ENERGY LOSS IN CONDENSED MATTER OF $^1\text{H}$ AND $^4\text{He}$

IN THE ENERGY RANGE $4 < E < 30$ keV
ENERGY LOSS IN CONDENSED MATTER OF $^1\text{H}$ AND $^4\text{He}$ IN THE ENERGY RANGE $4 \leq E \leq 30$ keV.

by

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Measurements are reported of the energy loss suffered by $^1H$ and $^4He$ particles, of 4-30 keV energy, in passing through thin films of carbon, aluminum-oxide and VYNS. Only those particles that emerged in the forward direction were studied. Evidence is presented for identifying the stopping cross-sections per atom observed in this way with $S_e$, the electronic component of the total stopping cross-section per atom. It appears that the calculated energy dependence of $S_e(\propto \sqrt{E})$ is somewhat in error, and that the magnitudes of the $S_e$'s for $^4He$ are systematically too small by 10-15%. 

(ii)
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CHAPTER I

Introduction

(1) The Need for Low-Energy Stopping Data

The manner in which heavy particles lose energy in passing through matter is of interest in several branches of physics.

First to take an interest in the subject were nuclear physicists who wished to employ range-energy curves to determine charged-particle energies. From this interest has grown an extensive body of literature which has recently been summarized by Whaling (1958) for the energy range $0 < E < 10$ MeV. Most of the data presented by Whaling relate to protons, deuterons and alpha-particles. As it happens, charged particle energy determinations are now made with much greater precision using magnetic and electrostatic analysers, with the result that the original interest in range-energy relationships has been replaced by a need to correct observed energies for the relatively small energies lost in the source. Thus, in Whaling's article, the old range-energy curves have been superseded by curves presenting the differential energy loss ($\frac{dE}{dx}$) as a function of energy. In the energy region below 50 keV the tabulated data come almost entirely from two experiments (Phillips, 1953; Reynolds, Dunbar, Wenzel and Whaling, 1953), in both of which protons lost energy to gases confined in gas cells. In Phillips' experiment the energy
range was $10 < E < 80$ keV while in that of Reynolds et al it was $30 < E < 600$ keV. The lack of agreement in the region of overlap has prompted Whaling to remark that "Further experiments in this energy region would be desirable".

A relatively new area of application of energy loss data is the study of nuclear reactions by the observation of recoiling nuclei. In many cases where more than one particle is emitted in a nuclear reaction, the only practicable way to follow the course of events is to observe the recoil of the nucleus. The importance of this type of approach has recently been stressed by Harvey (1960), who deplores the lack of energy-loss data for low-speed, heavy particles. Normally, of course, the "heavy" particle is heavier than a proton or alpha particle, but whatever knowledge is gained concerning lighter particles will presumably aid in rationalizing and predicting the behaviour of heavy ones.

The study of radiation damage to solids represents a third important area in which existing energy-loss information is utilized, and additional information is desired (Seitz and Koehler, 1956; Billington and Crawford, 1961).

Energy loss data are also important for the interpretation of secondary electron emission and luminescence under low speed ion bombardment. By using some of the experimental results presented in this thesis, the author has recently made progress, as yet unpublished, in inter-
preting these phenomena.

Finally, further experimental work is called for in order to subject to trial certain recent calculations of the differential energy losses suffered by heavy, low-speed particles. This matter has been the subject of a good deal of study (Bohr, 1948; Lindhard and Scharff, 1953; Lindhard, 1954; Nielsen, 1956; Seitz and Koehler, 1956; Holmes and Leibfried, 1960; Lindhard and Scharff, 1961; the present author (see section I,2,d), and others) and it has now reached the point where a comparison with experiment in the energy range $0 < E < 50$ keV is highly desirable. The experiments to be described go a long way towards providing this comparison for $\text{H}^1$ and $\text{He}^4$. Other experiments which are currently underway in this laboratory (Ormrod and Duckworth, 1962) utilize heavier particles, and should help further to clarify the picture.

(2) **Theory**

(a) **Introduction**

We shall outline briefly, in general following the recent summary by Lindhard and Scharff (1961), the mechanism by which low-speed, heavy ions are thought to lose energy.

For particles with velocity $v \lesssim v_o$ (where $v_o = \frac{2\pi e^2}{\hbar}$ is the velocity of the electron in the first Bohr orbit in hydrogen), energy is lost both to electrons in the stopping medium and to recoiling atoms, with the latter
effect becoming more important as the velocity of the particle decreases. Thus the total stopping cross-section per atom of stopping medium may be written as

\[ S = S_e + S_n \]  

(1)

where the subscripts e and n refer to the electronic and atomic (or nuclear) components, respectively.

The differential energy loss, \( \frac{dE}{dx} \), is related to the stopping cross-section per atom through the relationship

\[ \frac{dE}{dx} = -NS \]  

(2)

where \( N \) is the number of stopping atoms per unit volume. The quantity \( -\frac{dE}{dx} \), the negative of the differential energy loss, is known as the stopping power of the medium for the particles under study.

(b) Loss in Energy to Electrons

\( S_e \) may be calculated by assuming that the particle moves through an electron gas of constant density... a model first introduced by Fermi and Teller (1947) to study the slowing down of negative muons. The actual expression deduced by Lindhard and Scharff is valid only for particle velocities that are small compared to \( v_0 z_1^{2/3} \) (where \( z_1 \) is the atomic number of the particle) and has the value

\[ S_e = \int_e^E \left[ 8\pi e^2 a_0 \left\{ \frac{z_1 z_2}{\left( z_1^{2/3} + z_2^{2/3} \right)^{3/2}} \right\} \left[ \frac{V}{V_0} \right] \right] \]  

(3)

Here \( z_2 \) is the atomic number of the stopping atom, \( a_0 \) is

*See Appendix*
the Bohr radius and $\kappa_e$ is of the order $1-2$, possibly having the value $Z_1^{1/6}$. In fact, the value used by Powers and Whaling (1962) is

$$\kappa_e = Z_1^{1/6}$$

With this value equation (3) can be written in the form

$$S_e = Z_1^{1/6} \left[ \frac{8\hbar e^2 a_0}{\pi} \left[ \frac{z_1 z_2}{(z_1^{2/3} + z_2^{2/3})^{3/2}} \right] \sqrt{\frac{E}{E_{0v}}} \right]$$

where $E_{0v}$ is the kinetic energy of the projectile when its speed equals

$$v_0 \simeq 2.2 \times 10^8 \text{ cm sec}^{-1}$$

The values of $E_{0v}$ for $^1H$ and $^4He$ are 25 keV and 100 keV, respectively. For $v_0 \gtrsim v Z_1^{2/3}$, the expression no longer holds and $S_e$ gradually assumes a $(\ln E)/E$ dependence (Bloch 1933).

Thus the preceding equation can only be used at energies below about 250 keV for the slowing down of atomic helium and at energies below approximately 25 keV for atomic hydrogen. However, in the experiments to be described, we shall later apply the $S_e$ formula to the stopping of atomic hydrogen and of atomic helium in the energy interval $0 < E < 30$ keV. In this energy interval, equation (3) holds for the slowing down of atomic helium but does not hold for the stopping of atomic hydrogen in the energy interval $25 < E < 30$ keV.

Thus, we shall not be able to compare theory and experiment rigorously for atomic hydrogen in the energy
range studied. However, perfect agreement with theory cannot be anticipated anyway, since the formulas for \( S_\text{e} \) are merely approximations (the \( E^{1/2} \) dependence is only approximate and \( S_\text{e} \) is quite uncertain) to the cross-section for loss in energy to electrons.

(c) **Loss in Energy in Atomic Recoils**

The form of \( S_n \) was first studied by Bohr (1948) who assumed that atomic collisions occur in a potential

\[
P(r) = e^{2Z_1Z_2} \frac{e^{-r/a}}{r}
\]

where \( r \) is the distance between the nuclei of the colliding particles and \( a \) is the so-called screening parameter. The value of the screening parameter used by Lindhard and Scharff is

\[
a = 0.8853 a_0 \sqrt{\frac{2/3}{Z_1^{2/3} + Z_2^{2/3}}}
\]

In this type of potential the distance of closest approach, \( b' \), in a head-on collision is given by

\[
\frac{b}{a} = \frac{b'}{a} e^{(b'/a)}
\]

where \( b \), the distance of closest approach if the colliding particles were stripped of their planetary electrons, has the value

\[
b = 2Z_1Z_2e^2 \left[ \frac{M_1 + M_2}{M_1 M_2 v^2} \right]
\]

where \( M_1 \) is the mass of the projectile of velocity, \( v \), and
 $m_2$ is the mass of the target particle. From now on we shall denote the quotient $b'/a$ by

$$ s = \frac{b'}{a} $$

For the purpose of showing the behavior of $s$ with energy we have tabulated in Table I some values of $s$ corresponding to collisions of a number of different atomic projectiles with a carbon atom, initially at rest.

<table>
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<tr>
<td></td>
<td>H¹</td>
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<tr>
<td>1</td>
<td>0.28</td>
</tr>
<tr>
<td>5</td>
<td>0.073</td>
</tr>
<tr>
<td>10</td>
<td>0.037</td>
</tr>
<tr>
<td>20</td>
<td>0.018</td>
</tr>
<tr>
<td>60</td>
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Bohr expands equation (5) and finds that the assumed potential may be approximated by

$$ P_n(r) = Z_1 Z_2 e^2 a^{n-1} \left[ \frac{n-1}{2.71} \right]^{n-1} r^{-n} \quad (9) $$

where $n = 1 + \frac{r}{a}$.

The effect of different values of $r$ on the field can easily be seen from Equation (9); for instance, if $r << a$ the potential is with high approximation coulombic. It is
thus of importance to investigate in which part of the field the scattering effectively takes place. A measure of this is the value of the parameter $s$. For instance, at relatively high energies, $s \ll 1$, so that most of the scattering effectively takes place at a distance, $r$, which is small compared to $a$. Under these conditions, $n$ is only slightly larger than unity and the scattering, therefore, is predominantly of the Rutherford type (that is, in a coulombic potential). In this energy range, Rohr finds $S_n \sim 1$, the stopping cross-section for loss in energy to atomic recoils, to be given approximately by

$$S_n \sim 1 \approx 2\pi \frac{M_1}{M_2} \frac{e^4 Z_1^2 Z_2^2}{E} \ln \left[ \frac{2a}{Z_1 Z_2 e^2} \frac{M_2}{M_1 + M_2} \right]$$ \hspace{1cm} (10)

For lower energies of the impinging particle, $a$ increases in value, and the scattering takes place to an increasing extent in the screened part of the field. Thus, in the energy range where $s \sim 1$ the scattering takes place entirely in the screened part of the field, $(r \sim a)$, which falls off more rapidly than does the coulomb field. In this energy region, the $n$ value is approximately

$$n \sim 2$$

For the corresponding inverse square potential ($\Phi(r) \sim \frac{1}{r^2}$) Lindhard and scharff (1961) find, in good agreement with Nielsen (1956) that

$$S_n \sim 1 = 1.13 \left[ \frac{2}{2.71} \frac{Z_1 Z_2}{M_1} \frac{M_1}{M_1 + M_2} \right]$$ \hspace{1cm} (11)
represents a first order approximation to the actual cross-section for loss in energy in atomic recoils.

Approximations for $S_n$ for still lower energies are given by the following formula:

$$(S_n)_n = \left(\frac{1}{n-1}\right) \pi \left[\frac{n}{\pi}\right]^{2/n} \left[\frac{n-1}{2.71}\right] \frac{1}{e} \left[\left(\frac{M_1}{M_1+M_2}\right) \left(\frac{M_2E}{M_1+M_2}\right)^{1-2/n}\right]_1^{2/n}$$

where

$$I_n = \int_{-\infty}^{+\infty} \frac{dt}{(1+t^2)^{n+2}}$$

Equation (12), which will be derived in the next section, has been deduced by the author and is valid for any value of $n \geq 2$.

When $n = 2$ Equation (12) reduces to

$$S_{n=2} = 1.57 \left[\frac{2}{\pi e} \frac{a}{2.71} Z_1 Z_2 \frac{M_1}{M_1+M_2}\right]$$

which is in fair agreement with Equation (11).

Lindhard and Scharff state in their 1961 publication, without presenting a derivation for the following formula or Equation (11), that in a power potential of the form

$$F(r) \propto \frac{Z_1 Z_2^{a^n-1}}{r^n}$$

the energy loss cross-section behaves as
\[(S_n)_n \propto a^{2-2/n} (Z_1 Z_2)^{2/n} \left[ \frac{M_1}{M_1 + M_2} \right] \left[ \frac{M_2 E}{M_1 + M_2} \right]^{1-2/n}\]

in agreement with our result, Equation (12).

To summarize: we have Equation (10) representing a first-order approximation to \(S_n\) at high energies \((s \ll 1)\).

At lower energies, where \(s \sim 1\), \(S_n\) can be approximated by Equation (11) or by Equation (12) with \(n = 2\). At still lower energies \((s \gg 1)\), the approximation is given by Equation (12) with \(n \sim 3\); et cetera.

Physically we require that in a plot of \(S_n\) versus energy, the successive approximations smoothly join each other. From this, and from the energy dependence of the various approximations, it is seen that \(S_n\) qualitatively has the form as shown in Figure 1.

In the experiments to be described on the slowing down of atomic hydrogen and helium in the energy range \(5 \lesssim E \lesssim 30\) keV, the parameter \(s\) was always less than 0.5 and usually much less than 0.5 (see Table 1). Equation (10) is, therefore, the relevant expression to be used in computing \(S_n\) for these atomic particles. On the other hand, the slowing down of atomic argon in the same energy region (see Table 1) should be described by Equation (12) with \(n \geq 2\).

(d) Loss in Energy to Atomic Recoils in a Lower Potential.

In this section, we shall present a derivation of Equation (12)
Figure 1 - A Qualitative Presentation of the Energy Dependence of the Nuclear Component, $S_n$, of Total Stopping Cross Section per Atom
Figure 2 - The Scattering of a Projectile, Mass $M_1$, by an Arbitrary Central Force, Centered at the Target Particle of Mass $M_2$. 
In a collision between a projectile of mass $M_1$ and a target particle of mass $M_2$ the projectile will describe an orbit and suffer a net deflection angle $\phi$. This has been illustrated in Figure 2 where $p$, the impact parameter, would be the distance of closest approach if no forces were acting between the colliding particles.

Actually, the struck particle will also describe an orbit. For clarity in the figure, however, we have shown the target particle as infinitely heavy, even although we shall take its motion into account.

The differential cross-section, $d\sigma$, for collisions between impact parameters $p$ and $p+dp$ is

$$d\sigma = 2\pi p dp$$

(13)

This equation implies that distant collisions (large $p$) occur more frequently than the more violent collisions (small $p$) in which the particles involved are deflected through large angles. Because of this the loss in energy will be mainly determined by the frequent small angle deflections rather than by the occasional large angle events.

With this in mind, we shall deduce energy-loss expressions which are very nearly exact in the limit of small angle scattering (within the limitations of the assumptions involved), but which are only crude approximations to the large angle case. Roughly, however, all angular scattering events will be taken into account.

In distant collisions, the incident particle is
but little deflected. If all the momentum of the projectile is originally in the $X$ direction, the struck particle will be set in motion with momentum $M_y$ in a direction that is practically perpendicular to the direction of the incident particle. By Newton's second law

$$M_y = \int_{-\infty}^{\infty} F(y) \, dt$$

where

$$F(y) = \frac{v}{r} F(r)$$

and $r$ is the distance between the colliding particles.

For distant collisions, see Figure 2.

$$r \approx (p^2 + v^2 t^2)^{1/2}$$

where $v$ is the velocity of the projectile.

During the collision the incident particle also receives a $y$-component of momentum, which is just equal, but opposite, to $M_y$. Hence, the relative deflection angle, $\alpha$, of the incident particle equals

$$\alpha \approx \frac{M_y}{M}$$

where $M$, which remains practically unchanged in a distant collision, is the original momentum (of the projectile) in relative co-ordinates and is given by

$$M = \frac{M_1 M_2 \, v}{M_1 + M_2}$$

Evidently the relationship between the actual deflection angle, $\phi$, of the moving particle and the relative deflection angle is given by
\[ \tan \phi = \frac{M_2 \sin \alpha}{M_1 + M_2 \cos \alpha} \] (19)

By means of Equations (14), (15), (16) and (17), we find that

\[ \mathcal{R} = \int_{-\infty}^{+\infty} p \frac{F\left(\sqrt{\frac{2}{p^2 + v^2 t^2}}\right)}{M \sqrt{\frac{2}{p^2 + v^2 t^2}}} \, dt \] (20)

By introducing the new variable

\[ Z = \frac{vt}{p} \]

we see (see Equation (16)) that

\[ r = p \sqrt{1 + Z^2} \] (21)

Upon substitution of Equations (21) and (18) into (20) it follows that

\[ \mathcal{R} = \left(\frac{M_1 + M_2}{M_2}\right) \frac{p}{2E} \int_{-\infty}^{+\infty} F\left(\frac{1}{\sqrt{1 + Z^2}}\right) \, dZ \] (22)

where

\[ E = \frac{1}{2} M_1 v^2 \]

We shall now apply our equations to a field of force defined by the potential

\[ F_n = \frac{k_n}{r^n} \] (23)

which, with

\[ k_n = \frac{Z_1 Z_2 e^{\alpha n - 1}}{2.71} \] (24)

is the Bohr potential, Equation (9), for atomic collisions. Hence, the force, \( F(r) \), equals

\[ F(r) = -\frac{n}{r^{n+1}} \frac{k_n}{r^n} \] (25)

From Equations (25), (21) and (20) we find
\[ \alpha \propto - \left( \frac{M_1 + M_2}{v_2} \right) \frac{p^{-n} k_n I_n}{2E} \]  

(26)

where

\[ I_n = \int_{-\infty}^{+\infty} \frac{dZ}{(1+Z^2)^{\frac{n+2}{2}} \left( \frac{1}{1+n} \right)} \]

Evidently

\[ I_2 = \frac{\pi}{2} \]

The differential cross-section, \( d\sigma \), can be found now by substitution of Equation (26) into (13). This yields

\[ d\sigma = \frac{2\pi}{n} \left[ \left( \frac{M_1 + M_2}{M_2} \right) \frac{n k_n I_n}{2E} \right]^{2/n} \frac{1}{\alpha^{2+n}} d\alpha \]

(27)

or

\[ d\sigma = \frac{1}{n} \left[ \left( \frac{M_1 + M_2}{M_2} \right) \frac{n k_n I_n}{2E} \right]^{2/n} \frac{1}{\alpha^{2+2n}} d\omega \]

where for small angles the solid angle, \( d\omega \), equals

\[ d\omega = 2\pi \alpha d\alpha \]

The energy, \( T \), transferred to particle \( M_2 \) during the collision can easily be shown to be

\[ T = 2 \frac{M_1^2 M_2 v^2 \sin^2 \frac{\alpha}{2}}{(M_1 + M_2)^2} \]

which for small angles reduces to

\[ T \propto \frac{M_1 M_2 E}{(M_1 + M_2)^2} \]

(28)

The average cross-section for energy loss, taking all angular deflections into account, is given by
\[ S_n = \int_0^\pi T \, d\sigma \quad (29) \]

On substitution of Equations (27) and (28) into the above, it follows that

\[ (S_n)_n = \left( \frac{n-1}{n-1} \right)^{3-2/n} \pi^{2/n} \left( \frac{n \, k_n I_n}{2} \right)^{2/n} \left( \frac{M_1}{M_1 + M_2} \right) \left( \frac{M_2 E}{M_1 + M_2} \right)^{1-2/n} \]

Equation (12) can now be deduced by inserting the value for \( k_n \) (Equation 24) into the preceding formula.
CHAPTER II

Apparatus

(1) Principal Apparatus

A schematic diagram of the apparatus is shown in Figure 3. The principal parts of the apparatus are (1) a source of ions; (2) a ten-inch sector type mass spectrometer, which serves as the ion-analyser; (3) a thin film, in which the ions lose energy; (4) an electrostatic analyser for determining the energy of the ions that emerge from the film in the forward direction, and (5) an electron multiplier for detecting the ions that are transmitted by the electrostatic analyser. 

The output of the electron multiplier is connected to an amplifier feeding a scalar. Connected to the scalar is a rate-meter from which the number of ions emerging from the electrostatic analyser per unit time interval is read directly.

The film can be removed from the path of the ions (thereby permitting the initial energy of the ions to be determined), and can be rotated through an angle, thereby increasing its effective thickness to the value

\[ t' = t / \cos \theta \]

The acceptance angle of the electrostatic analyser is 1.2°, and the analyser itself is used as a direction focusing device.

The whole assembly is evacuated, to a pressure of a few times \( 10^{-6} \) mm of Hg, by standard vacuum techniques,
FIGURE 3 - Schematic Diagram of the Apparatus.
FIGURE 4 - Modification for Studying Angular Distribution of Transmitted Particles.
utilizing oil diffusion pumps and a rotary mechanical pump.

(2) **Apparatus for the Angular Distribution of the Transmitted Beam**

For certain experiments, the electrostatic analyser was removed and the electron multiplier was attached to the rest of the apparatus through a bellows system. A schematic diagram of this part of the apparatus is shown in Figure 4. In this way the angular distribution (but not the energy) of the ions emerging from the film could be studied.

(3) **Mounting of the Thin Films**

Preliminary experiments indicated that the VYNS and Al$_2$O$_3$ foils, when subjected to ion bombardment became positively charged to potentials of the order of 1000 volts. This charging-up effect, however, could be eliminated by mounting the thin film on a metal holder, in such a manner that part of the impinging ion beam bombarded the holder. Since the number of secondary electrons emitted by a metal per incident ion, of energies above a few keV, is generally larger than unity (this was investigated in detail in some unpublished work) the charge on the film was neutralized by these secondaries. In this thesis only results that were obtained with uncharged films will be presented.
The thin films are of three types: VYNS, aluminum oxide (Al$_2$O$_3$) and carbon. The VYNS films were prepared in a manner suggested by the work of Revell et al (1955) and although their actual thicknesses are not known, thin specimens could easily be prepared and were useful in establishing or confirming certain functional relationships. The aluminum oxide films were made by anodic oxidation of sheet aluminum. With a solution of ammonium citrate as the anodizing bath, the thickness of the oxide layer was taken to be 13.7 Å per volt. The aluminum substrate was then removed by hydrochloric acid. The anodic oxidation of aluminum in ammonium citrate has recently been studied by Davies (1960). The carbon films were produced by evaporation onto a clean glass slide. When the slide was dipped in water, the film floated free and could be picked up on the film holder. Because these carbon films are very fragile, they required a support which consisted of a finely-meshed metal screen. Owing to this metal backing, the unit when rotated through an appreciably large angle became almost opaque to an ion beam. Consequently, experiments in which Θ (see Figure 3) was varied were not carried out with a carbon film, but were only performed with VYNS and aluminum-oxide foils.

The values of N for the aluminum oxide and carbon are taken to be 1.03 x 10$^{23}$ atoms/cm$^3$ and 1.13 x 10$^{23}$ atoms/cm$^3$, respectively.
Experimental Results on Energy Loss

(1) Observations of the Energy Loss of H$^1$ in Thin Films

(a) Energy Loss of H$^1$ in Aluminum Oxide and VYNS.

A typical set of experimental results is shown in Figure 5. These data were obtained with protons of energy $E_0$ ( = 17.49 keV) incident upon a 205 Å film of aluminum oxide. The sharp peak at $E_0$ was observed by removing the film and reducing the ion current to a value much lower than was needed when the film was in position. The finite resolution of the electrostatic analyser accounts for practically the entire width of this peak. The $\theta = 2.5^0$ curve shows the energy distribution in the detected ion beam with the film in position and 2.5° (nominally 0°) from being normal to the incident beam. The $\theta = 42.5^0$, $\theta = 57.5^0$ and $\theta = 67.5^0$ curves were obtained by rotating the film to secure successively larger effective film thicknesses ($t' = t/\cos \theta$).

The difference, $\Delta E_0$, between the initial energy, $E_0$, and the maximum of the transmitted distributions is the most probable value of the energy lost by the particle in passing through that particular thickness of film. $\Delta E_0$, of course, increases with film thickness. It is assumed that the average energy, $\bar{E}$, of the particle whilst traversing the film is $\frac{E_0 - \Delta E_0}{2}$.
Figure 5 - Energy Distribution Amongst $^1$H Particles after Traversing Al$_2$O$_3$ Films of Various Thicknesses
Curves similar to those of Figure 5 were obtained for several different incident energies. From these, values of $\Delta E_0$ and $\bar{E}$ were obtained which form the basis for the $\ln \Delta E_0$ versus $-\ln \bar{E}$ plot shown in Figure 6.

We proceed now to obtain a stopping cross-section versus-energy curve from Figure 6.

As mentioned earlier, the stopping power of a material for a given particle is defined as $-\frac{dE}{dx}$, where $dE$ is an infinitesimal small energy loss suffered by the particle in travelling an infinitesimal distance, $dx$, in the stopping medium. How closely does the expression $-\frac{\Delta E_0}{t'}$ formed from our observables correspond to $-\frac{dE}{dx}$?

We answer this question by referring to Table II, in which for several values of $\bar{E}$, the computed values of $\frac{\Delta E_0}{t'}$ are tabulated for each of four film thicknesses. The necessary values of $\Delta E_0$ have been read off from the experimental curves plotted in Figure 6.

<table>
<thead>
<tr>
<th>$\bar{E}$ (keV)</th>
<th>$\frac{\Delta E_0}{t'}$ (eV/Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t' = 205Å$ ($\Theta = 2.5°$)</td>
<td>16.3 17.1 16.7 17.2</td>
</tr>
<tr>
<td>$t' = 278Å$ ($\Theta = 42.5°$)</td>
<td>13.8 14.4 14.2 14.4</td>
</tr>
<tr>
<td>$t' = 383Å$ ($\Theta = 57.5°$)</td>
<td>12.1 12.7 12.5 12.6</td>
</tr>
<tr>
<td>$t' = 536Å$ ($\Theta = 67.5°$)</td>
<td>10.3 10.6 10.7</td>
</tr>
</tbody>
</table>
Figure 6 - Log - Log Plot Showing, for Various Film Thicknesses, the Most Probable Energy Loss as a Function of Average Particle Energy. The Curves Relate to H\textsuperscript{+} Moving in Al\textsubscript{2}O\textsubscript{3}
It is evident from Table II that for a given average energy, $E$, the value $\frac{\Delta E}{t'}$ does not change much, if at all, as the thickness of the film changes. The constancy of the value $\frac{\Delta E}{t'}$ as $t'$ decreases from $536^\circ$ to $205^\circ$ strongly suggests that it is essentially the same in the range $205^\circ < t' < 0$ as well. This result, indicating that our $\frac{\Delta E}{t'}$ values actually correspond to stopping powers at energies $E = E$ was confirmed by using VYNS films, as we observe from Table III.

<table>
<thead>
<tr>
<th>$E$ (keV)</th>
<th>$\Delta E_{E} / t'$ (arbitrary units)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta = 4^\circ$</td>
</tr>
<tr>
<td>30</td>
<td>17.5</td>
</tr>
<tr>
<td>25</td>
<td>16.2</td>
</tr>
<tr>
<td>20</td>
<td>14.6</td>
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<tr>
<td>15</td>
<td>13.0</td>
</tr>
<tr>
<td>10</td>
<td>10.6</td>
</tr>
<tr>
<td>6</td>
<td>8.4</td>
</tr>
<tr>
<td>4</td>
<td>6.8</td>
</tr>
</tbody>
</table>

This table, which is similar to Table II, has been compiled from the observed energy loss measurements of a beam of $H^1$ in passing through a VYNS film. The confirmation is especially valuable at lower energies since the relative thinness of the VYNS films made it possible for all four values of $t'$ to be utilized even with average energies as low as $E = 5$ keV.
We have, therefore, obtained \( \frac{dE}{dx} \) values from the observed values (utilizing the \( \Theta = 2.5^\circ \) results as the most reliable, inasmuch as an error in the determination of the angle is least critical for these), and used Equation (2) to calculate corresponding values of the stopping cross-section per atom, \( S_0 \), as observed in our experiment. We shall later see that \( S_0 \) is practically identical to \( S_e \), the electronic component of the total stopping cross-section per atom, but for the time being shall refer to it as the "observed" stopping cross-section per atom. In Figure 7, \( S_0 \) is plotted against \( E \).

We might also have plotted \( \ln S_0 \) against \( \ln E \), but the reader will realize that such a curve would have had the same shape as the \( \Theta = 2.5^\circ \) curve in Figure 6. It will be noted that, in the energy range \( 6 < E < 30 \) keV, this latter curve is closely linear. Hence, we conclude that the \( \ln S_0 \)- versus- \( \ln E \) is also linear in this energy range. Furthermore, by using the known values of \( t' \) (205A) and \( N \) (1.03 \( \times \) \( 10^{23} \) atoms/cm \(^3 \)), we find the following empirical relationship:

\[
S_0 = 4.3 \times 10^{-15} E^{0.45} \text{ ev-cm}^2
\]  

in the energy range \( 6 \text{ keV} < E < 30 \text{ keV} \).

Using VYNS films, it was demonstrated that an empirical relationship similar to Equation (31) exists for this material as well. But since the film thicknesses were not known, it was not possible to calculate the
Figure 7 - Plot of Observed Stopping Cross Section per Atom as Function of Energy, for Case of H⁺ Moving in Al₂O₃
value of the proportionality constant. The value of the exponent, however, was precisely the same, namely, 0.45.

(b) **Energy Loss of \( H^1 \) in Carbon.**

Experiments similar to those described for aluminum oxide and VYNS were also done with a carbon film. This film was always kept normal to the incident ion beam and experiments in which \( \theta \) was varied were not carried out with it. The energy distribution curves, which were obtained were similar to those shown previously for aluminum oxide, and will be further discussed in Chapter IV.

The thickness of the film was not measured directly but was estimated in the following manner. Whaling (1958) gives a \( \ln S - \ln E \) curve for \( H^1 \) moving in carbon with energies greater than 100 keV. This curve is reproduced in Figure 8 as a solid smooth curve. Our \( \frac{dE}{dx} \) (that is, \( \Delta E_0 / t' \)) - versus - \( E \) (that is, \( E \)) curve, which extends from 4 keV to 30 keV, and which is marked in Figure 8 with circles, was matched to Whaling's curve by assuming a value for the thickness of the film which allowed the two curves to be connected, as shown in Figure 8, by a dotted line. Although there is considerable latitude in this matching, it appears that the film was about 250 Å units thick. Using this value, we have plotted for \( H^1 \) in carbon the stopping cross-section per
Figure 8 - Matching the Observed $S$ - versus - $E$ Results in the Energy Region $4 < E < 30$ keV to Those Given by Whaling (1958) for $E > 100$ keV. The Curves Relate to $H^1$ Moving in Carbon.
Figure 9 - Plot of Observed Stopping Cross Section, $S_0$, per Atom as Function of Energy for Case of H\textsuperscript{1} Moving in Carbon.
atom-versus-energy curve shown in Figure 9. In the energy interval $4 \text{ keV} < E < 30 \text{ keV}$ this curve is closely described by the empirical equation

$$S_0 = 3.5 \times 10^{-14} E^{0.41} \text{ eV-cm}^2 \quad (32)$$

(2) **Observations of the Energy Loss of He$^4$ in Thin Films.**

The experiments described in the preceding section were repeated with He$^4$ as the projectiles and very similar results were obtained. In the energy range $5 \text{ keV} < E < 30 \text{ keV}$, it was found that the stopping cross-section per atom of aluminum oxide varies with energy according to the following empirical relationship:

$$S_0 = 4.6 \times 10^{-15} E^{0.44} \text{ eV-cm}^2 \quad (33)$$

Further, in the energy interval $9 \text{ keV} < E < 30 \text{ keV}$, the stopping cross-section per atom of carbon is closely described by the empirical relationship

$$S_0 = 4.0 \times 10^{-15} E^{0.40} \text{ eV-cm}^2 \quad (34)$$

(3) **Experiments with Molecular Ions**

Some experiments were carried out with H$_2^+$, F$_2^+$, H$_3O^+$, and N$_2^+$ ions as projectiles. It was found that the results on the energy loss were consistent with the theory that the entire molecule breaks up, on impact, into its constituents, each penetrating the foil with initial velocity equal to the velocity of the impinging molecule.
It is well known that a particle, in passing through a material medium, changes its charge many times. It is not surprising, therefore, that many of the particles in our experiments emerged from the film with a charge that was different from that of the ions impinging upon the film. For example, when $O^{++}$ or $O^+$ ions were used as projectiles, $O^{++}$, $O^+$ and $O^-$ ions were detected in the emerging beam. Neutral oxygen atoms were undoubtedly also present, but escaped detection by the electrostatic analyser.

Likewise, with protons incident upon the film, $(H^1)^+$ and $(H^1)^-$ were both observed in the transmitted beam. The ratio $(H^1)^+/ (H^1)^-$ increased rapidly with the energy of the emerging particles. For example, with a VYNS film, this ratio was $\sim 3$ for an emerging energy of $5$ keV, $\sim 11$ for $10$ keV, and $\sim 30$ for $30$ keV. On the other hand, when $(He^4)^+$ was used as the projectile, the ratio $(He^4)^+/(He^4)^-$ did not vary rapidly with emerging energy, but had the approximate value $500$ in the energy range $10$ keV-$30$ keV.

The most significant observation for our purpose, however, was that, within experimental error, the energy distribution in the beam emerging from a film, and the most probable value of its energy loss ($\Delta E_0$), are, for a given atomic projectile, independent of the charge carried by the atom as it enters and leaves the film. This charge
independence is believed to imply that our observed stopping cross-sections are weighted averages that take into account the various possible charge states that the moving particle may assume during its passage. It is for this reason that, in this thesis, we have consistently designated the moving atom as $H^1$ or $He^4$, rather than $(H^1)^+$, $(He^4)^+$, et cetera, in order to indicate that within the stopping material, the charge state of the projectile is dependent upon its nuclear charge, its velocity and the nature of the stopping medium, but is independent of the charge with which it enters. Presumably, there is a transition period during which the entering ion comes to terms with the stopping material, but in our experiments this period must have been very brief indeed.
CHAPTER IV

Energy Spread in the Beam Transmitted by Thin Films

(1) The Shape of the Energy Distribution Curves

The reader will have noted in Figure 5 that the energy distribution amongst the $^1$H particles emerging from Al$_2$O$_3$ films is bell-shaped and practically symmetric. This was true of the other films as well, and also when He$^4$ was used as the projectile.

On the other hand, when much heavier projectiles of the same energy are used, the energy distribution among the particles emerging from the film is markedly asymmetric. Figure 10 shows the distribution curve for A$^{40}$ particles which have passed through the 205Å aluminum oxide film. These particles entered the film with an energy of 62.5 keV. The shape of the observed energy distribution curves for A$^{40}$ was practically independent in the energy range 10 keV $< E < 70$ keV investigated.

The tail of the distribution suggests that an appreciable portion of the energy lost by A$^{40}$ particles is being taken up by recoiling atoms, as one would expect (Harvey, 1960) for a low-speed heavy particle such as here. When appreciable energy is lost in such an atomic collision, the particle suffers a relatively wide angle scattering. For this particle to
reappear in the forward direction, and be observed in this experiment, it must experience additional wide angle scattering. Calculations by J.H. Ormrod (1962) indicate that multiple wide angle scattering in which a heavy particle emerges from the film in the forward direction is a frequent enough occurrence to account for the tail in Figure 10.

In short, we conclude that the presence of a tail at the low energy end of the energy distribution, as in Figure 10, is evidence that atomic collisions are taking place. Conversely, the absence of such a tail, as was observed for H\textsuperscript{1} and He\textsuperscript{4} in the 5 keV < E < 30 keV energy range, is regarded as evidence that the projectile loses its energy primarily to the electrons of the stopping medium and, consequently, suffers little deflection. Certain observations on the angular distribution of the emerging beam, to be described in the next chapter, also substantiate this conclusion.

(2) **The Width of the Energy Distribution**

(a) **Introduction**

In the following, we shall refer to the full width of the observed energy distribution curves at half height by $W_0$. This width is larger than the corresponding "true width", $W$. This is due to the following effect. When the electrostatic analyser is set to detect particles of a certain energy, it will
Figure 10 - Energy Distribution Amongst $^{40}$A$^\alpha$ Particles after Traversing an Al$_2$O$_3$ Film of 205Å Thickness
actually detect particles over a small energy interval, because of its finite resolution. It is for this reason that we observed (see Figure 5) that the energy distribution of the beam without a film in its path had a finite width. We shall denote the full width of these energy distribution curves by \( w \). Experimentally, it was observed that

\[
  w = \frac{E}{120} \quad (34a)
\]
gives a rough approximation for the magnitude of \( w \).

In this expression \( w \) and \( E \) should both be expressed in the same units; for example in keV.

When the actual energy distribution amongst the particles leaving the film is Gaussian and the energy distribution of the beam, without a film in its path, as observed by the electrostatic analyser, is also Gaussian, then it may easily be shown that the actual full width, \( W \), is given by

\[
  W = \sqrt{W_0^2 - w^2} \quad (34b)
\]

Here \( w \) may be estimated from Equation (34a). For the purpose of correcting our observed values of \( W_0 \) we shall assume Gaussian distribution curves for \( H^1 \) and \( He^4 \) as projectiles and we shall, therefore, use Equations (34a) and (34b) to evaluate the actual full width, \( W \).
Figure 11. Energy Distribution Amongst $^1$H Particles, for Various Incident Energies, after Traversing a Carbon Film of 250Å Thickness.
(b) The Straggling in the Energy Loss of H\textsuperscript{1} in Carbon

Figure 11 shows about half of the energy distribution curves (so-called straggling curves) that were observed for H\textsuperscript{1} particles passing through the 250 Å carbon film. The corresponding energy loss observations in this film have previously been discussed. (III, 1, b). The insets near the top of each curve gives the nominal values of the energy loss, in units of keV, and the observed width, W\textsubscript{0}. The values of the average energy $\bar{E} = E_0 - \frac{\Delta E_0}{2}$ can readily be evaluated from these data. They have been tabulated in the first column of Table IV, which has been compiled from the numbers in the insets of Figure 11.

**Table IV**

<table>
<thead>
<tr>
<th>$\bar{E}$</th>
<th>$W_0$</th>
<th>W</th>
<th>$\Delta E_0$</th>
<th>$W/\Delta E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>keV</td>
<td>keV</td>
<td>keV</td>
<td>keV</td>
<td></td>
</tr>
<tr>
<td>29.9</td>
<td>1.18</td>
<td>1.15</td>
<td>3.86</td>
<td>.30</td>
</tr>
<tr>
<td>25.6</td>
<td>1.05</td>
<td>1.03</td>
<td>3.69</td>
<td>.28</td>
</tr>
<tr>
<td>21.5</td>
<td>1.00</td>
<td>.98</td>
<td>3.40</td>
<td>.29</td>
</tr>
<tr>
<td>17.9</td>
<td>.89</td>
<td>.88</td>
<td>3.19</td>
<td>.28</td>
</tr>
<tr>
<td>13.7</td>
<td>.75</td>
<td>.74</td>
<td>2.87</td>
<td>.26</td>
</tr>
<tr>
<td>9.6</td>
<td>.63</td>
<td>.62</td>
<td>2.48</td>
<td>.25</td>
</tr>
<tr>
<td>6.8</td>
<td>.57</td>
<td>.57</td>
<td>2.12</td>
<td>.27</td>
</tr>
<tr>
<td>4.2</td>
<td>.33</td>
<td>.33</td>
<td>1.72</td>
<td>.19</td>
</tr>
</tbody>
</table>
From the Table, it appears that the value of the quotient formed by the corrected width, \( W \), (shown in the third column) and \( \Delta E_0 \) (shown in the next column) is almost independent of energy in the range \( 7 < E < 30 \) keV, even although \( W \) and \( \Delta E_0 \) are both markedly dependent upon energy. This, of course, means that the energy dependence of \( W \) and \( \Delta E_0 \) are similar. The average value of the quotient, taking also into account the values of \( W \) for other observed straggling curves, not shown in Figure 11, is

\[
(W/\Delta E_0)_{\text{ave}} = .26
\]

(c) The Straggling in the Energy Loss of \( \text{He}^4 \) in Carbon

Figure 12, which is similar to Figure 11, shows some of the straggling curves that were observed when \( \text{He}^4 \) traversed the carbon film. Table V, which is similar to Table IV, has been compiled from the numbers in the insets of Figure 12 and also from the observed energy distribution curves not shown in this Figure. From this Table we find that the average value of the quotient \( W/\Delta E_0 \) is

\[
\left( \frac{W}{\Delta E_0} \right)_{\text{ave}} = .29
\]

Again, as for \( \text{H}^1 \) (see Table IV), we observe that large deviations from this average value occur. However, these deviations appear to be random. This justifies our conclusion that, within experimental error, \( W/\Delta E_0 \)
Figure 12. Energy Distribution Amongst $^{4}\text{He}$ Particles, for Various Incident Energies, after Traversing a Carbon Film of 250° Thickness.

- $E_i = 32.58$, $\Delta E = 4.46$, $W_i = 1.41$
- $E_i = 28.17$, $\Delta E = 4.21$, $W_i = 1.30$
- $E_i = 24.87$, $\Delta E = 4.01$, $W_i = 1.15$
- $E_i = 19.20$, $\Delta E = 3.60$, $W_i = 1.41$
- $E_i = 14.90$, $\Delta E = 3.21$, $W_i = 0.93$
- $E_i = 10.41$, $\Delta E = 2.71$, $W_i = 0.80$
is constant for He$^4$ in carbon in the energy range studied.

### Table V

<table>
<thead>
<tr>
<th>$E$ (keV)</th>
<th>$W_0$ (keV)</th>
<th>$W$ (keV)</th>
<th>$\Delta E_0$ (keV)</th>
<th>$W/\Delta E_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.4</td>
<td>1.41</td>
<td>1.39</td>
<td>4.46</td>
<td>0.31</td>
</tr>
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<td>1.37</td>
<td>4.36</td>
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<td>1.28</td>
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</tr>
<tr>
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<td>1.23</td>
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<td>1.13</td>
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</tr>
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<td>1.14</td>
<td>3.75</td>
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<td>1.08</td>
<td>1.07</td>
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<td>0.92</td>
<td>0.91</td>
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<td>13.3</td>
<td>0.93</td>
<td>0.92</td>
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<tr>
<td>11.5</td>
<td>0.88</td>
<td>0.87</td>
<td>3.04</td>
<td>0.29</td>
</tr>
<tr>
<td>9.1</td>
<td>0.80</td>
<td>0.79</td>
<td>2.71</td>
<td>0.29</td>
</tr>
</tbody>
</table>

(d) The straggling in the energy loss of H$^1$ in Al$_2$O$_3$

In Table VI, which is similar to Table IV, we have tabulated some typical observations obtained with the 205° aluminum oxide foil for the θ = 2.5° film. The corresponding energy loss observations of H$^1$ in the θ = 2.5° film have previously been discussed (III, 1, a). The average value of $W/\Delta E_0$ for the θ = 2.5° film is

$$(W/\Delta E_0)_{ave} = 0.36$$
Roughly the same value of $W/\Delta E_0$ was also observed for the $\theta = 42.5^\circ$ ($t' = 278\text{Å}$), the $\theta = 57.5^\circ$ ($t' = 383\text{Å}$) and the $\theta = 67.5^\circ$ ($t' = 536\text{Å}$) films. However, we shall see later [Equation (49)] that $W/\Delta E_0$ should depend upon film thickness in the following manner:

$$\frac{W}{\Delta E_0} \propto \frac{1}{\sqrt{t'}}.$$  

This means that $W/\Delta E_0$ should decrease as $t'$ increases. This dependence was not observed and, unfortunately, we must conclude that our observed energy distributions for the larger angles $\theta$ are too wide. The reason for this discrepancy is thought to be due to the following effect.

Our Al$_2$O$_3$ films were not perfectly flat but possessed small ripples. The area of these
irregularities was of the order of the cross-sectional area of the bombarding ion beam. When such a film is normal to the incident ion beam, practically all particles traverse nearly the same film thickness. When the film is rotated through a certain angle $\theta_0$, the average film thickness, as seen by the ion beam, is approximately given by

$$t' = \frac{t}{\cos \theta_0}$$

However, owing to small ripples, variations in $\theta$, around the average value $\theta_0$, will occur. Thus some particles will traverse more film material and others will traverse less film thickness than the average value $t'$. This effect will noticeably broaden the width, $W_0$, only when $\theta$ is appreciably larger than zero.

The small ripples, however, will only slightly affect the observed energy loss, $\Delta E_0$. Thus, although we conclude that our observed widths, $W_0$, for the energy distribution curves corresponding to the larger angles $\theta$ are wrong, we still believe that the corresponding observed $\Delta E_0$ values are approximately correct.

(e) The Straggling in Energy Loss of He$^4$ in Al$_2$O$_3$

No detailed experiments in which $\theta$ was varied were carried out with He$^4$ in passing through Al$_2$O$_3$ films. In Table VII, which is similar to Table VI,
most of the observations pertaining to the energy loss measurements of \( \text{He}^4 \) in passing through the 205\( \AA \) film have been tabulated.

**Table VII**

<table>
<thead>
<tr>
<th>( E \text{(keV)} )</th>
<th>( W_0 \text{(keV)} )</th>
<th>( W \text{(keV)} )</th>
<th>( \Delta E_0 \text{(keV)} )</th>
<th>( W/\Delta E_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>1.61</td>
<td>1.59</td>
<td>4.2</td>
<td>0.38</td>
</tr>
<tr>
<td>26</td>
<td>1.34</td>
<td>1.32</td>
<td>4.0</td>
<td>0.33</td>
</tr>
<tr>
<td>24</td>
<td>1.40</td>
<td>1.39</td>
<td>3.8</td>
<td>0.37</td>
</tr>
<tr>
<td>21</td>
<td>1.12</td>
<td>1.10</td>
<td>3.6</td>
<td>0.31</td>
</tr>
<tr>
<td>19</td>
<td>1.24</td>
<td>1.23</td>
<td>3.4</td>
<td>0.36</td>
</tr>
<tr>
<td>16</td>
<td>1.10</td>
<td>1.09</td>
<td>3.3</td>
<td>0.33</td>
</tr>
<tr>
<td>14</td>
<td>1.05</td>
<td>1.04</td>
<td>3.0</td>
<td>0.35</td>
</tr>
<tr>
<td>11</td>
<td>1.08</td>
<td>1.08</td>
<td>2.8</td>
<td>0.39</td>
</tr>
<tr>
<td>9</td>
<td>0.90</td>
<td>0.90</td>
<td>2.5</td>
<td>0.36</td>
</tr>
<tr>
<td>7</td>
<td>0.78</td>
<td>0.78</td>
<td>2.3</td>
<td>0.34</td>
</tr>
</tbody>
</table>

The average value of \( \Delta E_0/W \), as obtained from Table VII is \( \Delta E_0/W = 0.35 \).

(f) **Summary of Results**

Table VIII summarizes the observed results for the full widths of the energy distribution curves for \( \text{H}^1 \) and \( \text{He}^4 \) in passing through a 205\( \AA \) \( \text{Al}_2\text{O}_3 \) film and a 250\( \AA \) carbon film respectively. The straggling data from this Table will be discussed in Chapter VI.
<table>
<thead>
<tr>
<th>Film</th>
<th>$\frac{W}{AE_0}$ for $H^1$</th>
<th>$\frac{W}{AE_0}$ for $He^{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>205Å $Al_2O_3$ Film</td>
<td>$0.36 \pm 0.02$</td>
<td>$0.35 \pm 0.02$</td>
</tr>
<tr>
<td>250Å Carbon Film</td>
<td>$0.26 \pm 0.02$</td>
<td>$0.29 \pm 0.01$</td>
</tr>
</tbody>
</table>
CHAPTER V

Observations On the Angular Distribution of the Beam Emerging From Thin Films

For this experiment, the apparatus was modified in the manner shown in Figure 4. Ideally, both $S_3$ and $S_5$ should have been circular apertures, or $S_3$ should have been an infinitely long narrow slit and $S_5$ a narrow slit of finite length. In the first case, one obtains easily interpretable results and in the latter case one observes "true" angular distribution curves directly. In practice, however, $S_3$ and $S_5$ were narrow slits whose long dimensions (0.125" for $S_3$ and 0.5" for $S_5$) were perpendicular to the plane of the paper and were, of course, finite. Consequently, in every position, $\Phi$, the detector accepted particles that had suffered net deflections over a complicated range of angles.

The results of this crude experiment, however, were adequate for our purpose which was to confirm our suspicion that the $H^1$ and $He^4$ particles under study suffered relatively few wide angle deflections; that is, relatively few atomic collisions. The film used in this experiment was a VYNS film, in which $\Delta E_0$ was about one-third of its value for the 205Å aluminum oxide foil.

Figure 13, which is a plot of the output of the electron multiplier as a function of its angular position, shows results obtained with $H^1$, $He^4$ and $A^{40}$, each at two
FIGURE 13 - Angular Distribution Amongst H\(^1\), He\(^4\) and A\(^{40}\) Particles after Emerging from a VYNS Film.
different values of incident energy. For a given projectile, the incident current was the same for the two curves. Evidently the $^{40}\text{A}$ particles have suffered wide-angle deflections, whilst the deflections suffered by $^1\text{H}$ and to a somewhat lesser extent, $^4\text{He}$, are primarily small-angle ones, especially at the higher value of the energy. It is a simple problem (VI-1) in mechanics to show that, if $^1\text{H}$ and $^4\text{He}$ were losing an appreciable amount of this energy in elastic collisions (with atoms of the stopping medium), the average scattering angle would be much larger than we have observed. Thus, it follows from this experiment that, in the energy range investigated, $^1\text{H}$ and $^4\text{He}$ lose their energy mainly to electrons in the stopping medium. This conclusion is consistent with the nearly symmetric energy distribution curves discussed in the preceding chapter and suggests that our observed $S_0$'s, which refer to particles moving in the forward direction, are practically identical with $S_e$, the electronic component of the total stopping cross-section per atom.

The broad angular distribution curves for $^{40}\text{A}$ are also consistent with the conclusions of the previous chapter. These curves are indicative of atomic collisions, as were the pronounced low-energy tails that characterize the energy distributions for this heavy particle.
CHAPTER VI

Interpretation of Results

(1) Calculation of the Energy Lost in Atomic Recoils for $H^1$.

In the following, a calculation will be made which will allow us to use our angular distribution data to make a quantitative estimate of the energy lost in atomic recoils. We shall find that, at energies greater than a few keV, only a small fraction of the total energy lost by $H^1$ is lost in this way.

The maximum energy transfer, $T_{\text{max}}$, of a particle of mass $M_1$ and a kinetic energy $E$ in a head-on collision with a target particle of mass $M_2$ at rest equals

$$T_{\text{max}} = \frac{4 \frac{M_1 M_2}{(M_1 + M_2)^2} E}{(M_1 + M_2)^2} \quad (35)$$

In a glancing collision, where the particles are deflected through a relative deflection angle $\alpha$, the energy transfer, $T$, equals

$$T = T_{\text{max}} \sin^2 \frac{\alpha}{2} \quad (36)$$

The relation between the deflection angle, $\phi$, of the projectile in the laboratory co-ordinates and the relative deflection angle is

$$\tan \phi = \frac{M_2 \sin \alpha}{M_1 + M_2 \cos \alpha} \quad (19)$$
For H\textsuperscript{1} travelling in VYNS (C\textsubscript{4}H\textsubscript{5}O\textsubscript{2}Cl)\n
\[ M_1 \leq M_2 \]

Hence, it may easily be shown that

\[ \frac{\alpha}{2} \leq \phi \leq \alpha \] (37)

Suppose the square of the resultant scattering angle, \( \Phi \), (in laboratory co-ordinates), after passing through a sheet of matter is small (as was the case for our measurements of H\textsuperscript{1} above a few keV). Then essentially all the individual scattering angles, \( \phi_i \), (and consequently \( \alpha_i \)) will also be small. Therefore (Bohr - 1948 - Equation 2.5.1)

\[ \frac{\Phi^2}{\Phi} = \sum_i \phi_i^2 \] (38)

where the summation extends over all scattering events in the process of traversal of the film.

The energy loss in a single scattering event for a small deflection angle, \( \alpha_i \), is given approximately by [see Equation (36)]

\[ T_i = T_{\text{max}} \left( \frac{\alpha_i}{2} \right)^2 \]

The total energy loss, \( \Delta E_n \), of the particles corresponding to the resultant scattering angle \( \sqrt{\frac{\Phi^2}{\Phi}} \) is

\[ \Delta E_n = \sum_i T_i = \sum_i T_{\text{max}} \left( \frac{\alpha_i}{2} \right)^2 \]

The subscript, \( n \), in this formula and others to follow, denotes that only the loss in atomic recoils has
been considered.

If the film is thin (as in our angular distribution experiments) the energy of the particle is approximately constant, in which case $T_{\text{max}}$ is seen to be approximately constant [Equation (35)]. Then the preceding equation may be re-written

$$\Delta E_n = T_{\text{max}} \sum_i \left( \frac{\alpha_i}{2} \right)^2$$

By utilizing Equation (37) we see that

$$\Delta E_n < T_{\text{max}} \sum_i \phi_i^2$$

Upon substitution of Equations (35) and (38), this expression becomes

$$\Delta E_n < \frac{4 M_1 M_2}{(M_1 + M_2)^2} \bar{E} \bar{\phi}^2$$

from which an upper limit to the energy lost in atomic collisions may be calculated, provided the angular distribution is known. This we shall now do.

The most probable energy loss, $\Delta E_0$, of $H^1$ in traversing the VYNS film, which was employed in our angular distribution experiments, was

$$\Delta E_0 \approx 0.6 \text{ keV at } \bar{E} \approx 10 \text{ keV}$$

and

$$\Delta E_0 \approx 0.3 \text{ keV at } \bar{E} \approx 4 \text{ keV}$$

The root mean square of a Gaussian distribution curve is equal to its half-width. Our angular distribution curves are approximately Gaussian. They are sharper, however, than the true angular distribution curves.
because slit $s_3$ (see Figure 4 and also Chapter V) had a finite rather than an infinite length. On the other hand, the half widths of the experimental curves are significantly broadened (by about 0.5 degrees) because of the finite acceptance angle of the detector. Taking these factors into account, it appears that no serious error is introduced by assuming that $\sqrt{\Phi^2}$ is equal to the half width of our observed angular distribution curves.

The half width of the distribution curve for, say, $E_0 (\Delta E = E_0 - \frac{\Delta E_0}{2}) = 10$ keV in Figure 13 is about 2 degrees. Thus

$$\sqrt{\Phi^2} = 2^\circ = \frac{2 \times 2\pi}{360} \text{ radians}.$$ 

Hence

$$\Phi^2 = \left(\frac{2 \times 2\pi}{360}\right)^2$$ \hspace{1cm} (41)

For VYNS ($C_4H_5O_2Cl$), $M_2$ in Equation (39) can have different values. The average value is $\approx 10$ amu. Substituting this value and $N_1 = 1$ amu for $H^1$, together with Equation (41) into Equation (39) we find for $E = 10$ keV

$$\Delta E_n < 0.004 \text{ keV}$$

Hence, compared to the total observed energy loss, $E$ of 0.6 keV (at $E \approx 10$ keV) it is indeed true that

$$\Delta E_n \ll \Delta E_0$$

This means that atomic collisions cannot account for the energy loss, $\Delta E_0$. Therefore, atomic hydrogen at $E = \ldots$
10 keV loses its energy mainly to the electrons of the stopping medium.

Similarly, at $E = 4$ keV, we find, by means of our observed angular distribution curves for $H^1$ at lower energies, which have not been shown in Figure 13, that

$$\Delta E_n < 0.064 \text{ keV}$$

The total energy loss, $\Delta E_0$, at this energy, however, equals (Equation 40)

$$\Delta E_0 \approx 0.3 \text{ keV}$$

Hence, we conclude that $H^1$ with a kinetic energy above a few keV loses its energy mainly to the electrons of the stopping medium.

At this point, a remark might be made concerning the energy loss of $He^4$. Previously, we concluded that, for energies greater than a few keV, both $He^4$ and $H^1$ lose their energy mainly to electrons, although, as we shall see in the next section, the theory does predict a small but finite loss of energy in atomic recoils for both of these atomic projectiles. The relative importance of this mechanism of energy loss is predicted to be appreciably larger (roughly five times) for $He^4$ than for $H^1$. The observed angular distribution curves of the transmitted beam are in qualitative agreement with the prediction, inasmuch as the relative widths of the

$\ast$ At higher energies, above $\sim 100$ keV, protons lose energy by Coulomb excitation and ionization of the electrons in the stopping medium.
observed angular distribution curves of the transmitted beam are larger for He\(^4\) than for H\(^1\).

(2) Comparison of Observed Cross-section, \(S_0\), with Theory

On the basis of the energy distributions in the transmitted beams of H\(^1\) and He\(^4\), we concluded that these particles lose their energy primarily to the electrons of the stopping medium rather than in atomic collisions. The results of the angular distribution experiments are in agreement with this conclusion. We should further note that the predictions of the theory are also in accord with this conclusion. In order to show this, we have plotted in Figure 14 (a) the \(S_e\) - versus - E curve [using Equation (1)]; (b) the \(S_n\) - versus - E curve [using Equation (10)], and (c) the experimentally observed \(S_0\) - versus - E curve, in each case for H\(^1\) passing through carbon. Figure 15 shows the corresponding curves for He\(^4\) passing through carbon. Figures 16 and 17, respectively, show these corresponding curves for H\(^1\) and He\(^4\) passing through Al\(_2\)O\(_3\). The \(S_e\) and the \(S_n\) curves for the slowing down process in Al\(_2\)O\(_3\) were obtained by taking as average values \(Z_2 = 10\) and \(M_2 = 20\).

In these figures, the calculated values of \(S_n\) are much less than those of \(S_e\), although the difference is not as marked for He\(^4\) as for H\(^1\). Moreover, in our experiments, only a small portion of \(S_n\) comes into the picture, inasmuch as most of the atoms that suffer appreci-
Figure 14 - Composite Plot Showing (a) Calculated Nuclear Component of Total Stopping Cross Section per Atom, (b) Calculated Electronic Component of Total Stopping Cross Section per Atom, and (c) Observed Stopping Cross Section per Atom, All as a Function of Energy, for H\textsuperscript{1} Passing through Carbon.
Figure 15 - Composite Plot Showing (a) Calculated Nuclear Component of Total Stopping Cross Section per Atom, (b) Calculated Electronic Component of Total Stopping Cross Section per Atom, and (c) Observed Stopping Cross Section per Atom, All as a Function of Energy, for He\(^4\) Passing through Carbon.
Figure 16 - Composite Plot Showing (a) Calculated Nuclear Component of Total Stopping Cross Section per Atom, (b) Calculated Electronic Component of Total Stopping Cross Section per Atom, and (c) Observed Stopping Cross Section per Atom, All as a Function of Energy, for H\textsuperscript{1} Passing through Al\textsubscript{2}O\textsubscript{3}
Figure 17 - Composite Plot Showing (a) Calculated Nuclear Component of Total Stopping Cross Section per Atom, (b) Calculated Electronic Component of Total Stopping Cross Section per Atom, and (c) Observed Stopping Cross Section per Atom, All as a Function of Energy, for He\textsuperscript{4} Passing through Al\textsubscript{2}O\textsubscript{3}. 
able energy loss in atomic collisions are scattered out of the beam and consequently, are not observed. The multiple wide-angle scattering event, mentioned earlier for A\(^{40}\), in which the particle regains its original direction, is relatively rare for H\(^1\) and He\(^4\) at the energies used.

Hence, on theoretical, as well as experimental, grounds, we appear justified in identifying our observed \(S_0\) with \(S_e\), the electronic component of the total stopping cross-section per atom. With this understanding, let us now compare the form of the \(S_0\)-versus-\(E\) curve with that predicted for \(S_e\).

We note that the empirical expressions for \(S_0\) \[Equations (31), (32), (33) and (34)\] do not display the same energy dependence that is predicted for \(S_e\), the electronic component of the stopping cross-section per atom. Whereas \(S_e\) is predicted to vary as \(E^{1/2}\), our observed \(S_0\)'s vary as \(E^{0.45}\) and \(E^{0.44}\) for H\(^1\) and He\(^4\) respectively, in Al\(_2\)O\(_3\), and as \(E^{0.41}\) and \(E^{0.40}\) for H\(^1\) and He\(^4\) in carbon, respectively.

Three explanations for the discrepancy suggest themselves:

(a) The electronic component \(S_e\) does not correspond closely to our observed \(S_0\)'s, as we have assumed. Instead, there is a not-insignificant \(S_n\) contribution which, since it increases with decreasing energy, causes the \(S_0\)-versus-\(E\) curve to be flatter than the \(S_e \propto \sqrt{E}\).
one, as is experimentally observed. This effect, at
any given energy, should be much more pronounced for
He\(^4\) than for \(H^1\) - a prediction which appears not to
be borne out by experiment.

(b) We are operating in a region where electronic
losses are the dominating ones all right, but the transi-
tion to the \((\ln E/V_E)\) region has already begun. This
effect, also, would cause the \(S_0\)-versus-\(E\) curve to be
flatter than the \(S_e \propto \sqrt{E}\) one. Moreover, the effect
should be more noticeable, at any given energy, for \(H^1\)
than for \(He^4\), a prediction not borne out by experiment.

(c) The predicted dependence of \(S_e\) upon \(E\) is
somewhat in error. We appear to be left with this
alternative.

Finally, let us compare the absolute magnitudes
of the observed \(S_0\)'s and the calculated \(S_e\)'s. For \(H^1\),
as can be seen from Figures 14 and 16, the ratio
\(S_0/S_e\) is never far from unity over the entire energy
range studied. Indeed, the uncertainties in the thick-
nesses of our film are such that the average value of
the ratio may be even closer to unity than appears to
be the case in the figures. For \(He^4\), however, (see
Figures 15 and 17), the absolute values appear to be
systematically in error.

At this point we quote from a recent private
communication from Professor Lindhard: "It should be
emphasized that Equation (4) is approximate in more
than one sense. The constant \( \frac{\xi}{Z_e} \sim Z^{1/6} \) is based on Thomas-Fermi arguments and it is to be expected that fluctuations around this constant can occur, especially for \( Z_1 \leq 10 \). Moreover, a precise proportionality to \( v \) (i.e. \( \sqrt{E} \)) will not be correct over the whole of the velocity region \( v < v_1 \), \( (v_1 = v_0 Z_1^{2/3}) \). It would appear that our observations of the energy dependence of \( S_0 \), and of the systematic difference between \( S_0 \) and \( S_e \) values for \( \text{He}_4 \), illustrate the relevance of Professor Lindhard's remarks.

(3) Comparison of \( W/\Delta E_0 \) with Theory

The energy distribution amongst the particles leaving the film arises out of two processes:

(a) Not all particles in traversing the film suffer the same number of collisions; some particles suffer more, others undergo fewer, collisions than some average number.

(b) Not all particles that undergo the same number of collisions suffer the same energy loss per collision.

Bohr (1948) investigates the straggling in some detail. He writes the total energy loss, \( \Delta E \), of a particle in traversing a sheet of matter of thickness, \( t \), as

\[
\Delta E = \sum_i T_i n_i \quad (4.2)
\]
Here $n_i$ is the number of collisions, for the particle under consideration, in which it transfers the energy $T_i$ per collision. $n_i$ will be distributed around its mean value, $\overline{n_i}$, according to a Poisson distribution. Thus

$$P(n_i) = \frac{n_i^{n_i} e^{-n_i}}{n_i!}$$

(43)

The average value of $\Delta E$, which we have denoted as $\Delta E_0$, is given by

$$\Delta E_0 = \sum_i T_i \overline{n_i}$$

(44)

with a mean square deviation.

$$\mathcal{N}^2 = \langle (\Delta E - \Delta E_0)^2 \rangle = \sum_i T_i^2 (n_i - \overline{n_i})^2 = \sum_i T_i^2 \overline{n_i}$$

(45)

since $(n_i - \overline{n_i})^2 = \overline{n_i}$ for a Poisson distribution.

Here $\mathcal{N}$ represents the half width of the energy distribution, amongst the particles leaving the sheet of matter, at half its maximum height. Thus

$$W = 2 \mathcal{N}$$

(46)

Equation (42) is equivalent to

$$\Delta E_0 = N \cdot \mathcal{N} \cdot Td\sigma$$

(47)

where $\int Td\sigma = S$ [see Equation (29)] represents the stopping cross-section per atom. Equation (45) is equivalent to

$$\mathcal{N}^2 = N \cdot \int T^2 d\sigma$$

(48)

We note from Equations (47) and (48) that
\[
\frac{\Omega}{E_0} \propto \frac{1}{\sqrt{t}}. \tag{49}
\]

From the foregoing it is evident that the relationship between \(\Omega\) and \(\Delta E_0\) can only be found if the energy transfer, \(T\), can be expressed as a function of the differential cross-section, \(d\sigma\). This, of course, involves a knowledge of the mechanism of the energy loss.

In our experiments with \(H^1\) and \(He^4\) as projectiles, the loss in energy to electrons was studied and the results compared with the theory of Lindhard and Scharff (1961). One of these author's Lindhard, has also studied the straggling in the energy loss and finds (Lindhard, 1954, Equation 4.16) for a particle moving in a degenerate Fermi gas that

\[
\Omega^2 = 4\pi z_1^2 e^4 \rho L_\text{ab} t \tag{50}
\]

Here \(\rho\) is the electron density. Lindhard has considered the form of \(L_\Omega\) and finds that over a wide region of densities \(L_\Omega\) is proportional to \(\rho^{-1/2}\). In an actual stopping medium, of course, the electron density will strongly vary as the particle moves a distance of the order of the interatomic distances in the absorber. However, since the form of \(L_\Omega\) does not depend upon the magnitude of \(\rho\) it is meaningful to take for \(\rho\) the average electron density in the stopping medium. In the calculations to follow, we shall, therefore, take for \(\rho\) the average electron density of the
stopping material.

For low velocities Lindhard obtains the following estimate for a degenerate electron gas

\[
\frac{\Omega^2}{\Delta E_o} \simeq (5mv^2 + \hbar \omega) \frac{1}{2} \tag{51}
\]

where

\[
\omega = \sqrt{\frac{4k_e^2 \rho}{m}}
\]

is the so-called plasma frequency. Here \(m\) is the electronic mass. \(\Omega/\Delta E_o\) can now be found by substitution of

\[
\Delta E_o = N_S e t
\]

(see section III, 1, a) where \(S_e\) is given by Equation (4). On carrying out the substitution and inserting the known values of the constants we find

\[
\frac{\Omega}{\Delta E_o} = 6.14 \times 10^4 \left[ \frac{(Z_1^{2/3} + Z_2^{2/3})^{3/2}}{Z_1^{7/6} Z_2} \cdot \frac{\rho^{1/4}}{N_t} \right]^{1/2}
\]

Thus, see Equation (46),

\[
\frac{W}{\Delta E_o} = 1.23 \times 10^5 \left[ \frac{(Z_1^{2/3} + Z_2^{2/3})^{3/2}}{Z_1^{7/6} Z_2} \cdot \frac{\rho^{1/4}}{N_t} \right]^{1/2} \tag{52}
\]

From Equation (52) we observe that \(W/\Delta E_o\) is independent of energy. This is in agreement with observation (Chapter IV).

We shall now compare our observed values of \(W/\Delta E_o\)
with Equation (52). The electron density can be estimated as follows. For our carbon film we assumed that

\[ N = 1.13 \times 10^{23} \text{ atoms/cm}^3 \]

Thus

\[ \rho = Z^2 N = 6 (1.13 \times 10^{23}) \text{ electrons/cm}^3 \]

For our \( \text{Al}_2\text{O}_3 \) film we assumed that

\[ N = 1.03 \times 10^{23} \text{ atoms/cm}^3 \]

and that the average value of \( Z^2 \approx 10 \). Thus

\[ \rho = Z^2 N = 10(1.02 \times 10^{23}) \text{ electrons/cm}^3. \]

In Table IX, we have tabulated the calculated values of \( W/\Delta E_0 \). This Table also presents the corresponding observed values of \( W/\Delta E_0 \) (see Table VIII).

<table>
<thead>
<tr>
<th>Film</th>
<th>( W/\Delta E_0 ) (Observed)</th>
<th>( W/\Delta E_0 ) (Theory)</th>
<th>( W/\Delta E_0 ) (Observed)</th>
<th>( W/\Delta E_0 ) (Theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>205( \text{Å} ) ( \text{Al}_2\text{O}_3 ) Film</td>
<td>.36 ± 0.02</td>
<td>.31</td>
<td>.35 ± 0.02</td>
<td>.22</td>
</tr>
<tr>
<td>250( \text{Å} ) Carbon Film</td>
<td>.26 ± 0.02</td>
<td>.27</td>
<td>.29 ± 0.01</td>
<td>.20</td>
</tr>
</tbody>
</table>

From this Table we notice that Equation (52) predicts the right order of magnitude for \( W/\Delta E_0 \).

(4) Conclusion

Thus we conclude that our measurements are in tolerable agreement with the Lindhard and Scharff theory, for loss in energy to electrons of low speed.
atomic particles, in more than one sense:

(a) In the energy range studied $H^1$ and $He^4$ lose their energy mainly to electrons.

(b) The theory correctly predicts the magnitude of the observed stopping cross-section, $S_0$, and predicts fairly accurately its energy dependence.

(c) The theory gives a reasonable estimate for the magnitude of $W/\Delta E_0$ and also correctly predicts that this quotient is independent of energy.

Our results, therefore, confirm that for the purpose of energy loss of low speed atomic particles the electrons in the stopping medium may, to a first approximation, be considered as a degenerate Fermi gas (see Appendix).
APPENDIX

The Energy Loss of an Atomic Particle in a Degenerate Electron Gas (Sect. I.2.b)

At high energies the speed of the moving particle is much larger than that of the orbital electrons in the stopping medium. These electrons may, therefore, to a first approximation be considered as stationary. Using arguments of this type Bloch (1933) showed that

$$ - \frac{dE}{dx} \propto \frac{1}{E} \ln E $$

At low energies, i.e. for velocities below the orbital velocities of the electrons the conventional stopping power formula, applicable to fast heavy particles, no longer represents a useful approximation.

Fermi and Teller (1947) deduced an expression for the energy loss at low speeds, for particles much heavier than the electron, in the following manner. In their theoretical considerations they replaced the stopping medium by a degenerate electron gas, in which the maximum electron speed is approximately equal to the velocity, $v_0$, of the outer electrons in the atoms comprising the stopping medium. In such a gas the moving particle cannot excite, i.e. cannot collide with all the electrons in the Fermi gas. Owing to the Pauli principle only those electrons whose resulting speed after the collision will be outside the occupied zone of the velocity space of the degenerate
electron gas will be scattered.

In a collision between the projectile and an individual electron the change in speed of the latter will be of the order of $v$, the speed of the moving particle. Fermi and Teller considered the case when $v \ll v_0$. Thus only the electrons near the Fermi level will be scattered. In particular, only the electrons within the range $v_0 - v$ to $v_0$ participate in such collisions. From an evaluation of the density of the electrons for which collisions are not forbidden by the Pauli principle and from an estimate of the collision cross-section Fermi and Teller estimated the order of magnitude of the energy loss per unit time to be

$$- \frac{dE}{dt} \approx \frac{m^2 e^4 E}{M_1 \hbar^2}$$

Here $m$ is the mass of the electron, $M_1$ is the mass of the projectile and $\hbar$ is Planck's constant divided by $2\pi$.

The stopping power, $- \frac{dE}{dx}$, can easily be deduced from the above equation. We find

$$- \frac{dE}{dx} \approx \frac{m^2 e^4}{\hbar^2} \sqrt{\frac{E}{2M_1}}$$

It will be noted that $- \frac{dE}{dx}$ is proportional to the velocity of the moving particle. This velocity dependence is in agreement with the velocity dependence of the Lindhard and Scharff formula [Equation (3)] for the stopping cross-section for loss in energy to electrons.
Their expression is a generalization of the Fermi and Teller result and is applicable to all systems.
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