# VISUAL POSITION DISCRIMINATION:: TEMPORAL AND SPATIAL FACTORS

# VISUAL POSITION DISCRIMINATION: A MODEL RELATING TEMPORAL AND SPATIAL FACTORS

By

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## A Thesis

Submitted to the Faculty of Graduate Studies in Partial Fulfilment of the Requirements

for the Degree

Doctor *ot* Philosophy

McMaster University

May 1968

DOCTOR OF PHILOSOPHY (1968)<br>(Psychology) MCMASTER UNIVERSITY Hamilton, Ontario TITLE: Visual Position Discrimination: A Model Relating Temporal and Spatial Factors AUTHOR:: Lorraine G. Allan, B.A. (University of Toronto) M.A. (University of Toronto) SUPERVISOR: Dr. R. A. Kinchla NUMBER OF PAGES: vi, 118

SCOPE AND CONTENTS:

This paper presents a decision theory model of the perceptual processes by which an observer compares two visual stimuli presented at different points in'time and at different locations in the visual field. The model specifies how information about the first stimulus is lost during the interstimulus delay and over the spatial translation required for the comparison. Emphasis is placed on the manner in which the effect of temporal separation combines with the effect of spatial separation in determining the observer <sup>i</sup>s sensitivity. Two experiments are reported. The observer was required to discriminate a difference in vertical position between two laterally separated points of light presented successively in the dark. The progressive loss in sensitivity with increasing temporal and spatial separations is consistent with the predictions of the model.

(ii)

#### ACKNOWLEDGMENT

I would like to express my sincere appreciation to Dr. Ronald Kinchla for the stimulation, encouragement, and guidance he provided during all phases of this investigation. I am indebted to him for creating an atmosphere conducive to learning and research. Thanks are also due to Dr. P. L. Newbigging for his comments regarding this work, and to Dr. Steve Link for his patience during conversations relating to the analysis of the data.

I would like to record my graditude to my husband for his co-operation, encouragement, and tolerance. He was unfailing in his understanding of the time and effort which this research demanded.

Finally, I would like to acknowledge Cy Dixon who prepared the graphs, Laurine Walker who typed the manuscript, and June Cooper who willingly assisted me in many ways.

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#### INTRODUCTION

Suppose an observer is presented with a stimulus at one point in his visual field and then, some time later, presented with a second stimulus at a different point in the field. Most readers would probably agree that his accuracy in judging the similarity of the two stimuli would be reduced by both the temporal delay between their occurrence and their spatial separation in his visual field. For example, consider the advantage one gains in comparing the colour of two objects if they are viewed in immediate temporal succession and/or close together in space. Increasing either the temporal delay or the spatial separation between the two objects makes it more difficult to perceive whether the two colours are the same or different. This suggests that an observer<sup>t</sup>s ability to judge the similarity or two objects is limited not only by their physical similarity but also by the manner in which they are observed.

In this paper, a decision theory model is developed to represent the effect of temporal and spatial factors on an obaerver':s ability to compare two visual stimuli. The model is applied to data from a visual position discrimination task. The observer was required to discriminate a difference

in vertical position between two laterally separated points of light presented successively in the dark. The variables of primary interest in this task were the length of the delay between successive presentations of the two stimuli, and the degree of lateral separation between them.

Decision theory models in psychophysics have largely addressed themselves to discrimination problems in which the two stimuli are temporally and spatially contiguous. Such discrimination tasks are generally referred to as detection problems. The most prominent of these models is the Theory of Signal Detection (see Green and Swets, 1966, for a summary of detection models). This is a two-process model in that an observer''s performance in a discrimination task is represented as the product of two subprocesses: an input process and a decision process. The input process relates the external stimulus event to hypothetical sensory states. The particular sensory state evoked is treated as a value of a random variable whose distribution depends on the physical properties of the actual stimulus value and the sensory capacities of the observer. A measure of the observer's "sensitivity" characterizes the input process. This measure is based on the relationship between the two distributions of sensory states evoked by the two stimulus events to be discriminated. The decision process relates the random variable representing the sensory states to the observer's overt response. An observer's tendency or "bias" to make a particular response is influenced

by such experimenter controlled events as the presentation probabilities or the stimuli, and the costs or gains associated with the various stimulus-response contingencies. The bias is characterized by a single variable, the "decision criterion". The observer's overt response is based on the relation between the value or the random variable evoked by a particular stimulus event and his decision criterion. While an observer may modify his performance by altering his decision criterion, the measure of his sensitivity will not be affected. It is the separation of previously confounded "sensitivity" and "response bias" measures which is the major accomplishment or the decision theory approach to the analysis or psychophysical data.

A two-process model like the Theory of Signal Detection makes no provision for specifying changes in an observer's sensitivity produced by variations in the temporal delay between the two stimuli. Nor does it specify changes in sensitivity produced by variations in the spatial separation between the two stimuli.

Kinchla and Smyzer (1967), Ronken (1967), Tanner, Haller and Atkinson (1967), and Tanner, (1961) have developed models which represent the perceptual processes by which an observer compares stimulus events presented at different points in time. These models are concerned with the problem of memory in psychophysics, and have been denoted as recogni tion models (Kinchla, 1966; Kinchla and Smyzer, 1967; and

Tanner, Haller and Atkinson, (1967) in order to distinguish them from detection models. Of special interest here is Kinchla's diffusion model of perceptual memory (Kinchla and  $Smyzer, 1967$ . This model is a three-process model in that recognition is represented as the product of three subprocesses: an input process, a memory process, and a decision process. As in the Theory of Signal Detection, the input process specifies the manner in which the physical properties of the stimulus event and the sensory capacity of the observer determine the hypothetical sensory state evoked by the stimulus event. The memory process considers the manner in which information about the initial stimulus is lost or degenerated during the interstimulus interval. The sensory state evoked by the first stimulus event is stored in memory where it is "diffused" through a random walk process during the interstimulus interval. The observer is represented as basing his response on the discrepancy between his memory of the sensory state evoked by the initial stimulus, and the sensory. state evoked by the second stimulus. The decision process specifies how a given discrepancy interacts with the observer's decision criterion to determine his overt response. The observer reports a stimulus difference only if the discrepancy exceeds his decision criterion.

Keller and Kinchla (1968), Kinchla and Allan (1968), and Kinchla and Smyzer (196?) applied this model to data from a visual recognition task. The observer was required

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to discriminate a lateral difference in position between two small points of light flashed successively in the dark with a variable time interval between flashes. The progressive loss in sensitivity predicted by the model for · increasing delays between successive presentations of the two lights, was shown to be consistent with the observed data.

Keller and Kinchla, Kinchla and Allan, and Kinchla and Smyzer suggested that the loss of position information represented by the theoretical random walk process may to a large extent reflect overt, involuntary eye movements during the interval between the two flashes. Support for this interpretation is found in studies concerned with direct measurement of eye movements in the dark. For example, Cornsweet (1956), and Matin (cited as personal communication by Kinchla and Smyzer, 1967} have reported that the drift components of involuntary movements of the eye in the dark suggest a random walk process. If the loss of position information in the experiments described by Kinchla and his associates is primarily a result of eye movements during the interstimulus interval, then the model proposed by Kincbla would allow a measure of eye drift to be derived from purely psychophysical data.

As mentioned earlier, it can readily be demonstrated that an observer<sup>®</sup>s efficiency in discriminating the colour *ot* two visual stimuli decreases as the spatial separation

between them increases. However, no quantitative theory of this relationship is available. The present paper presents an elaboration of Kinchla<sup>t</sup>'s model to represent the manner in which information about one stimulus is lost during a spatial translation required to compare it with another stimulus •. Furthermore, the model specifies the manner in which the effect of spatial separation combines with the effect of temporal separation in determining an observer's ability to discriminate two stimuli. The loss of information during the spatial translation is represented by the same mathematical process that is used to represent the loss of information over time in the Kinchla model. Therefore, since the loss due to time is attributed to imperfect perceptual memory in the Kinchla model, the loss due to space will be characterized as a perceptual memory effect in this model. On the basis of the distinction between detection and recognition made earlier, this model will be denoted as a recognition model.

One other aspect of the discrimination process will be considered empirically, although no theoretical treatment will be proposed. The majority of decision theory models in psychophysics treat the observer's performance as a series of stochastically independent trials. That is, it is usually assumed that the observer's response on any given trial is independent of the stimuli and responses which occurred on previous trials. However, several investigators (Atkinson, Carterette, and Kinchla, 1962; Atkinson and Kinchla, 1965;

Kinchla, 1964, 1966; Parducci and Sandusky, 1965; Tanner, Haller, and Atkinson, 1967) have reported data from a number of experimental tasks which do not support this assumption. They have shown that the observer's response tendencies on a given trial are correlated with the stimulus and the response on the previous trial. Of particular interest here are the large and orderly sequential effects found in recognition tasks (Kinchla, 1966; Parducci and Sandusky, 1965; Tanner et. al., 1967). These far exceeded the sequential effects typically found in detection experiments. In fact, it has been pointed out that the most striking disparity between recognition and detection data is the sequential structure of the observer<sup>1</sup>s performance (Kinchla, 1966). Since the perceptual memory process provides the primary distinction between detection and recognition models, the sequential structure of the. data collected in the present experiments was examined in order to further explore the relationship between the memory and decision processes •.

#### A MATHEMATICAL MODEL OF THE PERCEPTUAL PROCESS

In this section a model for the comparison of temporally separated stimuli (Kinchla and Smyzer, 1967) will be elaborated to represent the comparison of spatially separated stimuli as well. Before introducing the theoretical issues, it will be helpful to specify the experimental situation and introduce some notation.

The discrimination experiments we shall consider consist of a series of discrete trials. On each trial the observer is shown two serially presented points of light,  $L_0$  and  $L_1$  respectively, which may or may not differ along a particular stimulus dimension. The observer's task is to report a stimulus difference when one exists. We shall denote the relevant stimulus variable by Y, and the values of  $L_0$  and  $L_1$ , by  $y_0$  and  $y_1$  respectively. On some trials,  ${\tt y}_1$  equals  ${\tt y}_0$ ; on others,  ${\tt y}_1$  differs from  ${\tt y}_0$  by an amount y. The following notation will be used to describe the stimulus-response combinations on trial n:

> $S_1, n =$  the presentation of a stimulus difference on trial n,

 $S_{\text{O}}$ ,  $_{\text{n}}$  = the presentation of no stimulus difference on trial n,

 $A_1,$  = the observer's response that a stimulus difference occurred on trial n,  $A_{\Omega}$ , = the observer's response that no stimulus

difference occurred on trial n.

Thus, on each trial either  $S_1$  or  $S_0$  is presented, and the observer makes either an  $A_1$  or  $A_0$  response. Note that his performance can be summarized by two proportions: the proportion of  $S_i$  trials on which he makes an  $A_i$  response, for i equal to 1 or 0. These proportions are normally treated as estimates of corresponding conditional probabilities; respectively,  $P(A_1 | S_1)$  and  $P(A_1 | S_0)$ . In keeping with the decision theory literature, an  $A_1$  response made to an  $S_1$ stimulus pattern will be called a hit  $(H)$ , and an  $A_1$  response made to  $S_0$  will be called a false alarm (FA).

The purpose of the model is to account for changes in an observer's ability to discriminate a stimulus difference y produced by variations of two variables: the temporal inter-<u>val</u> between the offset of  $L_0$  and the onset of  $L_1$ , denoted t; and the spatial separation between  $L_0$  and  $L_1$ , denoted x. The basic structure of the model is shown schematically in Figure 1. Each time some value of the stimulus variable Y initiates the input process it evokes some value of the sensory variable V. The values of V evoked by  $y_0$  and  $y_1$  are denoted respectively by  $v_0$  and  $v_1$ . Since  $y_1$  occurs later in time and/ or spatially separated from  $y_0$ , the observer stores  $v_0$  in "memory" until  $v_1$  is available for comparison. He then makes



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a similarity decision regarding  $y_0$  and  $y_1$  on the basis of the size of the <u>discrepancy</u> between his memory of  $v_0$ , denoted  $m_{\tau,x}$ , and  $v_1$ . He reports a stimulus difference only if the observed discrepancy exceeds his decision criterion. Thus, three processes interact to determine the relationship between stimulus and response: input, memory, and decision. While these three processes can be defined in a rigorous. axiomatic manner, a simple, verbal presentation should suffice here.

#### The Input Process

Repeated inputs of the same stimulus value do not necessarily evoke the same sensory value. The distribution of evoked sensory values is assumed to be Gaussian with an expected value equal to the actual stimulus value. Thus, if the same stimulus values  $y_0$  and  $y_1$  were presented on every discrimination trial, the sensory values  $\mathbf{v}_{\mathbf{0}}$  and  $\mathbf{v}_{\mathbf{1}}$ on any particular trial could be treated as values of two corresponding, independent, Gaussian, random variables,  $V_{Q}$ and  $V_1$  respectively, where

$$
E(V_0) = y_0,
$$
 (1)  

$$
E(V_1) = y_1.
$$
 (2)

It is assumed that the variances of the two distributions,  $Var(V_0)$  and  $Var(V_1)$ , are equal. Their sum will be referred to as the input variance. (It will be shown later that the equality or variance assumption is reasonable for the experiments to be considered).

#### The Memory Process

Once the sensory value  $v_0$  is stored in memory, it is "diffused" or modified through two simultaneous but independent random walk processes until it is read into the decision process. One random walk occurs during the interstimulus interval; the other during the spatial translation of the information provided by the initial stimulus. Note that  $m_{t,x}$ , the memory of  $v_0$ , depends solely on the initial value in memory  $(v_0)$  plus the cumulative effect of each random walk. These cumulative effects, denoted  $d_t$  and  $d_x$ respectively, are simply the sum of all the incremental steps minus the sum of all the decremental steps. The value of the memory can be written as

$$
m_{tx} = v_0 + d_t + d_x.
$$
 (3)

Since the decision process operates on the difference between  $v_1$  and  $m_{tx}$ , it will be useful to denote this discrepancy by  $u_{tx}$ , where

$$
u_{tx} = v_1 - m_{tx},
$$

or by Eq. 3

$$
u_{tx} = v_1 - v_0 - d_t - d_x.
$$
 (4)

If the same stimulus values  $y_0$  and  $y_1$  are presented on every discrimination trial, the values  $d_t$ ,  $d_x$ ,  $m_{tx}$ , and  $u_{tx}$  on any particular trial can be treated as values of four corresponding, independent, random variables,  $D_t$ ,  $D_x$ ,  $M_{tx}$ , and  $U_{tx}$ . Since the expected value of the sum of n independent, random variables is the algebraic sum of their expected values, then from Eq. 4,

$$
E(U_{tx}) = E(V_1) - E(V_0) - E(D_t) - E(D_x).
$$
 (5)

Furthermore, since the variance of the sum of n independent, random variables equals the sum of their individual variances, then from Eq. 4,

$$
Var(U_{tx}) = Var(V_1) + Var(V_0) + Var(D_t) + Var(D_x).
$$
 (6)

If the random walks are symmetrical, then as the number of steps in the random walks increases, the distributions of  $D_t$  and  $D_x$  (actually binomial) will approach Gaussian distributions with the following means and variances $:$ <sup>1</sup>

$$
E(D_t) = 0, \qquad (7)
$$

$$
E(D_x) = 0, \qquad (8)
$$

$$
Var(D_t) = \phi_t t, \qquad (9)
$$

$$
Var(D_x) = \phi_x x, \qquad (10)
$$

where  $\phi_t$  and  $\phi_x$  are constants. The constant  $\phi_t$ , which we shall refer to as the temporal diffusion rate, is the rate at which the variance of  $D_t$  increases as a linear function of the interstimulus interval (t). Similarly,  $\phi_x$ , the spatial diffusion rate, is the rate at which the variance of  $D_x$  increases as a linear function of the spatial displacement between the two stimulus lights (x).

Since  $U_{tx}$  is defined as a linear combination of four . independent, Gaussian, random variables (Eq. 4)., it too willl

 $1$ The actual derivations are presented in Appendix A.

have a Gaussian distribution. The expected value of  $U_{\text{tx}}$ depends on the actual pair of stimulus values; that is, substituting in Eq. 5 using Eqs. 1, 2, 7, and 8 yields,

$$
E(U_{tx}) = y_1 - y_0.
$$
 (11)

Furthermore, substituting in Eq. 6 using Eqs. 9 and 10 yields,

$$
Var(U_{tx}) = Var(V_{1}) + Var(V_{0}) + \phi_{t}t + \phi_{x}x.
$$

Thus, the variance of the  $U_{tx}$  is the simple sum or the input variance plus the diffusion variance accrued in memory. It will be convenient to denote  $Var(U_{\text{tr}})$  by the symbol  $G^2_{tx}$ , and the input variance by the symbol K. Thus,  $\hat{\zeta}_{tx} = \phi_t t + \phi_x x + K.$  (12)

#### The Decision Process

The observer<sup>1</sup>'s task is to decide whether the discrepancy  $(u_{tx})$  on a particular trial was produced by an actual stimulus difference. For example, suppose  $y_{ij}$  equalled  $y_{0}$  (an  $S_0$  stimulus pattern) on a randomly determined 50 per cent of the trials, and equalled  $y_0$  plus y (an  $s_1$  stimulus pattern) on the remaining trials. The observer''s decision problem is illustrated in Figure 2, which presents two overlapping, Gaussian distributions of  $U_{\text{tr},\mathbf{v}}$ . The distribution with mean zero corresponds to  $s_0$  trials, while the mean y distribution corresponds to  $S_1$  trials (Eq. 11). The observer is assumed to establish some cutoff point or criterion value of  $U_{tx}$ , denoted  $C_{tx}$ , and to report a stimulus difference only if the





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 $\mathbf{1}$ 

observed discrepancy exceeds  $C_{tx}$ ; specifically,

$$
P(A_1, n | u_{tx} > C_{tx}) = 1,
$$
  

$$
P(A_1, n | u_{tx} \le C_{tx}) = 0.
$$

# Some Properties of the Model

From Figure 2 it can be seen that  $P(A_1|S_1)$  is the area to the right of  $C_{tx}$  under the  $S_1$  distribution; similarly,  $P(A_1 | S_0)$  is the area to the right of  $C_{tx}$  under the  $S_0$  distribution. Note that although an observer may produce many combinations of  $P(A_1 | S_1)$  and  $P(A_1 | S_0)$  by shifting his criterion, he can never change  $P(A_1 | S_1)$  without simultaneously changing  $P(A_1 | S_0)$ . The possible pairs of hit and false alarm rates available to the observer through variation in his decision criterion are his operating characteristic (OC). The OC can be specified by a single number, the sensitivity measure  $6_{tx}$ , defined as follows:

$$
\delta_{\text{tx}} = \frac{E_1(U_{\text{tx}}) - E_0(U_{\text{tx}})}{Var(U_{\text{tx}}) \frac{1}{2}},
$$
\n(13)

where  $E_0(U_{tx})$  is the expected value of  $U_{tx}$  on  $S_0$  trials, and  $E_1(U_{tx})$  is the expected value on  $S_1$  trials. Thus  $\delta_{tx}$  is simply the distance between the means of the two distributions of  $U_{t,x}$ expressed in standard deviation units. Substituting in Eq. 13 from Eqs. 11 and 12 yields,

$$
\delta_{tx} = \frac{y}{(\phi_t t + \phi_x x + K)\frac{1}{2}}.
$$
 (14)

Note that in detection,

$$
\delta_{\mathtt{tx}} = \frac{\mathtt{y}}{\mathtt{K}_{\xi}}.
$$

Thus, when the two stimuli are contiguous in time and in space, the present model is equivalent to the Theory of Signal Detection, and  $\delta_{tx}$  and d' (the sensitivity measure in that theory) are equivalent. In recognition, the variance introduced by an imperfect perceptual memory process  $(\phi_t t + \phi_x x)$  is added to the input variance and reduces  $\delta_{xx}$ .

#### AN EMPIRICAL TEST OF THE MODEL

We shall now consider two experiments which provide a test of the model. In order to increase the comparability of results, data for the two experiments were collected on alternate days with the same observers participating in both studies. During the experiments, the observer sat in complete darkness and tried to discriminate a difference in vertical position between two successively presented points of light. In terms of the model, the stimulus variable Y corresponds to the vertical position of each light. The first experiment was designed to provide information about the manner in which an observer"s sensitivity  $(\delta_{t\tau})$  is influenced by the size of the vertical displacement to be discriminated  $(y)$ , and by the extent of the spatial separation between the two stimulus lights  $(x)$ . Experiment II investigated the manner in which the effect of spatial separation (x) combines with the effect of temporal separation (t) in determining an observer "s sensitivity.

Since the two experiments were similar in several respects, we will first consider the common features.

#### APPARATUS

The stimuli were small points of light (Dialco #39,  $28$  volts, . O4 amps, operated at  $15$  volts D.C.) subtending

.18

.033 degrees of visual angle at a brightness of  $4$  millilamberts. They were presented at approximately eye level, 4.1 meters in front of an observer seated in a completely dark room. The observer sat in a normal chair with no special constraints on his head movements, and viewed the stimuli binocularly. The distance between the midpoints of two stimulus lights was specified in degrees or visual angle subtended by the lights. The timing of the stimulus presentations was electronically controlled to at least an accuracy of 1 msec., and the stimulus sequence was preprogrammed and fully automatic.

#### OBSERVERS

Six paid observers were used. Each observer had an uncorrected visual acuity of 20/20 or better in both eyes according to the conventional Snellen visual acuity test. The observers were informed of the physical structure of the stimulus display, the random method *tor* generating stimulus sequences, and the relative frequency of occurrence *ot* the various stimulus patterns.<sup>2</sup>

#### METHOD

Each discrimination trial began with a 1 second auditory warning signal followed immediately by a 1 second presentation of a point of light,  $L_0$ . After a t second delay,

<sup>2</sup>The actual instructions given to each observer are presented in Appendix B.

a second point of light,  $L_1$ , displaced x degrees to the right, was presented for  $1$  second.  $L_1$  was either at the same vertical height as  $L_0$  (an  $S_0$  stimulus pattern), or displaced y degrees down on the vertical axis (an  $S_1$ stimulus pattern). Finally, the observer was given 2 seconds to indicate (by pressing an appropriate pushbutton on the arm of his chair) one of two decisions:  $L_1$  occurred in the same vertical position as  $L_0$  (an  $A_0$  response); or  $L_1$ was below  $L_0$  (an  $A_1$  response). The experimental variables were t (the temporal interval between the two lights), x (the lateral separation between the two lights), and y (the size of the vertical displacement to be discriminated). The two stimulus patterns are presented schematically in Figure 3.

 $L_0$  occurred in the same position in space on every trial: a point at approximately eye level directly in front of the seated observer. It defined the origin of the twodimensional, stimulus space. It is implicit in the present form of the model that the observer has a perfect conception of the vertical and horizontal dimensions.. In an attempt to approximate this :condition, an additional light was presented simultaneously with  $L_0$ . This light was 2 degrees to the left of  $L_0$  and at the same vertical height as  $L_0$ . Naturally, in the future it will be necessary to evaluate the importance or this additional cue.

50 stimulus pattern:







Figure 3. Schematic *ot* .the two stimulus patterns

#### EXPERIMENT I

The sequence of stimulus presentations was separately determined for each block of 50 trials. In each such sequence an S<sub>1</sub> stimulus pattern was presented on a randomly determined 25 trials, while on the remaining 25 trials an  $S_0$  pattern was presented. The interstimulus interval, t, was zero seconds (the lights occurred in immediate temporal succession) throughout the experiment. During each test session  $x(2, 4, 4)$ 6, 8, or 10 degrees) and y (.26 or .13 degrees) were varied between blocks of 50 trials. The sequence of the ten experimental conditions was randomly determined within each session, *(*  and there was a one minute rest in the dark between blocks or trials. The experiment consisted or 12 such test sessions. In this way, 12 blocks of 50 trials each were collected for each of the ten conditions, for a total of 600 trials per experimental condition.

In addition, three preliminary days of testing were conducted to provide stable data *tor* analysis. Also, in order to control warm-up effects, and to allow sufficient time *tor* dark adaptation (about 10 minutes), two practice blocks of 50 trials each were conducted at the beginning of each session. The practice conditions were randomly determined with the limitation that every condition was used an equal number of times before any or the 10 conditions were repeated. The data from the three preliminary days and from the practice blocks were not included in the final data analysis.

#### EXPERIMENT I I

The size of the vertical displacement to be discriminated, y, was .26 degrees throughout the experiment. During each test session t (O, .333, .667, or 1 second) and x (6 or 10 degrees) were varied between blocks of 50 trials. The experiment consisted of 12 such sessions. Thus, 12 blocks of 50 trials each were collected for each of the . eight conditions for a total of 600 trials per experimental condition. In all other respects, Experiment II was identical to Experiment I.

#### RESULTS

In this section we will present the data and statistically evaluate the effect of the independent variables in each experiment. A deeper analysis of the results is provided by the model and is presented in the subsequent section.

#### EXPERIMENT I

Each observer's performance can be summarized by the average hit and false alarm proportions obtained for each of the ten experimental conditions. These proportions were treated as estimates of corresponding conditional probabilities, denoted as  $\hat{P}(A_1 | S_1)$  and  $\hat{P}(A_1 | S_0)$ , and are presented in Table 1.<sup>3</sup> An index of sensitivity frequently used in psychophysical experi- ·ments is the probability of a correct response, P(C). Estimates of  $P(C)$ , denoted  $\hat{P}(C)$ , for each experimental condition, can readily be obtained from each observer"s summary data in the following manner:

# $\hat{P}(C) = \hat{P}(A_1 | S_1) \times - \left[1 - \hat{P}(A_1 | S_0)\right] (1-\gamma),$

where  $\forall$  is the probability of an  $S_1$  stimulus pattern. These estimates are presented in Table 2 for each observer. Note that, in general,  $\hat{P}(C)$  tends to decrease with increases in

 $3$ The daily hit and false alarm frequencies for each observer under each experimental condition are presented in Appendix C.  $24$ 

## Table 1

Estimated values of  $P(A_1|S_1)$  and  $P(A_1|S_0)$  for each observer under each experimental condition in Experiment I

Obs. 1 Obs. 2 Obs. 3 Obs. 4 Obs. 5 Obs. 6  $x^{\circ}$   $\hat{P}(A_{1}|S_{1})$   $\hat{P}(A_{1}|S_{0})$   $\hat{P}(A_{1}|S_{0})$   $\hat{P}(A_{1}|S_{1})$   $\hat{P}(A_{1}|S_{0})$   $\hat{P}(A_{1}|S_{0})$   $\hat{P}(A_{1}|S_{1})$   $\hat{P}(A_{1}|S_{0})$  $.26<sup>2</sup>$  $.26 +$  $.266$  $.26 \ 8$ .26 10 .13 2  $.13 +$  $.13 \, 6$  $-13$   $8$ .13 10 .72 .67<br>.56  $•52$ .45' .20 .22 .29  $.34$ .32 .01 .01 .02  $.05.$ .10 .oo .03 .o; .13  $\overline{.11}$ .83 .82 • 70}  $.66$ .66 .67 .58<br>.55 .60<br>.55 .36  $.43$ -37 - 37<br>- 48 .44  $.41$ . 44<br>. 44 .86 .89  $.84$  $.82$ .88  $.54$ .;J. .61  $.64$  $.74$ .08 • 88 .19 • 82  $.23 - .76$  $.41$   $.78$  $.49$   $.72$  $.12$   $.58$ .19 .63 .26 .65<br>.44 .59<br>.47 .57 .24  $.32$  $\cdot \, \frac{38}{36}$ .50 .43 .29 .4o .41<br>.45 .49  $.90$ <br> $.85$  $\cdot 77$  $-75$ .71 .70 -73  $.66$  $.63$ <br> $.55$ .16 .23  $.27$ .30 .30 .28 .28  $•33$  $.38$ .36 .99 .98- .91  $\frac{.91}{.85}$  $-93$ .88  $.83$  $\frac{.77}{.74}$ .16 .26  $-33$ .39  $.41$ .21  $-35$ . 46<br>. 48

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Estimated values or P(C) *tor* each observer under each experimental condition in Experiment I

yo .26 .26  $.26$  $•26$ .26 .13 .13 .13 .13 .13 xo 2  $\frac{1}{2}$ 6 8 10 2 4  $\tilde{\mathsf{e}}$ 8 10 Obs. 1 .86  $.83$ -77  $.74$ .68 .6o .60 .62 .60 .6o Obs. 2 -73  $.70$  $.66$  $.65$ .59 .61  $• 58$  $.58$  $.58$ • 5'6 Obs. 3 Obs. 4  $.89$   $.82$  $.85$   $.74$  $.80$  .  $.69$  $-70$   $-64$  $•69$   $•64$  $.70 - .64$  $.66$   $.62$  $.68$   $.62$ • 60 • 5'7  $-63$   $-54$ Obs. 5  $.87$ .81  $\cdot \frac{25}{20}$ .72 .70 ·71 .72 .66 .62 .59 Obs. 6 .92 .86 -79  $.76$ .72 .86  $\cdot 77$ -72  $.66$ .63

> 1\)  $\bullet$   $\overline{\phantom{a}}$

the lateral separation between the two lights  $(x)$ , and is considerably greater for the larger vertical discrepancy (y).

The effect of each independent variable on each observer's performance was evaluated by Chi-square homogeneity tests (Anderson and Goodman, 1957, p.97). Under the null hypothesis the expected number of  $A_4$  responses made to  $S_4$ , for i and j equal to 1 or o, is the same for all values of the independent variable. Assuming the null hypothesis to be true, the best estimate of the expected frequency of an  $A_i$  response to  $S_i$  is the average frequency over all values of the variable. Given these expected frequencies, a Chisquare statistic may be calculated in the usual manner. Both the lateral separation between the two lights  $(x)$  and the size of the vertical discrepancy to be discriminated (y) had a statistically significant effect (P<.OOl) on each observer's performance.

#### EXPERIMENT II

For each observer,  $\hat{P}(A_1|S_1)$  and  $\hat{P}(A_1|S_0)$  for each of the eight experimental conditions are presented in Table  $3,$ <sup>5</sup> and  $\hat{P}(C)$  in Table 4. Note that  $\hat{P}(C)$  tends to decrease as the temporal interval between the two lights (t} is increased, and is generally greater for the smaller lateral separation  $(x)$ .

<sup>4</sup>The results of the Chi-square tests are presented in Appendix D.

 $5$ The daily hit and false alarm frequencies for each observer under each experimental condition are presented in Appendix C.

# Table 3

**Estimated values of P(A<sub>1</sub>|S<sub>1</sub>)** and P(A<sub>1</sub> $|S_0\rangle$ ) for each observer under each experimental condition in Experiment II

Obs. 1 Obs. 2 Obs. 3 Obs. 4 Obs. 5 Obs. 6  $\bar{x}^{\circ}$  tsec $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$ 6 .000 .60  $6 \t333 \t.54$  $6.667.47$ 6 .000 .60<br>6 .333 .54<br>6 .667 .47<br>6 1.000 .41 10 .000 .63<br>
10 .333 .42<br>
10 .667 .49<br>
10 1.000 .39 10 ·333 .42 10 .667 .49 10 1.000 .39 .04 .06 .06 .09 .16 .10 .12 .11 ' . .67  $.64$ .63 .56 .69 .66  $.63$ -57 .4o . 42<br>. 42 .41 .43 .41  $\frac{1}{50}$  $.42$ .85  $.69$ .56  $• 52$  $.87$ .83  $\cdot 7^{\mathsf{L}}$  $62$  $.29$   $.76$  $.26$   $.74$ -31 . • 65 .28 • *70*  • 56 • 75 • 55 • 77 .47 *,.7'5*   $.34.$   $.64.$ -37  $.52$  $-45$ .52 • *'50*   $-55$ . 62<br>. 54 .84 • 77  $.66$ .60 .69 .69  $.67$  $.62$ .22 .28  $.27$  $.34$ .40  $\cdot$ 39 .36 -93 .91  $.85$  $.74$  $.89$  $.82$ <br> $.78$ -70  $.34$  $\cdot$ 33  $.31$ ·37  $.43$  $.44$  $.38$ -37

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# Table 4

Estimated values of P(C) for each observer under each experimental condition in Experiment II



 $\frac{8}{3}$ 

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As in Experiment I, the effect of each independent variable on each observer''s performance was evaluated by Chisquare homogeneity tests. With one exception, both the temporal delay (t) and the lateral separation  $(x)$  had a statistically significant effect (P<.01) on each observer's performance.<sup>6</sup> Manipulation of the lateral displacement between the two lights did not produce a significant influence (P>.05) on the performance of Observer 2. However the overall Chi-square for x, computed by summing the values for each observer, was significant (P<.001).

<sup>6</sup>The results of the Chi-square tests are presented in Appendix D.

 $30<sub>l</sub>$ 

### THEORETICAL ANALYSIS AND DISCUSSION

## Effects Of The Independent Variables On Sensitivity  $(\delta_t, \lambda)$

For each observer an estimate of  $\delta_{tx}^{\phantom{\dag}},$  denoted  $\hat{\delta}_{tx}^{\phantom{\dag}},$ can be obtained for each experimental condition from a table of normal deviates. Specifically,

 $\hat{\delta}_{\text{tx}} = \hat{P}_{z}(A_{1} | S_{0}) - \hat{P}_{z}(A_{1} | S_{1})$ 

where  $\hat{P}_{z}(A_1 | S_0)$  is that value of a normal deviate which is exceeded with a probability  $\hat{P}(A_1|S_0)$ , and  $\hat{P}_z(A_1|S_1)$  is a similar transformation of  $\hat{P}(A_1|S_1)$ . These estimates of  $\delta_{tx}$ are presented as data points in Figure 4, and numerically in Tables  $5$  and  $6.$  It is clear that in general, the independent variables have a systematic effect on  $\delta_{tx}$  for each observer. In Experiment II, the relationship between the two independent variables, x and t, would have been more clearly demonstrated if the difference between the two values of x was greater. Note that  $\delta_{tx}$  appears to be a more sensitive measure than A<br>P(C) in that variations in the independent variables that *A* do not produce an apparent change in P(.C), clearly affect  $\hat{\delta}_{\texttt{tx}}$ . For example, examine the performance of Observer 1 in Experiment I, y equal to .13 degrees. As x is increased from 2 degrees to 10 degrees,  $\delta_{\text{tx}}$  decreases from 1.48 to .76 (Table 5), while  $\hat{P}(C)$  varies only slightly around .60 (Table 2)·.

. 31

deg $^2$ / sec.



Figure 4a. Estimated (points) and predicted (lines) values of  $\delta_{tx}$  for Observer 1 under each experimental condition in the two experiments {2-parameter form of the model)





Figure 4b. Same as la for Observer 2



Figure 4c. Same as 4a for Observer 3





#### Same as the for Observer 4 Figure 4d.







# Figure 4f. Same as 4a for Observer 6

 $\overline{\phantom{a}}$ 

**Estimated and predicted (2 parameters) values of**  $6$ **<sub>tx</sub> for each observer under each experimental condition in Experiment I** 



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**Estimated and predicted (2 parameters) values of**  $\delta_{tx}$  **for each observer under each experimental condition in** Experiment II

 $Obs. 1$ Obs. 2  $Obs.3$ Obs.  $4$ Obs.  $5$ Obs. 6 Pred  $\delta$ <sub>tx</sub>  $\hat{\delta}_{\mathbf{tx}}$  $\widetilde{\delta}_{\mathtt{tx}}$  $x^{\alpha}$  t sec.  $\delta_{tx}$ Pred  $\delta_{tx}$ Pred  $\delta_{tx}$ Pred  $\delta_{tx}$  $\hat{\delta}_{\mathbf{tx}}$  $\hat{\delta}_{\texttt{tx}}$ Pred  $\delta_{tx}$  $\hat{\delta}_{\texttt{tx}}$ Pred  $\delta_{tx}$  $1.88$ <br> $1.53$ <br> $1.32$ <br> $1.18$  $.70$ <br> $.56$ <br> $.53$ <br> $.38$  $1.60$ <br> $1.14$ <br>.66<br>.63  $1.53$ <br>.99<br>.79<br>.67  $1.04$ <br> $.59$ <br> $.52$ <br> $.48$  $.000$ 2.00 **666**  $-79$ <br> $-58$ <br> $-48$ <br> $+2$ 1.02 1.76 1.42 1.88  $2.90$  $: 333$ <br> $: 667$ <br>1.000  $1.65$ <br> $1.48$  $-61$ <br> $-47$ <br> $-40$  $1.78$ <br> $1.54$ <br> $.97$  $1.32$ 1.08  $1.53$  $.90$ <br> $.79$  $1.02$  $\frac{1}{1.29}$ <br>1.13 1.11  $.66$  $.68$ <br> $.64$ <br> $.33$ <br> $.38$  $1.45$ <br> $1.27$  $.000$ 1.32  $.61$ <br> $.50$ <br> $.38$  $.98  
.82  
.72$  $1.19$ <br>.88<br>.73<br>.73<br>.63  $.68$ <br> $.61$ <br> $.37$ <br> $.26$ 10  $-79$ <br> $-54$ <br> $-38$  $.94$ <br> $.76$ <br> $.72$ <br> $.66$ 1.10  $1.41$ 1.55  $.333$ <br>.667 1.08 10  $.92$ <br> $.80$ <br> $.72$ 1.06  $1.30$  $\overline{1.15}$ <br> $1.05$ 10<br>10<br>10 1.15 1.08  $1.14$ 1.000  $.95$  $.86$ 1.03

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In order to evaluate the extent to which the estimates of  $\delta_{tx}$  presented in Tables 5 and 6 are consistant with the values of  $\delta_{tx}$  predicted by the model (Eq. 14), estimates of the diffusion rates,  $\phi_t$  and  $\phi_x$ , and of the input variance, K, have to be determined for each observer. If an observer's sensitivity does not vary between the two experiments, then these three parameters can be estimated from his performance in both experiments. Note that two of the experimental conditions in Experiment I were duplicated in Experiment II (x equal to  $6^\circ$ , y equal to .26 $^\circ$ , t equal to 0 sec.; and x equal to  $10^{\circ}$ , y equal to .26 $^{\circ}$ , t equal to 0 sec.). A statistical test proposed by Gourevitch and Galanter (1967) was used to determine whether an observer's sensitivity under these two conditions in Experiment I differs significantly from his sensitivity under the corresponding conditions in Experiment II. Specifically, the null hypothesis tested was that an estimate of  $\delta_{tx}$  obtained in Experiment I does not differ significantly from the estimate of  $\delta_{tx}$  obtained for the same experimental condition in Experiment II. For each observer, two such tests were conducted. In only one case (Observer 5; x equal to 6°, y equal to .26°, t equal to 0 sec.) could the null hypothesis be rejected  $(p < 01)$ .<sup>7</sup> On the basis of these results, it can be argued that, in general, the variability in sensitivity between experiments is negligible.

7The results of all these tests are presented in Appendix B.

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Thus  $\phi_t$ ,  $\phi_x$ , and K were estimated from an observer's performance in both experiments. These estimates, denoted respectively as  $\hat{\phi}_t$ ,  $\hat{\phi}_x$ , and  $\hat{k}$ , are listed in Table 7 for each observer. They were chosen to minimize the sum of squared discrepancies between the 18 estimates of  $\delta_{\pm x}$  obtained from the two experiments for a particular observer, and the cor responding values predicted by Eq. 14 with  $\widehat{\phi}_t^*, \widehat{\phi}_x^*$ , and  $\widehat{\kappa}$ substituted for  $\phi_t$ ,  $\phi_x$ , and K.

One way to test how well the model accounts for changes in sensitivity under the various experimental conditions is in terms of the proportion of the variance in  $\delta_{tx}$ accounted for by the predicted  $\delta_{tx}$  values. The variance of the 18  $\hat{\delta}_{tx}$  values around their mean can be regarded as an estimate of the total variance in the dependent variable. An estimate of the unpredicted (or residual) variance in  $\delta_{\text{tx}}$ is simply the variance of the 18  $\hat{\delta}_{\texttt{tx}}$  values about their predicted values minus the variance that can be attributed to sampling. Given an estimate or the observer's theoretical hit and false alarm probabilities, denoted  $\widehat{P}(A_1|S_1)$  and  $\hat{P}(A_1|S_0)$ , an estimate of the sampling variance can be obtained using the method discussed by Gourevitch and Galanter (1967). The pair of hit and false alarm probabilities on the predicted operating characteristic most similar to the observer's performance was regarded as an estimate *ot* his theoretical hit



Estimated values of  $\phi_t$ ,  $\phi_x$ , and K<br>for each observer

 $\ldots$  8 and false alarm probabilities. The proportion of the total variance accounted for by the model was calculated for each observer, and indicated that the model accounts on the aver age for .97 of the total variance in  $\hat{\delta}_{\texttt{tx}}$ : the actual values obtained for Observers 1 through 6 are respectively .97, 1.00, .98, 1.00, .95, and .91. Thus, the model accounts for virtually all of the variance in  $\delta_{xx}$ .

Note that the input variance,  $\hat{\mathbf{k}}$ , is negligible for all observers. In fact for Observers 4 and 6 the input variance is actually zero. This suggests an appropriate simplification of the model: let K equal zero by assumption, and fit the data with two parameters,  $\phi_t$  and  $\phi_x$ . The estimates of  $\phi_t$  and  $\phi_x$  listed in Table 8 were chosen to minimize the sum of squared discrepancies between the estimated  $\delta_{tx}$  values and those predicted by Eq. 14 with  $\hat{\phi}_t$  and  $\hat{\phi}_x$  substituted for  $\phi_t$ and  $\phi_x$  and K equal to zero. These predicted  $\delta_{tx}$  values, denoted Pred. $\delta_{\text{tx}}$ , are presented graphically in Figure  $4$ , and numerically in Tables 5 and 6.<sup>9</sup> It seems clear from Figure  $\frac{11}{2}$ that the 2-parameter form of the model provides a reasonable interpretation of each observer's performance. On the average .96 of the total variance is accounted for: the actual values obtained for Observers 1 through 6 are respectively, .96, 1.00,

 $^{\circ}$ Estimates of the theoretical hit and false alarm<br>probabilities for each observer under each experimental condition in both experiments are presented in Appendix F.

<sup>9</sup>The values of  $\delta_t$ , predicted by the 3-parameter form of the model are presented in Appendix G.

43.

# Estimated values of  $\phi$ , and  $\phi$  (2 parameter<br>form of the model) for each observer

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 $\hat{\phi}_t$  $\hat{\phi}_{\mathbf{x}}$  $\text{(deg}^2\text{/deg)}$  $(\deg^2/\sec)$  $.003$ <br> $.018$ <br> $.005$ <br> $.011$ <br> $.006$ <br> $.003$  $.029$ <br> $.278$ <br> $.120$ 358  $.074$ <br> $.036$ 

.98, 1.00, .94 and .91. $^{10}$  Note that the 2-parameter form of the model accounts for essentially the same proportion of the total variance as the 3-parameter form. It is apparent that the two diffusion parameters allow a good prediction of  $6_{tx}$ under the 16 different experimental conditions.

In the development of the model it was assumed that Var(V<sub>0</sub>) and Var(V<sub>1</sub>), the variances of the two sensory distributions, were equal. Since the predictions of the two-parameter form of the model are consistent with the observed data, the equality of variance assumption appears to be reasonable, at least for the vertical position discrimination task considered.

## Estimates of Decision Criterion  $(C_{tx})$

Another theoretical question of interest is the relationship between variations in the values of the independent variables and an observer"s decision criterion,  $C_{\tau,\mathbf{x}^*}$ Note that  $\hat{P}(A_1|S_0)$  would by definition equal the area to the right of  $C_{tx}$  under the distribution of  $U_{tx}$  for an  $S_0$  stimulus pattern (See Figure 2).  $\hat{P}_z(A_1 | S_0)$ , that value of a normal deviate which is exceeded with a probability  $\hat{P}(A_1 | S_0)$ , can be obtained from a table of normal deviates. An estimate of the criterion, denoted  $\hat{G}_{tx}$ , in degrees visual angle is  $\operatorname{simply} \, \hat{\mathbb{P}}_{\mathbf{z}}(\mathbf{A}_1 | \, \mathbf{S}_Q)$ ) multiplied by the standard error of  $U_{\mathbf{t},\mathbf{y}}$ . That is,

10Estimates of the theoretical hit and false alarm probabilities (2-parameter form of the model) are presented<br>in Appendix F.

$$
\hat{c}_{tx} = \hat{P}_z (A_1 | s_0) G_{tx}
$$

These estimates may be thought of as the minimum discrepancy in degrees visual angle that must occur between the memory of the first stimulus and the position of the second stimulus in order for the observer to respond "different".

Estimates of each observer's criterion for each experimental condition are presented graphically in Figures 5 and 6, and numerically in Tables 9 and 10. Since an estimate of the sampling variance is not available, we can not determine whether the observed variance in  $\hat{c}_{tx}$  is greater than the variance that could be attributed to sampling. However, it is clear that, if part of the observed variance is attributable to variations in the independent variables, the relationship between the independent variables and  $\hat{c}_{tx}$  is idiosyncratic. For example, the decision criterion held by Observer 3 is apparently dependent upon the value of x, but independent of the value of y. Observer 5, however, seems to maintain a fairly stable criterion with variations in x, but holds a consistently smaller criterion for'the smaller value of y. Also note, that for Observer 3 the decision criterion is positively correlated with t, while for Observer  $4$  the correlation is negative. Thus the decision theory approach to the analysis of the data allows the removal of such idiosyncratic sources of variability from the estimates of the observer's sensitivity.



Figure 5. Estimated values of  $C_{tx}$  for each observer under each experimental condition in Experiment I



Figure 6. Estimated values of  $C_{tx}$  for each observer under each experimental condition in Experiment II

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Estimated values of  $C_{1}$  for each observer under each  ${\tt experimental\ condition}$ Xin Experiment I



*\$* 

**Estimated values of**  $C_{tx}$  **for each observer under each experimental condition in Experiment II** 



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### Sequential Properties Of The Data

Another feature of the data which is of interest are the sequential fluctuations in hits and false alarms. The sequential effects of particular interest concern the correlation between the response on trial n and the stimulus and response events on trial n-1. For each observer the proportion of  $A_1$  responses on trial n contingent upon the stimulus and the response on trial n-1 as well as upon the stimulus event on trial n was determined. These proportions were treated as estimates of the following conditional probabilities:

$$
P(A_1, n | S_1, n | A_1, n-1 | S_1, n-1)
$$
\n(15)

$$
P(A_1, n \mid S_0, n \mid A_j, n-1 \mid S_1, n-1)
$$
 (16)

for i and j equal to 1 or 0. The notation in Expressions 15 and 16 can be simplified by omitting the trial subscripts (n and n-1). In the remainder of the paper, the temporal order of events in the sequential statistics should be interpreted as corresponding to that in Expressions 15 and 16. Estimates of  $P(A_1 | S_1A_1S_1)$  and  $P(A_1 | S_0A_1S_1)$ , denoted as  $\hat{P}(A_1 | S_1A_jS_i)$  and  $\hat{P}(A_1 | S_0A_jS_i)$ , for each experimental condition averaged over the six observers are presented in Tables 11 and 12.<sup>11</sup> These estimates are presented onoperating-characteristic

 $^{11}$ The estimates for individual observers, as well as the number of observations each estimate is based on, are pre-<br>sented in Appendix H.

 $x=2^{\circ}$ 

Estimates of sequential statistics for each experimental condition in Experiment I

 $x = 4^\circ$  $x=8^\circ$  $x=6^\circ$  $x=10^\circ$  $\hat{P}(A_1|S_1 - \hat{P}(A_1|S_0 - \hat{P}(A_1|S_1 - \hat{P}(A_1|S_0 - \hat{P}(A_1|S_1 - \hat{P}(A_1|S_0 - \hat{P}(A_1|S_0$  $.95$ <br> $.81$ <br> $.86$ <br> $.91$ <br> $.82$  $.34$ <br> $.18$ <br> $.13$ <br> $.11$ <br> $.20$ <br> $.12$  $-88058$ <br> $-88658$ <br> $-8888$ <br> $-78$  $.30$ <br> $.39$ <br> $.12$ <br> $.33$ <br> $.17$  $.878$ <br> $.78$ <br> $.701$ <br> $.80$ <br> $.66$  $.49$ <br> $.23$ <br> $.14$ <br> $.22$  $.858$ <br> $.786$ <br> $.550$ <br> $.802$  $.44$  $.90$  $-50$ <br> $-28$ <br> $-23$ <br> $-14$ <br> $-26$  $.27$ <br> $.17$ <br> $.129$ <br> $.17$  $.81$ <br> $.73$ <br> $.51$ <br> $.82$  $\frac{4}{4}$  and set  $\frac{4}{4}$  and  $\frac{4}{4$  $y = .26°$  $.70$  $.81$ <br> $.70$ <br> $.754$ <br> $.726$ <br> $.56$  $.41$ <br> $.27$ <br> $.18$ <br> $.14$ <br> $.29$ <br> $.16$  $.7714521$  $.48$ <br> $.36$ <br> $.18$ <br> $.37$ <br> $.20$  $-37$ <br> $-37$ <br> $-140$ <br> $-22$ <br> $-162$  $A_{1S}^{S}$ <br>  $A_{1S}^{S}$ <br>  $A_{0S}^{S}$ <br>  $A_{0}^{S}$ <br>  $A_{1}^{S}$  $-73$ <br> $-59$ <br> $-52$ <br> $-70$ <br> $-50$  $-73$ <br> $-65$ <br> $-53$ <br> $-53$  $-50$ <br> $-128$ <br> $-28$ <br> $-128$ <br> $-28$  $-543$ <br> $-130$ <br> $-130$ <br> $-30$ <br> $-30$  $-21$ <br> $-69$ <br> $-47$ <br> $-70$ <br> $-48$  $y=.13^\circ$ 



 $\boldsymbol{\lambda}$ 

 $x=6^\circ$ 

 $x=10^{\circ}$ 

 $\mathfrak{L}$ 

 $(0)$  graphs plotted on double-probability paper<sup>12</sup> in Figures 7 and 8. Each point plotted for a given experimental condition characterizes performance on trials following each or the four possible combinations of stimulus and response. It is clear that the response on trial n is correlated with the stimulus and the response events on trial n-1. Note that the order of the four points in the OC space is consistent over all experimental conditions.

These trial-to-trial sequential effects are similar to those described by Kinchla (1966), and Tanner, Haller and Atkinson (1967) for an auditory recognition problem involving the comparison or stimulus events presented at different points in time. Parducci and Sandusky (1965) reported a study involving the comparison or the horizontal position or two vertically separated visual stimuli. Both the horizontal discrepancy and the vertical separation were constant throughout the experiment. According to Kinchla (1966)! a sequential analysis or the Parducci - Sandusky data revealed a similar ordering of points as in the auditory studies involving temporally noncontiguous stimuli. Thus, this relative ordering of the sequential

 $54.$ 

 $12$ Double-probability paper (also known as normalnormal paper) is graph paper in which the x and y co-ordinates have been transformed so that normal deviates are linearly spaced. If the underlying distributions are Gaussian, the set spaced. If the underlying distributions are Gaussian, the set of performances produced by changes in criterion with fixed sensitivity fall on a straight line. If the underlying distributions have equal variance, the straight line has unit slope.



Estimated values of  $P(A_1 | S_1 A_j S_1)$  and  $P(A_1 | S_0 A_j S_1)$  for Figure 7. each experimental condition in Experiment I

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**Figure 8.** Estimated values of  $P(A_1 | S_1 A_j S_j)$  and  $P(A_1 | S_0 A_j S_j)$ for each experimental condition in Experiment II

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statistics is observed in a number of recognition studies, suggesting that strong sequential effects occur consistently in tasks involving temporally or spatially noncontiguous stimuli.

Figures *7* and 8 suggest that, in general, the response on trial n is more highly correlated with the response on trial n-1 than the stimulus on trial n-1~ Tables 11 and 12 present estimates of  $P(A_1, n|S_1, nA_j, n-1)$ and  $P(A_1, n | S_0, n A_j, n-1)$ , denoted as  $\hat{P}(A_1 | S_1 A_j)$  and  $\widehat{P}(A_1 | S_0 A_1)$  for each experimental condition averaged over the six observers.<sup>13</sup> These estimates are plotted on double probability paper in Figures 9 and 10. Each point plotted for a given experimental condition characterizes performance on trials following each of the two possible responses on trial n-1. The order and the spacing of the two points in the OC space is consistent over all experimental conditions. For each experimental condition a straight line with unit slope provides a very good fit to the two points. Thus, the sequential effects can be interpreted as shifts in decision criterion along the same OC curve. The straight line, unit slope, operating characteristic suggests that the two distributions of  $U_{tx}$  (see Figure 2) are Gaussian and have equal variance. This is consistent with the assumptions of the model.

 $^{13}$ The estimates for individual observers, as well as the number of observations each estimate is based on, are pre-<br>sented in Appendix I.



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**Figure 10.** Estimated values of  $P(A_1 | S_1 A_j)$  and  $P(A_1 | S_0 A_j)$ for each experimental condition in Experiment II

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For each experimental condition the OC curve for the response contingent sequential effects is replotted in Figures 7 and 8, and provides an adequate fit to the four points. The most deviant points are generally the most unreliable estimates in that they are based on few observations. It appears then that the observer's decision criterion on trial n is contingent on both the stimulus and response events on trial n-1.

The model developed in this paper does not specify that the observer shifts his criterion from trial to trial. However, it is clear that since the model accounts for virt- *<sup>1</sup>* ually all of the variance in the estimated  $\delta_{tx}$  values, the variance due to fluctuations in criterion does not significantly influence  $\delta_{tx}$ . This point can be made more explicitly in terms of the model. Suppose the criterion was not constant but actually had a Gaussian distribution with mean zero and variance  $\phi_c$ . Then,

$$
\delta_{tx} = \frac{y}{(\phi_t t + \phi_x x + K + \phi_c)^{\frac{1}{2}}}.
$$

The results indicate that  $\phi_c$  is so small relative to the other components of the variance in the denominator of the preceeding equation that a good fit between model and data may be achieved with the assumption that  $\phi_c$  equals zero (as in Eq.  $14$ ). It would be of theoretical interest to elaborate the present model to include a dynamic decision process which specifies the manner in which an observer shifts his criterion from trial to trial.

### CONCLUSION

It is clear that the results of the two experiments are generally consistent with the predictions of the model. An observer's ability to compare the vertical position of two points of light presented successively in the dark is reduced.by both a temporal delay between their presentation and a lateral separation in their position. The loss of vertical position information during the temporal delay is similar to the loss over a lateral translation in that both can be represented by a random walk process. When the transformation is made simultaneously over both a temporal and a lateral separation, the random walk processes appear to proceed simultaneously but independently, and the cumulative loss of information is simply the sum of both walks. Note that two estimates,  $\hat{\phi}_t$  and  $\hat{\phi}_x$ , account on the average for 96 per cent of the variance in  $\delta_{tx}$  for each observer.

It should be emphasized that the discrimination problem considered is somewhat unique in that the loss or vertical position information represented by the theoretical random walk processes may to a large extent reflect overt, involuntary eye movements. The drift components or involuntary eye movements in the dark suggest a random walk process, and probably are the result or instability in the oculomotor system

(Cornsweet, 1956). Thus, it is conceivable that the loss of vertical position information during the interstimulus inter val,  $\hat{\phi}_+$ , is due, at least in part, to the instability of the oculomotor system. Similarly, the loss of vertical position information during the lateral translation,  $\widehat{\Phi}_{\mathbf{x}}$ , might also be attributable, in part, to the instability of the oculomotor system. Unfortunately, there are no reports of direct measurements of eye movements in an experimental situation similar to that used in the present experiments.

It would be of value to apply the model to data from a discrimination problem which is not affected by involuntary eye movements. For example, a task involving the comparison of the brightness of two objects would be appropriate. Specifically, the observer is required to discriminate a difference in brightness between two laterally separated points of light presented successively in the dark. On some trials the two lights would be equated in intensity, on others, they would differ by an amount y. In this way it would be possible to test the model in a situation where the variance introduced by a temporal delay and a lateral separation could not be attributed to involuntary eye movements.

Note that this approach would allow one to determine the relation between "brightness memory"' and stimulus intensity. According to Weber's Law, an observer's ability to discriminate a change in brightness is related in a systematic manner to the intensity of the stimulus being incremented. Specifically,

where I is the intensity of the original stimulus,  $\Delta I$  is the increment in intensity necessary for the observer to detect a change in brightness, and K is a constant. It has been shown that Weber's Law holds for a fairly wide range of intensities (Dember, 1960). Thus, on the basis of Weber's Law we would expect variations in the intensity of the initial stimulus ( $y_0$ ) to influence  $\delta_{tx}$ . This suggests that the rate at which brightness information is lost over time  $(\phi_t)$  and over space  $(\phi_x)$  is dependent on the value of  $y_0$ . That is, an observer's ability to "remember" the brightness of an object may be related to the intensity of the object.

An interesting application of the model would be in animal psychophysics. There has been a great deal of interest in an animal's ability to discriminate stimuli which are separated in time. For example, a number of investigators have studied the acquisition of a "delayed matching-to-sample" problem by the pigeon (Berryman, Cumming, and Nevin, 1963; Blough, 1959; Cumming and Berryman, 1965). In general, the animal in this problem situation is required to select that one of two "choice stimuli" which is the same as the "sample stimulus". A temporal delay can be introduced between the offset of the sample stimulus and the presentation of the choice stimuli. It would be interesting to determine whether the predictions of the model are consistent with the poorer

 $\frac{\Delta I}{I}$  = K

matching performance observed in these studies as the temporal delay is increased. Unfortunately, the data are in a form not amenable to a decision theory analysis.

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## APPENDIX A

Derivation of the expected value and variance of  $D_t$  and  $D_x$ .

# DERIVATION OF THE EXPECTED VALUE,<br>E(D<sub>t</sub>), AND VARIANCE, Var(D<sub>t</sub>), of D<sub>t</sub>.

The memory process specifies that during the t second interstimulus interval one step in the random walk occurs every  $^1\!k$  seconds. Each step increases the value in memory by an amount s with probability p, or decreases it by the same amount with probability (1-p). S is a random variable which represents the effect of the random walk after 1 step. Thus, I'

> $S = \begin{cases} +s & \text{if the} \end{cases}$ -s if the step is positive. step is negative.

Therefore,

$$
E(S) = \frac{2}{k} kP(S=k) \text{ where } k \text{ equals } +s \text{ or } -s
$$

$$
= sp - s(1-p)
$$

$$
= s(2p-1)
$$

and,

Var(S) = 
$$
\frac{2}{k}k^2P(S=k) - [E(S)]^2
$$
  
=  $s^2p+s^2(1-p)-s^2(2p-1)^2$   
=  $4s^2p(1-p)$ 

 $D_t$  was defined as a random variable representing the cumulative effect of the random walk after t seconds. Thus,  $E(D_t) = \frac{\mathcal{L}}{\zeta r t}$  is the number of steps in t seconds

 $=$  rts(2p-1)

and,

$$
Var(D_t) = \frac{2}{\tau r t}Var(S)
$$
  
=  $4rts^2p(1-p)$ 

If the random walk is symmetrical (that is, if  $p=5$ ), then,

$$
E(D_t) = 0
$$

and,

$$
Var(D_t) = rts^2
$$
  
=  $\phi_t t$  where  $\phi_t$  equals  $rs^2$ 

# DERIVATION OF THE EXPECTED VALUE,  $\overline{E(D_x)}$ , AND VARIANCE, Var(D<sub>x</sub>), of D<sub>x</sub>•

The memory process specifies that during the x degree translation one step in the random walk occurs every  $\frac{1}{\lambda}$  seconds. Each step increases the value in memory by an amount w with probability q, or decreases it by the same amount with probability  $(1-q)$ . The actual derivations of  $E(D_x)$  and Var(D<sub>x</sub>) are identical to those of  $E(D_t)$  and Var(D<sub>t</sub>).

### APPENDIX B

# Each observer read the following set of instructions.

#### INSTRUCTIONS

This experiment is one of a series of studies designed to contribute to our understanding of man''s perceptual system. The room will be darkened during the experiment. At the beginning of each trial you will hear a tone. This is to ensure that you will be maximally attentive for the stimuli about to be presented. At the offset of the tone two small dots of light (the Standard Stimulus) will be flashed at the front of the room. At the offset of the Standard Stimulus another small dot of light (the Comparison Stimulus). will appear to the right of the Standard. Your task is to indicate whether the Comparison Stimulus was on the SAME horizontal level as the Standard, or was LOWER than the Standard.

You'll notice that the arm of the chair in which you are sitting has four response buttons. During this experiment only two of these buttons will be used. You are to press the upper left button if the Comparison Stimulus was the same as the Standard, and the lower left button if the Comparison Stimulus was lower than the Standard. You are to make your decison and press one of the buttons immediately after the Comparison Stimulus has been terminated. It is essential that you make a response on each trial, even if this entails guessing.

This experiment will be conducted in blocks of 50 trials, with a one minute rest between blocks. The onset of

the tone will indicate that the rest period is over. An. equal number of SAME and LOWER Comparison Stimuli will be presented during each block of 50 trials. The order in which these Comparison Stimuli are presented is a random one.

The $^>$ spatial distance between the Standard Stimulus and the Comparison Stimulus, and the time interval between the offset of the Standard and the onset of the Comparison is constant within any block of'50 trials. A number of different spatial distances and time intervals may be used during an experimental session. However, these variations do not alter the basic task as far as you are concerned.

An intercom connects the dark room with the experimenter's room. Therefore, if you wish to tell me something that is relevant to the experiment, you merely have to speak. I'll be able to hear you and answer you.

Are there any questions? It is essential that you understand the instructions.

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#### APPENDIX C

This section presents the daily hit (H) and false alarm (FA) frequencies for each observer under each of the ten experimental conditions in Experiment I, and each of the eight experimental conditions in<br>Experiment II. As described earlier, the sequence of experimental conditions was randomly determined within each session. Each frequency indicates the number of  $A_1$  responses the observer made to the 25  $S_1$  stimulus patterns (hits), or to the 25  $S_0$  stimulus patterns (false alarms) in each 50 trial block under a specific experimental condition. Since failures to respond were infrequent, it can be assumed that if the observer did not make an  $A_1$  response, he made an  $A_0$  response.



Daily hit and false alarm frequencies for Observer 1 under each experimental condition in Experiment <sup>I</sup>

> $\overline{\phantom{a}}$ ~



 $\ddot{\phantom{a}}$ 

Daily hit and false alarm frequencies for Observer 1 under each experimental condition in Experiment II

> ~  $\mathbf{v}$



Daily hit and false alarm frequencies for Observer 2 under each experimental condition in Experiment I

 $\approx$ 



Daily hit and false alarm frequencies for Observer 2 party into and rathe dictm frequencies for esserver under each experimental condition in Experiment II

 $\boldsymbol{\mathcal{Z}}$ 



Daily hit and false alarm frequencies for Observer 3 under each experimental condition in Experiment I

 $\approx$ 



Daily hit and false alarm frequencies for Observer 3 under each experimental condition in Experiment II

 $\mathfrak{F}$ 

Daily hit and false alarm frequencies for Observer 4 under each experimental condition in Experiment I

 $y = .260$   $y = .130$ 



 $\infty$  $\mathsf{C}$ 



Daily hit and false alarm frequencies for Observer 4 under each experimental condition in Experiment II

CD *......* 



Daily hit and false alarm frequencies for Observer 5 under each experimental condition in Experiment I

82



Daily hit and false alarm frequencies for Observer 5 under each experimental condition in Experiment II

> $\infty$ ~



Daily hit and false alarm frequencies for Observer 6 under each experimental condition in Experiment I

> $\infty$  $\mathbf{f}$



Daily hit and false alarm frequencies for Observer 6 under each experimental condition in Experiment II

<u>8</u>

#### APPENDIX D

Summary of the Chi-square homogeneity tests on the effect of each independent variable on each observer's performance in the two experiments.

Summary of the Chi-square homogeneity tests on the effect of x and y on each observer's performance in Experiment I



 $6$  ...  $159.68**$  ...  $77.56**$ 

\*\*  $p<.001$ 

Summary of the Chi-square homogeneity tests on the effect of t and x on each observer's performance in Experiment II



 $p<.01$ 

## APPENDIX E

### Summary of the Gourevitch-Galanter (1967) tests for variability in sensitivity between the two experiments.

# Summary of the Gourevitch-Galanter tests



\*  $p < 01$ 

Condition A:  $x=6^\circ$ ,  $y=.26^\circ$ ,  $t=0$  sec. Condition B:  $x=10^{\circ}$ ,  $y=.26^{\circ}$ ,  $t=0$  sec. 90

#### APPENDIX F

Estimates of the theoretical hit and false alarm probabilities,  $\hat{P}(A_1 | S_1)$  and  $\hat{P}(A_1 | S_0)$ , for each observer under each experimental condition in the two experiments. Estimates based on both the 3 parameter and the 2-parameter forms of the model are presented.



r

Obs. 1  $\cosh 2$   $\cosh 3$   $\cosh 4$   $\cosh 5$   $\cosh 6$  $y^{\circ}$  x  $\hat{\mathcal{P}}(A_{1}|S_{1})$   $\hat{\mathcal{P}}(A_{1}|S_{0})$   $\hat{\mathcal{P}}(A_{1}|S_{0})$   $\hat{\mathcal{P}}(A_{1}|S_{1})$   $\hat{\mathcal{P}}(A_{1}|S_{0})$   $\hat{\mathcal{P}}(A_{1}|S_{1})$   $\hat{\mathcal{P}}(A_{1}|S_{1})$   $\hat{\mathcal{P}}(A_{1}|S_{0})$  $.26<sub>2</sub>$  $.26 +$  $.26 \t 6 =$ <br>.26 8 .26 10 .13 2  $\begin{array}{r} .13 \\ .13 \end{array}$  6 .13 8 .13 10 .78 .61  $.52$  $\frac{52}{1}$ -53 .22 .23  $.28$ ·37 .32 .01 .02 .03  $.05$  $-07$ .01 .03  $.06$ .12 .11 .83  $.80$ .69  $.65$ .69 .69 • 59  $\frac{54}{1}$ • 59 .56  $.96$  $\frac{1}{2}$ .  $.38$  $-38$ .. 45 .44  $.40$  $.38$ .1+5 .43 .86  $.87$  $.82$  $.85$ .88 .54  $-51$  $.58$  $.67$  $.72$ .07  $.22$  $.26$ - 37<br>- 48 .12 .18  $.28$ .41 .49 .87 .80 *.16*   $\cdot 79$ -73 .61  $.64$ .63 .61 .60 .26  $.34$  $.38$  $•<sup>47</sup>$ .43 .2? .40 . 44<br>. 44 .90  $.85$ • 79 .76 .72 .71  $.68$  $.63$  $.6\overline{3}$ • 56 .15  $\sqrt{24}$  $-25$ .29 .29 .27  $.3^{4}$  $-35$  $-38$ .35 .99 .97  $.93$  $.92$ .88 .88  $.84$  $\frac{.79}{.78}$  $.75$ .13  $.28$  $.30$ - 37<br>- 35 .29 •. 41  $-42$  $.46$  $.46$ 

 $\widehat{P}(A_1|S_1)$  and  $\widehat{P}(A_1|S_0)$  for each observer under each experimental condition in Experiment II (3-parameter form of the model)

Obs. 1 Obs. 2  $\,$  Obs. 3  $\,$  Obs. 4 .. Obs. 5 Obs. 6  $x^{\circ}$ t sec.  $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$   $\hat{P}(A_1|S_1)$   $\hat{P}(A_1|S_0)$ 6 ~000  $5 \cdot 333$  $6.667$ 6 1.000 10 .000<br>10 .333<br>10 .667 .!.0 • 333 10 • 667 101.000 .58 . 44<br>. 41 .67  $.47$ .49 .41 .04  $.07$  $.07$ .08 .14 .08 .12 .10 .69  $.65$  $.62$  $.56$ .68 .63  $.65$ .57 .38 .42  $.43$ .39 .44 - 48<br>- 42 .85 .66 .59 • 53 .89  $.84$  $.74$  $.60$ .30  $.28$ <br> $.29$  $.27$ .50  $-54$  $.47$ .36 .76  $.75$ <br> $.64$ .68 .76  $.25$ ·77 .• 66 .38 . 46<br>• 53  $.47$  $-55$  $.62$ .51 .80 .73  $.63$  $.62$ .73 .73 .68 .63 .27  $-32$  $.28$  $•32$ .30  $-37$ - 37<br>- 35  $.94$ .89  $.82$ .76 .90  $.85$  $• 79$ • 73  $.33$ <br> $.38$  $.35'$ <br> $.34'$ .4o .4o  $-37$ ·33

 $\mathfrak{F}$ 

 $\widehat{P}(A_1|S_1)$  and  $\widehat{P}(A_1|S_0)$  for each observer under each experimental condition in Experiment I (2-parameter<br>form of the model)



~



 $\widehat{P}(A_1|S_1)$  and  $\widehat{P}(A_1|S_0)$  for each observer under each  $P(A_1|S_1)$  and  $P(A_1|S_0)$  for each observer under each experimental condition in Experiment II (2-parameter form of the model)

> ~  $\mathbf{v}$

#### APPENDIX G

Predicted values of  $\delta_{\text{tx}}$  (3-parameter form of the model) for each observer under each experimental condition in the two experiments.



Predicted values of  $\delta_{\text{ex}}$  (3-parameter form of the model) for each observer under each experimental condition in Experiment I

 $\mathcal{L}6$ 



Predicted values of  $\delta_{t,x}$  (3-parameter form of the model) for<br>each observer under each experimental condition in Experiment II

 $\frac{8}{6}$ 

#### APPENDIX H

Estimates of  $P(A_1 | S_1 A, S_1)$  and  $P(A_1 | S_0 A, S_1)$  for individual observers under each experimental condition in the two experiments. The number of trials, N, each estimate is based on is also disted. Note that if  $N \leq 10$ , no value of the statistic was estimated  $(-).$ 

Experiment I

 $x=2^{\circ}$ 

 $y = .26°$ 

 $y = .13^{\circ}$ 



Experiment I

 $x=4^\circ$ 

 $y = .26^{\circ}$ 

 $y = .13^{\circ}$ 


Experiment I

 $x=6^\circ$ 

 $y = .26°$ 

 $\hat{\boldsymbol{\beta}}$ 

 $y=.13^\circ$ 



 $\ddot{\phantom{0}}$ 

 $x=8^{\circ}$ 

 $y = .26^{\circ}$ 

 $y = -13^{\circ}$ 



 $x=10^{\circ}$ 

 $\epsilon_{\rm{B}}$ 

 $y = .26^{\circ}$ 

 $y = .13^\circ$ 



 $t=0$  sec.

 $\epsilon_{\rm g}$ 

 $x=6^\circ$ 

 $x=10^{\circ}$ 



 $\spadesuit$ 

## $t = .333 sec.$

 $\bar{z}$ 

 $x=6^{\circ}$ 



 $t = .667 sec.$ 

 $x=6^{\circ}$ 



 $t=1$  sec.

 $\bar{\mathcal{A}}$ 

 $x=6^\circ$ 

 $x=10^\circ$ 

 $\bar{\gamma}$ 



 $\ddot{\phantom{a}}$ 

 $\mathcal{L}$ 

#### APPENDIX I

Estimates of  $P(A_1 | S_1 A_1)$  and  $P(A_1 | S_0 A_1)$  for individual observers under each experimental condition in the two experiments. The number of trials, N, each estimate experiments. The number of<br>is based on is also listed.

# $x=2^{\circ}$



 $y = .13^{\circ}$ 



 $x=4$ °



## $x=6^{\circ}$

 $y = .26°$ 

 $y = .13^{\circ}$ 



## $x=8^{\circ}$

 $y = .26°$ 

 $y = .13^{\circ}$ 



۰

# $x=10^\circ$

 $y = .26^\alpha$ 

 $y = .13^{\circ}$ 



#### $t=0$  sec.

 $x=6^\circ$ 



### $t = .333$  sec.

 $x=6^{\circ}$ 



# $t = .667 sec.$

 $x=6^{\circ}$ 

 $x=10^\circ$ 



## $t=1$  sec.

 $x=6^{\circ}$ 

 $x=10^{\circ}$ 



 $118.$