PRACTICAL FILTERS

USING DISTRIBUTED RC STRUCTURES

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By

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TITLE: Practical Filters Using Distributed RC Structures.

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Scope and Contents:

Theoretical and experimental investigations of distributed RC structures used as filters are described, both in isolation and as part of active filters. Novel lowpass and bandpass circuits are developed and a general design procedure for bandpass amplifiers presented.

ABSTRACT

Theoretical and experimental studies into practical filters using distributed RC structures are described. Various techniques to surmount the transcendental nature of the circuit parameters of such structures are considered and shown to be mostly too clumsy or restricted for practical use. The use of computer - aided analysis together with a sound physical understanding is suggested as an alternative.

Application of the structures to active filters is considered and the experimental developement of lowpass and bandpass amplifiers using them described. This leads to the presentation of a general design procedure for bandpass amplifiers using distributed RC null circuits.

It is concluded that distributed RC filters suffer from a number of limitations but have a part to play as a circuit element and are now ready to advance to the stage of practical implementation.

ACKNOWLEDGEMENTS

The author is indebted to Dr. C.K.Campbell for his help and encouragement during the course of this work, for the excellent laboratory facilities made available and for a grant to attend the December 1970 IEEE International Symposium on Circuit Theory.

Much is owed to my colleague Emerson Johnston for many fruitful discussions. Some of the work of Chapter 8 was carried out in conjunction with him.

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CONTENTS

LIST OF FIGURES

List of Figures (Cont.)

List of Figures (Cont.)

List of Figures (Cont.)

CHAPTER I

Introduction

I,I Integration Techniques

A major developement in electrical engineering during the last decade has been the growth of integrated circuit concepts and applications. This trend is expected to continue. Advantages commonly found are reduced size, increased reliability and eventual lower costs. These developements have led to the introduction of complex electronic systems that would not have been considered feasible a decade ago. Examples are mini-computers and auto-correlators for signal processing.

The term " Integrated Circuit " has been applied to several types of structure. The greatest advances so far have been with silicon monolithic circuits, which lend themselves to the use of large numbers of active elements for use in gates or operational amplifiers. Absolute resistor tolerances are generally low. In contrast resistors and capacitors of very high precision and stability can be formed using thin film techniques, while thick film circuits provide lower cost versions. Other techniques such as MOS are also under developement.

CHAPTER 1

Introduction

1.1 Integration Techniques

A major developement in electrical engineering during the last decade has been the growth of integrated circuit concepts and applications. This trend is expected to continue. Advantages commonly found are reduced size, increased reliability and eventual lower costs. These developements have led to the introduction of complex electronic systems that would not have been considered feasible on a large scale a decade ago. Examples are mini-computers and auto-correlators used for signal processing.

The term " Integrated Circuit " has been applied to several types of structure. The greatest advances so far have been with silicon monolithic circuits, which lend themselves to the use of large numbers of active elements for use in, for example, gates and operational amplifiers. Absolute resistor tolerances are generally low in this form. In contrast resistors and capacitors of very high precision and stability can be formed using thin film techniques, while thick film circuits provide lower cost versions. Other techniques such as MOS are also under developement.

1.2 Thin Film Technology

The term " Thin Film " is used to designate techniques using films of resistive, conducting and insulating material ranging roughly from 80 Å to 8,000 Å thick deposited on some suitable substrate [45, 18, 19]. Deposition may be by vacuum evaporation, sputtering or some other method. The area of film may be controlled by a mask during deposition or by photo-etching afterwards. Thickness may be monitored directly although the dependence of the resistivity of a film on its thickness is highly complex. Conducting contacts and paths may be laid down to interconnect resistive and capacitive elements to form an electrical network.

Thin films, usually requiring vacuum processes, are to be contrasted with " Thick Film " circuits produced by a process essentially similar to silk screen printing.

Discrete active devices, based on silicon monolithic technology for instance, may be bonded onto a substrate with thin film components already deposited on it to form a hybrid thin film integrated circuit. Such circuits find applications in military and aerospace systems; they may shortly be applied to consumer products[46].

1.3 Distributed RC Circuits

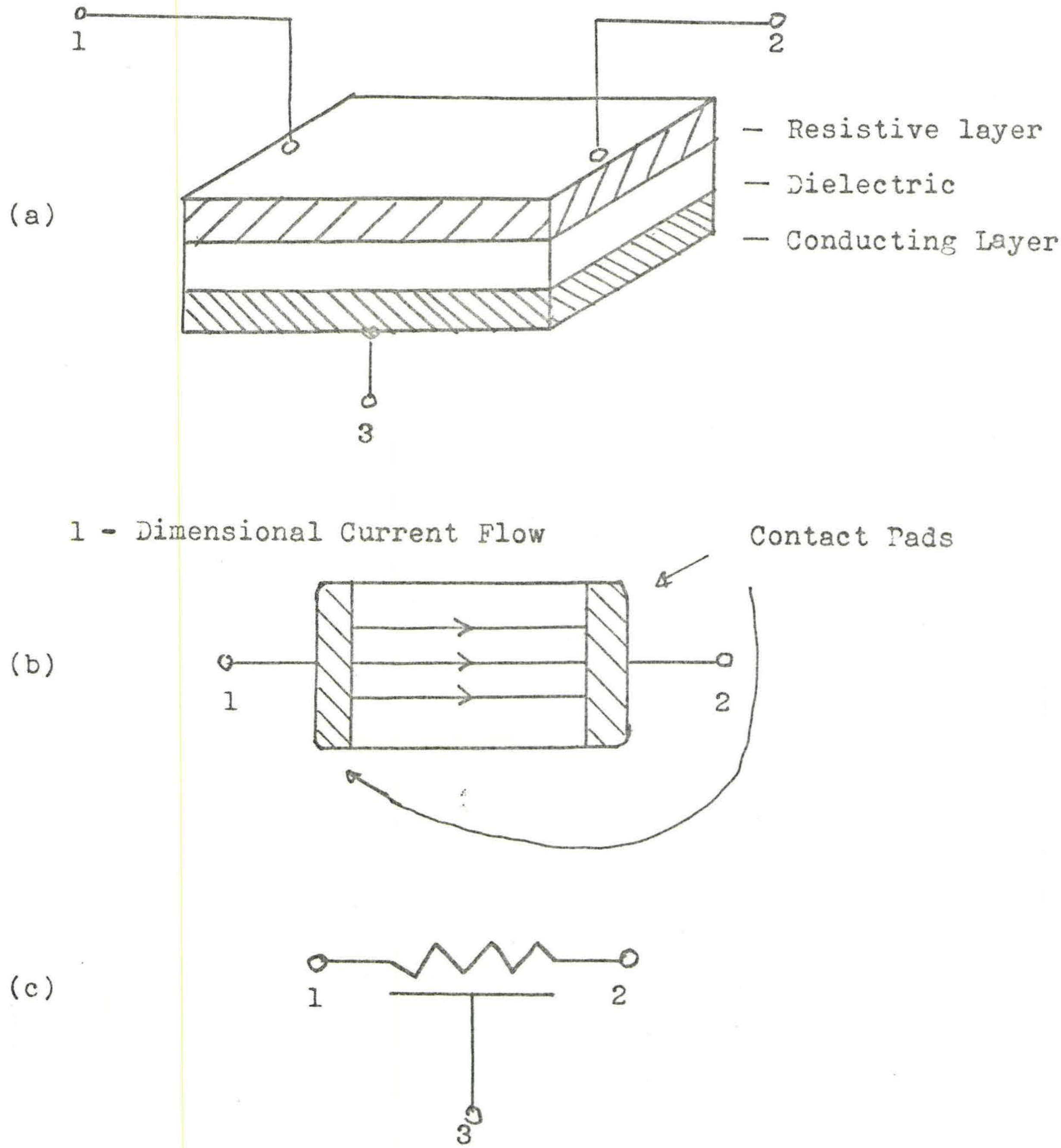
A natural form for thin film fabrication is the distributed RC element. This is built up in its simplest form from three layers of material, as shown in Fig, 1.1, which also shows its commonly accepted circuit symbol. Resistive and capacitive elements are so closely associated that the circuit must be considered an infinite cascade of infinitesimal elements, as shown in Figure 1.2. This may be analysed to give a version of the standard transmission line equations:-[3]

$$\frac{dV(s,x)}{dx} = -r(x)\, I(s,x)$$

$$\frac{dI(s,x)}{dx} = -sc(x)\, V(s,x)$$

This formulation assumes that series inductance and shunt conductance are negligible, which is reasonable.

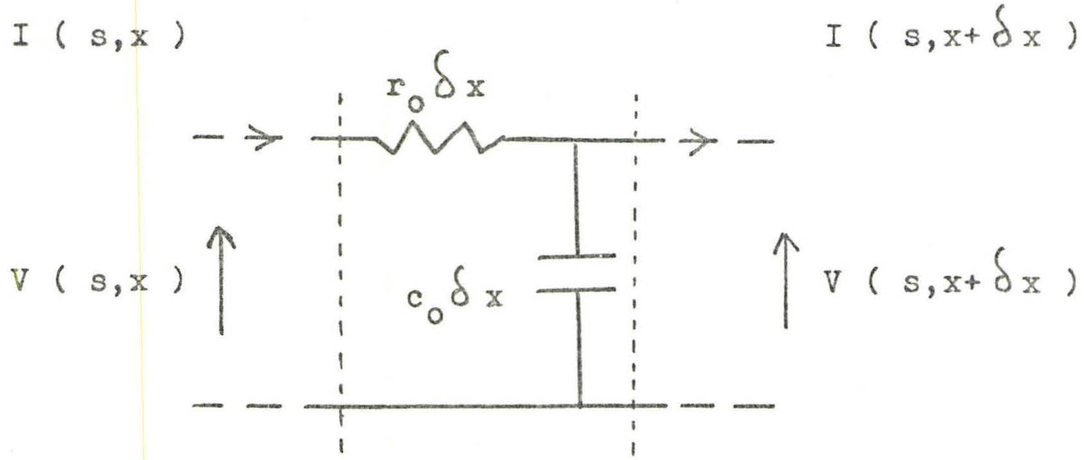
For uniform widths and thicknesses with isotropic materials the circuit is known as a Uniform Distributed RC network, or $\overline{\text{URC}}$ for short. Variations of taper will be considered later. The $\overline{\text{URC}}$ is a three terminal device and



PHYSICAL STRUCTURE AND CIRCUIT SYMBOL FOR A

UNIFORM $\overline{RC}$ NETWORK

<u>Figure 1.1</u>



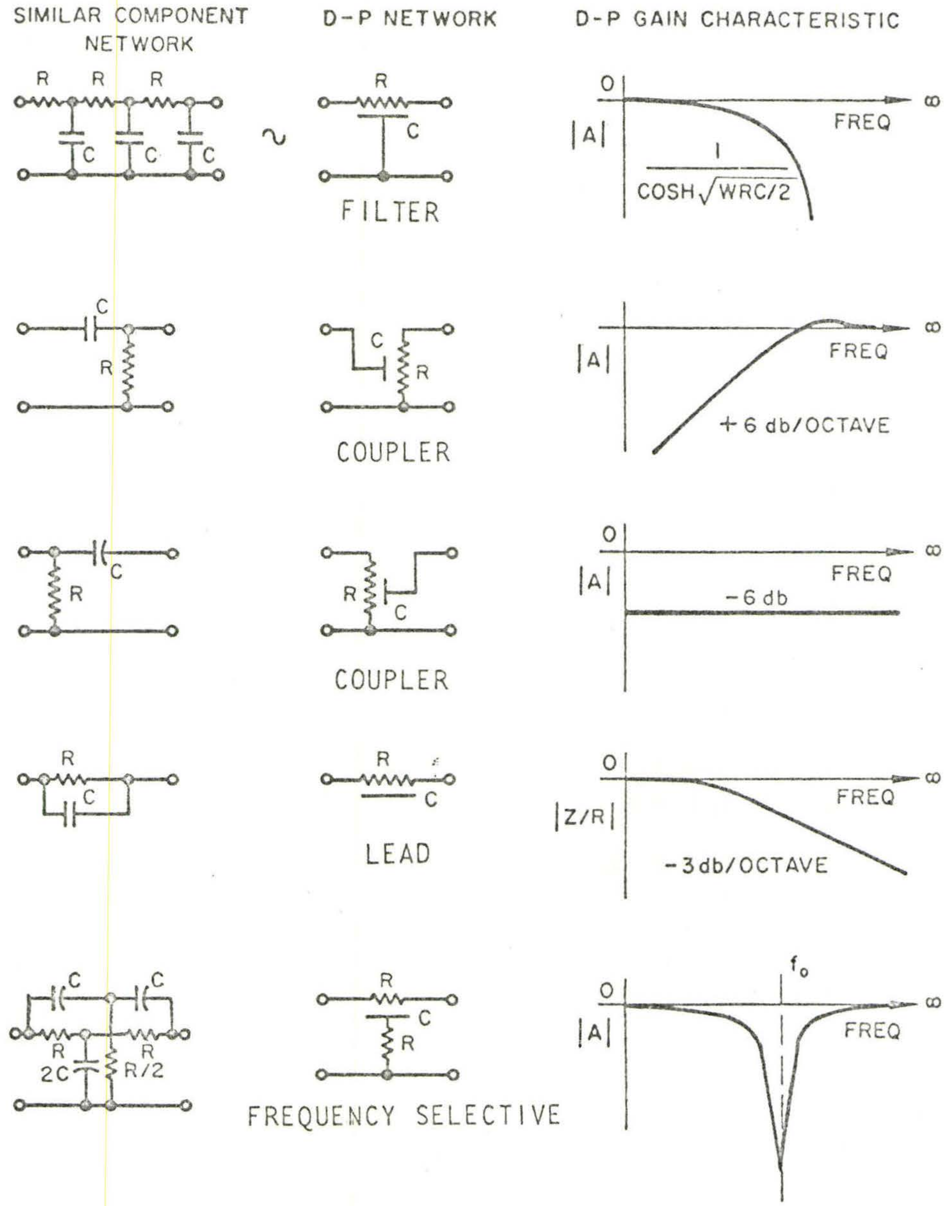
r_o = Series Resistance per Unit Length

c_o = Shunt Capacitance per Unit Length

TRANSMISSION LINE

ELEMENTAL EQUIVALENT CIRCUIT

Figure 1.2



Basic Connections For A Simple URC

(Ref. 8 page 602)

Figure 1.3

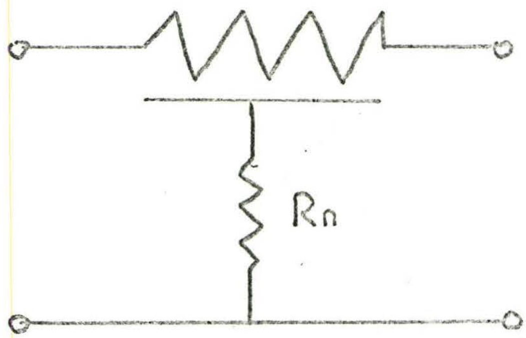


Figure I.4 DISTRIBUTED RC NOTCH FILTER

may be connected to form a two-port network in several ways, as indicated in Figure 1.3, which also indicates the corresponding responses.

More complicated morphologies and topologies are possible. These have been extensively investigated by Happ and others (see section 2.3). By connecting an external resistor as in Figure 1.4 a phase and amplitude cancellation can be arranged at the output for a particular frequency giving a notch in the transfer characteristics, Figure 1.5.[18, 42] Similar notches may be obtained in various ways as illustrated in Figure 1.6.

The early workers in the field (1959 - 1962 Refs. 1, 2, 5, 6, 7, 8, and 9) saw promise of the application of RC filters in the construction of oscillators, amplifiers and filters, perhaps in conjunction with the thin film transistors then under investigation. They were also attracted by the reduced number of interconnections among circuit elements required when using distributed circuits.

1.4 Scope of this Thesis

This thesis investigates the use of distributed RC circuits as filters both in isolation and also in relation to recent developements in active filters and integrated circuits.

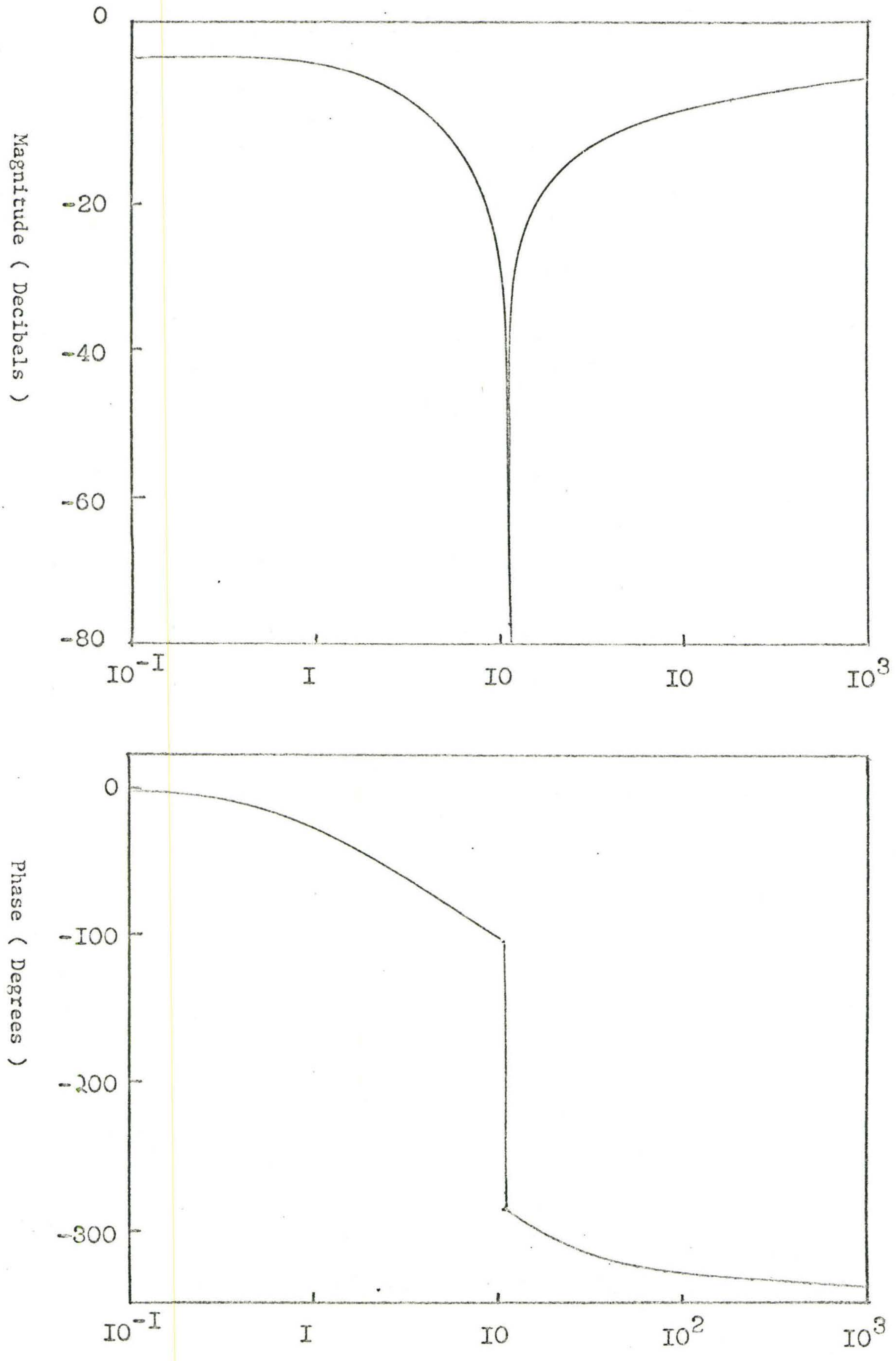


Fig.I.5 MAGNITUDE AND PHASE RESPONSES FOR URC NOTCH FILTER FROM Ref.I8)

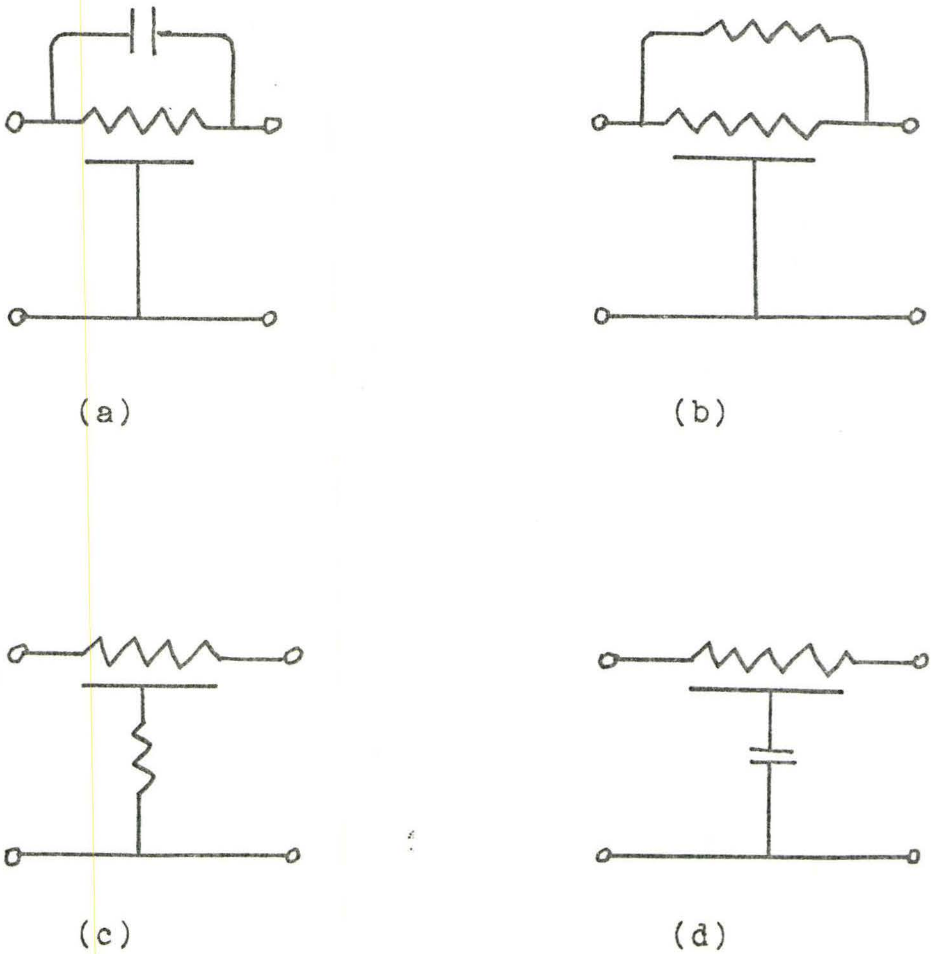


Figure 1.6 SOME $\overline{RC}$ ARRANGEMENTS GIVING NOTCH FILTERS

(See Ref. 8, page 605 for more details.)

CHAPTER 2

Analytical Treatment

2.I Solution for Uniform Taper

It was pointed out in section I.3 that distributed structures must be analysed as a series of incremental networks as in Fig. I.2 . Network analysis yields a version of the standard transmission line equations with series L and shunt G neglected.(see Eqn. (I)) This assumes for the sake of simplicity one-dimensional current flow. In practice it may be necessary to extend the treatment for non-uniform structures using transformations or a curvilinear-square approach, see Oehler, Ref.[I2] .

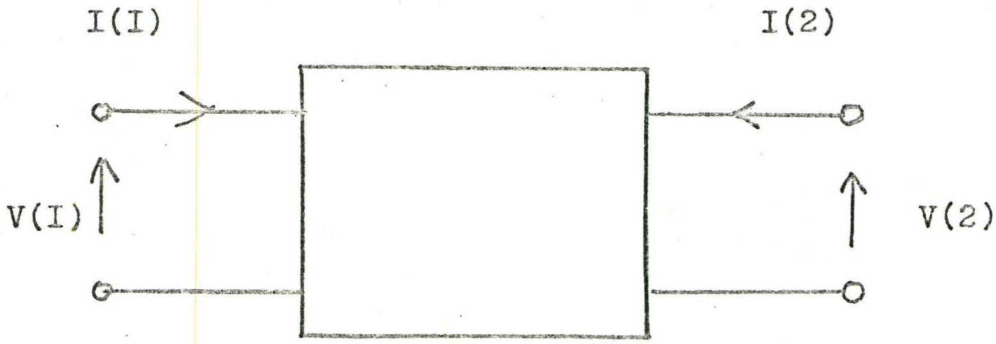
For the Uniform $\overline{RC}$, r_o and c_o the resistance and capacitance per unit length respectively are not functions of x so that Eqns. I may be differentiated[4] to give:-

$$\frac{d^2}{dx^2} V - (s r_o c_o) V = 0 \qquad \text{-- 2.I}$$

and

$$\frac{d^2}{dx^2} I - (s r_o c_o) I = 0 \qquad \text{-- 2.2}$$

These are second order linear homogeneous differential equations with solutions known to be :-



$$\begin{bmatrix} V(I) \\ I(I) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(2) \\ -I(2) \end{bmatrix}$$

Figure 2.I(a) <u>Basic Two Port</u>

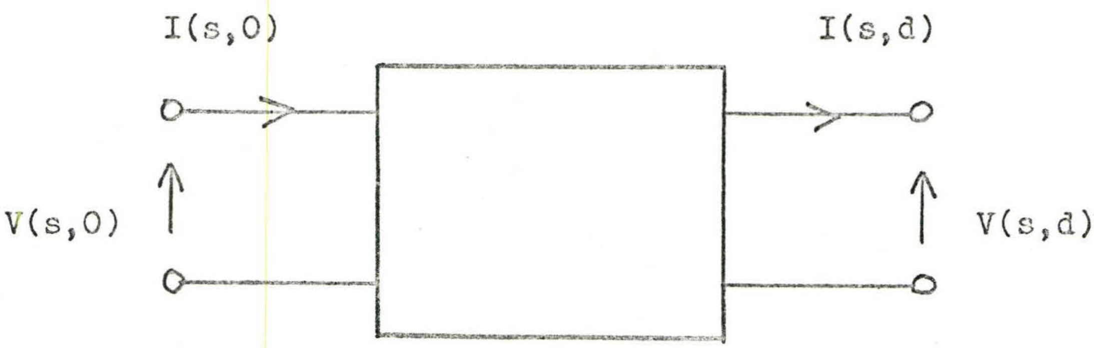


Figure 2.I(b) <u>Alternative Current Notation Used</u>

FIG. 2.I CHAIN MATRIX REPRESENTATIONS OF A TWO - PORT

$$V(s,x) = A_I \cosh \gamma x + A_2 \sinh \gamma x \qquad \text{--2.3}$$

$$I(s,x) = B_I \cosh \gamma x + B_2 \sinh \gamma x \qquad \text{--2.4}$$

where $\gamma = \sqrt{sr_o c_o}$ and is referred to as the Propagation Constant and may be split into parts $\gamma = \alpha + j\beta$ to represent the attenuation and phase shift per unit length of line. Then $\theta = \gamma d$ applies for a fixed length d of line.

Another parameter of interest is

$$Z_o = \sqrt{\frac{sr_o}{c_o}}$$ the Characteristic Impedance.

Applying the boundary conditions of Fig. I.2 to Eqns 2.I to 2.4 and then applying the results to the chain matrix formulation of Fig. 2.I yields :-[3]

$$\begin{bmatrix} V(s,0) \\ I(s,0) \end{bmatrix} = \begin{bmatrix} \cosh \gamma d & Z_o \sinh \gamma d \\ \dfrac{\sinh \gamma d}{Z_o} & \cosh \gamma d \end{bmatrix} \begin{bmatrix} V(s,d) \\ -I(s,d) \end{bmatrix}$$

The other basic two- port parameters may either be obtained from this result or from the equations by suitable manipulation.

All the parameters are non-rational and hyperbolic in form. Of particular interest is the open-circuit voltage transfer function :-

$$\frac{V_{out}}{V_{in}} = \frac{1}{A} = \frac{1}{\cosh \gamma d} = \frac{1}{\cosh \Theta}$$

Now the transcendental cosh term may be expanded [3]:-

$$\cosh \Theta = \prod_{n=1}^{\infty} \left[1 + \frac{4\Theta^2}{(2n-1)^2 \pi^2} \right]$$

Note that this gives $1/\cosh\Theta$ an infinity of poles along the negative real axis with all the zeroes at infinity. It is this infinity of poles that is responsible for many of the characteristics of distributed $\overline{RC}$ circuits.

The characteristics of $\overline{URC}$'s may readily be plotted from the parameters given above.[13] For example Fig. 2.3 shows the amplitude and the phase of the open-circuit voltage transfer function. The cumulative effect of poles as the frequency is increased can clearly be seen in both curves. This may also be seen in Fig. 2.4 which compares the low-pass characteristics of discrete and distributed circuits.

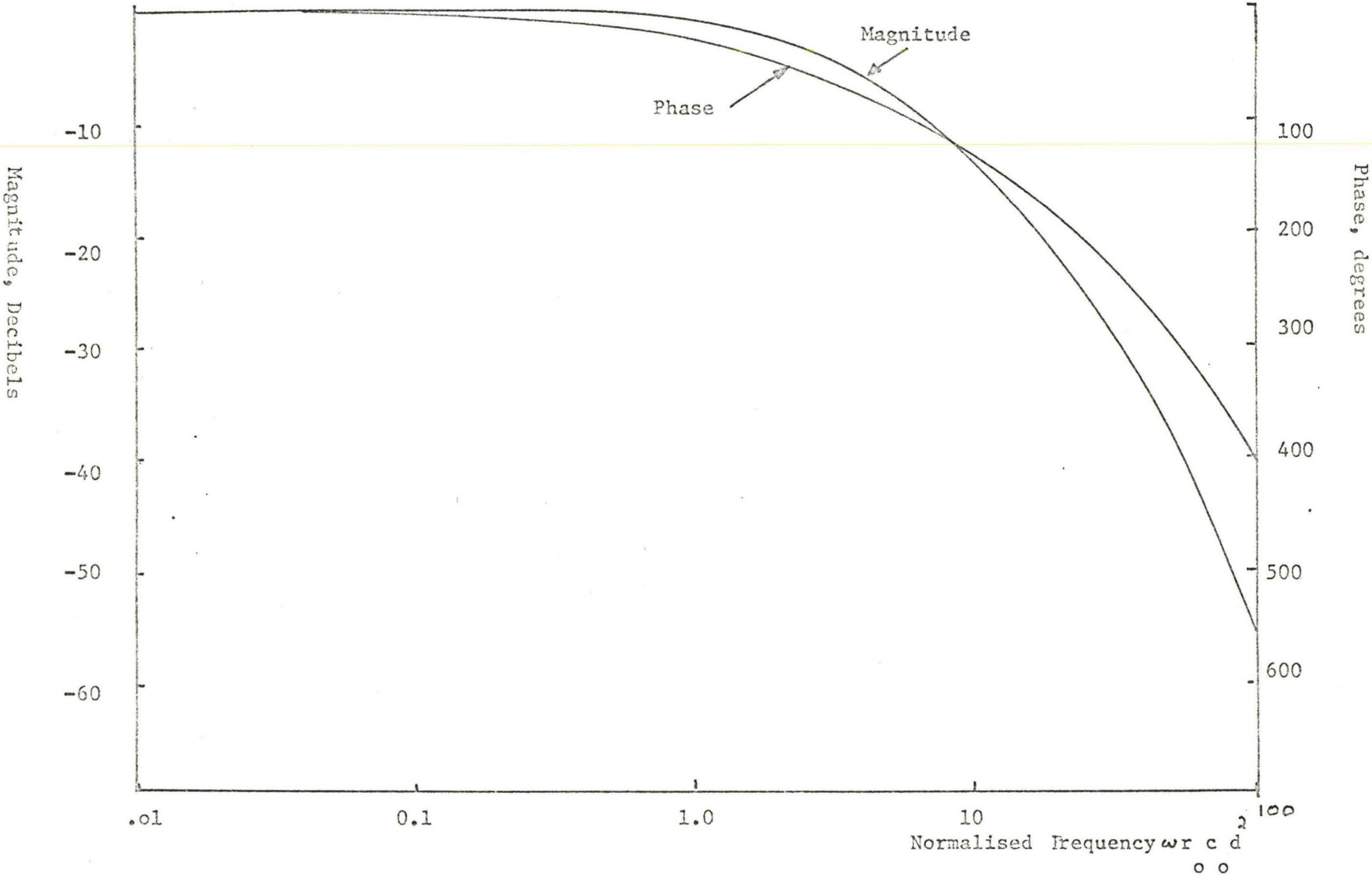


Figure 2.3 OPEN CIRCUIT VOLTAGE TRANSFER CHARACTERISTICS FOR URC

2.2 Other Closed Form Solutions

So far attention has been focused on uniform structures with r_o and c_o constant. It is possible to extend the treatment to cover exponential and linear tapers fairly readily[4]. After that the situation becomes complicated. Kelly and Ghausi[14] have shown that it is possible to define a class of networks having closed form solutions and similar pole-zero patterns. The class is shown to include uniform, exponential, hyperbolic, square and trigonometric tapers. The analysis is in each case based on a common basic set of solutions, giving a class of networks with similar immitance characteristics. More general non-analytical methods for treating arbitrary non-uniform tapers will be discussed in section 4.1.

In every case using the basic topology of Fig. 1.1 it will be found that there is an infinity of poles on the negative real axis with zeroes at infinity. This and other general analytical properties of $\overline{RC}$ transmission lines are given in a full treatment by Protonotarios and Wing[15].

2.3 Other Configurations

So far attention has been focused on circuits of the form shown in Fig. I.I, which are the most readily analysed. Other alternatives are possible and may be grouped under various catagories. Many of them have been investigated by Happ and associates.[5,6,7,8,II]

Alternative Morphologies. A 5-layer structure as in Figure 2.5 may be constructed and is the basis of much of the work of Heizer and others in this field.(see chapter 3) Other alternatives are a radial transmission line[II] and an arrangement with a number of nodes capacitively coupled to a common distributed resistive layer.[I7]

Alternative Interconnections. The basic $\overline{RC}$ is a three terminal device and may be connected to make a two port in different ways as shown in Fig. I.3. For a five layer structure the situation is more complicated as indicated in part in Fig. 2.6 .

Alternative Geometries. When both the above have been decided there still remains the problem of the particular geometry or taper to use. Figure 2.7 indicates some possibilities among analytically definable shapes.

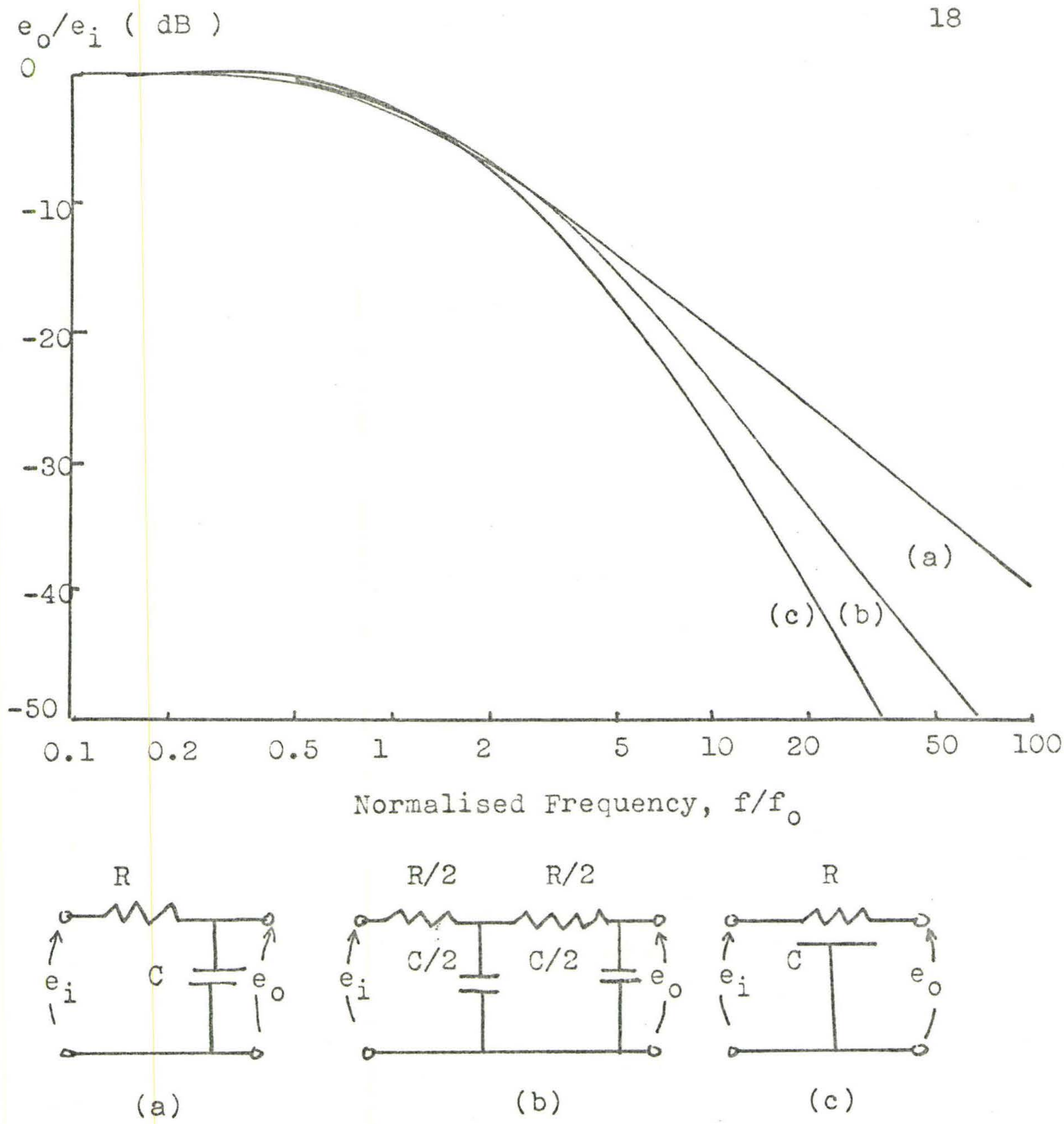
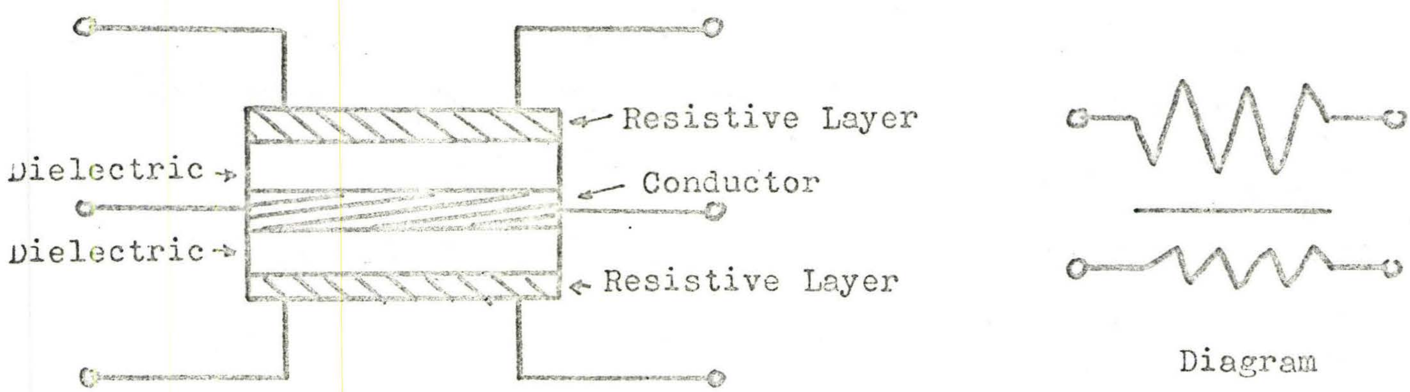


Figure 2.4 LOWPASS FILTER NETWORKS

(Open Circuit voltage transfer functions, matched at 3 dB points. From Ref. 5)



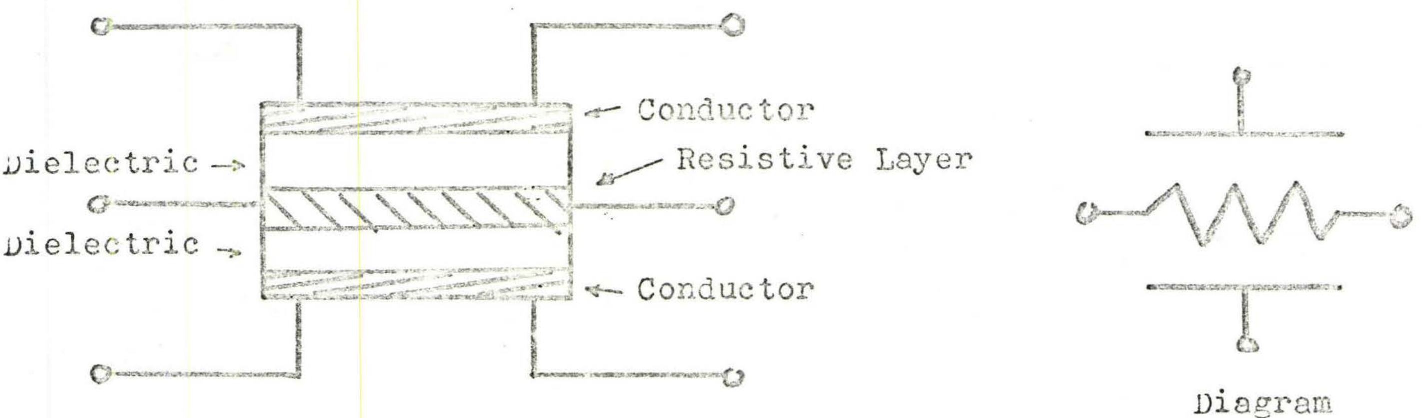


Figure 2.5

5 LAYER DISTRIBUTED STRUCTURES

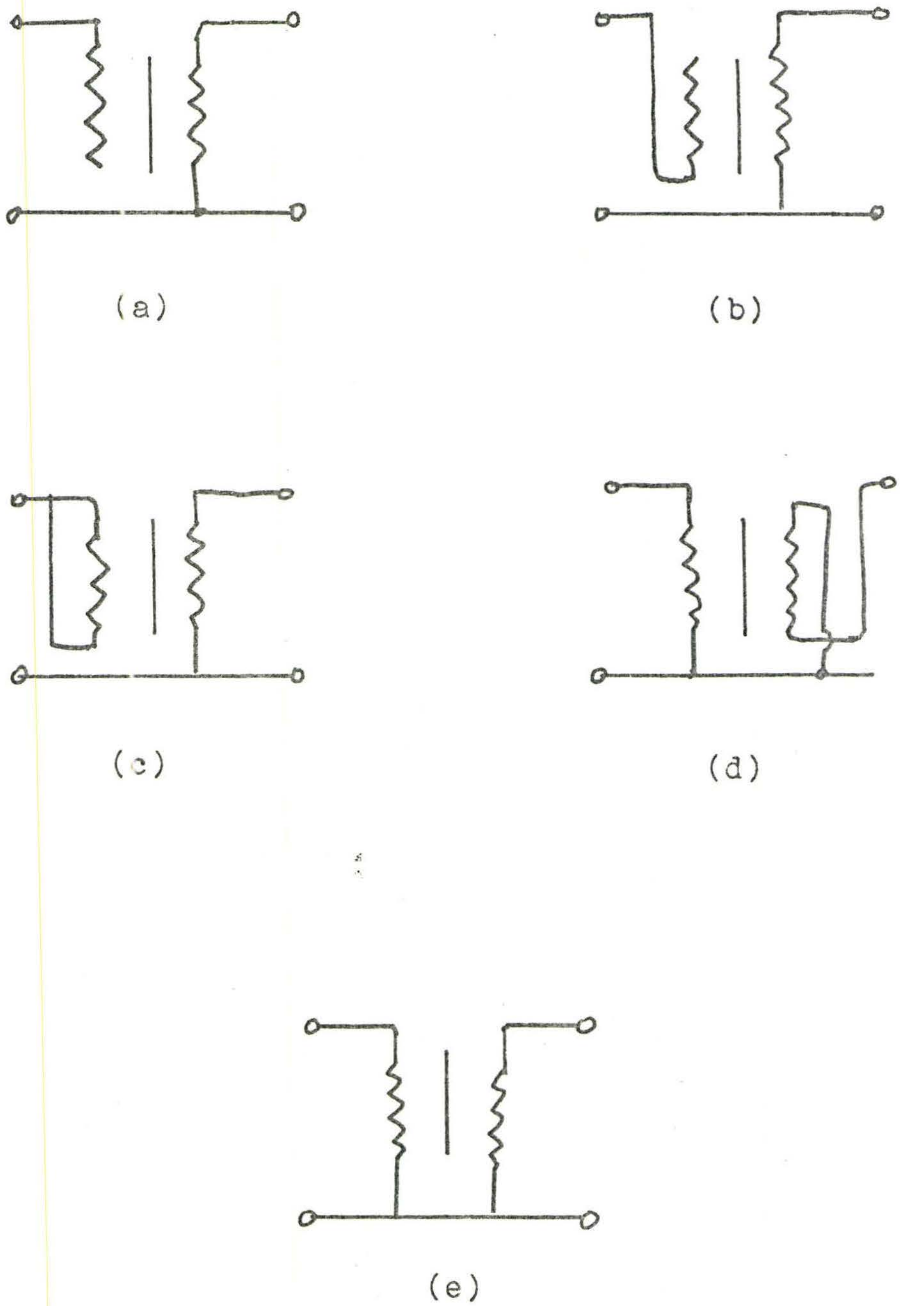
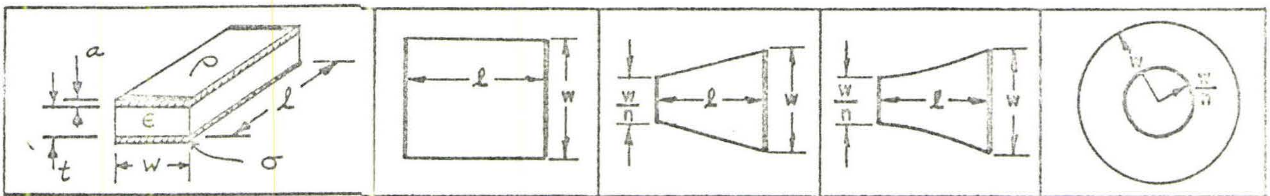


Figure 2.6 NON REDUNDANT TWO-PORTS WITH CAPACITIVE COUPLING

(See also Ref 6, page 458 for more details)



Normalized Parameter		Rectangular	Linear Taper Approximate	Exponential Taper Approximate	Resistive Disk
Resistance	$\frac{a}{\rho} R$	$\frac{\ell}{w}$	$\frac{(\ell)}{w} \frac{n \log}{(n-1)}$	$\frac{n-1}{\alpha w}$	$\frac{\log n}{2\pi}$
Capacitance	$\frac{t}{\epsilon} C$	ℓw	$(\ell w) \frac{n+1}{2n}$	$w \frac{(n-1)}{\alpha n}$	$\pi \frac{w^2}{n^2} (n^2 - 1)$
Time Constant	$\frac{at}{\rho\epsilon} RC$	ℓ^2	$(\ell^2) \frac{(n+1)\log n}{2(n-1)}$	$\frac{(n-1)^2}{n\alpha^2}$	$\frac{w^2}{2n^2} (n^2-1)\log n$
Impedance Level	$\frac{\epsilon}{\rho} \frac{R}{C}$	$\frac{1}{w^2}$	$(\frac{1}{w^2}) \frac{2n^2 \log n}{n^2 - 1}$	$\frac{n}{w^2}$	$\frac{1}{2\pi^2} \frac{n^2 \log n}{w^2(n^2-1)}$

Figure 2.7 SOME ANALYTICALLY DEFINABLE GEOMETRIES

(From Ref.[8], page 605)

2.4 Effects of Differing Exponential Tapers

2.4.I Introduction

It is possible to taper the incremental resistance and capacitance of an $\overline{RC}$ structure according to an exponential law giving an exponential $\overline{RC}$, or $\overline{ERC}$, as shown in figure 2.8 . It was mentioned in section 2.2 that analytical solutions are possible for such circuits. Such a treatment may be found in Carson[19] and Chirlian[4] . This section presents an approach to the physical understanding of the effects of taper.

2.4.2 Analysis and Characteristics

Exponential tapers may be characterised by

$$r(x) = r_0 e^{mx}$$

$$c(x) = c_0 e^{-mx}$$

which gives for a length λ

$$\frac{r(\lambda)}{r(0)} = e^{m\lambda}$$

or

$$m\lambda = \log_e \frac{r(\lambda)}{r(0)}$$

This factor $m\lambda$ is often put

$$m\lambda = 2D$$

and D is used as a parameter determining the degree of taper.

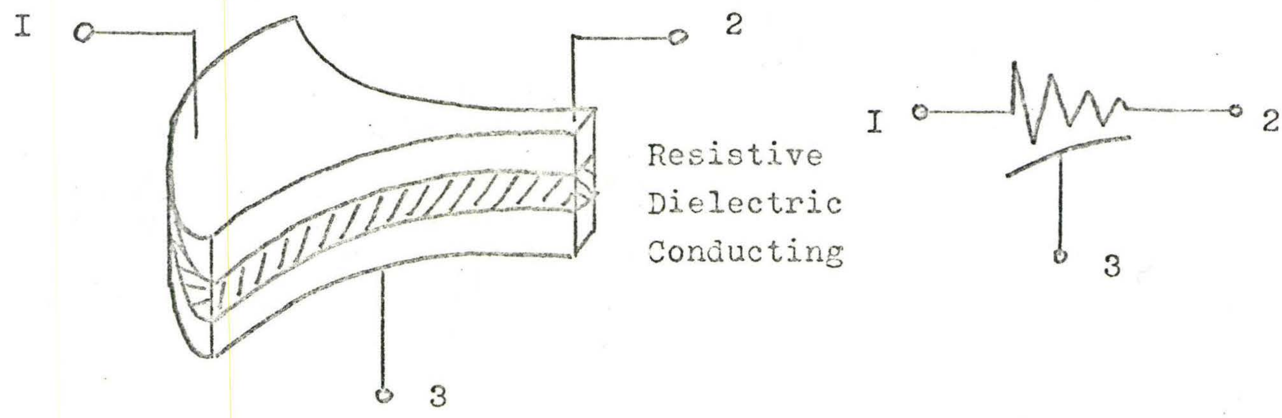


Figure 2.8 EXPONENTIALLY TAPERED $\overline{RC}$ STRUCTURE ($\overline{ERC}$)

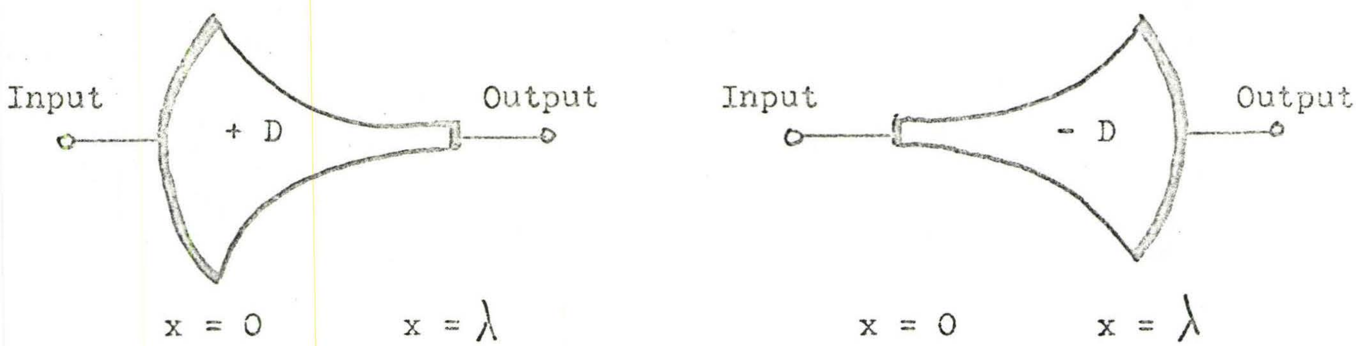


Figure 2.9 NOMENCLATURE FOR $\overline{ERC}$'s

D may be positive or negative depending on which port is chosen as the input, see figure 2.9.

It is interesting that since

$$Z_0 = \sqrt{\frac{r(x)}{sc(x)}}$$

if W is taken as the width of the section with uniform layers then:

$$\frac{Z_0 (x=\lambda)}{Z_0 (x=0)} = \frac{r(\lambda)}{r(0)} = \frac{W(0)}{W(\lambda)} = e^{m\lambda} = e^{2D}$$

Thus D determines the ratio of impedances at each end of the structure, which correspond to the inverse ratio of widths.

The various characteristics of exponential filters ($\overline{ERC}$'S) are obtained by a modification of the method of section 2.1 [4]. They are presented in figs 2.10 and 2.10. These curves show that a high positive value of D can give a sharper cut-off. The reasons for this will be discussed below.

2.4.3 $\overline{ERC}$ Notch Filters

An $\overline{ERC}$ may also be used to construct a notch filter as for any $\overline{RC}$. The amplitude of the Voltage Transfer Function for various degrees of taper is displayed in Figure 2.I2 . At first glance it appears that negative D values give a less selective

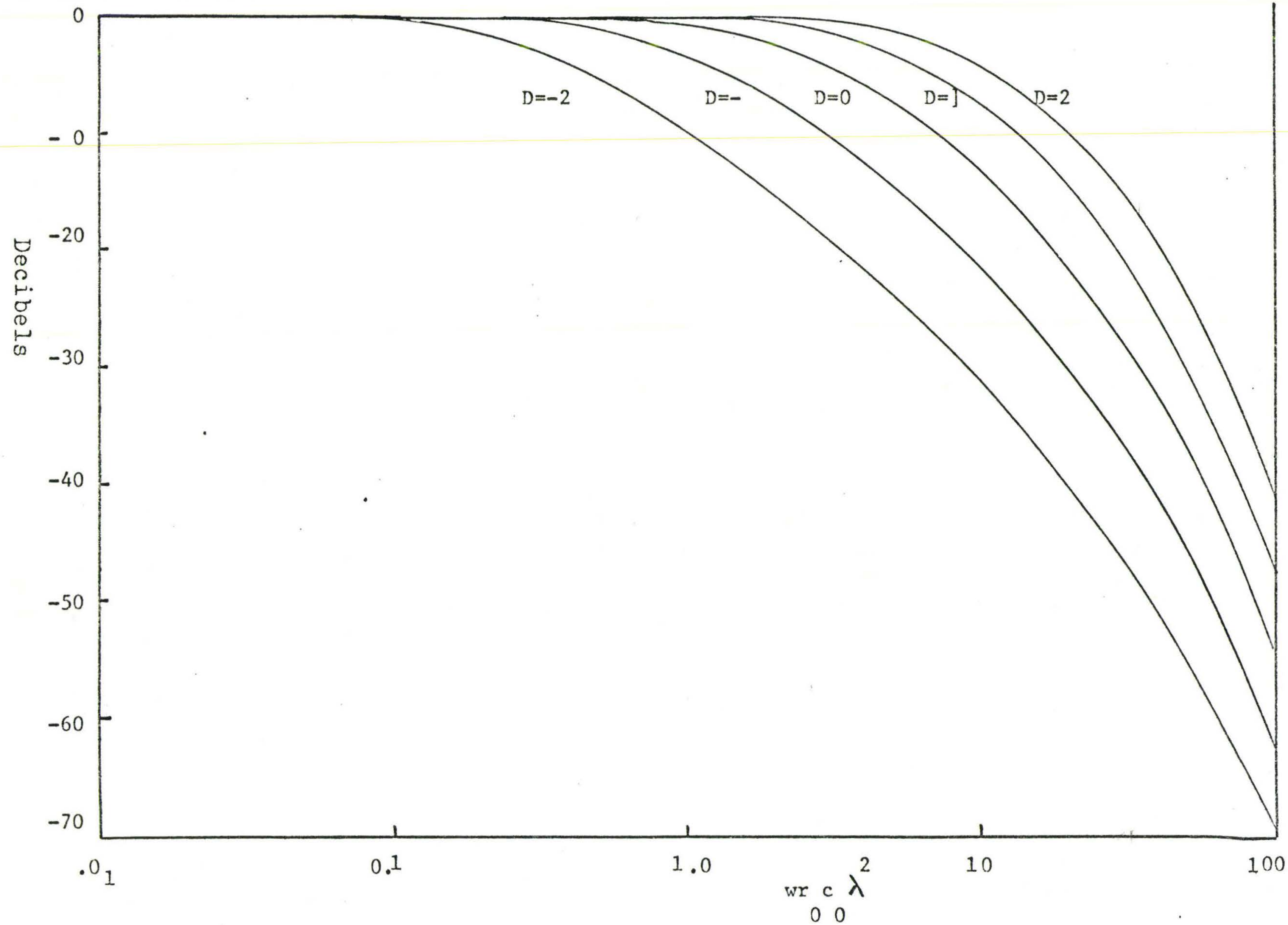


Figure 2.10 VOLTAGE TRANSFER FUNCTION MAGNITUDE OF AN ERC LOW PASS FILTER

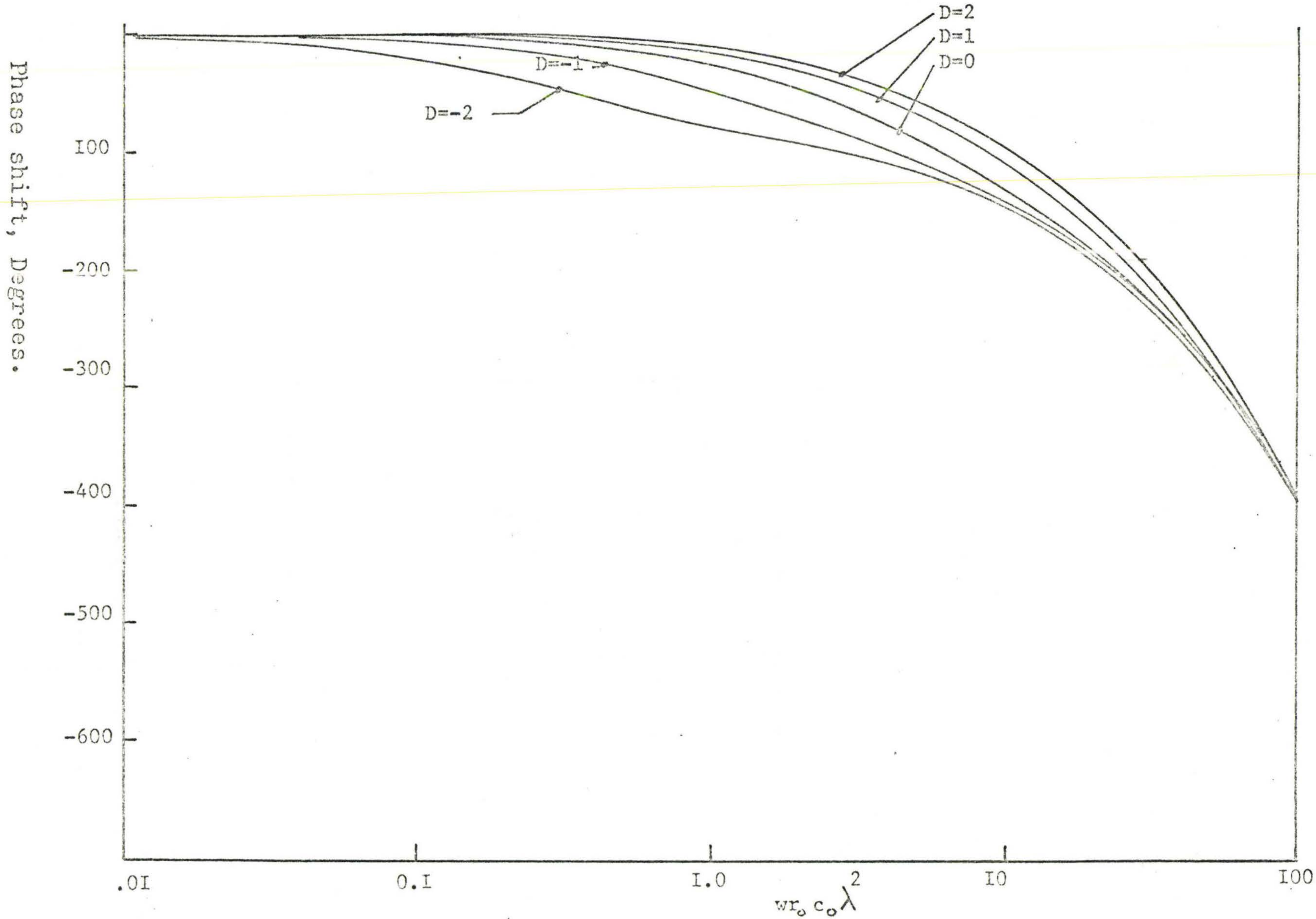
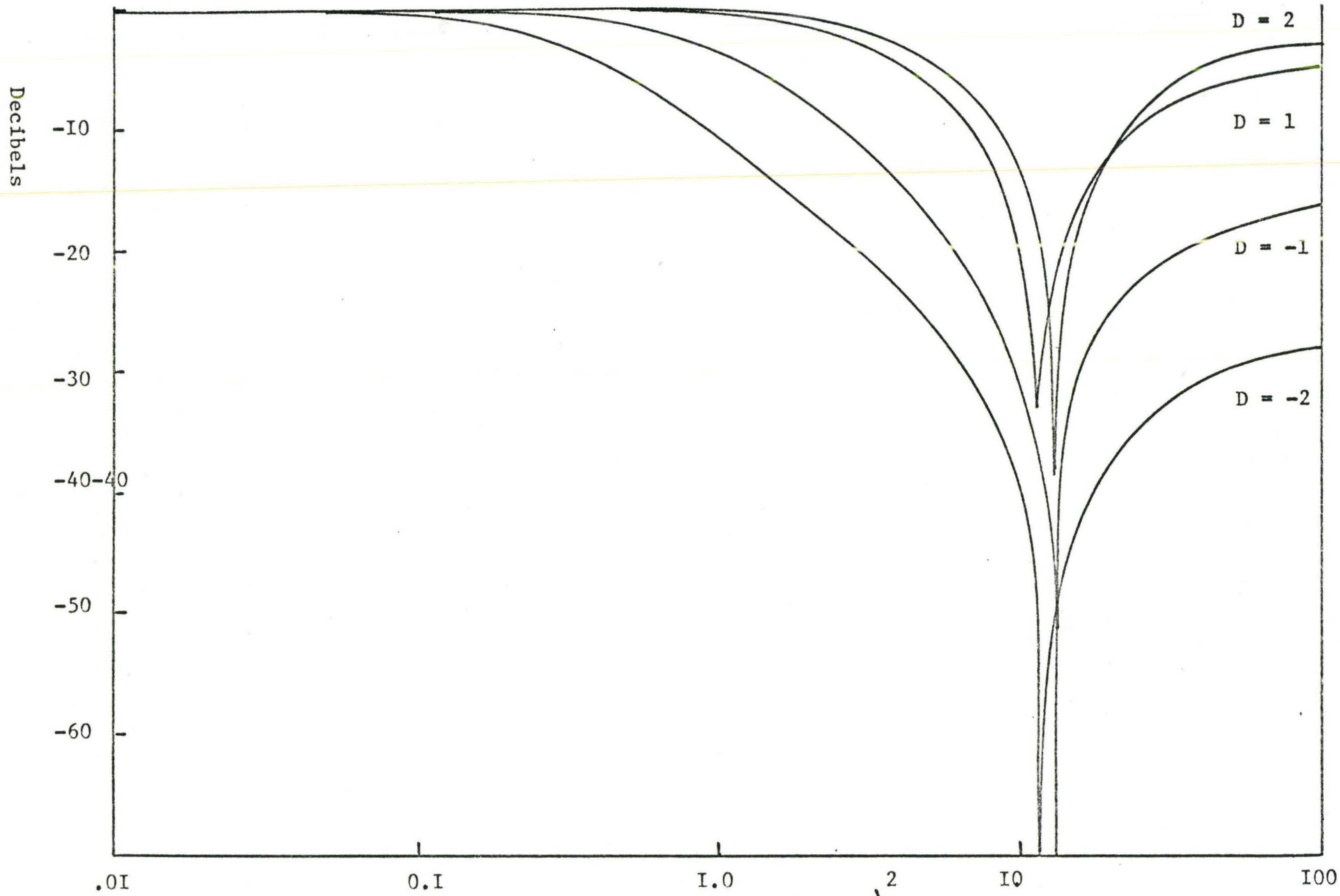


Figure 2.II VOLTAGE TRANSFER FUNCTION PHASE FOR AN ERC LOW PASS FILTER



$\omega r_0 c_0 \lambda^2$

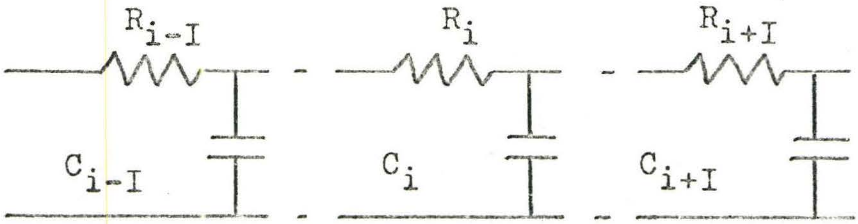
Figure 2.12 MAGNITUDE OF THE VOLTAGE TRANSFER FUNCTION OF AN ERC NOTCH FILTER WITH DIFFERENT DEGREES OF TAPER filter but close inspection will reveal a more complex mechanism.

The notch is formed by phase and amplitude cancellation as shown by Campbell.[42] For " optimum notch " conditions the phase cancellation determines the shape in the notch vicinity. Figure 2.12 shows similar shapes for the notch in this region for all tapers. However the low-pass amplitude characteristics Figure 2.10 show that for negative values of D significant attenuation occurs well before the frequency of the notch of Figure 2.12 is reached.

It is therefore postulated that the apparent lack of sensitivity for notch filters with negative values of D is merely a reflection of the curves of Figure 2.10. It remains to consider the mechanism behind the variations of Fig. 2.10.

2.4.4. Models For Tapered Structures

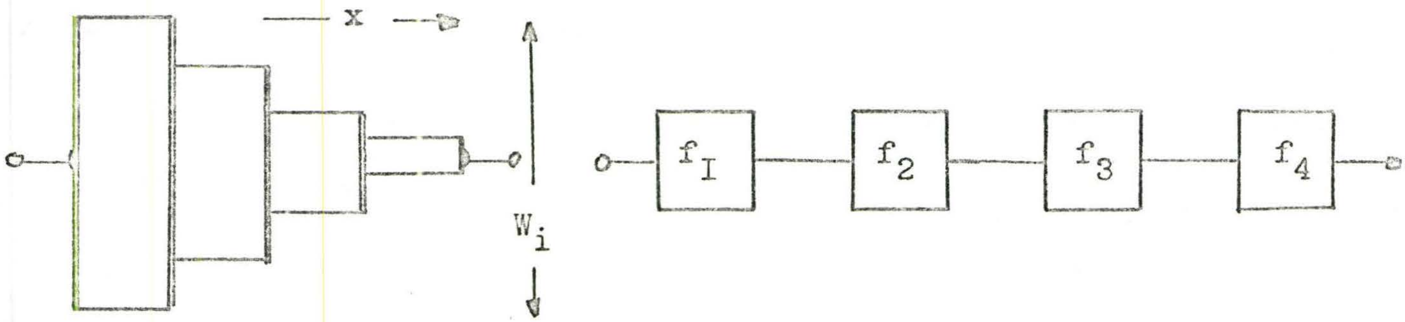
Various models of a distributed circuit are possible if the loading effects of adjacent sections are ignored. Figure 2.13(a) shows the traditional lumped approximation while Figure 2.13(b) shows a tapered structure represented by a cascade of uniform lines. The latter form has recently been treated rigorously by Walsh and Close.[27] For our purpose each segment may be considered a low-pass filter with frequency dependence



Voltage Transfer Function, Each Section $= \dfrac{1}{j\omega R_i C_i + 1}$

$R_i C_i$ = Const.

(a) <u>Cascaded Lumped RC Sections</u>



$\Theta_i = \Theta_i(\omega)$

$Z_{Oi} = Z_{Oi}(\omega, W_i)$

Transfer Function Each Section $= \dfrac{1}{\mathrm{Cosh}\,\Theta_i}$

(b) <u>Cascaded $\overline{\text{URC}}$ Transmision Lines (L.P. Filters)</u>

Figure 2.13
PHYSICAL MODELS FOR TAPERED $\overline{\text{RC}}$'s NEGLECTING LOADING

characterised by a " cut-off frequency " f .

2.4.5 Physical Explanation

The effect of taper is to vary the characteristic impedance of the transmission line and therefore the loading which each succesive elemental Lowpass filter imposes on the previous one. This is shown in Figure 2.14. It is surmised that this loading both increases the attenuation of each elemental filter and causes a lowering of " cut-off frequency ".

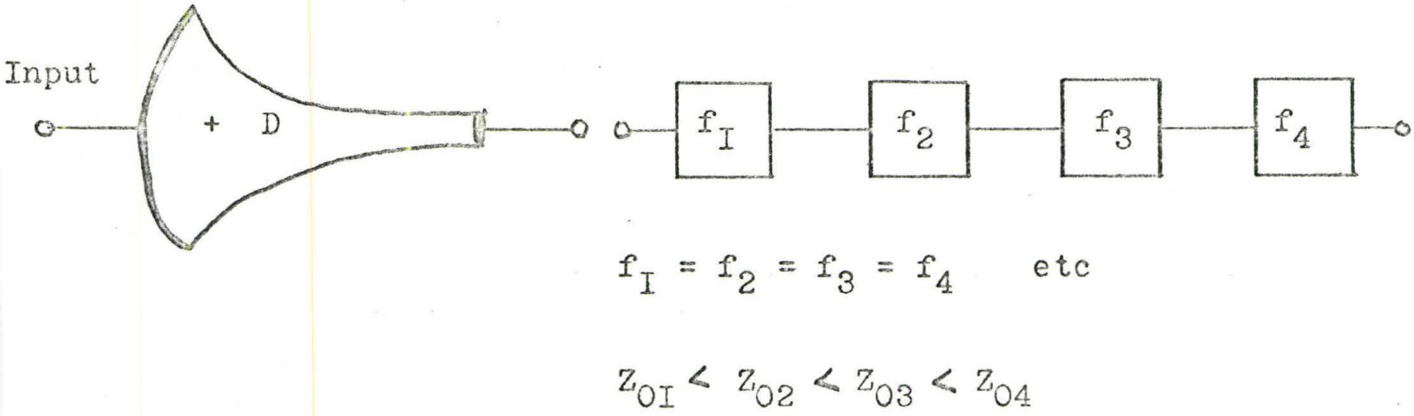
This may be demonstrated for the lumped RC model as shown in Figure 2.15. A passive non-inductive load will always increase the attenuation. If Z_1 is capacitive, as is the case, another $j\omega$ term is added to the $j\omega RC$ already present giving a lowered cut-off frequency.

2.4.6 Applications of Impedance Tapering

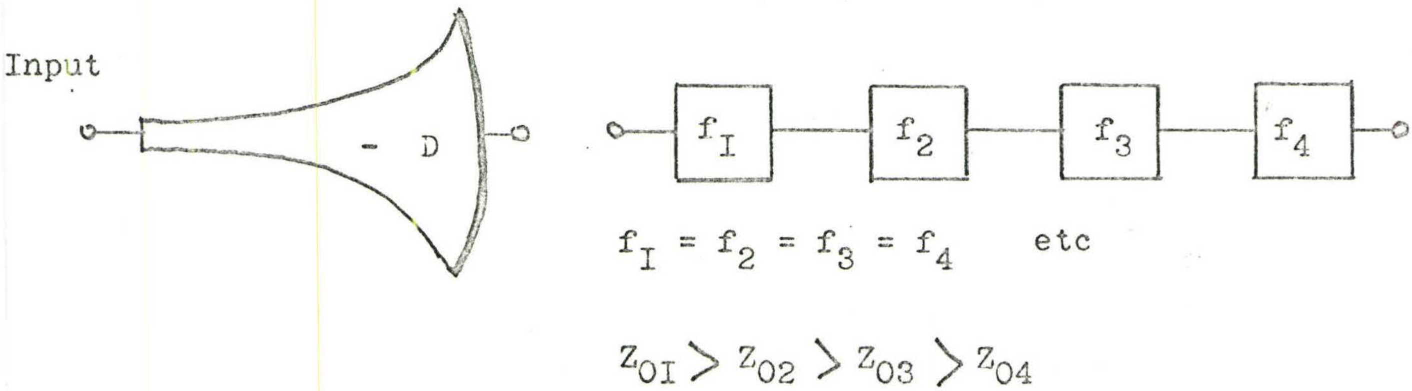
The effects of taper on the frequency domain transfer functions have already been mentioned. There are two other possible areas of application :-

(a) Loading or matching between a high and a low impedance combined with a lowpass filtering function. This might find uses in active filter circuits.

(b) Maximisation of the phase shift between two impedances for minimum loss. K.K.Pang [28] has shown that



(a) Slight Loading



(b) Heavy Loading

(c) Corresponding Characteristics

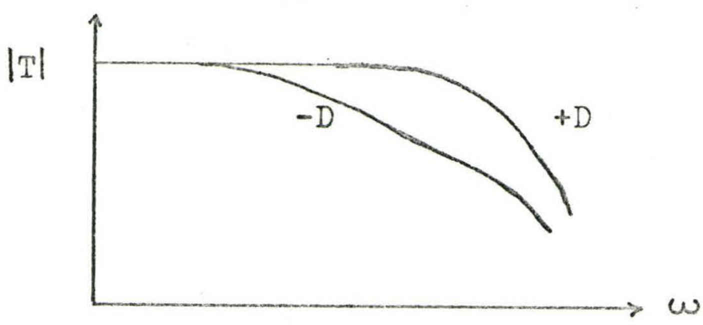
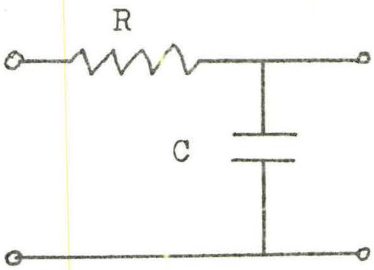
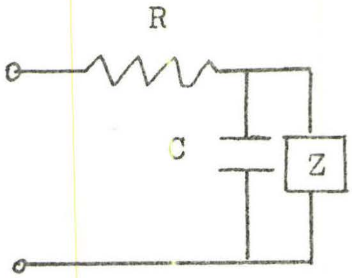


Figure 2.I4 THE EFFECTS OF IMPEDANCE LOADING OF SUCCESIVE SECTIONS FOR $\overline{ERC}$'s



$$\frac{V_o}{V_i} = \frac{1}{1 + sRC}$$



$$\frac{V_o}{V_i} = \frac{1}{1 + sRC + \frac{1}{Z}}$$

N.B. 1 Loading by Z causes attenuation.

N.B. 2 Capacitive Loading lowers cut-off frequency.

Figure 2.15 LOADING EFFECTS FOR SIMPLE RC CIRCUIT for the distributed case an exponentially tapered line is optimum.

CHAPTER 3

Transfer Function Synthesis

3.1 Statement of the Problem

The first stage in the treatment of a network lies in its analysis and characterisation. The second is consideration of what synthesis procedures may be used in order to achieve some desired result with the network. This result can normally for linear passive two-port networks be expressed as a transfer function. Interest in one-port characterisations usually stems from a desire to incorporate the results into a two-port synthesis procedure.

In traditional circuit theory transfer functions are specified and synthesised as ratios of polynomials in the s-plane. Comprehensive synthesis techniques based on such rational functions have been developed[3]. However these methods find themselves at a loss when faced with distributed circuits. It was shown in Chapter 2 that even when closed form solutions are possible, the parameters so obtained are transcendental in form. They may be expressed either in hyperbolic form or as infinite series.

Several methods have been proposed to surmount this problem, and these will now be examined in turn. Emphasis will be on the frequency domain but work has also been done in the time domain where the picture is similar in outline [44].

3.2 Empirical Approximations

A network approximating the desired response may be chosen on the basis of analytically derived response curves such as those given by W.W.Happ et al. [5, 6, 7 and8]. Such sections may be cascaded. This is a useful approach, especially for yielding an initial design prior to optimisation, but is essentially graphical and does not approach the general synthesis problem.

Alternatively it is possible under some circumstances to use a dominant pole approximation to model the device (see Refs. 3, page267, 4 pp 142, 408, Refs 14 and 16). This is often done for a notch filter [3]. The neglected poles can lead to instability problems if the device is used in an active filter circuit.

3.3 Positive - Real Transformations

These alter the problem from a distributed one in the s-plane to a lumped one in some transform plane.

Wyndrum's method is based on the transform

$$W = \tanh \sqrt{sRC}\, l$$

for R = Total series Resistance
C = Total shunt Capacitance
l = Section length

giving a lumped LC problem in the w plane. Figure 3.I illustrates the complete procedure necessary.

O'Shea[2I] has proposed an alternative transformation

$$W = \cosh \sqrt{sRC}\, l$$

which gives a lumped RL network amenable to Foster type realizations.

Much work has been done on one port, two port, active and passive synthesis using these transforms. (see the survey references 3 and 4) Most methods use $\overline{RC}$'s that are uniform or have simple tapers. Circuit topology becomes more complicated with increased transfer function complexity. Usually for each basic $\overline{RC}$ segment only one parameter is varied and realizations can involve awkward crossovers.

A more serious drawback is that the synthesis cannot proceed until the s-plane specification has been put in a particular form suited to the transform in use. Beyond that point though the methods are exact.

3.3.I Limitations on the class of Realizable Functions.

The above methods are both based on simple distributed circuit building blocks of the type shown in Fig I.I , usually with uniform or exponential taper. As mentioned in Chapter 2

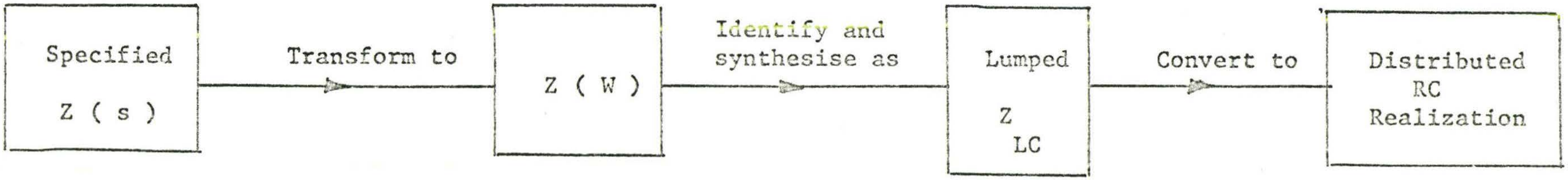


Figure 3.1 BLOCK DIAGRAM ILLUSTRATING WYNDRUM'S SYNTHESIS PROCEDURE

such structures have all the transmission zeros of the open-circuit voltage transfer function at infinity. Wyndrum proposed the use of stubs and ladder sections, but zeros are then still confined to the negative real axis. In O'Shea's scheme zeros may be realized anywhere in the left - half s-plane but parallel networks and lattice configurations are required. The resulting network is often too complex for practical realization.

These limitations have led to the consideration of other basic element structures as building blocks for synthesis procedures.

3.4 Rational Functions

One line of approach to surmount the transcendental nature of the network parameters has been to find some combination of taper and structure that yields a rational characterisation.

Heizer[22, 23] has started with the structure shown in Figure 3.2 He has shown that by using certain Fourier series for the capacitance and conductance tapers all the Y parameters can be rational. Two degrees of freedom are available to determine the pole positions and complex zeroes are possible. Physical construction of the resulting circuit can be very difficult.

A number of workers[52, 53, 54] have used Heizer's work as the basis for active network synthesis schemes. In general the arrangements are elaborate and it is necessary to contend with normal problems of sensitivity.

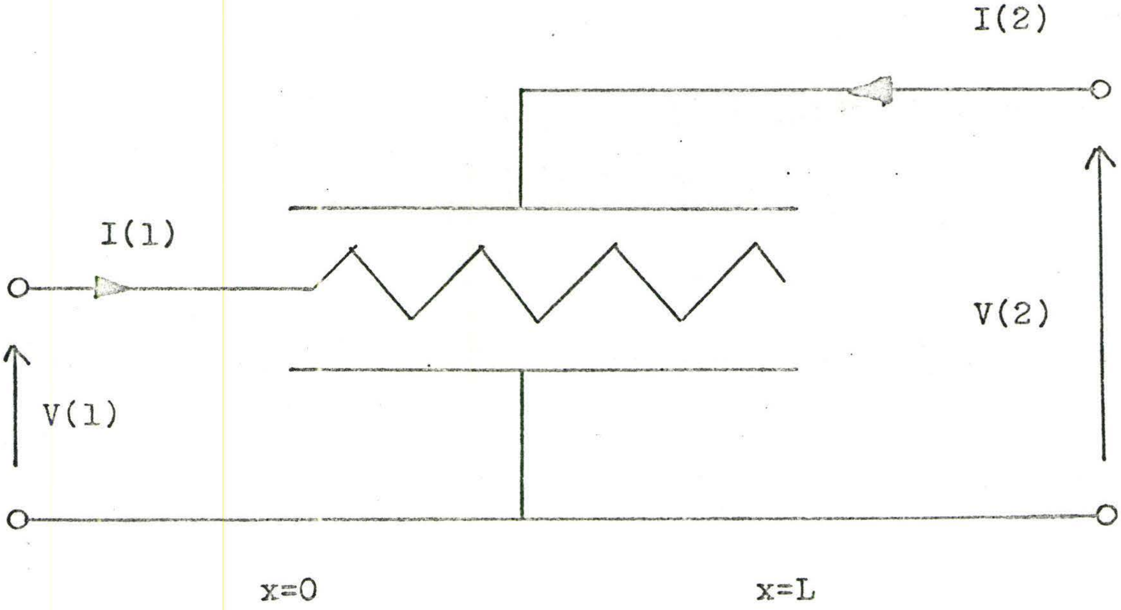


Figure 3.2

5 - LAYER DISTRIBUTED STRUCTURE ANALYZED BY HEIZER [22]

Heizer's original work has been generalised by Hwang and Duesterhoeft [25]. They have been able to obtain , in theory, poles anywhere in the left - half plane with rational functions. Heizer himself has extended his original work to a consideration of two-dimensional flow in a structure with shaped electrodes.[24] Neither of these is as general as the work of Walsh and Close detailed below.

3.5 Variable Taper Methods

So far schemes have been considered in which fairly simple basic distributed elements have been interconnected in a network to synthesise some transfer function. Usually only one or two degrees of freedom are assigned to each element. This does not exploit the full potential of differing topologies and arbitary tapers. The restrictions have mainly stemmed from a desire to continue using rational s-plane methods even though they are not naturally applicable.

Rohrer, Resh and Hoyt [26] broke new ground in devising an iterative computer program to adjust the taper of a single structure to minimise the least squares error between the specified and actual responses. This approach will be further investigated in Chapter 4. For now it is necessary to note that all the zeros lie at infinity and so its generality is limited.

This point led Walsh and Close[27,33] to consider multilayer structures of the type previously used by Heizer. A modification of Heizer's structure is used by them to realise any transfer function having distinct poles limited to the negative real axis i.e. any lumped RC - realizable function. The problem is treated in two parts. First the resistive taper is found by means of an iterative scheme to realize the required poles. Then the capacitive taper is adjusted to give a rational transfer function with the specified zeros. Modifications are then made to allow construction with uniform layers and homogeneous material.

Walsh has programmed the procedure completely on a digital computer which provides a print out of the geometrical dimensions required for any given transfer function. The resulting circuits have the same basic topology. The experimental results, with teledeltos paper filters, suggest that a " taper " consisting of twenty steps along the length of the transmission line is sufficiently close for the realization of an eighth order transfer function.

It would seem that this work represents the answer to the problem posed, that of synthesising rational transfer function specifications using $\overline{RC}$'s. The method would lend itself to automated thin film production. The computer output could be linked to a mask cutting or photo - printing setup used to delineate the topology of a standard basic thin film circuit. Two extensions would be necessary to the method for this to be practical ; a two dimensional curvilinear squares program and some sort of sensitivity study to determine what order of complexity of transfer function could be synthesised with given production tolerances.

This work was not undertaken for this program, mainly because interest shifted away from any consideration of rational s - plane specifications for reasons that will be examined in Chapter 4.

CHAPTER 4

Application of Computer-Aided Design Techniques

4.1 Goals

In Chapter 3 it was shown that even for a uniform three layer $\overline{RC}$ the network parameters are in transcendental form. It was also pointed out that no general analytical solution exists for arbitary tapers. These facts led to grave difficulties in using traditional network synthesis methods with $\overline{RC}$ s. Two further questions may be posed which also cannot be answered by traditional methods:-

Q1 What is the range of network frequency response types obtainable for a simple 3-layer $\overline{RC}$ structure if arbitary tapers within reasonable max/min ratios are allowed ?

Q2 What taper is required for an optimum Low Pass filter response from such a structure ?

These questions may be approached through empirical computer analysis of the effects of various tapers. An intuitive feel for the scope and behaviour of the circuits may be built up, answering question 1. This information may then be used as the basis for a guess at a starting point for a computer optimisation program to answer question 2.

This is a process familiar in the field of Computer-Aided Design. It is found generally that an optimisation algorithm starting from scratch will not produce as good a result as an experienced engineer using intuition and analysis. However if fed a starting point design based on the engineer's work the algorithm will often be able to show an appreciable improvement.

4.2 Characterisation of a $\overline{URC}$

In section 2.1 a treatment was given for an $\overline{RC}$ with uniform resistance and capacitance per unit length. The chain matrix characterisation was given :-

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \text{Cosh}\,\gamma d & Z_o \text{Sinh}\,\gamma d \\ \dfrac{\text{Sinh}\,\gamma d}{Z_o} & \text{Cosh}\,\gamma d \end{bmatrix}$$

This matrix uniquely determines the two-port behaviour of the $\overline{URC}$ given two parameters, Z_o and γd (d = length of structure), defined by :-

$$Z_o = \sqrt{\frac{sr_o}{c_o}} \quad , \qquad \gamma d = \sqrt{sr_oc_o d^2}$$

Thus the $\overline{URC}$ has two degrees of freedom available.

However in terms of a physical structure there are at least six parameters that might be specified :-

(a) Length of section = d

(b) Width of section = W

(c) Thickness of resistive layer = t_r

(d) Thickness of dielectric = t_c

(e) Bulk resistivity = k

(f) Dielectric Constant = ϵ

Relations between some of these, such as t_r and k cannot be defined for evaporated thin film structures. Nevertheless taking an idealised case they determine the restance and capacitance per unit length, r_o and c_o according to :-

$$r_o = \frac{k}{t_r W} \quad , \quad c_o = \frac{\epsilon W}{t_c}$$

giving :-

$$Z_o = \sqrt{\frac{k}{s\epsilon} \cdot \frac{t_c}{t_r} \frac{1}{W^2}}$$

and

$$\gamma d = \sqrt{\frac{s\epsilon k}{t_d \, t_c}} \cdot d$$

Two equations and six parameters leaves us free to fix four of the parameters. For homogeneous isotropic materials ϵ and k can be taken as constant if t_r is also constant. This is convenient because of the non-linear relationship of k and t_r for thin film and because these parameters are the least easy to vary from one structure to another during fabrication. Of the remaining three it is most convenient to fix t_c giving :-

$$\gamma d = \sqrt{s.\mathrm{Const}.d^2}$$

$$Z_o = \sqrt{\frac{\mathrm{Const.}}{s\ W^2}}$$

Thus d and W are chosen as the parameters to represent the two degrees of freedom available for a single section of uniform $\overline{RC}$. In fabrication they would be properties of mask dimensions that could be controlled accurately and conveniently.

4.3 Characterisation of Arbitary Tapers

Two methods are available for using URC analysis to characterise a structure with arbitary taper. The taper involved is primarily an impedance variation along the structure. It follows from the relationships given in sections 2.3 and 4.2 that this is directly related to $r_o(x)$ and inversely to $c_o(x)$ and $W(x)$.

4.3.1 Resistance Function

Consider the formulation :-

$$r_o = f(x)$$

using the conventions of figure 4.1. This function may be specified at certain discrete points:-

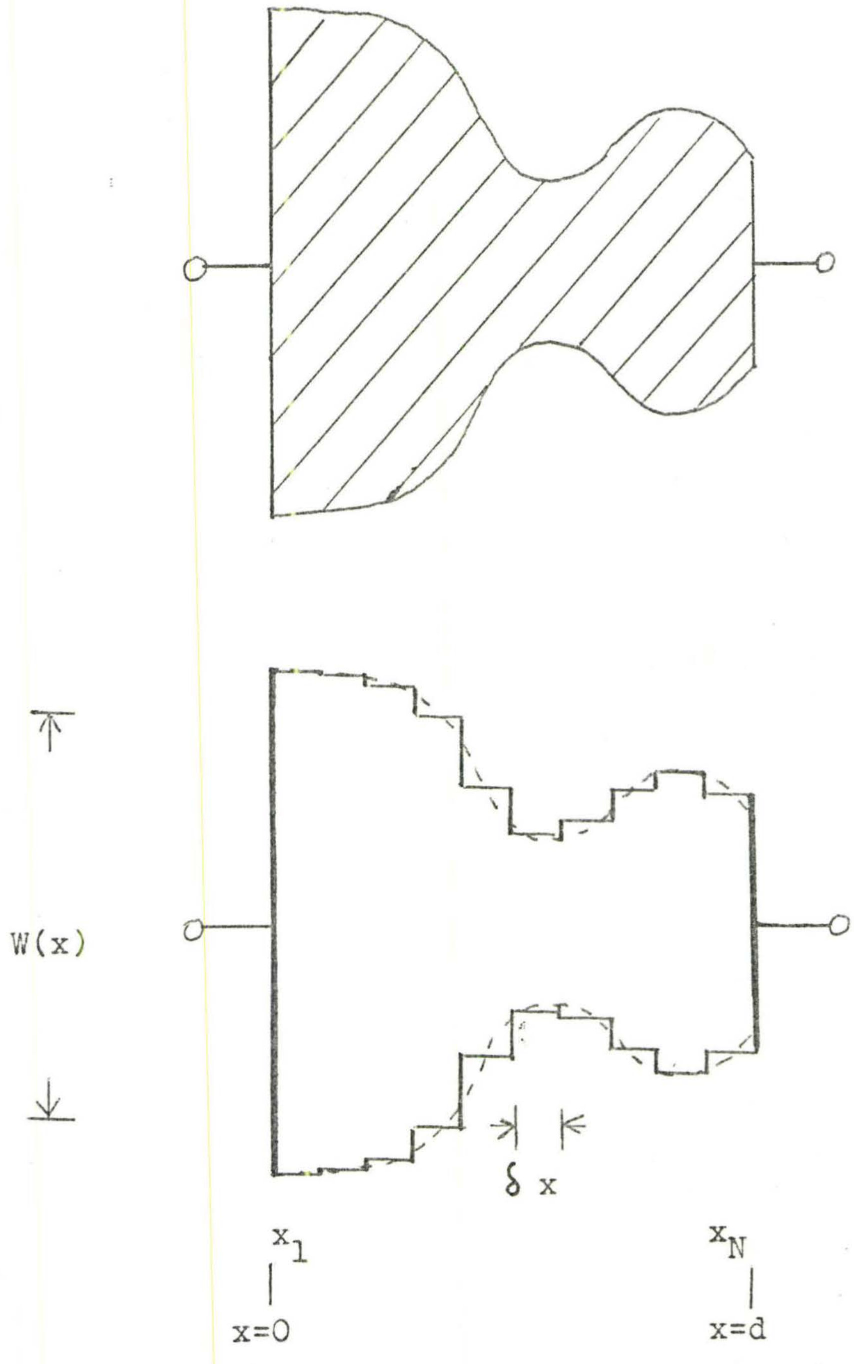
$$x = x_1, x_2, \ldots x_N$$

Walsh has shown[33] that twenty points can give a good approximation. For convenience the points may be equally spaced so that :-

$$x_n = n\,\delta x, \quad n=1, N$$

In analysis the structure is then considered as a cascade of N $\overline{URC}$'s, each of equal section length δx but varying $W(x)$. Because of this constraint there is only one degree of freedom available to optimise for each section, giving a total of N. The overall chain matrix is easily calculated from that of each section by multiplication.

However in practice the current-flow lines of a tapered structure are two-dimensional while our analysis has assumed one-dimensional flow over each section in turn. This means that after



N Sections, each specified by $W(x) = K/r_o(x)$

N Variables to Optimise

Curvilinear Square program necessary for implementation except for large values of N.

RESISTANCE FUNCTION ANALYSIS OF ARBITARY TAPER

<u>Figure 4.1</u>

a final design has been reached a curvilinear-square program or some sort of transform is necessary to derive physical values W'(x) for the mask layout. Work has been done on this with respect to exponential tapers.[12] The general case is more complicated and might involve a relaxation program.

Thus this method gives simple analysis but complicated implementation.

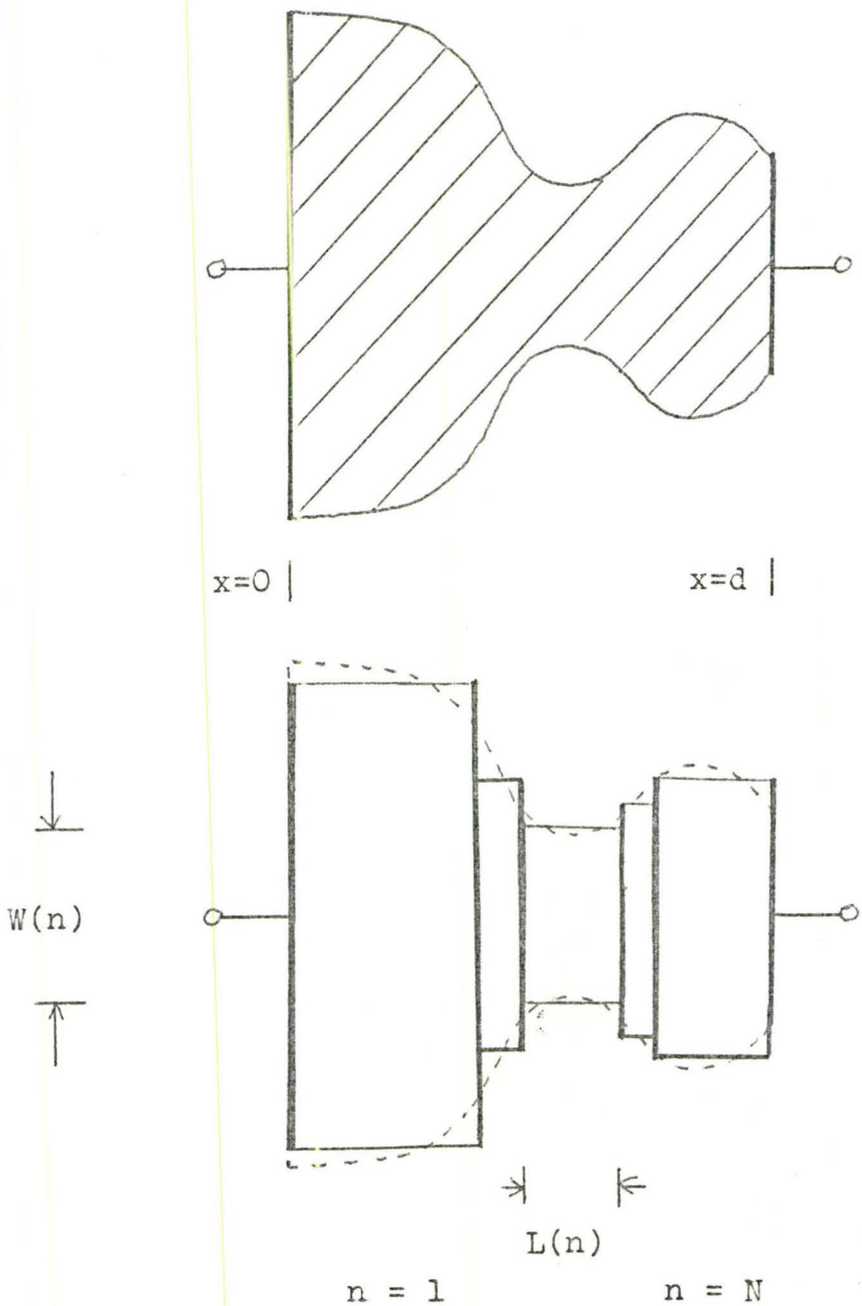
4.3.2 Cascaded $\overline{URC}$'s

Alternatively the arbitrarily tapered structure could be approximated by a cascade of URC's , of independent lengths and widths as shown in Figure 4.2. In this case for each of N sections we have :-

$$W(n), \quad n = 1, N \quad ; \qquad L(n), n = 1, N$$

giving two independent variables available for optimisation per section, 2N in total.

In this case a curvilinear square program would not be appropriate because of the varying section lengths . It is necessary to put conducting strips across the boundaries between adjacent URC's . This makes the current flow within each $\overline{URC}$ one-dimensional, and is quite feasible in thin film production provided there is sufficient mask alignment precision and that



N Sections, each specified by W(n) and L(n).

2N Variables to optimise

Conducting strips betwwen sections necessary.

CASCADED $\overline{\text{URC}}$ ANALYSIS OF ARBITARY TAPER

Figure 4.2

stray capacity effects are not significant. The latter could easily be included in the analysis if known.

Note that the second method makes use of twice as many variables for a given number of sections. However only half as many sections may be needed to provide an equally good approximation to a particular taper when length as well as width can be varied for each section. A comparison of Fig. 4.1, with 10 sections and Figure 4.2, with 5 sections illustrates this.

4.4 Analysis Program

The method of section 4.3.2 was chosen, with a structure of five basic $\overline{URC}$'s in series giving ten independent parameters. Accordingly an analysis program for cascaded sections was developed. It was designed to be flexible for possible later extension to include shunt arms. Various different types of elements were allowed for, as shown in figure 4.3. Each was to be characterised by a number of parameters sufficient to construct its ABCD or chain matrix at any frequency.

A one amp load current was postulated in a load resistor R_L. The chain matrices were then used to calculate the remaining V's and I's in the system using the method of figure 4.4. The input

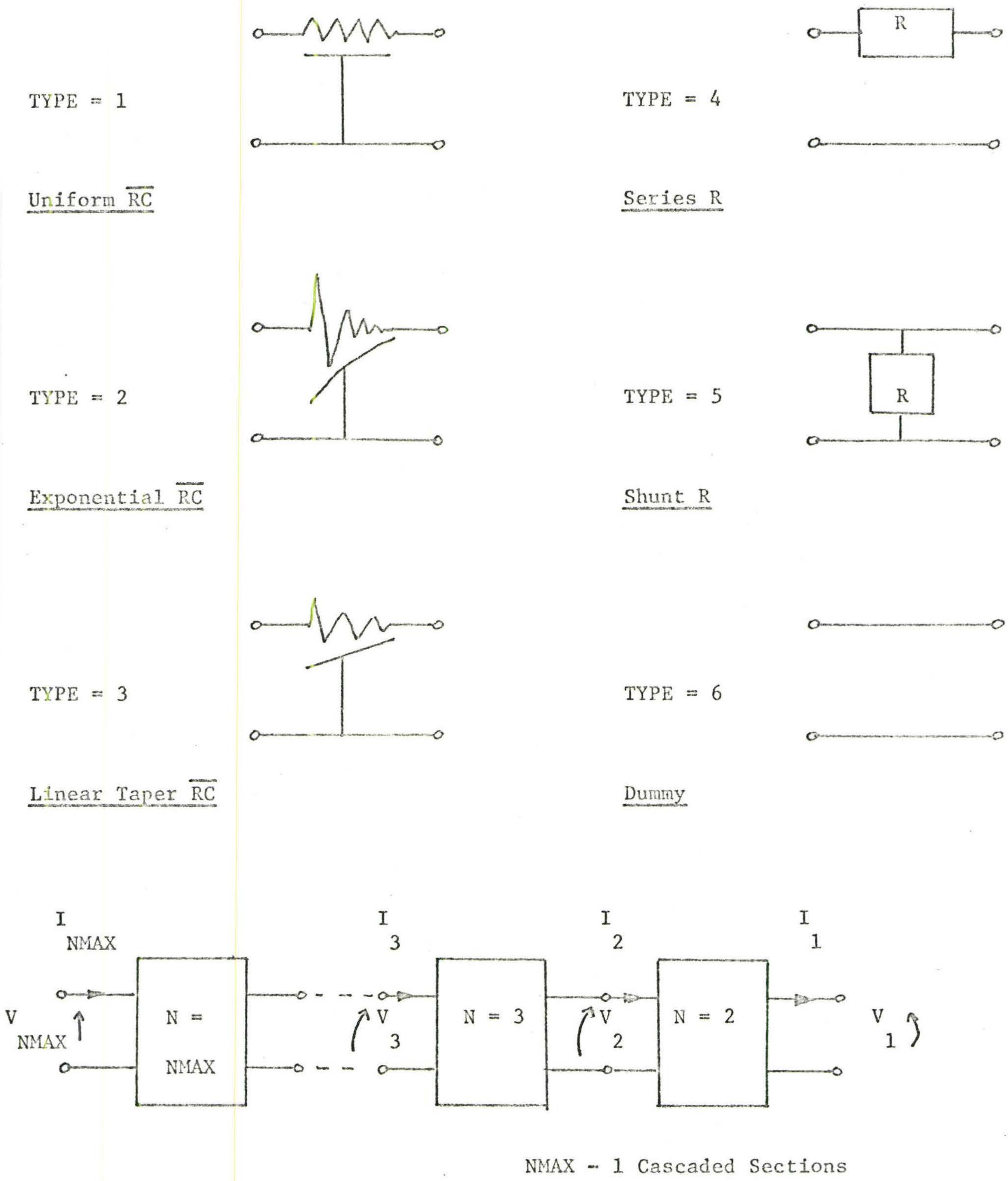
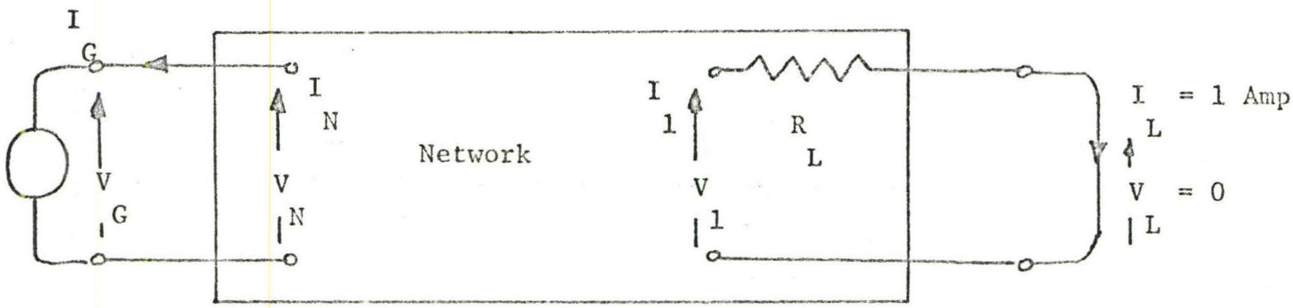


Figure 4.3 NUMBERING AND SIGN CONVENTIONs,ANALYSIS PROGRAM



$V_G = V_N$ $\qquad$ $V_1 = R_L$ $\qquad$ $V_L = 0$

$I_G = -I_N$ $\qquad$ $I_1 = 1$ $\qquad$ $I_L = 1$

$$\begin{bmatrix} V_N \\ I_N \end{bmatrix} = \begin{bmatrix} A_N \end{bmatrix} \cdot \begin{bmatrix} A_{N-1} \end{bmatrix} - - - \begin{bmatrix} A_2 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$\text{Voltage Transfer Function} = V_1 / V_N$$

Figure 4.4 Calculation of Transfer Functions

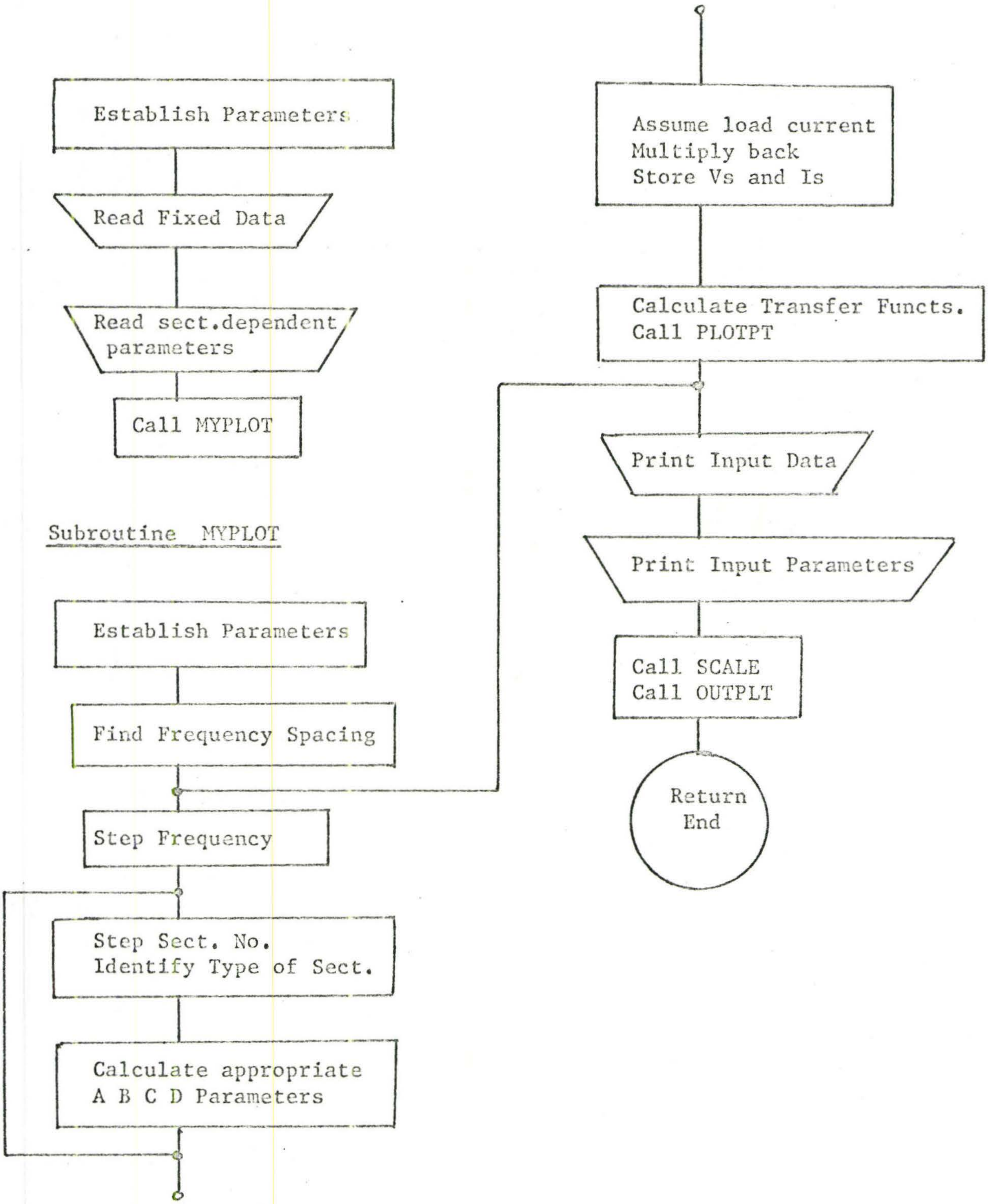


Figure 4.5 FLOW CHART OF CASCADE ANALYSIS PROGRAM

to output voltage ratio can then be taken for the transfer function. This procedure must be carried out over a range of frequency. The use of normalised frequency $2\pi fr_o c_o$ is convenient. The results may the be plotted on the computer printer using subroutine PLOTPT.

To facilitate later use of the analysis as part of an optimisation strategy if required a hierarchical structure was set up, with a main program calling a subroutine Myplot. This enabled the input parameters to be stepped between successive analysese automatically. Figure 4.5 shows a flow chart for the whole analysis program.

4.5 Analysis of Differing Topologies

To gain experience of the types of response available a series of analysis runs was carried out on arbitrary tapers, some of which are shown in figure 4.6. In several cases one parameter, such as a width or the load resistor, was stepped over a wide range. In all cases plots of the voltage transfer function against frequency were obtained from the computer printer.

All the characteristics decreased monotonically with frequency. Figure 4.8 shows a standard $\overline{URC}$ which may be compared with the two extreme cases of figures 4.7 and 4.9. Most other responses obtained lay in the region between these two. Structures with large variations in width (e.g. fig. 4.6(d)) behaved approximately as lumped RC

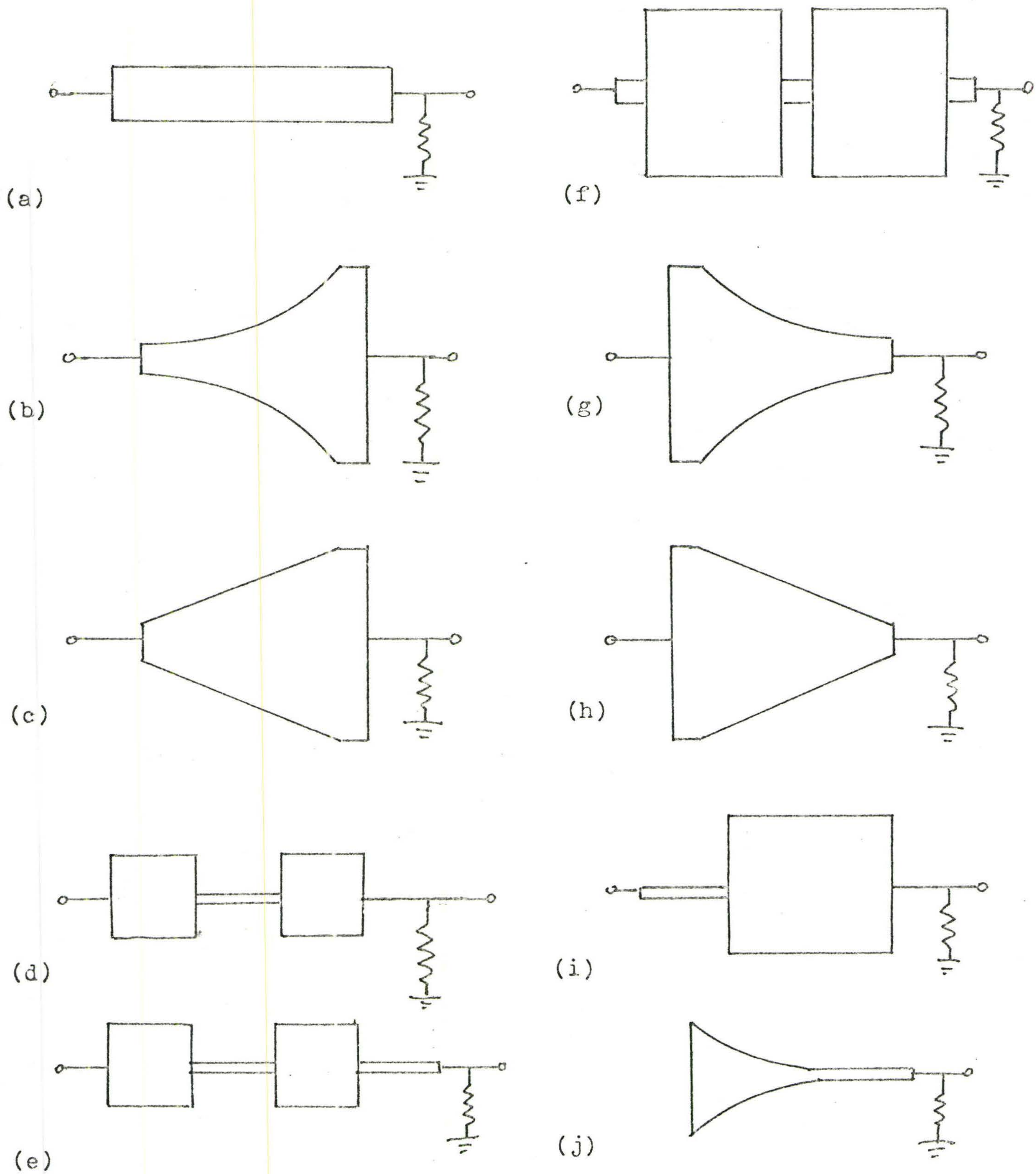


Figure 4.6 EXAMPLES OF TAPERED $\overline{RC}$ STRUCTURES ANALYSED
(Not to scale)

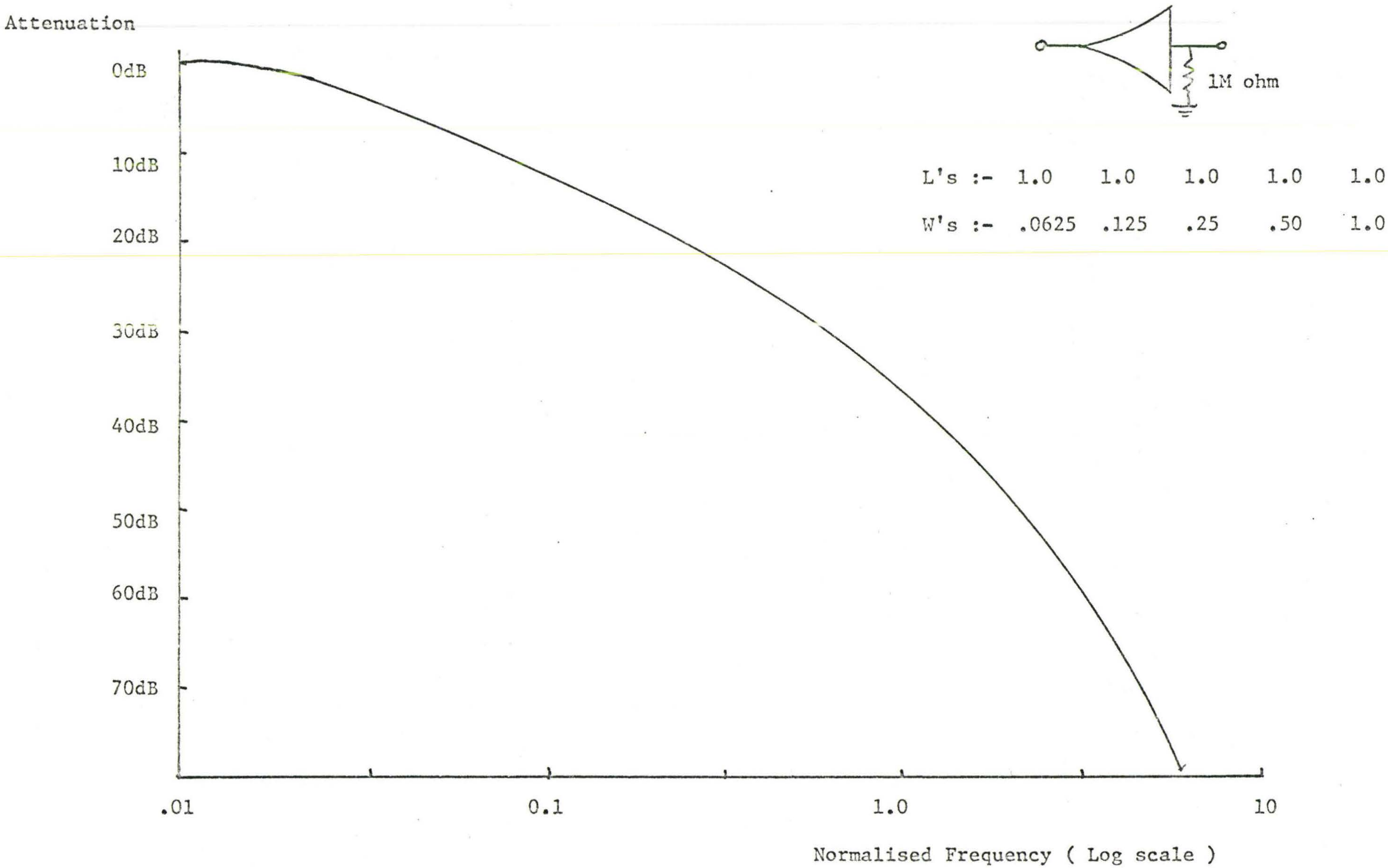


Figure 4.7 RESPONSE OF TAPERED FILTER STRUCTURE

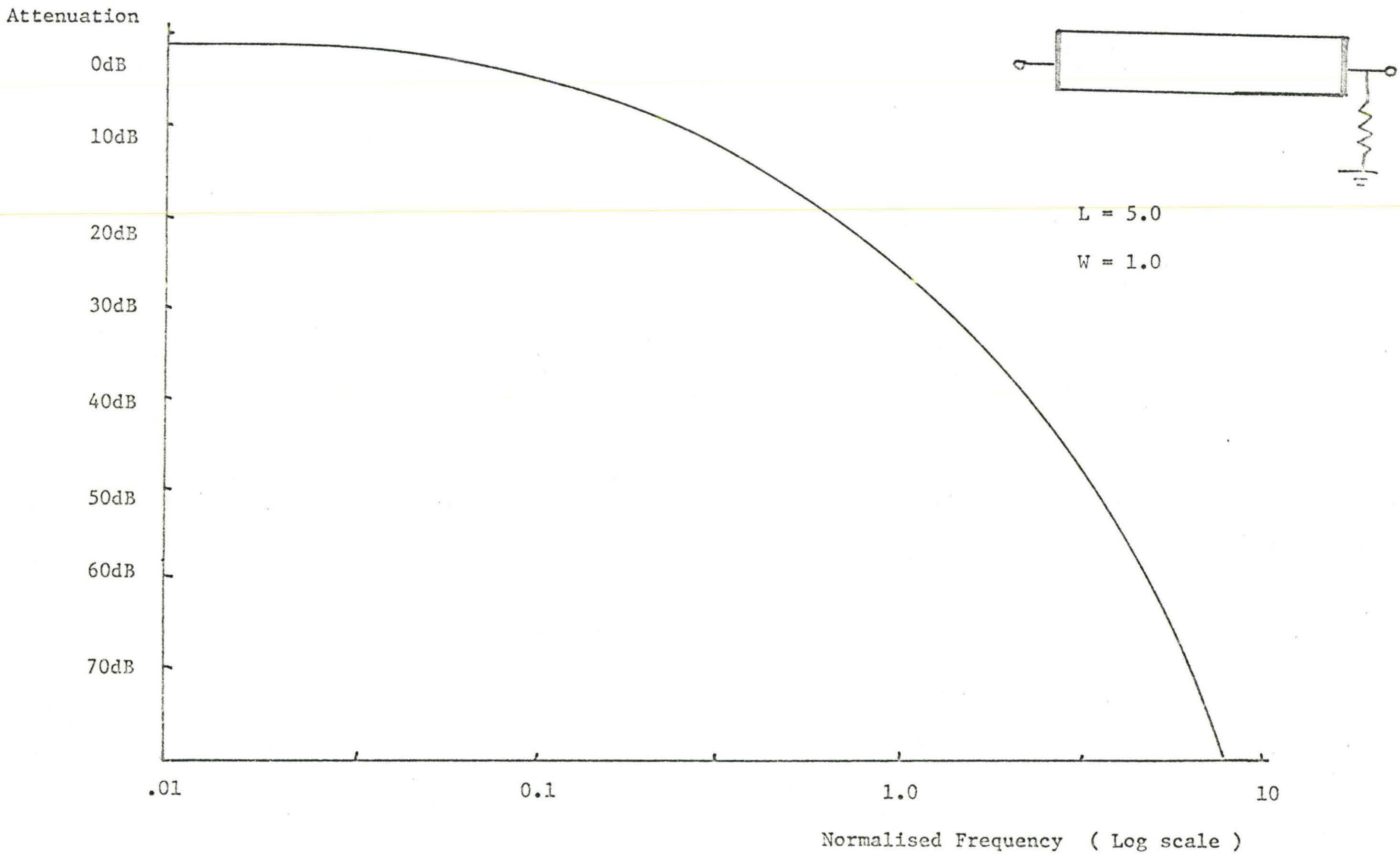


Figure 4.8 RESPONSE OF UNIFORM $\overline{RC}$

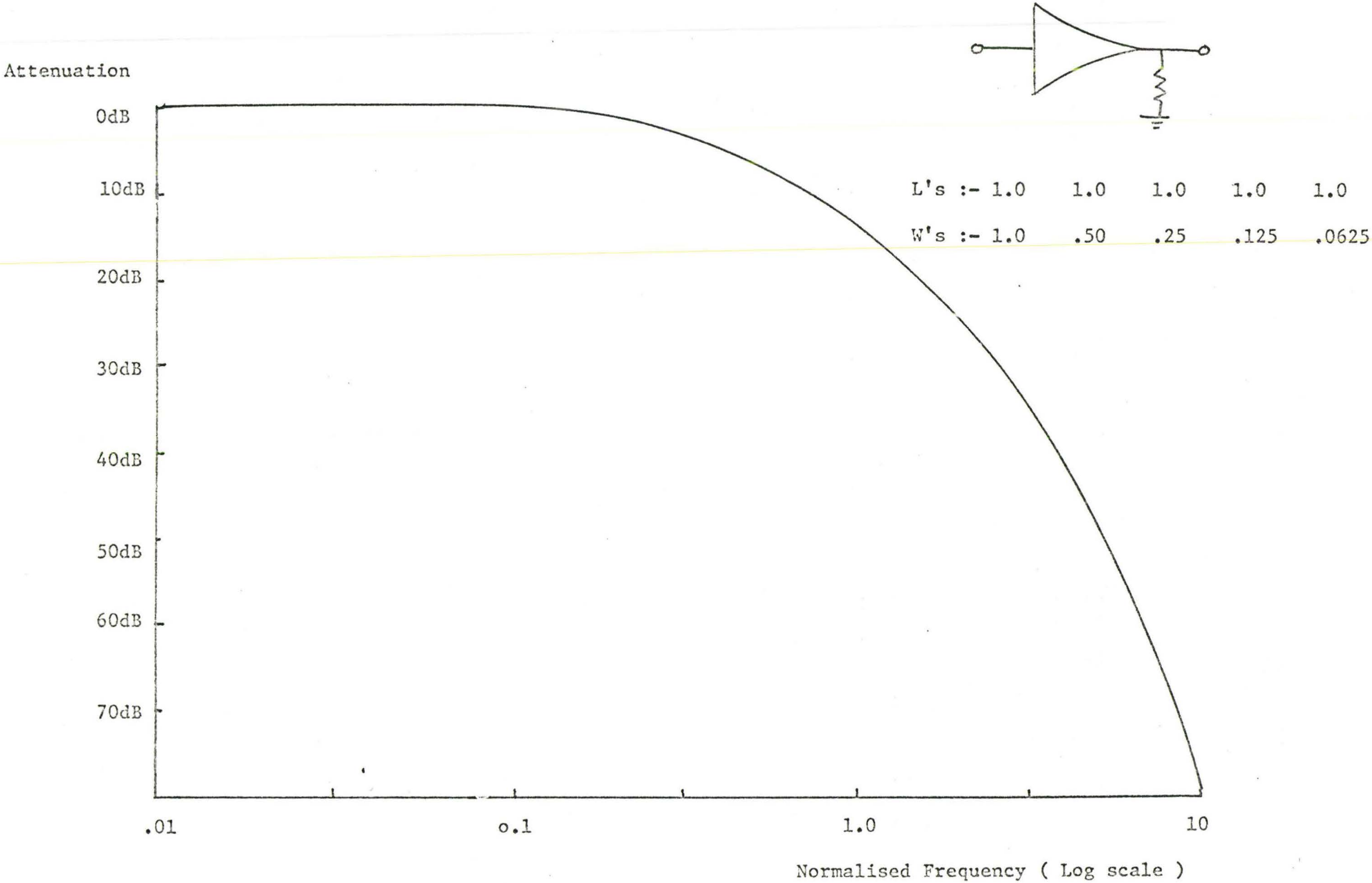


Figure 4.9 RESPONSE OF TAPERED $\overline{RC}$

ladder networks.

It is interesting that figure 4.7 provides an almost linear fall off of amplitude of 40 dB over two decades. This could probably be improved using optimisation but no application for it is known at present.

From the low pass filter viewpoint , figure 4.9 represents the best structure found, showing almost 50 dB of attenuation in the first decade. The response is similar to that of a positive exponentially tapered filter and is in accord with the physical understanding of taper effects developed in section 2.4. Whether the exponential taper is optimum in some respect is a question that might receive attention in the future. It does not appear to be critical provided there is a sharp decrease in width or increase in impedance.

4.6 Comments

This Chapter has been concerned with analysis, which represents the simplest and most basic technique of computer-aided design. The results obtained are useful in three respects :

(a) An answer has been provided to Q1 posed at the start of this chapter. $\overline{RC}$ filters of the topology considered provide low pass transfer characteristics broadly similar to those obtainable with series R, shunt C lumped ladder networks. The main difference is that the rate of attenuation always increases with frequency, due to the infinity of poles. Large variations in width merely provide approximations to lumped elements. Degree of steady taper increase or decrease appears the most important design parameter.

Thus there appear to be no radical new circuit properties to exploit. Advantages lie in terms of size and reduced number of interconnections in realization.

(b) The results reinforce the physical understanding of $\overline{RC}$ structures built up in section 2.4. This is a necessary base for the intuition required in original design work.

(c) The results go part of the way towards answering the second question in indicating the type of taper producing the sharpest cut off. This is very useful in the choice of an initial design for use in an optimisation program.

Such an optimisation strategy forms the next stage of com complexity for computer-aided design and will now be considered.

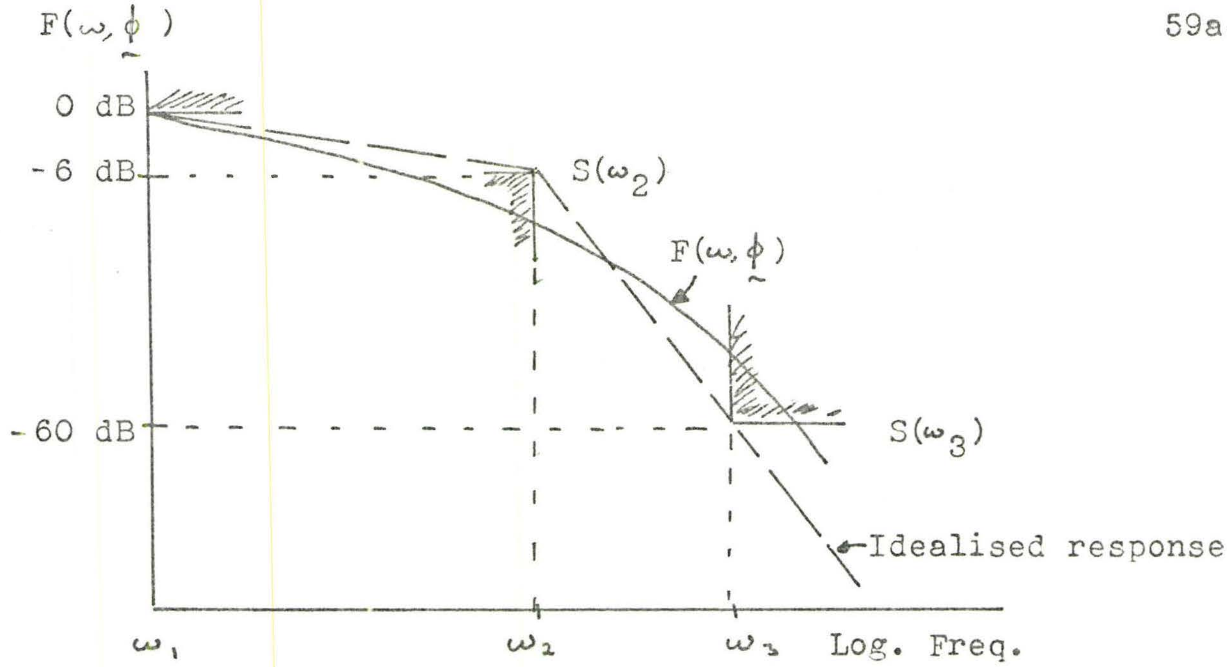
4.7 Extensions Using Optimisation Techniques

To use an optimisation algorithm it is necessary to have an objective function that embodies some desirable characteristic of the circuit performance. A suitable one for an $\overline{RC}$ low pass filter is suggested in Figure4.10. It is then necessary to choose some initial design. This may be a URC, or the analysis results discussed above may be utilised. This design must be specified by a number of independent parameters. An optimisation algorithm may then step these parameters in turn away from the nominal values and test for an improvement in the objective function. This is repeated until some local minimum is reached.

Many techniques are available for carrying out this process[34]. The most powerful utilise information about the gradient of the objective function with regard to the various parameters. This may be obtained using the Adjoint Network method of Director and Rohrer[35, 36].

The analysis program previously described is adapted for easy extension to include calculation of these gradients. An optimisation was not attempted for this thesis since primary interest was in active RC filters rather than on the consideration of the optimum taper for lowpass or notch filters. The results given in Chapters 6 and 7 for active $\overline{RC}$ filters may readily be extended to encompass the use of an $\overline{RC}$ with any taper.

In practice an $\overline{RC}$ structure would probably form part of some broader circuit such as an active filter. Therefore it would make sense to apply analysis and optimisation techniques to the circuit as a whole. It would also be possible to produce manufacturing masks directly from the computer for automated production in some cases.



Let $F(\omega, \underset{\sim}{\phi})$ = Response Function , for $\underset{\sim}{\phi}$ = the independent variables.

$$= 20 \log_{10}\left(V_{out} / V_{in} \right)$$

Define the specified function at ω_2 and ω_3 with respect to the value of F at the low frequency ω_1 in order to take care of any DC attenuation :-

$$S(\omega_2) = F(\omega_1, \underset{\sim}{\phi}) - 6 \text{ dB}$$

$$S(\omega_3) = F(\omega_1, \underset{\sim}{\phi}) - 60 \text{ dB}$$

Then <u>Objective Function</u>

$$U = \left| F(\omega_2, \underset{\sim}{\phi}) - S(\omega_2) \right| + \left| F(\omega_3, \underset{\sim}{\phi}) - S(\omega_3) \right|$$

Figure 4.10 SUGGESTED OBJECTIVE FUNCTION FOR LOWPASS $\overline{RC}$ FILTER

CHAPTER 5

Active Filters

5.1 Introduction

Filters may normally be defined as linear signal processing units. Their behaviour may be specified in the form of a transfer function, chosen to yield some desired characteristic in the frequency or time domain. Chapter 3 considered distributed circuit transfer function synthesis from a device point of view. This Chapter mentions some of the system approaches to filtering of current engineering practice.

In the days of vacuum tubes, most filters were passive RLC configurations, possibly in the form of loosely-coupled " resonant circuits ". However the growth of transistors and integrated circuits changed the relative costs of active elements, resistors, capacitors and inductors. These costs must include considerations of volume (or area), stability and tolerances as well as manufacturing costs. It is found extremely inconvenient, even if possible, to include a lumped inductor as part of an integrated circuit. Even on a printed circuit board it is often a nuisance. A gain element such as some form of transistor is frequently preferable to a high-value resistor. These and similar considerations have led to the developement of alternative implementations for filters.

5.2 Active Analog Filters

These use gain in conjunction with R and C elements, usually lumped, to synthesise all the common filter types (BP, HP etc.). Normally a rational transfer function must be broken down to biquadratic (bi-quad) expressions that may be individually realised and then cascaded. Designs are now commonly available in which pole and zero frequencies and the circuit " Q " may be tuned by adjusting resistor values. Two or three capacitors are needed. One to three operational amplifiers may be used, it being found that three amplifiers can give negligible sensitivity to gain variations. Such filters may be built for $5 - $10 (see Ref. 39).There is an extensive literature on their design and use [37, 40].

Other approaches that have been investigated are the use of Negative Impedance Convertors and Gyrators [37]. These are not used extensively in practical equipment at the moment though recent work by Leach and Chan [51] may alter the position.

5.3 Digital Filters

Digital filters deal normally with digital representations of a signal at discrete time intervals. The basic building blocks are shift registers, multipliers and adders [38]. Precision, noise and also cost depend to a great extent on the number of bits used to represent the signal. Performance is normally invariant with respect to temperature and aging provided there is no hardware malfunction In some systems A/D and D/A convertors are necessary. Alternatively a large part of the system may be digital as in many signal processing, switching and time-shared systems.

5.4 Design Evaluation Criteria

There are many factors to be considered when choosing between the implementation possibilities mentioned above or when evaluating some other technique.

The frequency range of operation must be considered. Current operational amplifier filters extend from a fraction of 1Hz to perhaps 50 kHz, at which point amplifier phase and frequency roll-offs become important, although the recent develpoment of an operational amplifier with a 50 MHz unity gain bandwidth [41] promises to raise this considerably. Possible applications of $\overline{RC}$'s in the range up to ~ 10 MHz are considered in Chapters 6 and 7.

Other performance criteria are absolute and relative stabilities, tolerances and sensitivities. These are functions of both circuit design and manufacturing costs and so depend on the implementation technique chosen. The chief independent parameters are gain and RC product. This is an area which has in the past often been neglected by circuit theoreticians.

It is also necessary to consider the filter in relation to the total design philosophy of the system of which it is to form part. Partitioning and interfacing decisions must be made on that level and may well determine the type of filter to be used in practice.

CHAPTER 6

Experiments with L.P. $\overline{RC}$ Active Filters

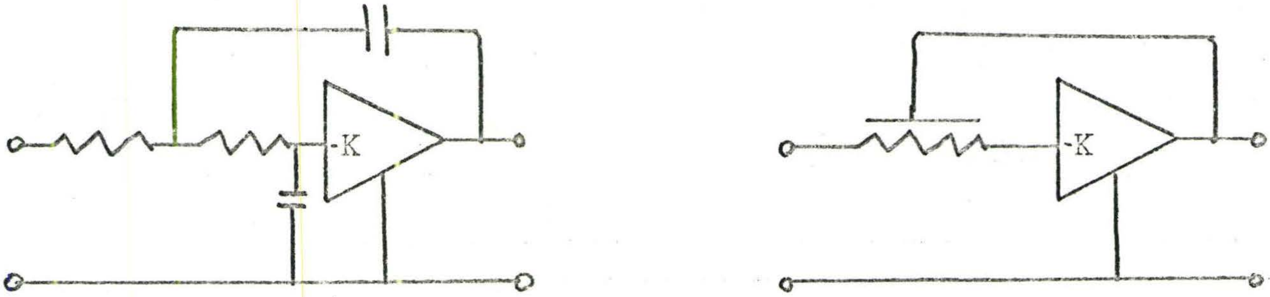
We are now in a position to investigate the application of distributed circuits, characterised by one of the methods of Chapter 3 or Chapter 4, to practical filters. This Chapter will be chiefly concerned with achieving the sharpest possible cut-off for a low pass $\overline{RC}$ active filter. Chapter 7 will be concerned with applications of notch filters.

6.I Second Order Simulations

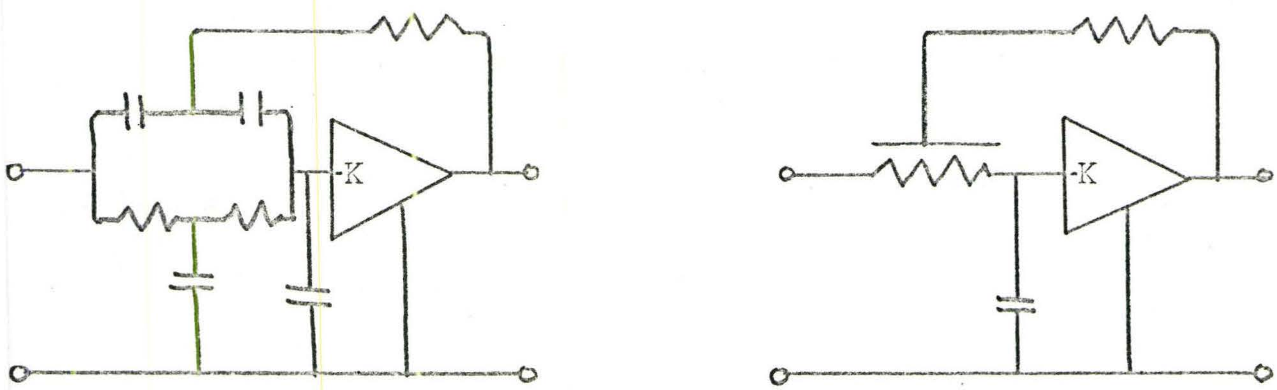
One approach, taken by Huelsman et al,[28] is to start with a lumped RC second order building block such as those shown in Figure 6.I. They have shown using computer analysis techniques that the distributed equivalents shown are a close approximation to the second order filters as expected.

The characteristics have cut off slopes of about 40 dB per decade. Higher orders are possible using cascaded sections. The circuit of figure 6.I(a) has K always less than 1. Similar circuits have been investigated by other workers.[29,37] It has been suggested that taper variations can be used to reduce the sensitivity.[30] Extensions to high pass are possible.[3I]

Advantages claimed over the lumped version are a reduced



(a) Low Pass Filters



(b) Band Pass Filters

Figure 6.I COMPARISON OF 2ndORDER LUMPED AND DISTRIBUTED / LUMPED ACTIVE FILTER CIRCUITS

complexity of physical realization and in some cases a lower sensitivity. Disadvantages are the necessity for computer analysis in any exact work and the paramount importance of controlling the RC ratio in production.

The method in effect is based on a dominant pole approximation, which was one of the simplest methods considered in Chapter 3. However it is sufficient to provide a good starting point for a computer optimisation algorithm. In this context the rational-parameter configurations and transformations are either restrictive or unwieldy. Extensions of those methods using NIC's or gyrators are in the same category.

6.2 Experimental Circuit

The circuits considered in the previous section provided second order cut-offs. At higher frequencies the slope of a $\overline{\text{URC}}$ approximates 50 dB/decade. The disadvantage is the soft fall off at lower frequencies. The idea was conceived that negative feedback could be used to reduce and straighten out the response below a certain frequency giving a sharper cut-off. This is shown in Figure 6.2(a). Circuit arrangements for trying this are shown in Fig. 6.2(b) and (c). The amount of negative feedback that can be applied depends on the phase characteristics of the $\overline{\text{URC}}$.

This Chapter describes the equipment used, the tests carried out and conclusions reached with respect to this circuit arrangement.

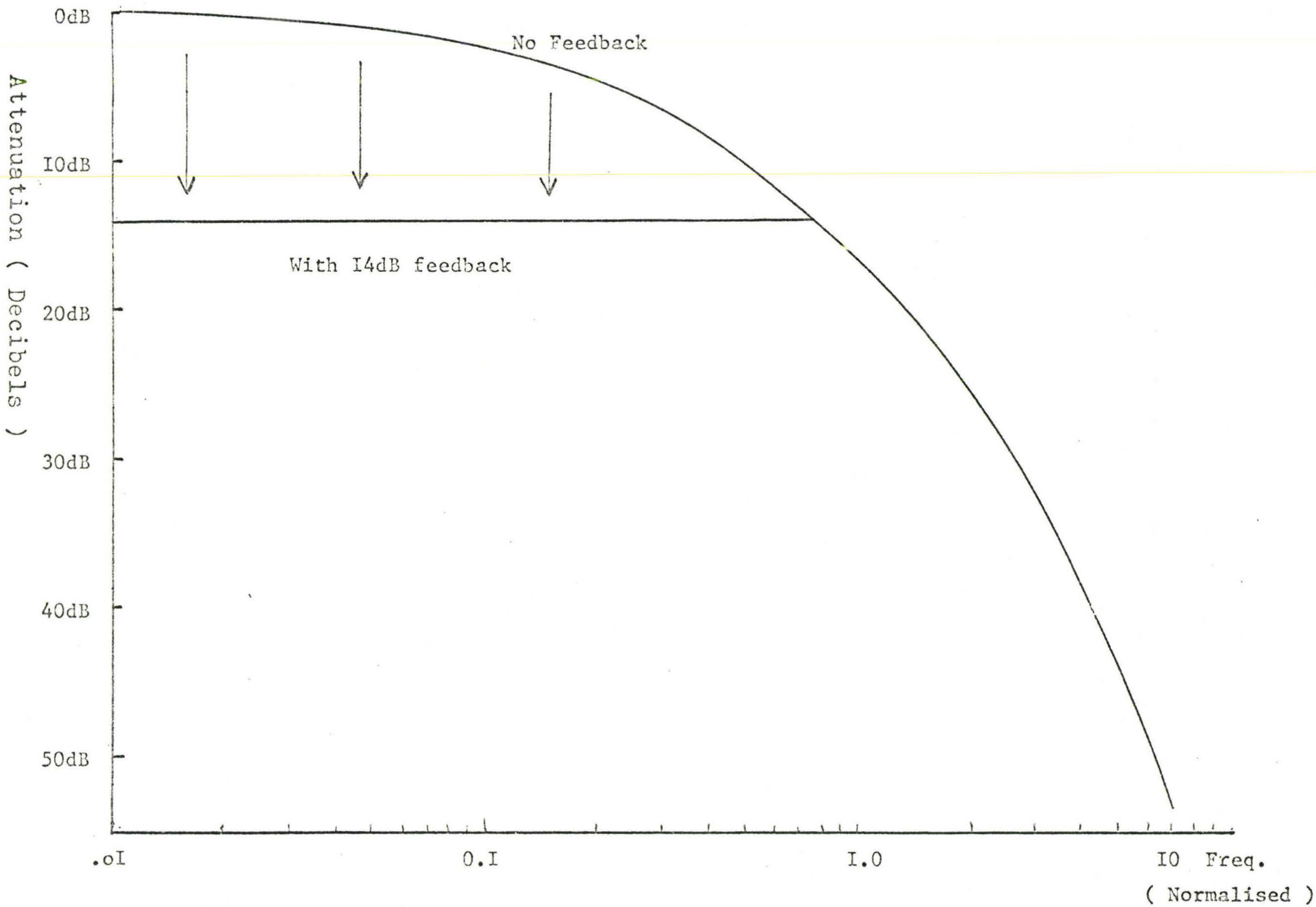


Figure 6.2(a) APPROXIMATION FOR THE EFFECT OF I4dB NEGATIVE FEEDBACK ON URC FREQUENCY RESPONSE

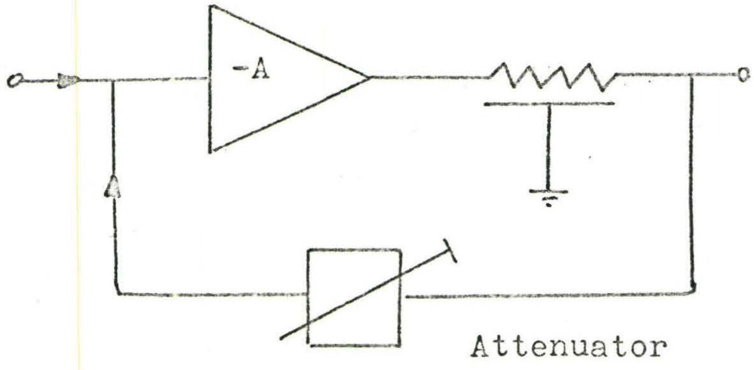


Figure 6.2(b) BASIC EXPERIMENTAL L.P. FILTER CIRCUIT

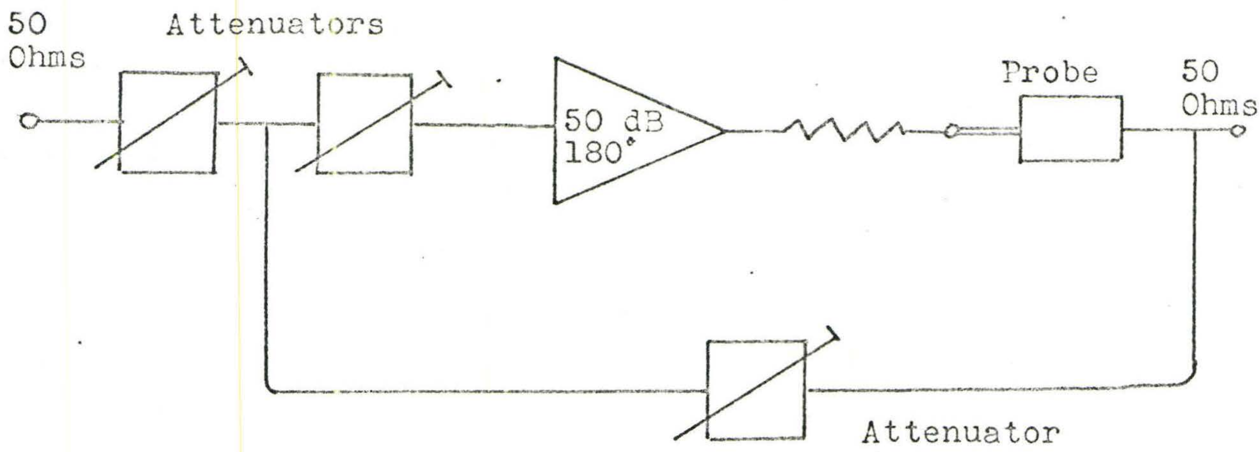


Figure 6.2(c) PRACTICAL EXPERIMENTAL L.P. CIRCUIT

6.3 Teledeltos Filters

Distributed RC circuits may be constructed in a number of ways. Commercially the main interest would be in thin and thick films. However in the laboratory it is possible to build a perfectly good model using teledeltos resistance paper with Mylar sheet for the dielectric.

Several experimental jigs for filters were built. It is necessary to provide a firm and uniform pressure pad to keep the teledeltos paper, Mylar film and ground plane in intimate contact. Any inductance in the ground circuit connections causes the LP response of a straight $\overline{URC}$ to rise at high frequency[32] The ratio of this break point to the " cut-off " frequency may be taken as an indication of the quality of a particular filter.

Various filters were built with cut off frequencies in the range 400kHz to 5 MHz. This was convenient both for construction and for display on a Hewlett Packard model 675A / 676A Network Analyzer that was available in the laboratory, with a sweep range of 10 kHz to 30 MHz. The $\overline{URC}$ that was used for the experiments described below had the following characteristics,

Dissipation Factor $D = \cdot 00239$, measured at 100Hz

Total Capacitance = 1,035 pF

Resistance 1,736 ohms

Dimensions 6 cms square

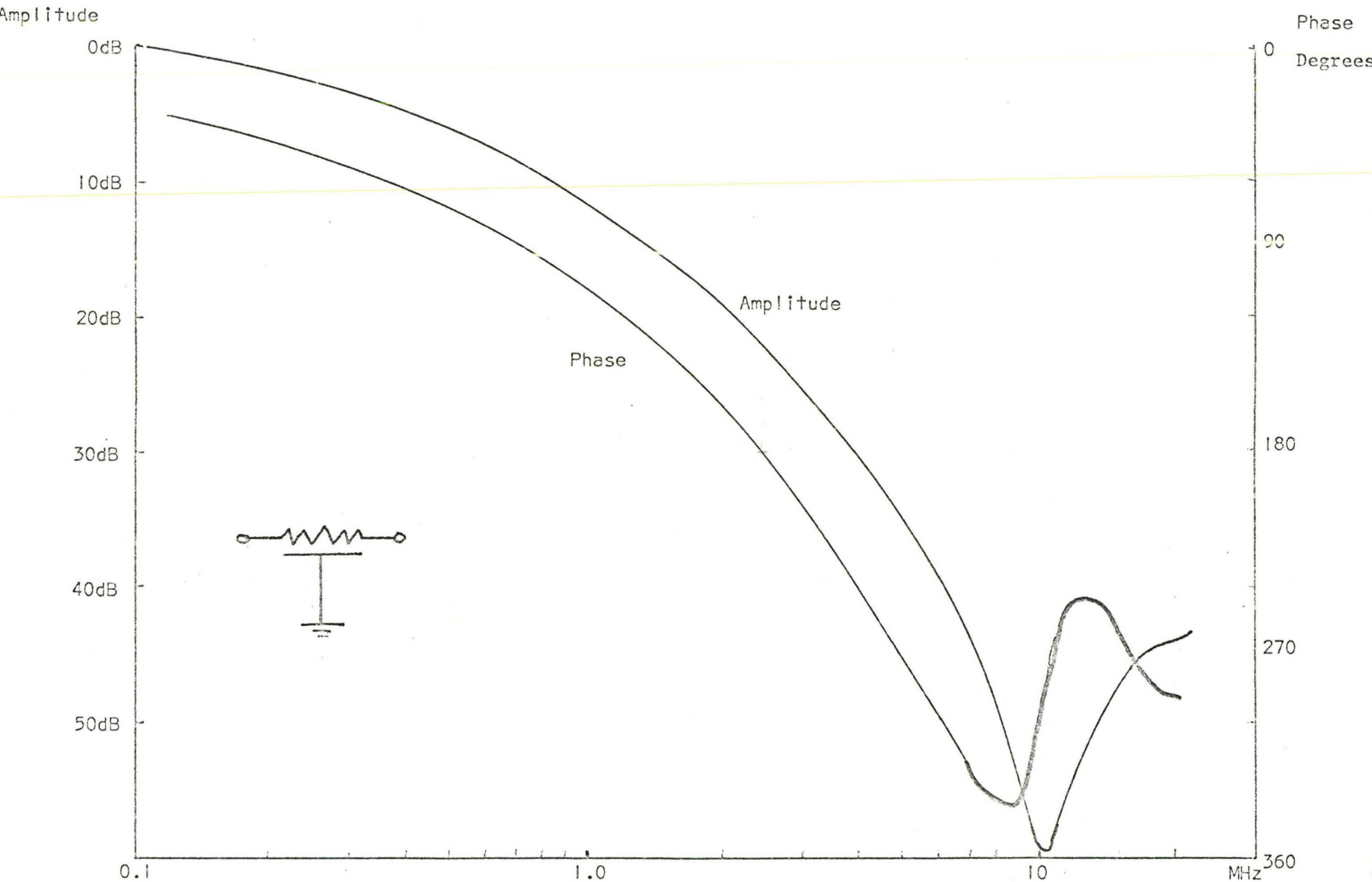


Figure 6.3 AMPLITUDE AND PHASE CHARACTERISTICS OF URC LOW PASS FILTER USING TELEDELTOS PAPER

(Open Circuit Voltage Transfer Function)

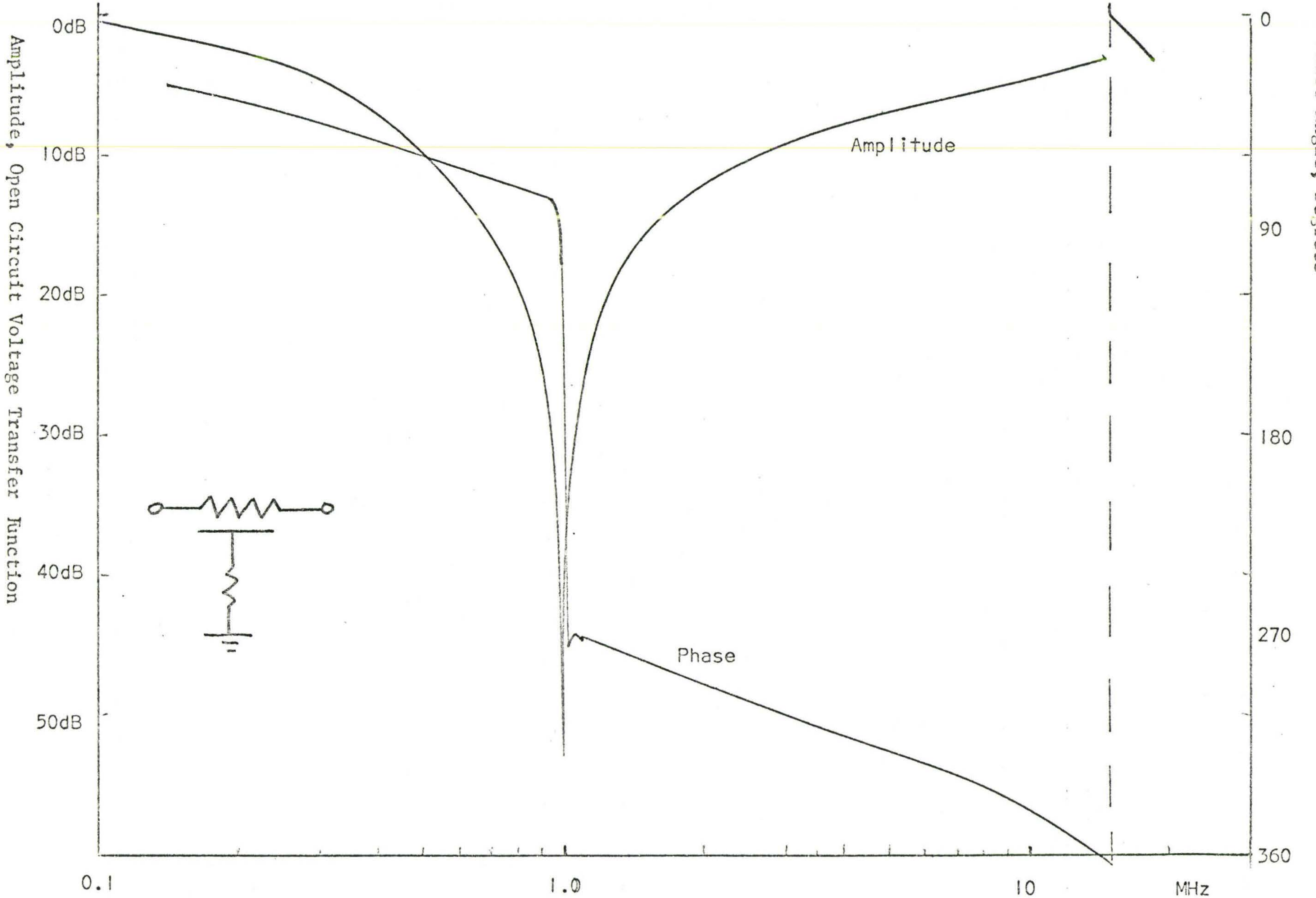


Figure 6.4 Amplitude and phase characteristics of teledeltos $\overline{URC}$ Notch Filter(" Optimum Notch")

Figures 6.3 and 6.4 show measured frequency characteristics for this URC in both straight LP and notch filter modes of operation. The notch resistor was made up from a lumped combination since no trimmers were available with sufficiently low inductance. It will be seen that parasitic effects are not significant for $f < 10MHz$. Below this frequency there is close agreement with the theoretical responses discussed in chapter 2.

The diagrams were traced from an X-Y recorder plot, driven by the network analyzer in a slow scan mode. Figure 6.5 gives a block diagram of the equipment used.

6.4 Experimental Amplifiers

For active filter experiments it is necessary to have available one or more amplifiers. The required characteristics are high gain, low output impedance and wide, flat phase and amplitude responses. Since such amplifiers were not available various standard integrated circuits were investigated and it was found that quite cheap units could be built using the Motorola #MC1590. Figure 6.6 shows the final arrangement. Construction was on a specially prepared printed circuit board. Two amplifiers were built, one with one stage and one with two stages cascaded. Emitter follower output stages were necessary to prevent slew-rate limiting at large signal levels when feeding capacitive loads

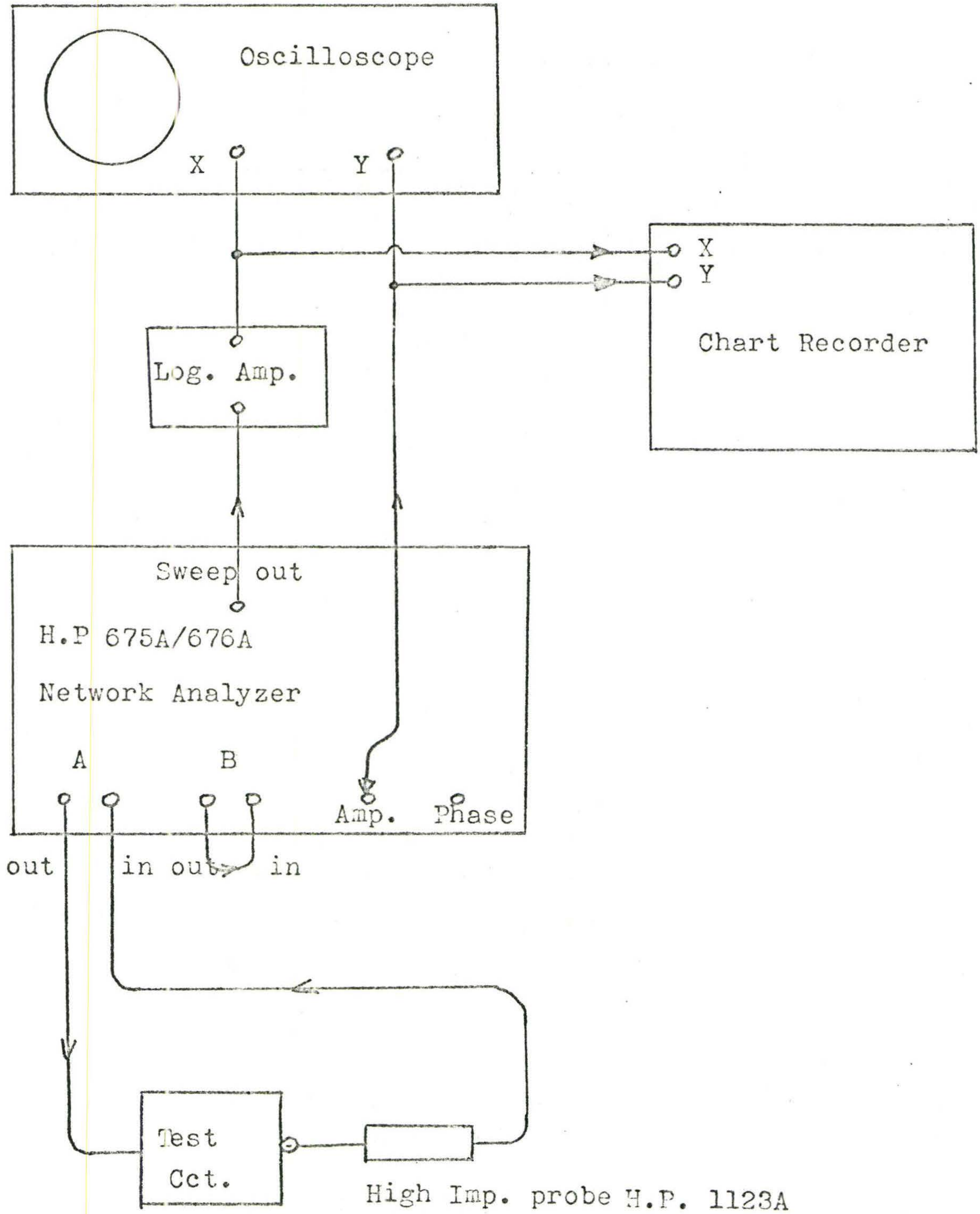


Figure 6.5 EQUIPMENT FOR PLOTTING PHASE AND AMPLITUDE OF VOLTAGE TRANSFER FUNCTIONS OVER .01 - 30 MHz

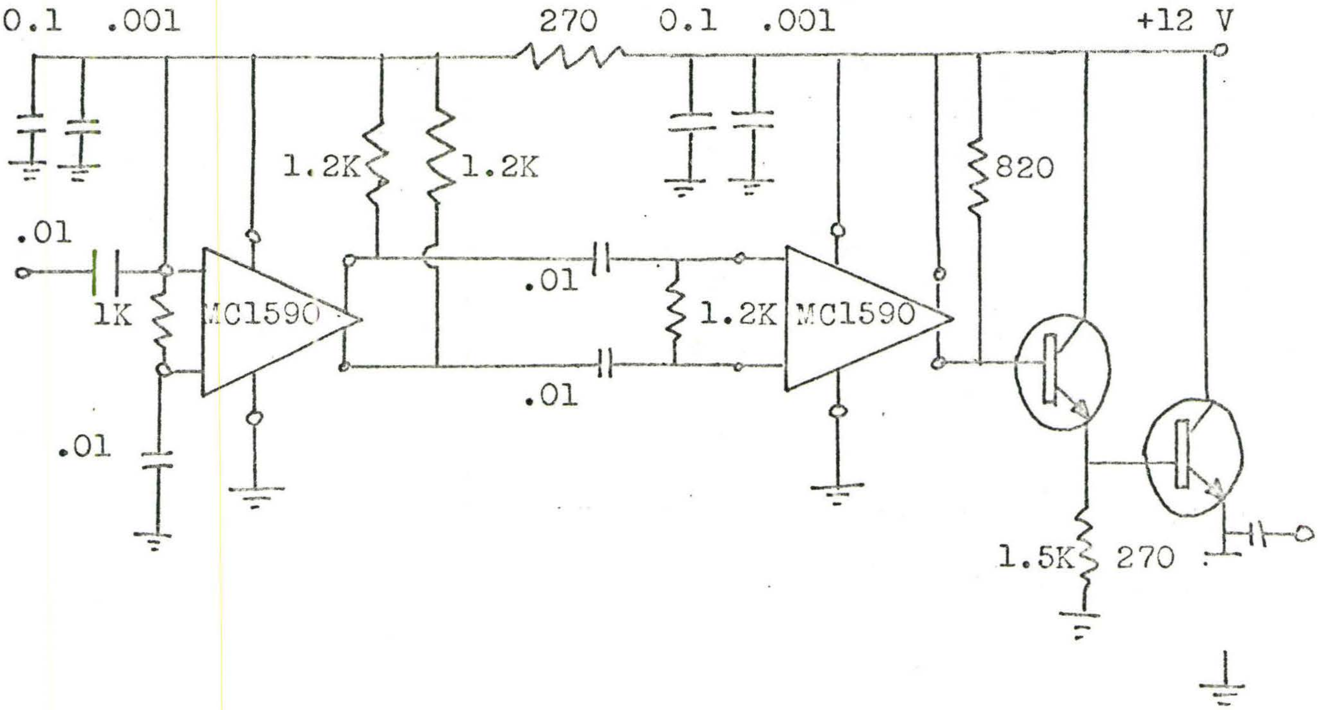


Figure 6.6(a) Integrated Circuit Amplifier Schematic

Overall LF phase shift	0 or 180°	180°
Forward Gain	30 dB	50 dB
Upper 3 dB point	14 MHz	12 MHz
Max. o/p swing	3 volts pk-pk	3 volts pk-pk
Dynamic range	60 dB	40 dB
o/p Impedance	33 ohms	33 ohms

Figure 6.6(b) Integrated Circuit Amplifier Performance

such as a short length of coaxial cable.

Figure 6.6 presents the measured performance characteristics in tabular form while Figure 6.7 shows the plotted phase and frequency responses.

6.5 Open Loop Characteristics

A cascade connection of amplifier and $\overline{URC}$ gave a combined characteristic , shown in Figure 6.8, of the form expected from Figs. 6.3 and 6.7. In plotting the phase the zero reference was established by momentarily connecting the probe to the input and adjusting the network analyzer controls for zero phase shift at low frequency. This was not a very exact process so the phase may have an absolute error of up to 30 degrees.

The forward signal path is virtually the same as the open loop for frequency response since the only difference is an attenuator, flat up to IIO MHz. Considering the curves in the light of Bode analysis it is apparent that the circuit will tend to oscillate at just over I MHz if the loop gain is greater than 21 dB.

6.6 Closed Loop Characteristics

The loop was closed and responses plotted with attenuator settings giving various low frequency gains. As predicted

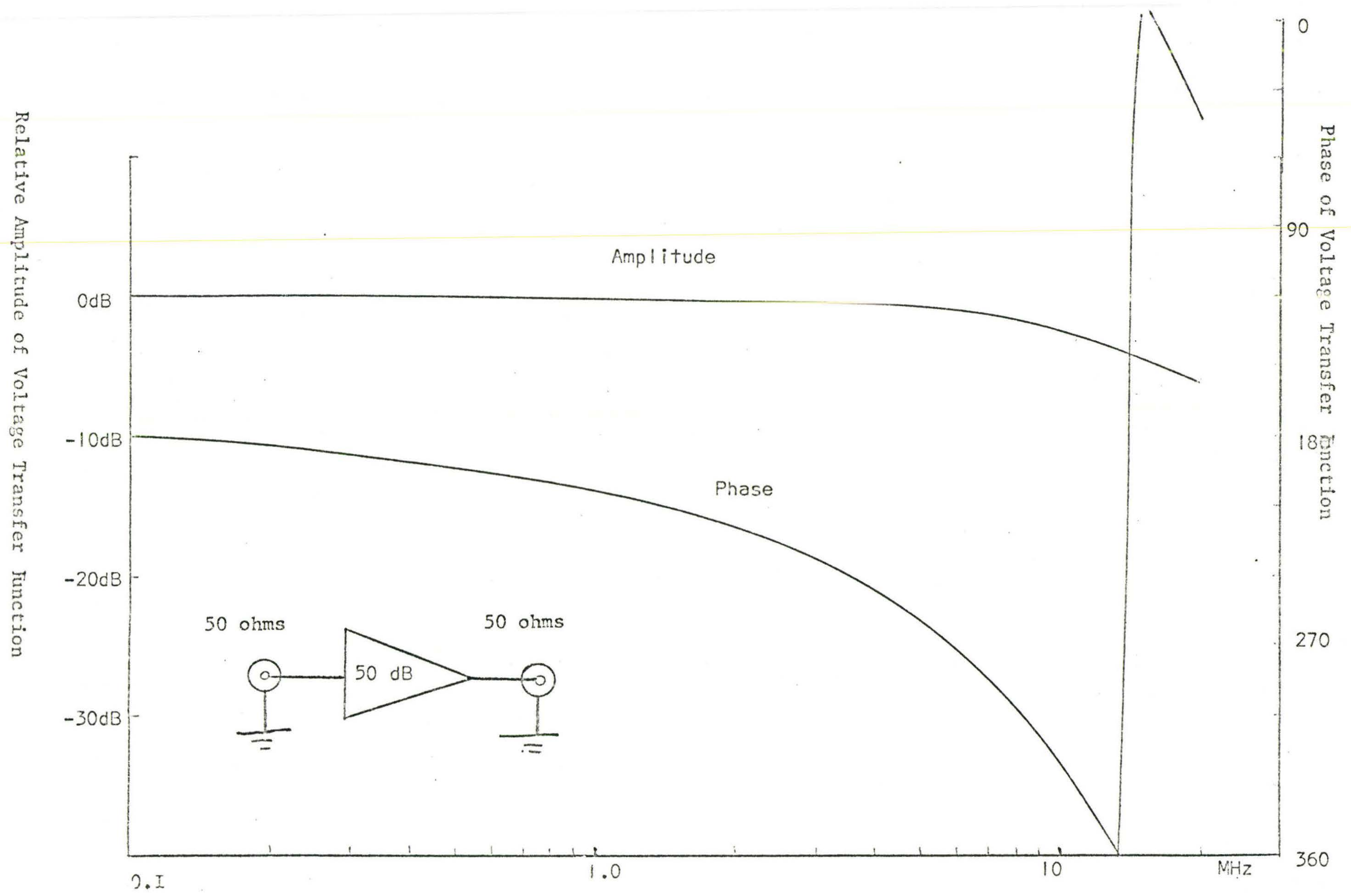


Figure 6.7 PLOTTED PHASE AND FREQUENCY RESPONSES OF 50dB EXPERIMENTAL AMPLIFIER

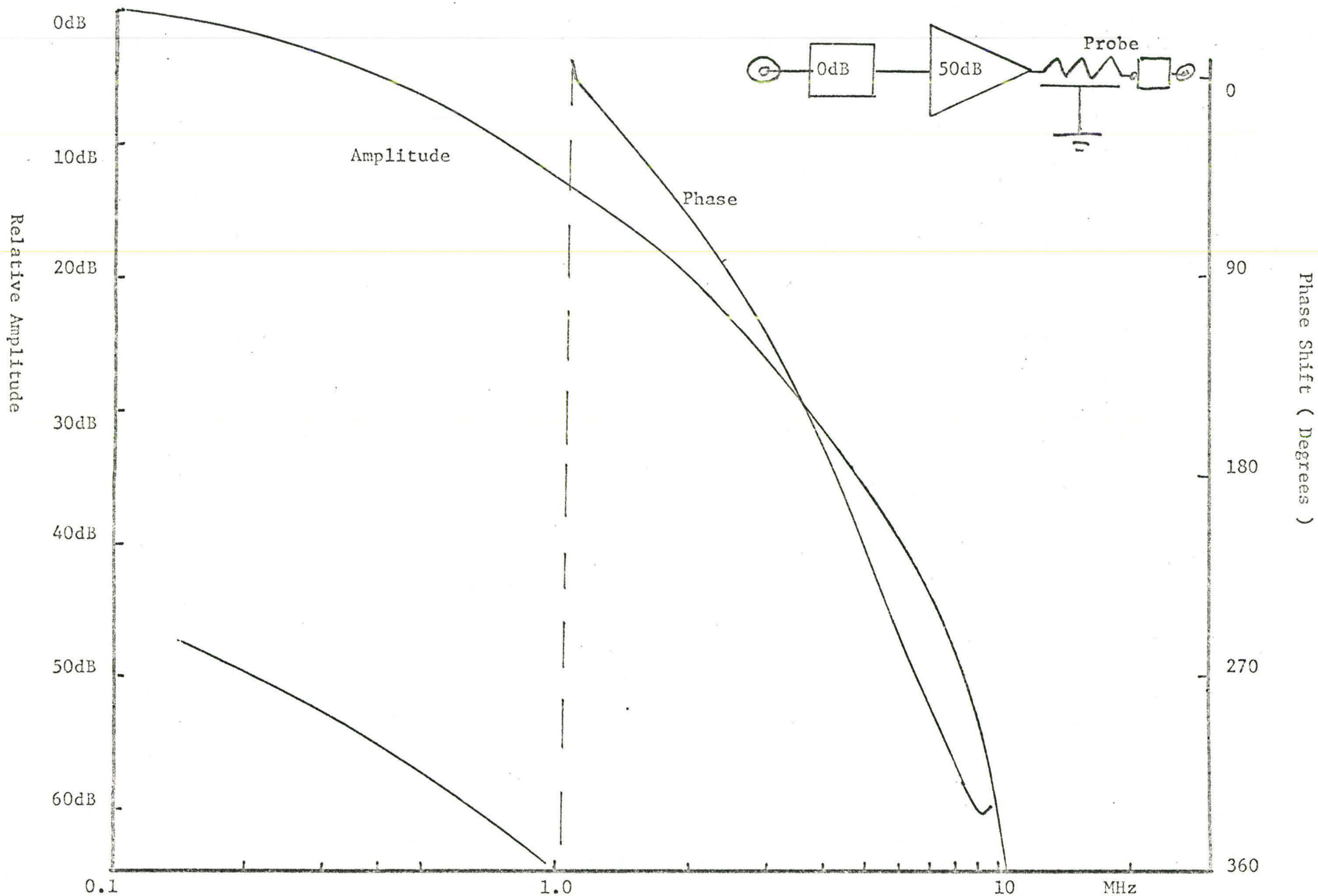


Figure 6.8 PHASE AND AMPLITUDE RESPONSES FOR FORWARD SIGNAL PATH (Open Circuit Voltage Transfer Function)

oscillation tends to occur in the region just above IMHz if the loop gain (= 50dB - K_2) exceeds I5dB. The curves bear a similarity to the standard second order system characterised by

$$\frac{V_{out}}{V_{in}} = \frac{I}{I + j\zeta 2\omega_n - \omega_n^2} \quad ; \quad \omega_n = \text{normalised frequency}, \quad \zeta = \text{damping factor}$$

as might be expected.

Figure 6.10 shows the optimum curve from the family of Figure 6.9 compared on the basis of normalised 3dB frequency with a standard $\overline{URC}$ low pass. It will be seen that there has been a significant improvement. Attenuation is now almost 60dB in the first decade.

The curve shown is with K_2 = 40dB, ie with only I0dB of negative feedback. Unfortunately the phase shift inherent in $\overline{RC}$ circuits prevents the application of more feedback with a URC. However positively exponentialy tapered filters have both a sharper cut off and less phase shift in the critical region allowing the application of more feedback. A physical explanation of these taper effects was given in section 2.3 and a full analysis may be found in Carson [I9]. Practical work with exponential filters was not undertaken since it would have involved a curvilinear square program for filter construction which would have been outside the scope of this thesis.

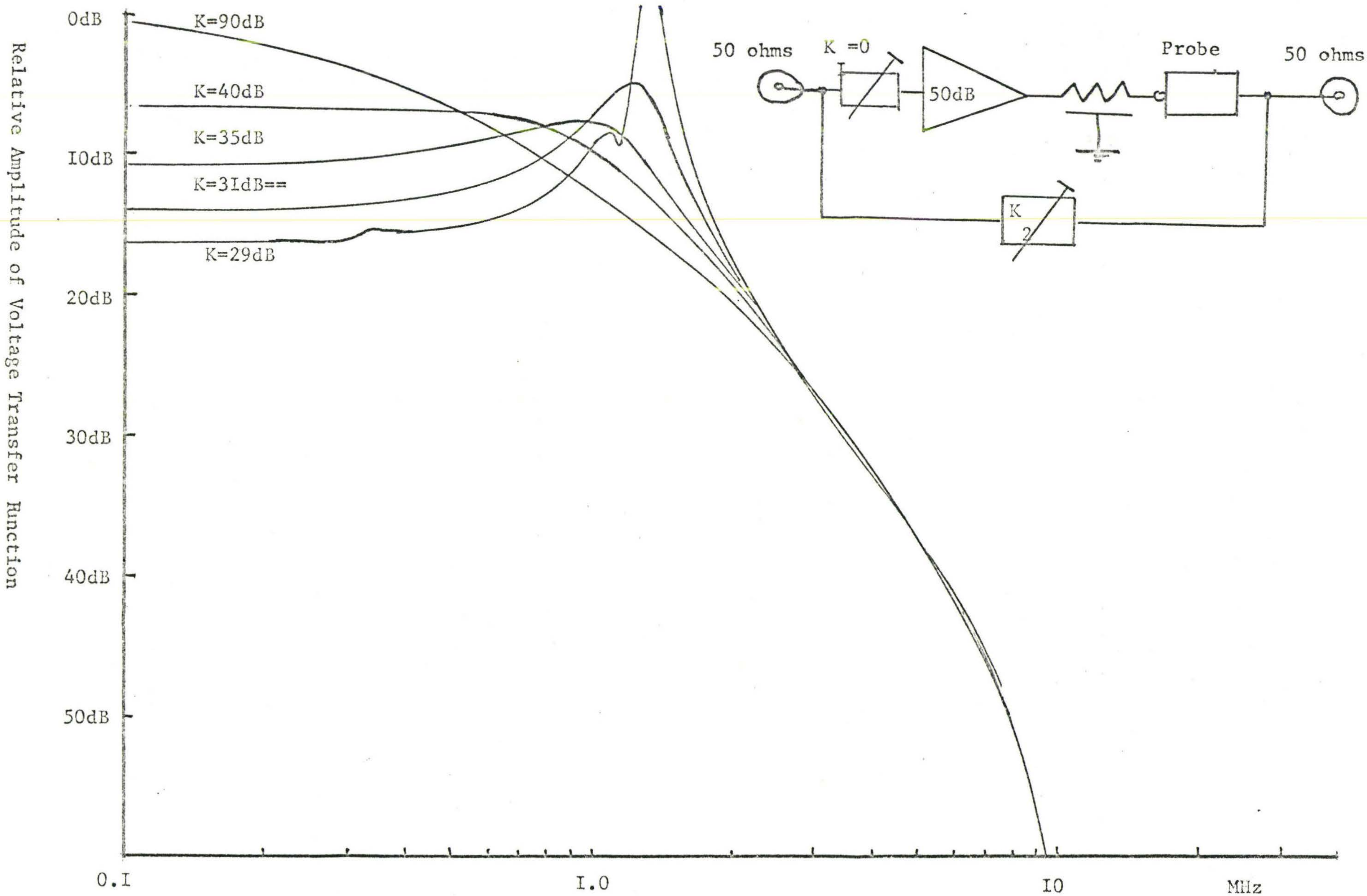


Figure 6.9 CLOSED LOOP CHARACTERISTICS (AMPLITUDE RESPONSE)

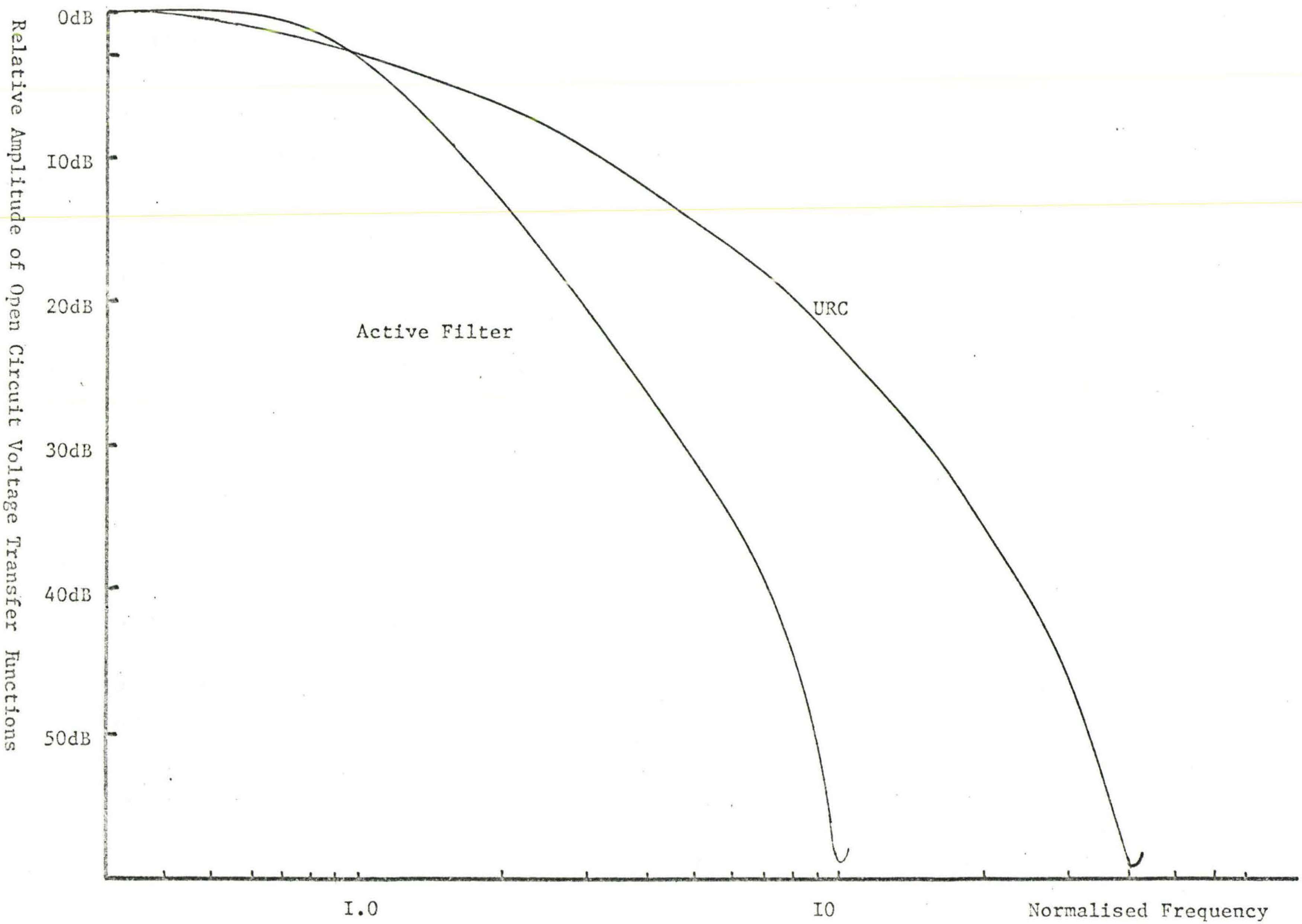


Figure 6.10 COMPARISON OF OPTIMUM L.P. CURVE AND A SIMPLE URC WITH THE FREQUENCY AXIS NORMALISED FOR COINCIDENT 3dB POINTS

6.7 Comparisons and Conclusions

The circuit arrangement proposed in this Chapter provides a better low pass characteristic than the others considered. The use of tapered $\overline{RC}$'s should make it even better. However the response is quite sensitive to amplifier gain and the circuit can become unstable.

The second order simulations (Sect. 6.1) generally have gains less than one and are therefore unconditionally stable but they are still sensitive to gain. The Bi-quad op-amp realizations (Sect.5.2) are practically invariant with respect to gain but they require more hardware.

One major point in favour of the feedback circuit is its potentially higher frequency range. The amplifier need only have a gain of 15 - 20 dB and phase shift reasonably constant to about five times cutoff frequency. These requirements are met by commonly available video I/C's. However common Op. Amps. as used in the Bi-quad circuits with very high forward gain have cutoffs restricted to around 1MHz.

CHAPTER 7

Experiments with Band Pass Notch Filters

7.1 Introduction

This Chapter is concerned with B.P. filters. While it has always been possible to obtain good L.P. filters with only resistive and capacitive elements, B.P. filters have traditionally required the use of inductors. This is not desirable with modern implementation techniques.

There are several alternative approaches. The system may be redesigned on a digital basis obviating the need for a band pass filter as such. A digital filter technique may be used, or an op-amp biquad realization of the required transfer function.

One class of methods sometimes considered is oscillatory circuits on the verge of oscillation. Such circuits exhibit a " B.P." response but sensitivity is very high for high Q and, of course, the circuit is liable to oscillate. An example, using the phase shift of a standard $\overline{URC}$, is the response of Figure 7.1. This was obtained by suitable adjustment of the apparatus used for the experiments of chapter 6. Such designs do not generally represent good engineering practice.

There exists an alternative that has been often mentioned but not developed to any extent. An $\overline{RC}$ together with a " notch resistor " can be used to give a transmission null (see section 1.2 and Figs. 1.4, 1.5). This notch circuit may then be placed in

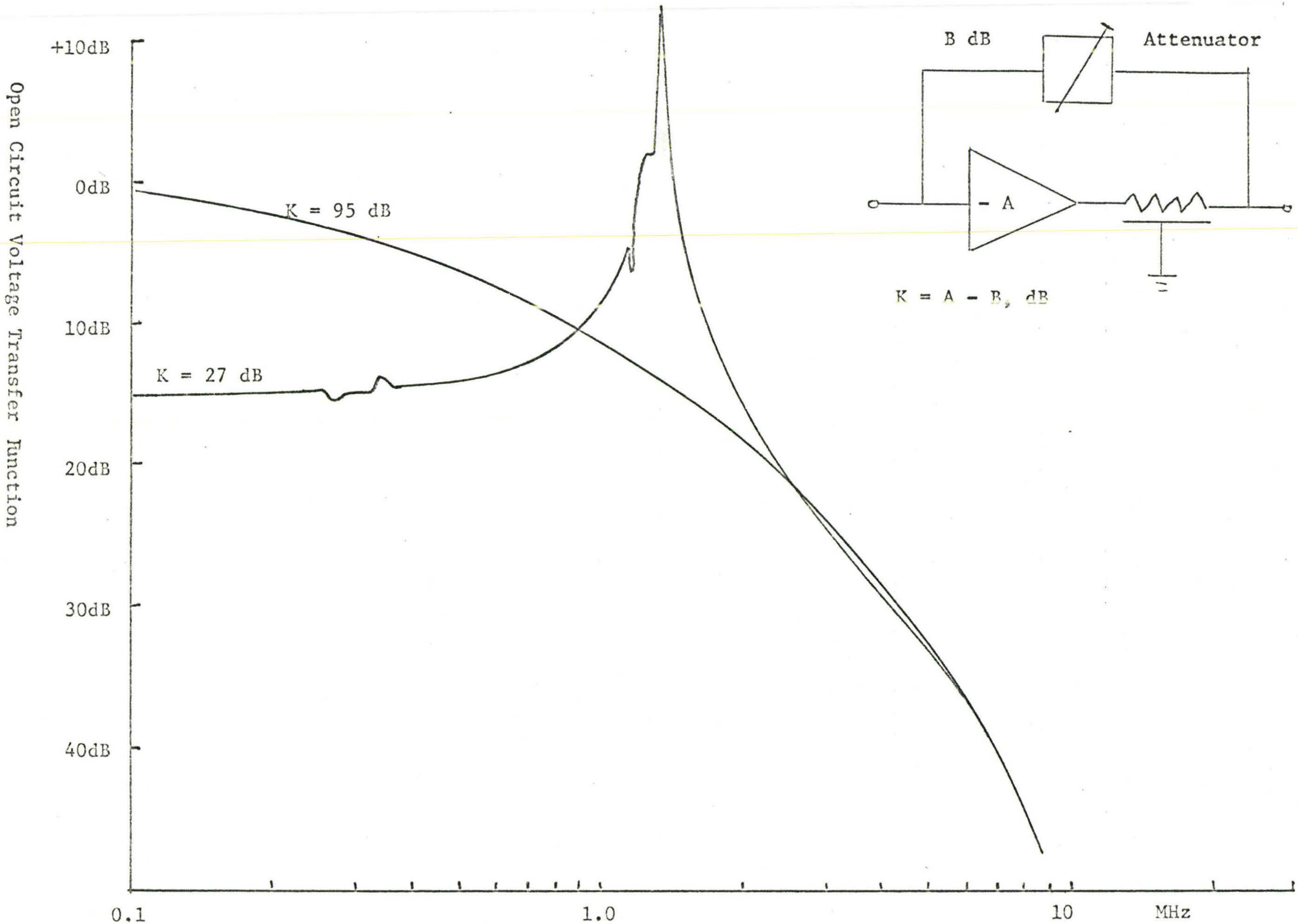


Figure 7·1 " BAND PASS " FILTER FORMED FROM A URC FEEDBACK CIRCUIT ON THE VERGE OF OSCILLATION the feedback path of an amplifier to invert the amplitude response, giving a Band Pass filter. This idea has been around for a long time but no satisfactory experimental results have come to light. Normally the importance of phase shift as well as amplitude response is not fully appreciated and the resulting circuit behaves as that of Figure 7.1. Notch resistor and centre frequency values will not correspond with the " optimum notch " case and the sensitivities will be high.

This Chapter describes the developement of a Band Pass filter for which the characteristics are a true reflection of the $\overline{RC}$ notch. Such a circuit has advantages in terms of sensitivity and stability.

7.2 Stability Problem

The basic circuit of a Band Pass amplifier using an $\overline{RC}$ notch filter is shown in Figure 7.2. For stability the loop phase shift must be 180 degrees at low frequency, achieved by the use of an inverting amplifier. This means that in the notch region the feedback is positive in phase but reduced in amplitude by the notch. Slightly off the notch frequency the amplitude increases rapidly but the phase still has a large component of positive feedback. The process is not just a simple one of inverting the amplitude characteristic.

Stability analysis is difficult since it is not sufficient to consider the dominant poles only. The other poles have an a appreciable effect on the crucial phase shift. Bode plots may easily be obtained but are not very helpful in the notch region. A Nyquist diagram has the form shown in Figure 7.3, which suggests stability. An accurate version could be plotted using computer analysis and suitable coordinate transforms. Computer analysis of the circuit over a range of frequencies does not show any effects that would be indicative of instability. In view of these facts the circuit was concluded to be prima facia stable.

7.3 The Non-Ideal Case

So far a " perfect " amplifier with very high forward gain, high input and low output impedances has been considered. E. Johnston has written a computer program to analyse the network, both with an ideal amplifier and also with various amplifier imperfections modelled. He has found that a forward gain of 30 - 40 dB is sufficient to give a reasonably sharp characteristic.(see Fig 7.4) Input and output impedances of the order found for the practical amplifiers described in section 6.4 are not significant.

Experiments were conducted, both in conjunction with E. Johnston and seperately, with a wide variety of amplifiers in different arrangements with appropriate attenuators and teledeltos notch filters. Initially no combination could be found to give a proper B.P. characteristic; that is one whose response was a

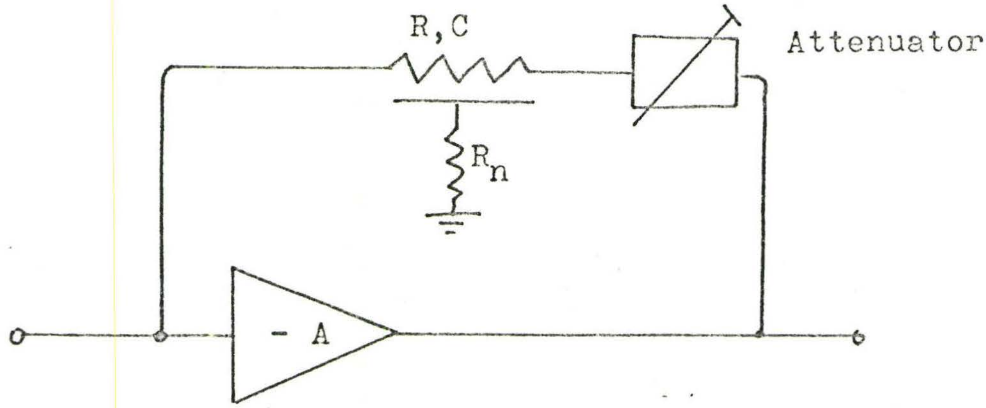


Figure 7.2 BASIC BANDPASS AMPLIFIER USING $\overline{RC}$ NOTCH CIRCUIT

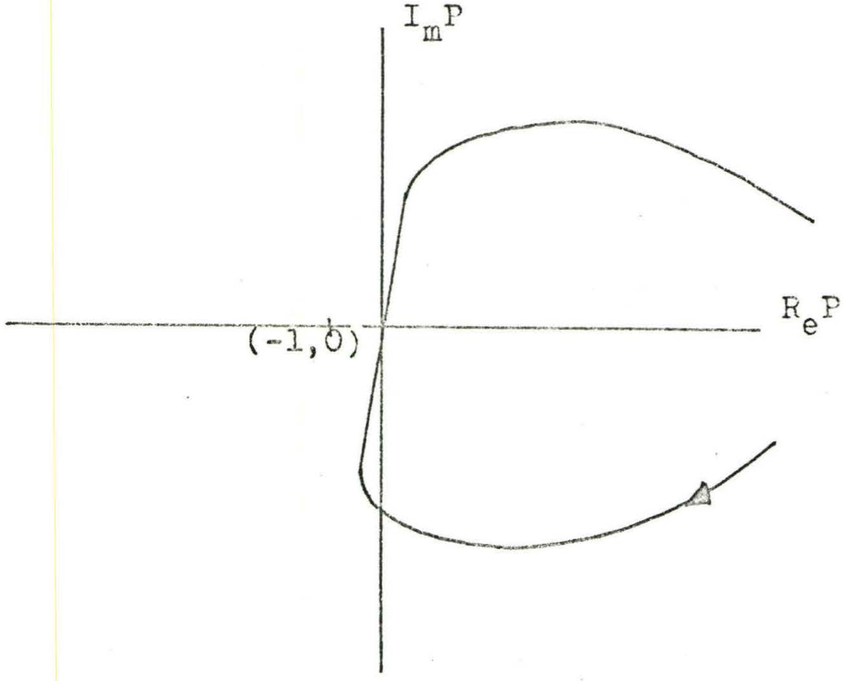


Figure 7.3 FORM OF THE NYQUIST DIAGRAM FOR BANDPASS FILTER

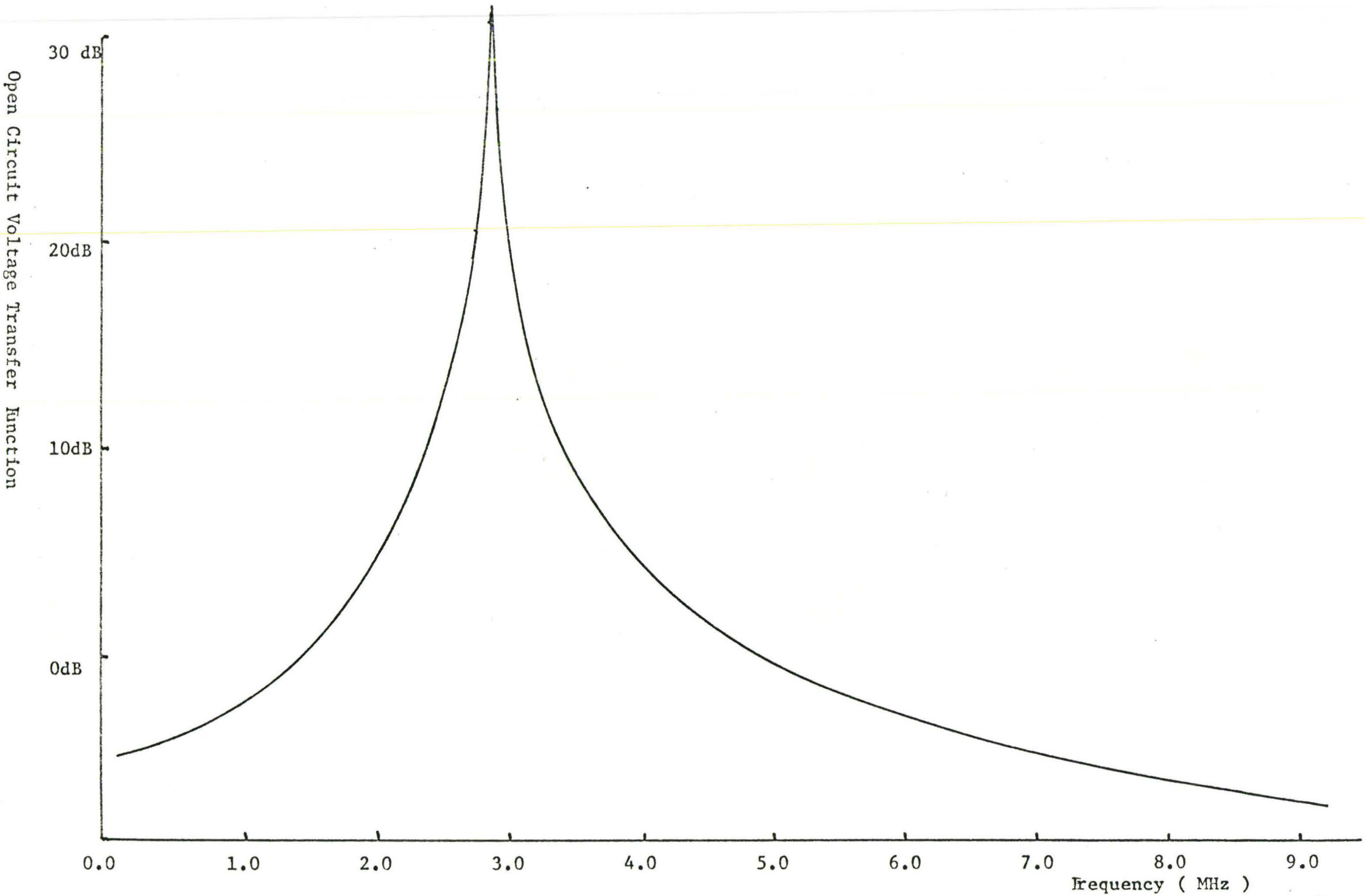


Figure 7.4 RESULT OF COMPUTER ANALYSIS OF $\overline{RC}$ notch BANDPASS AMPLIFIER USING 40dB OF FORWARD GAIN

reflection of the notch characteristic. It was easy to obtain a response of the type of figure 7.1 with a non-optimum notch resistor and centre frequency displaced from notch frequency. With proper conditions the circuit would invariably oscillate with less forward gain than was required for a reasonable B.P. notch.

7.4 Amplifier Compensation

To further investigate the trouble, Bode plots of the open loop response were taken photographically from the H.P. network analyzer. These indicated instability problems at high frequency in the amplifier cut-off region. To compensate for this a single pole lag network was included to roll off the amplifier amplitude at high frequency while, hopefully, not introducing excessive phase shift. Figure 7.5 shows the amplifier response with various time constants in the compensation network and the method of applying compensation. Figure 7.6 shows the corresponding open loop Bode plots. It can be seen that the gain and phase margins are much improved.

On closed loop the circuit was now stable , giving the response of Figure 7.7 which is a true reflection of the notch filter. Band-Pass Q and notch height were dependent on amplifier forward gain only in as much as the amplifier could not be considered perfect. This contrasts with the position in Figure 7.1 where

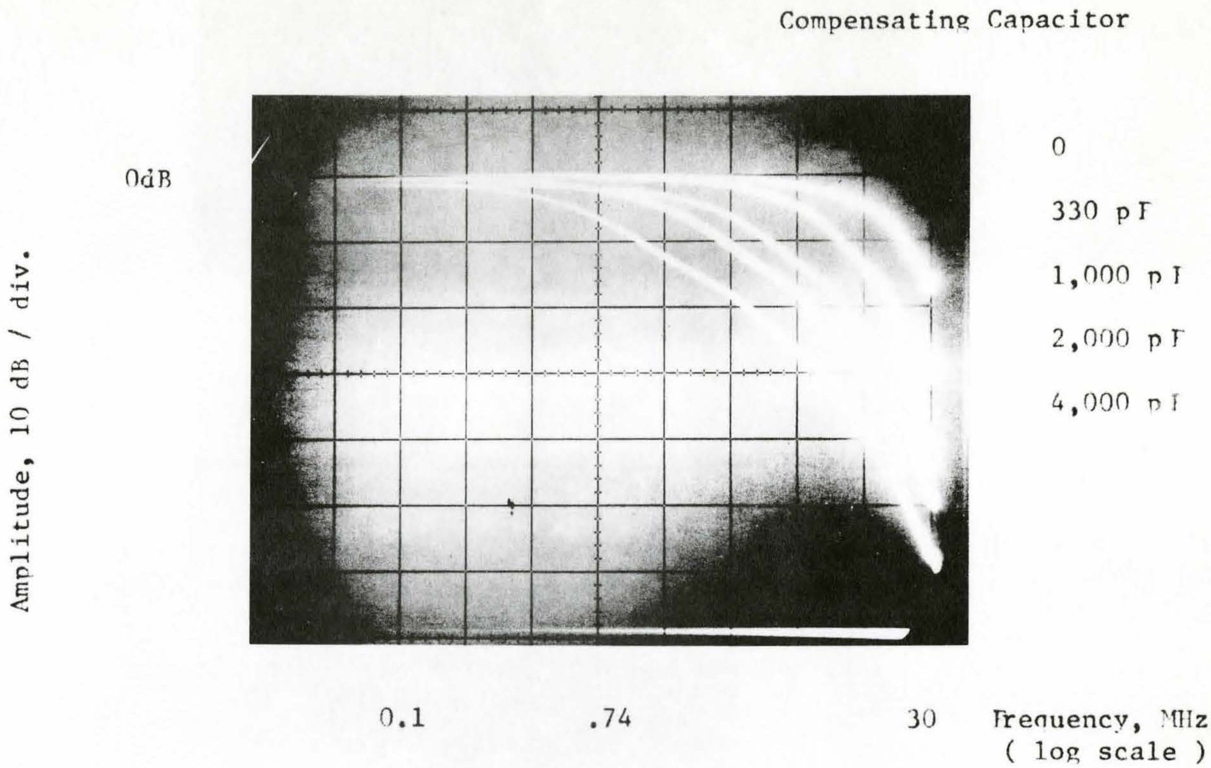


Figure 7.5(a) Amplifier Frequency Response with Differing Degrees of Taper

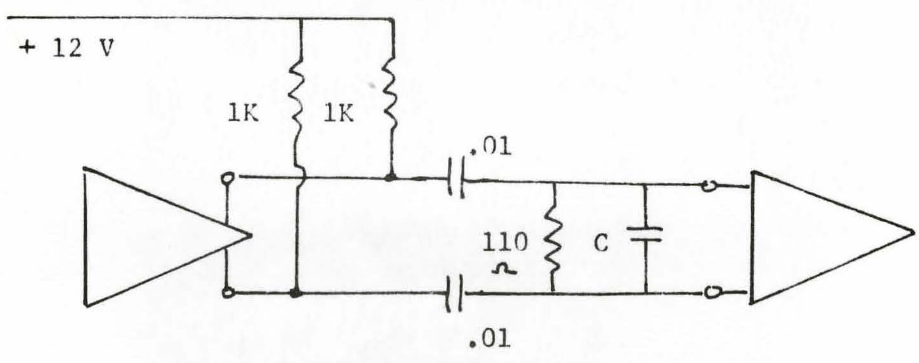


Figure 7.5(b) Method of Applying Compensation

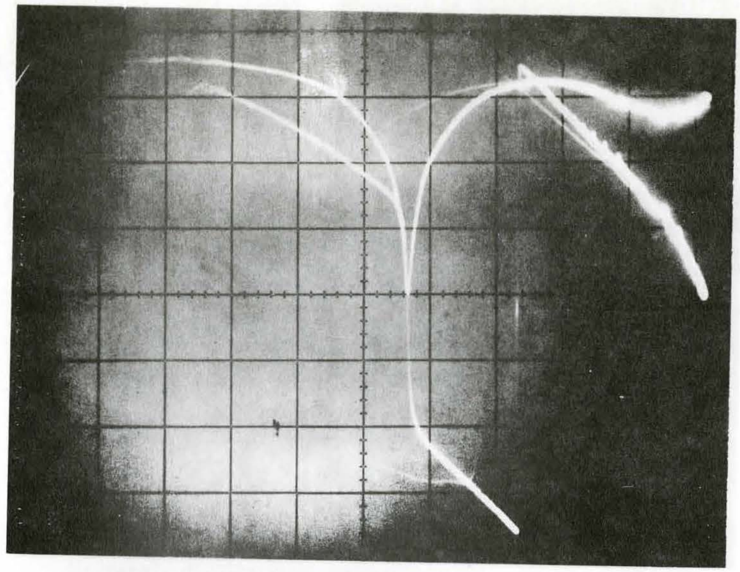


Figure 7.6(a) Bode Plots, No Compensation

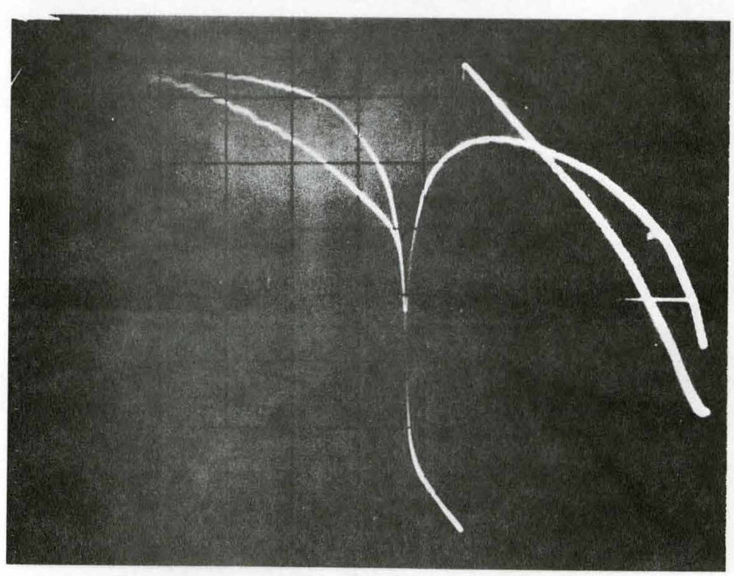


Figure 7.6(b) Bode Plots, 1,000 pF Compensation

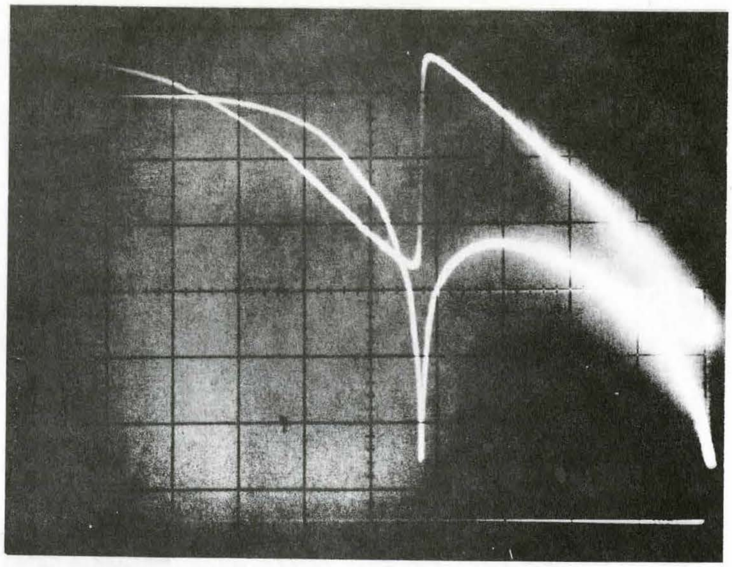


Figure 7.6(c) Bode Plots, Optimum Compensation

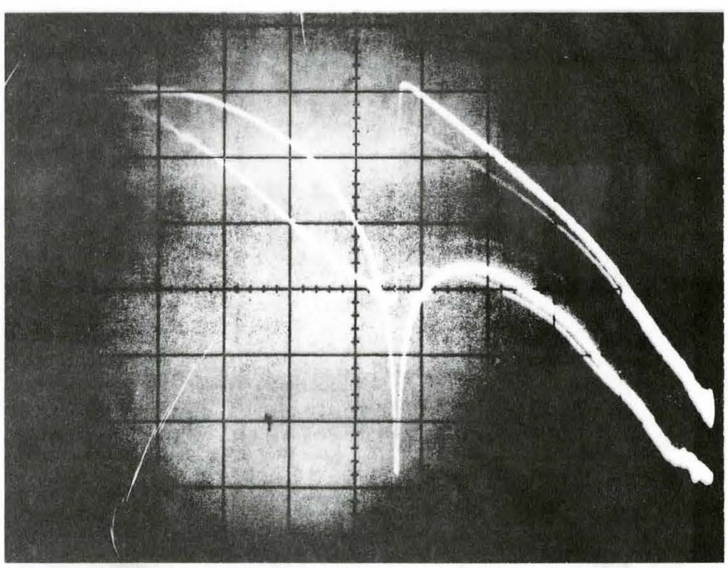


Figure 7.6(d) Bode Plots, Over Compensated Case

$\pm$ 2dB causes either the circuit to oscillate or the peak to flatten.

The compensation is required to reduce the open loop gain sufficiently to augment the natural amplifier roll off so that the gain becomes less than one by the time the 180° phase shift freqency is reached. Working backwards from that frequency with, eg, single pole 20 dB / decade compensation in mind gives a maximum usable gain at the notch frequency for a particular amplifier and ratio of notch and cut-off frequencies.

In the case of our experiments, using the amplifier described in section 6.4, Figure 6.7 about 40 dB at the notch was the most that could be achieved . This was with 10 dB gain margin, 60° phase margin and a cut off frequency equal to 8 times notch frequency (see Figure 7.7). Increased compensation allowed more forward gain which cancelled out the gain reduction at notch frequency due to the increased compensation, leaving the loop gain situation as before. Compensation was restricted to 20 dB per decade for simplicity. The uses of higher order filters are considered in the next section.

The experiments on bandpass amplifiers were carried out in conjunction with Emerson Johnston. He developed the experimental method for measuring open loop characteristics while the author conceived and implemented the compensation techniques.

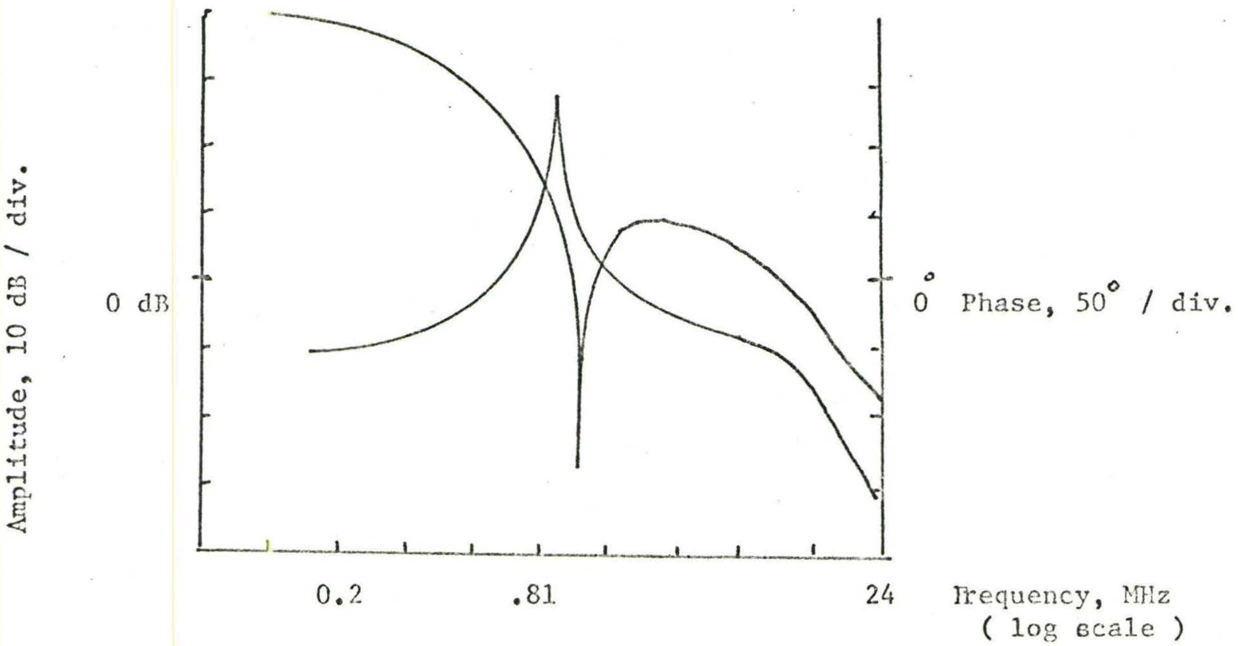


Figure 7.7 Closed Loop Bandpass Response Compared With Notch Filter Response

7.5 Bandpass (BP) Amplifier Design Procedure

It is possible on the basis of the above experimental work to give some general design principles for band pass amplifiers using distributed RC notch filters. The basic steps are :-

(a) Determine the required maximum centre frequency.

(b) Choose an amplifier with frequency response extending at least five times and preferably ten or more higher than this.

(c) Determine the required gain at the notch frequency. 30dB is needed for a reasonable approximation to the " perfect " amplifier case.

(d) From the amplifier characteristics determine the extra gain reduction from this by the amplifier cut-off frequency to assure satisfactory gain and phase margins.

(e) Design a compensation network to accomplish this. In critical cases lead-lag or other higher order filters may be necessary. They should not introduce enough phase shift to upset operation of the B.P. circuit at notch frequency.

(f) Choose the $\overline{RC}$ notch impedance level (length to width ratio) with regard to the amplifier input impedance and loading effects.

7.6 Comments

The basic $\overline{RC}$ BP amplifier developed in this work was a success. It represents a fuller treatment of the subject from the practical point of view than is generally available. Some conclusions may be drawn about the circuit's advantages, disadvantages and areas of applicability.

7.6.1 Advantages

The main advantage of the arrangement presented over a Bi-Quad realization, the chief rival in the analog field, lies in available frequency range. The requirement is for a video amplifier with some 30dB of gain. Such amplifiers are commonly available with cut-offs in the 50MHz region, giving the possibility of $\overline{RC}$ notch BP filters to 10MHz or so. $\overline{RC}$ notch filters may readily be fabricated in thin film form in this range. This compares with a few hundred kHz for filters using silicon I/C operational amplifiers that are presently available, though this limit is likely to be raised considerably with the advent of new high frequency operational amplifiers[41].

An $\overline{RC}$ BP filter should cost the same or less than an equivalent Bi-Quad arrangement in thin film hybrid form. It would likely occupy less area, and should be capable of providing higher Q in a single stage. Two or more filters may be cascaded to spread the pass band.

A voltage tunable BP filter is a possibility using the techniques for voltage tunable $\overline{RC}$ thin film structures developed by P.Swart[18] .

7.6.2 Disadvantages

The filter Q is determined by the quality of the notch and is not readily tunable, as with some other designs. The discussion given has tacitly assumed an optimum notch while a practical filter may diverge from this due to :

(1) Series Inductance

(2) Resistor value tolerances.

The main effects of a non-optimum notch are to smooth out the phase characteristic and reduce the Q. Experiments by Carson,[19] Swart[18] and Campbell[32] suggest that satisfactory distributed notch filters can be constructed in thin film form provided reasonable precautions are taken.

Peak frequency is dependent on $r_o c_o d^2$, where d is the section length. The $r_o c_o$ product may be trimmed by anodising if really necessary. d may be trimmed using etching or laser techniques. The success of a production run might well depend on the initial accuracy that could be maintained on these parameters.

7.6.3 Applications and extensions

The bandpass filters developed nicely cover the conven - tional video range of up to 5 MHz. Most communication systems whether binary or analog operate in this range at some stage of the process. Therefore applications for such things as signal processing suggest themselves. Tunable filters might be used in adaptive filtering systems.

Frequency Following Loop

A tunable band pass amplifier of the type described in conjunction with a phase detector may be used as the basis for a frequency following loop detector. The block diagram is shown in Fig. 7.8. Tracking depends on the phase characteristics of the amplifier and with suitable constant phase shift works to centre the band pass peak on the input signal frequency. The band pass function provides useful interference suppression. For a Frequency Modulated (FM) input the loop error voltage provides a demodulated output.

The error signal is derived from the amplifier phase characteristic but is in fact proportional to the difference between the input signal and BP amplifier centre frequencies. Therefore the error is constant for a constant frequency difference or "error". However for a Phase Locked Loop,[47, 48, 49, 50] the error is cumulative with time in the same case since it is based on the phase of the two signals themselves and phase is the integral of frequency.

Thus the performance of the circuit is not expected to equal that of the phase locked loop. Both systems require a phase detector, low pass filter and error amplifier. Instead of a voltage controlled oscillator the suggested circuit requires a tunable band pass

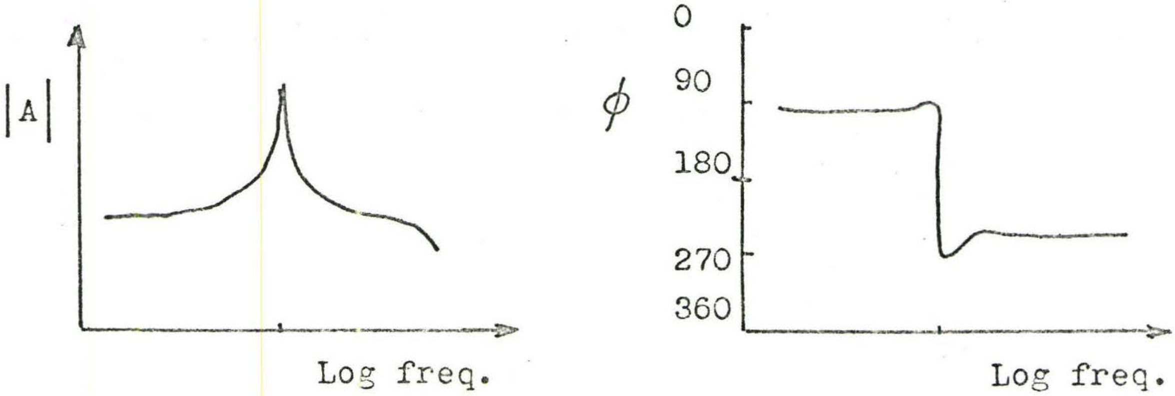
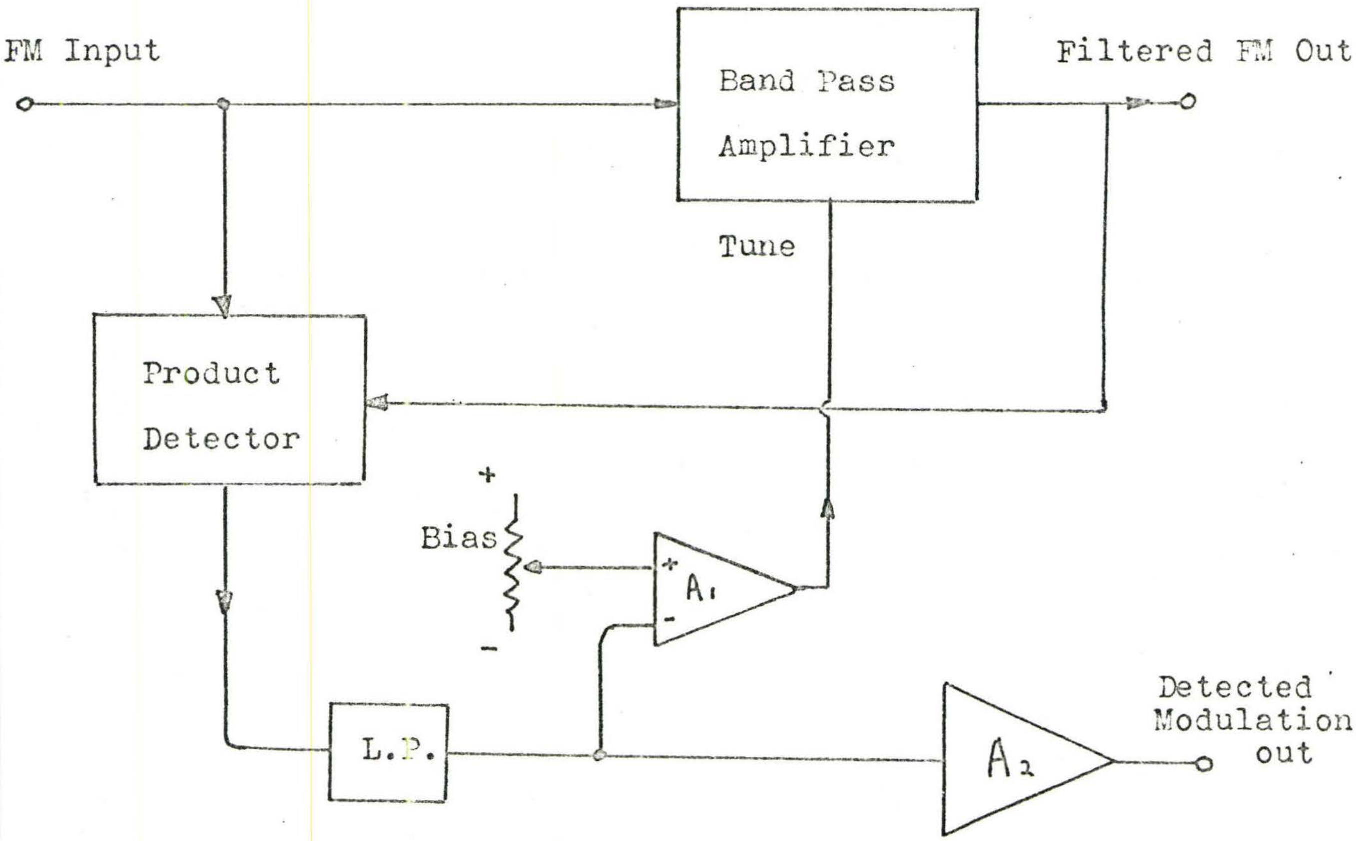


Figure 7.8(a) B.P.Amplifier Characteristics



7.8(b) Block Diagram

Figure 7.8 FREQUENCY FOLLOWING LOOP DETECTOR amplifier, which might be simpler and cheaper under some conditions. One possible advantage is that the Bandpass amplifier circuit will be less susceptible to undesired harmonic and subharmonic modes of operation to which square wave based voltage controlled oscillator circuits are particularly prone.

Preliminary experiments were carried out with this circuit using the phase characteristics of a tunable notch filter built by Swart [18]. Satisfactory locking and FM detection were achieved. It was not possible to continue the experiments using a proper Bandpass amplifier due to time considerations.

CHAPTER 8

Conclusions

8.1 Summary

This thesis has been a theoretical and experimental study of the use of simple three layer distributed RC structures in filtering applications.

Initial attention was focussed on the analysis of such structures for certain restricted cases. The parameters were found to be transcendental. Various techniques to surmount this were discussed and shown to be, for the most part, too clumsy or restricted for practical use.

It was shown, in Chapter 4, that the drawback of irrational parameters could be overcome through the use of computer analysis techniques. Many different configurations were analysed and found to lie within a certain range of lowpass responses. It was surmised that degree of taper is the most important parameter, sharper decreases in width providing sharper lowpass cut-offs. Possible extensions of the techniques to optimisation and automated design were discussed.

Thus far in the thesis, $\overline{RC}$ structures had been considered in isolation. Attention was therefore broadened to include their circuit applications and practical use. For this it was necessary first to consider current active filter design philosophies in Chapter 5.

Ways of achieving a sharp lowpass active filter were considered in Chapter 6 and a novel circuit proposed. This was tried out under various conditions and was found to posses advantages in some respects over the alternatives.

Chapter 7 was concerned with bandpass amplifiers using an $\overline{RC}$ notch null circuit as the frequency selective element. Experiments were described leading to the developement of a succesful circuit and the presentation of a general step by step design procedure for such an amplifier, which has an expected frequency capability of up to 5 MHz. This is considerably better than with most other active filter techniques. Possible undeveloped applications include a frequency following loop for FM filtering and detection.

8.2 Prognosis for Distributed RC Filters

Simple $\overline{RC}$ filters on their own will probably find uses as lowpass filters in applications where a very high attenuation at high frequency is required and a moderately sharp cutoff is acceptable. Unfortunately the high attenuation is accompanied by large increasing phase shift which makes $\overline{RC}$'s not very suitable for use in feedback circuits.

It appears that the range of frequency responses obtainable with the simple topology considered is somewhat limited. Such filters do not form a very worthwhile tool for transfer function synthesis which is anyway being superseded by computer aided design techniques. More complicated topologies bring implementation difficulties and still have transcendental parameters.

The situation is more hopeful with regard to $\overline{RC}$ bandpass amplifiers. These have been considered for some ten years but have not so far got off the ground. The results presented in this thesis show the way for the developement of practical circuits. Work is being done currently at McMaster University by E.Johnston on this topic.

A basic disadvantage impeding the acceptance of $\overline{RC}$ circuits has been the necessity to tightly control the RC product, coupled with the lack of single component adjustability. Recent improvements in thin film fabrication techniques and methods of trimming have now changed the situation for the better.

The main advantage of $\overline{RC}$ circuits is probably that they are likely to occupy less area than a lumped circuit. Since use in hybrid integrated circuits is envisaged, the reduction in the number of interconnections, previously suggested as a great advantage, is not so important.

In conclusion, it is hoped that as a result of this and other similar work distributed RC circuits will finally take their place along with the other components presently available to the designer.

8.3 Suggestions for Future Work

Uniform RC's were used in the experimental parts of this work for convenience. However, as was pointed out in Section 2.4 and Chapter 4, tapered filters have certain advantages and they should be seriously considered in any future work. Most of the material presented here is readily extendable to tapered rather than uniform filters.

Investigation is also needed into the optimum taper or geometry. A two-pronged attack, using computer analysis and a physical understanding of taper effects might indicate that exponential tapers are in fact the best. Some work on these lines is being carried out at McMaster by J.Kostynyk.

Any new consideration of active $\overline{RC}$ filters should be concerned with optimisation of the whole circuit. With suitable modelling this would take care of such things as amplifier input and output impedance loading effects.

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