EFFECTIVE INTERACTION IN FINITE NUCLEI

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SCOPE AND CONTENTS:

The solution of Bethe-Goldstone equation for finite nuclei is studied. Techniques of Nuclear Matter calculations are applied. A formalism is proposed to treat the low-lying discrete intermediate states. A shell-model calculation is then done with reduced matrix elements thus obtained and spectra obtained for O^{18} and F^{18} are compared with experiment. It is shown that the inclusion of the low-lying discrete states is significant.

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CHAPTER I

INTRODUCTION

Section 1.

In the last century, the advance of science and technology has been staggering. In little more than sixty years, man has progressed from the first fluttering flight to the conquest of the moon. The first explosion of the atomic bomb to the stock-piling of the nuclear powers took little more than two decades. In the realm of physics, there are discoveries, breakthroughs and inventions, too numerous to mention. However, there still are problems left unsolved. In this category, are the two basic questions of nuclear physics. What is the force that correlates the nucleons, and how are the nucleons correlated? Through the years, enormous efforts have been made in these directions and two distinct yet related fields of research are formed as a result. The first question led to the search for nuclear force, and the second one initiated efforts towards the solution of nuclear many-body problems.

Although no nuclear force which is free from phenomenology is available at the present time, some general features are now well-known about the two-nucleon interaction. The possibility of three-or-more-body forces cannot be denied. But, first of all, the two-nucleon interaction should be fully

understood before more ambitious projects are considered. We now understand that the origin of the nuclear force is from the exchange of different kinds of mesons, and attempts have been made to derive nuclear forces from meson theory. (See, Rev. of Mod. Phys. Vol. 39,#3, 1967, for general reference) However, meson theory has not yet reached a stage where a nuclear force may be derived completely, and we can only say something general about the force in reference to meson theory; the long ranges, for $r > 2$ fm, are dominated by the exchange of single π-mesons. This is the part that can be calculated accurately, since the coupling of π-mesons to nucleon is well-known. The intermediate ranges, for $r \simeq 1$ fm, are thought to be governed by the exchange of vector mesons ρ , ω , ϕ and something called the σ-meson, which may simply be a resonance of spin zero and isospin zero, or a contribution from multi-meson exchange. This intermediate range is still under intense investigation. Finally, at short distance, of about 0.5 fm, there is the core region about which little is known, except that it is repulsive, because experimentally, the S-wave phase-shifts change sign at high energy. So, we see that the two-nucleon interaction is by no means completely understood. One may wonder why the two-body force cannot be determined from the experimental two-nucleon data. The answer is that experimental data so far have provided on "on the energy shell" information when "off the energy shell" information is also required for the determination of the nucleon-nucleon inter-

action. We shall not go into any detail to explain this but only say that nucleon-nucleon scatterings in the free space involve energy-conserving processes whereas scatterings in a nuclear medium involve energy-nonconserving processes. Experimental studies of these energy-nonconserving processes, such as nucleon-nucleon bremstrahlung experiments have been done and more "off shell" information will be available in the future.

There are many other problems related to nucleon-nucleon interactions. We shall not be concerned with these in particular. Suffice it to say that enough is known about the interaction to make a calculation of the nuclear many-body problem realistic.

Section 2.

Even if the nuclear force can be derived from meson theory, many-body problems still face one almost insurmountable difficulty ------ the physicist himself. For he is incapable of solving problems involving more than two particles. Even the two-body problem, he is able solve only because it can be reduced to two one-body problems. With this intrinsic difficulty the many-body problem is really a study of approximations. The general procedure is to propose a theory for the problem, to test the theory with a simplified model of the system, and finally, to apply it to the physical system. Such is true for Brueckner's theory of finite nuclei. It has been tested in the

calculation of nuclear matter and is now being applied to finite nuclei. It is this application that we are interested in.

There are many difficulties in Brueckner's theory of finite nuclei. To mention a few, there are the double self-consistency, the particle spectrum, the solution of the Bethe-Goldstone equation and the treatment of the Pauli operator. In this thesis, we shall be concerned mainly with the solution of the Bethe-Goldstone equation and the treatment of the Pauli operator. The other difficulties are treated with the usual approximations. M. Baranger (1967) has made a very detailed summary on these in his lecture notes. We shall only make brief comments.

The double self-consistency refers to the Hartree-Fock self-consistency, and the Brueckner self-consistency. The former involves the choice of single particle occupied states and is a problem only in finite nuclei. The Brueckner self-consistency refers to the definition of the G-matrix in terms of the single particle potential U, which is unknown until G has been calculated. The treatment of this double self-consistency is still in a rather ambiguous state and we shall not go any further in this regard. The particle spectrum is really closely related to the Brueckner self-consistency and we shall also pursue no further, but shall bear in mind these difficulties in the application of Brueckner's theory.

With regard to the solution of B-G equation and the treatment of Pauli operator, we believe that these are insepa-

rable and should be dealt with together. The traditional methods of solution of the B-G equation are also summarized in the lecture notes of M. Baranger (1967). Very recently, S. Butler et al (1969) reported a new approximate method to the solution of B-G equation which seems very promising. We believe, however, that our approach to the problem brings out much more physical insight.

In Chapter II, we shall study the B-G equations. We shall propose a formalism which is particularly suited for finite nuclei, and which makes use of techniques used in nuclear matter calculations. Chapter III contains discussions on how the reduced integrals are calculated. We make a shell-model calculation with the two-body matrix elements obtained from Chapters II & III in Chapter IV. The results of calculations are then presented and comparisons made in Chapter V.

CHAPTER II

BETHE-GOLDSTONE EQUATIONS

Section 1.

In the theoretical study of nuclear properties with realistic forces, one is almost invariably faced with the problem of solving an integro-differential equation. In the case of free scattering, this equation is of the Lipmann-Schwinger type, and in the case of scattering in a medium, whether it be finite or infinite, this equation is of the Bethe-Goldstone type. The latter differs from the former in that it contains a 'Pauli' operator, which prevents certain intermediate states from entering the equation. Since we are not dealing with nucleon-nucleon scattering as such, our main concern is the solution of the Bethe-Goldstone equation. There are various methods of solving the equation; for example, the integral equation method of Brueckner and Gammel (1958), the separation method of Moszkowski and Scott (1960) and the reference spectrum method of Bethe et al (1963). The relative merits of these methods have been investigated in great detail by G. Dahl et al (1969). It is well-known that the Pauli operator and the energy denominator introduce great difficulties in solving the Bethe-Goldstone equation, and these methods are designed to treat certain characteristics of these operators. Most of these methods have been applied to nuclear matter

calculations, with varying degrees of success. However, when one is dealing with finite nuclei, further complications are introduced. The allowed intermediate states are no longer in the continuum; they are now discrete states and the treatment of the Pauli operator must be modified accordingly. C. W. Wong (1967) has made a very thorough study on the treatment of this operator for finite nuclei. He introduced the so-called 'local' and 'global' treatments of the operator. The 'local' operator is essentially a nuclear matter Pauli operator with an average density dependence. The 'global' operator is treated in configuration space in an attempt to include the discrete intermediate states correctly. We shall examine the physical situation, in which each of the treatments of the Pauli operator is good.

Nuclear matter, by definition, is an infinite medium with a density which closely approximates that in the interiors of heavy nuclei. It is then apparent that the 'local' treatment of Q, the Pauli operator, should be quite adequate for two nucleons interacting deep in the nuclear interior. For nucleons interacting near the nuclear surface, an effective density approach must adopted. In other words, a 'local' Q corresponding to a nuclear matter of a somewhat smaller density should be used. C. W. Wong obtained some reasonable results with this approach. However, we can see that in this treatment, the discrete nature of the intermediate states is completely ignored. The 'global' treatment, on the other hand

treats the operator (1 - Q) instead of Q. With the basis states forming a complete set, the operator corresponds to a finite summation of the discrete occupied states. Consider the Bethe-Goldstone equation:

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \frac{Q}{e} V |\psi_{ij}\rangle$$

where $|\phi_{ij}\rangle$ is the unperturbed two-particle wave function,

$|\psi_{ij}\rangle$ is the correlated two-particle wave function,

and V is the interaction, e the energy denominator. We shall concentrate on Q for the time being.

The explicit form of Q, for a finite system, is

$$Q = \sum_{\substack{m\,n \\ \text{unoccupied}}} |\phi_{mn}\rangle\langle\phi_{mn}|$$

(Note: Sum over mn is used as short-hand for sum over ϕ_{mn}) which is an infinite summation. To avoid this, the equation is re-arranged as:

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \frac{1}{e} V |\psi_{ij}\rangle + \frac{(1-Q)}{e} V |\psi_{ij}\rangle$$

If the basis states $|\phi_{ij}\rangle$ form a complete set, then, (1 - Q) corresponds to a finite summation over the occupied states. In the case of A=18 nuclei, with harmonic oscillator basis states, this corresponds to the 1S and 1P shells. At first sight, this seems to be a reasonably small set to sum over. However, in solving the B-G equation, a transformation to centre-of-mass and relative co-ordinates is always necessary. It is this transformation that increases the number of possible

intermediate states in terms of the C.O.M. and relative quantum numbers. This number increases all the more drastically when heavier nuclei are treated, the single particle occupied states increasing in number. C. W. Wong has found that 'global' treatment converges well for light nuclei in which the occupied states are relatively few. For heavy nuclei, the magnitude of the summation becomes almost unmanageable.

Section 2.

It appears apparent that the 'local' treatment of the Pauli operator is incomplete from the physical point of view, while the 'global' treatment becomes calculationally inconvenient with heavier nuclei. We shall now introduce a method which retains the 'virtues' of the nuclear matter operator, while, at the same time, making the inclusion of the discrete nature of the intermediate states possible. To do this, we have to examine more closely, some of the well-known physical features of an interacting pair of nucleons.

One of the well-known features of realistic forces is the short-range strong repulsion. This implies that two nucleons will come very close together only when they possess high momenta, and similarly, the scattered states will be states of high relative momenta, which are well approximated by plane wave states. Imagine two nucleons interacting in the nuclear interior, their short-range correlation corresponding

to the repulsive part of the interaction, will be very similar to that of two nucleons in an infinite medium. A very trivial example is the infinite hard core potential which produces zero correlated wave functions in nuclear matter as well as in finite nuclei, inside the hard core region. Thus, the difference between the 'finite' and 'infinite' Pauli operators, has little effect on the short-range part of the two-particle wave function. This is the basic assumption of our formalism. It enables us to treat most of the intermediate states of high energy as continuous, and only the low-lying states as discrete. Such a separation is not always clearcut, and certain criteria have to be met. But, we shall assume for the time being that such a separation is possible and proceed.

Let us consider the two-particle B-G equation for a finite nucleus,

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \frac{Q}{e} V |\psi_{ij}\rangle$$

where $|\phi_{ij}\rangle$'s are some basis states,

$|\psi_{ij}\rangle$'s are the correlated wave functions,

and V is the realistic potential.

Writing out the operator Q explicitly, we have:

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \sum_{\substack{mn \\ \text{unoccupied}}} \frac{|\phi_{mn}\rangle\langle\phi_{mn}| V |\psi_{ij}\rangle}{e_{ij}^{mn}}$$

where $e_{ij}^{mn} = E(m) + E(n) - E(i) - E(j)$.

We shall consider how to handle the energy denominator in more detail later on. Let us, further, make an arbitrary separation of the states mn to be summed, into those belonging to a subspace we call D and those not within D. Then, we can write the two-body B-G equation for finite nuclei in the following way:

$$|\psi_{ij}\rangle = |\phi_{ij}\rangle - \sum_{mn\in D} \frac{|\phi_{mn}\rangle\langle\phi_{mn}|V|\psi_{ij}\rangle}{e_{ij}^{mn}}$$

$$- \sum_{ab\notin C,D} \frac{|\phi_{ab}\rangle\langle\phi_{ab}|V|\psi_{ij}\rangle}{e_{ij}^{ab}} \qquad \text{(II.1)}$$

where C represents the so-called core states which are occupied. The nature of the subspace D depends on the nuclei under consideration. It can be a partially filled shell such as the S-D shell in the case of O^{18} and F^{18}, or it can be a completely empty shell when a closed shell nucleus such as O^{16} is considered. Here, we have used the language of harmonic oscillator basis states. In fact, any complete set of states, whether it be deformed or not, can be used. This again depends on what nuclei are under consideration. We shall restrict ourselves in actual application of this formalism to harmonic oscillator basis states. To solve equation (II.1), there are many methods as mentioned previously. What we want to achieve, is to make use of the 'known' short-range behavior of the wave function to render an easier solution and to gain more physical insight. To do this, let us imagine a hypothetical problem. Consider

two nucleons originally in a discrete state $|\phi_{ij}\rangle$ being 'embedded' in an infinite medium of Fermi momentum k_c. The behavior of this pair of nucleons in the medium is described by the followong B-G equation:

$$|\psi_{ij}\rangle_N = |\phi_{ij}\rangle - \left(\frac{Q}{e}\right)_N V |\psi_{ij}\rangle_N$$

where the subscript N denotes the similarity between this and the nuclear matter equation. The main difference is in $|\phi_{ij}\rangle$ which is now a discrete state, instead of a plane wave state as in nuclear matter. We shall call this the 'semi-infinite' equation. Writing Q explicitly, we have,

$$|\psi_{ij}\rangle_N = |\phi_{ij}\rangle - \int_{k_c}^{\infty}\int_{k_c}^{\infty} d^3k_1\, d^3k_2\, |k_1 k_2\rangle \frac{\langle k_1 k_2 | V | \psi_{ij}\rangle_N}{e_{ij}^{k_1 k_2}} \qquad \text{(II.2)}$$

where $|k_1 k_2\rangle$ is a two-particle plane wave state,

$$e_{ij}^{k_1 k_2} = E(k_1) + E(k_2) - E(i) - E(j)$$

and k_c is the cutoff or Fermi momentum, below which all states are occupied. Note that we have suppressed the spin quantum numbers S and m_S in the plane wave state $|k_1 k_2\rangle$. A summation over these quantum numbers is understood unless otherwise noted. Again, we defer the discussion on the energy denominators.

Equation (II.2) is in exactly the same form as the nuclear matter equation and can be solved by the various methods previously mentioned. By a suitable choice of the subspace D and the Fermi momentum k_c , $|\psi_{ij}\rangle_N$ and $|\psi_{ij}\rangle$ will have similar short-range correlations and one is justified in

approximating $(|\psi_{ij}\rangle - |\psi_{ij}\rangle_N)$ by a truncated expansion:

$$|\psi_{ij}\rangle - |\psi_{ij}\rangle_N = \sum_{\alpha\beta \in D} A_{\alpha\beta}^{ij} |\psi_{\alpha\beta}\rangle_N \qquad \text{(II.3)}$$

Note that we have used $|\psi_{\alpha\beta}\rangle_N$ in the expansion instead of $|\phi_{\alpha\beta}\rangle$ in order to satisfy the boundary condition on the correlated wave function $|\psi_{\alpha\beta}\rangle$, at the core radius of the potential. Here $A_{\alpha\beta}^{ij}$ are the coefficients of the expansion which we shall have to derive.

Let us now see what physical insight one can gain by considering equations (II.1), (II.2) and (II.3). Equation (II.1) represents the 'finite' situation. A set of single particle basis states is chosen. This set is divided into the core states C, the subspace D and higher states, as in Fig.(II.1).

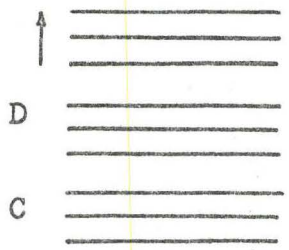


Fig. (II.1)

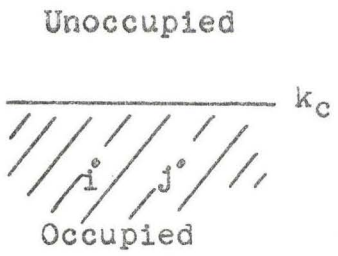


Fig. (II.2)

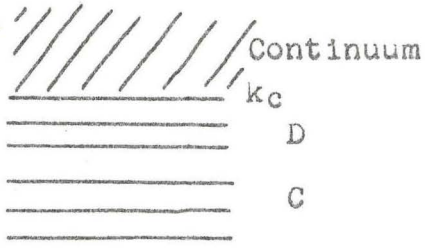


Fig. (II.3)

The B-G equation describes the behavior of the two nucleons when they are allowed to interact and scatter into D and higher states. Equation (II.2) may be represented by Fig.(II.2). This is the situation, when both the occupied and the unoccupied states are in the continuum. Only the initial state $|\phi_{ij}\rangle$ is discrete. The composite picture of Fig.(II.1) and

Fig.(II.2) is shown in Fig.(II.3). This represents the truncated expansion of (II.3).

In the composite picture, we are in fact saying that we shall treat single particle states with momentum components lower than k_c as discrete and those with momentum components greater than k_c are treated as continuous, i.e. represented by plane wave states. However, it is not possible to make a one-to-one correspondence between k_c and the uppermost state in D. A state above D has momentum components smaller than k_c , and states within D also have momentum components greater than k_c. It is therefore necessary to have some criteria by which k_c may be chosen. To accomplish this, we shall have to look into the structures of the coefficients $A^{ij}_{\alpha\beta}$ more closely.

Aside from this difficulty of determining the cutoff momentum, a much more basic problem arises when we consider the 'semi-infinite' problem as represented by equation (II.2). One of the basic requirements of perturbative method is that the perturbed wave function should be normalized to unity with the unperturbed wave function, i.e.,

$$\langle \Phi_{ij} / \Psi_{ij} \rangle_N = 1$$

This is clearly not satisfied by equation (II.2), and the reaction matrix one obtains corresponding to $/ \Psi_{ij} \rangle_N$ is clearly not a suitable quantity because of the incorrect normalization. We shall present in the following section a

formalism which provides for both the choice of k_c as well as the proper normalization for the perturbed waves.

We note here also that although we have been using k_c interchangeably as the cutoff momentum and as the Fermi momentum, it will emerge in the next section as being quite distinct from the Fermi momentum of the nuclear system.

Section 3.

Let us define a two-particle plane wave state by:

$$|k_1 k_2^*\rangle = |k_1 k_2\rangle - \sum_{ij \in C \text{ or } D} \langle \phi_{ij} | k_1 k_2 \rangle \cdot |\phi_{ij}\rangle \qquad \text{(II.4)}$$

for $k_1, k_2 > k_c$, such that $\langle \phi_{ij} | k_1 k_2^* \rangle = 0$ for $ij \in C$ or D. In other words, $|k_1 k_2^*\rangle$ has been orthogonalized to all the discrete states in the core and the subspace D.

The normalization of $|k_1 k_2^*\rangle$ is given by:

$$\langle k_1 k_2^* | k_1' k_2'^* \rangle = \delta(k_1 - k_1')\, \delta(k_2 - k_2') - \sum_{ij \in C \text{ or } D} \langle k_1 k_2 | \phi_{ij} \rangle \langle \phi_{ij} | k_1' k_2' \rangle \qquad \text{(II.5)}$$

in which the second term becomes negligible when k_1, k_2, k_1' and k_2' are greater than k_c and the momentum components of $|\phi_{ij}\rangle$, greater than k_c appear to only a small extent. This will bear significance in our formalism later on. With the plane wave states defined by (II.4), let us recast our semi-infinite problem as:

$$|\psi_\alpha^*\rangle = |\phi_\alpha\rangle - \int_{k_c}^{\infty} d\underline{k}\; |k^*\rangle \frac{\langle k^*|V|\psi_\alpha^*\rangle}{e_{\alpha k}^*} \qquad \text{(II.6)}$$

where the following simplifications of notations have been introduced:

i) $|k^*\rangle$ stands for $|k_1 k_2{}^*\rangle$.

ii) $\int_{k_c}^{\infty} d\underline{k}$ stands for $\int_{k_c}^{\infty}\int_{k_c}^{\infty} d^3k_1\, d^3k_2$.

iii) α stands for a pair of indices such as ij in (II.5).

iv) $e_{\alpha k}^*$ stands for $e_{ij}^{k_1 k_2 *}$ which will be treated in more detail later.

v) $\underline{k} > \underline{k_c}$ means $\underline{k_1}, \underline{k_2} > \underline{k_c}$, and $\alpha \in C$ means $(i,j) \in C$.

We shall use this set of simplified notations whenever convenient, and shall specify so.

Because of (II.4), $|\psi_\alpha^*\rangle$ is now properly normalized with $|\phi_\alpha\rangle$ such that,

$$\langle \phi_\alpha | \psi_\beta^* \rangle = \delta_{\alpha\beta} \quad \text{for} \quad \alpha \in C \text{ or } D,$$

and the perturbative developments may be correctly applied to the problem. We can also define a reaction matrix G* corresponding to equation (II.6),

$$G^* = V - V\left(\frac{Q}{e}\right)^* G^* \qquad \text{(II.7)}$$

where $$\left(\frac{Q}{e}\right)^* = \int_{k_c}^{\infty} d\underline{k}\; \frac{|k^*\rangle\langle k^*|}{e_{\alpha k}^*}$$

We may now again apply our argument of the truncated

expansion to obtain the true wave function $|\psi_\alpha\rangle$; only this time we shall use $|\psi_\alpha^*\rangle$ as basis states. The three basic equations of our problem now can be written in simplified notations as:

$$|\psi_\alpha\rangle = |\phi_\alpha\rangle - \sum_{\beta\in D} |\phi_\beta\rangle \frac{\langle\phi_\beta|V|\psi_\alpha\rangle}{e_{\alpha\beta}} - \sum_{\gamma\notin C,D} |\phi_\gamma\rangle \frac{\langle\phi_\gamma|V|\psi_\alpha\rangle}{e_{\alpha\gamma}} \tag{II.8}$$

$$|\psi_\alpha^*\rangle = |\phi_\alpha\rangle - \int_{k_c}^{\infty} dk\, |k^*\rangle \frac{\langle k^*|V|\psi_\alpha^*\rangle}{e_{\alpha k}^*} \tag{II.9}$$

and $$|\psi_\alpha\rangle - |\psi_\alpha^*\rangle = \sum_{\substack{\beta\in D\\ \neq\alpha}} A_{\alpha\beta}^* |\psi_\beta^*\rangle \tag{II.10}$$

Assuming solution of (II.9), our task reduces to evaluation of coefficients $A_{\alpha\beta}^*$ in (II.10).

Using the fact that $\langle\phi_\alpha|\psi_\beta^*\rangle = \delta_{\alpha\beta}$ for $\alpha\in$ C or D, $A_{\alpha\beta}^*$ may be obtained from (II.10) as:

$$A_{\alpha\beta}^* = \langle\phi_\beta|\psi_\alpha - \psi_\alpha^*\rangle \qquad \text{for } \beta\in C \text{ or } D.$$

It is easy to see that for $\alpha\neq\beta$, we have,

$$A_{\alpha\beta}^* = \langle\phi_\beta|\psi_\alpha\rangle = -\frac{\langle\phi_\beta|G|\phi_\alpha\rangle}{e_{\alpha\beta}} \simeq -\frac{\langle\phi_\beta|G^*|\phi_\alpha\rangle}{e_{\alpha\beta}} \tag{II.11}$$

where we have used equation (II.8) and where the G-matrix is replaced by G* as a first order approximation.

Let us examine more closely the truncated expansion of (II.10). This approximation is good if the magnitude of

components $|\psi_\gamma^*\rangle$, where $\gamma \notin$ C or D, in $|\psi_\alpha\rangle$ is small as compared to components in the space D. The quantity we wish to study is therefore:

$$I \equiv |\psi_\alpha\rangle - \left(|\psi_\alpha^*\rangle + \sum_{\beta \in D, \neq \alpha} A_{\alpha\beta}^* |\psi_\beta^*\rangle\right)$$

From equations (II.8), (II.9), and (II.11), we get,

$$I = + \sum_{\beta \in D} \frac{\langle \phi_\beta | G | \phi_\alpha \rangle}{e_{\alpha\beta}} \cdot |\psi_\beta^*\rangle - \sum_{\beta \in D} |\phi_\beta\rangle \cdot \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} \qquad \text{(II.12)}$$

$$- \sum_{\gamma \notin C,D} |\phi_\gamma\rangle \frac{\langle \phi_\gamma | V | \psi_\alpha \rangle}{e_{\alpha\gamma}} + \int_{k_c}^{\infty} d\underset{\sim}{k} |k^*\rangle \frac{\langle k | V | \psi_\alpha^* \rangle}{e_{\alpha k}^*}$$

Applying equation (II.9) to the first term, and inserting a complete set of plane waves and a complete set of discrete states $|\phi_\delta\rangle$, in the third and fourth terms respectively, we can further reduce the expression (II.12) to:

$$I = - \sum_{\substack{\beta \in D \\ \neq \alpha}} \frac{\langle \phi_\beta | G | \phi_\alpha \rangle}{e_{\alpha\beta}} \cdot \int_{k_c}^{\infty} d\underset{\sim}{k} |k^*\rangle \frac{\langle k^* | V | \psi_\beta^* \rangle}{e_{\beta k}^*}$$

$$- \sum_{\beta \notin C \text{ or } D} \sum_{\gamma \in C,D} \int_{k_c}^{\infty} |\phi_\gamma\rangle \langle \phi_\gamma | k \rangle \langle k | \phi_\beta \rangle \frac{\langle \phi_\beta | V | \psi_\alpha^* \rangle}{e_{\alpha\beta}^*} \cdot d\underset{\sim}{k}$$

$$- \sum_{\beta \notin C \text{ or } D} \int_0^{k_c} d\underset{\sim}{k} \langle k | \phi_\beta \rangle \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}}$$

$$+ \sum_{\beta \notin C \text{ or } D} \int_{k_c}^{\infty} d\underset{\sim}{k} |k\rangle \langle k | \phi_\beta \rangle \left\{ \frac{\langle \phi_\beta | V | \psi_\alpha^* \rangle}{e_{\alpha\beta}^*} - \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} \right\}$$

where we have introduced the same cutoff momentum k_c for the complete set of plane waves $|k\rangle$'s and the discrete states have been separated into those in C or D and those that are not. The last term appears because we have retained the

difference between $\langle \phi_\beta | V | \psi_\alpha^* \rangle$ and $\langle \phi_\beta | V | \psi_\alpha \rangle$ for completeness. We have also used the fact that $\langle K^* | \psi_\beta \rangle \equiv \langle K | \psi_\beta \rangle$ for $\beta \notin$ C or D, and $\langle K^* | \psi_\beta \rangle \equiv 0$ for $\beta \in$ C or D. To further simplify the expression, we again introduce a complete set of discrete states in the first and the third terms,

$$
\begin{aligned}
I = & -\sum_{\substack{\gamma \in D \\ \neq \alpha}} \sum_{\beta \notin C \cup D} \frac{\langle \phi_\gamma | G | \phi_\alpha \rangle}{e_{\alpha\gamma}} \int_{k_c}^{\infty} d\underline{k} \; |\phi_\beta\rangle\langle\phi_\beta | K\rangle \frac{\langle K^* | V | \psi_\gamma^* \rangle}{e_{\gamma K}^*} \\
& - \sum_{\beta \notin C \cup D} \sum_{\gamma \in C, D} \int_0^{k_c} d\underline{k} \; |\phi_\gamma\rangle\langle\phi_\gamma | K\rangle\langle K | \phi_\beta\rangle \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} \\
& - \sum_{\beta \notin C \cup D} \sum_{\gamma \notin C \cup D} \int_0^{k_c} d\underline{k} \; |\phi_\gamma\rangle\langle\phi_\gamma | K\rangle\langle K | \phi_\beta\rangle \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} \\
& + \sum_{\beta \notin C \cup D} \int_{k_c}^{\infty} d\underline{k} \Big[|K\rangle\langle K | \phi_\beta\rangle \Big\{ \frac{\langle \phi_\beta | V | \psi_\alpha^* \rangle}{e_{\alpha\beta}^*} - \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} \Big\} \\
& + \sum_{\gamma \in C, D} |\phi_\gamma\rangle\langle\phi_\gamma | K\rangle\langle K | \phi_\beta\rangle \Big\{ \frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} - \frac{\langle \phi_\beta | V | \psi_\alpha^* \rangle}{e_{\alpha\beta}^*} \Big\} \Big]
\end{aligned}
$$

The second term in the expression vanishes after integration over the K space. Therefore,

$$
\begin{aligned}
I = & -\sum_{\beta \notin C \cup D} |\phi_\beta\rangle \cdot \Big\{ \sum_{\gamma \in D} \int_{k_c}^{\infty} d\underline{k} \; \frac{\langle \phi_\gamma | V | \psi_\alpha \rangle}{e_{\alpha\gamma}} \cdot \langle \phi_\beta | K \rangle \frac{\langle K^* | V | \psi_\gamma^* \rangle}{e_{\gamma K}^*} \\
& + \sum_{\gamma \notin C \cup D} \int_0^{k_c} d\underline{k} \; \langle \phi_\beta | K \rangle \langle K | \phi_\gamma \rangle \frac{\langle \phi_\gamma | V | \psi_\alpha \rangle}{e_{\alpha\gamma}} \Big\} \\
& - \sum_{\beta \notin C \cup D} \Big(\frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} - \frac{\langle \phi_\beta | V | \psi_\alpha^* \rangle}{e_{\alpha\beta}^*} \Big) \cdot \int_0^{k_c} d\underline{k} \; \cdot \\
& \cdot \sum_{\gamma \notin C \cup D} |\phi_\gamma\rangle\langle\phi_\gamma | K\rangle\langle K | \phi_\beta\rangle \qquad \text{(II.13)}
\end{aligned}
$$

The first term in (II.13) may be further simplified as follows:

$$
\begin{aligned}
& -\sum_{\beta \notin C \cup D} |\phi_\beta\rangle \cdot \sum_{\gamma \in D} \int_{k_c}^{\infty} \frac{\langle \phi_\gamma | V | \psi_\alpha \rangle}{e_{\alpha\gamma}} d\underline{k} \; \langle \phi_\beta | K \rangle \frac{\langle K^* | V | \psi_\gamma^* \rangle}{e_{\gamma K}^*} \\
& = -\sum_{\beta \notin C \cup D} |\phi_\beta\rangle \cdot \sum_{\gamma \in D} \frac{\langle \phi_\gamma | V | \psi_\alpha \rangle}{e_{\alpha\gamma}} \cdot \sum_{\delta \notin C \cup D} \frac{\langle \phi_\delta | V | \psi_\gamma^* \rangle}{e_{\gamma\delta}} \cdot \int_{k_c}^{\infty} d\underline{k} \; \langle \phi_\beta | K \rangle \langle K | \phi_\delta \rangle
\end{aligned}
$$

$$= -\sum_{\beta \notin C \cup D} |\phi_\beta\rangle \cdot \sum_{\gamma \in D} \frac{\langle \phi_\gamma | V | \psi_\alpha \rangle}{e_{\alpha\gamma}} \cdot \sum_{\delta \notin C \cup D} \frac{\langle \phi_\delta | V | \psi_\gamma^* \rangle}{e_{\gamma\delta}^*} \cdot$$

$$\cdot \left[\delta_{\beta\delta} - \int_0^{k_c} d\underline{k} \, \langle \phi_\beta | k \rangle \langle k | \phi_\delta \rangle \right]$$

where we have again used the properties of $|k^*\rangle$.

Putting this in, we finally obtain, after some re-arrangements, an expression for the difference we want to study:

$$I = -\sum_{\beta \notin C \cup D} |\phi_\beta\rangle \left[\sum_{\gamma \notin C \cup D} \int_0^{k_c} d\underline{k} \, \langle \phi_\beta | k \rangle \langle k | \phi_\gamma \rangle \cdot \right.$$

$$\cdot \left\{ \frac{\langle \phi_\gamma | V | \psi_\alpha^* \rangle}{e_{\alpha\gamma}^*} - \sum_{\delta \in D} \frac{\langle \phi_\delta | V | \psi_\alpha \rangle \langle \phi_\gamma | V | \psi_\delta^* \rangle}{e_{\alpha\delta} \quad e_{\delta\gamma}^*} \right\} \qquad \text{(II.14)}$$

$$\left. + \left(\frac{\langle \phi_\beta | V | \psi_\alpha \rangle}{e_{\alpha\beta}} - \frac{\langle \phi_\beta | V | \psi_\alpha^* \rangle}{e_{\alpha\beta}^*} \right) + \sum_{\delta \in D} \frac{\langle \phi_\delta | V | \psi_\alpha \rangle \langle \phi_\beta | V | \psi_\delta^* \rangle}{e_{\alpha\delta} \quad e_{\delta\beta}^*} \right]$$

Expression (II.14) appears as a summation over states outside of D and C, which is as expected. The magnitude of the coefficients is indicated by the three terms inside the square brackets. The first term is small when k_c is chosen such that $\langle \phi_\beta | k \rangle$ is small for $\beta \notin$ C or D and $k < k_c$. The second term is of the same order as the term we have ignored in the evaluation of $A_{\alpha\beta}^*$, i.e. we replaced G by G*. The last term is of second order and has at least one energy denominator $e_{\delta\beta}^*$ which contains energy difference of at least $\hbar\omega$ bigger than $e_{\alpha\beta}$ in $A_{\alpha\beta}^*$.

Expression (II.14) indicates, therefore, that the truncated expansion (II.10) is consistent with our order of approximation. It also implies that the important criterion

for k_c is that $\langle \kappa | \phi_\beta \rangle$ should be small for $\kappa < k_c$ and $\beta \notin C$ or D. Therefore, the ideal separation of momentum components by k_c for the states in C and D and for the states outside is not necessary. Fig.(II.4) shows the momentum transforms of the single particle states used in the calculation. For $n = 0$, the $\ell = 2$ and 4 states are plotted. Although these states are separated by energy gaps in the simple harmonic spectrum, we see that there is too much overlap in the momentum space between these states to make a unique determination of k_c. The situation for $n = 1$ is even worse, because there is a peak in its momentum transform inside the maxima of the $n = 0$ states.

A choice of zero for k_c is not desirable, although it eliminates the first term in (II.14). The reason is simply that our formalism will also reduce to the same footing as the 'global' approach in which all of the occupied states have to be summed. As previously mentioned, this will make the magnitude of computation prohibitive when heavier nuclei are encountered.

Let us consider the problem from a different point of view by asking how well the true wave function is approximated by (II.10) at different ranges. For intermediate and long ranges, we know from experience with shell-model that the wave function is adequately represented by a suitably chosen D of reasonable size. Also we know that asymptotically,

the wave function $|\psi_\alpha\rangle$ goes to $|\phi_\alpha\rangle$. So except for the extreme tail, which is not very important anyway, a truncatedexpansion in a suitable subspace D of the true wave function, seems adequate for the intermediate and long ranges.

What about the short ranges? We have argued in qualitative terms, that the wave function $|\psi_\alpha^*\rangle$ associated with the semi-infinite problem has the same short-range correlation as that for the $|\psi_\alpha\rangle$. This, however, is valid only when a suitable k_c is chosen for equation (II.9). If k_c is too large, we will be leaving out Fourier components that are not re-introduced by the truncated expansion (II.10) and, if k_c is too small, we shall be double-counting some of the Fourier components which are already well represented by states in D. Our approximation, which appears later in the discussion of Pauli correction terms, requires that $|\phi_i\rangle$'s in D have small components $\langle\phi_i|\kappa\rangle$ for $\kappa > k_c$. So we see that we must choose k_c so that for $\alpha \in D$, $\langle\phi_\alpha|\kappa\rangle$ is small for $\kappa > k_c$, but not so large that there is a group of wave numbers less than k_c which are not adequately represented by our truncated expansion (II.10). The latter condition indicates that k_c should not be too far from the Fermi momentum associated with average nuclear density. In other words, the restrictions for a complete separation of the momenta can be relaxed. However, this is by no means a unique choice of k_c and we shall have to test the sensitivity of our formalism to different values of k_c.

Section 4.

So far, we have made the bold assumption that equation (II.9) can be solved without difficulties. This is not the case in actual practice. For equation (II.9), we still have to use approximate methods, such as the 'reference spectrum' method, the 'separation' method and the 'integral equation' method. As remarked by G. Dahl et al (1969), the choice of a method should be made by considering its accuracy and the labour involved. We have chosen the reference spectrum method, partly because of the ease in extending the method to all partial waves, and partly because of the author's previous experience with this method. We shall mention some of the salient features of this approximation.

The basic procedure is to replace the actual intermediate state energy spectrum, which is unknown, by an assumed 'reference spectrum' e^R. The B-G equation can then be reduced to a differential equation when the assumed spectrum e^R is a quadratic function of the momentum. The differential equation is then solved numerically by first ignoring the Pauli operator, Q. This leads to a 'reference' wave function or 'reference' reaction matrix G^R. The effects of the Pauli operator Q and of the difference between e^R and the true spectrum e* are then calculated as the Pauli and Spectral

corrections respectively. This idea of an approximate solution to the B-G equation may be summarized in the following way. Consider a B-G equation represented by an operator equation,

$$G_A = V_A - V_A \frac{Q_A}{e_A} G_A \qquad \text{(II.15)}$$

where G_A is the reaction matrix corresponding to the interaction V_A and Q_A/e_A is the propagator appropriate for the particular problem. Formally, let us consider another such equation with subscript B,

$$G_B = V_B - V_B \frac{Q_B}{e_B} G_B \qquad \text{(II.16)}$$

and try to get an expression for G_A in terms of G_B. Let us define another operator Ω by,

$$\Omega_A = 1 - \frac{Q_A}{e_A} G_A \qquad \text{and} \qquad \Omega_B = 1 - \frac{Q_B}{e_B} G_B$$

The following identity holds:

$$G_A = G_A - G_B^{\dagger}\left[\Omega_A + \left(\frac{Q_A}{e_A}\right) G_A - 1\right] + \left[\Omega_B^{\dagger} + G_B^{\dagger}\left(\frac{Q_B}{e_B}\right)^{\dagger} - 1\right] G_A$$

This again simplifies to:

$$G_A = G_B^{\dagger} + \Omega_B^{\dagger}(V_A - V_B)\Omega_A + G_B^{\dagger}\left[\left(\frac{Q_B}{e_B}\right)^{\dagger} - \left(\frac{Q_A}{e_A}\right)\right] G_A \qquad \text{(II.17)}$$

The † indicates Hermitian conjugates.

The operator relations tell us that if equation (II.16) represents approximate situation of equation (II.15) and has the virtue that it can be handled more easily, then equation

(II.17) tells us how to get the exact G_A from G_B. Here, we have followed very closely the treatment of Bethe et al (1963) as presented by G. E. Brown (1967).

Let us assume that all the operators involved, except Ω are Hermitian @. Then equation (II.17) reduces to:

$$G_A = G_B + \Omega_B^{\dagger}(V_A - V_B)\Omega_A + G_B\left[\left(\frac{Q_B}{e_B}\right) - \left(\frac{Q_A}{e_A}\right)\right] G_A$$

The reference spectrum method is to assume that the only difference between the exact and the approximate reaction matrix is in their propagators. Further, the approximate propagator is taken to be simply $1/e^R$, where e^R is the so-called 'reference' spectrum. Bearing these in mind and remembering that $V_A = V_B$ in this case, we have,

$$G_A = G_B + G_B \cdot \left\{\frac{1}{e^R} - \frac{Q_A}{e_A}\right\} \cdot G_A$$

$$= G_B + G_B\left\{\frac{1}{e^R} - \frac{1}{e_A}\right\} G_A + G_B\left\{\frac{1 - Q_A}{e_A}\right\} G_A$$

@ Note that G^R is not strictly Hermitian because of the state-dependence of e^R; however, we can make it Hermitian if the same starting energies are used. In other words, for $\langle ab | G^R | cd \rangle$ and $\langle cd | G^R | ab \rangle$, if we use $\mathcal{E} = \frac{1}{2}[E(a)+E(b)+E(c)+E(d)]$ as starting energy, they will be equal. See Bethe et al (1963) or Chapter III.

or, $$G_A \simeq G_B + G_B \left\{ \frac{1}{e^R} - \frac{1}{e_A} \right\} G_B + G_B \left\{ \frac{1 - Q_A}{e_A} \right\} G_B \qquad \text{(II.18)}$$

where we have replaced G_A on the R.H.S. by G_B as a first order approximation. The second and last terms on the right are respectively the Spectral and Pauli corrections.

The reference spectrum e^R is usually defined as:

$$e^R = E(i) + E(j) - E(l) - E(m)$$

where i and j are single particle intermediate states, with energies given by:

$$E(i) = A + \frac{\hbar^2 k_i^2}{2m^* M}$$

and E(1) and E(m) are initial state energies obtained from experiment.

A and m* are the reference spectrum parameters which should be calculated by a self-consistent process (See for example, M. Razavy,1963). It was found, however, that in nuclear matter, using $E(i) = \hbar^2 k_i^2/2M$ seems to produce reasonable results. (H. Bethe, 1965 and S. Mozskowski, 1965). Kuo and Brown (1966) used this approximation to e^R. In fact, in their calculation, e^R is treated as identical to the actual spectrum e*; so, there is no Spectral correction. This is reasonable, since nuclear matter calculations indicate results of using a flat particle spectrum and of self-consistent calculations of m* and A (the two parameters of the particle spectrum)

are comparable.

We shall follow the same procedure here in treating the free particle spectrum as exact and in using experimental values for E(l) and E(m). Therefore, we have,

$$e^{ij}_{lm} \equiv e^{R} = \frac{\hbar^2 k_i^2}{2M} + \frac{\hbar^2 k_j}{2M} - E(l) - E(m)$$

Assuming that we have solved for the reaction matrix G* for the 'semi-infinite' problem, we need an expression for the actual reaction matrix G, in terms of the coefficients $A^{*}_{\alpha\beta}$ and G*. Again, using the simplified notations, we have:

$$\langle \phi_\alpha | G | \phi_\beta \rangle = \langle \phi_\alpha | V | \psi_\beta \rangle$$

$$= \langle \phi_\alpha | V | \psi_\beta^* \rangle + \sum_{\substack{\gamma \in D \\ \neq \beta}} A^{*}_{\beta\gamma} \langle \phi_\alpha | V | \psi_\gamma^* \rangle \qquad \text{(II.19)}$$

$$= \langle \phi_\alpha | G^* | \phi_\beta \rangle - \sum_{\substack{\gamma \in D \\ \neq \beta}} \frac{\langle \phi_\gamma | G^* | \phi_\beta \rangle}{e_{\beta\gamma}} \cdot \langle \phi_\alpha | G^* | \phi_\gamma \rangle$$

Since G is Hermitian only if we use a fix starting energy and calculate up to all orders, we shall follow the practice of taking,

$$\langle \phi_\alpha | G | \phi_\beta \rangle = \frac{1}{2} \left\{ \langle \phi_\alpha | G | \phi_\beta \rangle + \langle \phi_\beta | G | \phi_\alpha \rangle \right\} \qquad \text{(II.20)}$$

In actual calculation, $\langle \phi_\alpha | G | \phi_\beta \rangle$ and $\langle \phi_\beta | G | \phi_\alpha \rangle$ are found to be close enough for such an average, to our order of approximation.

CHAPTER III

THE CALCULATION OF REDUCED INTEGRALS

Section 1.

In this chapter, we shall be mainly concerned with the solution of the reference spectrum equation. However, before doing that, we should mention that it is the shell-model matrix elements we are after. In other words, we want to obtain effective interactions for shell-model type calculation. Let us denote a two-particle state by the j-j coupling scheme. Thus, we may write an anti-symmetric two-particle wave function of a given set of quantum numbers, JM_J , and TM_T as:

$$|j_1 m_1, j_2 m_2; JM_J, TM_T\rangle_A$$

$$= \frac{1}{\sqrt{2(1+\delta_{12})}} \cdot \left\{ |j_1 m_1, j_2 m_2; JM_J, TM_T\rangle - (-1)^{1-T+j_1+j_2-J} |j_2 m_2, j_1 m_1; JM_J TM_T\rangle \right\}$$

$$= N_{12}(j_1 m_1 j_2 m_2 | JM_J)(\tfrac{1}{2}\sigma_1 \tfrac{1}{2}\sigma_2 | TM_T) \cdot \left[\phi_{j_1 m_1}^{(1)} \phi_{j_2 m_2}^{(2)} \cdot \right.$$

$$\left. \cdot \chi_{\frac{1}{2}\sigma_1}^{(1)} \chi_{\frac{1}{2}\sigma_2}^{(2)} - \phi_{j_2 m_2}^{(1)} \phi_{j_1 m_1}^{(2)} \chi_{\frac{1}{2}\sigma_2}^{(1)} \chi_{\frac{1}{2}\sigma_1}^{(2)} \right]$$

We have written out the wave function completely, in terms of the products of single-particle wave functions. $\phi_{j_1 m_1(1)}$ denotes the space and spin wave function of particle 1,

and the χ's are isospin wave functions. N_{12} is just the normalization factor and the first two factors are Clebsch-Gordan coefficients for the spin and isospin couplings respectively. The j-j coupling scheme is not convenient when the interaction is decomposed into different angular momentum states. We, therefore, have to transform to the L-S coupling scheme. Using notations of Edmonds (1960), we note the following identities:

$$|l_1 S_1 (j_1);\ l_2 S_2 (j_2);\ JM_J\rangle = \sum_{L,S} \sqrt{(2L+1)(2S+1)(2j_1+1)(2j_2+1)} \cdot \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix} \cdot |l_1 l_2 (L), S, S_2 (S), JM_J\rangle \qquad \text{(III.1)}$$

and,

$$\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix} = (-1)^{l_1+l_2+L+S+1+j_1-j_2+J} \cdot \begin{Bmatrix} l_2 & \frac{1}{2} & j_2 \\ l_1 & \frac{1}{2} & j_1 \\ L & S & J \end{Bmatrix} \qquad \text{(III.2)}$$

In (III.1) the L.H.S. is a two-body wave function in j-j coupling scheme. $\begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix}$ is the nine-j symbol for the transformation and $|l_1 l_2 (L), S, S_2 (S), JM_J\rangle$ is a two-body wave function in the L-S coupling scheme. With these, we get,

$$|j_1 M_1, j_2 M_2;\ JM_J,\ TM_T\rangle_A = N_{12} \cdot \sum_{L,S} \sqrt{(2L+1)(2S+1)(2j_1+1)(2j_2+1)} \cdot \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix} \cdot \left[|l_1 l_2 (L), S, S_2 (S); JM_J, TM_T\rangle - (-1)^{l_1+l_2+L+S-T} |l_2 l_1 (L), S, S, (S);\ JM_J,\ TM_T\rangle \right]$$

Note that we have consistently suppressed the principal quantum number n in the basis wave functions, since it is, so far, not involved in any recoupling. Our interaction V is basically a function of the relative distance between the two particles. Thus, we shall reduce the independent pair state into C.O.M. and relative co-ordinates. For this, we use the Brody-Moshinsky transformation (1960). A pair state can then be written as:

$$|n_1 l_1, n_2 l_2 (L), S, JM_J\rangle$$

$$= \sum_{\substack{n,l \\ N,\mathcal{L}}} \langle nl, N\mathcal{L}, L | n_1 l_1, n_2 l_2, L\rangle \cdot |nl, N\mathcal{L}, (L), S, JM_J\rangle$$

where n,l and N,$\mathcal{L}$ are the relative and C.O.M. quantum numbers respectively. $\langle nl, N\mathcal{L}, L | n_1 l_1, n_2 l_2, L\rangle$ is the Moshinsky transformation bracket. We have suppressed the isospin quantum numbers. With these, we may write the anti-symmetric pair state in C.O.M. and relative co-ordinates:

$$|n_1 l_1 j_1, n_2 l_2 j_2, JM_J, TM_T\rangle_A$$

$$= N_{12} \cdot \sum_{L,S} (2L+1)(2S+1)(2j_1+1)(2j_2+1) \cdot \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix} \cdot \sum_{\substack{n,l \\ N,\mathcal{L}}} \cdot$$

$$\langle nl, N\mathcal{L}, L | n_1 l_1, n_2 l_2, L\rangle \cdot [1 - (-1)^{l+S+T}] \cdot$$

$$|nl, N\mathcal{L}, (L), S, JM_J, TM_T\rangle$$

We have to recouple $\underline{l}$ and $\underline{S}$ to a relative $\underline{j}$, because the interaction, after being analysed into angular momentum states, depends on this quantum number $\underline{j}$. This recoupling may be achieved by noting that,

$$|nl, N\mathcal{L}, (L), S, JM_J\rangle$$

$$= \sum_j (-1)^{\mathcal{L}+S+L+j} \sqrt{(2L+1)(2j+1)} \cdot \begin{Bmatrix} l & S & j \\ J & \mathcal{L} & L \end{Bmatrix} \cdot$$

$$\cdot |nl, S(j), N\mathcal{L}, JM_J\rangle$$

where $\begin{Bmatrix} l & S & j \\ J & \mathcal{L} & L \end{Bmatrix}$ is the six-j symbol.

Thus,

$$|n, l, j, ; n_2 l_2 j_2 ; JM_J, TM_T\rangle_A$$

$$= N_{12} \sum_{L,S} (-1)^{L+S} (2L+1) \sqrt{(2S+1)(2j_1+1)(2j_2+1)} \cdot \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & S & J \end{Bmatrix} \cdot$$

$$\cdot \sum_{\substack{n,l \\ N,\mathcal{L}}} (-1)^{\mathcal{L}} \langle nl, N\mathcal{L}, L | n_1 l_1, n_2 l_2, L\rangle \cdot [1 - (-1)^{l+S+T}] \cdot$$

$$\cdot \sum_j (-1)^j \sqrt{(2j+1)} \begin{Bmatrix} l & S & j \\ J & \mathcal{L} & L \end{Bmatrix} \cdot |nl, S(j), N\mathcal{L}, JM_J, TM_T\rangle$$

Therefore, the matrix element of V taken between these states is:

$$\langle (ab JT)_A | V | (cd JT)_A \rangle \equiv \langle n_a l_a j_a, n_b l_b j_b, JT | V | n_c l_c j_c, n_d l_d j_d, JT \rangle$$

$$= N_{ab} \cdot N_{cd} \cdot \sum_{S,L,L'} (-1)^{L+L'} (2L+1)(2L'+1)(2S+1) \cdot (2j_a+1)(2j_b+1)(2j_c+1)(2j_d+1)$$

$$\cdot \begin{Bmatrix} l_a & \frac{1}{2} & j_a \\ l_b & \frac{1}{2} & j_b \\ L & S & J \end{Bmatrix} \cdot \begin{Bmatrix} l_c & \frac{1}{2} & j_c \\ l_d & \frac{1}{2} & j_d \\ L' & S & J \end{Bmatrix} \cdot \sum_{\substack{N,\mathcal{L} \\ n,l \\ n',l'}} \langle nl, N\mathcal{L}, L | n_a l_a, n_b l_b, L\rangle \cdot$$

$$\cdot \langle n'l', N\mathcal{L}, L' | n_c l_c, n_d l_d, L'\rangle \cdot [1 - (-1)^{l+S+T}] \cdot [1 - (-1)^{l'+S+T}] \cdot$$

$$\cdot \sum_j (2j+1) \cdot \begin{Bmatrix} l & S & j \\ J & \mathcal{L} & L \end{Bmatrix} \cdot \begin{Bmatrix} l' & S & j \\ J & \mathcal{L} & L' \end{Bmatrix} \langle nlS(j) | V | n'l'S(j)\rangle$$

(III.3)

This expression is identical with expression (2.1) of Kuo and Brown (1966). So far, we have implicitly assumed that the interaction V is the so-called residual interaction and contains no singularity. However, the Brueckner theory tells us that the effective interaction one should use in such a calculation is the reaction matrix G, derived from the 'realistic' two-body force V. We shall make a mental adjustment here that all the V's appearing in the expression (III.3) are to be replaced by G. So, our basic aim is to evaluate $\langle nlS(j) | G | n'l'S(j) \rangle$ from some realistic interaction.

Section 2.

Let us now recall the 'semi-infinite' equation and proceed to solve it. We have, in full notations,

$$|\psi_{ij}^*\rangle = |\phi_{ij}\rangle - \left(\frac{Q}{e}\right)^* V |\psi_{ij}^*\rangle$$

Define a defect wave function,

$$|\zeta_{ij}^*\rangle = (|\phi_{ij}\rangle - |\psi_{ij}^*\rangle)$$

We can then write the equation as:

$$|\zeta_{ij}^*\rangle = \left(\frac{Q}{e}\right)^* V (|\phi_{ij}\rangle - |\zeta_{ij}^*\rangle)$$

or,
$$\left(1/\left(\frac{Q}{e}\right)^* + V\right) \cdot |\zeta_{ij}^*\rangle = V|\phi_{ij}\rangle$$

To set up the reference spectrum equation corresponding to this problem, we replace Q* by 1 and e* by e^R of Chapter II. Therefore,

$$\left\{ \frac{\hbar^2 k_1^2}{2M} + \frac{\hbar^2 k_2^2}{2M} - E(i) - E(j) + V \right\} |\chi_{ij}^R\rangle = V |\phi_{ij}\rangle$$

where k_1 and k_2 refer to the momenta of the two particles. Also $|\chi_{ij}^R\rangle$ denotes the reference wave defect. Let us make a transformation to the C.O.M. and relative co-ordinates,

$$\underset{\sim}{r} = (\underset{\sim}{r}_1 - \underset{\sim}{r}_2)/\sqrt{2} \quad ; \quad \underset{\sim}{R} = (\underset{\sim}{r}_1 + \underset{\sim}{r}_2)/\sqrt{2}$$

and

$$\underset{\sim}{k} = (\underset{\sim}{k}_1 - \underset{\sim}{k}_2)/\sqrt{2} \quad ; \quad \underset{\sim}{K} = (\underset{\sim}{k}_1 + \underset{\sim}{k}_2)/\sqrt{2}$$

Then,

$$\left[\frac{\hbar^2 k^2}{2M} + \frac{\hbar^2 K^2}{2M} - E(i) - E(j) + V \right] |\phi_{N\mathcal{L}}\rangle |\chi_{n\ell}^R\rangle = V |\phi_{N\mathcal{L}}\rangle |\phi_{n\ell}\rangle$$

where $(2N + \mathcal{L} + 2n + \ell) = (2n_i + \ell_i + 2n_j + \ell_j)$ and $|\phi_{N\mathcal{L}}\rangle$ denotes the C.O.M. harmonic oscillator function. Multiplying from the left of this equation by $\langle \phi_{N\mathcal{L}}|$ and integrating over $\underset{\sim}{R}$ space, we get,

$$\left[\frac{\hbar^2 k^2}{2M} + \langle T_{N\mathcal{L}} \rangle - E(i) - E(j) + V \right] |\chi_{n\ell}^R\rangle = V |\phi_{n\ell}\rangle \qquad \text{(III.4)}$$

Here, $\langle T_{N\mathcal{L}} \rangle$ is the expectation value of the kinetic energy operator of the C.O.M. part of the wave function. We have also assumed that the separation of the reference wave defect $|\chi_{ij}^R\rangle$ into C.O.M. and relative co-ordinates is complete. However, equation (III.4) contains centre-of-mass dependence implicitly in $\langle T_{N\mathcal{L}} \rangle$, and such a separation may only be regarded as approximate. This discrepency may be compensated by including state-dependence in the final reaction matrix, as done by Kuo (1967). Let us define a

parameter γ^2 as:

$$\gamma^2 = [\langle T_{N\mathcal{L}} \rangle - E(i) - E(j)] \cdot \frac{2M}{\hbar^2}$$
$$= [\tfrac{1}{2} E_{N\mathcal{L}} - E(i) - E(j)] \cdot \frac{2M}{\hbar^2}$$

where we have replaced $\langle T_{N\mathcal{L}} \rangle$ by the average kinetic energy of the oscillator in state $N\mathcal{L}$. We see that γ^2 contains all of the state-dependence, and it is through γ^2 that we are able to include the proper C.O.M. dependence. Note also that owing to the symmetric transformation of Moshinsky, the reduced mass remains the nucleon mass M.

The equation for the radial part of the wave function may be obtained by defining:

$$\langle r | \mathcal{S}^R_{n\ell} \rangle = \frac{\chi^R_{n\ell}(r)}{r} \mathcal{Y}^m_{\ell s j}(\hat{r})$$

$$\langle r | \phi_{n\ell} \rangle = \frac{R_{n\ell}(r)}{r} \mathcal{Y}^m_{\ell s j}(\hat{r})$$

and

$$\langle r | \psi^R_{n\ell} \rangle = \frac{u^R_{n\ell}(r)}{r} \mathcal{Y}^m_{\ell s j}(\hat{r}) \qquad \text{(III.5)}$$

We have written out, explicitly, the spin quantum numbers S, j and m, which have been suppressed previously. Also, the radial harmonic oscillator function differs from the usual one by a factor of r. $\mathcal{Y}^m_{\ell s j}(\hat{r})$'s are the total angular momentum wave function. For uncoupled states, the radial wave defect $\chi^R_{n\ell}(r)$ satisfies the following equation:

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \gamma^2 - \frac{2M}{\hbar^2} V(r)\right] \chi^R_{n\ell}(r) = -\frac{2M}{\hbar^2} V(r) R_{n\ell}(r)$$

(III.6)

This equation is true for uncoupled partial waves only, and it is implicit that V(r) used is of the proper angular momentum. We shall generalize this equation to include coupled states in Appendix A.

Equation (III.6) is to be solved numerically with the boundary conditions that,

$$\chi^R_{n\ell}(r_c) = R_{n\ell}(r_c)$$

and

$$\chi^R_{n\ell}(\infty) = 0$$

where r_c is the core radius of the infinite-repulsive region of the potential used. For soft-core potentials, we can put in an infinitesimal hard core and the first boundary condition will still hold. To solve this second order differential equation, we use Ridley's method (1957). We shall illustrate the salient features by the solution of an uncoupled equation. The coupled equations are readily obtained as in Appendix A.

Consider the differential equation for partial wave ℓ,

$$\left[\frac{d^2}{dr^2} + g_{n\ell}\right]\chi^R_{n\ell}(r) = h_{n\ell}$$

where $g_{n\ell}$, $\chi^R_{n\ell}$, $h_{n\ell}$ are functions of r.
Rewriting this as,

$$\left(\frac{d}{dr} - S_{n\ell}\right)\left(\frac{d}{dr} + S_{n\ell}\right)\chi^R_{n\ell} = h_{n\ell}$$

gives us the following equations:

$$\frac{dS_{nl}}{dr} = S_{nl}^2 + g_{nl} \qquad \text{(III.7a)}$$

$$\frac{d\chi^R_{nl}}{dr} = -S_{nl}\chi^R_{nl} + W_{nl} \qquad \text{(III.7b)}$$

where $W_{nl}(r)$ satisfies,

$$\frac{dW_{nl}}{dr} = S_{nl}W_{nl} + h_{nl} \qquad \text{(III.7c)}$$

The procedure is to integrate equation (III.7a) and equation (III.7c) inward from $r \to \infty$ with the proper initial conditions for functions S_{nl} and W_{nl}, up to $r = r_c$. Then equation (III.7b) is integrated outward from $r = r_c$ with the correct initial condition for χ^R_{nl} and the tabulated values of functions S_{nl} and W_{nl}. Integration is done by the Runge-Kutta-Gill method proposed by Gill (1951).

To determine the initial conditions for S_{nl} and W_{nl}, consider the homogeneous equation as $r \longrightarrow \infty$ and $V(r) = 0$,

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \gamma^2\right] H_l = 0$$

For positive γ^2, the solution is simply proportional to the Spherical Henkel functions,

$$H_l^{(\pm)}(r) = i^{(l+1)}(\mp i\gamma\cdot r)\,h_l^{(1)}(\mp i\gamma\cdot r)$$

$$= i^{-(l+1)}(\pm i\gamma\cdot r)\,h_l^{(2)}(\pm i\gamma\cdot r)$$

Using only the decaying function, we have then,

$$\frac{d\chi^R_{nl}(r)}{dr} - \gamma \frac{H_l^{(-)'}(\gamma\cdot r)}{H_l^{(-)}(\gamma\cdot r)} \cdot \chi^R_{nl}(r) = 0$$

as $r \to \infty$ (III.8)

where the prime denotes differentiation with respect to the argument. Comparing this with equation (III.7b) we see that for $r \to \infty$, if we set,

$$S_{nl} = -\gamma \cdot \frac{H_l^{(-)'}(\gamma\cdot r)}{H_l^{(-)}(\gamma\cdot r)} \quad \text{and} \quad W_{nl} = 0,$$

the boundary condition (III.8) is satisfied.

Results of some of the partial waves using the Hamada-Johnston potential are plotted in Fig.(III.1)

After the wave defects χ^R_{nl}'s are evaluated, the reference G-matrix may be obtained without too much difficulty. The reduced radial integrals are of the form,

$$\langle n'l'S(j) \,/\, G^R \,/\, nl\, S(j)\rangle \equiv \int_0^\infty R_{n'l'}(r)\, V(r)\, U^R_{nl}(r)\, dr$$

where $U^R_{nl}(r)$ is the radial part of the perturbed wave function. Here, we shall limit ourselves to integrals diagonal in n and l. Off-diagonal ones may easily be obtained. Consider the radial wave equation for the harmonic oscillator of parameter $\hbar\omega$, and that for the reference wave defect,

$$\left\{\frac{d^2}{dr^2} - \frac{l(l+1)}{\gamma^2} + \frac{2M}{\hbar^2}\left(E_{nl} - \tfrac{1}{2}M\omega^2\gamma^2\right)\right\} R_{nl}(r) = 0$$

and

$$\left\{\frac{d^2}{dr^2} - \frac{l(l+1)}{\gamma^2} - \gamma^2\right\} \chi^R_{nl}(r) = -\frac{2M}{\hbar^2} V(r)\, U^R_{nl}(r) \quad \text{(III.9)}$$

Pre-multiplying these equations by χ^R_{nl} and R_{nl}

respectively, we integrate from $r = r_c$ to $r \to \infty$, and subtract. Then we have,

$$\int_{r_c}^{\infty} R_{n\ell}(r)\, V(r)\, U^R_{n\ell}(r)\, dr$$

$$= \int_{r_c}^{\infty} \left(E_{n\ell} + \frac{\hbar^2}{2M}\gamma^2\right) \cdot R_{n\ell}(r)\, \chi^R_{n\ell}(r)\, dr - \frac{1}{2}\int_{r_c}^{\infty} M\omega^2 R_{n\ell}(r)\, \chi^R_{n\ell}(r)\, r^2 dr$$

$$+ \frac{\hbar^2}{2M}\int_{r_c}^{\infty} \left[\chi^R_{n\ell}(r)\, \frac{d^2}{dr^2} R_{n\ell}(r) - R_{n\ell}(r)\, \frac{d^2}{dr^2}\chi^R_{n\ell}(r)\right] dr$$

Integrating the last integral by parts and using the boundary conditions of $\chi^R_{n\ell}(r_c) = R_{n\ell}(r_c)$ and $\chi^R_{n\ell}(\infty) = R_{n\ell}(\infty) = 0$, we have,

$$\int_{r_c}^{\infty} R_{n\ell}(r)\, V(r)\, U^R_{n\ell}(r)\, dr$$

$$= \int_{r_c}^{\infty} \left(E_{n\ell} + \frac{\hbar^2}{2M}\gamma^2\right) R_{n\ell}(r)\, \chi^R_{n\ell}(r)\, dr - \frac{1}{2}\int_{r_c}^{\infty} M\omega^2 R_{n\ell}(r)\, \chi^R_{n\ell}(r)\, r^2 dr$$

$$- \frac{\hbar^2}{2M}\left[R_{n\ell}(r)\, U^{R\,\prime}_{n\ell}(r)\right]_{r=r_c} \qquad \text{(III.10)}$$

Now, because of the hard core, the wave function $U^R_{n\ell}(r)$ is discontinuous in its first derivative at $r = r_c$. One can then separate the reduced integral into three terms:

$$\int_0^{\infty} R_{n\ell}(r)\, V(r)\, U^R_{n\ell}(r)\, dr$$

$$= \int_0^{r_c} \left[\left(E_{n\ell} + \frac{\hbar^2}{2M}\gamma^2\right) - \frac{1}{2}M\omega^2 r^2\right] R^2_{n\ell}(r)\, dr$$

$$+ \frac{\hbar^2}{2M}\left[R_{n\ell}(r)\, U^{R\,\prime}_{n\ell}(r)\right]_{r=r_c} + \int_{r_c}^{\infty} R_{n\ell}(r)\, V(r)\, U^R_{n\ell}(r)\, dr$$

Here, we have used the fact that $\chi^R_{n\ell}(r) = \mathcal{R}_{n\ell}(r)$ for $r \leq r_c$. The terms on the R.H.S. are usually called the core volume term, the surface term and the outer term, respectively.

Substituting expression (III.10) for the outer term, we have finally,

$$\int_0^\infty \mathcal{R}_{n\ell}(r)\, V(r)\, U^R_{n\ell}(r)\, dr$$

$$= \int_0^{r_c} \left[\left(E_{n\ell} + \frac{\hbar^2}{2M}\gamma^2\right) - \frac{1}{2}M\omega^2 r^2 \right] \mathcal{R}^2_{n\ell}(r)\, dr$$

$$+ \int_{r_c}^\infty \left[\left(E_{n\ell} + \frac{\hbar^2}{2M}\gamma^2\right) - \frac{1}{2}M\omega^2 r^2 \right] \mathcal{R}_{n\ell}(r)\, \chi^R_{n\ell}(r)\, dr$$

$$= \int_0^\infty \left[\left(E_{n\ell} + \frac{\hbar^2}{2M}\gamma^2\right) - \frac{1}{2}M\omega^2 r^2 \right] \mathcal{R}_{n\ell}(r)\, \chi^R_{n\ell}(r)\, dr \qquad \text{(III.11)}$$

For off-diagonal cases, we have similarly:

$$\int_0^\infty \mathcal{R}_{n'\ell'}(r)\, V(r)\, U^R_{n\ell}(r)\, dr$$

$$= \int_0^{r_c} \left[\left(E_{n'\ell'} + \frac{\hbar^2}{2M}\gamma^2\right) - \frac{1}{2}M\omega^2 r^2 \right] \mathcal{R}_{n'\ell'}(r)\, \mathcal{R}_{n\ell}(r)\, dr \cdot \delta_{\ell\ell'}$$

$$+ \int_{r_c}^\infty \left\{ \left(E_{n'\ell'} + \frac{\hbar^2}{2M}\gamma^2\right) - \frac{1}{2}M\omega^2 r^2 - \frac{1}{r^2}\left[\ell'(\ell'+1) - \ell(\ell+1)\right] \right\} \mathcal{R}_{n'\ell'}(r)\, \chi^R_{n\ell}(r)\, dr$$

(III.12)

These expressions can be generalized to include coupled states.

Section 3.

The reference reaction matrix G^R is only a first approximation to G*, the reaction matrix for the 'semi-

infinite' problem. We have to include Pauli corrections,

$$\langle n'l', N'L', S(j) | G^R \frac{1-Q^*}{e^R} G^R | nl, NL, S(j) \rangle$$

$$= - \langle f^R(n'l', N'L', S(j)) | e^R (Q^* - 1) | f^R(nl, NL, S(j)) \rangle \quad \text{(III.13)}$$

where $\langle r R | f^R(nl, NL, S(j)) \rangle = \phi_{NL}(\underline{R}) \frac{\chi^R_{nl}(r)}{r} \cdot \mathcal{Y}^{m_j}_{lsj}(\hat{r})$

is the total wave defect. e^R is the reference energy denominator given by:

$$e^R = \frac{\hbar^2}{2M}(k^2 + K^2) \quad \text{(III.14)}$$

The Pauli operator Q* is an integral operator in the momentum space. It differs from the ordinary Q of nuclear matter in that the intermediate states are $|k_1 k_2^* \rangle$'s instead of $|k_1 k_2\rangle$'s. However, the space in which the momentum vector must be integrated does not differ and we can use the same angular-averaging technique of Brown, Shappert and Wong (1964), when transformation is made to C.O.M. and relative co-ordinates. It is found that for a single particle momentum cutoff of k_c the angular average of Q* is:

$$\bar{Q}^*(k, K, k_c) = 0 \quad \text{for} \quad (k^2 + K^2) > 2k_c^2$$

$$= 1 \quad \text{for} \quad |k - K| > \sqrt{2} k_c$$

$$= \frac{k^2 + K^2 - 2k_c^2}{2kK} \quad \text{otherwise} \quad \text{(III.15)}$$

This angular averaged $\bar{Q}^*$ differs from Kuo and Brown's value because we have used a symmetric definition for k and K.

Before writing out expression (III.13) more explicitly, let us examine the Pauli correction in terms of the shell-model states. From equation (II.7) and the definition of the reference reaction matrix G^R, it is easy to see that (c.f. (II.12)),

$$G^* = G^R + G^R \left(\frac{1}{e^R} - \frac{Q^*}{e^*} \right) G^* \simeq G^R + G^R \left(\frac{1 - Q^*}{e^R} \right) G^R \qquad \text{(III.16)}$$

where we have assumed $e^R = e^*$ and that G^R is close enough to G^* to replace it in the second term. It is this term that we have labelled 'Pauli' correction. Writing more explicitly, this Pauli correction is given by:

$$\langle \phi_{\alpha\beta} / G^R \frac{(1-Q^*)}{e^R} G^R / \phi_{\gamma\delta} \rangle$$

$$= \langle \chi^R_{\alpha\beta} / e^R (1 - Q^*) / \chi^R_{\gamma\delta} \rangle$$

$$= \langle \chi^R_{\alpha\beta} / e^R / \chi^R_{\gamma\delta} \rangle - \int_{k_c}^{\infty} \int_{k_c}^{\infty} d^3k_1 \, d^3k_2 \, e^R \langle \chi^R_{\alpha\beta} / k_1 k_2^* \rangle \langle k_1 k_2 / \chi^R_{\gamma\delta} \rangle$$

(III.17)

Here, we have expressed Q* explicitly as an integral operator over the momentum space. $\chi^R_{\alpha\beta}$ is the reference wave defect corresponding to an unperturbed state $\phi_{\alpha\beta}$.

By the definition of $/k_1 k_2^* \rangle$ we have,

$$\langle k_1 k_2^* / \chi^R_{\alpha\beta} \rangle = \langle k_1 k_2 / \chi^R_{\alpha\beta} \rangle - \sum_{\gamma,\delta \in C,D} \langle k_1 k_2 / \phi_{\gamma\delta} \rangle \langle \phi_{\gamma\delta} / \chi^R_{\alpha\beta} \rangle \qquad \text{(III.18)}$$

Putting this expansion in (III.17) and using the simplified notations, we have,

$$\langle \phi_\alpha | G^R \frac{1-Q^*}{e^R} G^R | \phi_\beta \rangle$$
$$= \langle f_\alpha^R | e^R | f_\beta^R \rangle - \int_{k_c}^{\infty} d\underset{\sim}{k} \langle f_\alpha^R | K \rangle \langle K | f_\beta^R \rangle e^R(K,\alpha,\beta)$$
$$+ \sum_{\gamma \in C,D} \int_{k_c}^{\infty} d\underset{\sim}{k}\, e^R(K,\alpha,\beta) \Big\{ \langle f_\alpha^R | \phi_\gamma \rangle \langle K | f_\beta^R \rangle \langle \phi_\gamma | K \rangle$$
$$+ \langle f_\alpha^R | K \rangle \langle \phi_\gamma | f_\beta^R \rangle \langle K | \phi_\gamma \rangle \Big\}$$
$$- \sum_{\delta,\gamma \in C,D} \int_{k_c}^{\infty} d\underset{\sim}{k}\, e^R(K,\alpha,\beta) \langle f_\alpha^R | \phi_\gamma \rangle \langle \phi_\delta | f_\beta^R \rangle \langle K | \phi_\delta \rangle \langle \phi_\gamma | K \rangle$$

(III.19)

To determine the importance of each term, we shall consider the following. For $K > k_c$,

a) $\langle f_\alpha^R | K \rangle$, the overlap of wave defect with high momentum, is important.

b) $\langle f_\alpha^R | \phi_\beta \rangle$, is not negligible since both f_α^R and ϕ_β are finite inside the core.

c) $\langle \phi_\alpha | K \rangle$, for $\alpha \in$ C or D is negligible if k_c is chosen to make it so.

Expression (III.19) is a fairly complicated one; but, it is easily simplified if k_c is chosen according to condition c). We have referred to this criterion before. Assuming that c) is satisfied, we see that the first summation in (III.19) is of the order of importance of the Fourier components greater than k_c in the $|\phi\rangle$'s in C and D. The second summation is a further order smaller. If need arises, the first summation in (III.19) may be evaluated. However, since we already may

have been making error of this order in (II.11), it seems consistent to leave out this correction when condition c) is satisfied. We shall only retain the first two terms in (III.19) as the Pauli correction. In terms of C.O.M. and relative co-ordinates, and using the complete notations, we have,

$$-\int d^3k\, d^3K\, e^R\, |\langle kK|J^R(N\mathcal{L}, n\ell, S(j))\rangle|^2 (\bar{Q}^+ - 1)$$

Here, we have again restricted ourselves to diagonal terms. However, off-diagonal terms may be obtained readily. The overlap $\langle kK|J^R(N\mathcal{L}, n\ell, S(j))\rangle$ is obtained as follows:

$$|kK\rangle \equiv |k, K, S'\rangle = 4\pi \frac{1}{\sqrt{2}} e^{i\underline{K}\cdot\underline{R}} \sum_{m_{\ell'}, \ell'} (i)^{\ell'} j_{\ell'}(kr)\, Y_{\ell'}^{m_{\ell'}}(\hat{k})$$

$$\cdot \sum_{j'} (\ell' S' m_{\ell'}, m_{s'} | j' m_{j'})\, \mathcal{Y}_{\ell' S' j'}^{m_{j'}}(\hat{r})$$

where $\Omega = (2\pi)^3$ is the volume normalization of the plane wave. Therefore,

$$\langle kK|J^R(N\mathcal{L}, n\ell, S(j))\rangle$$

$$= \frac{4\pi}{\sqrt{2}} \int e^{i\underline{K}\cdot\underline{R}}\, \phi_{N\mathcal{L}}(\underline{R}) \cdot \sum_{m_{\ell'}, \ell'} (-i)^{\ell'} j_{\ell'}(kr)\, Y_{\ell'}^{m_{\ell'}}(k) \sum_{j'} (\ell' S' m_{\ell'}, m_{s'} | j' m_{j'})$$

$$\cdot \mathcal{Y}_{\ell' S' j'}^{m_{j'}}(\hat{r})\, \frac{\chi_{n\ell}^R(r)}{r}\, \mathcal{Y}_{\ell S j}^{m_j}(\hat{r})\, d^3R\, d^3r$$

$$= \frac{4\pi}{\sqrt{2}} (2\pi)^{3/2} \phi_{N\mathcal{L}}(\underline{K}) \int \sum_{m_{\ell'}} (-i)^{\ell} j_{\ell}(kr)\, Y_{\ell}^{m_{\ell}}(\hat{k})$$

$$\cdot (\ell S m_{\ell}, m_{s'} | j m_{j'}) \cdot \chi_{n\ell}^R(r)\, r\, dr$$

where $\phi_{N\mathcal{L}}(\underline{K})$ is the harmonic oscillator wave function in $\underline{K}$ space. In simplifying the expressions we have used the ortho-normality condition for the total angular momentum wave functions. Therefore, $|\langle kK|J^R(N\mathcal{L}, nl, S(j))\rangle|^2$ is given by :

$$(4\pi)^2 \frac{1}{\sqrt{2}} \cdot (2\pi)^3 \cdot \phi^*_{N\mathcal{L}}(\underline{K})\, \phi_{N\mathcal{L}}(\underline{K}) \cdot$$

$$\cdot \left[\int_0^\infty \chi^R_{nl}(r)\, j_l(kr)\, r\, dr\right]^2 \cdot \sum_{m_{l'}, m_{l''}} Y_l^{m_{l'}*}(\hat{k})\, Y_l^{m_{l''}}(\hat{k})$$

$$\cdot (l S m_{l'} m_{s'} | j m_j)(l S m_{l''} m_{s''} | j m_j)$$

Inserting this into the expression for Pauli correction, and integrating out the angular parts of $\underline{k}$ and $\underline{K}$, we get: $-\langle J^R(N\mathcal{L}, nl, S(j))|e^R(Q-1)|J^R(N\mathcal{L}, nl, S(j))\rangle$

$$= -\frac{2}{\pi} \int_0^\infty e^R\, R^2_{N\mathcal{L}}(K)\, [I_{nl}(k)]^2 \cdot [\bar{Q}^* - 1]\, dk\, dK \qquad \text{(III.20)}$$

where $I_{nl}(k) = \int_0^\infty \chi^R_{nl}(r)\, j_{nl}(kr)\, r\, dr$.

e^R is given by equation (III.14) and Q* satisfies condition (III.15).

It is seen that expression (III.20) differs from the similar expression in Kuo's calculation (1967) by a factor of 2. This factor appears if one chooses to use anti-symmetric plane wave intermediate states. However, a simple normalization test, by setting $e^R(Q - 1) = 1$, shows that $|kK\rangle$ used, should not be anti-symmetrized. (See, Law and Bhaduri, 1969). We shall discuss later in more detail, the consequence of this factor on the results obtained by Kuo and Brown, and Kuo.

Equation (III.20) may be generalized to include off-diagonal terms, in which case,

$$-\langle J^R(N\mathcal{L}';n'\mathcal{L}';S(j)) \,|\, e^R(Q-1) \,|\, J^R(N\mathcal{L}';n'\mathcal{L}';S(j'))\rangle$$

$$= -\frac{2}{\pi}\int_0^\infty\int_0^\infty e^R(\bar{Q}^*-1)\, R_{N\mathcal{L}}(K)\, R_{N'\mathcal{L}'}(K)\, I_{n\mathcal{L}}(k)\, I_{n'\mathcal{L}'}(k)\, dK\, dk\, \delta_{\mathcal{L}\mathcal{L}'}\,\delta_{\mathcal{L}\mathcal{L}'}$$

For coupled states, the expression for Pauli correction involves summation over the coupled channels. This makes the expression slightly more complicated, but still easily derived.

Section 4.

In the calculation of the reaction matrix G*, γ^2 appears as a parameter. It is necessary to study the dependence of G* on γ^2 because γ^2 contains the state-dependence of the matrix elements as given by:

$$\gamma^2 = \left[\tfrac{1}{2}E_{N\mathcal{L}} - E(i) - E(j)\right]\cdot\frac{2M}{\hbar^2}$$

We see that not only is there a centre-of-mass dependence but also a dependence on the initial state ij. This initial state-dependence brings in certain amount of ambiguity. Should one use the same γ^2 for both $\langle ab \,|\, G^R \,|\, mn\rangle$ and $\langle mn \,|\, G^R \,|\, ab\rangle$? In the first case, the initial state is mn, and in the latter case, ab. If different γ^2's are used, the two matrix elements will not be equal. This is a reflection of the fact that the reaction matrix G is not Hermitian even

for Hermitian interaction V, unless the propagator Q/e is Hermitian. In reference spectrum calculation, we are dealing with $1/e^R$, and unless e^R is made Hermitian, G^R will not be Hermitian. To make e^R Hermitian, we shall have to use the same starting energy for the two matrix elements. In other word, for both $\langle ab|G^R|mn\rangle$ and $\langle mn|G^R|ab\rangle$, the γ^2 is,

$$\gamma^2 = \frac{1}{2}\left\{ E_{NL} - [E(a) + E(b) + E(m) + E(n)] \right\} \frac{2M}{\hbar^2} \qquad \text{(III.21)}$$

where we have taken an average of the two starting energies $E(a) + E(b)$ and $E(m) + E(n)$.

The same approach has been used by Kuo.

The dependence of the reduced integrals on the values of γ^2 has been studied. Fig.(III.1) and Fig.(III.2) show the variation of the $^3S_1$ and $^1S_0$ reduced integrals with respect to γ^2. With momentum cutoff set at 1.4 fm^{-1}, we see that for both S states, the dependence is nearly linear with γ^2. This agrees with Kuo's results qualitatively. Our matrix elements are seen to be more sensitive to the values of γ^2.

Table (III.1) also shows reduced matrix elements for other partial waves. We have calculated states up to $\ell = 4$, and made no approximations for the higher partial waves. Since our application is to A = 18 nuclei, in which there are two nucleons in the S-D shell, $(2n_1 + \ell_1 + 2n_2 + \ell_2) = 4$, which leads to a maximum ℓ of 4. Thus, $\ell = 4$ is the highest partial wave needed. In this respect, we differ from Kuo and Brown, who made linear approximations for the wave defects

of $\ell \geq 2$. Fig.(III.3) makes a comparison between the wave defect χ^R we get for $^3D_2$ at $\gamma = 4$ and the corresponding linear approximation of Kuo and Brown. This approximation is clearly not adequate. However, mutual cancellation between high partial wave contributions may overshadow this. Since it does not take much more computation time to get the $\ell \geq 2$ states in a reference spectrum calculation, we shall include these contributions without approximations to the reference wave defects.

Fig.(III.4) and Fig.(III.5) show reference wave defects for the P states. They compare well with results from Kuo and Brown.

CHAPTER IV

APPLICATION TO SHELL MODEL

Section 1.

Nuclear shell theory has been responsible for a large number of successful interpretations of experimental data, and has been invaluable in the understanding of nuclear structure. In this respect, it is purely phenomenological in the sense that it provides a framework within which observed properties of nuclei are interpreted and correlated. (See, for example, A. deShalit and I. Talmi, 1963). A much more fundamental aspect is when shell model theory is studied as a link between the properties of complex nuclei and the free nucleon interactions. A large body of calculations under the general heading of "Shell Model Calculations with Realistic Interactions" has been devoted for this purpose. (See, for example, C. W. Wong, 1967; Köhler and McCarthy, 1967; Kalio and Day, 1967). No matter which approach we wish to take, the assumptions and limitations of the shell model theory must be borne in mind. (See M. Macfarlane, 1967). There are two basic assumptions in the shell theory. First, only the nucleon co-ordinates need be considered. This restricts the theory to low energy regions, where the possibility of real meson creations is ignored. Secondly, the interaction between

nucleons is a two-body one, meaning that effects due to higher order forces are neglected. So, we see that the premises in which the nuclear shell theory operates, is a rather restricted. However, it is still possible to form a theory of shell model and its validity, within the framework of these assumptions (Brandow, 1967), and then, a comparison with experimental results would be a test for the forces used.

We shall restrict ourselves to the more fundamental aspect of nuclear shell theory, and study the so-called 'realistic' type calculations. In this regard, we have mentioned earlier that the first basic step is to replace the free-nucleon interaction by a Brueckner reaction matrix. We shall now show why the reaction matrix is a good first order approximation to the 'effective' shell model matrix.

Consider the shell model Hamiltonian of an A-particle system,

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j}^{A} V_{ij}$$

where V_{ij} is the two-body interaction. We may rewrite this Hamiltonian as:

$$H = H_0 + V$$

where

$$H_0 = \sum_{i=1}^{A} (T_i + U_i)$$

$$V = \sum_{i<j}^{A} V_{ij} - \sum_{i=1}^{A} U_i$$

Here, U_i is a single particle potential which, hopefully, will contain most of the effects of the many-body system. From H_o, one can get a set of single particle eigenfunctions with which to construct the shell model wave function. This set of eigenfunctions will span the whole of the Hilbert space $\mathcal{H}$ and exceed all bounds. So, by the very nature of the problem, diagonalization in space $\mathcal{H}$ is impossible and a truncation of the space is inevitable.

Suppose we have chosen a finite space D in which the problem becomes tractable. We must then define an 'effective' Hamiltonian in this space, which will reproduce the same eigen-energies as in $\mathcal{H}$. Let us define this by:

$$H_{eff} = H_o + V_{eff}$$

such that $$H_{eff} |\psi_D\rangle = H |\psi\rangle = E |\psi_D\rangle \quad \text{in D,}$$

where E is the eigen-value of the Hamiltonian H in $\mathcal{H}$ space. V_{eff} is the effective interaction we alluded to earlier. In this process of projection into a finite space, some information is bound to be lost. However, we have retained at least the nuclear level information .

We shall now study the relation between V, the realistic interaction, and V_{eff} . The condition that V_{eff} must satisfy is that it should produce the same low-lying energy levels in the model space D, as V does in $\mathcal{H}$. However, this is not enough, since we do not have any criterion for the eigen-

vectors in D. One simple way is just to define the eigen-vectors for the effective interaction as the eigen-vectors of the true Hamiltonian projected onto the model space D. Therefore the conditions for V_{eff} are:

$$\left.\begin{array}{l}(H_0 + V)|\psi\rangle = E|\psi\rangle \\ \text{in } \mathcal{H}\end{array}\right\} \quad \left\{\begin{array}{l}(H_0 + V_{eff})|\psi_D\rangle = E|\psi_D\rangle \\ \text{with } |\psi_D\rangle = P|\psi\rangle\end{array}\right. \tag{IV.1}$$

where P is the projection operator defined by:

$$P = \sum_{i \in D} |\phi_i\rangle\langle\phi_i| \tag{IV.2}$$

We may also define the complementary operator Q as:

$$Q = 1 - P = \sum_{i \notin D} |\phi_i\rangle\langle\phi_i|$$

where $|\phi_i\rangle$'s are the eigen-vectors for the single particle Hamiltonian H_0. It is evident from (IV.1) and (IV.2) that we may write,

$$|\psi\rangle = \sum_{i \in \mathcal{H}} a_i|\phi_i\rangle \quad \text{and} \quad |\psi_D\rangle = \sum_{i \in D} a_i|\phi_i\rangle$$

We note here that the model wave function $|\psi_D\rangle$ has the same coefficients of expansion as $|\psi\rangle$ for i ∈ D. If the true wave function contains much contribution from single particle states outside of D, then $|\psi_D\rangle$ will not resemble the

true wave function. However, we have only one condition for $|\psi_D\rangle$ and that is that it should produce true low-lying eigen-energies when acted upon by H_{eff}. There is no requirement that $|\psi_D\rangle$ should resemble $|\psi\rangle$. In calculations where the wave function is more critical, we should use expansion (II.10) in which the model wave function is expressed in basis states with built-in short-range correlations.

The Schrödinger equations for $\mathcal{H}$ and D spaces are then:

$$H|\psi\rangle = E|\psi\rangle \quad \text{or} \quad (E-\varepsilon_i)a_i = \sum_{j\in\mathcal{H}} \langle\phi_i|V|\phi_j\rangle a_j \qquad i\in\mathcal{H}$$

and (IV.3)

$$H_{eff}|\psi_D\rangle = E|\psi_D\rangle \quad \text{or} \quad (E-\varepsilon_i)a_i = \sum_{j\in D} \langle\phi_i|V_{eff}|\phi_j\rangle a_j \qquad i\in\mathcal{H}$$

where we have assumed the normalization $\langle\psi|\psi_D\rangle = \langle\psi_D|\psi_D\rangle = 1$.

Comparing expressions in (IV.3) we see that for any operator V_{eff} defined by

$$V_{eff}|\psi_D\rangle = V|\psi\rangle \qquad \text{(IV.4)}$$

equations in (IV.3) will be satisfied. Therefore, (IV.4) is the condition for V_{eff} . To derive an expression for V_{eff} we note that,

$$\frac{Q}{(E-H_0)}V|\psi\rangle = \sum_{i\notin D} |\phi_i\rangle \frac{\langle\phi_i|V|\psi\rangle}{(E-\varepsilon_i)} \qquad \text{(IV.5)}$$

$$= \sum_{i\notin D} a_i|\phi_i\rangle = Q|\psi\rangle$$

by virtue of (IV.3) and the definition of Q.

Combining equations (IV.3), (IV.4) and (IV.5), we have,

$$V_{eff}\,|\psi_D\rangle = V|\psi\rangle = V(P+Q)|\psi\rangle$$

$$= V|\psi_D\rangle + V\frac{Q}{(E-H_0)}V|\psi\rangle$$

$$= V|\psi_D\rangle + V\frac{Q}{(E-H_0)}V_{eff}\,|\psi_D\rangle$$

or more simply,

$$V_{eff} = V + V\frac{Q}{E-H_0}V_{eff} \qquad \text{in D space.} \qquad \text{(IV.6)}$$

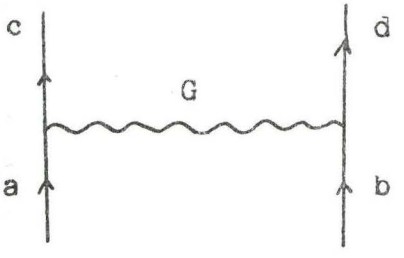
This, then, is the relation we want between V and V_{eff}. It is clear that V_{eff} depends on the eigen-value E. With certain restrictions on the operator Q, equation (IV.4) reduces to the Brueckner reaction matrix equation. For a system with two nucleons outside of a completely inert core, V_{eff} is equal to G. The corresponding restriction on Q is that it should prevent any type of excitation of the core. Thus, for systems such as O^{18} and F^{18}, the reaction matrix G is a good first order approximation to V_{eff}. It is only a first order approximation, because core excitations must be taken into account as renormalizations.

Equation (IV.6) and second equation in (IV.3) are the two basic equations in nuclear shell theory. The latter looks very much like the ordinary secular equation. However,

there are two main points of difference. First, the matrix element of V_{eff} is E-dependent, as shown by Bloch and Horowitz (1958), and secondly, the eigen-functions obtained from this 'secular' equation are not necessarily orthogonal since they are eigen-functions of different effective Hamiltonians $H_{eff}(E)$. This dependence on E is characteristic of the Brillouin-Wigner perturbation theory. (See, for example, March, Young, and Sampanthar, 1967). As we have mentioned in the previous chapters, we shall make no attempt at self-consistency of this nature. A suitable average will be used for E in the energy denominator in (IV.6). (See, M. Macfarlane, 1967)

Section 2.

It is apparent from discussion in the last section, that core excitations must somehow be taken into account. What we have included in the reaction matrix is a diagram of the type:



where a, b, c and d are single particle states in the model space D. In the case of O^{18} and F^{18} , these are states in the S-D shell. There are two important diagrams which are second order in G and involve states inside the core. These are:

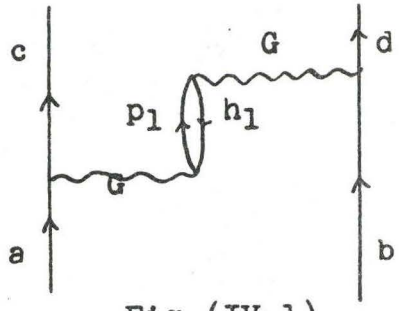


Fig.(IV.1)

and

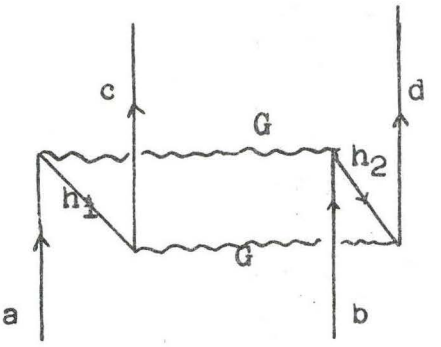


Fig.(IV.2)

Fig.(IV.1) is usually called the core-polarization diagram, in which a particle inside the core is excited out, creating a particle-hole pair. This diagram involves 3p-1h intermediate states and because of parity consideration it usually represents an energy excitation of at least $2\hbar\omega$ in harmonic oscillator language. Fig.(IV.2) represents core-excitation of two particles and also involves a smallest energy excitation of $2\hbar\omega$. However, whether these excitations can be treated by perturbative methods, is now a question. For example, in O^{18} and F^{18}, 4p-2h configurations are believed to contribute heavily to their low-lying states. (See, Federman and Talmi, 1965). One remedy, of cource, is to include 4p-2h configurations explicitly in the model space D. This increases the difficulties of the shell model problem and until a better method is found, we shall treat 4p-2h excitations by perturbative methods. There are other diagrams which we have not included. Among these is the 2p intermediate state excitation represented by:

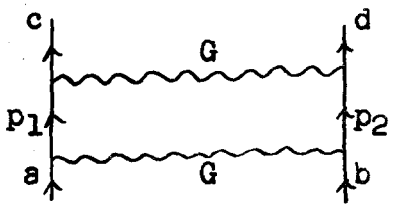


Fig. (IV.3)

This was included in the works by Kuo and Brown (1966), and by Kuo (1967), in an attempt to include some of the discrete nature of the intermediate states. We do not need to include this diagram because we have taken this into account through our formalism in Chapter II.

Bertsch (1965) has studied effects of core-polarizations in finite nuclei and derived expressions for 3p-1h excitations. However, we shall follow notations used by Kuo (1967) which appear more convenient for our computations.

Assuming an excitation energy of $2\hbar\omega$, the 3p-1h contribution to the effective interaction is given by:

$$-(ab\,JT\,|\,G\,\frac{Q_{3p-1h}}{2\hbar\omega}\,G\,|\,cd\,JT)$$

$$\equiv -\sum_{2\hbar\omega}\frac{1}{2\hbar\omega}(ab\,JT\,|\,G\,|\,3p\text{-}1h\,JT)(3p\text{-}1h\,JT\,|\,G\,|\,cd\,JT)$$

$$=\frac{1}{\sqrt{(1+\delta_{ab})}}\frac{1}{\sqrt{(1+\delta_{cd})}}(-1)^{j_a+j_b+J+T}\cdot\sum_{ph\,J''T''}\frac{(-1)^{J''+T''}}{\hbar\omega}\cdot W(\tfrac{1}{2}TT''\tfrac{1}{2};\tfrac{1}{2}\tfrac{1}{2})$$

$$\cdot\Big\{\sqrt{(2j_b+1)(2j_d+1)}\cdot W(j_aJJ''j_c;j_bj_d)\cdot L(j_c,j_b)\,L(j_a,j_d)$$

$$+\sqrt{(2j_b+1)(2j_c+1)}\cdot W(j_aJJ''j_d;j_bj_c)\cdot L(j_d,j_b)\,L(j_a,j_c)\cdot(-1)^{J+T+1}$$

$$+\sqrt{(2j_a+1)(2j_d+1)}\cdot W(j_bJJ''j_c;j_aj_d)\cdot L(j_c,j_a)\,L(j_b,j_d)\cdot(-1)^{J+T+1}$$

$$+ (2j_a+1)(2j_c+1) \cdot W(j_b J T J'' j_a ; j_a j_c) \cdot L(j_a, j_a) L(j_b, j_c) \Big\}$$

(IV.5)

where the L-matrix elements are defined by:

$$L(j_1, j_2) \equiv L(ph\, J'' T''; j_1, j_2)$$

$$= (-1)^{j_p + j_h + J'' + T'' + 1} \left\{ \frac{(2T''+1)(2J''+1)}{2(2j_2+1)} \right\}^{\frac{1}{2}} \cdot \sum_{J''' T'''} [(2J'''+1)(2T'''+1)]$$

$$\cdot W(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}; T''' T'') \cdot W(j_1 j_p j_2 j_h ; J''' J'') \cdot$$

$$\cdot (j_1 j_p J''' T''' / G / j_2 j_h J''' T''') \qquad \text{(IV.6)}$$

In expression (IV.5), a, b, c and d denote the sets of quantum numbers for the respective particles; $|3p\text{-}1h\ JT\rangle$ represents a 3p-1h intermediate state with spin J and isospin T. W's are the Racah coefficients. We have also defined $Q_{3p\text{-}1h}$ as the projection operator for all 3p-1h intermediate states with $2\hbar\omega$ excitation energy.

For 4p-2h excitations, assuming again an excitation energy of $2\hbar\omega$, we have:

$$(ab JT / -G \frac{Q_{4p-2h}}{2\hbar\omega} G / cd JT)$$

$$= \frac{1}{2\hbar\omega} \sum_{h_1 \geq h_2} (ab JT / G / h_1 h_2 JT)(h_1 h_2 JT / G / cd JT)$$

(IV.7)

where $|h_1 h_2 JT\rangle$ represents 4p-2h intermediate state with the two hole states h_1 and h_2 coupled to spin J and isospin T.

In both expressions, the two-body reaction matrix used should contain the proper 'starting' energy as given by

equation (III.21).

Section 3.

So far, we have no distinction between the discrete subspace and the model space for shell theory. We have even used the same notation for both. Let us now examine if there is any relation between these two subspaces.

The model space for shell theory must satisfy the following conditions:

a) It must be finite and small enough for diagonalization.

b) It must produce the correct eigen-values for low-lying states when operated upon by an effective Hamiltonian H_{eff} .

c) It must not contain too much impure configuration (collective, deformed). (See, M. Macfarlane, 1967)

The discrete subspace in our formalism has to satisfy only the one criterion that it should contain discrete states with significant momentum components up to the cut-off k_c. This automatically makes the subspace finite. It is apparent that the shell theory model space is more restrictive. As we have mentioned in Chapter II, our choice of the discrete subspace is not unique. In principle, the larger the subspace, the better is the approximation to the perturbed wave function. Therefore, it seems reasonable to make it at least as big as the shell theory model space. As a matter

of fact, it may even be possible to include configurations not included in the shell model space, in this discrete subspace.

We shall be able to make more conclusions regarding this after we have presented results where the size of the discrete subspace is changed along with the cutoff momentum k_c .

CHAPTER V

RESULTS AND CONCLUSIONS

Section 1.

As an application of our formalism, we have chosen the $A = 18$ nuclei, where two nucleons interact outside of a doubly closed core of O^{16}. Also, we are only considering even parity states. The low-lying states of both O^{18} and F^{18} have been determined experimentally by Polletti and Warburton (1965). We have chosen the Hamada-Johnston potential for calculation (See Appendix B), mainly because of the readily available works of Kuo and Brown (1966) and Kuo (1967), and Wong (1967) for comparison. It also fits the two-body data well at moderate energies.

The single particle basis for our calculation are the harmonic oscillator states of $\hbar\omega = 14$ MeV, which reproduces the r.m.s. radius of O^{16}. (Elton, 1961). This oscillator constant may also be treated as a parameter of the calculation.

We did calculation of the semi-infinite G*-matrix at three different values of the cutoff momentum k_c. These values are chosen to be close to the realistic nuclear density as required by our formalism. Tables (V.1) to (V.8)

show the reduced integrals of G* as k_c is varied. Each table corresponds to a particular value of γ^2, the parameter that contains the state-dependence of the reduced integrals. G*(0) represents the reaction matrix with no Pauli correction. We see that Pauli corrections are important in both singlet and triplet S-states. They are small but still significant for states of higher angular momenta. The variation with respect to k_c within the range of realistic nuclear density, however, has very little effect on the reduced integrals. A variation of no more than a few percent exists in the S-states, whereas it is almost negligible in the higher momentum states. This seems to justify our relaxing the ideal criterion for the choice of k_c . We note here that our bare G* (with no Pauli corrections) compares well with both C. W. Wong and Kuo & Brown results, except for the triplet S-states, where the incorrect numerical factor in Kuo and Brown's expression has doubled their off-diagonal tensor contributions. We cannot make valid comparison with C. W. Wong's Pauli-corrected results because we have used a free particle spectrum, and our formalism of the discrete low-lying states, is really another form of Pauli correction, which is included only in the two-particle representation and not the C.O.M. and relative representation. Both "local" and "global" Pauli corrections are included in the reduced integrals by Wong.

The variation of the S-states integrals with respect to γ^2 are shown in Fig.(III.1) and Fig.(III.2). It is seen

that the singlet states agree with well with Kuo's results whereas we see the disagreement in the triplet case as mentioned earlier. We have checked also our P-states contributions and they are in very close agreement with Kuo and Brown's results.

The resulting shell-model matrix elements are listed in table (V.9), for $k_c = 1.4$ fm^{-1} and the discrete subspace D is chosen to be the S-D shell. G* is the Pauli-corrected semi-infinite reaction matrix; G_{DS} represents correction from the discrete low-lying states in D; $G_{3p\text{-}1h}$ and G_{2h} are, respectively, contributions from three-particle-one-hole and four-particle-two-hole excitations. The reaction matrix we used for shell-model diagonalization comes under "SUM". A comparison of $G_{3p\text{-}1h}$ with Kuo's values whows the effect of the incorrect $T = 0$ reduced integrals. $G_{3p\text{-}1h}$ consists of summation over both $T = 0$ and $T = 1$ matrix elements as shown in Chapter IV, and the incorrect $T = 0$ matrix elements made the $G_{3p\text{-}1h}$ calculation an over-estimation. G_{2h}, on the other hand, agrees well with Kuo's results for the $T = 1$ matrix elements and is again in general disagreement with Kuo's $T = 0$ results. This may also be attributed to the incorrect numerical factor in the triplet S tensor contribution.

G_{DS}, the discrete state corrections, are seen to be in general, smaller than the other two renormalizations; but, compared to G*, the corrections are clearly signigicant. Variation of G_{DS} with respect to different choices of D is

shown in Table (V.10). G_{DS}(S-D) denotes the discrete contribution when the S-D shell is chosen to be the subspace D. G_{DS}(S-D, P-F) is the contribution when D is chosen to include both S-D and P-F shells. We see that the difference is significant. This is reasonable, since we see from Fig. (II.4) that even for $k_c = 1.4$ fm^{-1} , there is a significant amount of $n = 1$, $\ell = 1$ components which are excluded by our Pauli operator. However, the second S-D shell will not contribute so much. We see here that there is a distinct advantage in our formalism. In including the P-F shell as the discrete subspace, we need only evaluate, in addition to matrix elements of the S-D shell, off-diagonal matrix elements of the type $\langle (\text{P-F})^2 JT \,/\, G \,/\, (\text{S-D})^2 JT \rangle$, whereas, when the P-F shell is included in the shell-model space, then matrix elements of the type $\langle (\text{P-F})^2 JT \,/\, G \,/\, (\text{P-F})^2 JT \rangle$ will also be required. $\langle (\text{P-F})^2 JT \,/\, G \,/\, (\text{S-D})^2 JT \rangle$'s have been calculated at the different k_c's, using the reduced integrals with proper state-dependence, i.e. the proper γ^2's.

Fig.(V.1) and Fig.(V.2) show the resulting spectra of O^{18} and F^{18} . We have also included Kuo's results for comparison.

In the diagonalization of the shell-model effective interaction, the single-particle energies are taken to be

$$E(d_{5/2}) = \epsilon_0 + 0.0 \quad \text{(MeV)}$$

$$E(1S_{1/2}) = \epsilon_0 + 0.87$$

$$E(d_{3/2}) = \epsilon_0 + 5.08$$

where ϵ_o is set equal to zero so that the spectrum obtained is a relative one. In calculating the effective interaction ϵ_o = -4.14 MeV for a single neutron state and ϵ_o = -0.60 MeV for a single proton state.

With a fixed D, which in this case is the S-D shell, k_c = 1.3, 1.4 and 1.5 fm^{-1} produce almost identical spectra. In the T = 1 spectrum, an increasing k_c tends to shift the whole spectrum slightly upwards. In the T = 0 spectrum, increasing k_c shifts all levels up, except the first 3^+ level, which is shifted slightly downwards. The inclusion of the P-F shell as discrete states has little effect on the T = 1 spectrum. For the F^{18} spectrum it tends to shift the levels down. A more detailed study of these variation should include the evaluation of the third term in equation (III.19).

A general comparison of our spectra with the experimental results shows agreement in ordering, but the quantitative agreement is not very good. We were not able to reproduce the experimental ordering for the second 0^+ and 2^+ levels in O^{18}; also, the first 2^+ and 0^+ levels are far too high. Kuo's results indicated that agreement for the first 0^+ state in O^{18} is due largely to the 3p-1h contribution. We have examined this core polarization, and found it to be an over-estimate as mentioned earlier.

Section 2.

We have presented a formalism in which the B-G

equation for finite nuclei may be solved, such that one is able to include the discrete nature of the low-lying states allowed by the Pauli operator. We have made approximations in our actual application of the formalism, by leaving out the last two terms in equation (III.19), but with a little more time and labour, these terms may also be evaluated.

We have studied the effective interaction obtained from the Hamada-Johnston potential in three respects. First, the state-dependence of the effective interaction is studied through the parameter γ^2. We have seen that state-dependence affects the reduced integrals almost linearly with γ^2, in agreement with Kuo; but, we have found that the dependence is much stronger than is reported by Kuo. As a result, we have carefully included this state-dependence in our calculation by using the proper value of γ^2 for each reduced integral.

Secondly, we have varied one of our relatively free parameters, the cutoff momentum k_c. This does not seem to have too much effect when k_c is kept within the range of realistic nuclear density. Finally, we have increased our finite subspace to include the P-F shell. This again does not have drastic effect on our spectral calculations. From these, we may conclude that our formalism is not very sensitive to the variations of k_c and D in the energy level calculations, as long as they satisfy our criteria.

One avenue of improvement might be to use a linear combination of oscillators as basis states rather than just

one oscillator. This is believed to improve the treatment of the centre-of-mass dependence of the reaction matrix elements.Also, calculations have been performed with a oscillator well depth as a parameter (Becker, McKellar, and Morris, 1968). However, this is believed to be important in the binding energy calculations, and not in a spectral calculation.

With our formalism, a calculation of the effective interaction in finite nuclei becomes only slightly more difficult than a nuclear matter calculation and it may then be possible to test a variety of forces, with relative ease.

APPENDIX A

For triplet states, the $\ell = j + 1$ and $\ell = j - 1$ channels are coupled. In generalizing equation (III.6) we shall suppress the principal quantum number n in the relative co-ordinates.

Thus, for coupled triplet states, we have:

$$\left\{ \nabla_j^2 - \gamma^2 - \frac{2M}{\hbar^2} V_j(r) \right\} \chi_j^R(r) = - \frac{2M}{\hbar^2} V_j(r) R_j(r) \qquad (A.1)$$

where,

$$\nabla_j^2 \equiv \begin{pmatrix} \frac{d^2}{dr^2} - \frac{j(j-1)}{r^2} & 0 \\ 0 & \frac{d^2}{dr^2} - \frac{(j+1)(j+2)}{r^2} \end{pmatrix}$$

$$R_j(r) \equiv \begin{pmatrix} R_{j-1}(r) & 0 \\ 0 & R_{j+1}(r) \end{pmatrix}$$

$$V_j(r) \equiv \begin{pmatrix} \langle j, j-1 | V | j, j-1 \rangle & \langle j, j-1 | V | j, j+1 \rangle \\ \langle j, j+1 | V | j, j-1 \rangle & \langle j, j+1 | V | j, j+1 \rangle \end{pmatrix}$$

with $|j, \ell\rangle$ representing the angular momentum wave function, $\mathcal{Y}_{\ell S j}^{m_j}(r)$; and,

$$\chi^R_{j(r)} \equiv \begin{Bmatrix} \chi^R_{j-1,j-1}(r) & \chi^R_{j-1,j+1}(r) \\ \chi^R_{j+1,j-1}(r) & \chi^R_{j+1,j+1}(r) \end{Bmatrix}$$

with

$$\begin{bmatrix} \chi^R_{j-1,j-1}(r) \\ \chi^R_{j+1,j-1}(r) \end{bmatrix} \equiv \begin{bmatrix} R_{j-1}(r) - U_{j-1,j-1}(r) \\ -U_{j+1,j-1}(r) \end{bmatrix}$$

and

$$\begin{bmatrix} \chi^R_{j-1,j+1}(r) \\ \chi^R_{j+1,j+1}(r) \end{bmatrix} \equiv \begin{bmatrix} -U_{j-1,j+1}(r) \\ R_{j+1}(r) - U_{j+1,j+1}(r) \end{bmatrix}$$

where the U's are the radial parts of the coupled perturbed wave functions.

Both equations (III.6) and (A.1) can be solved by the Ridley method (1957). We shall write out the detail of each in turn. The basic equations are (III.7a), (III.7b) and (III.7c). Suppressing the quantum number n again, we rewrite these equations as:

$$\frac{dS_l}{dr} = S_l^2 + g_l$$

$$\frac{d\chi^R_l}{dr} = -S_l \chi^R_l + W_l \qquad \text{(A.2)}$$

where, $$\frac{dW_\ell}{dr} = S_\ell W_\ell + \mathcal{L}_\ell$$

With the uncoupled states, where $\ell = j$, we have,

$$\mathcal{Q}_\ell = -\gamma^2 - j(j+1)/r^2 - \frac{2M}{\hbar^2}\left[V_c - 2j(j+1)\,V_{LL}\right]$$

$$\mathcal{L}_\ell = -\frac{2M}{\hbar^2}\left[V_c - 2j(j+1)\,V_{LL}\right] R_\ell(r)$$

for singlets, and

$$\mathcal{Q}_\ell = -\gamma^2 - j(j+1)/r^2 - \frac{2M}{\hbar^2}\left[V_c + 2V_T - V_{LS} + \{2j(j+1)-1\}\,V_{LL}\right]$$

$$\mathcal{L}_\ell = -\frac{2M}{\hbar^2}\left[V_c + 2V_T - V_{LS} + \{2j(j+1)-1\}\,V_{LL}\right] R_\ell(r)$$

for triplets. Also, V_C, V_T, V_{LS} and V_{LL} are, respectively, the central, tensor, spin-orbit and quadratic spin-orbit parts of the potential.

The boundary conditions for the above differential equations are:

$$\chi_\ell^R(r_c) = R_\ell(r_c)$$

$$S_\ell(r) = -\gamma \cdot \frac{H_\ell^{(-)\prime}(\gamma \cdot r)}{H_\ell^{(-)}(\gamma \cdot r)} \quad \text{and} \quad W_\ell(r) = 0 \quad \text{as } r \to \infty .$$

The prime denotes differentiation with respect to the argument and $H_\ell^{(-)}$ is the decaying Spherical Henkel function.

For the coupled states $\ell = j \pm 1$, we have to generalize the Ridley equations (A.2) as follows:

$$\frac{dS_j}{dr} = S_j^{+} S_j + \mathcal{G}_j$$

$$\frac{dX_j^R}{dr} = -S_j^{+} X_j^R + W_j$$

where,

$$\frac{dW_j}{dr} = S_j^{+} W_j + \mathcal{L}_j$$

The functions are now expressed as matrices. Combining these with equation (A.1), we have then:

$$\mathcal{G}_j \equiv -\begin{pmatrix} \frac{j(j-1)}{r^2} + \gamma^2 + \frac{2M}{k^2}(j,j-1|V|j,j-1); & \frac{2M}{k^2}(j,j-1|V|j,j+1) \\ \frac{2M}{k^2}(j,j+1|V|j,j-1); & \frac{(j+2)(j+1)}{r^2} + \gamma^2 + \frac{2M}{k^2}(j,j+1|V|j,j+1) \end{pmatrix}$$

$$\mathcal{L}_j \equiv -\frac{2M}{k^2}\begin{pmatrix} R_{j-1}(j,j-1|V|j,j-1); & R_{j+1}(j,j-1|V|j,j+1) \\ R_{j-1}(j,j+1|V|j,j-1); & R_{j+1}(j,j+1|V|j,j+1) \end{pmatrix}$$

where,

$$(j,j-1|V|j,j-1) \equiv V_c - \frac{2(j-1)}{2j+1} V_T + (j-1) V_{LS} + (j-1) V_{LL}$$

$$(j,j+1|V|j,j+1) \equiv V_c - \frac{2(j+2)}{2j+1} V_T - (j+2) V_{LS} - (j+2) V_{LL}$$

and,

$$(j,j+1|V|j,j-1) \equiv (j,j-1|V|j,j+1) \equiv 6\sqrt{j(j+1)}/(2j+1)\, V_T$$

We have, in deriving these, used the following

identities for $S = 1$ states:

$$\langle j, j \mid S_{12} \mid j, j \rangle \equiv 2$$

$$\langle j, j+1 \mid S_{12} \mid j, j+1 \rangle \equiv -2(j+2)/(2j+1)$$

$$\langle j, j-1 \mid S_{12} \mid j, j-1 \rangle \equiv -2(j-1)/(2j+1)$$

$$\langle j, j-1 \mid S_{12} \mid j, j+1 \rangle \equiv \langle j, j+1 \mid S_{12} \mid j, j-1 \rangle \equiv 6\sqrt{j(j+1)}/(2j+1)$$

and

$$\langle j, l \mid L \cdot S \mid j, l \rangle \equiv \frac{1}{2}\left[j(j+1) - l(l+1) - 2\right].$$

APPENDIX B

THE HAMADA-JOHNSTON POTENTIAL

Core radius $r_c = 0.343 \frac{\hbar}{\mu c}$ fm , for all states.

$$V = V_C + V_T S_{12} + V_{LS} (\underline{L}\cdot\underline{S}) + V_{LL} L_{12}$$

where S_{12} , L S, and L_{12} are the tensor, spin-orbit and quadratic spin-orbit operators respectively. L_{12} is defined by:

$$L_{12} = [\delta_{ij} + (\underline{\sigma}_1 \cdot \underline{\sigma}_2)]\,\underline{l}^2 - (\underline{l}\cdot\underline{S})^2$$

$$V_C = 0.08\,(\tfrac{1}{3}\mu)\,(\underline{\tau}_1\cdot\underline{\tau}_2)(\underline{\sigma}_1\cdot\underline{\sigma}_2)\,Y(x)\,[1 + a_c Y(x) + b_c Y^2(x)]$$

$$V_T = 0.08\,(\tfrac{1}{3}\mu)\,(\underline{\tau}_1\cdot\underline{\tau}_2)\,Z(x)\,[1 + a_T Y(x) + b_T Y^2(x)]$$

$$V_{LS} = \mu\, G_{LS}\, Y^2(x)\,[1 + b_{LS} Y(x)]$$

$$V_{LL} = \mu\, G_{LL}\, x^{-2} Z(x)\,[1 + a_{LL} Y(x) + b_{LL} Y^2(x)]$$

with $Y(x) = e^{-x}/x$, $Z(x) = (1 + \frac{3}{x} + \frac{3}{x^2})\cdot Y(x)$.

μ = 139.4 MeV is the pion reduced mass and x is in units of $\frac{\hbar}{\mu c}$ = 1.415 fm.

THE PARAMETERS

State	a_c	b_c	a_T	b_T	G_{LS}	b_{LS}	G_{LL}	a_{LL}	b_{LL}
S.E.	8.70	10.6					-0.000891	0.20	-0.20
T.O.	-9.07	3.48	-1.29	0.55	0.1961	-7.12	-0.000891	-7.26	6.92
T.E.	6.00	-1.00	-0.50	0.20	0.0743	-0.10	0.00267	1.80	0.40
S.O.	-8.00	12.0					-0.00267	2.00	6.00

TABLE CAPTION

Table (V.1) Reduced integrals of uncoupled states, $\gamma^2 = 1$, with $\hbar\omega$=14 MeV at $k_c = 0$, 1.3, 1.4 and 1.5 fm^{-1}.

Tables (V.2)--(V.4)

Reduced integrals of uncoupled states, with same parameters as in Table (V.1), for $\gamma^2 =$ 1.5, 2.0 and 2.5 respectively.

Tables (V.5)--(V.8)

Same as the above tables, for coupled states.

Table (V.9) Shell-model matrix elements calculated with $\hbar\omega$ = 14 MeV, k_c=1.4 fm^{-1}.

G* is the Pauli-corrected semi-infinite G-matrix;

G_{DS} is the contribution from the discrete states in D;

$G_{3p\text{-}1h}$ is the contribution from 3p-1h intermediate states of $2\hbar\omega$ excitation;

G_{2h} is the contribution from 4p-2h intermediate states of $2\hbar\omega$ excitation;

SUM is the total effective interaction.

Table (V.10) Comparison of contribution from S-D shell discrete states and S-D & P-F shell discrete states with $\hbar\omega$ = 14 MeV.

TABLE(V.1)

Reduced Integrals at $\gamma^{\lambda} = 1$

$\Sigma^{\dagger}$	n	n'	ℓ	ℓ'	S	J	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	0	0	-7.9559	-6.7906	-6.6618	-6.5793
2	1	1	0	0	0	0	-5.1067	-4.2712	-4.2308	-4.2062
0	2	2	0	0	0	0	-1.6774	-1.3173	-1.3083	-1.3042
3	0	0	1	1	0	1	1.6392	1.6731	1.6787	1.6846
1	1	1	1	1	0	1	2.5424	2.6005	2.6135	2.6290
2	0	0	2	2	0	2	-0.5288	-0.5256	-0.5247	-0.5238
0	1	1	2	2	0	2	-0.8348	-0.8308	-0.8287	-0.8266
1	0	0	3	3	0	3	0.4306	0.4333	0.4338	0.4343
0	0	0	4	4	0	4	-0.0982	-0.0981	-0.0981	-0.0980
3	0	0	1	1	1	0	-1.8291	-1.7979	-1.7964	-1.8073
1	1	1	1	1	1	0	-0.9416	-0.9278	-0.9274	-0.9272
3	0	0	1	1	1	1	1.7598	1.7980	1.8040	1.8239
1	1	1	1	1	1	1	2.4818	2.5397	2.5508	2.5633
2	0	0	2	2	1	2	-2.3225	-2.2581	-2.2437	-2.2285
0	1	1	2	2	1	2	-3.1125	-3.0379	-3.0137	-2.9840
1	0	0	3	3	1	3	0.2838	0.2850	0.2852	0.2855
0	0	0	4	4	1	4	-0.5046	-0.5014	-0.5005	-0.4994

UNCOUPLED STATES

$\dagger$ $\Sigma = 2N + \mathcal{L}$

TABLE (V.2)

Reduced Integrals at $\gamma^2 = 1.5$

Σ	n	n'	ℓ	ℓ'	S	J	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	0	0	-7.2677	-6.4981	-6.4101	-6.3512
2	1	1	0	0	0	0	-4.5771	-4.0318	-4.0040	-3.9871
0	2	2	0	0	0	0	-1.3440	-1.1084	-1.1029	-1.1007
3	0	0	1	1	0	1	1.6739	1.7146	1.7101	1.7160
1	1	1	1	1	0	1	2.6090	2.6629	2.6756	2.6911
2	0	0	2	2	0	2	-0.5267	-0.5240	-0.5233	-0.5225
0	1	1	2	2	0	2	-0.8298	-0.8261	-0.8247	-0.8228
1	0	0	3	3	0	3	0.4317	0.4340	0.4345	0.4349
0	0	0	4	4	0	4	-0.0983	-0.0982	-0.0982	-0.0982
3	0	0	1	1	1	0	-1.8001	-1.7754	-1.7741	-1.7735
1	1	1	1	1	1	0	-0.9127	-0.9020	-0.9017	-0.9106
3	0	0	1	1	1	1	1.7888	1.8228	1.8288	1.8345
1	1	1	1	1	1	1	2.5303	2.5808	2.5935	2.6058
2	0	0	2	2	1	2	-2.2931	-2.2409	-2.2286	-2.2151
0	1	1	2	2	1	2	-3.0654	-3.0047	-2.9836	-2.9675
1	0	0	3	3	1	3	0.2843	0.2853	0.2855	0.2858
0	0	0	4	4	1	4	-0.5041	-0.5013	-0.5006	-0.4996

UNCOUPLED STATES

TABLE (V.3)

Reduced Integrals at $\gamma^2 = 2.0$

Σ	n	n'	ℓ	ℓ'	S	j	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	0	0	-6.8238	-6.2601	-6.1945	-6.1505
2	1	1	0	0	0	0	-4.2047	-3.8115	-3.7911	-3.7791
0	2	2	0	0	0	0	-1.0565	-0.9060	-0.9004	-0.8963
3	0	0	1	1	0	1	1.7038	1.7321	1.7375	1.7434
1	1	1	1	1	0	1	2.6696	2.7201	2.7325	2.7478
2	0	0	2	2	0	2	-0.5205	-0.5228	-0.5221	-0.5224
0	1	1	2	2	0	2	-0.8258	-0.8226	-0.8214	-0.8197
1	0	0	3	3	0	3	0.4325	0.4333	0.4349	0.4353
0	0	0	4	4	0	4	-0.0983	-0.0982	-0.0982	-0.0981
3	0	0	1	1	1	0	-1.7781	-1.7579	-1.7567	-1.7561
1	1	1	1	1	1	0	-0.8877	-0.8791	-0.8789	-0.8788
3	0	0	1	1	1	1	1.8123	1.8430	1.8486	1.8544
1	1	1	1	1	1	1	2.5721	2.6202	2.6305	2.6425
2	0	0	2	2	1	2	-2.2710	-2.2273	-2.2089	-2.2045
0	1	1	2	2	1	2	-3.0289	-2.9779	-2.9594	-2.9360
1	0	0	3	3	1	3	0.2846	0.2854	0.2857	0.2859
0	0	0	4	4	1	4	-0.5037	-0.5013	-0.5006	-0.4997

UNCOUPLED STATES

TABLE (V.4)

Reduced Integral at $\gamma^2 = 2.5$

Σ	n	n'	ℓ	ℓ'	S	J	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	0	0	-6.4951	-6.0576	-6.0062	-5.9720
2	1	1	0	0	0	0	-3.9077	-3.6081	-3.5926	-3.5838
0	2	2	0	0	0	0	-0.8122	-0.6998	-0.6972	-0.6965
3	0	0	1	1	0	1	1.7307	1.7570	1.7623	1.7680
1	1	1	1	1	0	1	2.7256	2.7734	2.7855	2.8005
2	0	0	2	2	0	2	-0.5236	-0.5041	-0.5211	-0.5204
0	1	1	2	2	0	2	-0.8225	-0.8197	-0.8186	-0.8171
1	0	0	3	3	0	3	0.4332	0.4350	0.4353	0.4357
0	0	0	4	4	0	4	-0.0983	-0.0982	-0.0982	-0.0982
3	0	0	1	1	1	0	-1.7602	-1.7431	-1.7422	-1.7417
1	1	1	1	1	1	0	-0.8651	-0.8581	-0.8579	-0.8578
3	0	0	1	1	1	1	1.8323	1.8604	1.8658	1.8714
1	1	1	1	1	1	1	2.6092	2.6538	2.6636	2.6158
2	0	0	2	2	1	2	-2.2537	-2.2162	-2.2065	-2.1957
0	1	1	2	2	1	2	-2.9997	-2.9559	-2.9394	-2.9916
1	0	0	3	3	1	3	0.2849	0.2858	0.2858	0.2860
0	0	0	4	4	1	4	-0.5034	-0.5013	-0.5006	-0.4998

UNCOUPLED STATES

TABLE (V.5)

Reduced Integrals at $\gamma^2 = 1.0$

Σ	n	n'	ℓ	ℓ'	S	J	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	1	1	-16.0690	-10.7675	-10.4534	-10.2472
2	1	1	0	0	1	1	-10.7254	-6.9716	-6.8521	-6.7794
0	2	2	0	0	1	1	-4.6597	-2.8386	-2.8056	-2.7878
2	0	0	2	2	1	1	-0.5157	-0.4937	-0.4856	-0.4762
0	1	1	2	2	1	1	-2.3601	-2.3126	-2.2944	-2.3160
2	1	0	0	2	1	1	-5.3847	-4.7076	-4.5302	-4.3399
0	2	1	0	2	1	1	-6.0105	-5.7683	-5.6745	-5.5613
3	0	0	1	1	1	2	-0.9656	-0.9447	-0.9418	-0.9392
1	1	1	1	1	1	2	-1.7302	-1.6908	-1.6853	-1.6934
1	0	0	3	3	1	2	-0.2119	-0.2109	-0.2108	-0.2106
1	1	0	1	3	1	2	0.4063	0.4140	0.4141	0.4141
2	0	0	2	2	1	3	0.0073	0.0073	0.0073	0.0073
0	1	1	2	2	1	3	-0.0238	-0.0238	-0.0238	-0.0238
0	0	0	4	4	1	3	0.2164	0.2173	0.2175	0.2177
0	1	0	2	4	1	3	-0.6713	-0.6477	-0.6464	-0.6539

COUPLED STATES

(Off-diagonal matrix elements are averages)

TABLE (V.6)

Reduced Integrals at $\gamma^2 = 1.5$

Σ	n	n'	ℓ	ℓ'	S	j	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	1	1	-12.7719	-10.1124	-9.9497	-9.8419
2	1	1	0	0	1	1	-8.3617	-6.5086	-6.4462	-6.4084
0	2	2	0	0	1	1	-3.2912	-2.4347	-2.4194	-2.4106
2	0	0	2	2	1	1	0.0619	0.0670	0.0691	0.0716
0	1	1	2	2	1	1	-1.1069	-1.0990	-1.0948	-1.0889
2	1	0	0	2	1	1	-4.4020	-3.9743	-3.8568	-3.7304
0	2	1	0	2	1	1	-4.9694	-4.8228	-4.7626	-4.6882
3	0	0	1	1	1	2	-0.9465	-0.9284	-0.9265	-0.9234
1	1	1	1	1	1	2	-1.6979	-1.6639	-1.6289	-1.6542
1	0	0	3	3	1	2	-0.2089	-0.2081	-0.2080	-0.2078
1	1	0	1	3	1	2	0.3992	0.4055	0.4056	0.4057
2	0	0	2	2	1	3	0.0164	0.0164	0.0164	0.0164
0	1	1	2	2	1	3	-0.0107	-0.0107	-0.0107	-0.0107
0	0	0	4	4	1	3	0.2216	0.2224	0.2226	0.2228
0	1	0	2	4	1	3	-0.6707	-0.6508	-0.6496	-0.6487

COUPLED STATES

(Off-diagonal matrix elements are averages)

TABLE (V.7)

Reduced Integrals at $\gamma^* = 2.0$

Σ	n	n'	ℓ	ℓ'	S	j	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	1	1	-10.9358	-9.3157	-9.2151	-9.1481
2	1	1	0	0	1	1	-6.9768	-5.8730	-5.8349	-5.8116
0	2	2	0	0	1	1	-2.4109	-1.9290	-1.2905	-1.9147
2	0	0	2	2	1	1	0.3472	0.3502	0.3509	0.3517
0	1	1	2	2	1	1	-0.4802	-0.4789	-0.4778	-0.4760
2	1	0	0	2	1	1	-3.9297	-3.6113	-3.5214	-3.4241
0	2	1	0	2	1	1	-4.4515	-4.3509	-4.3058	-4.2491
3	0	0	1	1	1	2	-0.9310	-0.9151	-0.9128	-0.9105
1	1	1	1	1	1	2	-1.6715	-1.6417	-1.6371	-1.6327
1	0	0	3	3	1	2	-0.2069	-0.2062	-0.2060	-0.2060
1	1	0	1	3	1	2	0.3944	0.3998	0.3999	0.3999
2	0	0	2	2	1	3	0.0243	0.0243	0.0243	0.0243
0	1	1	2	2	1	3	0.0011	0.0011	0.0011	0.0011
0	0	0	4	4	1	3	0.2254	0.2262	0.2263	0.2265
0	1	0	2	4	1	3	-0.6705	-0.6533	-0.6521	-0.6514

COUPLED STATES

(Off-diagonal matrix elements are averages)

TABLE (V.8)

Reduced Integrals at $\gamma^* = 2.5$

Σ	n	n'	ℓ	ℓ'	S	J	G*(0)	G*(1.3)	G*(1.4)	G*(1.5)
4	0	0	0	0	1	1	-9.6956	-8.6070	-8.5389	-8.4935
2	1	1	0	0	1	1	-5.9952	-5.2734	-5.2448	-5.2323
0	2	2	0	0	1	1	-1.7372	-1.4420	-1.4365	-1.4317
2	0	0	2	2	1	1	0.5193	0.5224	0.5228	0.5231
0	1	1	2	2	1	1	-0.0987	-0.0980	-0.0977	-0.0971
2	1	0	0	2	1	1	-3.6546	-3.3982	-3.3244	-3.2441
0	2	1	0	2	1	1	-4.1480	-4.0699	-4.0335	-3.9817
3	0	0	1	1	1	2	-0.9179	-0.9037	-0.9016	-0.8995
1	1	1	1	1	1	2	-1.6490	-1.6234	-1.6182	-1.6141
1	0	0	3	3	1	2	-0.2054	-0.2048	-0.2046	-0.2045
1	1	0	1	3	1	2	0.3908	0.3955	0.3956	0.3956
2	0	0	2	2	1	3	0.0314	0.0314	0.0314	0.0314
0	1	1	2	2	1	3	0.0119	0.0119	0.0119	0.0119
0	0	0	4	4	1	3	0.2282	0.2289	0.2291	0.2293
0	1	0	2	4	1	3	-0.6705	-0.6553	-0.6542	-0.6535

COUPLED STATES

(Off-diagonal matrix elements are averages)

TABLE (V.9)

Shell-model matrix elements in the S-D shell, with $k_c = 1.4$ fm^{-1} and S-D shell as the finite subspace D.

Symbols: $0d_{5/2} \longrightarrow 4$, $1s_{1/2} \longrightarrow 5$, $0d_{3/2} \longrightarrow 6$.

$\langle abJT \mid G \mid cdJT \rangle$

T = 1 :

J	a	b	c	d	G*	G_{DS}	$G_{3p\text{-}1h}$	G_{2h}	SUM
0	4	4	4	4	-1.2196	-0.1467	-0.2097	-0.2356	-1.8117
0	4	4	5	5	-0.5962	-0.0253	0.0701	-0.0126	-0.5644
0	4	4	6	6	-2.8377	-0.0057	-0.4460	-0.1348	-3.4249
0	5	5	5	5	-1.9688	-0.0110	0.4418	-0.0008	-1.5387
0	5	5	6	6	-0.5020	-0.0331	-0.0156	-0.0103	-0.5637
0	6	6	6	6	-0.1724	-0.1762	-0.2288	-0.1806	-0.7580
1	4	6	4	6	-0.4333	-0.0001	0.4373	-0.0025	0.0014
1	4	6	5	6	-0.0585	------	0.1207	0.0009	0.0631
1	5	6	5	6	-0.3556	-0.0001	0.1797	-0.0004	-0.1764
2	4	4	4	4	-0.9636	-0.0194	-0.0218	-0.0468	-1.0516
2	4	4	4	5	-0.5590	0.0031	0.0235	-0.0217	-0.5540
2	4	4	4	6	-0.3253	-0.0029	0.0038	-0.0305	-0.3551
2	4	4	5	6	-0.5216	0.0179	0.0277	0.0116	-0.4634
2	4	4	6	6	-0.6487	-0.0124	-0.2526	-0.0284	-0.9434
2	4	5	4	5	-1.1818	-0.0489	0.1395	-0.0106	-1.1017
2	4	5	4	6	-0.1471	-0.0298	0.0775	-0.0132	-0.1115
2	4	5	5	6	1.3640	-0.0040	0.0197	0.0036	1.3831
2	4	5	6	6	-0.7498	-0.0083	0.0215	-0.0104	-0.7657

TABLE (V.9) cont'd

T = 1 : $\langle abJT \mid G \mid cdJT \rangle$

J	a	b	c	d	G*	G_{DS}	G_{3p-1h}	G_{2h}	SUM
2	4	6	4	6	-0.5180	-0.0201	0.4343	-0.0216	-0.1254
2	4	6	5	6	0.6817	0.0003	-0.1214	0.0109	0.5715
2	4	6	6	6	-0.6734	-0.0061	-0.0249	-0.0234	-0.7282
2	5	6	5	6	-0.6362	-0.0506	0.1969	-0.0091	-0.4991
2	5	6	6	6	-0.0214	0.0228	0.0479	0.0164	0.0672
2	6	6	6	6	-0.2580	-0.0305	-0.1090	-0.0311	-0.4268
3	4	5	4	5	-0.3088	------	0.3506	------	0.0418
3	4	5	4	6	-0.0333	------	0.0260	------	-0.0073
3	4	6	4	6	-0.4305	------	0.3167	------	-0.1138
4	4	4	4	4	-0.4354	-0.0180	0.1701	------	-0.2833
4	4	4	4	6	-1.0152	------	0.0742	------	-0.9409
4	4	6	4	6	-1.9594	-0.0198	0.7757	------	-1.2035

TABLE (V.9) cont'd

T = 0 : $\langle abJT \mid G \mid cdJT \rangle$

J	a	b	c	d	G*	G_{DS}	G_{3p-1h}	G_{2h}	SUM
1	4	4	4	4	-0.8667	-0.2398	-0.4192	-0.0791	-1.6049
1	4	4	4	6	2.2780	0.0139	-0.1456	0.2019	2.3489
1	4	4	5	5	-0.4524	-0.0470	0.0450	0.0038	-0.4514
1	4	4	5	6	-0.5688	-0.1135	0.1434	-0.1099	-0.6552
1	4	4	6	6	2.8319	-0.0081	-0.0370	-0.0744	2.7395
1	4	6	4	6	-4.0437	-0.1540	-0.0506	-1.0967	-5.3450
1	4	6	5	5	1.2667	0.0097	0.2300	0.0697	1.5760
1	4	6	5	6	1.1049	0.0098	-0.0990	0.6458	1.6614
1	4	6	6	6	0.1548	-0.1606	-0.0632	0.2358	0.1586
1	5	5	5	5	-3.0670	-0.0363	-0.0559	-0.1805	-3.3397
1	5	5	5	6	0.4551	-0.0314	-0.0205	-0.2143	0.1877
1	5	5	6	6	0.0085	0.0092	-0.0218	-0.0377	-0.0410
1	5	6	5	6	-4.1901	-0.0698	0.3259	-0.5521	-4.4862
1	5	6	6	6	1.3537	0.0291	0.1621	-0.1637	1.3789
1	6	6	6	6	-1.5831	-0.2096	-0.0722	-0.0808	-1.9458
2	4	5	4	5	-0.6763	-0.1508	0.3321	-0.0007	-0.4956
2	4	5	4	6	-1.3380	-0.0789	0.0246	0.0215	-1.3678
2	4	5	5	6	-2.5901	-0.0411	-0.0147	-0.0008	-2.6487
2	4	6	4	6	-2.9274	-0.0865	0.0377	-0.6974	-3.6736
2	4	6	5	6	-1.6513	-0.0669	0.0879	0.0264	-1.6035
2	5	6	5	6	-1.7682	-0.1839	0.2376	-0.0010	-1.7155

TABLE(V.9) cont'd

T = 0 : $\langle abJT \mid G \mid cdJT \rangle$

J	a	b	c	d	G*	G_{DS}	G_{3p-1h}	G_{2h}	SUM
3	4	4	4	4	-1.0294	-0.0697	-0.1596	-0.0313	-1.2899
3	4	4	4	5	-1.1763	-0.0255	0.0743	-0.0352	-1.1629
3	4	4	4	6	1.3751	-0.0050	-0.0202	0.0658	1.4155
3	4	4	6	6	0.8469	-0.0314	-0.0959	-0.0671	0.6495
3	4	5	4	5	-2.9403	-0.0391	0.1611	-0.0397	-2.8579
3	4	5	4	6	0.8354	0.0202	0.3304	0.0741	1.2594
3	4	5	6	6	0.3520	-0.0047	-0.2611	-0.0755	0.0111
3	4	6	4	6	-0.9283	-0.0915	0.0742	-0.1385	-1.0839
3	4	6	6	6	1.4784	0.0295	0.2631	0.1411	1.8516
3	6	6	6	6	-2.8703	-0.0643	0.1474	-0.1438	-2.9310
4	4	6	4	6	-4.0218	------	0.5712	------	-3.4506
5	4	4	4	4	-3.3191	------	0.3652	------	-2.9540

TABLE (V.10)

COMPARISON OF D(S-D) AND D(S-D, P-F)

Symbols: $0d_{5/2} \longrightarrow 4$, $1s_{1/2} \longrightarrow 5$, $0d_{3/2} \longrightarrow 6$, and at $k_c = 1.4$ fm^{-1}

T	J	a	b	c	d	G*	G_{DS}(S-D)	G_{DS}(S-D, P-F)
1	0	4	4	4	4	-1.2196	-0.1467	-0.2060
1	0	4	4	5	5	-0.5962	-0.0253	-0.0461
1	0	4	4	6	6	-2.8377	-0.0057	-0.0108
1	0	5	5	5	5	-1.9688	-0.0110	-0.0223
1	0	5	5	6	6	-0.5020	-0.0331	-0.0548
1	0	6	6	6	6	-0.1724	-0.1762	-0.2435
1	1	4	6	4	6	-0.4333	-0.0001	-0.0011
1	1	4	6	5	6	-0.0585	------	-0.0007
1	1	5	6	5	6	-0.3556	-0.0001	-0.0008
1	2	4	4	4	4	-0.9636	-0.0194	-0.0320
1	2	4	4	4	5	-0.5590	0.0031	-0.0083
1	2	4	4	4	6	-0.3253	-0.0029	-0.0063
1	2	4	4	5	6	-0.5216	0.0179	0.0066
1	2	4	4	6	6	-0.6487	-0.0124	-0.0226
1	2	4	5	4	5	-1.1818	-0.0489	-0.0739
1	2	4	5	4	6	-0.1471	-0.0289	-0.0406
1	2	4	5	5	6	1.3640	-0.0040	-0.0179
1	2	4	5	6	6	-0.7498	-0.0083	-0.0156
1	2	4	6	4	6	-0.5180	-0.0201	-0.0333

TABLE (V.10) cont'd

T	J	a	b	c	d	G*	G_{DS}(S-D)	G_{DS}(S-D, P-F)
1	2	4	6	5	6	0.6817	0.0003	-0.0033
1	2	4	6	6	6	-0.6734	-0.0061	-0.0109
1	2	5	6	5	6	-0.6362	-0.0506	-0.0720
1	2	5	6	6	6	-0.0214	0.0228	0.0132
1	2	6	6	6	6	-0.2580	-0.0305	-0.0388
1	3	4	5	4	5	-0.3088	------	-0.0011
1	3	4	5	4	6	-0.0333	------	-0.0005
1	3	4	6	4	6	-0.4305	------	-0.0013
1	4	4	4	4	4	-0.4354	-0.0180	-0.0242
1	4	4	4	4	6	-1.0152	------	-0.0072
1	4	4	6	4	6	-1.9594	-0.0198	-0.0378
0	1	4	4	4	4	-0.8667	-0.2398	-0.3688
0	1	4	4	4	6	2.2780	0.0139	-0.1634
0	1	4	4	5	5	-0.4524	-0.0470	-0.1273
0	1	4	4	5	6	-0.5688	-0.1135	-0.0845
0	1	4	4	6	6	2.8319	-0.0081	0.0367
0	1	4	6	4	6	-4.0437	-0.1540	-0.5277
0	1	4	6	5	5	1.2667	0.0097	-0.1505
0	1	4	6	5	6	1.1049	0.0098	-0.0644
0	1	4	6	6	6	0.1548	-0.1606	-0.2108
0	1	5	5	5	5	-3.0670	-0.0363	-0.1935

TABLE(V.10) cont'd

T	J	a	b	c	d	G*	G_{DS}(S-D)	G_{DS}(S-D, P-F)
0	1	5	5	5	6	0.4551	-0.0314	0.0085
0	1	5	5	6	6	0.0085	0.0092	0.0471
0	1	5	6	5	6	-4.1901	-0.0698	-0.2210
0	1	5	6	6	6	1.3537	0.0291	-0.4132
0	1	6	6	6	6	-1.5831	-0.2096	-0.2917
0	2	4	5	4	5	-0.6763	-0.1508	-0.2170
0	2	4	5	4	6	-1.3380	-0.0789	-0.1584
0	2	4	5	5	6	-2.5901	-0.0411	-0.1133
0	2	4	6	4	6	-2.9274	-0.0865	-0.2852
0	2	4	6	5	6	-1.6513	-0.0669	-0.1738
0	2	5	6	5	6	-1.7682	-0.1839	-0.2840
0	3	4	4	4	4	-1.0294	-0.0697	-0.1155
0	3	4	4	4	5	-1.1763	-0.0255	-0.0848
0	3	4	4	4	6	1.3751	-0.0050	0.0345
0	3	4	4	6	6	0.8469	-0.0314	-0.0439
0	3	4	5	4	5	-2.9403	-0.0391	-0.1404
0	3	4	5	4	6	0.8354	0.0202	0.0681
0	3	4	5	6	6	0.3520	-0.0047	-0.0004
0	3	4	6	4	6	-0.9283	-0.0915	-0.1601
0	3	4	6	6	6	1.4784	0.0295	-0.0156
0	3	6	6	6	6	-2.8703	-0.0643	-0.1408

TABLE (V.10) cont'd

T	J	a	b	c	d	G*	G_{DS}(S-D)	G_{DS}(S-D, P-F)
0	4	4	6	4	6	-4.0218	------	-0.1601
0	5	4	4	4	4	-3.3191	------	-0.0624

FIGURE CAPTIONS

Fig. (II.4) Fourier Transforms of harmonic oscillator wave functions with $\hbar\omega = 14$ MeV. Radial part $R_{n\ell}(k)$ plotted in arbitrary units and k in units of fm^{-1}.

Fig. (III.1) $^3S_1$ reduced integrals plotted as function of γ^2. Solid lines represent values obtained by Kuo. γ^2 in units of fm^{-2}.

Fig. (III.2) $^1S_0$ reduced integrals. Conventions and units similar to Fig. (III.1).

Fig. (III.3) $^3D_2$ reference wave defect in arbitrary unit; r in fm; $n = 0$, $\gamma^2 = 4$. Solid line represents actual reference wave defect and dotted line represents linear approximation of Kuo and Brown.

Fig. (III.4) $^1P_1$ reference wave defect for $n = 1$, $\gamma^2 = 4$.

Fig. (III.5) Triplet P-state reference wave defects.

Fig. (V.1) Spectra of O^{18} and F^{18}.

& Fig. (V.2) First column shows the experimentally determined energy levels of the nucleus. Columns (a), (b) and (c) show spectra obtained for $D \equiv$ (S-D) shell and $k_c = 1.3$, 1.4 and 1.5 fm^{-1} respectively. Column (d)

shows spectrum for D ≡(S-D, P-F) shells and $k_c = 1.4$ fm^{-1}. Column (e) shows results of Kuo.

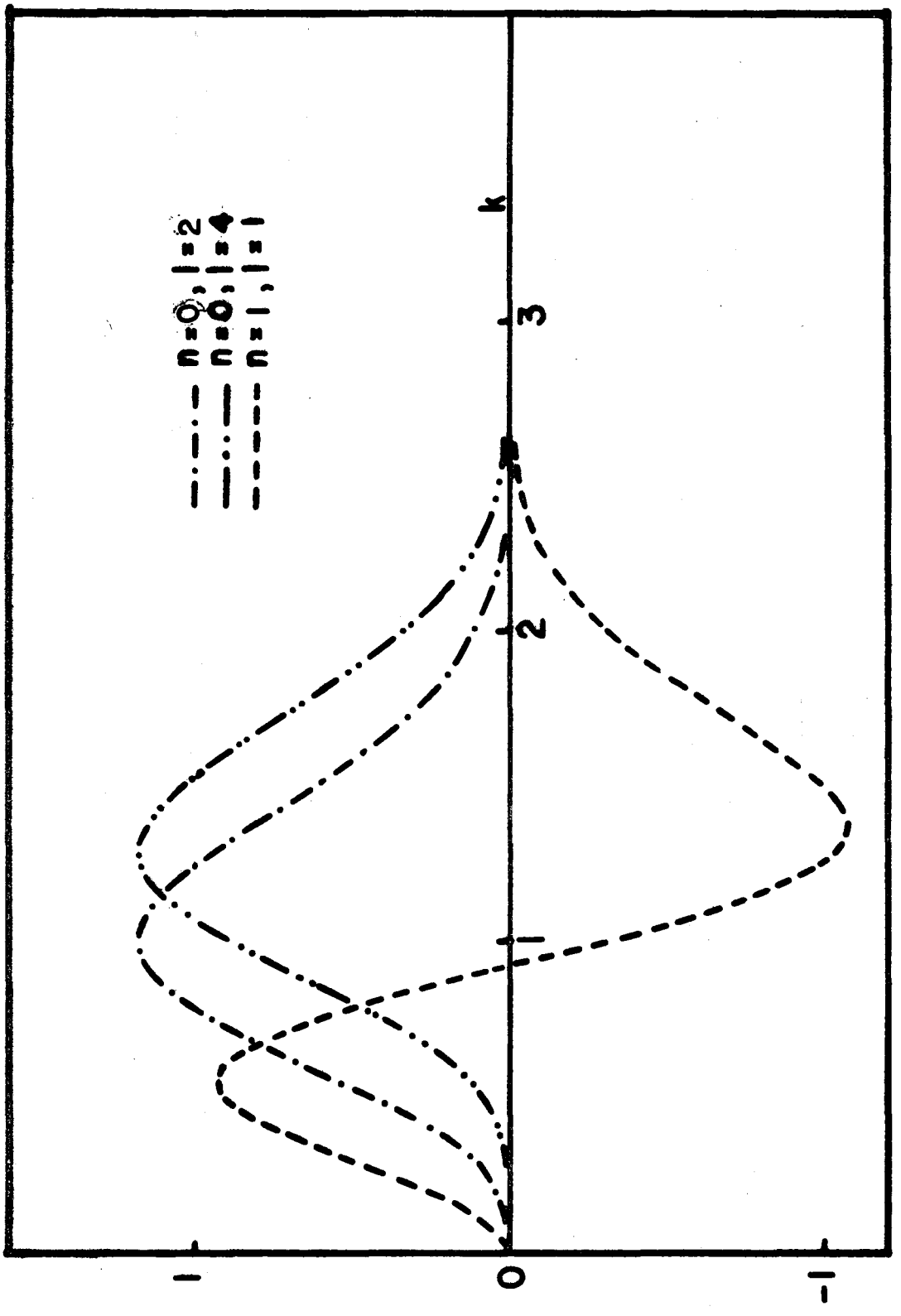


Fig. (II.4)

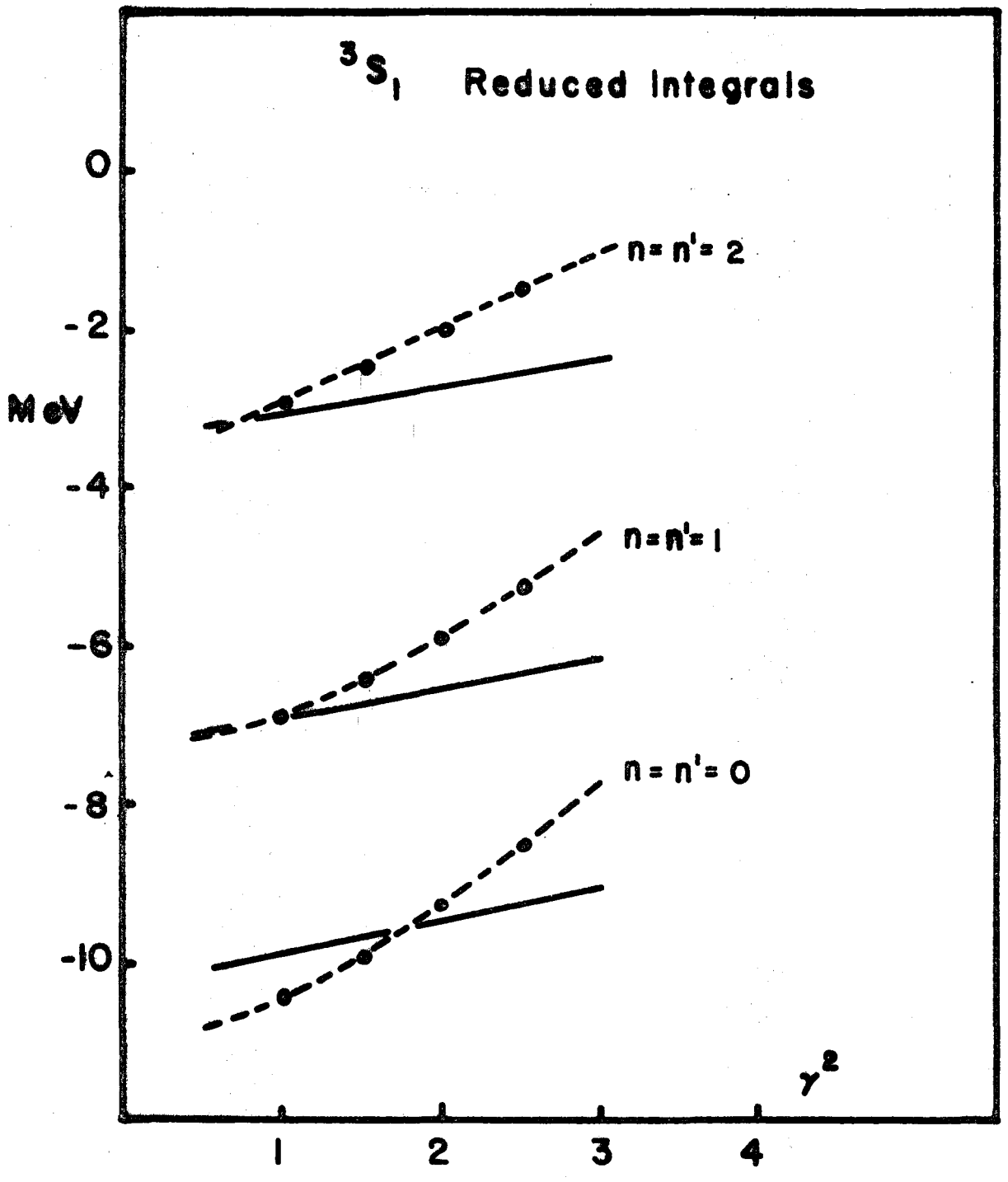


Fig. (III.1)

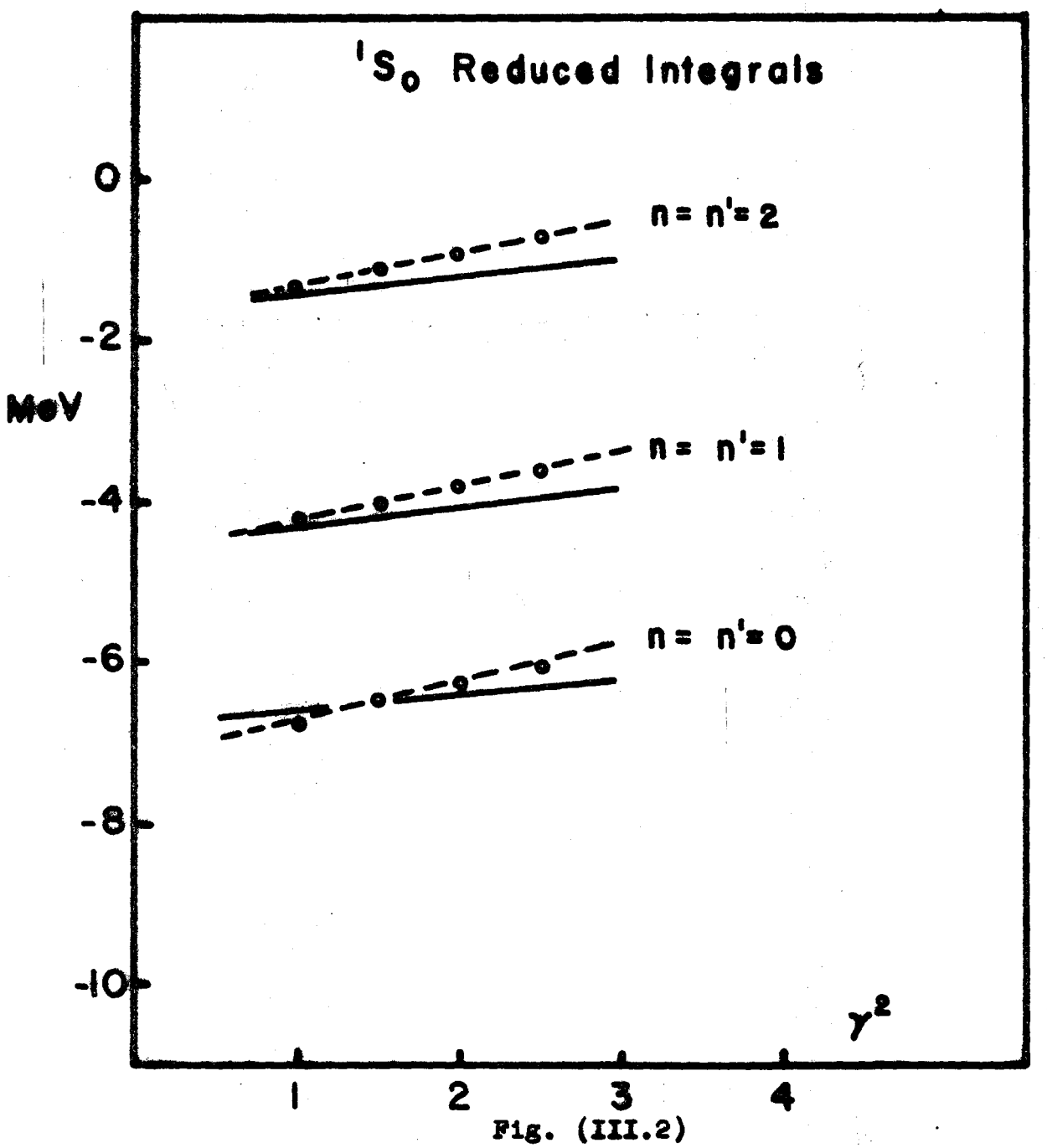


Fig. (III.2)

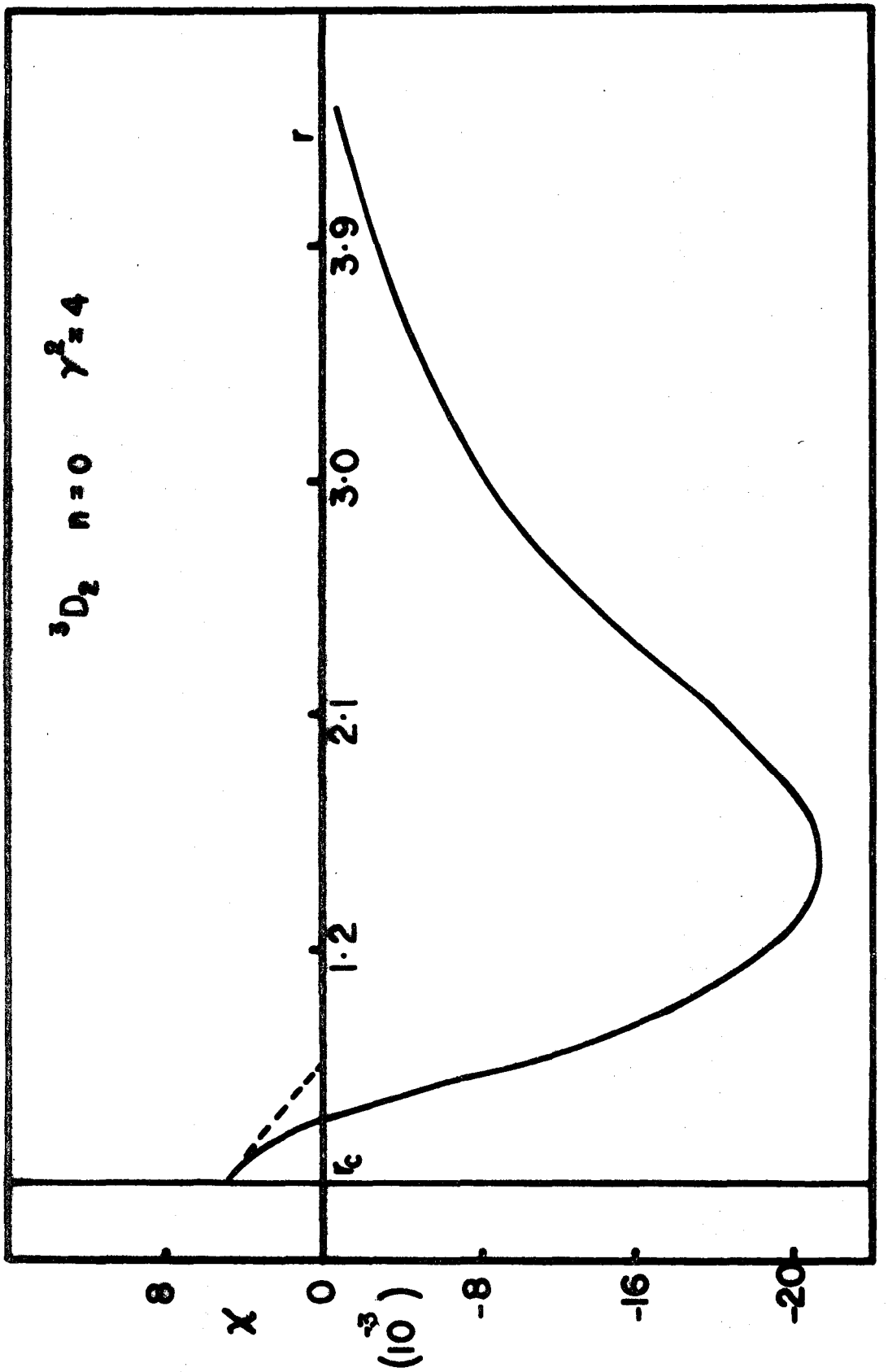


Fig. (III.3)

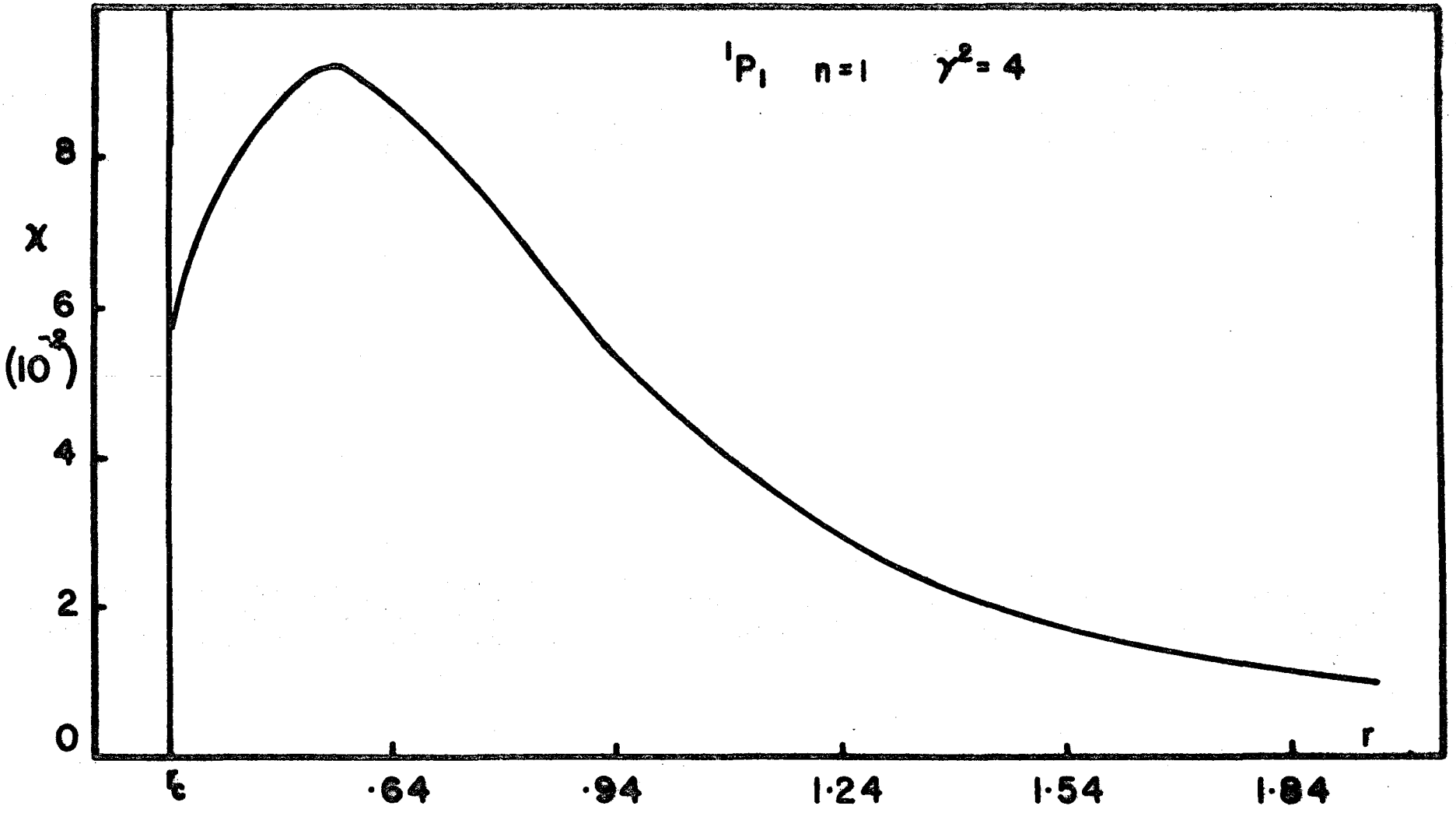


Fig. (III.4)

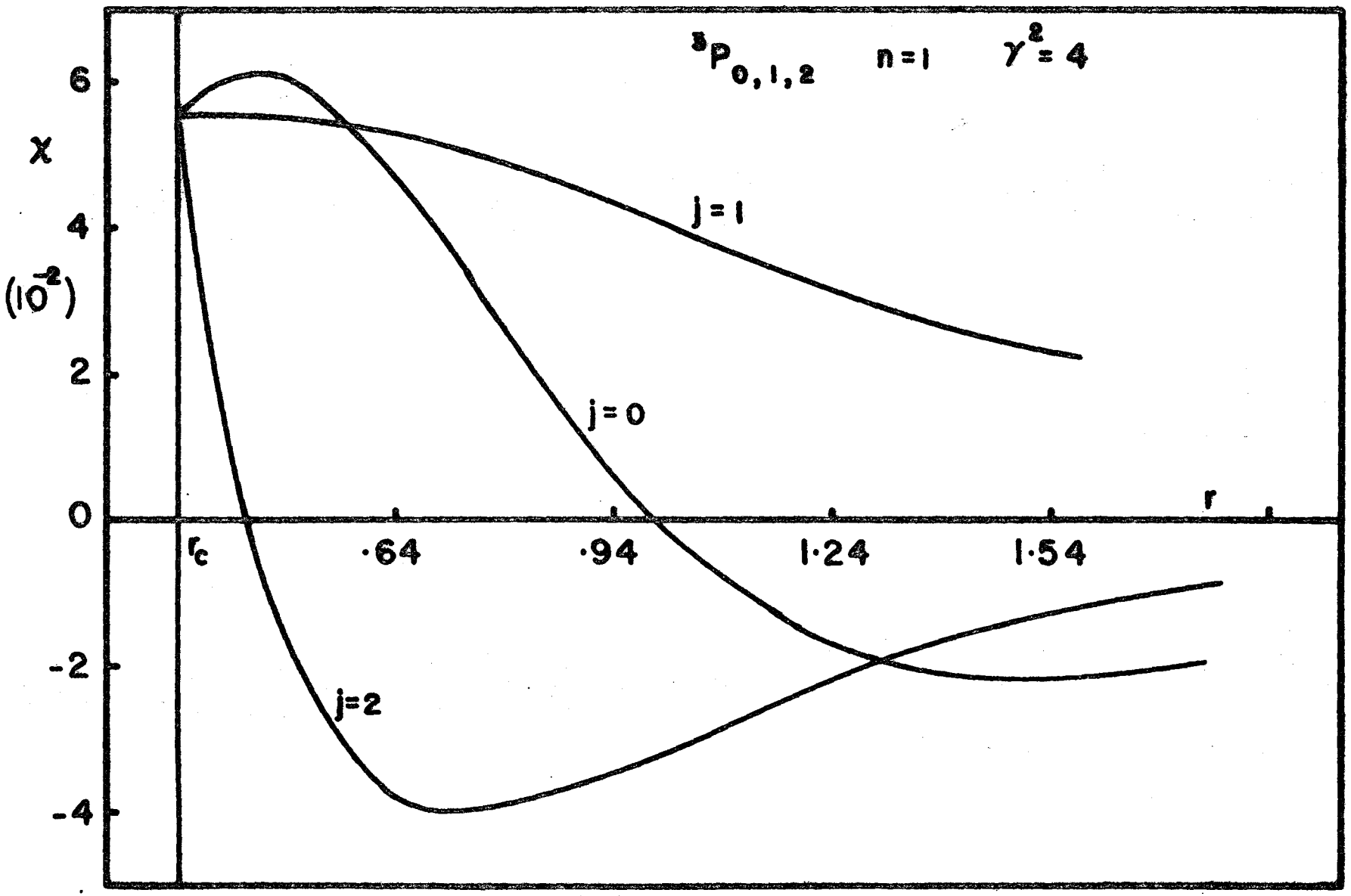


Fig. (III.5)

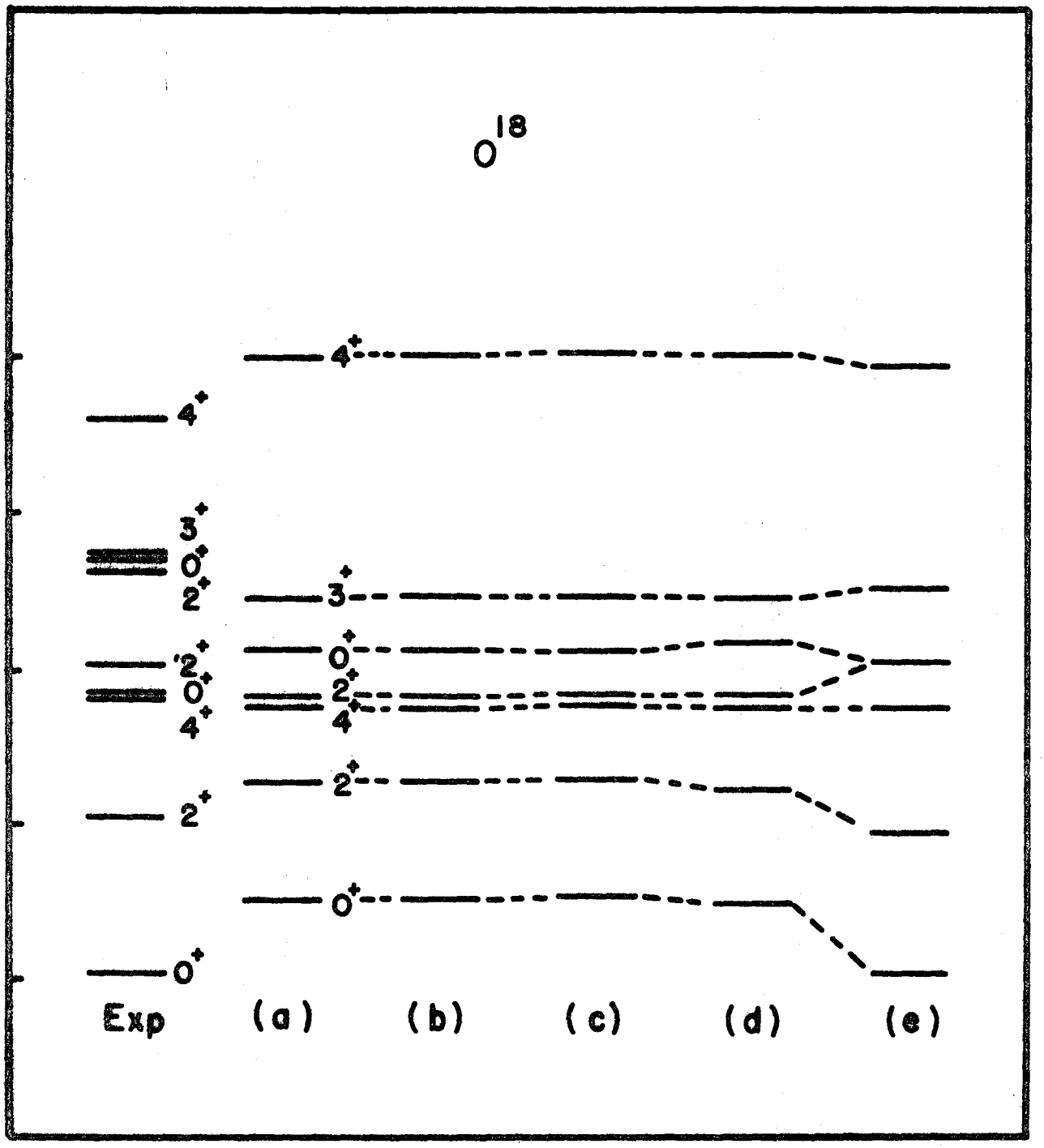


Fig. (V.1)

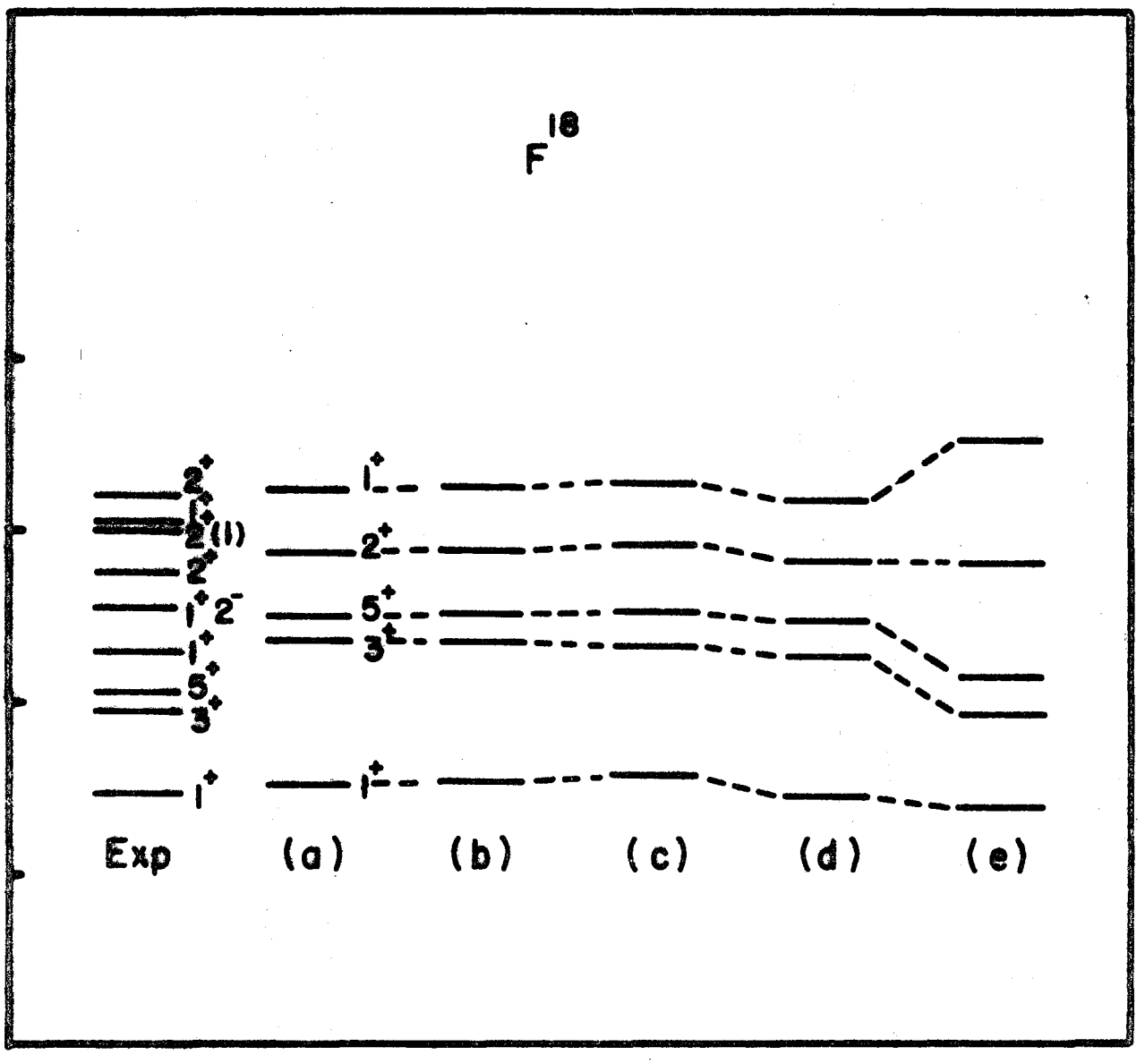


Fig. (V.2)

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