MATHEMATICAL MODELING AND COMPUTER CONTROL

OF A TWO-PHASE PERMANENT-MAGNET STEPPING MOTOR

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OF A TWO-PHASE PERMANENT-MAGNET STEPPING MOTOR

By

RICHARD C. S. WONG,   B.Eng.

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AUTHOR: Richard Chau-Sheung Wong,
B. Eng. (McMaster University)

SUPERVISORS: Professor A. R. Elliott,
B. Sc., M. Sc. (Queen's)
Ph.D. (Case Western Reserve)

Professor N. K. Sinha,
B. Sc. Eng(Banaras)
Ph.D. (University of Manchester)

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ABSTRACT:

The analysis of stepping motors using linear models has been simplified through identifications of the constants. Though the existing nonlinear models assuming a smooth air-gap machine may, in some cases, yield a fairly close prediction of the characteristics of a stepping motor, the models do not represent the actual motors which are essentially salient-pole motors. A new salient-pole nonlinear model is introduced. The analysis of permanent-magnet stepping motors using the nonlinear models has been simplified by assuming constant current sources. Dynamic behaviors of a permanent-magnet stepping motor are shown by phase-plane plots and step by step transient response plots.

Computer control of stepping motors in both open-loop and closed-loop is discussed. The open-loop control has been demonst-

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rated to be successful and a closed-loop control system using light-sensors as feedback has been designed.

ACKNOWLEDGEMENTS

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CHAPTER ONE

THE STEPPING MOTOR

1. Introduction

A stepping motor may be defined as an electromagnetic device which, when actuated by pulses in the right sequence, rotates the motor shaft by a fixed angle called a step such that the number of steps is equal to the number of input pulses with a stepping rate equal to the input pulse rate provided that it is operated in the stable region. The actual beginning of stepping motors is unknown, however in the early 1930s the British Navy used them in remote position control. The U.S. Navy used them widely in the second world war[6]. Stepping motors have many apparent advantages in position control applications. For instance, the digital nature of their driving sequence permits them to interface with many types of digital transducers, networks, and computers with a minimum of interface hardware.

Applications of stepping motors in fine positioning control have become more and more popular. Unlike servo-motors, stepping motors can be operated without loss of synchronism in open-loop control. In fact, the majority of the present applications of stepping motors are in open-loop control though in some sophisticated equipment closed-loop control techniques are used. The fast growth of stepping motor applications is attributed to the rapid

development in the digital control field as well as to improved modern technology.

2. Types of stepping motors

There are numerous types of stepping motors manufactured and being used. They may be classified either according to their step sizes (i.e. angular displacement in degrees per step) or according to their internal structures. In general they are classified into two major types - magnetic and mechanical stepping motors. The mechanical stepping motors are primarily rotary solenoid-operated indexing devices. They require some type of mechanical linkage to translate the sequential d.c. pulses into discrete, predetermined angles of shaft rotation. At the end of each step, the shaft is secured by a mechanical device. Both Proctor [6] and Veinott [39, chapter 15] have given detailed descriptions of this type of stepping motor and Hong[33] gives both a dynamic and static analysis of a solenoid actuated stepping motor. Magnetic stepping motors are mostly derived from the conventional alternate current motors. They may further be classified into variable reluctance and permanent-magnet motors. The variable reluctance stepping motors are mostly three-phase with a soft iron rotor. They resemble the conventional reluctance motors and are normally analysed by using smooth air-gap machine principles [25] . The permanent-magnet stepping motors are essentially synchronous motors with permanent-magnet rotors.

3. General characteristics of stepping motors

The general characteristics of stepping motors are usually described by a plot of their torque vs stepping rates. Figure 1-1 gives a general concept of this kind of plot. The magnitudes are not given because they vary over a very large range. Typical torque ranges vary from 1 oz-in to 2000 oz-in and the stepping rate for the stable region varies from as few as 10 steps per second to over 2000 steps per second. The torque - speed characteristics of a stepping motor may be divided into three regions [28] . They are the stable, slew, and unstable regions. In the stable region a stepping motor can start, stop and reverse on command with faithful response. The slew range is entered while the motor is running and within this region the motor cannot reverse or stop without losing steps, consequently the motor must be brought back to the stable region before it can be accurately stopped or reversed. A stepping motor may not be operated in the unstable region and when it enters this region, it oscillates.

The field windings of stepping motors have relatively high resistance, therefore when the terminal voltage is switched from a positive voltage to a negative voltage and vice versa there is a transient phenomenon similar to that of a R-L circuit which may give a nearly hundred percent overshoot in terminal voltage. This effect may damage the motor and may cause erroneous operation, consequently a transient voltage suppression circuit has to be used.

Another important feature of stepping motors is the deviation of rotor position under load from its no-load position. This angular deviation is directly proportional to the load torque.

unstable region

slew range

stable region

torque

no. of steps/second
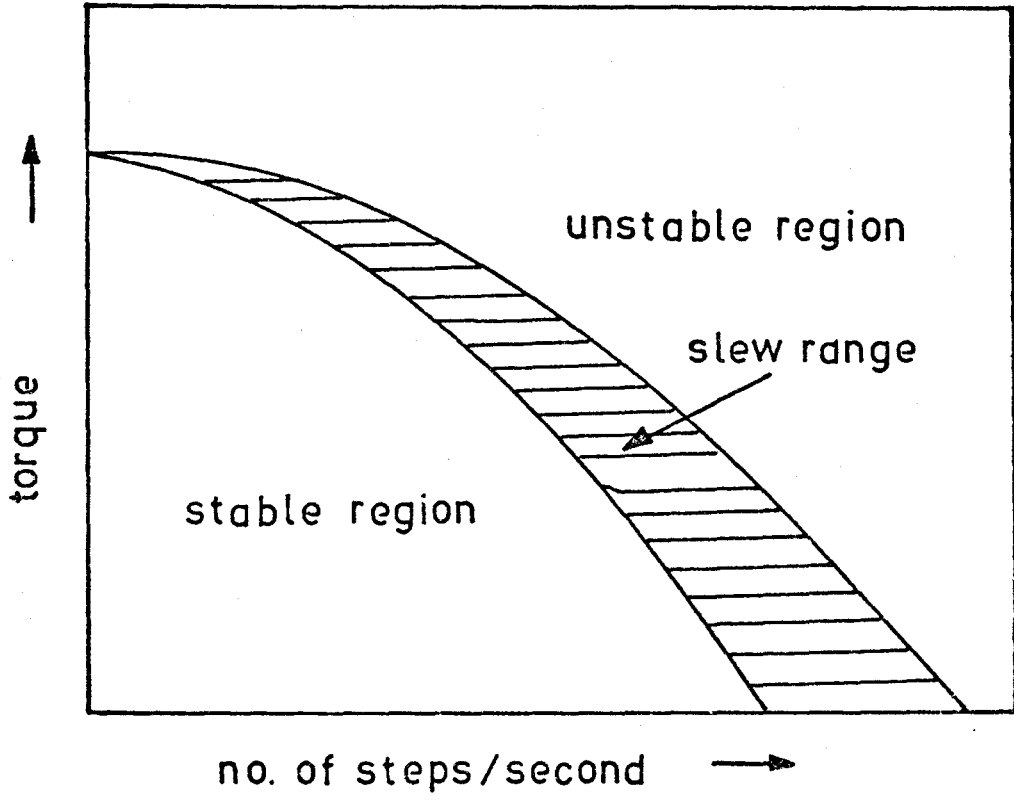
Figure 1 - 1

Torque - Speed Characteristics of Stepping Motors

% of holding torque

100

50

0

1 STEP

deviation from no-load position
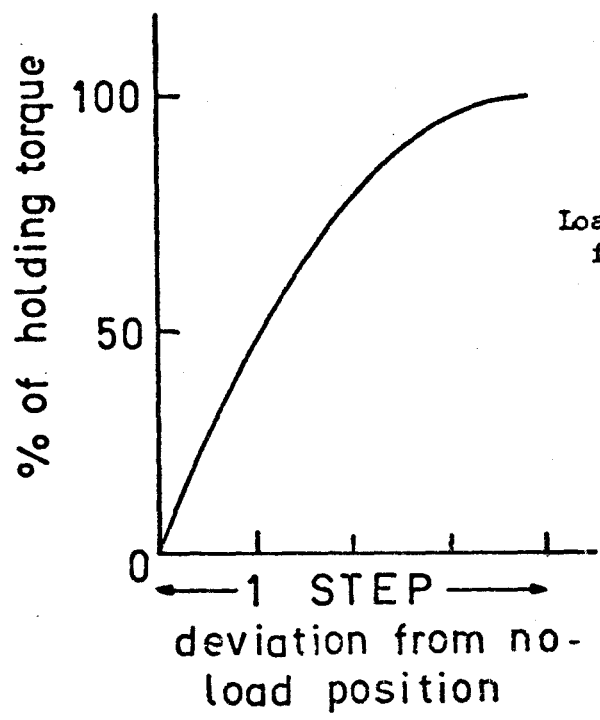
Figure 1 - 2

Load Torque vs Deviation from No-load

A typical curve is shown in Figure 1 - 2. It has been verified by many authors [4,7,24,36] that this deviation remains constant for a fixed load, therefore this deviation will not in general affect the accuracy of the rotor position very seriously provided that the load torque does not excede the pull-in torque given by the motor. However, for fine positioning and dynamic analysis, this deviation will become important because of the damping effects.

The synchronous speed of the magnetic type of stepping motors follows the same formula as the one used in large synchronous machines. The formula is simply:

$$v = 120 \ f/p \quad \text{rpm} \tag{1 - 1}$$

where v = synchronous speed, f = field current frequency in hertz, and p = number of poles, but in stepping motors , this number refers to the number of teeth in the rotor. For a permanent-magnet stepping motor with two rotors mounted on the same shaft equation (1-1) may be reduced to

$$v = 60 \ f/p_1 \quad \text{rpm} \tag{1 - 2}$$

where p_1 = number of teeth in one rotor. Since the time constant (L/R) of most stepping motors is very small, synchronism is achievable within milli-seconds.

4. Principles of operation

The principles of operation that are discussed in this section are mainly concerned with the two-phase permanent-magnet stepping motors. This type of motor is operated on a push-pull (or attract-repel) magnetic force [4] . Figure 1-3 gives a simplified

cross-section of this type of motor . The permanent-magnet flux path
in the figure illustrates the state at which the field windings are
unexcited.

A two-phase stepping motor may be operated by connecting the
terminals to a two-phase power supply as shown in Figure 1-4. Under
this operation the stepping motor resembles that of a self-started
synchronous motor having the synchronous speed given by equation 1-1.
For a stepping motor having fifty teeth in its rotor operating at a
power frequency of 60 Hz. the synchronous speed is 72 rpm. Since a
two-phase power supply is not commonly available, a modification
of this operation so that the motor may be operated with a single-
phase power supply is shown in Figure 1-5. This modification is
simply achieved by adding a resistor and a capacitor in series with
one of the phase windings. The main purpose of this external
connection is to give a 90 degree phase shift to the corresponding
phase-winding. The resistor in Figure 1-5 may be shorted and with
this operational technique the stepping motor has different speed
vs torque characteristics and may be considered as a permanent-split
-capacitor motor with its auxiliary winding identical to that of
the main winding [40, page 532-536] .

The direct current operation of a stepping motor is normally
referred to as pulse actuated operation. The motor responds to
each pulse input by advancing its shaft to a new angular position.
It is under this d.c. operation that the motor demonstrates its high-
est capabilities. The fundamental stepping sequence is directly
derived from the two-phase a.c. operation as shown in Figure 1-6.

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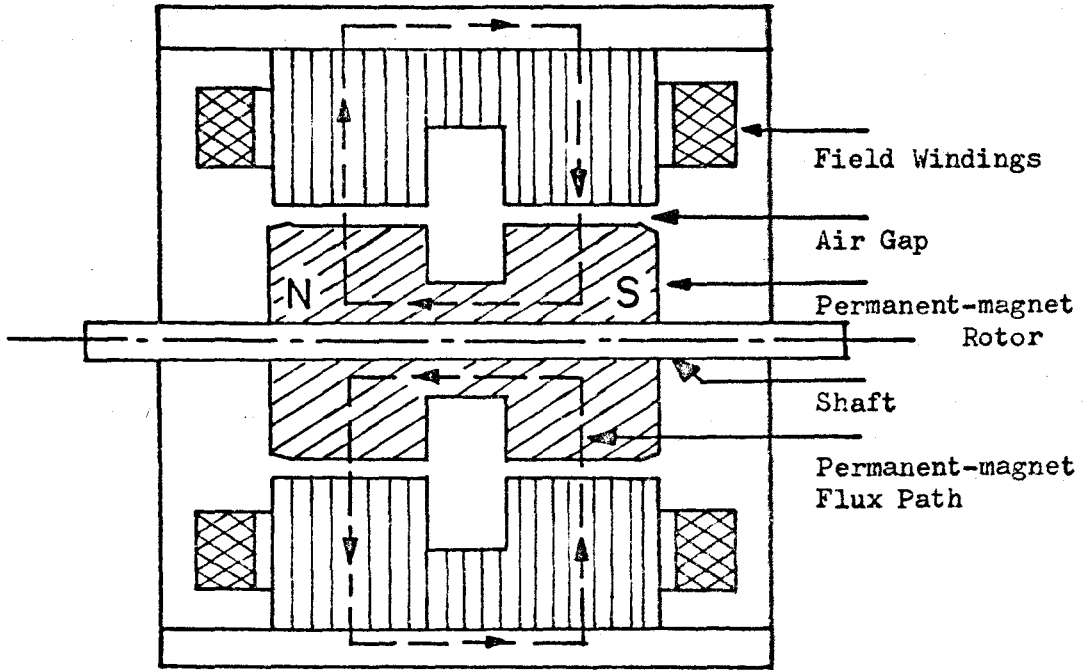
Figure 1 - 3

Simplified Cross-section of a Stepping Motor
Parallel to its Shaft
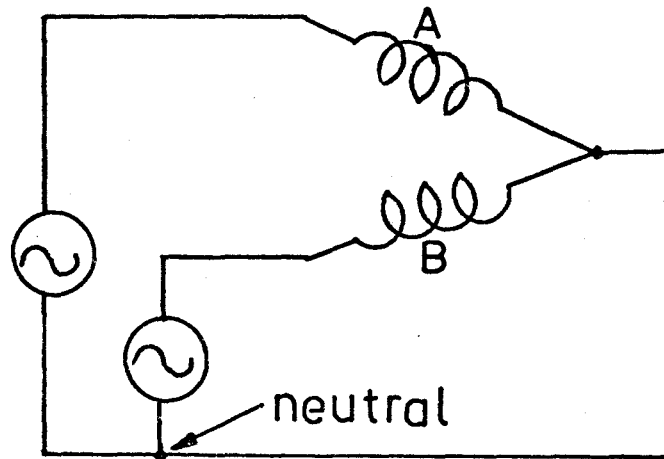
Figure 1 - 4

Connection for A.C. Operation (Two-phase
Power Supply)

90° Phase Shift

cw
ccw

A

B
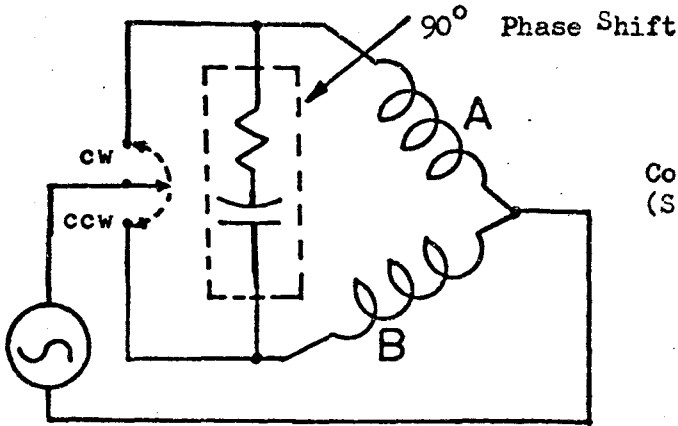
Figure 1 - 5

Connection for A.C Operation
(Single Phase Power Supply)

Phase A

Phase B

V

0

π/2 π

$\frac{3\pi}{2}$ 2π

t

←t=0

-V

Step 1 Step 2 Step 3 Step 4
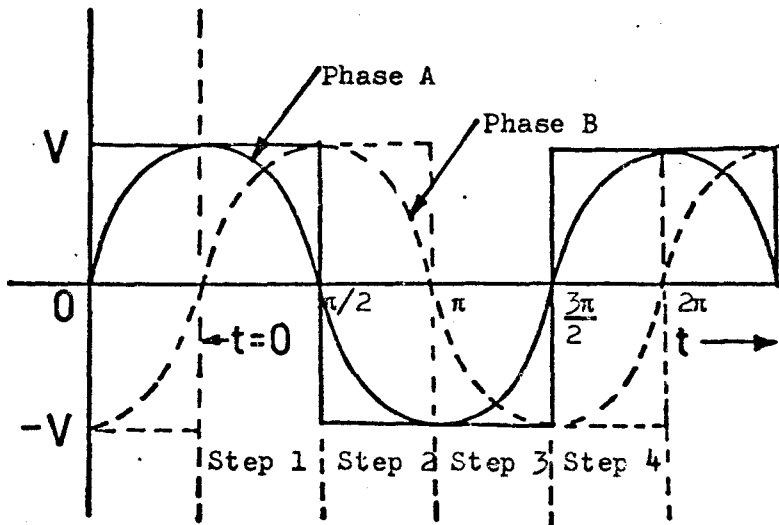
Figure 1 - 6

Representation of
Sinusoidal Waves
By Square Waves

TIME(t)	PHASE A	PHASE B
$0 < t \leqslant \pi/2$	V	V
$\pi/2 < t \leqslant \pi$	-V	V
$\pi < t \leqslant \frac{3\pi}{2}$	-V	-V
$\frac{3\pi}{2} < t \leqslant 2\pi$	V	-V

Table 1

Representation of Sinusoidal
Waves By Constant Voltages
(One Cycle of the Sine Wave)

The positive half cycle of the sine wave is represented by a constant positive voltage and the negative half cycle of the sine wave is represented by a constant negative voltage. It can be seen from this figure that there are four different combinations in one cycle of the rectangular waves as listed in Table 1. Each combination represents one step as indicated in Figure 1-6. The stepping operation is illustrated in Figure 1-7 for this sequence of 4 steps using a two-phase, 8 salient poles, permanent-magnet stepping motor [5]. Step 1 is chosen as the reference position. The polarities of the stator poles are shown in the figure and the cross-section of the rotor is the south-pole portion of the permanent magnet. If the polarities of the poles of phase B are unchanged, and the polarities of the poles of phase A are reversed, the rotor advances one step clockwise (equal to half rotor tooth width) and assumes a position as shown in Step 2. Taking step 2 as reference and keeping the polarity of phase A unchanged but with phase B switched to reverse its polarity , the rotor will advance another step to a new position Step 3, from Step 3 to Step 4 is accomplished by reversing the polarities of the poles in phase A. The correct pulse train operation can be obtained from a d.c. supply by using switches such as shown in Figure 1-8, using the switch actions listed in Table 2. By means of simple digital logic this switching sequence may be implemented as shown in Figure 1-9, and the corresponding output wave-forms from the flip-flops are shown in Figure 1-10. For reverse direction of rotation of the motor, terminals a and b have to be interchanged. Single phase operation may easily be achieved by using the connections as shown in Figure 1-11. Under this operation the motor combines two steps into one,

Step 1   Step 2

Step 3   Step 4
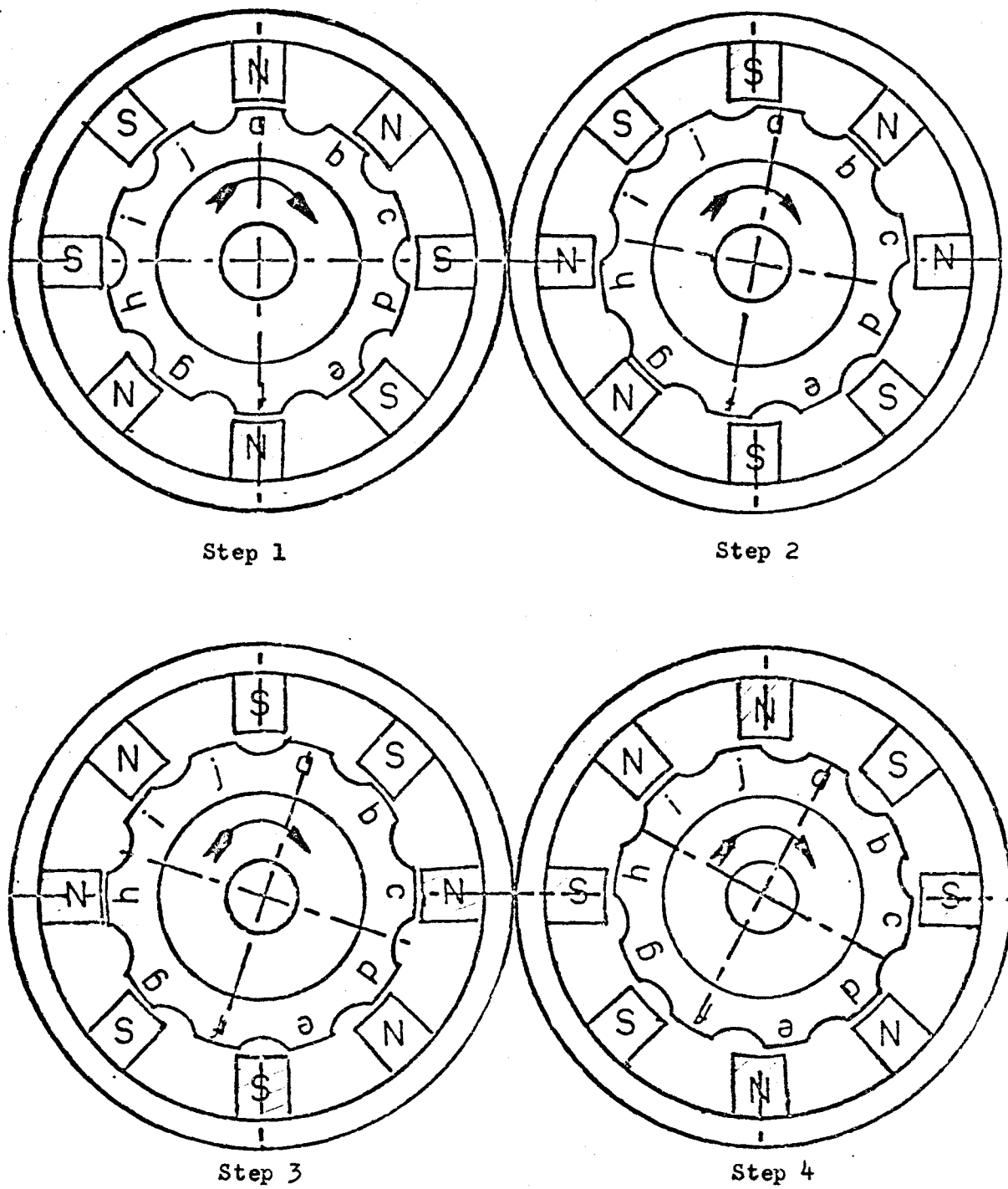
Figure 1 - 7

Step by Step Analysis of a PM Stepping Motor
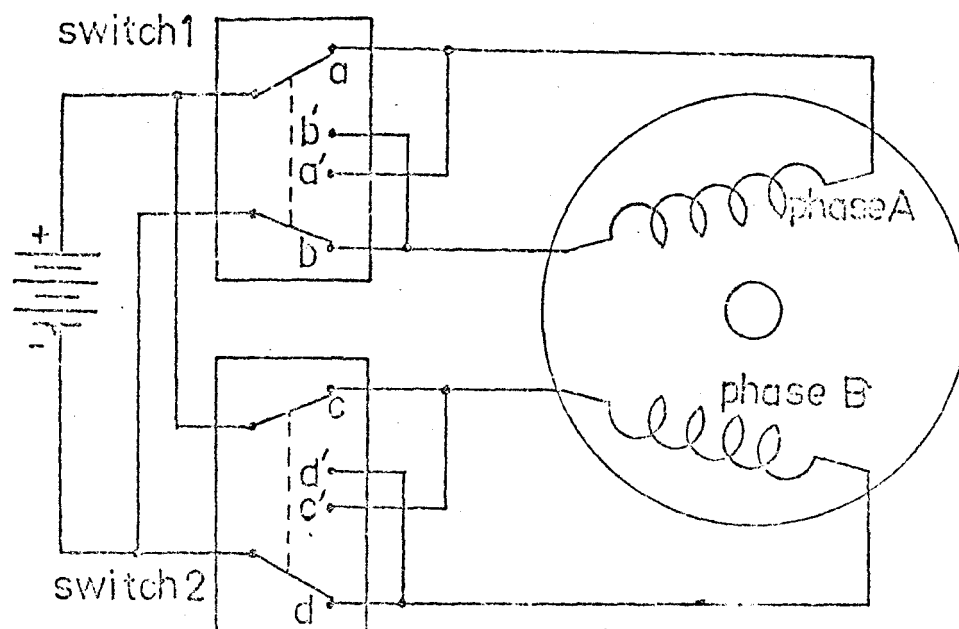
Connection for phase-switch operation with both windings energized

Figure 1 - 8

STEP	SWITCH 1	SWITCH 2
1	a , b	c , d
2	a , b	c' , d'
3	a' , b'	c' , d'
4	a' , b'	c , d
1	a , b	c , d

(To reverse direction, read chart up from bottom)

Table 2

SWITCHING SEQUENCE

giving an angular displacement twice that shown in Figure 1-9. However, since only one phase is being energized for each step, the electromagnetic torque produced by the motor is accordingly reduced and the maximum stepping rate is lowered. The capacitor acts as a directional biasing medium which produces a transient current through the unexcited phase winding. Fractional step operation is also possible but this operation is seldom used because smaller step intervals may be made through gear ratios and the logic control circuits required for fractional operation is normally very complex.

A stepping motor may be operated in the slew range with a much higher stepping rate. Under suitable operation an ordinary stepping motor having a maximum stepping rate of several hundred steps per second may be operated up to several thousand steps per second without loss of accuracy. The basic principle underlying this technique is the minimization of the rise time of the currents in the phase windings and the suitable acceleration of the stepping rate. The rise time of the currents in the phase windings may be shortened by adding external resistors to the windings, or by using constant current sources. The constant current source eliminates the effect of impedance at high frequency and enables the motor to produce a constant torque to overcome its load. The input pulses are adjusted so that the pulse rate is increased slowly, either manually or automatically. An alternate approach which leads to a better performance is to give a variable acceleration. For automatic control a small digital computer like the PDP8/L, which may be programmed to generate pulse trains at various frequencies, may be
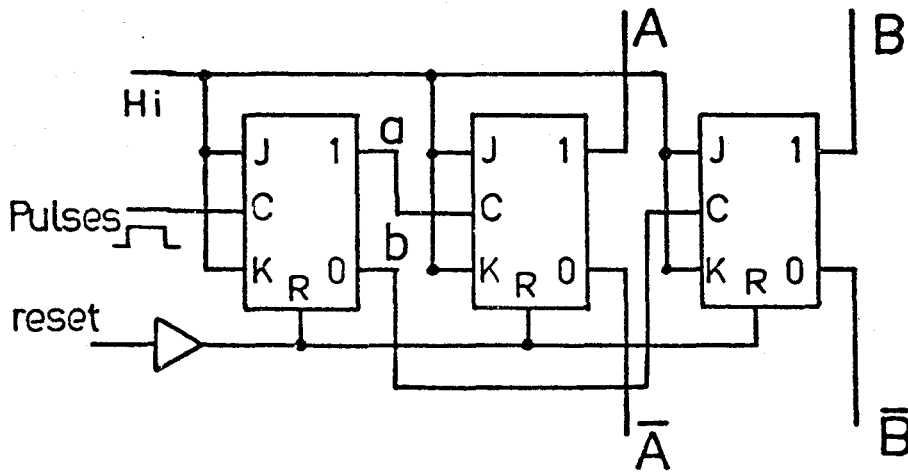
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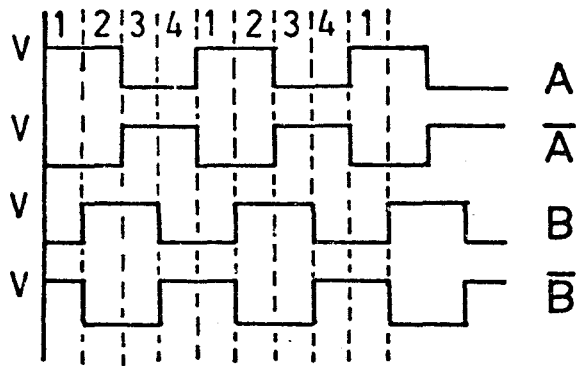
Figure 1 - 10
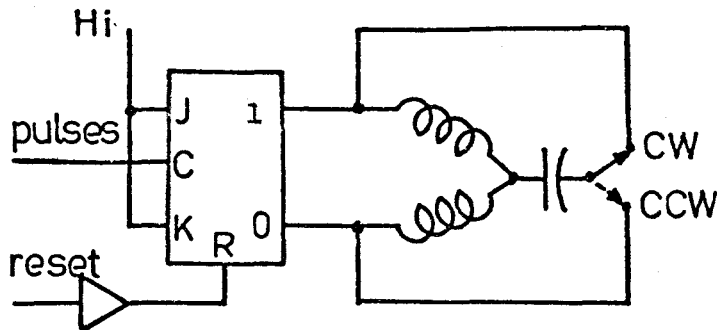
Figure 1 - 11

Figure 1-9:   Digital Implementation for two phase operation
Figure 1-10: Output waveforms from figure 1-9
Figure 1-11: Digital Implementation for single phase operation

found to be very effective. Theoretically a stepping motor may be driven in the slew range to a speed which gives the maximum temperature increment which the motor can tolerate.

Permanent-magnet stepping motors may be driven as generators though this operation has seldom been used. By coupling the shaft of a low speed d.c. shunt motor to the shaft of a stepping motor and with variation of the shunt field resistance the stepping motor acts as a variable frequency power supplier. Using this method, a stepping motor of higher ratings may be used to provide sufficient power for lower power rating stepping motors at various frequencies.

CHAPTER TWO

MATHEMATICAL MODELS FOR A TWO-PHASE PERMANENT-MAGNET STEPPING MOTOR

PART I : LINEAR MODELS

1. General Derivation

The induced voltage (v) due to a rotating magnetic medium

on a conductor of surface s is given by Lenz's Law

$$v = \frac{d}{dt}\int_s \vec{B} \cdot \vec{N} \; ds \qquad \text{volts} \qquad (2 - 1)$$

where $\vec{B}$ = magnetic flux density in webers/meter^2 (vector quantity)

and $\vec{N}$ is the normal vector defined by the right-hand rule. If the

flux linkage λ for the conductor is defined as

$$\lambda = \int_s \vec{B} \cdot \vec{N} \; ds \qquad \text{webers} \qquad (2 - 2)$$

then

$$v = \frac{d\lambda}{dt} \qquad \text{volts.}$$

In an angular rotational electromechanical system the flux linkage

may be written as [47]

$$\lambda = \lambda (i, \Theta) \qquad (2 - 3)$$

where i=the conductor current, and Θ= the angular position of the

rotating conductor. Taking the partial derivative of λ yields

$$d\lambda = \frac{\partial\lambda}{\partial i}di + \frac{\partial\lambda}{\partial\Theta}d\Theta \qquad (2-4)$$

therefore,

$$v = \frac{d\lambda}{dt} = \frac{\partial\lambda}{\partial i}\frac{di}{dt} + \frac{\partial\lambda}{\partial\Theta}\frac{d\Theta}{dt} \qquad (2-5)$$

The first term of equation 2-5 is the result of a time varying current when the system is mechanically stationary. This term is referred to as a transformer voltage. The second term of equation 2-5 is conventionally referred to as a speed voltage. For a system with N electrical terminal pairs and M mechanical variables that are functions of time, then there will be N current elements:

$$i_1, \ i_2, \ i_3, \ \dots i_N \qquad (2-6)$$

and M mechanical displacements

$$\Theta_1, \ \Theta_2, \ \Theta_3, \ \dots \Theta_M \qquad (2-7)$$

The flux linkage for any terminal pair (e.g. the k th) is given by

$$\lambda_k = \int_{sk} \vec{B} \cdot \vec{N} \ ds \qquad (2-8)$$

Equation 2-8 may be generalized according to equation 2-3 as

$$\lambda_k = \lambda_k (\ i_1, \ i_2, \ \dots i_N \ ; \ \Theta_1, \ \Theta_2, \ \dots \Theta_M \ ) \qquad (2-9)$$

where $k = 1, 2, 3, \dots N$. Therefore,

$$v_k = \frac{d\lambda_k}{dt} \qquad (2 - 10)$$

$$= \sum_{j=1}^{N} \frac{\partial \lambda_k}{\partial i_j} \frac{di_j}{dt} + \sum_{j=1}^{M} \frac{\partial \lambda_k}{\partial \theta_j} \frac{d\theta_j}{dt} \qquad (2 - 11)$$

$$k = 1, 2, 3, \ldots N.$$

For an electrically linear system with N pairs of terminals the flux linkage is a linear function of current, therefore λ may be expressed in terms of an inductance L as

$$\underline{\lambda} = \underline{L}(\theta) \underline{i} \qquad (2 - 12)$$

where $\underline{\lambda}$ and $\underline{i}$ are column vectors of order N and $L(\theta)$ is an N x N matrix. The induced voltages may be expressed as

$$\underline{v} = \underline{L}(\theta) \frac{d\underline{i}}{dt} + \underline{i} \frac{d\underline{L}(\theta)}{d\theta} \frac{d\theta}{dt} \qquad (2 - 13)$$

Most electromechanical devices that involve mechanical motion normally have air-gaps [40] where the material is electrically linear and the field in the air-gap predominates. Therefore in the analysis of these machines, it may be assumed that the system is electrically linear. In electrical machines there is usually a single rotational element, thus M = 1. Under these assumptions, the generalized equation may then be rewritten as

$$v_k = \sum_{j=1}^{N} \frac{\partial \lambda_k}{\partial i_j} \frac{di_j}{dt} + \frac{\partial \lambda_k}{\partial \theta} \frac{d\theta}{dt} \qquad (2 - 14)$$

For the electrically linear system, it will be seen from 2-13 that

$$\frac{\partial \lambda_k}{\partial i_j} = L_{jk}(\Theta) \qquad\qquad (2 - 15)$$

and $\qquad \dfrac{\partial \lambda_k}{\partial \Theta} = \displaystyle\sum_{j=1}^{N} i_j \frac{\partial L_{jk}(\Theta)}{d\Theta} \qquad\qquad (2 - 16)$

Equation 2-14 may now be expressed as

$$v_k = \sum_{j=1}^{N} \left\{ L_{jk}(\Theta) \frac{di_j}{dt} + i_j \frac{\partial L_{jk}(\Theta)}{d\Theta} \frac{d\Theta}{dt} \right\} \qquad (2 - 17)$$

Accordingly it will be seen that $L_{jk}\big|_{j \neq k}$ is the mutual inductance between coil j and coil k, and that $L_{jk}\big|_{j=k}$ is the self-inductance of coil k. Losses in electrical energy are normally accounted for by a series resistor. For a two terminal system of the type discussed here, the equivalent circuit is as shown in Figure 2-1. For a two-phase salient-pole synchronous machine with one rotor winding, the magnetic circuit is as shown in Figure 2-2. There are three pairs of terminals, one is the rotor terminals, the other two pairs constitute the two stator fields. According to equation 2-12 with N = 3, the flux linkage is given by

$$\lambda = \sum_{j=1}^{3} L_{jk}(\Theta) i_j \quad , \quad k = 1,2,3. \qquad (2 - 18)$$

Therefore,

$$\lambda_{as} = L_{aa}(\Theta) i_a + L_{ba}(\Theta) i_b + L_{ra}(\Theta) i_r$$

$$\lambda_{bs} = L_{ab}(\Theta) i_a + L_{bb}(\Theta) i_b + L_{rb}(\Theta) i_r \qquad (2 - 19)$$

$$\lambda_r = L_{ar}(\Theta) i_a + L_{br}(\Theta) i_b + L_{rr}(\Theta) i_r$$
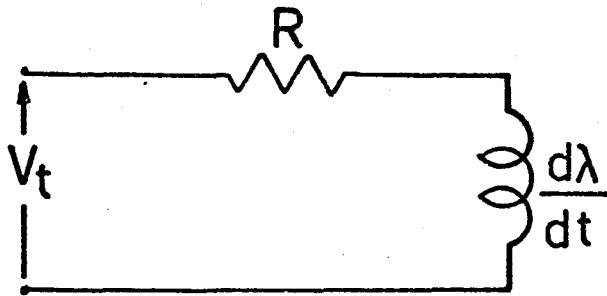
Figure 2-1

Equivalent Circuit For
A Two Terminal System
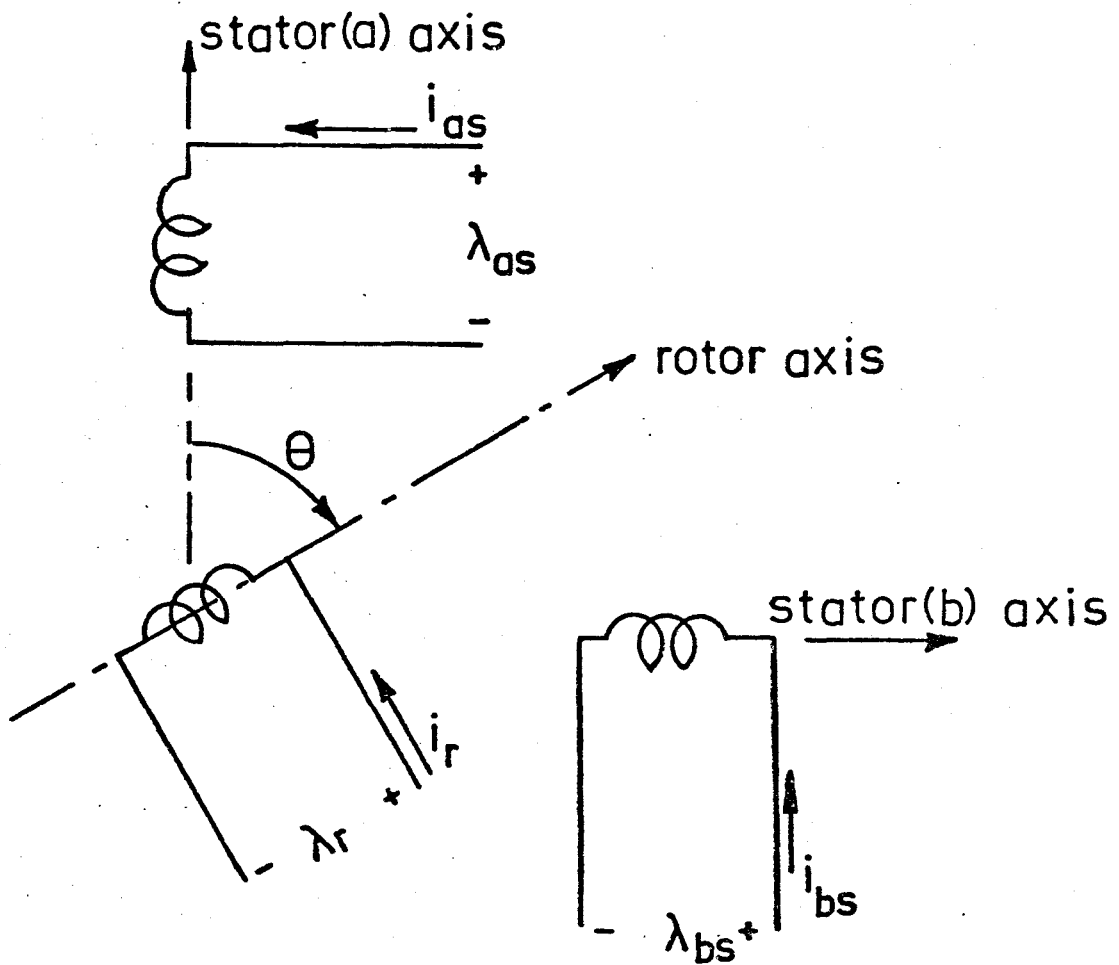
Figure 2-2:   Magnetic Circuit for a Two-phase Machine

All inductances in equations 2-19 are function of Θ. The magnetic field energy is defined in Fitzgerald [40] as

$$W = \sum_{j=1}^{N} i_j \lambda_j \qquad \text{joule} \qquad (2-20)$$

where the λ_j's are restricted to mechanically stationary terminals. Substituting 2-19 into 2-20

$$W = L_{aa}(\Theta)i_a^2 + L_{ab}(\Theta)i_a i_b + L_{ra}(\Theta)i_r i_a +$$

$$L_{ab}(\Theta)i_a i_b + L_{bb}(\Theta)i_b^2 + L_{rb}(\Theta)i_r i_b \qquad (2-21)$$

The electromagnetic torque produced by the motor is defined as

$$T^e = \frac{dW}{d\Theta} \qquad \text{newton - meters}$$

$$= i_a^2 \frac{dL_{aa}(\Theta)}{d\Theta} + i_b i_a \frac{dL_{ba}(\Theta)}{d\Theta} + i_r i_a \frac{dL_{ra}(\Theta)}{d\Theta} +$$

$$i_b^2 \frac{dL_{bb}(\Theta)}{d\Theta} + i_a i_b \frac{dL_{ab}(\Theta)}{d\Theta} + i_b i_r \frac{dL_{rb}(\Theta)}{d\Theta}$$

$$(2-22)$$

The induced voltages are derived from equation 2-17 as

$$v_a = L_{aa}(\Theta) \frac{di_a}{dt} + L_{ba}(\Theta) \frac{di_b}{dt} + L_{ra}(\Theta) \frac{di_r}{dt} +$$

$$\left\{ i_a \frac{dL_{aa}(\Theta)}{d\Theta} + i_b \frac{dL_{ba}(\Theta)}{d\Theta} + i_r \frac{dL_{ra}(\Theta)}{d\Theta} \right\} \frac{d\Theta}{dt}$$

$$v_b = L_{ab}(\Theta) \frac{di_a}{dt} + L_{bb}(\Theta) \frac{di_b}{dt} + L_{rb}(\Theta) \frac{di_r}{dt} +$$

$$\left\{ i_a \frac{dL_{ab}(\Theta)}{d\Theta} + i_b \frac{dL_{bb}(\Theta)}{d\Theta} + i_r \frac{dL_{rb}(\Theta)}{d\Theta} \right\} \frac{d\Theta}{dt}$$

$$v_r = L_{ar}(\Theta) \frac{di_a}{dt} + L_{br}(\Theta) \frac{di_b}{dt} + L_{rr}(\Theta) \frac{di_r}{dt} +$$

$$\left\{ i_a \frac{dL_{ar}(\Theta)}{d\Theta} + i_b \frac{dL_{br}(\Theta)}{d\Theta} + i_r \frac{dL_{rr}(\Theta)}{d\Theta} \right\} \frac{d\Theta}{dt} \qquad (2-23)$$

The terminal voltages are:

$$V_{at} = i_a R_a + v_a$$

$$V_{bt} = i_b R_b + v_b$$

$$V_{rt} = i_r R_r + v_r \qquad (2-24)$$

where R_j represents coil resistance loss. In a permanent-magnet rotor stepping motor, the flux density is nearly constant [41] thus the current i_r may be approximated by a constant current source I_r. With this modification the terminal voltages may be rewritten as:

$$V_{at} = i_a R_a + L_{aa}(\Theta)\frac{di_a}{dt} + L_{ba}(\Theta)\frac{di_b}{dt} +$$

$$\left\{ i_a \frac{dL_{aa}(\Theta)}{d\Theta} + i_b \frac{dL_{ba}(\Theta)}{d\Theta} + I_r \frac{dL_{ra}(\Theta)}{d\Theta} \right\} \frac{d\Theta}{dt}$$

$$V_{bt} = i_b R_b + L_{bb}(\Theta)\frac{di_b}{dt} + L_{ab}(\Theta)\frac{di_a}{dt} +$$

$$\left\{ i_b \frac{dL_{bb}(\Theta)}{d\Theta} + i_a \frac{dL_{ab}(\Theta)}{d\Theta} + I_r \frac{dL_{rb}(\Theta)}{d\Theta} \right\} \frac{d\Theta}{dt}$$

$$V_{rt} = I_r R_r + L_{ar}(\Theta)\frac{di_a}{dt} + L_{br}(\Theta)\frac{di_b}{dt} +$$

$$\left\{ i_a \frac{dL_{ar}(\Theta)}{d\Theta} + i_b \frac{dL_{br}(\Theta)}{d\Theta} + I_r \frac{dL_{rr}(\Theta)}{d\Theta} \right\} \frac{d\Theta}{dt} \qquad (2-25)$$

The electromagnetic torque is given by equation 2-22 as

$$T^e = i_a^2 \frac{dL_{aa}(\Theta)}{d\Theta} + i_a i_b \frac{dL_{ba}(\Theta)}{d\Theta} + I_r i_a \frac{dL_{ra}(\Theta)}{d\Theta} +$$

$$i_b^2 \frac{dL_{bb}(\Theta)}{d\Theta} + i_a i_b \frac{dL_{ab}(\Theta)}{d\Theta} + I_r i_b \frac{dL_{rb}(\Theta)}{d\Theta} \qquad (2-26)$$

The balanced mechanical torque differential equation [40] is

$$T^e + T_m = J\frac{d^2\Theta}{dt^2} + B\frac{d\Theta}{dt} + T_{or}(d\Theta/dt)/|d\Theta/dt| \qquad (2-27)$$

where $J\dfrac{d^2\Theta}{dt^2}$ = inertia torque, including the inertia of motor and the

inertia of any load imposed on the stepping motor shaft; $B(d\Theta/dt)$ = viscous damping torque, including the motor damping and any viscous damping on the motor shaft; T_m = external mechanical torque; and T_{or} $(d\Theta/dt)/|d\Theta/dt|$ = torque due to coulomb friction. The exact solution of equations 2-25, 2-26 and 2-27 with Θ in terms of terminal voltages is not an easy one and as a consequence the exact solution has seldom been used. In the rest of this chapter some of the approximate solutions are discussed.

2. Second Order Linear Model

The second order model discussed in this section was proposed by Kieburtz [7] which is a modification of an earlier second order model [3] . This model neglects the effect of self-inductance of the field windings and the mutual inductance between the stators. The mutual inductances between the rotor and stator windings may be taken as the same since only the root-mean squared value is considered. This model of Kieburtz can be derived from the equations above as follows. According to equation 2-25, and assuming $L_{aa}= L_{ab} = L_{bb} = 0$, one of the terminal voltages may be expressed as

$$V(t) = R\ i(t) + K_e\frac{d\Theta}{dt} \qquad (2-28)$$

where K_e is a voltage constant of the motor in volts/radian/sec. Taking the Laplace transform of equation 2-28

$$V(s) = R\ I(s) + K_e s\Theta(s) \qquad (2-29)$$

therefore,

$$I(s) = \left[V(s) - K_e s\Theta(s) \right]/R \qquad (2-30)$$

The differential equation for a motor including terms for acceleration and restoring force [7] expressed in Laplace transform format is

$$T^e(s) = K_t I(s)$$

$$= Js^2\Theta(s) + Bs\Theta(s) + K_r\Theta(s) \qquad (2-31)$$

where $K_r\Theta(s)$ is the linearized restoring torque in newton-meter/radian and K_t is a torque constant of the motor in newton-meter per ampere. Substituting equation 2-30 into equation 2-31 and solving for the transfer function yields

$$\frac{\Theta(s)}{V(s)} = \frac{\dfrac{K_t}{RK_r}\dfrac{K_r}{J}}{s^2 + \dfrac{BR + K_t K_e}{JR}s + \dfrac{K_r}{J}}$$

$$= \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (2-32)$$

where $\omega_n = (K_r/J)^{\frac{1}{2}}$ is known as the natural frequency and

$$\xi = \frac{K_t K_e + BR}{2R(JK_r)^{\frac{1}{2}}}$$

is known as the damping ratio and $K = K_t / RK_r$. The step response of this transfer function may easily be observed by the analog simulation as shown in Figure 2-4 or using the inverse Laplace transform with the assumption that V is a step voltage. The inverse Laplace transform for equation 2-32 is

$$\Theta(t) = KV \left\{ 1 - \frac{1}{(1 - \xi^2)^{1/2}} e^{-\xi \omega_n t} \sin\left[\omega_n (1 - \xi^2)^{1/2} t\right] + \phi \right\} \quad (2 - 33)$$

where $\phi = \tan^{-1}(1 - \xi^2)^{1/2}/\xi$. The experimentally obtainable constants K_r, K_t and K_e introduced by Kieburtz [7] and O'Donohue [3] complicate this model. However, according to equation 2-25 with $L_{aa} = L_{bb} = L_{ab} = 0$, it will be seen that $K_e = I_r \, dL_{ra}/d\Theta$. Therefore K_e is obtainable by driving the stepping motor as a generator in an open circuit test to obtain a plot of the rms terminal voltage versus the speed of the generator in radians per second. K_e is the slope of this plot having the dimension volts-sec/radian as shown in Figure 2-3. Further, from equation 2-22 it may be seen that the electromagnetic torque produced by the motor is expressed as

$$T^e = i_a I_r \frac{dL_{ra}(\Theta)}{d\Theta} + i_b I_r \frac{dL_{rb}(\Theta)}{d\Theta}$$

Assume that $L_{ra}(\Theta) = M \cos(N_r \Theta)$, $L_{rb}(\Theta) = M \sin(N_r \Theta)$ where Θ is in mechanical radians and with a balanced two-phase d.c. supply such that $i_a = i_b = I$, then
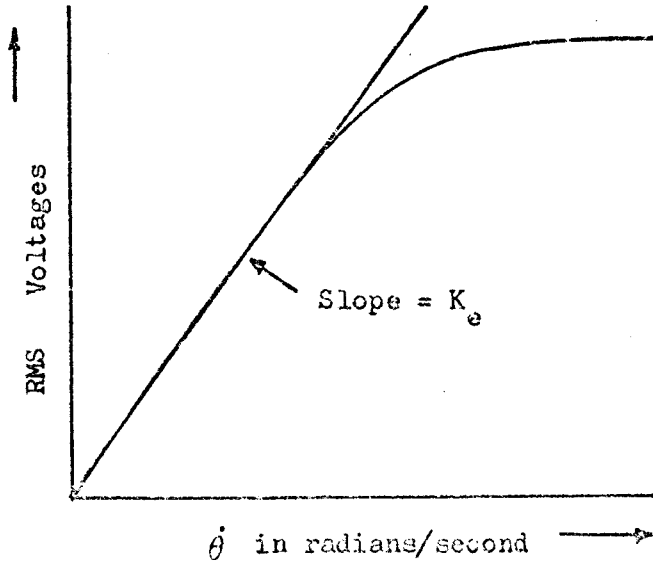
Figure 2-3: Open-circuit Characteristics of Stepping Motor
Driven as Generator
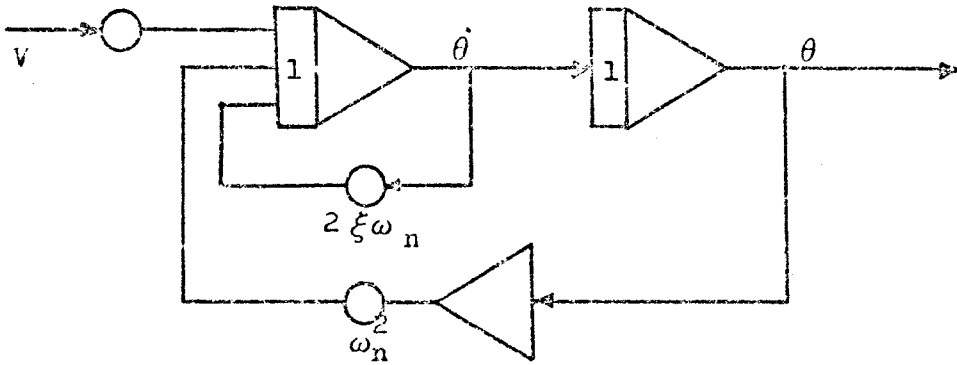
Figure 2-4: Analog Simulation of Second Order Model

$$T^e = II_r MN_r \left[\cos(N_r\Theta) - \sin(N_r\Theta)\right]$$

$$= \sqrt{2}K_e I \left[\cos(N_r\Theta) - \sin(N_r\Theta)\right]$$

$$= T_1^e + T_2^e$$

where $K_e = MN_r I_r /\sqrt{2}$, $T_1^e = \sqrt{2}K_e I \sin(N_r\Theta)$ and $T_2^e = \sqrt{2}K_e I \cos(N_r\Theta)$. It may be

seen that T_1^e is the torque that tends to restore the rotor to its

original position and T_2^e is the torque that tends to move the rotor to

a new position. For small angular displacement, $\sin(N_r\Theta)$ may be appro-

ximated by $N_r\Theta$ and $\cos(N_r\Theta)$ approaches unity. Under this assumption

$$T_1^e = -\sqrt{2}IK_e N_r\Theta \; ,$$

$$T_2^e = \sqrt{2}K_e I \qquad\qquad (2-34)$$

and

$$T^e = \sqrt{2}K_e(I - IN_r\Theta) \qquad\qquad (2-35)$$

Therefore it may be seen that K_r is numerically equal to $\sqrt{2}N_r I K_e$

and K_t is numerically equal to $\sqrt{2}K_e$, and that K_r will be constant only

if I is constant. For the simplicity of analysis, I may be considered

to be equal to the rated current I_o of the motor if it is operated

at the rated voltages so that T_1^e may be linearized as

$$T_1^e = -K_r\Theta \; , \quad \text{where } K_r = \sqrt{2}I_o N_r K_e$$

and

$$T^e = \sqrt{2}K_e I - K_r\Theta \qquad\qquad (2-36)$$

The derivations from equation 2-34 to equation 2-36 make the second

order model much easier to handle than that suggested by [7] because

R and K_e are the only parameters that are required to be determined

experimentally.

3. Third Order Linear Model

This model was suggested by Delgado [24] for fine positioning operation of stepping motors. The basic assumptions are:

1. The self-inductances are constant (not a function of angular displacement).

2. The inertia torque includes applied load torque.

3. There is no mutual inductance between the stator phase - windings.

4. The induced voltage is a linear function of angular speed.

From equation 2-25 the equation for any one phase terminal voltage is given by

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + K_e \frac{d\Theta(t)}{dt} \qquad (2 - 37)$$

Equation 2-37 is expressed in Laplace transform as

$$V(s) = RI(s) + LsI(s) + K_e s\Theta(s) \qquad (2 - 38)$$

therefore,

$$I(s) = \left[V(s) - K_e s\Theta(s) \right] / (R + Ls) \qquad (2 - 39)$$

According to Delgado [24] the electromagnetic torque is given by

$$T(s) = K_r \left[K_i I(s) - \Theta(s) \right] \qquad (2 - 40)$$

where K_r and K_i are constants obtainable experimentally. The motor

motion equation is approximated to be

$$T(s) = Js^2\Theta(s) \tag{2 - 41}$$

Substituting equation 2-39 into 2-40 and equating the result to 2-41 yields

$$Js^2\Theta(s) = K_r\left\{K_i\left[V(s) - K_e s\Theta(s)\right]/(R+Ls) - \Theta(s)\right\} \tag{2 - 42}$$

Rearranging 2-42 to obtain

$$\frac{\Theta(s)}{V(s)} = \frac{K_i/R}{\dfrac{J\varepsilon}{K_r}s^3 + \dfrac{J}{K_r}s^2 + (\varepsilon + \dfrac{K_e K_i}{R})s + 1} \tag{2 - 43}$$

where $\varepsilon = L/R$, or

$$\frac{\Theta(s)}{V(s)} = \frac{K_i K_r/JL}{s^3 + \dfrac{R}{L}s^2 + (\dfrac{K_r}{J} + \dfrac{K_i K_e K_r}{JL})s + \dfrac{K_r R}{JL}} \tag{2 - 44}$$

There is not much difference between this model and the second order one, except that the self inductances are considered. The constants K_r and K_i again create difficulties in applying this model, but with a second look at this model, it will not be too difficult to appreciate that equation 2-40 is of the same form as equation 2-36. Thus it may be seen that $K_r K_i = K_t$ of the second order model and that K_r is

the same for both models. An interesting proof of these relationships may be taken from the example given by Delgado [24] with consideration of the stepping motor SLO-SYN HS-50 which has 50 pairs of teeth, therefore $N_r=50$. Some of the experimentally determined values given by the author according to the methods suggested in the paper are

$$K_r = 4880 \text{ ounce-inches per radian}$$

$$= 4880 \times .007061 \text{ newton-meter/radian}$$

$$= 34.456 \text{ newton-meter/radian}$$

$$K_i = .00837 \text{ radians per ampere}$$

$$K_e = .21 \text{ volt per radian per second}$$

$$\varepsilon = .000909 \text{ second} = L/R \text{ second}$$

and

$$J = .01326 \text{ ounce-inches second}^2.$$

However, from the experimental transfer function given by equation 10 of [24] the following results are obtained:

$$\varepsilon + K_e K_i/R \quad = 1285 \times 10^{-12} \qquad\qquad (2-45)$$

$$J/K_r \quad = 2512 \times 10^{-9} \qquad\qquad (2-46)$$

$$J\varepsilon/K_r \quad = 2227 \times 10^{-12} \qquad\qquad (2-47)$$

where ε, J and R are constants experimentally obtainable without difficulties and are less subject to mistakes. K_i/R has been proved by the author to be the same for both the theoretically derived and experimentally determined transfer functions. From equation 2-45 the constant K_e may be computed as

$$K_e = \frac{1285 \times 10^{-12} - \varepsilon}{K_i/R}$$

$$= .2574 \text{ volt/rad/sec}$$

and from equation 2-46

$$K_r = .01326/2512 \times 10^{-9} = 5278 \text{ ounce-inches/rad.}$$

The method to find K_e suggested by Delgado is rather awkward and is likely to have experimental errors. According to Delgado the back EMF voltage is defined by the crossing point on the screen of an oscilloscope of the two sine waves obtained from the motor when it is operated as a generator. For a two-phase generator the outputs may be expressed as

$$V_1 = V \cos \alpha \qquad (2-48)$$

$$V_2 = V \sin \alpha \qquad (2-49)$$

The crossing-points are computed by equating 2-48 to 2-49 such that

$$\sin \alpha = \cos \alpha$$

$$\alpha = \pi/4 + n\pi \qquad \text{where } n = 0, 1, 2, \ldots n,$$

therefore $V_1 = V_2 = V/\sqrt{2}$ at the crossing-points which is essentially the rms voltage of a single sine wave. Consequently it may be more accurately to record the voltage by using voltmeters rather than getting the crossing points of two sine waves. With $K_e = .2574$ volts/rad./sec., the constants K_t and K_r may be computed according to the relationship shown on page 27 as

$$K_t = 2K_e = .3640 \text{ volt/sec.}$$

and $\qquad K_r = 2IK_e N_r = 5154 \text{ ounce-inches/rad.}$

The current I_o is taken as 2 amperes according to [24]. A comparison of the data is shown in Table 3. From this table it may be seen that only K_e need be accurately determined and all other constants may be computed. From theory and experimental data it may be concluded that the measurement of K_r and K_i are redundant. One of the difficulties

Table 3

Comparison of Evaluated and Experimentally Determined

Values of K_r, K_t and K_e

Constants	K_e volt/rad/sec	K_t newton-m/a	K_r oz-in/rad
Experimentally det- ermined (Direct Method) (A)	0.21	NO*	4820
Experimentally determined (from transfer function) (B)	0.2574	NO*	5278
Computed according to equations 2-34 and 2-35 with refer- ence to the values shown in (A)	0.21	$K_t = \sqrt{2}K_e$ $= .2969$ $K_t = K_i K_r$ $= .2884$	$K_r = \sqrt{2} I N_r K_e$ $I = \sqrt{2} A$ $N_r = 50$ $K_r = 4205$
Computed according to equations 2-34 and 2-35 with refer- ence to the values shown in (B)	0.2574	$K_t = \sqrt{2}K_e$ $= .3639$ $K_t = K_i K_r$ $= .3119$	5154

* NO indicates that there was no value available for this constant

that may be encountered while determining K_r and K_i experimentally is the accuracy in the measurement of angular displacement. Another interesting result given by Delgado which proved the previous statement that K_r is constant only if the current applied is constant is given in figure 8 of his paper.

With the substitution of K_e for K_i, K_r and K_t the transfer function 2-44 may be expressed as

$$\frac{\Theta(s)}{V(s)} = \frac{\sqrt{2}\, K_e/JL}{s^3 + (R/L)\, s^2 + \frac{K_e}{J}\left(\sqrt{2}N_r I_o + \frac{\sqrt{2}}{L}K_e\right)s + \frac{\sqrt{2}I_o N_r K_e R}{JL}}$$

$$(2-51)$$

where $I_o = V/R$, this d.c. current has to be determined before substituting into the equation. The step response of this transfer function may be plotted by using the analog simulation as shown in Figure 2-5. A block diagram representation of this third order model is shown in Figure 2-6.
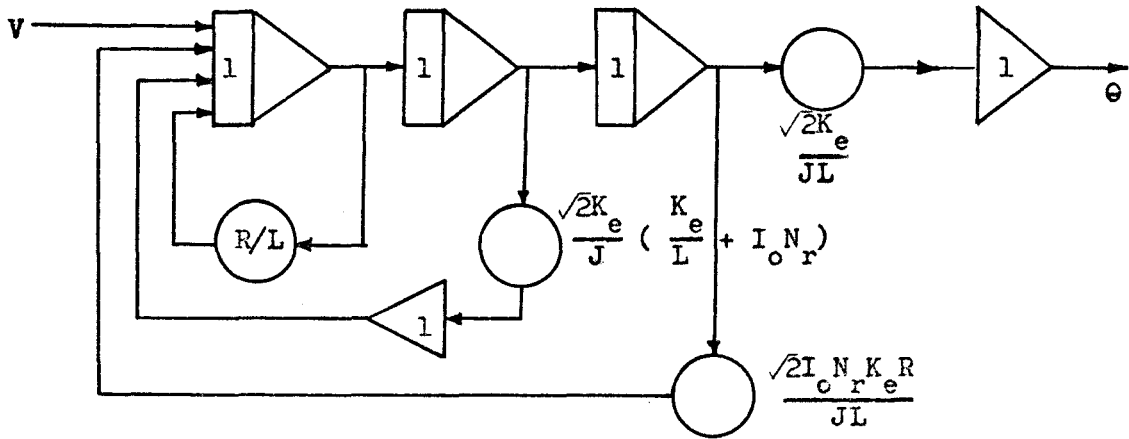
Figure 2-5:   Analog Simulation of Third Order Model
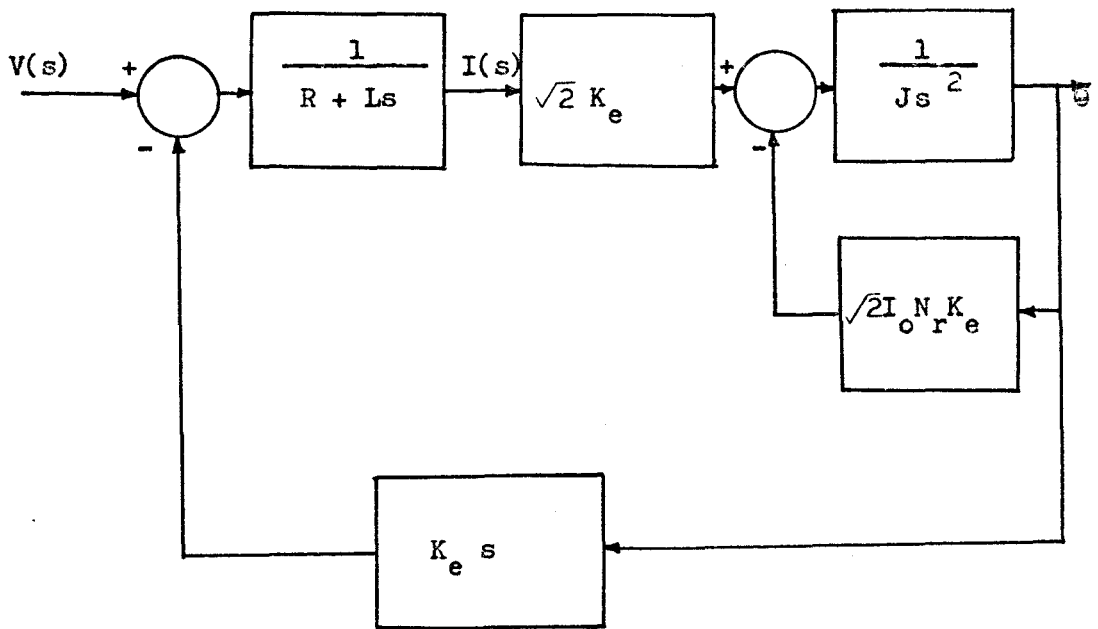
Figure 2-6.   Block Diagram Representation of Third Order Model

CHAPTER THREE

MATHEMATICAL MODELS FOR A TWO-PHASE PERMANENT-MAGNET STEPPING MOTOR

PART II: NONLINEAR MODELS

1. Smooth air-gap synchronous machine model

Magnetic stepping motors like most electromechanical devices
are non-linear devices though approximation methods may lead to a fairly
close analysis of their characteristics. In control systems, every
method aims at achieving a simpler mathematical model so that it may
be easily applied. However, in the case of stepping motors the
linear models may be too crude for fine positioning analysis.
Venkataratam [27, 30] and Taft [35] treated stepping motors as non-
linear devices by considering the motors as a kind of smooth air-gap
synchronous machines. In fact, all linear models so far developed are
based on the concept of smooth air-gap synchronous machines. With
this approach, the self-inductance of the stator windings is consid-
ered to be independent of the rotor position, and there is no mutual
inductance between the stator windings. Under these conditions and
by assuming the Fourier transform of the mutual inductance between
the rotor and stator as to be of the form

$$L_{ar} = M \cos(N_r \theta)$$

$$L_{br} = M \sin(N_r \theta)$$

$$(3 - 1)$$

- 35 -

where N_r = number of pairs of teeth in rotor, the terminal voltages

given by equation 2-25 may be expressed as

$$V_{at} = R_a i_a + L_a \frac{di_a}{dt} - I_r M N_r \sin(N\,\Theta)\frac{d\Theta}{dt}$$

$$V_{bt} = R_b i_b + L_b \frac{di_b}{dt} + I_r M N_r \cos(N_r\Theta)\frac{d\Theta}{dt} \qquad (3-2)$$

The electromagnetic torque developed is obtained from 2-26 as

$$T^e = I_r i_a \frac{dL_{ra}(\Theta)}{dt} + I_r i_a \frac{dL_{rb}(\Theta)}{dt}$$

$$= I_r M N_r \left[i_b \cos(N_r\Theta) - i_a \sin(N_r\Theta) \right] \qquad (3-3)$$

Equations 3-2 and 3-3 specify the smooth air-gap synchronous machine

model.

   2. Analysis of the smooth air-gap synchronous
       machine model

      In general it is not possible to solve for T^e explicitly in

terms of the excitation voltages V_{at}, V_{bt} and the shaft velocity

$d\Theta/dt$ [27]. Fortunately, it is not necessary to anticipate the

characteristics of a permanent-magnet stepping motor by expressing

T^e in terms of the excitation voltages. Three of the analytical

techniques are :

      1. Step by step analysis with assumption that the currents

in the phase windings are constant.

2. Analysis using the assumption that the inductance L of the phase windings is very small compared to the resistance of the windings [27].

3. Analysis using constant current source.

The step by step analysis given by Robinson and Taft [35] using phase-plane techniques lacks the capability to illustrate the actual step by step response of the motor. The step by step analysis used here is a modification of that derived by Robinson and Taft. Runge Kutta formulas are used in the analysis (appendix A). The initial conditions and the corresponding excitation currents are as follows:

Step	$N_r \Theta_o$	$d\Theta/dt$	I_a	I_b
1	45°	$= 0$, at $t=0$	I	I
2	Θ_1	$=v_1$, at $t=\Delta$	-I	I
3	Θ_2	$=v_2$, at $t=2\Delta$	-I	-I
4	Θ_3	$=v_3$, at $t=3\Delta$	I	-I
5	Θ_4	$=v_4$, at $t=4\Delta$	I	I

Θ_i is the rotor position at the end of step i, v_i is the rotor velocity at the end of step i and Δ is the step duration. For very low frequency analysis it may be assumed that the motor gives a perfect step each time. Thus $\Theta_i = \Theta_{i-1} + 90^\circ$ may be used and all v_i's are set to zero. The stepping duration Δ is calculated by dividing the input pulse period by four. For instance, for 60 hertz pulse chain Δ is equal to $(1/240)$ second.

With no load applied to the shaft of the motor and neglecting the torque due to coulomb friction, the equation to be solved is obtained by equating 2-27 and 3-3 as:

$$\frac{d^2\Theta}{dt^2} + \frac{B}{J}\frac{d\Theta}{dt} + \frac{K_1}{J}\left[i_a \sin(N_r\Theta) - i_b \cos(N_r\Theta)\right] = 0 \qquad (3-4)$$

where $K_1 = I_r M N_r$. For step 1, equation 3-4 may be written as

$$\frac{d^2\Theta}{dt^2} + \frac{B}{J}\frac{d\Theta}{dt} + \frac{K_1 I}{J}\left[\sin(N_r\Theta) - \cos(N_r\Theta)\right] = 0 \qquad (3-5)$$

Phase-plane plots and step response plots are obtainable by solving equation 3-5 with the Runge Kutta method. The programming may be simplified by using the digital simulated analog computer language MIMIC [48, 49] which uses a higher order Runge Kutta method.

The assumption that the inductance of the windings are negligible compared with the resistance of the windings is acceptable for very low frequency analysis. The prominent feature of this assumption is that one may express Θ in terms of the excitation terminal voltages. From equation 3-2 with $L_a = L_b = 0$, the currents i_a and i_b may be expressed as:

$$i_a = \left[V_{at} + K_1 \sin(N_r\Theta)\frac{d\Theta}{dt}\right] / R_a$$

$$i_b = \left[V_{bt} - K_1 \sin(N_r\Theta)\frac{d\Theta}{dt}\right] / R_b \qquad (3-6)$$

Substituting i_a and i_b into equation 3-3 the torque equation is

$$T^e = \frac{K_1}{R}\left[-V_{at}\sin(N_r\Theta) + V_{bt}\cos(N_r\Theta) - K_1\frac{d\Theta}{dt}\right] \qquad (3-7)$$

The electromechanical torque balance with no load applied is

$$J \frac{d^2\theta}{dt^2} + (B + \frac{K_1^2}{R}) \frac{d\theta}{dt} + \frac{K_1}{R} \left[V_{at}\sin(N_r\theta) - V_{bt}\cos(N_r\theta) \right] = 0$$

$$(3 - 8)$$

Equation 3-8 may be solved in the same way as the step by step method with V_{at} and V_{bt} equal to balanced two-phase power supply voltages. However the constant voltage source and constant current source are not the same. The term $L(di/dt)$ may be much larger than the term iR in equation 3-2 especially at high frequencies. Consequently this approach is valid only at very low frequencies when the term $L(di/dt)$ is really very small compared to iR.

The constant current source analysis is basically the same as the step by step method. The excitation currents are considered to be square waves with 90° phase difference. By means of Fourier series expansion the square waves may be expressed as:

$$I_a = \frac{4I}{\pi} (\cos \omega t - \frac{1}{3} \cos 3\omega t + \frac{1}{5} \cos 5\omega t - \frac{1}{7} \cos 7\omega t + \ldots)$$

$$I_b = \frac{4I}{\pi} (\sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \ldots)$$

where I = rated current or magnitude of current to be used in the operation of PM stepping motors and $\omega = 2\pi f$ is the natural frequency of the square wave which in turn is equal to four times the stepping rate of the stepping motor in steps/second. The step by step response plots and phase-plane plots may be done in the same way as shown in appendix A with an added consideration for the time variable t. The

flow diagram shown in figure 3-1 may be used to analysis a PM stepping motor according to equation 3-4.

3. Salient pole synchronous machine model

Most PM stepping motors are synchronous machines with salient poles though they have mostly been analysed by using the smooth air-gap models. The self-inductance of the stator windings is a function of the position of the rotor. The self-inductance may be considered as representing the superposition of a smooth air-gap inductance and a periodically varying air-gap inductance due to saliency. As a first approximation, the inductances may be expressed as:

$$L_{aa} = L_1 + L_2 \cos(2N_r\theta)$$

$$L_{bb} = L_1 - L_2 \sin(2N_r\theta)$$

$$L_{ar} = M\cos(N_r\theta)$$

$$L_{br} = M \sin(N_r\theta)$$

$$L_{ab} = L_{ba} = L_2 \sin(2N_r\theta) \tag{3 - 10}$$

The values of L_1 and L_2 are defined such that

$$L_1 = (L_{max} + L_{min})/2$$

$$L_2 = (L_{max} - L_{min})/2 \tag{3 - 11}$$

L_{max} and L_{min} are obtainable from a locked-rotor test (appendix B). The mutual inductance is obtainable by driving the stepping motor as a generator at different speeds (see figure 2-3). By substituting equation 3-10 into 2-25 the terminal excitation voltages may be expressed as

(1)
```
Read in: 1. J, K₁, Nr, I, w, B.
         2. Initial rotor position Θₒ and initial rotor
            velocity Vₒ
         3. Incremental time constant DT and time limit for
            iteration TM.
```

$$t = 0$$

(2)

Compute the currents:

$$I_a = \frac{4I}{\pi}\left[\cos(wt) - \frac{1}{3}\cos(3wt) + \frac{1}{5}\cos(5wt) - \frac{1}{7}\cos(7wt) + \ldots\right]$$

$$I_b = \frac{4I}{\pi}\left[\sin(wt) + \frac{1}{3}\sin(3wt) + \frac{1}{5}\sin(5wt) + \frac{1}{7}\sin(7wt) + \ldots\right]$$

(3)

Compute the first constant of Runge Kutta formulas

$$C_1 = DT \times V_o$$

$$CL_1 = -C_1 B/J + DT \times K_1/J\left[I_b\cos(N_r\Theta) - I_a\sin(N_r\Theta)\right]$$

$$T1 = t + DT/2$$

(4) Compute I_a and I_b as shown in (2) with T1=t

(5)

Compute the second constants of Runge Kutta formulas

$$C_2 = DT \times (V_o + CL_1/2)$$

$$CL_2 = -C_1 B/J + DT \times K_1/J\left[I_b\cos(N_r\Theta_o + N_r C_1/2) - I_a\sin(N_r\Theta_o + N_r C_1/2)\right]$$

(6)

Compute the third constants of Runge Kutta formulas

$$C_3 = DT \times (V_o + CL_2/2)$$

$$CL_3 = -C_3 B/J + DT \times K_1/J\left[I_b\cos(N_r\Theta_o + N_r C_2/2) - I_a\sin(N_r\Theta_o + N_r C_2/2)\right]$$

$$T2 = t + DT \qquad \longrightarrow (7)$$

Figure 3-1 : Flow Diagram for the Analysis of
PM Stepping Motors Using the Smooth
Air-Gap Model

$\boxed{6}$

(7) Compute I_a and I_b as shown in (2) with $t = T2$

(8) Compute the fourth constants of Runge Kutta formulas

$$C_4 = DT \times (V_o + CL_3)$$

$$CL_4 = -C_4 B/J + DT \times K_1/J \left[I_b \cos(N_r \Theta_o + N_r C_3) - I_a \sin(N_r \Theta + N_r C_3) \right]$$

(9) $$V_1 = V_o + (CL_1 + 2CL_2 + 2CL_3 + CL_4)/6$$

$$\Theta_1 = \Theta_o + (C_1 + 2C_2 + 2C_3 + C_4)/6$$

(10) Step by step plot: store or plot Θ_o vs t

Phase-plane plot :   store or plot Θ_o vs V_o

Is $t > TM$?   yes → Stop

no

(11) $t = t + DT;$ $V_o = V_1;$ $\Theta_o = \Theta_1$ ⟶ (2)
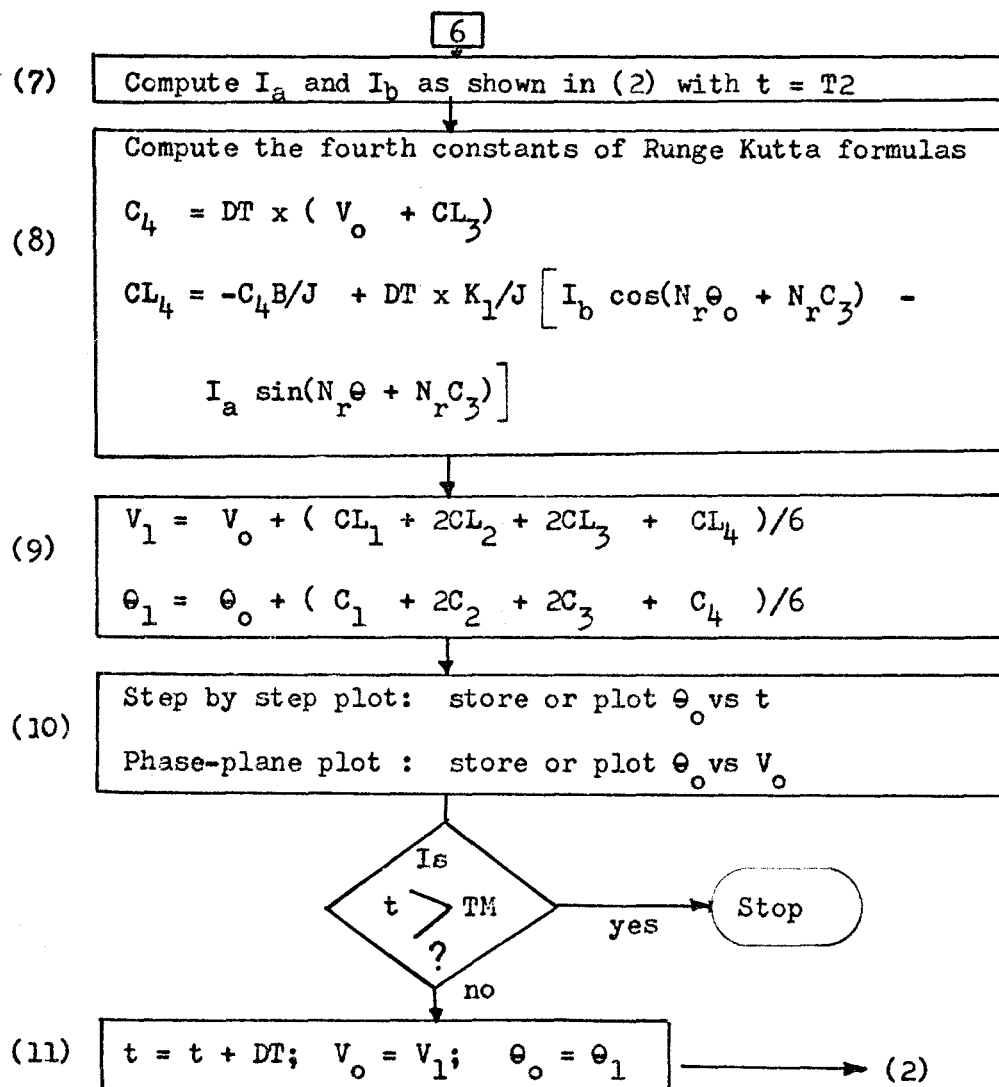
Figure 3-1 continued

$$V_{at} = i_a R_a + \left[L_1 + L_2 \cos(2N_r\theta) \right] \frac{di_a}{dt} + L_2 \sin(2N_r\theta) \frac{di_b}{dt} +$$

$$\left[-2i_a L_2 N_r \sin(2N_r\theta) + 2i_b L_2 N_r \cos(2N_r\theta) - K_1 \sin(N_r\theta) \right] \frac{d\theta}{dt}$$

$$V_{bt} = i_b R_b + \left[L_1 - L_2 \cos(2N_r\theta) \right] \frac{di_b}{dt} + L_2 \sin(2N_r\theta) \frac{di_a}{dt} +$$

$$\left[-2i_b L_2 N_r \sin(2N_r\theta) + 2i_a L_2 N_r \cos(2N_r\theta) + K_1 \cos(N_r\theta) \right] \frac{d\theta}{dt}$$

$$(3 - 12)$$

The electromagnetic torque equation from 2-26 is

$$T^e = 2L_2 N_r \sin(2N_r\theta)(i_b^2 - i_a^2) + 4 i_a i_b L_2 N_r \cos(2N_r\theta) +$$

$$K_1 \left[i_b \cos(N_r\theta) - i_a \sin(N_r\theta) \right] \qquad (3 - 13)$$

An equivalent circuit for this model is shown in Figure 3-2.

4. Analysis of salient pole synchronous machine model

Many numerical methods may be used to analyse the salient pole model. The main object of this analysis is to obtain a phase-plane plot and a step by step response plot from the model. Three of the methods are discussed in this section.

a. The Runge-Kutta formulas - this method is the same as discussed in section 2. The constant current source is again assumed as shown in equations 3-9. The first term in equation 3-13 may be set to be equal to zero because $(i_a^2 - i_b^2)$ is always equal to zero with
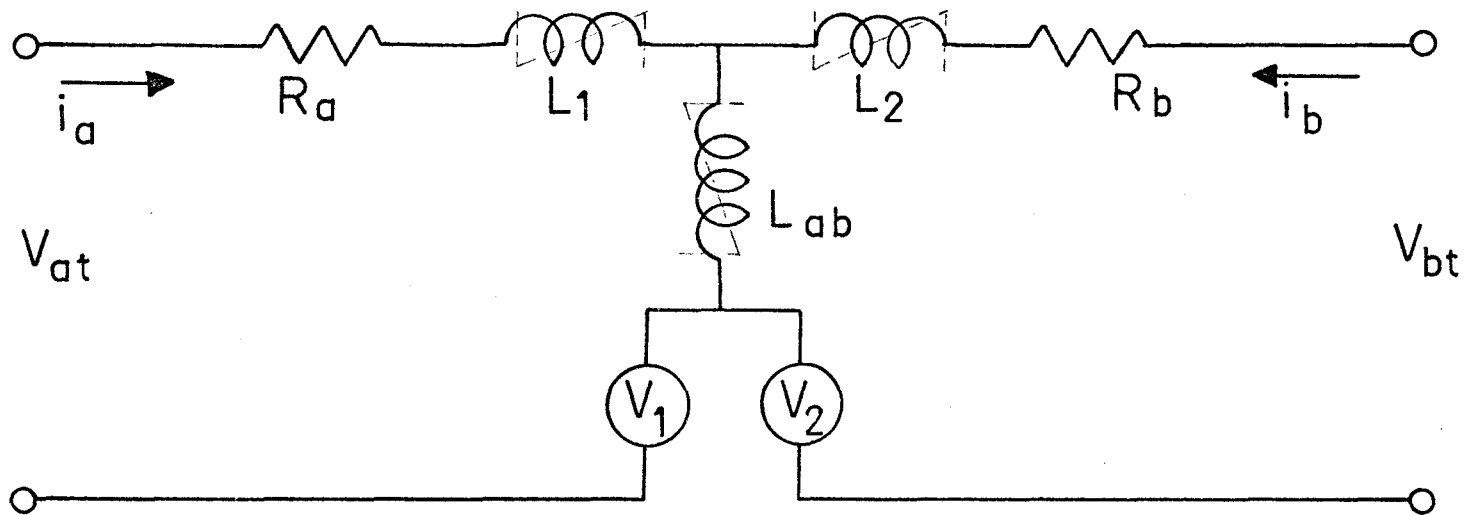
Figure 3-2

Equivalent Circuit For A Two-Phase Permanent Magnet Stepping Motor

$(L_1 = L_{aa} - L_{ab} \; ; \; L_2 = L_{bb} - L_{ab}; \; V_1 = I_r \dfrac{dL_{ra}(\theta)}{d\theta} \dfrac{d\theta}{dt} \; ; \; V_2 = I_r \dfrac{dL_{rb}(\theta)}{d\theta} \dfrac{d\theta}{dt} \;)$

i_a and i_b of the same magnitude. The equation may be written as

$$T^e = 4i_a i_b L_2 N_r \cos(2N_r\theta) + K_1\left[i_b\cos(N_r\theta) - i_a\sin(N_r\theta)\right] \qquad (3 - 14)$$

Consequently the no-load torque balanced equation may be expressed as

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} - 4i_a i_b L_2 N_r \cos(2N_r\theta) - K_1\left[i_b\cos(N_r\theta) - i_a\sin(N_r\theta)\right] = 0$$
$$(3 - 15)$$

Equation 3-15 may be analysed by using the flow diagram shown in Figure 3-1 with minor modification of the constants CL_1, CL_2, CL_3 CL_4, so that the equation may include the term $4i_a i_b L_2 N_r \cos(2N_r\theta)$.

b. MIMIC - as discussed before MIMIC may be used to analyse equation 3-15. The computation time for using MIMIC is more than that with Fortran IV and the Runge Kutta formulas, but the convenience and easy management of MIMIC makes it a very attractive tool for this kind of continuous nonlinear analysis. A typical MIMIC program written in accordance with equation 3-15 is shown on the following page.

c. Analog computer simulations - compared with the preceding methods, the analog computer simulation technique is the most time consuming in the preparation and the scaling techniques depend largely on experiences. However, the continuous outputs, the fast parameter variations and adjustments which one may expect to obtain from an analog computer makes this method favorable to those designers and system analysts who want to have an immediate answer. An analog computer simulation scheme for equation 3 - 15 is shown in

MIMIC LANGUAGE PROGRAM
SOLUTION USING CONSTANT CURRENT SOURCES


```
READ IN THE CONSTANTS.
C
1          DT=INCREMENTAL TIME CONSTNAT
2         ICX=INITIAL ROTOR POSITION
3         ICY=INITIAL ROTOR VELOCITY
4          TF=UPPER TIME LIMIT
5     L2, K1,NR,B,BJ=J ARE MOTOR CONSTNATS
6         W =FREQUENCY IN HERTZ, I =MAGNITUDE OF CURRENT IN AMPERES
                 CON(K1,NR,B,J,I,L2)
                 PAR(DT,ICX,ICY,TF,W )
         I1       I*4.0/3.14159
         IA1      COS(W*T)-(COS(3.*W*T))/3.0
         IA2      .2*COS(5.*W*T)-(COS(7.*W*T))/7.
         IA3      (COS(9.*W*T))/9.-(COS(W*T*11.))/11.
         IA       (IA1+IA2+IA3)*I1
         IB1      SIN(W*T)+(SIN(3.*W*T))/3.
         IB2      .2*SIN(5.*W*T)+(SIN(7.*W*T))/7.
         IB3      (SIN(9.*W*T))/9.+(SIN(11.*W*T))/11.
         IB       (IB1+IB2+IB3)*I1
         C1       4.*L2*NR/J
         C2       K1/J
         C3       B/J
         R1       C1*IA*IB*COS(100.*X)
         R2       C2*IB*COS(50.*X)
         R3       C2*IA*SIN(50.*X)
         X        INT(Y,ICX)
         Y        INT(R1+R2-R3-C3*Y,ICY)
         TORQ     R1+R2-R3
                  FIN(T,TF)
PRINT THE RESULTS, TIME, VELOCITY, DISPLACEMENT, CURRENTA
CURRENT B, AND TORQUE
                 HDR(TIME, TORQUE,VELOCITY,POSITION, IA,IB)
                 OUT(T,TORQ,Y,X,IA,IB)
DISPLACEMENT-TIME PLOT
                 LBL(TIME,POSITION)
                 PLO(T,X)
PHASE-PLANE PLOT
                 LBL(POSITION,VELOCITY)
                 PLO(X,Y)
TORQUE-DISPLACEMENT PLOT
                 LBL(TORQUE,POSITION)
                 PLO(TORQ,X)
                 END
```

Figure 3-3.

5. Application of the Salient-pole Model

A permanent-magnet stepping motor (SS25) had been driven as a generator and a plot of the no-load terminal full wave voltage is shown in Figure 3-4. The slope of the linear portion of the curve was found to be .537 volt/rad/sec = K_1. The motor gave 200 steps per revolution, therefore for each step the rotor rotated 1.8°= $.01\pi$ radian. The salient-pole model was used to study its characteristics at low and high frequencies. The positional accuracy is determined by comparing the number of steps which the rotor is supposed to have advanced to the actually computed displacement. For example, initially the rotor is at θ_o =$.005\pi$ radian, and after ten steps the motor is in the position which is $.105\pi$ radian from the reference position. This means that the rotor has rotated $.1\pi$ radian which is equal to 10 steps (1 step = $.01\pi$ radian). The motor is then said to be positionally accurate. The stability of the motor is determined from the phase-plane plot. The conditions for instability are:

1. the phase-trajectory crosses the θ-axis before completing a step;

2. the phase-trajectory never crosses the θ-axis;

3. the phase-trajectory crosses the θ-axis after many steps have gone by. In this case the motor oscillates.

The motor will run stably if the phase-trajectory crosses the θ-axis each step at a cross-over point slightly larger than a step interval.
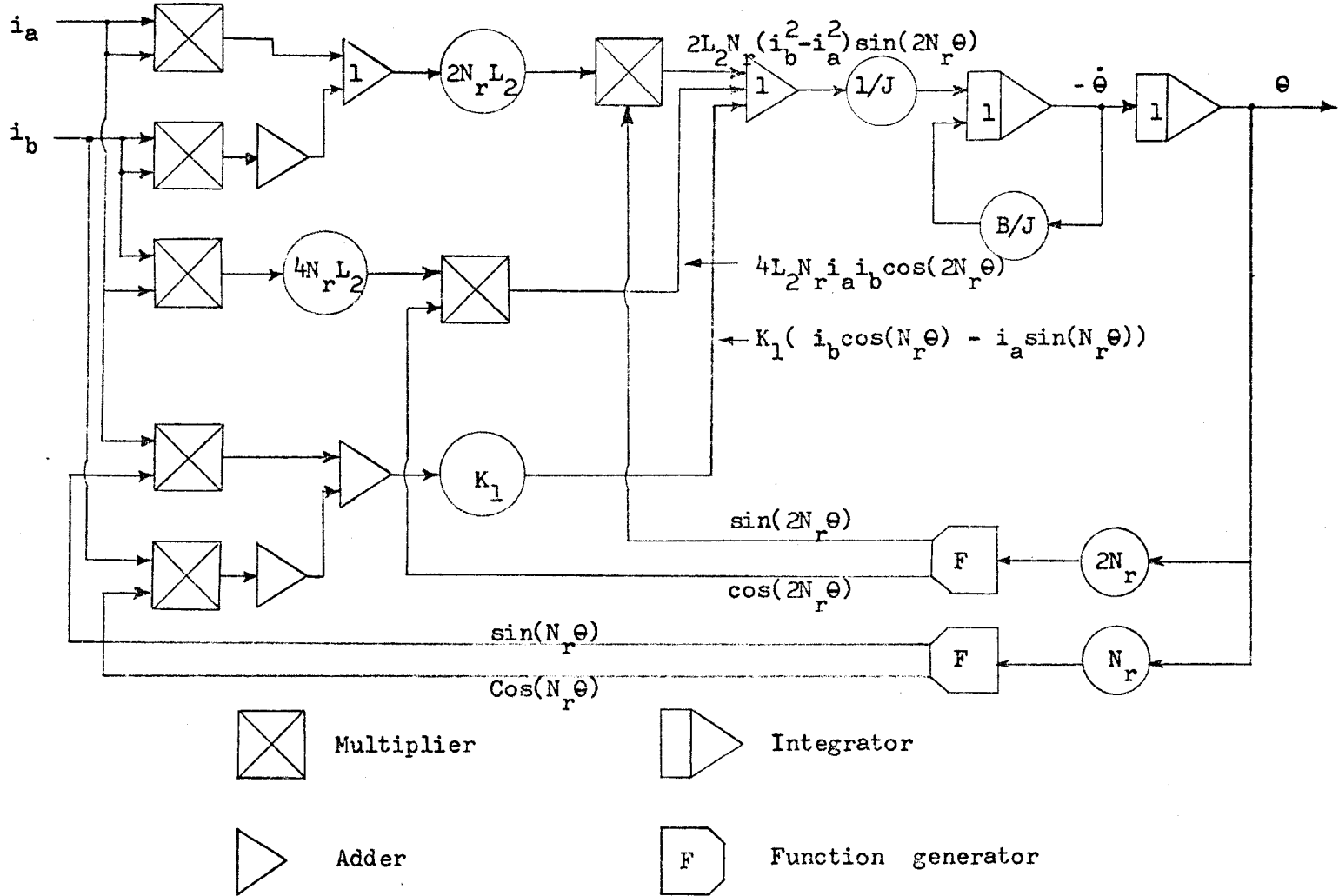
Figure 3-3: Analog Simulation of PM Stepping Motors with Constant Current Sources
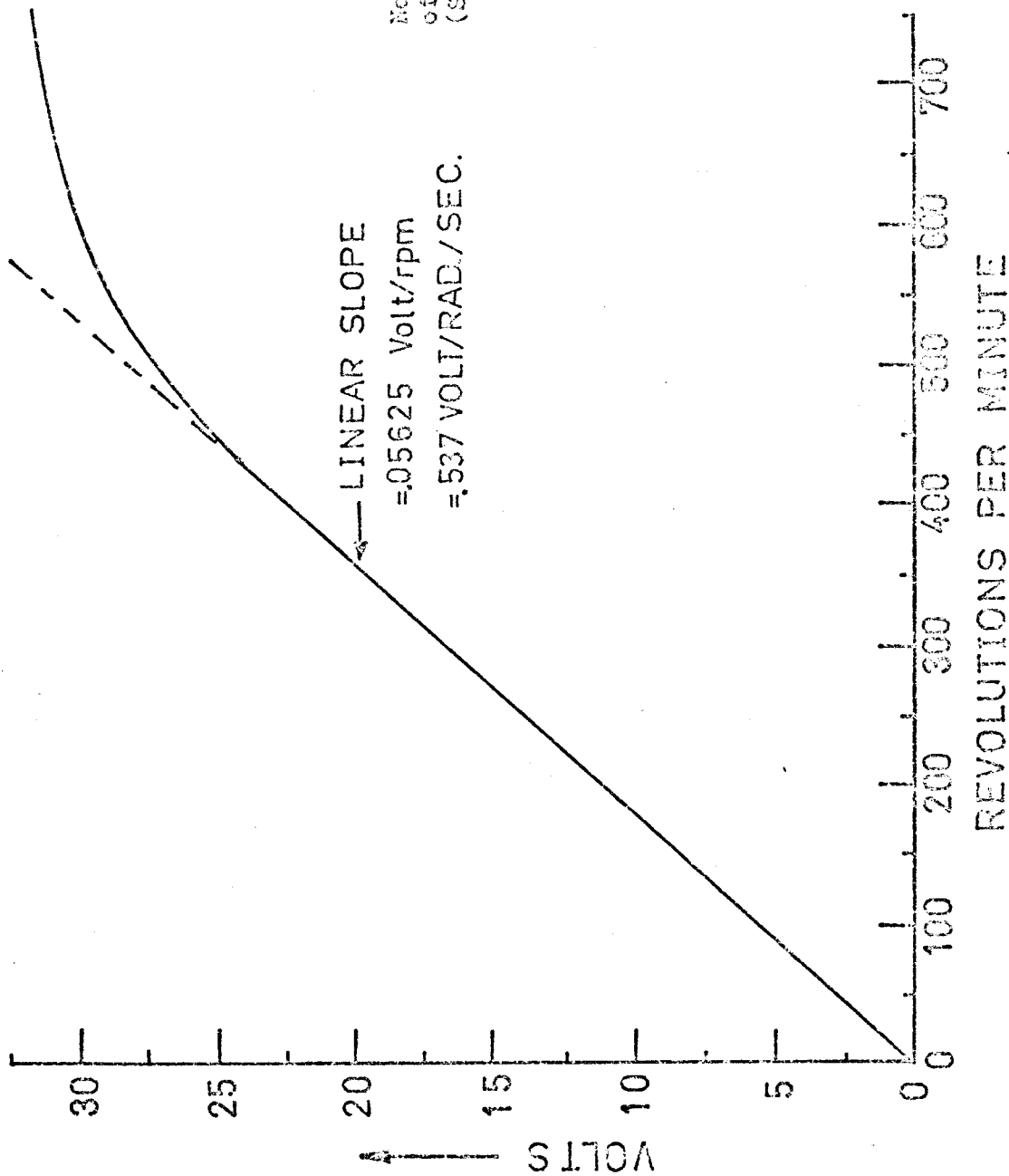
Figure 3-4

No-load Characteristics
of a PM Stepping Motor
(SS-25) Driven as Generator

A program written to faciliate this study is shown in appendix
C. The following data are used in the analysis:

$$J = .000025 \qquad \text{newton-meter-sec}^2/\text{rad.}$$

$$B = .0125 \qquad \text{newton-meter-sec /rad.}$$

$$N_r = 50$$

$$I = .35 \qquad \text{ampere}$$

$$L_2 = .0011 \qquad \text{henry}$$

$$K_1 = .537 \qquad \text{volt-sec/rad.}$$

A number of initial conditions have been attempted. The phase-plane
and step-by-step response plots are shown in Figures 3-5, 3-6 and 3-7.
Figure 3-5 illustrates the behavior of the PM stepping motor at low
frequency (30 hertz = 120 steps per second). It may be seen that, irre-
spective of the initial conditions of the rotor positions and velocit-
ies, the motor can reach its synchronous speed in a couple of steps
and is stable with accurate stepping response after the first step.
However, as indicated by this figure, there is only one initial cond-
ition (initial rotor position = .01π) which will enable the motor
to give the most accurate displacement with the given stepping sequence.
This phenomenon can be explained by referring to Figure 1-7. If the
unexcited motor has its initial condition as shown in step 1, then, when
the phase-windings are so excited that the polarities of the poles are
also as shown by step 1 of the figure, there will be no displacement
at all. Under this condition the motor will miss the initial step.
Other initial conditions which lead to a discrepancy in the angular
displacement of the first step may similarly be explained. This type
of inaccuracy is not caused by the motor and is essentially the

responsibility of the stepping sequence. When very accurate positioning is desired, it is important to know the initial rotor position. Figure 3-6 illustrates the behavior of the permanent-magnet (PM) stepping motor at a medium frequency (150 hz = 600 steps per second). The phase-plane plots indicate that the motor is not stable, they also indicate that with zero velocity initial condition the motor oscillates. The step-by-step response however in some cases(with initial rotor velocity greater than zero)indicates that the motor gives accurate stepping responses. The most accurate one is found to be with initial rotor velocity equal to 12 rad/sec, and the initial rotor position equal to .01π radian. The indication that the motor may not be started from rest means that the motor is operated in the slew range. One or two steps may be missed during the acceleration period of the motor. Operating the motor in this region normally requires closed-loop control which may yield a suitable compensation for the missing steps. Figure 3-7 illustrates operation in the unstable region for this motor (at 200 hz = 800 steps/sec). It is seen that regardless of the starting conditions the motor oscillates. Consequently,it may not be operated at this stepping rate. This does not mean that the optimal stepping rate of the motor is limited to a stepping rate lower than 200 hertz. It only illustrates that, with a repetitive stepping sequence, unaltered motor constants, and at this stepping rate, the motor oscilates and may not be operated. With some complicated circuitry and multistep operations the motor may be operated at a higher stepping rate. Fredriksen [10] used closed-loop control to run this type of motor at a stepping rate over 2000 steps per second.
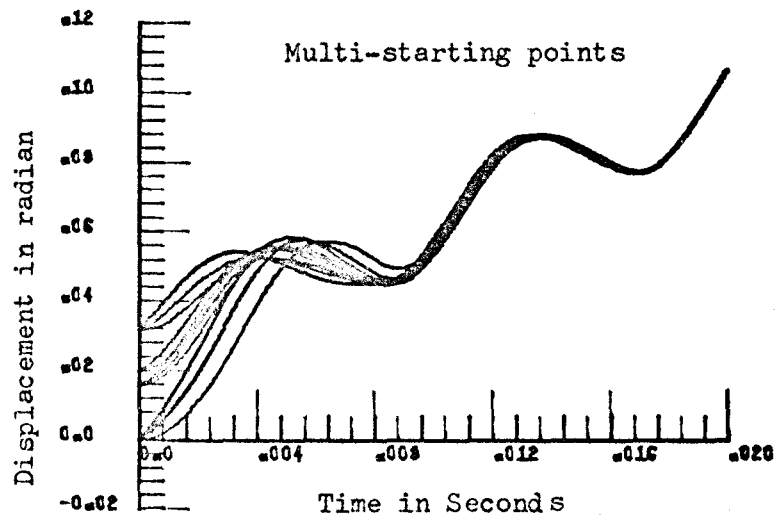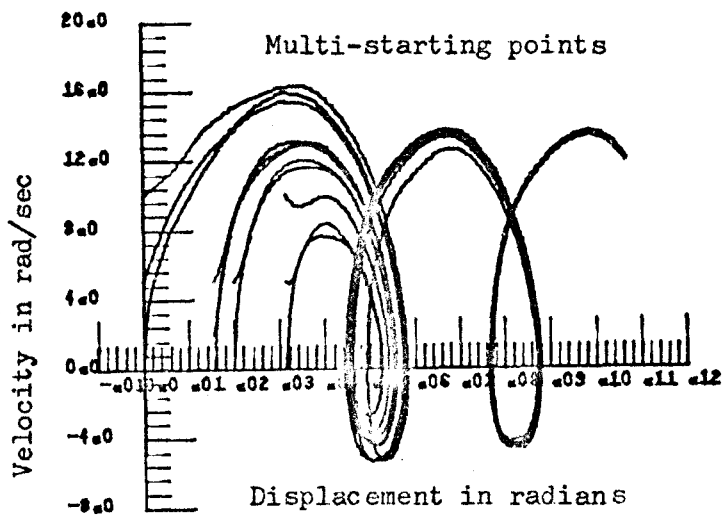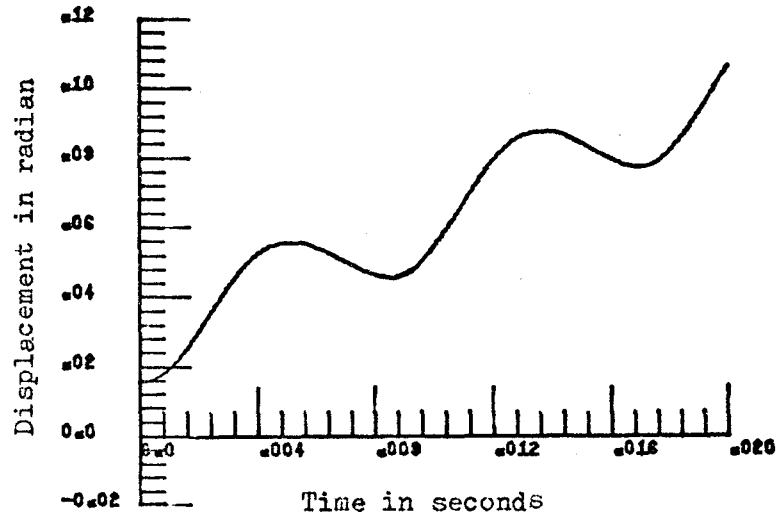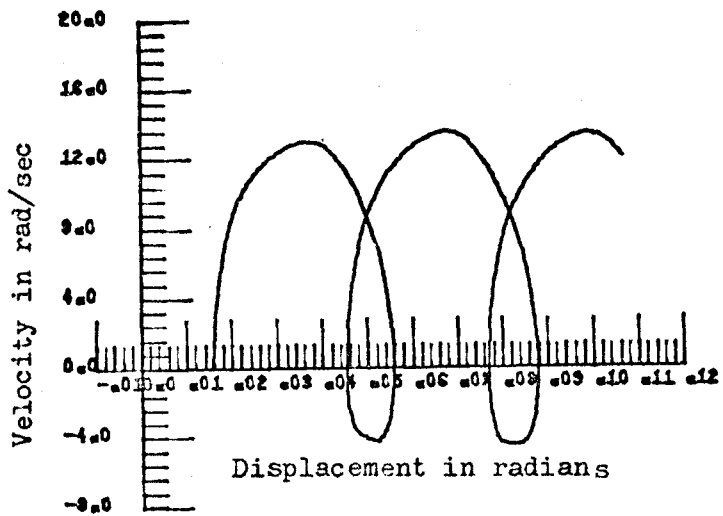
Figure 3-5: Phase-Plane Plots and Displacement Time Plots (30 Hz)
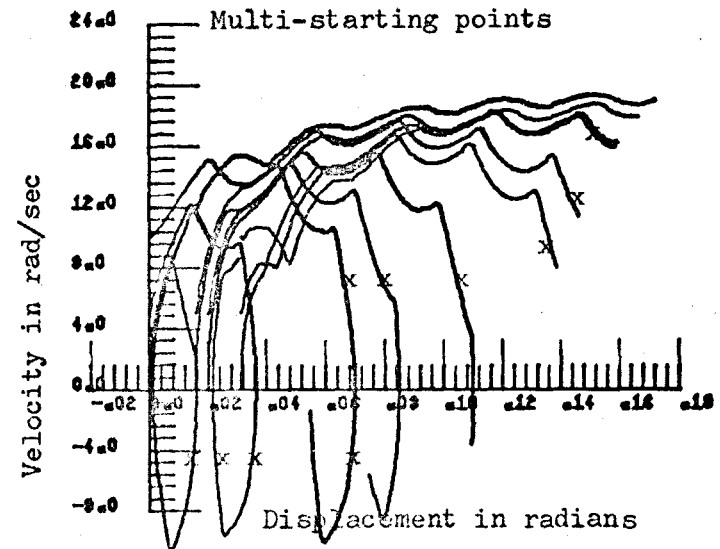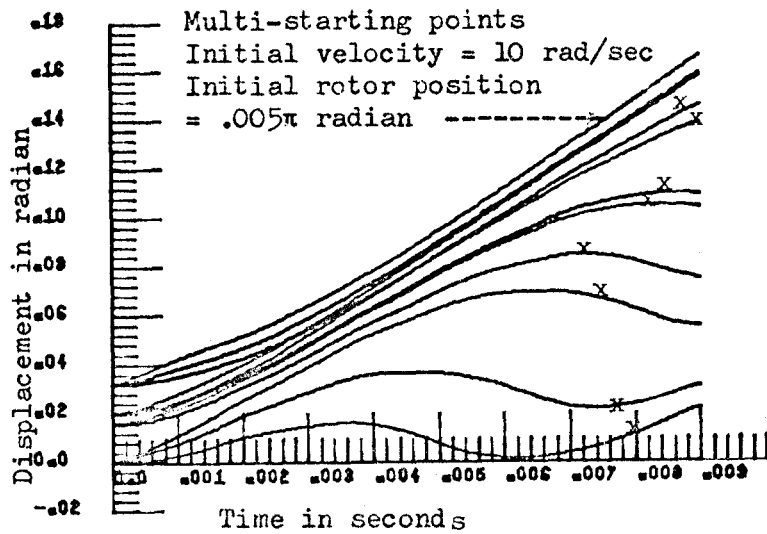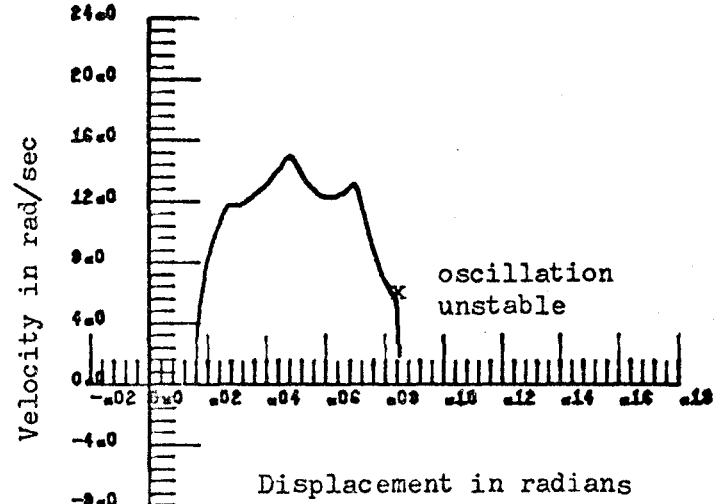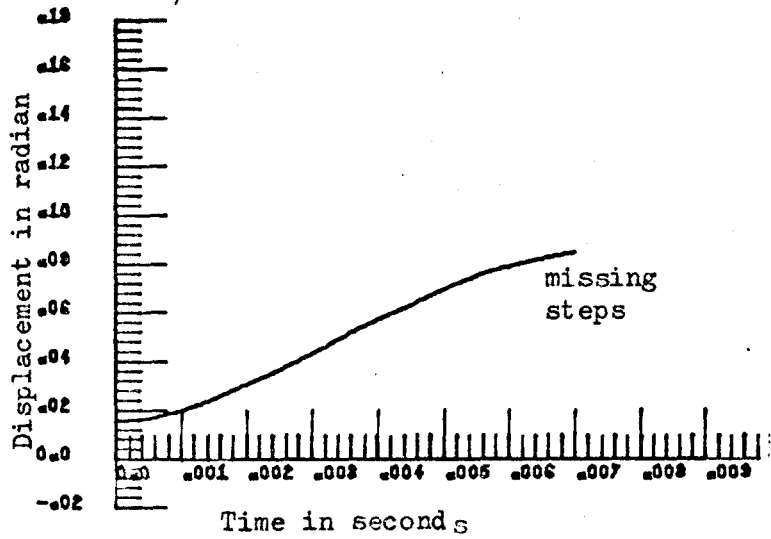
Figure 3-6: Phase-Plane Plots and Displacement Time Plots (150 Hz)

Figure 3-7

Phase-Plane Plots and
Displacement Time Plots
(200 Hz)
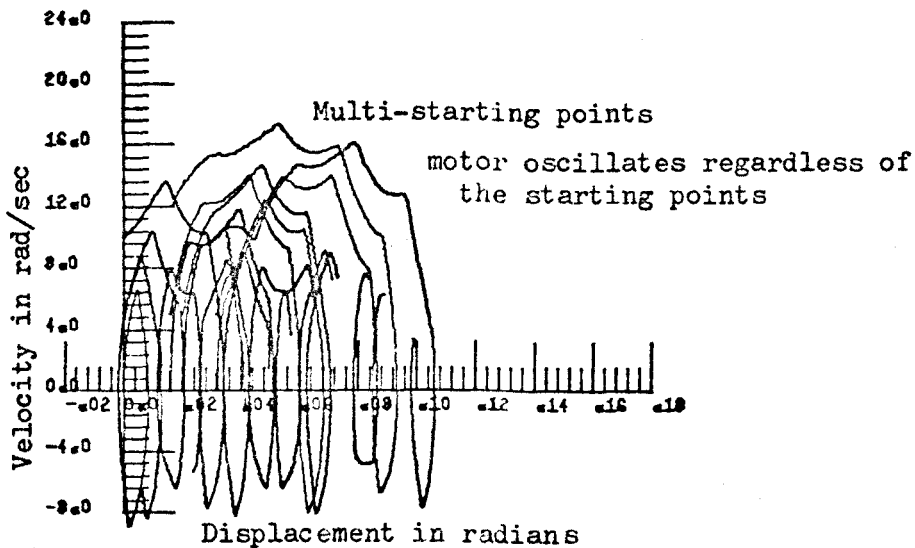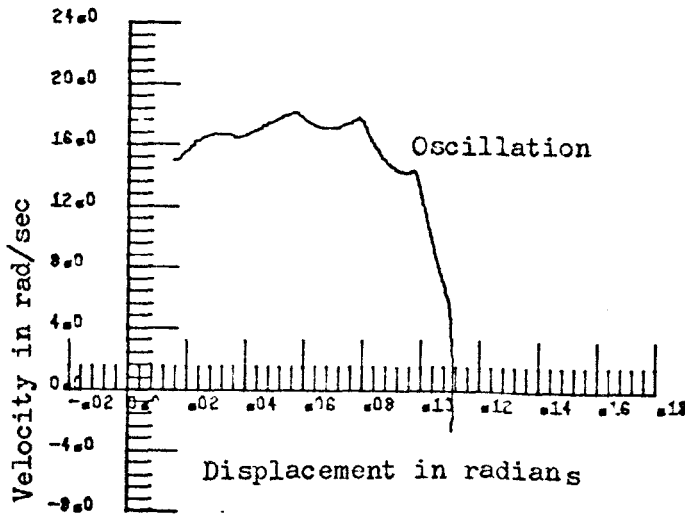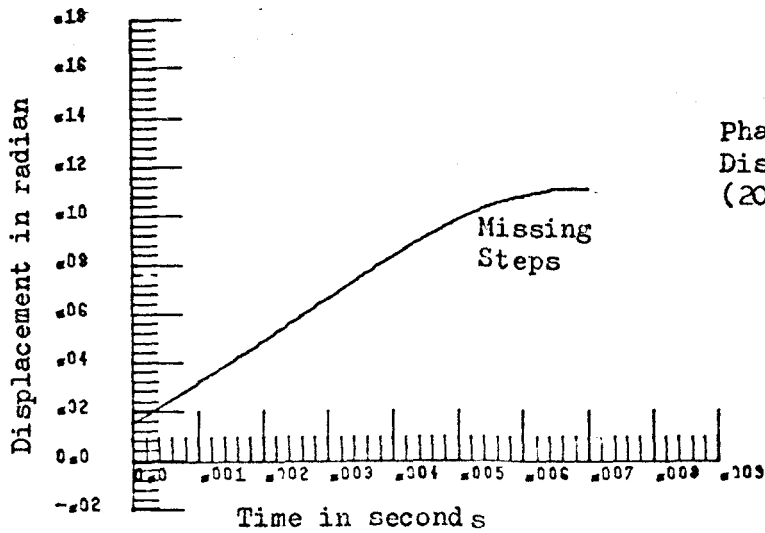
The time required to decelerate the motor from a certain velocity to rest is often a very interesting problem. The decelerating behavior of this PM stepping motor can be computed from equation 3-15 by assuming that the currents in the phase-windings are direct currents. The initial position and velocity of the rotor will be set equal to the instantaneous position and velocity. A plot of velocity versus time will give a clear idea of the stopping behavior. The time taken by the rotor to decelerate from the given velocity to rest may be estimated from the plot. Figure 3-8 illustrates two such plots. The top plot illustrates the stopping response of the motor from 600 steps/sec. It is seen that it takes about 20 milliseconds in order to have the motor completely stopped. The displacement of the rotor during this period may be found by summing the area under the curve. The bottom one illustrates the stopping response from 120 steps/sec. It is seen that the deceleration time is shorter. The general shape of this type of response graphs has been verified by Fredriksen [10] experimentally.

Sometimes the power supply to the PM stepping motor may be completely shut off. The analysis of the stopping response under this condition is much simpler. Consider equation 3-15 with $i_a = i_b = 0$, then the equation may be written as

$$\frac{d^2\theta}{dt^2} + \frac{B}{J}\frac{d\theta}{dt} = 0 \qquad (3-16)$$

By taking the Laplace transform, equation 3-16 may be expressed as

$$\Theta(s) = \frac{\Theta_o}{s} + \frac{\dot{\Theta}_o}{s(s + B/J)} \qquad (3-17)$$

where Θ_o and $\dot{\Theta}_o$ are the instantaneous position and velocity of the rotor at the moment when the power is turned off. The inverse Laplace transform of equation 3-17 yields

$$\Theta(t) = \Theta_o + \dot{\Theta}_o (1 - e^{-Bt/J})/(B/J) \qquad (3-18)$$

and

$$\frac{d\Theta}{dt} = \dot{\Theta}_o e^{-Bt/J} \qquad (3-19)$$

Equation 3-19 indicates that under the sudden elimination of power supply the velocity of the motor decays exponentially.

The maximum stepping rate may be estimated by estimating the maximum instantaneous velocity from equation 3-15. The maximum or minimum velocity occurs at the condition that $\frac{d}{dt}(\frac{d\Theta}{dt})=0$. Under this condition the maximum or minimum velocity at a given rotor position is given by

$$\frac{d\Theta}{dt} = \frac{4 i_a i_b N_r}{B} \cos(2N_r\Theta) + \frac{K_1}{B} (i_b \cos(N_r\Theta) - i_a \sin(N_r\Theta)) \qquad (3-20)$$

In order to eliminate the time variable from 3-20 and for the convenience of estimation it may be assumed that the stepping sequence is given as
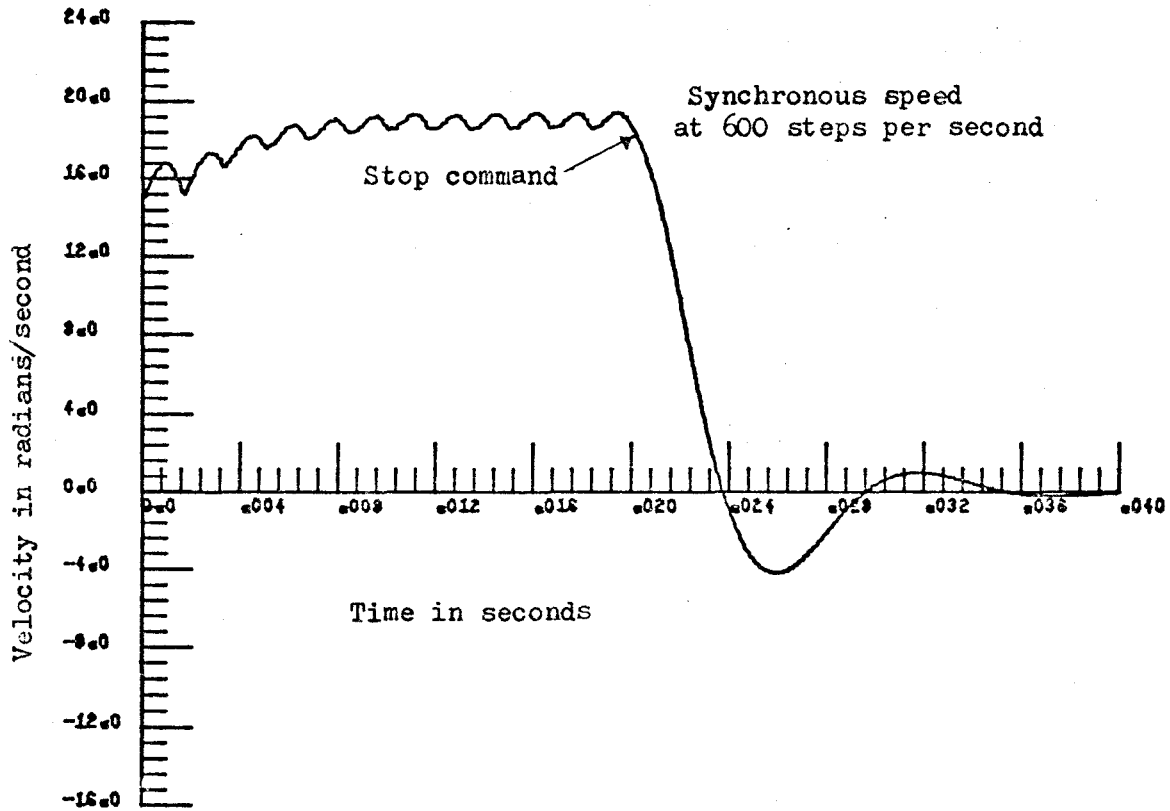
Figure 3-8 : Stop Responses of PM Stepping Motor
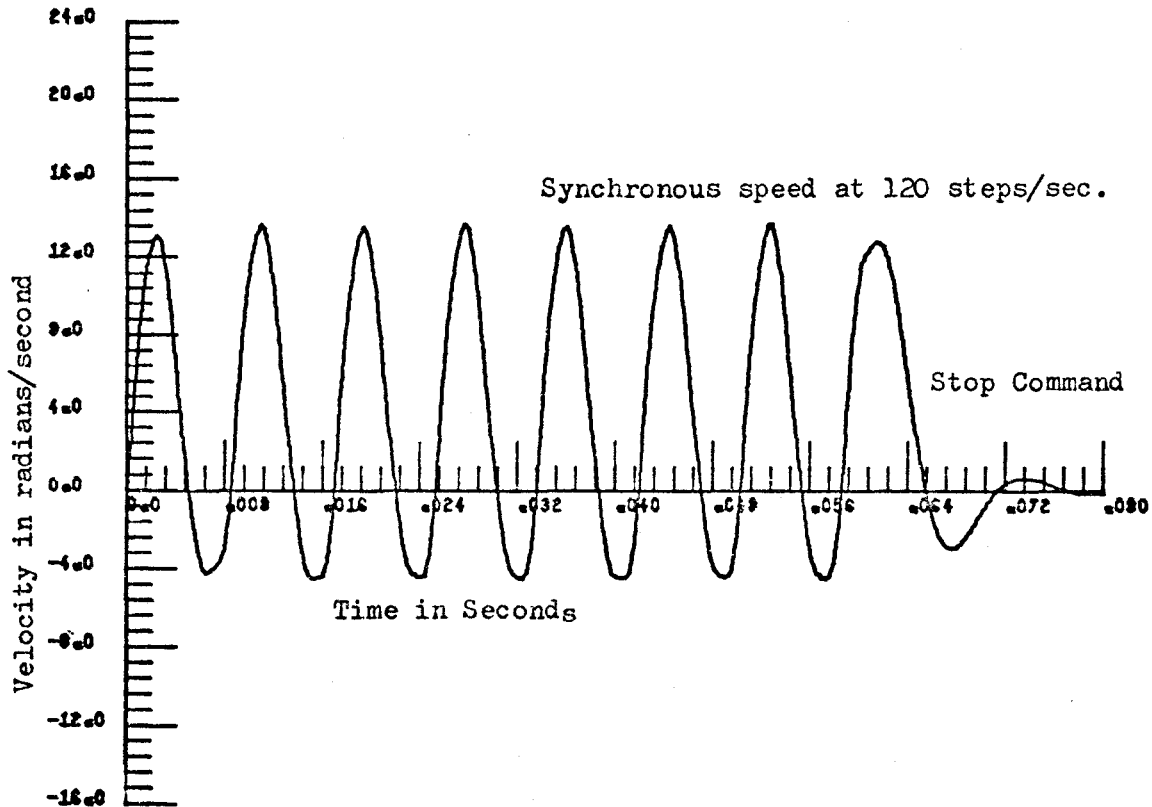
Step 1 $i_a = I = i_b$

Step 2 $-i_a = I = i_b$

Step 3 $-i_a = I = -i_b$

Step 4 $i_a = I = -i_b$

The maximum value of $d\theta/dt$ may be estimated from 3-20 by plotting $d\theta/dt$ versus $N_r\theta$ with $N_r\theta$ varies from zero to 360 degrees for each step. Each of the following equations may be used.

Step 1

$$\frac{d\theta}{dt} = \frac{4I^2 N_r}{B} \cos(2N_r\theta) + \frac{K_1 I}{B} (\cos(N_r\theta) - \sin(N_r\theta)) \qquad (3-21)$$

Step 2

$$\frac{d\theta}{dt} = \frac{-4I^2 N_r}{B} \cos(2N_r\theta) + \frac{K_1 I}{B} (\cos(N_r\theta) + \sin(N_r\theta)) \qquad (3-22)$$

Step 3

$$\frac{d\theta}{dt} = \frac{4I^2 N_r}{B} \cos(2N_r\theta) + \frac{K_1 I}{B} (\sin(N_r\theta) - \cos(N_r\theta)) \qquad (3-23)$$

Step 4

$$\frac{d\theta}{dt} = \frac{-4I^2 N_r}{B} \cos(2N_r\theta) - \frac{K_1 I}{B} (\cos(N_r\theta) + \sin(N_r\theta)) \qquad (3-24)$$

The maximum velocity is estimated from the plot where $d\theta/dt$ is maximum. However, since the velocity so obtained is in radians per

second it has to be divided by the step interval to be converted to steps per second. For instance, the maximum velocity is estimated as 22 radians per second and the motor displaces .01π radian per step, then the maximum stepping rate is about 700 steps per second.

Equation 3-15 may also be expressed as

$$J\frac{dv}{dt} = \frac{1}{v}\left[4i_a i_b L_2 N_r \cos(2N_r\theta) + K_1(i_b\cos(N\,\theta) - i_a\sin(N_r\theta))\right] - B$$

$$(3 - 25)$$

where $v = d\theta/dt$. It will be seen from 3-25 that with increasing v, Jdv/dt can never be zero which confirms that there is a maximum v provided that B is not equal to zero. However, the maximum v may be increased by increasing i_a and i_b .

6. Comparison of the Non-linear Models

From a comparison of equation 3-3 and equation 3-14 it will be seen that, with the assumption of constant current sources, the difference between the nonlinear models is given by the term $4\,i_a i_b L_2 N_r\cos(2N_r\theta)$. The value of L_2 according to the definition given by 3-11 is usually very small, and if I_a and I_b are less than unity, then the difference between these two models is not significant. However, for small angular displacement per step N_r has to be large and for higher torque, i_a and i_b have to be large as well. A typical PM stepping motor (HS-50) has a rated current of four amperes and N_r equals 50. The maximum torque predicted by the models will differ by a significant figure. Consequently the stepping responses predicted from these models are also

different.

As an illustration of the differences, a PM stepping motor described by the following parameters is analysed.

K_1 = .30   volt/rad/sec

N_r = 50

J = .000094   newton-meter sec^2/rad

B = .06   newton-meter sec /rad

L = .00518   henry

L_2 = .00025   henry

R = 5.7   ohms

I = 2   amperes

The stepping rate is set at 60 steps per second. The phase-plane plots are shown in figure 3-9 while the torque-displacement and transient responses are shown in figure 3-10. The greatest difference between the models is shown on the torque displacement plots, the salient-pole model indicates that the motor can give a greater useful torque than that predicted by the smooth air-gap model. The phase-plane plots and transient response plots given by the salient-pole model indicates that the motor is far from being critically damped.

In the estimation of maximum stepping rate the difference between these two models may be approximated by the ratio of the terms $4I^2 N_r L_2$ to $2 K_1 I$. In this case this ratio is approximately equal to twenty-five per cent .

Figure 3-9: Phase-Plane Plots (15 Hz)
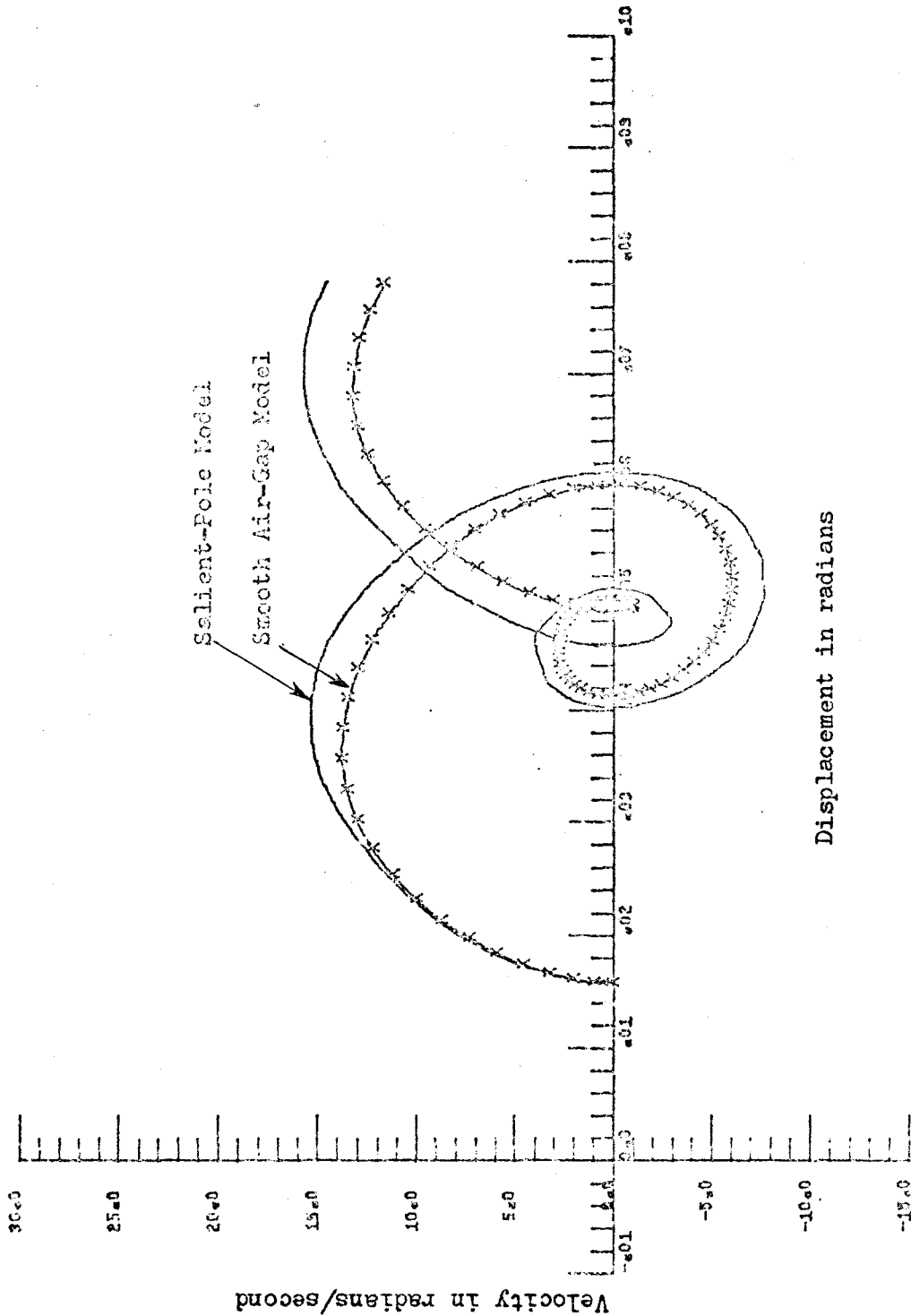
Figure 3-10: a. Torque-Displacement Plot
b. Displacement-Time Plot
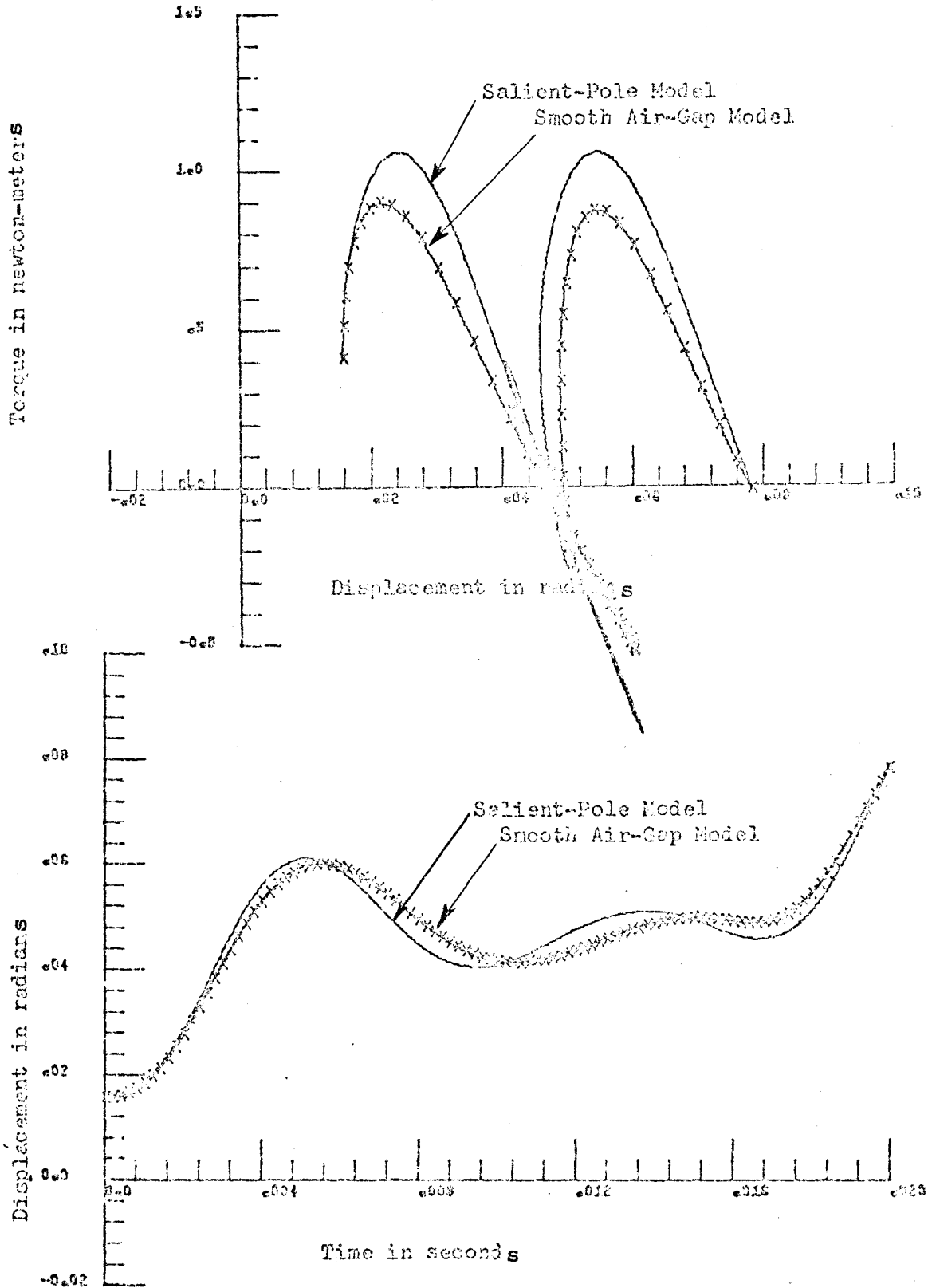
7. Effects of the motor constants on the performance

of a stepping motor

From the constant current source analysis it is found that each constant has its effect on the performance of the motor. Increasing J will bring the same result as decreasing B, L_2 and K_1 by the same factor, but as seen from 3-25, the maximum stepping rate will not be affected directly. The settling time for the step response will be larger due to increasing J which results in decreasing B. B/J resembles the damping ratio of a second order linear system and K_1/J accounts for most of the natural frequency though L_2/J contributes some of the effects. The excitation currents have a direct effect on the maximum stepping rate. Increasing currents results in increasing the maximum stepping rate, increasing maximum electromagnetic torque and increasing the natural frequency. The effect of an external load applied to the shaft of the motor may be accounted for by increasing J or B depending on the way it is attached to the shaft. A heavy load will result in over-damping which will slow down the speed of the motor and eventually the motion may be completely damped. The maximum load torque applied to the shaft of the motor should not excede .707 times the maximum torque produced by the motor. With a constant voltage source supply, a stepping motor will not behave in the same way as it does with constant current source. The resistance (R) and self-inductance (L) of the windings have a definite effect on the motor. The time constant which is the ratio of inductance and resistance (L/R) limits the rise time of the current. For low stepping rates the currents have

sufficient time to rise up to the rated or desired values, however, as the stepping rate increases there will be insufficient time for the currents to rise up to the rated or desired values. As mentioned before, decreasing the current will decrease the output torque and the maximum stepping rate, and in such a case, if a motor cannot produce enough torque to overcome the inertia torque of its rotor, then the motor simply does not step at all. Consequently, in many operations of stepping motors, external resistors are added in series with the windings to decrease the time constant. The current is then compensated for by using higher voltages.

8. Conclusion

In order to analyse a PM stepping motor either the linear or nonlinear models may be used depending on the accuracy and results desired. The second order linear model can be handled easily. There are fixed formulas for calculating the settling time, and plotting the step responses. The third order linear model is not so simple and factorization of the transfer function usually is tedious. Consequently, it is difficult to obtain a step response, however, the frequency response plots may be obtained easily. The non-linear models requires some numerical analysis while being able to describe the dynamic behavior of a PM stepping motor. They may be used to illustrate the stable, slew and unstable regions. The effect of unbalanced excitation currents may also be shown by substituting the corresponding values of the currents into the equations. The salient-pole nonlinear model includes a second harmonic to account

for the effect of saliency since most PM stepping motors are salient-pole synchronous motors. The smooth air-gap model may yield a relatively close approximation but does not really represent the motor. The assumption of constant current sources in the analysis of the nonlinear models is highly reasonable since constant current sources are not difficult to obtain and in practical applications of stepping motors constant current sources have popularly been used.

CHAPTER FOUR

COMPUTER CONTROL OF STEPPING MOTORS

1. Introduction

There are numerous ways of controlling stepping motors [8, 9, 10, 16, 21, 22, 23, 26, 29]. The most fundamental idea is to get, by some means, a pulse chain or single pulse, then apply these pulses to a digital logic circuit which in turn goes to the stepping motor driver circuit. The pulse generator may be as simple as an on-off switch which generates one pulse at a time or an astable circuit which generates a pulse train at a fixed or variable frequency. The digital logic circuit may simply be a bistable circuit or a flip-flop, while the driver circuit may be constructed according to the type of motors used. Beling [29] had given a relatively useful presentation of the driver circuits. Since stepping motors are stable in open-loop applications, they do not require feedback though the latter may be added to improved the performance of the motor.

The applications of a stepping motor can generally be classified into the following categories:

1. Single stepping: The motor is required to advance one step at a time, within a prescribed time interval. Occasionally oscillation and overshoots are allowed.

2. Multiple stepping : The rotor is to travel a prescribed number of steps in the shortest possible time.

3. Constant speed drive (in the slew range): The motor is to reach a prescribed constant speed within a certain time interval.

In computer control of stepping motors the digital computer serves as a pulse generator and controller which generates pulses on command. The main difference between a digital processor and a regular pulse generator in this sense is that the computer is programmed to generate the desired number of pulses at a certain prescribed frequency, whereas the regular pulse generator may not perform the operation automatically. Most digital computers may generate single pulses, or pulse trains upon programming, however the programming languages used are quite different from the popular Fortran language. Machine or assembler languages are normally required.

2. Existing problems in computer control

When a digital computer is used to control a slow motion device such as the stepping motor, the designer has to consider first of all the frequency of the pulse train generated by the computer. Most computers generate a continous pulse train with a frequency so high that none of the existing stepping motors can stay in the stable region while being operated. One means of solving this problem is to pass the pulse train generated by the computer through a frequency divider circuit For instance, a BCD counter may be used to divide the frequency by ten. The second method to overcome this difficulty is to use a program delay technique. The programmer

writes a program so that the computer has to perform a group of func-
tionless commands over and over again before generating a pulse, which
in essence resembles the dummy DO loop in Fortran language. The third
method is to use a peripheral system which receives digital signals
from the computer and transforms them into pulses. In this sense the
computer is not a pulse generator but a controller and translator.
Comparatively the program delay method uses the least external elements,
but sometimes the pulse duration may be so short that very fast switch-
ing logic gates have to be used. The frequency divider method requires
more external elements and consumes the same amount of computer time
as the program delay method, therefore it is not an economical method.
The third method is relatively expensive but will save much of the
computer time, particularly when the digital computer is equipped with
a program interrupt facility. By using the program interrupt facility
the computer will not be fully occupied by the stepping motor, thus
it may control more devices. The program interrupt facility is to
be accomplished by a main program which contains many subroutines to
control various devices. A signal from the device which requests
action through the interrupter to thecomputer informs the computer
that the corresponding device subroutine is to be performed. After
finishing the subroutine the computer returns to its main program to
wait for interruptions from other devices. In a typical interrupt
operation, while the computer is executing one subroutine, it will
not start performing another subroutine until it has finished its
current subroutine. The peripheral system may contain a memory device
which accepts a command word from the computer indicating the number
of steps to be incremented, a pulse generator to generate pulses at the

prescribed frequency, an up-counter which counts the number of pulses
generated and a comparator which compares the data in the up-counter
with that shown in the memory device. When the numbers are equal, the
comparator will generate a signal to stop thepulse generator and to
inform the interrupt facility. The block diagram in figure 4-1
illustrates further how this program interrupt facility may be imple-
mented.

The second problem that one might encounter is the output
voltages from the computers. Since most digital computers use IC
gates, their output voltages may be expected to vary between three to
five volts. This voltage range will energize most digital gates,
however if the designer wants to obtain a higher voltage he may do
so by passing the output from the computer through an amplifier.

The third problem concerns the maximum distance between the
peripheral device and the computer. It is understood that pulses
take time to travel from the computer to the device. The delay time
would be appreciable if the distance between the computer and the
device is too great. Also the impedance of the transmission lines
thus used may attenuate the amplitudes of the pulses and modify their
rise time, thus there is a limiting distance between the computer and
the device under control. In the case that the delay time is non-
critical, remote control may be achieved by using substations.
The function of the sub-station is to amplify the signals from the
computer before transferring them to the device.

In addition to those existing problems in the computer just
mentioned, there are several other problems to be considered:
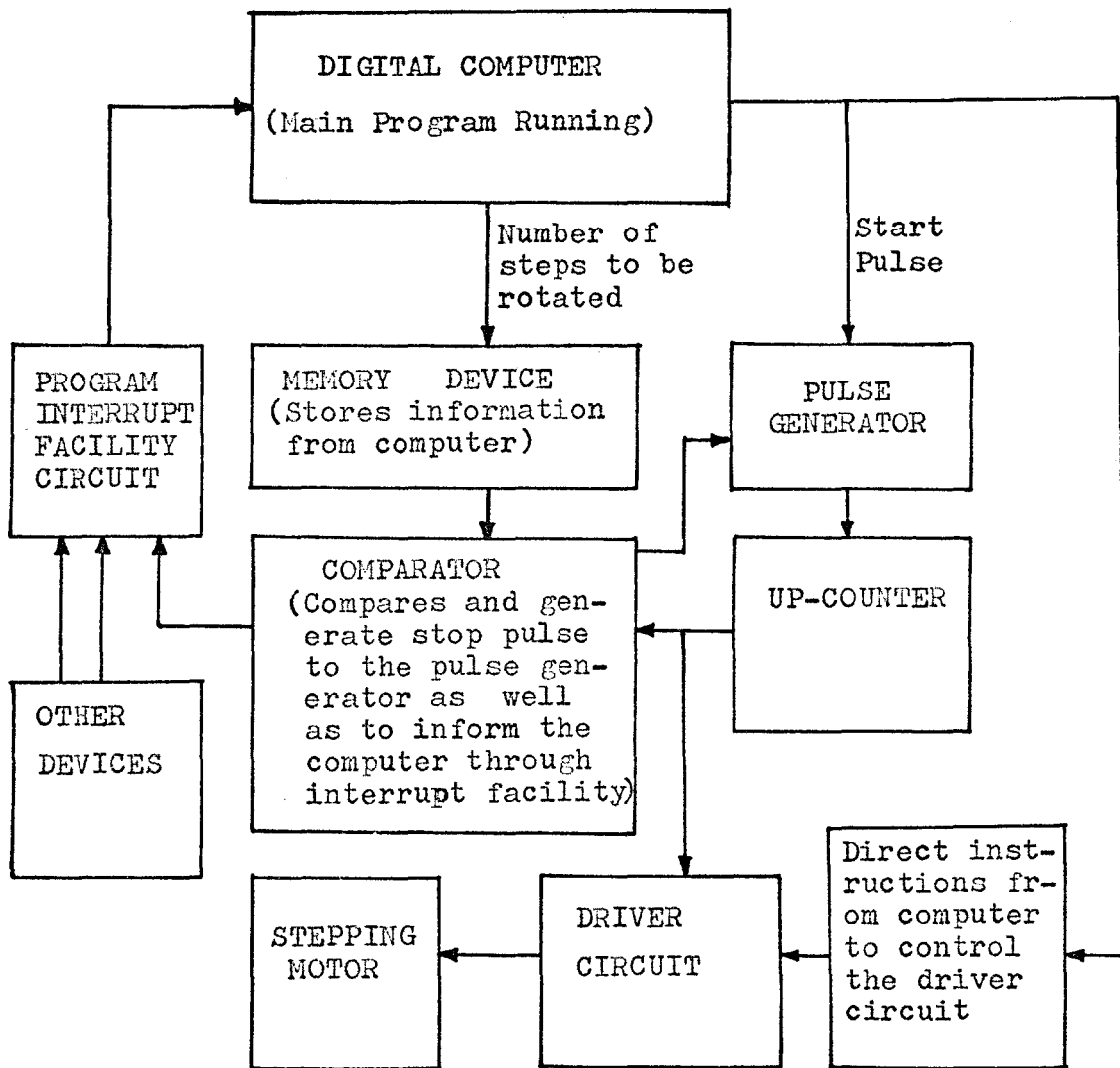
Figure 4-1

AN ILLUSTRATION OF OPERATING PROGRAM INTERRUPT
FACILITY

1. Means to start and stop the computer from transferring signals.

2. Means for initializing the motor to a known position in case of incompleted command and power failures.

3. Techniques to allow the motor to complete the execution of one command before the next is issued.

4. The motor under control must bear no accumulative positional errors.

Problem 1 may be solved easily by using programming techniques. The program controls the start signal and end signal completely. A solution for problem 2 is not so easily achieved. It is difficult to solve this problem by programming techniques alone. In this case there must be some positional feedback control to allow the computer to know where the position of the motor was when the system failed. Problem 3 may be solved using feedback signals. A signal is fed back to the computer when the device is ready for the next execution. According to the information given by many stepping motor manufacturers, many of their stepping motors have a characteristic of non-accumulative positional error. Thus a proper choice of stepping motor may solve problem 4.

3. Advantages of using the Digital Computer as a Stepping Motor controller

It appears that using a digital computer which costs more than ten thousand dollars to control a stepping motor is extravagant since stepping motors may be controlled easily by using other circuits which probably cost less than a hundred dollars. The

versatility of using the computer as a controller is that whenever the
user wants to change the switching rate, number of steps to be
rotated either clockwise or counterclockwise, duration of complete
working period etc., all he has to do is to modify his program with-
out changing his circuit, in addition, the control may be made fully
automatic. Optimal or sub-optimal control of stepping motors with
varying stepping rates may be achieved through suitable program delay
methods. Simultaneous operation of many stepping motors will require
only one computer operator whose work is to turn the computer on and
load the program into the computer.

4. A practical application of a Digital Computer as

a controller of a stepping motor

The computer used in this application is the PDP 8/L computer
manufactured by the Digital Equipment Corporation, and the stepping
motor under control is the SS25-1014 manufactured by the American
Superior Electric Company. Attempts were made to use the digital
computer to control the synchronous stepping motor in an open-loop
scheme. The following controls have been achieved:

1. Number of steps per revolution -- limited to two
choices: 100 steps/rev. and 200 steps/rev.

2. Stepping rate (steps/sec) -- from one step per second
to 250 steps per second with high accuracy.

3. Directions -- namely clockwise or counterclockwise
drive.

4. Number of steps -- a demonstration program limits a

maximum of 999 steps for either direction. Test showed that there were no missing steps.

The SLO-SYN SS25-1014 motor takes 200 steps to make one revolution of the rotor shaft. Thus for each step the rotor advances by 1.8^{o}. The manufacturer claims that the stepping motor has an accuracy of $\pm 0.09^{o}$ per step non-accumulative. The PDP-8/L is a twelve bit computer with an unextended memory of 4096 core locations. Assembler languages used are either PAL III or MACRO. Control of peripheral devices is through an input/output device selector code. One device code may control many devices provided that they are connected in parallel and are to be operated simulataneously. Other facilities such as program interrupt andprogrammed data transfer are also available [42, 43, 44,45]. In the stepping motor control that follows, only assembler language programming and the device selectors are involved. The assembler language being used is essentially the PAL III language.

The computer is used as a pulse generator as well as a controller in controlling the stepping motor. The speed control is achieved by using the program delay method. The output pulses are compatible with most digital gates and transistors that may be used in the digital circuit and the driver circuit. A continuous pulse train is obtainable through programming by using a single IOP pulse. Such a pulse train has a maximum frequency of approximately 171 KHz. The program shown below has been found to be very effective for testing the frequency of the pulse train generated.

/PROGRAM 1

*200

      CLA CLL

      6771

      JMP .-1

The starting address of this program is 200 (Octal). To perform the

command 6771 the computer takes 4.25 microseconds and to perform the

statement JMP .-1 it takes 1.6 microseconds. Thus a total of 5.85

microseconds is required for every cycle. The frequency may be

evaluated by the simple formula

$$f = 1/T$$ where f = frequency in hertz and T = period

                     in seconds

$$= 1/(5.58 \times 10^{-6}) \text{ hertz}$$

$$= 171 \text{ KHz.}$$

The pulse width is fixed at about 500 nanoseconds. By using the

program delay method the frequency may be lowered. Program 2

illustrates how the program delay method has been applied.

/PROGRAM 2

*200

      CLA CLL

      6771

      TAD A

      DCA COUNT

      ISZ COUNT

      JMP .-1

```
        JMP  .-5

        NOP

        NOP

A,      -2300

COUNT,  0

$
```

To perform each of the commands TAD, DCA, ISZ it takes the computer 3.2 µseconds, and to perform each direct JMP, NOP commands it takes 1.6 µseconds. The computer then will take 4.8 µseconds to perform the two commands ISZ COUNT and JMP .-1. Since A is set at -2300 (octal, equivalent to 1216 decimal) these two statements will be performed 1216 times before another pulse is generated, resulting in a delay of 5836.2 µseconds. A simple arithmetic shows that the output pulse train will have a frequency of 171 Hz. The two NOP commands are placed in the program to obtain a more accurate result. This method may be used to generate any other frequency which is lower than the maximum frequency by adjusting the value of A. The accuracy of this evaluation may be slightly different from one PDP 8 computer to another, therefore some compensation may have to be done to the calculated value. This may be achieved by trial and error.

The instructions to the computer as to the number of steps and directions for the stepping motor to rotate is accepted from the teletype. A decimal number typed on the teletype is converted by the program into its corresponding octal number and stored. On command the computer will generate the desired number of pulses. The driver circuit for the motor is designed according to the voltage and current

ratings of the motor using a constant voltage source supply. A block
diagram representation of this open-loop control system is shown in
Figure 4-2 and detailed driver circuit and digital logic circuits are
shown in appendix D and appendix E respectively.

5. Design of a closed-loop control system

In the design of a stepping motor system, it is necessary to
guarantee that the rotor actually advances one position before a signal
for another step is initiated to avoid ambiguity between the actual
and desired positions. This can be done as mentioned before in an
open-loop system by setting the input rate low enough to assure that
the motor can follow the input signal sequence. Smoother, faster
operation can be obtained, however, by closing the loop and sensing
the present position before supplying another input. The reaction of
a motor to a particular input change depends on the position of the
rotor at the time of the change. It is the tendency of the rotor to
move to the nearest steady-state position corresponding to the input,
thus to predict motor action it is only necessary to know where the
rotor is. Also, if the rotor position is known, the adjacent steady-
state positions are also known and any change in input calls for a
unique motor action.

This design of closed-loop stepping motor system aims at
minimizing the positional error while maximizing the stepping rate.
The loop is closed through light sensors and the positional accuracy
is interpreted by the computer through the inputs from the accumulator.

READER & TELETYPE

Input
Program and
Data

PDP 8/L
COMPUTER

Executes the Program
in accordance to the
input data. Gives
signals to the
Digital Circuit
through device
selector

DIGITAL CIRCUIT

200 Steps per
revolution

DRIVER CIRCUIT
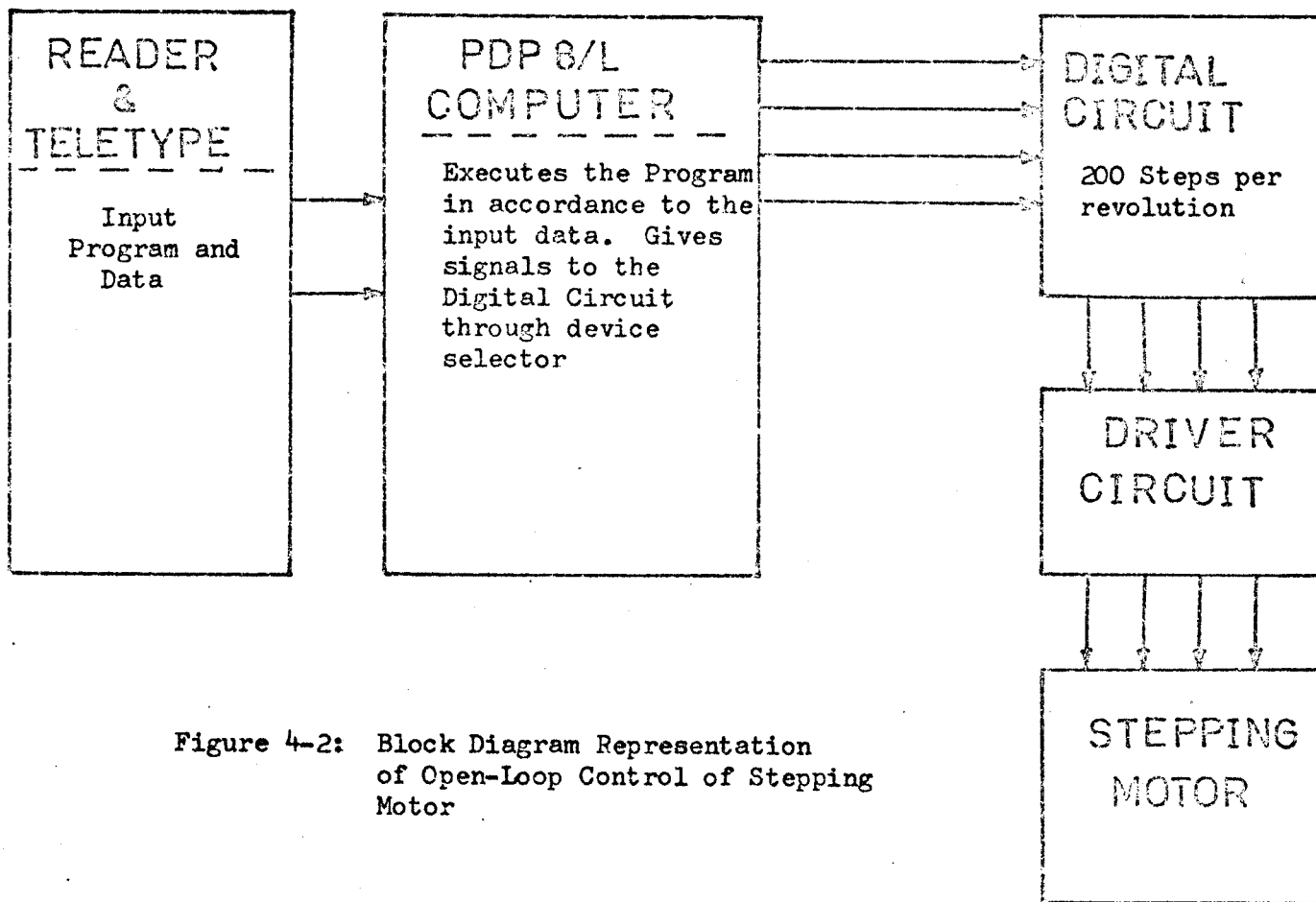
STEPPING MOTOR

Figure 4-2: Block Diagram Representation
of Open-Loop Control of Stepping
Motor

The complete system is represented by the block diagrem shown in Figure 4-3. The functions and constructions of each of the blocks are discussed in the following.

1. The PDP 8/L computer - The computer accepts and stores input data from the teletype through a machine language program. The computer accepts digital signals from the feedback sensors through an accumulator inputting device. The computer controls the translator by sending its command through the device selectors using the IOP pulses. The necessary machine language program is written such that data may be stored and modified easily. The program will enable the computer to perform the following actions:

a. To generate IOP pulses in an increasing or decreasing frequency. These pulses are used to actuate the motor.

b. To generate IOP pulses to energize clockwise or counter-clockwise operation of the motor.

c. To generate IOP pulses to control repetitive single or multistepping.

d. To enable the accumulator input to accept data from the feedback sensors so that the position of the rotor may be checked from step to step, meanwhile recording the total number of steps gone by. This total number of steps is then compared to the number of steps required and the discrepancy is then compensated. Figure 4-4 shows the designed flowchart for this closed-loop system.

2. The translator - it receives all the IOP pulses from the computer and translates them into a stepping sequence to the driver circuit. It contains the following components:
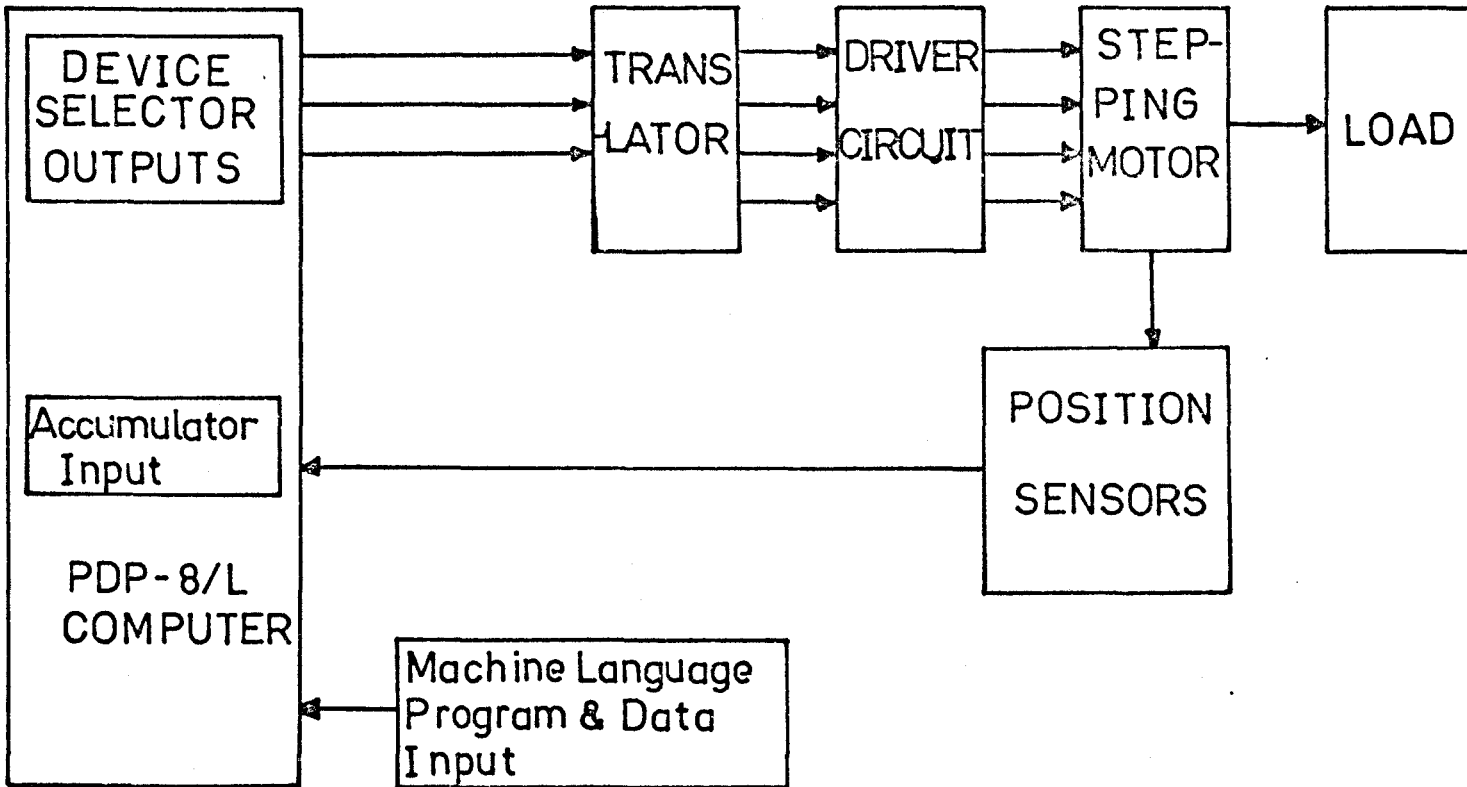
Figure 4 - 3

CLOSED-LOOP COMPUTER CONTROL OF A
PERMANENT-MAGNET STEPPING MOTOR
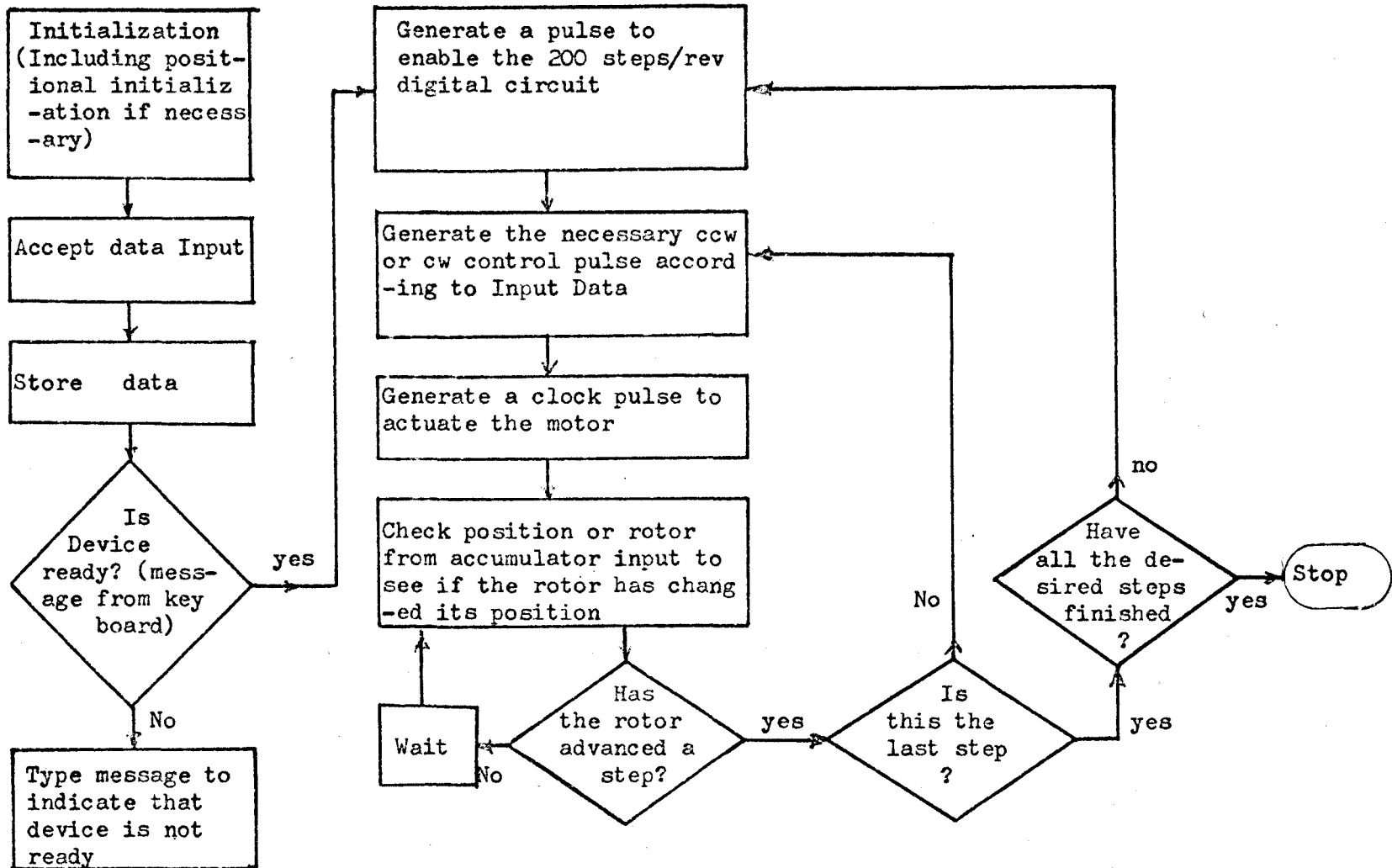
Figure 4-4: Fundamental Flowchart for Closed-Loop
Computer control of a Stepping Motor

a. A directional controller which may be easily set up by using two J-K flip-flops.

b. A 200 steps/rev. and a 100 steps/rev. logic circuits. The 100 steps/rev. circuit is to enable the motor to advance two steps on one command. This is one form of the multistepping circuits.

c. Double stepping or single stepping selector. This may also be done easily by using J-K flip-flops. A designed translator circuit is shown in appendix F.

3. Motor driver circuit - the main function of this circuit is to provide the necessary power to operate the rotor by following the commands from the translator faithfully. In order to enable the stepping motor to be driven in the slew range without continuously changing the voltage sources, the circuit is to be energized by using constant current sources. The circuit shown in appendix D with the switch connected to the constant current source supply has been designed for this purpose.

4. The position sensors - these position sensors were first proposed and tested by Fredriksen [8, 10] . The position sensoring system uses four photo-transistors and four light sources located consecutively at 23.4° apart. A cardboard with fifty holes evenly distributed at 7.2° apart on the circumference is secured to the shaft of the motor. This cardboard is to be large enough to keep the holes far enough from each other avoid ambiguity. This special arrangement ensures that for each step the rotor rotates, only one photo-transistor may be energized. Figure 4-5 illustrates this light sensoring principle as well as the general layout of this feed-

0°   23.4°   46.8°   70.2°

4 light sensors located at 23.4° apart

0° 7.2° 14.4° 21.6° 28.8° 36° 43.2° 50.4° 57.6° 64.8° 72°

Step 1

0° 7.2° 14.4° 21.6° 28.8° 36° 43.2° 50.4° 57.6° 64.8° 72°

Step 2

0° 7.2° 14.4° 21.6° 28.8° 36° 43.2° 50.4° 57.6° 64.8° 72°

Step 3

0° 7.2° 14.4° 21.6° 28.8° 36° 43.2° 50.4° 57.6° 64.8° 72°

Step 4

50 Holes on Cardboard evenly distributed at 7.2° apart

0°   10°   20°   30°   40°   50°   60°   70°
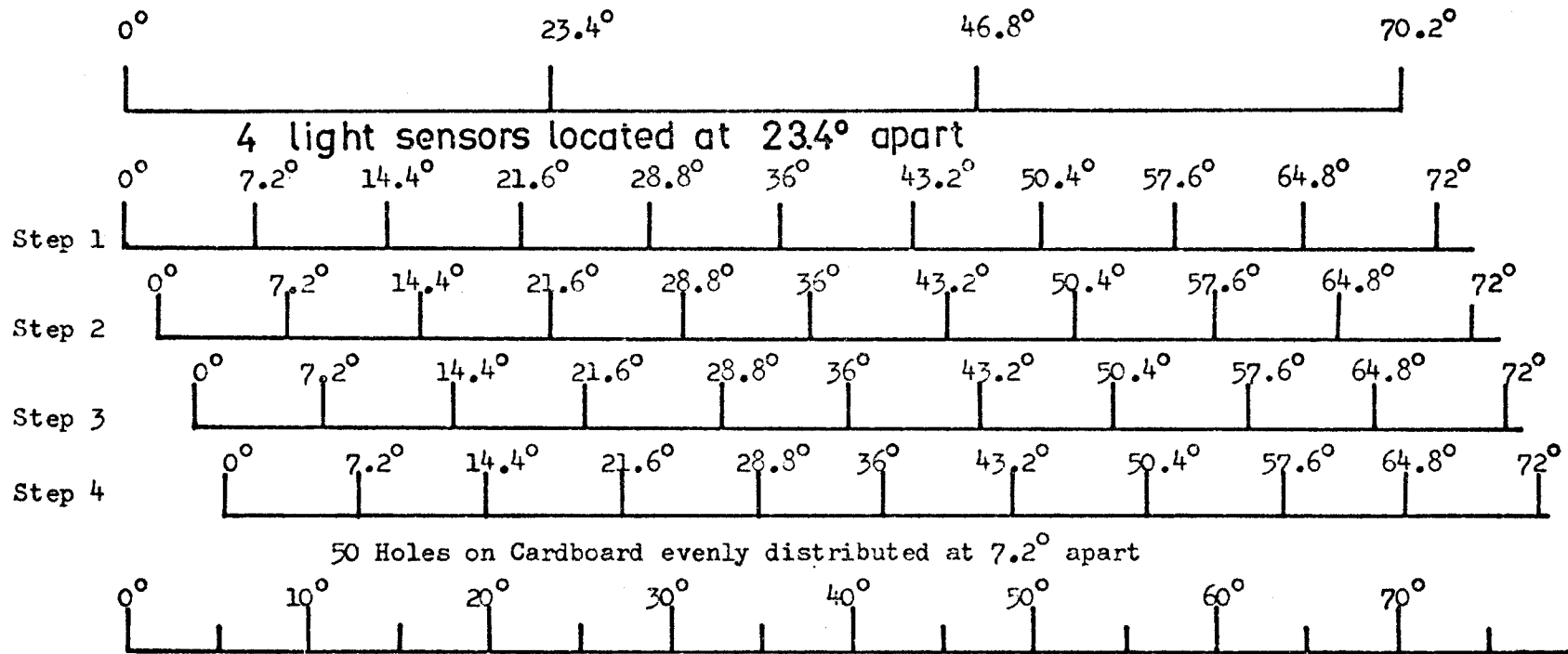
SCALE

4 light sources   4 photo-transistors

STEPPING MOTOR

connected to driver circuit

shaft

cardboard with 50 holes
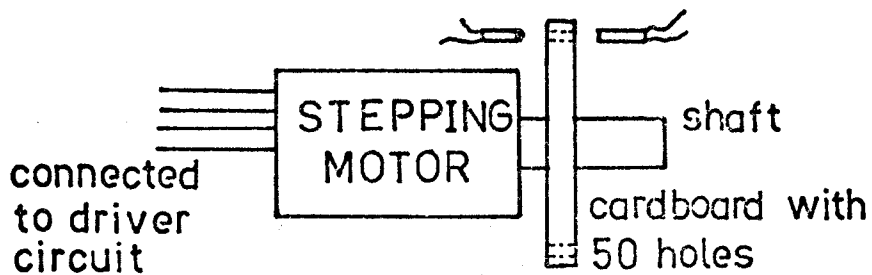
GENERAL ARRANGEMENT OF FEEDBACK SENSORS

Figure 4-5: Light Sensoring Principle and General Layout of Feedback System

back system. Assume that initially the first light source passes through the zero degree hole on the cardboard as shown in step one. When the rotor has rotated 1.8° forward, the second light source will pass through the 21.6° hole on the cardboard. The photo-transistors will be turned on when they are lighted. The signals from these photo-transistor are then amplified before going into the accumulator of the computer. The accumulator accepts these signals and decodes them into the step numbers. Basically there are only four different steps, thus the next position of the rotor may be predicted when the present step number is known. Referring back to figure 1-7, if the position of the rotor is known as shown in step one, single stepping sequence in the clockwise direction will bring the second step to step 2. However, after the rotor has established the tendency of clockwise rotation double stepping may be initiated. With the rotor in step 2 (figure 1-7) and the phase windings energized so that the polarities of the poles are as shown in step 4, then the rotor will also assume a position as shown in step 4 and under this operation the rotor will have completed, on a single command, an angular displacement equal to two step intervals.

This design of a closed-loop system will depend largely on the software programming. The external equipment suggested is probably the minimum. This design serves to demonstrate the efficiency and convenience of computer control of stepping motors. Practical applications may require more than one stepping motor but one computer will be enough and the same design may be extended to control all of them.

6. Conclusion

The control of stepping motors may be very simple as well as very sophisticated. Computer control of stepping motors as discussed in this chapter appears to be relatively easy. In fact, computer control may be considered to be the most sophisticated control. The whole system is made simple at the expense of the sophisticated design of the computer. The computer designers after many years of hard work have contributed all these conveniences. Economically, computer control in many cases is cheaper than numerical device control of stepping motors. The open-loop and closed-loop controls discussed in this chapter demonstrate the way that stepping motors may be controlled. Further, the efficiency of the motors as stepping devices may be maximized by closing the loop. Optimal computer control has not yet been attempted but in the near future by following the closed-loop control technique it may become true.

CHAPTER FIVE

CONCLUSIONS

A basic introduction to the definition, characteristics,
types and general operation principles was given in chapter one.
From the study of permanent-magnet stepping motors, a unique
mathematical model has been derived from the existing salient-pole
synchronous machine model. The difficulties involved in applying
the linear models can be eliminated as illustrated in chapter
two, and only simple tests are required. The nonlinear models are
simplified in chapter three by using square wave constant current
sources. Though only the Runge Kutta iteration method of analysing
the nonlinear models has been used, many other methods may be
applied. It was shown that the smooth air-gap model, irrespective of
its possible accuracy, fails to realize the actual motor.

Computer control of stepping motors in both open-loop
and closed-loop schemes has been discussed. The open-loop control
has been tested and satisfactory results were obtained. A closed-
loop control system has been designed using a minimum number of
peripheral devices. As is seen from chapter four, stepping
motor control is simplified by using the facilities provided by
a digital computer.

The manufacture of stepping motors tends to be concentrated
on the production of high torque and high stepping rate motors.

Optimal design of stepping motors through parameter optimization may up-grade the performance of stepping motors. The open-loop stability of stepping motors will enable the stepping motors to replace the servo motors in many applications. Future applications of stepping motors in remote control and communication will increase rapidly. Optimal computer control of stepping motors using Bang-Bang control may be developed.

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APPENDIX A

Runge Kutta Formulas for Solving Simultaneous First Order

Differential Equations

1. The Formulas

Let the first order equations be

$$\frac{dy}{dt} = f_1(t,y,p)$$

and

$$\frac{dp}{dt} = f_2(t,y,p)$$

with initial conditions $y(t_o) = y_o$, and $p(t_o) = p_o$. The Runge Kutta
formulas are:

$$k_1 = hf_1(t_n , y_n , p_n)$$

$$\ell_1 = hf_2(t_n , y_n , p_n)$$

$$k_2 = hf_1(t_n + \tfrac{1}{2}h, \quad y_n + \tfrac{1}{2}k, \quad p_n + \tfrac{1}{2}\ell_1)$$

$$\ell_2 = hf(t + \tfrac{1}{2}h, \quad y + \tfrac{1}{2}k, \quad p + \tfrac{1}{2}\ell_1)$$

$$k_3 = hf_1(t_n + \tfrac{1}{2}h, \quad y_n + \tfrac{1}{2}k_2, \quad p_n + \tfrac{1}{2}\ell_2)$$

$$\ell_3 = hf_2(t_n + \tfrac{1}{2}h, \quad y_n + \tfrac{1}{2}k_2, \quad p_n + \tfrac{1}{2}\ell_2)$$

$$k_4 = hf_1(t_n + h, \quad y_n + k_3, \quad p_n + \ell_3)$$

$$\ell_4 = hf_2(t_n + h, \quad y_n + k_3, \quad p_n + \ell_3)$$

$$y_{n+1} = y_n + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

$$p_{n+1} = p_n + (\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4)/6$$

where h = incremental constant.

2. Example

Consider equation 3-5

$$\frac{d^2\Theta}{dt^2} + \frac{B}{J}\frac{d\Theta}{dt} + \frac{K_1 I}{J}(\sin(N_r\Theta) - \cos(N_r\Theta)) = 0$$

with $p = d\Theta/dt$, therefore $dp/dt = d^2\Theta/dt^2$, or

$$\frac{dp}{dt} = f_2(t, \Theta, p)$$

$$= -\frac{K_1 I}{J}(\sin(N_r\Theta) - \cos(N_r\Theta)) - Bp/J$$

and $\quad d\Theta/dt = f_1(t, \Theta, p) = p.$

For the first iteration, with initial condition P_o and Θ_o, the constants may be calculated as follows:

$$k_1 = hP_o$$

$$\ell_1 = -\frac{K_1 I}{J}h(\sin(N_r\Theta) - \cos(N_r\Theta)) - BP_o/J$$

$$k_2 = h(P_o + \tfrac{1}{2}\ell_1)$$

$$\ell_2 = \frac{K_1 I}{J}h\left[\sin N_r(\Theta_o + \tfrac{1}{2}k_1) - \cos N_r(\Theta_o + \tfrac{1}{2}k_1)\right]$$

$$\qquad - h\frac{B}{J}(P + \tfrac{1}{2}\ell_1)$$

$$k_3 = h(P_o + \tfrac{1}{2}\ell_2)$$

$$\ell_3 = -\frac{K_1 I}{J} h \left[\sin N_r(\Theta_0 + \tfrac{1}{2}k_2) - \cos N_r(\Theta_0 + \tfrac{1}{2}k_2) \right]$$

$$- h\frac{B}{J}(P_0 + \tfrac{1}{2}\ell_2)$$

$$k_4 = h(P_0 + \ell_3)$$

$$\ell_4 = -\frac{K_1 I}{J} h \left[\sin N_r(\Theta_0 + k_3) - \cos N_r(\Theta_0 + k_3) \right]$$

$$- h\frac{B}{J}(P_0 + \ell_3)$$

$$p_1 = P_0 + (\ell_1 + 2\ell_2 + 2\ell_3 + \ell_4)$$

$$\Theta_1 = \Theta_0 + (k_1 + 2k_2 + 2k_3 + k_4)$$

The second iteration is calculated by substituting P_0 by p_1 and Θ_0 by Θ_1 into the equations.

APPENDIX B

Locked-rotor Test of PM Stepping Motors

The inductance of the stator windings of a permanent magnet stepping motor is recommended to be measured using the following procedure.

The shaft of the motor is to be coupled to a gear-box which may give a step-up ratio of about fifty (Figure B-1). The shaft is further locked in position by using a clamp. The self-inductances of the windings are measured using an inductance meter. The shaft is then rotated through a very small angle through the disc which is attached to the gear-box. The inductances of the windings with the rotor locked in this new position are again measured. Similar procedures are repeated. A curved may be plotted, and the maximum and minimum values of the inductances are then recorded.

Figure B-1: Schematic Representation for the Inductance Measurements of a PM Stepping Motor
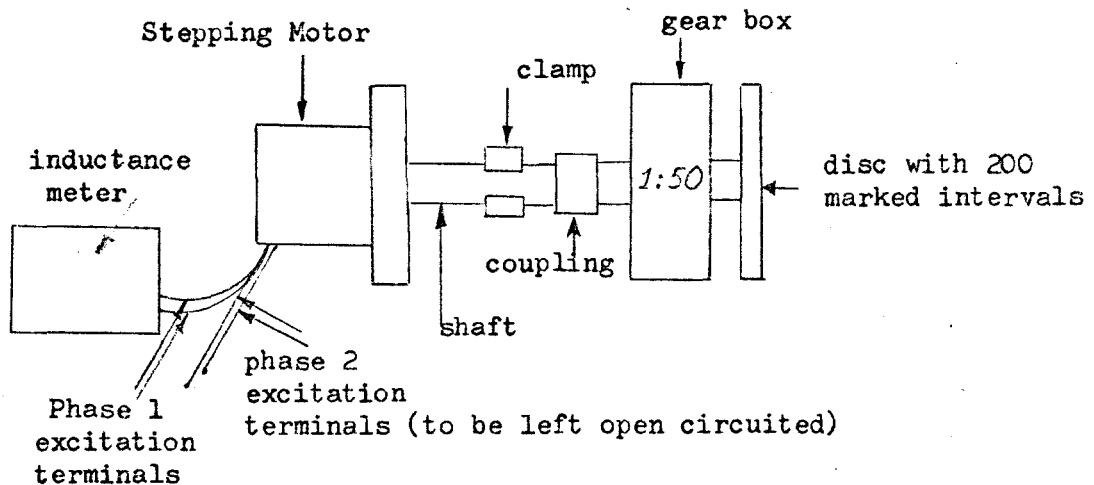
A permanent-magnet stepping motor had been tested using this
method and the experimental values are shown in Figure B-2. The
inductance meter used is able to give four figure readings with an
accuracy up to one-tenth of a milli-henry. The dotted line in the
figure is the interpreted curve . The estimated values for L_{max} and
L_{min} are 98.6 and 96.5 milli-henries respectively. Therefore L_1 =
(98.6+96.5)/2 = 97.6 milli-henries and L_2 =(98.6 - 96.5)/2 = 1.05
milli-henries.

Estimated L_{max} = 98.6 milli-henries

O indicates measured
values

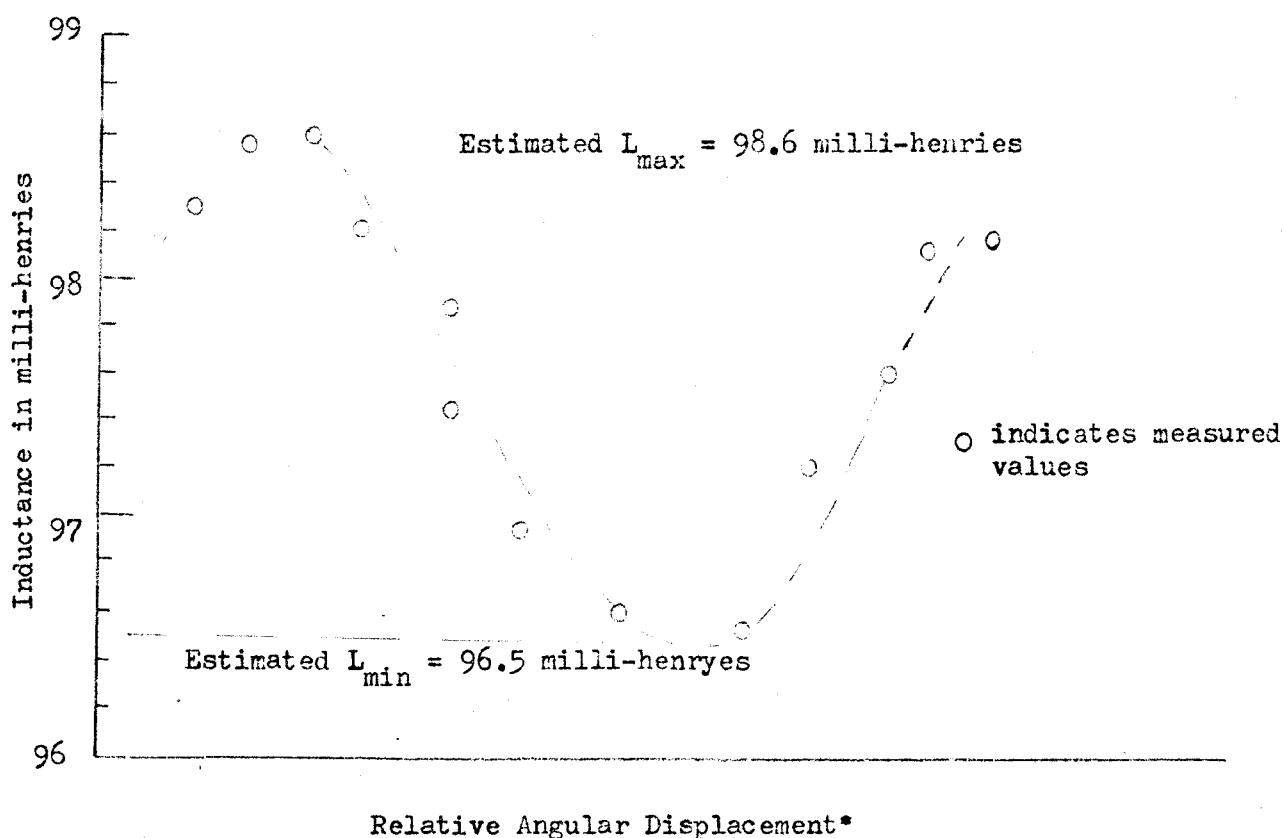Estimated L_{min} = 96.5 milli-henryes

Relative Angular Displacement*

Figure B-2 : Inductance Measurements

* The actual angles were not measured during the experiment but
the rotor was rotated through fixed intervals until maximum and
minimum values had occurred.

APPENDIX C

A Program for the Analysis of PM Stepping Motors

The numerical method used in this program is mainly the Runge Kutta method. Constant square wave current sources are assumed. The program will provide three plots : namely 1. Phase-plane plot, 2. Step-by-step plot, and 3. Torque displacement plot. The definitions of the variables in addition to those given in the program are: TL2 = L_2 in henry, MAGI =I = magnitude of current source in ampere, W = frequency of square wave current source, BJ = J, in newton-meter sec^2/rad, DAMP = B, in newton-meter sec/rad, and EMF = K_1 in volt-sec/rad. Some of the definitions of the plotter section of the program are given in the following diagram.
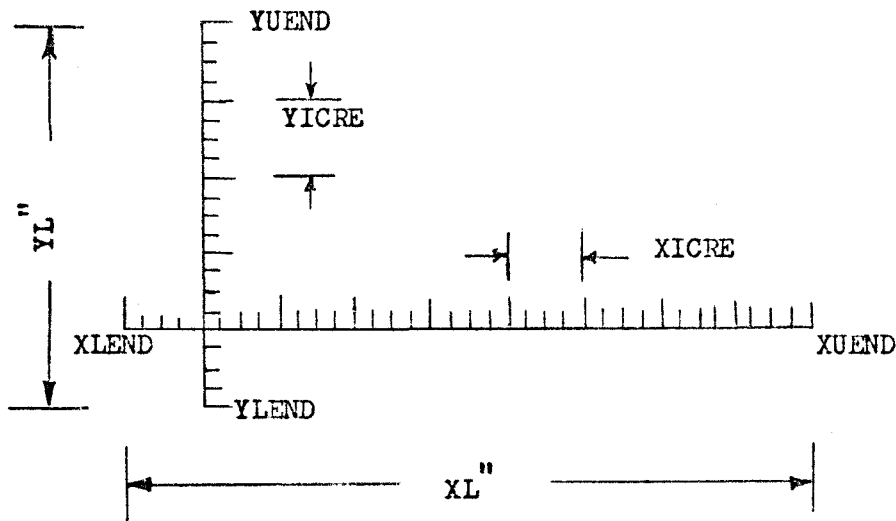
```
C*****USE RUNGEKUTTA METHOD TO PLOT PHASE TRAJECTORY
C*****SQUARE WAVE, CONSTANT CURRENT SOURCER IS ASSUMED
      REAL MAGI
      DIMENSION A100(2), B100(2)
      DIMENSION Y(1000), P(1000),TT(1000),TOR(1000)
C*****READ IN THE INITIAL CONDITIONS PNO.YNO WHICH CORRESPOND  TO
C      THE INITIAL ROTOR POSITION AND ITS ANGULAR VELOCITY, AND
C      THE INCREMENTAL TIME, UPPER LIMIT TIME
      READ (5,1) PNO,YNO,DT,TL,TL2,MAGI
    1 FORMAT(6F12.7)
      READ (5,2) W,BJ,DAMP,EMF,NR
    2 FORMAT (4F15.9,I3)
      A1=DAMP/BJ
      A2 = 4.0*NR*TL2/BJ
      A3 =EMF/BJ
      W=W*6.28318
      T=0.0
      L=1
C*****CALCULATE THE 1ST CONSTANT OF R.K. FORMULA
    3 CK1=PNO*DT
      CALL WONG2(W,T,MAGI,AI)
      CALL WONG3(W,T,MAGI,BI)
C*****CALCULATE THE 2ND CONSTANT
      CL1=(-A1*PNO+A2*BI*AI*COS(NR*2.*YNO))*DT
    1 +A3*(BI*COS( NR*YNO)-AI*SIN( NR*YNO))*DT
C*****CALCULATE KTHE 3RD CONSTANT
      G=T+DT/2.
      CK2=(PNO+CL1/2.)*DT
      CALL WONG2(W,G,MAGI,AL)
      CALL WONG3(W,G,MAGI,BL)
C*****FOURTH CONSTANT
      CL2=(-A1*(PNO+CL1/2.)+A2*BL*AL*COS(NR*2.*(YNO+CL1/2.)))*DT
    1 +A3*(BL*COS( NR*(YNO+CK1/2.))-AL*SIN( NR*(YNO+CK1/2.)))*DT
      TOR1=4.*NR*TL2*AI*BI*COS(2.*NR*YNO)
      TOR2=EMF*(BI*COS(NR*YNO)-AI*SIN(NR*YNO))
      TOR(L)=TOR1+TOR2
C*****FIFTH CONSTANT
      CK3=(PNO+CL2/2.)*DT
C*****SIXTH CONSTNAT
      CL3=(-A1*(PNO+CL2/2.)+A2*BL*AL*COS(2.*NR*(YNO+CK2/2.)))*DT
    1 +A3*(BL*COS( NR*(YNO+CK2/2.))-AL*SIN( NR*(YNO+CK2/2.)))*DT
C*****SEVENTH CONSTANT
      CK4=DT*(PNO+CL3)
      G1=T+DT
      CALL WONG2(W,G1,MAGI,AK)
      CALL WONG3(W,G1,MAGI,BK)
C*****EIGHTH CONSTANT
      CL4=(-A1*(PNO+CL3)+A2*BK*AK*COS(NR*2.*(YNO+CK3)))*DT
    1 +A3*(BK*COS( NR*(YNO+CK3))-AK*SIN( NR*(YNO+CK3)))*DT
      Y(L)=YNO
      P(L)=PNO
      TT(L)=T
      YN1=YNO+(CK1+2.*CK2+2.*CK3+CK4)/6.
      PN1=PNO+(CL1+CL2*2.+CL3*2.+CL4)/6.
      PNO=PN1
```

```
      YNO=YN1
      L=L+1
      T=T+DT
      IF (T-TL) 3,3,4
    4 NEND = 1
      KK=L-1
   10 READ (5,11)    XMIN,XMAX,YMIN,YMAX,V,R,XL,YL
   11 FORMAT (8F8.4)
      XSCALE =(XMAX-XMIN)/XL
      YSCALE = (YMAX-YMIN)/YL
      CALL PLTIN (XSCALE,YSCALE,V,R,XMIN,XMAX,YMIN,YMAX)
      READ (5,12)    XUEND,XLEND,YUEND,YLEND,XICRE,YICRE
   12 FORMAT (6F10.5)
C  DRAW THE X-AXIS
      A100(1)=XLEND
      A100(2)=XUEND
      B100(1)=0.0
      B100(2)=0.0
      CALL PLTMPL (A100,B100,2)
C  DRAW THE Y - AXIS
      A100(1)=0.0
      A100(2)=0.0
      B100(1)=YLEND
      B100(2)=YUEND
      CALL PLTMPL(A100,B100,2)
C       DRAW THE SCALES OF X-AXIS
      XLABEL = -.1*YSCALE
      DD1 = XLEND
      READ (5,13) ICRE,LINEX,JCRE,LINEY
   13 FORMAT (4I3)
      DO 14 I=1,ICRE
      A100(1)=DD1
      A100(2)=DD1
      B100(1)=0.0
      B100(2) =.12*YSCALE
      CALL PLTMPL (A100,B100,2)
      DD1 = DD1 + XICRE/5.
   14 CONTINUE
      DO 15 I=1,LINEX
      READ (5,16) MVA
   16 FORMAT (A6)
      CALL PLTLET (6,.06,0.0,XLEND,XLABEL,MVA)
      A100(1) = XLEND
      A100(2) = XLEND
      B100(1)=0.0
      B100(2) =.25*YSCALE
      CALL PLTMPL (A100,B100,2)
      XLEND = XLEND + XICRE
   15 CONTINUE
C       DRAW THE SCALES OF Y-AXIS
   17 YLABEL = -.4*XSCALE
      GD1 = YLEND
      DO 19 I=1,JCRE
      A100(1) =0.0
      A100(2) = .12*XSCALE
```

```
      B100(1) =GD1
      B100(2) =GD1
      CALL PLTMPL (A100,B100,2)
      GD1 = GD1 + YICRE/5.
   19 CONTINUE
      DO 20 I=1,LINEY
      READ (5,21) KVA
   21 FORMAT (A6)
      CALL PLTLET (6,.06,0.0,YLABEL,YLEND,KVA)
      A100(1) = 0.0
      A100(2) = .25*XSCALE
      B100(1)= YLEND
      B100(2)= YLEND
      CALL PLTMPL (A100, B100, 2)
      YLEND = YLEND + YICRE
   20 CONTINUE
   22 IF (NEND - 2) 23,24,25
   23 CALL PLTMPL (Y,P,KK)
      CALL PLOT (15.,0.0,-3)
      NEND = NEND + 1
      GO TO 10
   24 CALL PLTMPL (TT,Y,KK)
      CALL PLOT (15.,0.0,-3)
      NEND = NEND + 1
      GO TO 10
   25 CALL PLTMPL (Y,TOR,KK)
      CALL PLOT (15.,0.0,-3)
      CALL PLOT (0.0,0.0,999)
      STOP
      END
      SUBROUTINE WONG2(D,C,B,CUR )
C     THIS SUBROUTINE CALCULATES THE SQUARE WAVE (SINE) CURRENT
C      AT A GIVEN TIME OF A SPECIFIED FREQUENCY USING FOURIER SERIES
C      D=FREQUENCY IN RAD./SEC
C      C=TIME IN SECOND
C      B =ABSOLUTE MAX.VALUE OF THE CURRENT
C      CUR=VALUE OF CURRENT AT THAT SPECIFIED TIME
      AI1=SIN(D*C)+SIN(3.*D*C)/3.
      AI2=SIN(D*C*5.)/5.+ SIN(D*C*7.)/7.
      AI3=SIN(9.*D*C)/9.+SIN(D*C*11.)/11.
      AI4=SIN(13.*D*C)/13.+SIN(15.*D*C)/15.
      AI5=SIN(17.*D*C)/17.+SIN(19.*D*C)/19.
      DUM=4.0/3.1415926
      CUR=- B*DUM*(AI1+AI2+AI3+AI4+AI5)
      RETURN
      END
      SUBROUTINE WONG3(W1,T1,BI, CURR)
C      THIS SUBROUTINE CALCULATES THE SQUARE WAVE (COS) CURRENT
C      W1=FREQUENCY IN RAD/SEC
C      T1=TIME  IN SEC
C      BI =MAXABSOLUTE CURRENT MAGNITUDE
      BI1=COS(W1*T1)-COS(3.*W1*T1)/3.
      BI2=COS(5.*W1*T1)/5.-COS(7.*W1*T1)/7.
      BI3=COS(9.*W1*T1)/9.-COS(11.*W1*T1)/11.
```

```
BI4=COS(13=*W1*T1)/13.-COS(15.*W1*T1)/15.
BI5=COS(17.*W1*T1)/17.-COS(19.*W1*T1)/19.
DUMM=4.0/3.1415926
CURR=BI*DUMM*(BI1+BI2+BI3+BI4+BI5)
RETURN
END
```

```
BI4=COS(13=*W1*T1)/13.-COS(15.*W1*T1)/15.
BI5=COS(17.*W1*T1)/17.-COS(19.*W1*T1)/19.
DUMM=4.0/3.1415926
CURR=BI*DUMM*(BI1+BI2+BI3+BI4+BI5)
```
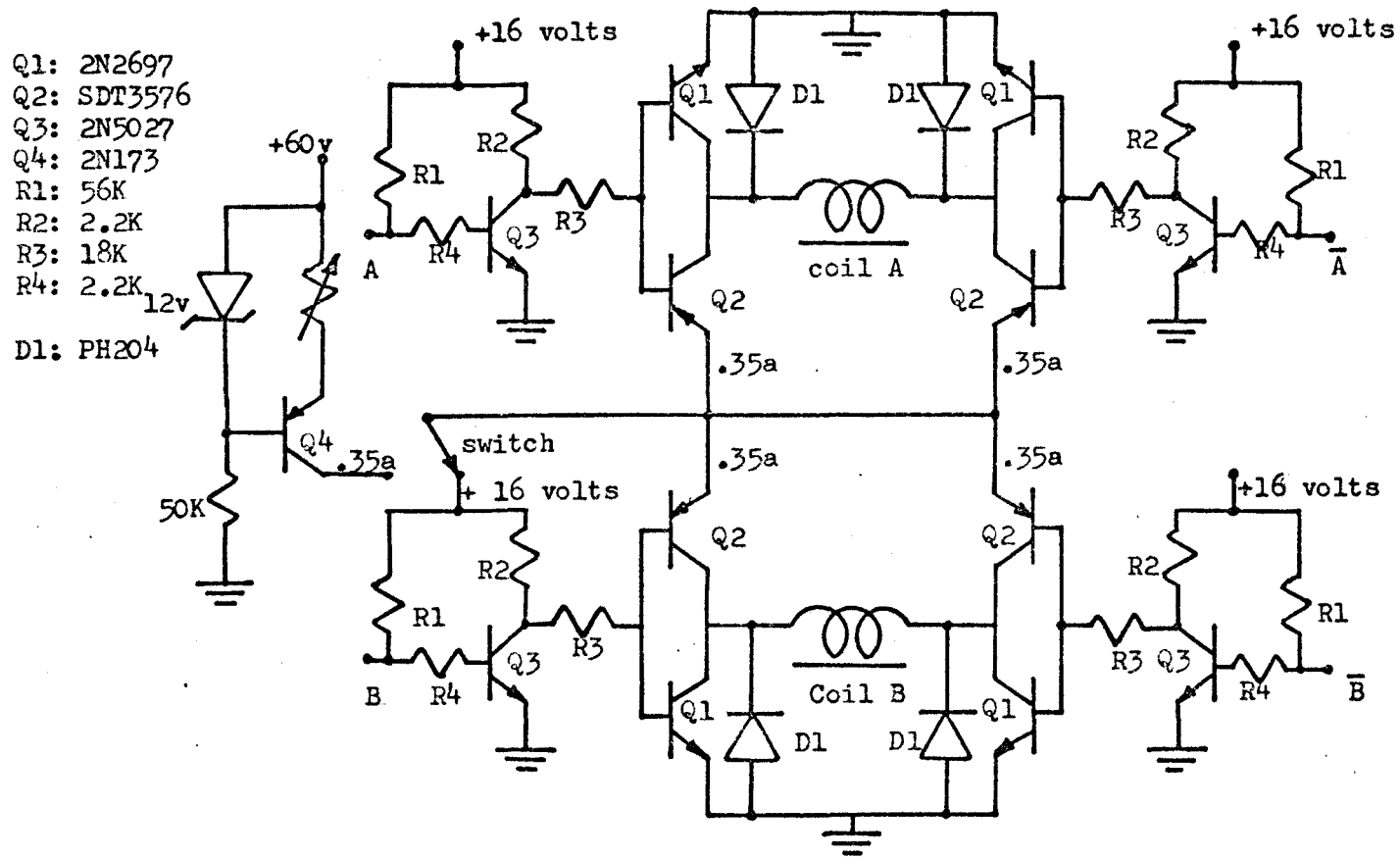
APPENDIX D
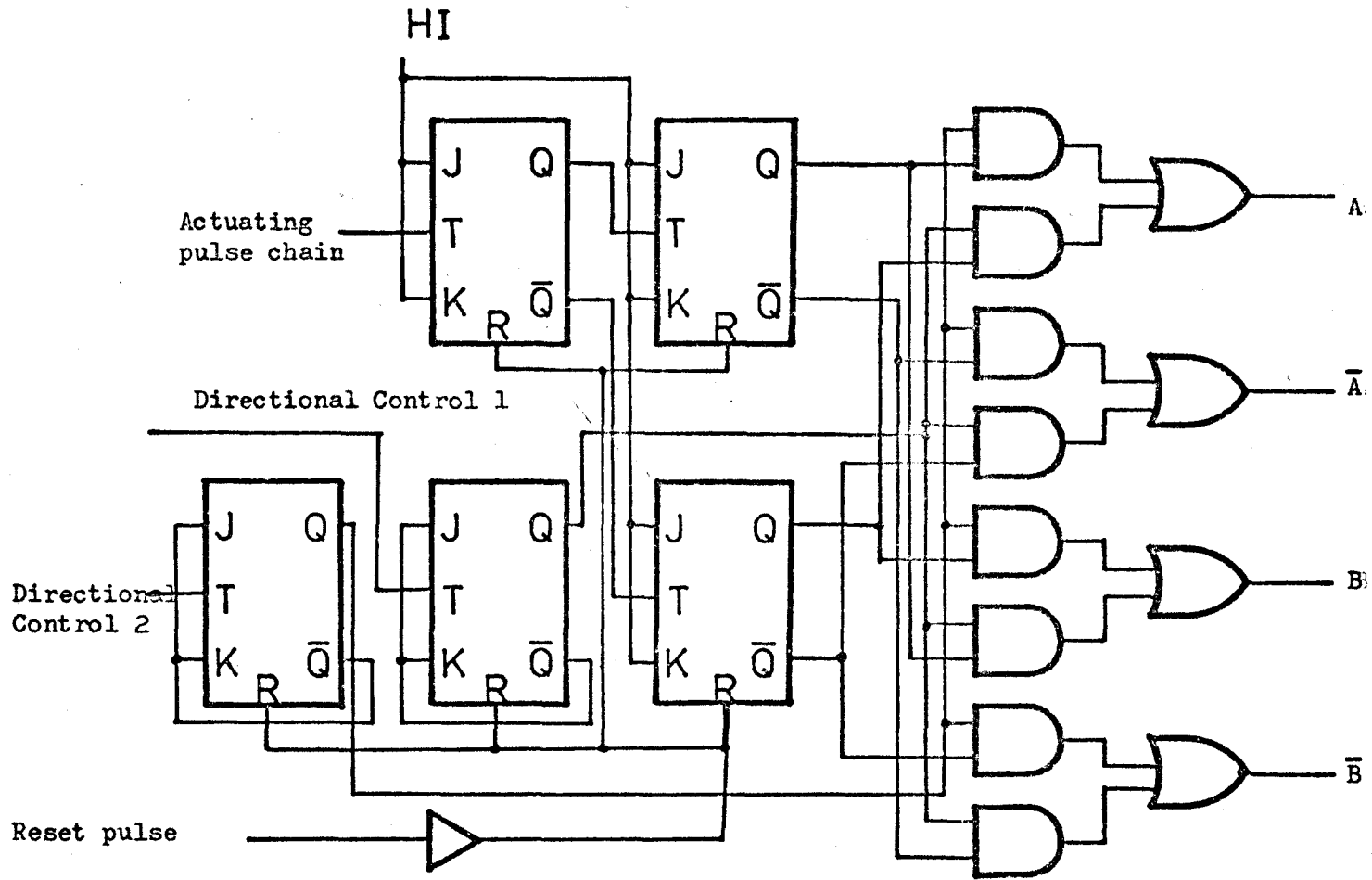
Figure D-1:   DRIVER CIRCUIT

APPENDIX E

Figure E-1: 200 Steps Per Revolution Digital Logic Circuit

APPENDIX F

Figure F-1 :   TRANSLATOR CIRCUIT
             1: Directional Control 1; 2: Directional Control 2; 3: 100 Steps/rev Enabler;
             4: 200 Steps/rev Enabler; 5: Complete Circuit Reset; 6: 200 Steps/rev Disabler;
             7: Actuating pulse chain