OPTIMAL SYNTHESIS AND BALANCING OF LINKAGES

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By

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SCOPE AND CONTENTS:

The problems of dimensional synthesis and of balancing of linkages are formulated as multifactor optimization problems. Using the new techniques developed in the thesis to solve these problems, a general computer program has been written to be a design aid for such problems. A guide to usage and a complete documentation for this computer program are included in the thesis.

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I INTRODUCTION

In recent years the computer has become a very valuable business and research tool. However, its application to mechanism design has not been as widespread as it should be. The primary stumbling block to such applications has been the lack of easy to use programs to deal with the complexities of mechanisms. An efficient, reliable, and sufficiently comprehensive program dealing with mechanisms is not an overnight undertaking; thus it has been spurned by practicing Engineers in favor of existing intuition-based trial-and-error methods which oft-times lead to many unacceptable prototypes before a sub-optimum creation is deemed acceptable for current needs. This thesis presents a complete and tested technique for synthesizing, balancing, and analyzing the particular mechanisms included ((namely the planar four-bar (R-R-R-R*), planar slider-crank (R-R-R-P), and spatial four-bar (R-G-G-R) linkages)).

* R-R-R-R is the symbolic name given to a linkage with four pin-connected (revolute) joints. The order of naming is clockwise around the linkage from the crank-frame joint. Other joint symbols are P for a parallel-sliding (prismatic) joint and G for a ball-and-socket (globular) joint.

The technique requires that the designer need only specify the particular design requirements as program input, select the appropriate method for the problem, and relax for a period of about two minutes while the computer determines the optimum mechanism design parameters for his problem and displays, through different plots, how his optimum mechanism will behave. He may then indicate his desire to see how his optimum mechanism parameters compare with other possible mechanism parameters as a check on their optimality - forty seconds will pass as a meaningful graphical contour plot is displayed. After examining one or more such contour plots, the designer may either reject the "optimum" design and redefine the problem conditions as a result of the enlightenment provided by the computer output, or he may accept the parameters and ask the computer to do a complete velocity and acceleration analysis of his optimum mechanism. Almost instantly appropriate tables and plots of his mechanism's performance will be displayed. If this data is acceptable, then the designer can "finish off" his mechanism by asking the computer to determine the optimum counterweights and their positions required to balance his mechanism with respect to the horizontal and vertical shaking forces, and the shaking moment about the crankshaft axis. Within seconds the optimum counterweight parameters and appropriate tables and plots showing the before and after effect of the balancing will be displayed. As in the synthesis optimization, a contour map showing the effect of the design parameter changes can be displayed if desired. The analysis and balancing procedures are not restricted to the computer synthesized mechanism, but may be applied to any mechanism parameters the designer may conceive. Thus the program is designed to have complete flexibility with respect to a designer's needs.

But what does this all mean in terms of time and money. The total synthesis, analysis and balancing usually requires about three minutes of computer time (on the medium-sized CDC 6400 computer) which, at the computer rate of \$600./hour, means a cost of about \$30. for the average mechanism design problem. However, what is more important is that the designer has been released from the arduous task of trial-and-error synthesis - a very expensive procedure which has been eliminated. The designer man-hours consist only of those required to think up and specify the problem conditions and examine the computer output.

Thus, in comparison with present mechanism design techniques, the user-orientated computer program which has been developed using the techniques embodied in this thesis, many of which are new to the field of mechanism design, not only provides a better design, but also produces this design in less time and at a reduced overall cost.

II GENERAL THEORY OF OPTIMIZATION AS APPLIED TO MECHANISMS

In engineering work it is often necessary to determine the optimum dimensions of the independent design variables in order to minimize a particular dependent design variable such as cost or weight. There are numerous well-developed techniques for accomplishing such a task. OPTIPAC [4] is an attempt to provide a user-orientated system for such general optimization problems. A number of people [5] [6] [7] [8] [9] [10] [11] [12] have dealt with the problem of optimizing the independent mechanism design variables to minimize the mechanism structural error (the difference between the desired and actual positions of the mechanism output) evaluated for given values of an operating variable such as time or the crank angle. However, as is explained in Chapter III, in mechanism design it is paramount to consider more than a single factor in a meaningful optimization; thus a multi-factor optimization technique is required.

A survey of available techniques for multifactor optimization shows that the most meaningful way to tackle the simultaneous optimization of more than one dependent design variable is to use what are termed in the literature as utility functions. The utility functions are merely functions which convert the dissimilar units of different dependent variables into general units of utility which may be directly combined together. The magnitude of these units indicates the relative importance of the particular dependent variables. As an example, for a particular design, a cost of one dollar may have a utility value of two utility units, and a weight of one pound a utility value of four utility units; thus, if maximum utility is the optimization criterion, a weight of one pound is twice as desirable as a cost of one dollar. Therefore, utility functions convert noncompatible units such as dollars and pounds weight into universal utility units which can then be combined to form a total desirability function which is to be maximized by optimizing the independent design variables.

Siddall [13] gives a summary of the available techniques for developing utility function relationships. Unfortunately, none of these techniques are appropriate for the multifactor optimization problem developed in this thesis. Therefore a meaningful, general, and easy-to-use technique for establishing utility function relationships for multifactor optimization problems has been developed.

This new technique involves the use of "inverse utility functions". Inverse utility, as its name may indicate, can be defined as the reciprocal of the conventional utility value previously described. That is to say, a dependent variable's inverse utility is the reciprocal of its relative desirability for a particular design.

The use of a graph of the dependent design variable (horizontal axis) versus its inverse utility (vertical axis) clearly depicts the functional relationship in graphical form - henceforth the graph will be referred to as an inverse utility curve. This inverse utility curve is developed by using the following five-step procedure.

1. An inverse utility of zero is assigned to the most desirable or ideal value of the dependent design variable (e.g. a value of zero would be assigned to a manufacturing cost of zero dollars).
2. An inverse utility of positive infinity is assigned to the least desirable value of the dependent design variable (e.g. a value of positive infinity would be assigned to a manufacturing cost of infinite dollars).
3. An inverse utility of plus one is assigned to the dependent design variable value which approximately defines the line between acceptable and unacceptable dependent design variable values (e.g. a manufacturing cost of two thousand dollars for a compact automobile).
4. An inverse utility curve is sketched to satisfy steps 1, 2, and 3, and also the optional requirement that the slope of the curve for an inverse utility of zero be zero (see Figure 2.1 for the compact automobile example).

5. A simple mathematical relationship which best fits the curve sketched in step 4 is derived.

Note that step 3 is the key step in establishing the proper scaling of the utility functions. It is essential, for these inverse utility functions to work properly, that, for a given problem, all the dependent design variable values (which are to be combined into the objective function for minimization) have values of equal importance to the designer corresponding to inverse utilities of one. The values which mark the border between unacceptable and acceptable values of the dependent design variables usually provide values of equal desirability to the designer - thus these values are all assigned an inverse utility of one for a particular problem. However, a designer is not restricted to using these unacceptable - acceptable values to provide the relative scaling for the dependent design variables. If, for a given problem, the designer knows of another complete set of variable values, all of which are of equal desirability, then he is free to give each member of this set of variable values an inverse utility of one. Thus, step 3 is just a reasonable step to provide the relative scaling for the component dependent design variables which are included in the objective function for minimization.

The first advantage of this inverse utility technique is that after the above five steps have been done once, if the number decided on in step 3 is left as a variable, x, then a whole family of inverse utility curves is defined (Figure 2.2), each particular curve identified with a particular x value. Thus the curve derived for the compact automobile in the example could also be used for a medium-sized automobile if x were changed to a higher value, say $2500.

The second advantage of the inverse utility technique is apparent when the utilities of the various dependent design variables being considered in the optimization procedure are combined to form the total desirability function. Using conventional utility curves there is some question as to whether the utilities should be added, multiplied, or combined in some more complicated way, and also whether the utilities should be weighted. However, using the inverse utility concept, simple addition of the component utilities is meaningful and certainly superior to their multiplication. Addition of inverse utilities appears to have the advantages of both the addition and multiplication of conventional utilities without their inherent disadvantages. For example, in the addition of conventional utilities, if one of the utilities is zero (implying a very undesirable variable value), the total desirability function is still high if the other utility values are high. However, the inverse utility corresponding to a conventional utility of zero is infinity, and infinity plus any positive number is infinity.

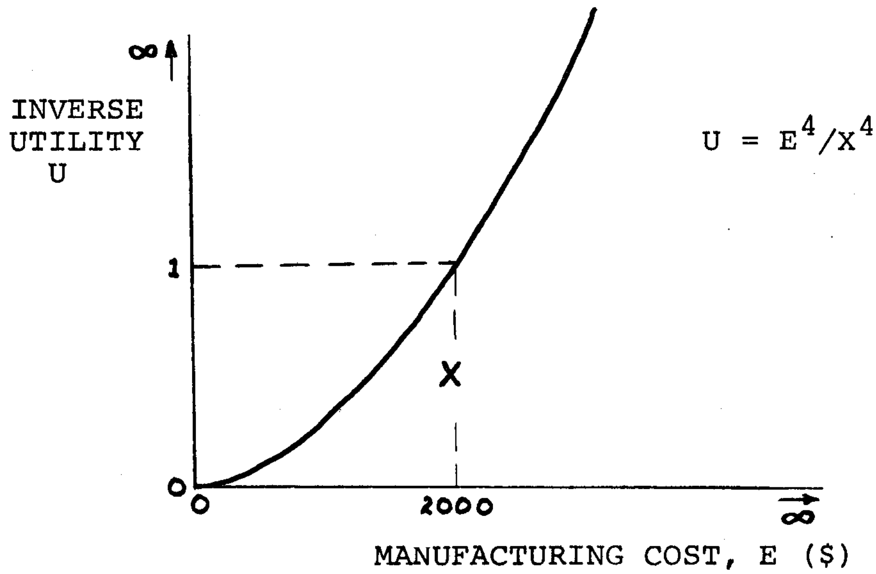


FIG. 2-1 INVERSE UTILITY CURVE FOR THE MANUFACTURING COST OF A COMPACT CAR

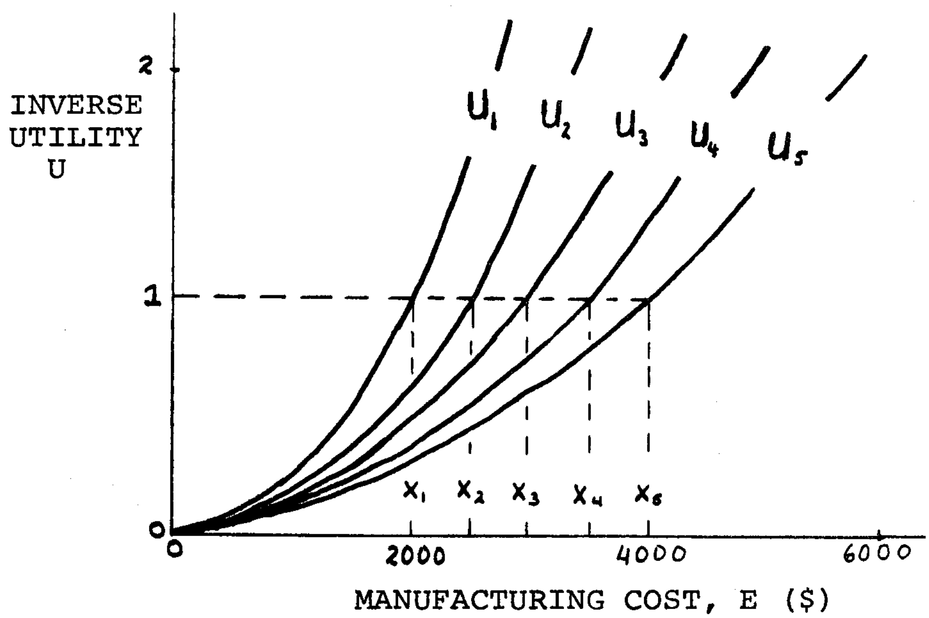


FIG. 2-2 FAMILY OF MANUFACTURING COST UTILITY CURVES

Thus, a high value of the inverse utility total desirability function (which corresponds to a low value of the conventional utility total desirability function) is calculated no matter what the other utility values are. (This is one of the desirable features of the multiplication of conventional utilities.)

Unfortunately not all the conditions affecting an optimization problem can be handled using the inverse utility concept. These are usually explicit constraints on one or more of the independent design variables (e.g. the thickness of a door cannot be negative) and implicit constraints which are functions of the independent design variables (e.g. the frequency of lateral vibration of a beam cannot be negative). Explicit constraints are best handled using variable transformations, and implicit constraints are usually best handled using some type of penalty function transformation.

To handle explicit constraints variables may be transformed* using the following methods.

* The transformation is applied before the variable is used in the objective function. Thus a nonacceptable variable value is never evaluated in the objective function.

Let x be a constrained independent design variable, x_t^{**} be the transformed unconstrained variable corresponding to x, and L and U be scalar constants.

1. For constraint $x \geq L$,

   let $x = L + |x_t - L|$ ***

2. For constraint $x \leq L$,

   let $x = L - |x_t - L|$

3. For constraint $L \leq x \leq U$, where $U > L$,

   let $x = L + ((U-L)/\pi)$ ARCCOS(COS

   $(\pi (x_t - L)/(U-L)))$

   where $0 \leq$ ARCCOS $(\ldots) \leq \pi$

Transformations 1 and 2 (above) work well, but transformation 3 can cause some difficulties. The difficulties arise from the necessary periodicity of the optimization surface with respect to the untransformed variable x. If the minimization routine used develops a step size in the direction of variable x which has a magnitude approximately

** In practice x_t is not distinguished from x. Hence transformation 1 would be $x = L + |x-L|$.

*** Single vertical lines on each side of an expression indicate that the absolute value of that expression is to be used.

equal to a multiple of $(U-L)/\pi$, and is also a type of minimization routine which will accelerate in a particular direction, then there is a chance of an unnecessary large number of useless steps being taken. This situation seems unlikely, but it is aggravated by a relatively flat optimization surface, and has occurred in practice. Thus it is necessary to place some form of weak constraint on the variable to prevent such cases from causing troubles. (Note that this constraint is only necessary for certain minimization techniques, many being satisfied completely with only transformation 3.) For the minimization technique developed in Chapter V this constraint, C, takes the following form:

$$C = M\ (U-L)\ /\ 2 - \left| x - (U-L)\ /\ 2 \right|$$

Where M is an odd integer between 1 and 45, and $C \geq 0$ in the feasible region and $C < 0$ in the infeasible region with respect to constraint C.

M indicates the degree of constrictiveness of the constraint - 1 the most constrictive and 45 the least. Eleven is a suitable value for M. Constraint C is then considered as one of the implicit constraints discussed in the following paragraphs.

Direct variable transformation is the most efficient technique for handling constraints when it can be used. However, implicit constraints can usually not be handled in this way. The penalty function technique is the most general technique available which handles constraints which cannot be eliminated through direct variable transformations. Fiacco and McCormick [14] provide a complete description of various exterior-point and interior-point transformations which they rigorously prove to be mathematically valid.

The basis of the penalty transformation is the addition of a function of the constraint function to the original constrained objective function to form a new unconstrained objective function which can be minimized using one of the many efficient techniques available for minimizing an unconstrained objective function.

An interior-point transformation requires an initially feasible starting point for the minimization sequence. The transformation is such that as a constraint boundary is approached from the interior-feasible region, the constraint function term of the objective function increases smoothly towards infinity at the constraint boundary. Thus the optimization surface of the original constrained objective function is disturbed so as to form a bowl of infinite sides which the minimization technique can theoretically not escape from. Interior-point transformation methods rely on successive decreases of a perturbation parameter to effectively reduce the bottom of the bowl to the original constrained optimization surface except at points very close to the constraint boundaries where the "high sides" are retained.

Exterior-point transformation methods, unlike interior-point transformation methods, do not require a feasible starting point and do not disturb the optimization surface in the feasible region. The constraint function term merely adds infinitely long upward-sloping "sides" to the constrained objective function starting from the constraint boundaries, much like the sides of a gold pan extended to infinity. Conventional methods either start with a very small slope to the sides and successively increase it or they use one very large fixed slope throughout the minimization sequence.

It so happens that the optimization surface for the mechanism synthesis problem already has a bowl shape with sides approximately 10^{10} utility units high at the boundary of the constraint which insures mechanism closure at all points in the desired range of motion. Thus there is no sense in using an interior-point transformation since the constrained problem, as originally posed, already restricts any minimization method to the feasible region once it gets there. However, initially identifying the feasible region for a given problem may be difficult, so that the use of an exterior-point transformation, which enables a minimization method to reach a feasible region from an infeasible region, is desirable.

Figures 2.3 to 2.5 illustrate the importance of the proper formation of the magnitude of the constraint function term of the objective function. Thus it is apparent that one must either devise a scheme for proper scaling of the constraint function term for the general case, or rely on the time wasting procedure of starting with a very small term and successively increasing it. (Starting with an originally very large term is generally unacceptable due to the unnecessary stalling it causes most minimization techniques.)

Thus the following exterior-point transformation has been devised to provide a general transformation technique which accounts for scaling of the constraint term.

$$S(\bar{x}, e, t) = f(\bar{x}) - et \sum_{i=1}^{m} \min(o, C_i(\bar{x}))$$

where $e = a$ if $|f(\bar{x})| < a$

$= |f(\bar{x})|$ if $|f(\bar{x})| \geq a$,

a and t are positive nonzero scalar parameters,

m is the number of constraints C_i of the form

$C_i(\bar{x}) \geq o$,

f is the original constrained function,

$\bar{x}$ is the vector of independent design variables, and

S is the new transformed objective function.

Appendix A shows that it is necessary that scalar a be greater than zero, and that a minimum of S is equal to a local (feasible) minimum of f for a sufficiently large value

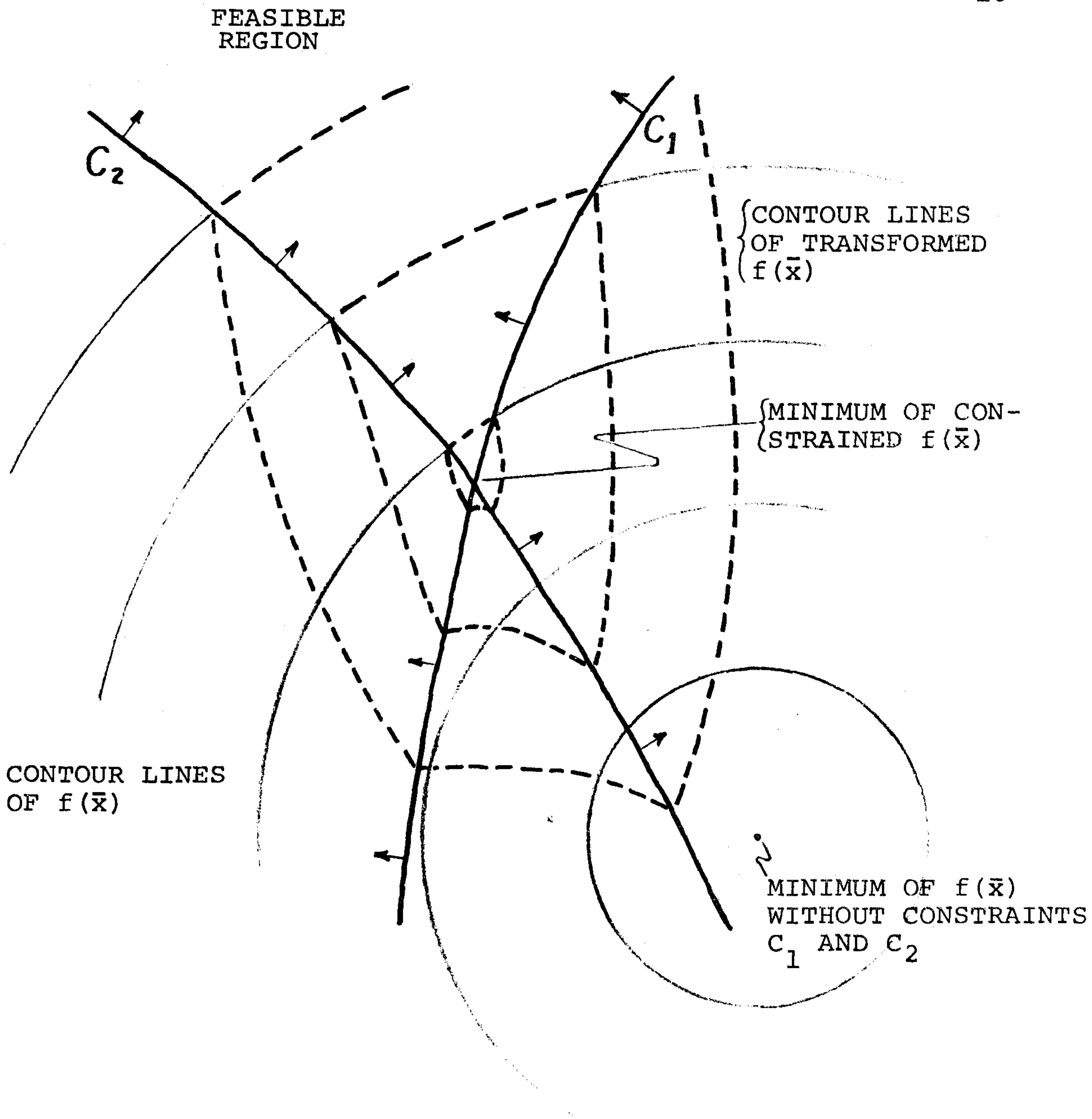
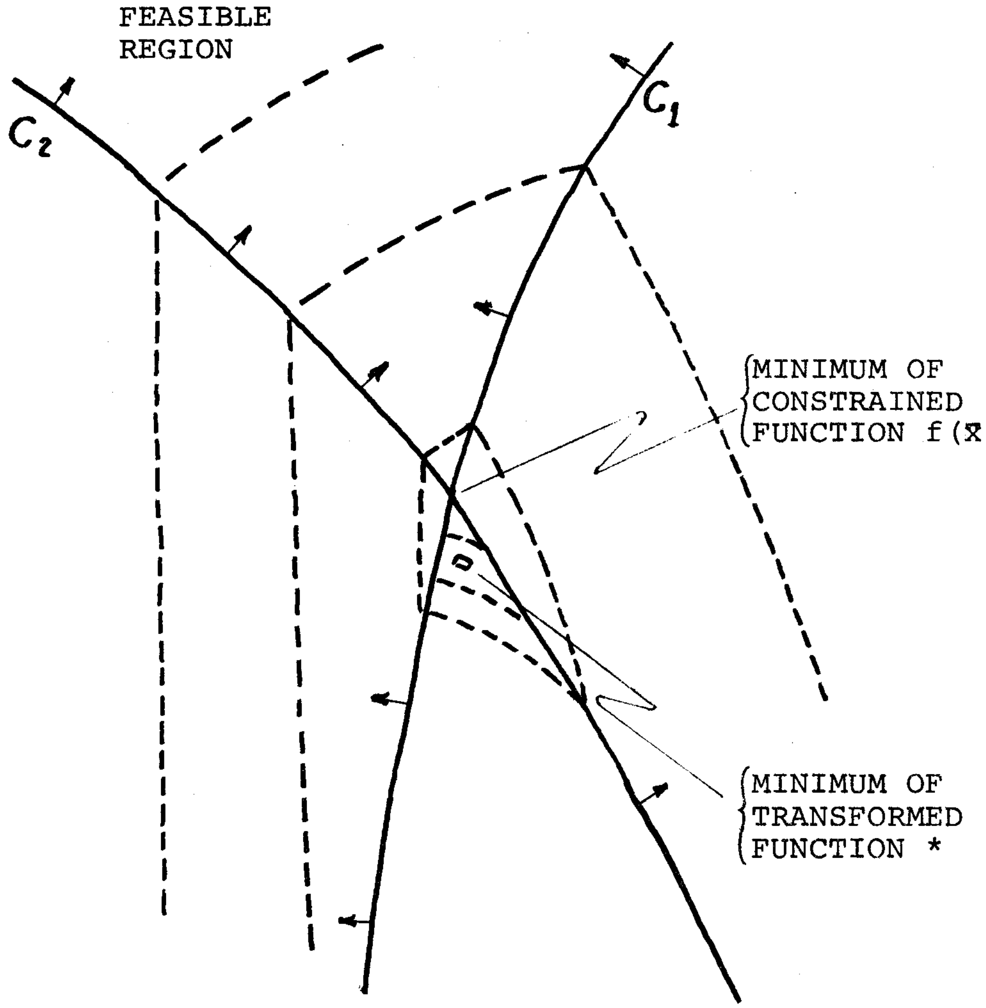


FIG. 2-3 OPTIMIZATION SURFACE FOR EXTERIOR-POINT TRANSFORMED FUNCTION IN WHICH CONSTRAINT TERM IS OF CORRECT MAGNITUDE



* Note that the minimum of the transformed function is not the correct minimum of $f(\bar{x})$.

FIG. 2-4 OPTIMIZATION SURFACE IN WHICH CONSTRAINT TERM HAS TOO LOW A MAGNITUDE

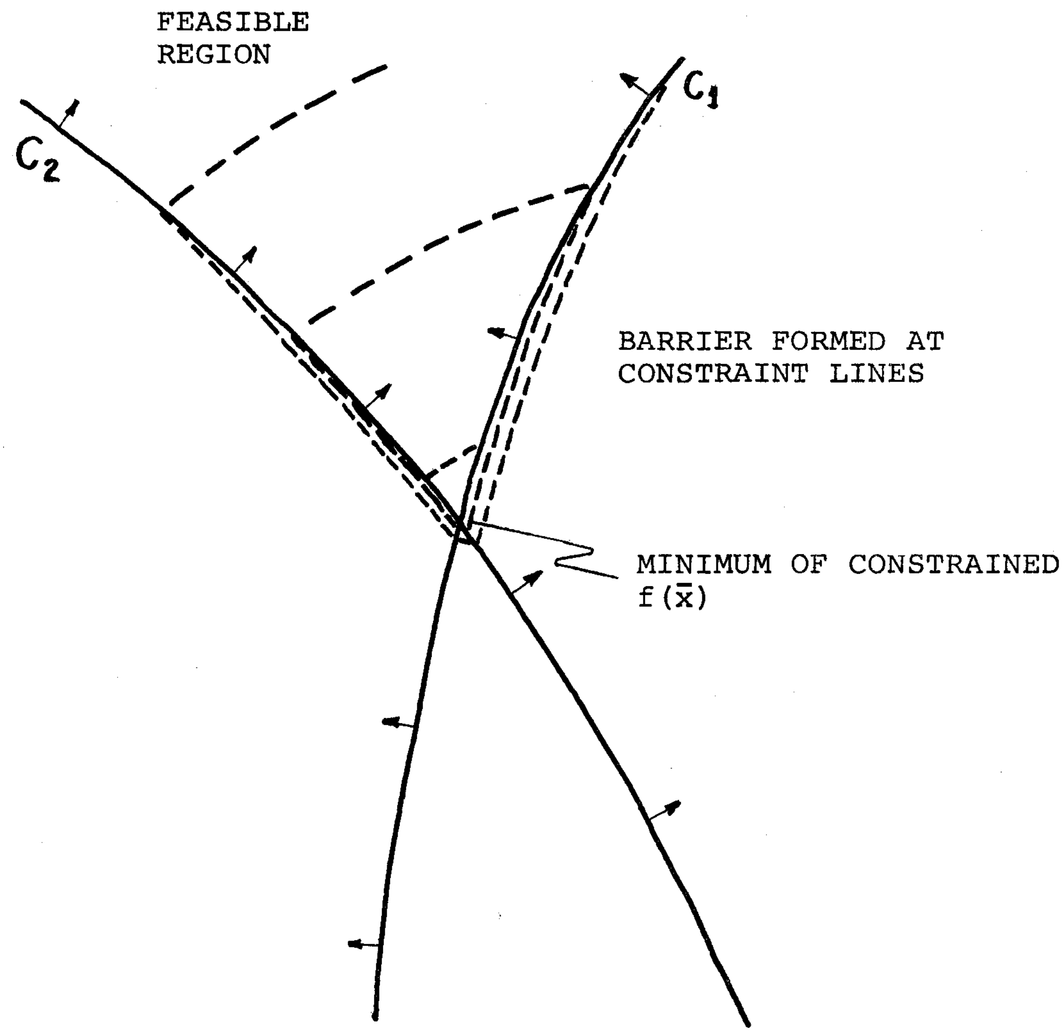


FIG. 2-5 OPTIMIZATION SURFACE IN WHICH CONSTRAINT TERM HAS TOO HIGH A MAGNITUDE

of the parameter t.

The multiplier product et is the key to the scaling problem. Parameter e insures that the constraint term is of the same order of magnitude as the constrained objective function, f. Thus, for the case $f(\bar{x}) > a$, which is the most common one encountered in the minimization problems of this thesis,

$$S = f(\bar{x}) \left[1 - t \sum_{i=1}^{m} \min\left(o, C_i(\bar{x})\right)\right] .$$

From the above relation it becomes apparent that the amount that the objective function S is increased as a constraint is violated is proportional to the size of the constrained objective function. Thus, if the constrained objective function, f, is of magnitude 10^{10}, then the unconstrained transformed objective function, S, will have increases which are significant with respect to 10^{10} if one of the constraints, C, is violated. Similarly, if f is of magnitude 10, then S will have increases which are significant with respect to 10 if one of the constraints is violated. The value of parameter t also affects the magnitude of the constraint term. In fact, parameter t can be considered to be the fine scaling value for a particular problem, e being the parameter that gets the constraint term in the magnitude ballpark. In some problems, the value of t required to produce a reasonably shaped optimization surface also introduces a false optimum in the infeasible zone.

When a minimization method converges to this false optimum, it is desirable to be able to increase t, and start a new minimization sequence from this point. If a method has this restarting feature, then it can use an initially low value of t (between .1 and 10.), which is all that is necessary for most methods, without worrying about the exceptions where an infeasible optimum is created.

The general minimization sequence now takes the following form.

1. Introduce the necessary direct independent variable transformations into the constrained objective function f.
2. Choose suitable values for a and t, and perform transformation S (values of 1 and 10 for a and t respectively have worked well in practice).
3. Minimize the transformed function S.
4. Check for negative values of C_i:
   (a) if any C_i are negative, then increase t (multiplying by 10000. has worked well in practice) and go back to step 3;
   (b) if all $C_i \geq o$, then optimization is completed.

In this chapter the tools necessary to optimize the independent mechanism design variables with respect to one or more dependent mechanism design variables have been developed. In the following chapters it will be shown how these tools can be effectively put to use.

III MECHANISM SYNTHESIS

Chapter II deals with the development of a technique for including the effect of one or more dependent design variables to establish a suitable objective function for minimization. The optimum values of the independent design variables which are used in the objective function are those values which make the objective function a minimum. Thus, if all the significant independent design variables are included in an objective function which properly assesses the relative values of the important dependent design variables (using the inverse utility curves of Chapter II), then the optimum values of the independent design variables can be obtained by minimization of the objective function. It is the purpose of this Chapter to develop the appropriate objective functions for the following five general synthesis problems:

(1) planar four-bar function generation;

(2) planar four-bar coupler-point curve generation;

(3) planar slider-crank function generation;

(4) planar slider-crank coupler-point curve generation; and

(5) spatial (RGGR) four-bar function generation.

Freudenstein [1] is the developer of the traditional analytical precision-point method based on the evaluation of only the theoretical structural error term determined at precision points with Chebychev spacing. In his work the theoretical structural error is made equal to zero at these precision points* (input values for which the output values are evaluated); however, the number of such precision points is limited to the number of independent design variables in a particular linkage synthesis. Thus Freudenstein relies on the optimality of Chebychev spacing, or modifications thereof [2] [3], to control the magnitude of the structural error in between the precision points. The answers obtained using this traditional precision point technique are difficult to improve on from a theoretical structural error standpoint. However, the inclusion of mechanism constraints (such as the actual existence of the mechanism between precision points) and additional dependent design variables (such as the transmission angle) in the objective function are not possible. Thus the technique developed in this thesis is not an alternate technique for minimizing the structural error of a linkage (such as those

* Strictly speaking, the precision points used in this thesis are not true precision points, since the structural error need not necessarily be zero at such points.

in references [6] [7] [8] [9] [10] [11] [12]), but is a new technique which is intended to aid in the design of linkages which will be truly optimum for a given purpose.

In order to determine the optimum link lengths for a given linkage one must stipulate the input to the linkage and what sort of output is desired. (A linkage is only a contrivance for transferring motion, being a passive object with no source of energy unto itself.) The structural error is then, for a given input motion, the difference between the desired output motion and the actual output motion.

It is often desirable for control instruments and mechanical calculators used in industry to have a device that converts motion from one form to another with the scales of both motions being linear. For example, for a certain automatically controlled water acidity control system, it is necessary to add cupric chloride at a rate proportional to the common logarithm of the water pH. If a mechanical link connects a pH indicator machine to a hopper of cupric chloride, the linkage output must be linearly proportional to the common logarithm of the linkage input. Thus we have an example of the classic four-bar function generator problem.

To obtain a linear scale for both the crank and follower links of a planar four-bar linkage (Figure 3.1) for a particular desired output, the following relationships are used:

$$x = x_s + (x_f - x_s)\ (\phi - \phi_s)\ /\ (\phi_f - \phi_s)\ , \text{ and}$$

$$y_a = y_s + (y_f - y_s)\ (\psi - \psi_s)\ /\ (\psi_f - \psi_s) \qquad \ldots\ldots 3.1$$

where x is the functional input variable,

y_a is the actual linkage functional output,

ϕ is the crank angle,

ψ is the follower angle,

s is the subscript referring to the starting position, and

f is the subscript referring to the finishing position.

The maximum structural error, ϵ_{max}, is given by

$$\epsilon_{max} = \max\ (|y_a - y_d|) \qquad \ldots\ldots 3.2$$

where y_d, the desired functional output, and y_a are evaluated over the input variable range.

Minimizing expression 3.2 will minimize the theoretical structural error of the linkage in producing the desired relationship between the input and output links of a four-bar linkage (both R-R-R-R and R-G-G-R types). The required expressions for a slider-crank functional synthesis are identical except that ψ is replaced by the linear distance, s, moved by the slider (Figure 3.2).

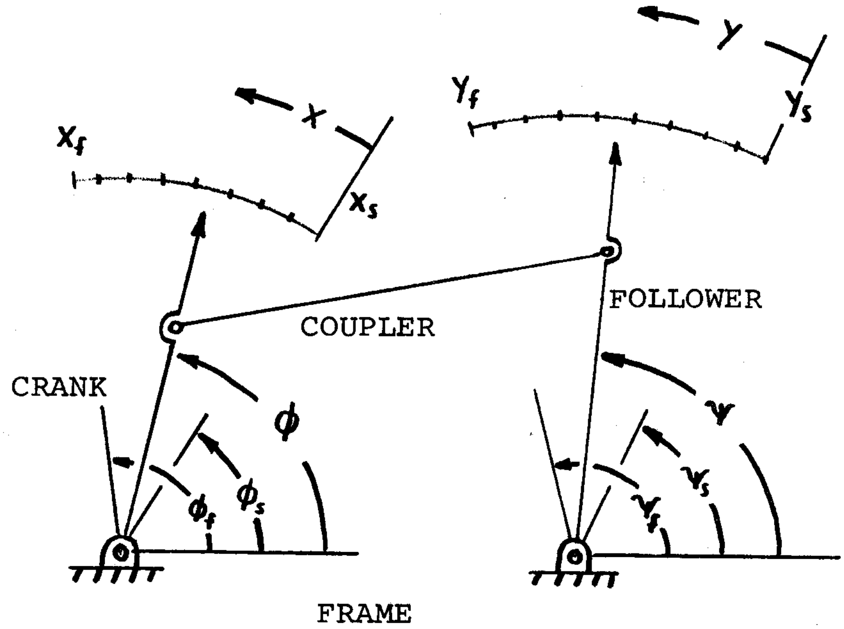


FIG. 3-1 FOUR-BAR FUNCTION GENERATOR

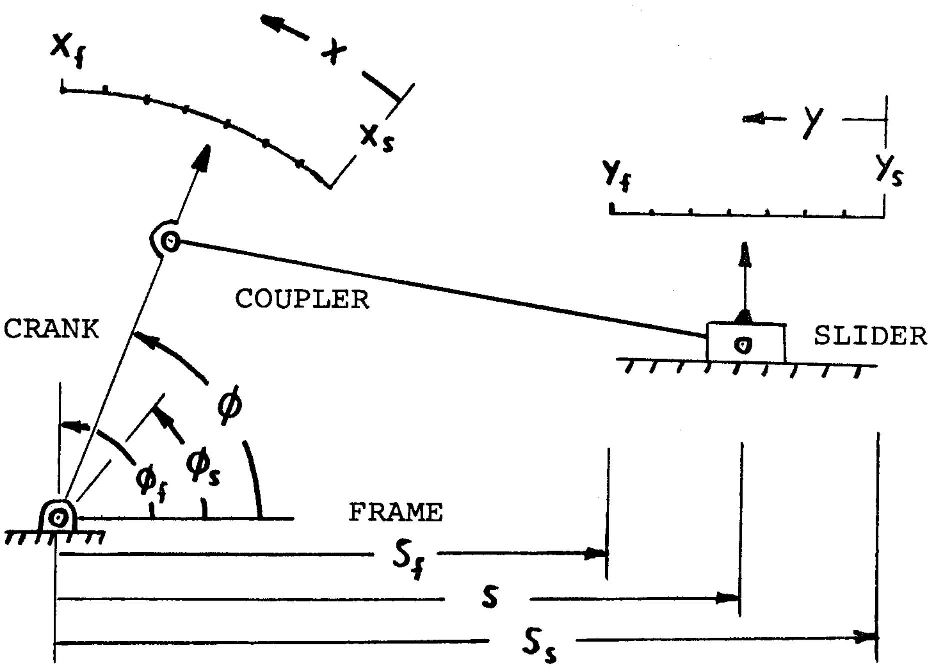


FIG. 3-2 SLIDER-CRANK FUNCTION GENERATOR

The mathematical equations which express the follower angle as a function of the crank angle for the planar and spatial four-bar linkages are derived in Appendices B-1 and B-5 respectively. The equations which express the slider distance as a function of the crank angle are derived in Appendix B-3.

In a computer simulation of a linkage, the values of $(y_a - y_d)$ cannot be evaluated continuously from ϕ_s to ϕ_f, but must be evaluated at discrete positions of the crank in the input range of motion. Freudenstein [1] [2] shows that Chebychev spacing of the precision points provides near optimum spacing for minimizing the maximum structural error from both rigorous mathematical and purely intuitive points of view. The precision points, x_i, separated by Chebychev spacing are the following:

$$x_i = a - h \cos\left((2i-1)\pi/2n \right), \quad i=1,\ldots,n,$$

where $a = (x_s + x_f)/2$,

$h = (x_f - x_s)/2$, and

n = number of precision points required.

Thus the expression for maximum structural error, 3.2, becomes,

$$\epsilon_{max} = \{ \max(|y_{a_i} - y_{d_i}| \quad , \quad i=1,\ldots,n) \} \qquad \ldots\ldots 3.3$$

where y_{a_i} and y_{d_i} are evaluated at each of the n precision points with Chebychev spacing.

The expression for the structural error for the problem of coupler-point curve synthesis is not treated in the same manner as for the functional synthesis. The strict precision-point method (namely, specifying successive desired horizontal and vertical coupler-point curve components at specified input (crank) angles is too restrictive a problem, especially if more than five precision points are selected. That is to say, it is very easy to specify coupler curve conditions that simply cannot be satisfactorily met using a planar slider-crank or planar four-bar linkage.

The problem with the strict precision point method is that an implicit time factor is involved - namely, the linkage must not only produce a specified coupler curve, but also must lie on that curve at specified positions for given crank angles. (The input (crank) angle is assumed to be directly proportional to the time function.) There are many examples, such as synthesizing a straight-line mechanism, where such restrictive specifications are not required. Thus a more general method for coupler curve synthesis is required which must have the strict precision-point method as a particular case.

Such a method has been developed for this thesis. The method depends on the designer specifying acceptable bilateral tolerances on both the horizontal and vertical co-ordinates of each desired coupler point position associated with a specified crank angle. These bilateral tolerances are used to define elliptical zones of acceptability (symmetrical with respect to the horizontal and vertical axes) which are defined by the following relation:

$$\epsilon_i^2 = (x_{a_i} - x_{d_i})^2 \,/\, x_{tol_i}^2 + (y_{a_i} - y_{d_i})^2 \,/ y_{tol_i}^2 \qquad i=1,\ldots,n \qquad \ldots\ldots 3.4$$

where x_{a_i} is the actual mechanism coupler point horizontal component, x_{d_i} is the desired mechanism coupler point horizontal component, y_{a_i} is the actual mechanism coupler point vertical component, y_{d_i} is the desired mechanism coupler point vertical component, x_{tol_i} is the largest acceptable bilateral tolerance in the horizontal direction, y_{tol_i} is the largest acceptable bilateral tolerance in the vertical direction, ϵ_i is the magnitude of the "transformed" structural error, and the subscript i refers to a given crank angle.

If the magnitude of the transformed or scaled structural error, ϵ_i, is evaluated at the crank angle (precision points) specified by the designer, then the appropriate structural error term for minimization is

$$\epsilon_{max} = \{ \max(\epsilon_i \;,\; i=1,\ldots,n) \} \qquad \ldots\ldots 3.5$$

The mathematical equations which express the horizontal and vertical co-ordinates of a given coupler as a function of the crank angle for the planar four-bar and planar slider-crank linkages are derived in Appendices B-2 and B-4 respectively.

Garrett and Hall [15], in their statistical analysis of the function generating properties of four-bar linkages, taking into account possible manufacturing tolerances in the link lengths and clearances in the link connections (i.e. accounting for sources of mechanical error), indicate that mechanism design from the standpoint of minimizing the theoretical structural error only, may, in fact, not be optimum in minimizing structural error from a statistical point of view. The statistical point of view is the realistic point of view; therefore the additional factors required to determine the truly optimum linkage must be established. Essentially, what Garrett and Hall show is that some linkage designs are more sensitive to statistical changes in their independent variables than other linkage designs. Hartenberg and Denavit [16], in their excellent text on mechanism synthesis, indicate that the key parameter in determining a linkage's sensitivity to mechanical error is the transmission angle (Figure 3.3)*.

* The transmission angle of a planar four-bar linkage is defined as the smallest angle between the coupler link (or its extension) and the follower link (or its extension).

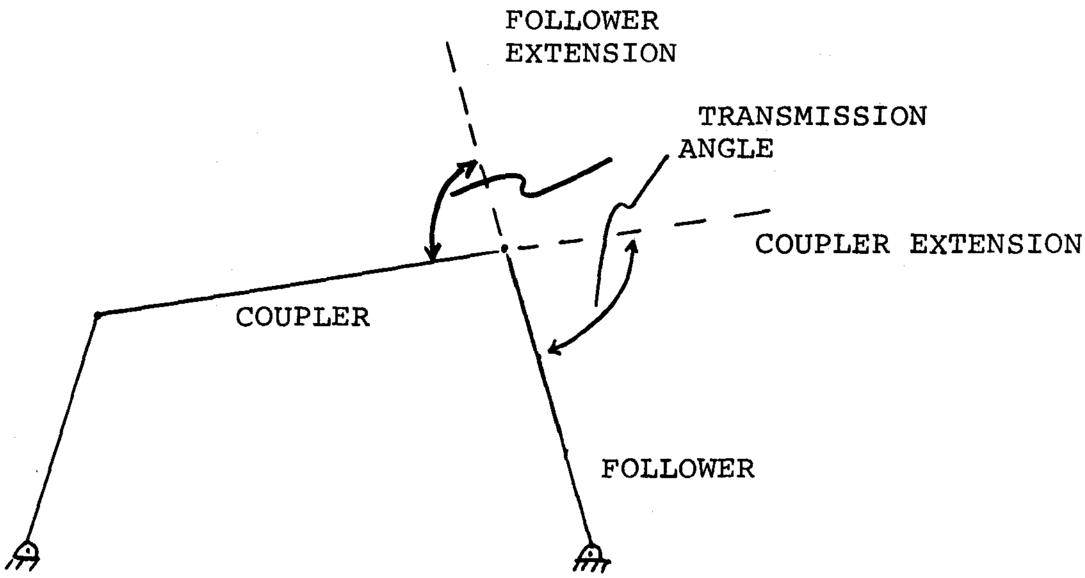


FIG. 3-3 TRANSMISSION ANGLE FOR A PLANAR FOUR-BAR LINKAGE

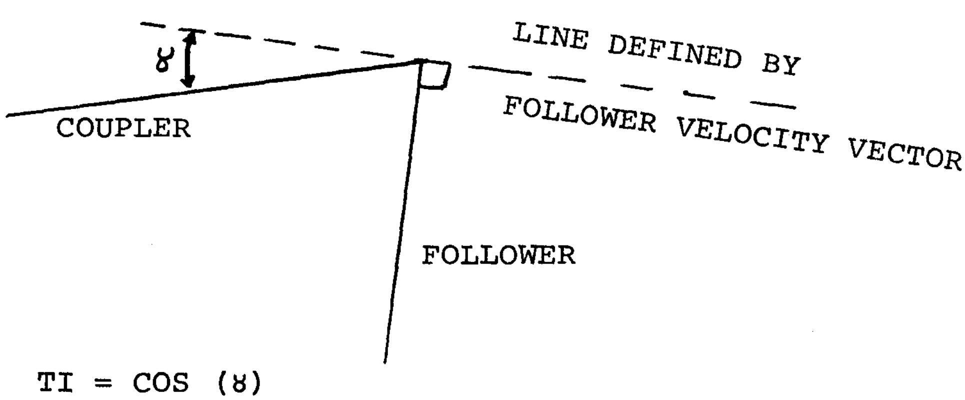


FIG. 3-4 TRANSMISSIBILITY INDEX (TI) FOR A LINKAGE

They show that the mechanical error of a four-bar linkage is directly proportional to the cosecant of the transmission angle. Thus, if the minimum transmission angle is reduced from thirty degrees to five degrees, with all other things being held constant, the maximum mechanical error increases almost six times. A preliminary analysis of Garrett and Hall's work indicates that the maximum sensitivities to mechanical error that they establish can be almost wholly attributed to variances in the value of the minimum transmission angle from one linkage design to the next. It then follows that the minimum value of the transmission angle* should be maximized to obtain a linkage which is least sensitive to mechanical errors.

Unfortunately the present definition of transmission angle cannot be extended with any meaning from planar linkages to spatial linkages. In fact, the spatial angle between the follower and coupler of a general spatial R-G-G-R mechanism can be ninety degrees (the most desirable angle for the planar condition) while the mechanism is at a "dead point" in follower motion (the case of poorest static force

* The transmission angle is also an indicator of the aptness of the static force transmission from the coupler (driving) link to the follower (driven) link - ninety degrees being the optimum value for good static force transmission.

transmission corresponding to theoretically infinite mechanical error). Thus a new indicator of force transmissibility, which is general in its application to both planar and spatial mechanisms, is required.

Just such an index, called the transmissibility index (TI), is developed in this thesis. The TI is the ratio of the force being transmitted from the coupler to the follower which does useful work to the total force in the coupler. Physically this index reduces to the absolute value of the cosine of the spatial angle, γ, between the coupler link (or the coupler force vector) and the direction of motion of the follower (i.e. the follower velocity vector) (see Figure 3.4). It can be obtained by taking the absolute value of the scalar product of the unit vector in the direction of the total coupler force (inertia forces not included since it is not desirable that they be transmitted) and the unit vector in the direction of the follower motion. Hence, for the general case, one must obtain the following three expressions:

(1) a general vector expression for the position of the coupler link;

(2) a general vector expression for the position of the follower link; and

(3) a general vector expression for the instantaneous axis of rotation of the follower link

(i.e. the direction of the follower angular velocity vector).

The direction of the follower velocity vector must then be perpendicular to expressions (2) and (3) above, and is thus uniquely defined (except for its sign which is unimportant since the absolute value of the cosine of the angle γ is used).

A general expression of the transmissibility index for only the most simple planar and spatial mechanisms can be obtained using simple trigonometric analysis. The techniques of general vector analysis and rigid-body mechanics must be used for the more complex spatial cases. In Appendices B-6, B-7, and B-8 the general transmissibility index expression is derived for the planar four-bar (RRRR), planar slider-crank (RRRP), and spatial four-bar (RGGR) mechanisms respectively. The spatial four-bar TI is derived using both a short-cut trigonometric analysis and a rigid-body mechanics vector analysis in order to illustrate both techniques on the same problem.

Harrisberger [17] presents a more complicated expression than the TI to indicate the force transmission characteristics of a special case of the RGGR mechanism with two of the independent design variables fixed at zero. His more complex expression is really only a special case of the general expression presented here.

Thus the general mechanism force transmission and mechanical error characteristics are taken into account by using a single index, the TI. A value of 1 is the greatest and most desirable value that this index can obtain; a value of 0 is the smallest and least desirable. Therefore some value between 0 and 1 indicates the smallest acceptable value of TI for a particular design problem. Since the linkage mechanical error is directly proportional to the reciprocal of TI, as is shown in Appendix C, the designer can easily see the effect of lowering the acceptable value of TI on the reliability of his design.

The structural error and TI expressions are not enough in themselves to completely define the total constrained mechanism synthesis objective function. Some control over the link lengths is required in order to limit the link masses and moments of inertia, which are proportional to the link lengths and the cube of the link lengths respectively. This control is required to reduce the inertia forces and torques on the mechanism as much as possible as is consistent with satisfactory maximum structural error and minimum TI requirements. It is also a well known rule of thumb that linkages in which the ratio of the longest to shortest link is high do not generally make satisfactory linkages. Thus an expression restricting the link lengths would be a valuable addition to the previously derived expressions for the structural error and transmissibility index in forming the total objective function.

Three factors, the maximum structural error, ϵ_{max}, the minimum transmissibility index, TI_{min}, and the maximum link length, L_{max}, are now designated as the significant factors to be included in the objective function for minimization. Figures 3.5, 3.6, and 3.7 illustrate the inverse utility curves derived for each factor according to the procedure developed in Chapter II. For the planar four-bar function generation problem, the ratio of the longest link to the frame length is used for L_{max}; for the spatial four-bar function generation problem the ratio of the longest link to the coupler link is used for L_{max}. The longest link length is used for L_{max} for the other three general problems. Note that the contribution of the maximum link length term is zero if L_{max} is less than one. This is sensible since for function generation one link is fixed at one - therefore it would not be meaningful to penalize the particular linkage design if L_{max} were less than one.

The utility functions, U_1, U_2, and U_3, associated with the utility curves shown in figures 3.5, 3.6, and 3.7 respectively are

$$U_1 = \epsilon^2_{max} / \epsilon^2_{all} \qquad \text{......3.6}$$

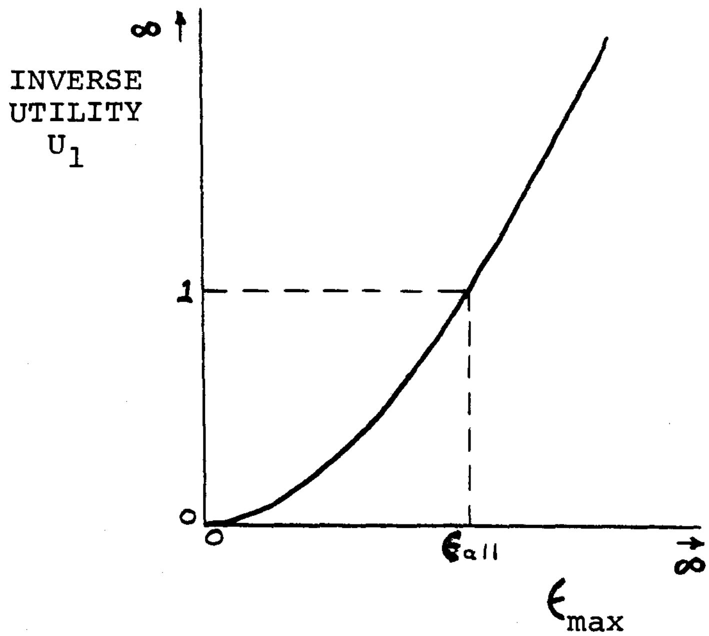


FIG. 3-5 INVERSE UTILITY CURVE FOR ϵ_{max}

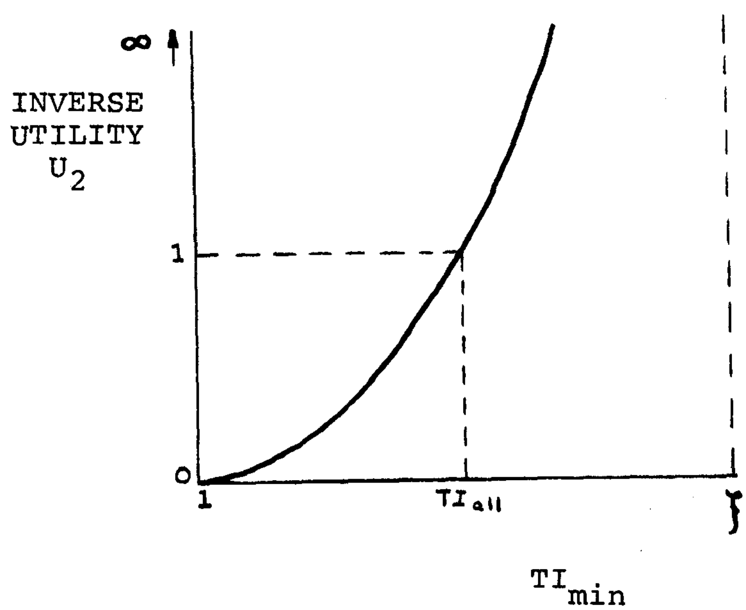


FIG. 3-6 INVERSE UTILITY CURVE FOR TI_{min}

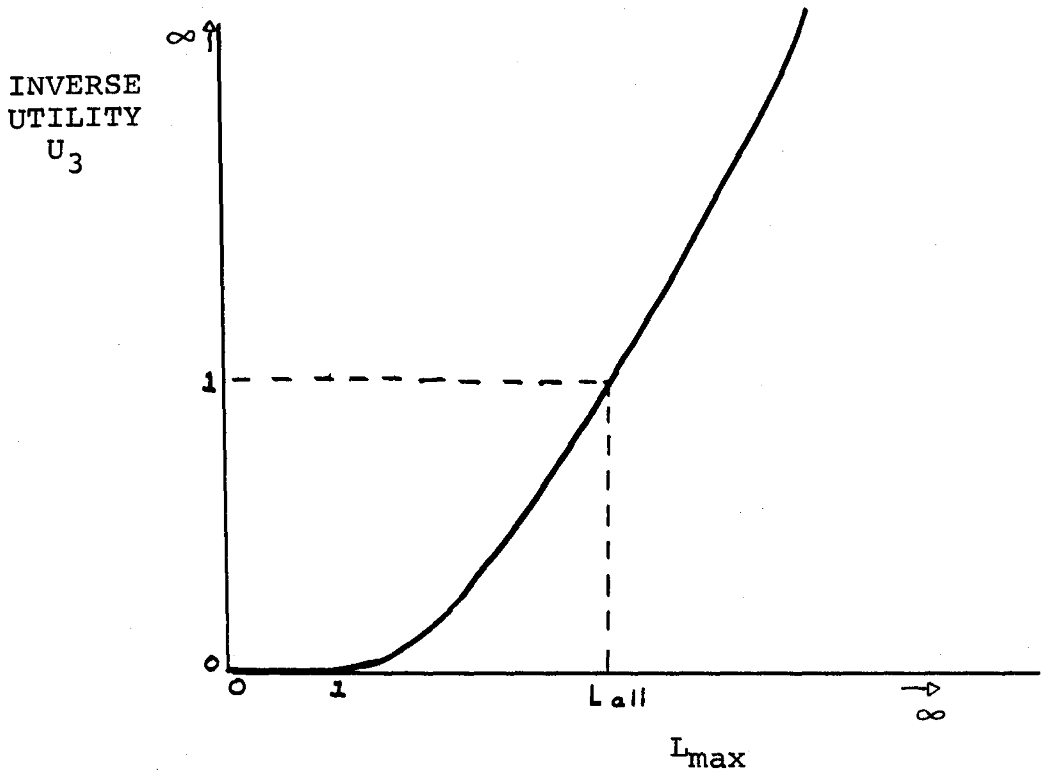


FIG. 3-7 INVERSE UTILITY CURVE FOR L_{max}

for function generation synthesis* where $_{all}$ is the largest acceptable structural error and $_{max}$ is given by equation 3.3;

$$U_2 = \{(1-TI_{min})\ (TI_{all}-\xi)\ /\ (TI_{min}-\xi)(1-TI_{all})\}^2 \quad \ldots.3.7$$

where TI_{all} is the lowest acceptable TI, and ξ is a parameter which is zero for the planar and spatial four-bar linkages and is $\mu/(1+\mu^2)^{1/2}$**, where μ is the coefficient of friction between the slider and its sleeve, for the planar slider-crank linkage; and

$$\begin{aligned} U_3 &= 0 \quad \text{if } L_{max} \leq 1 \\ &= (L_{max}-1)^2\ /\ (L_{all}-1)^2 \quad \text{if } L_{max} > 1 \end{aligned} \quad \ldots\ldots.3.8$$

where L_{all} is the largest acceptable link length or link length ratio.

* Note that for coupler curve synthesis no scaling is required and $U_1 = \epsilon^2_{max}$, where ϵ_{max} is given by equation 3.5.

** This expression is derived from the fact that the slider-crank linkage will "lock up" at a TI greater than zero as a result of the friction force between the slider and its sleeve (friction forces in the revolute joints being ignored). The friction force in the slider is $\mu F \sin\gamma$, and the component of the coupler force, F, in the direction of sliding is $F\cos\gamma$ (see Figure 3.8). When the linkage locks up ($\tan\gamma = 1/\mu$)

$$TI = \cos\gamma = \mu\sin\gamma = \mu/(1+\mu^2)^{1/2} = \xi\ .$$

Thus the worst (lowest) possible value of TI where motion is impending is ξ, not zero.

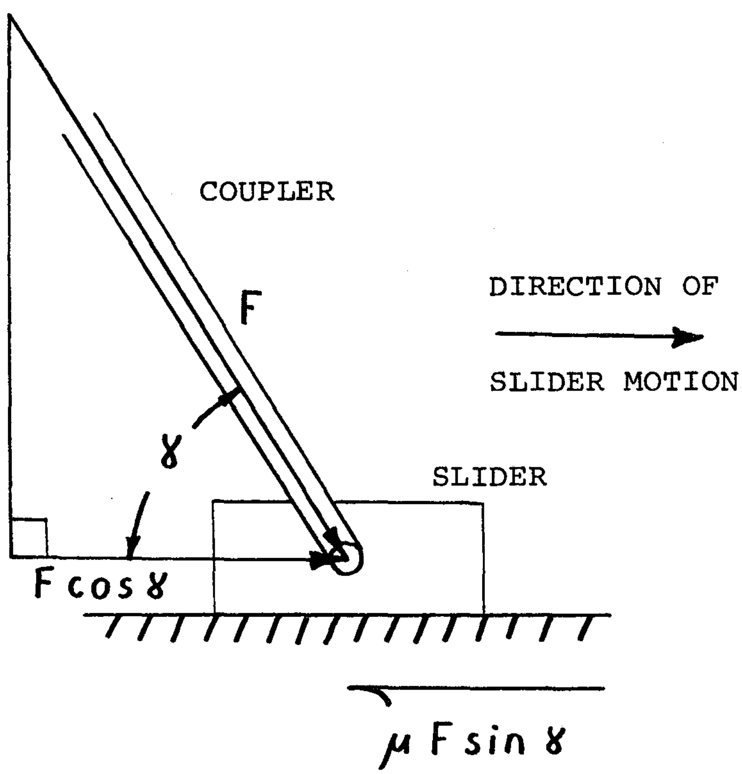


FIG. 3-8 STATIC FORCES ON THE SLIDER-CRANK SLIDER

Therefore the total constrained objective function for minimization, U, to optimize the linkage independent design variables can be expressed as

$$U = U_1 + U_2 + U_3 \qquad \text{......3.9}$$

where U_1, U_2, and U_3 are as defined above.

In this chapter a suitable expression for the constrained objective function required for the realistic evaluation of a mechanism's performance has been developed. This expression does not represent the complete function for minimization because, as is explained in Chapter II, the effects of various mechanism constraints must be added in the transformation of this constrained objective function into an unconstrained objective function.

The only constraint which is common to all the linkage synthesis problems, that which ensures linkage closure at all the precision points, is evaluated in Appendix B for the three types of linkages discussed. Thus the basic unconstrained objective function for the linkage synthesis problem is

$$\{\, U_1 + U_2 + U_3 + et\,[\min(0,C)]\} \qquad \text{.....3.10}$$

where U_1, U_2, and U_3 are respectively given by equations 3.6, 3.7, and 3.8, e and t are the scaled exterior-point transformation parameters discussed in Chapter II and Appendix A, and C is the mechanism closure constraint given in Appendix B. This unconstrained objective function is the quantity to be minimized by the minimization technique developed in Chapter V.

IV MECHANISM ANALYSIS AND BALANCING

Forces in mechanisms arise from various sources. There are forces due to the weight of parts, forces of assembly, forces from applied loads, forces from energy transmitted, friction forces, spring forces, impact forces, and forces due to change in temperature. However, the forces which are usually undesirable, and most often the source of problems, are inertia forces. The purpose of the dynamic balancing of mechanisms, and of machinery in general, is the reduction of these unwanted time varying inertia forces by the addition of suitable counterweights.

The shaking force of a mechanism is defined as the resultant of all the inertia forces acting on the frame of the mechanism. A consideration of this force is important because the frame must be strong enough to withstand it. Also the time varying shaking force may set up troublesome vibrations in the frame, and, if the mechanism is placed in a building, this force will be transmitted to the floor, and may have disturbing effects.

Although the shaking force may be zero, a shaking moment (or couple) may exist. Proper dynamic balancing of a mechanism consists of reducing both the shaking force and the shaking moment. The reduction of the shaking moment is important because the individual frame bearing stresses are directly related to the magnitude of this moment. Current techniques do not account for this shaking moment. For example, an elegant analytical procedure has been recently developed [18] using the "method of linearly independent vectors" to add counterweights to two mechanism links to make the total centre of mass of a planar four-bar mechanism stationary. This technique completely eliminates the mechanism shaking force, but does not take into account the shaking moment. As a result the individual bearing forces on the frame are as high, if not higher, than they were before balancing. Thus, not only would a large, alternating, vibration inducing shaking moment be applied to the frame, but also the critical bearing stresses might be exceeded. Therefore a dynamic balancing technique is required which will take into account both the shaking forces and shaking moments.

In Appendix D the mathematical relationships are derived (using complex number vector analysis techniques) which express the link angular accelerations and link mass centre linear accelerations in terms of the crank angular velocity and acceleration, link lengths, and the position of each link's centre of mass with respect to the link joint axes. Thus, if the mass and polar moment of inertia of each link about its centre of mass are known, then the inertia forces on each link (and thus the total inertia force or shaking force), and the shaking moment about the crankshaft axis can be directly calculated. Hence, we can now calculate the shaking force and shaking moment of an unbalanced mechanism for a given crank velocity and acceleration.

It is common industrial practice to only add counterweights to mechanism links which are pin (R) connected to the frame [19]. Thus a practical mechanism balancing technique should rely on only adding a counterweight to the crank link of a planar slider-crank mechanism, and to both the crank and follower links of a planar four-bar mechanism.

The new balancing technique developed in this thesis calculates the optimum mass, the moment of inertia about the centre of mass, and the position of the centre of mass of the counterweight(s) to be added to the crank (for both the four-bar and slider crank) and the follower (for the four-bar only) links to minimize the total shaking force and shaking moment resulting from the inertia forces and torques due to the counterweights and the original links. The total shaking force is broken down into its horizontal and vertical components, and, along with the total shaking moment, make up three dependent variables which can be combined into a constrained objective function for minimization using the inverse utility function technique developed in Chapter II.

From Appendix D-3 we obtain the relations required to evaluate the total horizontal shaking force (TSF_h), vertical shaking force (TSF_v), and shaking moment about the crankshaft axis (TSM). These relations make up the following equations for the planar four-bar linkage (see Figure 4.1).

$$\overset{+}{\rightarrow} \quad TSF_h = SFH - X_7\, a_{h_1} - X_3\, a_{h_3} \qquad \text{.....4.1}$$

$$+\uparrow \quad TSF_v = SFV - X_7\, a_{v_1} - X_3\, a_{v_3} \qquad \text{.....4.2}$$

$$\begin{aligned} + \quad TSM = SMO &+ X_7\, a_{h_1} (X_1 \sin\theta_1 + X_2 \cos\theta_1) \\ &- X_7\, a_{v_1} (X_1 \cos\theta_1 - X_2 \sin\theta_1) - X_8\, \alpha_1 \\ &+ X_3\, a_{h_3} (X_5 \sin\theta_3 + X_6 \cos\theta_3) \\ &- X_3\, a_{v_3} (f + X_5 \cos\theta_3 - X_6 \sin\theta_3) - X_4\, \alpha_3 \quad \text{...4.3} \end{aligned}$$

where SFH, SFV, and SMO are given in Appendix D-3, a_{h_1} and a_{v_1} are, respectively, the horizontal and vertical components of acceleration of the centre of mass of the crank counterweight, a_{h_3} and a_{v_3} are, respectively, the horizontal and vertical components of acceleration of the centre of mass of the follower counterweight, α_1 is the crank angular acceleration, α_3 is the follower angular acceleration, X_1 and X_2 are the co-ordinates of the crank counterweight centre of mass as shown in Figure 4.1, X_3 is the follower counterweight mass, X_4 is the follower counterweight polar moment of inertia about its centre of mass, X_5 and X_6 are the follower counterweight centre of mass components as shown in Figure 4.1, X_7 is the crank counterweight mass, and X_8 is the crank counterweight polar moment of inertia about its centre of mass. For the planar slider-crank balancing equations only the first two terms of equations 4.1 and 4.2 and the first four terms of equation 4.3 are required.

The appropriate inverse utility curves are illustrated in Figures 4.2, 4.3, and 4.4. The corresponding inverse utility functions are

$$U_1 = TSF^2_{h_{max}} \;/\; TSF^2_{h_{all}} \qquad .....4.4$$

where $TSF_{h_{all}}$ is the largest allowable value for the maximum horizontal shaking force, and

$$TSF_{h_{max}} = \{ \max(TSF_{h_i} \;, i=1,...,n) \},$$

where TSF_{h_i} is the horizontal shaking force for a given crank angle, and n is the total number of points at which TSF_h is calculated;

$$U_2 = TSF^2_{v_{max}} \;/\; TSF^2_{v_{all}} \qquad .....4.5$$

where $TSF_{v_{all}}$ is the largest allowable value for the maximum vertical shaking force, and

$$TSF_{v_{max}} = \{ \max(TSF_{v_i} \;, i=1,...,n) \},$$

where TSF_{v_i} is the vertical shaking force for a given crank angle, and n is the total number of points at which TSF_v is calculated; and

$$U_3 = TSM^2_{max} \;/\; TSM^2_{all} \qquad .....4.6$$

where TSM_{all} is the largest allowable value for the maximum shaking moment about the crankshaft axis, and

$$TSM_{max} = \{ \max(TSM_i \;, i=1,...,n) \} \;,$$

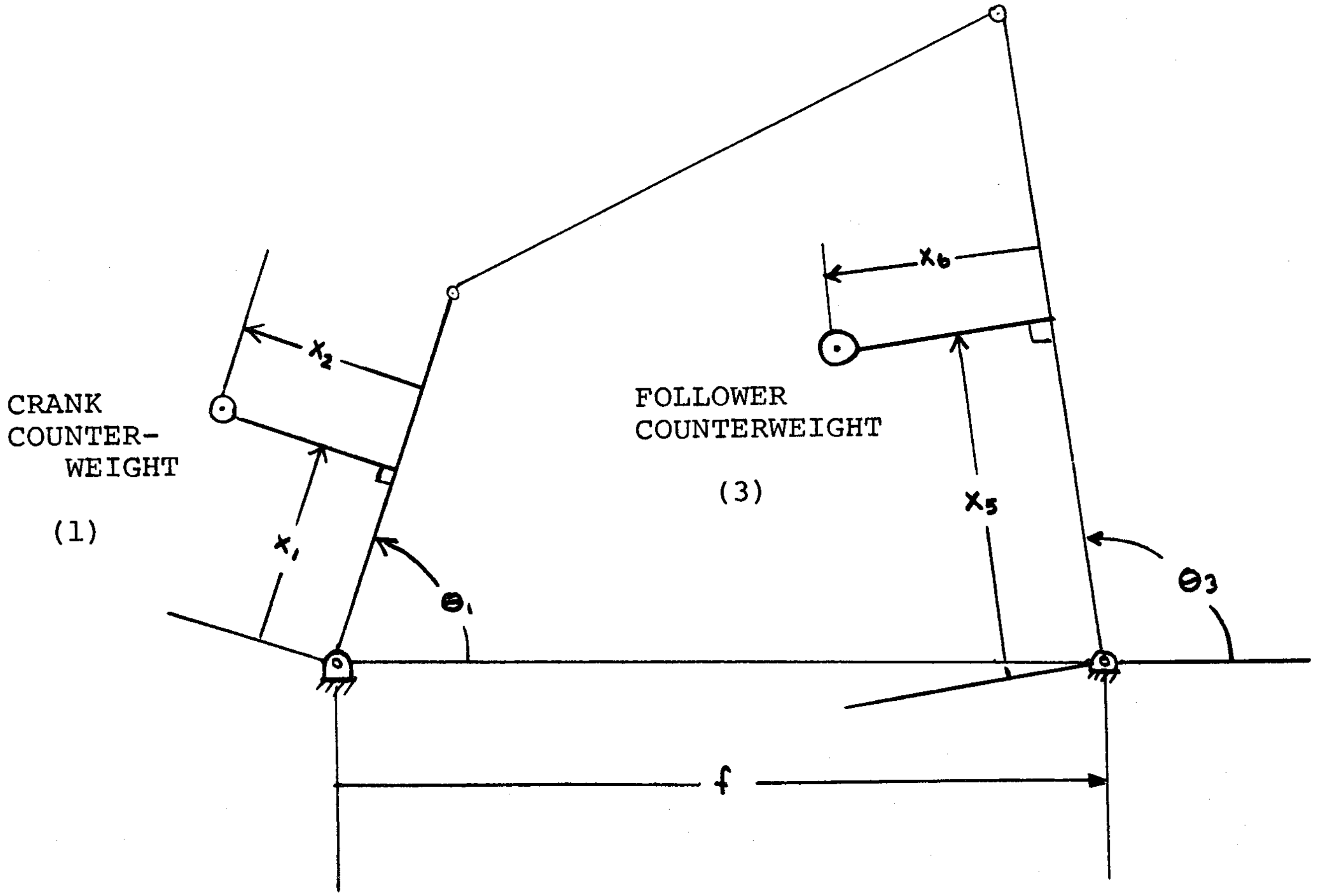


FIG. 4-1 BALANCING COUNTERWEIGHTS FOR PLANAR FOUR-BAR LINKAGE

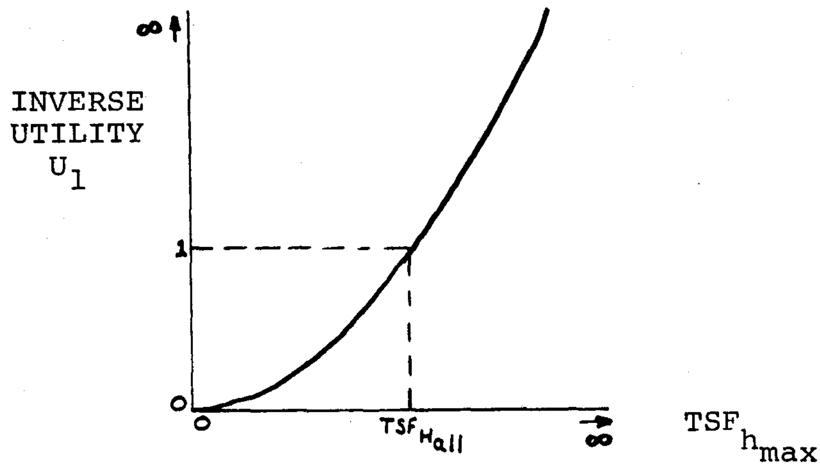


FIG. 4-2 INVERSE UTILITY CURVE FOR $TSF_{h_{max}}$

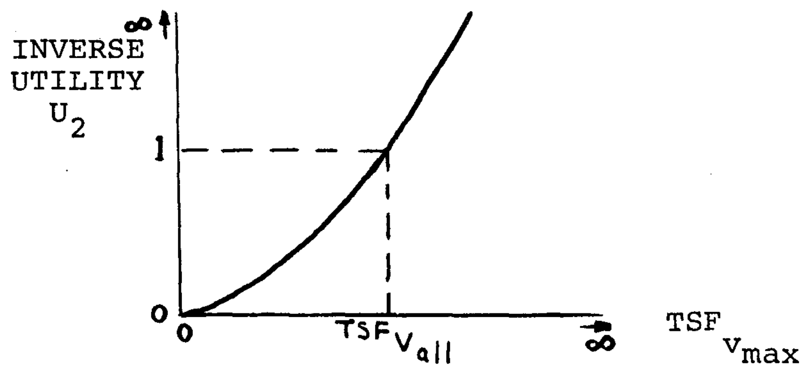


FIG. 4-3 INVERSE UTILITY CURVE FOR $TSF_{v_{max}}$

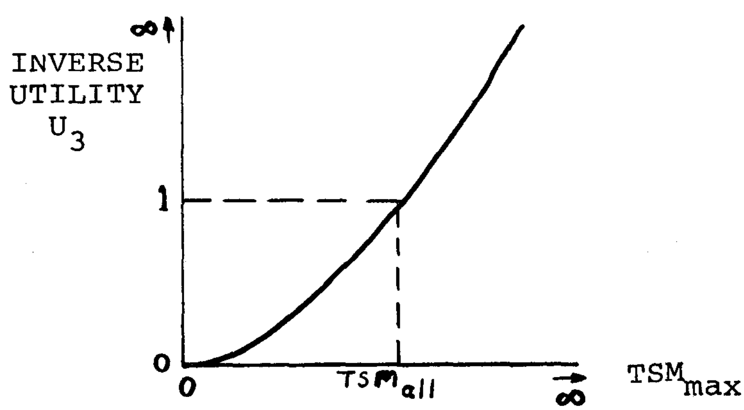


FIG. 4-4 INVERSE UTILITY CURVE FOR TSM_{max}

where TSM_i is the shaking moment about the crankshaft axis for a given crank angle, and n is the total number of points evaluated. The points i are at equal intervals of crank rotation within the range of crank motion. The total constrained objective function for minimization is then

$$U = U_1 + U_2 + U_3 \qquad .....4.7$$

where U_1, U_2, and U_3 are as defined above.

All the constraints inherently contained in the dynamic balancing problem are of the explicit type - namely that the counterweight masses and moments of inertia cannot be negative. Thus, for the general case, equation 4.7 represents the unconstrained objective function ready for minimization by the techniques of Chapter V, if the counterweight masses and moments of inertia variables are transformed according to transformation type number 1 in Chapter II. However, the balancing method used in the balancing computer program developed does allow for the addition of implicit constraints and additonal explicit constraints, if an individual balancing problem requires such constraints. (This option is also available for the mechanism synthesis problems.)

Thus a balancing technique has been developed which accounts for both shaking forces and shaking moments (as well as additional constraining factors if necessary) in the balancing of mechanisms. This method is completely general and could easily be extended to spatial mechanisms and multi-planar mechanisms (e.g. multi-cylinder automobile engines) using the vector analysis techniques of general rigid-body vector mechanics.

V OPTIMIZATION TECHNIQUE

Kowalik and Osborne [20] give an excellent description of the most important unconstrained optimization techniques currently available. These methods may be broken down into three categories - zeroth order methods which require no functional derivatives, first order methods which require the first derivative of the objective function, and second order methods which require both the first and second derivatives of the objective function. The second order methods are probably the most efficient methods when they can be used. However, they require not only that the second derivative of the objective function be known, but that it be a continuous function at all points. This differentiability requirement rules out not only the second order methods, but also the first order methods for the mechanism synthesis problem because, for the reasons stated in Chapter II, an exterior-point transformation technique is employed. The exterior-point transformation used introduces first and second derivative discontinuities at the constraint boundaries which can cause both first and second order methods considerable difficulties. Thus the choice of minimization technique is among those available zeroth order methods.

The method of successive linear approximations using truncated-to-linear Taylor series expansions of the nonlinear objective function and constraints as inputs to Dantzig's classical linear simplex method modified for use in OPTIPAC [4], fails to give solutions for mechanism synthesis problems. Its failure is probably due to the highly non-linear and discontinuous (at constraint boundaries) optimization surface of the mechanism synthesis problem.

The method of Hooke and Jeeves using a conventional exterior-point transformation (SEEK1 in OPTIPAC) works occasionally, depending much on the starting point, but tends to "hang up" easily or give poor answers. The same method using an interior-point transformation (SEEK3 in OPTIPAC) is more reliable than SEEK1, but still tends to give poor answers and takes up to three hundred per cent longer than SEEK1 when SEEK1 works.

A random-direction method developed by the author, which uses the scaled exterior-point transformation developed in Chapter II, works on all the synthesis problems it has been tried on, and obtains better answers than SEEK3 when SEEK3 works. However, a modified version of Powell's conjugate direction method without derivatives [21] has been found to be superior to this random method in all respects, and is thus the technique choosen for the minimization requirements of this thesis.

Powell's technique, as originally stated in 1964, takes the following form for the general case of an n variable minimization problem.

Initially choose $\bar{\xi}_1, \ldots, \bar{\xi}_n$ to be the n co-ordinate directions, and $\bar{p}_o$ to be the starting point.

Step (1) For r=1,2,...,n calculate λ_r to minimize $f(\bar{p}_{r-1} + \lambda_r \bar{\xi}_r)$ and set $\bar{p}_r = \bar{p}_{r-1} + \lambda_r \bar{\xi}_r$

Step (2) For r=1,2,...,n-1 replace $\bar{\xi}_r$ by $\bar{\xi}_{r+1}$, and replace $\bar{\xi}_n$ by $(\bar{p}_n - \bar{p}_o)$

Step (3) Choose λ to minimize $f\{\bar{p}_n + (\bar{p}_n + \lambda(\bar{p}_n - \bar{p}_o)\}$, replace $\bar{p}_o$ by $\bar{p}_o + \lambda(\bar{p}_n - \bar{p}_o)$, and start the next iteration from Step (1).

In general terms, the idea of the method is to calculate $\bar{p}_1, \ldots, \bar{p}_n$ by successive minimization in the directions $\bar{\xi}_1, \ldots, \bar{\xi}_n$. Then a new set of directions is defined by deleting the old $\bar{\xi}_1$, letting the new $\bar{\xi}_r$ be the old $\bar{\xi}_{r+1}$ for r=1,...,n-1, and finally defining the new $\bar{\xi}_n$ by $\bar{\xi}_n = \bar{p}_n - \bar{p}_o$. Then the new $\bar{p}_o$ is found by minimizing from $\bar{p}_n$ using the new $\bar{\xi}_n$ direction. The entire cycle from one $\bar{p}_o$ to the next $\bar{p}_o$ comprises one iteration, the new $\bar{\xi}_n$ direction being theoretically conjugate to the other (n-1) directions.

The definitions, theorems, and advantages of conjugate directions for minimization are developed in [20]. The chief advantage of a method using conjugate directions is that the minimum of a quadratic objective function can always be reached in a finite number of steps, thus indicating the probably efficiency of the method on functions of higher degree.

In his 1964 paper [21], Powell introduces a modification to his method which he claims increases his method's efficiency for problems of dimensionality five or greater. However, in a 1968 paper [22], Zangwill shows that Powell's modification is not just a nice refinement, but is necessary for problems of any dimensionality. Zangwill's proofs are quite satisfying, his paper providing considerable insight into Powell's modified algorithm for function minimization.

Powell's modified algorithm takes the following step-by-step form.

Let the co-ordinate directions $\bar{\xi}_1^1$, $\bar{\xi}_2^1, \ldots, \bar{\xi}_n^1$, an initial point $\bar{p}_o^1$, and a scalar ϵ, $1 \leq \epsilon \leq 0$ be given. Also assume the directions are normalized to unit length, so that $\| \bar{\xi}_r^1 \| = 1$*, $r=1,\ldots,n$. Set $\delta^1=1$. Go to iteration k with k=1.

* Double bars on each side of an expression indicate that the Euclidian norm of its components is to be taken.

Iteration k:

Step (1) For $r=1,2,\ldots,n$ calculate λ_r^k to minimize $f(\bar{p}_{r-1}^k + \lambda_r^k \bar{\xi}_r^k)$, and define $\bar{p}_r^k = \bar{p}_{r-1}^k + \lambda_r^k \bar{\xi}_r^k$

Step (2) Define $\alpha^k = \| \bar{p}_n^k - \bar{p}_o^k \|$ and $\bar{\xi}_{n+1}^k = (\bar{p}_n^k - \bar{p}_o^k)/\alpha^k$.

Calculate λ_{n+1}^k to minimize $f(\bar{p}_n^k + \lambda_{n+1}^k \xi_{n+1}^k)$

and set $\bar{p}_o^{k+1} = \bar{p}_{n+1}^k = \bar{p}_n^k + \lambda_{n+1}^k \xi_{n+1}^k$

Step (3) Let $\lambda_s^k = \{ \max(\lambda_r^k, r=1,\ldots,n) \}$

Case (a) If $\lambda_s^k \delta^k / \alpha^k \geq \epsilon$, let $\bar{\xi}_r^{k+1} = \bar{\xi}_r^k$ for $r \neq s$, $\bar{\xi}_s^{k+1} = \xi_{n+1}^k$, and set

$$\delta^{k+1} = \lambda_s^k \delta^k / \alpha^k$$

Case (b) If $\lambda_s^k \delta^k / \alpha^k < \epsilon$, let

$\bar{\xi}_r^{k+1} = \bar{\xi}_r^k$, $r=1,\ldots,n$, and set

$$\delta^{k+1} = \delta^k$$

Go to iteration k with (k+1) replacing k.

In the above procedure δ^k is the determinant of the matrix in which the column vectors are the set of directions ξ_r^k, $r=1,\ldots,n$. (Zangwill [22] uses a neat inductive proof to show this fact.) Thus the expression $\lambda_s^k \delta^k / \alpha^k$ is the determinant of the matrix of the new set of directions

$\bar{\xi}_r^{k+1}$, r=1,...,n, where $\bar{\xi}_{n+1}^{k}$ has replaced $\bar{\xi}_s^{k}$. It is common knowledge [23][24] that if the determinant is non-zero of a matrix in which the columns represent given vectors, then these vectors are linearly independent. Also, if these vectors are all normalized with respect to one, then the magnitude of the determinant of the matrix reaches its maximum of one when all the vectors are mutually orthogonal. Thus the magnitude of the determinant of the matrix made up of any general set of linearly independent normalized vectors will lie between zero and one. Step (3) of the above procedure is then a test to insure a certain amount of linear independency (or nearness to orthogonality) in the directions $\bar{\xi}_r^{k+1}$, r=1,...,n. Note that if two directions, $\bar{\xi}_r^{k+1}$, were allowed to become linearly dependent, then it would be quite possible that the full optimization hyperspace could not be spanned in a minimization sequence; hence, the optimum point would not be reached even in an infinite number of steps.

In general it is an advantage to be able to search for a minimum in both the negative and positive $\bar{\xi}_r^{k}$ directions. However, the admission of negative values of λ_r^k, which correspond to minimums in the negative $\bar{\xi}_r^{k}$ directions, will cause step (3) in the Powell-Zangwill algorithm to be unsatisfactory, since the maximum of λ_r^k, r=1,...,n, will not necessarily maximize the magnitude of the determinant of the $\bar{\xi}_r^{k+1}$ directions. Thus the method's efficiency is significantly reduced unless the following modification is introduced: replace the first statement of Step (3) by

$$\lambda_s^k = \{\max(|\lambda_r^k| \ , \ r=1,\ldots,n)\}$$

where $|\lambda_r^k|$ is the absolute value of λ_r^k. (Note that in checking for linear dependancy, the negative of a direction has the same meaning as the direction itself.)

This modification creates an efficient minimization routine which uses both the positive and negative $\bar{\xi}_r^k$ directions. This decreases the computation time for convergence by a minimum at ten per cent, and can even mean the difference between convergence and non-convergence for many problems. Thus λ_r^k has been replaced by $|\lambda_r^k|$ in the minimization technique used in the computer program of Appendix G.

As a result of experimentation with the minimization method, a value of $.5/n^{.5}$ for ϵ, where n is the number of independent design variables, has been found to be efficient.

A very important part of the modified Powell-Zangwill method is the minimization along a vector, $\bar{\xi}_r^k$, procedure. The following procedure is presented as an efficient method to accomplish this.

Let $.5 \leq \rho_0 \leq 1.5$, $\mu \geq 1$, and $\delta_o > 0$ be given.

Step (1) Set $\rho=\rho_o$ and $\delta=\delta_o$ and let $\eta= \max(\|\bar{p}_r^k\|, 1)$.

Calculate $\bar{t}_i = \bar{p}_r^k + \eta\, a_i\ \bar{\xi}_r^k$, $i=1,2,3$

where a_i is a random number between -3 and +3.

Then $f_1 = \{\min[g(\bar{p}_r^k), (g(\bar{t}_i)\ , i=1,2,3)]\}$

where g is the objective function to be minimized. Let $\bar{x}$ be the vector corresponding to f_1.

Step (2)

Case (a) If $f_1 \geq g(\bar{x} + 2\ \delta\ \bar{\xi}_r^k)$, then g is evaluated at $\bar{x} + 4\ \delta\ \bar{\xi}_r^k$, $\bar{x} + 8\ \bar{\delta}_r^k$, $\ldots$, $\bar{x} + 2^i\ \delta\ \bar{\xi}_r^k$ until $g(\bar{x} + 2^{(i-1)}\delta\ \bar{\xi}_r^k) <$ $g(\bar{x} + 2^i\ \delta\ \bar{\xi}_r^k)$.

Case (b) If $f_1 \geq g(\bar{x} - 2\ \delta\ \bar{\xi}_r^k$, then g is evaluated at $\bar{x} - 4\ \delta\ \bar{\xi}_r^k$, $\ldots$, $\bar{x} - 2^i\ \delta\ \bar{\xi}_r^k$

until $g(\bar{x} - 2^{i-1}\ \delta\ \bar{\xi}_r^k) < g(\bar{x} - 2^i\ \delta\ \bar{\xi}_r^k)$

Case (c) If $g(\bar{x} - 2\ \delta\ \bar{\xi}_r^k) > f_1 < g(\bar{x} + 2\ \delta\ \bar{\xi}_r^k)$

Case (i) If $2\ \delta_o\ /\ \delta < \mu$ go back to the start of Step (2) with $\rho = 1$

Case (ii) If $2\ \delta_o\ /\ \delta \geq \mu$ set $f_2 = f_1$,

$f_1 = g(\bar{x} - 2\ \delta\ \bar{\xi}_r^k)$,

$f_3 = g(\bar{x} + 2\ \delta\ \bar{\xi}_r^k)$, $\alpha = o$, and

$\kappa = 2\,\delta$, and go directly to Step (4).

Step (3) Set $\kappa = 2^{i-2}\,\delta$

For Case (a) of Step (2): Let

$f_3 = g(\bar{x} + 3\,\delta\,2^{i-2}\,\bar{\xi}_r^k)$, and

$f_4 = g(\bar{x} + 2^{i-1}\,\delta\,\bar{\xi}_r^k)$

Case (i) If $f_3 > f_4$, then $f_2 = f_4$,

$f_1 = g(\bar{x} + 2^{i-2}\,\delta\,\bar{\xi}_r^k)$, and

$\alpha = 2^{i-1}\,\delta$

Case (ii) If $f_3 \leq f_4$, then $f_1 = f_4$, $f_2 = f_3$,

$f_3 = g(\bar{x} + 2^{i}\,\delta\,\bar{\xi}_r^k)$, and $\alpha = 3\,\delta\,2^{i-2}$

For Case (b) of Step (2): Let

$f_1 = g(\bar{x} - 3\,\delta\,2^{i-2}\,\bar{\xi}_r^k)$ and

$f_4 = g(\bar{x} - 2^{i-1}\,\delta\,\bar{\xi}_r^k)$

Case (i) If $f_1 > f_4$, $f_2 = f_4$,

$f_1 = g(\bar{x} - 2^{i-2}\,\delta\,\bar{\xi}_r^k)$ and $\alpha = -\,2^{i-1}\,\delta$

Case (ii) If $f_1 \leq f_4$, then $f_2 = f_1$, $f_3 = f_4$,

$f_1 = g(\bar{x} - 2^{i}\,\delta\,\bar{\xi}_r^k)$ and $\alpha = -\,3\,\delta\,2^{i-2}$

Step (4) Let

$\beta = \alpha - (\kappa/2)\,(f_3 - f_1)\,/\,(f_3 + f_1 - 2\,f_2)$

Then $\bar{p}_{r+1}^k = \bar{x} + \rho\,\beta\,\bar{\xi}_r^k$

The above procedure is, roughly speaking, a bracketing procedure which ultimately uses a quadratic approximation to locate the minimum. Step (1) starts from point $\bar{p}_r^k$ and generates random steps along the direction $\bar{\xi}_r^k$ in order to avoid isolated local optimums and to save computation time if the initial point $\bar{p}_r^k$ is a long distance from the minimum along $\bar{\xi}_r^k$. Step (2) involves accelerated steps in the positive (Case (a)) and negative (Case (b)) $\bar{\xi}_r^k$ direction to evaluate three points (the last three points calculated) which bracket the minimum. Case (c) of Step (2) occurs when steps of magnitude plus and minus 2δ bracket the minimum. In this case the relaxation multiplier ρ is set equal to 1 and the initial step interval, δ, is halved and the method returns to the evaluation of Cases (a) and (b) unless $2\ \delta_o/\delta$ already exceeds the specified number μ, in which case the method goes directly to Step (4). Step (3) is a combination of reducing the size of the bracket and equalizing the intervals between the three points in the bracket. (Note that if i=2, then the intervals are already of equal size 2δ and Step (3) is not necessary.) Step (4) is an application of a quadratic interpolating polynomial to the three equally spaced bracketing points to determine the approximate minimum point within the bracket which will be the starting point for the next directional search in the $\bar{\xi}_{r+1}^k$ direction. Note that Step (4) locates the minimum only if $\rho = 1$. Setting ρ_o, the initially relaxation multiplier, less than one corresponds to underrelaxing, and greater than one to overrelaxing. The advantage of the successive halving of δ in Case (c) of Step (2) is that an initially large value of δ, δ_o, can be used to speed up convergence to the minimum without losing accuracy when the minimum is reached. Thus a "minimization along a line" technique has been developed which has many features not available in other existing techniques.

The minimization stopping criterion is a combination relative error-absolute error test on the variable with the largest change from one minimization iteration to the next. The procedure takes the following form.

Set $q > o$

Step (1) Calculate $d_i = | p_{o_i}^{k+1} - p_{o_i}^{k} |$, $i=1,\ldots,n$ where n is the number of independent design variables.

Step (2) Calculate $s_i = \{ d_i/\max(1,p_{o_i}^{k+1}) \}$, $i=1,\ldots,n$

Step (3) $T = \{\max(S_i \ , \ i=1,\ldots,n)\}$

Step (4) If $T < q$, then convergence is assumed; If $T \geq q$, then a new minimization iteration is required.

This procedure gives a precision of approximately $\log_{10}(.1/q)$ decimal places if $p_{o_i}^{k+1}$ is less than one, and $\log_{10}(.1/q)$ significant figures if $p_{o_i}^{k+1}$ is greater than one.

Examination of the contour plots of the optimization hypersurface for many mechanism synthesis problems reveals that the surface is irregular, and that there are many regions with local optimums isolated completely by zones of infeasibility from regions with local optimums of lower value. Thus it is quite possible that the optimum point determined by the minimization technique from a given starting point does not represent the local optimum of lowest value, the global optimum. Hence, a valuable feature in any minimization technique would be the self generation of new starting points, and comparing the minimums obtained from these starting points to determine the lowest local optimum. The optimization technique developed for this thesis does just that; it keeps generating new random starting points (which are based on perturbed values of the previous minimum point) until the minimum corresponding to the last starting point exceeds or equals the lowest previous minimum. This procedure is an attempt to give the designer greater confidence in the optimality of the computer chosen optimum values for the independent design variables.

Thus, a procedure has been developed for efficiently minimizing an objective function which expresses, using inverse utility curves, the design requirements of a given mechanism. The independent design variables corresponding to the local minimum value of the objective function may, or may not, actually represent the true optimum values, depending on whether the local minimum obtained is a global minimum or not. However, the method presented does do a good job at trying to obtain the global minimum of an unconstrained objective function for a mechanism synthesis problem.

VI USER-ORIENTED MECHANISM DESIGN PROGRAM

It is basic human nature that the easier to use something is, the more it will be used. Hence, in designing a computer program which embodies the principles developed in the previous five chapters, an attempt has been made to reduce as much as possible the basic input requirements of the program, as well as trying to present the output in its most meaningful form.

The program, which consists of seventeen FORTRAN source subroutines which are interrelated, but may be called separately if desired, requires approximately 7700 storage locations in the central memory of the CDC 6400 computer. Since the program can be easily segmented into its synthesis, analysis, balancing, and output plotting routines, the program can be run efficiently with overlays on even small computers like the IBM 1130 (which is a common small computer in business and industry).

There are basically sixteen things that this program can do. It can

1. synthesize a planar four-bar linkage for function generation;
2. synthesize a planar four-bar linkage for coupler-point curve generation;
3. synthesize a planar slider-crank mechanism for function generation;

4. synthesize a planar slider-crank mechanism for coupler-point curve generation;
5. synthesize a spatial four-bar linkage for function generation;
6. determine the angular velocities and accelerations of a planar four-bar's coupler and follower links for up to thirty-six positions of the crank link;
7. determine the linear velocities and accelerations of any four-bar coupler-point, and the linear acceleration of any four-bar crank-point or follower-point at up to thirty-six positions of the crank link;
8. determine the linear velocities and accelerations of any planar slider-crank coupler-point (including the slider) or crank-point at up to thirty-six positions of the crank link;
9. determine the angular velocities and accelerations of a planar slider-crank coupler link at up to thirty-six positions of the crank link;
10. determine the optimum crank and follower counter-weights to dynamically balance a planar four-bar linkage;
11. determine the optimum crank counterweight to balance a planar slider-crank linkage;
12. determine the feasibility of any planar four-bar, planar slider crank, or spatial four-bar linkage;

13. plot the structural error versus the crank angle for any planar four-bar, planar slider-crank, or spatial four-bar linkage;
14. plot the coupler curve for any planar four-bar or planar slider-crank coupler-point;
15. plot the horizontal shaking force, vertical shaking force, and shaking moment about the crankshaft axis versus the crank angle for the planar four-bar and planar slider-crank mechanisms; and
16. produce a two-dimensional contour map of the optimization hypersurface (including the intersections of constraint surfaces) with respect to any two independent design variables for the mechanism synthesis and balancing problems.

The number of input computer cards required for any given problem should rarely exceed ten. Thus many problems can be solved with a minimum of input using this completely self-sufficient computer program.

A complete program documentation, the required procedures for making up program input and interpreting program output, and a program FORTRAN source listing are included in Appendices E and F.

The prime considerations in the development of this computer program have been the simplification of input and the completeness of output. Unfortunately this has meant the sacrifice of computer time and space in some of the subroutines, especially with respect to providing data for the output plotting routines. However, considerable time has been spent trying to avoid such wastes as much as possible, so that the program produced is as efficient as possible while consistent with the primary aims stated above.

VII SAMPLE PROBLEMS

In this chapter three sample problems which can be handled by the computer program developed for this thesis are discussed*. These problems are designed to illustrate some of the capabilities and advantages of the concepts developed in this thesis rather than illustrate short run times or dramatic convergence from a given starting point. Thus, though the run times are longer than normal and the improvements quite small in some cases, the results are interesting, and show the importance of the new ideas presented in this thesis.

PROBLEM NUMBER 1 - Slider-Crank Linkage as a Function Generator

This test problem is designed to specifically show the interplay of two of the factors which make up the objective function for minimization. The problem also shows up some of the special features in the optimization routine which take over if an extremely poor starting point is given or generated.

* See Appendices E and F for an explanation of the required input and output variables and format.

The problem is that of generating the cosine function from zero to 2π for a range of crank motion of 360 degrees and a range of slider motion of 4. ± .1 inches using a slider-crank linkage where the upper sign (see Appendix D) in the expression for the link angles is used (ICASE equals +1). This problem can be satisfied exactly from a structural error standpoint by using a crank length (X_1) of 1.95 inches, a coupler length (X_2) of infinite length, a slider eccentricity (X_3) of zero, and a mid-range crank angle* (X_4) of π radions (see Figure 7.1). However, for the multifactor synthesis program, if an output function structural error scaling factor (SCAL2) of .01 in the cosine function and a link length scaling factor (SCAL3) of 10. inches are used, it is interesting to see the interplay of the structural error and link length factors in the objective function. (The transmissibility index has little effect on the problem except to provide a slight bias towards increasing the coupler length.) The FORTRAN input cards required for this problem are shown in Figure 7.2.

* The mid-range crank angle is defined as the crank starting angle plus one half the crank range of motion. Note that the crank starting angle is printed out at the bottom of Figure 7.4; the range of crank motion is fixed by the user in the program input.

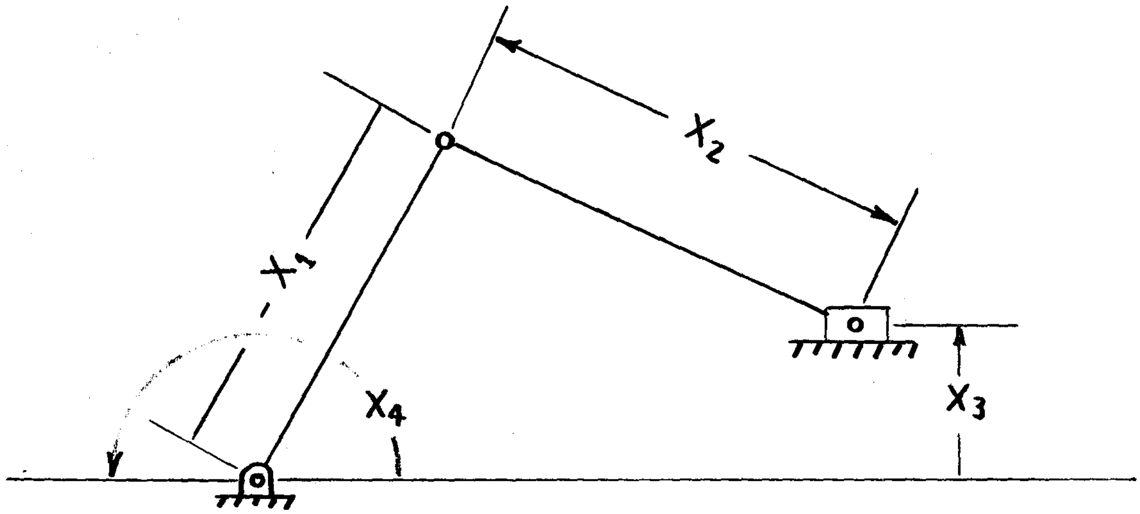


FIG. 7.1 PLANAR SLIDER-CRANK

INPUT FOR SLIDER-CRANK FUNCTION SYNTHESIS

```
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON /STRTPT/STRTPT(10)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      DATA RNGI/360./,RNGO/4./,METHOD/3/,ICASE/1/,NPP/15/,TITE/.1/,
     1IEXCO/1/,XMIN/0./,XMAX/6.2831853072/,CFRIC/.3/,ISYM/0/,
     2SCAL1/.6428/,SCAL2/.01/,SCAL3/10./,STRTPT/2.,10.,0.,3.14159/
      CALL LINK(1,.0003,1.,2.5,.0001)
      STOP
      END
      FUNCTION FUNSYN(X)
      FUNSYN=COS(X)
      RETURN
      END
```

FIG. 7.2

The actual computer output is shown in Figures 7.3 to 7.5. Figures 7.3 and 7.4 show the results of the optimization procedure. Since three-figure accuracy is requested in the input (PREC = .0001), the optimum crank length, coupler length, slider eccentricity, and mid-range crank angle are respectively 1.95 inches, 29.0 inches, 0. inches, and 3.14 radians. The values of X_1, X_3, and X_4 are exactly the expected optimum values; the value of X_3 represents the optimum compromise (to three significant figures) between the maximum acceptable structural error and the maximum acceptable link length, if a structural error of .01 and a link length of 10. represent equal utility values of 1.

Note that the minimization routine has considerable difficulty in starting from the new randomly generated starting point, which is a poor starting point since both the mechanism closure (C_1) and range of output motion (C_3) constraints are violated at this point. The method then unfortunately hangs up in the infeasible region and, after a couple of trials at increasing the constraint term multiplier (t in Appendix A), decides that a new starting point should be generated. The minimization then proceeds smoothly from this new point, and finds a local optimum of higher objective function value than the previous local optimum determined. This then terminates the minimization process.

```
SYNTHESIS OF A PLANAR SLIDER-CRANK FUNCTION GENERATOR MECHANISM
WHERE ICASE=  1

THE STARTING DESIGN VARIABLE VALUES ARE
          2.0000          10.000          0.              3.1416

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   19.388

DESIGN VARIABLE VALUES ARE
          1.9501          28.962          -6.15806E-05    3.1416

THE CONSTRAINT VALUES ARE
          8.35291E+05     942.48          1.08516E-04


THE NEW STARTING DESIGN VARIABLE VALUES ARE
          3.3020          1.2618          1.89796E-05     4.20375E-02

THE OPTIMUM VALUE DETERMINED IS INFEASIBLE
CONSTRAINT NUMBER  3  HAVING THE VALUE    -3.6791
THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM STARTED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
          -.11035         7.0004          .29267          4.17941E-02

THE OPTIMUM VALUE DETERMINED IS INFEASIBLE
CONSTRAINT NUMBER  3  HAVING THE VALUE    -3.6785
THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM STARTED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
          -.11066         6.9979          .29262          4.17941E-02

MINIMIZATION ROUTINE HAS HUNG UP IN AN INFEASIBLE REGION
INDICATIONS ARE THAT THE CONSTRAINTS ARE SUCH THAT A FEASIBLE REGION
CANNOT BE REACHED FROM THE CURRENT LOCATION
THEREFORE A NEW STARTING POINT HAS BEEN GENERATED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
          -.25516         -4.8972         .87304          .11929

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   231.49

DESIGN VARIABLE VALUES ARE
          -1.9359         6.3625          .77114          .12729

THE CONSTRAINT VALUES ARE
          33297           1243.9          4.95230E-04


     TRANSMISSIBILITY INDEX FUNCTION IS  8.65760E-06
```

FIG. 7-3

```
     STRUCTURAL ERROR FUNCTION IS    9.7352
     LINK LENGTH FUNCTION IS    9.6529
     FINAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS    19.388
     DESIGN VARIABLE VALUES ARE
              1.9501            28.962            -6.15806E-05       3.1416
  NUMBER OF FUNCTION CALLS WAS    1408
  TOTAL EXECUTION TIME FOR METHOD  3 IS     12.940  SECONDS
```

FIG. 7-4

STRUCTURAL ERROR PLOT FOR PLANAR SLIDER-CRANK FUNCTION SYNTHESIS

MAXIMUM STRUCTURAL ERROR (YA-YD) IS 2.70006E-13
MINIMUM STRUCTURAL ERROR IS -3.37030E-02

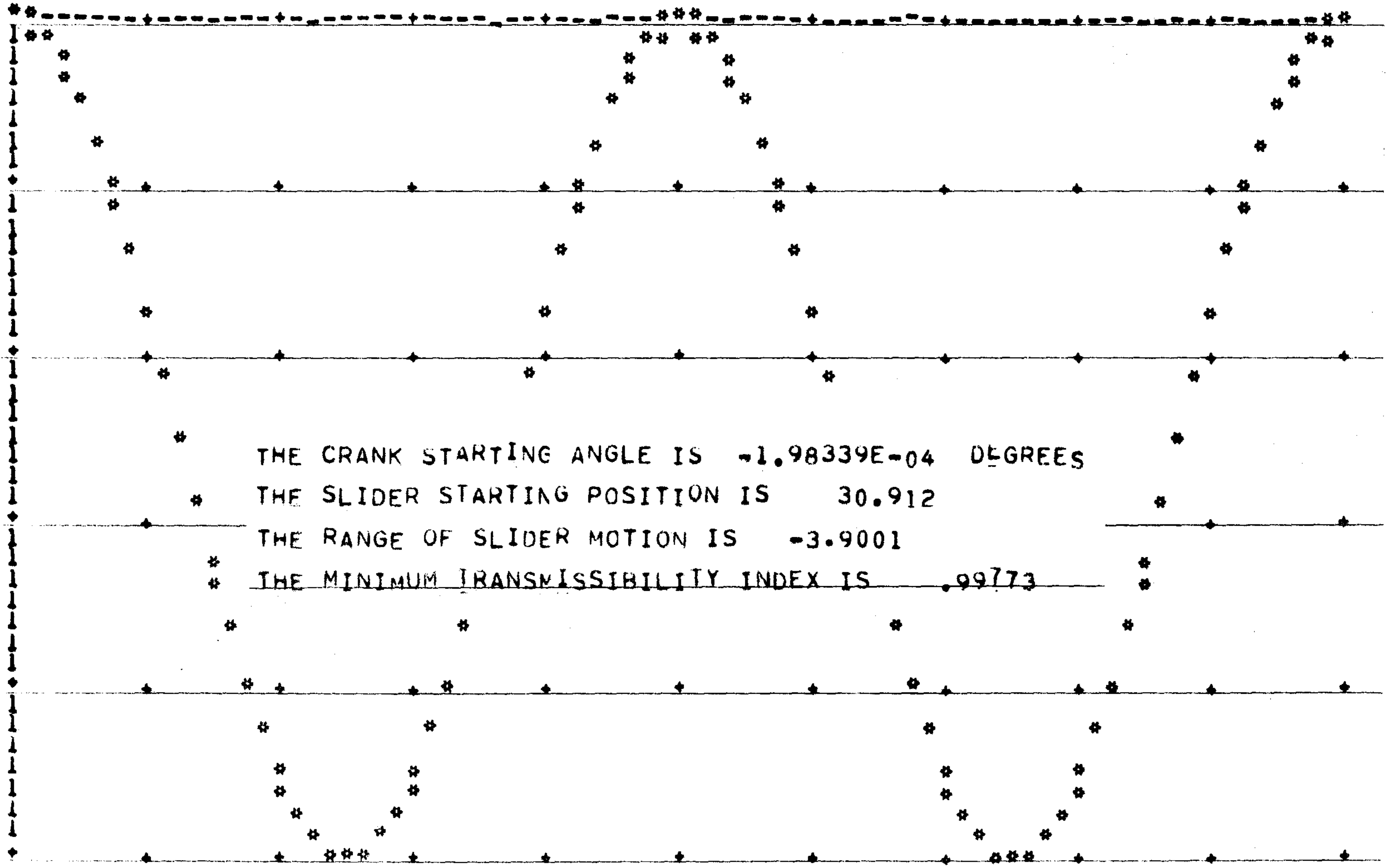


FIG. 7-5

A preliminary analysis of the optimum mechanism is then automatically done, the results being shown in Figure 7.5. Note that the structural error shown is relative to the mid-range point ($X=\pi$) being exact. If, in setting up the actual mechanism, the one-eighth range point ($X=\pi/4$) were made exact, then the structural error plot would be equally disposed on both sides of the horizontal axis. This simple change in the scale base point enables a halving of the structural error in this example. Thus a similar investigation should always be made in the setting up of the scales of any actual function generating mechanism.

PROBLEM NUMBER 2 - Symmetrical Function Generation

This is a problem to show the advantages of the spatial four-bar linkage over the planar four-bar linkage for the generation of a symmetrical function ($y=x^2$) where x varies from -1 to +1). It also shows what improvements can be made in the linkage design by including the maximum link length and transmissibility index factors, as well as the structural error factor, in the optimization objective function.

Figure 7.6 shows the FORTRAN input cards required for both the planar and spatial four-bar optimizations. Figure 7.7 shows the preliminary computer analysis results for the planar four-bar function generator using the optimum linkage parameters determined by Freudenstein [2] using only the structural error

INPUT FOR SPATIAL FOUR-BAR FUNCTION SYNTHESIS

```
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON /STRTPT/STRTPT(10)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      DATA METHOD/5/,ICASE/-1/,NPP/ 9/,IEXCO/1/,ISYM/0/,
     1XMIN/-1./,XMAX/1./,RNGI/200./,RNGO/100./,TITE/30./,SCAL1/.5/,
     2SCAL2/.01/,SCAL3/10./,STRTPT/-.449,.0929,0.,0.,4.71,.1797,-.822/
      CALL LINK(1,.0003,1.,2.5,.0001)
      STOP
      END
      FUNCTION FUNSYN(X)
      FUNSYN=X*X
      RETURN
      END
```

INPUT FOR PLANAR FOUR-BAR FUNCTION SYNTHESIS

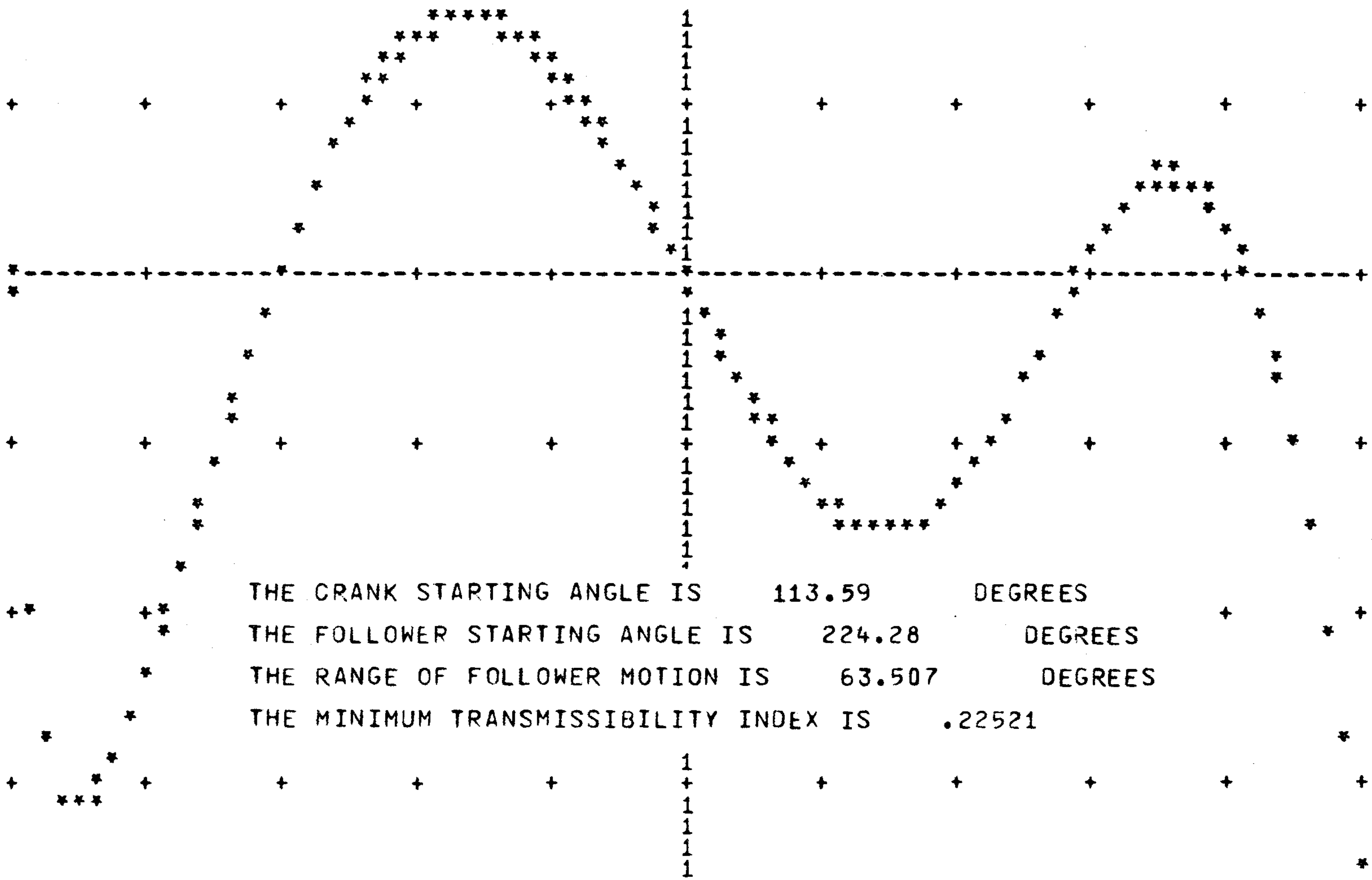
```
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON /STRTPT/STRTPT(10)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      DATA METHOD/1/,ICASE/-1/,NPP/ 9/,IEXCO/1/,ISYM/0/,
     1XMIN/-1./,XMAX/1./,RNGI/90./,RNGO/60./,TITE/20./,SCAL1/.5/,
     2SCAL2/.01/,SCAL3/10./,STRTPT/-.6102,.5656,.3804,2.768/
      CALL LINK(1,.0003,1.,2.5,.0001)
      STOP
      END
      FUNCTION FUNSYN(X)
      FUNSYN=X*X
      RETURN
      END
```

FIG. 7-6

FIG. 7-7

SYNTHESIS OF A PLANAR 4-BAR FUNCTION GENERATOR MECHANISM
WHERE ICASE= -1

MAXIMUM STRUCTURAL ERROR (YA-YD) IS 3.44648E-02
MINIMUM STRUCTURAL ERROR IS -7.66839E-02



factor. Figures 7.8 to 7.10 show the results of the program synthesis and preliminary analysis of the computer synthesized optimum linkage. Amazingly, not only does the program increase the minimum transmissibility index from .225 to .471 (effectively more than halving the mechanical error - see Appendix C), but also reduces the maximum structural error in the output function, x^2, from .077 to .037 (i.e. approximately by one half). Thus considerable improvement is made on Freudenstein's "optimum" linkage parameters for this problem. (See Appendix F for the meanings of the variables in the computer output listing.)

However, for the spatial four-bar the optimum linkage parameters determined by Hartenberg [16] (preliminary analysis shown in Figure 7.11), using extensive hand calculations, cannot be significantly improved by the computer program. In fact, because the maximum allowable structural error scaling factor (SCAL2) is set at only .01 for this problem, and the minimum allowable transmissibility index scaling factor (SCAL1) is set at .5, the maximum structural error is increased from .0013 to .0023 in order to effect an improvement in the transmissibility index from .586 to .608 (see Figures 7.12 to 7.14). (If SCAL2 were set equal to .001, then the minimum transmissibility index would probably decrease in order to effect a lowering in the maximum structural error.) Thus, for this spatial four-bar example little improvement can be

```
SYNTHESIS OF A PLANAR 4-BAR FUNCTION GENERATOR MECHANISM
WHERE ICASE= -1

THE STARTING DESIGN VARIABLE VALUES ARE
          -.61020           .56560            .38040            2.7680

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   10.238

DESIGN VARIABLE VALUES ARE
          -.54228           .57659            .37849            2.7671

THE CONSTRAINT VALUES ARE
           85.498           979.93            .15275


THE NEW STARTING DESIGN VARIABLE VALUES ARE
          -1.2258           .30798            1.0626            6.5558

THE OPTIMUM VALUE DETERMINED IS INFEASIBLE
CONSTRAINT NUMBER  3  HAVING THE VALUE    -.68531
THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM STARTED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
        -1.26526E-02        .31288            1.3128            .27900

THE OPTIMUM VALUE DETERMINED IS INFEASIBLE
CONSTRAINT NUMBER  3  HAVING THE VALUE    -.68429
THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM STARTED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
        -2.23409E-02        .32296            1.3088            .49501

MINIMIZATION ROUTINE HAS HUNG UP IN AN INFEASIBLE REGION
INDICATIONS ARE THAT THE CONSTRAINTS ARE SUCH THAT A FEASIBLE REGION
CANNOT BE REACHED FROM THE CURRENT LOCATION
THEREFORE A NEW STARTING POINT HAS BEEN GENERATED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
         1.23429E-02    -5.34240E-02          .50434            .23656

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   93.959

DESIGN VARIABLE VALUES ARE
           .72239           .56706           -.44145            .41936

THE CONSTRAINT VALUES ARE
           15.604           1214.7           7.75631E-02


     TRANSMISSIBILITY INDEX FUNCTION IS   1.2664
```

FIG. 7-8

```
STRUCTURAL ERROR FUNCTION IS   8.9718

LINK LENGTH FUNCTION IS  0.

FINAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   10.238

DESIGN VARIABLE VALUES ARE
     -.54228          .57659          .37849          2.7671
NUMBER OF FUNCTION CALLS WAS  10448

TOTAL EXECUTION TIME FOR METHOD  1 IS   124.768  SECONDS
```

FIG. 7-9

STRUCTURAL ERROR PLOT FOR PLANAR 4-BAR FUNCTION SYNTHESIS

MAXIMUM STRUCTURAL ERROR (YA-YD) IS 3.35830E-02
MINIMUM STRUCTURAL ERROR IS -3.73173E-02

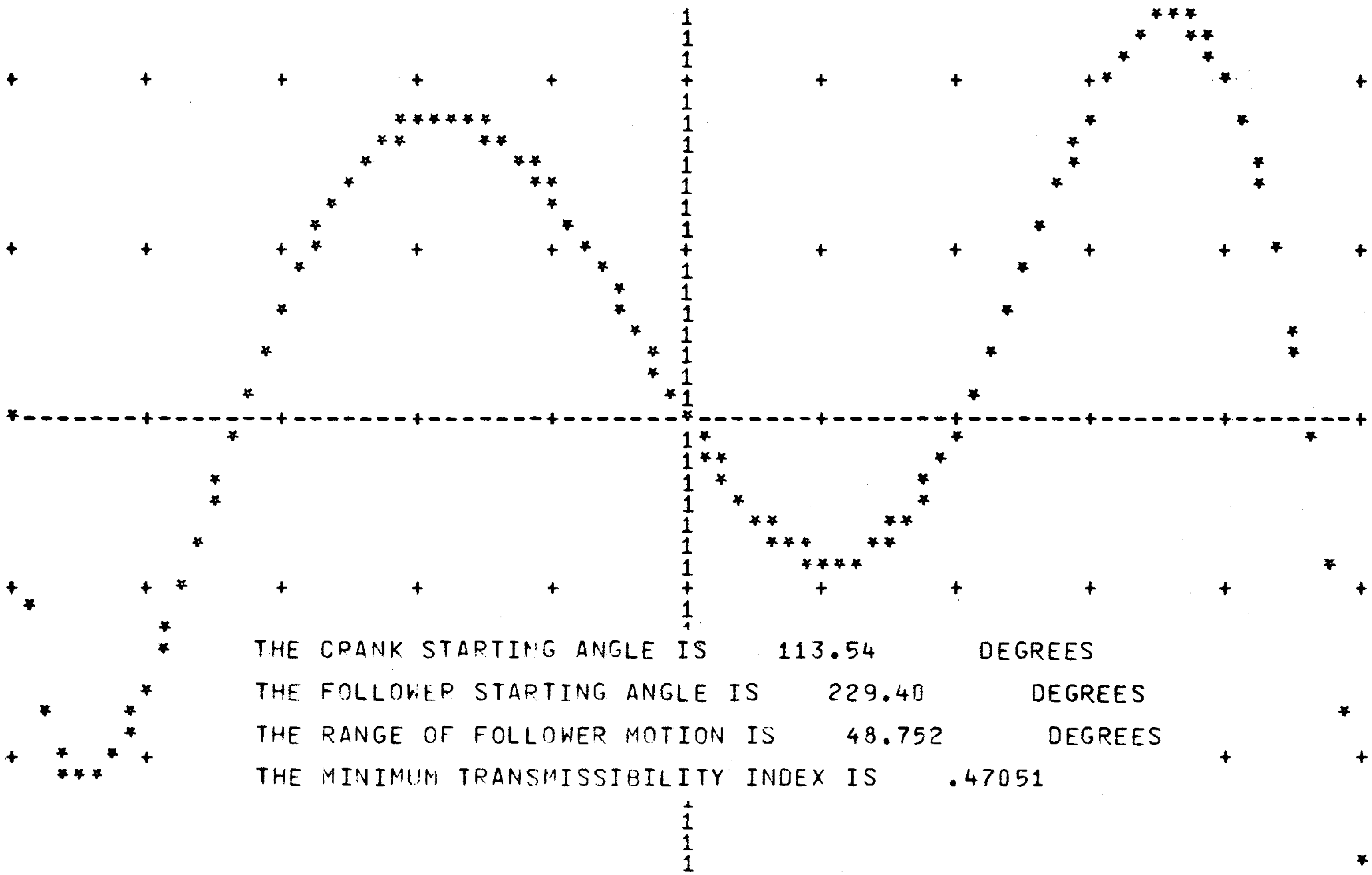
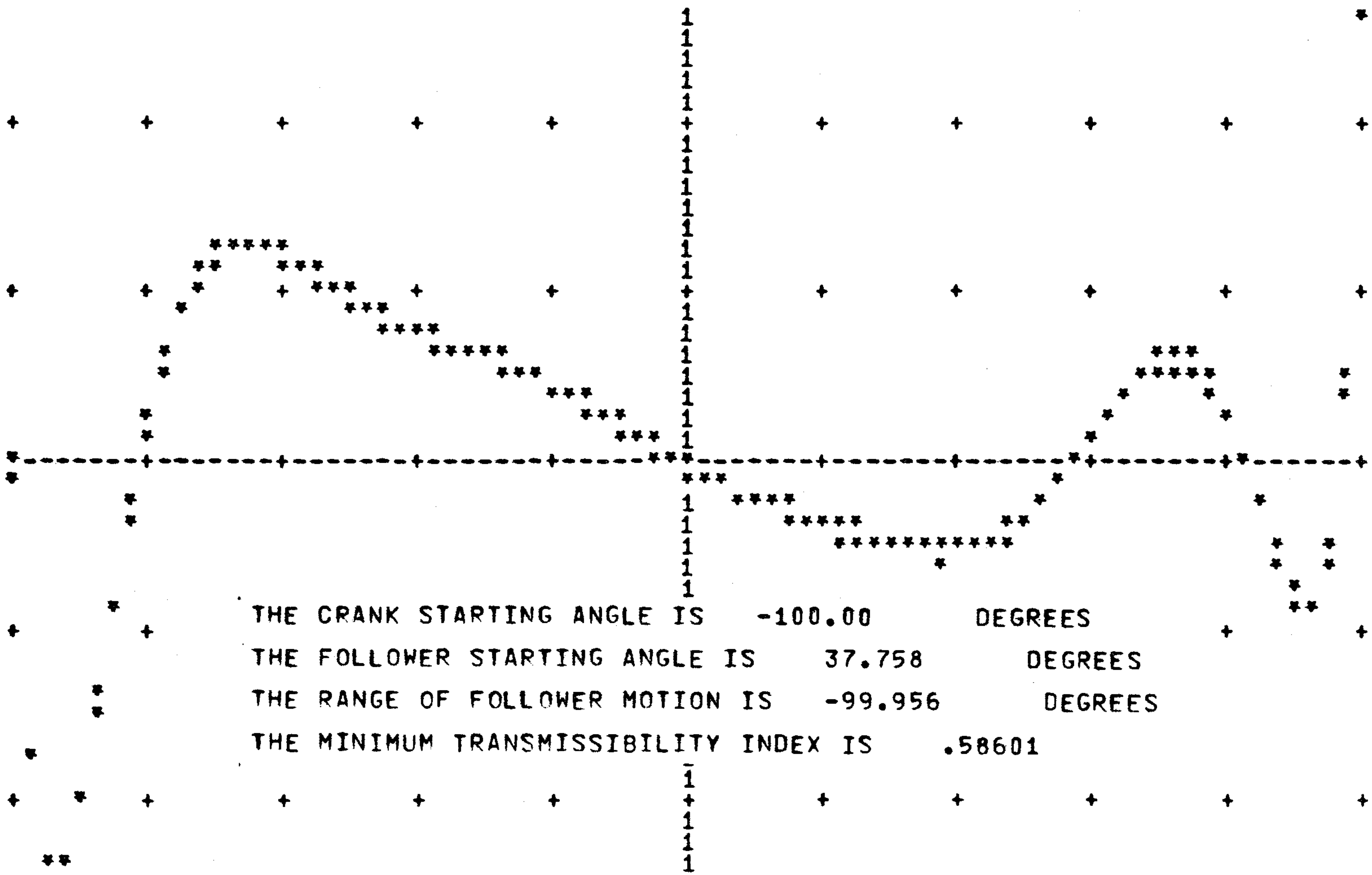


FIG. 7-10

FIG. 7-11

SYNTHESIS OF A SPATIAL 4-BAR (RGGR) FUNCTION GENERATOR MECHANISM
WHERE ICASE= -1

MAXIMUM STRUCTURAL ERROR (YA-YD) IS 1.26402E-03
MINIMUM STRUCTURAL ERROR IS -1.10265E-03



THE CRANK STARTING ANGLE IS -100.00 DEGREES

THE FOLLOWER STARTING ANGLE IS 37.758 DEGREES

THE RANGE OF FOLLOWER MOTION IS -99.956 DEGREES

THE MINIMUM TRANSMISSIBILITY INDEX IS .58601

```
SYNTHESIS OF A SPATIAL 4-BAR (RGGR) FUNCTION GENERATOR MECHANISM
WHERE ICASE= -1

THE STARTING DESIGN VARIABLE VALUES ARE
           -.44900          9.29000E-02       0.                0.
           4.7100            .17970          -.82270

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS    .44293

DESIGN VARIABLE VALUES ARE
           -.44803          9.60506E-02      1.41680E-03     -5.56209E-0
           4.7147            .18070          -.82293

THE CONSTRAINT VALUES ARE
           545.12           1256.1            .47828           785.17


THE NEW STARTING DESIGN VARIABLE VALUES ARE
           .18525            .19332          4.01042E-03     -9.09401E-0
           11.814            .42263          -1.2181

THE OPTIMUM VALUE DETERMINED IS INFEASIBLE
CONSTRAINT NUMBER  3  HAVING THE VALUE    -1.2130
THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM STARTED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
         -2.98946E-03        .94481          2.24080E-02       1.9868
           5.5655            .46363          -1.2183

THE OPTIMUM VALUE DETERMINED IS INFEASIBLE
CONSTRAINT NUMBER  3  HAVING THE VALUE    -1.2130
THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM STARTED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
         -2.98760E-03        .94481          2.24108E-02       1.9868
           5.5655            .46371          -1.2183

MINIMIZATION ROUTINE HAS HUNG UP IN AN INFEASIBLE REGION
INDICATIONS ARE THAT THE CONSTRAINTS ARE SUCH THAT A FEASIBLE REGION
CANNOT BE REACHED FROM THE CURRENT LOCATION
THEREFORE A NEW STARTING POINT HAS BEEN GENERATED


THE NEW STARTING DESIGN VARIABLE VALUES ARE
         -4.11441E-03       2.1630           4.16946E-02       1.8402
           10.825            .35712          -2.5004

MINIMIZATION ROUTINE HAS HUNG UP IN AN INFEASIBLE REGION
INDICATIONS ARE THAT THE CONSTRAINTS ARE SUCH THAT A FEASIBLE REGION
CANNOT BE REACHED FROM THE CURRENT LOCATION
THEREFORE A NEW STARTING POINT HAS BEEN GENERATED
```

FIG. 7-12

```
THE NEW STARTING DESIGN VARIABLE VALUES ARE
          1.42838E-03      -10.549           -.70355          3.7472
          8.7824           -.29429           3.2455

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS     27897

DESIGN VARIABLE VALUES ARE
          1.7720           -2.4089           -5.3313          -4.9854
          2.9911           -.38170           6.1904

THE CONSTRAINT VALUES ARE
          2.8718           758.10            218.06           957.53

     TRANSMISSIBILITY INDEX FUNCTION IS   .41476

     STRUCTURAL ERROR FUNCTION IS  2.81712E-02

     LINK LENGTH FUNCTION IS  0.

     FINAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   .44293

     DESIGN VARIABLE VALUES ARE
          -.44803          9.60506E-02      1.41680E-03     -5.56209E-03
          4.7147           .18070           -.82293

NUMBER OF FUNCTION CALLS WAS  17921

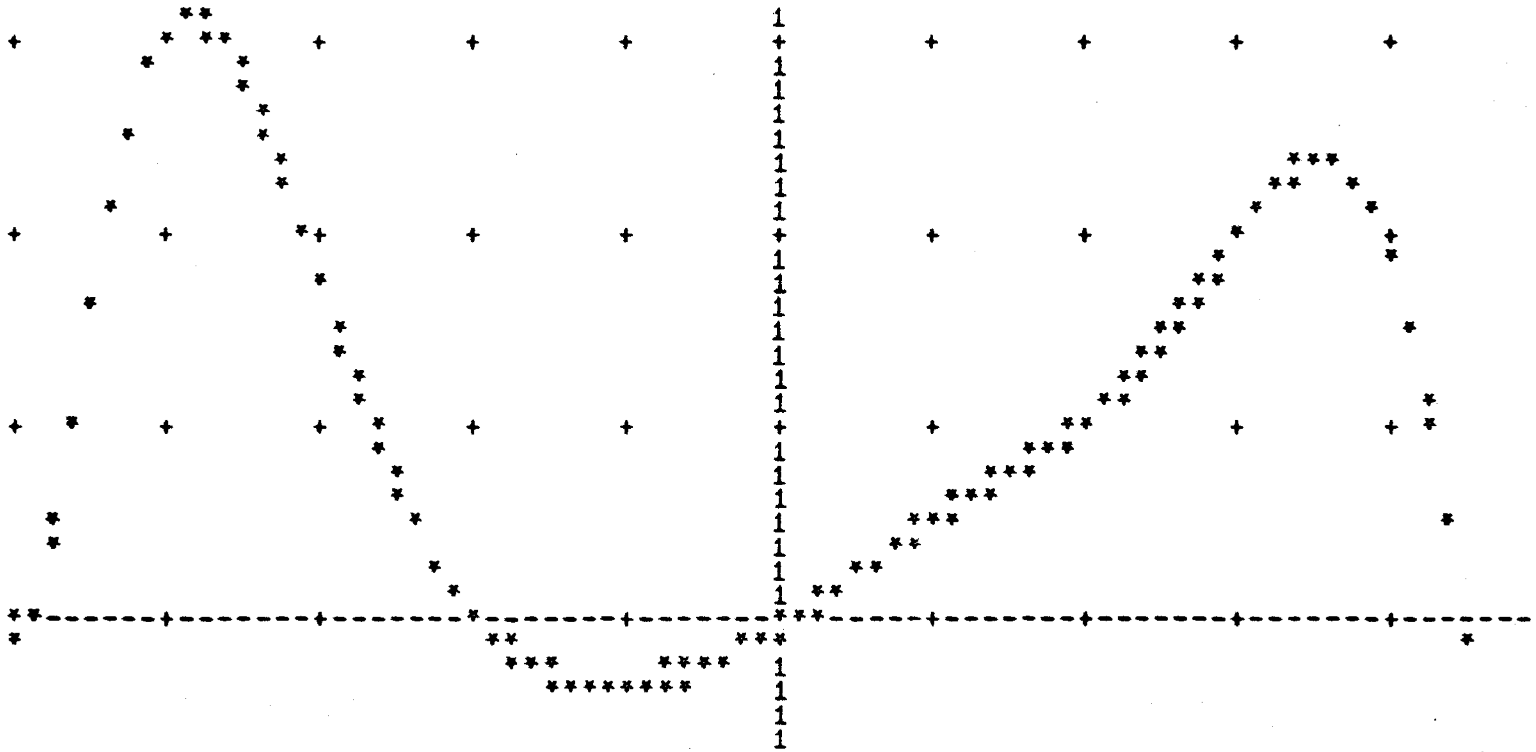
TOTAL EXECUTION TIME FOR METHOD  5 IS   217.114  SECONDS
```

FIG. 7-13

FIG. 7-14

STRUCTURAL ERROR PLOT FOR SPATIAL FOUR-BAR (RGGR) SYNTHESIS

MAXIMUM STRUCTURAL ERROR (YA-YD) IS 2.28304E-03
MINIMUM STRUCTURAL ERROR IS -1.34055E-03



THE CRANK STARTING ANGLE IS -100.32 DEGREES

THE FOLLOWER STARTING ANGLE IS 35.759 DEGREES

THE RANGE OF FOLLOWER MOTION IS -97.403 DEGREES

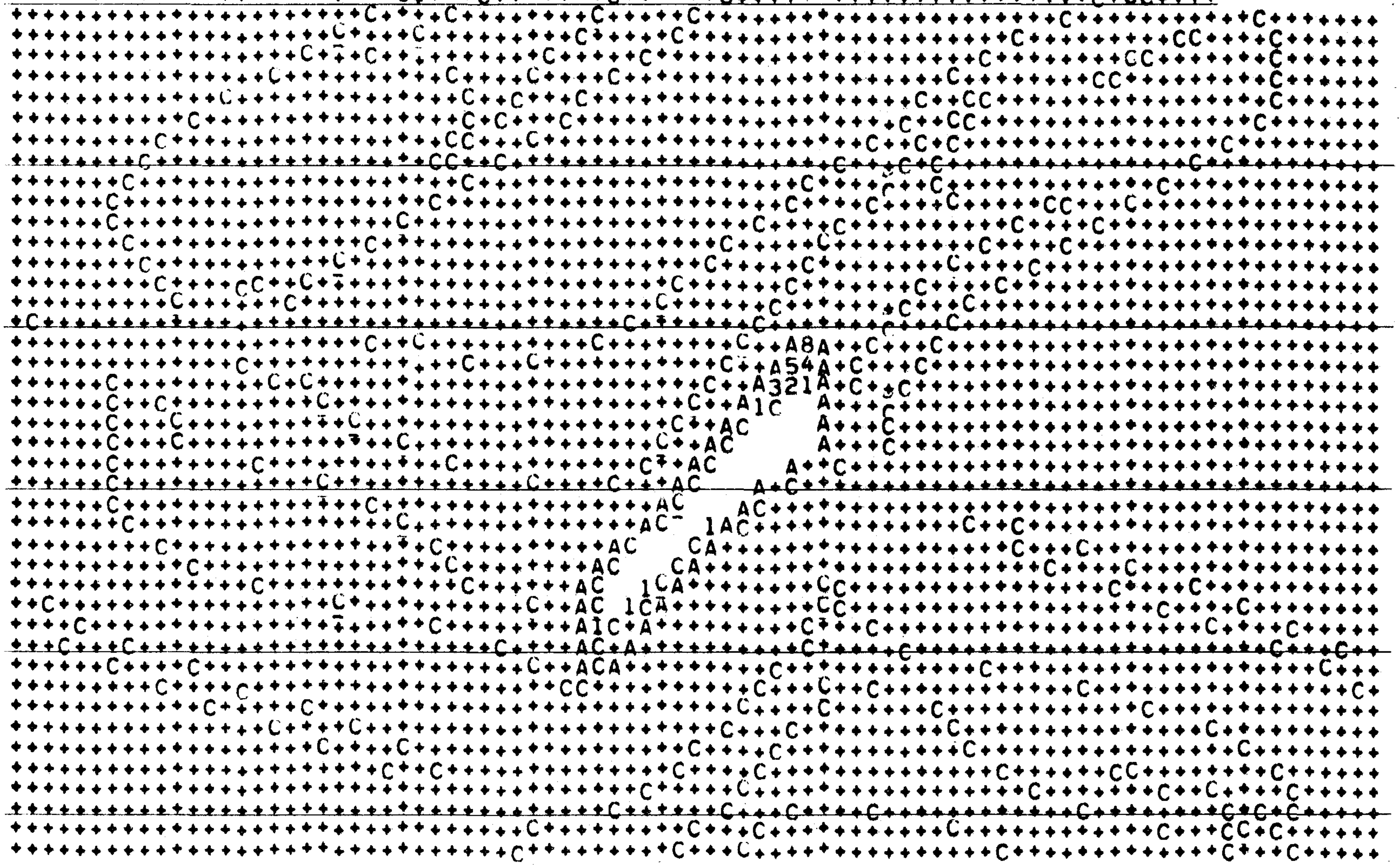
THE MINIMUM TRANSMISSIBILITY INDEX IS .60826 made to optimum parameters determined considering only the structural error. However, through use of this program considerably more information is revealed about this linkage than can be possibly attained through the synthesis technique employed by Hartenberg.

Why little improvement can be made on Hartenberg's values becomes clearer when the two-dimensional contour-constraint plot of the optimization hypersurface shown in Figure 7.15 is examined. (A detailed explanation of how to interpret such plots is contained in Appendix E.) However, for present purposes, it is enough to know that the blank region in the centre of the plot represents the only feasible region in the two-dimensional subspace of variables X_3 (quotient of parameters d over b in Figure B-6 of Appendix B) and X_4 (the crank mid-range angle), all other parameters being fixed at their optimum values. It is clear that there is little range for movement in the feasible region, and since no minus signs are printed, the local optimum in this region, with respect to variables X_3 and X_4, has been reached.

This example thus shows that the inclusion of more than one factor in the optimization objective function can usually, but not always, provide a better basis for optimization. Note also that for this example, not only is the maximum structural error for the spatial four-bar symmetrical function generator a magnitude less than for its planar counterpart, but also the

FIG. 7-15

PARTIAL CONTOUR PLOT OF FINAL OPTIMIZATION HYPER-SURFACE
X(3) IS THE HORIZONTAL AXIS VARIABLE, X(4) IS THE VERTICAL AXIS VARIABLE
+ IS PRINTED WHERE FUNCTION VALUES EXCEED 30000
- IS PRINTED WHEN FUNCTION VALUES ARE LESS THAN .44300
X(3) VARIES FROM -1.0000 TO 1.0000 , X(4) VARIES FROM -3.1416 TO 3.1416



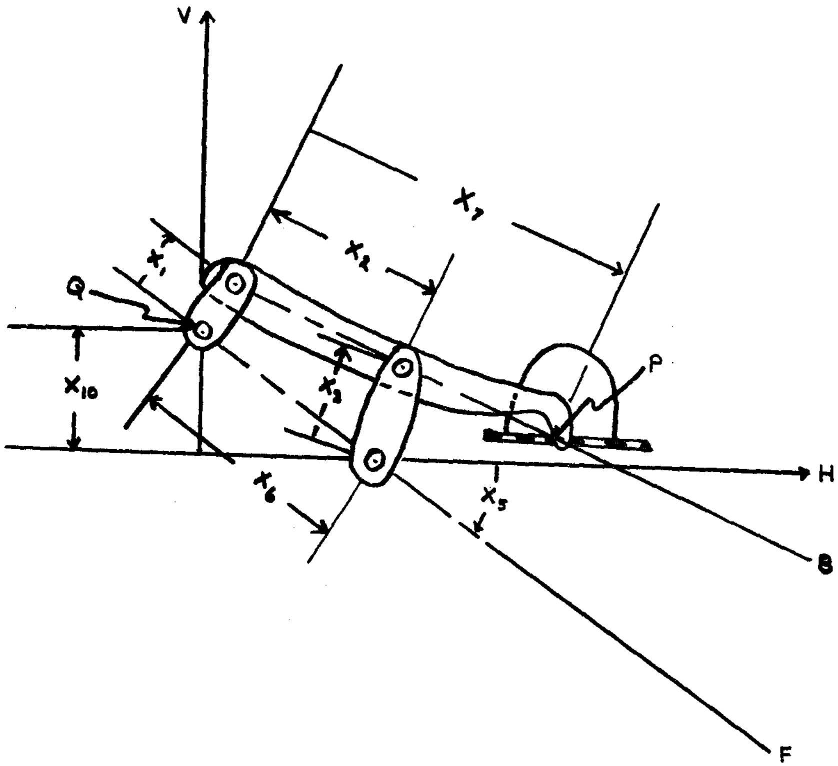
minimum transmissibility index is significantly greater for the spatial four-bar.

PROBLEM NUMBER 3 - Planar Four-Bar Coupler-Point Curve Synthesis and Balancing

Figure 7.16 shows to full scale an actual four-bar linkage used in a motion picture projector [19] to give the film intermittent motion. Point P on the coupler link traces the coupler curve shown. As the driving crank rotates, the catcher moves down into a film slot, pulls the film across one frame, moves up out of the slot, and then moves back across preparatory to engaging the film again. The original design has been obtained using a combination of Hrone's and Nelson's atlas of four-bar coupler-point curves [26] and trial-and-error synthesis.

The problem is to use the computer program developed to confirm the shape of the curve for coupler-point P (on the film catcher) for the linkage dimensions listed in Figure 7.16; use these same dimensions as a starting point for the computer program to see if the design can be improved; and balance the resulting optimum mechanism assuming a constant crank angular velocity of 24 rps.

In order to use the computer program as a coupler-point curve synthesis tool, the desired coupler-point curve



X_1 = .30"

X_2 = .96"

X_3 = .54"

X_4 = crank starting angle measured counterclockwise from frame centreline F
= .6 $\dot{r}$

X_5 = −.58 $\dot{r}$

X_6 = 1.09"

X_7 = 1.89"

X_8 = distance from point p to coupler centreline B
= 0."

X_9 = horizontal distance crankshaft axis Q is from V-axis
= 0."

X_{10} = .6"

FIG. 7-16 FILM INTERMITTENT MOTION MECHANISM co-ordinates and associated bilateral tolerances and crank angles must be defined. To accomplish this the problem is further specified as follows. The desired coupler point, P, moves horizontally for .65 inches (the catcher moving the film over one frame) while the crank link rotates through 140 degrees; the point then moves straight up for .15 inches (enabling the catcher to clear the film), moves back over the film clearing it by at least .15 inches, and moves down (vertical for the last .15 inches) to the starting point (to re-engage the film catcher) while the crank rotates through the remaining 220 degrees of its motion. To meet these basic requirements the values shown in Table 7.1 are assumed. (Note that nine precision points are used.) These numbers are included in the program input in SUBROUTINE COUPLER as shown in Figure 7.17. The rest of the required input is included in PROGRAM MAIN, also shown in Figure 7.17.

To illustrate the use of a direct variable transformation as an additional explicit constraint, transformation type 2 of Chapter II is used to limit variable X_7 to values less than three inches. Thus the variable transformation

$$X_{7_t} = 3. - | X_7 - 3. |$$

is used in SUBROUTINE EXCON (see Figure 7.17). Note that since IEXCO is set equal to zero to initiate the call to EXCON, NC, the number of implicit constraints (three is the number of basic implicit constraints for this problem - see

TABLE 7.1 - DESIRED COUPLER-POINT CO-ORDINATES WITH THEIR ASSOCIATED TOLERANCES

POINT NO.	CRANK ANGLE (radians)	H-CO-ORD. (inches)	V-CO-ORD. (inches)	H-TOL. (inches)	V-TOL. (inches)
1	0.00	2.20	.20	.01	.05
2	0.61	2.04	.20	.10	.05
3	1.22	1.87	.20	.20	.05
4	1.83	1.71	.20	.10	.05
5	2.44	1.55	.20	.01	.05
6	2.79	1.55	.35	.05	.05
7	4.19	1.75	.70	.20	.40
8	5.24	2.00	.70	.20	.40
9	5.93	2.20	.35	.05	.05

```
INPUT FOR PLANAR FOUR-BAR COUPLER-POINT CURVE SYNTHESIS
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON /STRTPT/STRTPT(10)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      DATA METHOD/2/,IEXCO/0/,NC/3/,NPP/9/,ICASE/1/,SCAL1/.5/,
     1SCAL3/2.5/,RNGI/360./,STRTPT/.3,.96,.54,.6,-.58,1.09,1.89,0.,0.,
     2.6/
      CALL LINK(1,.0003,1.,2.5,.0001)
      STOP
      END
      SUBROUTINE COUPLER
      COMMON /DESIRE/X(21),Y(81),ANG(81),XTOL(21),YTOL(21)
      DATA X/2.2,2.04,1.87,1.71,1.55,1.55,1.75,2.,2.2/,
     1Y/.2,.2,.2,.2,.2,.35,.7,.7,.35/,XTOL/.01,.1,.2,.1,.01,.05,.2,.2,
     2.05/,YTOL/6*.05,.4,.4,.05/,ANG/0.,.61,1.22,1.83,2.44,2.79,4.19,
     35.24,5.93/
      RETURN
      END
      SUBROUTINE EXCON(X,C,NC)
      DIMENSION X(1)
      X(7)=3.-ABS(X(7)-3.)
      RETURN
      END
```

FIG. 7-17

comments cards for FUNCTION G of Appendix F), must be defined in PROGRAM MAIN.

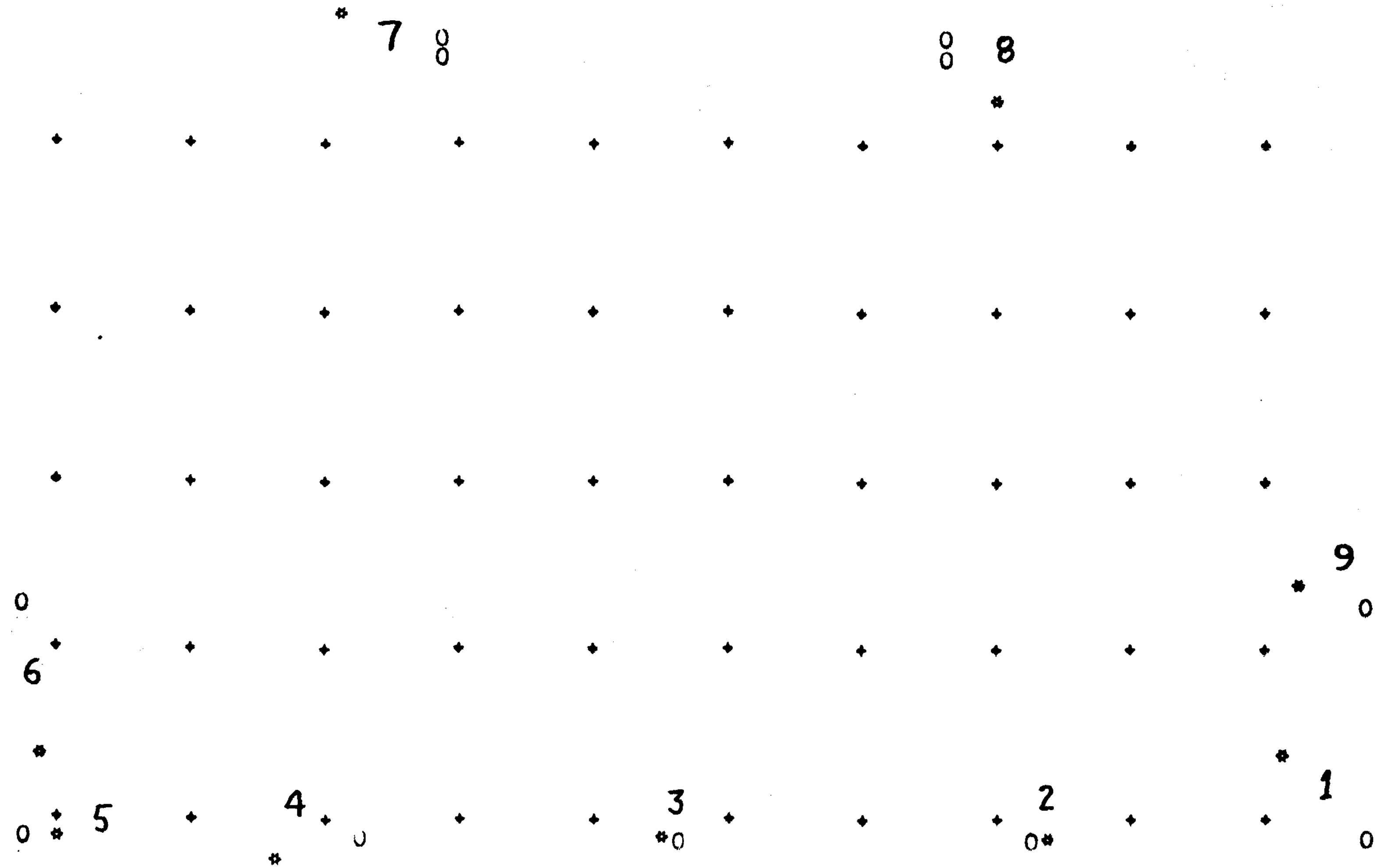
The coupler-point co-ordinates of the original mechanism for the nine precision point crank angles are shown in Figure 7.18 (as *'s). The coupler curve that these co-ordinate points outline is similar to that shown in Figure 7.16; thus the original mechanism performs the job expected of it. (Also the computer program appears to be working properly!) The desired coupler-point co-ordinates defined by the data in Table 7.1 are also shown in Figure 7.18 (as O's)* for comparison. The magnitude of the difference between these desired and actual coupler points versus the crank angle is shown in Figure 7.19.

* The double O's and *'s directly above each other, which occur in the plots from time to time, indicate that the actual point lies between one quarter and three quarters of a line-space between the lines that the two symbols are printed on. If only one symbol is printed, then the actual point lies within one quarter of a line-space above or below the line that the symbol is printed on. This technique increases the accuracy with which the vertical co-ordinate of a plotted point can be read. (See Figure 7.11 for a good example.)

FIG. 7-18

PLOT OF DESIRED (0) AND ACTUAL (*) COUPLER POINTS FOR THE PLANAR FOUR-BAR

MINIMUM X VALUE IS 1.5500 MAXIMUM X VALUE IS 2.2000
MINIMUM Y VALUE IS .18832 MAXIMUM Y VALUE IS .71708



PLOT OF COUPLER CURVE ERROR VS. CRANK ANGLE CHANGE FROM STARTING ANGLE
CRANK ANGLE CHANGE IS 360.00 DEGREES
CRANK STARTING ANGLE IS 34.377 DEGREES
MINIMUM VALUE OF ERROR IS 2.35226E-03, MAXIMUM VALUE OF ERROR IS 9.06813E-02

THE MINIMUM TRANSMISSIBILITY INDEX IS .72368

FIG. 7-19

The results of the linkage synthesis are shown in Figures 7.20 to 7.22. Note that Figure 7.22 is a plot of the actual structural error, not the scaled structural error discussed in Chapter III (which is used in the objective function for minimization). Thus, though the magnitude of the actual maximum structural error is not reduced, the scaled structural error, which is based on the bilateral tolerances at each precision point given in Table 7.1, is reduced since the synthesized points which deviate greatest, points 7 and 8, are those for which the tolerances are largest. Also the minimum transmissibility index is increased from .724 to .752. Therefore, if the data contained in Table 7.1 truly represents the designer's problem requirements, then the given four-bar projector mechanism has been improved by the computer program.

The mechanism should also be balanced. In order to do this the masses, polar moments of inertia about the mass centre, and positions of the mass centre of each link must be known. These values are determined assuming that the links are made of uniform 1/4 inch diameter solid steel rods (density .283 lb/in^3) and are listed in Table 7.2. For lack of better information, the scaling factors for the horizontal shaking force, vertical shaking force, and shaking moments utility curves are assumed equal to .01 lb., .01 lb., and .01 in.-lb., respectively.

```
SYNTHESIS OF A PLANAR 4-BAR MECHANISM TO PRODUCE A GIVEN COUPLER CURV
WHERE ICASE=  1

THE STARTING DESIGN VARIABLE VALUES ARE
          .30000          .96000          .54000          .60000
         -.58000         1.0900          1.8900          0.
         0.               .60000

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS    1.4996

DESIGN VARIABLE VALUES ARE
          .33826          .96894          .56852          .51518
         -.59258         1.0640          1.8850         -5.46849E-03
         1.93029E-03      .60278

THE CONSTRAINT VALUES ARE
          288.97          1205.1          1197.4


THE NEW STARTING DESIGN VARIABLE VALUES ARE
         8.90261E-02     1.4328           .33830         -.39447
         -1.2249        -6.59062E-02     3.9072         -6.18700E-03
         3.45496E-04      .67099

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS    14.975

DESIGN VARIABLE VALUES ARE
         -2.1044         2.1329           .48389         2.69725E-03
         -1.2232          .34857         2.1249          6.74849E-02
          1.7992          .16400

THE CONSTRAINT VALUES ARE
          486.04          1256.4          1134.3

     TRANSMISSIBILITY INDEX FUNCTION IS    .10845

     STRUCTURAL ERROR FUNCTION IS    1.0430

     LINK LENGTH FUNCTION IS    .34809

     FINAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS    1.4996

     DESIGN VARIABLE VALUES ARE
          .33826          .96894          .56852          .51518
         -.59258         1.0640          1.8850         -5.46849E-03
         1.93029E-03      .60278

NUMBER OF FUNCTION CALLS WAS    4277

TOTAL EXECUTION TIME FOR METHOD  2 IS    71.866  SECONDS
```

FIG. 7-20

FIG. 7-21

PLOT OF DESIRED (O) AND ACTUAL (*) COUPLER CURVES FOR THE PLANAR FOUR-BAR SYNTHESIS

MINIMUM X VALUE IS 1.5274 MAXIMUM X VALUE IS 2.2000
MINIMUM Y VALUE IS .18072 MAXIMUM Y VALUE IS .78013

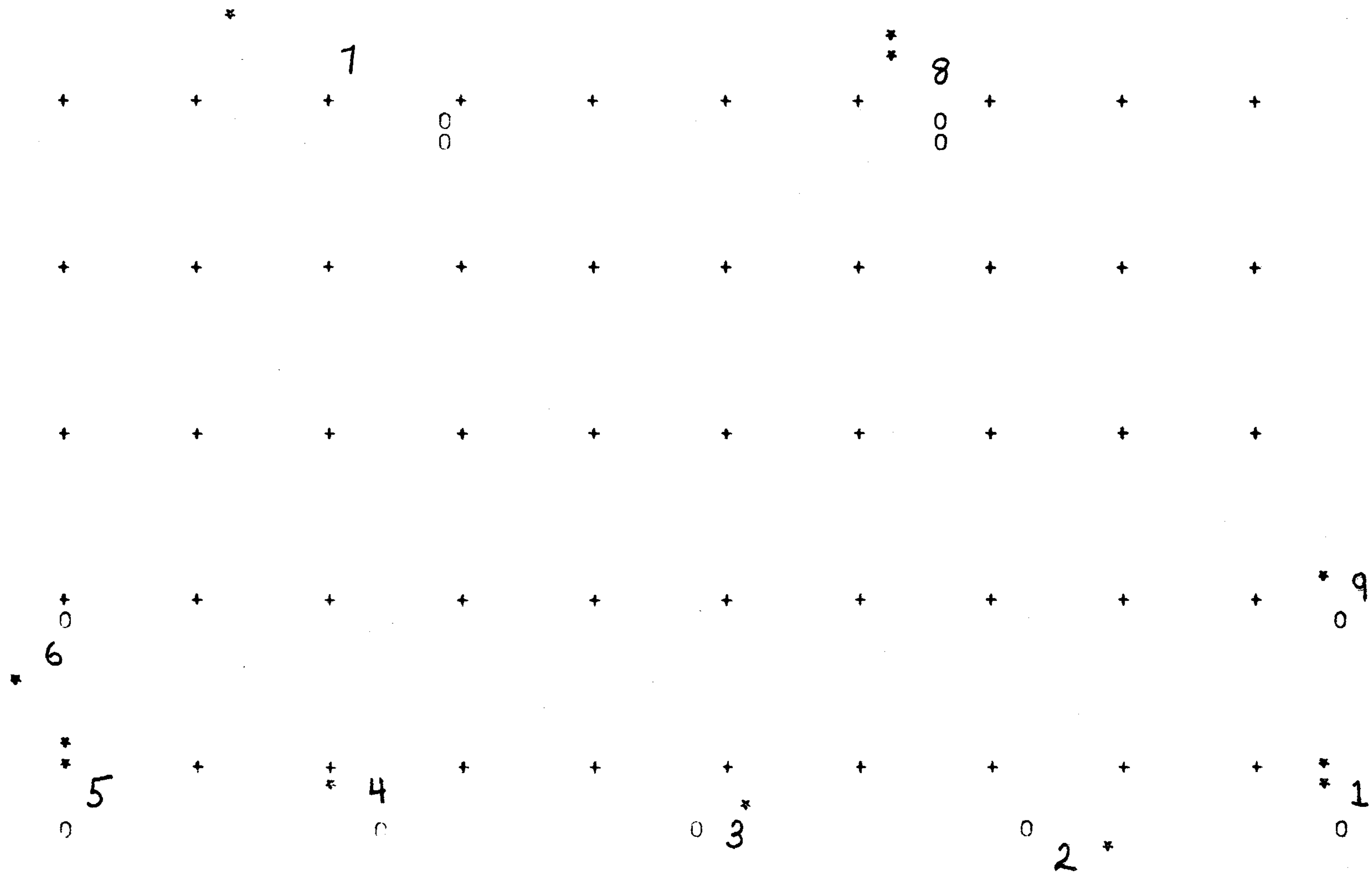


FIG. 7-22

```
PLOT OF COUPLER CURVE ERROR VS. CRANK ANGLE CHANGE FROM STARTING ANGLE
CRANK ANGLE CHANGE IS     360.00       DEGREES
CRANK STARTING ANGLE IS     29.518       DEGREES
MINIMUM VALUE OF ERROR IS   2.54880E-02, MAXIMUM VALUE OF ERROR IS    .13621
```


```
THE MINIMUM TRANSMISSIBILITY INDEX IS    .75227
```

TABLE 7.2 - PROJECTOR LINKAGE PARAMETERS

LINK NAME	MASS RM ($lbf\text{-}sec^2/in$)	POLAR M.I. RJ ($lbf\text{-}sec^2\text{-}in$)	Rel. Co-ord.* of C.M. CM (in,in)
Crank (1)	.0000122	.000000163	(.169,0.)
Coupler (2)	.0000679	.0000205	(.943,0.)
Follower (3)	.0000205	.000000634	(.284,0.)

* relative to each link - see Appendix D for explanation.

Figure 7.23 shows the computer input cards required for balancing. Figures 7.24 to 7.28 show the computer output obtained (where the design variables correspond to those X_i discussed at the end of Chapter IV). Note that the maximum and minimum values printed in the headings for each plot (Figures 7.26 to 7.28) are for both the balanced and unbalanced values, and are thus used to establish the scales for the plots. The exact balanced and unbalanced shaking forces and moments at the eighteen crank positions are shown in Figure 7.25.

The optimum counterweight parameters are shown in Figure 7.29. Note that since the crank link rotates at a constant angular velocity, the crank counterweight's polar moment of inertia does not enter into the balancing equations. Also only the product of the crank counterweight mass and the distance from its centre of mass to the crankshaft axis is important for this problem. Since NOA1 is set equal to zero, the computer program accounts for these facts by not including the crank counterweight polar moment of inertia, X_8, in the optimization, and by leaving the crank counterweight mass, X_7, fixed at the user designated starting value (.0001 lbf-sec^2/in for this example).

The optimum counterweight parameters may be difficult to obtain in practice; thus the closest available counterweight prarmeters can be checked before using by calling BALANCE (O)

with these available parameters in COMMON / SAVOPT /.
(See Appendix E for further details.)

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /STRTPT/STRTPT(10)
 COMMON /SCLFAC/SCLSFH,SCLSFV,SCLSMO
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
1RM(3),RJ(3)
 DATA METHOD/10/,NPP/18/,ICASE/+1/,IEXCO/1/,RNGA/360./,
1STRTA/0./,PAR/.338,.969,.569,.515,-.593,1.064/,NOA1/0/,
2W1/18*25.15/,A1/18*0./,CM/.169,0.,.943,0.,.284,0./,
3RM/1.22E-5,6.79E-5,2.05E-5/,RJ/1.63E-7,2.05E-5,6.34E-7/,
4SCLSFH/.01/,SCLSFV/.01/,SCLSMO/.01/,STRTPT/-.2,0.,1.E-4,
51.E-5,-.3,0.,1.E-4,1.E-5/
 CALL BALANCE(1,1.E-5)
 STOP
 END
```

FIG. 7-23 FOUR-BAR BALANCING INPUT

```
BALANCING OF A PLANAR 4-BAR MECHANISM
WHERE ICASE=  1

THE STARTING DESIGN VARIABLE VALUES ARE
        -.20000          0.                  1.00000E-04      1.00000E-05
        -.30000          0.

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   5.0355

DESIGN VARIABLE VALUES ARE
        -2.51000E-02     -.20273             6.30413E-04      2.36836E-09
        -7.14094E-02     3.55405E-02


THE NEW STARTING DESIGN VARIABLE VALUES ARE
        2.41154E-02      -.47170             2.42332E-04      3.27480E-09
        3.44080E-03      1.45435E-02

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   4.2851

DESIGN VARIABLE VALUES ARE
        -1.42152E-02     -.10077             3.93922E-03      1.78665E-10
        -1.11159E-02     3.78283E-03


THE NEW STARTING DESIGN VARIABLE VALUES ARE
        -1.71924E-02     4.41076E-02         3.17342E-03      2.88422E-11
        1.03954E-02      6.73140E-03

LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS   13.287

DESIGN VARIABLE VALUES ARE
        -6.86686E-02     -.26472             1.21953E-04      2.46917E-09
        -.36420          .22830


    HORIZONTAL SHAKING FORCE FUNCTION IS    .52685

    VERTICAL SHAKING FORCE FUNCTION IS    1.3311

    SHAKING MOMENT FUNCTION IS    2.4271

    FINAL VALUE OF BALANCING OBJECTIVE FUNCTION IS    4.2851

    BALANCING VARIABLE VALUES ARE
        -1.42152E-02     -.10077             3.93922E-03      1.78665E-10
        -1.11159E-02     3.78283E-03

NUMBER OF FUNCTION CALLS WAS   8636

TOTAL EXECUTION TIME FOR METHOD 10 IS   241.943  SECONDS
```

FIG. 7-24

FIG. 7-25

TABLE OF UNBALANCED (UB) AND BALANCED(B) SHAKING FORCES AND MOMENTS VS. CRANK ANGLE IN DEGREES

CRANK ANG	UBSFH	BSFH	UBSFV	BSFV	UBSMO
0.	5.386270E-02	2.209735E-04	3.808120E-04	1.153750E-02	-3.312198E-02
21.1765	4.544711E-02	3.785842E-03	-8.476057E-03	9.484961E-03	-4.078815E-02
42.3530	2.705118E-02	5.525246E-03	-1.395813E-03	4.707564E-03	-2.123778E-02
63.5294	1.391803E-02	4.087544E-03	6.908603E-03	2.345556E-03	-4.003493E-03
84.7059	4.320941E-03	3.111535E-03	1.119253E-02	1.551425E-03	5.231336E-03
105.882	-3.416905E-03	2.676356E-03	1.110420E-02	1.410514E-03	8.060215E-03
127.059	-9.222412E-03	2.740491E-03	6.971761E-03	1.605322E-03	6.036585E-03
148.235	-1.103470E-02	3.123420E-03	-1.552126E-04	2.234800E-03	7.116262E-04
169.412	-1.007918E-02	2.653837E-03	-6.079296E-03	3.568278E-03	-4.134413E-03
190.588	-4.779905E-03	-4.407264E-04	-4.629081E-03	5.076506E-03	-3.198828E-03
211.765	-5.755303E-04	-4.599752E-03	7.642646E-04	5.275109E-03	1.278739E-03
232.941	3.547913E-04	-6.954371E-03	2.689434E-03	3.639452E-03	3.866646E-03
254.118	-1.829141E-03	-7.169717E-03	8.302489E-04	4.889157E-04	4.795782E-03
275.294	-6.856719E-03	-6.211520E-03	-1.662859E-03	-3.783936E-03	7.042293E-03
296.471	-1.228020E-02	-5.719181E-03	-9.691356E-05	-8.031874E-03	1.348494E-02
317.647	-6.864212E-03	-7.253229E-03	9.410964E-03	-8.111371E-03	2.065844E-02
338.824	2.459060E-02	-7.258439E-03	1.542790E-02	1.544229E-03	6.104065E-03
360.000	5.386283E-02	2.210370E-04	3.806782E-04	1.153752E-02	-3.312221E-02

CRANK ANG	BSMO
0.	-1.512848E-02
21.1765	-1.557906E-02
42.3530	-9.419784E-03
63.5294	-5.147746E-03
84.7059	-3.521274E-03
105.882	-3.197470E-03
127.059	-2.946354E-03
148.235	-1.898658E-03
169.412	-2.238841E-04
190.588	3.626687E-04
211.765	-2.049990E-04
232.941	-1.742629E-05
254.118	1.646599E-03
275.294	4.520778E-03
296.471	7.329167E-03
317.647	6.421194E-03
338.824	-3.008418E-03
360.000	-1.512953E-02

PLOT OF BALANCED(*) AND UNBALANCED (O) HORIZONTAL SHAKING FORCES
VS. CRANK ANGLE IN DEGREES
THE PLOTTING SCALES ARE DETERMINED FROM THE FOLLOWING DATA
MINIMUM SHAKING FORCE IS -1.228020E-02, MAXIMUM SHAKING FORCE IS 5.386283E-02
CRANK ANGLE VARIES FROM 0. TO 360.000 DEGREES

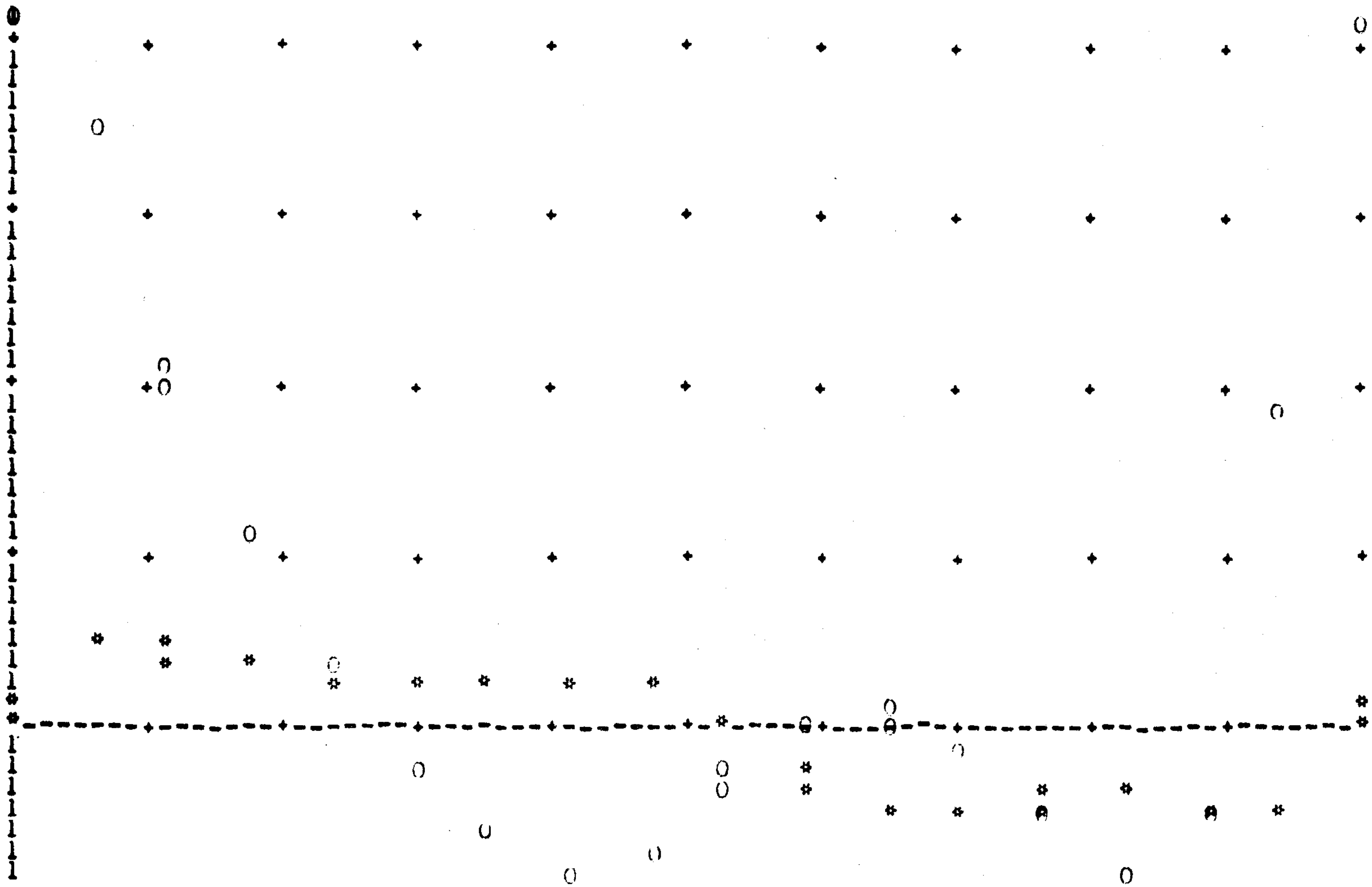


FIG. 7-26

PLOT OF BALANCED (*) AND UNBALANCED (O) VERTICAL SHAKING FORCES
VS. CRANK ANGLE IN DEGREES
THE PLOTTING SCALES ARE DETERMINED FROM THE FOLLOWING DATA
MINIMUM SHAKING FORCE IS -8.476057E-03, MAXIMUM SHAKING FORCE IS 1.542790E-02
CRANK ANGLE VARIES FROM 0. TO 360.000 DEGREES

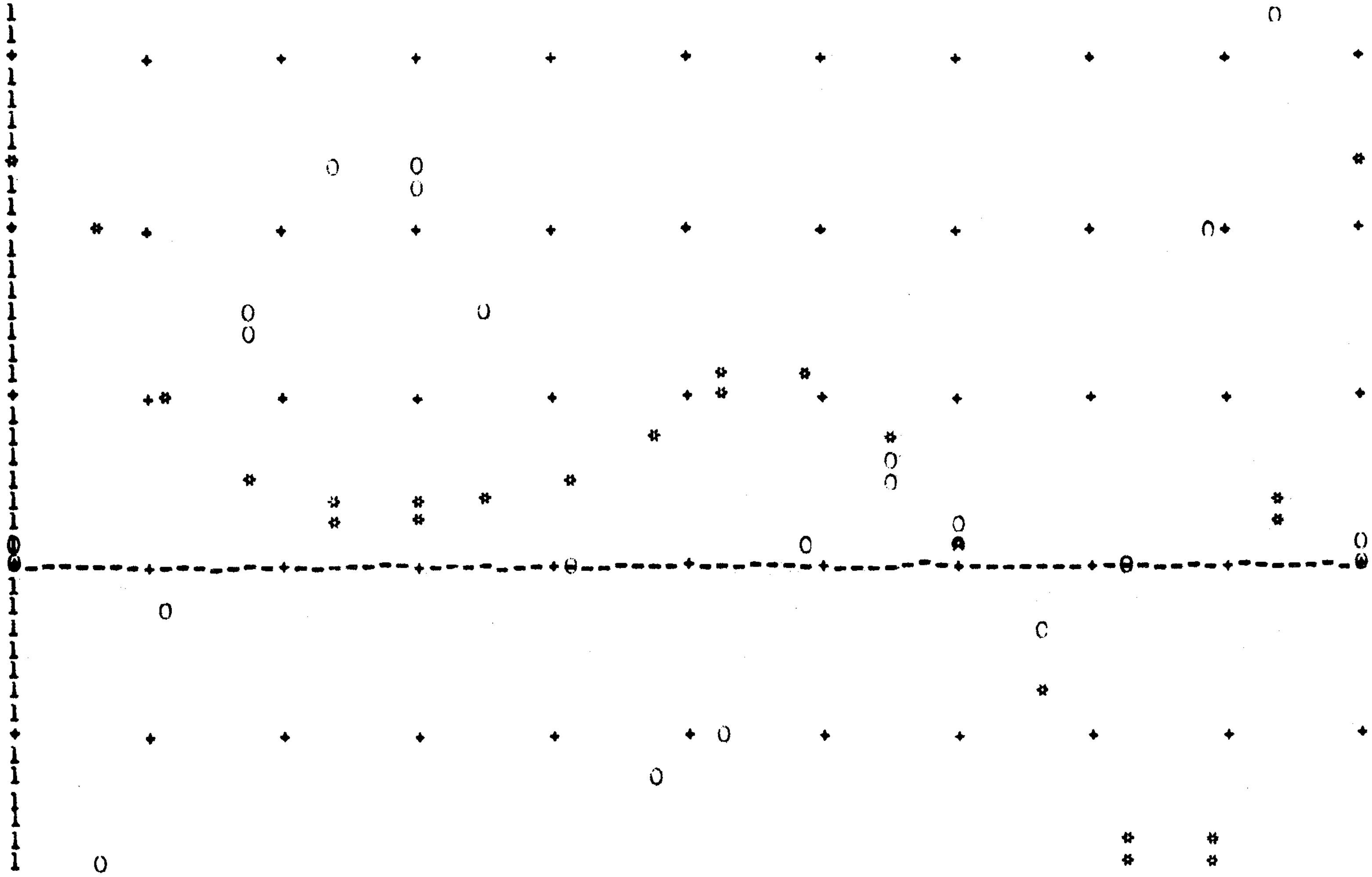


FIG. 7-27

```
PLOT OF BALANCED (*) AND UNBALANCED (O) SHAKING MOMENTS
VS. CRANK ANGLE IN DEGREES
THE PLOTTING SCALES ARE DETERMINED FROM THE FOLLOWING DATA
MINIMUM SHAKING MOMENT IS  -4.078815E-02, MAXIMUM SHAKING MOMENT IS   2.065844E-02
CRANK ANGLE VARIES FROM   0.          TO    360.000     DEGREES
```

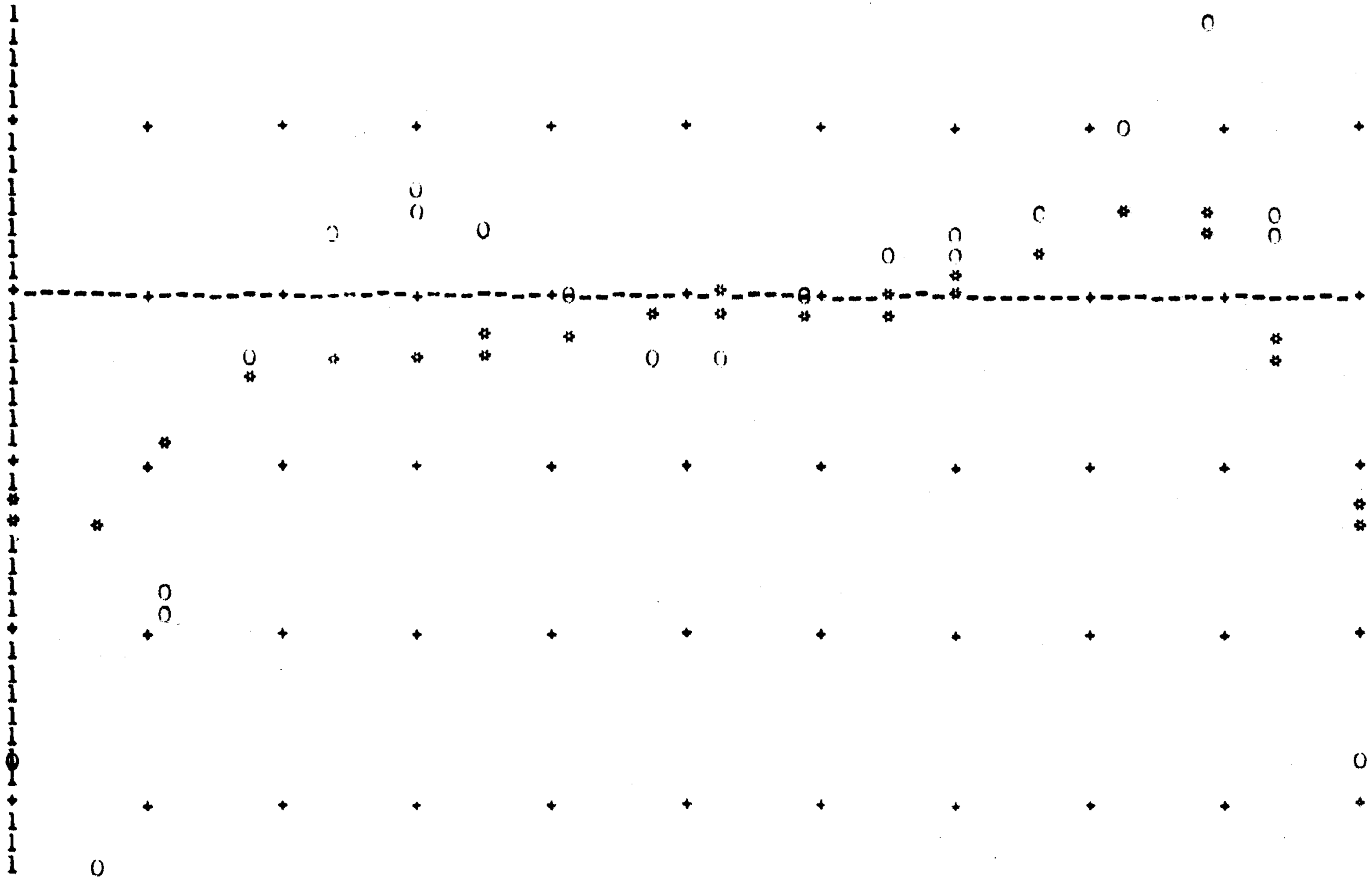


FIG. 7-28

X_1 = .0142 in.

X_2 = .1008 in.

X_3 = Follower Counterweight Mass
= .0039 lb-sec^2/in.

X_4 = Follower Counterweight Polar M.I.
= .0000 lb-in-sec^2

X_5 = .0111 in.

X_6 = .0038 in.

Crank Counterweight Mass =
.0001 lb-sec^2/in.

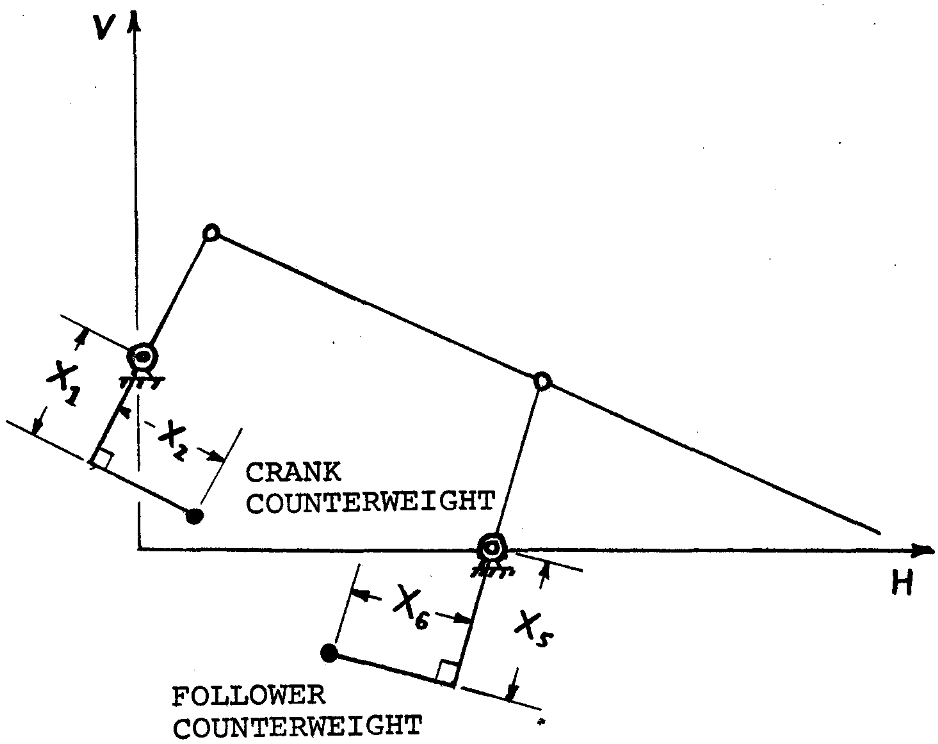


FIG. 7-29 OPTIMUM COUNTERWEIGHT PARAMETERS FOR FOUR-BAR PROJECTOR MECHANISM

VIII CONCLUSIONS

The concept of the minimization of the structural error of a mechanism to produce a desired result is not new. However, the concept of simultaneously minimizing the maximum structural error, maximizing the minimum transmissibility index, and minimizing the maximum link length to produce an optimum mechanism is a new concept. Examples 1 and 2 of Chapter VII illustrate the need for all these three factors in a general mechanism synthesis objective function.

The general transmissibility index for planar and spatial mechanisms, as stated in this thesis, is also a new concept. The transmissibility index (which varies from 0 to 1) indicates the fraction of the coupler force which is doing useful work (i.e. causing the follower to move). However, its more useful purpose is that of being an indicator of the general sensitivity of the mechanism to changes in any of the link dimensions. In fact, the TI is inversely proportional to the linkage mechanical error, as is shown in Appendix C. Hence, the size of a linkage's minimum transmissibility index is a good indicator of the mechanism's inherent reliability in producing a desired output.

The planar four-bar function generating problems for which Freudenstein has published the optimum link lengths based on his five precision-point computerized synthesis technique [2], which accounts for only the theoretical structural error, have been tried using the computer program developed for this thesis. In general, Freudenstein's dimensions represent the optimal dimensions considering theoretical structural error only (example 2 of Chapter VII being one of the exceptions). However, because the linkage dimensions obtained using Freudenstein's technique do not necessarily produce a linkage with a high transmissibility index, some of these linkage dimensions are quite sensitive to small changes, and thus must require extremely small tolerances (as small as ± .0001) in order to achieve reasonable statistical output accuracy. The cost of producing a linkage is inversely proportional to the size of the tolerances required, and the tolerances required are proportional to the minimum transmissibility index. Thus, as can be seen from sample problem 2, equal or better structural accuracy with larger required manufacturing tolerances (and thus lower production cost) can be obtained from a linkage synthesized with respect to both the TI and structural error, than with a linkage synthesized with respect to only the structural error.

The inclusion of the shaking moment in the calculations for the optimum balancing counterweights represents an important improvement over current non-cut-and-try methods of mechanism balancing. Accurate non-experimental methods for balancing have been previously limited to only the simplest slider-crank configurations. Now an unusually shaped complex machanism can be balanced just as easily as a uniform linked non-eccentric slider-crank linkage. Example 3 of Chapter VII clearly illustrates how effective this balancing procedure is in simultaneously reducing both the shaking forces and moments for a particular mechanism. Thus, one need not have to reduce the shaking force at the expense of increasing the shaking moment, but can significantly reduce both by adding the proper counterweights (determined by the computer program).

The balancing and synthesis techniques could not have been properly developed were it not for the prior conception of the inverse utility technique for the combining of more than one factor into an objective function for minimization. The inverse utility technique is designed specifically for computer minimization routines, but is not limited to the field of mechanism design (the automobile example in Chapter II illustrating this point).

The constraint transformations presented in Chapter II, and the minimization algorithms presented in Chapter V, are completely general, not specifically relating to mechanism design problems. Thus these algorithms can be used without modification for any minimization problem.

The new ideas presented in this thesis - the inverse utility curve, the scaled exterior-point transformation, the transmissibility index, the minimization-along-a-line technique, the multifactor linkage synthesis technique, and the multifactor mechanism balancing technique - are all employed in the general computer program listed in Appendix F. Though these ideas have been developed for this program, many of them are general, and can be easily applied to other fields.

IX APPENDICES

APPENDIX A - PROOF OF CONVERGENCE OF SCALED EXTERIOR-POINT TRANSFORMATION

Theorem I

If given a continuous single-valued objective function $f(\bar{x})$ subject to continuous single-valued constraints $C_i(\bar{x}) \geq 0$, $i=1, \ldots, m$ (where m is the total number of constraints), scalar $a > 0$, and scalar $t > t_o$, then a value of t_o can always be chosen such that the unconstrained transformed function,

$$S(\bar{x}, e, t) = f(\bar{x}) - e\ t \sum_{i=1}^{m} \min \left(0,\ C_i(\bar{x})\right) \ldots \qquad \text{A.1}$$

where $\quad e = a \text{ if } |f(\bar{x})| < a$

$\quad\quad = |f(\bar{x})| \text{ if } |f(\bar{x})| \geq a$,

will converge, in a given minimization sequence, to a local minimum of $f(\bar{x})$.

Corollary of Theorem I

If only one local constrained minimum of $f(\bar{x})$ exists, then for $a > 0$ and $t > t_o$, the transformed function, S, will converge to the constrained global minimum of $f(\bar{x})$.

Proof of Theorem I*

If all $C_i(\bar{x}) \geq 0$, $i = 1, \ldots, m$, then $S(\bar{x}, e, t)$ is identically equal to $f(\bar{x})$. Thus for any point satisfying the condition that all $C_i(\bar{x}) \geq 0$, convergence of S to a local minimum of f is guaranteed.

However, in order to fully prove Theorem I it must be shown to be valid for the case of any $C_i(\bar{x})<0$, i=1, ..., m. Thus it must be shown that

$$S(\bar{x}_\beta + \bar{\varepsilon},\ e,\ t) - f(\bar{x}_\beta) > 0 \ . \ . \ . \qquad \text{A.2}$$

for all $\bar{\varepsilon}$ such that any $C_i(\bar{x}_\beta + \bar{\varepsilon}) < 0$, i=1, ..., m, where $\bar{x}_\beta$ is the value of $\bar{x}$ at a boundary of the infeasible region. (The feasible region is defined as that region where any $C_i(\bar{x}) < 0$, i=1, ..., m.)

Relation A.2 must be proven for two distinct cases: (1), when $|f(\bar{x}_\beta + \bar{\varepsilon})| \leq a$, and (2), when $|f(\bar{x}_\beta + \bar{\varepsilon})| \geq a$.

* This proof is not flawless, but does provide considerable mathematical justification for the confident usage of transformation S.

Case (1):

Replacing $\bar{x}$ by $\bar{x}_\beta + \bar{\varepsilon}$ in A.1 and substituting in A.2, we get

$$f(\bar{x}_\beta + \bar{\varepsilon}) - a\ t \sum_{i=1}^{m} \min\left(0,\ C_i(\bar{x}_\beta + \bar{\varepsilon})\right) - f(\bar{x}\ ) > 0 \quad .$$

Replacing $\sum_{i=1}^{m} \min\left(0,\ C_i(\bar{x}_\beta + \bar{\varepsilon})\right)$ by G, where $G < 0$, we get

$$f(\bar{x}_\beta + \bar{\varepsilon}) - a\ t\ G - f(\bar{x}_\beta) > 0 \qquad \text{A.3}$$

From inspection of A.3, it is clear that the relation will be the most difficult to satisfy if $f(\bar{x}_\beta + \bar{\varepsilon}) = -a$ and $f(\bar{x}_\beta)$ is some arbitrarily large number L. A.3 then becomes

$$-a - a\ t\ G - L > 0 \quad ,$$

$$\text{or} \qquad t > (L + a)/a|G|$$

to satisfy A.2. The t_o which is required by Theorem I for Case (1) is therefore

$$(L + a)/a|G| \quad .$$

Case (2):

Replacing $\bar{x}$ by $\bar{x}_\beta + \bar{\varepsilon}$ in A.1 and substituting in A.2, we get

$$f(\bar{x}_\beta + \bar{\varepsilon}) - |f(\bar{x}_\beta + \bar{\varepsilon})|t \sum_{i=1}^{m} \min \left(0,\ C_i(\bar{x}_\beta + \bar{\varepsilon})\right) - f(\bar{x}_\beta) > 0$$

or $\quad f(\bar{x}_\beta + \bar{\varepsilon}) - |f(\bar{x}_\beta + \bar{\varepsilon})|t\ G - f(\bar{x}_\beta) > 0$

Dividing by $|f(\bar{x}_\beta + \bar{\varepsilon})|$ we get

$$1\left(SGN\left(f(\bar{x}_\beta + \bar{\varepsilon})\right)\right) - t\ G - f(\bar{x}_\beta)/|f(\bar{x}_\beta + \bar{\varepsilon})| > 0 \ldots \qquad A.4$$

From inspection of A.4, it is clear that the relation is the most difficult to satisfy if $f(\bar{x}_\beta + \bar{\varepsilon}) = -a$ and $f(\bar{x}_\beta)$ is some large positive number L. A.4 then becomes

$$-1 - t\ G - L/a > 0 \quad ,$$

or $\qquad t > (L + a)/a|G|$

to satisfy A.2.

Thus the value of

$$t_o = (L + a)/a|G| \qquad A.5$$

satisfies relation A.2 for both cases. Since L, a, and G are finite numbers, a value of $t > t_o$ can always be chosen that will satisfy A.2. Thus Theorem I is proven.

Note that if $a = 0$ in A.5, then Theorem I would not be valid since t_o would be infinity. Hence it is imperative tha $a > 0$, not $a \geq 0$, be used in the transformation A.1.

APPENDIX B - MECHANISM SYNTHESIS EQUATIONS

B-1 Follower Angle as a Function of the Crank Angle for a Planar Four-bar Linkage

Let the following parameters be defined (see Figure B-1):

a = crank link length

b = coupler link length

c = follower link length

e = frame link length

ϕ_s = crank starting angle

ϕ_i = crank angle relative to ϕ_s

Ψ = follower angle

ε = angle between coupler and follower

γ = angle between coupler and direction of follower motion.

Using plane trigonometry the following relations are derived:

$$\phi = \phi_s + \phi_i$$

$$\mu = \arctan\left(a \sin \phi/(e - a \cos \phi)\right)$$

$$d = (a^2 + e^2 - 2\, a\, e \cos \phi)^{1/2}$$

$$\eta = \arccos\left((c^2 + d^2 - b^2)/2\, c\, d\right)$$

$$\Psi_+ = \pi - (\mu + \eta) \text{ or } \Psi_- = \pi - \mu + \eta \qquad \text{B.1}$$

Note that there are two distinct values of Ψ (Ψ_+ and Ψ_-) for any set of values for a, b, c, e, and ϕ .

The constraint to insure mechanism closure for any given value of ϕ is

$$\left|(c^2 + d^2 - b^2)/2\, c\, d\right| \leq 1 \quad . \qquad \text{B.2}$$

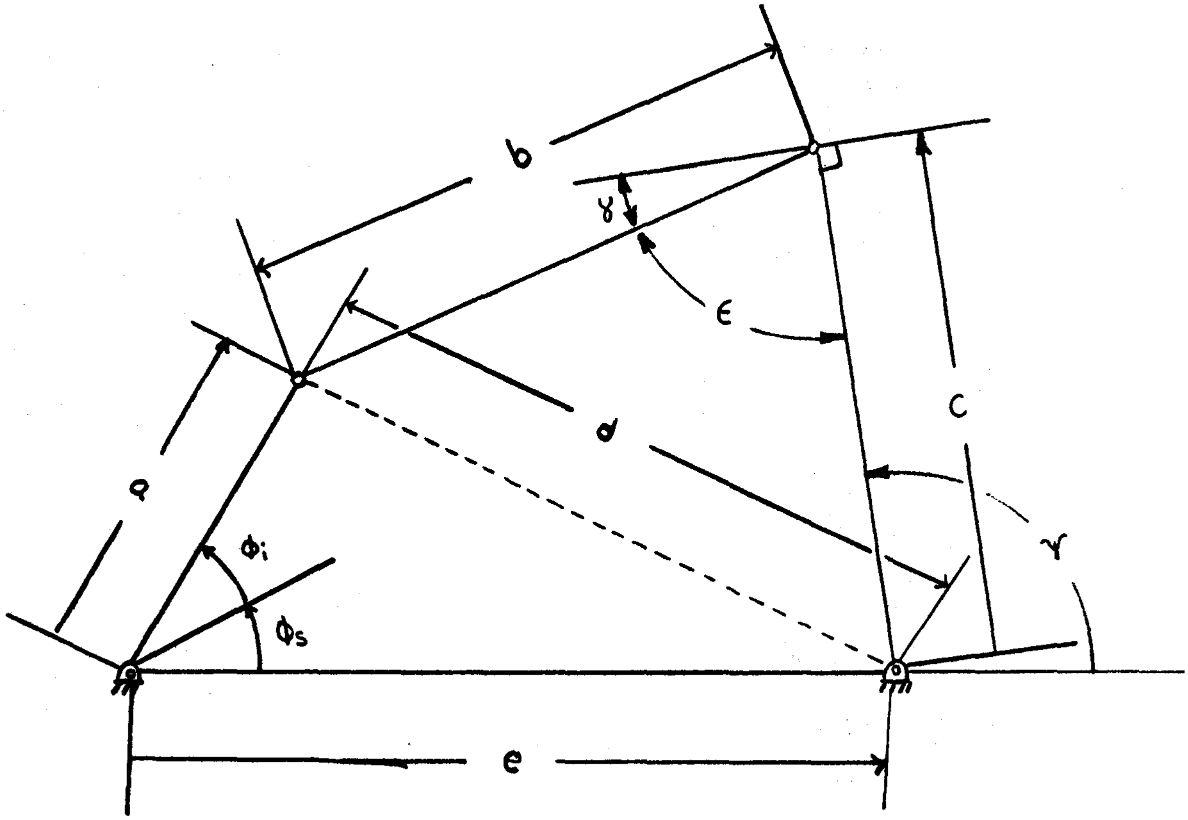


FIG. B-1 FOUR-BAR LINKAGE PARAMETERS

B-2 Coupler Point Position as a Function of Crank Angle for a Planar Four-bar Linkage

Let the following parameters be defined (see Figure B-2):

h, ℓ = horizontal and vertical components, respectively, at the crankshaft axis with respect to the arbitrary H-V coordinates

ξ = angle counterclockwise from horizontal that frame link e is oriented

x,y = horizontal and vertical components, respectively, of an arbitrary coupler-point with respect to the H-V coordinates

ε,ρ = horizontal and vertical components, respectively, of a coupler-point a distance f along the coupler link from the crankpin (with respect to the H-V coordinates)

g = perpendicular distance from coupler link b to coupler-point (x, y)

f = distance along coupler link from crankpin to intersection with g

Using plane trigonometry the following relations are derived:

$$\phi = \phi_s + \phi_i$$

$$M = e \cos \xi - a \cos (\phi + \xi) + c\left(\cos(\Psi + \xi)\right)$$

$$N = e \sin \xi + C \sin (\Psi + \xi) - a \sin(\phi + \xi)$$

$$\varepsilon = h + a \cos(\phi + \xi) + f\, M/b$$

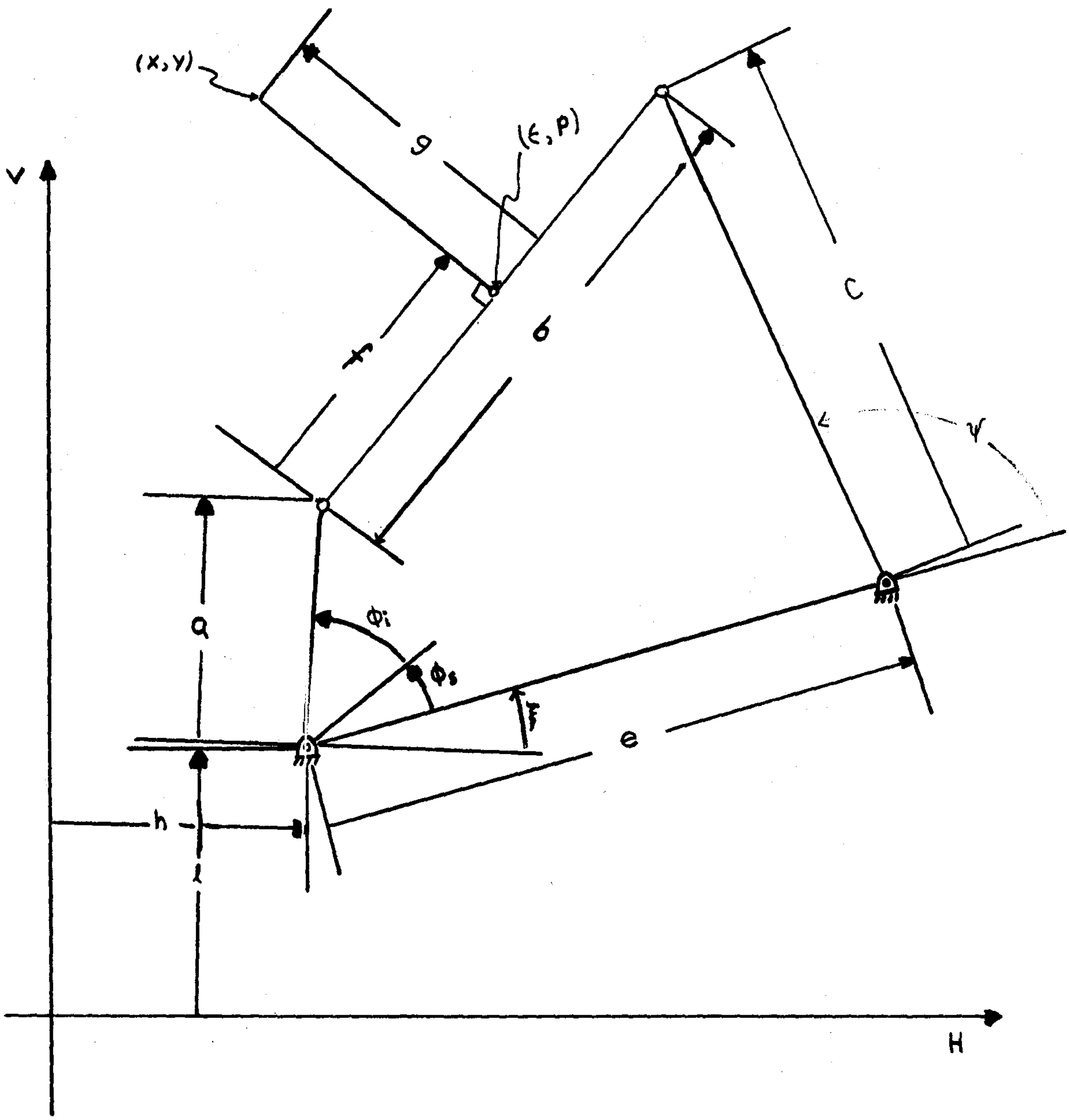


FIG. B-2 PARAMETERS FOR FOUR-BAR COUPLER POINT

$\rho = \ell + a \sin(\phi + \xi) + f\ N/b$

By similar triangles (see Figure B-3)

$x = \varepsilon - g\ N/b$, and B.3

$y = \rho + g\ M/b$. B.4

B-3 Slider Distance as a Function of the Crank Angle for a Planar Slider-crank Linkage

Let the following parameters be defined (see Figure B-4):

a = crank link length

b = coupler link length

c = distance from a line parallel to the slider motion through the crankshaft axis to the slider axis

ϕ_s = crank starting angle

ϕ_i = crank angle relative to ϕ_s

s = distance from the crankshaft axis to the slider axis measured parallel to the slider motion

α = angle between coupler link and direction of slider motion.

Using plane trigonometry the following relations are derived:

$\phi = \phi_s + \phi_i$

$M = a \cos \phi$

$N = b^2 - (a \sin \phi - c)^2$

$s_+ = M + N^{1/2}$, $s_- = M - N^{1/2}$ (2 possibilities) B.5

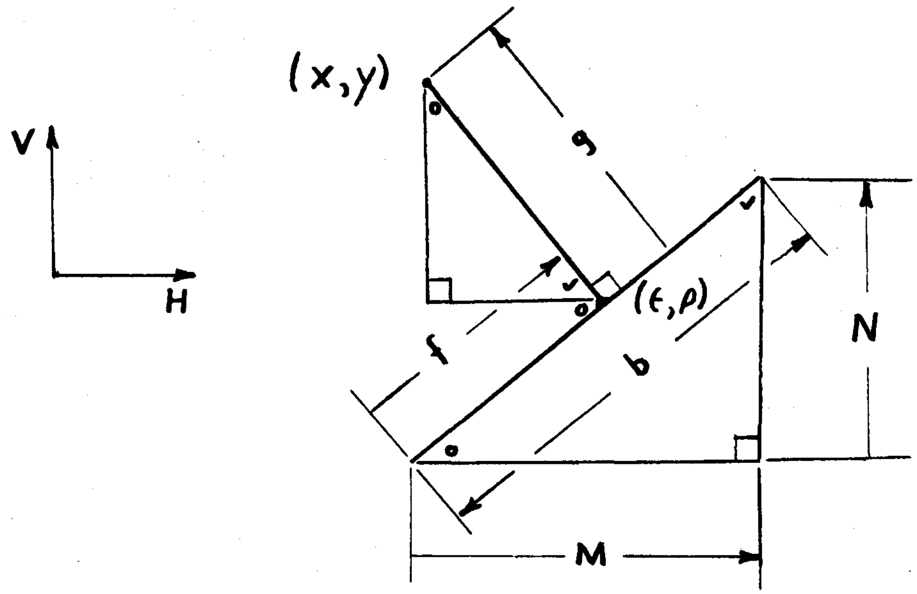


FIG. B-3 SIMILAR TRIANGLES FOR COUPLER POINT

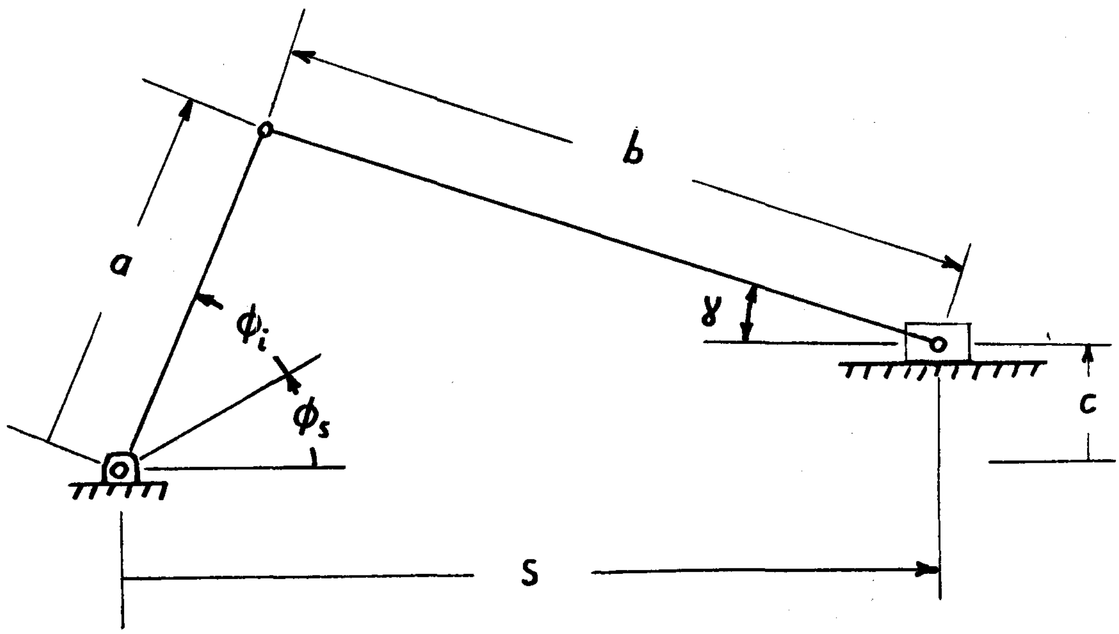


FIG. B-4 SLIDER-CRANK PARAMETERS

The constraint to ensure mechanism closure for any given value of ϕ is

$$N \geq 0 \qquad \text{B.6}$$

B-4 Coupler Point Coordinates as a Function of the Crank Angle for a Planar Slider-crank Linkage

The parameters (see Figure B-5) have the same meanings as in Sections B-2 and B-3.

Using plane trigonometry the following expressions are derived:

$$\phi = \phi_i + \phi_s$$

$$M = s \cos \xi - a \cos(\phi + \xi) - c \sin \xi$$

$$N = s \sin \xi + c(\cos \xi) - a \sin (\phi + \xi)$$

$$\varepsilon = h + a \cos(\phi + \xi) + f\ M/b$$

$$\rho = \ell + a \sin(\phi + \xi) + f\ N/b$$

Using similar triangles (see Figure B-3)

$$x = \varepsilon - g\ N/b \qquad \text{B.7}$$

$$y = \rho + g\ M/b \qquad \text{B.8}$$

B-5 Follower Angle as a Function of the Crank Angle for a Spatial (RGGR) Linkage

Let the following parameters be defined (see Figure B-6):

a = crank link length

b = coupler link length

c = follower link length

Z = crankshaft axis

Z' = followershaft axis

X = coordinate axis perpendicular to Z and Z' axes

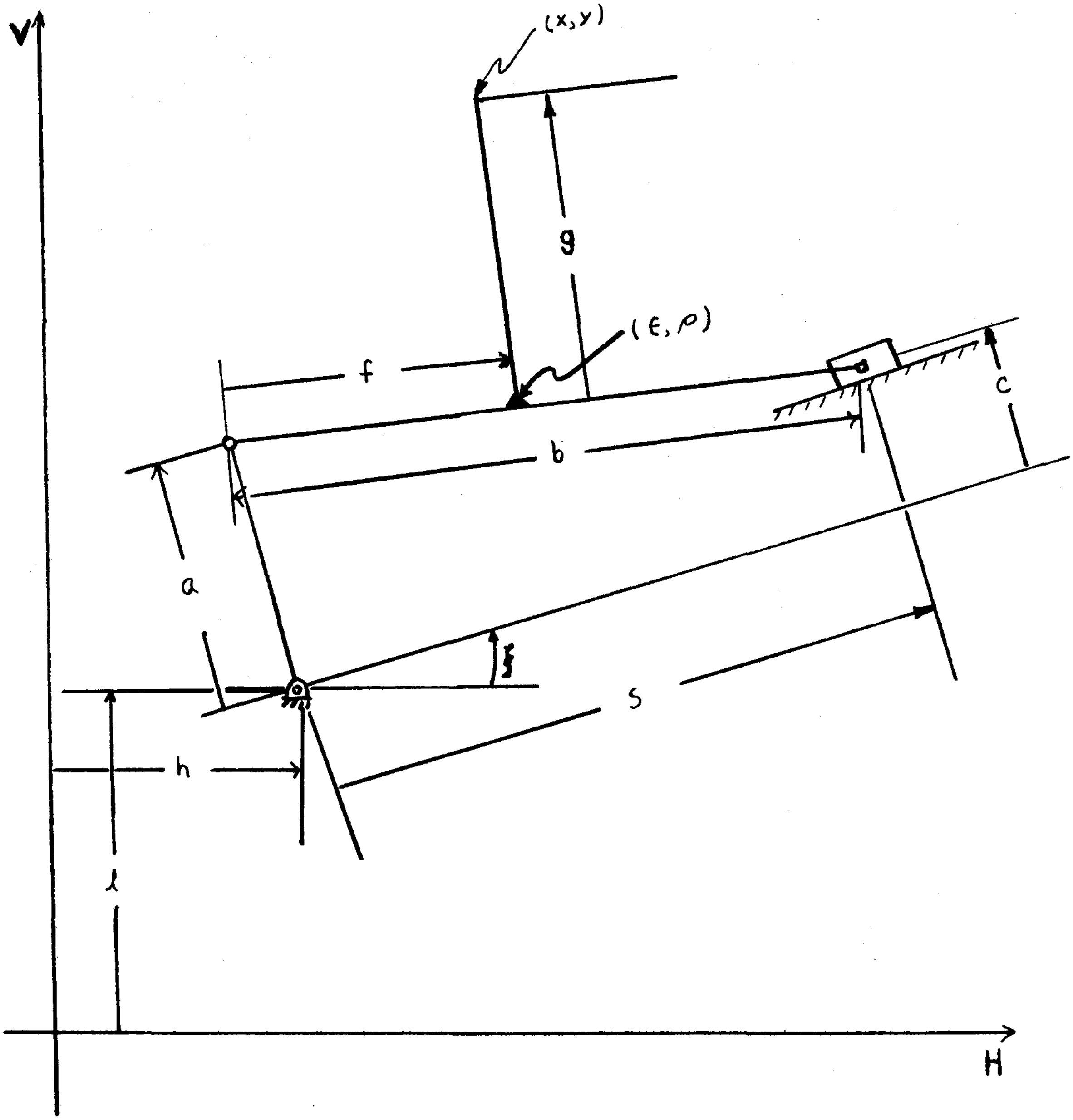


FIG. B-5 SLIDER-CRANK COUPLER POINT PARAMETERS

Y = coordinate axis perpendicular to X and Z axes so as to form a righthanded triad

ξ = angle between Z and Z$'$ axes looking in at the X axis measured clockwise from the Z axis

d = distance from X axis to followershaft axis measured along Z$'$ axis

e = distance from Z axis to Z$'$ axis measured along X axis

f = distance from X axis to crankshaft axis measured along Z axis

ϕ_s = crank starting angle

ϕ_i = crank angle relative to ϕ_s

Ψ = follower angle

Both $\phi = \phi_s + \phi_i$ and Ψ are measured counterclockwise about the Z axis from the X axis looking in at the Z axis. The positioning of the spatial four-bar linkage as shown in Figure B-6 is attributed to Hunt [25].

The required equations are derived using vectors identified by the small numbers (Figure B-6) at the linkage joints. For example, 01 means a vector from point 0 to point 1. $\bar{i}$, $\bar{j}$ and $\bar{k}$ are the unit vectors in the X, Y, and Z coordinate directions respectively.

$$01 = f\,\bar{k}$$

$$12 = a\cos\phi\,\bar{i} + a\sin\phi\,\bar{j}$$

$$\therefore \quad 02 = a\cos\phi\,\bar{i} + a\sin\phi\,\bar{j} + f\,\bar{k}$$

$$05 = e\,\bar{i}$$

$$54 = d\sin\xi\,\bar{j} + d\cos\xi\,\bar{k}$$

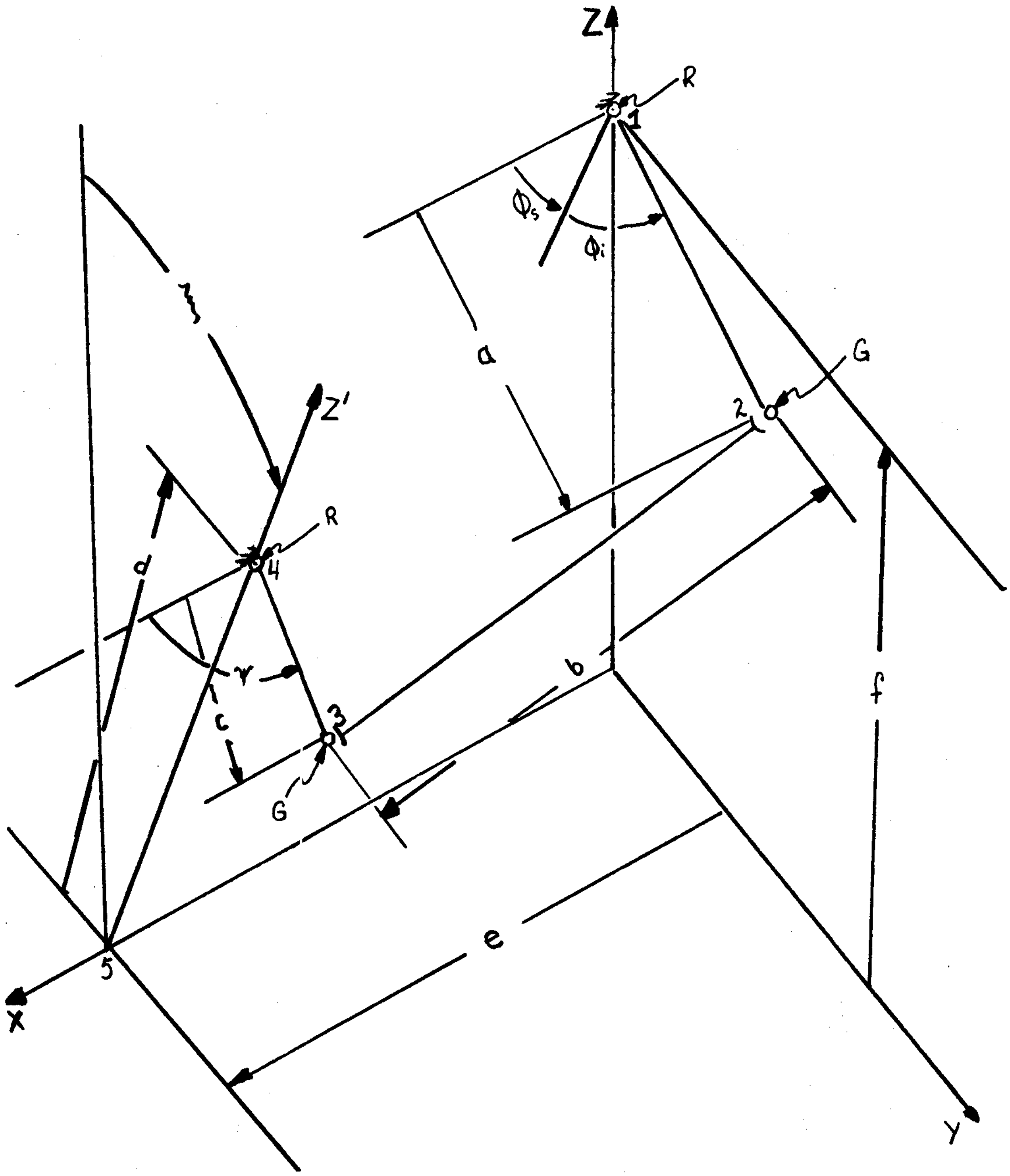


FIG. B-6 SPATIAL FOUR-BAR PARAMETERS

$$43 = c(\cos \Psi)\ \bar{i} + c \sin \Psi(\cos \xi\ \bar{j} - \sin \xi\ \bar{k})$$

$$\therefore \quad 03 = \left(e + c(\cos \Psi)\right)\ \bar{i} + (d \sin \xi + c \sin \Psi \cos \xi)\ \bar{j}$$
$$+ (d \cos \xi - c \sin \Psi \sin \xi)\ \bar{k}$$

$$|23| = |03 - 02| = b$$

$$23 = (e + c(\cos \Psi) - a \cos \phi)\ \bar{i}$$
$$+ (d \sin \xi + c \sin \Psi \cos \xi - a \sin \phi)\ \bar{j}$$
$$+ (d \cos \xi - c \sin \Psi \sin \xi - f)\ \bar{k}$$

$$\therefore \quad b^2 = |23|^2 = (e + c(\cos \Psi) - a \cos \phi)^2$$
$$+ (d \sin \xi + c \sin \Psi \cos \xi - a \sin \phi)^2$$
$$+ (d \cos \xi - c \sin \Psi \sin \xi - f)^2$$

which reduces to

$$b^2 = a^2 + c^2 + d^2 + e^2 + f^2 - 2\ a\ e \cos \phi$$
$$+ 2 \cos \Psi\ (c\ e - c\ a \cos \phi) - 2\ a\ d \sin \phi \sin \xi$$
$$+ 2 \sin \Psi\ (c\ f \sin \xi - a\ c \sin \phi \cos \xi)$$
$$- 2\ d\ f \cos \xi \qquad \text{B.9}$$

Rewriting B.9 by separating the follower angle terms from the other linkage independent variables we obtain

$$F_1\ (\phi) + F_2\ (\phi) \cos \Psi - \sin \Psi = 0 \qquad \text{B.10}$$

where

$$F_1\ (\phi) = P_1 + P_2 \cos \phi + P_3 \sin \phi)/(P_6 + P_7 \sin \phi),$$

$$F_2\ (\phi) = (P_4 + P_5 \cos \phi)/(P_6 + P_7 \sin \phi)$$

$$P_1 = d\ f \cos \xi - 1/2\ (a^2 + c^2 + d^2 + e^2 + f^2 - b^2)$$

$$P_2 = a\ e$$

$$P_3 = a\ d \sin \xi$$

$$P_4 = -\ c\ e$$

$$P_5 = a\ c$$

$$P_6 = c\ f \sin \xi$$

$$P_7 = - P_5 \cos \xi$$

Substituting

$$\sin \Psi = 2 \tan \left(\frac{\Psi}{2}\right) / \left(1 + \tan^2 \left(\frac{\Psi}{2}\right)\right), \text{ and}$$

$$\cos \Psi = \left(1 - \tan^2 (\Psi/2)\right) / \left(1 + \tan^2 (\Psi/2)\right)$$

in equation B.10, and solving the resulting quadratic in tan $(\Psi/2)$, we get

$$\tan \left(\frac{\Psi}{2}\right) = \left(1 \pm (1 + F_2^2 (\phi) - F_1^2(\phi))^{1/2}\right) / \left(F_1(\phi) - F_2(\phi)\right)$$

$$\therefore \quad \Psi = 2 \arctan \{ (1 \pm (1 + F_2^2(\phi) - F_1^2(\phi))^{1/2}) / (F_1(\phi) - F_2(\phi)) \} \qquad \text{B.11}$$

The constraint to ensure mechanism closure for any given value of ϕ is

$$1 + F_2^2(\phi) - F_1^2(\phi) \geq 0 \qquad \text{B.12}$$

B-6 Transmissibility Index for a Planar Four-bar Linkage

See Section B-1 and Figure B-1 for parameter definitions. The following relations are derived using plane trigonometry:

$$\gamma = |\varepsilon - \pi/2|$$

from the sine law

$$\sin \varepsilon = d \sin \eta / b$$

but $\cos \gamma = \sin \varepsilon$

$$\therefore \quad TI = |\cos \gamma| = |d \sin \eta / b| \qquad \text{B.13}$$

B-7 Transmissibility Index for a Planar Slider-crank Linkage

See Section B-3 and Figure B-4 for parameter definitions. The following relation is derived using plane trigonometry

$$TI = |\cos \gamma| = |(s - a \cos \phi)/b| \qquad \text{B.14}$$

B-8 Transmissibility Index for Spatial Four-bar Linkage

See Section B-5 and Figure B-6 for parameter definitions.

As is explained in Section III of this thesis, the transmissibility index is the absolute value of the cosine of the angle between the coupler and the direction of follower motion. For this linkage the coupler is represented by the vector (see Section B-5)

$$\begin{aligned} 32 = &\left(a \cos \phi - e - c(\cos \Psi)\right)\bar{i} \\ &+ (a \sin \phi - d \sin \xi - c \sin \Psi \cos \xi)\bar{j} \\ &+ (f + c \sin \Psi \sin \xi - d \cos \xi)\,\bar{k} \quad . \end{aligned}$$

Since the magnitude of the follower velocity vector, $\bar{V}$, is unimportant, only its direction being required, let

$$\begin{aligned} \bar{R} &= (V_1/V_3)\,\bar{i} + (V_2/V_3)\,\bar{j} + \bar{k} \\ &= R_1\,\bar{i} + R_2\,\bar{j} + \bar{k} \quad . \end{aligned}$$

Vector $\bar{V}$ is perpendicular to both the follower link, 43, and the follower axis, 54;

$$\therefore \quad \bar{R} \cdot 43 = 0 \quad , \text{ and}$$

$$\bar{R} \cdot 54 = 0 \quad .$$

Thus using the expressions for 43 and 54 derived in Section B-5,

$$R_1\, c \cos \Psi + R_2\, c \sin \Psi \cos \xi - c \sin \Psi \sin \xi = 0 \ldots \qquad \text{B.15}$$

and $$R_2\, d \sin \xi + d \cos \xi = 0 \qquad \text{B.16}$$

From equation B.16,

$$R_2 = -\cos \xi / \sin \xi \quad .$$

Substituting the above expression for R_2 in equation B-15, we get

$$\begin{aligned} R_1 &= (1/\cos \Psi)(\sin \Psi \sin \xi + \sin \Psi \cos^2 \xi / \sin \xi) \\ &= \sin \Psi / \cos \Psi \sin \xi \end{aligned}$$

$$\therefore \quad \bar{R} = (\sin \Psi/\cos \Psi \sin \xi)\, \bar{i} - (\cos \xi/\sin \xi)\, \bar{j} + \bar{k}$$

$$= \tan \Psi\, \bar{i} - \cos \xi\, \bar{j} + \sin \xi\, \bar{k}$$

$$|\bar{R}| = (\tan^2 \Psi + \cos^2 \xi + \sin^2 \xi)^{1/2}$$

$$= \sec \Psi$$

Let $$\bar{U} = \frac{\bar{R}}{|\bar{R}|}^{*}$$

$$= -\sin \Psi\, \bar{i} + \cos \xi \cos \Psi\, \bar{j} - \sin \xi \cos \Psi\, \bar{k} \quad .$$

$\bar{U}$ is the unit vector along the line defined by the follower velocity vector.

The unit vector $\bar{U}$ can also be derived by using plane trigonometric inspection of Figure B-6 by expressing 43 initially in the X Y´ Z´ coordinate system as follows:

unit vector in 43 direction is

$$\cos \Psi\, \bar{i}' + \sin \Psi\, \bar{j}' \qquad \text{B.17}$$

$\therefore$ a unit vector perpendicular to $\bar{k}'$ and expression B-5 is

$$- \sin \Psi\, \bar{i}' + \cos \Psi\, \bar{j}' \qquad \text{B.18}$$

Transforming B-18 into the XYZ coordinate system, we get

$$\bar{U} = -\sin \Psi\, \bar{i} + \cos \xi \cos \Psi\, \bar{j} - \sin \xi \cos \Psi\, \bar{k}$$

which is identical to the previous expression for $\bar{U}$. Thus the inspection method is much quicker for this problem, but can be very difficult for problems where the follower-frame joint is a globular (ball-and-socket) joint. In such a case the vector mechanical technique described earlier is the best means of obtaining the required expressions to evaluate the transmissibility index.

---

* Sign change is for later comparisons, and does not affect the results, since the absolute value of the cosine is used for the TI.

Taking the scalar product of 32 and $\bar{U}$ to obtain TI,

$$
\begin{aligned}
TI &= |(32 \cdot \bar{U})/b| \\
&= |(-a \cos\phi \sin\Psi + e \sin\Psi + c \sin\Psi \cos\Psi \\
&\quad + a \sin\phi \cos\xi \cos\Psi - d \sin\xi \cos\xi \cos\Psi \\
&\quad - c \sin\Psi \cos^2\xi \cos\Psi - f \sin\xi \cos\Psi \\
&\quad - c \sin\Psi \sin^2\xi \cos\Psi + d \cos\xi \sin\xi \cos\Psi)/b| \\
&= |\big(\sin\Psi(e - a\cos\phi) + \cos\Psi(a \sin\phi \cos\xi \\
&\quad - f \sin\xi)\big)/b|
\end{aligned}
\qquad \text{B.19}
$$

APPENDIX C - RELATION BETWEEN TI AND MECHANICAL ERROR

Hartenberg [16] shows that the mechanical error in the output angle Ψ of a planar four-bar linkage due to small perturbations of one or more of the link lengths (which result, in practice, from manufacturing tolerances) yields an expression the denominator of which is (see Section B-1 and Figure B-1 for meanings of parameters).

$$2\,a\,c\,\sin\phi\,\cos\Psi - \sin\Psi\,(2\,c\,e + 2\,a\,c\,\cos\phi) \quad .$$

Hartenberg also shows that the above expression can be equated to

$$\pm\, 2\,b\,c\,\sin\varepsilon$$

where ε is the transmission angle previously defined.

For the planar four-bar linkage the transmissibility index is equivalent to the cosine of the complement of the transmission angle, or

$$TI = \cos\,(\pi/2 - \varepsilon) = \sin\varepsilon \qquad C.1$$

Therefore, the mechanical error of a planar four-bar linkage is proportional to 1/TI. (The same relation can be also shown for the planar slider-crank and spatial four-bar linkages.)

APPENDIX D - BALANCING AND ANALYSIS EQUATIONS

D-1 Velocity and Acceleration Equations for the Planar Four-bar

The appropriate parameters to be used in the development of the required equations are illustrated in figure D-1.

The following relations are derived using plane trigonometry:

$$d = (a^2 + f^2 - 2\ a\ f \cos\theta_1)^{1/2}$$

$$\alpha = \arccos\ \left((b^2 + d^2 - c^2)/2\ b\ d\right)$$

$$\beta = \arctan\ \left(a \sin\theta_1/(f - a\cos\theta_1)\right)$$

$$\lambda = \arccos\ \left((c^2 + d^2 - b^2)/2\ c\ d\right)$$

$$\theta_2 = -\beta \pm \alpha$$

$$\theta_3 = 2\pi - \beta \mp \lambda$$

$$\theta_4 = \pi$$

Expressing the links as complex number vectors,

$$a\ e^{i\theta_1} + b\ e^{i\theta_2} + c\ e^{i\theta_3} + f\ e^{i\theta_4} = 0$$

or

$$a\ e^{i\theta_1} + b\ e^{i\theta_2} + c\ e^{i\theta_3} = f \qquad \text{D.1}$$

Taking the derivative of D.1 w.r.t. time, t, setting $\frac{d\theta_i}{dt} = \omega_i$ we get

$$i\ a\ \omega_1\ e^{i\theta_1} + i\ b\ \omega_2\ e^{i\theta_2} + i\ c\ \omega_3\ e^{i\theta_3} = 0 \qquad \text{D.2}$$

Noting that $e^{i\theta_i} = \cos\theta_i + i \sin\theta_i$, and that both real and imaginary parts of both sides of a complex number equation must be equivalent, we get,

Re: $\quad - a\ \omega_1 \sin\theta_1 - b\ \omega_2 \sin\theta_2 - c\ \omega_3 \sin\theta_3 = 0$, and

Im: $\quad a\ \omega_1 \cos\theta_1 + b\ \omega_2 \cos\theta_2 + c\ \omega_3 \cos\theta_3 = 0$.

Solving the two above equations for ω_1 and ω_2 we get

$$\omega_2 = \omega_1\ a \sin(\theta_1 - \theta_3)/b \sin(\theta_2 - \theta_3), \text{ and}$$

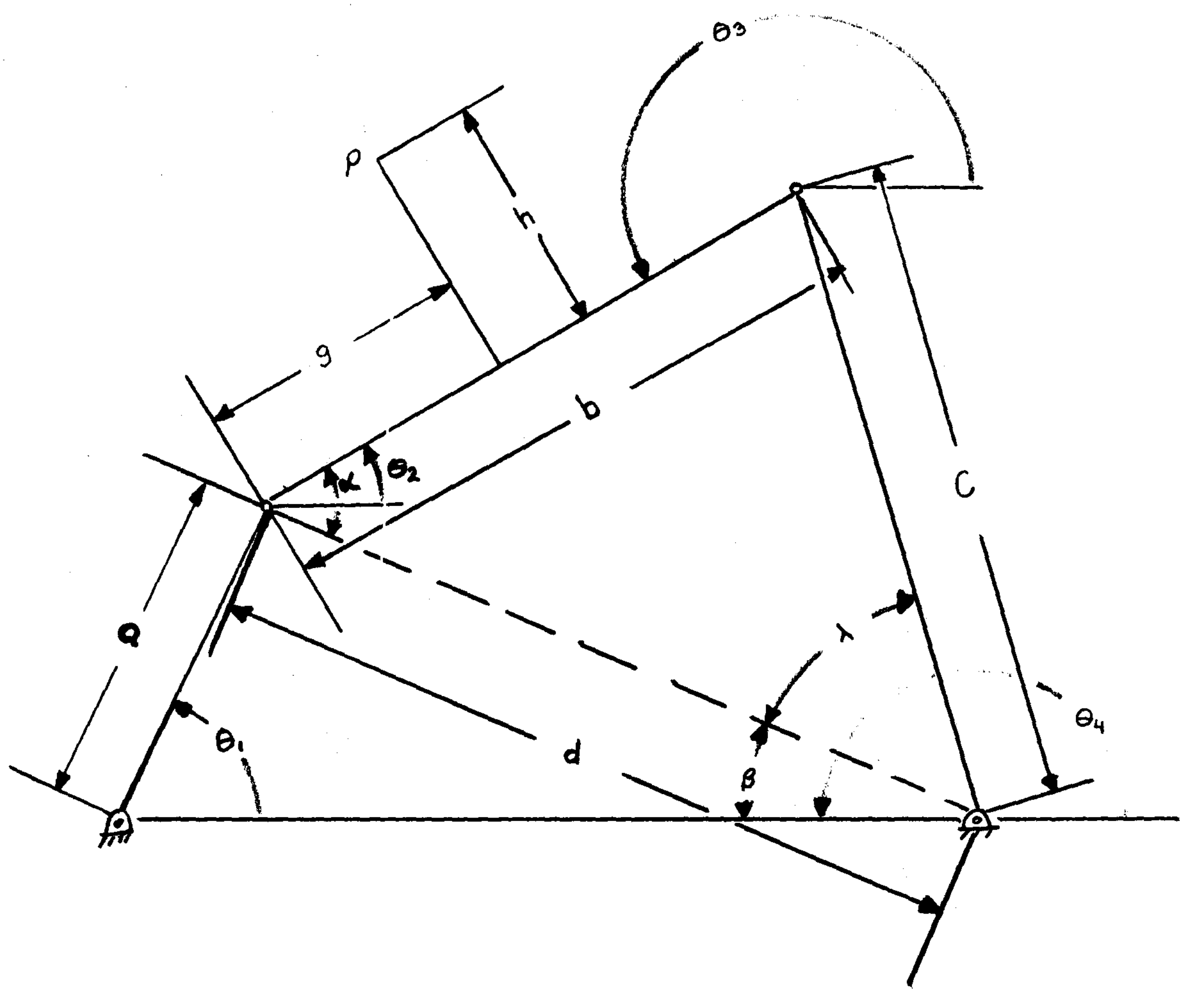


FIG. D-1 FOUR-BAR PARAMETERS

$$\omega_3 = \omega_1 \, a \sin(\theta_1 - \theta_2)/c \sin(\theta_2 - \theta_3) \quad .$$

Taking the derivative of D.2 w.r.t. time, t, and setting $\frac{d\omega_i}{dt} = \alpha_i$ we get

$$(\omega_1^2 - i\,\alpha_1)\, a\, e^{i\theta_1} + (\omega_2^2 - i\,\alpha_2)\, b\, e^{i\theta_2} + (\omega_3^2 - i\,\alpha_3)\, c\, e^{i\theta_3} = 0$$

from whence

Re: $a\,\omega_1^2 \cos\theta_1 + a\,\alpha_1 \sin\theta_1 + b\,\omega_2^2 \cos\theta_2 + b\,\alpha_2 \sin\theta_2 + c\,\omega_3^2 \cos\theta_3 + c\,\alpha_3 \sin\theta_3 = 0$, and

Im: $a\,\omega_1^2 \sin\theta_1 - a\,\alpha_1 \cos\theta_1 + b\,\omega_2^2 \sin\theta_2 - b\,\alpha_2 \cos\theta_2 + c\,\omega_3^2 \sin\theta_3 - c\,\alpha_3 \cos\theta_3 = 0 \quad .$

Solving the two above equations for α_2 and α_3 we get

$$\alpha_2 = \omega_2\,\alpha_1/\omega_1 - (a\,\omega_1^2 \cos(\theta_1 - \theta_3) + b\,\omega_2^2 \cos(\theta_2 - \theta_3) + c\,\omega_3^2)/b \sin(\theta_2 - \theta_3) \quad \text{, and}$$

$$\alpha_3 = \omega_3\,\alpha_1/\omega_1 + \left(a\,\omega_1^2 \cos(\theta_1 - \theta_2) + b\,\omega_2^2 + c\,\omega_3^2 \cos(\theta_2 - \theta_3)\right)/c \sin(\theta_2 - \theta_3) \quad .$$

Expressing the position of point p as a complex number vector, $\bar{p}$,

$$\bar{p} = a\,e^{i\theta_1} + g\,e^{i\theta_2} + i\,h\,e^{i\theta_2} \quad .$$

Taking the derivative of $\bar{p}$ w.r.t. time we get,

$$\frac{d\bar{p}}{dt} = \bar{V}_p = a\,i\,\omega_1\,e^{i\theta_1} + g\,i\,\omega_2\,e^{i\theta_2} - h\,\omega_2\,e^{i\theta_2} \quad ,$$

from whence,

Re $(\bar{V}_p) = -\,a\,\omega_1 \sin\theta_1 - g\,\omega_2 \sin\theta_2 - h\,\omega_2 \cos\theta_2$ (horizontal component), and

Im $(\bar{V}_p) = a\,\omega_1 \cos\theta_1 + g\,\omega_2 \cos\theta_2 - h\,\omega_2 \sin\theta_2$ (vertical component).

Therefore

$$|\bar{V}_p| = \left((\mathrm{Re}(\bar{V}_p))^2 + (\mathrm{Im}(\bar{V}_p))^2\right)^{1/2} \quad \text{, and}$$

$$\theta_{\bar{V}_p} = \arctan\left(\mathrm{Im}(\bar{V}_p)/\mathrm{Re}(\bar{V}_p)\right)$$

Taking the derivative of $\bar{V}_p$ w.r.t. time we get

$$\frac{d\bar{V}_p}{dt} = \bar{a}_p = (-a\,\omega_1^2 + a\,i\,\alpha_1)\,e^{i\theta_1}$$
$$+(-g\,\omega_2^2 + g\,i\,\alpha_2)\,e^{i\theta_2}$$
$$-(h\,\alpha_2 + h\,i\,\omega_2^2)\,e^{i\theta_2} ,$$

from whence,

$$\mathrm{Re}\,(\bar{a}_p) = -a\,\alpha_1 \sin\theta_1 - a\,\omega_1^2 \cos\theta_1 - g\,\alpha_2 \sin\theta_2$$
$$-g\,\omega_2^2 \cos\theta_2 - h\,\alpha_2 \cos\theta_2 + h\,\omega_2^2 \sin\theta_2 \text{ (horizontal component),}$$

and

$$\mathrm{Im}\,(\bar{a}_p) = a\,\alpha_1 \cos\theta_1 - a\,\omega_1^2 \sin\theta_1 + g\,\alpha_2 \cos\theta_2$$
$$-g\,\omega_2^2 \sin\theta_2 - h\,\alpha_2 \sin\theta_2 - h\,\omega_2^2 \cos\theta_2 \text{ (vertical component).}$$

Therefore,

$$|\bar{a}_p| = \left((\mathrm{Re}\,(\bar{a}_p))^2 + (\mathrm{Im}\,(\bar{a}_p))^2\right)^{1/2}, \text{ and}$$
$$\theta_{\bar{a}_p} = \arctan\left(\mathrm{Im}\,(\bar{a}_p)/\mathrm{Re}\,(\bar{a}_p)\right)$$

D-2 Velocity and Acceleration Equations for the Planar Slider-crank

The appropriate parameters to be used in the development of the required equations are illustrated in Figure D.2

The following relations are derived using plane trigonometry:

$$\theta_2 = \arctan\left((c - a\sin\theta_1)/ \pm (b^2 - (c - a\sin\theta_1)^2)^{1/2}\right)$$
$$\theta_3 = 3\,\pi/2$$
$$\theta_4 = \pi$$

Expressing the links in complex number notation,

$$a\,e^{i\theta_1} + b\,e^{i\theta_2} + c\,e^{i\theta_3} + s\,e^{i\theta_4} = 0$$

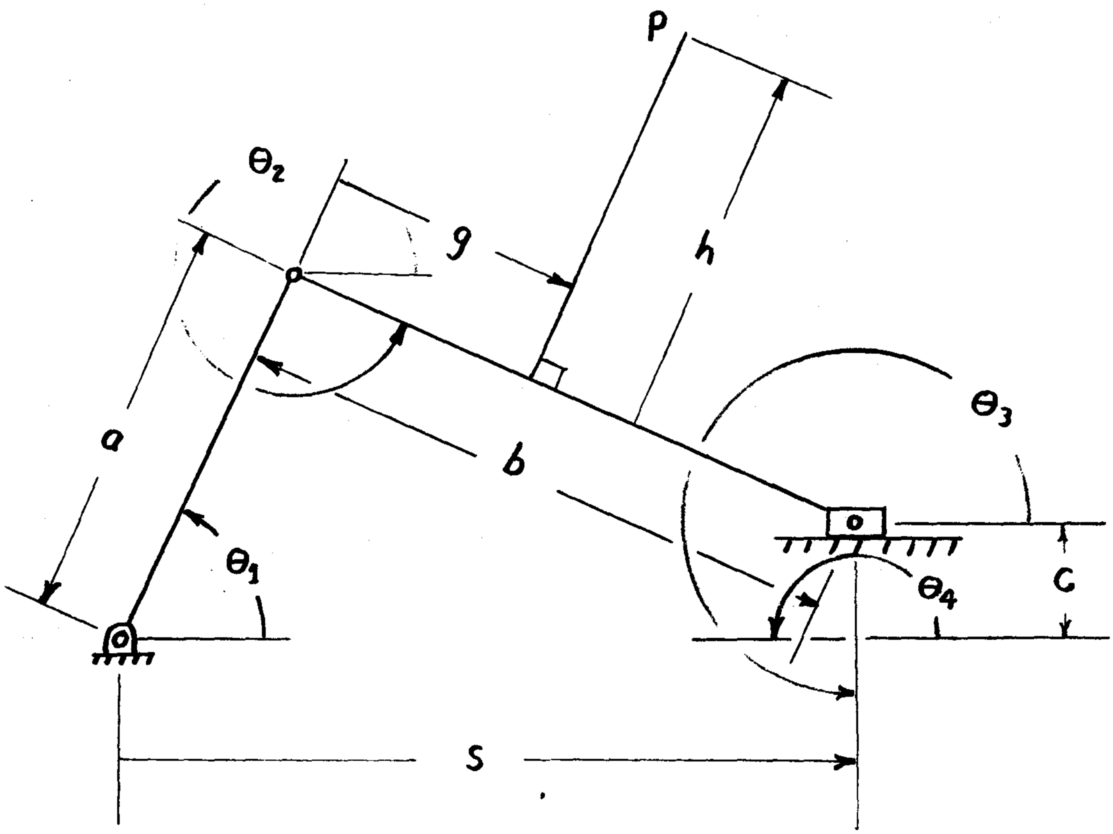


FIG. D-2 SLIDER-CRANK PARAMETERS

Differentiating the above with respect to time, and setting $\omega_i = \frac{d\theta_i}{dt}$,

we get

$$a\, i\, \omega_1\, e^{i\theta_1} + b\, i\, \omega_2\, e^{i\theta_2} + \frac{ds}{dt}\, e^{i\theta_4} = 0 \qquad \text{D.3}$$

from whence

Re: $\quad - a\, \omega_1 \sin\theta_1 - b\, \omega_2 \sin\theta_2 + \frac{ds}{dt} \cos\theta_4 = 0$, and

Im: $\quad a\, \omega_1 \cos\theta_1 + b\, \omega_2 \cos\theta_2 + \frac{ds}{dt} \sin\theta_4 = 0$.

Solving the above two equations for ω_2 we get

$$\omega_2 = - a\, \omega_1 (\cos\theta_1 \cos\theta_4 + \sin\theta_1 \sin\theta_4)/b\, (\cos\theta_2 \cos\theta_4 + \sin\theta_2 \sin\theta_4)$$

Since $\theta_4 = \pi$, ω_2 reduces to

$$\omega_2 = - a\, \omega_1 \cos\theta_1/b \cos\theta_2 \quad ;$$

also

$$\frac{ds}{dt} = - a\, \omega_1 \sin\theta_1 - b\, \omega_2 \sin\theta_2 \quad .$$

Differentiating D.3 w.r.t. time, t, and setting $\alpha_i = \frac{d\omega_i}{dt}$, we get

$$- a\, \omega_1^2\, e^{i\theta_1} + i\, a\, \alpha_1\, e^{i\theta_1} - b\, \omega_2^2\, e^{i\theta_2} + i\, b\, \alpha_2\, e^{i\theta_2} + \frac{d^2s}{dt^2}\, e^{i\theta_4} = 0 \quad ,$$

from whence

Re: $\quad -a\, \omega_1^2 \cos\theta_1 - a\, \alpha_1 \sin\theta_1 - b\, \omega_2^2 \cos\theta_2 - b\, \alpha_2 \sin\theta_2 + \frac{d^2s}{dt^2} \cos\theta_4 = 0$, and

Im: $\quad - a\, \omega_1^2 \sin\theta_1 + a\, \alpha_1 \cos\theta_1 - b\, \omega_2^2 \sin\theta_2 + b\, \alpha_2 \cos\theta_2 + \frac{d^2s}{dt^2} \sin\theta_4 = 0$.

Substituting $\theta_4 = \pi$ into the above imaginary equation we get

$$\alpha_2 = (a\, \omega_1^2 \sin\theta_1 - a\, \alpha_1 \cos\theta_1 + b\, \omega_2^2 \sin\theta_2)/b \cos\theta_2 \quad ,$$

but

$$- a \cos \theta_1 / b \cos \theta_2 = \omega_2/\omega_1 \quad .$$

Therefore

$$\alpha_2 = \omega_2 \, \alpha_1/\omega_1 + (a \, \omega_1^2 \sin \theta_1 + b \, \omega_2^2 \sin \theta_2)/b \cos \theta_2$$

$\frac{d^2 s}{dt^2}$ can be obtained by substituting $\theta_4 = \pi$ and α_2 into the real expression above, but it is more convenient to obtain it by considering a coupler point with g = b and h = 0. The resulting expression for $\bar{a}_p$ is then equivalent to $\frac{d^2 s}{dt^2}$.

The expressions for $|\bar{V}_p|$ and $\theta_{\bar{V}_p}$, and $|\bar{a}_p|$ and $\theta_{\bar{a}_p}$ for a planar slider-crank coupler point p are identical to those for the planar four-bar coupler point except that θ_2 is defined differently.

D-3 Balancing Equations

As shown in Figure D-3, any point t on the crank link can be expressed in terms of an m and n, and similarly, any point u on the follower link can be expressed in terms of a q and r.

The horizontal and vertical components of acceleration of a point t on the crank link (derived in a manner similar to that for point p on the coupler link in Section D-1) are respectively

$$|\bar{a}_t|_h = - m \, \omega_1^2 \cos \theta_1 - m \, \alpha_1 \sin \theta_1 + n \, \omega_1^2 \sin \theta_1$$
$$- n \, \alpha_1 \cos \theta_1 \quad , \text{and}$$

$$|\bar{a}_t|_V = - m \, \omega_1^2 \sin \theta_1 + m \, \alpha_1 \cos \theta_1 - n \, \omega_1^2 \cos \theta_1$$
$$- n \, \alpha_1 \sin \theta_1 \quad .$$

Similarly the horizontal and vertical components of the

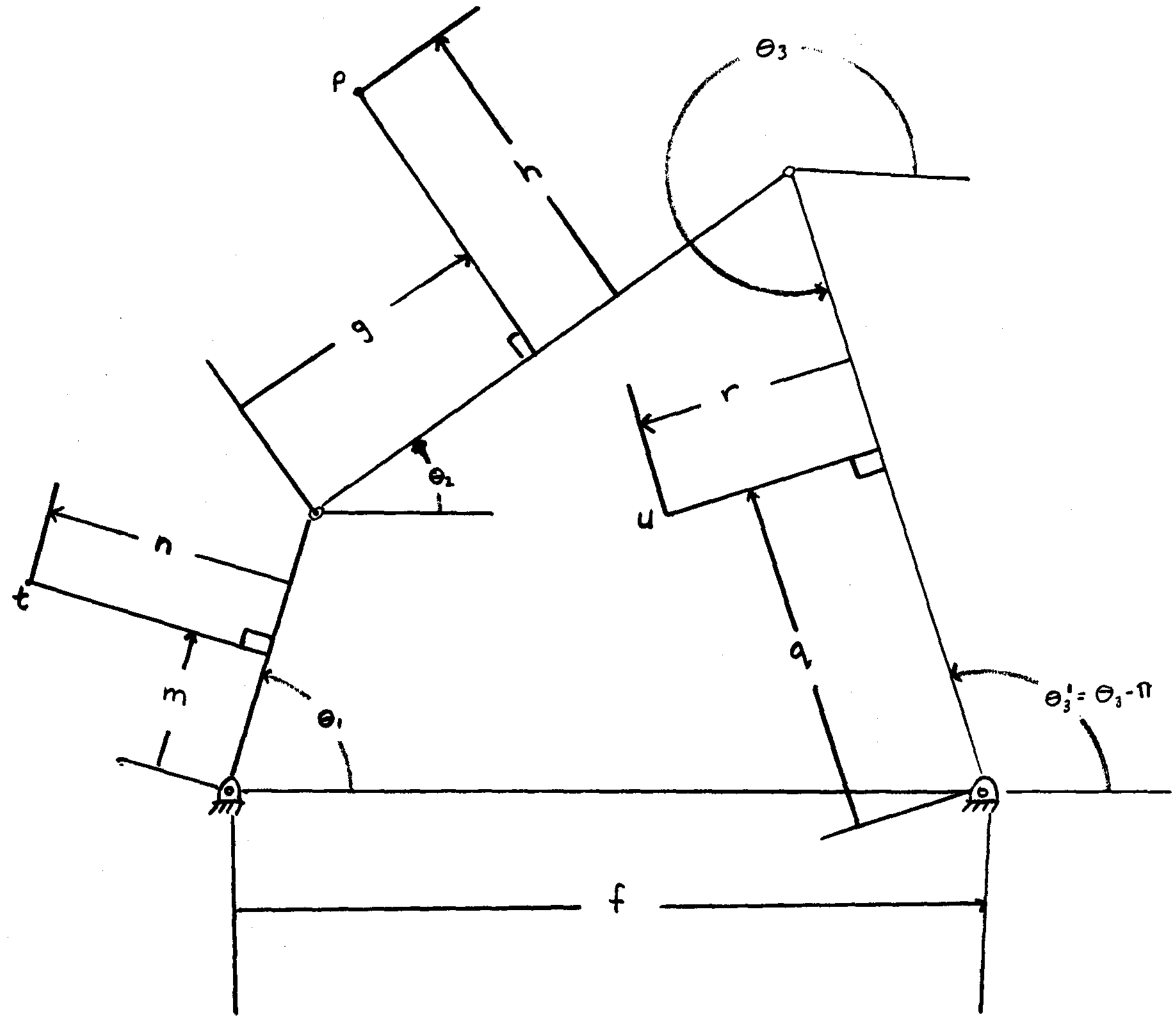


FIG. D-3 FOUR-BAR LINK POINTS

acceleration of a point u on the follower link of a planar four-bar linkage are respectively

$$|\bar{a}_u|_h = - q\,\omega_3^2 \cos\theta_3' - q\,\alpha_3 \sin\theta_3' + r\,\omega_3^2 \sin\theta_3' - r\,\alpha_3 \cos\theta_3' \quad , \text{ and}$$

$$|\bar{a}_u|_v = - q\,\omega_3^2 \sin\theta_3' + q\,\alpha_3 \cos\theta_3' - r\,\omega_3^2 \cos\theta_3' - r\,\alpha_3 \sin\theta_3' \quad .$$

Thus the linear accelerations of any points t, p, and u on the crank, coupler, and follower links respectively can be analytically determined in terms of the angular displacement, velocity, and acceleration of the crank link.

If t, p, and u are the locations of the centres of mass of the crank, coupler, and follower links of masses and moments of inertia M_i and J_i, i=1, 2, 3, respectively, then the horizontal shaking force (SFH), vertical shaking force (SFV), and the counterclockwise shaking moment (SMO) about the crankshaft axis can be calculated as follows:

$$\stackrel{+}{\rightarrow} \quad SFH = - M_1\,a_{t_h} - M_2\,a_{p_h} - M_3\,a_{u_h} \quad ,$$

$$+\uparrow \quad SFV = - M_1\,a_{t_v} - M_2\,a_{p_v} - M_3\,a_{u_v} \quad , \text{ and}$$

$$\begin{aligned} SMO = & + M_1\,a_{t_h}\,(m \sin\theta_1 + n \cos\theta_1) - M_1\,a_{t_v}\,(m \cos\theta_1 - n \sin\theta_1) \\ & + M_2\,a_{p_h}\,(a \sin\theta_1 + g \sin\theta_2 + h \cos\theta_2) \\ & - M_2\,a_{p_v}\,(a \cos\theta_1 + g \cos\theta_2 - h \sin\theta_2) \\ & + M_3\,a_{u_h}\,(q \sin\theta_3 + r \cos\theta_3) - M_3\,a_{u_v}\,(f + q \cos\theta_3 - r \sin\theta_3) \\ & - J_1\,\alpha_1 - J_2\,\alpha_2 - J_3\,\alpha_3 \end{aligned}$$

The above expressions represent the unbalanced shaking forces and moments for a given crank angle, crank angular velocity and crank angular acceleration of a completely defined planar four-bar linkage. The expressions for a planar slider-crank are similar except that the vertical inertia force and inertia torque of the slider are nonexistent. Thus the horizontal inertia force of the slider is the only additional term to the inertia forces and torques of the crank and coupler links.

The expressions for the inertia force and moment effects of the counterweights are identical to those for the crank and follower links except that M_1, J_1, m, n, M_3, J_3, q, and r have the values corresponding to the crank and follower counterweight design variables which are being optimized.

APPENDIX E - PROGRAM USER'S GUIDE

CONTENTS Page

1. PURPOSE

A generalized (system) FORTRAN computer program has been created to handle an assortment of common linkage analysis, dimensional synthesis, and balancing problems. The linkage types used are the planar four-bar (RRRR), planar slider-crank (RRRP), and spatial four-bar (RGGR). The problem types that can be solved using this system program are discussed under their respective headings in Section 3.

The system program is arranged so that a designer can solve both easy and complex linkage design problems with a minimum understanding of the methods of solution used and a minimum computer programming knowledge.

The computer output, which consists of both plots and tabulated values, is printed out at key stages of the design process so that a designer obtains considerable insight into the actual performance of his linkage as it relates to his design requirements.

The method used for the dimensional synthesis of the linkages for coupler-point curve and function generating problems simultaneously minimizes the maximum structural error*,

* See Chapter III for an explanation of the structural error.

maximizes the minimum transmissibility index*, and minimizes the maximum link length. For such problems the method also ensures that linkage closure is possible at all points within the designated range of linkage motion.

The linkage analysis method used is an exact one based on complex number vector analysis techniques - the only limit on accuracy being that of the particular computer used.

The linkage balancing method used adds optimum counterweights to the crank and follower links of the planar four-bar linkage, and to the crank link only of the planar slider-crank linkage. The size and position of these optimum counterweights are optimized by simultaneously minimizing the total horizontal shaking force**, the total vertical shaking force, and the total counterclockwise shaking moment about the crankshaft axis.

2. HOW TO USE

The input required for all the problems which can be handled by this program is minimal. This input is contained in

* See Chapter III for an explanation of the transmissibility index.

** See Chapter IV for definitions of the shaking forces and moments, and for reasons for their use.

a small user defined MAIN program and up to two small additional (service) subroutines. The particular format of the main program and the service subroutines is described for each problem type in Section 3.

The objective function and the basic required constraints for each synthesis problem are completely defined within the system program except for the scaling factors (see Section 4) and any additional constraints required for a specific problem (see Section 7).

The meanings of the particular independent design variables and the basic constraints for optimization are explained under each problem type heading in Section 3.

After defining the required input parameters in the main program, which is most easily done by using a FORTRAN* DATA statement, the user need only make the appropriate call to the required subroutine as outlined in Section 3.

The required program input parameter values are transferred from the user's main program to the system program through labelled COMMON blocks which must be included in the

* The user supplied main program and service subroutines must be written in the FORTRAN II, FORTRAN IV, or FORTRAN VI programming language.

main program and service subroutines exactly as shown in the examples shown in Section 3. (The meanings of all the variables in the labelled COMMON blocks are listed in Section 9.) It is suggested that the user duplicate a number of these labelled COMMON blocks using the correct formats shown in Figure 9-1 to avoid simple, but costly, mistakes in future use.

3. PROBLEM TYPES

3.1 Planar Four-bar Function Synthesis (METHOD=1)

Purpose

The system program determines the optimum linkage dimensions and starting position to generate a particular functional relation (FUNSYN) between the crank (input) angle and the follower (output) angle within a designated range of crank rotation (RNGI).

Input Parameters (see Section 9 for meanings)

METHOD, ICASE, NPP, IEXCO, ISYM, XMIN, XMAX, RNGI, RNGO, TITE, SCAL1, SCAL2, SCAL3, STRTPT

Input COMMON Blocks

STRTPT, SCLFAC, NUMBERS, SYNIN

Input Routines

MAIN, FUNSYN

Program Set-up

Figure 3-1 shows a typical input for this problem type. The main program consists of the required input COMMON blocks, the input parameters defined in a DATA statement, and a call to subroutine LINK. Function subprogram FUNSYN contains the desired functional relation between the input and output link angles.

Basic Output*

Optimum independent design variables (see Figure 3-2):

X(1) - the crank length divided by the frame length**;

X(2) - the coupler length divided by the frame length;

X(3) - the follower length divided by the frame length; and

X(4) - the mid-point angle of the crank range of rotation (in radians counterclockwise positive).

* If user added constraints are employed in subroutine EXCON (see Section 7), then these constraint values at the optimum are printed out following the basic constraints.

** The frame length is thus always equal to one; this is because only the link angles are required to determine the linkage input/output functional relations.

FIG. 3-1 PLANAR FOUR-BAR FUNCTION SYNTHESIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /STRTPT/STRTPT(10)
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
 COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
 DATA METHOD/1/,ICASE/-1/,NPP/ 9/,IEXCO/1/,ISYM/0/,
1XMIN/-1./,XMAX/1./,RNGI/90./,RNGO/60./,TITE/20./,SCAL1/.5/,
2SCAL2/.01/,SCAL3/10./,STRTPT/-.6102,.5656,.3804,2.768/
 CALL LINK(1,.0001,1.,2.5,.0001)
 STOP
 END

 FUNCTION FUNSYN(X)
 FUNSYN=X*X
 RETURN
 END
```

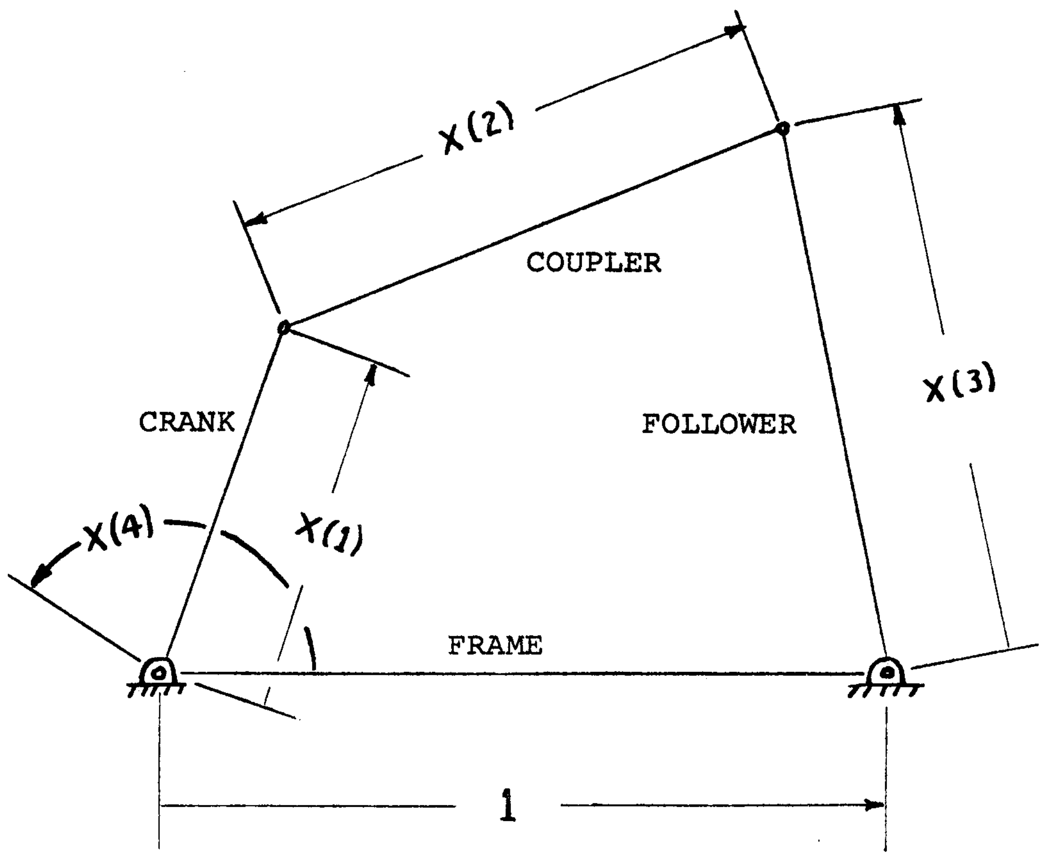


FIG. 3-2 PLANAR FOUR-BAR FUNCTION GENERATING VARIABLES

Basic constraints*:

C(1) - ensures linkage closure;

C(2) - prevents X(4) from increasing to infinity; and

C(3) - keeps the actual range of output motion within the desired range of putput motion (RNGO ± TITE).

Typical output includes a minimization from at least two starting points, the optimum independent design variable parameters, the minimum value of the objective function with its contributing factors itemized, the constraint values at the optimum point, the total number of objective function calls, and the total execution time. A preliminary analysis is automatically done on the optimum linkage; the output from this analysis includes a structural error plot, the optimum crank starting angle, the corresponding follower starting angle, the actual range of output (follower) motion, and the minimum transmissibility index.

* All constraint values should be positive at the optimum point for a feasible solution.

3.2 Planar Four-bar Coupler-point Curve Synthesis (METHOD=2)

Purpose

The system program determines the optimum linkage dimensions and position to generate a given coupler-point curve as a function of the crank (input) angle. The coupler-point curve is defined relative to a given co-ordinate system by specifying particular desired positions (with tolerances) on the curve for given crank angles. (See COUPLER in Section 10 and DESIRE in Section 9 for more details on specifying the desired coupler curve.) The optimum position of the four-bar linkage is determined relative to the same (H-V in Figure 3-4) co-ordinate system that is used to define the desired coupler-point positions.

Input Parameters (see Section 9 for meanings)

METHOD, IEXCO, NPP, ICASE, SCAL1, SCAL3, RNGI, STRTPT, XD, YD, ANG, XTOL, YTOL

Input COMMON Blocks

STRTPT, NUMBERS, SCLFAC, SYNIN, DESIRE

Input Routines

MAIN, COUPLER

Program Set-up

Figure 3-3 shows a typical input for this problem type. The first eight input parameters listed above are defined in MAIN along with the first four COMMON blocks listed above.

The last five input parameters are defined, along with COMMON block DESIRE, in COUPLER. A call to subroutine LINK initiates the system program.

Basic Output

Optimum independent design variables (see Figure 3-4):

X(1) - crank link length;

X(2) - coupler link length;

X(3) - follower link length;

X(4) - crank link mid-range angle relative to the frame (in radians counterclockwise positive);

X(5) - frame link angle relative to the horizontal (in radians counterclockwise positive);

X(6) - frame link length;

X(7) and

X(8) - co-ordinates of the desired coupler point relative to the coupler link; and

X(9) and

X(10)- co-ordinates of the crankshaft axis relative to the H-V co-ordinate system.

Basic constraints:

C(1) - ensures linkage closure;

C(2) - prevents X(4) from going to infinity; and

C(3) - prevents X(5) from going to infinity.

FIG. 3-3 PLANAR FOUR-BAR COUPLER-POINT CURVE SYNTHESIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /STRTPT/STRTPT(10)
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
 COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
 DATA METHOD/2/,IEXCO/1/,NPP/9/,ICASE/+1/,SCAL1/.5/,
1SCAL3/2.5/,RNGI/360./,STRTPT/.3,.96,.54,.6,-.58,1.09,1.89,0.,0.,
2.6/
 CALL LINK(1,.0001,1.,2.5,.0001)
 STOP
 END

 SUBROUTINE COUPLER
 COMMON /DESIRE/X(21),Y(81),ANG(81),XTOL(21),YTOL(21)
NOTE THAT X IS USED INSTEAD OF XD AND Y INSTEAD OF YD
THIS IS O.K. SINCE THERE IS CONSISTENCY BETWEEN DATA AND COMMON
STATEMENTS   I.E. A GIVEN VARIABLE NAME CAN BE CHANGED FROM THAT
SUGGESTED BUT THE ORDER OF THE VARIABLES IN THE COMMON BLOCKS
CANNOT BE CHANGED
 DATA X/2.2,2.04,1.87,1.71,1.55,1.55,1.75,2.,2.2/,
1Y/.2,.2,.2,.2,.2,.35,.7,.7,.35/,XTOL/.01,.1,.2,.1,.01,.05,.2,.2,
2.05/,YTOL/6*.05,.4,.4,.05/,ANG/0.,.61,1.22,1.83,2.44,2.79,4.19,
35.24,5.93/
 RETURN
 END
```

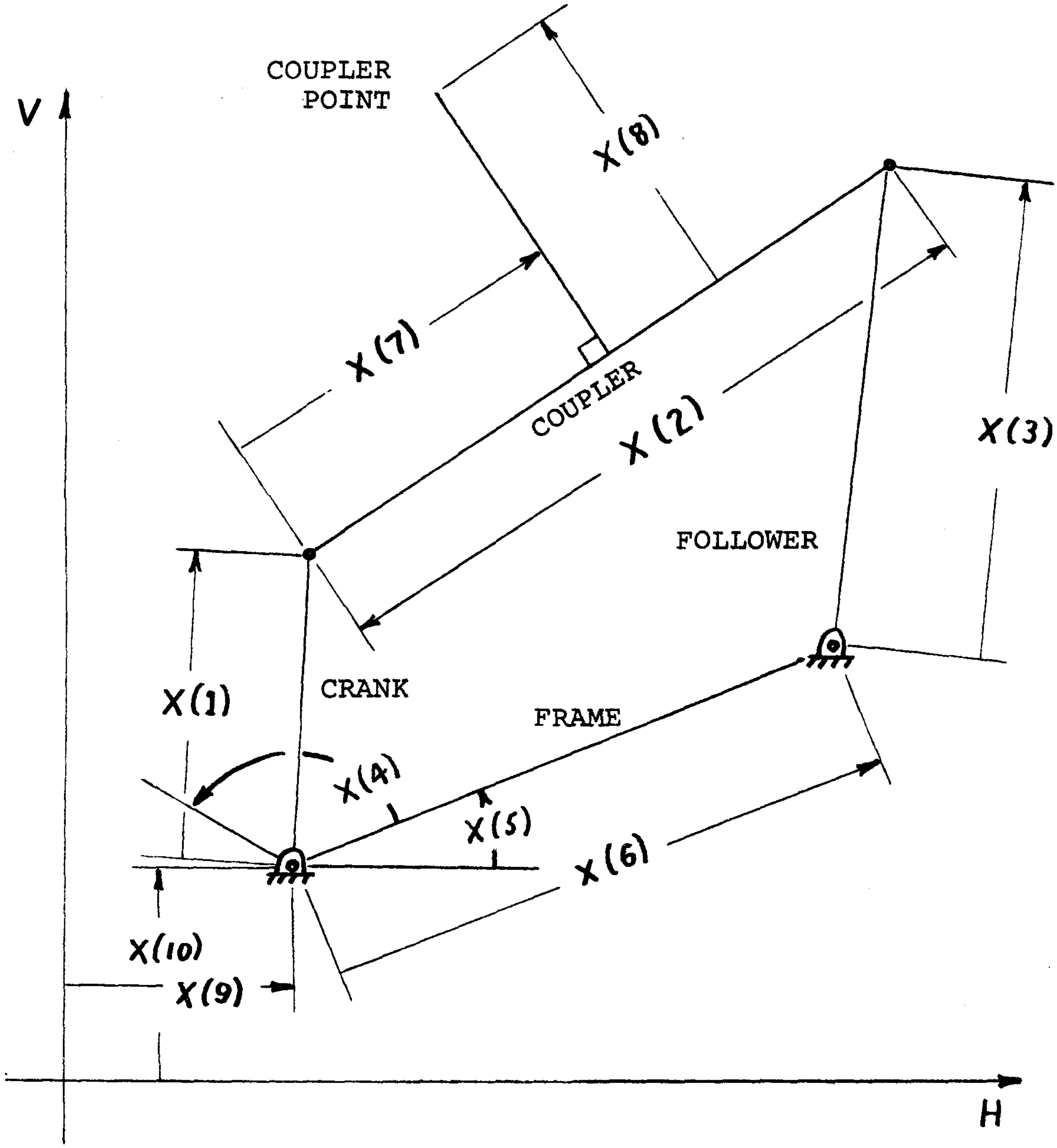


FIG. 3-4 PLANAR FOUR-BAR COUPLER-POINT CURVE GENERATING VARIABLES

The rest of the output is similar to that for Section 3-1 except that the desired and actual coupler point positions, as well as the differences between them (the structural errors), are plotted.

3.3 Planar Slider-crank Function Synthesis (METHOD=3)

Purpose

The system program determines the optimum linkage dimensions and starting position to generate a given functional relation (FUNSYN) between the crank (input) angle and the slider (output) position within a designated range of crank rotation.

Input Parameters (see Section 9 for meanings)

METHOD, ICASE, NPP, RNGI, RNGO, TITE, IEXCO, XMIN, XMAX, CFRIC, ISYM, SCAL1, SCAL2, SCAL3, STRTPT

Input COMMON Blocks

STRTPT, SCLFAC, NUMBERS, SYNIN

Input Routines

MAIN, FUNSYN

Program Set-up

See Figure 3-5 for a sample input. All the input parameters are defined in MAIN; the desired functional relation is defined in function subprogram FUNSYN. A call to subroutine

FIG. 3-5 PLANAR SLIDER-CRANK FUNCTION SYNTHESIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /STRTPT/STRTPT(10)
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
 COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
 DATA RNGI/360./,RNGO/4./,METHOD/3/,ICASE/1/,NPP/15/,TITE/.1/,
1IEXCO/1/,XMIN/0./,XMAX/6.2831853072/,CFRIC/.3/,ISYM/0/,
2SCAL1/.6428/,SCAL2/.01/,SCAL3/10./,STRTPT/2.,10.,0.,3.14159/
 CALL LINK(1,.0001,1.,2.5,.0001)
 STOP
 END

 FUNCTION FUNSYN(X)
 FUNSYN=COS(X)
 RETURN
 END
```

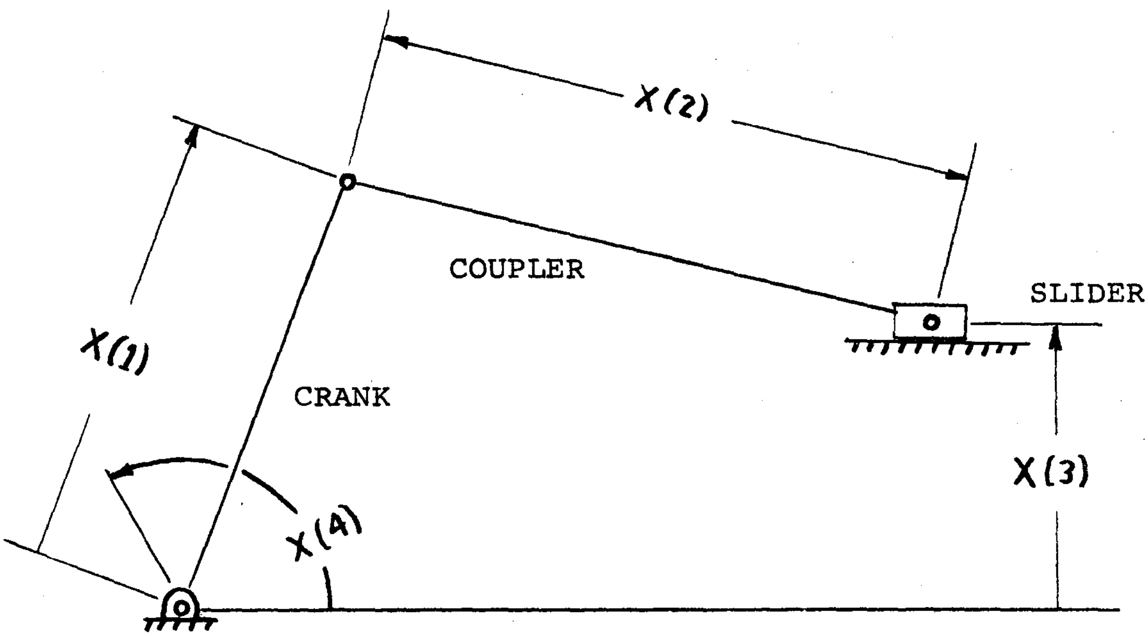


FIG. 3-6 PLANAR SLIDER-CRANK FUNCTION GENERATING VARIABLES

LINK initiates the system program.

Basic Output

Optimum independent design variables (see Figure 3-6):

X(1) - crank link length;

X(2) - follower link length;

X(3) - slider eccentricity (upwards positive); and

X(4) - crank mid-range angle (in radians positive counterclockwise).

Basic constraints:

C(1) - ensures linkage closure;

C(2) - prevents X(4) from going to infinity; and

C(3) - keeps the actual range of output motion within the desired range of output motion (RNGO ± TITE)

The rest of the computer output is similar to that for Section 3-1.

3.4 Planar Slider-crank Coupler-point Curve Synthesis (METHOD=4)

Purpose

The system program determines the optimum linkage dimensions and position to generate a given coupler-point curve as a function of the crank (input) angle. (The comments in the Purpose for Section 3-2 also apply to this section.)

Input Parameters (see Section 9 for meanings)

METHOD, ICASE, NPP, IEXCO, RNGI, SCAL1, SCAL3, CFRIC, STRTPT, XD, YD, ANG, XTOL, YTOL

Input COMMON Blocks

STRTPT, SCLFAC, NUMBERS, SYNIN, DESIRE

Input Routines

MAIN, COUPLER

Program Set-up

Figure 3-7 shows a typical input for this problem type. The first nine input parameters listed above are defined in MAIN along with the first four COMMON blocks listed above. The last five input parameters are defined, along with COMMON block DESIRE, in COUPLER. A call to subroutine LINK initiates the system program.

Basic Output

Optimum independent design variables (see Figure 3-8)

X(1) - crank link length;

X(2) - coupler link length;

X(3) - slider ecentricity;

X(4) - crank mid-range angle (in radians counter-clockwise positive);

X(5) - frame link angle (in radians counter-clockwise positive);

FIG. 3-7 PLANAR SLIDER-CRANK COUPLER-POINT CURVE SYNTHESIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /STRTPT/STRTPT(10)
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
 COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
 DATA RNGI/270./,IEXCO/1/,NPP/4/,METHOD/4/,ICASE/1/,SCAL1/.5/,
1SCAL3/20./,CFRIC/.3/,STRTPT/2.,8.,0.,3.14,0.,5.,0.,0.,2./
 CALL LINK(1,.0001,1.,2.5,.0001)
 STOP
 END

 SUBROUTINE COUPLER
 COMMON /DESIRE/X(21),Y(81),ANG(81),XTOL(21),YTOL(21)
NOTE THAT X IS USED INSTEAD OF XD AND Y INSTEAD OF YD
THIS IS O.K. SINCE THERE IS CONSISTENCY BETWEEN DATA AND COMMON
STATEMENTS   I.E. A GIVEN VARIABLE NAME CAN BE CHANGED FROM THAT
SUGGESTED BUT THE ORDER OF THE VARIABLES IN THE COMMON BLOCKS
CANNOT BE CHANGED
 DATA X/1.,3.,5.,3./,Y/2.,1.,2.,3./,
1ANG/0.,1.5707963,3.1415926,4.7123889/,
2XTOL/4*.1/,YTOL/4*.1/
 RETURN
 END
```

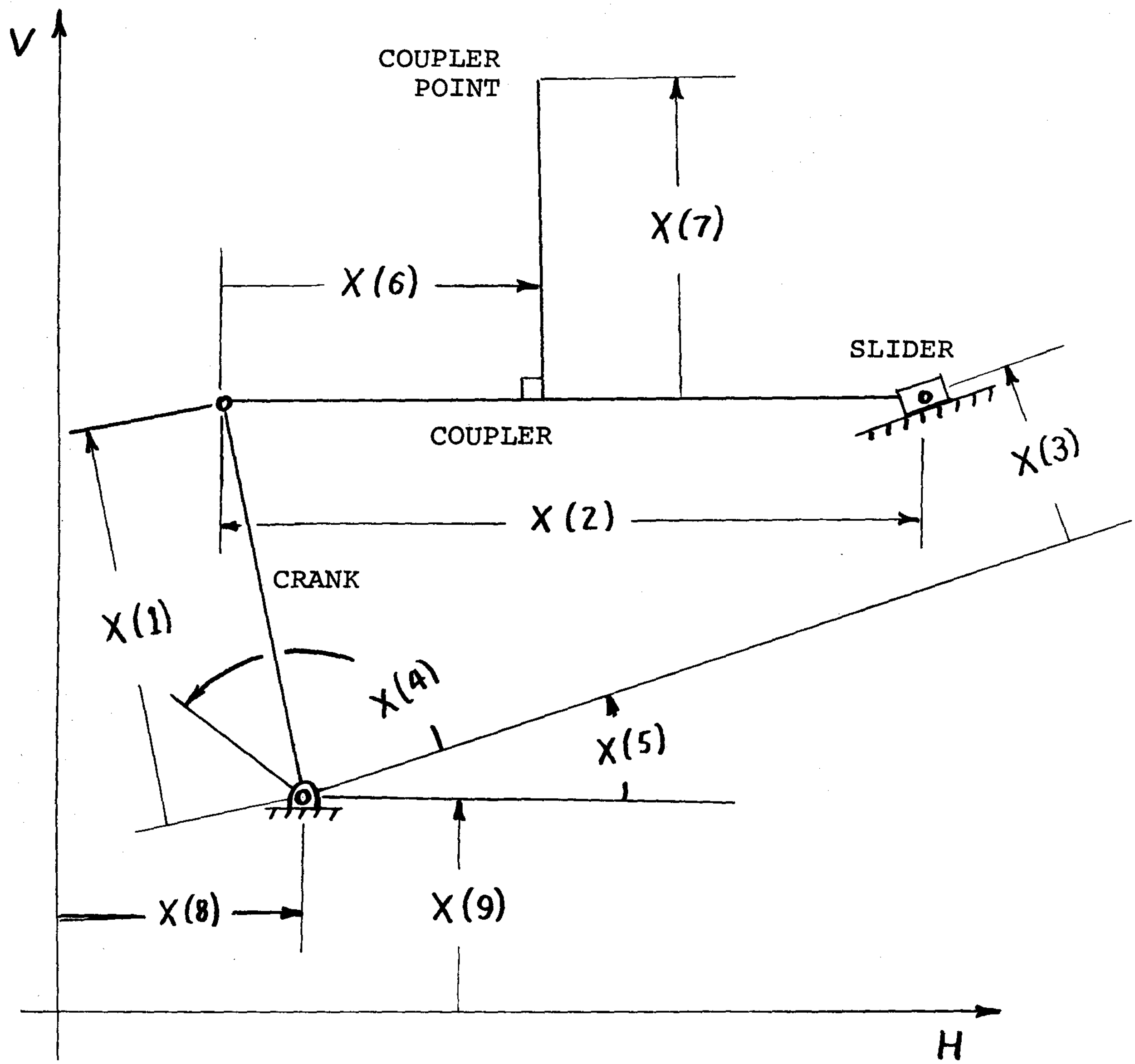


FIG. 3-8 PLANAR SLIDER-CRANK COUPLER-POINT CURVE GENERATING VARIABLES

X(6) and

X(7) - co-ordinates of the desired coupler point relative to the coupler link; and

X(8) and

X(9) - co-ordinates of the crankshaft axis relative to the H-V co-ordinate system.

Basic constraints:

C(1) - ensures linkage closure;

C(2) - prevents X(4) from going to infinity; and

C(3) - prevents X(5) from going to infinity

The rest of the computer output is similar to that for Section 3-2.

3.5 Spatial Four-bar Function Generation (METHOD=5)

Purpose

The system program determines the optimum linkage dimensions and starting position to generate a particular functional relation (FUNSYN) between the crank (input) angle and the follower (output) angle within a designated range of crank rotation (RNGI).

Input Parameters (see Section 9 for meanings)

METHOD, ICASE, NPP, IEXCO, ISYM, XMIN, XMAX, RNGI, RNGO, TITE, SCAL1, SCAL2, SCAL3, STRTPT

Input COMMON Blocks

STRTPT, SCLFAC, NUMBERS, SYNIN

Input Routines

MAIN, FUNSYN

Program Set-up

See Figure 3-9 for a sample input. All the input parameters are defined in MAIN; the desired functional relation is defined in function subprogram FUNSYN. A call to subroutine LINK initiates the system program.

Basic Output

Optimum independent design variables (see Figure 3-10):

X(1) - the crank length divided by the coupler length*;

X(2) - the follower length divided by the coupler length;

X(3) - the distance along the follower shaft axis from the X-axis to the follower pin divided by the coupler length;

X(4) - the crank mid-range angle (in radians counterclockwise positive);

* The coupler length is thus always equal to one; this is because only the link angles are required to determine the linkage input/output functional relations.

FIG. 3-9 SPATIAL FOUR-BAR FUNCTION SYNTHESIS INPUT

```
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON /STRTPT/STRTPT(10)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      DATA METHOD/5/,ICASE/-1/,NPP/ 9/,IEXCO/1/,ISYM/0/,
     1XMIN/-1./,XMAX/1./,RNGI/200./,RNGO/100./,TITE/30./,SCAL1/.5/,
     2SCAL2/.01/,SCAL3/10./,STRTPT/-.449,.0929,0.,0.,4.71,.1797,-.8227/
      CALL LINK(1,.0001,1.,2.5,.0001)
      STOP
      END

      FUNCTION FUNSYN(X)
      FUNSYN=X*X
      RETURN
      END
```

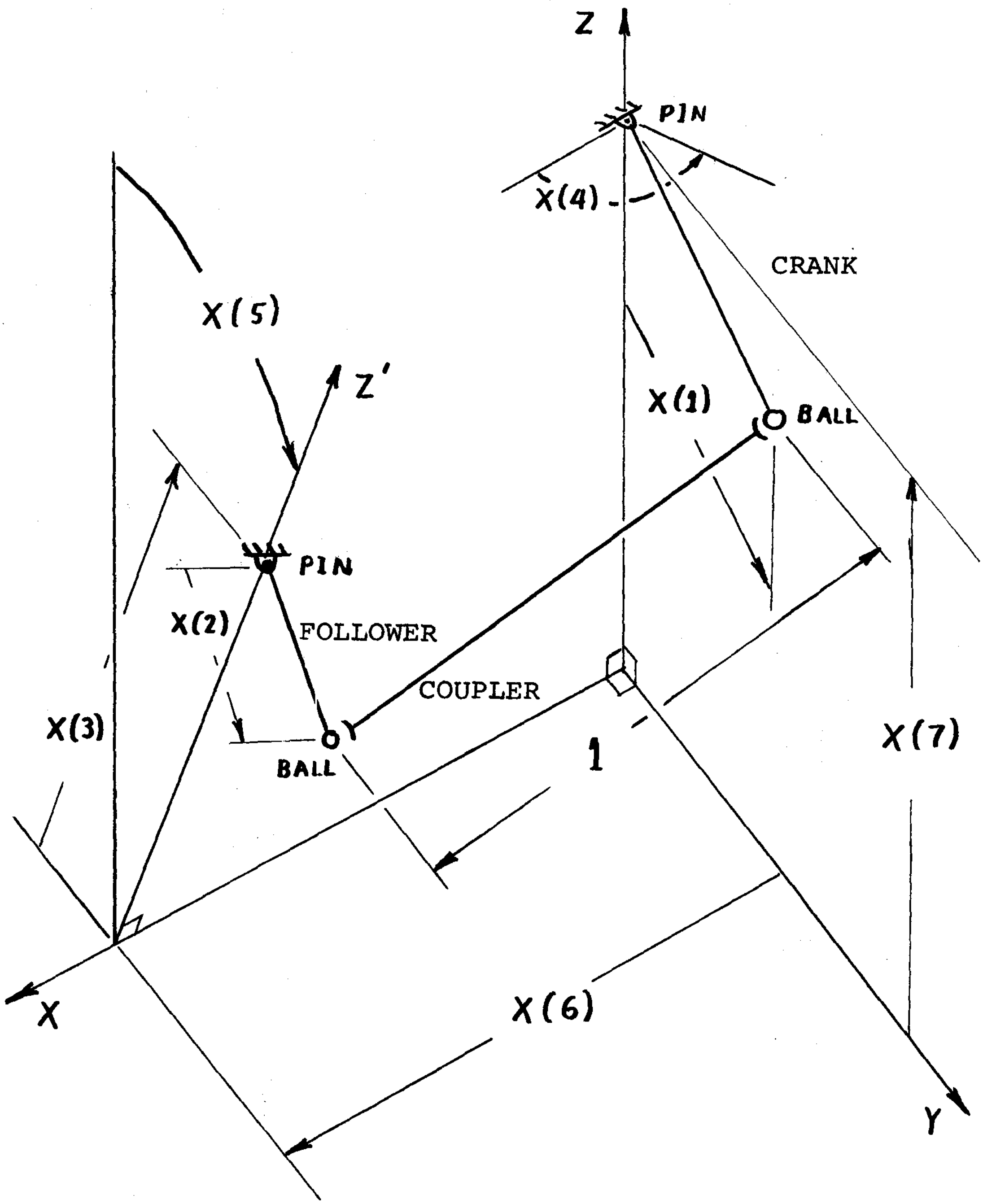


FIG. 3-10 SPATIAL FOUR-BAR FUNCTION GENERATING VARIABLES

X(5) - the angle from the crankshaft axis to the followershaft axis measured clockwise positive looking in at the X-axis;

X(6) - the perpendicular distance from the crankshaft axis to the followershaft axis divided by the coupler length (this distance vector establishes the X-axis in Figure 3-10);

X(7) - the distance from the X-axis to the crank pin measured along the crankshaft axis divided by the coupler length

Basic constraints:

C(1) - ensures linkage closure;

C(2) - prevents X(4) from increasing to infinity;

C(3) - keeps the actual range of output motion within the desired range of output motion (RNGO ± TITE); and

C(4) - prevents X(5) from increasing to infinity.

The rest of the computer output is similar to that for Section 3-1.

3.6 Preliminary Linkage Analysis

The analysis output, including the plots, obtained for each synthesis problem (METHOD=1 to 5) can be obtained for any set of independent design variables. The set of independent linkage design variables (SV) for which an analysis is desired is defined in MAIN along with the identical input for the corresponding synthesis problem except that COMMON block SAVOPT (where the SV are contained) replaces COMMON block STRTPT in MAIN, and the statement

```
CALL PLTERR(0)
```

for METHOD equal to 1, 3, or 5, or the statement

```
CALL PLTCUP(0)
```

for METHOD equal to 2 or 4, replaces the CALL LINK statement. See Figure 3-11 for a sample four-bar coupler-point curve analysis (METHOD=2) input.

3.7 Acceleration and Velocity Analysis (METHOD=6 to 9)

Purpose

The system program determines the angular velocities and accelerations of a planar four-bar's coupler and follower links (METHOD=6), the linear velocities and accelerations of a given planar four-bar coupler point (METHOD=7), the angular velocities and accelerations of a planar slider-crank's coupler link (METHOD=8), and the linear velocities and accelerations of a given planar slider-crank's coupler point (METHOD=9) for NPP equispaced positions of the crank link in the designated range of crank motion (RNGA degrees). The first evaluations are for a crank angle of STRTA degrees.

FIG. 3-11 PLANAR FOUR-BAR COUPLER-POINT CURVE ANALYSIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
 COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
 COMMON /SAVOPT/SV(10)
 DATA METHOD/2/,IEXCO/1/,NPP/9/,ICASE/+1/,SCAL1/.5/,
1SCAL3/2.5/,RNGI/360./,STRTPT/.3,.96,.54,.6,-.58,1.09,1.89,0.,0.,
2.6/
 DATA SV/.3,.96,.54,.6,-.58,1.09,1.89,0.,0.,.6/
 CALL PLTCUP(0)
 STOP
 END

 SUBROUTINE COUPLER
 COMMON /DESIRE/X(21),Y(81),ANG(81),XTOL(21),YTOL(21)
NOTE THAT X IS USED INSTEAD OF XD AND Y INSTEAD OF YD
THIS IS O.K. SINCE THERE IS CONSISTENCY BETWEEN DATA AND COMMON
STATEMENTS   I.E. A GIVEN VARIABLE NAME CAN BE CHANGED FROM THAT
SUGGESTED BUT THE ORDER OF THE VARIABLES IN THE COMMON BLOCKS
CANNOT BE CHANGED
 DATA X/2.2,2.04,1.87,1.71,1.55,1.55,1.75,2.,2.2/,
1Y/.2,.2,.2,.2,.2,.35,.7,.7,.35/,XTOL/.01,.1,.2,.1,.01,.05,.2,.2,
2.05/,YTOL/6*.05,.4,.4,.05/,ANG/0.,.61,1.22,1.83,2.44,2.79,4.19,
35.24,5.93/
 RETURN
 END
```

Input Parameters (see Section 9 for meanings)

METHOD, NPP, ICASE, STRTA, RNGA, PAR, W1, A1

Input COMMON Blocks

NUMBERS, BALIN

Input Routines

MAIN

Program Set-up

Figure 3-12 shows a typical input for METHOD=9. A call to LINCUP initiates the system program for METHOD=7 and 9; a call to FBANG initiates the system program for METHOD=6; and a call to SCANG initiates the system program for METHOD=8. Note that AR(NPP) and AI(NPP) must be put in a DIMENSION statement if PLTCUP is called in MAIN.

Basic Output

The program output consists of a table and plots of the designated velocities and accelerations. Note that the coupler point linear velocity and acceleration vector angles (for METHOD=7 and 9) are given relative to the horizontal as defined by PAR(5)* (see Figures 3-4 and 3-8).

* This parameter corresponds to independent design variable X(5) in Sections 3.2 and 3.4

FIG. 3-12 PLANAR SLIDER-CRANK COUPLER-POINT VELOCITY AND ACCELERATION ANALYSIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 DIMENSION AR(13),AI(13)
*** NOTE THAT AR AND AI MUST BE DIMENSIONED IN MAIN WHEN
PLTCUP IS CALLED        THIS ALSO IS TRUE IF PLTCUP IS CALLED
BY EXCON
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOJ1,CM(6),
1RM(3),RJ(3)
 DATA PAR/2.,6.,0.,0.,0./,STRTA/0./,RNGA/360./,NPP/13/,NOJ1/0/,
1W1/13*6.2831853/,A1/13*0./,ICASE/1/,METHOD/9/,CR/6./,CT/0./
 CALL LINCUP(1,1,CR,CT,AR,AI)
 STOP
 END
```

3.8 Planar Four-bar Balancing Synthesis (METHOD=10)

Purpose

The system program determines the optimum crank and follower counterweights for a given four-bar linkage to minimize the maximum shaking force and moment on the linkage due to inertia forces and torques on the links and counterweights.

Input Parameters (see Section 9 for meanings)

METHOD, ICASE, NPP, IEXCO, RNGA, STRTA, PAR, NOA1, W1, A1, CM, RM, RJ, SCLSFH, SCLSFV, SCLSMO, STRTPT

Input COMMON Blocks

STRTPT, SCLFAC, NUMBERS, BALIN

Input Routines

MAIN

Program Set-up

Figure 3-13 shows a typical input.

If NOA1 equals zero (meaning that the crank angular velocity is constant), then there are only six independent design variables: X(7) is fixed at the value given to STRTPT (7) by the user in MAIN, and X(8) is ignored by the optimization routine. Thus only the starting points (STRTPT) for the first seven independent design variables need be specified in MAIN if NOA1 equals zero.

FIG. 3-13 PLANAR FOUR-BAR BALANCING SYNTHESIS INPUT

```
      PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
      COMMON /STRTPT/STRTPT(10)
      COMMON /SCLFAC/SCLSFH,SCLSFV,SCLSMO
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
     1RM(3),RJ(3)
      DATA METHOD/10/,NPP/18/,ICASE/+1/,IEXCO/1/,RNGA/360./,
     1STRTA/0./,PAR/.338,.969,.569,.515,-.593,1.064/,NOA1/0/,
     2W1/18*25.13/,A1/18*0./,CM/.169,0.,.943,0.,.284,0./,
     3RM/1.22E-5,6.79E-5,2.05E-5/,RJ/1.63E-7,2.05E-5,6.34E-7/,
     4SCLSFH/.01/,SCLSFV/.01/,SCLSMO/.01/,STRTPT/-.2,0.,1.E-4,
     51.E-5,-.3,0.,1.E-4,1.E-5/
      CALL BALANCE(1,1.E-5)
      STOP
      END
```

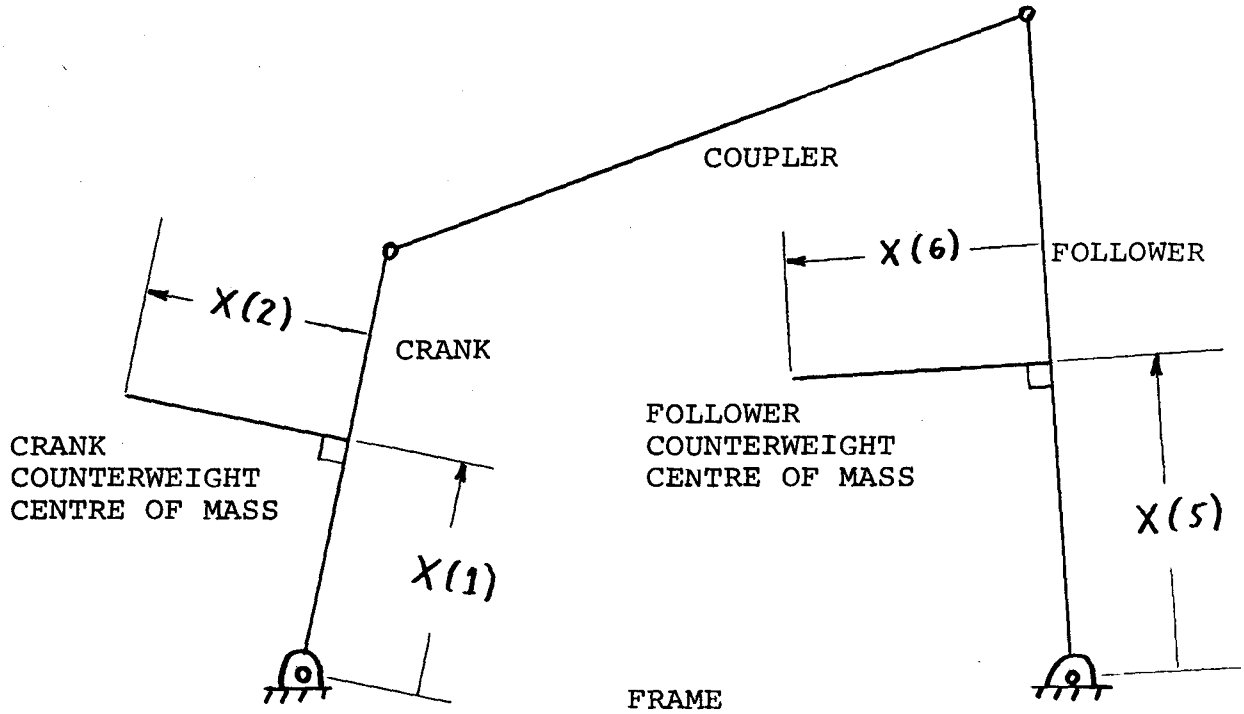


FIG. 3-14 FOUR-BAR BALANCING VARIABLES

Basic Output

Optimum independent design variables (Figure 3-14):

X(1) and

X(2) - co-ordinates of the crank counterweight centre of mass relative to the crank link;

X(3) - follower counterweight mass;

X(4) - follower counterweight polar moment of inertia about its centre of mass

X(5) and

X(6) - co-ordinates of the follower counterweight centre of mass relative to the follower link;

X(7) - crank counterweight mass; and

X(8) - crank counterweight polar moment of inertia about its centre of mass.

Basic constraints:

none

The rest of the output is in the same form as the basic synthesis output for the linkage dimensional synthesis problems. However, instead of the preliminary analysis plots obtained for the linkage synthesis problems, the balancing synthesis output routines produce a table of values and plots of the balanced and unbalanced horizontal and vertical shaking forces and counterclockwise shaking moments about the crankshaft axis at the NPP equispaced precision points specified.

3.9 Planar Slider-crank Balancing Synthesis (METHOD=11)

Purpose

The system program determines the optimum crank counterweight, for a given slider-crank linkage, required to minimize the maximum shaking force and moment on the linkage due to inertia forces and torques on the links, the slider, and the counterweight.

Input Parameters (see Section 9 for meanings)

METHOD, ICASE, NPP, IEXCO, RNGA, STRTA, PAR, NOA1, W1, A1, CM, RM, RJ, SCLSFH, SCLSFV, SCLSMO, STRTPT

Input COMMON Blocks

STRTPT, SCLFAC, NUMBERS, BALIN

Input Routines

MAIN

Program Set-up

Figure 3-15 shows a typical input for this problem type.

If NOA1 equals zero, then there are only two independent design variables: X(3) is fixed at the value given to SRTRPT (3) in MAIN, and X(4) is ignored by the optimization routine. Thus, if NOA1 equals zero, then only the first three starting point values (STRTPT) for the independent design variables need to be specified in MAIN.

Basic Output

Optimum independent design variables (Figure 3-16):

X(1) and

X(2) - co-ordinates of the crank counterweight centre of mass relative to the crank link;

X(3) - crank counterweight mass; and

X(4) - crank counterweight polar moment of inertia about its centre of mass.

Basic constraints:

none

The rest of the computer output is similar to that for Section 3.8.

3.10 Balancing Analysis

The effectiveness of any given set of independent counterweight design variables for balancing can be evaluated by using the same input as for Section 3.8 for the four-bar linkage and Section 3.9 for the slider-crankage with the independent design variable values (SV) placed in COMMON block SAVOPT. If NOA1 is zero, then STRTPT (3), for the slider-crank balancing analysis, and STRTPT (7), for the four-bar balancing analysis, must be set equal to SV(3) and SV(7) respectively in MAIN; otherwise the vector STRTPT need not be defined. The statement

CALL BALANCE (0)

FIG. 3-15 PLANAR SLIDER-CRANK BALANCING SYNTHESIS INPUT

```
 PROGRAM MAIN (OUTPUT,TAPE6=OUTPUT)
 COMMON /STRTPT/ STRTPT(10)
 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
 COMMON /SCLFAC/SCLSFH,SCLSFV,SCLSMO
 COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
1RM(3),RJ(3)
 DATA NOA1/0/,STRTA/0./,RNGA/360./,CM/.125,  0.,.33333,0./,
1RM/.31056,1.0559,.62112/,RJ/.00288,.11977/,W1/18*104.6/,
2A1/18*0./,PAR/.33333,1.1667,0./,STRTPT/-.25,0.,.5/,
3NPP/18/,ICASE/1/,METHOD/11/,SCLSFH/1000./,SCLSFV/1000./,
4SCLSMO/1000./
 CALL BALANCE(1,.0001)
 STOP
 END
```

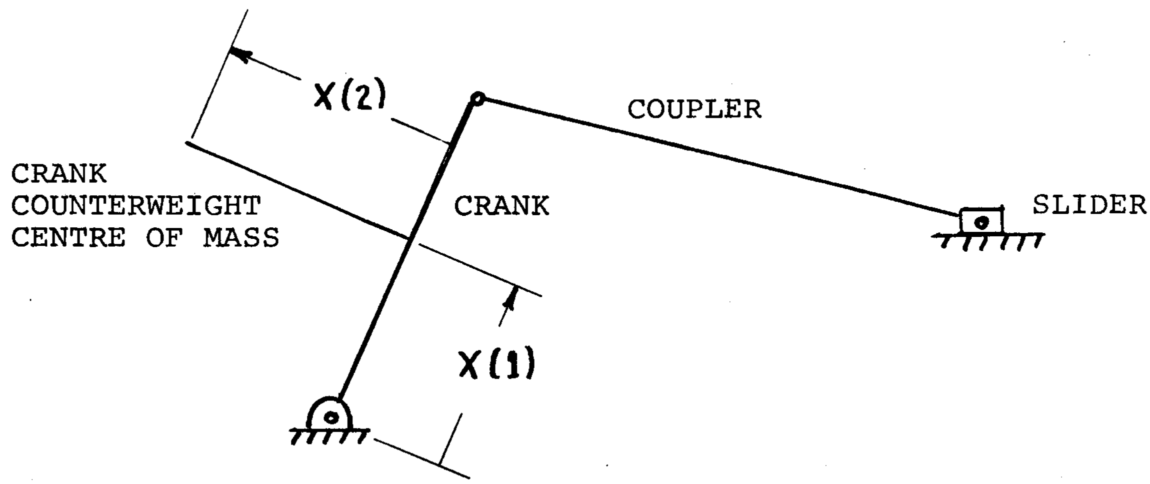


FIG. 3-16 SLIDER-CRANK BALANCING VARIABLES

replaces

```
CALL BALANCE (1, PREC)
```

in MAIN.

The computer output table and plots obtained for this problem type are similar to those for the corresponding balancing synthesis problems.

3.11 Optimization Surface Plotting

Purpose

The system program produces a contour plot of a two-dimensional subspace of an optimization hypersurface with respect to two of the independent design variables used to calculate the optimization hypersurface. This plot can be interpreted to show the sensitivity of the objective function to changes in given independent design variables. Since the intersections of the optimization problem constraint surfaces with the optimization hypersurface are also shown on the plot, the linkage dimensional synthesis optimization surface plots can be interpreted to show the mobility ranges*

* These are ranges of values for a set of independent linkage design variables for which the linkage is continuously closed (mobile) for a certain range of input and output motion.

of a given linkage with respect to two of the independent linkage design variables.

Input and Output

For balancing synthesis optimization surface plotting, the input is identical to that for Section 3.10 except that

```
CALL OPTSURF (NX, NY, GMAX, GMIN, XMAX, XMIN, YMAX,
                   YMIN, ISKIP)
```

replaces

```
CALL BALANCE (0)
```

in MAIN.

For linkage synthesis optimization surface plotting the input is identical to that for Section 3.6 except that the call to OPTSURF replaces

```
CALL PLTCUP (0)
```

or

```
CALL PLTERR (0)
```

in MAIN.

The subroutine input parameters for OPTSURF have the following meanings:

NX - the number of the variable to be varied along the horizontal axis of the plot (i.e. SV(NX) is varied);

NY - the number of the variable to be varied along the vertical axis of the plot (i.e. SV(NY) is varied);

GMAX - the maximum value of the unconstrained objective function to be included in the calculation of the contour lines;

GMIN - the smallest value of the unconstrained objective function to be included in the calculation of the contour lines;

XMAX - the largest value of SV(NX) for plotting;

XMIN - the smallest value of SV(NX) for plotting;

YMAX - the largest value of SV(NY) for plotting;

YMIN - the smallest value of SV(NY) for plotting; and

ISKIP- set equal to one if this is the first call to OPTSURF, LINK, or BALANCE in MAIN, or set equal to zero if OPTSURF, LINK, or BALANCE are previously called in MAIN.

Thus, if we want a plot of the contour lines of the unconstrained objective function, G, between the values of .1 and 5. with respect to variable number one and variable number three, which are varied from -1. to +1. and -5. to +10. respectively, then the following statement in MAIN is used:

```
CALL OPTSURF (1, 3, 5., .1, 1., -1., 10., -5., 1)
```

Contour lines are represented by numbers from one to nine. These lines indicate equal increments in the function G from GMIN to GMAX. For exampke, the value of G along contour line number four is

GMIN + 4/9 (GMAX-GMIN).

The intersections of the optimization surface with the implicit constraint surfaces are also plotted. These intersections are printed as letters starting with the letter A representing the first constraint. For example, the intersection of the unconstrained optimization surface with constraint C(2) is plotted as a series of B's.

If the unconstrained objective function evaluated for a given set of variable values* is greater than GMAX, then a + is printed; if the function value is less than GMIN, then a - is printed. If a function value is neither greater than GMAX, nor less than GMIN, nor on a contour line, then a blank space appears in the plot corresponding to the particular horizontal (NX) and vertical (NY) axis variable values for which it has been evaluated.

* Note that only number NX and NY variables are varied in the plot; the other variable values remaining fixed at their values specified in COMMON block SAVOPT.

4. SCALING FACTORS

The three scaling factors contained in COMMON block SCLFAC have a great effect on the magnitude of the objective function for minimization. It thus is important that these values be properly defined.

The dependent design variable value corresponding to each particular scaling factor is given an inverse utility of one (zero being the lowest and most desirable inverse utility, and positive infinity the highest and least desirable inverse utility). Hence the dependent design variable value which each scaling factor represents, each has the same amount of desirability as far as the program is concerned. Thus it is imperative that the user make sure that SCAL1, SCAL2, and SCAL3 for linkage synthesis, and SCLSFH, SCLSFV, and SCLSMO for linkage balancing, each have the same importance to him. It is suggested that each scaling value represent the dependent design variable value corresponding to the line between acceptability and unacceptability of the besign based on that variable alone; however, any other workable scheme can be used if desired.

5. OPTIMIZATION ROUTINE PARAMETERS

The input parameters which directly affect the optimization routine are SM, RF, RAT, and PREC. The meanings of these parameters are found in the documentation for subroutines LINK and UNIMIN in Section 10.

The expression $\log_{10}(.1/PREC)$ gives the number of significant figures or decimal places (depending on the size of the variable - see LINK in Section 10) expected from the optimization routine. SM and RAT should be chosen such that SM/RAT approximately equals PREC/2 for problems for which the average variable magnitude, AV, is of the order one of less, and approximately equals one half PREC times AV for problems for which AV is greater than one. For example, for a given four-bar function generating problem in which three significant figure precision is required, the expected independent design variable values might be 2, 10, 7 and 1. Thus

$$PREC = .1/10^3 = .0001$$

$$AV = (2 + 10 + 7 + 1)/4 = 5, \text{ and therefore}$$

$$SM/RAT = (5 \times .0001)/2 = .00025 \qquad 5.1$$

The minimization procedure works best if SM is within an order of magnitude of PREC. Thus, for the above example, an SM of .0005 and a RAT of 2.5 should work well. Note that .5 has been added to the expected RAT from equation 5.1; this is to ensure that a reduction ratio of two in SC is made by the program. The addition of .5 to the desired reduction ratio to obtain RAT should be done for all problems.

The relaxation factor, RF, should be set to 1. for all problems on the first trial. If convergence to a solution cannot be made, or a solution is suspect, then values of RF between .5 and 1.5 can be tried. However, using an RF other than one is an emergency measure, and should only be used as a last resort.

If the user has no idea of the expected size of the optimum independent design variables, then the following values for the optimization routine parameters can be used:

SM = .0003,

RF = 1. ,

RAT = 2.5 , and

PREC = .0001.

6. STARTING POINTS

The initial starting point, STRTPT, for all optimization problems must be defined by the user in MAIN. It is not necessary that this starting point be feasible, but a reasonable starting point will reduce the computer execution time and thus the design cost. Using the results of a simple three precision point geometric or algebraic synthesis, as outlined by Hartenberg in reference [16], will provide a reasonable starting point for the function generating problems. For four-bar coupler-point curve problems, Hrones' and Nelson's atlas [26] provides a good source for reasonable starting points. Slider-crank coupler-point curve reasonable starting points can be easily estimated by inspection. For the balancing problems, the starting counterweights should have about the same mass and moment of inertia as the links they are attached to. However, the starting point position of the centre of each link counterweight mass should be the reflected position of the link centre of mass with respect to the linkshaft axis.

7. ADDING EXTRA CONSTRAINTS

The user has complete freedom to add an unlimited number of explicit constraints (which require direct variable transformations), and up to twenty implicit (C) constraints. To do this, he must set IEXCO equal to zero (it should otherwise be set equal to one), and set NC equal to the number of basic implicit constraints (see NC in Section 9) plus the number of implicit constraints to be added. The direct variable transformations* and/or implicit constraints must be defined in subroutine EXCON. Note that the implicit constraints added in EXCON must be numbered starting from one plus the number of basic constraints (see Figure 7-1).

Though the added implicit constraints, C, are of the form

$$C(I) \geq 0 \ , \ I=M,\ldots,NC \qquad 7.1$$

where M is the number of basic constraints, they are written

* See Chapter II for transformation formats.

in EXCON in the form

C(I) = expression

where "expression" is the FORTRAN expression for C(I) in relation 7.1.

All the analysis subroutines - SCANG, FBANG, LUNCUP, and LINIO - are available to subroutine EXCON in order to form an implicit constraint using a link angular velocity or acceleration or a link point velocity or acceleration. The output and input for each subroutine is passed through the appropriate labelled COMMON blocks and subroutine argument lists which are found in the FORTRAN program listing*. The input parameters required are the same as that for the direct linkage analysis in Section 3.7. METHOD is not to be changed when calling these analysis routines from EXCON, but is to be left at its synthesis value (1, 2, 3, 4, 5, 10, or 11).

8. SYMMETRIC FUNCTION GENERATION

The computer program operates slightly differently for symmetric function generation between input limits which are symmetrically placed about the functional axis of symmetry: for example, generating the function $y=(x-5)^2$ for x varying from 3 to 7 (the axis of symmetry lying at x=5). For such

* See Appendix F for the listing.

symmetric problems ISYM must be set equal to zero (otherwise it must be set equal to one). Since the function is symmetric, the follower link will end up at its starting point; thus the value for RNGO will be for half the functional output range (the second half being the negative of the first half), rather than for the full functional output range as it is for other types of function generating problems.

9. COMMON BLOCK VARIABLES

Most of the program and internal variables are contained in the blank and labelled COMMON blocks listed at the beginning of each program subroutine*. Knowing the meanings of the variables in each of these blocks is essential to the detailed understanding of the program. Thus these variable names and meanings are now listed as they appear in each COMMON block**. (See Figure 9-1 for a summary list of the labelled COMMON blocks.)

* See Appendix F for the program listing.

** The number of storage locations required by the CDC 6400 computer for each block is in parentheses after the block name.

FIG. 9-1 LABELLED COMMON BLOCKS

```
 COMMON /STRTPT/STRTPT(10)

 COMMON /SCLFAC/SCAL1,SCAL2,SCAL3

 COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO

 COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM

 COMMON /SAVOPT/ SV(10)

 COMMON /DESIRE/XD(21),YD(81),PH(81),XTOL(21),YTOL(21)

 COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
1CYA(21),ERRMAX(21),TRI,ZERO

 COMMON /MODULO/IMD,IHELP

 COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
1RM(3),RJ(3)

 COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
2TSFH(36),TSFV(36),TSMO(36),VR(36),VI(36)
```

Blank COMMON (1795)

For Subroutines LINK and UNIMIN

P - a 10 x 12 array for which each column represents a set of independent design variable values.

XI - a 10 x 11 array for which each column represents a vector search direction.

DIR - a 10 element multipurpose vector for subroutine LINK.

X - a 10 element dummy vector for subroutine UNIMIN.

For Subroutine OPTSURF and PLOTCN

X - a vector for which the elements represent the independent design variable values.

FUNC or F - a vector of unconstrained objective function values for plotting; each element corresponds to a different set of independent design variable values.

CONS or C - an array for which each column element represents an implicit constraint value corresponding to an element of F; each column (up to 20) represents a set of values for a different implicit constraint.

FILL - a dummy vector in subroutine PLOTCN.

STRTPT* (10)

* See Section 6 for details on defining the starting points.

STRTPT - a vector of input starting point values for the independent design variables.

SCLFAC* (3)

For Linkage Synthesis

SCAL1 - the scaling value for minimum transmissibility index** control; it can vary from .0175 to .9998.

SCAL2 - the scaling value for theoretical maximum structural error control; it can vary from 10^{-7} to positive infinity.

SCAL3 - the scaling value for maximum link length control; it can vary from 1.1 to positive infinity.

For Balancing Synthesis

SCLSFH - the scaling value for maximum horizontal shaking force magnitude control; it can vary from 10^{-7} to positive infinity.

SCLSFV - the scaling value for maximum vertical shaking force magnitude control; it can vary from 10^{-7} to positive infinity.

* See Section 4 for details on establishing the scaling factors.

** See Chapter III for the meaning of "transmissibility index".

SCLSMO - the scaling value for maximum counterclockwise shaking moment about the crankshaft axis magnitude control; it can vary from 10^{-7} to positive infinity.

NUMBERS (6)

NPP - the number of precision points (positions at which the linkage is evaluated) for the synthesis, analysis, and balancing problems; it must be less than 22 for METHOD=2 or 4, less than 82 for METHOD=1, 3, or 5, and less than 37 for METHOD=6 to 11; also, for METHOD=1, 3, or 5 NPP must be an odd number. NPP should be at least 5 for most problems.

METHOD - equals 1 for planar four-bar function generation;
equals 2 for planar four-bar coupler-point curve generation;
equals 3 for planar slider-crank function generation;
equals 4 for planar slider-crank coupler-point curve generation;
equals 5 for spatial four-bar function generation;
equals 6 for planar four-bar link angular velocity and acceleration analysis;
equals 7 for planar four-bar coupler-point linear velocity and acceleration analysis;
equals 8 for planar slider-crank link angular velocity and acceleration analysis;

equals 9 for planar slider-crank coupler-point linear velocity and acceleration analysis; equals 10 for planar four-bar balancing; and equals 11 for planar slider-crank balancing.

ICASE - equals plus or minus one depending on which of two possible ways the linkage is to be closed*.

N - the number of independent design variables (defined internally by the program).

NC - the number of implicit constraints; it is defined internally if IEXCO is not equal to zero, otherwise it is user defined, where NC equals the number of basic implicit constraints (see Table 9-1) plus the number of user added implicit constraints.

* +1 corresponds to the subscript + cases in Appendix B, and the upper signs, where two signs are given in Appendix D; -1 corresponds to the subscript - cases in Appendix B, and the lower signs, where two signs are given, in Appendix D.

Table 9-1: NUMBER OF BASIC CONSTRAINTS

METHOD	Number of Basic Constraints
1	3
2	3
3	3
4	3
5	4
10	0
11	0

IEXCO - set equal to zero if the user adds implicit or explicit constraints in subroutine EXCON.

SYNIN (7) For METHOD=1 to 5

XMIN - the starting (smallest) value of the function input variable for function generation (METHOD=1, 3, and 5).

XMAX - the finishing (largest) value of the function input variable for function generation.

RNGI - the range of the desired crank (input) rotation in degrees for linkage synthesis (METHOD=1 to 5).

RNGO - the range of the desired follower (output) rotation in degrees (for METHOD=1 and 5), or slider (output) motion (for METHOD=3).

TITE - the bilateral tolerance, in degrees for METHOD=1 and 5 or in linear units (same units as for RNGO) for METHOD=3, allowed for RNGO.

CFRIC - the coefficient of friction between the slider and its sleeve for slider-crank synthesis (METHOD=3 and 4)

ISYM - set equal to zero if the function to be generated (for METHOD=1, 3, and 5) is symmetrical and the input limits (XMIN and XMAX) are symmetrically placed with respect to the function. (See Section 8 for further details.)

SAVOPT (10)

SV - a vector of the optimum or final independent design variable values.

DESIRE (225)

XD - a vector of the desired horizontal coupler-point co-ordinates at the precision points (for METHOD=2 and 4); also referred to as X.

YD - a vector of the desired vertical coupler-point co-ordinates at the precision points (for METHOD=2 and 4); or a vector of the desired functional values at the precision points (for METHOD=1, 3, and 5); also referred to as Y.

PH - a vector of the crank angles at the precision points; also referred to as ANG.

XTOL - a vector of the horizontal bilateral tolerances at the precision points (for METHOD=2 and 4).

YTOL - a vector of the vertical bilateral tolerances at the precision points (for METHOD=2 and 4).

INTERN (173)

RPSI - the desired range of functional output for function generation

CG - the constrained objective function for minimization

C - a vector of the implicit constraint values (up to 20 implicit constraints allowed).

U1 - the transmissibility index factor in CG for linkage synthesis; or the horizontal shaking force factor in CG for linkage balancing.

U2 - the structural error factor in CG for linkage synthesis; or the vertical shaking force factor in CG for linkage balancing.

U3 - the link length control factor in CG for linkage synthesis; or the shaking moment factor in CG for linkage balancing.

PM - the constraint term scalar multiplier in the scaled exterior-point objective function transformation*.

ICOUNT - the counter index which contains the current number of objective function evaluations.

PS - a vector of the actual follower angles at the precision points for METHOD-1, 2, and 5, and the

* PM corresponds to parameter t in Appendix A.

actual slider distances at the precision points for METHOD=3 and 4.

CXA - a vector of the actual coupler-point horizontal co-ordinates at the precision points (for METHOD=2 and 4).

CYA - a vector of the actual coupler-point vertical co-ordinates at the precision points (for METHOD=2 and 4).

ERRMAX - a vector of the distances between the desired and actual coupler-point positions at the precision points (for METHOD=2 and 4).

TRI - the minimum transmissibility index* in the range of linkage motion.

ZERO - the minimum possible transmissibility index for a given problem: equals 0 for METHOD=1, 2, and 5, and equals

$$CFRIC/(1 \pm CFRIC^2)^{1/2}$$

for METHOD=3 or 4.

MODULO (2)

IMD - an index which equals zero if METHOD=2, 4, or 5.

IHELP - an index which equals zero if two consecutive unsuccessful starting points are generated by subroutine LINK.

* See Chapter III for the meaning of "transmissibility index".

BALIN (93) For METHOD=6 to 11

W1 - a vector of the crank angular velocities at the precision points.

A1 - a vector of the crank angular accelerations at the precision points.

PAR - a vector of the required linkage parameter values corresponding to the independent design variable values discussed in Section 3.4 for the slider-crank and in Section 3.2 for the four-bar: the first three independent design variables are required for the slider-crank for balancing and the first six independent design variable values are required for the four-bar for balancing and analysis. The first five parameters are required for the slider-crank for analysis.

STRTA - the crank starting angle in degress for the range of crank motion.

RNGA - the range of crank motion in degrees. (Note that the precision points are spaced at RNGA/(NPP-1) degree intervals starting at STRTA degrees.)

NOA1 - an index set equal to zero if all the A1 are zero (i.e. for constant crank angular velocity).

CM - a vector of the link centre of mass positions relative to each link (Figure 9-2); for the slider-crank CM(5) and CM(6) are assumed to be zero - thus placing the slider centre of mass at its connection with the

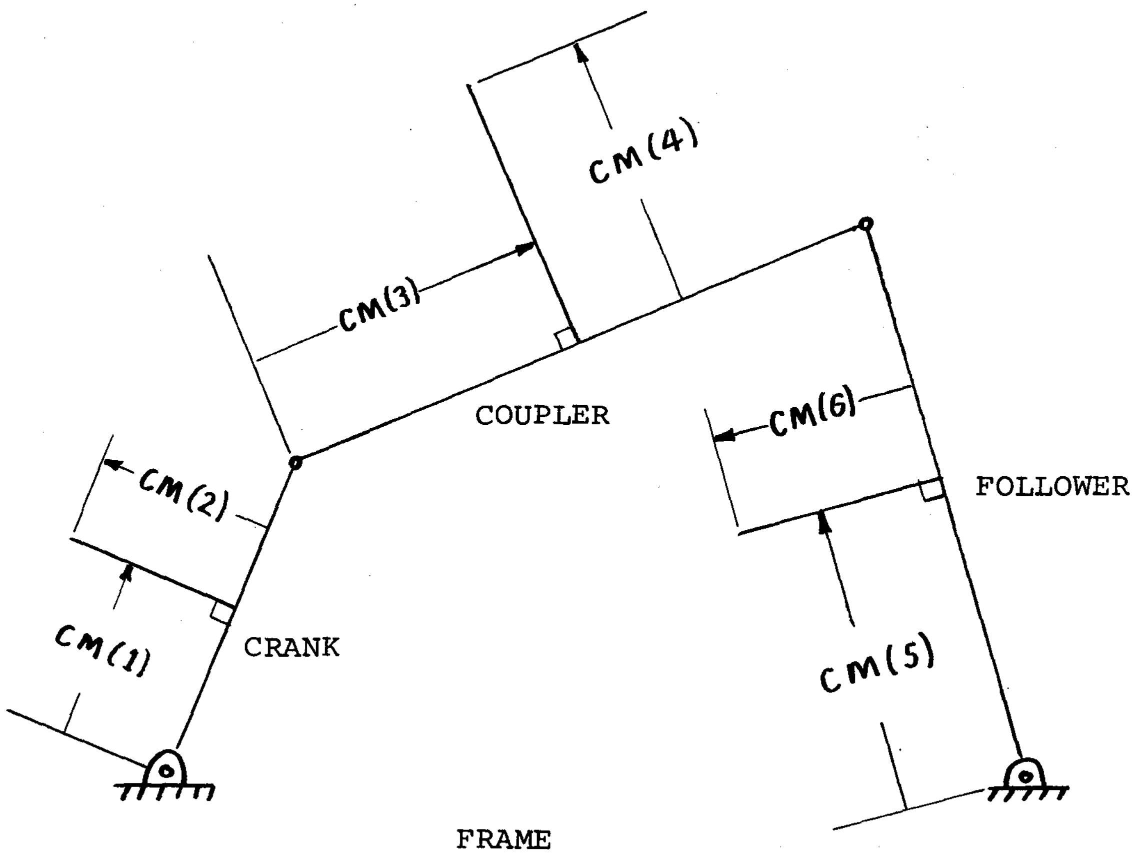


FIG. 9-2 LINK CENTRE OF MASS POSITION PARAMETERS

coupler link.

RM - a vector of the link masses.

RJ - a vector of the link polar moments of inertia about their centres of mass. (Note that CM, RM, and RJ must have compatible units with W1, A1, and PAR since the units for the shaking forces and moments are derived from these units.)

BALVAL (720) For METHOD=6 to 11

T1 - a vector of the crank angles* in radians at the precision points.

T1D - a vector of the crank angles in degrees at the precision points.

T2 - a vector of the coupler angles* in degrees at the precision points.

T3 - a vector of the follower angles* in radians at the precision points for the four-bar only.

W2 - a vector of the coupler angular velocities in radians per second at the precision points.

W3 - a vector of the follower angular velocities in radians per second at the precision points (for the four-bar only).

* See Appendix D for the definitions of these angles.

A2 - a vector of the coupler angular accelerations in radians/sec/sec at the precision points.

A3 - a vector of the follower angular accelerations in rad/sec/sec at the precision points (for the four-bar only).

AH1 - a vector of the horizontal acceleration components of a given crank point at the precision points.

AV1 - a vector of the vertical acceleration components of a given crank point at the precision points.

AH3 - a vector of the horizontal acceleration components of a given follower point at the precision points.

AV3 - a vector of the vertical acceleration components of a given follower point at the precision points.

SFH - a vector of the unbalanced horizontal shaking forces at the precision points

SFV - a vector of the unbalanced vertical shaking forces at the precision points.

SMO - a vector of the unbalanced shaking moments at the precision points.

TSFH - a vector of the balanced horizontal shaking forces at the precision points.

TSFV - a vector of the balanced vertical shaking forces at the precision points.

TSMO - a vector of the balanced shaking moments at the precision points.

VR - a vector of the horizontal velocity components of a given coupler point at the precision points.

VI - a vector of the vertical velocity components of a given coupler point at the precision points.

10. SUBROUTINE DESCRIPTIONS

Now that the COMMON block variables have been identified, a brief description of each FORTRAN subroutine is presented. The subroutine arguments are in parentheses after the subroutine name. The number of storage locations required for the instructions in each subroutine is enclosed in brackets after the subroutine argument list.

MAIN [variable]

This is the user supplied main program which initiates the calls to the proper subroutines and defines the necessary input data. See Section 3 for further details.

FUNSYN (X) [variable]

This user supplied function subprogram evaluates the desired output function (for METHOD=1, 3, and 5) for input values of X at the computer determined precision points. No COMMON or DIMENSION statements are required.

COUPLER [variable]

This user supplied subroutine (for METHOD=2 and 4) defines the horizontal (XD) and vertical (YD) co-ordinates of up to 21 desired coupler point positions corresponding to the crank angles (PH) which also must be specified. The bilateral horizontal (XTOL) and vertical (YTOL) tolerances of the co-ordinates of each desired coupler point must be specified. Labelled COMMON block DESIRE, which includes the necessary variable dimensioning, must be included.

EXCON* (X, C, NC) [variable]

This user supplied subroutine evaluates any additional implicit or explicit constraints that the user may want to include in a given problem. IEXCO must be set equal to zero in MAIN for this subroutine to be called. X(1) and/or C(1) must be placed in a DIMENSION statement if X (for explicit constraints) and/or C (for implicit constraints) are used in this subroutine. NC must also be defined in MAIN if this subroutine is used. See Figure 10-1 for an example.

CHEBSP [53]

This subroutine spaces the precision points at Chebychev spacing** of the input function variable values for function generation (METHOD=1, 3, and 5). It also determines the crank angle (PH) and the desired function output value (YD) at the precision points. The midpoint of the crank rotation is used as a base point for establishing the scales for the input and output motions. Thus the crank angle varies from -RNGI/2 to +RNGI/2 relative to this base angle.

* See Section 7 for further details on adding constraints.

** See Chapter III for the meaning of Chebychev spacing.

FIG. 10-1 SAMPLE ADDED CONSTRAINTS IN EXCON

```
 EXCON(X,C,NC)
NOTE USE OF VARIABLE DIMENSIONING
 DIMENSION X(1),C(1)
METHOD=3, THEREFORE NUMBER OF BASIC IMPLICIT CONSTRAINTS=3
THUS THE FIRST ADDED IMPLICIT CONSTRAINT MUST BE C(4)
EXPLICIT CONSTRAINT RESTRICTING VARIABLE X(3) TO LIE
BETWEEN -1 AND +1 IS REQUIRED
THEREFORE DIRECT VARIABLE TRANSFORMATION 3
IN CHAPTER II IS USED
 PI=3.14159265
 ARG=COS(PI*(X(3)+1.)/2.)
 X(3)=-1.+2./PI*ACOS(ARG)
SINCE TRANSFORMATION 3 CAN CAUSE TROUBLES,
THE IMPLICIT CONSTRAINT INDICATED IN CHAPTER II IS ADDED
NOTE THAT M IN THE CONSTRAINT IS SET TO 11
 C(4)=11.-ABS(X(3)-1.)
 RETURN
 END
```

LINK (ISURF, SM, RF, RAT, PREC)* [706]

This is the basic subroutine for the modified Powell-Zangwill minimization process**. It is also the organizational subroutine for the linkage synthesis problems (METHOD=1 to 5), making the appropriate calls to the precision point set-up subroutines (CHEBSP for function generation and COUPLER for coupler-point curve generation) and the output plotting subroutines (PLTERR for function generation and PLTCUP for coupler-point curve generation).

If ISURF equals zero, meaning only a linkage analysis is being done, then the subroutine only calls the appropriate precision-point set-up routine (CHEBSP or COUPLER), makes some elementary checks for input data errors, and sets up the scaling factors. If ISURF is not equal to zero, then an objective function minimization problem is assumed. The input variables SM, RF, and RAT are input parameters for subroutine UNIMIN and are discussed under that heading. PREC is the parameter which determines the precision of the optimum independent design variables. If the optimum variable value is greater than one, then its precision is approximately

* See Section 5 for further details on the input parameters.

** See Chapter V for a detailed explanation of this process.

$\log_{10}(.1/PREC)$ significant figures; if the variable value is less than one, then its precision is approximately $\log_{10}(.1/PREC)$ decimal places.

This subroutine also determines the number of independent design variables (N) for METHOD=1 to 5, and the number of basic implicit constraints (NC) if IEXCO is not equal to zero.

The user supplied starting point is iteratively improved using the modified Powell-Zangwill minimization technique until the greatest variable change* (for a variable value less than one) or the greatest variable relative change* (for a variable value greater than one) is less than PREC. An iteration consists of successive minimizations along N distinct directions (corresponding to each column vector of XI), and then a minimization along a direction which is conjugate to the previous N directions. The minimization along the directions is done by subroutine UNIMIN. The technique defines this (N+1)th (conjugate) direction and checks the degree of its linear dependency with respect to the other directions as follows. If the magnitude of the

* change = $p_i - p_{i-1}$;

relative change = $(p_i - p_{i-1})/p_i$;

where p_i is the variable value at the end of the current minimization iteration, and p_{i-1} is the variable value at the end of the previous minimization iteration.

determinant (TST1) of the matrix for which the columns are the vector search directions - the new direction (XI(I,M±1), I=1,...,N) replacing the old direction along which the maximum independent design variable changes occurred (XI(I,JS), I=1,...,N) - is greater than EPS($=.5/N^{.5}$), then this new direction replaces the JSth direction for the next minimization iteration. Otherwise, the same set of directions is used for the next minimization iteration.

The "optimum" point is checked to see if it violates any constraints. If it does, then a new minimization sequence (a set of minimization iterations starting with the co-ordinate directions as the initial search vectors) is started from this point with the constraint term multiplier (PM) multiplied by 10000. This is done until either PM is greater than $-10^6/C$ (a hang-up), where C is the violated (less than zero) constraint value, or C becomes positive. If the first case occurs, then a new starting point is generated which is a random perturbation of the "optimum" point. If the second case occurs, then the optimun independent design variables are put in COMMON block SAVOPT, and the optimum values for CG (OPTNEW), U1, U2, and U3 are set equal to CGBST, U1BST, U2BST, and U3BST respectively. A random starting point is then generated for a new minimization sequence unless OPTNEW is greater than the previous value for OPTNEW (OPTOLD), where OPTOLD is initially set to 10^{50}, in which case the optimum point is assumed to be reached and the output routines are set into action.

If two consecutive hang-ups occur IHELP is set equal to zero. This causes CG in FUNCTION G(X) to be set to 10^{20} and only the constraint terms allowed to vary until all the implicit constraints (C) are satisfied, in which case CG is again allowed to assume its actual value evaluated in subroutine CONFUNC, FBBAL, or SCBAL. This procedure should drive the variable values into the feasible region if it is possible. If, after this procedure is invoked, two more consecutive hang-ups occur (making four consecutive hang-ups), then the problem is abandoned by the subroutine and an appropriate diagnostic is printed.

The minimization execution time (TIME), and the number of function calls (ICOUNT), are printed out after the optimization is completed.

PLTERR (ICK) [137]

This subroutine initiates the plotting of the structural error in the output function (Y) versus the function input (X) for function generation. The actual value of the function generated (YA) minus the desired value (YD) is evaluated at 81 equally spaced precision points in the range of crank motion (RNGI). The linkage closure constraint (C(1)) is checked at all 81 crank positions. If it is violated, then the plotting is not done, and an appropriate diagnostic is printed. The minimum transmissibility index and the actual range of output motion are printed out after the plotting is done.

For a function synthesis problem ICK is set equal to one. However, for a direct analysis of a given linkage ICK is set equal to zero, and PLTERR is called directly from program MAIN, the linkage parameters (SV) being put in labelled COMMON block SAVOPT, as they also are for function synthesis problems, except by subroutine LINK instead of manually by the user.

Essentially, PLTERR sets up the required input for the general plotting subroutine, COMPARE.

PLTCUP (ICK) [130]

This subroutine initiates the plotting of the desired (XD, YD) and actual (CXA, CYA) coupler points, as well as the structural error (ERRMAX) versus the crank angle (PH) for coupler-point curve generation. This subroutine plays the same role as PLTERR does for the function generation problems.

COMPARE (N, YA, YD, XA, XD, YMAX,UYMIN, XMAX, XMIN, IFLG3) [323]

This subroutine plots one (IFLG3=0) or two (IFLG3=1) functions given by pairs of points ((XA,YA), (XD,YD)) to the same scale. The (XA,YA) pairs are the ones printed if IFLG3=0. The (XA,YA) points are represented by *'s and the (XD,YD) points are represented by O's.

The meanings of the input variables are as follows:

N - the number of pairs of points.

YA - a vector (of length N) of the vertical components of the first set of points to be plotted;

YD - a vector (of length N) of the vertical components of the second set of points to be plotted;

XA - a vector (of length N) of the horizontal components of the first set of points to be plotted;

XD - a vector (of length N) of the horizontal components of the second set of points to be plotted;

YMAX - the greatest value of YA and YD to be plotted;

YMIN - the smallest value of YA and YD to be plotted;

XMAX - the greatest value of XA and XD to be plotted;

XMIN - the smallest value of XA and XD to be plotted; and

IFLG3 - equals 0 if one set of points is to be plotted, or
equals 1 if two sets of points are to be plotted.

The subroutine sorts the pairs according to their Y values - the largest values being first - after converting

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the X and Y values to integers corresponding to their position in the plot (which is of size 81 columns by 41 rows), indexed with respect to the top left-hand corner of the plot. This process destroys the original values contained in XA, XD, YA, and YD. The horizontal (Y=0) and vertical (X=0) axes are printed using l's and -'s respectively. Grid points represented by +'s are printed every eight horizontal and vertical positions from the vertical and horizontal axes respectively. A function-pair point takes precedence over all other symbols; however, points from the two different sets of points being plotted (a * and an O) can be printed over top of each other.

UNIM (NS, NF, RMAX, JS, N, SC, RF, RAT) [399]

This subroutine finds the minimum of an unconstrained objective function (G) along a series of directions (XI) for the modified Powell-Zangwill minimization method in subroutine LINK. The meanings of the subroutine arguments are as follows:

NS - the value of index J for the first search direction (XI(I,J), I=1,...,N);

NF - the value of index J for the last search direction;

RMAX - the magnitude of the vector change along the search direction in which the largest variable changes occurred;

JS - the value of index J for the search direction yielding RMAX;

N - the number of independent design variables for optimization;

SC - one half the initial step-size for the search for a minimum in a search direction (XI);

RF - the initial value for the relaxation factor;

RAT - the final reduction ratio for SC (plus .5) for the bracketing of a minimum point in a search direction.

Initially a random step size search is made in the search direction in order to locate a point giving a lower function value. Then the method systematically searches in the positive and then the negative, if necessary, XI direction by successively doubling the step size until three consecutive points bracket the minimum. Three equally spaced points in this bracket are then determined in order to form a quadratic approximation of the function for which the distance (RMIN) from the starting point to the minimum function value is determined.

If the starting point is bracketed within 2 SC of the minimum point, SM (which is initially set equal to SC) is halved unless SC/SM is greater than RAT, in which case the quadratic approximation is immediately made. If SM is halved, then a new search to bracket the minimum is made from the current starting point.

If desired, the relaxation factor can be set less than one (underrelaxing) or greater than one (overrelaxing). The starting point for the next directional search is then set at RF x RMIN (instead of at the distance from the minimum, RMIN) from the current starting point. However, if the current starting point is within 4 SC of the minimum point, then RF is set temporarily equal to one to ensure proper convergence. This relaxation procedure may be advantageous in avoiding hang-ups for certain problems.

G(X) [61]

This function subprogram transforms, using the scaled exterior-point transformation*, the constrained objective function (CG) subject to the basic mechanism implicit constraints (C) evaluated in CONFUNC (for METHOD=1 to 5), or FBBAL (for METHOD=10) or SCBAL (for METHOD=11), as well as any user added constraints evaluated in EXCON, into the total unconstrained objective function for minimization (G).

For the special case when IHELP equals zero, CG is set to 10^{20} if any C(I), I=1,...,NC, is negative, and C(3) (the basic mechanism constraint to ensure the desired range of output motion for METHOD=1, 3, and 5) is multiplied by 1000

* See Chapter II for an explanation of this transformation.

to normalize it with respect to the other constraints (especially the mechanism closure constraint, C(1), which is always multiplied by 1000 in subroutine CONFUNC). The scaled exterior-point transformation is then applied to this special case.

For the balancing synthesis (METHOD=10 or 11), if IEXCO is not equal to zero, then no transformation is required, since there are no implicit constraints; thus G equals CG.

CONFUNC [634]

This subroutine evaluates the constrained objective function (CG) and the basic mechanism constraints (C(I), I=1,...,3 or 4 - see NC in Section 9) for the linkage synthesis problems (METHOD=1 to 5).

CG is the sum of inverse utility U1, which accounts for the minimum transmissibility index, TRI, inverse utility U2, which accounts for the maximum structural error, EM, and inverse utility U3, which accounts for the maximum link length, RAL. Scaling factors SCAL1, SCAL2, and SCAL3, which are defined in LINK from the user input in MAIN, are used to define the inverse utilities U1, U2, and U3 respectively.

Constraint C(1) is common to all five linkage synthesis problems. If C(1) is negative, then the linkage cannot be closed (i.e. all the links do not touch each other) at one or more precision points.

Constraint C(3) is used to ensure that the linkage has the desired range of output motion for function generation. Essentially C(3) is a "loose" equality constraint defined from the user input RNGO and TITE, which have been converted from degrees to radians in subroutine LINK for METHOD=1 and 5. This constraint takes the form

$$C(3) = TITE - |RGABS - RNGO|$$

where RGABS is the absolute value of the actual range of output motion. It is thus possible to have a feasible range of output motion of plus <u>or</u> minus RNGO ± TITE.

The other basic implicit constraints (C(2) for METHOD=1 to 5, C(3) for METHOD=2 and 4, and C(4) for METHOD=5) directly depend on the independent design variable values and are thus explained in Section 3.

A direct variable transformation (for an explicit constraint) restricts the coupler link length (independent design variable number two) to be positive for METHOD equal to 2, 3, and 4.

BALANCE (ICWAN) [356]

This subroutine calculates the unbalanced horizontal (SFH) and vertical (SFV) shaking forces and the unbalanced counterclockwise shaking moment (SMO) about the crankshaft axis at NPP equispaced positions of the crank link for a given planar four-bar (METHOD=10) or planar slider-crank (METHOD=11) linkage.

If ICWAN equals -1, then only the unbalanced forces and moments are calculated. This is required by subroutine OPTSURF for setting up the contour plotting. If ICWAN equals 0, then the balancing counterweight independent design variables are not optimized, and the current variable values (SV) in COMMON block SAVOPT are used to calculate the balanced shaking forces and moments. If ICWAN equals 1, then the balancing counterweight independent design variables are optimized by calling subroutine LINK.

For ICWAN equal to 0 and 1, a table and plots of the unbalanced and balanced shaking forces and moments are automatically printed out.

PREC retains the same meaning it had for subroutine LINK. PREC also doubles as the value for SC in the call to subroutine LINK. The other subroutine LINK input parameters, ISURF, RF, and RAT, are fixed at 1, 1., and 2.5 respectively.

The number of independent design variables (N) for the balancing synthesis problems are defined in subroutine BALANCE.

SCBAL (X, ICWAN) [79]

This subroutine determines the additional inertia forces and moments due to the added counterweight to the crank link for slider-crank balancing (METHOD=11) for up to

36 equispaced positions of the crank link. It also adds the unbalanced shaking forces and moments (SFH, SFV, and SMO) calculated in subroutine BALANCE to the counterweight forces and moments (FCWH, FCWC, and FCWM) to form the total balanced shaking forces and moments (TSFH, TSFV, and TSMO) for a particular set of counterweight independent design variables, X. The maximum values of TSFH, TSFV, and TSMO at the precision point, and the scaling factors SCLSFH, SCLSFV, and SCLSMO determined in subroutine BALANCE from the user input, are then used to form the inverse utilities U1, U2, and U3. These inverse utilities are summed to form the constrained objective function CG.

This subroutine also applies a direct variable transformation to independent design variables X(3) (the crank counterweight mass) and X(4) (the crank counterweight polar moment of inertia) to prevent them from becoming negative.

If ICWAN equals zero, then the subroutine skips evaluating the objective function CG. If ICWAN is not equal to zero, then CG is calculated.

FBBAL (X, ICWAN) [113]

This subroutine is the equivalent of subroutine SCBAL for the planar four-bar balancing problem (METHOD=10). In addition to the inertia forces resulting from the crank counterweight (FCWH1 and FCWV1), the inertia forces from the follower counterweight (FCWH3 and FCWV3) are calculated. Also the total inertia torque resulting from both the crank and follower counterweights (FCWM) is calculated. Thus,

TSFH = SFH + FCWH1 + FCWH3

TSFV = SFV + FCWV1 + FCWV3, and

TSMO = SMO + FCWM.

In a way similar to that for SCBAL, the calculations are based on a particular set of independent counterweight design variables, X. This subroutine applies a direct variable transformation to variables X(3) (the follower counterweight mass), X(4) (the follower counterweight polar moment of inertia), X(7) (the crank counterweight mass), and X(8) (the crank counterweight polar moment of inertia) to prevent these variables from becoming negative.

If ICWAN equals zero, then FBBAL skips the evaluation of CG.

SCANG (JFLAG1, JFLAG2) [195]

This subroutine calculates the angular velocities (W2) and accelerations (A2) of the coupler link of a planar slider-crank linkage for NPP equispaced positions of the crank link. This subroutine can be called directly by the user (METHOD=8), or by other subroutines (see Section 11).

If JFLAG1 equals zero, then a table of the coupler link angular velocities and accelerations at the precision-point crank angles is printed out. If JFLAG2 is equal to zero, then plots of the tabulated values are produced.

FBANG (JFLAG1, JFLAG2) [296]

This subroutine calculates the angular velocities (W2 and W3) and accelerations (A2 and A3) of the coupler and follower links respectively of a planar four-bar linkage for NPP equispaced positions of the crank link. This subroutine can be called directly by the user (METHOD=6), or by other subroutines (see Section 11).

JFLAG 1 and JFLAG 2 have the same meanings as for subroutine SCANG.

LINCUP (IPLOT, ISKIP, CR, CT, AR, AI) [385]

This subroutine determines the horizontal (VR and AR) and vertical (VI and AI) velocities and accelerations of a point on the coupler link of a planar four-bar or slider-crank linkage for NPP equispaced positions of the crank link. This subroutine can be called directly by the user (with METHOD=7 for the planar four-bar or METHOD=9 for the planar slider-crank), or by other subroutines (see Section 11).

For METHOD equal to 7 or 9 a table of the magnitudes and angles of the velocities and accelerations is automatically printed out. If IPLOT equals zero, then subroutine COMPARE is called to make plots of the tabulated values.

This subroutine calls SCANG (for METHOD=9) or FBANG (for METHOD=7) to obtain the angular velocities and accelerations of the links unless ISKIP equals zero, in which case the subroutine assumes that these feeder subroutines have been previously called by the user, and thus need not be called again.

CR and CT (Figure 10-2) are the parameters which define the coupler-point position relative to the coupler link.

LINIO (X) [86]

This subroutine calculates the acceleration of given points on the crank and follower links for NPP positions of the crank link. Vector X contains the parameters which determine the position of the crank (X(1) and X(2)) and the follower (X(5) and X(6)) points relative to their respective links and corresponds to the balancing independent design vector (Figure 3-14). This subroutine is used for balancing problems, but can be called by the user in subroutine EXCON if desired.

OPTSURF (NX, NY, GMAX, GMIN, XMAX, XMIN, YMAX, YMIN, ISKIP) [128]

This subroutine, which is called directly by the user in MAIN, sets up values for subroutine PLOTCN to produce a contour plot of a two dimensional subspace of an optimization hypersurface with respect to the independent design variables

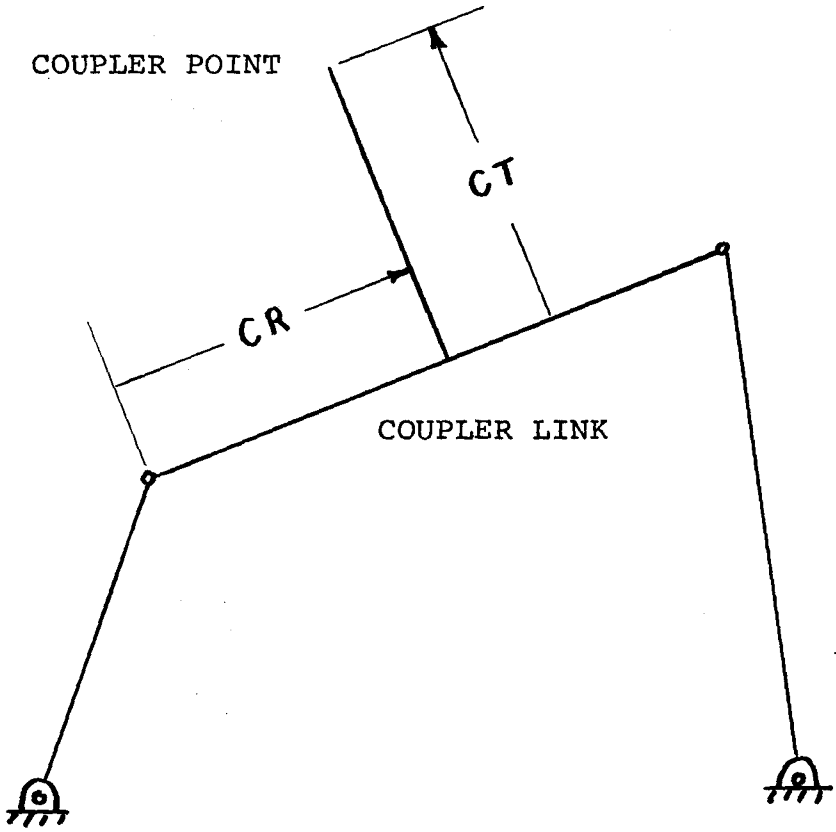

FIG. 10-2 COUPLER-POINT PARAMETERS

of index NX and NY. The intersections of the implicit constraint hypersurfaces with the optimization hypersurface are also plotted. Details on the input and output of this subroutine are found in Section 3-11.

PLOTCN (FMAX, FMIN, NC) [89]

This is the subroutine which plots the values set up in subroutine OPTSURF. FMAX and FMIN correspond to GMAX and GMIN respectively in OPTSURF. NC is the number of implicit constraints to be included in the plotting.

11. SYSTEM FLOWCHARTS

Figures 11-1 to 11-14 show the subroutine relationships (in flowchart form) for the problems which can be handled by the computer program.

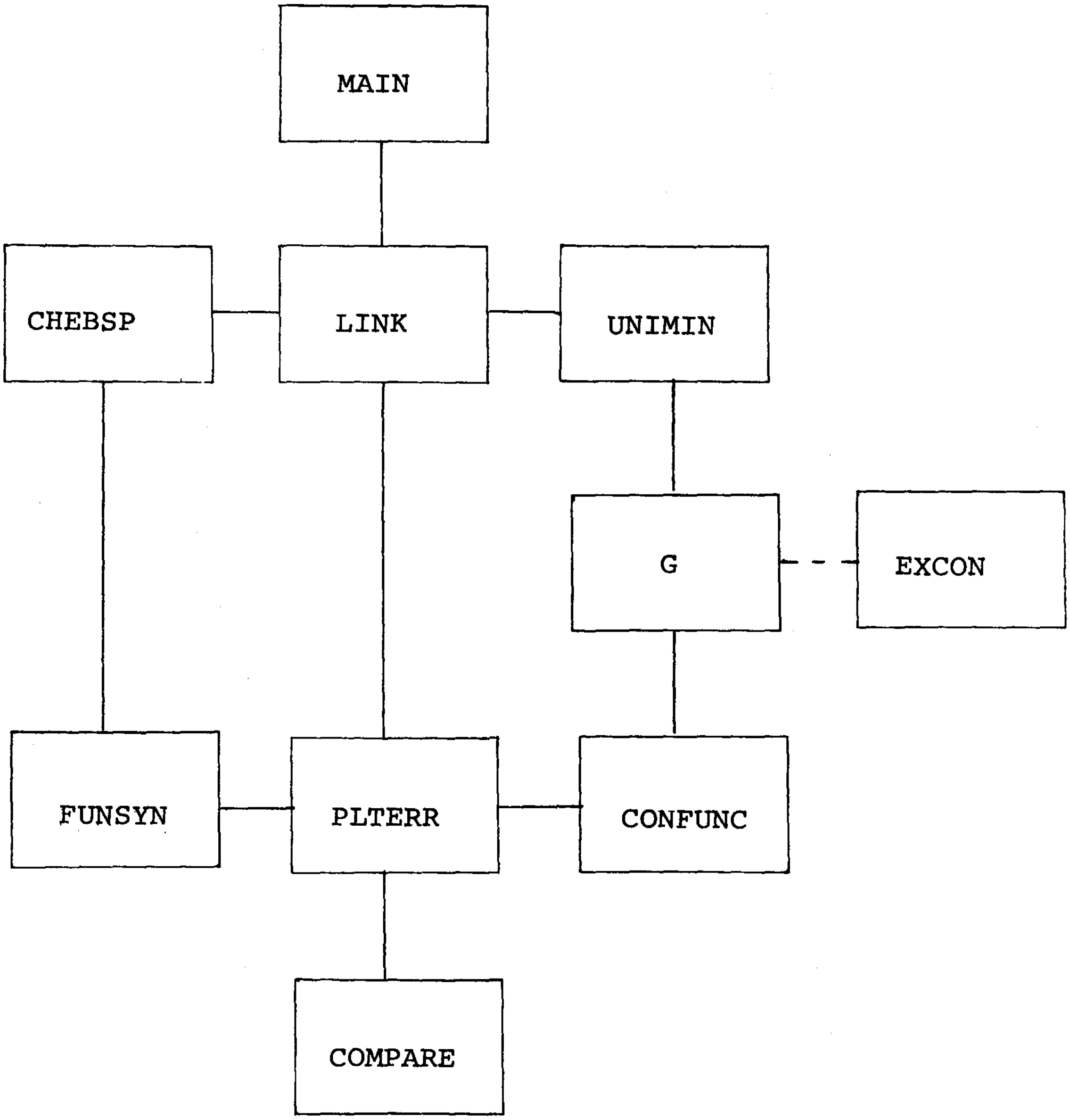

FIG. 11-1 SYSTEM FLOWCHART FOR FUNCTION SYNTHESIS

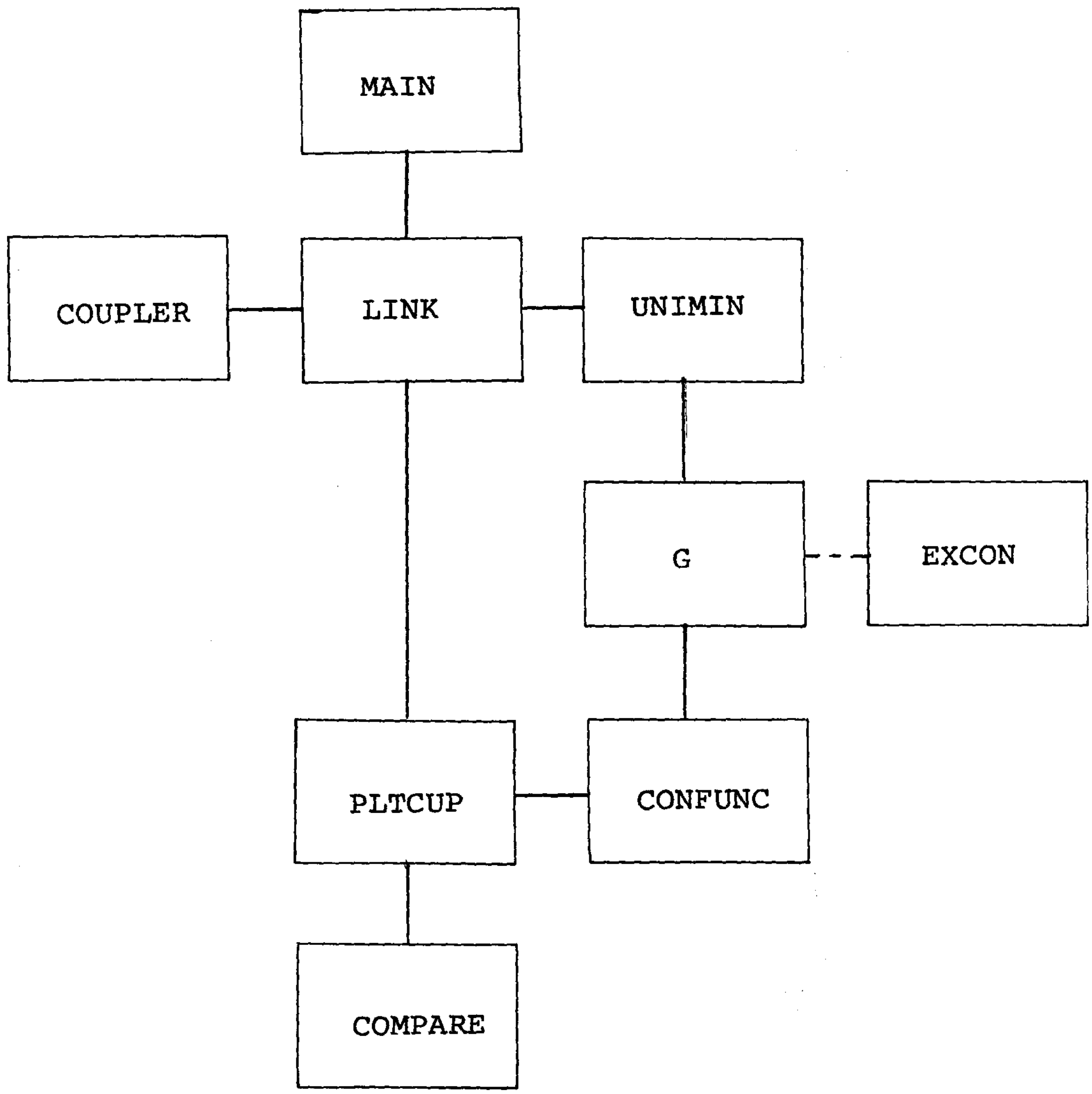

FIG. 11-2 SYSTEM FLOWCHART FOR COUPLER-POINT CURVE SYNTHESIS

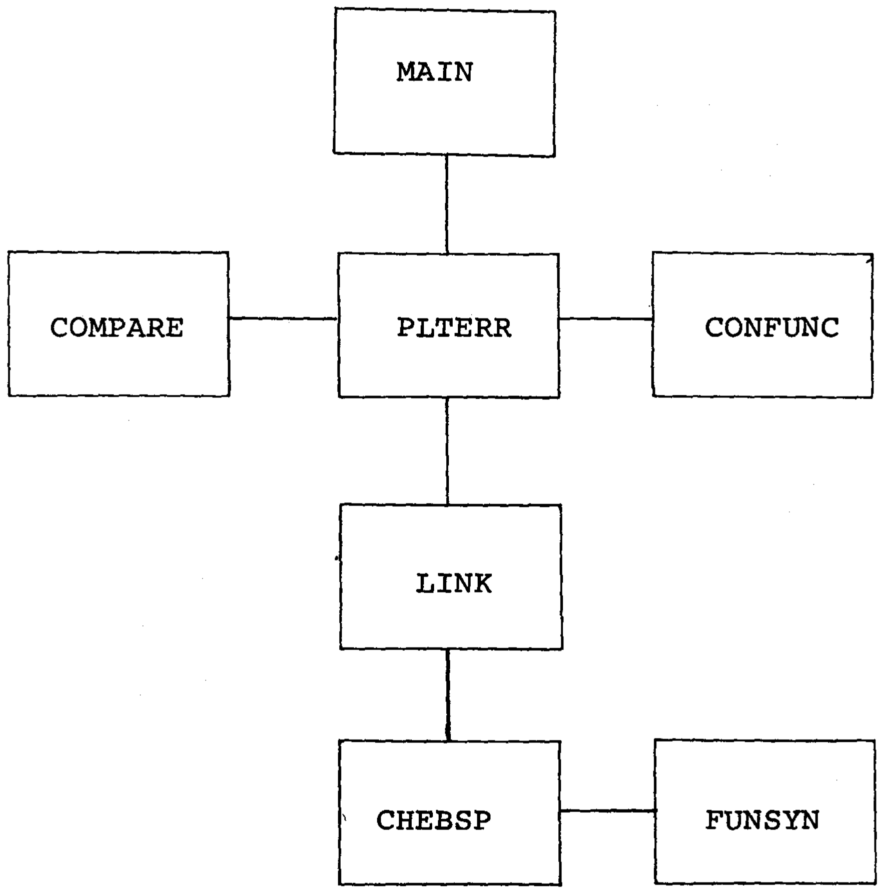

FIG. 11-3 SYSTEM FLOWCHART FOR FUNCTION ANALYSIS

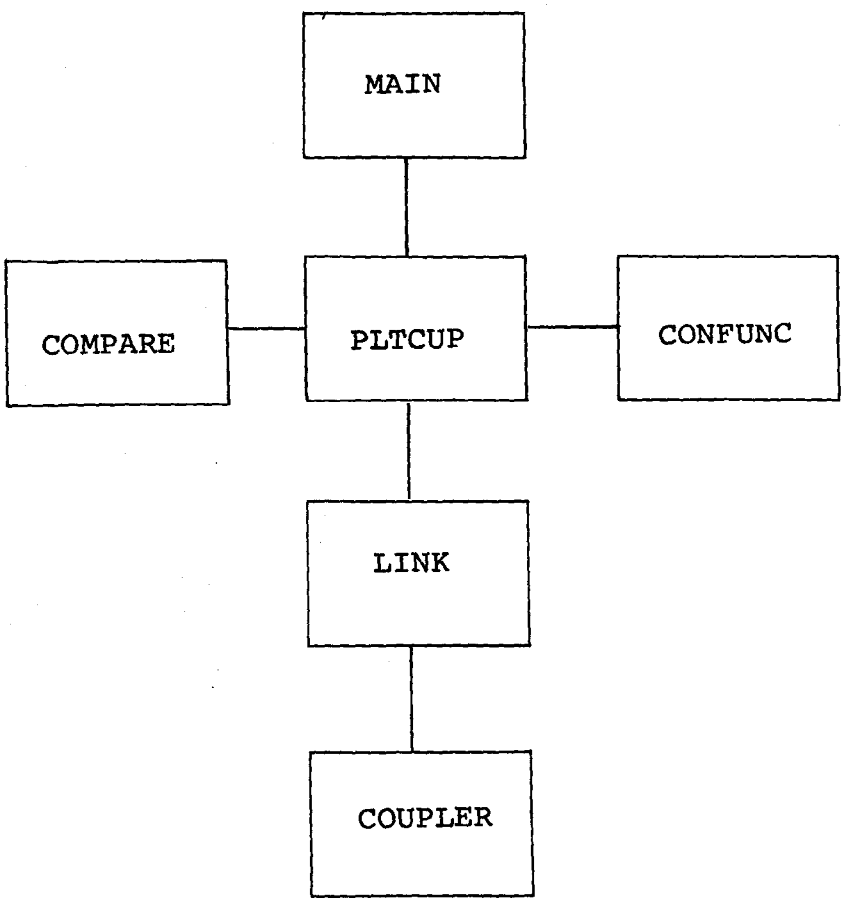

FIG. 11-4 SYSTEM FLOWCHART FOR COUPLER-POINT CURVE ANALYSIS

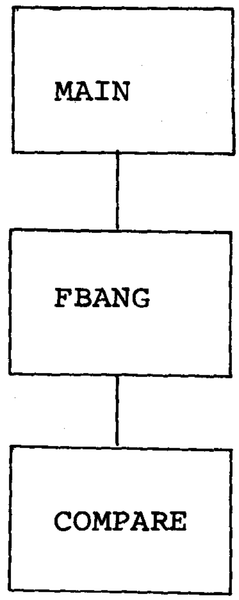

FIG. 11-5 SYSTEM FLOWCHART FOR THE FOUR-BAR ANGULAR VELOCITY AND ACCELERATION ANALYSIS

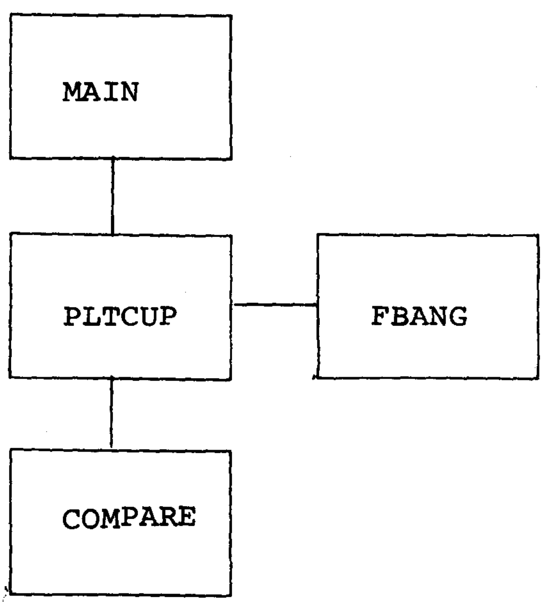

FIG. 11-6 SYSTEM FLOWCHART FOR THE FOUR-BAR COUPLER-POINT VELOCITY AND ACCELERATION ANALYSIS

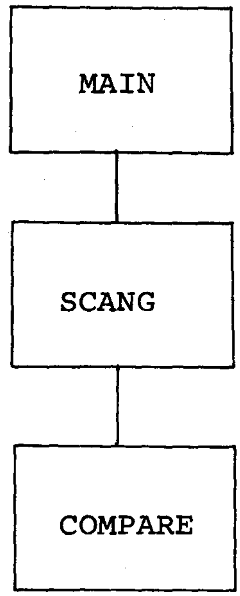

FIG. 11-7 SYSTEM FLOWCHART FOR SLIDER-CRANK ANGULAR VELOCITY AND ACCELERATION ANALYSIS

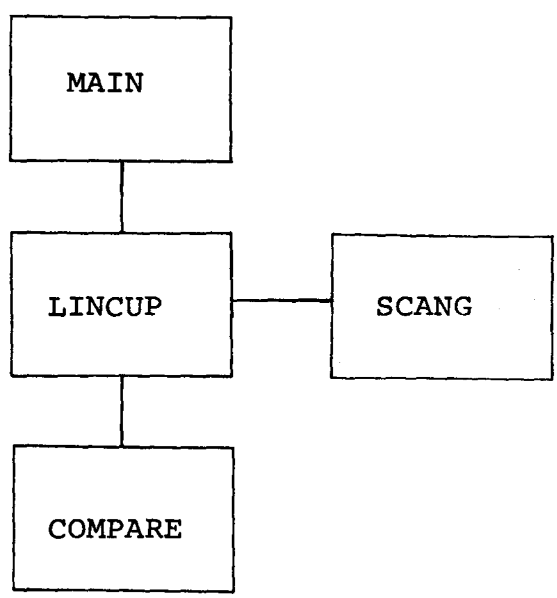

FIG. 11-8 SYSTEM FLOWCHART FOR SLIDER-CRANK COUPLER-POINT VELOCITY AND ACCELERATION ANALYSIS

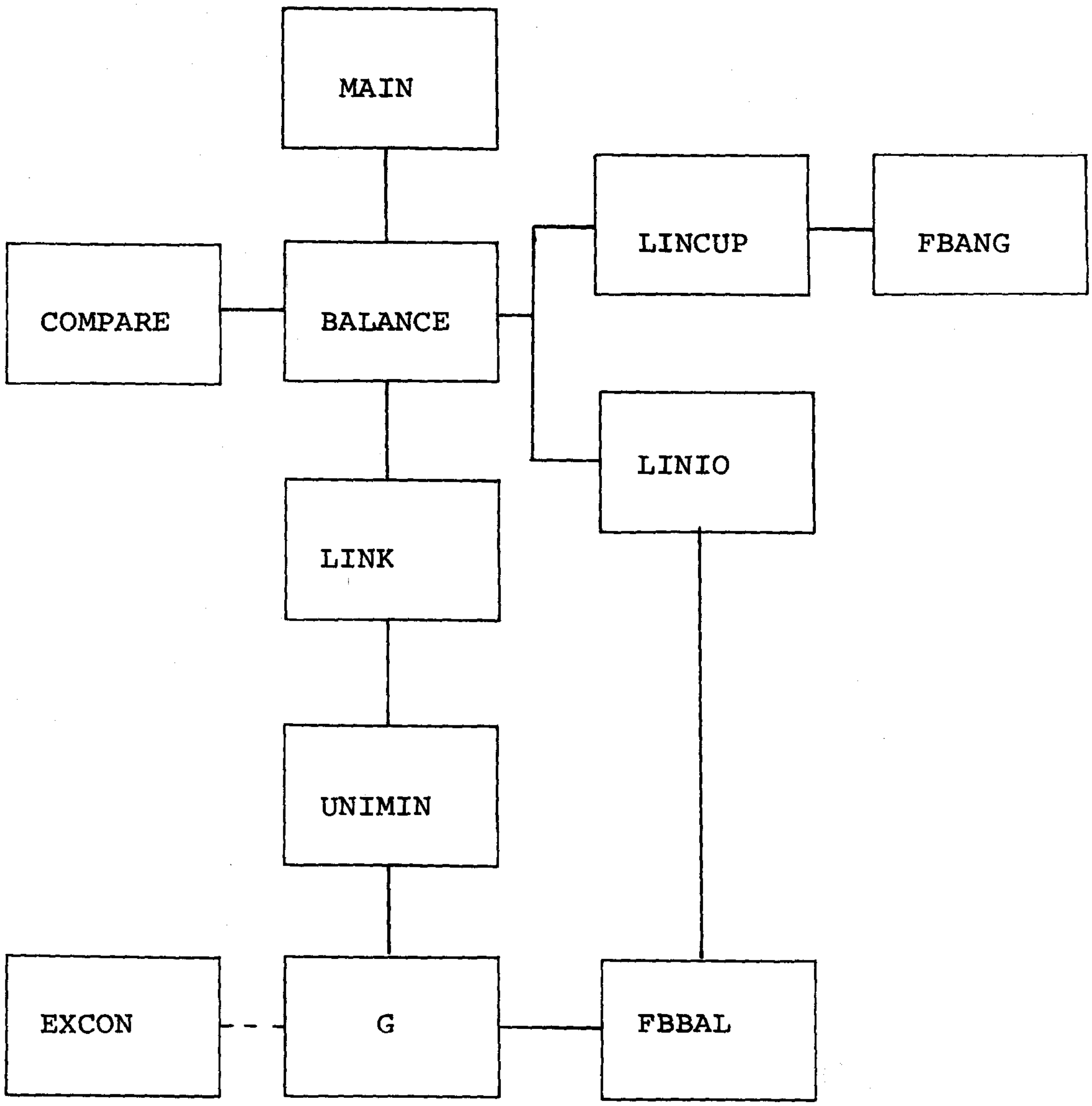

FIG. 11-9 SYSTEM FLOWCHART FOR THE FOUR-BAR BALANCING SYNTHESIS AND ANALYSIS

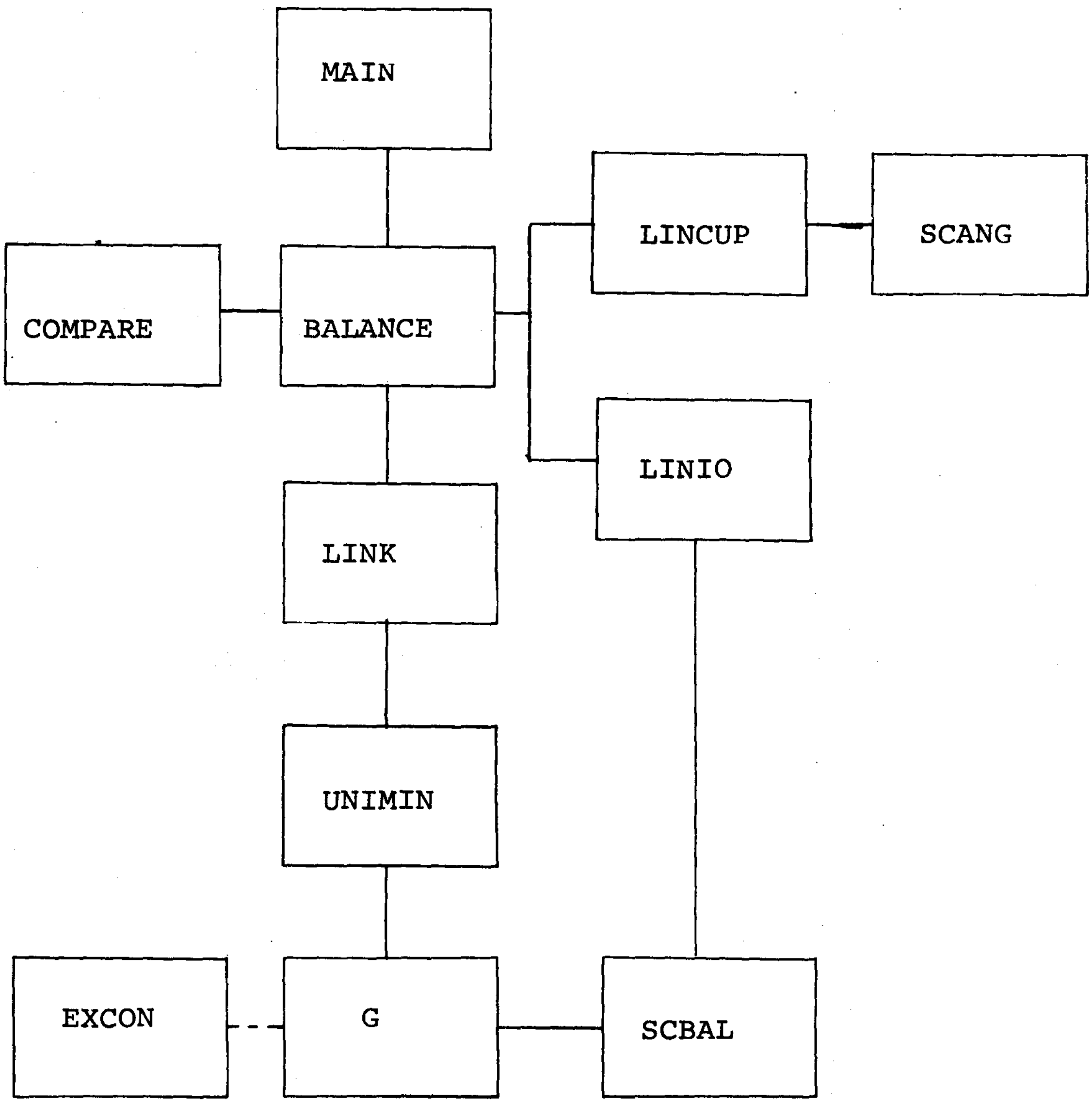

FIG. 11-10 SYSTEM FLOWCHART FOR THE SLIDER-CRANK BALANCING SYNTHESIS AND ANALYSIS

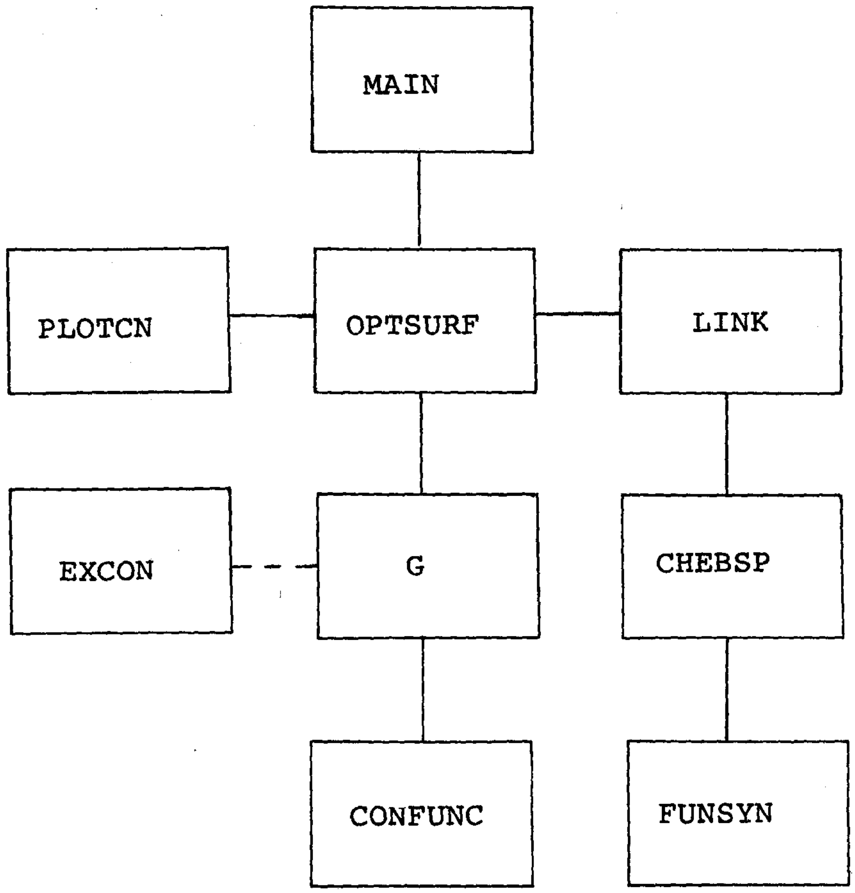

FIG. 11-11 SYSTEM FLOWCHART FOR THE FUNCTION SYNTHESIS OPTIMIZATION SURFACE PLOTTING

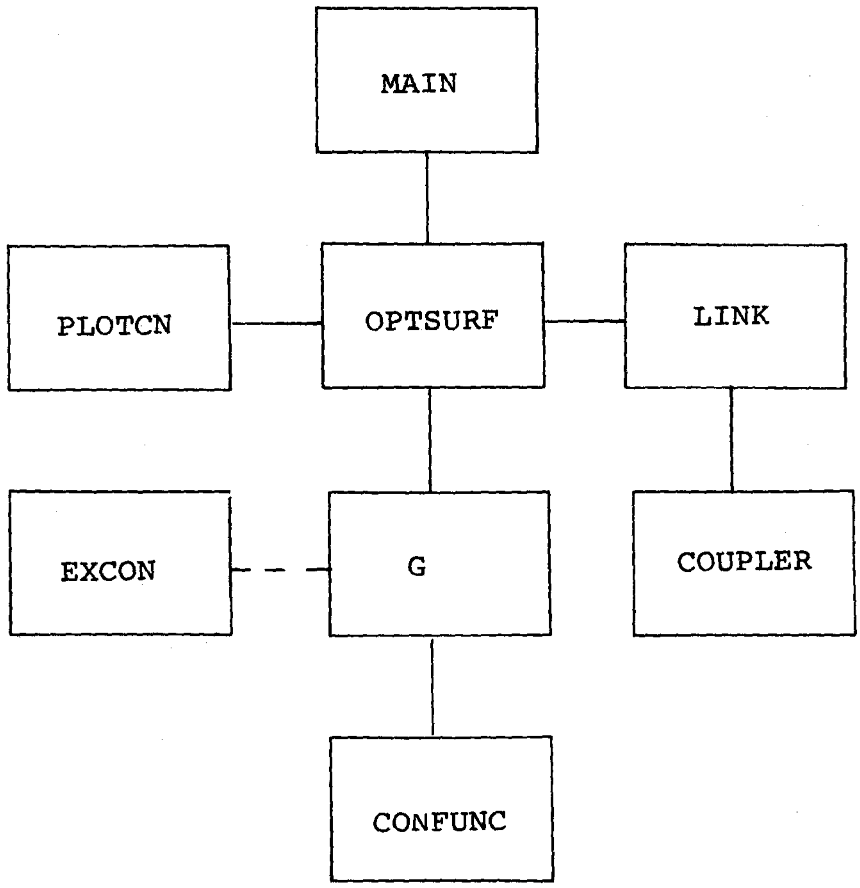

FIG. 11-12 SYSTEM FLOWCHART FOR THE COUPLER-POINT CURVE SYNTHESIS OPTIMIZATION SURFACE PLOTTING

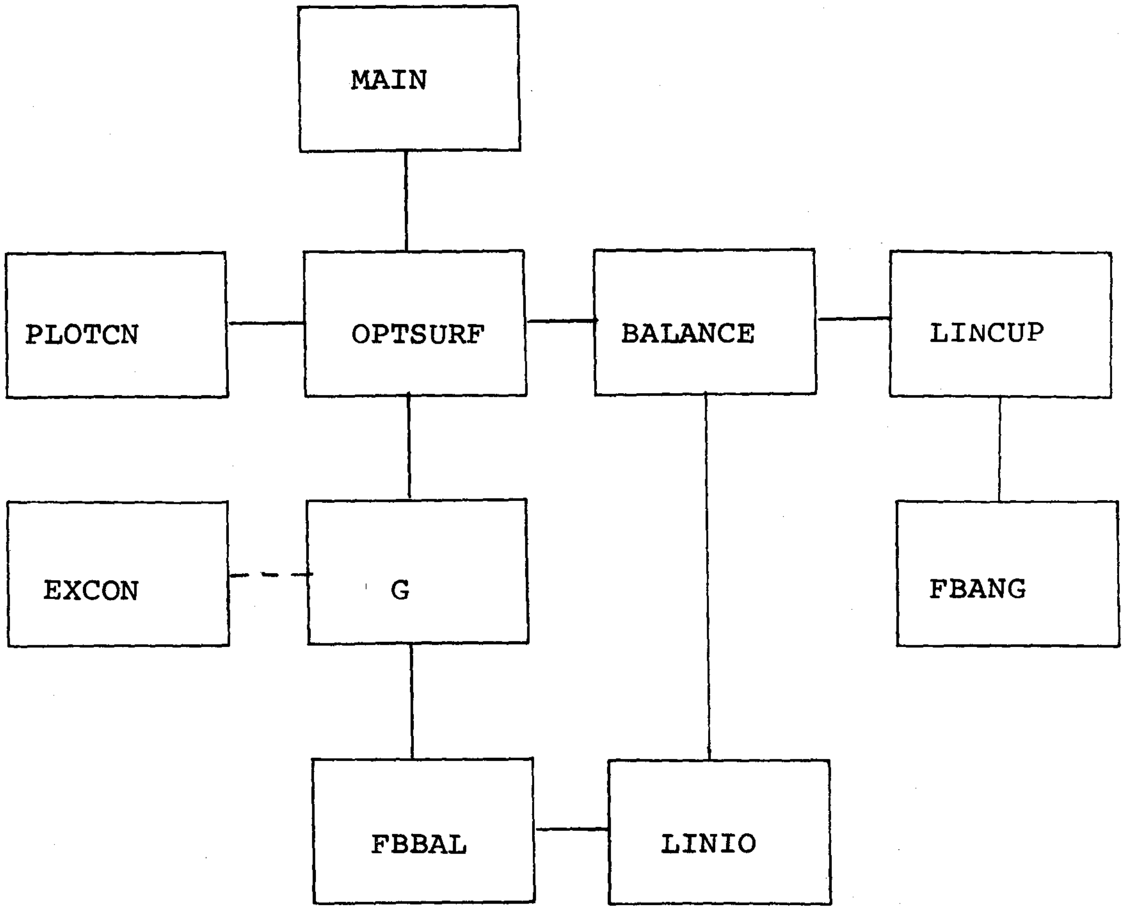

FIG. 11-13 SYSTEM FLOWCHART FOR THE FOUR-BAR BALANCING SYNTHESIS OPTIMIZATION SURFACE PLOTTING

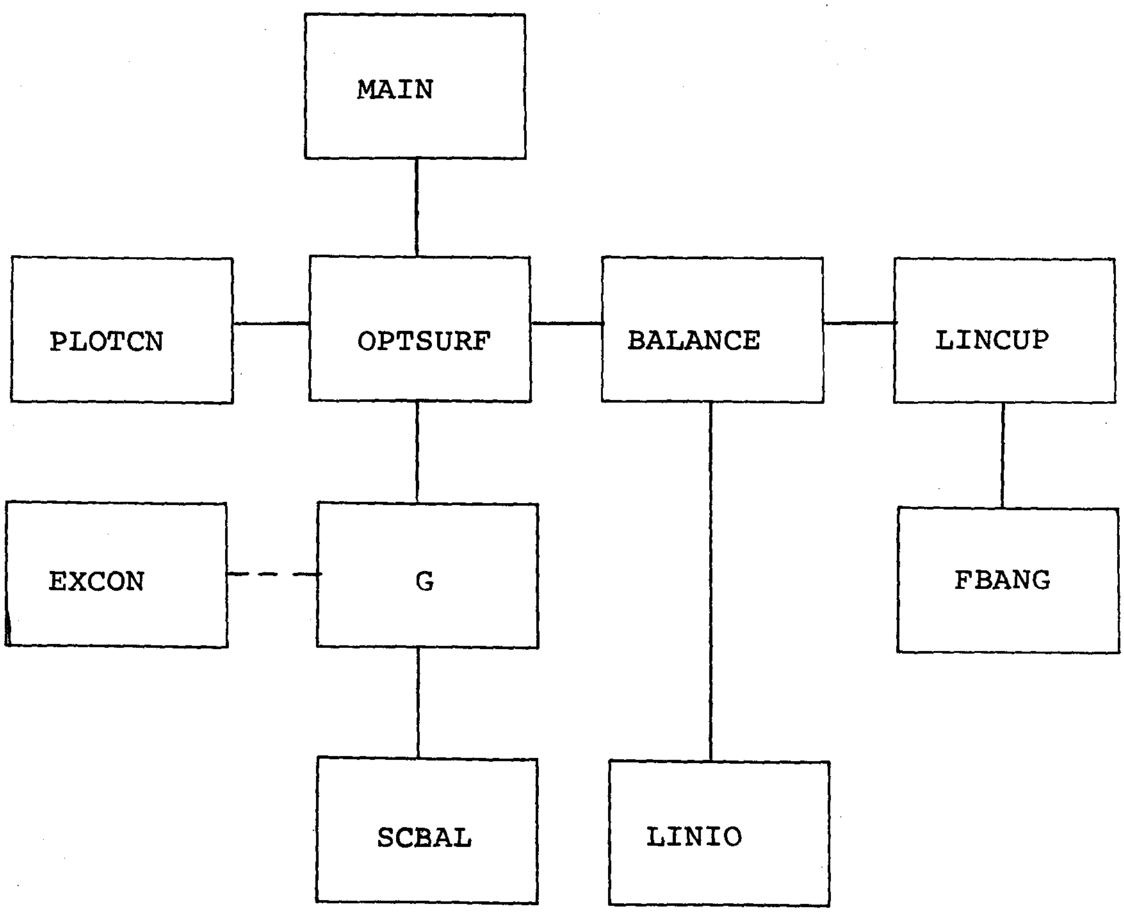

FIG. 11-14 SYSTEM FLOWCHART FOR THE SLIDER-CRANK BALANCING SYNTHESIS OPTIMIZATION SURFACE PLOTTING

APPENDIX F - PROGRAM LISTING

CONTENTS

```
      SUBROUTINE CHEBSP
      COMMON VERBOT(1795)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      COMMON /DESIRE/XD(21),YD(81),PH(81),XTOL(21),YTOL(21)
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
C SUBROUTINE TO DETERMINE THE PRECISION POINTS WITH CHEBYCHEV SPACING
C TO MINIMIZE THE STRUCTURAL ERROR FOR FUNCTION GENERATION
C ALSO DETERMINES THE INPUT ANGLES, PH(I), AT THE PRECISION POINTS AND
C THE DESIRED OUTPUT FUNCTION, YD(I), AT THE PRECISION POINTS
C NOTE- THE MIDPOINT OF THE INPUT RANGE IS ALWAYS AN EXACT PRECISION
C POINT
C THE OTHER POINTS ARE ONLY PSEUDO PRECISION POINTS
C PH(1) AND PH(NPP+2) ARE NOT AT PSEUDO PRECISION POINTS, BUT ARE
C USED ONLY TO OBTAIN THE ACTUAL OUTPUT RANGE OF MOTION IN CONFUNC
      PH(1)=-RNGI/2.
      PH(NPP+2)=-PH(1)
      IF(XMAX.LE.XMIN) GOTO 3
      A=(XMAX+XMIN)/2.
      H=(XMAX-XMIN)/2.
      TN=1.5707963268/FLOAT(NPP)
      SCAL=RNGI/(2.*H)
      DO 1 I=1,NPP
C DEFINING FIXED VALUES OF INPUT VARIABLE FOR PRECISION POINTS
      X= -H*COS(FLOAT(2*I-1)*TN)
      PH(I+1)=X*SCAL
      FILL=X+A
C DEFINING DESIRED VALUES OF SCALED OUTPUT VARIABLE AT PRECISION POINTS
    1 YD(I)=FUNSYN(FILL)
C DETERMINING RANGE OF DESIRED FUNCTION OUTPUT
      FXM=FUNSYN(XMIN)
      IF(ISYM.EQ.0) GOTO 2
      RPSI=FUNSYN(XMAX)-FXM
      RETURN
    2 RPSI=YD(NPP/2+1)-FXM
      RETURN
    3 WRITE(6,100)
      STOP
  100 FORMAT(1H0,10X,*IMPOSSIBLE SITUATION- XMIN .GE. XMAX/
     111X,*PROGRAM HAS BEEN ABORTED*)
      END
```

```
      SUBROUTINE LINK(ISURF,SM,RF,RAT,PREC)
C SUBROUTINE TO FIND THE MINIMUM OF A FUNCTION SUBJECT TO INEQUALITY
C CONSTRAINTS
C EQUALITY CONSTRAINTS CAN BE USED IF THEY ARE PUT IN THE FORM OF
C A SUITABLE INEQUALITY CONSTRAINT
C THE BASIS OF THIS PROGRAM IS POWELL'S CONJUGATE DIRECTION METHOD
C WITHOUT DERIVATIVES I.E. THIS PROGRAM USES A ZEROTH ORDER METHOD
C THE TEST FOR EFFIENCY OF NEW CONJUGATE DIRECTIONS IS BASED ON A
C MODIFICATION OF POWELL'S EFFICIENCY TEST BY ZANGWILL IN 1968
C THE MINIMIZATION ALONG A LINE AND TRANSFORMATION OF THE CONSTRAINED
C PROBLEM INTO AN UNCONSTRAINED PROBLEM ARE NEW FEATURES DESIGNED
C ESPECIALLY FOR THIS PROGRAM
C SET ISURF=0 IF ONLY DESIRING SET-UP FOR CALLING FOR SURFACE
C OPTIMIZATION PLOT
      COMMON P(10,12),XI(10,11),DIR(10)
      COMMON /STRTPT/STRTPT(10)
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /SAVOPT/ SV(10)
      COMMON /MODULO/IMD,IHELP
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /DESIRE/XD(21),YD(81),PH(81),XTOL(21),YTOL(21)
C CALLING CDC CLOCK TO GET STARTING TIME
      CALL SECOND(TIME)
C MAKING SURE ICASE = + OR - 1
      ICASE=ISIGN(1,ICASE)
      IF(METHOD.GT.9) GOTO 16
      IMD=2
      IF(METHOD.NE.1.AND.METHOD.NE.3) IMD=0
C CONVERTING INPUT RANGE OF MOTION TO RADIANS FROM DEGREES
      RNGI=RNGI*.0174533
      IF(METHOD.NE.3.AND.METHOD.NE.4) CFRIC=0.
      NNC=3
      GOTO(7,8,9,10,11),METHOD
C DETERMINE PRECISION POINTS AND OUTPUT FUNCTION AT THESE POINTS
C AND DEFINE NUMBER OF DESIGN VARIABLES AND CONSTRAINTS
    7 CALL CHEBSP
      N=4
      WRITE(6,99)
      GOTO 62
    8 CALL COUPLER
      N=10
      WRITE(6,100)
      GOTO 18
    9 CALL CHEBSP
      N=4
      WRITE(6,110)
      GOTO 18
```

```
   10 CALL COUPLER
      N=9
      WRITE(6,111)
      GOTO 18
   11 CALL CHEBSP
      N=7
      NNC=4
      WRITE(6,118)
C CONVERTING OUTPUT RANGE OF MOTION TO RADIANS FROM DEGREES
   62 RNGO=RNGO*.0174533
C INSURING A POSITIVE RANGE OF OUTPUT MOTION
C CHECKING FOR INPUT DATA ERRORS
      IF(RNGO.LT.0) WRITE(6,128)
      RNGO=ABS(RNGO)
      TITE=TITE*.0174533
      GOTO 18
   16 IF(METHOD.EQ.10) WRITE(6,119)
      IF(METHOD.EQ.11) WRITE(6,120)
   18 WRITE(6,98) ICASE
      IF(METHOD.GT.9) GOTO 17
C SET NO. OF CONSTRAINTS (NC) EQUAL TO NNC DEFINED ABOVE IF USER
C IS NOT ADDING ANY CONSTRAINTS (IEXCO .NE. 0)
      IF(IEXCO.NE.0) NC=NNC
C CONVERTING SCALING PARAMETERS TO A SUITABLE FORM
      ZERO=CFRIC/SQRT(1.+CFRIC*CFRIC)
C CHECKING FOR INPUT DATA ERRORS
      IF(SCAL1.GT..9998) WRITE(6,122)
      IF(SCAL1.LT..0175) WRITE(6,123)
      SCAL1=AMIN1(SCAL1,.9998)
      SCAL1=AMAX1(SCAL1,.0175)
      IF(ZERO.GE.SCAL1) GOTO 51
      SCAL1=(SCAL1-ZERO)/(1.-SCAL1)
      IF(MOD(METHOD,2)) 12,13,12
C ROUTE FOR FUNCTION GENERATION
C CHECKING FOR INPUT DATA ERRORS
   12 IF(SCAL2.LT.1.0E-07) WRITE(6,124)
      SCAL2=AMAX1(SCAL2,1.0E-07)
      SCAL2=1./(SCAL2*SCAL2)
      GOTO 14
C ROUTE FOR COUPLER CURVE SYNTHESIS
   13 DO 15 I=1,NPP
      XTOL(I)=XTOL(I)*XTOL(I)
   15 YTOL(I)=YTOL(I)*YTOL(I)
C CHECKING FOR INPUT DATA ERRORS
   14 IF(SCAL3.LT.1.1) WRITE(6,125)
      SCAL3=AMAX1(SCAL3,1.1)
      SCAL3=(SCAL3-1.)*(SCAL3-1.)
C INITIALIZE COUNTER
```

```
   17 ICOUNT=0
C DEFINE INITIAL PENALTY MULTIPLIER,PM
      PM=10.
C TESTING TO SEE IF ONLY SCALING VALUES WANTED FOR A HYPER-SURFACE
C EXPLORATION
      IF(ISURF.EQ.0) RETURN
C INITIALIZING STARTING POINT
      DO 35 I=1,N
   35 P(I,1)=STRTPT(I)
C PRINT OUT THE STARTING POINT
      WRITE(6,103)(P(I,1),I=1,N)
      IVIOL=0
C INITIALIZE SPECIAL INDEX FOR FUNCTION LEVELER IN INFEASIBLE REGION
      IHELP=2
C DEFINING LINEAR DEPENDANCY PARAMETER
      EPS=.5/SQRT(FLOAT(N))
C SEEDING RANDOM NUMBER GENERATOR FOR PERTURBATION OF OPTIMUM
      SEED=RANF(.123456789)
      N1=N+1
      N2=N+2
C INITIATING OPTOLD COMPARISON VARIABLE FOR LOCAL OPTIMUM CHECK
      OPTOLD=1.0E+50
C SET CONJUGATE DIRECTION VECTORS INITIALLY TO COORDINATE DIRECTIONS
   30 DO 1 I=1,N
      DO 2 J=1,N
    2 XI(I,J)=0.
    1 XI(I,I)=1.
C SET EFFICIENCY PARAMETER, DEL, INITIALLY TO 1.
      DEL=1.
   24 A=0.
C CALL SUBROUTINE UNIMIN TO FIND THE MINIMUM OF THE UNCONSTRAINED
C FUNCTION ALONG EACH OF THE N CONJUGATE DIRECTIONS AND DETERMINE THE
C LARGEST SCALAR MULTIPLIER,A (AND ITS INDEX,JS), OF THE N SCALAR
C MULTIPLIERS OF THE CONJUGATE DIRECTIONS
      CALL UNIMIN(1,N,A,JS,N,SM,RF,RAT)
C DEFINING A NEW CONJUGATE DIRECTION
C AND DEFINING THE NORMALIZING CONSTANT, ALP, FOR THE EFFICIENCY TEST
C OF THE NEW CONJUGATE DIRECTION
      DIR(1)=P(1,N1)-P(1,1)
      TST1=DIR(1)*DIR(1)
      DO 19 I=2,N
      DIR(I)=P(I,N1)-P(I,1)
   19 TST1=TST1+DIR(I)*DIR(I)
      ALP=SQRT(TST1)
C NORMALIZING THE NEW CONJUGATE DIRECTION VECTOR
      DO 21 I=1,N
      XI(I,N1)=DIR(I)/ALP
   21 CONTINUE
      B=0.
```

```
C FINDING THE MINIMUM OF THE FUNCTION ALONG THE NEW CONJUGATE DIRECTION
C VECTOR
      CALL UNIMIN(N1,N1,B,I,N,SM,RF,RAT)
C COMBINATION ABSOLUTE-RELATIVE ERROR TEST ON VARIABLE WITH LARGEST
C CHANGE
C ABSOLUTE TEST IF VARIABLE .LE. 1, RELATIVE TEST IF VARIABLE .GT.1
      TST4=0.
      DO 5 I=1,N
      TST5=ABS(P(I,N2)-P(I,1))
      CKSZ=ABS(P(I,N2))
      IF(CKSZ.GT.1.) TST5=TST5/CKSZ
C DEFINING THE STARTING POINT FOR THE NEXT MINIMIZATION SEQUENCE
      P(I,1)=P(I,N2)
    5 TST4=AMAX1(TST4,TST5)
      IF(TST4.LT.PREC) GOTO 20
C TESTING TO SEE IF NEW CONJUGATE DIRECTION IS EFFICIENT
C ALL OLD DIRECTIONS RETAINED IF NEW CONJUGATE DIRECTION IS NOT
C EFFICIENT
C INEFFIENCY IS A SIGN THAT THE NEW CONJUGATE DIRECTION IS NOT LINEARLY
C INDEPENDENT AND THUS NOT REALLY CONJUGATE TO THE OTHER DIRECTIONS
C HOPEFULLY A NEW SEQUENCE WILL PRODUCE A MORE EFFICIENT DIRECTION
      TST1=A*DEL/ALP
      IF(TST1.LT.EPS) GOTO 24
C ROUTE IF NEW DIRECTION IS EFFICIENT
C THE DIRECTION WITH THE MULTPLIER OF HIGHEST MODULUS IS REPLACED WITH
C THE NEW CONJUGATE DIRECTION
      DO 25 I=1,N
      XI(I,JS)=XI(I,N1)
   25 CONTINUE
C DEFINING A NEW EFFIENCY PARAMETER
      DEL=TST1
      GOTO 24
C ROUTE WHEN CONVERGENCE TO A SOLUTION HAS BEEN REACHED
C OBTAINING FINAL VALUES OF THE CONSTRAINED OBJECTIVE FUNCTION AND THE
C CONSTRAINTS THROUGH LABELLED COMMON
   20 OPTNEW=G(P)
C CHECKING TO SEE IF PERTURBED STARTING POINT HAS LED TO A LOWER LOCAL
C UNCONSTRAINED OPTIMUM OBJECTIVE FUNCTION VALUE
C ON FIRST RUN OPTOLD=1.0E+50
      IF(OPTNEW-OPTOLD) 28,29,29
C PRINT LOCAL OPTIMUM
   28 WRITE(6,106) CG,(P(I,1),I=1,N)
      IF(NC.EQ.0) GOTO 60
      WRITE(6,102) (C(I),I=1,NC)
C CHECKING FOR NEGATIVE CONSTRAINT VALUES AT FINAL OPTIMUM
      DO 3 I=1,NC
      IF(C(I)) 32,3,3
    3 CONTINUE
C SAVE PARAMETER VALUES FOR BEST POINT
   60 CGBST=CG
```

```
      U1BST=U1
      U2BST=U2
      U3BST=U3
C SETTING OPTOLD FOR NEXT COMPARISON
      OPTOLD=OPTNEW
      PM=10.
      IHELP=2
      IVIOL=0
      DO 47 I=1,N
   47 SV(I)=P(I,1)
   38 DO 27 I=1,N
C RANDOM PERTURBATION OF OPTIMUM POINT DESIGN VARIABLES TO START A NEW
C MINIMIZATION SEQUENCE
   27 P(I,1)=P(I,1)*(3.-4.*RANF(0.))
C PRINT OUT STARTING VALUES FOR MINIMIZATION IN THE NEXT SEQUENCE
      WRITE(6,104) (P(I,1),I=1,N)
      GOTO 30
C ROUTE IF METHOD HAS TERMINATED AT A SUPPOSED OPTIMUM AFTER AT LEAST
C ONE PERTURBATION OF THE DESIGN VARIABLES
C CHECKING FOR CONSTRAINT VIOLATIONS
   29 IF(NC.EQ.0) GOTO 33
      DO 31 I=1,NC
      IF(C(I)) 32,31,31
   31 CONTINUE
      GOTO 33
C ROUTE IF OPTIMUM DETERMINED IS INFEASIBLE
C CHECK TO SEE IF AN INFEASIBLE HANG-UP HAS OCCURRED
   32 IF(PM.LT.(-1.0E+06/C(I))) GOTO 37
C INCREASING UNSUCCESSFUL STARTING POINT COUNTER
      IVIOL=IVIOL+1
C PROGRAM ABORTED IF 4 CONSECUTIVE UNSUCCESSFUL STARTING POINTS
C GENERATED
      IF(IVIOL.EQ.4) GOTO 39
C IF 2 CONSECUTIVE UNSUCCESSFUL STARTING POINTS GENERATED
C SET INDEX FOR FUNCTION LEVELING
      IF(IVIOL.EQ.2) IHELP=0
      WRITE(6,126)
      GOTO 38
   37 WRITE(6,107) I,C(I)
      WRITE(6,104) (P(I,1),I=1,N)
C INCREASING PENALTY MULTIPLIER FOR NEXT LOOP
      PM=PM*10000.
      GOTO 30
C CALCULATING TOTAL EXECUTION TIME
   33 CALL SECOND(TF)
      TIME=TF-TIME
C PRINTING FINAL VALUES OF CONSTRAINED OBJECTIVE FUNCTION, DESIGN
C VARIABLES, AND THE CONSTRAINTS
      WRITE(6,106) CG,(P(I,1),I=1,N)
```

```
      IF(NC.EQ.0) GOTO 45
      WRITE(6,102) (C(J),J=1,NC)
      IF(METHOD.GT.9) GOTO 45
C PRINTING PARAMETERS AT BEST POINT
      WRITE(6,109) U1BST,U2BST,U3BST,CGBST,(SV(I),I=1,N)
      GOTO 46
   45 WRITE(6,121) U1BST,U2BST,U3BST,CGBST,(SV(I),I=1,N)
C PRINTING NUMBER OF FUNCTION EVALUATIONS
   46 WRITE(6,108) ICOUNT
C PRINTING OUT TOTAL EXECUTION TIME
      WRITE(6,105) METHOD,TIME
      IF(METHOD.GT.9) RETURN
C DETERMINING WHICH PLOTTING ROUTINE TO USE
C THE ROUTINE TO BE USED DEPENDS ON METHOD
      GOTO(40,41,40,41,40),METHOD
   40 CALL PLTERR(1)
      RETURN
   41 CALL PLTCUP(1)
      RETURN
   51 WRITE(6,117)
      STOP
   39 WRITE(6,127)
      STOP
   98 FORMAT(11X,*WHERE ICASE=*,I3)
   99 FORMAT(1H1,10X,*SYNTHESIS OF A PLANAR 4-BAR FUNCTION GENERATOR MEC
     1HANISM*)
  100 FORMAT(1H1,10X,*SYNTHESIS OF A PLANAR 4-BAR MECHANISM TO PRODUCE A
     1 GIVEN COUPLER CURVE*)
  102 FORMAT(1H0,10X,*THE CONSTRAINT VALUES ARE*/(16X,4G16.5))
  103 FORMAT(1H0,10X,*THE STARTING DESIGN VARIABLE VALUES ARE*/
     1(16X,4G16.5))
  104 FORMAT(1H0//11X,*THE NEW STARTING DESIGN VARIABLE VALUES ARE*/
     1(16X,4G16.5))
  105 FORMAT(1H0,10X,*TOTAL EXECUTION TIME FOR METHOD*,I3,* IS*,
     1F10.3,*  SECONDS*)
  106 FORMAT(1H0,10X,*LOCAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS*,
     1G13.5//11X,*DESIGN VARIABLE VALUES ARE*/(16X,4G16.5))
  107 FORMAT(1H0,10X,*THE OPTIMUM VALUE DETERMINED IS INFEASIBLE*/
     111X,*CONSTRAINT NUMBER*,I3,*  HAVING THE VALUE*,G14.5/
     211X,*THEREFORE THE PENALTY MULTIPLIER HAS BEEN MULTIPLIED BY 10000
     3*/11X,*AND A NEW MINIMIZATION SEQUENCE FROM THE CURRENT OPTIMUM ST
     4ARTED*)
  108 FORMAT(1H0,10X*NUMBER OF FUNCTION CALLS WAS*,I7)
  109 FORMAT(1H0//16X,*TRANSMISSIBILITY INDEX FUNCTION IS*,G13.5//
     116X,*STRUCTURAL ERROR FUNCTION IS*,G13.5//16X,
     2*LINK LENGTH FUNCTION IS*
     3,G13.5//16X,*FINAL OPTIMUM VALUE OF OBJECTIVE FUNCTION IS*,
     4G13.5//16X,*DESIGN VARIABLE VALUES ARE*/(16X,4G16.5))
  110 FORMAT(1H1,10X,*SYNTHESIS OF A PLANAR SLIDER-CRANK FUNCTION GENERA
```

```
      1TOR MECHANISM*)
  111 FORMAT(1H1,10X,*SYNTHESIS OF A PLANAR SLIDER-CRANK MECHANISM TO PR
      1ODUCE A GIVEN COUPLER CURVE*)
  117 FORMAT(1H0,10X,*THE OPTIMIZATION FUNCTION IS ILL-FORMED SINCE THE
      1WORST POSSIBLE CASE (ZERO)/11X,*FOR THE INDEX OF TRANSMISSIBILITY
      2EXCEEDS ITS SMALLEST ACCEPTABLE VALUE (SCAL1)*/11X,*THEREFORE INCR
      3EASE SCAL1 AND/OR, IF DOING A SLIDER-CRANK SYNTHESIS, DECREASE CFR
      4IC*/11X,*CURRENT RUN HAS BEEN ABORTED*)
  118 FORMAT(1H1,10X,*SYNTHESIS OF A SPATIAL 4-BAR (RGGR) FUNCTION GENER
      1ATOR MECHANISM*)
  119 FORMAT(1H1,10X,*BALANCING OF A PLANAR 4-BAR MECHANISM*)
  120 FORMAT(1H1,10X,*BALANCING OF A PLANAR SLIDER-CRANK MECHANISM*)
  121 FORMAT(1H0//16X,*HORIZONTAL SHAKING FORCE FUNCTION IS*,G14.5//
      116X,*VERTICAL SHAKING FORCE FUNCTION IS*,G14.5//16X,
      2*SHAKING MOMENT FUNCTION IS*,G14.5//
      316X,*FINAL VALUE OF BALANCING OBJECTIVE FUNCTION IS*,G14.5//16X,
      4*BALANCING VARIABLE VALUES ARE*/(16X,4G16.5))
  122 FORMAT(1H0,*WARNING, SCAL1 GREATER THAN .9998 WAS DETECTED BY LINK
      1 AND WAS SET EQUAL TO .9998*)
  123 FORMAT(1H0,*WARNING, SCAL1 LESS THAN .0175 WAS DETECTED BY LINK AN
      1D WAS SET EQUAL TO .0175*)
  124 FORMAT(1H0,*WARNING, SCAL2 LESS THAN 1.0E-07 WAS DETECTED BY LINK
      1AND WAS SET EQUAL TO 1.0E-07*)
  125 FORMAT(1H0,*WARNING, SCAL3 LESS THAN 1.1 WAS DETECTED BY LINK AND
      1WAS SET EQUAL TO 1.1*)
  126 FORMAT(1H0,10X,*MINIMIZATION ROUTINE HAS HUNG UP IN AN INFEASIBLE
      1REGION*/11X,*INDICATIONS ARE THAT THE CONSTRAINTS ARE SUCH THAT A
      2FEASIBLE REGION*/11X,*CANNOT BE REACHED FROM THE CURRENT LOCATION*
      3/11X,*THEREFORE A NEW STARTING POINT HAS BEEN GENERATED*)
  127 FORMAT(1H0,10X,*INDICATIONS ARE THAT NO FEASIBLE SOLUTION EXISTS*/
      111X,*RECHECK CONSTRAINTS IN SUBROUTINE EXCON, AND METHOD INPUT DAT
      2A*/11X,*PROGRAM HAS BEEN ABORTED*)
  128 FORMAT(1H0,10X,*RNGO MUST BE POSITIVE*/11X,*THEREFORE THE ABSOLUTE
      1 VALUE OF THE RNGO GIVEN HAS BEEN USED*/11X,*IF THE OUTPUT RANGE O
      2F MOTION OBTAINED IS UNSATISFACTORY CHANGE ICASE OR VECTOR STRPT*)
       END
```

```
      SUBROUTINE PLTERR(ICK)
C SUBPROGRAM TO INITIATE PLOTTING OF THE STRUCTURAL ERROR OF PLANAR
C 4-BAR, PLANAR SLIDER-CRANK, AND SPATIAL 4-BAR MECHANISMS FOR FUNCTION
C GENERATION (METHOD=1,3,5)
C THE VALUE OF THE ACTUAL FUNCTION GENERATED MINUS THE DESIRED FUNCTION
C IS EVALUATED AT 81 POINTS WITHIN THE RANGE OF THE INPUT VARIABLE
C AND PLOTTED AS THE ORDINATE
C THE ACTUAL FUNCTION INPUT,X, IS REPRESENTED BY THE ABSCISSA
C THE MINIMUM TRANSMISSION ANGLE, ACTUAL RANGE OF FOLLOWER MOTION, AND
C THE INPUT STARTING ANGLE ARE ALSO CALCULATED
C THE MECHANISM PRODUCED IS ALSO CHECKED FOR CLOSURE AT ALL POSITIONS
C WITHIN THE DESIGNATED RANGES OF MOTION
      DIMENSION Y(81),X(81)
      COMMON /SAVOPT/ SV(10)
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /NUMBERS/NPP,METHOD
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
      COMMON /DESIRE/XD(21),YD(81),PH(81),XTOL(21),YTOL(21)
C OBTAINING SET-UP VALUES IF NECESSARY (ICK=0)
      IF(ICK.EQ.0) CALL LINK(0)
C PRINTING TITLES
      IF(METHOD.EQ.1) WRITE(6,112)
      IF(METHOD.EQ.3) WRITE(6,114)
      IF(METHOD.EQ.5) WRITE(6,116)
      PH(1)=-RNGI/2.
      YD(1)=FUNSYN(XMIN)
      X(1)=XMIN
      ADD=RNGI/80.
      RNGDIV=(XMAX-XMIN)/80.
      DO 4 I=2,81
      X(I)=X(I-1)+RNGDIV
      PH(I)=PH(I-1)+ADD
    4 YD(I)=FUNSYN(X(I))
C OBTAINING ACTUAL OUTPUT USING OPTIMUM VARIABLES
      CALL CONFUNC(SV,81,0)
C CHECKING FOR CLOSURE OF THE MECHANISM AT ALL POSITIONS
C NON-CLOSURE IF CONSTRAINT C(1) IS VIOLATED
      IF(C(1)) 2,3,3
    3 YMAX=-1.0E+50
      YMIN=1.0E+50
      IF(ISYM.EQ.0) GOTO 6
C ROUTE FOR NON-SYMMETRIC FUNCTIONS
      RNGOA=PS(81)-PS(1)
      RPSI=(YD(81)-YD(1))/RNGOA
      GOTO 7
C ROUTE FOR SYMMETRIC FUNCTIONS (ISYM=0)
    6 RNGOA=PS(41)-PS(1)
      RPSI=(YD(41)-YD(1))/RNGOA
```

```
    7 DO 1 I=1,81
      YA=YD(41)+RPSI*(PS(I)-PS(41))
      Y(I)=YA-YD(I)
C DETERMINING THE MAXIMUM AND MINIMUM VALUES OF THE STRUCTURAL ERROR
      YMAX=AMAX1(YMAX,Y(I))
    1 YMIN=AMIN1(YMIN,Y(I))
C DETERMINING THE INPUT STARTING ANGLE AND THE RANGE OF OUTPUT MOTION
C IN DEGREES
      STAR=57.2957795*(SV(4)+PH(1))
      WRITE(6,100) YMAX,YMIN
      CALL COMPARE(81,Y,Y,X,X,YMAX,YMIN,XMAX,XMIN,0)
      IF(METHOD.EQ.3) GOTO 5
C ROUTE FOR 4-BAR LINKAGES
      FINA=PS(1)*57.2957795
      RNGPR=RNGOA*57.2957795
C PRINTING CRANK STARTING ANGLE, FOLLOWER STARTING ANGLE
C AND RANGE OF OUTPUT MOTION
      WRITE(6,102) STAR,FINA,RNGPR
      GOTO 8
C ROUTE FOR SLIDER-CRANK LINKAGE
    5 WRITE(6,103) STAR,PS(1),RNGOA
      GOTO 8
C ROUTE IF C(1) VIOLATED
    2 WRITE(6,101)
    8 WRITE(6,104) TRI
      RETURN
  100 FORMAT(1H0,5X,*MAXIMUM STRUCTURAL ERROR (YA-YD) IS*,G14.5/
     16X,*MINIMUM STRUCTURAL ERROR IS*,G14.5//)
  101 FORMAT(1H0,10X,*WARNING, MECHANISM SYNTHESIZED DOES NOT CLOSE AT C
     1ERTAIN POSITIONS*/11X,*IN THE DESIGNATED RANGE OF MOTION*/
     2  11X,*SUGGEST INCREASING THE NUMBER OF PRECISION POINTS OR DECREA
     3SING*/11X,*THE DESIRED RANGE OF MOTION*)
  102 FORMAT(1H0,10X,*THE CRANK STARTING ANGLE IS*,G14.5,*  DEGREES*//
     111X,*THE FOLLOWER STARTING ANGLE IS*,G14.5,*  DEGREES*//11X,
     2*THE RANGE OF FOLLOWER MOTION IS*,G14.5,*  DEGREES*)
  103 FORMAT(1H0,10X,*THE CRANK STARTING ANGLE IS*,G14.5,*  DEGREES*//
     111X,*THE SLIDER STARTING POSITION IS*,G14.5//11X,
     2*THE RANGE OF SLIDER MOTION IS*,G14.5)
  104 FORMAT(1H0,10X,*THE MINIMUM TRANSMISSIBILITY INDEX IS*,G14.5)
  112 FORMAT(1H1,5X,*STRUCTURAL ERROR PLOT FOR PLANAR 4-BAR FUNCTION GEN
     1ERATION*)
  114 FORMAT(1H1,5X,*STRUCTURAL ERROR PLOT FOR PLANAR SLIDER-CRANK FUNCT
     1ION GENERATION*)
  116 FORMAT(1H1,5X,*STRUCTURAL ERROR PLOT FOR SPATIAL FOUR-BAR (RGGR) F
     1UNCTION GENERATION*)
      END
```

```
      SUBROUTINE PLTCUP(ICK)
C SUBROUTINE SUBPROGRAM TO INITIATE PLOTTING OF THE DESIRED AND ACTUAL
C SYNTHESIZED COUPLER CURVES FOR THE PLANAR 4-BAR AND SLIDER-CRANK
C MECHANISMS (METHOD=2,4)
C THE MAGNITUDE OF THE STRUCTURAL ERROR OF THE COUPLER CURVE AT THE
C PRECISION-POINTS IS ALSO PLOTTED VS. THE CRANK ANGLE IN THE RANGE OF
C DESIRED MOTION
      COMMON /SAVOPT/ SV(10)
      COMMON /NUMBERS/NPP,METHOD
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE
      COMMON /DESIRE/CXD(21),CYD(81),PH(81),XTOL(21),YTOL(21)
C OBTAINING SET-UP VALUES
      IF(ICK.EQ.0) CALL LINK(0)
C OBTAING COUPLER POINTS AND STRUCTURAL ERROR FOR
C OPTIMUM VARIABLE VALUES
      CALL CONFUNC(SV,NPP,0)
C PRINTING PLOT TITLES
      IF(METHOD.EQ.2) WRITE(6,113)
      IF(METHOD.EQ.4) WRITE(6,115)
C DETERMINING MAXIMUM AND MINIMUM X AND Y VALUES FOR COUPLER CURVE PLOT
      YMAX=-1.0E+50
      XMAX=-1.0E+50
      YMIN=1.0E+50
      XMIN=1.0E+50
      DO 6 I=1,NPP
      YMAX=AMAX1(YMAX,CYA(I),CYD(I))
      YMIN=AMIN1(YMIN,CYA(I),CYD(I))
      XMAX=AMAX1(XMAX,CXA(I),CXD(I))
    6 XMIN=AMIN1(XMIN,CXA(I),CXD(I))
C PRINTING TITLES FOR COUPLER-POINT PLOT
      WRITE(6,104) XMIN,XMAX,YMIN,YMAX
      CALL COMPARE(NPP,CYA,CYD,CXA,CXD,YMAX,YMIN,XMAX,XMIN,1)
C DETERMINING MAXIMUM AND MINIMUM Y VALUES FOR STRUCTURAL ERROR PLOT
      YMIN=1.0E+50
      YMAX=-1.0E+50
      PMAX=RNGI*57.2957795
      STARA=SV(4)*57.2957795
      DO 1 I=1,NPP
      CYA(I)=SQRT(ERRMAX(I))
      YMIN=AMIN1(YMIN,CYA(I))
    1 YMAX=AMAX1(YMAX,CYA(I))
C PRINTING TITLES FOR STRUCTURAL ERROR PLOT
      WRITE(6,103) PMAX,STARA,YMIN,YMAX
      XMAX=AMAX1(0.,RNGI)
      XMIN=AMIN1(0.,RNGI)
      CALL COMPARE(NPP,CYA,CYA,PH,PH,YMAX,YMIN,XMAX,XMIN,0)
      WRITE(6,101) TRI
```

```
      RETURN
  101 FORMAT(1H0,10X,*THE MINIMUM TRANSMISSIBILITY INDEX IS*,G14.5)
  103 FORMAT(1H1,5X,*PLOT OF COUPLER CURVE ERROR VS. CRANK ANGLE CHANGE
     1FROM STARTING ANGLE*/6X,*CRANK ANGLE CHANGE IS*,G14.5,* DEGREES*/
     26X,*CRANK STARTING ANGLE IS*,G14.5,* DEGREES*/
     36X,*MINIMUM VALUE OF ERROR IS*,G14.5,*, MAXIMUM VALUE OF ERROR IS*
     4,G14.5/)
  104 FORMAT(1H0,5X,*MINIMUM X VALUE IS*,G14.5,*MAXIMUM X VALUE IS*,
     1G14.5/6X,*MINIMUM Y VALUE IS*,G14.5,*MAXIMUM Y VALUE IS*,G14.5/)
  113 FORMAT(1H1,5X,*PLOT OF DESIRED (O) AND ACTUAL (*,1H*,*) COUPLER PO
     1INTS FOR THE PLANAR FOUR-BAR*)
  115 FORMAT(1H1,5X,*PLOT OF DESIRED (O) AND ACTUAL (*,1H*,*) COUPLER PO
     1INTS FOR THE PLANAR SLIDER-CRANK*)
      END
```

```
      SUBROUTINE COMPARE(N,YA,YD,XA,XD,YMAX,YMIN,XMAX,XMIN,IFLG3)
C SUBROUTINE SUBPROGRAM TO PLOT ONE (IFLG3=0) OR TWO (IFLG3=1) FUNCTIONS
C GIVEN BY PAIRS OF POINTS TO THE SAME SCALE
C X AND Y VALUES ARE CONVERTED TO INTEGERS CORRESPONDING TO THEIR
C POSITION IN THE PLOT (THUS DESTROYING THEIR ORIGINAL VALUES)
C THE PAIRS OF POINTS ARE SORTED ACCORDING TO THEIR Y VALUES BEFORE
C PLOTTING (EVEN FURTHER DESTROYING THEIR ORIGINAL VALUES)
C THE FIRST FUNCTION IS PLOTTED WITH * SYMBOLS
C THE SECOND FUNCTION IS PLOTTED WITH O SYMBOLS
      DIMENSION YD(N),YA(N),XD(N),XA(N),K(81),KB(6)
      DATA KB/1H1,1H-,1H+,1H ,1H*,1HO/
C DETERMINING SCALING FACTORS
      XFAC=80./(XMAX-XMIN)
      YFAC=40./(YMAX-YMIN)
C DETERMINING FACTORS TO DETERMINE COORDINATE AXES AND GRID POINTS
      KX=1.5+YMAX*YFAC
      KY=1.5-XMIN*XFAC
      L1=MOD(KY,8)
      IF(L1.EQ.0) L1=8
      LL=(81-L1)/8
C INCREASING PRECISION OF Y RANGE
      YFAC=3.*YFAC
C GETTING YA VALUES IN DECREASING ORDER
      DO 10 I=1,N
C NOTE YA(I) ARE PUT IN REVERSE ORDER
C THIS IS SO INDEXING CAN BE DONE FROM THE PLOT
C TOP LEFT-HAND CORNER
      YA(I)=1.5+YFAC*(YMAX-YA(I))
   10 XA(I)=1.5+XFAC*(XA(I)-XMIN)
      ILO=2
      IHI=N
   19 IND=0
C SORTING FORWARDS
      DO 20 I=ILO,IHI
      IF(YA(I).GE.YA(I-1)) GOTO 20
      IND=I
C SWITCHING ELEMENTS
      T=YA(I)
      YA(I)=YA(I-1)
      YA(I-1)=T
      T=XA(I)
      XA(I)=XA(I-1)
      XA(I-1)=T
   20 CONTINUE
      IF(IND.EQ.0) GOTO 40
      IHI=IND
      II=IHI+ILO
      IND=0
C SORTING BACKWARDS
```

```
      DO 30 J=ILO,IHI
      I=II-J
      IF(YA(I).GE.YA(I-1)) GOTO 30
C SWITCHING ELEMENTS
      IND=I
      T=YA(I)
      YA(I)=YA(I-1)
      YA(I-1)=T
      T=XA(I)
      XA(I)=XA(I-1)
      XA(I-1)=T
   30 CONTINUE
      ILO=IND
C TESTING TO SEE IF SORTING OF YA IS COMPLETED
      IF(IND.NE.0) GOTO 19
C TESTING TO SEE IF YD TO BE USED ALSO
   40 IF(IFLG3.EQ.0) GOTO 41
C LOOP TO SORT YD VALUES IN ORDER OF DECREASING VALUES
C SORTING FORWARDS
      DO 11 I=1,N
C NOTE YD(I) ARE PUT IN REVERSE ORDER
      YD(I)=1.5+YFAC*(YMAX-YD(I))
   11 XD(I)=1.5+XFAC*(XD(I)-XMIN)
      ILO=2
      IHI=N
   18 IND=0
C SORTING BACKWARDS
      DO 21 I=ILO,IHI
      IF(YD(I).GE.YD(I-1)) GOTO 21
C SWITCHING ELEMENTS
      IND=I
      T=YD(I)
      YD(I)=YD(I-1)
      YD(I-1)=T
      T=XD(I)
      XD(I)=XD(I-1)
      XD(I-1)=T
   21 CONTINUE
      IF(IND.EQ.0) GOTO 41
      IHI=IND
      II=IHI+ILO
      IND=0
      DO 31 J=ILO,IHI
      I=II-J
      IF(YD(I).GE.YD(I-1)) GOTO 31
C SWITCHING ELEMENTS
      IND=I
      T=YD(I)
      YD(I)=YD(I-1)
```

```
      YD(I-1)=T
      T=XD(I)
      XD(I)=XD(I-1)
      XD(I-1)=T
   31 CONTINUE
      ILO=IND
C TESTING TO SEE IF SORTING OF YD IS COMPLETE
      IF(IND.NE.0) GOTO 18
   41 IND=2
C GETTING THE FIRST VALUE OF YA WITHIN THE PRINTING RANGE
      KQA=0
   45 IF(YA(KQA+1).GT.0) GOTO 47
      KQA=KQA+1
      GOTO 45
   47 IF(IFLG3.EQ.0) GOTO 48
C GETTING THE FIRST VALUE OF YD WITHIN THE PRINTING RANGE
      KQD=0
   46 IF(YD(KQD+1).GT.0) GOTO 48
      KQD=KQD+1
      GOTO 46
   48 CONTINUE
C LOOP TO PRINT ONE LINE OF THE PLOT AT A TIME
      DO 50 I=1,41
C SETTING ALL ELEMENTS TO BLANK OR MINUS FOR THE X-AXIS
      II=I-KX
      K(1)=KB(4)
      IF(II.EQ.0) K(1)=KB(2)
      DO 55 J=2,81
   55 K(J)=K(J-1)
      IF(MOD(II,8).NE.0) GOTO 58
C SETTING UP GRID-POINT ELEMENTS
      K(L1)=KB(3)
      L2=L1
      DO 57 J=1,LL
      L2=L2+8
   57 K(L2)=KB(3)
      GOTO 59
C CHECKING TO SEE IF Y-AXIS (ONES) CAN BE PRINTED
   58 IF(KY.GE.1.AND.KY.LE.81) K(KY)=KB(1)
C DETERMINING YA POSITIONS
   59 KR=KQA+1
   65 IF(KR.GT.N) GOTO 62
      IY=IFIX(YA(KR))
      IF(IY-IND) 60,61,62
   60 KQA=KR
   61 IX=IFIX(XA(KR))
      IF(IX.GE.1.AND.IX.LE.81) K(IX)=KB(5)
      KR=KR+1
      GOTO 65
```

```
   62 WRITE(6,100) K
      IF(IFLG3.EQ.0) GOTO 51
C DETERMINING YD POSITIONS
C RESETTING ALL ELEMENTS TO BLANKS
      DO 80 LOOP=1,81
   80 K(LOOP)=KB(4)
      KR=KQD+1
   75 IF(KR.GT.N) GOTO 72
      IY=IFIX(YD(KR))
      IF(IY-IND) 70,71,72
   70 KQD=KR
   71 IX=IFIX(XD(KR))
      IF(IX.GE.1.AND.IX.LE.81) K(IX)=KB(6)
      KR=KR+1
      GOTO 75
   72 WRITE(6,101) K
   51 IND=IND+3
   50 CONTINUE
      RETURN
  100 FORMAT(1X,81A1)
  101 FORMAT(1H+,81A1)
      END
```

```
      SUBROUTINE UNIMIN(NS,NF,RMAX,JS,N,SC,RF,RAT)
      DIMENSION F(48)
      COMMON P(10,12),XI(10,11),X(10)
      COMMON /MODULO/IMD
C SUBROUTINE TO MINIMIZE A FUNCTION ALONG ONE CONJUGATE DIRECTION AT A
C TIME BY VARYING A SCALAR MULTIPLIER, RMIN
C THE FUNCTION IS MINIMIZED ALONG EACH VECTOR IN SUCCESSION USING THE
C LAST MINIMUM POINT AS A STARTING POINT FOR THE EACH MINIMIZATION
C FIRST A REGION IN WHICH THE FUNCTION HAS A MINIMUM IS BRACKETED BY
C USING AN ACCELERATED UNIVARIATE SEARCH TECHNIQUE
C THEN THREE EQUALLY SPACED POINTS WITHIN THE BRACKET ARE USED TO DEFINE
C QUADRATIC INTERPOLATING FORMULA FOR WHICH THE MINIMUM IS OBTAINED BY
C SETTING ITS FIRST DETIVATIVE EQUAL TO ZERO
C INSTRUCTIONS USE 517 (OCTAL) STORAGE LOCATIONS IN C.M.
C LIST OF INPUT PARAMETERS
C             NS      INDEX OF FIRST MINIMUM POINT TO BE DETERMINED
C             NF      INDEX OF LAST MINIMUM POINT TO BE DETERMINED
C                     (NF-NS= NO. OF LOOPS THROUGH SUBROUTINE REQUIRED)
C             G       EXTERNAL FUNCTION SUBPROGRAM WHICH EVALUATES THE
C                     UNCONSTRAINED OBJECTIVE FUNCTION AT A PARTICULAR
C                     POINT, P.  ANY CONSTRAINT CONSIDERATIONS MUST BE
C                     INCLUDED IN G
C             RMAX    LARGEST SCALAR MULTIPLIER USED IN THE MINIMIZATION
C                     ALONG ALL DIRECTIONS
C             JS      INDEX, J, OF THE CONJUGATE DIRECTION ASSOCIATED
C                     WITH RMAX
C             N       NUMBER OF DESIGN VARIABLES TO BE OPTIMIZED
C             SC      INITIAL MAGNITUDE PARAMETER FOR SCALAR MULTIPLIER
C             RF      INITIAL VALUE OF RELAXATION FACTOR
C             RAT     FINAL REDUCTION RATIO (+.5) FOR SYMMETRICAL
C                     BRACKETING OF THE MINIMUM
C APPROPRIATE VALUES FOR SC, RF, AND RAT ARE .0001, 1., AND 2.5 RESPECT.
      DO 20 J=NS,NF
C RESETTING RELAXATION FACTOR
      RFF=RF
C DEFINING MULTIPLIERS
      SM=SC
      S2=SM*2.
      S3=SM*3.
      J1=J+1
      DO 26 I=1,N
   26 X(I)=P(I,J)
C OBTAINING FUNCTION VALUE AT INITIAL POINT
      F(1)=G(X)
      PNORM=0.
      DO 1 I=1,N
      P(I,J)=X(I)
    1 PNORM=PNORM+P(I,J)*P(I,J)
      PNORM=SQRT(PNORM)
```

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      PNORM=AMAX1(PNORM,1.)
C ATTEMPT TO AVOID LOCAL MINIMUM
C INITIAL SCALAR MULTIPLIER, RINT, USED FOR MINIMIZATION ALONG A LINE IS
C SET EQUAL TO NORM OF POSITION VECTOR, PNORM, TIMES A RANDOM NUMBER
C BETWEEN -3. AND +3.
C IF NEW PONT ALONG THE LINE , A DISTANCE RINT*XI(I,J) FROM THE INITIAL
C STARTING POINT, YIELDS A LOWER VALUE THEN THIS NEW POINT BECOMES THE
C INITIAL STARTING POINT FOR UNIMIN
C NOTE THAT CONJ STILL THINKS P(I,J) IS THE STARTING POINT
      DO 4 LO=1,3
      RINT=PNORM*(3.-6.*RANF(0.))
      DO 3 I=1,N
      X(I)=P(I,J)+XI(I,J)*RINT
    3 CONTINUE
      B=G(X)
      IF(B-F(1)) 19,19,4
    4 CONTINUE
C ROUTE IF ORIGINAL STARTING POINT ACCEPTED
      RINT=0.
      GOTO 2
C ROUTE IF A NEW STARTING POINT GENERATED
   19 F(1)=B
      DO 22 I=1,N
   22 P(I,J)=X(I)
C INITIAL TEST SECTION
    2 K=0
    6 K=K+1
C LOOP TO DEFINE TEST POINTS IN POSITIVE DIRECTION
C NOTE, X IS A DUMMY POSITION VECTOR USED THROUGHOUT SUBROUTINE
C TEST FOR UNBOUNDED OPTIMUM
      IF(K.GT.47) GOTO 21
      DO 5 I=1,N
      X(I)=P(I,J)+SM*(2**K)*XI(I,J)
    5 CONTINUE
      F(K+1)=G(X)
C TEST TO SEE IF LIMIT IN POSITIVE DIRECTION HAS BEEN REACHED
      IF(F(K+1).LE.F(K)) GOTO 6
C TESTS TO CHECK ON SPECIAL CASES WHEN LIMIT IN POSITIVE DIRECTION HAS
C BEEN REACHED
      IF(K.EQ.1) GOTO 7
      IF(K.EQ.2) GOTO 17
C DEFINING A NEW TEST POINT TO GIVE EQUAL SPACING
      EQ=S3   *(2**(K-2))
      DO 8 I=1,N
      X(I)=P(I,J)+EQ*XI(I,J)
    8 CONTINUE
      F3=G(X)
C FINDING A COMBINATION OF 3 EQUALLY SPACED POINTS TO EVALUATE THE
C FUNCTION TO DEFINE THE QUADRATIC INTERPOLATING FORMULA FOR THE
```

```
C POSITIVE CASE
C DEFINE THE QUADRATIC INTERPOLATING FORMULA FOR THE POSITIVE CASE
C TWO COMBINATIONS ARE POSSIBLE
C RL IS THE MIDDLE POINT
      IF(F3.LE.F(K)) GOTO 9
C FIRST POSSIBLE COMBINATION
      F1=F(K-1)
      F2=F(K)
      RL=SM   *(2**(K-1))
      GOTO 10
C SECOND POSSIBLE COMBINATION
    9 F1=F(K)
      F2=F3
      F3=F(K+1)
      RL=EQ
      GOTO 10
C CASE WHERE ONLY FIRST THREE POSITIVE POINTS ARE EVALUATED
   17 F1=F(1)
      F2=F(2)
      F3=F(3)
      RL=S2
      RINC=S2
      RFF=1.
      GOTO 16
C NEGATIVE DIRECTION BRACKETING SECTION
    7 F3=F(2)
      K=0
   11 K=K+1
C LOOP TO DEFINE TEST POINTS IN NEGATIVE DIRECTION
C TEST FOR UNBOUNDED OPTIMUM
      IF(K.GT.47) GOTO 21
      DO 12 I=1,N
      X(I)=P(I,J)-SM*(2**K)*XI(I,J)
   12 CONTINUE
      F(K+1)=G(X)
C TEST TO CHECK IF BRACKET HAS BEEN REACHED IN NEGATIVE DIRECTION
      IF(F(K+1).LE.F(K)) GOTO 11
C TESTS FOR SPECIAL CASES WHEN LIMIT IN NEGATIVE DIRECTION HAS BEEN
C REACHED
      IF(K.EQ.1) GOTO 13
      IF(K.EQ.2) GOTO 18
C ROUTE FOR NORMAL TERMINATION IN NEGATIVE DIRECTION
C A FOURTH EQUIDISTANT POINT WITHIN THE BRACKET IS DETERMINED AND THE
C BEST OF TWO POSSIBLE COMBINATIONS OF THREE POINTS IS PICKED
      EQ=S3   *(2**(K-2))
      DO 14 I=1,N
      X(I)=P(I,J)-EQ*XI(I,J)
   14 CONTINUE
      F1=G(X)
```

```
      IF (F1.LE.F(K)) GOTO 15
C FIRST POSSIBLE COMBINATION
      F2=F(K)
      F3=F(K-1)
      RL=-SM   *(2**(K-1))
      GOTO 10
C SECOND POSSIBLE COMBINATION
   15 F2=F1
      F1=F(K+1)
      F3=F(K)
      RL=-EQ
      GOTO 10
C SPECIAL CASE WHEN LIMITS SYMMETRICAL ABOUT STARTING POINT
C REDUCE MAGNITUDE PARAMETER FOR SCALAR MULTIPLIER BY 1/2
   13 S2=SM
      SM=S2/2.
      S3=SM*3.
C SETTING RELAXATION FACTOR TO 1. FOR ACCURACY
      RFF=1.
C TEST TO SEE IF RESTARTING IS NECESSARY
      IF((SC/SM).LT.RAT ) GOTO 2
C ROUTE IF SYMMETRICAL LIMITS MUST BE USED
      F2=F(1)
      F1=F(2)
      RL=0.
      RINC=S2*2.
      GOTO 16
C SPECIAL CASE WHEN ONLY FIRST 3 NEGATIVE  POINTS DEFINE THE BRACKET
   18 F1=F(3)
      F2=F(2)
      F3=F(1)
      RL=-S2
      RINC=S2
      RFF=1.
      GOTO 16
C NORMAL ROUTE INTO QUADRATIC MINIMIZING STATEMENT
   10 RINC=EQ/3.
C ROUTE FOR SPECIAL CASES WHERE RINC HAS BEEN DEFINED PREVIOUSLY
   16 DEN=F3+F1-2.*F2
      RMIN=RL-(RINC/2.)*(F3-F1)/DEN
C APPLYING RELAXATION FACTOR TO SCALAR MULTIPLIER
      RMIN=RFF*RMIN
C CHECKING FOR THE LARGEST SCALAR MULTIPLIER AND ITS INDEX
      CKMAX=ABS(RMIN)
      IF(CKMAX.LT.RMAX) GOTO 23
      JS=J
      RMAX=CKMAX
C DEFINING A NEW POINT FOR THE START OF THE NEXT LOCAL MINIMIZATION
C SEQUENCE
```

```
   23 DO 24 I=1,N
   24 P(I,J1)=P(I,J)+RMIN*XI(I,J)
      IF(METHOD.GT.9) GOTO 20
      P(4,J1)=AMOD(P(4,J1),6.2831853072)
      IF(IMD.EQ.0) P(5,J1)=AMOD(P(5,J1),6.2831853072)
   20 CONTINUE
      RETURN
C ROUTE IF AN UNBOUNDED OPTIMUM HAS BEEN DETECTED
   21 WRITE(6,100)
      STOP
  100 FORMAT(1H0,10X,*UNIMIN HAS DETECTED AN UNBOUNDED OPTIMUM*/11X,
     1*THEREFORE THE PROGRAM HAS BEEN ABORTED, RECHECK ALL INPUT PARAMET
     2ERS AND RELATIONSHIPS*)
      END
```

```
      FUNCTION G(X)
C FUNCTION SUBPROGRAM TO EVALUATE UNCONSTRAINED OBJECTIVE FUNCTION FOR
C ALL METHODS (1,2,3,4,5,10,11) WHICH REQUIRE OPTIMIZATION OF DESIGN
C VARIABLES (UP TO 10)
C UP TO 20 CONSTRAINTS CAN BE HANDLED
C FOR ALL OPTIMIZING METHODS UP TO 16 CONSTRAINTS CAN BE ADDED THROUGH
C SUBROUTINE EXCON(C,NC) WHERE C IS THE CONSTRAINT VECTOR OF SIZE NC
C WHERE NC IS THE TOTAL NUMBER OF CONSTRAINTS INCLUDING THOSE BASIC
C CONSTRAINTS DEFINED IN SUBROUTINE CONFUNC (SEE FOLLOWING TABLE)
C     METHOD      NO. OF BASIC CONSTRAINTS
C        1                 3
C        2                 3
C        3                 3
C        4                 3
C        5                 4
C       10                 0
C       11                 0
      DIMENSION X(10)
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /MODULO/IMD,IHELP
C INCREMENT FUNCTION COUNTER
      ICOUNT=ICOUNT+1
C CALLING FOR ADDITIONAL USER ADDED CONSTRAINTS IF NECESSARY
C (IEXCO=0)
      IF(IEXCO.EQ.0) CALL EXCON(X,C,NC)
C CALL SUBROUTINE TO EVALUATE CONSTRAINED OBJECTIVE FUNCTION AND
C APPROPRIATE CONSTRAINTS FOR METHODS 1 TO 5
      IF(METHOD.LE.5) CALL CONFUNC(X,NPP,1)
C CALL SUBROUTINE TO EVALUATE CONSTRAINED OBJECTIVE FUNCTION AND
C APPROPRAIATE CONSTRAINTS FOR PLANAR FOUR-BAR BALANCING (METHOD=10)
      IF(METHOD.EQ.10) CALL FBBAL(X,1)
C CALL SUBROUTINE TO EVALUATE CONSTRAINED OBJECTIVE FUNCTION AND
C APPROPRAIATE CONSTRAINTS FOR PLANAR SLIDER-CRANK BALANCING (METHOD=11)
      IF(METHOD.EQ.11) CALL SCBAL(X,1)
C CHECK FOR NO CONSTRAINTS
      IF(NC.EQ.0) GOTO 3
C TRANSFORMING A CONSTRAINED OBJECTIVE FUNCTION INTO AN UNCONSTRAINED
C RELATIONSHIP WHICH WILL CONVERGE TO A LOCAL MINIMUM OF THE CONSTRAINED
C FUNCTION GIVEN A SUFFICIENTLY LARGE PM
C THE TRANSFORMATION IS OF A SCALED EXTERIOR-POINT FORM USING THE
C PENALTY FUNCTION CONCEPT
C SF IS THE SCALING FACTOR
      PEN=0.
C NORMALIZING OUTPUT MOTION RANGE CONSTRAINT
      IF(METHOD.LT.6.AND.IHELP.EQ.0) C(3)=C(3)*1000.
      DO 1 I=1,NC
```

```
      PEN=PEN-AMIN1(C(I),0.)
    1 CONTINUE
      IF(IHELP.NE.0) GOTO 2
      IF(PEN.LE.1.0E-13) GOTO 2
C LEVELING CONSTRAINED OBJECTIVE FUNCTION AT 1.0E+20 IN THE
C INFEASIBLE REGION IF TROUBLES ENCOUNTERED (IHELP=0)
      G=1.0E+20*(1.+PEN)
      RETURN
C NORMAL ROUTE FOR CONSTRAINED PROBLEMS
    2 SF=ABS(CG)
      IF(SF.LT.1.) SF=1.
      G=CG+PM*SF*PEN
      RETURN
C NORMAL ROUTE FOR UNCONSTRAINED PROBLEMS
    3 G=CG
      RETURN
      END
```

```
      SUBROUTINE CONFUNC(X,NPP,IFLG1)
      DIMENSION X(10)
      COMMON /NUMBERS/NOO,METHOD,ICASE
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /SCLFAC/SCAL1,SCAL2,SCAL3
      COMMON /DESIRE/XD(21),YD(81),PH(81),XTOL(21),YTOL(21)
      COMMON /SYNIN/XMIN,XMAX,RNGI,RNGO,TITE,CFRIC,ISYM
C SUBROUTINE SUBPROGRAM TO EVALUATE CONSTRAINED OBJECTIVE FUNCTION
C FOR PLANAR 4-BAR FUNCTION SYNTHESIS (METHOD=1) AND COUPLER-CURVE
C SYNTHESIS (METHOD=2), PLANAR SLIDER-CRANK FUNCTION SYNTHESIS
C (METHOD=3) AND COUPLER CURVE SYNTHESIS (METHOD=4), AND SPATIAL 4-BAR
C (R-G-G-R) FUNCTION SYNTHESIS (METHOD=5)
C THE ESSENTIAL MECHANISM CONSTRAINTS ARE ALSO EVALUATED
C THE OBJECTIVE FUNCTION IS THE SUM OF THE INVERSE UTILITY FUNCTIONS
C ASSOCIATED WITH THE MAXIMUM TRANSMISSION ANGLE, MINIMUM STRUCTURAL
C ERROR, AND MINIMUM MASS AND MASS MOMENTS OF INERTIA REQUIREMENTS
C (I.E. LINK LENGTH CONTROL)
      EM=0.
      C(1)=1.0E+10
C CONSTRAINT TO PREVENT INFINITE ANGLE CHANGES
      C(2)=(12.5664-ABS(X(4)))*100.
      TRI=1.
C INSURING MINIMUM REASONABLE VALUES FOR VARIABLES
      IF(ABS(X(1)).LT.1.0E-20) X(1)=SIGN(1.0E-20,X(1))
      IF(ABS(X(2)).LT.1.0E-20) X(2)=SIGN(1.0E-20,X(2))
      IF(ABS(X(3)).LT.1.0E-20) X(3)=SIGN(1.0E-20,X(3))
C MAKING X(2) POSITIVE FOR METHODS 2 TO  4 TO GIVE A REFLECTED
C OPTIMIZATION SURFACE IN THE INFEASIBLE REGION
      IF(METHOD.NE.1.AND.METHOD.NE.5) X(2)=ABS(X(2))
C DETERMINING MAXIMUM LENGTH OF LINKS COMMON TO ALL METHODS
      RAL=AMAX1(ABS(X(1)),ABS(X(2)))
      IF(METHOD.LT.5) RAL=AMAX1(RAL,ABS(X(3)))
      X2SQ=X(2)*X(2)
      LOOP=NPP+2
      IF(IFLG1.EQ.0.OR.METHOD.EQ.2.OR.METHOD.EQ.4) LOOP=NPP
      IF(METHOD.GT.2.AND.METHOD.NE.5) GOTO 3
      X1SQ=X(1)*X(1)
      X3SQ=X(3)*X(3)
      IF(METHOD.EQ.1) X(6)=1.
      X6SQ=X(6)*X(6)
      IF(METHOD.EQ.5) GOTO 8
C LOOP TO DETERMINE ACTUAL MECHANISM FOLLOWER ANGLE, PS, AT EACH PSEUDO
C ACCURACY POINT
      DO 5 I=1,LOOP
      ANG=PH(I)+X(4)
      SAV=X(1)*COS(ANG)
      D2=X1SQ+X6SQ-2.*X(6)*SAV
C D IS LINKAGE DIAGONAL
```

```
      D=SQRT(D2)
      T=X3SQ+D2-X2SQ
      DEN=2.*D*X(3)
      IF(ABS(DEN).LT.1.0E-50) DEN=SIGN(1.0E-50,DEN)
      ARG=T/DEN
      CK=1.-ARG*ARG
C CHECK TO PREVENT AN IMPOSSIBLE MECHANISM POSITION
      C(1)=AMIN1(C(1),CK)
C SET ARG=1. AS A DEFAULT VALUE TO PREVENT BEING OUT OF RANGE
      IF(CK.LT.0.) ARG=SIGN(1.,ARG)
      B=ACOS(ARG)
      A=ATAN2(X(1)*SIN(ANG),X(6)-SAV)
C CHECK TO INSURE CONTINUOUS VALUES OF VARIABLE A
C I.E. NO DISCONTINUITY AT A=180 DEGREES
      IF(I.EQ.1) GOTO 7
      IF((ISIGN(1,INT(ALAST)).NE.ISIGN(1,INT(A))).AND.
     1(ABS(ALAST).GT.1.5708)) A=SIGN(6.2831853072,ALAST)+A
    7 ALAST=A
C DETERMINING MINIMUM TRANSMISSIBILITY INDEX,TRI
      TRT=D*SIN(B)/X(2)
      TRI=AMIN1(ABS(TRT),TRI)
C VALUE OF ICASE DETERMINES WHETHER B IS ADDED OR SUBTRACTED
    5 PS(I)=3.1415926535898-A-FLOAT(ICASE)*B
      GOTO(20,21),METHOD
C ROUTE FOR DETERMINING STRUCTURAL ERROR FOR METHOD=1,3,5
C RETURN IF ONLY DETERMINING POINTS FOR STRUCTURAL ERROR PLOT
   20 IF(IFLG1.EQ.0) RETURN
C SUBSCRIPT FOR MIDPOINT OF INPUT RANGE
      MIDPP=NPP/2+2
      IF(ISYM.NE.0) GOTO 10
C ROUTE FOR SYMMETRICAL CASE
      RNGOA=PS(MIDPP)-PS(1)
      GOTO 11
C DETERMINING ACTUAL RANGE OF OUTPUT MOTION
   10 RNGOA=PS(LOOP)-PS(1)
C CONSTRAINT ON OUTPUT RANGE OF MOTION-RANGE + OR - TITE DEGREES
   11 RGABS=ABS(RNGOA)
      C(3)=TITE-ABS(RGABS-RNGO)
C STRUCTURAL ERROR IS DETERMINED USING THE ACTUAL OUTPUT RANGE OF MOTION
      IF(RGABS.LT.1.0E-10) RNGOA=SIGN(1.0E-10,RNGOA)
      DMSC=RPSI/RNGOA
      MIDPL=MIDPP-1
C LOOP TO DETERMINE STRUCTURAL ERRORS AT PSEUDO PRECISION POINTS
      DO 12 I=1,NPP
      YA=YD(MIDPL)+DMSC*(PS(I+1)-PS(MIDPP))
      ERROR=YA-YD(I)
C THIS TERM OF THE OBJECTIVE FUNCTION IS THE SUM OF THE SQUARES OF THE
C STRUCTURAL ERRORS AT EACH PRECISION POINT
   12 EM=AMAX1(EM,ERROR*ERROR)
```

```
      GOTO 2
C ROUTE IF SYNTHESIZING PLANAR FOUR-BAR COUPLER CURVE (METHOD=2)
C DETERMINING THE COORDINATES OF THE COUPLER POINT AT EACH PRECISION
C POINT
C THE COORDINATES ARE OBTAINED USING PLANE GEOMETRIC TECHNIQUES
   21 RA=X(7)/X(2)
C CONSTRAINT TO PREVENT INFINITE ANGLE CHANGES
      C(3)=(12.5664-ABS(X(5)))*100.
      RSM=X(8)/X(2)
      SVTM=X(6)*SIN(X(5))
      CVTM=X(6)*COS(X(5))
      DO 1 I=1,NPP
      ANG2=X(4)+PH(I)+X(5)
      ANG3=PS(I)+X(5)
      CANG=X(1)*COS(ANG2)
      SANG=X(1)*SIN(ANG2)
      DEL=SVTM+X(3)*SIN(ANG3)-SANG
      SI=CVTM-CANG+X(3)*COS(ANG3)
C EPS AND RHO ARE THE COORDINATES OF THE PERPENDICULAR PROJECTION OF
C THE COUPLER POINT ON THE COUPLER LINK
      EPS=X(9)+CANG+RA*SI
      RHO=X(10)+SANG+RA*DEL
      CXA(I)=EPS-RSM*DEL
      CYA(I)=RHO+RSM*SI
      ERRX=CXA(I)-XD(I)
      ERRY=CYA(I)-YD(I)
      ERRX=ERRX*ERRX
      ERRY=ERRY*ERRY
C OBTAINING ACTUAL STRUCTURAL ERROR SQUARED AT EACH PRECISION POINT FOR
C PLOTTING
      ERRMAX(I)=ERRX+ERRY
C OBTAINING THE SCALED STRUCTURAL ERROR AT EACH PRECISION POINT FOR
C MINIMIZATION
      ERROR=ERRX/XTOL(I)+ERRY/YTOL(I)
C EVALUATING THE SUM OF THE SQUARES OF THE STRUCTURAL ERRORS AT EACH
C PRECISION POINT
    1 EM=AMAX1(EM,ERROR)
C DETERMINING MAXIMUM LINK LENGTH
      RAL=AMAX1(RAL,ABS(X(7)),ABS(X(8)))
      GOTO 2
C ROUTE FOR SLIDER CRANK SYNTHESIS (METHODS 3 AND 4)
    3 DO 4 I=1,LOOP
      ANG=X(4)+PH(I)
      SANG=X(1)*SIN(ANG)-X(3)
      A=X(1)*COS(ANG)
      ARG=X2SQ-(SANG**2)
C TEST TO PREVENT ATTEMPTING TO TAKE THE SQRT OF A NEGATIVE NUMBER
C WHICH WOULD INDICATE MECHANISM NONCLOSURE FOR A PARTICULAR CRANK
C POSITION
```

```
      C(1)=AMIN1(C(1),ARG)
C SET ARG=0 AS A DEFAULT VALUE FOR MECHANSIM NONCLOSURE
      IF(ARG.LT.0.) ARG=0.
C DETERMINING ACTUAL SLIDER POSITION FOR A GIVEN INPUT CRANK ANGLE
C ICASE DETERMINES WHICH WAY THE MECHANISM IS CLOSED
      PS(I)=A+ICASE*SQRT(ARG)
C DETERMINING MINIMUM TRANSMISSIBILITY INDEX,TRI
      TRT=ABS((PS(I)-A)/X(2))
    4 TRI=AMIN1(TRT,TRI)
      IF(METHOD.EQ.3) GOTO 20
C ROUTE IF SLIDER CRANK COUPLER POINT CURVE SYNTHESIS IS DESIRED
C (METHOD=4)
C DEFINING TIME SAVING CONSTANTS
      RA=X(6)/X(2)
      RSM=X(7)/X(2)
      CANG=COS(X(5))
      SANG=SIN(X(5))
C CONSTRAINT TO PREVENT INFINITE ANGLE CHANGES
      C(3)=(12.5664-ABS(X(5)))*100.
C LOOP TO DETERMINE ACTUAL COUPLER POINT COORDINATES AT PSEUDO
C PRECISION POINTS
      DO 6 I=1,NPP
      ANG=PH(I)+X(4)+X(5)
      FILL1=X(1)*COS(ANG)
      FILL2=X(1)*SIN(ANG)
      SI=PS(I)*CANG-FILL1-X(3)*SANG
      DEL=PS(I)*SANG+X(3)*CANG-FILL2
      EPS=X(8)+FILL1+RA*SI
      RHO=X(9)+FILL2+RA*DEL
      CXA(I)=EPS-RSM*DEL
      CYA(I)=RHO+RSM*SI
      ERRX=CXA(I)-XD(I)
      ERRY=CYA(I)-YD(I)
      ERRX=ERRX*ERRX
      ERRY=ERRY*ERRY
C OBTAINING ACTUAL STRUCTURAL ERROR SQUARED AT EACH PRECISION POINT FOR
C PLOTTING
      ERRMAX(I)=ERRX+ERRY
C OBTAINING THE SCALED STRUCTURAL ERROR AT EACH PRECISION POINT FOR
C MINIMIZATION
      ERROR=ERRX/XTOL(I)+ERRY/YTOL(I)
C EVALUATING THE SUM OF THE SQUARES OF THE STRUCTURAL ERRORS AT EACH
C PRECISION POINT
    6 EM=AMAX1(EM,ERROR)
C DETERMINING MAXIMUM LINK LENGTH
      RAL=AMAX1(RAL,ABS(X(6)),ABS(X(7)))
      GOTO 2
C ROUTE FOR SPATIAL FOUR BAR FUNCTION GENERATION (METHOD=5)
C DEFINING TIME SAVING PARAMETERS
```

```
    8 X7SQ=X(7)*X(7)
C PENALIZING X(5) FOR EXCEEDING + OR - 720 DEGREES
      C(4)=(12.5664-ABS(X(5)))*100.
      SS=X7SQ+X1SQ+X6SQ+X3SQ+X2SQ-1.
      SNX5=SIN(X(5))
      CSX5=COS(X(5))
      P1=X(7)*X(3)*CSX5-.5*SS
      P2=X(1)*X(6)
      P3=X(1)*X(3)*SNX5
      P4=-X(2)*X(6)
      P5=X(2)*X(1)
      P9=X(7)*SNX5
      P6=X(2)*P9
      P7=-P5*CSX5
      P8=X(1)*CSX5
C LOOP TO DEFINE OUTPUT ANGLE AND THE TRANSMISSIBILITY INDEX FOR
C EACH VALUE OF PH
      DO 9 I=1,LOOP
      ANG=X(4)+PH(I)
      CANG=COS(ANG)
      SANG=SIN(ANG)
      P2C=P2*CANG
      P3S=P3*SANG
      P7S=P7*SANG
      P5C=P5*CANG
      DEN=P6+P7S
      F1=(P1+P2C+P3S)/DEN
      F2=(P4+P5C)/DEN
C CHECKING TO INSURE ARGUMENT OF SQRT FUNCTION IS POSITIVE
C A NEGATIVE ARGUMENT MEANS THAT THE MECHANISM WILL NOT CLOSE
C IN ITS CURRENT POSITION
C CONSTRAINT C(1) PREVENTS CONTINUALLY PRODUCING SUCH IMAGINARY
C MECHANISMS
      ARG=1.+F2*F2-F1*F1
      C(1)=AMIN1(C(1),ARG)
C SET ARG=0 AS A DEFAULT VALUE FOR AN IMAGINARY MECHANISM
      IF(ARG.LT.0.) ARG=0.
C DEFINING A PARTICULAR OUTPUT ANGLE
C ITS POSITION DEPENDS ON ICASE
      PS(I)=2.*ATAN((1.+ICASE*SQRT(ARG))/(F1-F2))
      IF(I.EQ.1) GOTO 16
C CHECK TO INSURE CONTINUOUS VALUES OF OUTPUT ANGLE AT + OR - 180 DEGREE
      IF((ISIGN(1,INT(PLAST)).NE.ISIGN(1,INT(PS(I)))).AND.
     1(ABS(PLAST).GT.1.5708))PS(I)=SIGN(6.2831853072,PLAST)+PS(I)
   16 PLAST=PS(I)
C DETERMINING MINIMUM TRANSMISSIBILITY INDEX,TRI
      CPS=COS(PS(I))
      SPS=SIN(PS(I))
      TRT=SPS*(X(6)-X(1)*CANG)+CPS*(P8*SANG-P9)
```

```
      TRT=ABS(TRT)
    9 TRI=AMIN1(TRI,TRT)
      GOTO 20
    2 TRI=AMAX1((ZERO+1.0E-10),TRI)
C MINIMUM TRANSMISSION INDEX CONTROL FUNCTION
      U1=(1.-TRI)*SCAL1/(TRI-ZERO)
      U1=U1*U1
C SCALING CONSTRAINT C(1) TO BE COMPARABLE TO OTHER CONSTRAINTS
      C(1)=C(1)*1000.
C MAXIMUM STRUCTURAL ERROR CONTROL FUNCTION
      IF(MOD(METHOD,2)) 13,14,13
   13 U2=SCAL2*EM
      GOTO 15
   14 U2=EM
C MAXIMUM LINK LENGTH CONTROL FUNCTION
   15 U3=0.
      IF(RAL.GT.1.) U3=(RAL-1.)*(RAL-1.)/SCAL3
C TOTAL CONSTRAINED OBJECTIVE FUNCTION
      CG=U1+U2+U3
      RETURN
      END
```

```
      SUBROUTINE BALANCE(ICWAN,PREC)
C SUBROUTINE TO CALCULATE THE VERTICAL AND HORIZONTAL SHAKING FORCES
C AND THE SHAKING MOMENT ABOUT THE CRANKSHAFT AXIS FOR THE PLANAR 4-BAR
C AND SLIDER-CRANK MECHANISMS
C IT THEN CALLS SUBROUTINE CONJ TO CALCULATE THE OPTIMUM POSITION
C (OF THE C.M. OF THE C.W.) AND SIZE (MASS AND M.I. ABOUT THE C.M. OF
C THE C.W.) OF THE BALANCING COUNTERWEIGHTS REQUIRED
C FOR THE PLANAR 4-BAR (METHOD=10) COUNTERWEIGHTS ARE ADDED TO BOTH
C THE CRANK AND FOLLOWER LINKS (LINKS 1 AND 3)
C FOR THE PLANAR SLIDER-CRANK (METHOD=11) A COUNTERWEIGHT IS ADDED
C TO THE CRANK LINK ONLY
C THE COUNTERWEIGHT PARAMETERS (POSITION, MASS, AND M.I.) ARE DETERMINED
C TO GIVE AN OPTIMUM COMPROMISE (ACCORDING TO THE SCALING FACTORS
C SCLSFH, SCLSFV, AND SCLSMO) BETWEEN COMPLETELY BALANCING THE
C VERTICAL SHAKING FORCES, THE HORIZONTAL SHAKING FORCES, AND THE
C SHAKING MOMENTS ABOUT THE CRANKSHAFT AXIS
C THE DESIGNER MUST INPUT THE FOLLOWING PARAMETERS
C     NPP      NUMBER OF CHECK POINTS AT WHICH SHAKING FORCES AND
C              SHAKING MOMENTS ARE TO BE EVALUATED
C     ICASE    EQUALS + OR - ONE DEPENDING ON MECHANISM CONFIGURATION
C     METHOD   =10 FOR PLANAR 4-BAR BALANCING,
C              =11 FOR PLANAR SLIDER-CRANK BALANCING
C     STRTPT   VECTOR OF STARTING VALUES FOR BALANCING PARAMETERS
C              1 AND 2 CRANK C.W. C.M. POSITION (RADIAL AND TANG. RESP.)
C              3 CRANK C.W. MASS FOR S.C. OR FOLLOWER C.W. MASS FOR F.B.
C              4 FOLLOWER C.W. M.I. FOR F.B. BAL. OR CRANK C.W. M.I.
C              FOR S.C. BALANCE
C              5 AND 6 FOLLOWER C.W. C.M. POSITION FOR F.B. BAL
C              7 CRANK C.W. MASS FOR F.B. BAL
C              8 CRANK C.W. M.I. FOR F.B. BAL
C     NOA1     SET EQUAL TO 0 IF ALL A1(I)=0.
C     STRTA    STARTING ANGLE, IN DEGREES, OF CRANK LINK (C.C.W. W.R.T.
C              MECHANISM HORIZONTAL REFERENCE LINE)
C     RNGA     CRANK ANGLE MOTION RANGE, IN DEGREES, OVER WHICH
C              BALANCING IS TO BE DONE
C     CM       VECTOR OF LOCATIONS OF C.M. OF EACH MOVING LINK W.R.T.
C              THAT LINK
C     RM       VECTOR OF MAGNITUDES OF MASSES OF EACH MOVING LINK
C     RJ       VECTOR OF MAGNITUDES OF M.I. OF EACH MOVING LINK ABOUT
C              ITS C.M.
C     W1       VECTOR OF MAGNITUDES OF CRANK ANGULAR VELOCITY (RAD/SEC
C              C.C.W. +VE) AT EACH OF NPP CHECK POINTS
C     A1       VECTOR OF MAGNITUDES OF CRANK ANGULAR ACCELERATION
C              (RAD/SEC/SEC C.C.W. +VE) AT EACH OF NPP CHECK POINTS
C     PAR      LINKAGE PARAMETERS SAME AS FOR SYNTHESIS (FIRST 3 REQD
C              FOR SLIDER-CRANK, FIRST 6 REQD FOR 4-BAR)
C     SCLSFH   SCALING FACTOR FOR HORIZONTAL SHAKING FORCES
C     SCLSFV   SCALING FACTOR FOR VERTICAL SHAKING FORCES
C     SCLSMO   SCALING FACTOR FOR SHAKING MOMENTS ABOUT CRANKSHAFT AXIS
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C      IEXCO     SET=0 IF EXTRA CONSTRAINTS ARE ADDED THROUGH EXCON
C      ICWAN     SET=1 FOR BALANCING SYNTHESIS
C                SET=0 FOR BALANCING ANALYSIS
C                SET=-1 FOR SEPARATE PLOTTING ANALYSIS PRELIMINARIES
C 10/PREC IS THE NO. OF SIG FIG DESIRED IF VARIABLE GREATER THAN 1
C 10/PREC IS THE NO. OF SIGNIFICANT DECIMAL PLACES REQUIRED IF VARIABLE
C LESS THAN 1
      DIMENSION AH2(36),AV2(36)
      COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
     1RM(3),RJ(3)
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36),VR(36),VI(36)
      COMMON /SCLFAC/SCLSFH,SCLSFV,SCLSMO
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SAVOPT/ SV(10)
C SETTING NC=0 IF NO CONSTRAINTS ADDED (IEXCO .NE. 0)
      IF(IEXCO.NE.0) NC=0
C CALCULATING SHAKING FORCES AND MOMENTS WITHOUT C.W'S
C GETTING ACCELERATIONS OF C.M. OF COUPLER LINK
      CALL LINCUP(1,1,CM(3),CM(4),AH2,AV2)
C GETTING ACCELERATIONS OF SLIDER FOR S.C. BALANCING
      IF(METHOD.EQ.11) CALL LINCUP(1,0,PAR(2),0.,AH3,AV3)
C GETTING ACCELERATIONS OF C.M. OF CRANK LINK FOR BOTH SC AND FB
C AND THE FOLLOWER LINK FOR THE F.B.
      CALL LINIO(CM)
C LOOP TO CALCULATE SF AND SMO AT EACH POSITION
      DO 1 I=1,NPP
C CALCULATING INERTIA FORCES
      RMAV1=-RM(1)*AV1(I)
      RMAH1=-RM(1)*AH1(I)
      RMAV2=-RM(2)*AV2(I)
      RMAH2=-RM(2)*AH2(I)
      RMAV3=-RM(3)*AV3(I)
      RMAH3=-RM(3)*AH3(I)
C CALCULATING INERTIA MOMENTS
      RJA1=-RJ(1)*A1(I)
      RJA2=-RJ(2)*A2(I)
C CALCULATING SHAKING FORCES (SFV AND SFH) AND SHAKING MOMENTS (SMO)
      ST1=SIN(T1(I))
      CT1=COS(T1(I))
      ST2=SIN(T2(I))
      CT2=COS(T2(I))
      IF(METHOD.EQ.11) GOTO 5
      ST3=SIN(T3(I))
      CT3=COS(T3(I))
    5 SFV(I)=RMAV1+RMAV2+RMAV3
      SFH(I)=RMAH1+RMAH2+RMAH3
      SMO(I)=-RMAH1*(ST1*CM(1)+CT1*CM(2))
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     1+RMAV1*(CT1*CM(1)-ST1*CM(2))
     2-RMAH2*(PAR(1)*ST1+ST2*CM(3)+CT2*CM(4))
     3+RMAV2*(PAR(1)*CT1+CT2*CM(3)-ST2*CM(4))
      IF(METHOD.EQ.10) SMO(I)=SMO(I)-RMAH3*(ST3*CM(5)+CT3*CM(6))
     1+RMAV3*(PAR(6)+CT3*CM(5)-ST3*CM(6))+RJA1+RJA2-RJ(3)*A3(I)
    1 IF(METHOD.EQ.11) SMO(I)=SMO(I)-RMAH3*PAR(3)+RJA1+RJA2
C SKIPPING IF DOING ANALYSIS
      IF(ICWAN.EQ.0) GOTO 11
C CHECK ON SCALING FACTORS
      SCLSFH=AMAX1(ABS(SCLSFH),1.0E-07)
      SCLSFV=AMAX1(ABS(SCLSFV),1.0E-07)
      SCLSMO=AMAX1(ABS(SCLSMO),1.0E-07)
C SETTING SCALE FACTORS
      SCLSFH=1./(SCLSFH*SCLSFH)
      SCLSFV=1./(SCLSFV*SCLSFV)
      SCLSMO=1./(SCLSMO*SCLSMO)
   11 IF(METHOD.EQ.10) GOTO 2
C ROUTE FOR S.C. BALANCING
      N=4
      IF(NOA1.EQ.0) N=2
      IF(ICWAN.EQ.-1) RETURN
C SKIP OPTIMIZATION PROCEDURE IF C.W. ANALYSIS ONLY IS DESIRED (ICWAN=0)
      IF(ICWAN.EQ.0) GOTO 9
C CALL LINK TO OBTAIN OPTIMUM S.C. C.W. BALANCING PARAMETERS, SV(I)
      CALL LINK(1,PREC,1.,2.5,PREC)
C CALL SCBAL TO OBTAIN TOTAL SHAKING FORCES AND MOMENTS FOR THE S.C.
C MECHANISM WITH ITS OPTIMUM C.W.
    9 CALL SCBAL(SV,0)
      GOTO 3
C ROUTE FOR 4-BAR BALANCING
    2 N=8
      IF(NOA1.EQ.0) N=6
      IF(ICWAN.EQ.-1) RETURN
C SKIP OPTIMIZATION PROCEDURE IF C.W. ANALYSIS ONLY IS DESIRED (ICWAN=0)
      IF(ICWAN.EQ.0) GOTO 10
C CALL LINK TO OBTAIN OPTIMUM F.B. C.W. BALANCING PARAMETERS, SV(I)
      CALL LINK(1,PREC,1.,2.5,PREC)
C CALL FBBAL TO OBTAIN TOTAL SHAKING FORCES AND MOMENTS FOR THE F.B.
C MECHANISM WITH ITS OPTIMUM C.W.'S
   10 CALL FBBAL(SV,0)
C OBTAINING PLOTS OF BALANCED AND UNBALANCED SHAKING FORCES AND MOMENTS
    3 VMIN=1.0E+50
      VMAX=-1.0E+50
      HMIN=1.0E+50
      HMAX=-1.0E+50
      SMIN=1.0E+50
      SMAX=-1.0E+50
C OBTAINING LARGEST AND SMALLEST VALUES FOR PLOTS
      DO 4 I=1,NPP
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      VMIN=AMIN1(VMIN,TSFV(I),SFV(I))
      VMAX=AMAX1(VMAX,TSFV(I),SFV(I))
      HMIN=AMIN1(HMIN,TSFH(I),SFH(I))
      HMAX=AMAX1(HMAX,TSFH(I),SFH(I))
      SMIN=AMIN1(SMIN,TSMO(I),SMO(I))
    4 SMAX=AMAX1(SMAX,TSMO(I),SMO(I))
C PRINTING OUT TABLE OF UNBALANCED AND BALANCED SHAKING FORCES AND
C MOMENTS
      WRITE(6,100) ((T1D(I),SFH(I),TSFH(I),SFV(I),TSFV(I),SMO(I),TSMO(I)
     1),I=1,NPP)
      XMAX=T1D(NPP)
      XMIN=T1D(1)
C PRINTING TITLES FOR HORIZONTAL SHAKING FORCE PLOT
      WRITE(6,101) HMIN,HMAX,XMIN,XMAX
C REINITIALIZING PRINTING VARIABLES SINCE COMPARE DESTROYS ITS INPUT
C ARRAYS
      DO 6 I=1,NPP
      A2(I)=SFH(I)
      T2(I)=T1D(I)
    6 W2(I)=T1D(I)
      CALL COMPARE(NPP,TSFH,A2 ,W2,T2,HMAX,HMIN,XMAX,XMIN,1)
C PRINTING TITLES FOR VERTICAL SHAKING FORCE PLOT
      WRITE(6,102) VMIN,VMAX,XMIN,XMAX
      DO 7 I=1,NPP
      A2(I)=SFV(I)
      T2(I)=T1D(I)
    7 W2(I)=T1D(I)
      CALL COMPARE(NPP,TSFV,A2 ,W2,T2,VMAX,VMIN,XMAX,XMIN,1)
C PRINTING TITLES FOR SHAKING MOMENT PLOT
      WRITE(6,103)SMIN,SMAX,XMIN,XMAX
      DO 8 I=1,NPP
      A2(I)=SMO(I)
    8 W2(I)=T1D(I)
      CALL COMPARE(NPP,TSMO,A2 ,T1D,W2 ,SMAX,SMIN,XMAX,XMIN,1)
      RETURN
  100 FORMAT(1H1,5X,*TABLE OF UNBALANCED (UB) AND BALANCED(B) SHAKING FO
     1RCES AND MOMENTS*/6X,*VS. CRANK ANGLE IN DEGREES*//3X,*CRANK ANG*,
     27X,*UBSFH*,9X,*BSFH*,9X,*UBSFV*,9X,*BSFV*,
     310X,*UBSMO*,10X,*BSMO*//(7G14.6))
  101 FORMAT(1H1,5X,*PLOT OF BALANCED(*,1H*,*) AND UNBALANCED (O) HORIZO
     1NTAL SHAKING FORCES*/6X,*VS. CRANK ANGLE IN DEGREES*/
     26X,*THE PLOTTING SCALES ARE DETERMINED FROM THE FOLLOWING DATA*/
     26X,*MINIMUM SHAKING FORCE IS*,G15.6,*, MAXIMUM SHAKING FORCE IS*,
     3 G15.6/6X,*CRANK ANGLE VARIES FROM*,G15.6,* TO*,G15.6,* DEGREES*/)
  102 FORMAT(1H1,5X,*PLOT OF BALANCED (*,1H*,*) AND UNBALANCED (O) VERTI
     1CAL SHAKING FORCES*/6X,*VS. CRANK ANGLE IN DEGREES*/
     26X,*THE PLOTTING SCALES ARE DETERMINED FROM THE FOLLOWING DATA*/
     26X,*MINIMUM SHAKING FORCE IS*,G15.6,*, MAXIMUM SHAKING FORCE IS*,
     3 G15.6/6X,*CRANK ANGLE VARIES FROM*,G15.6,* TO*,G15.6,* DEGREES*/)
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103 FORMAT(1H1,5X,*PLOT OF BALANCED (*,1H*,*) AND UNBALANCED (O) SHAKI
   1NG MOMENTS*/6X,*VS. CRANK ANGLE IN DEGREES*/
   26X,*THE PLOTTING SCALES ARE DETERMINED FROM THE FOLLOWING DATA*/
   26X,*MINIMUM SHAKING MOMENT IS*,G15.6,*, MAXIMUM SHAKING MOMENT IS*
   3,G15.6/6X,*CRANK ANGLE VARIES FROM*,G15.6,* TO*,G15.6,* DEGREES*/)
    END
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      SUBROUTINE SCBAL(X,ICWAN)
C SUBROUTINE SUBPROGRAM TO DETERMINE ADDITIONAL INERTIA FORCES AND
C MOMENTS DUE TO THE ADDED COUNTERWEIGHT ON THE CRANK LINK FOR THE
C PLANAR SLIDER-CRANK MECHANISM
C THE SUBPROGRAM ALSO ADDS IN THE ORIGINAL UNBALANCED SHAKING FORCES
C AND MOMENTS TO GET THE TOTAL HORIZONTAL (TSFH) AND VERTICAL (TSFV)
C SHAKING FORCES AND TOTAL SHAKING MOMENT (TSMO) ABOUT THE CRANK-
C SHAFT AXIS AT ALL NPP POSITIONS OF THE CRANK LINK
C IT THEN COMBINES THESE FORCES AND MOMENTS TOGETHER ACCORDING TO
C THE SCALING FACTORS (SCLSFH, SCLSFV, AND SCLSMO) TO OBTAIN THE
C CONSTRAINED OBJECTIVE FUNCTION, CG, WHICH IS RETURNED TO FUNCTION
C SUBPROGRAM G TO BE COMBINED WITH THE CONSTRAINTS, C(I), TO FORM
C THE TOTAL UNCONSTRAINED OBJECTIVE FUNCTION TO BE MINIMIZED
      DIMENSION X(1)
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
     1RM(3),RJ(3)
      COMMON /SCLFAC/SCLSFH,SCLSFV,SCLSMO
      COMMON /STRTPT/STRTPT(10)
      COMMON /NUMBERS/NPP
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36)
      U1=0.
      U2=0.
      U3=0.
C DEFINING FIXED C.W. MASS FOR SPECIAL 2-VARIABLE CASE
      IF(NOA1.EQ.0) X(3)=STRTPT(3)
C INSURING POSITIVE C.W. MASS AND M.I.
      X(3)=ABS(X(3))
      IF(NOA1.NE.0) X(4)=ABS(X(4))
C OBTAINING ACCELERATIONS OF C.M. OF C.W.
      CALL LINIO(X)
      DO 1 I=1,NPP
C DETERMINING INERTIA FORCES AND INERTIA TORQUES DUE TO C.W.
      FCWH=-X(3)*AH1(I)
      FCWV=-X(3)*AV1(I)
      ST1=SIN(T1(I))
      CT1=COS(T1(I))
      FCWM=-FCWH*(X(1)*ST1+X(2)*CT1)+FCWV*(X(1)*CT1-X(2)*ST1)
      IF(NOA1.NE.0) FCWM=FCWM-X(4)*A1(I)
C CALCULATING TOTAL SHAKING FORCES AND MOMENTS
      TSFH(I)=SFH(I)+FCWH
      TSFV(I)=SFV(I)+FCWV
      TSMO(I)=SMO(I)+FCWM
C PRELIMINARY CALCULATIONS IN DETERMINING OBJECTIVE FUNCTION
C (MAXIMUM UNBALANCED SHAKING FORCES AND MOMENT SQUARED)
      U1=AMAX1(U1,TSFH(I)*TSFH(I))
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      U2=AMAX1(U2,TSFV(I)*TSFV(I))
    1 U3=AMAX1(U3,TSMO(I)*TSMO(I))
C RETURN IF DOING AN ANALYSIS
      IF(ICWAN.EQ.0) RETURN
C OBTAINING CONSTRAINED OBJECTIVE FUNCTION IN FINAL FORM
      U1=U1*SCLSFH
      U2=U2*SCLSFV
      U3=U3*SCLSMO
      CG=U1+U2+U3
      RETURN
      END
```

```
      SUBROUTINE FBBAL(X,ICWAN)
C COMMENTS SIMILAR TO SUBROUTINE SCBAL EXCEPT C.W. IS ADDED TO THE
C FOLLOWER LINK AS WELL AS THE CRANK LINK
      DIMENSION X(1)
      COMMON /NUMBERS/NPP
      COMMON /STRTPT/STRTPT(10)
      COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOA1,CM(6),
     1RM(3),RJ(3)
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      COMMON /SCLFAC/SCLSFH,SCLSFV,SCLSMO
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36)
      U1=0.
      U2=0.
      U3=0.
C DEFINING FIXED CRANK C.W. MASS FOR SPECIAL 6-VARIABLE CASE
      IF(NOA1.EQ.0) X(7)=STRTPT(7)
C INSURING POSITIVE VALUES FOR C.W. MASSES AND M.I.
      X(3)=ABS(X(3))
      X(4)=ABS(X(4))
      X(7)=ABS(X(7))
      IF(NOA1.NE.0) X(8)=ABS(X(8))
C OBTAINING ACCELERATIONS OF C.M. OF C.W.
      CALL LINIO(X)
      DO 1 I=1,NPP
C OBTAINING INERTIA FORCES AND TORQUES OF C.W.
      FCWH1=-X(7)*AH1(I)
      FCWH3=-X(3)*AH3(I)
      FCWV1=-X(7)*AV1(I)
      FCWV3=-X(3)*AV3(I)
      ST1=SIN(T1(I))
      CT1=COS(T1(I))
      ST3=SIN(T3(I))
      CT3=COS(T3(I))
      FCWM=-FCWH1*(X(1)*ST1+X(2)*CT1)
     1         +FCWV1*(X(1)*CT1-X(2)*ST1)
     2         -FCWH3*(X(5)*ST3+X(6)*CT3)
     3         +FCWV3*(PAR(6)+X(5)*CT3-X(6)*ST3)
     4         -X(4)*A3(I)
C INCLUDING EFFECT OF INERTIA TORQUE OF CRANK C.W. IF AT LEAST ONE
C A1(I) IS .NE. 0
      IF(NOA1.NE.0) FCWM=FCWM-X(8)*A1(I)
C OBTAINING TOTAL SHAKING FORCES AND MOMENTS
      TSFV(I)=SFV(I)+FCWV1+FCWV3
      TSFH(I)=SFH(I)+FCWH1+FCWH3
      TSMO(I)=SMO(I)+FCWM
C OBTAINING MAXIMUM VALUE OF UNBALANCED FORCES AND MOMENTS SQUARED
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      U1=AMAX1(U1,TSFH(I)*TSFH(I))
      U2=AMAX1(U2,TSFV(I)*TSFV(I))
    1 U3=AMAX1(U3,TSMO(I)*TSMO(I))
C RETURNING IF DOING ANALYSIS
      IF(ICWAN.EQ.0) RETURN
C OBTAINING CONSTRAINED OBJECTIVE FUNCTION IN FINAL FORM
      U1=U1*SCLSFH
      U2=U2*SCLSFV
      U3=U3*SCLSMO
      CG=U1+U2+U3
      RETURN
      END
```

```
      SUBROUTINE SCANG(JFLAG1,JFLAG2)
C SUBROUTINE TO CALCULATE ANGULAR VELOCITIES AND ACCELERATIONS OF THE
C COUPLER LINK OF A PLANAR SLIDER-CRANK MECHANISM
C THIS SUBROUTINE MAY BE CALLED DIRECTLY BY THE USER SETTING METHOD=8
C INTRODUCTORY COMMENTS FOR FBANG ALSO APPLY TO SCANG
      COMMON /NUMBERS/NPP,METHOD,ICASE
      COMMON /BALIN/W1(36),A1(36),X(6),  STRTA,RNGA,NOJ1,CM(6),
     1RM(3),RJ(3)
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36),VR(36),VI(36)
      X2SQ=X(2)*X(2)
      ADD=.0174533*RNGA/FLOAT(NPP-1)
      T1(1)=.0174533*STRTA
      DO 1 I=1,NPP
      IF(I.EQ.1) GOTO 3
C CALCULATING CRANK ANGLE IN RADIANS
      T1(I)=T1(I-1)+ADD
C CALCULATING CRANK ANGLE IN DEGREES FOR OUTPUT
    3 T1D(I)=T1(I)*57.2957795
      X1S=X(1)*SIN(T1(I))
      X1C=X(1)*COS(T1(I))
      S1=X(3)-X1S
      S2=FLOAT(ICASE)*SQRT(X2SQ-S1*S1)
C CALCULATING COUPLER ANGLE
      T2(I)=ATAN2(S1,S2)
      X2S=X(2)*SIN(T2(I))
      X2C=X(2)*COS(T2(I))
C CALCULATING COUPLER ANGULAR VELOCITY
      W2(I)=-W1(I)*X1C/X2C
      S3=W1(I)*W1(I)*X1S
      S4=W2(I)*W2(I)*X2S
C CALCULATING COUPLER ANGULAR ACCELERATION
    1 A2(I)=W2(I)*A1(I)/W1(I)+(S3+S4)/X2C
C TESTING TO SEE IF TABLE PRINT OUT IS DESIRED (JFLAG1=0)
      IF(JFLAG1.EQ.0) WRITE(6,100) ((T1D(I),W1(I),A1(I),W2(I),A2(I)),
     1I=1,NPP)
C TESTING TO SEE IF PLOTS ARE DESIRED (JFLAG2=0)
      IF(JFLAG2.NE.0) RETURN
C OBTAINING MAXIMUM AND MINIMUM VALUES OF VELOCITY AND ACCELERATION
      WMIN=W2(1)
      WMAX=W2(1)
      AMIN=A2(1)
      AMAX=A2(1)
      DO 2 I=2,NPP
      WMIN=AMIN1(WMIN,W2(I))
      WMAX=AMAX1(WMAX,W2(I))
      AMIN=AMIN1(AMIN,A2(I))
    2 AMAX=AMAX1(AMAX,A2(I))
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      XMAX=T1D(NPP)
      XMIN=T1D(1)
C PRINTING TITLES FOR ANGULAR VELOCITY PLOT
      WRITE(6,101) WMAX,WMIN,XMIN,XMAX
C REINITIALIZING PRINTING VARIABLES SINCE COMPARE DESTROYS ITS INPUT
C ARRAYS
      DO 4 I=1,NPP
      VI(I)=T1D(I)
    4 VR(I)=W2(I)
      CALL COMPARE(NPP,VR,VR,VI,VI,WMAX,WMIN,XMAX,XMIN,0)
C PRINTING TITLES FOR ANGULAR ACCELERATION PLOT
      WRITE(6,102) AMAX,AMIN,XMIN,XMAX
      DO 5 I=1,NPP
      VI(I)=T1D(I)
    5 VR(I)=A2(I)
      CALL COMPARE(NPP,VR,VR,VI,VI,AMAX,AMIN,XMAX,XMIN,0)
      RETURN
  100 FORMAT(1H1,1X,*TABLE OF ANGULAR VELOCITIES AND ACCELERATIONS OF PL
     1ANAR SLIDER CRANK LINKS*//3X,*CRANK ANG*,7X,*CRANK W*,
     27X,*CRANK A*,8X,*COUPLER W*,6X,*COUPLER A*/3X,*(DEGREES)*,6X,
     3*(RAD/SEC)*,4X,*(RAD/SEC/SEC)*,4X,*(RAD/SEC)*,5X,*(RAD/SEC/SEC)*//
     4(5G15.6))
  101 FORMAT(1H1,5X,*PLOT OF THE ANGULAR VELOCITY OF THE COUPLER LINK IN
     1 RAD/SEC*/6X,*VS. THE CRANK ANGLE IN DEGREES FOR THE PLANAR SLIDER
     2-CRANK MECHANISM*/6X,*MAXIMUM ANGULAR VELOCITY IS*,G14.5,
     3*RAD/SEC*/6X,*MINIMUM ANGULAR VELOCITY IS*,G14.5,*RAD/SEC*/
     46X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
  102 FORMAT(1H1,5X,*PLOT OF THE ANGULAR ACCELERATION OF THE COUPLER LIN
     1K IN RAD/SEC*/6X,*VS. THE CRANK ANGLE IN DEGREES FOR THE PLANAR SL
     2IDER-CRANK MECHANISM*/6X,*MAXIMUM ANGULAR ACCELERATION IS*,
     3G14.5,*RAD/SEC/SEC*/6X,*MINIMUM ANGULAR ACCELERATION IS*,
     4G14.5,*RAD/SEC/SEC*/
     56X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
      END
```

```
      SUBROUTINE FBANG(JFLAG1,JFLAG2)
C SUBROUTINE SUBPROGRAM TO CALCULATE ANGULAR VELOCITIES AND ACCELERATION
C COUPLER AND FOLLOWER LINKS OF A PLANAR FOUR-BAR LINKAGE
C AT NPP POSITIONS OF THE CRANK
C THE FIRST POSITION IS AT STRTA DEGREES, THE FOLLOWING POSITIONS AT
C RNGA/(NPP-1) DEGREE INTERVALS FOR RNGA DEGREES
C THE ANGULAR VELOCITY,W1(I), AND ACCELERATION,A1(I), OF THE CRANK LINK
C MUST BE DEFINED BY THE USER AT EACH OF NPP POSITIONS
C THE FIRST 6 LINKAGE PARAMETERS, PAR(I) (X(I) IN THIS SUBROUTINE), MUST
C BE USER DEFINED
C THIS SUBROUTINE MAY BE CALLED DIRECTLY BY THE USER SETTING METHOD=6
C SET JFLAG1=0 IF A TABLE PRINT-OUT OF THE ANGULAR VELOCITIES AND
C ACCELERATIONS VS. THE CRANK ANGLE IN DEGREES ARE DESIRED
C SET JFLAG2=0 IF PLOTS OF TABULAR VALUES ARE DESIRED
      COMMON /NUMBERS/NPP,METHOD,ICASE
      COMMON /BALIN/W1(36),A1(36),X(6),  STRTA,RNGA,NOJ1,CM(6),
     1RM(3),RJ(3)
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36)
      S1=X(6)*X(6)+X(1)*X(1)
      S2=2.*X(6)*X(1)
      S3=X(2)*X(2)-X(3)*X(3)
      S4=2.*X(2)
      S5=2.*X(3)
      ADD=RNGA*.0174533/FLOAT(NPP-1)
      T1(1)=STRTA*.0174533
      DO 1 I=1,NPP
      IF(I.EQ.1) GOTO 2
C DEFINING CRANK ANGLE IN RADIANS
      T1(I)=T1(I-1)+ADD
C CONVERTING CRANK ANGLE TO DEGREES FOR PRINTOUTS
    2 T1D(I)=T1(I)*57.2957795
      DSQ=S1-S2*COS(T1(I))
      D=SQRT(DSQ)
      RAM=ACOS((DSQ+S3)/(S4*D))
      A=ATAN2(X(1)*SIN(T1(I)),X(6)-X(1)*COS(T1(I)))
      B=ACOS((DSQ-S3)/(S5*D))
C DEFINING COUPLER ANGLE
      T2(I)=FLOAT(ICASE)*RAM-A
C DEFINING FOLLOWER ANGLE
      T3(I)=6.2831853-A-FLOAT(ICASE)*B
      DEL=T1(I)-T3(I)
      EPS=T2(I)-T3(I)
      GAM=T1(I)-T2(I)
      CEPS=COS(EPS)
      SEPS=SIN(EPS)
      TOP=X(1)*W1(I)
      DEN=X(3)*SEPS
```

```
C DEFINING COUPLER ANGULAR VELOCITY
      W2(I)=-TOP*SIN(DEL)/DEN
C DEFINING FOLLOWER ANGULAR VELOCITY
      W3(I)=TOP*SIN(GAM)/DEN
      W1SQ=X(1)*W1(I)*W1(I)
      W2SQ=X(2)*W2(I)*W2(I)
      W3SQ=X(3)*W3(I)*W3(I)
C DEFINING COUPLER ANGULAR ACCELERATION
      A2(I)=W2(I)*A1(I)/W1(I)-(W1SQ*COS(DEL)+W2SQ*CEPS+W3SQ)/(X(2)*SEPS)
C DEFINING FOLLOWER ANGULAR ACCELERATION
      A3(I)=W3(I)*A1(I)/W1(I)+(W1SQ*COS(GAM)+W2SQ+W3SQ*CEPS)/DEN
C REDEFINING T3 FOR CONVENIENCE IN BALANCING ROUTINES
    1 T3(I)=T3(I)-3.14159265
C PRINTING TABLE OF VALUES IF DESIRED
      IF(JFLAG1.EQ.0) WRITE(6,100) ((T1D(I),W1(I),A1(I),W2(I),A2(I),
     1                           W3(I),A3(I)),I=1,NPP)
C TEST TO SEE IF PLOTTING DESIRED
      IF(JFLAG2.NE.0) RETURN
C ROUTE IF PLOTTING DESIRED
      WMIN=1.0E+50
      WMAX=-1.0E+50
      AMIN=1.0E+50
      AMAX=-1.0E+50
C DETERMINING MAXIMUM AND MINIMUM VALUES FOR PLOTTING
      DO 3 I=1,NPP
      WMIN=AMIN1(W2(I),W3(I),WMIN)
      WMAX=AMAX1(W2(I),W3(I),WMAX)
      AMIN=AMIN1(A2(I),A3(I),AMIN)
    3 AMAX=AMAX1(A2(I),A3(I),AMAX)
      XMAX=T1D(NPP)
      XMIN=T1D(1)
C PRINTING TITLES FOR ANGULAR VELOCITY PLOTS
      WRITE(6,101) WMAX,WMIN,XMIN,XMAX
C REINITIALIZING PRINTING VARIABLES SINCE COMPARE DESTROYS ITS INPUT
C ARRAYS
      DO 4 I=1,NPP
      AV1(I)=T1D(I)
      AV3(I)=W2(I)
      AH3(I)=W3(I)
    4 AH1(I)=T1D(I)
      CALL COMPARE(NPP,AV3,AH3,AH1,AV1,WMAX,WMIN,XMAX,XMIN,1)
C PRINTING TITLES FOR ANGULAR ACCELERATION PLOTS
      WRITE(6,102) AMAX,AMIN,XMIN,XMAX
      DO 5 I=1,NPP
      AV1(I)=T1D(I)
      AV3(I)=A2(I)
      AH3(I)=A3(I)
    5 AH1(I)=T1D(I)
      CALL COMPARE(NPP,AV3,AH3,AH1,AV1,AMAX,AMIN,XMAX,XMIN,1)
```

```
      RETURN
  100 FORMAT(1H1,5X,*TABLE OF ANGULAR VELOCITIES AND ACCELERATIONS OF PL
     1ANAR FOUR-BAR LINKS*//3X,*CRANK ANG*,7X,*CRANK W*,
     27X,*CRANK A*,8X,*COUPLER W*,6X,*COUPLER A*,5X,*FOLLOWER W*,
     35X,*FOLLOWER A*/3X,*(DEGREES)*,6X,*(RAD/SEC)*,4X,*(RAD/SEC/SEC)*,
     44X,*(RAD/SEC)*,4X,*(RAD/SEC/SEC)*,4X,*(RAD/SEC)*,4X,
     5*(RAD/SEC/SEC)*//(7G15.6))
  101 FORMAT(1H1,5X,*PLOT OF THE ANGULAR VELOCITIES OF THE COUPLER (*,
     11H*,*) AND FOLLOWER (O) IN RAD/SEC*/6X,*VS. CRANK ANGLE IN DEGREES
     2 FOR THE PLANAR FOUR-BAR LINKAGE*/6X,*MAXIMUM ANGULAR VELOCITY IS*
     3,G14.5,* RAD/SEC*/6X,*MINIMUM ANGULAR VELOCITY IS*,G14.5,* RAD/SEC
     4*/6X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
  102 FORMAT(1H1,5X,*PLOT OF ANGULAR ACCELERATIONS OF THE COUPLER(*,
     11H*,*) AND FOLLOWER IN RAD/SEC/SEC*/6X,*VS. THE CRANK ANGLE IN DEG
     2REES FOR THE PLANAR FOUR-BAR LINKAGE*/6X,*MAXIMUM ANGULAR ACCELERA
     3TION IS*,G14.5,* RAD/SEC/SEC*/6X,*MINIMUM ANGULAR ACCELERATION IS
     4*,G14.5,* RAD/SEC/SEC*/
     56X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
      END
```

```
      SUBROUTINE LINCUP(IPLOT,ISKIP,CR,CT,AR,AI)
C SUBROUTINE SUBPROGRAM TO DETERMINE THE ACCELERATION OF POINTS
C ON THE COUPLER LINK FOR THE PLANAR FOUR-BAR AND SLIDER-CRANK
C MECHANISMS
C THE VELOCITIES OF COUPLER POINTS ARE ALSO DETERMINED
C THIS SUBROUTINE IS AVAILABLE FOR GENERAL USE AS WELL AS FOR
C PART OF THE BALANCING SYSTEM
C FOR GENERAL USE METHOD MUST BE EITHER 7 FOR A 4-BAR COUPLER-POINT
C OR 9 FOR A SLIDER-CRANK COUPLER-POINT
C FOR METHOD EQUAL TO 7 OR 9, A TABLE OF VELOCITIES AND ACCELERATIONS
C OF THE COUPLER-POINT AT NPP POSITIONS OF THE CRANK LINK IS GIVEN
C IF IPLOT .EQ.0 PLOTS OF THE ACCELERATIONS AND VELOCITIES VS.
C CRANK ANGLE IN THE RANGE OF MOTION, RNGA DEGREES, STARTING FROM
C STRTA DEGREES, ARE GIVEN (OTHERWISE SET IPLOT .EQ.1)
C UNLESS LINCUP HAS BEEN PREVIOULSY CALLED USING THE SAME LINKAGE
C PARAMETERS SET ISKIP=1 (IF LINCUP HAS BEEN PREVIOULSY CALLED SET ISKIP
C =0 TO SAVE COMPUTER TIME - RESULTS WILL BE IDENTICAL TO THOSE WITH
C ISKIP=1)
C IF JFLG2 WAS SET EQUAL TO 0 IN A PREVIOUS CALL TO FBANG OR SCANG
C ISKIP MUST BE SET=1 ON THE FIRST CALL TO LINCUP
C CR AND CT GIVE THE X AND Y COMPONENTS OF THE COUPLER-POINT W.R.T.
C THE CENTRAL AXIS OF THE COUPLER LINK MEASURED FROM ITS CRANK END
C ( THESE MUST BE DEFINED BY THE USER)
C AR AND AI MERELY HAVE TO BE DIMENSIONED OF SIZE NPP IN THE CALLING
C PROGRAM (PLEASE REMEMBER THIS)
C FIRST 6 LINKAGE PARAMETERS MUST BE DEFINED FOR METHO=7
C FIRST 5 LINKAGE PARAMETERS MUST BE DEFINED FOR METHOD=9
C NOTE- AS FOR ALL BALANCING-ANALYSIS SUBROUTINES, PAR(4) NEED NOT
C BE DEFINED
      DIMENSION AR(1),AI(1),VM(36),VANG(36),AM(36),AANG(36)
      COMMON /NUMBERS/NPP,METHOD
      COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOJ1,CM(6),
     1RM(3),RJ(3)
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36),
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36),VR(36),VI(36)
C SKIPPING CALLS IF LINCUP PREVIOUSLY CALLED
      IF(ISKIP.EQ.0) GOTO 1
C OBTAINING ANGULAR VELOCITIES AND ACCELERATIONS FOR 4-BAR
      IF(METHOD.EQ.7.OR.METHOD.EQ.10) CALL FBANG(1,1)
C OBTAINING ANGULAR VELOCITIES AND ACCELERATIONS FOR SLIDER-CRANK
      IF(METHOD.EQ.9.OR.METHOD.EQ.11) CALL SCANG(1,1)
    1 DO 2 I=1,NPP
      W1SQ=W1(I)*W1(I)
      W2SQ=W2(I)*W2(I)
      ST2=SIN(T2(I))
      CT2=COS(T2(I))
      X1S=PAR(1)*SIN(T1(I))
      X1C=PAR(1)*COS(T1(I))
```

```
      XRS=CR*ST2
      XRC=CR*CT2
      XTS=CT*ST2
      XTC=CT*CT2
C OBTAINING HORIZONTAL COMPONENT OF VELOCITY
      VR(I)=-X1S*W1(I)-XRS*W2(I)-XTC*W2(I)
C OBTAINING VERTICAL COMPONENT OF VELOCITY
      VI(I)=X1C*W1(I)+XRC*W2(I)-XTS*W2(I)
C OBTAINING HORIZONTAL COMPONENT OF ACCELERATION
      AR(I)=-X1S*A1(I)-X1C*W1SQ-XRS*A2(I)-XRC*W2SQ
     1     -XTC*A2(I)+XTS*W2SQ
C OBTAINING VERTICAL COMPONENT OF ACCELERATION
    2 AI(I)=X1C*A1(I)-X1S*W1SQ+XRC*A2(I)-XRS*W2SQ
     1        -XTS*A2(I)-XTC*W2SQ
C RETURN IF DOING A SYNTHESIS
      IF(METHOD.LT.6.OR.METHOD.GT.9) RETURN
C DETERMINING MAGNITUDE AND ANGLE OF VELOCITIES AND ACCELERATIONS
C FOR TABLES AND PLOTTING
      DO 3 I=1,NPP
      VM(I)=SQRT(VR(I)*VR(I)+VI(I)*VI(I))
      VANG(I)=(ATAN2(VI(I),VR(I))+PAR(5))*57.2957795
      AM(I)=SQRT(AR(I)*AR(I)+AI(I)*AI(I))
    3 AANG(I)=(ATAN2(AI(I),AR(I))+PAR(5))*57.2957795
C PRINTING TABLES
      WRITE(6,100) ((T1D(I),VM(I),VANG(I),AM(I),AANG(I)),I=1,NPP)
C TEST TO SEE IF PLOTTING REQUIRED
      IF(IPLOT.NE.0) RETURN
C OBTAINING MAXIMUM AND MINIMUM VALUES FOR PLOTTING
      VMIN=VM(1)
      VMAX=VM(1)
      VAMIN=VANG(1)
      VAMAX=VANG(1)
      AMIN=AM(1)
      AMAX=AM(1)
      AAMIN=AANG(1)
      AAMAX=AANG(1)
      DO 4 I=2,NPP
      VMIN=AMIN1(VMIN,VM(I))
      VMAX=AMAX1(VMAX,VM(I))
      VAMIN=AMIN1(VAMIN,VANG(I))
      VAMAX=AMAX1(VAMAX,VANG(I))
      AMIN=AMIN1(AMIN,AM(I))
      AMAX=AMAX1(AMAX,AM(I))
      AAMIN=AMIN1(AAMIN,AANG(I))
    4 AAMAX=AMAX1(AAMAX,AANG(I))
      XMAX=T1D(NPP)
      XMIN=T1D(1)
C PRINTING VELOCITY PLOTS
      WRITE(6,101) VMAX,VMIN,XMIN,XMAX
```

```
C REINITIALIZING PRINTING VARIABLES SINCE COMPARE DESTROYS ITS INPUT
C ARRAYS
      DO 5 I=1,NPP
    5 AH1(I)=T1D(I)
      CALL COMPARE(NPP,VM,VM,AH1,AH1,VMAX,VMIN,XMAX,XMIN,0)
      WRITE(6,102) VAMAX,VAMIN,XMIN,XMAX
      DO 6 I=1,NPP
    6 AH1(I)=T1D(I)
      CALL COMPARE(NPP,VANG,VANG,AH1,AH1,VAMAX,VAMIN,XMAX,XMIN,0)
C PRINTING ACCELERATION PLOTS
      WRITE(6,103) AMAX,AMIN,XMIN,XMAX
      DO 7 I=1,NPP
    7 AH1(I)=T1D(I)
      CALL COMPARE(NPP,AM,AM,AH1,AH1,AMAX,AMIN,XMAX,XMIN,0)
      WRITE(6,104) AAMAX,AAMIN,XMIN,XMAX
      DO 8 I=1,NPP
    8 AH1(I)=T1D(I)
      CALL COMPARE(NPP,AANG,AANG,AH1,AH1,AAMAX,AAMIN,XMAX,XMIN,0)
      RETURN
  100 FORMAT(1H1,1X,*TABLE OF COUPLER POINT LINEAR VELOCITIES AND ACCELE
     1RATIONS- MAGNITUDES AND ANGLES*//3X,*CRANK ANG*,7X,*VEL MAG*,8X,
     2*VEL ANG*,8X,*ACC MAG*,8X,*ACC ANG*/3X,*(DEGREES)*,5X,*(UNITS/SEC)
     3*,5X,*(DEGREES)*,3X,*(UNITS/SEC/SEC)*,3X,*(DEGREES)*//
     4(5G15.6))
  101 FORMAT(1H1,5X,*PLOT OF COUPLER POINT VELOCITY MAGNITUDE IN UNITS/S
     1EC*/6X,*VS. THE CRANK ANGLE IN DEGREES*/6X,*THE MAXIMUM VELOCITY I
     2S*,G14.5,*,  THE MINIMUM VELOCITY IS*,G14.5/
     36X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
  102 FORMAT(1H1,5X,*PLOT OF COUPLER POINT VELOCITY ANGLE IN DEGREES*/
     16X,*VS. THE CRANK ANGLE IN DEGREES*/6X,*THE MAXIMUM ANGLE IS*,
     2G14.5,*,  THE MINIMUM ANGLE IS *,G14.5/
     36X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
  103 FORMAT(1H1,5X,*PLOT OF THE COUPLER-POINT ACCELERATION MAGNITUDE IN
     1 UNITS/SEC/SEC*/6X,*VS. THE CRANK ANGLE IN DEGREES*/6X,
     2*THE MAXIMUM ACCELERATION IS*,G14.5,*,  THE MINIMUM ACCELERATION I
     3S*,G14.5/
     46X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
  104 FORMAT(1H1,5X,*PLOT OF THE COUPLER-POINT ACCELERATION ANGLE IN DEG
     1REES* /6X,*VS. THE CRANK ANGLE IN DEGREES*/6X,*THE MAXIMUM ANGLE I
     3S*,G14.5,*,  THE MINIMUM ANGLE IS*,G14.5/
     46X,*CRANK ANGLE VARIES FROM*,G14.5,* TO*,G14.5,* DEGREES*//)
      END
```

```
      SUBROUTINE LINIO(X)
C SUBROUTINE SUBPROGRAM TO CALCULATE THE ACCELERATIONS OF POINTS ON
C THE CRANK AND FOLLOWER LINKS, THEIR POSITIONS GIVEN BY VECTOR X
C THE ACCELERATIONS ARE GIVEN IN TERMS OF THEIR HORIZONTAL AND
C VERTICAL COMPONENTS
      DIMENSION X(1)
      COMMON /NUMBERS/NPP,METHOD
      COMMON /BALIN/W1(36),A1(36),PAR(6),STRTA,RNGA,NOJ1,CM(6),
     1RM(3),RJ(3)
      COMMON /BALVAL/T1(36),T1D(36),T2(36),T3(36),W2(36),W3(36),A2(36)
     1A3(36),AH1(36),AV1(36),AH3(36),AV3(36),SFH(36),SFV(36),SMO(36),
     2TSFH(36),TSFV(36),TSMO(36)
      DO 1 I=1,NPP
      ST1=SIN(T1(I))
      CT1=COS(T1(I))
      W1SQ=W1(I)*W1(I)
      W1S=W1SQ*ST1
      W1C=W1SQ*CT1
      A1S=A1(I)*ST1
      A1C=A1(I)*CT1
C DEFINING HORIZONTAL COMPONENT OF ACCELERATION ON CRANK LINK
      AH1(I)=-X(1)*(W1C+A1S)+X(2)*(W1S-A1C)
C DEFINING VERTICAL COMPONENT OF ACCELERATION ON CRANK LINK
    1 AV1(I)=X(1)*(A1C-W1S)-X(2)*(W1C+A1S)
C RETURN IF ONLY ACCELERATION OF POINT ON CRANK LINK DESIRED
C (METHOD=11)
      IF(METHOD.EQ.11) RETURN
C ROUTE FOR FOLLOWER POINT ACCELERATIONS
      DO 2 I=1,NPP
      ST3=SIN(T3(I))
      CT3=COS(T3(I))
      W3SQ=W3(I)*W3(I)
      W3S=W3SQ*ST3
      W3C=W3SQ*CT3
      A3S=A3(I)*ST3
      A3C=A3(I)*CT3
C DEFINING HORIZONTAL COMPONENT
      AH3(I)=-X(5)*(W3C+A3S)+X(6)*(W3S-A3C)
C DEFINING VERTICAL COMPONENT
    2 AV3(I)=X(5)*(A3C-W3S)-X(6)*(W3C+A3S)
      RETURN
      END
```

```
      SUBROUTINE OPTSURF(NX,NY,GMAX,GMIN,XMAX,XMIN,YMAX,YMIN,ISKIP)
C SUBROUTINE SUBPROGRAM TO PLOT THE INTERSECTION OF A N DIMENSIONAL
C HYPER-PLANE WITH THE (N+1) DIMENSIONAL OPTIMIZATION HYPER-SURFACE
C THE RESULT IS A CONTOUR PLOT WITH X(NX) REPRESENTING THE
C HORIZONTAL AXIS AND X(NY) REPRESENTING THE VERTICAL AXIS
C 9 CONTOUR LINES OF THE TOTAL UNCONSTRAINED OBJECTIVE FUNCTION
C G ARE PLOTTED AT EQUAL INCREMENTS OF G EQUAL TO (GMAX-GMIN)/9.
C BETWEEN THE GIVEN VALUES OF GMAX AND GMIN
C E.G. NO. 3 CONTOUR LINE HAS FUNCTION VALUE OF GMIN+3.*(GMAX-GMIN)/9.
C THE INTERSECTION OF THE CONSTRAINT HYPER-SURFACES WITH THE
C INTERSECTING HYPER-PLANE ARE ALSO PLOTTED AS LETTERS - A REPRESENTING
C THE INTERSECTION WITH CONSTRAINT 1, B WITH CONSTRAINT 2, ETC.
C (UP TO 20 CONSTRAINTS)
C + IS PRINTED WHERE THE G VALUE EXCEEDS GMAX
C - IS PRINTED WHERE THE G VALUE IS LESS THAN GMIN
      DIMENSION LABEL(10)
      COMMON X(10),FUNC(85),CONS(85,20)
      COMMON /NUMBERS/NPP,METHOD,ICASE,N,NC,IEXCO
      COMMON /SAVOPT/SV(10)
      COMMON /INTERN/RPSI,CG,C(20),U1,U2,U3,PM,ICOUNT,PS(81),CXA(21),
     1CYA(21),ERRMAX(21),TRI,ZERO
      DATA LABEL/4HX(1),4HX(2),4HX(3),4HX(4),4HX(5),4HX(6),4HX(7),
     14HX(8),4HX(9),5HX(10)/
C INITIALIZE DESIGN VARIABLES TO FINAL OPTIMUM VALUES
C SKIPPING IF PREVIOUSLY CALLED
      IF(ISKIP.EQ.0) GOTO 4
C CALLING DATA INITIALIZATION PROGRAMS
      IF(METHOD.LT.6) CALL LINK(0)
      IF(METHOD.GT.9) CALL BALANCE(-1)
    4 DO 1 I=1,N
    1 X(I)=SV(I)
C PRINTING TITLES FOR CONTOUR PLOT
      WRITE(6,100) LABEL(NX),LABEL(NY),GMAX,GMIN,LABEL(NX),XMIN,XMAX,
     1LABEL(NY),YMIN,YMAX
C OBTAINING VARIABLE INCREMENTS FOR PLOTTING
      YINC=(YMAX-YMIN)*.02
      XINC=(XMAX-XMIN)/84.
C LOOP TO CALCULATE FUNCTION VALUES AND CONSTRAINT VALUES
C AND CALL PLOTTING ROUTINE WHICH PRINTS ONE LINE AT A TIME
      DO 2 I=1,51
      X(NY)=YMIN+FLOAT(I-1)*YINC
      DO 3 J=1,85
      X(NX)=XMIN+FLOAT(J-1)*XINC
      FUNC(J)=G(X)
      DO 3 L=1,NC
    3 CONS(J,L)=C(L)
    2 CALL PLOTCN(GMAX,GMIN,NC)
      RETURN
  100 FORMAT(1H1,5X,*PARTIAL CONTOUR PLOT OF FINAL OPTIMIZATION HYPER-SU
```

```
1RFACE*/6X,A6,*IS THE HORIZONTAL AXIS VARIABLE, *,A6,
2*IS THE VERTICAL AXIS VARIABLE*/6X,*+ IS PRINTED WHERE FUNCTION VA
3LUES EXCEED*,G14.5/6X,*- IS PRINTED WHEN FUNCTION VALUES ARE LESS
4THAN*,G14.5/6X,A6,*VARIES FROM*,G14.5,* TO*,G14.5,*, *,A6,*VARIES
5FROM*,G14.5,* TO*,G14.5/)
 END
```

```
      SUBROUTINE PLOTCN(FMAX,FMIN,NC)
C SUBROUTINE SUBPROGRAM TO PLOT ONE LINE OF A FUNCTION CONTOUR AND
C CONSTRAINT PLOT
      DIMENSION K(85),KB(5),ICMP1(20),ICMP2(20)
      COMMON FILL(10),F(85),C(85,20)
      DATA KB/55B,33B,46B,45B,01B/
C DETERMINING INCREMENT FOR FUNCTION CONTOURS
      A=(FMAX-FMIN)/9.
      DO 1 I=1,NC
    1 ICMP1(I)=SIGN(1.1,C(1,I))
      DO 2 I=1,85
      LNOW=((F(I)-FMIN)/A)+1.
      K(I)=KB(1)
      IF(I.EQ.1) GOTO 3
C TESTING IF A CONTOUR HAS BEEN REACHED AND PRINTING THE APPROPRIATE
C NUMBER
      IF(LNOW.NE.LAST) K(I)=MIN0(LNOW,LAST)+KB(2)
    3 IF(K(I).LE.KB(2)) K(I)=KB(1)
      IF(LNOW.GT.9) K(I)=KB(4)
      IF(LNOW.LE.0) K(I)=KB(3)
C TESTING TO SEE IF A CONSTRAINT SURFACE HAS BEEN INTERSECTED AND
C PRINTING THE APPROPRIATE LETTER
      DO 4 J=1,NC
      ICMP2(J)=SIGN(1.1,C(I,J))
      IF(ICMP1(J)+ICMP2(J)) 4,5,4
    5 K(I)=KB(5)+J-1
      ICMP1(J)=ICMP2(J)
      GOTO 2
    4 CONTINUE
    2 LAST=LNOW
      WRITE(6,100) K
      RETURN
  100 FORMAT(5X,85R1)
      END
```

X REFERENCES

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