TURBULENT DRAG REDUCTION
IN TUBES WITH POLYMER SOLUTIONS

TURBULENT DRAG REDUCTION IN TUBES WITH POLYMER SOLUTIONS

by

S. K. VERMA

A Thesis
Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Engineering

McMaster University
April 1970

MASTER OF ENGINEERING (1970)
(Mechanical Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario.

TITLE: Turbulent Drag Reduction in Tubes with Polymer Solutions

AUTHOR: S. K. Verma, B. Tech. (Indian Inst. of Tech., Kharagpur, India)

SUPERVISOR: Professor J.H.T. Wade

NUMBER OF PAGES: vi, 78

SCOPE AND CONTENTS:

This thesis is a study of drag reduction with polymer solutions in tubes using a simple rheometer. The drag reduction was studied in five different small diameter tubes at Reynold's number varying from 1000 to 25000.

A polyacrylamide, MRL-402, manufactured by Stein Hall Ltd. was chosen for the above study because of its good drag reducing ability and resistance to degradation. The concentrations of polymer solutions used for the experiment ranged from 1 to 200 wppm.

An attempt has been made to correlate the drag reduction data so that drag can be predicted from a single curve. The correlations are given by equations $\beta = \beta(s'T)$ and $\beta = \beta'(s'/s'_{0.50})$ following the work of Astarita (2).

ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to his supervisor, Professor J.H.T. Wade, who provided constant assistance throughout the course of this work.

Thanks are also extended to Dr. B. Latto, from whom much valuable advice and information has been frequently sought.

Technical help from Mr. R. Brown and Mr. J. Crookes is gratefully acknowledged.

The research was financially supported by National Research Council of Canada under grant number A1585.

TABLE OF CONTENTS

NOMENCLATURE

SYMBOL	DESCRIPTION	UNITS
D	diameter of the tube	Ft
L	length of the tube	Ft
R	Reynolds number $(\frac{\rho VD}{\mu})$	
V	average velocity in the tube	Ft/sec
Δp	pressure difference across the tube	lbf/ft^2
τ_o	wall shear stress	lbf/ft^2
τ	shear stress at any point across the cross section	lbf/ft^2
u	axial velocity at any point across the cross section	Ft/sec
y	radial distance from the wall	Ft
g_c	gravitational constant	$\frac{lbm\ Ft}{lbf\ sec^2}$
f	friction factor defined by equation 4.2	
ρ	density	$lb\ /ft^3$
ν	kinematic viscosity	ft^2/sec
f_{pv}	purely viscous friction factor	
f_ℓ	laminar friction factor	
F	$\frac{f_{pv}-f}{f_{pv}-f_\ell}$	
N_{Re}	generalized Reynolds number defined by 2.3	
n	flow behaviour index	
u_*	friction velocity	Ft/sec
u_{*cr}	critical friction velocity at onset of drag reduction	Ft/sec

N_{Deb}	$\frac{V}{D} R^{0.75} \cdot T$	
N'_{Deb}	$\frac{V}{D} R^{x} \cdot T$	
T	constant relaxation time of a solution	sec
S	$\frac{V}{D} R^{0.75}$	1/sec
s'	$\frac{V}{D} R^{x}$	1/sec
t	dimensionless time ratio $\frac{\theta u_*^2}{\nu}$	
W_n^*	characteristic wave number $\frac{u_*}{\nu}$	1/ft
ℓ	development length	ft
θ	non constant relaxation time	sec
β	friction factor ratio $\frac{f}{f_{pv}}$	
α	defined by eqn. 2.7	
ξ_ℓ	dimensionless sublayer thickness	

CHAPTER I

INTRODUCTION

It has been established by numerous experimental studies (7,11,13,15,16,22,24,29,31) that small quantities of synthetic organic polymers in solution reduce turbulent skin friction drag drastically in pipe flow. This property of high molecular weight polymer solutions could be used in the future to reduce drag in pumping oils, water and other liquids.

The concentrations of polymers in polymer solutions are so low that the dilute aqueous polymer solutions are assumed to retain the original physical properties of the solvent. The polymer solutions are still Newtonian fluids in character but contain long coiled chains of molecules which alter the flow by some undefined mechanism. There appears to be no rigorous satisfactory explanation for this flow phenomenon up till now. A widely held view, at present, is that the drag reducing solutions thicken the viscous sublayer which causes drag reduction.

This work gives an empirical correlation for pressure drop data for flow of polymer solutions in small diameter tubes , based on the Reynold number and concentrations of solutions. However, no attempt has been made to explain the actual mechanism of drag reduction by polymer solutions.

CHAPTER 2

LITERATURE SURVEY

2.1 The reduction of pressure drop caused by the introduction of a small quantity of some additive to a solvent in turbulent flow as first noticed by Toms(27)*. He observed it in 1948 when he was studying the flow of polymethyl methacrylate in monochlorobenzene through straight tubes. Subsequently this anomalous behaviour of certain solutions drew the attention of others.

During the last twelve years, a great deal of literature has been published on this subject much of which is experimental.

2.2 Purely Viscous Non-Newtonian Fluids

The behaviour of drag reducing solutions was suspected to be non-Newtonian. The first attempts were, therefore, to develop a general theory for non-Newtonian fluids.

It has been found experimentally that the relationship between shear stress and shear rate for a great many fluids may be represented by a two constant power function of the

---

*Numbers indicate references in reference list form

$$\tau = k\left(\frac{du}{dy}\right)^n \qquad 2.1$$

Dodge and Metzner (5) gave the correlation between the friction factors and a generalized Reynolds number for such fluids. The velocity in the boundary layer was assumed to be a function of two additional parameters k and n as compared to a Newtonian fluid. The final expression derived was of the form

$$\sqrt{\frac{1}{f}} = A_{in} \log\left[N_{Re}(f)^{\frac{1-n}{2}}\right] + C_n \qquad 2.2$$

in which the Reynolds number was defined by the following expression

$$N_{Re} = \frac{D^n \, V^{2-n} \, \rho}{g_c \, k' \, (8)^{n-1}} \qquad 2.3$$

where $k' = k[\frac{3n+1}{4n}]^n$ and A_{in} and C_n are experimental constants as in the case of Prandtl's equation for Newtonian fluids. These constants are dependent upon n only. The above correlation also includes the Newtonian fluids (n=1).

Equation 2.2 was verified experimentally from Dodge and Metzner's experiments with various solutions. There was, however one solution which was an exception; that of carboxymethyl cellulose (CMC). It did not fit in equation 2.2

Shaver and Merril's (21) experiments with pseudoplastic fluids showed that the behaviour of these fluids fitted the power law very well. They also observed that these fluids have poor overall mixing and thicker non turbulent boundary layers at the wall than Newtonian fluids. The drag, under the same conditions, was found to be reduced .

With the work of Dodge and Metzner the analysis of purely viscous fluids can be said to be complete.

2.3 Fluids not Conforming to Purely Viscous Behaviour

As mentioned earlier in Dodge and Metzner's experiments CMC was an exception. It did not fit to equation 2.2 but showed much greater drag reduction. The authors suggested that the fluid was not purely viscous but possessed viscoelastic properties. There are many other solutions, most commonly long chained polymer solutions which show similar behaviour. This can be observed from many experiments done in this field (7,11,13,15,16,22,24,29,31).

The following general behaviour of polymer solutions can be observed from the different experiments:

(i) a very dilute polymer solution gives a remarkable drag reduction (anywhere up to 60% and even more) and the viscosity of the solution could be considered the same as that of the solvent.

(ii) The flow is turbulent in all cases where drag reduction is observed.

(iii) The drag reduces with increase of concentration up to a certain limit. After this limit the drag starts increasing again.

(iv) For the same concentration of polymer and Reynold number of flow, drag reduction is greater for smaller diameter pipes.

(v) Drag reducing effectiveness reduces as the solution is repeatedly used. The phenomenon is known as degradation. Some polymer solutions degrade faster than others.

(vi) With very high concentrations drag increases in the laminar flow region.

Hoyt and Fabula (10) performed experiments in pipes and rotating discs to relate certain properties of polymers, in general, to their drag reducing ability. The best drag reducers were found to possess a) higher molecular weight b) higher solubility and c) linear molecular structure as compared to other drag reducers. The authors gave a table listing various drag reducers such as guar gum, polyhall 27, polyox WSR-205 etc. and the concentrations which would give the same drag reduction for a particular Reynolds number.

Sherman (22) tried to relate the molecular structure and concentration of polymers to their drag reducing ability. He showed experimentally that the extent of drag reduction can be predicted by knowing the molecular chain length to diameter ratio and the concentration of the polymer. An effective drag reducing factor defined as

$$F = \frac{\text{Mol. wt. of Polymer}}{\text{Mol.wt. of a unit} \times \frac{\text{maxm. length of the unit}}{\text{maxm.diameter of the unit}}}$$

could be used to assess the relative drag reducing capability.

Onset of Drag Reduction

Virk and Merril (28) studied the onset phenomenon. The onset point was defined as the condition where the drag reduction started and was characterized by the wave number defined as

$$W_n^* = \frac{u_*}{\nu} \qquad 2.4$$

They concluded that for a certain polymer there is one particular wave number for all concentrations whence the wave number appeared to vary a little with the tube diameter. For some cases there was no distinct wave number. In these cases the drag reduction started before the fully developed turbulent flow was realized.

2.5 Suggested Explanations of Drag Reduction

Dodge and Metzner (5) suggested viscoelastic effects as an explanation of the failure of CMC data to fit in their general equation. Most of the authors afterwards have supported this explanation. A brief review of other hypotheses is given below.

The earliest attempt to explain drag reduction was by Oldroyd who offered a wall effect hypothesis for Toms' data (27). He proposed the existence of an abnormally mobile sublayer whose thickness was comparable to molecular dimensions and which caused apparent slip at the wall. Toms later showed that this failed to explain his data. Comparison of velocity profiles in ordinary fluids with velocity profiles in drag reducing fluids is the most direct method of determining the presence of wall effects. If there were slip at the wall, the velocity profiles would have to be more blunt.

Velocity profiles have been reported by Ernst (8), Wells (30), and others. Hershey compared Shaver's Newtonian and non-Newtonian profiles at similar Reynold's numbers and found that there was no significant difference. It must, however, be remembered that pitot tube, hot wires and tracer particle velocity measurement techniques are subject to certain limitations if drag reduction and viscoelasticity occur together. Under these conditions it may not be possible to measure true local velocities in these solutions (3,25).

Shaver and Merril (21) proposed that drag reduction may be a result of the non-Newtonian viscosity gradient. Since the shear rate is maximum at the tube wall and zero at the tube centre, a turbulent vortex must encounter an ever increasing viscosity in a pseudoplastic fluid. However, this fails to explain the absence of drag reduction in highly shear thinning carbopol or the presence of drag reduction in dilute polymer solutions.

2.6 Viscoelasticity

Savins (17) explained drag reduction with viscoelastic solutions as follows.

A viscoelastic fluid can be considered as a combination of an elastic solid and a purely viscous fluid. The elastic part conserves energy while the viscous part dissipates energy whenever subjected to oscillating stresses.

Under laminar conditions the elastic energy stored in a viscoelastic fluid is independent of time. Therefore, as observed, the pressure drop in a tube is not affected by the elasticity. With the onset of turbulence there is considerable flexing, stretching and shrinking of 'lumps' of fluid. In such a flow the elastic energy is stored and released continuously. The viscous part, however, dissipates energy continuously. Comparatively the pure viscous fluid would dissipate all energy in turbulence. With increasing turbulence the motion would be more strongly influenced by elastic energy while the viscosity would contribute a second order effect. The various experimental results could be physically explained in the light of this view.

2.7 Suggested Correlations

Metzner and Park (13) correlated the normal pressure difference to the drag co-efficient ratio defined as

$$F = \frac{f_{pv} - f}{f_{pv} - f_{\ell}}$$

where f - drag co-efficient for the polymer solution

f_{pv} - drag co-efficient for a purely viscous fluid which has the same flow behaviour index as the above polymer solution

f_{ℓ} - laminar friction factor

It was deduced that at constant generalised Reynolds number and flow behaviour index the drag co-efficient is directly proportional to $\frac{P_{11}-P_{22}}{\tau_{12}}$ $\left(\frac{\text{Normal Pressure Difference}}{\text{Shear stress}}\right)$ or to the ratio of elastic to viscous stresses.

This relationship was derived from a simplified constitutive relation and the equation of motion was verified experimentally. The one fluid tested was in quite good agreement with the theory. The fluid used had high concentrations (3000-20,000 wppm) so that elastic properties could be measured.

Elata and Tirosh (7) concluded from their experiments that the drag reduction effects nearly coincided with a decrease in the universal mixing length constant. This violates the Reynold's principle of similarity. However, later Ernst (9) and Meyer (14) disproved this conclusion. Ernst in his experiments found that the major effect of the CMC solutions is a linear shift of velocity parameter in the universal law of the wall relationship. This indicated that the mixing length constant was not affected by viscoelastic properties of the solution.

Meyer (14) gave a correlation between the friction factor f and the generalised Reynold number based on the experiment of Elata and Tirosh (7), Ernst (8) and Wells (30). He defined two new elastic parameters of the viscoelastic fluid (u_{*cr} and α). The co-relation had the form

$$\frac{1}{\sqrt{f}} = (4 + \frac{\alpha}{\sqrt{2}}) \log N_{Re}\sqrt{f} - 0.394 - \frac{\alpha}{\sqrt{2}} \log \frac{\sqrt{2}\ Du_{*cr}}{\nu} \qquad 2.5$$

The mixing length was assumed to be constant. From the relation

$$\frac{u}{u_*} = A \log \frac{yu_*}{\nu} + B \qquad 2.6$$

a plot between B and u_* showed that B remained constant till a certain critical value of u_* is reached and then starts increasing. Thereafter it is represented by the relation

$$B = B_N + \alpha \log (\frac{u_*}{u_{*cr}}) \ ; \ (u_* \geq u_{*cr}) \qquad 2.7$$

It was argued that, as k remains constant, the turbulent portion of the flow is entirely unaffected by viscoelastic properties. This indicated that the laminar sublayer became less sensitive to disturbances impressed upon it from above and thus became thicker.

Ernst's (9) experiments, later, verified this correlation. His results fitted well to equation 2.7. Experiments performed by Elata, Lehrer and Kahanovitz (6) also indicated that drag reduction was accompanied by the development of an increased laminar sublayer thickness. They concluded that frictional drag reduction in turbulent flow of polymer solutions will appear when the time scale $(1/\frac{du}{dy})$ of the laminar sublayer becomes smaller than the maximum relaxation time of the polymer molecules. As in the turbulent region, the time scale is much higher than the relaxation time of the molecules, that region is not affected at all by the polymer solutions.

Patterson and Zakin (16) gave an approximate theoretical derivation for drag reduction based on the assumption that energy is conserved in the turbulent core by the viscoelastic properties of the fluid. They derived the ratio of energy dissipated by a purely viscous fluid to that by a viscoelastic fluid. The viscoelastic fluid was represented by a simple Maxwell model. On the assumption that the relation between turbulent energy dissipation and shear stress for purely viscous fluid also holds for viscoelastic fluid, the shear stress ratio was calculated from this energy dissipation ratio. The experimental results did not show complete agreement with the theoretical model. This was attributed to the various approximations made in the theory.

Seyer and Metzner (20) pointed out that the flow of polymer solutions depended upon another dimensionless group in addition to the Reynolds number. This dimensionless group or the Deborah number was defined as the product of relaxation time (θ) of the fluid and a characteristic frequency of the turbulent field S.

$$N_{Deb} = S\theta \qquad 2.8$$

θ was measured experimentally. It is not a constant but varies with shear rate (19). Hinze (10) gave an estimation for S by dimensional analysis

$$S = \frac{V}{D} (N_{Re})^{3/4} \qquad 2.9$$

Drag reduction was defined as

$$F = \frac{f_{pv} - f}{f_{pv} - f_{\ell}} \qquad 2.10$$

and therefore $F = \phi(N_{Re}, N_{Deb})$ 2.11

for a particular solution.

In the experiment performed N_{Re} was kept constant and the variation of F with N_{Deb} was plotted. A set of eight curves were obtained for eight different values of N_{Re}. As θ

varied with shear rate it had to be found experimentally for a certain range of shear rate.

Recently, in another paper, Seyer and Metzner (19) included the relaxation time in their equations in a different and more practical way.

Assuming that the viscous sublayer thickens and the mixing length remained constant when polymer solutions were used, the logarithmic law was written as

$$\frac{u}{u_*} = A \ln \frac{yu_*}{\nu} + B(t) - C_3 \qquad 2.12$$

Here t was defined as a dimensionless ratio

$$t = \frac{\theta (u_*)^2}{\nu} \qquad 2.13$$

while the friction factor was defined as

$$f = \frac{\tau_o}{\frac{1}{2} \rho V^2} = 2 (\frac{u_*}{V})^2 \qquad 2.14$$

and the average velocity was defined as

$$V = 2 \int_0^1 uxdx \quad (x = \frac{y}{R}) \qquad 2.15$$

Equation 2.15 was integrated with the aid of equation 2.12 and the final equation obtained was

$$\sqrt{\frac{2}{f}} = A(1 - \xi_\ell)^2 \ln N_{Re}\sqrt{f} + (1 - \xi_\ell)^2 [B(t) - A\ln 2\sqrt{2}] - G \qquad 2.16$$

The function B(t), which depends upon t and hence the relaxation time of the fluid, was obtained experimentally. A plot between B(t) and t showed that there is a single curve for all concentrations of a particular fluid. The friction factor predicted by equation 2.16 agreed very well with the experimental values. However, to obtain B(t), concentrated solutions were used. Also an appropriate value of B(t) had to be selected for a particular shear rate.

Astarita, Greco and Nicodemo (2) carried this work further and attempted to avoid the relaxation time of the fluid which is difficult to determine experimentally for dilute solutions.

The friction factor was defined as

$$\beta = \frac{f}{f_{pv}} \qquad 2.17$$

It was assumed that β depends only on the Deborah number. Thus

$$\beta = \beta\ (ST) \qquad 2.18$$

where T is the relaxation time and is constant for a particular solution. According to the above equation β should be a unique function of S for a particular solution. Their experiment with three different tube diameters showed that the results agreed well with equation 2.18.

Furthermore, it was deduced that

$$\beta = \beta'\ (\frac{S}{S_{0.5}}) \qquad 2.19$$

where $S_{0.5}$ is the value of S at 50 percent drag reduction. According to this relation β and $S/S_{0.5}$ should co-relate with a single smooth curve for different tube diameters and different concentrations. This was verified experimentally.

2.8 Some Studies on Rough Pipes

There are two important aspects of flow of polymer solutions in rough pipes.

a) As pointed out by Brandt, McDonald and Boyle (4) many surfaces in engineering practice are not hydrodynamically smooth. The requirements on smoothness become more stringent as Reynold's number increases, and as a result, the designer of a large scale system can seldom afford to provide a smooth surface.

b) The concept of a thickened viscous sublayer has been accepted by many investigators as an explanation for drag reduction by polymer solutions in smooth pipes. Rough pipes can be used to verify this statement. There should be little or no drag reduction with polymer solutions if the size of roughness elements is comparable to the thickness of the viscous sublayer.(35)

White (32) found that guar gum had little significant effect on the pressure drop along the rough pipe up to a concentration of 480 wppm. The pipe selected had a threaded bore of 5/8 inch Whitworth thread form so that the roughness height projected well beyond the sublayer thickness. On the other hand fresh polyox solutions showed drag reductions for all except the very lowest concentrations. The drag reduction was, however, much reduced when compared to that in a smooth pipe. Again, an aged Polyox solution showed no drag reduction. It was concluded that only those additives which produced measurable elastic effects were successful in reducing drag in rough pipes by partially suppressing the core turbulence.

Brandt, McDonald and Boyle (4) used CMC solutions in their experiment with pipes roughened with silicon carbide particles. They found that CMC was more effective in reducing turbulent skin friction on smooth surfaces than on rough surfaces. Also CMC had little or no effect on flow in the fully rough regime. (The régime where the rough particles protruded out of the sublayer). However, the onset of this régime was considerably delayed.

Spangler (26) used P-295 (a polyacrylamide) in rough pipes and found similar effects as above. He also found that the onset of roughness was the same for Newtonian and drag reducing fluids, if the onset of roughness preceded the onset of drag reduction. Also, the drag reduction onset was the same in smooth and rough pipes if flow was not in the fully rough régime. He defined a roughness function for a drag reducing fluid which could be used to predict friction factors in rough pipes. The roughness function was defined to account for the deviation between the law of the wall velocity profiles in smooth and rough pipes. The equations 2.5 and 2.7 given by Meyer were used for drag reducing fluids and the roughness function was determined experimentally. The definition of roughness was similar to that of Newtonian fluids that accounts for deviation in the law of the wall for smooth and rough pipes in Newtonian flow.

The studies in rough pipes supports the view of a thickened viscous sublayer as an explanation for drag reduction in smooth pipes with polymer solutions.

A review on reduction of skin friction drag was made by Lumley (12). It was a survey of the various principles which have been suggested to reduce the skin friction drag.

Despite intensive efforts made by many researchers, an overall explanation for the drag reduction mechanism is still difficult over the completed range of concentration. Even if the study is restricted to very dilute polymer solutions in which the viscosity is but little affected by the additive, the problem is complicated by the fact that various additives can behave differently in different systems. Certainly, the method of mixing, shear rate, the age of the solution can all play a part in the effectiveness of an additive.

CHAPTER 3

EXPERIMENTAL APPARATUS AND PROCEDURES

3.1 The apparatus is essentially a simple rheometer, the design of which was obtained by private communication between Dr. A. Millward, of the Department of Aeronautics and Astronautics, University of Southampton, and Dr. J.H.T. Wade of McMaster University.

The experimental apparatus used is shown in figures 1 and 2. The solution was passed through the tube from a calibrated glass reservoir of capacity 500 c.c. The calibration on the reservoir served as a measure of volume flow through the tube. The tube was fixed vertically below the reservoir. A valve was fitted at the bottom of the tube for starting or stopping the flow. Five tubes (Table 3.1) of different diameters were used in the experiment.

Flow rate through the tube was varied by means of air pressure on the top of the solution in the reservoir. A compressed air cylinder was connected to the top of the glass reservoir through an air filter and a pressure regulator. The air pressure in the reservoir could be maintained to any desired value by the regulator. A pressure gauge was used to measure the air pressure. The entire

TABLE 3.1

DESCRIPTION OF TUBES

No.	Mean Internal Diameter (inches)	Material	Length of Tube (inches)	Remarks
1.	.0264	Stainless Steel	15.0	Hypodermic Tubing
2.	.0410	Stainless Steel	15.0	Hypodermic Tubing
3.	.0628	Stainless Steel	15.0	Hypodermic Tubing
4.	.0372	Glass	15.0	Capillary Tube
5.	.0799	Glass	15.0	Capillary Tube

apparatus was mounted on a wooden frame.

The apparatus was designed to cover both laminar and turbulent régimes by simply varying the air pressure and the size of the tube. Also it is completely portable when supplied with an air cylinder. Use of air avoided any degradation of polymer solution before it reached the tube. (Comparatively, when a pump is used, the polymer solution degrades due to shear force exerted by the pump.)

3.2 Calibration of Apparatus

The tube diameters were determined by filling the experimental tubes with mercury and then weighing the mercury

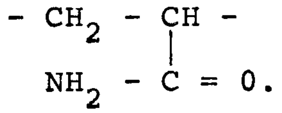
(Appendix V). As the mercury was visible in the glass tubes, it could be made sure that there was no air gap in the whole length of the tube. The absence of an air gap in the steel tubes was insured by taking repeated measurements of the weight of mercury. The measurements which showed a maximum constant weight were finally chosen to calculate the diameter. This method of measurement gave a mean value of the internal diameter along the whole length of the tube.

The pressure gauge was calibrated using a precision dial manometer (Appendix I). The calibration was found to be linear with the maximum percentage gauge error at the low pressure end of the scale.

The calibrated reservoir was checked before starting the experiment. 200 c.c. of water as indicated in reservoir, was weighed and its volume was calculated. This was repeated five times using different volumes. There was no significant error in the calibration and hence the reservoir did not need any further calibration.

3.3 Preparation of Polymer Solution

The polymer was obtained from Stein Hall Ltd. It was a polyacrylamide (MRL-402) in general. The general chemical formula for a monomer of polyacrylamide is as follows:



$$\begin{matrix} -\mathrm{CH_2} - & \mathrm{CH} - \\ & | \\ \mathrm{NH_2} - & \mathrm{C} = \mathrm{0}. \end{matrix}$$

MRL-402 is in a fine granular form and is hygroscopic. Therefore care was taken that the sample was not unduly exposed to moisture from the environment.

The solution was prepared in water in a zinc coated can which had a capacity of 140 lbs. A polyethylene bag was used to cover the inside of the can. Every time a new solution was prepared the bag was replaced. A sample of the polymer was baked in a refractory furnace at a temperature of 110°C for two to three hours till the moisture was driven out. The moisture content was found to be 4.5 percent. The baked polymer, however, was not used to prepare the solution (Appendix II) but the percentage error was taken into account during the preparation of the aqueous solution.

A solution of concentration 200 wppm (parts per million by weight) was prepared in the container. This was done by weighing an appropriate quantity of polymer in a crucible and adding it gradually to the water in the can. (The polymer should not be added to water all at once because then it does not disperse and becomes hard to dissolve). The residue left in the crucible was washed with water into the bulk of the solution. The mixture was then stirred with a pneumatic stirrer placed on the top of the can. All other concentrations (1,2,5,10,20,50 and 100 wppm) were prepared by diluting the 200 ppm solution with water.

3.4 Degradation of Polymer Solutions

The chains of polymer molecules are known to break up in solution under certain conditions. This results in a reduction of the molecular weight of the polymer. The phenomenon is known as degradation. Degradation causes a reduction in their effectiveness as drag reducers. The following main parameters cause degradation.

a) Exposure to sunlight
b) Shear forces
c) Temperature
d) Time .

MRL-402 (a polyacrylamide) was selected because of its good drag reducing ability and greater resistance to degradation as compared to other polymers such as polyox. Polyox degrades much faster than MRL-402 due to factors (a), (b) and (d). In general the first factor is not very important unless the solution is exposed to sunlight over long periods of time.

The only shear force that was exerted on the solutions prepared for the experiment was that by the stirrer which could not be avoided. To obtain repeatability all solutions prepared were stirred for approximately the same time.

As an indication of how temperature may degrade the polymer solutions, a sample set of solutions was heated to various temperatures between 75°F to 100°F and passed through the .0628" diameter tube. The results are shown as a plot between Reynolds number and friction factor for 81, 90 and 95°F (Fig. 3). It was found that a solution showed no significant degradation up to 95°F. All solutions prepared for the experiment were kept at room temperature which varied between 70 and 82°F.

Degradation of solutions with time was examined in the same way as with temperature. The solutions used were stored for 27, 48 and 120 hours before tests were made. It was found that the solutions degraded continuously with time. The degradation was faster in the beginning and then slowed down with time (Fig. 4). It was decided that a period of 65 to 75 hours preparation time would give satisfactory repeatability.

3.5 Procedure

The solution was poured in the calibrated reservoir and filled up to the 500 c.c mark. The pressure in the reservoir was then adjusted with the regulator. The solution was passed through the tube by opening the valve at the bottom. Time for a certain volume through the tube was measured by a stop watch. Care was taken to maintain the temperature constant during one particular run. This was done by preparing the solution at room temperature. The pressure was varied from 2 to 26 psig in steps to vary the Reynolds number of flow. The procedure was repeated for different concentrations. The tube was then replaced by another one and the same procedure repeated. The following were the variables in the experiment.

a) Tube diameter

Internal diameters used were .0264", .0372", .0410", .0628" and .0799".

b) Concentration of solutions.

1, 2, 5, 10, 20, 50, 100 and 200 ppm.

c) Reynolds number range as a function of cylinder pressure

1000 to 25,000 .

CHAPTER 4

RESULTS AND DISCUSSION

The Reynold's number and friction factor for water and polymer solutions both were calculated using the following equations:

$$R = \frac{VD}{\nu} \qquad 4.1$$

$$f = \frac{\tau_o}{\frac{1}{2}\rho V^2} = [\Delta p-(1+k)\ \frac{\rho V^2}{2g}]\ \frac{Dg}{2\rho LV^2}\ \text{(Appendix III)} \qquad 4.2$$

The results are presented as a plot between Reynold's number and friction factor (Figs. 5 to 9). The curves for pure water for all tubes show good agreement with the Blasius curve (15) given by the equation

$$f_\ell = \frac{16}{R}\ \text{(laminar)} \qquad 4.3$$

and

$$f = \frac{.0794}{R^{0.25}}\ \text{(turbulent)} \qquad 4.4$$

A review of the experimental friction factor curves (Figs. 5-9) shows that the friction factor decreases as the concentration increases. This trend is followed till a certain optimum concentration is reached. Further increase of concentration results in an increase of friction factor.

The curves in the graphs for various concentrations do not intersect at one point on the turbulent curve for water for any of the tubes used. In other words there is no critical shear stress or a characteristic wave number observed in the results. Virk (28) showed an existence of a critical shear stress for the onset of drag reduction for drag reducing fluids in general. This was true only when the drag reduction started in the turbulent region. For small diameter tubes the drag reduction starts at lower Reynold's number. In this experiment the tube diameters were small enough for drag reduction to start in the transition region for which Virk (28) commented that there was no onset point observed or can be reliably estimated.

The drag reduction data was analysed with the aid of Astarita's paper (2). It was assumed that β, the ratio of friction factor of polymer solution to water was a function of Deborah number only. The Deborah number was taken as

$$N'_{Deb} = \frac{V}{D} R^{x} \cdot T = s'T \qquad 4.5$$

Therefore

$$\beta = \beta(s'T) \qquad 4.6$$

The value of x (Equation 4.5) was taken as 0.5 by Astarita (1). Seyer and Metzner (19) in their derivation showed x equal to 0.75. Astarita (2) in another recent paper analysed his drag reduction data with x equal to 0.75 in equation 4.5.

Three different values of x were chosen in this work. β was plotted against $\frac{V}{D} R^{2.0}$ (Fig. 10), $\frac{V}{D} R^{2.25}$ (Fig. 11) and $\frac{V}{D} R^{2.50}$ (Fig. 12) for concentrations 20, 50, 100 and 200 wppm. It would be difficult to choose an exact value of x (Equation 4.5) from these figures as the scatter points is insensitive to small variations in x. It is expected that further increase or decrease in the value of x would slowly increase the scatter of points.

Equation 4.6, which was assumed, is justified as the data for different tubes lies on the same curve. The four curves for the four concentrations have a maximum scatter of 5 to 7 percent. Equation 4.6 thus gives the diameter correlation.

If different solutions are to be compared, the value of T should be known according to equation 4.6. It has been difficult to determine values of T for dilute solutions. Seyer and Metzner (20) defined relaxation time in a different way and determined its values experimentally for concentrated solutions only (2000-6000 wppm). The relaxation time varied with shear rate and therefore was not consistent for a particular solution. There has been no success in determining the relaxation time for dilute solutions. In the present work T has been avoided.

Let $s'_{0.50}$ be the value of s at a friction factor ratio $\beta = 0.50$.

Then for different concentrations

$$0.50 = \beta(s'_{0.5}T) = \beta(K_1)$$

$$0.50 = \beta(s'_{0.5}T) = \beta(K_2)$$

$$0.50 = \beta(s'_{0.5}T) = \beta(K_3)$$

$$0.50 = \beta(s'_{0.5}T) = \beta(K_4) \ .$$

From the above equation it can be seen that $K=K_1=K_2=K_3=K_4$ or

$$0.50 = \beta(s'_{0.50}T) = \beta(K) \qquad 4.7$$

where $K = s'_{0.5}T$ is a constant for all concentrations. Then

$$\beta = \left(\frac{s'K}{s'_{0.50}}\right) = \beta'\left(\frac{s'}{s'_{0.50}}\right) \qquad 4.8$$

The behaviour of different solutions can thus be compared with the aid of equation 4.8.

If equation 4.8 is valid drag reduction data for different concentrations should be correlated by a single curve in the form of β vs $\frac{s'}{s'_{0.50}}$. This plot is shown in fig. 13, and figs. 14 and 15. The scatter in figs. 10, 11 and 12 would be inherent in figs. 13 and 14 and 15 respectively because the same data is represented in the latter figures except for dividing the x-coordinate by a constant $(s_{0.50})$. The maximum dispersion in figs. 13, 14 and 15 is about 6-7 percent from the mean line.

The correlation (Equation 4.8) has an obvious drawback. The drag reduction cannot be predicted without knowing the value of $s'_{0.50}$. It is necessary to obtain enough experimental pressure drop data to evaluate the value of $s'_{0.50}$.

Equation 4.7 can be further analysed for a possible method of determining relaxation times of dilute solutions if relaxation time of concentrated solutions are known. As $K = s'_{0.5}T$ is a constant, a relationship between $s'_{0.5}$ and concentration can be used to determine T for various concentrations. $1/\frac{V}{D}\, R^{2.0}$ at $\beta = 0.5$ or $1/s'_{0.5}$ was plotted against concentration on a log log scale (fig. 16). A straight line seemed to connect the four points for the concentrations 20, 50, 100 and 200 wppm. The equation to the straight line can be written as

$$\frac{1}{s'_{0.5}} = B_1\, C^{B_2} \qquad 4.9$$

where C is the concentration and B_1 and B_2 are constants. B_1 and B_2 were found from fig. 16 as

$$B_1 = 5.8 \times 10^{-11}$$

$$B_2 = -\,0.44$$

If equation 4.9 extends over to higher concentrations, relaxation times for lower concentrations could be calculated if relaxation time for a high concentration is known.

The above analysis is confined to small diameter tubes only. It may or may not be valid for large diameters. Attempts to use the correlation (Eq. 4.6) for concentrations lower than 20 wppm. were unsuccessful because of the large scatter and it was considered that much more experimental evidence was required in the lower concentration range before further correlation can be attempted.

It is fortunate that optimum polymer concentrations for drag reduction generally lie in the range from 20 to 200 wppm so that for industrial use the correlation given by equation 4.6 and 4.8 are in the useful range.

The data has been presented in a different manner in figs. 20 to 28. Reynolds number was plotted against friction factor for all diameters and one particular concentration. Fig. 21 and fig. 22 show the plots for 1 and 2 wppm concentration solutions. The points for different diameters seem to lie on a straight line for low concentrations. However, as the concentration increases the data for different diameters disperses as seen in the later figures.

CHAPTER 5

DISCUSSION OF ERRORS

The following assumptions were made in the experiment and calculations:

a) The viscosity and density of all polymer solutions were assumed to be the same as that of pure water.

b) The flow behaviour index for all polymer solutions was assumed to be unity ($n=1$). The generalized Reynold number for a polymer solution for n not equal to unity would be given by equation 2.3. The solutions being dilute, the above two assumptions were reasonable.

c) The average velocity in the tube was assumed constant during one particular run.

The pressure difference across the tube was taken as the air pressure plus the average vertical head of water above the bottom of the tube. The height of solution in the reservoir varied during a particular run. The variation was 2" for the three smaller tubes, 6" for .0628" diameter and 8" for .0799" diameter tubes. Mean pressures were therefore used in the calculations. The pressure variation gave rise to velocity variation in the tube. The variation of velocity for the three small tubes is estimated as about 1% and about 3% for the larger diameter tubes at a pressure of 6 psi. At higher pressures the errors reduce to much lower values. The pressures used to obtain the co-relations were above 6 psi.

d) The flow was assumed as fully developed at the entrance of the tube.

The flow fully develops only after a certain development length. The development length can be estimated from different expressions given by various investigators. An estimation showed that the development lengths could be neglected without significant error in the results (Appendix IV). There was no need of estimating development lengths accurately and hence only one of the many expressions was used for estimation.

5.2 Human Errors

Human errors were mainly in observing the solution level in the reservoir and reading the pressure gauge for a particular adjustment. The percentage error in reading the gauge is greater at lower pressures. These errors were examined by repeating the experiment with one particular solution under the same conditions. Fig. (18) shows the repeatability of the experiment for the two runs with the same solution. The maximum error is of the order of 2% at low pressures.

The concentration of solutions vary with the method of preparation, the way the solutions are handled and the grade of polymer. The repeatability of concentrations has been one of the factors creating difficulty in comparison of results of two independent investigators. The concentrations prepared for this experiment could involve personal errors and other errors because all concentrations were not prepared from the same solution. The percentage error would be greater for low concentrations. Fig. (19) shows the variation of friction factor with the same concentration (20 wppm) prepared from two different solutions. The higher concentrations were finally used in correlating the drag reduction data.

Tube Diameter

The accuracy in calculating the tube diameter depends upon

a) The accuracy of measurement of weight of mercury and length of the tube

b) Human errors involved in the above measurements.

The microscope used to measure the length of the tubes could measure to a minimum of .001 cm. However, the observations made are expected to be accurate to .01 cm as the vernier scale on the microscope could not be read accurately beyond .01 cm. As the tube length was of the order of 38 cm, the percentage error would be of the order of .025.

The precision balance used to weigh the mercury could weigh a minimum of .0005 gms. The weight of mercury varied from 1.8 gms, for the smallest tube to 16 gms for the largest diameter tube. Repeated weight measurements reflected upon the extent of human errors involved in the weight (Appendix V). The values of weight varied from approximately 2.0 percent for the smallest tube to 0.4 percent for the largest tube. However this variation should not be confused with the error in weight. The error in weight could be judged as half of the variation in weight. As the diameter is proportional to square root of the weight of mercury the maximum error in diameter was expected to be less than 1 percent for the smallest diameter tube and even less for the other tubes.

The friction factor calculated in the results is roughly proportional to the fifth power of diameter. It is thus important to examine the change in value of the friction factor for a small change in the diameter. The change in friction factor for 1 percent change in diameter is shown in Table 5.1.

TABLE 5.1

Effect of Small Error in Tube Diameter on Friction Factor

S. No.	Gauge Pressure (psi)	Tube diameter d(in)	Original value of f × 10^3	f (For 0.5% change in D)	f (For-0.5% change in D)
1	25.0	.0264	10.11	10.39	9.86
2	25.0	.0372	9.42	9.68	9.18
3	25.0	.0410	8.96	9.16	8.72
4	24.0	.0628	7.36	7.61	7.14
5	26.0	.0799	6.52	6.76	6.32

The error in friction factor for one percent error in tube diameter is about 4.5 - 5% for all the diameters used. In the experiment this error is expected to vary from one percent for the largest diameter tube to 4% for the smallest diameter tube.

CHAPTER 6

CONCLUSIONS

1. There was an optimum concentration for which the friction factor was a minimum. It varied with the tube diameter.
2. No characteristic wave number values were observed for the onset of drag reduction.
3. The drag reduction data for small diameter tubes was correlated for a particular concentration for all the tubes used by the correlation $\beta = \beta(s'T)$.
4. The drag reduction data for small diameter tubes was correlated for various concentrations and all tube diameters used by the correlation $\beta = \beta'(s'/s'_{0.50})$.

The simple apparatus gives very satisfactory re-results. A large amount of data can be obtained in a relatively short time. Another advantage is the portability of the apparatus. However, at very low pressures (∿ 4 psi and lower) the results would not be quite satisfactory because of the variation of the vertical height of fluid in the reservoir.

CHAPTER 7

RECOMMANDATIONS

1. A future suitable work would be to make an experimental set up for larger tube diameters than used presently. The correlations (equations 4.6 and 4.8) could then be examined for larger tube diameters.
2. Other polymers may or may not fit into the correlations given by equations 4.6 and 4.8. Further similar work is recommended for other drag reducing polymer solutions.
3. The instrument allows simple shear degradation to take place in, say, one pass. It would be useful to examine shear degradation of the polymer over a number of passes.
4. Since only one length of tube has been used, it would seem logical to repeat the experiment for tubes of different lengths to obtain a possible L/D correlation.

REFERENCES

1. Astarita, G.: "Possible Interpretation of the Mechanism of Drag Reduction in Viscoelastic Liquids", I and EC Fundamentals, Vol. 4, No. 3, 1965, pp. 354-356.
2. Astarita, G., Greco Guido Jr. and L. Nicodemo : "A Phenomenological Interpretation and Correlation of Drag Reduction", A. I. Ch. E. Journal, Vol. 15, No. 4, 1969, p. 564.
3. Astarita, G. and L. Nicodemo, :"Velocity Distributions and Normal Stresses in Viscoelastic Turbulent Pipe Flow", A.I. Ch.E. Journal, Vol. 12, No. 3, May 1966, pp. 478-482.
4. Brandt, H., A. T. McDonald and F. W. Boyle: "Turbulent Skin Friction of Dilute Polymers in Rough Pipes", Viscous Drag Reduction, Edited by C. S. Wells, Plenum Press, New York, 1969.
5. Dodge, D.W. and A. B. Metzner: "Turbulent Flow of Non-Newtonian Systems", A.I.Ch.E. Journal, Vol. 5, No. 2, p. 189.
6. Elata, C., J. Lehrer and A. Kahanovitz: "Turbulent Shear Flow of Polymer Solutions", Israel Journal of Technology, Vol. 4, No. 1, 1966, pp. 87-95.
7. Elata, C. and J. Tirosh: "Frictional Drag Reduction" Israel Journal of Technology, Vol. 3, No. 1, 1965, pp. 1-6.

8. Ernst, W.D.: "Investigation of the Turbulent Shear Flow of Dilute Aqueous CMC Solutions", A.I.Ch.E. Journal, Vol. 12, No. 3, 1966, p. 581.

9. Ernst, W.D.: "Turbulent Flow of an Elasticoviscous Non-Newtonian Fluid", A.I.A.A. Journal, Vol. 5, No. 5, 1967.

10. Hinze, J.O.: "Turbulence", McGraw Hill, New York.

11. Hoyt, J.W. and A. G. Fabula: "The Effect of Additives on Fluid Friction", Proc. 5th Symposium on Naval Hydronamics, Bergen (1964).

12. Lumley, J.L.: "The Reduction of Skin Friction Drag", Proc. 5th Symposium on Naval Hydronynamics, Bergen (1964).

13. Metzner, A.B. and M. Grahm Park: "Turbulent Flow Characteristics of Viscoelastic Fluids", Journal of Fluid Mechanics, Vol. 20, Part 2, 1964, pp. 291-303.

14. Meyer, W.A.: "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Viscoelastic Non-Newtonian Fluids in Pipes", A.I.Ch.E. Journal, Vol. 12, No. 3, 1966, p. 522.

15. Middleton, J.A.: "An Experimental Study of the Boundary Layer Formed on a Flat Plate in a Dilute Homogeneous Polymer Solution", M.Eng. Thesis, McMaster University, June 1969.

16. Patterson, G.K. and J. L. Zakin: "Prediction of Drag Reduction with a Viscoelastic Model", A.I.Ch.E. Journal, May 1968, p. 434.

17. Savins, J.G.: "Drag Reduction Characteristics of Solutions of Macromolecules in Turbulent Pipe Flow", Society of Petroleum Engineers Journal, September 1964.

18. Schlichting, H.: Boundary Layer Theory, Fourth Edition, McGraw Hill, New York.

19. Seyer, F.A. and A. B. Metzner: "Turbulence Phenomena in Drag Reducing Systems", A.I.Ch.E. Journal, Vol. 15, No. 3, May 1969, pp. 426-434.

20. Seyer, F.A. and A. B. Metzner: "Turbulent Flow Properties of Viscoelastic Fluids", The Canadian Journal of Chemical Engineering, Vol. 45, June 1967, pp. 121-126.

21. Shaver, R. G. and E. W. Merril: "Turbulent Flow of Pseudoplastic Polymer Solutions in Straight Cylindrical Tubes", A.I.Ch.E. Journal, Vol. 5, No. 2, 1959, p. 181.

22. Shen, Chi-Hung: "An Experimental Study of the Effects of Aqueous Polymer Solutions on a Liquid Boundary Layer", M. Eng. Thesis, McMaster University, May 1968.

23. Sherman, B.A.: "Effect of Concentration and Molecular Structure on the Drag Reduction of Dilute Polymer Solutions", A.U.W.E. Tech. Note, 246/67, July 1967.

24. Smallman, John R.:"The Influence on Hydrodynamic Drag of High Molecular Weight Compounds in the Turbulent Boundary Layer", M. Eng. Thesis, McMaster University, May 1967.

25. Smith, K.A., E.W. Merril, H.S. Mickley and P. S. Virk,: Anomalous Pitot Tube and Hot Film Measurements in Dilute Polymer Solutions", Chem. Eng. Science, Vol. 22, 1967, pp. 619-626.

26. Spangler,J.J.: "Studies of Viscous Drag Reduction Including Turbulence Measurements and Roughness Effects", Viscous Drag Reduction, Edited by C. S. Wells, Plenum Press, New York, 1969.

27. Toms, B.A.: "Some Observations of the Flow of Linear Polymer Solutions Through Straight Tubes at Large Reynold's Number", Proc. 1st Int. Congress on Rheology, Vol. II, pp. 135-141, 1948.

28. Virk, P. S. and F. W. Merril: "The Onset of Dilute Polymer Solution Phenomena", Viscous Drag Reduction, Edited by C. S. Wells, Plenum Press, New York, 1969.

29. Vogel, W. M. and A. M. Patterson: "An Experimental Investigation of the Effect of Additives Injected Into the Boundary Layer of an Underwater Body", Proc. 5th Symposium on Naval Hydrodynamics, Bergen (1964).

30. Wells, C.S.Jr.:"Anomalous Turbulent Flow of Non-Newtonian Fluids", A.I.A.A. Journal, Vol. 3, No. 10, 1965.

31. White, A.: "Turbulent Drag Reduction with Polymer Additives", Journal of Mechanical Engineering Science, Vol. 8, No. 4, 1966.

32. White, A.: "Some Observations on the Flow Characteristics of Certain Dilute Macromolecular Solutions"-Viscous Drag Reduction, Edited by C.S. Wells, Plenum Press, New York, 1969.

33. Streeter, V.L.: Handbook of Fluid Dynamics, McGraw Hill, New York.

34. Technical Paper No. 410-C, "Flow of Fluids Through Valves, Fittings and Pipe", Crane Canada Ltd., Montreal, Que.

35. Von Karman: "Turbulence and Skin Friction", J.A.S., Vol. 1, No. 1, 1934.

Fig. 1 Experimental Apparatus

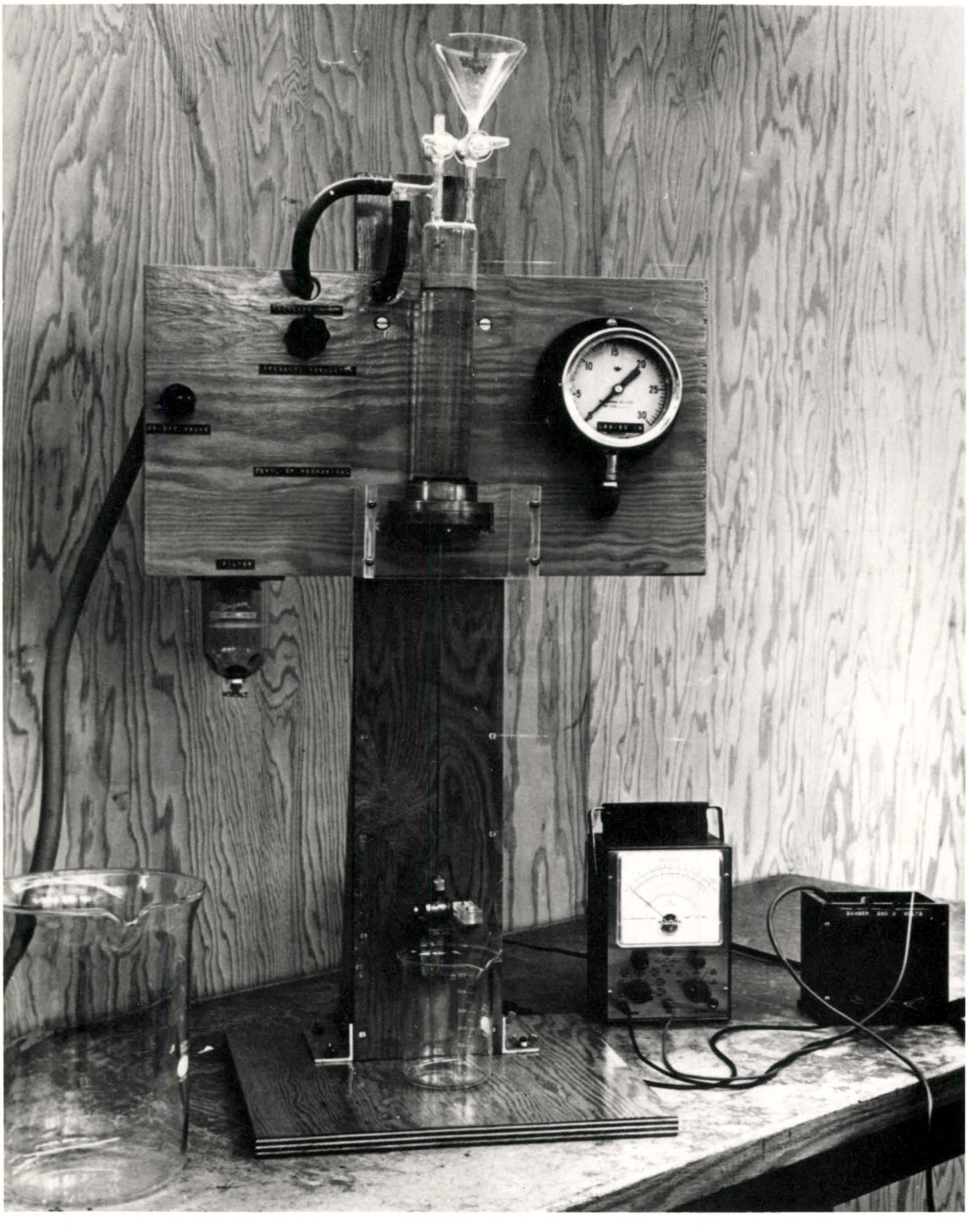


Fig. 2 Experimental Apparatus

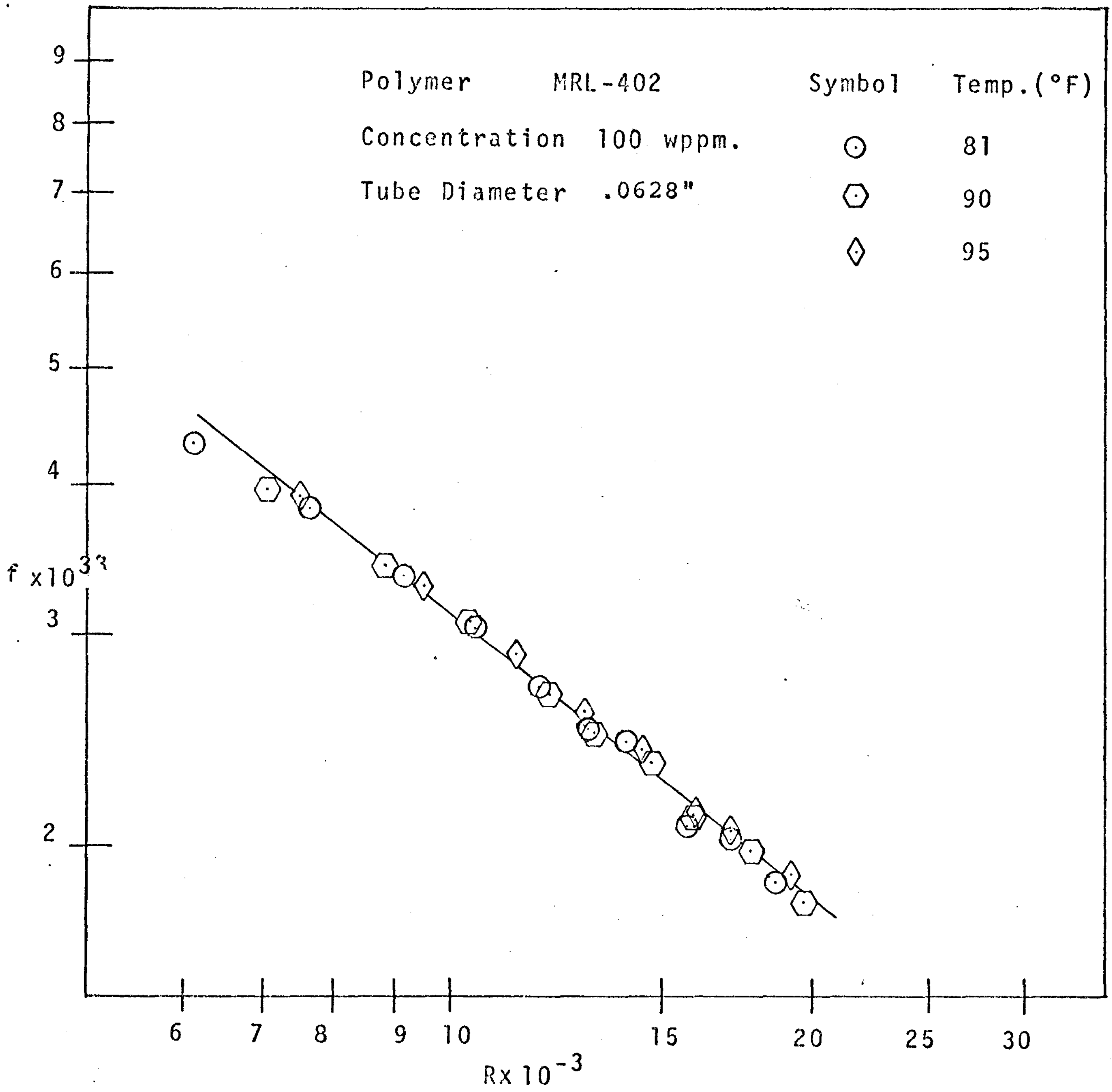


Fig. 3. Effect of Temperature on Polymer Solution.

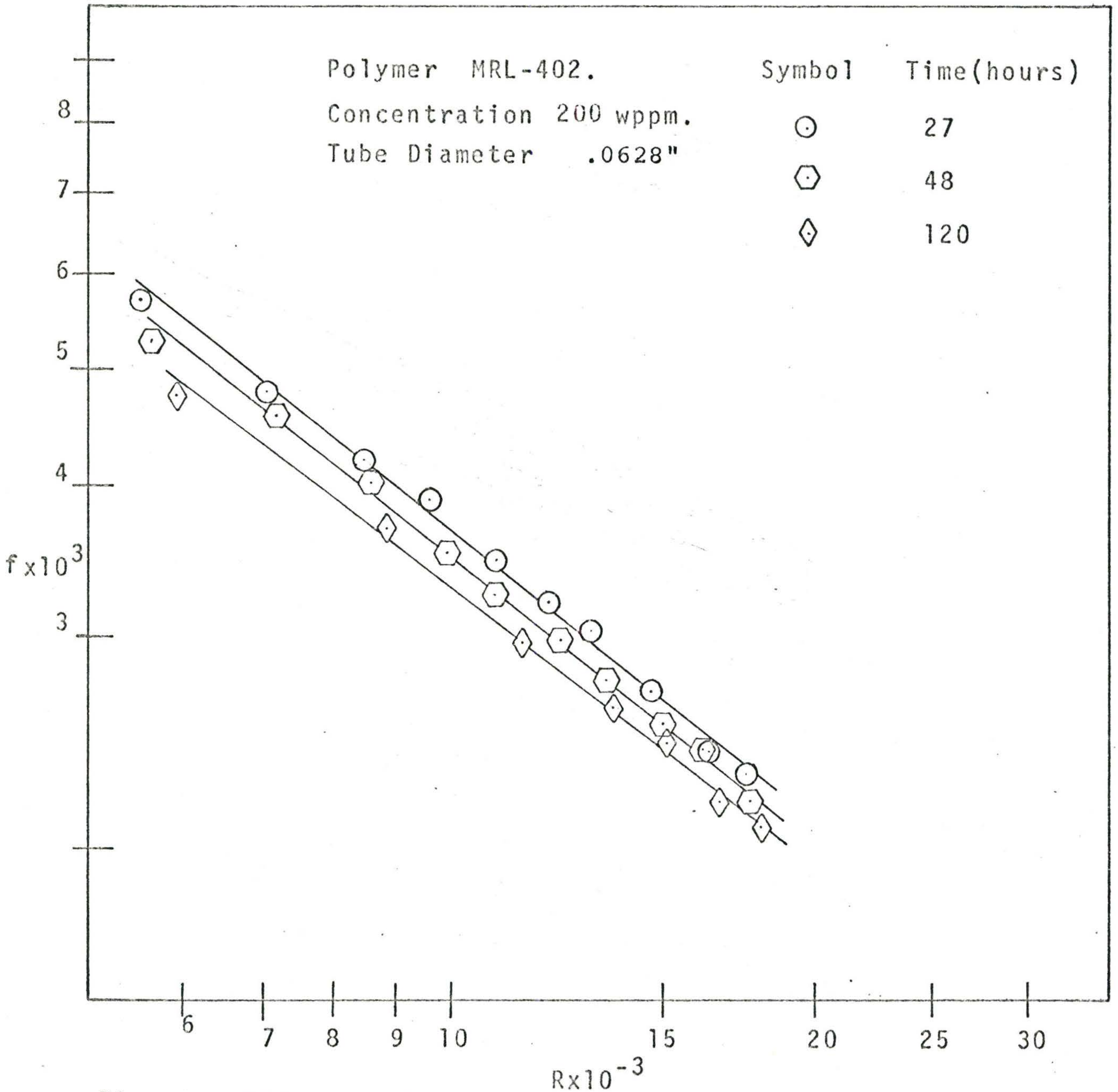
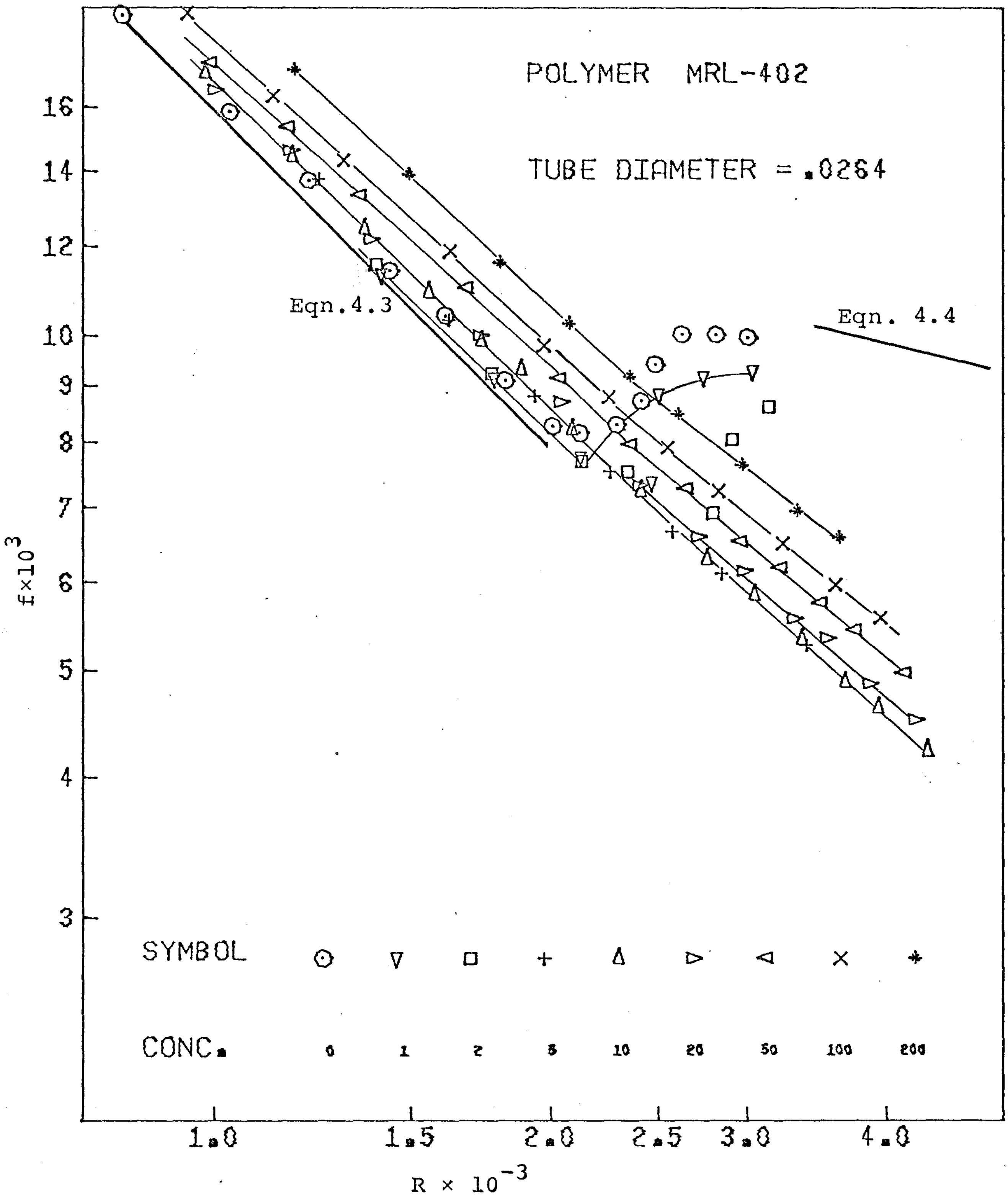
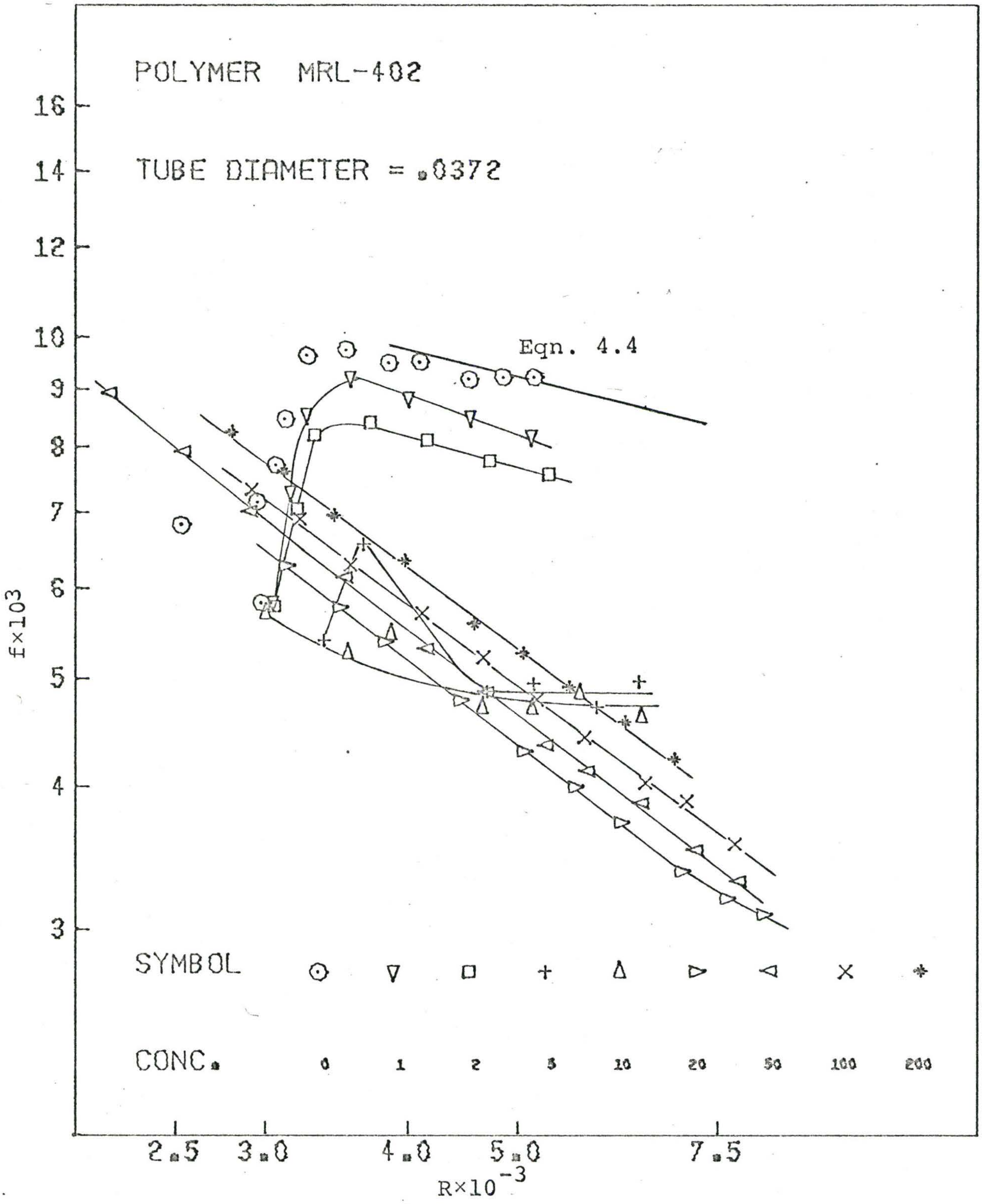


Fig. 4 Effect of Time on Polymer Solution.



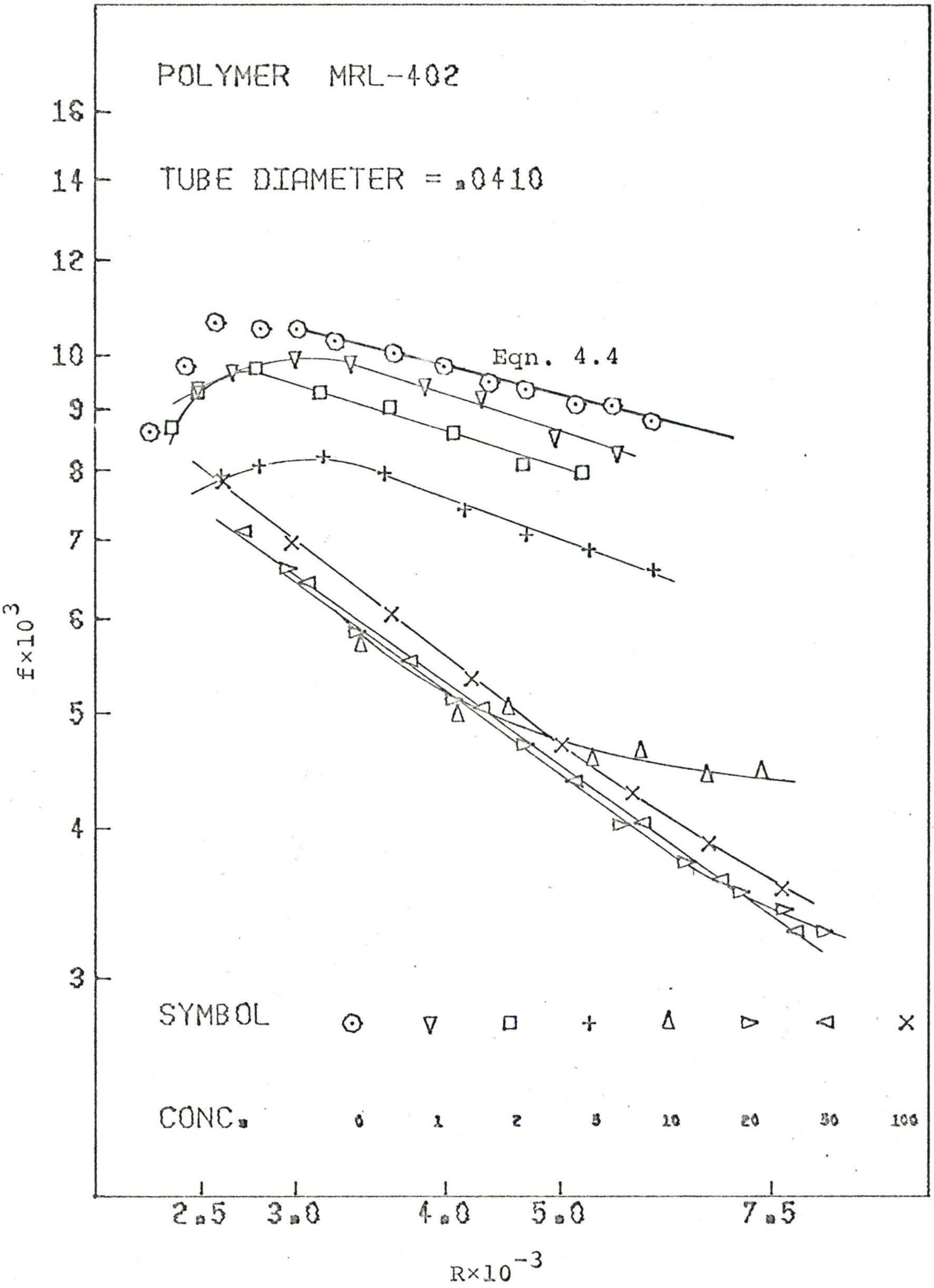
REYNOLD NUMBER VS. FRICTION FACTOR

Fig. 5



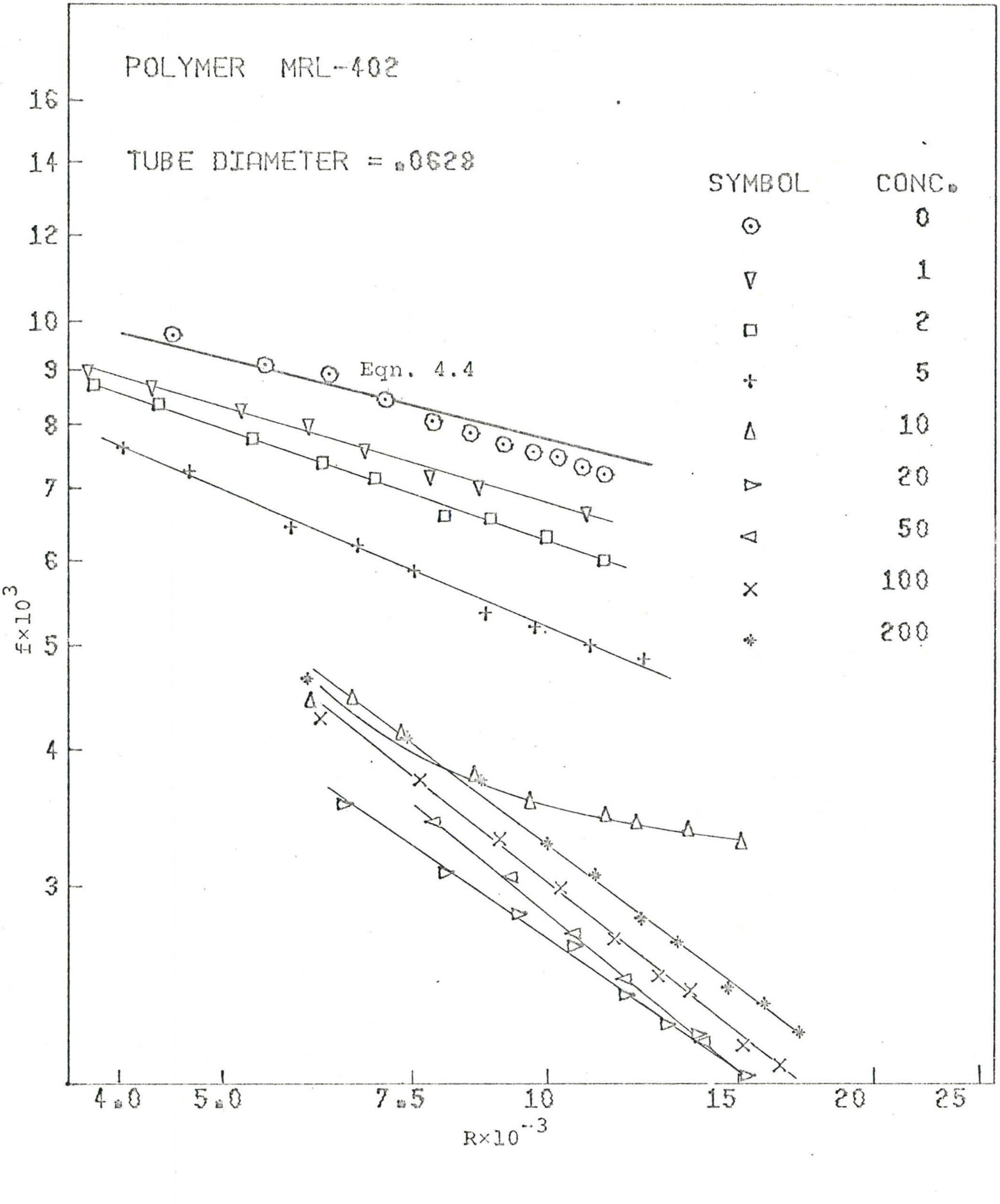
REYNOLD NUMBER VS. FRICTION FACTOR

Fig. 6



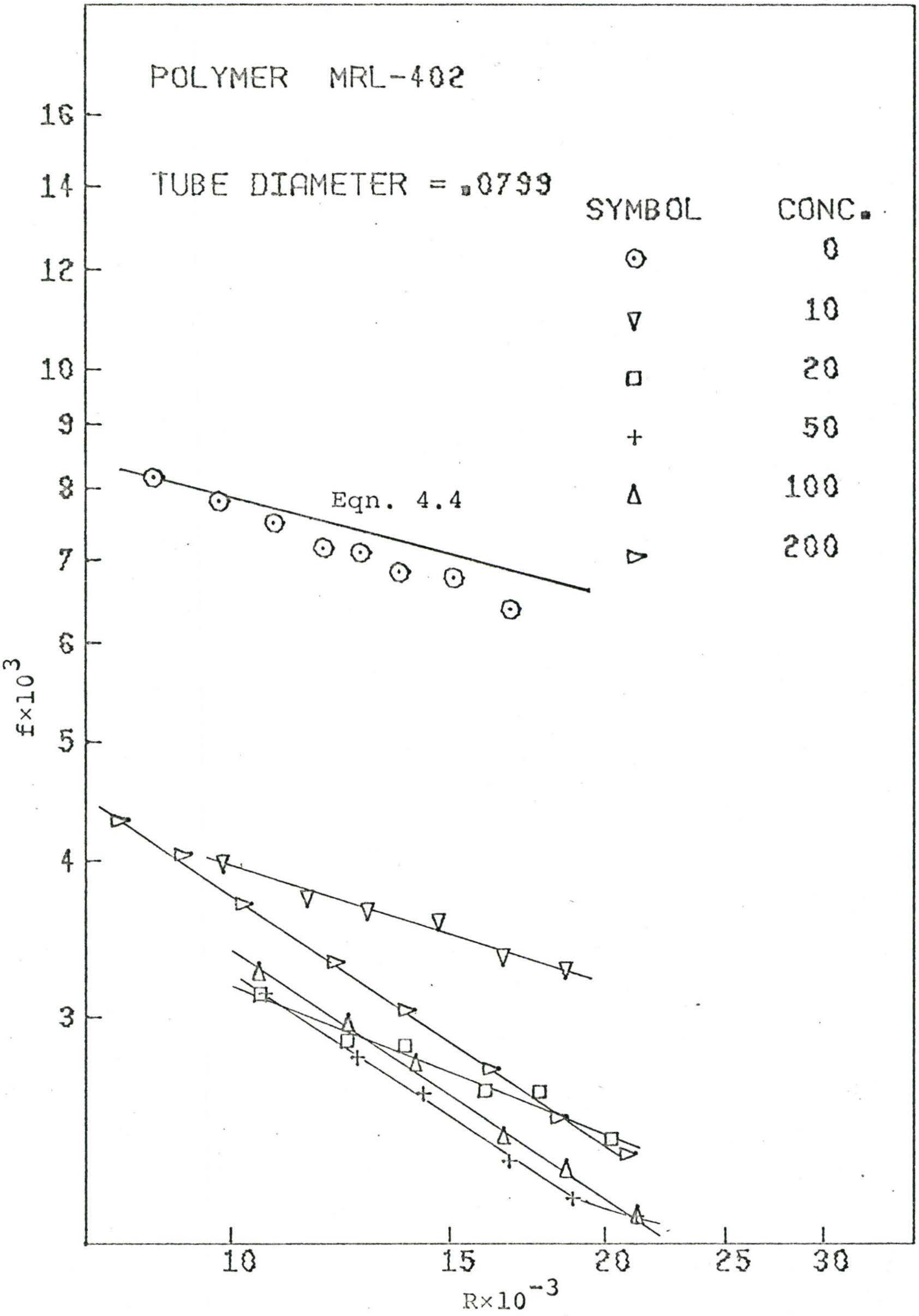
REYNOLD NUMBER VS. FRICTION FACTOR

Fig. 7



REYNOLD NUMBER VS. FRICTION FACTOR

Fig. 8



REYNOLD NUMBER VS. FRICTION FACTOR

Fig. 9

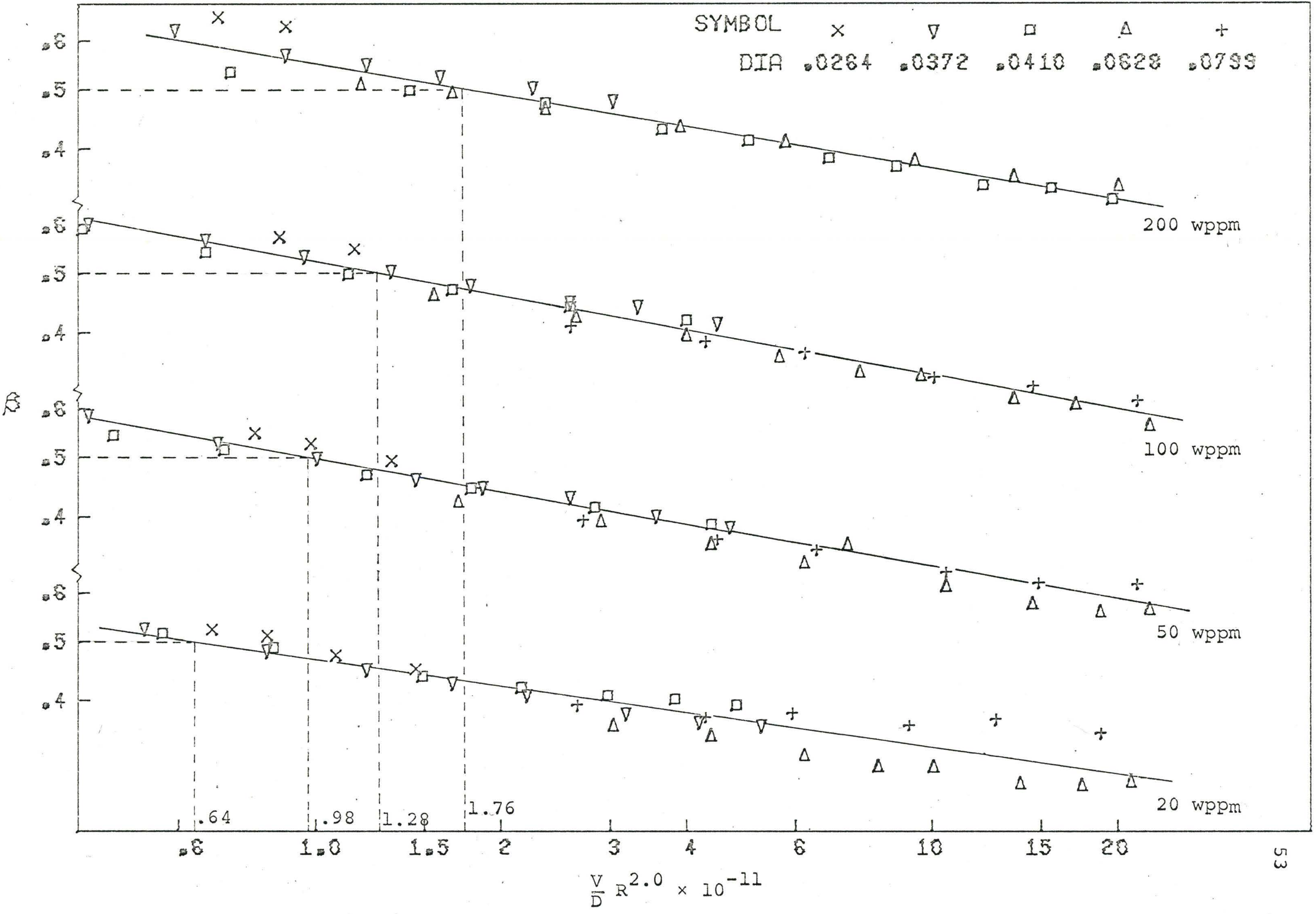


Fig. 10 TUBE DIAMETER CORRELATION (EQ. 4.6)

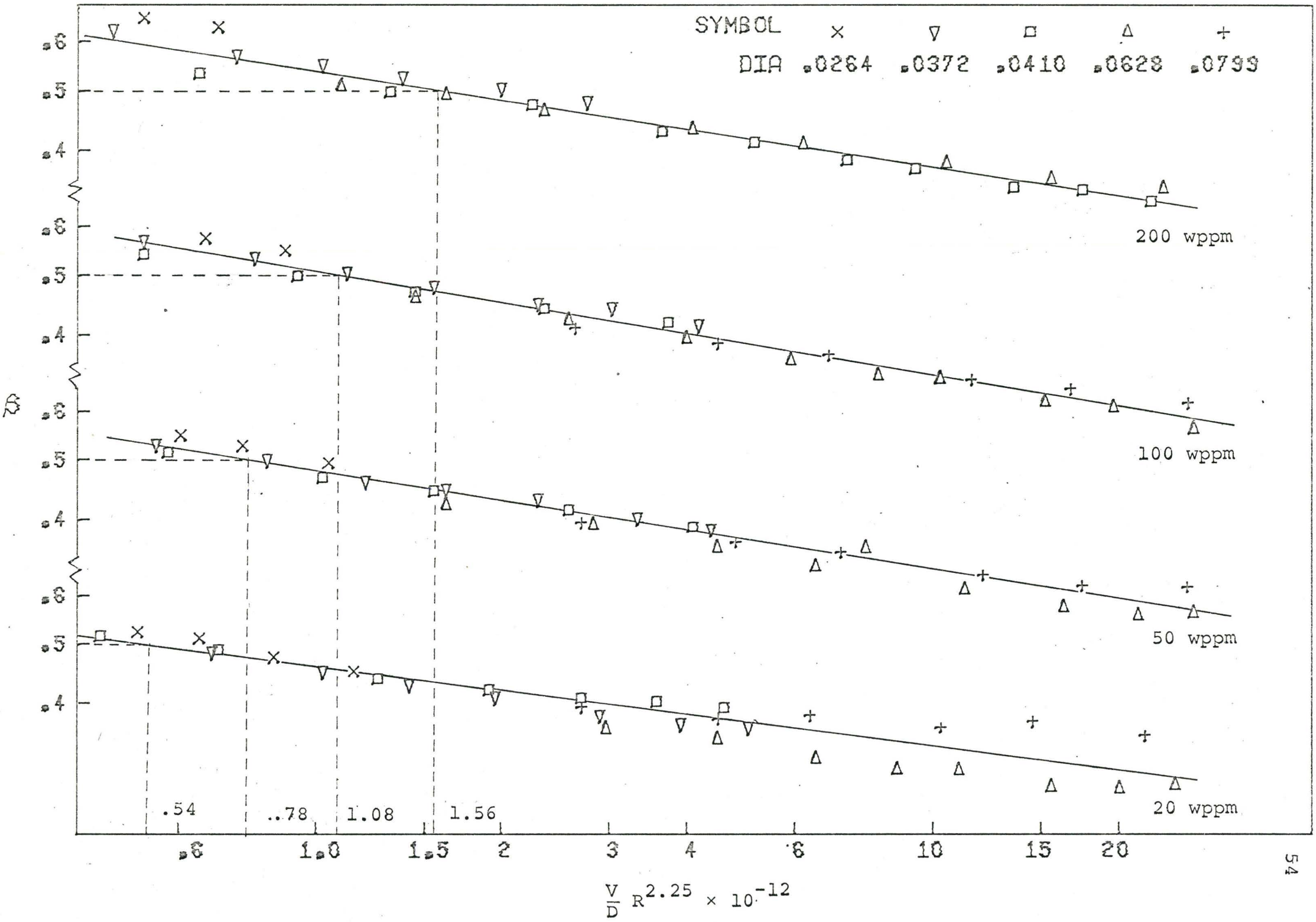


Fig. 11 TUBE DIAMETER CORRELATION (EQ. 4.6)

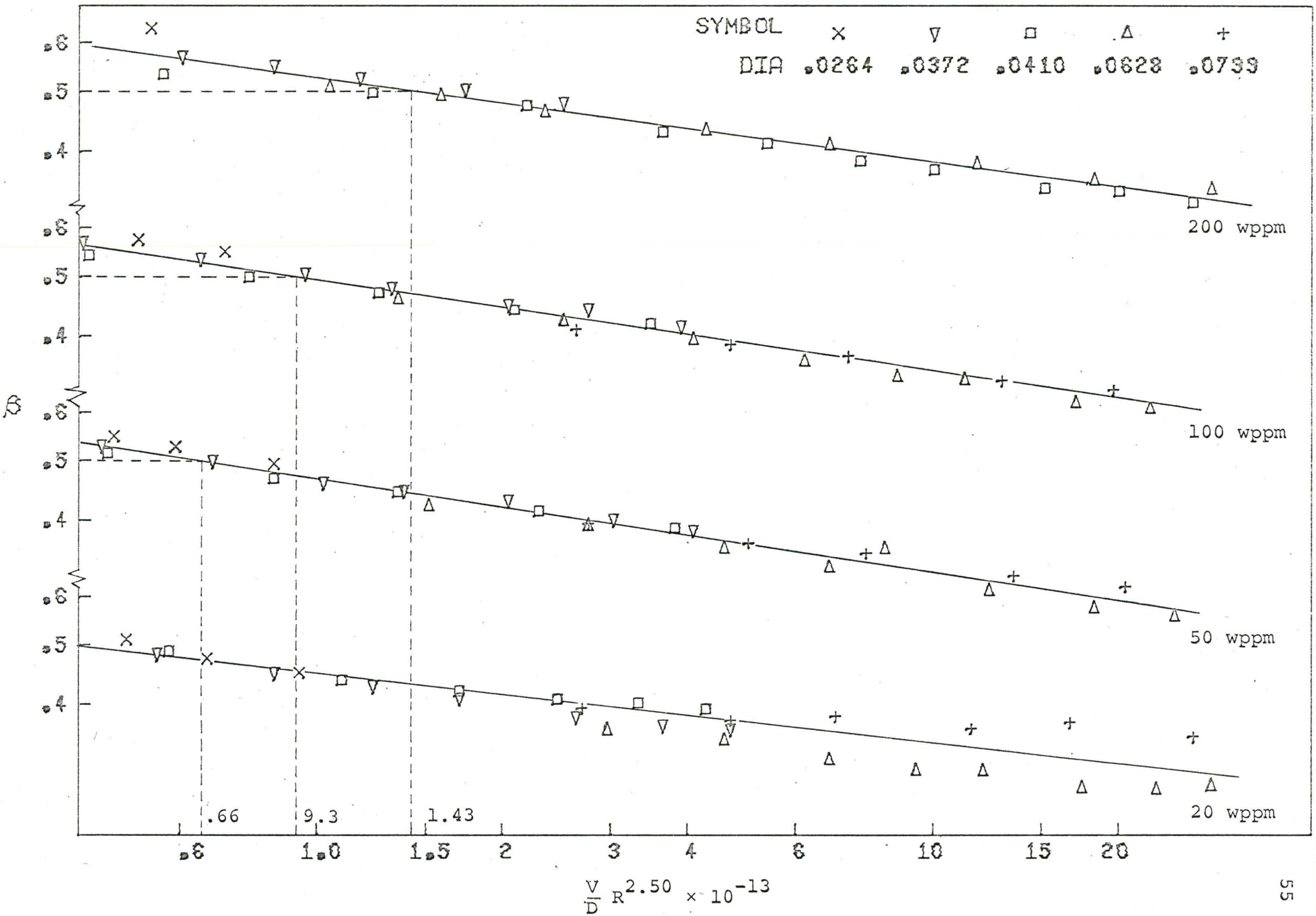
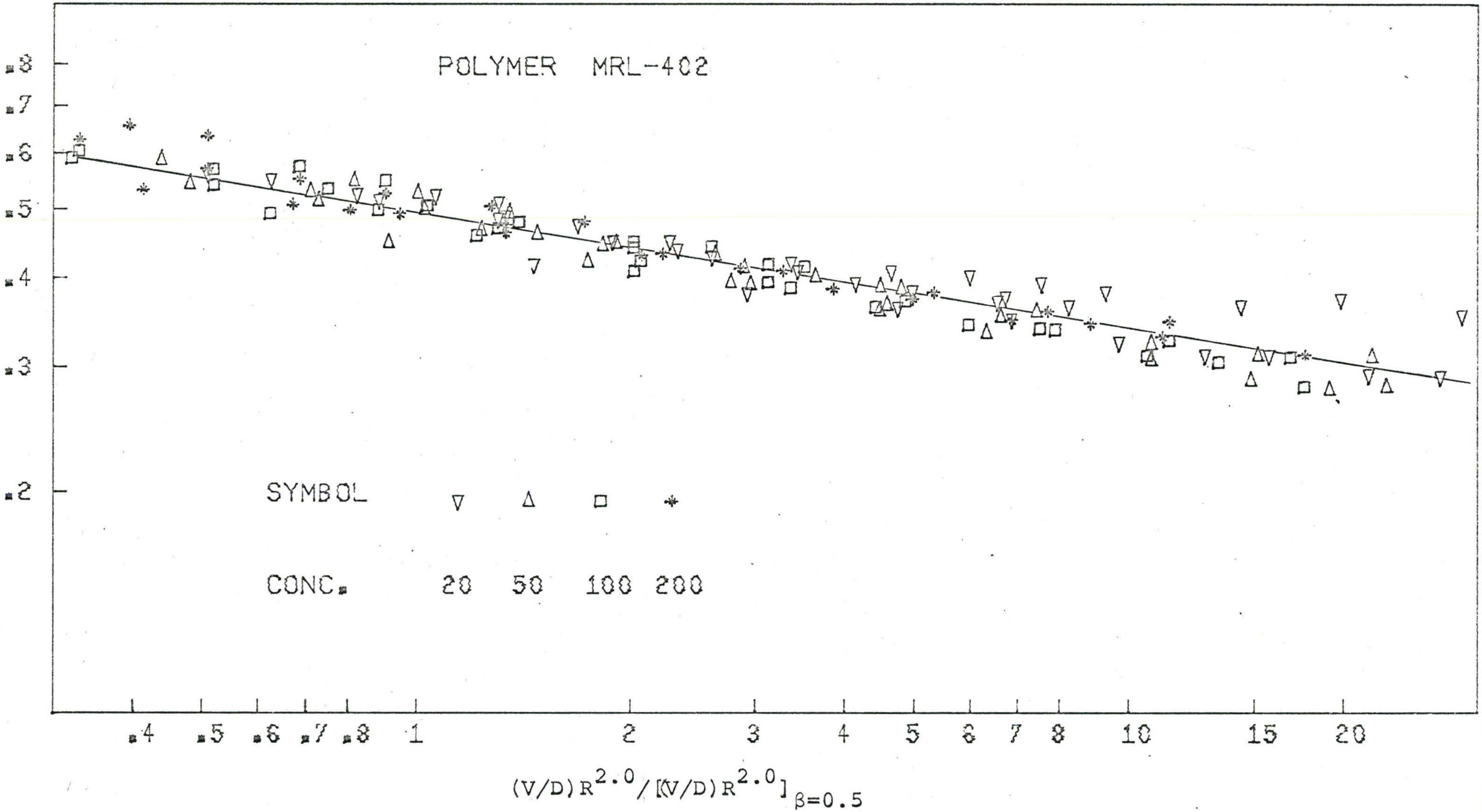
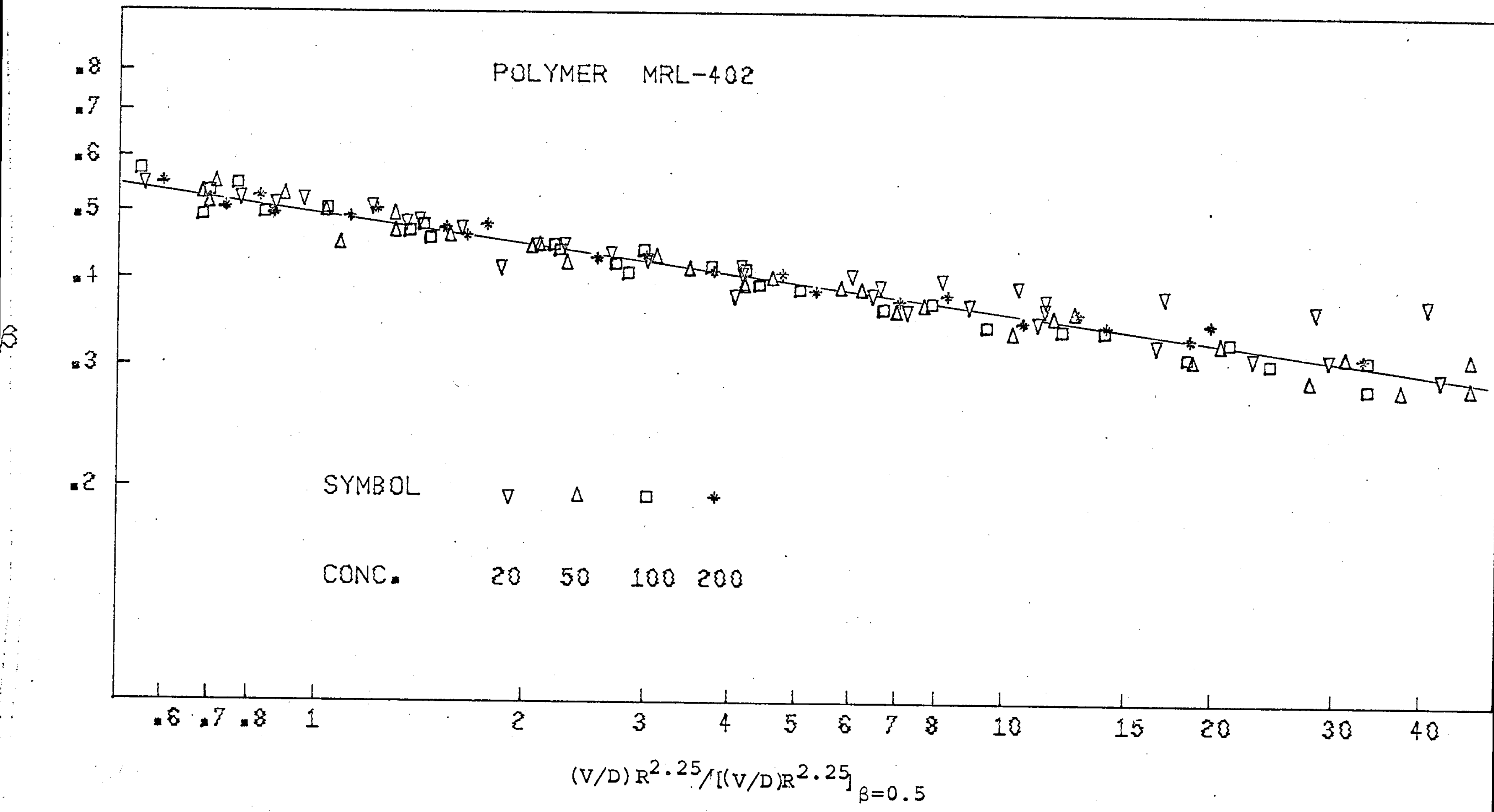


Fig. 12 TUBE DIAMETER CORRELATION (EQ. 4.6)



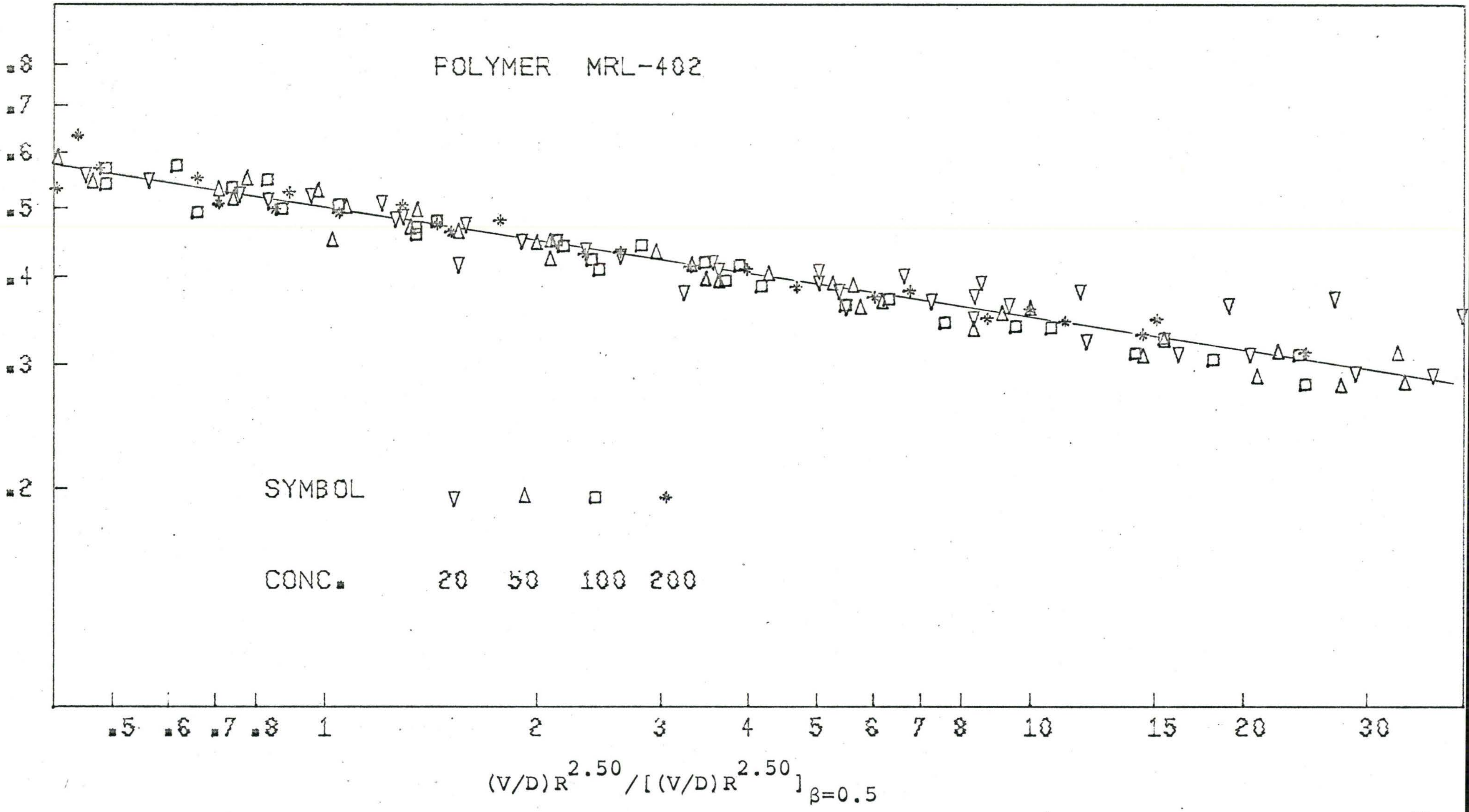
CORRELATION FOR CONC. AND TUBE DIAMETER

Fig. 13



CORRELATION FOR CONC. AND TUBE DIAMETER

Fig. 14



CORRELATION FOR CONC. AND TUBE DIAMETER

Fig. 15

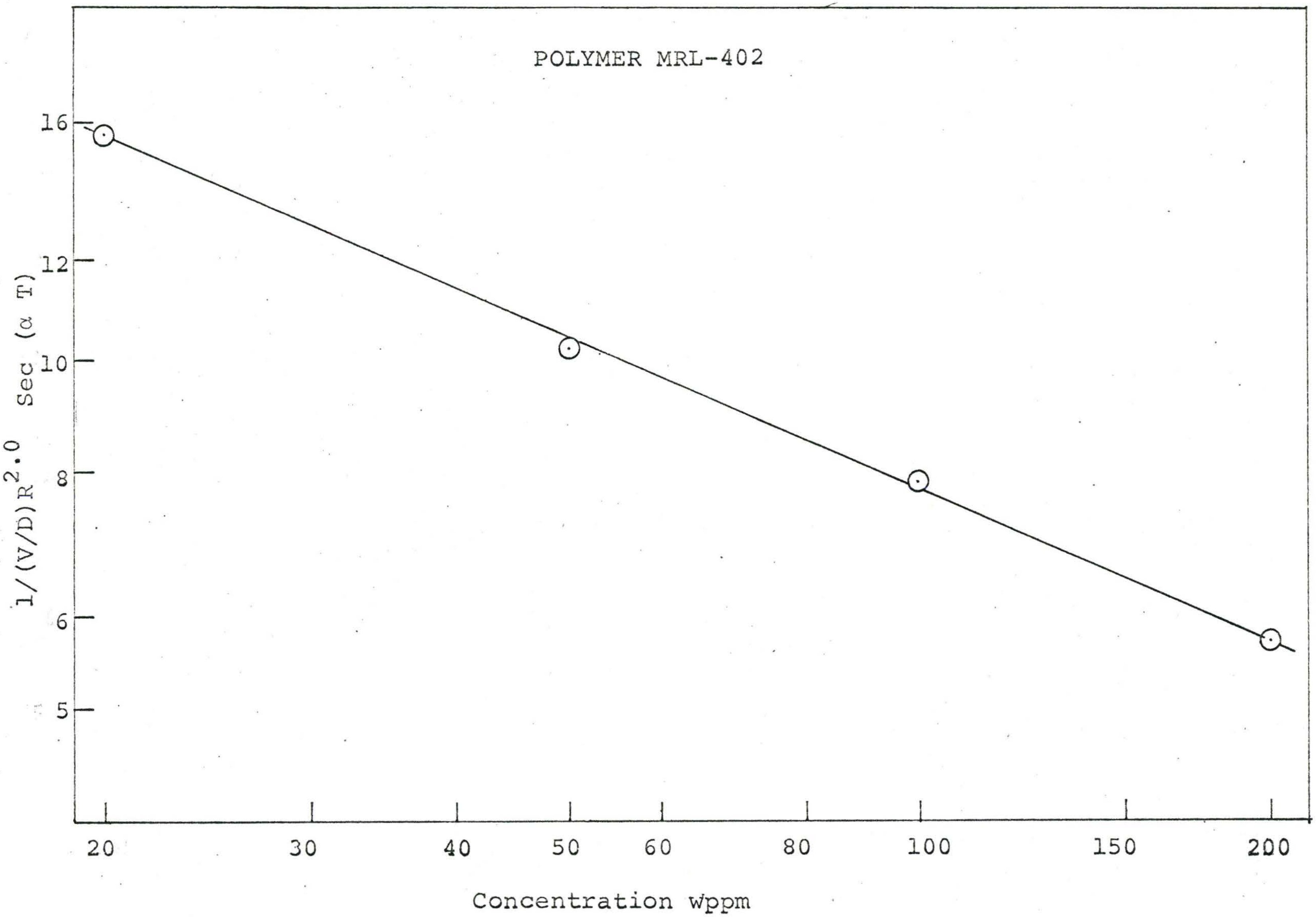


Fig. 16 RELATION BETWEEN $(V/D)R^{2.0}$ AND CONCENTRATION

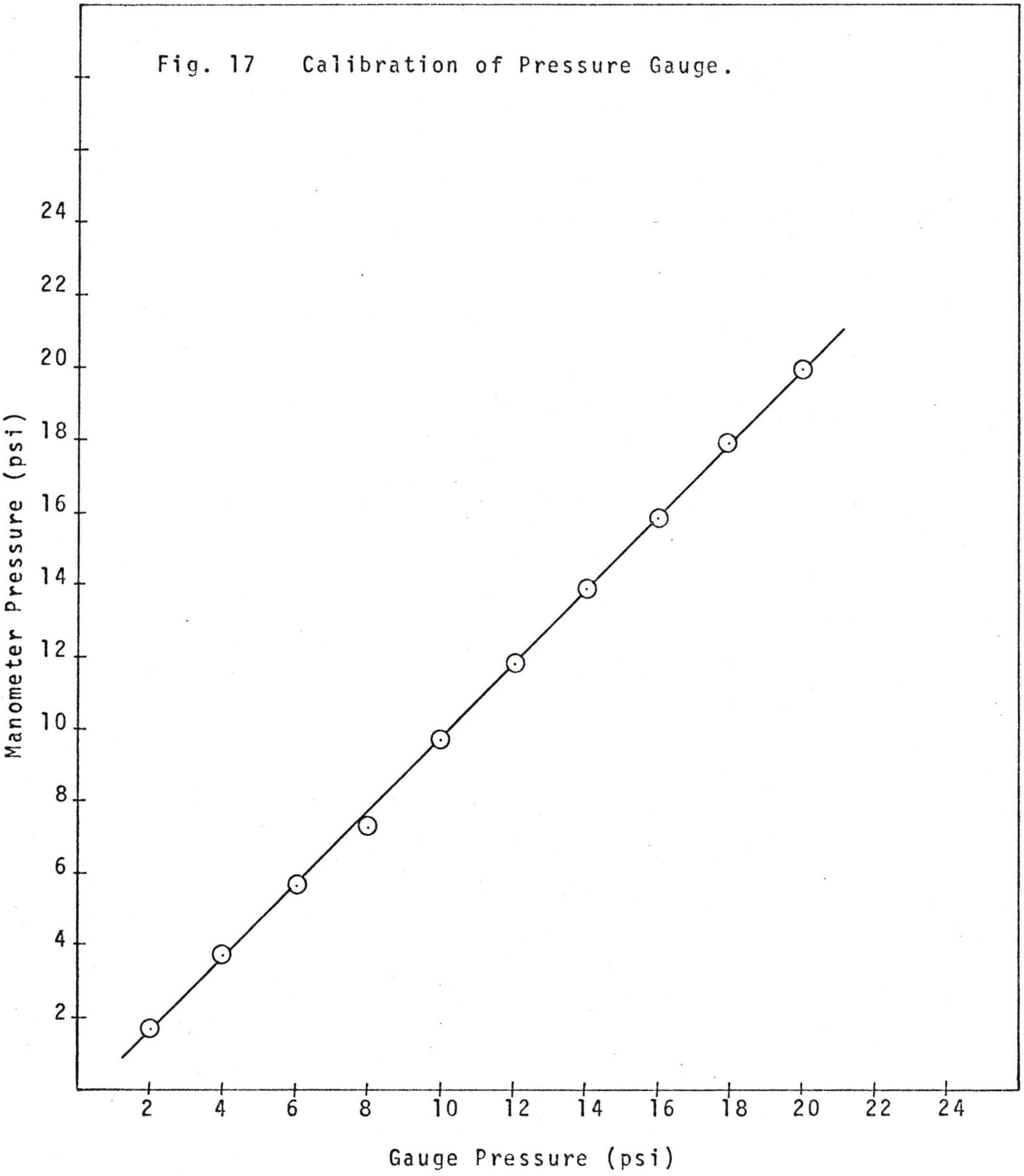


Fig. 17 Calibration of Pressure Gauge.

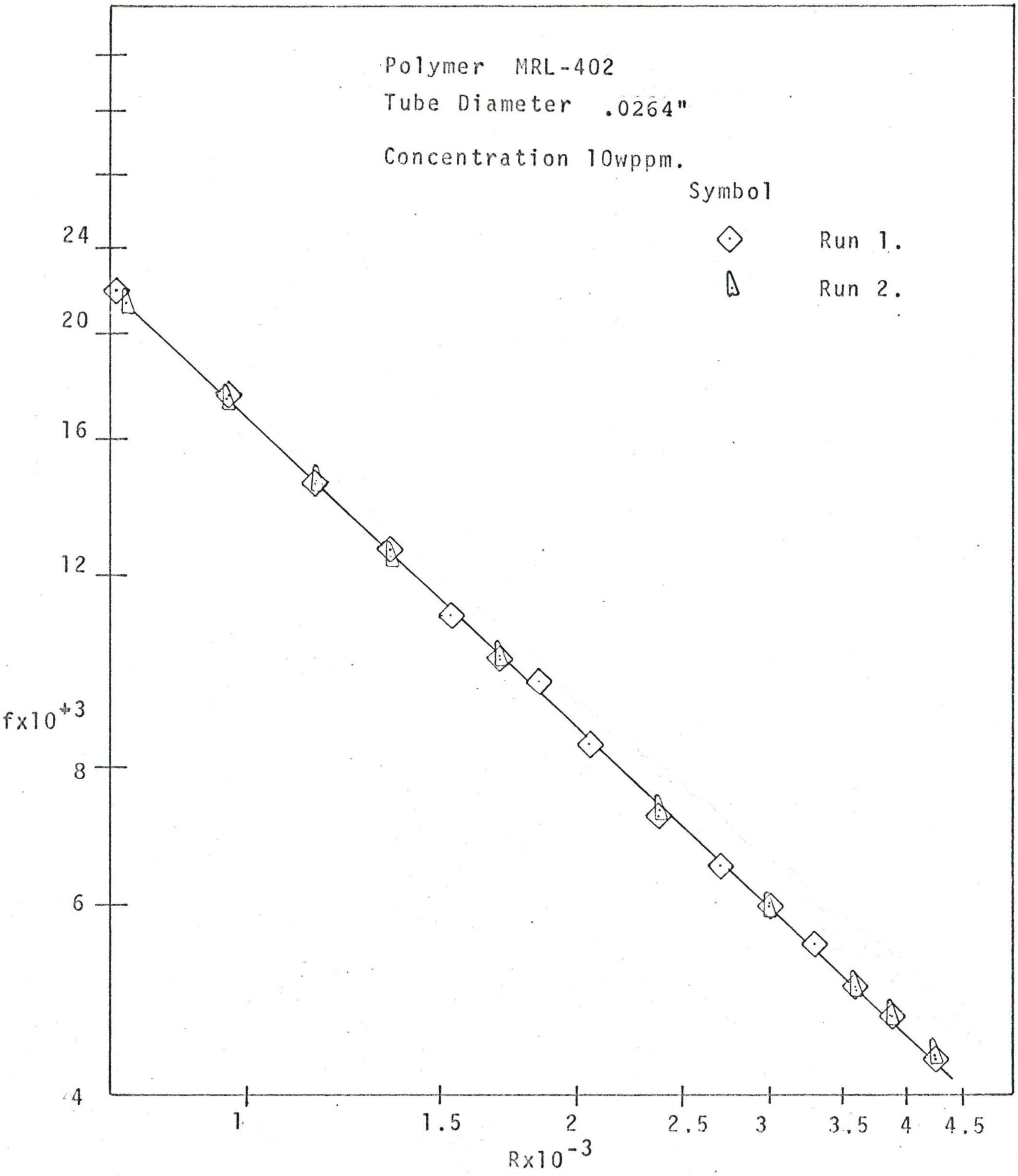


Fig. 18 Tests Showing the Repeatability of Results.

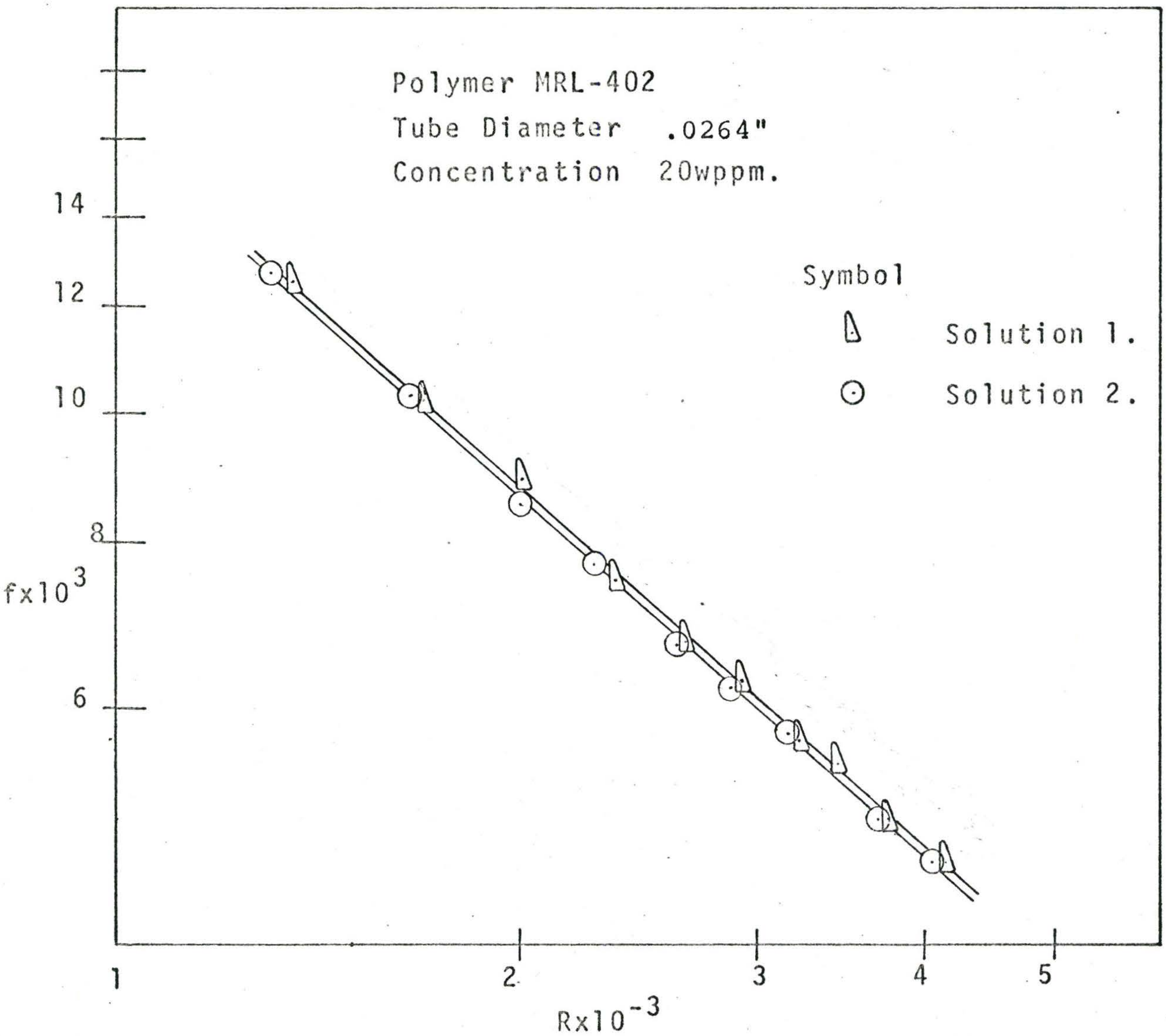


Fig. 19 Repeatability of Polymer Solutions.

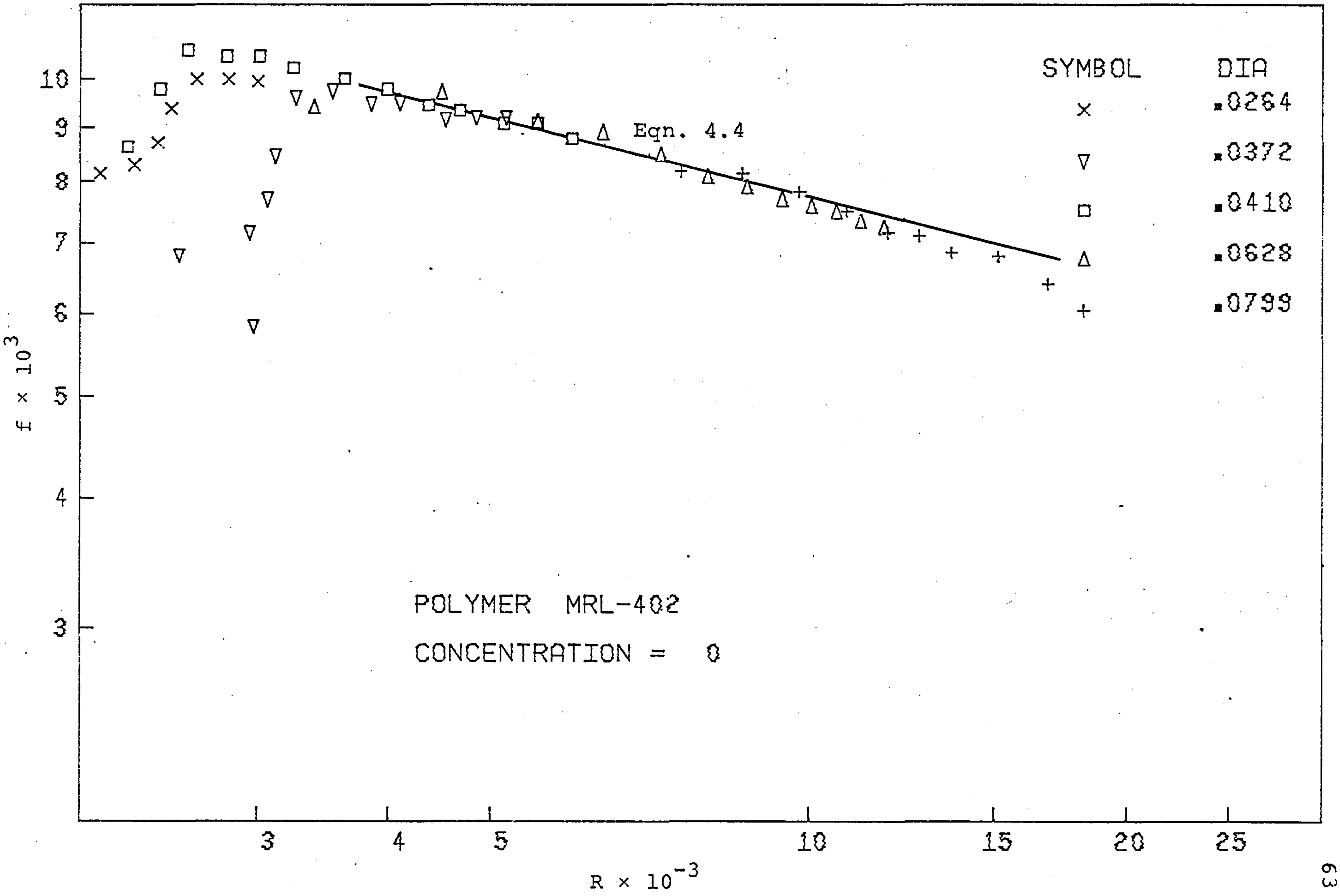


Fig. 20 REYNOLD NUMBER VS. FRICTION FACTOR

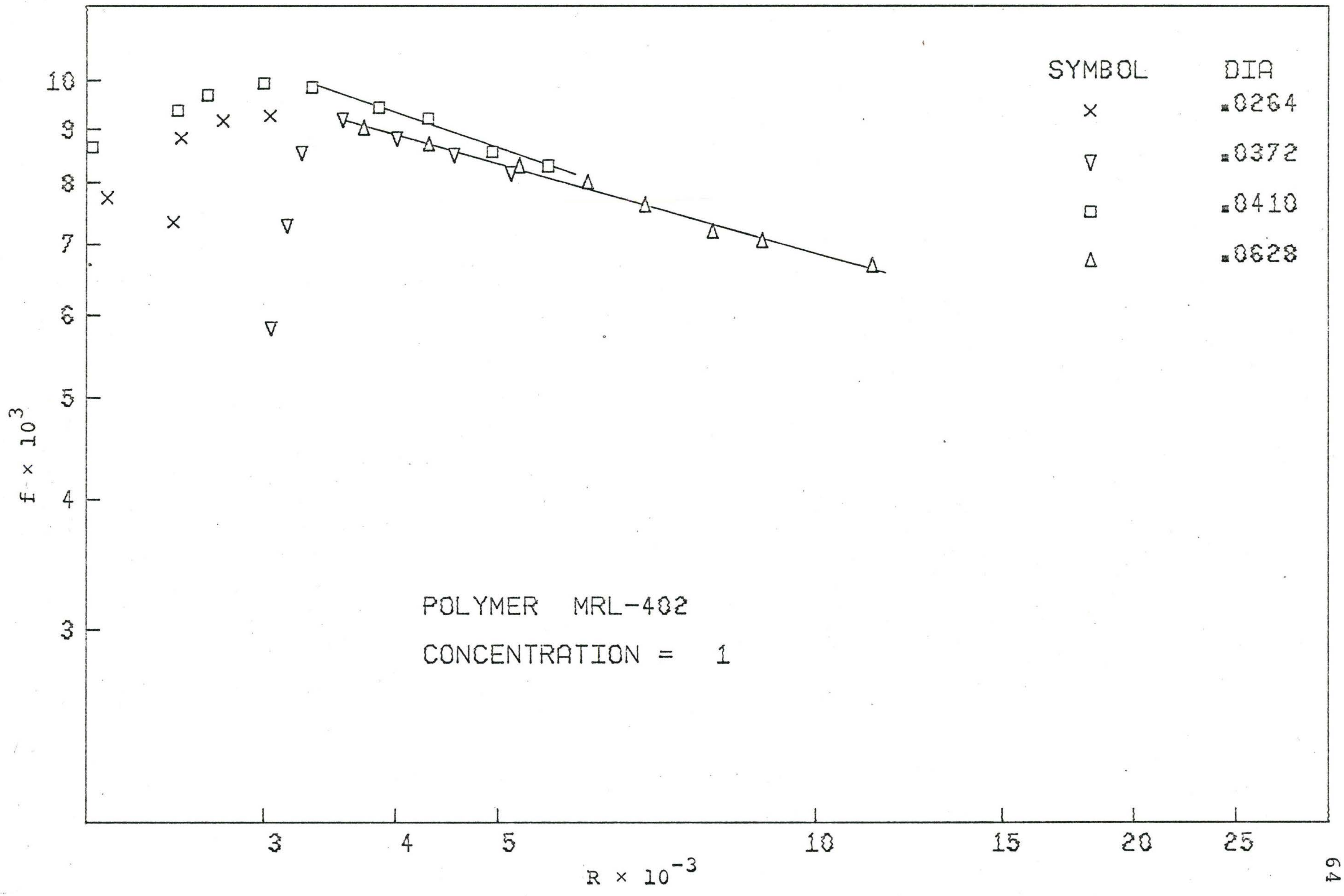


Fig. 21 REYNOLD NUMBER VS. FRICTION FACTOR

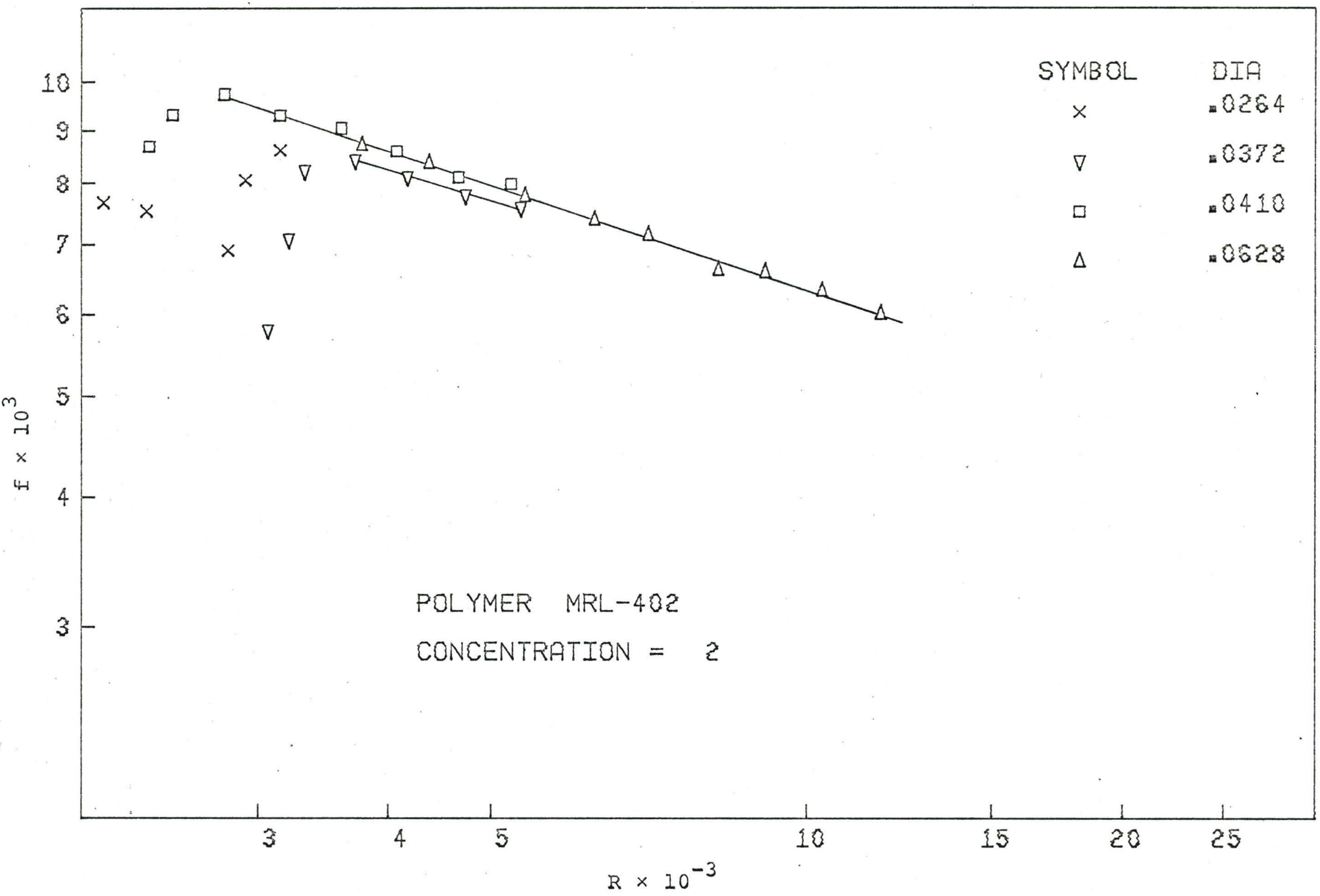


Fig. 22 REYNOLD NUMBER VS. FRICTION FACTOR

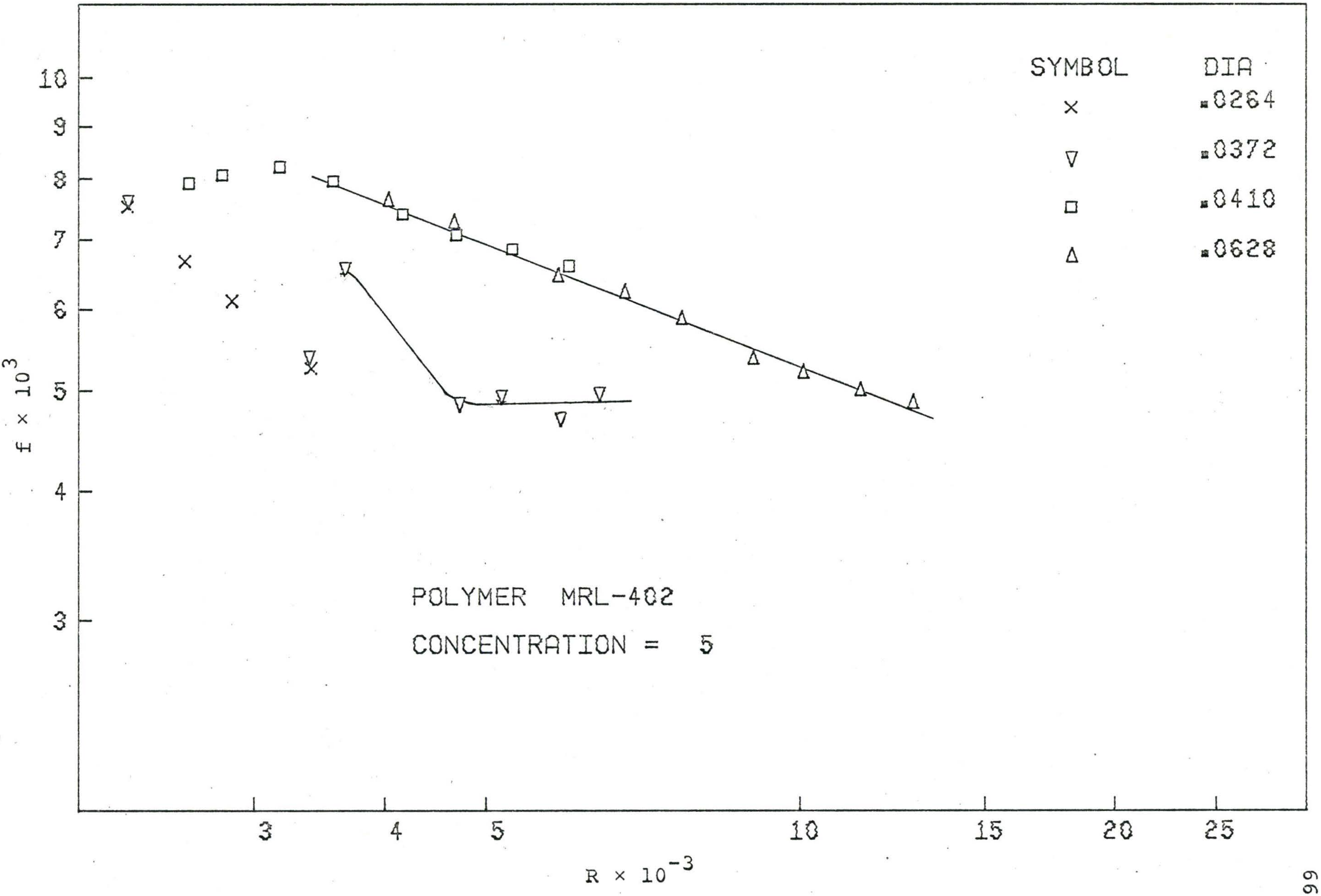


Fig. 23 REYNOLD NUMBER VS. FRICTION FACTOR

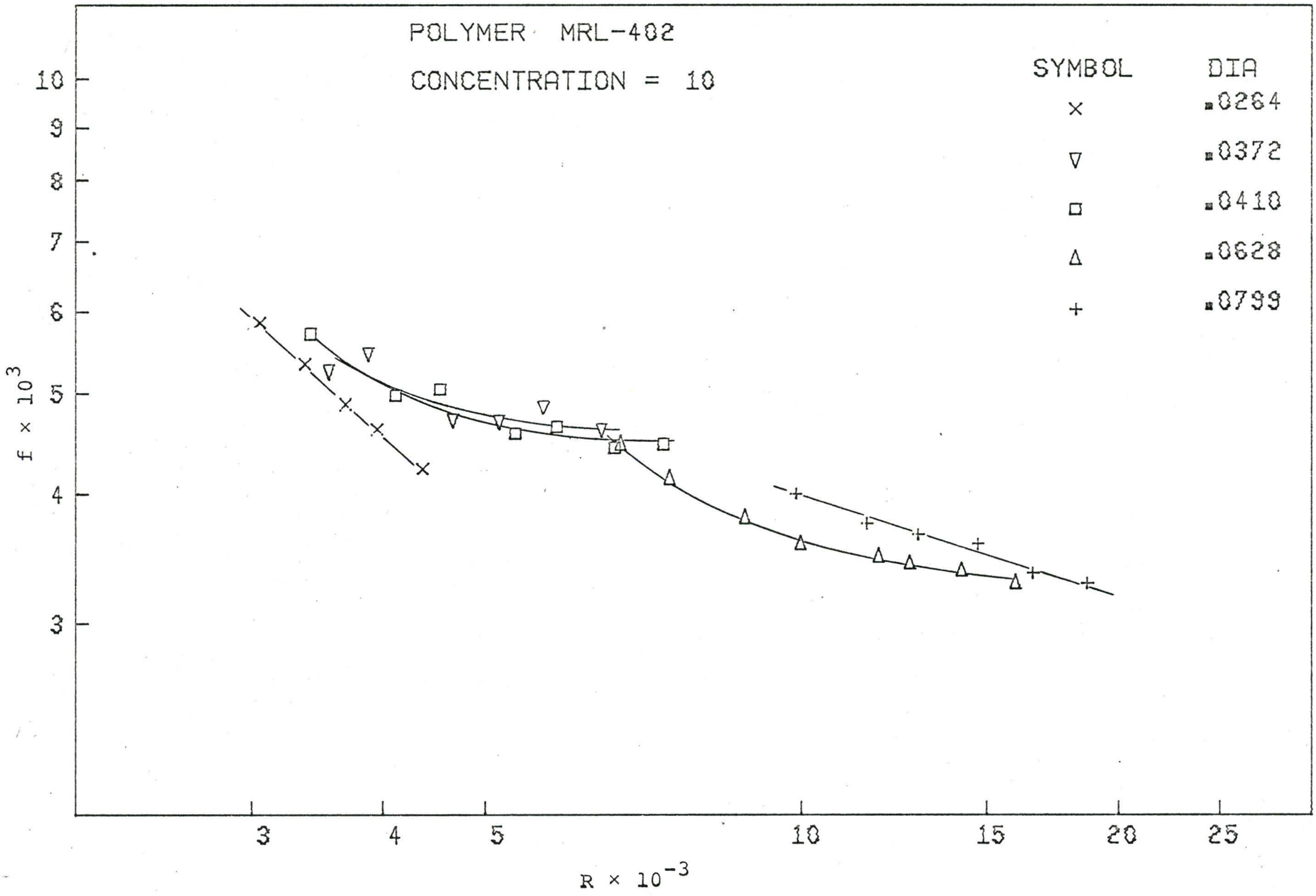


Fig. 24 REYNOLD NUMBER VS. FRICTION FACTOR

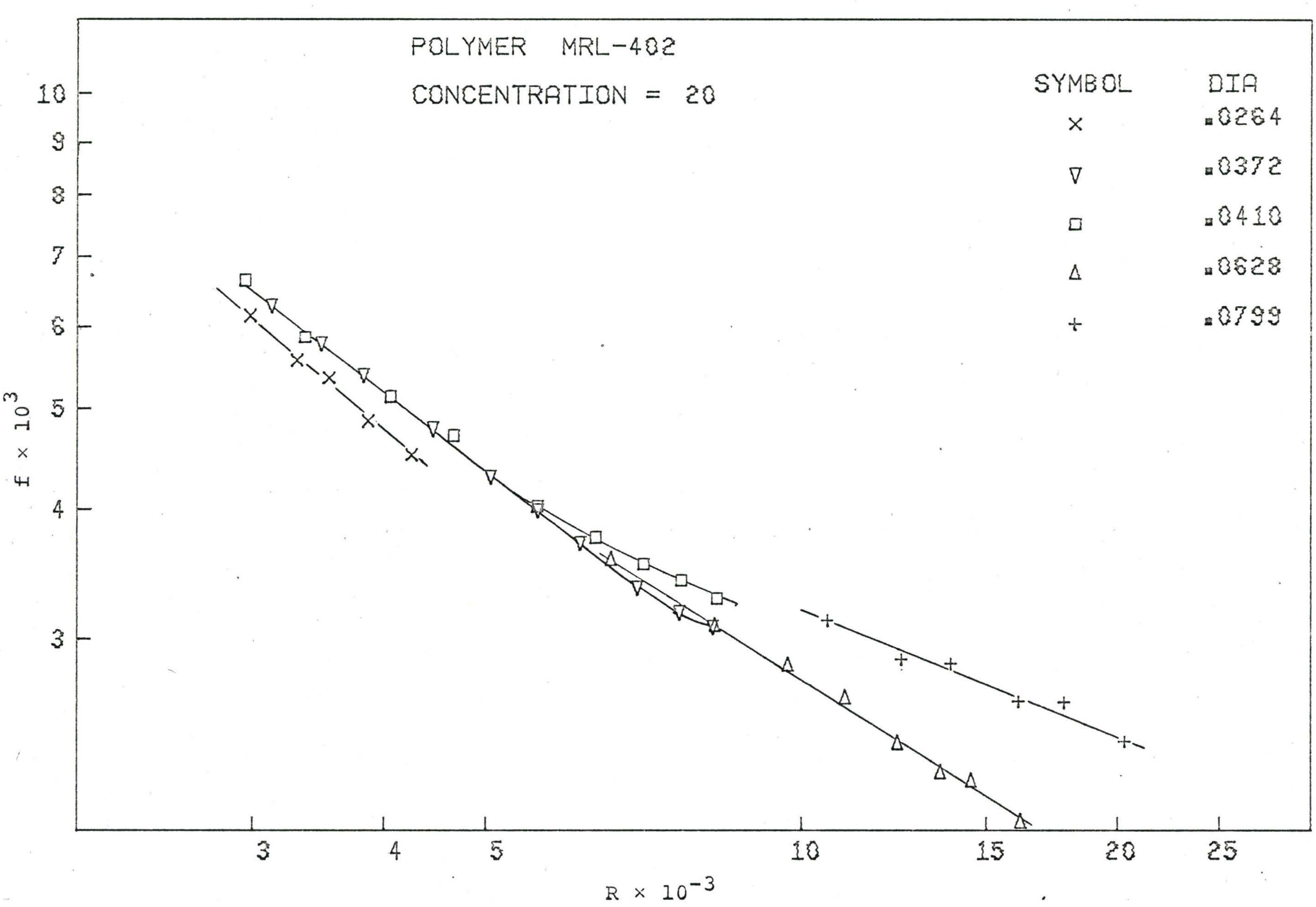


Fig. 25 REYNOLD NUMBER VS. FRICTION FACTOR

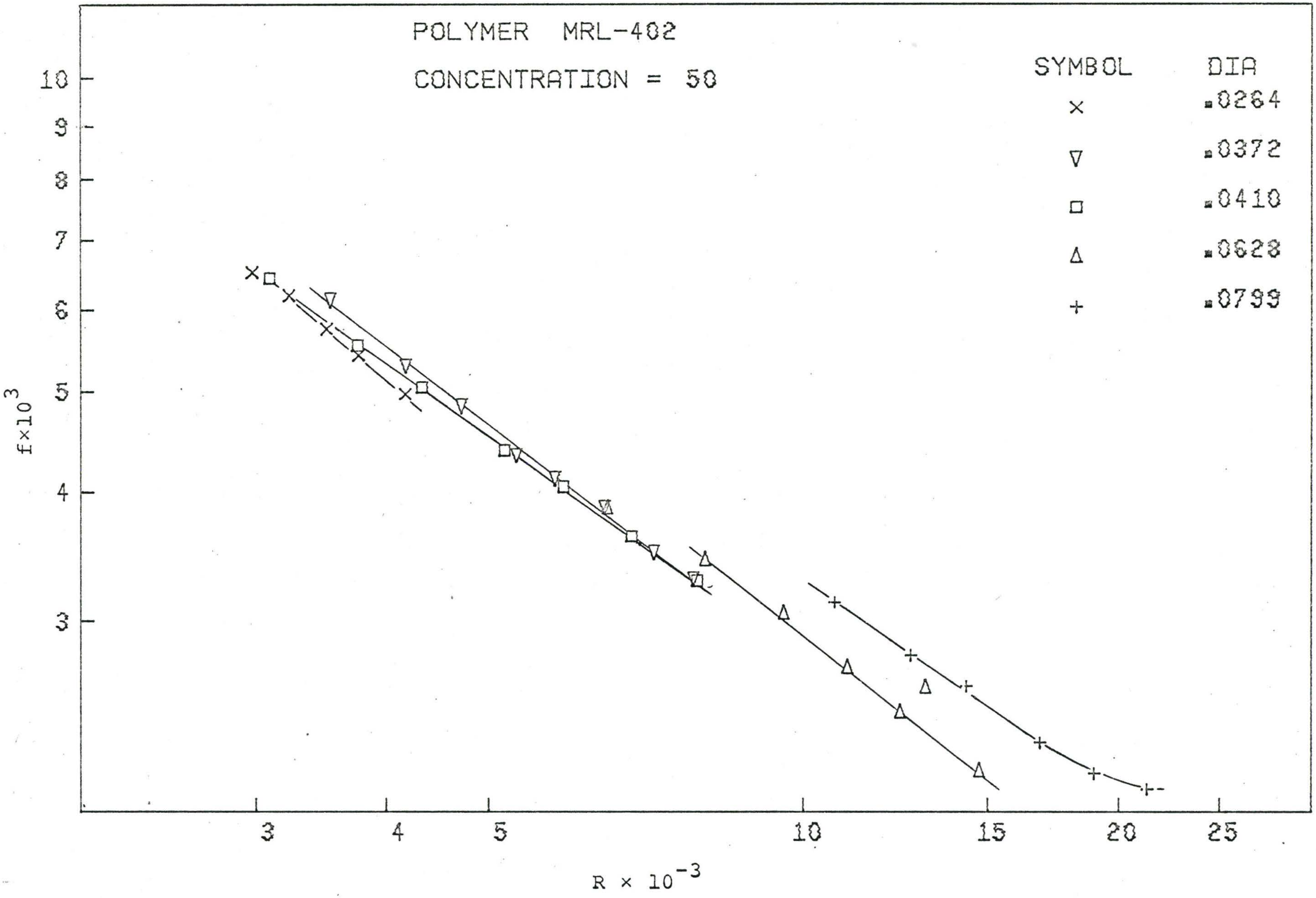


Fig. 26 REYNOLD NUMBER VS. FRICTION FACTOR

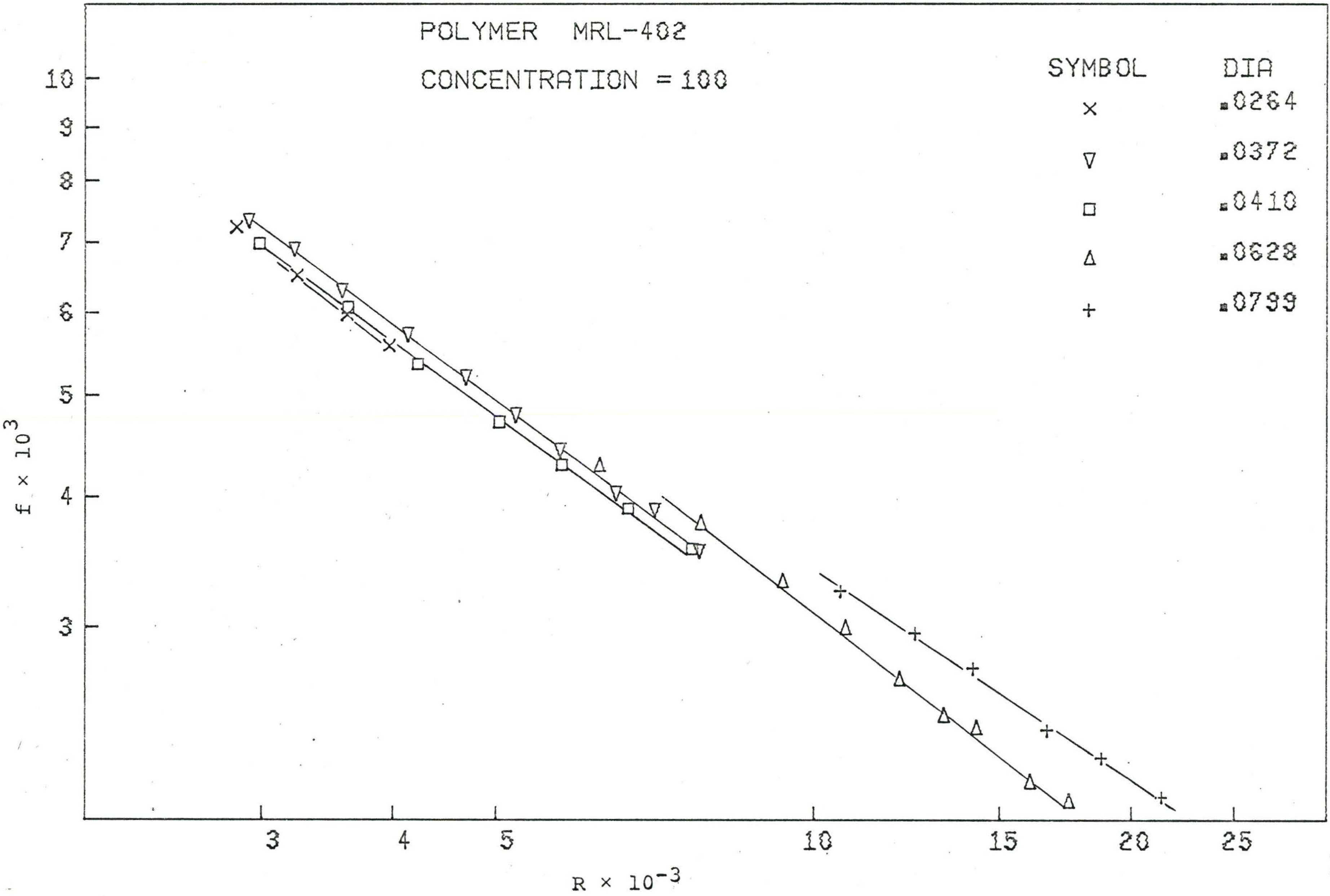


Fig. 27 REYNOLD NUMBER VS. FRICTION FACTOR

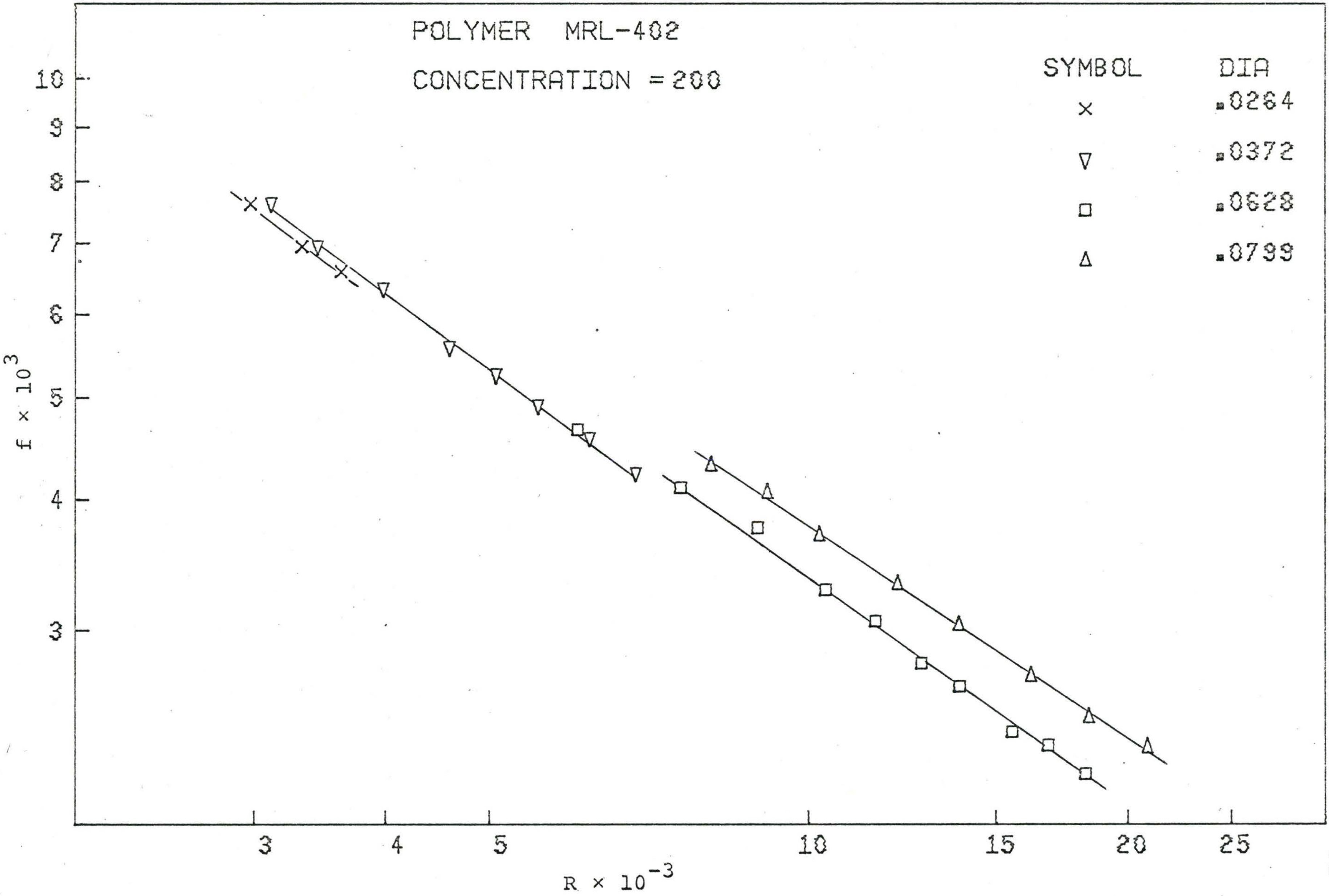


Fig. 28 REYNOLD NUMBER VS. FRICTION FACTOR

APPENDIX I

CALIBRATION OF PRESSURE GAUGE

The pressure gauge was calibrated with the aid of a precision dial manometer (Wallace and Tiernan, Type FA-145).

The observations recorded are given below.

OBSERVATIONS

S. No.	Pressure Gauge Reading (psi)	Dial Manometer Reading	
		Inches of Mercury	psi
1	2	3.4	1.67
2	4	7.45	3.75
3	6	11.55	5.67
4	8		7.66
5	10	19.75	9.73
6	12	24.0	11.79
7	14	28.1	13.80
8	16	32.2	15.82
9	18	36.45	17.91
10	20	40.55	19.92

A calibration curve drawn with the aid of the above data is shown in Fig. (17).

APPENDIX II

MOISTURE CONTENT IN MRL-402

A small quantity of polymer was weighed in a crucible and heated in a refractory furnace. The temperature of the furnace was maintained at approximately 110°C. The polymer was weighed again and again at certain intervals of time till a constant reading was obtained. A sample observation is recorded below.

1. Initial Weight of polymer = 4.69 gms
2. Weight of polymer after heating at 110°C for
    - a) 30 minutes = 4.60 gms
    - b) 1 hour = 4.55 gms
    - c) 90 minutes = 4.53 gms
    - d) 2 hours = 4.50 gms
    - e) 3 hours = 4.50 gms

Percentage moisture in the sample = $\left(\frac{4.69-4.50}{4.69}\right) \times 100$

= 4.51

It was suspected that the properties of the polymer no longer remained the same after heating at high temperatures. Thus a fresh sample was used to prepare solutions for the experiment.

APPENDIX III

DERIVATION OF EQUATION 4.2

Equation 4.2 relates the friction factor (f) to the pressure different (Δp) across the tube for a flow of an incompressible fluid in the tube.

Applying Bernouli's equation at the top level of the fluid in the reservoir and to the bottom of the tube (Fig. III)

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + H_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + H_2 + \frac{kV_2^2}{2g} + \frac{2fV_2^2L}{Dg} \quad (33) \qquad \text{III.1}$$

where subscript 1 is used for top section and 2 for the bottom section and

$\frac{kV_2^2}{2g}$ is the head loss at the inlet of the tube

f is the friction factor defined as $\frac{\tau_o}{\frac{1}{2}\rho V_2^2}$

$\frac{2fV_2^2L}{Dg}$ is the head loss due to skin friction

H_2 is zero and V_1 is assumed to be negligible.

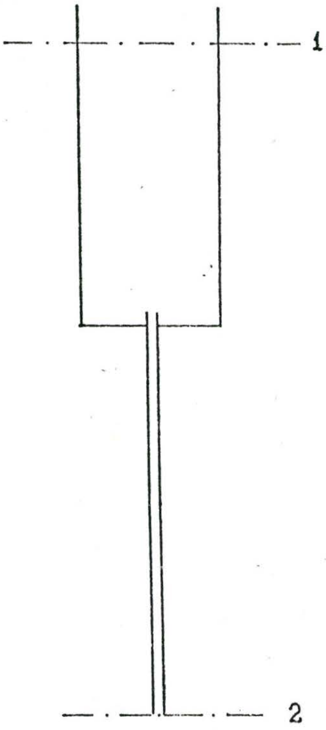


Fig. III

The velocity in the tube or V_2 was calculated from the mass flow knowing the size of the tube and the time of flow.

Let $V_2 = V$

and

$$\Delta p = (p_1 + \rho H_1 - p_2) .$$

Equation III.1 can be rewritten as

$$\Delta p = (1 + k) \frac{\rho V^2}{2g} + \frac{2fV^2L}{Dg}$$

or

$$f = [\Delta p - (1 + k) \frac{\rho V^2}{2g}] \frac{Dg}{2\rho LV^2} \qquad \text{III.2}$$

The value of k or the factor for inlet losses was taken as 0.78 (34)

Δp for the experiment was evaluated from the following equation

$$\Delta p = (p_g + p_{atm} + p_e) + \rho H_1 - p_{atm}$$

$$= p_g + p_e + \rho H_1$$

where p_g - gauge pressure

p_{atm} - atmospheric pressure

p_e - correction for gauge pressure

H_1 - height of the fluid above the exit end of the tube.

APPENDIX IV

ESTIMATION OF DEVELOPMENT LENGTH

An expression given in Hinze (10) was used to estimate the development lengths in the various tubes. The expression is

$$\frac{\ell}{D} = 0.693\ R^{1/4}$$

where ℓ is the development length. The following table gives the calculated results.

Reynold's Number = 15,000
Length of all Tubes = 15 inches

Tube No.	Tube Diameter D (inches)	Development Length ℓ (inches)
1	.0264	0.20
2	.0372	0.28
3	.0410	0.31
4	.0628	0.47
5	.0779	0.59

The above are the values for maximum development lengths in the different tubes. Normally, in the experiments, they would be shorter than these values.

APPENDIX V

DETERMINATION OF TUBE DIAMETERS

The tube diameters were determined by filling the whole lengths of tubes with mercury with the aid of a hypodermic needle. The mercury was taken out and weighed on a precision balance. The length of the tube was measured with a microscope.

$$D = \left(\frac{4W}{\pi \ell \rho}\right)^{\frac{1}{2}}$$

where W was the weight of mercury, ℓ the length of the tube and ρ the density of mercury.

The observations are shown in Table V.

TABLE V

Obs. No.	Weight of Mercury (gms)	Length of Tube (cm)	Tube Diameter (inches)	Mean Diameter (inches)
1	1.8225	38.085	.02640	.0264
2	1.8350	38.085	.02649	
3	1.8200	38.085	.02639	
4	1.8145	38.085	.02634	
1	3.6210	37.983	.03727	.0372
2	3.5995	37.983	.03716	
3	3.6100	37.983	.03720	

Obs. No.	Weight of Mercury (gms)	Length of Tube (cm)	Tube Diameter (inches)	Mean Diameter (inches)
1	4.3250	37.365	.04107	.0410
2	4.3100	37.365	.04100	
3	4.2950	37.365	.04093	
4	4.3125	37.365	.04100	
1	10.3150	38.080	.06282	.0628
2	10.2855	38.080	.06274	
3	10.3305	38.080	.06287	
4	10.3010	38.080	.06278	
1	16.7055	38.070	.07995	.0799
2	16.6810	38.070	.07990	
3	16.6525	38.070	.07983	