5, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,6,5,5,8,12,12,7,7,6,12,5,5,6,7,11,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,8,10,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,7,5,5,13,11,5,5,10,8,8,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,10,6,12,9,9,12,4,12,6,10,7,13,5,5,11,13,5,5,8,12,6,8,8,6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 8, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{12} (2, 3, 4, 4, 4, 5, 6, 8, 5, 4, 4, 6, 3, 8, 6, 5, 5, 7, 8, 3, 5, 8, 8, 5, 8, 4, 8, 7, 8, 4, 3, 8, 4, 5, 3, 8)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,8,12,6,12,9,9,4,6,12,6,10,13,9,5,5,13,9,5,5,4,8,8,6,12,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,6,12,12,6,10,13,7,5,5,13,7,5,5,6,10,8,12,6,6)
```

```
\frac{1}{15} (3, 4, 4, 4, 7, 7, 8, 8, 5, 5, 7, 10, 10, 5, 9, 8, 8, 5, 5, 10, 4, 8, 9, 11, 6, 4, 7, 3, 6, 4, 10, 5, 5, 5, 7, 10)
\frac{1}{15} (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 9, 8, 6, 8, 4, 11, 9, 10, 8, 4, 5, 5, 6, 10, 5, 5, 4, 3, 7, 7, 5, 10)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 5, 8, 5, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 8, 7, 7, 8, 3, 5, 5, 8, 5, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,4,8,10,13,13,8,6,10,6,5,5,8,12,12,11,11,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{6}(1, 1, 2, 2, 3, 3, 3, 4, 2, 3, 3, 2, 2, 2, 4, 3, 3, 2, 4, 4, 3, 4, 4, 2, 2, 2, 4, 2, 2, 2, 2, 2, 2, 4, 2, 2)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,6,6,12,9,9,4,12,12,6,10,13,7,5,5,13,11,5,5,8,8,8,12,6,6)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 11, 5, 7, 5, 5, 9, 10, 10, 8, 6, 4, 4, 5, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 9, 3, 10)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 7, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{9} (2, 2, 3, 3, 4, 4, 4, 6, 4, 3, 3, 6, 6, 6, 4, 5, 5, 2, 4, 6, 4, 6, 3, 3, 6, 4, 3, 3, 6, 4, 6, 4, 2, 3, 5, 2)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,6,12,12,6,10,13,7,5,5,13,11,5,5,6,8,8,12,6,6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,10,10,5,5,10,10,6,10,13,13,8,12,12,7,11,6,12,7,7,6,5,5,6,10,5,5)
\frac{1}{18} (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 9, 8, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 9, 13, 5, 5, 9, 8, 8, 4, 6, 6, 12)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,5,9,5,13,5,9,5,8,4,8,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,10,8,8,6,12,6)
\frac{1}{18}(4,4,4,8,8,8,9,12,6,8,6,6,12,5,10,8,12,12,6,5,10,6,12,12,13,8,12,6,7,12,6,9,4,11,6,5)
\frac{1}{15}(3, 4, 4, 6, 8, 8, 9, 11, 7, 7, 5, 5, 7, 10, 10, 8, 6, 4, 4, 5, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 9, 3, 10)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 8, 9, 5, 5, 7, 5, 10, 9, 10, 8, 8, 6, 11, 4, 9, 8, 10, 5, 4, 5, 8, 3, 4, 7, 5, 10, 5, 7, 10, 5)
\frac{1}{15} (3, 4, 4, 4, 6, 7, 7, 8, 5, 7, 7, 5, 10, 10, 9, 8, 8, 10, 5, 5, 4, 8, 6, 9, 11, 4, 8, 7, 3, 4, 5, 5, 10, 10, 5, 7)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,8,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,7,5,5,10,8,8,6,12,6)
\frac{1}{15} (3, 4, 4, 4, 6, 7, 7, 8, 5, 5, 7, 5, 10, 10, 9, 8, 8, 6, 9, 11, 4, 8, 10, 5, 5, 4, 8, 7, 3, 4, 5, 5, 10, 10, 5, 7)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 7, 8, 5, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,6,5,5,10,12,12,7,11,6,12,5,5,6,11,7,6,10,5,5)
\frac{1}{15} (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 8, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 6, 10, 5, 5, 4, 9, 3, 5, 7, 10)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,10,6,10,8,13,13,8,12,12,7,11,6,12,7,7,6,5,5,6,10,5,5)
\frac{1}{9}(2, 2, 3, 3, 4, 4, 6, 6, 4, 3, 5, 2, 6, 4, 4, 4, 4, 6, 2, 4, 4, 6, 5, 6, 3, 3, 3, 6, 3, 3, 4, 2, 6, 6, 3, 6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,12,6,12,9,9,10,6,12,6,10,13,7,5,5,13,11,5,5,8,8,8,6,12,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,10,8,8,6,12,6)
\frac{1}{15}(3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 10, 4, 5, 9, 10, 8, 5, 5, 6, 5, 5, 9, 5, 10)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 10, 4, 5, 5, 5, 5, 10, 10, 8, 5, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 7, 8, 5, 7, 7, 5, 10, 10, 9, 8, 8, 10, 5, 5, 4, 8, 6, 9, 11, 4, 8, 9, 3, 4, 5, 5, 10, 10, 5, 7)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 8, 5, 5, 7, 10, 10, 5, 9, 8, 8, 5, 5, 10, 4, 8, 9, 11, 6, 4, 9, 3, 6, 4, 10, 5, 5, 5, 7, 10)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 8, 9, 6, 5, 5, 9, 10, 5, 10, 7, 5, 9, 4, 11, 6, 10, 6, 5, 4, 5, 4, 5, 6, 5, 7, 10, 3, 7, 10, 7)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,10,12,6,12,9,9,4,6,12,6,10,13,9,5,5,13,9,5,5,4,8,8,6,12,6)
\frac{1}{18} (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 9, 6, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 5, 7, 5, 13, 5, 7, 5, 8, 8, 8, 12, 6, 6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,10,8,8,6,12,6)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 10, 4, 5, 5, 5, 9, 10, 10, 6, 5, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 7, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\tfrac{1}{G}(2,2,3,3,4,4,4,6,4,3,3,6,6,6,4,5,5,2,6,6,4,6,3,3,6,4,3,3,6,4,6,4,2,3,4,2)
```

```
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,10,12,7,11,6,6,5,5,6,6,10,11,5,7,5,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,9,5,5,13,11,5,5,4,8,8,6,12,6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{15} (3, 4, 4, 4, 6, 7, 8, 9, 5, 5, 7, 5, 10, 9, 10, 8, 8, 6, 11, 4, 9, 8, 10, 5, 4, 5, 6, 3, 4, 9, 5, 10, 5, 7, 10, 5)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 6, 4, 5, 5, 5, 9, 10, 10, 8, 5, 6, 5, 9)
\frac{1}{18} (4, 4, 6, 6, 9, 9, 12, 12, 8, 8, 10, 13, 13, 8, 8, 10, 10, 5, 5, 8, 10, 12, 5, 5, 6, 6, 7, 7, 6, 6, 10, 7, 5, 7, 5, 12)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,10,10,13,13,8,10,10,10,5,5,10,12,12,7,7,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,5,7,5,13,5,11,5,8,6,8,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,7,5,5,13,7,5,5,6,10,8,12,6,6)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,10,13,13,8,8,10,10,5,5,8,10,12,7,7,6,6,5,5,6,6,10,7,5,7,5,12)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,9,13,5,5,11,8,8,4,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,8,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,9,5,5,4,8,8,6,12,6)
\frac{1}{15}(3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 5, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 6, 10)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 5, 8, 3, 8, 4, 4, 7, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 9, 9, 4, 6, 6, 12, 9, 9, 10, 12, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 6, 8, 8, 12, 6, 6)
\frac{1}{10} (4, 4, 6, 6, 6, 9, 9, 12, 8, 4, 8, 8, 13, 13, 8, 6, 6, 10, 5, 5, 8, 12, 12, 11, 11, 6, 12, 7, 7, 6, 5, 5, 6, 10, 5, 5)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 5, 8, 3, 8, 4, 4, 5, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{9} (2, 2, 3, 3, 4, 4, 6, 6, 4, 3, 5, 2, 6, 4, 4, 4, 5, 6, 2, 4, 4, 6, 5, 6, 3, 3, 3, 6, 3, 3, 4, 2, 6, 6, 3, 6)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,10,5,5,8,12,12,7,7,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,10,8,10,6,12,6)
\frac{1}{15} (3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 5, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 6)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 9, 4, 4, 5, 10, 9, 4, 4, 5, 5, 10, 6, 5, 5, 5, 10, 6, 5, 9)
\frac{1}{18}(3, 3, 5, 7, 7, 8, 9, 12, 6, 6, 6, 6, 11, 12, 9, 8, 6, 10, 5, 6, 11, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 11, 11, 6, 11)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 10, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 5, 11, 5, 4, 5, 9, 5, 7, 5, 3, 8, 10, 6)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 7, 8, 5, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,6,10,8,13,13,8,12,12,7,7,6,12,7,11,6,5,5,6,10,5,5)
\frac{1}{18} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 6, 10, 6, 12, 9, 9, 12, 4, 12, 6, 10, 7, 13, 5, 5, 7, 13, 5, 5, 8, 12, 6, 8, 8, 6)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,10,5,5,8,10,10,8,13,13,8,8,12,7,11,6,6,5,5,6,6,10,7,5,7,5,12)
\frac{1}{2} (3, 6, 7, 7, 8, 10, 12, 12, 9, 8, 10, 7, 7, 9, 15, 13, 13, 14, 14, 6, 6, 14, 7, 7, 7, 7, 7, 7, 7, 7, 14, 14, 8, 14, 8, 6)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,8,10,6,6,12)
\frac{1}{21}(3,6,7,7,8,8,12,12,9,10,10,7,7,9,15,13,13,14,14,6,6,14,7,7,7,7,7,7,7,7,14,14,8,14,8,6)
\frac{1}{18}(3, 3, 5, 7, 7, 8, 9, 12, 6, 6, 6, 6, 11, 12, 9, 8, 6, 10, 5, 6, 11, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 7, 11, 6, 11)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 7, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)
\frac{1}{9}(2,2,3,3,4,4,4,6,4,3,3,6,6,6,4,5,5,2,4,6,4,6,3,3,6,5,3,3,6,5,6,4,2,3,5,2)
\frac{1}{18} (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 7, 10, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 5, 7, 5, 13, 5, 7, 5, 8, 8, 8, 12, 6, 6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,10,6,12,9,9,12,4,12,6,10,11,13,5,5,7,13,5,5,8,12,6,8,8,6)
\frac{1}{15} (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 9, 9, 4, 4, 10, 5, 9, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 5, 8)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 9, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{12} (3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 4, 9, 4, 8, 8, 8, 3, 4, 4, 8, 7, 8, 4, 7, 8, 4, 7, 7, 4)
\frac{1}{18}(4,4,4,8,8,8,9,12,6,8,6,6,12,5,10,8,12,12,6,5,10,6,12,12,13,8,12,6,7,12,6,7,4,11,6,5)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 7, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)
```

```
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,11,13,5,5,9,8,8,4,6,6,12)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 8, 9, 5, 7, 7, 5, 10, 9, 10, 8, 8, 10, 5, 4, 5, 8, 6, 3, 4, 9, 6, 11, 4, 9, 5, 10, 5, 7, 10, 5)
\frac{1}{9}(2, 2, 3, 3, 4, 4, 4, 6, 4, 3, 3, 6, 6, 6, 4, 5, 5, 2, 4, 6, 4, 6, 3, 3, 6, 4, 3, 3, 6, 5, 6, 4, 2, 3, 4, 2)
\frac{1}{10} (4, 4, 6, 6, 9, 9, 12, 12, 8, 6, 8, 13, 13, 8, 8, 10, 10, 5, 5, 8, 8, 12, 7, 7, 6, 6, 5, 5, 6, 6, 10, 5, 7, 5, 11, 12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,6,8,10,6,12,6)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,8,12,7,7,6,6,5,5,6,6,10,5,7,5,7,12)
\frac{1}{10} (4, 4, 6, 6, 6, 9, 9, 12, 8, 4, 8, 8, 13, 13, 8, 6, 6, 10, 5, 5, 10, 12, 12, 9, 11, 6, 12, 7, 7, 6, 5, 5, 6, 10, 5, 5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,11,8,8,6,6,6,12)
\frac{1}{10} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 8, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 9, 5, 5, 13, 9, 5, 5, 4, 8, 8, 6, 12, 6)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 5, 9, 10, 10, 6, 5, 6, 5, 9)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 8, 10, 13, 13, 8, 10, 10, 6, 5, 5, 10, 12, 12, 7, 11, 6, 12, 5, 5, 6, 7, 11, 6, 10, 5, 5)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,10,8,8,6,12,6)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 8, 5, 7, 7, 10, 10, 5, 9, 8, 8, 5, 5, 10, 4, 8, 3, 7, 8, 4, 11, 9, 6, 4, 10, 5, 7, 5, 5, 10)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 7, 8, 5, 5, 7, 5, 10, 10, 9, 8, 8, 6, 9, 11, 4, 8, 10, 5, 5, 4, 8, 9, 3, 4, 5, 5, 10, 10, 5, 7)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,11,8,8,6,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,11,5,5,10,8,10,6,12,6)
\frac{1}{12} (2, 3, 3, 4, 4, 6, 6, 6, 3, 5, 4, 4, 4, 6, 8, 6, 5, 7, 3, 3, 7, 7, 7, 9, 3, 7, 8, 8, 8, 4, 8, 4, 4, 6, 4, 4)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,10,12,7,11,6,6,5,5,6,6,10,7,5,11,5,12)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 9, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{12} (2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 5, 8, 3, 8, 4, 4, 5, 8, 4, 4, 5, 3, 7, 8, 4, 8)
\frac{1}{12} (2, 3, 3, 4, 4, 6, 6, 6, 3, 5, 4, 4, 4, 6, 8, 6, 5, 7, 3, 3, 7, 7, 7, 9, 3, 5, 8, 8, 8, 4, 8, 4, 4, 6, 4, 4)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,6,12,12,6,10,13,5,7,5,13,5,7,5,10,6,8,12,6,6)
\frac{1}{15}(3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 9, 8, 6, 8, 4, 11, 9, 10, 8, 4, 5, 5, 8, 10, 5, 5, 4, 3, 7, 7, 5, 10)
\frac{1}{15}(3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 10, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 5, 11, 5, 4, 5, 9, 5, 9, 5, 3, 6, 10, 8)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,6,6,12,9,9,4,12,12,6,10,13,7,5,5,13,11,5,5,8,8,10,12,6,6)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,8,6,12,12,9,9,4,12,6,6,10,13,5,5,9,13,5,5,11,8,8,4,6,6,12)
\frac{1}{15}(3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 9, 4, 4, 9, 10, 9, 4, 4, 5, 5, 10, 6, 5, 5, 5, 10, 8, 5, 9)
\frac{1}{14}(2, 3, 4, 5, 5, 7, 8, 8, 5, 6, 5, 7, 9, 6, 10, 5, 6, 4, 4, 9, 5, 9, 9, 5, 4, 4, 10, 4, 5, 5, 6, 5, 5, 5, 9, 4)
\frac{1}{21}(4, 5, 6, 10, 11, 11, 11, 13, 9, 10, 6, 7, 7, 15, 9, 11, 15, 14, 14, 6, 12, 14, 7, 7, 5, 7, 7, 7, 9, 7, 14, 8, 14, 8, 14, 6)
\frac{1}{9} (2, 2, 3, 3, 4, 4, 4, 6, 4, 3, 3, 6, 6, 6, 4, 5, 5, 2, 4, 6, 4, 6, 3, 3, 6, 5, 3, 3, 6, 5, 6, 4, 2, 3, 4, 2)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,10,10,10,8,13,13,8,12,12,7,7,6,12,7,11,6,5,5,6,10,5,5)
\frac{1}{18} (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 8, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 9, 5, 5, 13, 11, 5, 5, 4, 8, 8, 6, 12, 6)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 7, 8, 5, 5, 7, 5, 10, 10, 9, 8, 8, 6, 9, 11, 4, 8, 10, 5, 5, 4, 6, 7, 3, 4, 5, 5, 10, 10, 5, 7)
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 7, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)
\frac{1}{9} (2, 2, 3, 3, 4, 4, 4, 6, 4, 3, 3, 6, 6, 6, 4, 5, 5, 2, 4, 6, 4, 6, 3, 3, 6, 4, 3, 3, 6, 5, 6, 4, 2, 3, 5, 2)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,6,10,5,5,8,10,12,7,11,6,6,5,5,6,6,10,11,5,7,5,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,8,12,6,12,9,9,4,6,12,6,10,13,9,5,5,13,11,5,5,4,8,8,6,12,6)
\frac{1}{9}(2,2,3,3,4,4,4,6,4,3,3,6,6,6,4,5,5,2,4,6,4,6,3,3,6,4,3,3,6,4,6,4,2,3,4,2)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,10,12,5,5,6,6,7,11,6,6,10,7,5,11,5,12)
\frac{1}{18}(4, 4, 6, 6, 6, 9, 9, 12, 8, 4, 8, 10, 13, 13, 8, 6, 10, 6, 5, 5, 10, 12, 12, 9, 11, 6, 12, 5, 5, 6, 7, 7, 6, 10, 5, 5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,11,13,5,5,11,8,8,4,6,6,12)
```

 $\frac{1}{21}$ (3, 6, 7, 7, 8, 8, 12, 12, 9, 8, 8, 7, 7, 9, 15, 13, 13, 14, 14, 6, 6, 14, 7, 7, 7, 7, 7, 7, 7, 7, 14, 14, 8, 14, 8, 6) $\frac{1}{15} \left(3, 3, 4, 6, 6, 7, 8, 9, 6, 5, 5, 9, 10, 5, 10, 7, 5, 9, 4, 11, 6, 10, 6, 5, 4, 5, 4, 5, 6, 5, 9, 10, 3, 7, 10, 7\right)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 6, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 8, 8, 8, 12, 6, 6)  $\frac{1}{10}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 9, 5, 5, 13, 11, 5, 5, 4, 8, 8, 6, 12, 6)  $\frac{1}{15}$  (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 9, 8, 6, 8, 4, 11, 9, 10, 8, 4, 5, 5, 6, 10, 5, 5, 4, 3, 9, 7, 5, 10)  $\frac{1}{6} \left(1, 1, 2, 2, 3, 3, 3, 4, 2, 3, 3, 2, 2, 2, 4, 3, 3, 2, 4, 4, 4, 4, 4, 2, 2, 2, 4, 2, 2, 2, 2, 2, 2, 4, 2, 2\right)$  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,8,6,12,12,9,9,4,12,6,6,10,13,5,5,9,13,5,5,9,8,8,4,6,6,12)$  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,9,5,5,4,8,8,6,12,6)$  $\frac{1}{15}$  (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 8, 5, 9)  $\frac{1}{15}(3, 4, 4, 4, 6, 7, 7, 8, 5, 7, 7, 5, 10, 10, 9, 8, 8, 10, 5, 5, 4, 8, 6, 3, 7, 4, 6, 11, 9, 4, 5, 5, 10, 10, 7, 5)$  $\frac{1}{21}$ (4, 5, 6, 9, 10, 11, 11, 13, 9, 10, 7, 6, 7, 15, 9, 11, 14, 15, 14, 6, 12, 7, 14, 7, 5, 7, 7, 14, 8, 14, 7, 9, 7, 8, 14, 6) $\frac{1}{15}$  (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 10, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 5, 11, 5, 4, 5, 9, 5, 7, 5, 3, 6, 10, 8)  $\frac{1}{12}$  (3, 3, 3, 3, 5, 6, 7, 7, 4, 6, 6, 4, 5, 4, 8, 6, 6, 8, 9, 8, 4, 6, 4, 3, 4, 4, 8, 3, 4, 4, 7, 8, 4, 7, 7, 4)  $\frac{1}{12}$  (2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 6, 7, 7, 8, 3, 5, 5, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,6,12,12,6,10,13,5,7,5,13,5,11,5,8,6,8,12,6,6)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 10, 8, 8, 6, 12, 6)  $\frac{1}{18}(4,4,6,6,9,9,12,12,8,4,8,13,13,8,8,10,10,5,5,8,10,12,9,9,6,6,5,5,6,6,10,7,5,7,5,12)$  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,10,10,10,5,5,8,12,12,7,7,6,12,11,11,6,5,5,6,10,5,5)$  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,7,5,5,6,8,8,6,12,6)$  $\frac{1}{15}$  (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 6, 5, 9, 4, 4, 10, 5, 9, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 5, 8)  $\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 4, 6, 3, 8, 6, 8, 5, 7, 8, 3, 5, 5, 8, 5, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)$  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,10,10,8,13,13,8,12,12,7,7,6,12,11,11,6,5,5,6,10,5,5)$  $\frac{1}{10}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 8, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 11, 5, 5, 13, 11, 5, 5, 4, 8, 8, 6, 12, 6)  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,9,13,5,5,9,8,8,4,6,6,12)$  $\frac{1}{12}$  (2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 5, 8, 5, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)  $\frac{1}{21}(3, 6, 7, 7, 10, 10, 12, 12, 9, 10, 10, 7, 7, 9, 15, 13, 13, 14, 14, 6, 6, 14, 7, 7, 7, 7, 7, 7, 7, 7, 14, 14, 8, 14, 8, 6)$  $\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 8, 5, 5, 7, 10, 10, 5, 9, 8, 8, 5, 5, 10, 4, 8, 9, 11, 6, 4, 7, 3, 8, 4, 10, 5, 5, 5, 7, 10)$  $\frac{1}{15}(3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 6, 10, 5, 5, 4, 7, 3, 5, 7, 10)$  $\frac{1}{15}(3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 10, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 5, 11, 5, 4, 5, 9, 5, 9, 5, 3, 8, 10, 6)$  $\frac{1}{12}$  (3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 8, 9, 4, 8, 8, 8, 3, 4, 4, 4, 5, 8, 4, 5, 4, 8, 7, 7, 4)  $\frac{1}{12}$  (2, 3, 4, 4, 4, 5, 6, 8, 5, 4, 4, 6, 3, 8, 6, 5, 7, 7, 8, 3, 5, 8, 8, 7, 8, 4, 8, 5, 8, 4, 3, 8, 4, 5, 3, 8)  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,5,7,5,13,5,7,5,10,6,8,12,6,6)$  $\frac{1}{15}$  (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 6, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 8, 10, 5, 5, 4, 9, 3, 5, 7, 10)  $\frac{1}{15}$  (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 9, 4, 4, 5, 5, 10, 6, 5, 5, 5, 10, 8, 5, 9)  $\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 8, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)$  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,8,12,12,6,10,13,9,5,5,13,9,5,5,4,8,8,12,6,6)$  $\frac{1}{18}$ (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 6, 8, 8, 6, 12, 6)  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,10,10,6,6,12)$  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,7,5,5,13,7,5,5,10,8,8,12,6,6)$  $\frac{1}{15}(3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 9, 9, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 10)$  $\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 4, 4, 5, 5, 4, 4, 9, 5, 9, 10, 8, 5, 9)$ 

```
\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 5, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 7, 8, 4, 8)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 9, 5, 5, 13, 9, 5, 5, 4, 8, 8, 6, 12, 6)
\frac{1}{21}(3, 6, 7, 7, 8, 10, 12, 12, 9, 10, 10, 7, 7, 9, 15, 13, 13, 14, 14, 6, 6, 14, 7, 7, 7, 7, 7, 7, 7, 7, 14, 14, 8, 14, 8, 6)
\frac{1}{15} (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 8, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 8, 10, 5, 5, 4, 7, 3, 5, 7, 10)
\frac{1}{15} (3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 10, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 5, 11, 5, 4, 5, 9, 5, 9, 5, 3, 6, 10, 6)
\frac{1}{12} (3, 3, 3, 3, 5, 6, 7, 7, 4, 4, 6, 4, 5, 4, 8, 6, 6, 4, 3, 4, 4, 6, 8, 9, 8, 4, 8, 3, 4, 4, 7, 8, 4, 7, 7, 4)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,9,13,5,5,11,8,8,4,6,6,12)
\frac{1}{18} (4,4,5,5,8,8,10,10,6,7,7,6,6,6,12,9,9,4,12,12,6,10,13,7,5,5,13,7,5,5,8,8,10,12,6,6)
\frac{1}{21}(3,6,7,7,8,8,12,12,9,8,10,7,7,9,15,13,13,14,14,6,6,14,7,7,7,7,7,7,7,7,14,14,8,14,8,6)
\frac{1}{15}(3, 4, 4, 4, 6, 7, 8, 9, 5, 5, 7, 5, 10, 9, 10, 8, 8, 6, 11, 4, 9, 8, 10, 5, 4, 5, 6, 3, 4, 7, 5, 10, 5, 7, 10, 5)
\frac{1}{15}(3, 3, 4, 6, 6, 7, 7, 9, 6, 5, 5, 9, 10, 10, 10, 7, 5, 9, 4, 4, 6, 10, 6, 5, 11, 5, 4, 5, 9, 5, 7, 5, 3, 6, 10, 6)
\frac{1}{12} (3, 3, 3, 3, 5, 6, 7, 7, 4, 4, 6, 4, 7, 4, 8, 6, 6, 4, 3, 4, 4, 6, 8, 9, 8, 4, 8, 3, 4, 4, 7, 8, 4, 7, 7, 4)
\frac{1}{9}(2, 2, 3, 3, 3, 4, 4, 6, 4, 3, 5, 5, 2, 6, 4, 4, 4, 4, 6, 2, 4, 6, 6, 5, 6, 3, 6, 3, 6, 3, 6, 3, 6, 3, 4, 2, 6)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 10, 5, 5, 10, 6, 10, 10, 13, 13, 8, 12, 12, 7, 7, 6, 12, 7, 11, 6, 5, 5, 6, 10, 5, 5)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,10,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,6,8,10,6,12,6)
\frac{1}{21}(3, 6, 6, 7, 7, 11, 12, 12, 9, 9, 10, 10, 8, 15, 15, 12, 7, 7, 7, 6, 14, 13, 13, 5, 6, 6, 14, 14, 7, 7, 14, 7, 7, 11, 7, 12)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 8, 5, 7, 7, 10, 10, 5, 9, 8, 8, 5, 5, 10, 4, 8, 3, 9, 6, 4, 11, 9, 6, 4, 10, 5, 7, 5, 5, 10)
\frac{1}{9}(2,2,3,3,4,4,4,6,4,3,3,6,6,6,4,5,5,2,6,6,4,6,3,3,6,4,3,3,6,5,6,4,2,3,4,2)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,11,13,5,5,7,8,8,6,6,6,12)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 11, 5, 5, 13, 11, 5, 5, 4, 8, 8, 6, 12, 6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,10,12,7,11,6,6,5,5,6,6,10,7,5,7,5,12)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,10,6,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,6,6,6,12,9,9,4,12,12,6,10,13,11,5,5,13,7,5,5,8,8,8,12,6,6)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 6, 4, 4, 5, 5, 10, 4, 4, 5, 9, 10, 6, 5, 5, 8, 5, 5, 9, 5, 10)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,6,10,5,5,8,10,12,7,11,6,6,5,5,6,6,10,7,5,11,5,12)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,6,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,10,12,6,6)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,9,10,12,6,12,9,9,4,6,12,6,10,13,11,5,5,13,7,5,5,10,8,8,6,12,6)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{18}(3, 3, 5, 7, 7, 8, 9, 12, 6, 6, 6, 6, 11, 12, 11, 8, 6, 10, 5, 6, 11, 12, 12, 13, 6, 7, 12, 11, 6, 5, 5, 6, 11, 11, 6, 11)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,12,12,9,9,10,12,6,6,10,13,5,5,7,13,5,5,7,8,8,8,6,6,12)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,8,10,13,13,8,8,10,10,5,5,8,10,12,5,5,6,6,7,11,6,6,10,11,5,7,5,12)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,8,10,10,13,13,8,10,10,10,5,5,10,12,12,5,5,6,12,7,7,6,7,7,6,10,5,5)
\frac{1}{18} (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 9, 10, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 7, 13, 5, 5, 7, 8, 10, 6, 6, 6, 12)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 5, 4, 4, 5, 10, 7, 4, 4, 5, 5, 10, 6, 5, 5, 3, 10, 8, 5, 9)
\frac{1}{15} (3, 4, 4, 4, 6, 7, 8, 9, 5, 7, 7, 5, 10, 9, 10, 8, 8, 10, 5, 4, 5, 8, 6, 3, 4, 7, 6, 11, 4, 9, 5, 10, 5, 7, 10, 5)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,10,13,13,8,8,10,10,5,5,8,10,12,7,7,6,6,5,5,6,6,10,7,5,11,5,12)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 6, 4, 5, 5, 5, 9, 10, 10, 6, 5, 10, 5, 9)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 11, 7, 7, 5, 5, 7, 10, 10, 8, 10, 4, 4, 5, 7, 10, 4, 4, 5, 7, 10, 8, 5, 5, 8, 5, 5, 7, 3, 10)
```

 $\frac{1}{12}$  (3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 8, 9, 4, 8, 8, 8, 3, 4, 4, 4, 7, 8, 4, 7, 4, 8, 7, 7, 4)  $\frac{1}{21}(4, 5, 6, 9, 9, 10, 11, 13, 9, 10, 7, 7, 6, 15, 9, 11, 14, 14, 15, 6, 12, 7, 7, 14, 5, 7, 14, 7, 8, 14, 7, 8, 14, 9, 7, 6)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 7, 10, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 9, 13, 5, 5, 9, 8, 8, 4, 6, 6, 12)  $\frac{1}{15}$  (3, 3, 5, 5, 7, 7, 10, 10, 4, 4, 8, 4, 8, 7, 7, 8, 6, 8, 4, 9, 11, 10, 8, 4, 5, 5, 6, 10, 5, 5, 4, 7, 3, 5, 7, 10)  $\frac{1}{12}(2, 3, 4, 4, 5, 6, 8, 8, 5, 6, 6, 3, 8, 6, 6, 5, 7, 8, 3, 5, 5, 8, 7, 8, 4, 4, 3, 8, 4, 4, 5, 3, 5, 8, 4, 8)$  $\frac{1}{9}(2, 2, 3, 3, 4, 4, 4, 6, 4, 3, 3, 6, 6, 6, 4, 5, 5, 2, 6, 6, 4, 6, 3, 3, 6, 5, 3, 3, 6, 5, 6, 4, 2, 3, 4, 2)$  $\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 4, 4, 5, 9, 4, 4, 5, 5, 9, 10, 8, 5, 9)$  $\frac{1}{18}(4, 4, 5, 5, 8, 8, 8, 10, 6, 9, 9, 4, 6, 12, 6, 9, 9, 4, 12, 6, 12, 10, 13, 7, 5, 5, 13, 11, 5, 5, 8, 8, 8, 6, 12, 6)$  $\frac{1}{15}$  (3, 4, 4, 4, 7, 7, 7, 8, 5, 7, 7, 4, 10, 10, 5, 8, 8, 3, 5, 5, 10, 8, 11, 5, 5, 4, 11, 5, 9, 4, 6, 6, 7, 10, 5, 5)  $\frac{1}{15}(2, 3, 5, 5, 7, 8, 8, 11, 5, 5, 5, 5, 10, 10, 9, 6, 8, 4, 5, 5, 8, 10, 10, 5, 5, 6, 10, 5, 5, 6, 9, 9, 4, 10, 5, 5)$  $\frac{1}{6}(1, 1, 2, 2, 2, 3, 3, 4, 2, 1, 3, 3, 4, 4, 3, 3, 3, 3, 2, 2, 4, 2, 2, 4, 4, 2, 4, 2, 2, 2, 2, 2, 2, 2, 4, 2, 2)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 8, 8, 10, 6, 9, 9, 4, 6, 12, 6, 9, 9, 4, 12, 6, 12, 10, 13, 7, 5, 5, 13, 7, 5, 5, 8, 8, 8, 6, 12, 6)  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,6,10,10,5,5,8,12,12,11,11,6,12,7,7,6,5,5,6,10,5,5)$  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,5,7,5,13,5,7,5,8,6,8,12,6,6)$  $\frac{1}{15}(2, 3, 4, 5, 7, 8, 8, 9, 5, 6, 5, 5, 10, 10, 7, 7, 8, 4, 5, 5, 10, 9, 11, 6, 6, 5, 10, 5, 5, 4, 5, 5, 6, 10, 5, 5)$  $\frac{1}{18}(4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 7, 6, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 5, 7, 5, 13, 5, 7, 5, 8, 8, 8, 12, 6, 6)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 9, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 6, 8, 8, 6, 12, 6)  $\frac{1}{18}(4,4,5,5,8,8,8,10,6,9,9,4,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,11,5,8,8,8,12,6,6)$  $\frac{1}{15}(1, 4, 5, 5, 6, 6, 8, 10, 5, 6, 6, 5, 5, 7, 11, 9, 9, 10, 10, 4, 8, 10, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 4)$  $\frac{1}{10}$  (4, 4, 5, 5, 8, 8, 10, 10, 6, 9, 9, 4, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 8, 8, 10, 12, 6, 6)  $\frac{1}{15}(3, 4, 4, 4, 7, 7, 7, 8, 5, 7, 7, 4, 10, 10, 5, 8, 8, 3, 5, 5, 10, 8, 11, 5, 5, 4, 11, 9, 9, 4, 6, 6, 7, 10, 5, 5)$  $\frac{1}{15}$  (3, 4, 4, 6, 7, 7, 8, 8, 7, 7, 5, 10, 10, 5, 11, 8, 10, 5, 7, 4, 4, 10, 5, 7, 4, 4, 5, 5, 10, 6, 10, 5, 9, 5, 3, 8)  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,6,5,5,8,12,12,7,7,6,12,5,5,6,11,11,6,10,5,5)$  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,6,6,12,9,9,4,12,12,6,10,13,7,5,5,13,11,5,5,8,8,8,12,6,6)$  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,10,5,5,8,6,10,8,13,13,8,12,12,7,7,6,12,11,11,6,5,5,6,10,5,5)$  $\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)$  $\frac{1}{18}(4,4,5,5,8,8,8,10,6,9,9,4,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,10,8,12,6,6)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 8, 8, 10, 6, 9, 9, 4, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 5, 7, 5, 13, 5, 11, 5, 8, 10, 8, 12, 6, 6)  $\frac{1}{12}$  (3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 4, 3, 4, 4, 8, 8, 9, 4, 4, 8, 7, 4, 4, 7, 4, 4, 5, 7, 8)  $\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,10,12,12,6,10,13,7,5,5,13,7,5,5,6,8,8,12,6,6)$  $\frac{1}{18}$ (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 11, 5, 5, 13, 11, 5, 5, 6, 8, 8, 6, 12, 6)  $\frac{1}{18}(4,4,5,5,8,8,8,10,6,9,9,4,6,12,6,9,9,4,12,6,12,10,13,7,5,5,13,7,5,5,10,8,8,6,12,6)$  $\frac{1}{18}$  (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 7, 10, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 7, 13, 5, 5, 7, 8, 8, 6, 6, 6, 12)  $\frac{1}{18}$  (4, 4, 5, 5, 8, 10, 10, 10, 6, 7, 7, 6, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 11, 13, 5, 5, 11, 8, 8, 6, 6, 6, 12)  $\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,6,6,10,5,5,8,12,12,7,7,6,12,11,11,6,5,5,6,10,5,5)$  $\frac{1}{18}$ (4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 6, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 11, 5, 5, 10, 8, 8, 6, 12, 6)  $\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 8, 5, 10, 10, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)$  $\frac{1}{18}(4,4,5,5,8,8,8,10,6,9,9,4,6,12,6,9,9,4,12,6,12,10,13,7,5,5,13,11,5,5,10,8,8,6,12,6)$  $\frac{1}{15}(2, 3, 5, 5, 7, 8, 8, 11, 5, 5, 5, 5, 10, 10, 9, 6, 6, 4, 5, 5, 8, 10, 10, 5, 5, 6, 10, 5, 5, 6, 9, 9, 4, 10, 5, 5)$  $\frac{1}{15}(2, 3, 4, 5, 7, 8, 8, 9, 5, 6, 5, 5, 10, 10, 9, 7, 8, 4, 5, 5, 10, 9, 11, 6, 6, 5, 10, 5, 5, 4, 5, 5, 6, 10, 5, 5)$  $\frac{1}{18}(4,4,5,5,8,10,10,10,6,9,9,4,6,6,12,9,9,6,12,12,6,10,13,5,7,5,13,5,7,5,8,6,8,12,6,6)$ 

```
\frac{1}{10} (4, 4, 5, 5, 8, 8, 10, 10, 6, 9, 9, 4, 6, 12, 12, 9, 9, 4, 12, 6, 6, 10, 13, 5, 5, 7, 13, 5, 5, 7, 8, 8, 8, 6, 6, 12)
\frac{1}{16} (4, 4, 5, 5, 8, 8, 10, 10, 6, 9, 9, 4, 6, 6, 12, 9, 9, 4, 12, 12, 6, 10, 13, 5, 7, 5, 13, 5, 7, 5, 8, 10, 8, 12, 6, 6)
\frac{1}{15} (2, 3, 5, 5, 7, 8, 8, 11, 5, 5, 5, 5, 10, 10, 9, 8, 8, 4, 5, 5, 8, 10, 10, 5, 5, 6, 10, 5, 5, 6, 9, 9, 4, 10, 5, 5)
\frac{1}{15} (3, 4, 5, 5, 6, 7, 8, 11, 7, 6, 8, 9, 4, 11, 8, 5, 5, 10, 5, 4, 7, 10, 5, 10, 5, 6, 5, 10, 9, 6, 5, 10, 7, 7, 4, 3)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 5, 9, 10, 10, 8, 5, 6, 5, 9)
\frac{1}{15} (3, 4, 4, 4, 6, 7, 9, 9, 5, 7, 7, 5, 4, 10, 10, 8, 8, 10, 3, 5, 5, 8, 4, 11, 5, 5, 4, 11, 5, 9, 7, 5, 5, 6, 8, 10)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,8,12,6,6)
\frac{1}{15} (3, 4, 4, 6, 7, 8, 8, 9, 7, 7, 5, 10, 5, 11, 10, 8, 6, 9, 4, 4, 5, 10, 9, 4, 4, 5, 5, 10, 6, 5, 5, 5, 10, 8, 5, 9)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4, 4, 5, 5, 8, 8, 10, 10, 6, 7, 7, 10, 12, 6, 12, 9, 9, 4, 6, 12, 6, 10, 13, 7, 5, 5, 13, 7, 5, 5, 6, 8, 8, 6, 12, 6)
\frac{1}{12}(3,3,3,3,5,5,6,7,4,6,6,4,4,3,8,6,6,8,8,3,4,6,4,4,9,4,4,8,9,4,8,5,4,5,4,5)
\frac{1}{10} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 10, 5, 5, 10, 6, 10, 8, 13, 13, 8, 12, 12, 7, 7, 6, 12, 7, 11, 6, 5, 5, 6, 10, 5, 5)
\frac{1}{15} (2, 3, 4, 5, 7, 8, 8, 9, 5, 6, 5, 5, 10, 10, 7, 7, 8, 4, 5, 5, 10, 9, 11, 6, 10, 5, 10, 5, 5, 4, 5, 5, 6, 10, 5, 5)
\frac{1}{18} (4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,10,12,7,7,6,6,5,5,6,6,10,7,5,11,5,12)
\frac{1}{15}(2, 3, 4, 5, 7, 8, 8, 9, 5, 6, 5, 5, 10, 10, 9, 7, 8, 4, 5, 5, 10, 9, 11, 6, 10, 5, 10, 5, 5, 4, 5, 5, 6, 10, 5, 5)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,10,5,5,10,12,12,7,7,6,12,5,5,6,7,7,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,7,6,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,8,10,6,6,12)
\frac{1}{15} (3, 4, 4, 6, 8, 8, 9, 9, 7, 7, 9, 11, 11, 6, 6, 8, 6, 4, 4, 5, 9, 10, 4, 4, 5, 5, 6, 10, 5, 5, 8, 5, 9, 5, 5, 10)
\frac{1}{0}(1, 2, 3, 3, 3, 4, 4, 4, 3, 2, 4, 4, 3, 3, 5, 3, 5, 5, 6, 6, 6, 6, 6, 6, 5, 5, 3, 6, 3, 3, 3, 3, 3, 3, 6, 6, 6)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 9, 5, 7, 7, 4, 10, 5, 10, 8, 8, 3, 5, 10, 5, 8, 11, 5, 4, 5, 11, 9, 4, 9, 6, 7, 8, 5, 10, 5)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,8,10,13,13,8,10,10,6,5,5,10,12,12,7,7,6,12,5,5,6,7,11,6,10,5,5)
\frac{1}{15} (3, 4, 5, 5, 6, 7, 8, 11, 7, 6, 6, 9, 4, 11, 8, 5, 5, 10, 5, 4, 7, 10, 5, 10, 5, 6, 5, 10, 9, 6, 5, 10, 7, 7, 4, 3)
\frac{1}{18}(4,4,6,6,9,9,12,12,8,6,8,13,13,8,8,10,10,5,5,8,10,12,7,7,6,6,5,5,6,6,10,7,5,7,5,12)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 5, 9, 10, 10, 6, 5, 10, 5, 9)
\frac{1}{15}(3, 4, 4, 7, 7, 8, 8, 8, 7, 7, 10, 10, 5, 5, 11, 8, 5, 5, 4, 6, 4, 5, 5, 4, 6, 4, 10, 5, 5, 9, 5, 5, 9, 10, 8, 10)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 9, 5, 7, 7, 4, 10, 5, 10, 8, 8, 3, 5, 10, 5, 8, 11, 5, 4, 5, 11, 5, 4, 9, 6, 7, 8, 5, 10, 5)
\frac{1}{15} (3, 4, 4, 5, 5, 6, 7, 7, 7, 7, 6, 6, 9, 4, 4, 8, 5, 5, 10, 5, 11, 9, 9, 6, 11, 3, 10, 5, 10, 6, 5, 10, 10, 5, 5, 8)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,6,6,12,9,9,4,12,12,6,10,13,7,5,5,13,7,5,5,8,8,8,12,6,6)
\frac{1}{15}(3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 10, 6, 5, 5, 6, 5, 5, 9, 9, 10)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 8, 8, 5, 10, 6, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{18}(4,4,5,5,8,10,10,10,6,7,9,10,6,12,12,9,9,4,12,6,6,10,13,5,5,7,13,5,5,7,8,8,6,6,6,12)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,9,9,4,6,6,12,9,9,6,12,12,6,10,13,7,5,5,13,7,5,5,6,8,8,12,6,6)
\frac{1}{15}(3, 4, 4, 6, 8, 8, 9, 9, 7, 7, 9, 11, 11, 6, 6, 8, 6, 4, 4, 9, 9, 10, 4, 4, 5, 5, 6, 10, 5, 5, 8, 9, 9, 5, 5, 10)
\frac{1}{15} (3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 6, 5, 9)
\frac{1}{15} (3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 9, 4, 6, 4, 5, 5, 5, 9, 10, 10, 8, 5, 10, 5, 9)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,10,10,10,5,5,10,12,12,7,7,6,12,11,11,6,5,5,6,10,5,5)
\frac{1}{18}(4,4,5,5,8,8,10,10,6,7,7,6,12,6,12,9,9,4,6,12,6,10,13,7,5,5,13,7,5,5,10,8,8,6,12,6)
\frac{1}{18}(4,4,5,5,8,8,8,10,6,9,9,4,6,6,12,9,9,4,12,12,6,10,13,5,7,5,13,5,7,5,8,8,8,12,6,6)
\frac{1}{12}(1, 3, 4, 4, 5, 7, 7, 7, 4, 5, 5, 4, 6, 8, 8, 7, 7, 8, 4, 4, 4, 6, 3, 3, 7, 7, 9, 3, 5, 5, 6, 4, 4, 4, 4, 8)
\frac{1}{15}(3, 4, 4, 7, 8, 8, 8, 9, 7, 7, 10, 5, 5, 11, 10, 8, 5, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 5, 9, 10, 10, 8, 5, 10, 5, 9)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 7, 8, 5, 7, 7, 4, 10, 10, 5, 8, 8, 3, 5, 5, 10, 8, 11, 5, 5, 4, 11, 5, 5, 4, 6, 6, 7, 10, 5, 5)
```

```
\frac{1}{12} (2, 3, 3, 4, 4, 6, 6, 6, 5, 5, 4, 4, 4, 8, 8, 6, 5, 7, 3, 3, 7, 5, 7, 9, 3, 5, 8, 8, 8, 4, 8, 4, 4, 6, 4, 4)
\frac{1}{12}(3, 3, 3, 5, 5, 6, 7, 7, 6, 6, 4, 4, 3, 4, 4, 6, 4, 4, 3, 4, 4, 8, 8, 9, 4, 4, 8, 7, 4, 4, 7, 4, 4, 7, 7, 8)
\frac{1}{16}(2, 4, 5, 5, 6, 8, 8, 11, 6, 7, 7, 4, 6, 10, 9, 5, 9, 10, 6, 4, 11, 10, 11, 11, 9, 6, 11, 3, 5, 6, 10, 6, 5, 4, 5, 7)
\frac{1}{12} (3, 3, 3, 5, 6, 7, 7, 7, 6, 6, 4, 3, 4, 4, 4, 6, 4, 3, 4, 4, 4, 8, 9, 4, 4, 8, 7, 4, 4, 8, 5, 7, 7, 8, 4, 4)
\frac{1}{12}(2, 3, 3, 4, 6, 6, 6, 6, 5, 5, 4, 4, 8, 8, 8, 6, 7, 3, 3, 7, 7, 7, 9, 3, 5, 5, 8, 4, 4, 4, 6, 4, 4, 4, 4, 8)
\frac{1}{16}(2, 4, 5, 5, 6, 8, 8, 11, 6, 7, 7, 4, 6, 10, 9, 5, 9, 10, 6, 4, 11, 10, 11, 11, 7, 6, 11, 3, 5, 6, 10, 6, 5, 4, 5, 7)
\frac{1}{15} (3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 10, 8, 5, 5, 6, 5, 5, 9, 9, 10)
\frac{1}{12} (1, 3, 4, 4, 5, 7, 7, 7, 4, 5, 5, 4, 6, 8, 8, 7, 7, 8, 4, 4, 4, 6, 3, 3, 7, 7, 9, 3, 5, 7, 6, 4, 4, 4, 4, 8)
\frac{1}{12} (2, 3, 3, 4, 6, 6, 6, 6, 5, 5, 4, 4, 8, 8, 8, 6, 7, 3, 3, 7, 7, 7, 9, 3, 7, 7, 8, 4, 4, 4, 6, 4, 4, 4, 4, 8)
\frac{1}{12} (1, 3, 4, 4, 5, 7, 7, 7, 4, 5, 5, 4, 6, 8, 8, 7, 7, 8, 4, 4, 4, 6, 3, 3, 7, 7, 9, 3, 7, 7, 6, 4, 4, 4, 4, 8)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 6, 8, 13, 13, 8, 6, 10, 10, 5, 5, 10, 12, 12, 11, 11, 6, 12, 7, 7, 6, 5, 5, 6, 10, 5, 5)
\frac{1}{15} (3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 10, 6, 5, 5, 10, 5, 5, 9, 9, 10)
\frac{1}{15}(1, 4, 5, 5, 6, 6, 8, 10, 5, 6, 6, 5, 5, 7, 11, 9, 9, 10, 10, 4, 6, 10, 5, 5, 5, 5, 5, 5, 5, 5, 5, 10, 10, 6, 10, 6, 4)
\frac{1}{15}(3, 4, 4, 4, 7, 7, 8, 9, 5, 7, 7, 4, 10, 5, 10, 8, 8, 3, 5, 10, 5, 8, 11, 5, 4, 5, 11, 5, 4, 5, 6, 7, 8, 5, 10, 5)
\frac{1}{15}(3,4,4,6,7,8,8,9,7,7,5,10,5,11,10,8,10,7,4,4,5,10,7,4,4,5,5,10,6,5,5,3,10,8,5,9)
\frac{1}{16}(2, 4, 5, 5, 6, 8, 8, 11, 6, 7, 7, 4, 6, 10, 9, 5, 9, 10, 6, 4, 11, 10, 11, 11, 7, 6, 11, 3, 5, 6, 10, 6, 5, 4, 5, 9)
\frac{1}{18}(4,4,6,6,6,9,9,12,8,6,6,8,13,13,8,6,6,10,5,5,10,12,12,7,7,6,12,11,11,6,5,5,6,10,5,5)
\frac{1}{15} (3, 4, 4, 4, 6, 7, 9, 9, 5, 7, 7, 5, 4, 10, 10, 8, 8, 10, 3, 5, 5, 8, 4, 11, 5, 5, 4, 11, 5, 5, 7, 5, 5, 6, 8, 10)
\frac{1}{12} (2, 3, 3, 4, 4, 6, 6, 6, 5, 5, 4, 4, 4, 8, 8, 6, 5, 7, 3, 3, 7, 5, 7, 9, 3, 7, 8, 8, 8, 4, 8, 4, 4, 6, 4, 4)
\frac{1}{15}(3, 4, 4, 4, 7, 8, 8, 9, 5, 7, 7, 10, 5, 11, 10, 6, 6, 5, 10, 10, 5, 8, 5, 4, 4, 5, 5, 4, 4, 5, 5, 9, 10, 8, 5, 9)
\frac{1}{9}(1, 2, 3, 3, 3, 4, 4, 4, 3, 2, 4, 4, 3, 3, 5, 5, 5, 5, 6, 6, 6, 6, 6, 5, 5, 3, 6, 3, 3, 3, 3, 3, 3, 6, 6, 6)
\frac{1}{18} (4, 4, 6, 6, 6, 9, 9, 12, 8, 6, 8, 10, 13, 13, 8, 10, 10, 6, 5, 5, 10, 12, 12, 7, 7, 6, 12, 5, 5, 6, 11, 11, 6, 10, 5, 5)
\frac{1}{12} (2, 3, 3, 4, 6, 6, 6, 6, 5, 5, 4, 4, 8, 8, 8, 6, 7, 3, 3, 7, 7, 7, 9, 3, 5, 7, 8, 4, 4, 4, 6, 4, 4, 4, 4, 8)
\frac{1}{9} (1, 2, 2, 3, 3, 4, 4, 5, 3, 3, 4, 4, 3, 5, 6, 4, 5, 5, 6, 2, 3, 5, 5, 6, 6, 3, 6, 3, 3, 6, 3, 3, 6, 6, 3, 3)
\frac{1}{12} (3, 3, 3, 5, 6, 7, 7, 7, 6, 6, 4, 3, 4, 4, 4, 6, 4, 3, 4, 4, 4, 8, 9, 4, 4, 8, 7, 4, 4, 8, 7, 7, 7, 8, 4, 4)
\frac{1}{15}(3, 4, 4, 8, 8, 8, 9, 9, 7, 7, 5, 5, 11, 10, 10, 8, 4, 6, 4, 5, 5, 4, 6, 4, 5, 5, 10, 8, 5, 5, 10, 5, 5, 9, 9, 10)
\frac{1}{12} (1, 3, 4, 4, 5, 6, 8, 8, 4, 3, 5, 4, 7, 7, 7, 7, 3, 6, 3, 7, 7, 8, 7, 4, 4, 4, 9, 6, 4, 4, 3, 5, 5, 4, 4, 8)
\frac{1}{21}(3,4,6,6,9,10,11,11,7,9,9,6,7,14,14,10,10,5,14,7,7,12,15,6,7,7,15,6,7,7,9,8,8,7,7,14)
\frac{1}{12} (1, 3, 4, 4, 5, 6, 8, 8, 4, 3, 5, 4, 7, 7, 7, 7, 3, 6, 3, 7, 7, 8, 7, 4, 4, 4, 9, 6, 4, 4, 3, 7, 7, 4, 4, 8)
\frac{1}{12} (1, 3, 4, 4, 5, 6, 8, 8, 4, 3, 5, 4, 7, 7, 7, 7, 3, 6, 3, 7, 7, 8, 7, 4, 4, 4, 9, 6, 4, 4, 3, 5, 7, 4, 4, 8)
\frac{1}{12} (3, 3, 3, 6, 7, 7, 7, 7, 6, 6, 3, 4, 4, 4, 4, 6, 3, 4, 4, 4, 4, 9, 4, 4, 8, 8, 7, 7, 7, 7, 8, 4, 4, 4, 4, 8)
\frac{1}{9} (1, 2, 3, 3, 4, 4, 4, 6, 3, 4, 4, 3, 3, 5, 7, 5, 5, 6, 6, 2, 4, 6, 3, 3, 3, 3, 3, 3, 3, 3, 6, 6, 4, 6, 4, 2)
```

## Appendix C

## Software Publication

The source code of the ANSI C software implementation of the orbitwise vertex enumeration algorithm based on the Message Passing Interface (MPI) was made available online at: http://www.cas.mcmaster.ca/~deza/mpiowe.html. The input and output of metric polytopes  $met_n$  is provided for further testing and benchmarking of the software.

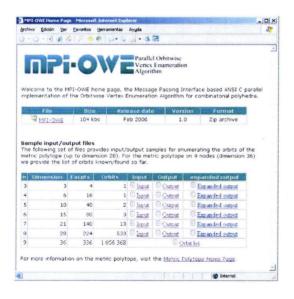


Figure C.1: MPI-OWE Home Page

M.Sc. Thesis - Gabriel Indik

McMaster - Computing and Software

## **Bibliography**

- D. Avis, D. Bremner and R. Seidel: How Good are Convex Hull Algorithms? Computational Geometry: Theory and Applications 7 (1997), 265–301.
- [2] D. Avis and F. Fukuda: A pivoting algorithm for convex hulls and vertex enumeration of arrangements and polyhedra. Discrete and Computational Geometry 8 (1992) 295–313.
- [3] D. Barnette. The minimum number of vertices of a simple polytope. *Israel Journal of Mathematics* 8 (1971) 304–308.
- [4] M. Bayer and C. Lee: Combinatorial aspects of convex polytopes. In P. Gruber and J. Wills (eds.) Handbook on Convex Geometry North Holland (1994) 485–534.
- [5] A. Brøndsted: An Introduction to Convex Polytopes. Graduate Texts in Mathematics 90 Springer-Verlag (1983).
- [6] A. Brouwer, A. Cohen and A. Neumaier: *Distance-Regular Graphs*. Springer-Verlag (1989).

- [7] D. Chand and S. Kapur: An algorithm for convex polytopes. *Journal of Association for Computing Machinery* 17 (1970) 78–86.
- [8] T. Christof and G. Reinelt: Decomposition and Parallelization Techniques for Enumerating the Facets of 0/1-Polytopes. Preprint, Heidelberg University (1998).
- [9] A. Deza and M. Deza: The ridge graph of the metric polytope and some relatives. In: T. Bisztriczky, P. McMullen, R. Schneider and A. I. Weiss (eds.) Polytopes: Abstract, Convex and Computational (1994) 359–372.
- [10] A. Deza and M. Deza: The combinatorial structure of small cut and metric polytopes. In: T. H. Ku (ed.) Combinatorics and Graph Theory World Scientific (1995) 70–88.
- [11] A. Deza, M. Deza and F. Fukuda: On skeletons, diameters and volumes of metric polyhedra. Lecture Notes in Computer Science 1120 Springer-Verlag (1996) 112–128.
- [12] A. Deza, K. Fukuda, T. Mizutani and C. Vo: On the face-lattice of the metric polytope. Lecture Notes in Computer Science 2866 Springer-Verlag (2003) 118–128.
- [13] A. Deza, K. Fukuda, D. Pasechnik and M. Sato: On the skeleton of the metric polytope. Lecture Notes in Computer Science 2098 Springer-Verlag (2001) 125–136.
- [14] A. Deza, B. Goldengorin and D. Pasechnik: The isometries of the cut, metric and hypermetric cones. *Journal of Algebraic Combinatorics* 23 (2006) 197–203.

- [15] M. Deza., V. Grishukhin and M. Laurent: The symmetries of the cut polytope and of some relatives. In: P. Gritzmann and B. Sturmfels (eds.) Applied Geometry and Discrete Mathematics, the Victor Klee Festschrift DIMACS Series in Discrete Mathematics and Theoretical Computer Science 4 (1991) 205–220.
- [16] A. Deza and G. Indik: A counterexample to the dominating set conjecture.

  Optimization Letters (to appear).
- [17] M. Deza and M. Laurent: Geometry of cuts and metrics. Algorithms and Combinatorics 15 Springer-Verlag (1997).
- [18] A. Deza, T. Mizutani and D. Pasechnik: Enumerating faces of polyhedra with large symmetry group. Optimization - Modeling and Algorithms 15 Institute of Statistical Mathematics, Tokyo (2002) 179–185.
- [19] M. E. Dyer: The complexity of vertex enumeration methods. *Mathematics of Operations Research* 8 (3) (1983) 381–402.
- [20] K. Fukuda: cddlib reference manual, version 0.86. IFOR, ETHZ, Zürich, Switzerland (1999).
- [21] K. Fukuda and A. Prodon: Double description method revisited. *Lecture Notes in Computer Science* 1120 Springer-Verlag (1996) 91–111.
- [22] V. P. Grishukhin: Computing extreme rays of the metric cone for seven points. *European Journal of Combinatorics* 13 (1992) 153–165.
- [23] B. Grünbaum: Convex Polytopes. Wiley (1967).

- [24] M. Laurent: Graphic vertices of the metric polytope. *Discrete Mathematics* 151 (1996) 131–153.
- [25] M. Laurent and S. Poljak: The metric polytope. In E. Balas, G. Cornuejols and R. Kannan (eds.) Integer Programming and Combinatorial Optimization Carnegie Mellon University (1992) 247–286.
- [26] P. McMullen: The maximal number of faces of a convex polytope. *Mathematika* 17 (1970) 179–184.
- [27] P. McMullen and G. C. Shephard: Convex Polytopes and the Upper Bound Conjecture. London Mathematical Society Lecture Note Series 3 Cambridge University Press (1971).
- [28] T. S. Motzkin, H. Raiffa, G. L. Thompson and R. M. Thrall: The double description method. In H. W. Kuhn and A. W. Tucker (eds.) Annals of Mathematics Studies 28 Princeton University Press (1953) 51–73.
- [29] S. Poljak and Zs. Tuza: Maximum Cuts and Large Bipartite Subgraphs. In W. Cook, L. Lovász and P. D. Seymour (eds.) DIMACS 20 (1995) 181–244.
- [30] R. Seidel: A convex hull algorithm optimal for point sets in even dimensions. University of British Columbia, Vancouver, BC, Canada (1981).
- [31] R. Seidel. Output-size sensitive algorithms for constructive problems in computational geometry. Ph.D. thesis, Cornell Univ., Ithaca, NY (1986).
- [32] G. M. Ziegler: Lectures on Polytopes. *Graduate Texts in Mathematics* 152 Springer-Verlag (1995).