MORPHOMETRIC PROPERTIES OF DRAINAGE BASINS

MORPHOMETRIC PROPERTIES OF SELECTED DRAINAGE BASINS

IN

SOUTHERN ONTARIO

By

EIRLYS HEDDWEN VANDERHOFF, B.Sc.

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Arts

McMaster University

October 1966

MASTER OF ARTS (1966)
(Geography)

MCMASTER UNIVERSITY
Hamilton, Ontario.

TITLE: Morphometric Properties of Selected Drainage Basins in Southern Ontario

AUTHOR: Eirlys Heddwen Vanderhoff, B.Sc. (University of Wales)

SUPERVISOR: Dr. D. C. Ford

NUMBER OF PAGES:

SCOPE AND CONTENTS: Eighteen small stream basins were chosen in Southern Ontario. With the aim of a quantification of the characteristics of these drainage basins, numerous basin parameters were measured from topographic maps. The degree of application of the 'Horton Laws of Stream Composition' to the stream segment numbers, lengths, and drainage areas of these third order networks was critically examined. Further, values of the morphometric properties measured in each basin were also tested for their variation with lithology and with map scale. Finally, an attempt was made to evaluate the inter-relationships among the parameters which were used to characterise these drainage basins.

ACKNOWLEDGEMENTS

Sincere thanks are most particularily due to Dr. D. C. Ford, who guided this study from its inception. Without his continued moral support and helpful criticism little would have been achieved. I also wish to express my gratitude to all other members of the Department of Geography at McMaster who assisted me in any way.

Finally, I am extremely grateful to Miss Joan Winegarden for the time and effort she expended on the typing of this thesis.

TABLE OF CONTENTS

Page

LIST OF MAPS AND DIAGRAMS

FIGURE

FIGURE

LIST OF TABLES

CHAPTER I

INTRODUCTION

This thesis presents a study in quantitative drainage basin morphology, based on the principles of what is colloquially known as "Horton Analysis"; certain morphometric properties of 18 drainage basins in the Niagara Peninsula and adjacent areas of South West Ontario were measured and analysed. The method of investigation, originally devised by R. E. Horton (1945), has been applied to numerous drainage basins in well scattered and highly contrasting areas of the United States, and to some extent has been modified in this application. The writer, however, has noted only one paper, (Roberts, 1963), which has attempted a quantitative examination of the morphology of Canadian streams developed on glacial materials. Thus the selection and study of stream basins in glaciated Southern Ontario seems both appropriate and a valuable contribution to the wider application of quantitative morphological analysis. The importance of this quantitative approach can be illustrated by the fact that, as a result of the tabulation of the values of parameters which must be incorporated in such a study, direct, accurate and statistically satisfactory comparisons will be permitted between this study of Canadian stream basins and other such studies carried out in different areas.

Using the National Topographic Series of maps of Canada, published by the Department of Mines and Technical Survey on a scale of 1:25,000 where these are available, and similar maps on a scale of 1:50,000 in all other areas, linear and areal measurements of planimetric aspects of the chosen basins were measured directly from the maps. Other parameters expressing the characteristics of the basins were derived from the measured quantities. Certain statistical tests were next employed to estimate the significance of these parameters and their degree of variation, and many of the inter-relationships between parameters were examined and tested statistically. For a number of the basins a check and extension of the map measurements was made by means of an analysis of aerial photographs on a scale larger than that of the maps, (usually a scale of approximately 1:15,000).

This study was conceived as a test of the degree of application of the laws of drainage basin arrangement discovered and first tested by Horton (1945), and further tested by his successors in the United States, to the selected streams in Canada. The streams are developed in areas which have only recently emerged from beneath an ice-sheet of considerable magnitude; control of drainage by the highly variable glacial materials cannot possibly have been eliminated in the 9,000 to 10,000 years (Hough, 1958) that have elapsed since the final disappearance of glacial ice from Southern Ontario. Examples of divergence from the "Horton Laws of Drainage Composition" were examined as closely as possible and alternative explanations are offered for the observed discrepancies. Finally the data and results have been tabulated (Appendix A and B) in a form which would permit a comparison between parameters in Canadian drainage basins and typical values of these parameters as they are quoted by various workers in this field for different areas in the United States.

Review of Literature

Since 1945, the purely qualitative and descriptive approach to the study of land-form development which characterized the work of geomorphologists both in Europe and North America has been superceded by a system which "has as it's aim the expression of quantitative laws relating process to form." (Strahler, 1950a, p. 210). Neither of the two early leaders of geomorphic thought, W. M. Davis and Walter Penck, gave any regard to the systematic measurement of land-form elements, nor to the establishment of such quantitative laws as Strahler mentions. Both of these workers, their contemporaries, and their disciples merely formulated theories on land form type and development by a process of deductive reasoning. However, in 1945, R. E. Horton published a treatise which, in his own words, described "two sets of tools which permit an attack on the problems of the development of land forms, particularly drainage basins and their stream nets, along quantitative lines", (Horton, 1945, p. 281).

Horton proposed three empirical laws describing the regularity of the numbers, lengths and slopes of streams within any basin. These laws, termed respectively the Law of Stream Numbers, the Law of Stream Slopes, and the Law of Stream Lengths, indicate that the numbers, lengths and slopes of stream channels change in geometric progression with the change from one order of stream to another within the drainage basin. The scheme of ordering is an attempt "to classify stream systems on the basis of branching or bifurcation" (Horton, 1945, p. 281), and Horton proposed a scheme under which the smallest tributary is of first order, and the trunk stream is of the highest order in the basin. Horton found that stream numbers and average channel slopes tend to form an inverse geometric series with order, whilst the average lengths of streams tend to form a direct geometric series with order. He also postulated that the geometric patterns of stream systems are developed as a result of the action of two forces, which he termed micro-piracy and cross-grading, and he offered a 'hydro-physical' basis for his laws. According to A. N. Stahler, however, "this paper is now emerging as a document of great importance, not as much for the validity of it's conclusions on drainage and slopes, as for the forceful manner in which it has brought to the attention of geomorphologists the application of quantitative and dynamic methods to land-form study", (Strahler, 1950a, p. 211).

The three "Laws of Drainage Composition" which Horton published in 1945 became the impetus for and the basis of an ever increasing tendency towards quantification in geomorphology, and most particularily in fluvial geomorphology. The challenge he issued when he stated ".... the time has now come when such quantitative interpretation can be undertaken", (Horton, 1945, p. 280) was accepted and variously interpreted by numerous other geomorphologists. As a result, three overlapping and frequently mutually interdependent lines of study have emerged in the literature.

In the first place, considerable effort has been expended on the extension and testing of these Horton Laws in areas far removed from Horton's own proving grounds in the Appalachians. Much of the early work along these lines came from geomorphologists working with A. N. Strahler at Columbia University under a contract from the United States Office of Naval Research Geography Branch which specified the "study of the basic principles of erosional topography", (Strahler, 1957b, p. 913). Miller (1953), for example, studied the morphological characteristics of drainage basins in mountain areas of Virginia and Tennessee, and discovered that Horton's Drainage Density is there dependent on the resistivity of the ground surface to sheet erosion, on the intensity of run-off, and on ground slope. The resistivity of the surface, according to Miller, depends on the nature of the bed-rock only if there is no vegetation or soil cover; where a soil cover is present, the resistivity is a function of the soil grain size and structure, and there is a 'superimposed' resistivity dependent on the density and type of plants.

Coates (1958) presented a statistical study of the morphometric properties of drainage basins in Southern Indiana and he analysed the relationships between some of the form factors such as drainage area and stream lengths, and causative factors such as the length of overland flow, lithology, and the percentage of unconsumed upland. He found that the length of overland flow is closely related to the length of first order streams, and there is a low degree of correlation between the length of overland flow and the percentage of unconsumed upland. Coates also found a significant linear relationship between the length of the basin perimeter and the square root of the area of third order basins. Melton (1957) moved further afield, publishing a report on the correlations of various geomorphological and climatological characteristics of numerous drainage basins in the Western states of the United States of America. This Ph.D. dissertation includes considerable data on the stream networks of New Mexico, Arizona, Colorado, and Utah; Melton published, for example, evidence of statistically significant correlations between channel frequency and the infiltration capacity, the precipitation-evaporation index, the percentage of bare area, and the wet strength of the soil. Similarily, he found that valley-side slope angles are correlated with infiltration, ruggedness, and relative relief.

In 1962, Morisawa returned to Horton's home territory and studied 15 drainage basins within various physiographic provinces of the Appalachians; she found that the horizontal or planimetric aspects of drainage basins show the expected exponential relationship with order, but that control by structure or lithology governs the vertical or gradient properties much more closely. A multiple regression of peak intensity of run-off on basin area, rainfall intensity and frequency, and topography was shown to be significant at the 0.001 level for these basins. Gray (1961), on the other hand, demonstrated various regressions between stream length and basin area for basins of an intermediate size range, as well as regional patterns of regression between the length of the main stream and the channel slope for that basin; this he attributed to the predominant effect of surface resistivity.

Each of these workers provided a wealth of data on the geometric characteristics of drainage basins, and most of them also began to seek interrelationships and patterns of cause and effect in an analysis of their information. As a result of much of this information, Melton (1958c) was able to publish a summary of the data already available which "provides estimates of variance, means, and coefficients of variation for populations that have been studied in the recent past, and will be of continued interest", (Melton, 1958c, p. 5).

Parallel to this initial accumulation of data, and the search for relationships in planimetric morphology, numerous researchers began to examine more and more of the landscape of drainage basins and to quantify them. Efforts to inter-relate measured variables became more sophisticated, and equally advanced techniques for characterising and representing morphological variables were presented. Perhaps one of the more important aspects of this type of work was Schumm's proposition of a fourth Law of Drainage Composition relating drainage area to order, (Schumm, 1956). Schumm found that drainage areas obey a direct geometric series law, similar to the laws of stream numbers and stream lengths, and also that there are limiting areal ranges of drainage basin size for each order of stream he studied in badlands at Perth Amboy, New Jersey.

Increasing emphasis was also laid on the vertical aspects of drainage basin morphology. Strahler's hypsometric analysis, for example, (Strahler, 1952b) has since frequently been employed to exemplify the area-altitude relationships of water-sheds. Schumm (1956) found that the geomorphic development of his badland stream basins is illustrated by Strahler's hypsometric integral. When more than 25% of the mass above the basal plane in second order basins has been removed, he demonstrated that the basin parameters remain essentially constant, (Schumm, 1956). Further A. N. Strahler (1954b) demonstrated that since gravitational acceleration varies with the sine of the slope angle, an isosinal map of the drainage basin will illustrate the distribution of the downslope components of gravity. A percentage frequency distribution histogram showing "the distribution of area in the drainage basin with respect to the sine of surface slope" indicated that "this distribution appears to be symmetrical for mature topography with smooth straight slopes in homogeneous materials", (Strahler, 1954b, p. 352).

Numerous standards for the estimation of basin shape have also been proposed, from Miller's Circularity Ratio, (Miller, 1953), and Schumm's Elongation Ratio, (Schumm, 1956), to the lemniscate equation devised by Chorley, Malm, and Pogorzelski, (1957). Schumm (1956) suggested that a correlation exists between his Relief Ratio and measures of basin circularity such that the basin becomes more elongate as the Relief Ratio increases; he also found a relationship between this Relief Ratio and valley side slope angles. Lubowe (1964) on the other hand, published evidence that local relief correlates to a high degree with stream junction angles, and she further demonstrated that these junction angles are related to the order of the receiving stream.

Strahler and Schumm may be credited with devising two new frameworks for quantitative landscape analysis. Strahler (1954a) proposed that the morphometric properties of drainage basins and many of the hydraulic characteristics of stream channels might be studied by means of the concepts of dimensional analysis. Stream order is, for example, a dimensionless number, stream lengths are characterized by the single dimension of length, and basin area by the dimension of L^2, (Strahler, 1954a). He stated that all geometric properties are expressed in the dimension of length, or by dimensionless ratios of length; kinematic properties such as velocity and acceleration can be reduced to the two dimensional plane of Time and Length, whereas mechanical characteristics which involve an element of mass are expressed in the three dimensions of Mass, Length, and Time.

Melton (1958b) on the other hand, suggested that only four parameters are essential for the complete determination of all other parameters. Using these particular parameters, the total length of stream segments, the total area of the basin, basin relief, and the length of the basin perimeter, he simulated a form of dimensional analysis. Under this scheme, each of these four properties represents one dimension. All other measured variables can, according to Melton, be expressed in one or more of these 'dimensions' depending on how many of the basic parameters are necessary to give a complete expression of that variable. This system Melton termed expression in "E_4 phase space", but he added the provision that for other than mature drainage networks a fifth 'dimension' to represent the degree of maturity might be necessary.

This review of the literature of fluvial geomorphology would be incomplete without a mention of studies of other features of drainage basins which have not yet been specifically related to the morphological variables embraced by the present study. However, it is to be hoped that in the very near future the ultimate relationships between climatic factors, drainage basin and channel morphology, and hydraulic characteristics may be discovered, and the entire drainage basin expressed in terms of a small number of equations. Towards this end, numerous geologists, especially those working under the auspices of the United States Geological Survey, have sought to extend the essentially geometric studies promoted by Horton.

The work of Leopold and Maddock (1953), Leopold and Miller (1956), and Wolman and Brush (1961) may be mentioned as examples of the search to relate the hydraulic and geometric characteristics of drainage basins both from field and laboratory experiments. Leopold and Maddock (1953) offered a concentrated attempt to link details of channel morphology to hydraulic factors; they found, for example, a logarithmic relationship between channel width, load, and velocity, and rates of discharge. Leopold and Miller (1956) attempted to link even more closely factors such as discharge and velocity to features of the channel geometry such as the width and slope; these particular authors also produced evidence for an exponential relationship between discharge and order number, and between channel width and order number. Other relationships have been the subject of research in laboratory experiments, and as a result of this type of work, Wolman and Brush (1961) enumerated some of the factors controlling the size and shape of stream channels. They found both in experimental work and in natural examples that the cross-sectional area of a channel is very closely related to discharge as a logarithmic function. Similarily, there has been a noticeable tendency for geomorphologists to unravel the complexities of 'grade', 'equilibrium', and 'quasi-equilibrium' as Langbein and Leopold (1964), Mackin (1948), and Howard (1965) have done.

An interesting addition to the early work on drainage basin morphology serves as an example of recent research in this field. Leopold, Wolman, and Miller (1964) have simulated the form of drainage systems through the use of a computer. Using a technique known as Random Walk, these authors have produced a hypothetical drainage net which exhibits a close fit to the Horton Laws of Stream Numbers and Stream Lengths. This, and other considerations have led certain authors to speculate on the precise nature of these laws. It has been suggested that these laws are merely the expression of a statistical relationship which results from "the random development of drainage networks rather than from orderly evolution as generally assumed" (Shreve, 1963 p. 44). Bowden and Wallis (1964 p. 768) on the other hand, consider the Law of Stream Numbers "a result of the definition of stream order rather than being due to either orderly evolution or random development".

Further 'theoretical' considerations in the literature have centered on the correct use of statistics. Strahler (1954a) offered a blue-print for the use of the more common techniques of parametric statistics such as the t-test. Melton (1958b) suggested that since the channel ordering system is rarely considered an interval scale, non-parametric statistics such as the Spearman Rank Correlation Coefficient should be employed for an accurate interpretation of the significance of the results.

The final topic in this review of the relevant literature must be an emphasis of the paucity of Canadian examples of this type of work. Although Mackay of the University of British Columbia, working on the Mackenzie Delta, has studied some of the channel and hydraulic characteristics of this river, he has not published an analysis of the morphometric properties of this system; this may possibly be due to the complications afforded by the Mackenzie's distributary channels. Only one other paper dealing with the characteristics of Canadian stream basins is known to the present writer. In a very brief article, Roberts discussed his application of the Horton Laws to the Humber river system west of Toronto (Roberts, 1963). Although the Strahler scheme of ordering was employed, Roberts does not specify which of the modifications of the

Law of Stream Lengths he adopted to overcome the discrepancies which arise as a result of this system of segment ordering. Further, the actual points on his plots of average stream segment length against order are not very well expressed by the line of best fit he inserted; were these points to be joined together, they would in fact produce a curve which is markedly concave upwards.

CHAPTER II

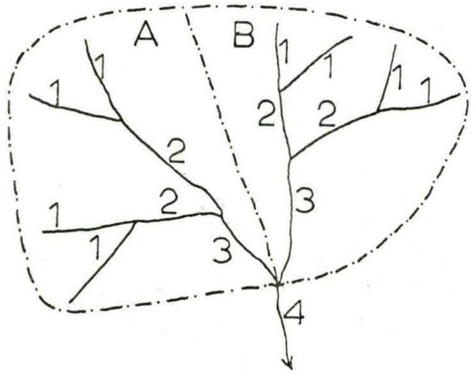
SELECTION OF THE BASINS

It was originally intended to restrict this study to basins of comparable size and relief in parts of the Niagara Peninsula and South West Ontario where map coverage on a scale of 1:25,000 is available. These maps are as yet, however, issued for only a very minor portion of this area, and as a result, the study was expanded to include similar basins in areas which are covered only by maps on a scale of 1:50,000. With the aim of a quantitative analysis of the morphometric characteristics of these drainage basins, certain criteria for selection were outlined before the basins were chosen. The purpose here was to hold constant as far as possible some of the more obvious variables which might cause significant constrasts within other geometric parameters of the chosen basins. It was felt that this technique might aid in the unravelling of the complex relationships and inter-relationships which are characteristic of all drainage basins, and which are not yet fully understood.

In the first place it was decided to restrict the areal size of these basins; in this way it was hoped that variations between basins in both causative and affected variables could be held to a minimum. Although it was recognized that the entire elimination of the areal variable was impossible, two successive procedures were introduced to ensure that the variation in this parameter was as low as possible; the first method had the added advantage that it's application enabled more reasonable inter-basin relationships to be recognized. Only basins which were of third order on the relevant 1:25,000 or 1:50,000 map sheets were selected. It was further stipulated that these basins should meet other basins of third or higher order, so that the chosen basin always included the entire length and drainage area of it's particular third order segement. The application of the Horton Laws could be faithfully tested only when such a condition was satisfied. An illustration of what was intended by this stipulation can be seen in figure 1.

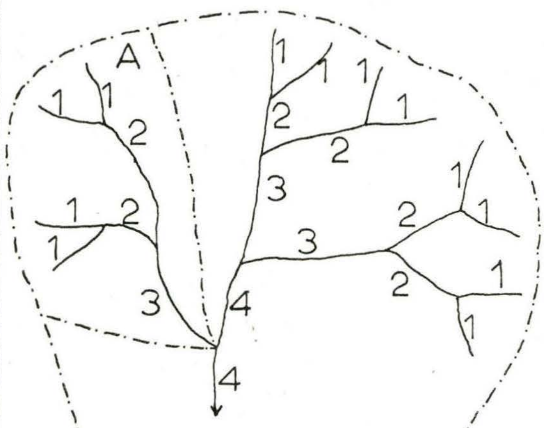
It was hoped that this condition alone would tend to limit sufficiently the areal size range of possible basins, but this faith proved unfounded. Third order basins varied from less than one square mile to more than 18 square miles, and great difficulty was experienced in finding a sufficiently large number of basins in which the areal size range was not too great. A large enough sample of basins was necessary that a statistical analysis would not only be possible, but would prove reasonably informative and reliable. As a final compromise, 18 basins were selected; they vary in total area from .92 square miles to 13.40 square miles. However, it is unlikely that a size range of this order of magnitude will have undesirable effects on the analysis, since, to quote Strahler, "....because order number is dimensionless, two drainage basins differing greatly in linear scale can be equated or compared with respect to corresponding points in their geometry", (Strahler, 1957b, p. 914). If two basins of the same order are in the scale ratio λ:1, and are developed on identical, homogeneous surfaces, their stream lengths will be in the ratio of λ:1, while their drainage

FIG 1
DEFINITION OF THIRD ORDER BASINS



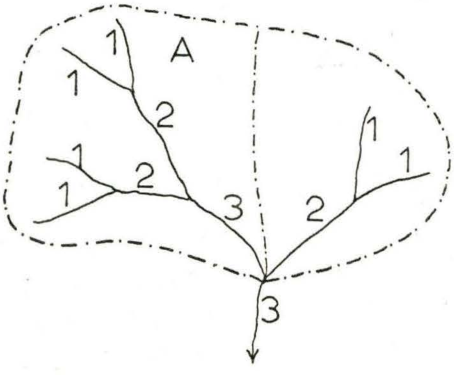
In basins A and B, the watershed delimits the entire third order basin area.

1. TWO THIRD ORDER SEGMENTS MEETING



In basin A, the watershed delimits the entire third order basin area.

2. THIRD ORDER SEGMENT MEETING A SEGMENT OF HIGHER ORDER



In basin A, the watershed does not delimit the entire third order basin area. The third order basin will not be complete until the stream meets another of third or higher order.

3. THIRD ORDER SEGMENT MEETING A SEGMENT OF LOWER ORDER

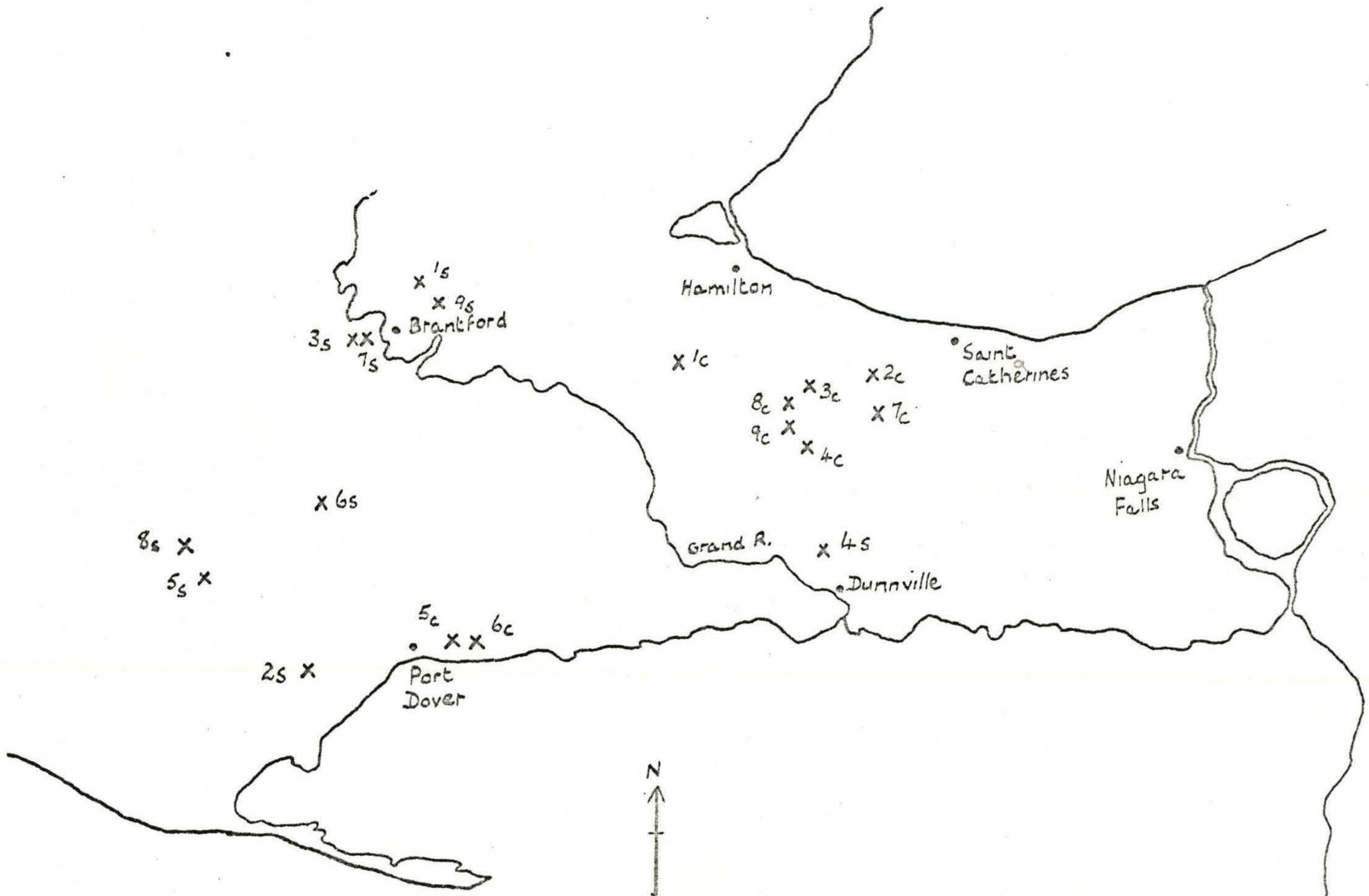
areas will satisfy the ratio λ^2:1, (Strahler, 1957b). It is this stated condition of identical homogeneity which promoted the idea that a limitation should be imposed on basin size variation as the condition is by no means satisfied by the chosen basins.

It was also decided to hold the relief of the basins as constant as possible. In this way, variation in the Relief Ratio could be limited. This Relief Ratio has been proved to have a high degree of correlation with both stream gradient and drainage density; Schumm (1956) for example, found that the drainage density is a power function of the Relief Ratio for streams developed on badlands at Perth Amboy, New Jersey. With the exception of the Escarpment zone, which was ignored when basins were being selected, the altitudinal range of most of this area of south west Ontario is relatively low. It did not prove difficult to limit the Local Relief of the basins to a minimum of 27 feet and a maximum of 165 feet, a range of only 138 feet. This factor was nevertheless still tested statistically for it's correlation with other measured parameters, as it was not found that the attempt to hold basin relief constant had been entirely sucessful. On the other hand, extreme values of relief, and therefore presumably effects of the parameter, had been avoided.

This study was also envisaged as a demonstration of the variation of morphometric characteristics within a limited range of geologic environments. To quote Horton once more, "one may naturally ask whether stream systems in similar terrain and which are genetically similar should not have identical or nearly identical stream composition", (Horton, 1945, p. 302). A further constraint was imposed on the basins to accomplish this limitation of geologic control. Nine of the basins were selected in areas which Chapman and Putnam termed 'sand plain', and the other nine were chosen in the areas which these authors called 'clay plain', (Chapman and Putnam, 1951).

These particular 'physiographic regions' were chosen because they would provide a fairly extreme range of lithology and soil development without too great a variation of topographic expression. Within these regions, features of glacial erosion and deposition which might cause specific dislocation of the drainage pattern were avoided; basins were not chosen where maps, aerial photographs, or Chapman and Putnam's text (1951) indicated that such controlling features as drumlins, moraines, eskers, kames, or glacial lake shorelines tended to disrupt drainage or had any obvious topographic expression. This precaution was taken despite Horton's own experience that "the laws of stream numbers and stream lengths" hold good "even with such pronounced geologic control of topography as that afforded by the drumlin areas in the Ganargua Creek drainage basin", (Horton, 1945 p. 300). Horton's statement was noticeably based on the evidence of only four Fourth Order stream basins, and Schumm (1956) on the other hand, favours a wary approach to relatively large altitudinal variation within a short distance such as many of these glacial features may produce. In his discussion of the calculation of the Relief Ratio, Schumm suggests that topographic residuals or abnormally high points should be ignored. In the selected areas, the topography undulates rather uniformly, with relatively gentle slopes.

FIG 2
LOCATION OF BASINS



It was unfortunately discovered that a coverage of maps on a scale of 1:25,000 was only available for parts of the 'clay plain' areas, and no suitable maps on this scale have yet been published for the 'sand plain' areas of the study region. As a result later sections of the analysis break down unexpectedly into three groups, instead of the anticipated division into two. All of the so-called sand basins were studied from maps on the 1:50,000 scale, but six of the clay basins were studied on a scale of 1:25,000 and the remaining three clay basins are in areas covered only by maps on the 1:50,000 scale. Differences in the values of the morphometric parameters of each of these three groups were tested statistically, and the results of this testing are discussed in a later section.

Statistical Tests

This dissertation has been deliberately divided into two sections. It was not considered sufficient to collect and present raw data to illustrate the morphometric properties of the selected basins, and an attempt was made to analyse the variations of this data within and between certain categories. When the selection of the basins was discussed, it was mentioned that two possible bases for subdivision immediately presented themselves: half of the basins were chosen in areas which Chapman and Putnam termed 'clay plain', and, of these, six basins were mapped on a scale of 1:25,000. The remaining three clay basins and nine sand basins were all on a scale of 1:50,000. Thus it was possible to divide the basins both on the basis of the underlying lithology, and according to map scale.

One of the first considerations was whether parametric or non-parametric statistical tests should be employed in the analysis. To justify the use of parametric tests, two assumptions must be made; the correct application of these tests requires that the data has been measured on an interval scale, and also that the population from which the samples have been chosen is normally distributed about the population mean value. All of the properties which were tested in the analysis were measured on an interval scale, and the data is recorded in feet, miles or in square miles; the only exceptions to this are the various Horton ratios, and these have generally been omitted from the statistical analysis. With such small samples, however, it was on the whole impossible to test for normality of the population from which the samples were drawn. To overcome this problem, normality was assumed in all cases, and this stated assumption underlies the use of all the statistical tests which are recorded below. The first step in the statistical analysis was the calculation of the mean, mean square, variance and standard deviation of the morphometric properties of all basins in each of the four overlapping groups. For the calculation of the mean and mean square, the following formulae were utilized.

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \qquad (\bar{x})^2 = \frac{\sum_{i=1}^{N} x_i^{\;2}}{N}$$

where $\bar{x}$ = mean, x_i = ith variate, and N = number of variates in the sample, (Dixon and Massey, 1957).

The variance and standard deviation of a sample is not a true estimate of the value of these statistical parameters for the population from which the sample is drawn. Moroney states that ".......while the mean value of a sample of n items is an unbiased estimate of the mean value in the population from which the sample is drawn, the standard deviation is biased, tending to underestimate the population value. This bias is especially marked in small samples", (Moroney, 1951, p. 225). The variance of a sample is given by the formula:

$$s^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N}$$

and the standard deviation s is the square root of this variance, (Moroney, 1951). To obtain a 'best estimate' of the value of these parameters for the entire population, a correction known as Bessel's Correction is applied to the formula to correct the underestimation which occurs when only a small sample of the population is available: Thus

$$\hat{\sigma}^2 = \left(\frac{n}{n-1} \right). s^2$$

where $\hat{\sigma}^2$ = best estimate of the population variance, s^2 = sample variance, n/n-1 = Bessel's correction, (Moroney, 1951). The resultant formula used in calculation (Dixon and Massey, 1957) is:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

The second stage in this statistical analysis was a test of the difference between the means of selected properties in the populations represented by the four sample groupings. This was accomplished by the use of Student's "t" test which is given by the formula (Gregory, 1963):

$$t = \frac{|\bar{x} - \bar{y}|}{\sqrt{\left(\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}\right)}}$$

For a discussion of the theory behind the use of this statistic, reference was made to Dixon and Massey (1957), and Blalock (1960). The degrees of freedom are given by the formula:

$$D.F. = n_x + n_y - 2$$

When the calculated value of t exceeded the value quoted in the t distribution tables (Dixon and Massey, 1957) for t with the given degrees of freedom at the 0.5 level of probability, the null hypothesis that the samples were drawn from the same population was rejected.

Where it was necessary to test for the differences among means of more than 2 samples from supposedly different populations, an Analysis of Variance test was conducted (Blalock, 1960). The between sample and within sample sums of squares were found from the formula:

$$\text{Total SS} = \sum_i \sum_j x_{ij}^2 - \frac{\left(\sum_i \sum_j x_{ij}\right)^2}{N}$$

$$\text{Between SS} = \sum_j \frac{\left(\sum_i x_{ij}\right)^2}{N_j} - \frac{\left(\sum_{ij} x_{ij}\right)^2}{N}$$

$$\text{Within SS} = \text{Total SS} - \text{Between SS}$$

where i = number of individuals, j = number of groups, N = total number of individuals in all groups.

From these values, the between groups and within groups estimates of variance were calculated, given that the between groups degrees of freedom D.F. = k - 1, where k = number of groups, and the within groups degrees of freedom = N - k, where N = total number of individuals.

The null hypothesis that the population means were equal was rejected when

$$F = \frac{\text{Greater Variance Estimate}}{\text{Lesser Variance Estimate}}$$

was greater than the tabulated value for F at the .05 level of probability, (Dixon and Massey, 1957).

The inter-relationships among morphometric characteristics of the drainage basins were tested for their degree of correlation as this correlation is estimated by the value of the coefficient r, the sample correlation coefficient, (Dixon and Massey, 1957):

$$b = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{N}}{\sum x_i^2 - \frac{(\sum x_i)^2}{N}}, \qquad b' = \frac{\sum x_i \sum y_i - \frac{\sum x_i \sum y_i}{N}}{\sum y_i^2 - \frac{(\sum y_i)^2}{N}}, \qquad r^2 = bb'$$

When the calculated value of r exceeded the value of r at the .025 level for the given number of individuals (N), it was assumed that the correlation between the 2 properties was statistically significant for the purposes of this analysis. In calculation, the following combinations were tested for significant correlations, where x and y represent the 2 variables,

x and y
x and log y
log x and log y
y and log x

Where 2 or more of the combinations resulted in values of r greater than $r_{.975}$, the highest r value was chosen as being indicative of the most significant correlation.

The application and results of these statistical tests as well as the conclusions drawn from them will be discussed in a later section.

CHAPTER III

TECHNIQUES, MEASUREMENTS, AND INSTRUMENTS

"The drainage area may be defined as the area which contributes water to a particular channel or set of channels. It is the 'source' area of the precipitation eventually provided to the stream channels by various paths", (Leopold, Wolman, and Miller, 1964, p. 131). On the basis of this definition, the drainage area of each of the basins under consideration in this dissertation was delimited on the relevant topographic map sheet as closely as the contour information permitted. On maps derived from sets of aerial photographs, the line of the watershed was more accurately determined through a stereoscopic analysis. All parameters which were investigated and analysed during the course of this study were thus measured within the limits of drainage basins according to the above definition.

The first step in the measurement and analysis which comprised this study was the completion of the channel system or drainage net. Detailed representation of the channel system is dependent on the scale of the map used, and on the accuracy of the cartography involved in the production of the maps. In general, it seems that the representation of smaller channels on a map sheet is a matter of pure chance, and there is no set standard for the inclusion or omission of small tributaries. For these reasons an attempt was made to add certain small channels which had apparently been omitted in the production of the official map sheets. Where two or more successive contours on the topographic sheet showed marked and acute inflections suggestive of a reasonably well defined valley, it was assumed that such a valley did in fact include a distinct stream course. This was added to the drainage net already marked on the map, and included in all further measurements and calculations. The line of this channel was drawn as far as the mid point between the highest sharply indented contour and the contour immediately above which gave no pronounced indication of the existence of such a valley. If the contour nearest the watershed was sharply inflected and still indicated the presence of a valley, the channel was extended to a point which expressed the average distance from the watershed at which other channels within the basin were initiated.

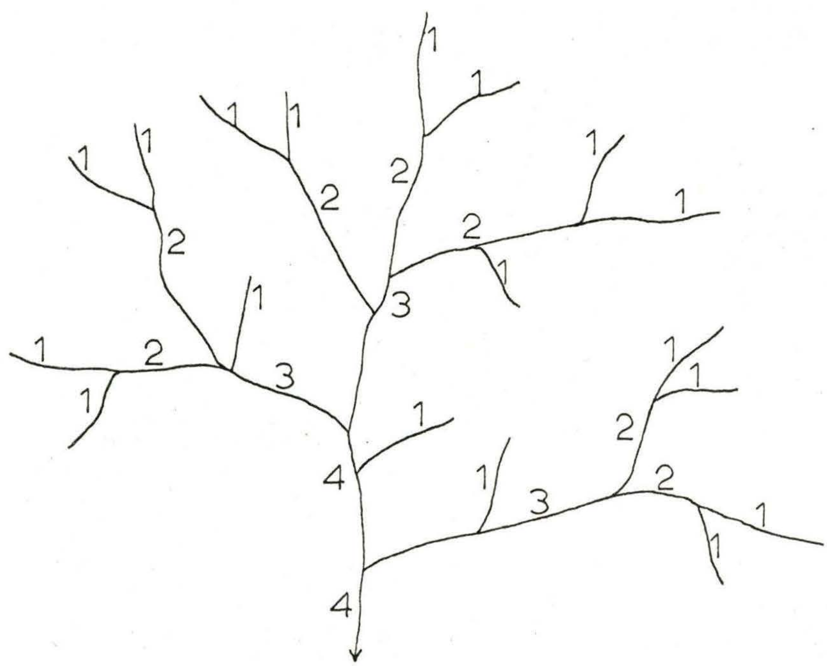
The inaccuracies of this method of completing the given channel system may well be criticized. However, some comparisons showed that this rough and ready method, when applied with care and forethought, does extend the channel system in a manner which correlates well with the stream net as it can be drawn from the much greater detail of aerial photographs. The air photograph checks were well scattered, and governed only by the bias of the ready availability of such photographs, so there is no reason to suggest that this is not the case in all basins.

When the channel net of each basin was completed, the entire system was indexed by an ordering system along the lines of that proposed by A. N. Strahler (1954b). This system of ordering differs somewhat from the method employed by Horton in his original paper (Horton, 1945), and the significance of this difference with respect to the "Horton Laws" will be discussed later. Using the Strahler system, all unbranched tributaries within the drainage basin are designated order 1; where two first order channels unite they form a second order stream segment which may receive further first order unbranched tributaries. At the confluence of two second order stream segments a third order channel is formed, and two third order channels unite to form a fourth order stream. The order of the basin is equivalent to the highest channel segment order in the basin, but unlike the Horton ordering system, the head-waters of the master or dominant stream of each order are not redesignated to the order of the main stream. Thus the highest order of stream in the basin does not extend to the limits of the major unbranched tributary at it's head as is the case with the Horton ordering system. The differences between the two systems of ordering are illustrated in figure 3, in which the same channel net is ordered according to both systems.

The geometric characteristics of the basins examined may be divided into three categories, namely those which illustrate the linear properties of the basins, those which demonstrate areal aspects and certain parameters which illustrate the relief or vertical characteristics. The choice of the parameters to be studied was made so that the geometry of the channel net and the watershed would be as fully characterized as possible by the value of the parameters, and so that the actual value of the parameters could be measured as accurately as possible from topographic maps or from maps made from aerial photographs. Further, the intention was to discover how closely these parameters were inter-related. Thus a constant scale unit was employed to minimize calculations and conversion of units. All planimetric measurements were made in inches and converted to miles to allow comparisons to be drawn between maps of different scales,

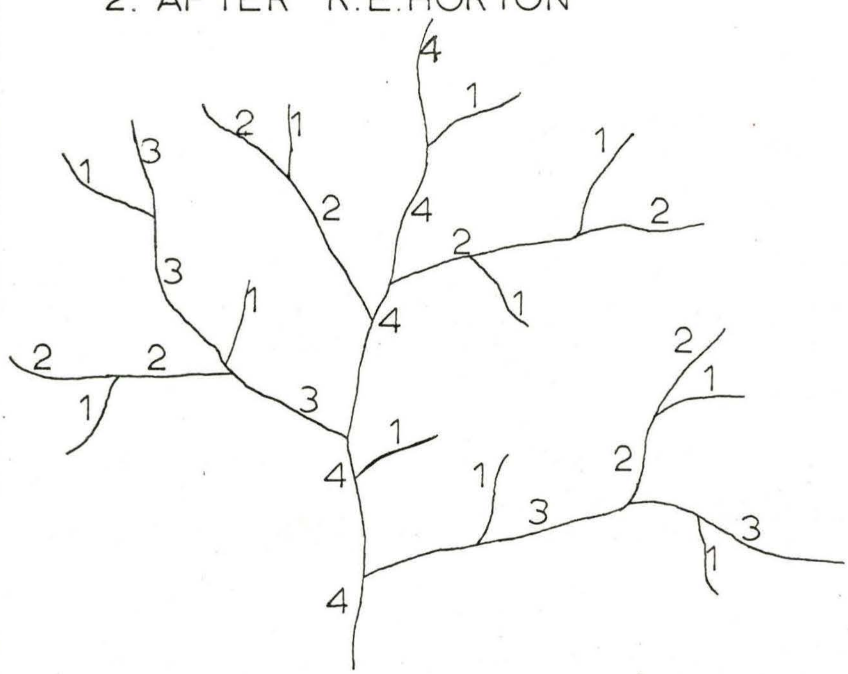
FIG 3 ORDERING SYSTEMS

1. AFTER A.N. STRAHLER



$n_1 = 18$
$n_2 = 7$
$n_3 = 3$
$n_4 = 1$

2. AFTER R.E. HORTON



$n_1 = 11$
$n_2 = 4$
$n_3 = 2$
$n_4 = 1$

and all vertical parameters were measured in feet.

Certain other parameters were obtained by mathematical combinations of measured properties; thus for example, in this study Drainage Density is defined as the sum of total channel length of all orders in miles and the reciprocal of the total area of the basin in square miles:

$$Dd = \frac{\Sigma L}{A}$$

Table I is a list of all the parameters included in this study, the symbol used to represent each parameter, and, for the derived properties, the mathematical derivation is given. Conventions followed in the table and throughout this dissertation include the use of subscripts to denote order, for example: n_1 = number of first order streams,

ℓ_2 = average length of second order streams:

Σ, (capital sigma), is used to indicate the sum of, or the total value of, any particular parameter, for example: $\Sigma\, a_2$ = total area drained by second order streams: Capital letters are used wherever the parameter quoted represents the total value for the entire basin, for example: N = total number of stream segments of all orders within the basin.

Linear measurements were in most cases obtained with an opisometer, a simple measuring wheel from which the linear distance traced by the wheel can be read in inches; these figures were then multiplied by a scale factor which varied according to the map scale, and all such values of linear parameters are thus given in miles. Total stream segment length for each order was measured separately and the average calculated. Total channel length within the basin was found by summing

TABLE I

TABLE OF SYMBOLS AND PARAMETERS

PARAMETERS	SYMBOLS	EQUATION	FIRST DEFINITION
I Dimensionless:			
1. Stream Order	$1,2,3,\ldots u$		Strahler, 1954, (b) p. 344
2. Basin Order	u		Horton, 1945, p. 281
3. Number of Streams of order o.	n_o		Horton, 1945, p. 279
4. Total number of	N	$=\sum_{o=1}^{u} n_o$	Horton, 1945, p. 285
II Linear:			
5. Average length of streams of order o	ℓ_o		Horton, 1945, p. 279
6. Total length of stream of order o	$\sum \ell_o$		
7. Total length of streams, all orders	L	$=\sum_{o=1}^{u} \ell_o$	Horton, 1945, p. 283
8. Perimeter	P		
9. Greatest length of basin	L_m		Schumm, 1956, p. 612
III Areal:			
10. Average drainage area, order o	a_o		Schumm, 1956, p. 606
11. Total drainage area, order o	$\sum a_o$		
12. Total area of given basin	A	$=\sum_{o=1}^{u} a_o$	Horton, 1945, p. 279
13. Area drained exclusively by segment order o	$i.a._o$		Schumm, 1956, p. 607

TABLE I CONTINUED

PARAMETERS	SYMBOLS	EQUATION	FIRST DEFINITION
IV Derived:			
14. Bifurcation ratio	$r_b(o\text{-}1:o)$	$= \frac{n_{o-1}}{n_o}$	Horton, 1945, p. 286
15. Length ratio	$r_\ell(o:o\text{-}1)$	$= \frac{\ell_o}{\ell_{o-1}}$	Horton, 1945, p. 280
16. Area ratio	$r_a(o:o\text{-}1)$	$= \frac{a_o}{a_{o-1}}$	Schumm, 1956, p. 606
17. Area of circle with same perimeter as basin	A_c	$= \frac{p^2}{4\pi}$	Miller, 1953, p. 8
18. Diameter of circle with same area as basin	d	$= 2\sqrt{\frac{A}{\pi}}$	Schumm, 1956, p. 612
19. Drainage Density	Dd	$= \frac{\sum L}{A}$	Horton, 1945, p. 283
20. Channel Frequency	F	$= \frac{N}{A}$	Horton, 1945, p. 285
21. Relief Ratio	R_r	$= \frac{\text{Local Relief}}{L_m}$	Schumm, 1956, p. 612
22. Circularity Ratio	R_c	$= \frac{A}{A_c}$	Miller, 1953, p. 8
23. Elongation Ratio	R_e	$= \frac{d}{L_m}$	Schumm, 1956, p. 612
24. Average Bifurcation Ratio	$\bar{r}_b$	$= \sum_{o=2}^{u} \frac{r_b(o\text{-}1:o)}{u-1}$	
25. Average Length Ratio	$\bar{r}_\ell$	$= \sum_{o=2}^{u} \frac{r_\ell(o:o\text{-}1)}{u-1}$	
26. Average Area Ratio	$\bar{r}_a$	$= \sum_{o=2}^{u} \frac{r_a(o:o\text{-}1)}{u-1}$	

the total length of stream of each order. Each measurement was repeated three times with the tracing wheel, and the linear measurements used here are the averages of these three measurements. Similarily, in each basin, the length of the perimeter P was taken as the average of three tracings with the opisometer.

The accuracy of this instrument leaves much to be desired, and the deviation between successive readings on the wheel is shown in Table II, which gives the results when a typical stream segment was repeatedly measured; the opisometer was rolled down the length of this stream segment as when an actual stream length was being measured, and the results were tabulated. The variation within this series of measurements was calculated as lying within a range of 3 standard deviations of the mean value. The dial of the opisometer wheel is graduated to only .5 inches. Estimations to .1 inches are the best that can be achieved with this device in the opinion of the writer. To improve somewhat on this situation, wherever the parameter to be measured was a straight line distance the length was read from a ruler graduated to 1/50 of an inch and subsequently converted to miles as with the opisometer measurements. Thus L_m, the length from the mouth to the most distant point on the perimeter, was measured by ruler, whereas stream segment and perimeter lengths were not suitable for this method. No such linear measurements, were corrected for gradient as the basins were chosen in areas of comparably low relief, and it was felt that inaccuracies introduced by not correcting for gradient were of a very minor nature and could be disregarded.

TABLE II

ACCURACY OF THE OPISOMETER

PERIMETER, BASIN NO. 1, SUCCESSIVE MEASUREMENTS

1. 13.7"
2. 13.5"
3. 13.5"
4. 13.1"
5. 13.6"
6. 13.6"
7. 13.7"
8. 13.4"
9. 13.8"
10. 13.6"

$\bar{x} = 13.55$

$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$\sigma = 0.1857$

$\bar{x} \pm \sigma = 13.7357, 13.3643$

$\bar{x} \pm 2\sigma = 13.9214, 13.1786$

PERIMETER, BASIN NO. 2, SUCCESSIVE MEASUREMENTS

1. 3.6"
2. 3.8"
3. 3.6"
4. 3.7"
5. 3.7"
6. 3.9"
7. 3.7"
8. 3.7"
9. 3.7"
10. 3.9"

$\bar{x} = 3.73$

$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$= 0.1$

$\bar{x} \pm \sigma = 3.83, 3.63$

$\bar{x} \pm 2\sigma = 3.93, 3.53$

The accuracy of linear measurements may be considered an important drawback to the credibility of results quoted in this dissertation, but it will also be shown that most of these linear measurements are statistically very closely related to certain areal parameters which were also measured. Thus it is possible to base almost all conclusions on areal measurements which, as will be shown, were taken to a much higher degree of accuracy.

A compensating polar planimeter, Keuffel and Esser model 4236 was used for all area measurements; this was considerably more accurate than the opisometer. The manufacturers claim that this instrument has a precision of better than 1:1,000 in measuring a 10 sq. in area. However, operator errors, particularily errors in reading the vernier, are such that in practice the precision is slightly less. The area drained exclusively by each stream segment was measured and the individual parts summed to give the total area drained by each order (Fig. 4). For the area drained by higher order basins, the area drained by it's lower order tributaries was included with and added to the area feeding directly into the particular channel segment. Thus the area drained by successive orders of channel segments is necessarily cumulative and corresponds each time to the definition of a drainage basin as it is given at the head of this chapter. The average area drained by segments of each order was then found by dividing the sum of this cumulative total for each of the basins of the order by the number of segments of that order. A second group of parameters was found by calculating the average area drained exclusively by channel segments of a particular order. This was termed the inter-area and is represented by the symbol i.a. subscripted by the number of the order of the segments. (Fig. 4).

In all cases the technique of measurement was the same: the vernier reading of the planimeter was noted when the head of the tracer arm was placed at a specific point on the circumference of the area to be measured; the boundary line was traced out as carefully as possible with the tracer arm, and the vernier reading was again noted when the starting point was re-attained. The characteristics of this particular polar planimeter are such that the difference between the two readings is a direct measurement of the area in 1/100 of a square inch. The outline of each areal unit was traced three times, and the average of the three measurements was taken as the areal parameter desired and converted to square miles. An indication of the accuracy of this instrument in use is given in Table III. As with the linear measurements made with the opisometer, one areal unit was chosen and measured repeatedly; it can be seen that the deviations between successive measurements are less than 7/100 of a square inch, and all values lie within 2 standard deviations of the calculated mean value. This same test was repeated on a smaller and somewhat simpler outline which was closer to the shape of the majority of areal units measured, and the deviations between successive measurements made of this outline, as can be seen from Table III, were always less than 3/100 of a square inch. The outlines used for these tests of the accuracy of the polar planimeter are shown in figure 5.

TABLE III

ACCURACY OF THE POLAR PLANIMETER

BASIN NO. 1, SUCCESSIVE MEASUREMENTS

1. 5.48
2. 5.54
3. 5.52
4. 5.48
5. 5.53
6. 5.49
7. 5.50
8. 5.51
9. 5.47
10. 5.49
11. 5.47
12. 5.50

$$\bar{x} = 5.498$$

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$= 0.022$$

$$\bar{x} \pm \sigma = 5.522,\ 5.478$$

$$\bar{x} \pm 2\sigma = 5.544,\ 5.456$$

BASIN NO. 2, SUCCESSIVE MEASUREMENTS

1. 0.92
2. 0.90
3. 0.91
4. 0.90
5. 0.91
6. 0.91
7. 0.91
8. 0.91
9. 0.91
10. 0.91
11. 0.92
12. 0.91

$$\bar{x} = 0.91$$

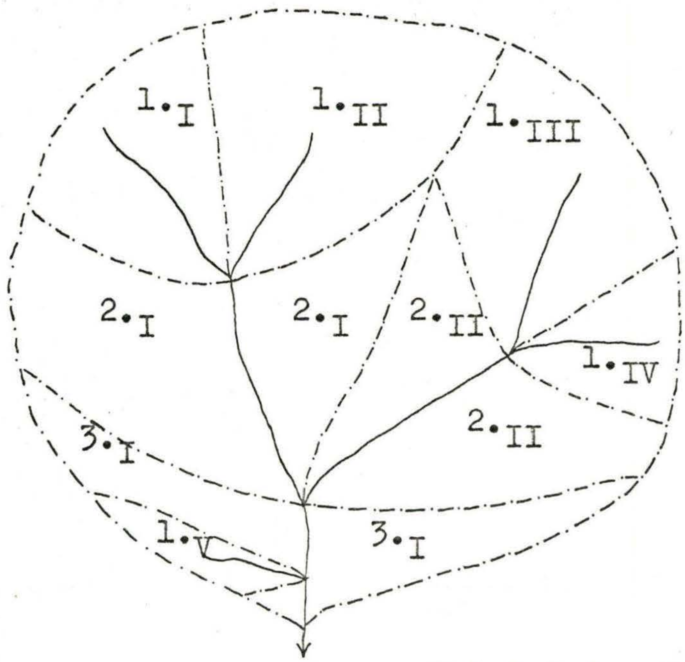
$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$= 0.0058$$

$$\bar{x} \pm \sigma = .9158,\ .9042$$

$$\bar{x} \pm 2\sigma = .9216,\ .8984$$

FIG 4
EXCLUSIVE AND ACCUMULATED DRAINAGE AREAS



Third Order Basin

Areas $1._{I}$ to $1._{V}$ - Exclusive first order areas

Areas $2._{I}$ to $2._{II}$ - Exclusive second order areas = $i.a._{2}$

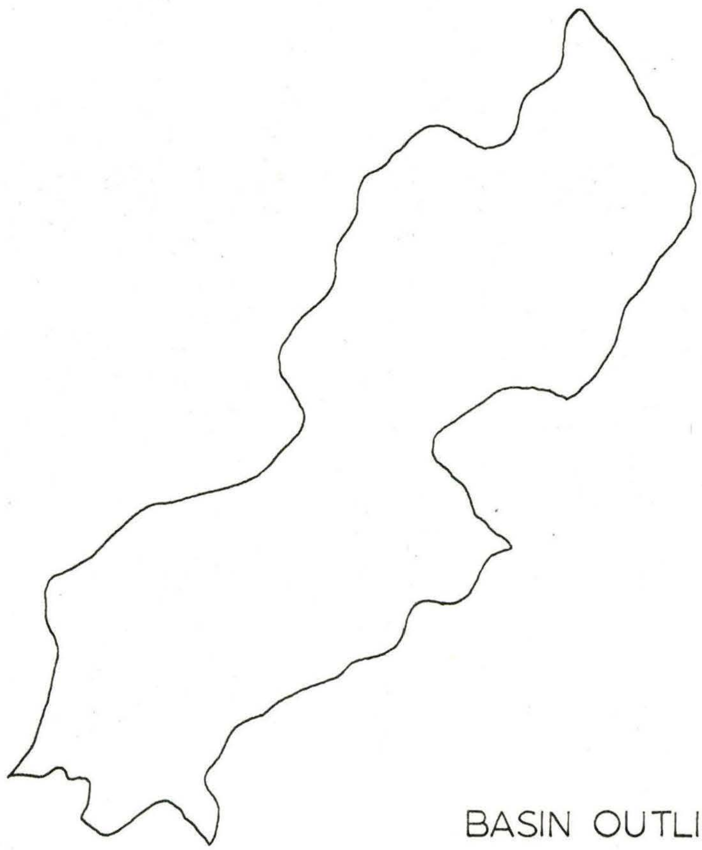
Area $3._{I}$ - Exclusive third order area = $i.a._{3}$

Average area, order 1 $= \dfrac{1._{I} + 1._{II} + 1._{III} + 1._{IV} + 1._{V}}{5} = a_1$

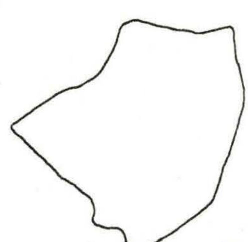
Average area, order 2, accumulated. $= \dfrac{1._{I}+1._{II}+1._{III}+1._{IV}+1._{V}+2._{I}+2._{II}}{2.} = a_2$

Average area, order 3, accumulated. $= 1._{I}+1._{II}+1._{III}+1._{IV}+1._{V}+2._{I}+2._{II}+3._{I} = a_3$

FIG 5
BASIN OUTLINES
(instrument accuracy tests)



BASIN OUTLINE No. 1



BASIN OUTLINE No. 2

As a further check of the areal measurements, the areas of all individual components of the drainage basins were totalled, and this figure was compared with a direct measurement of the total area; in all cases, the correlation was very high. Certain basins measured from aerial photographs were too large for one single measurement of the area to be made by one revolution of the planimeter, and in such cases the basin was subdivided. Straight lines were drawn across the basin with a ruler to form units of a convenient size, and the area of each of these units was summed to give the total area of the basin.

The only vertical parameter included in this study is the Local Relief of the basin; this figure was divided by the maximum length of the basin, L_m, to give a derived parameter which was termed the Relief Ratio by Schumm (1956). This parameter gives a reasonable representation of the average fall per unit distance or gradient of the entire basin, although not of any particular portion of the channel system. Local Relief is here defined as the difference between the highest point on the water shed and the mouth of the basin. The actual height of either of these points could only be determined where they precisely intersected a contour line, and in all other cases the height was estimated by comparing the relative linear distance of the point in question between the contour immediately above and the contour immediately below. For example, if the mouth of the basin was in a position between the upper and lower contours in the distance ratio, lower contour - mouth: mouth - upper contour, .4: .6, it was assumed that the point was in the same height ratio between the two contours, i.e., with a contour interval of 10 feet, in this example the mouth was taken as being 4 feet above the lower contour, and of course 6 feet below the upper contour.

The actual height of the highest point on the perimeter was estimated in a similar manner. The average distance between the contours in the upper parts of the basins was measured, then the distance of the highest point in the basin above it's nearest contour was expressed as a proportion of this between-contour distance. As with the mouth of the basin, this distance ratio was equated with a height ratio, and the height of the point found in feet above the nearest contour. (Fig. 6).

Most of the derived parameters in this study are so-called ratios, although not all are quoted as true mathematical ratios. The most obvious examples of these are the Horton ratios; in the case of the bifurcation ratios, the number of streams of any given order is divided by the number of stream segments of the next higher order:

e.g. $r_b\ (o\text{-}1:o) = n_{o-1}/n_o$,

or $r_b\ (2:3) = n_2/n_3$

The reverse relationship is the case for the length and area ratios since in the calculation of these ratios the average area drained by a stream segment of particular order is divided by the average area drained by stream segments of the next lower order:

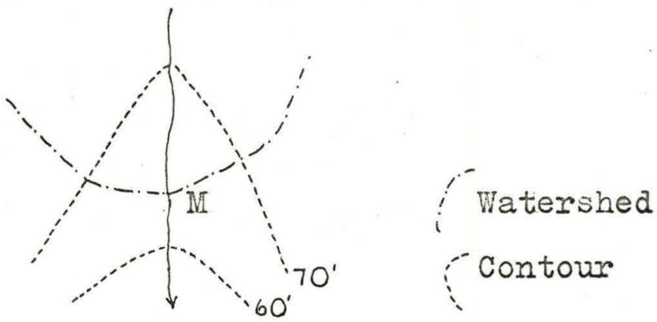
e.g. $r_a\ (o:o\text{-}1) = a_o/a_{o-1}$,

or $r_a\ (3:2) = a_3/a_2$,

and $r_\ell\ (o:o\text{-}1) = \ell_3/\ell_2$.

FIG 6
ESTIMATION OF HEIGHT

1. Estimation of Height of Basin Mouth

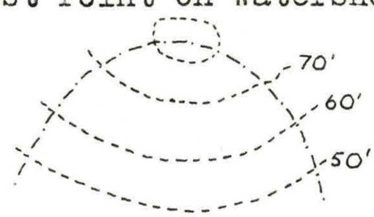


Horizontal Distance:
60' contour - M : 70' contour - M = 1 : 2

Vertical Distance:
60' contour - M : 70' contour - M = 1 : 2

Estimated Height of M: 63.3 feet

2. Estimation of Highest Point on Watershed



Horizontal Distance:
Average distance between contours : highest contour - highest point = 2 : 1

Vertical Distance:
Average distance between contours : highest contour - highest point = 2 : 1

Estimated Highest Point: 83.3 feet

The ratios used in this study are similar to those formulated by Horton (1945), but the subscript refers to the order of the stream segments derived through the Strahler ordering system. Average ratios of stream numbers, lengths, and areas for each basin have been derived in the following manner:

$$\bar{r}_b = \frac{r_b(1:2) + r_b(2:3) + \ldots\ldots\ldots r_b(u-1:u)}{u-1}$$

where u is the basin order.

Similarily,

$$\bar{r}_a = \frac{r_a(2:1) + r_a(3:2) + \ldots\ldots\ldots r_a(u:u-1)}{u-1}$$

and,

$$\bar{r}_\ell = \frac{r_\ell(2:1) + r_\ell(3:2) + \ldots\ldots\ldots r_\ell(u:u-1)}{u-1}$$

Exceptions to the rule that derived parameters are ratios of actual measurements are the derived properties A_c and d which are themselves used to formulate other derived parameters or ratios. The measurement A_c was first proposed by Miller, (1953) and he defined A_c as the area of a circle with the same perimeter as the drainage basin in question. This is compared with the actual area of the drainage basin to give a measure of the shape of the basin as compared to a circle, and is termed by Miller the Circularity Ratio,

$$R_c = A/A_c,$$

where A is the total area of the drainage basin.

d was first defined by S. A. Schumm (1956) as the diameter of a circle whose area is the same as the area of the drainage basin. Schumm's

Elongation Ratio R_e compares the length of this diameter with the greatest length of the basin by the formula:

$$R_e = d/L_m, \quad \text{where } d = 2\sqrt{\frac{A}{\pi}} .$$

This formula also expresses the shape of the drainage basin, but in slightly different terms to Miller's Circularity Ratio.

Two derived parameters are used to express the way in which the drainage net covers the area of the basin, and these will be compared later. Both of these ratios were initially proposed by Horton (1945). Drainage Density, represented by the symbol Dd, expresses the length of channel per unit area by the formula: $Dd = \Sigma L/A$.

Stream or Channel Frequency as it is variously termed expresses the number of stream segments of all orders per unit area. Since the headwaters of the main channel are always designated the order of the main channel under the Horton system of ordering and this is not the case under the Strahler system, slight differences occur in the value of the Stream Frequency. This Frequency is given by the formula:

$$F = N/A$$

where N is the total number of stream segments in the basin, i.e., $N = n_1 + n_2 + \ldots\ldots n_u$.

CHAPTER IV

HORTON ANALYSIS

It was stated in Chapter III that one of the initial assumptions in this study was the adoption of the Strahler (1954b) ordering system as opposed to that of Horton, (1945). Figure 3 illustrated the differences when the two schemes were applied to the same hypothetical drainage basin. At this point it seems relevant to discuss reasons for this choice, and the discrepancies which were anticipated in the application of the Horton Laws as a result of it. An application of the laws restated to encompass the characteristics of the Strahler system of ordering, will then be attempted.

The concept of order and the usefulness of an ordering system depends on the premise that ".... on the average, if a sufficiently large sample is treated, order number is directly proportional to watershed dimensions, channel system, and stream discharge at that place in the system." (Strahler, 1957b, p. 914). Ordering systems are, in theory at least, dimensionless and with reference to equivalent locations in the ordering system, drainage nets which are geometrically similar may be directly compared regardless of their relative size. The Strahler scheme of ordering has certain advantages over that of Horton, particularly with regard to its dimensionless properties. Not only is the system somewhat easier to use, but it does not require renumbering of the head-waters of all the major tributary streams. The designation of the order of stream segments is restricted from the beginning by

Strahler, and as a result, many of the computations involved in the Horton Laws are simpler. Further, the Strahler scheme has been proved to have more meaning in a mathematical sense. Melton derived the system from concepts of combinatorial analysis "...without the introduction of any arbitrary or non-mathematical concepts" (Melton, 1959, p. 345). Thus the Strahler system is "...a simply defined mathematical concept ..... (and)....is probably unique in this respect as all others involve from the beginning the notions of a downstream direction, entrance angles, or size of channels" (Melton, 1959, p. 346). To overcome a certain tendency for confusion between the 2 ordering schemes, stream channels delimited by their order number are frequently referred to as 'segments' when the Strahler ordering system has been adopted. This practice has been followed in this dissertation wherever it was felt that confusion might arise. Horton originally proposed 3 "Laws of Drainage Composition" in which he related stream numbers, lengths, and gradients to order (Horton, 1945). Only the laws of stream numbers and lengths were tested in the course of this study, but a fourth law, devised by S. A. Schumm (1956) and relating drainage area to order, was also tested for its degree of application to the chosen Ontario streams. Horton's Law of Stream Numbers states that the numbers of streams decrease in geometrical progression with increasing order, and was expressed in the following terms:

$n_o = r_b^{(s-o)}$, where n_o = number of streams of given order o,

s = order of the main stream,

o = order of a given class of tributaries,

r_b = bifurcation ratio = n_1/n_2

According to Leopold, Wolman, and Miller, "the laws relating to stream order and number are little affected" by the use of the Strahler scheme of ordering (Leopold, Wolman and Miller, 1964, p. 135). This fact is confirmed by R. L. Shreve (1964) who maintained that segment ordering resulted in a better fit of this law for 210 out of 246 networks he tested. The applicability of this law is however affected both by map scale and map quality, and this fact was demonstrated by Morisawa in her study of stream basins in the Appalachian Plateau (Morisawa, 1962). It was for this reason that an attempt was made to extend the channel net in the chosen basins from the contour information given on the topographic map sheets. It has also been stated that this law "....is a statistical relationship resulting from random development of drainage networks rather than from orderly evolution as generally assumed" (Shreve, 1963, p. 44). In contrast, Bowden and Wallis (1964, p. 768) are of the opinion that although the law represents a statistical relationship, it is "a result of the definition of stream order rather than being due to either orderly evolution or random development." It does not seem necessary to accept or refute either of these arguments here, although it may be pointed out that a hypothetical stream network produced by the random walk technique on a computer has been shown to parallel the Horton Law of Stream Numbers, as well as the Law of Stream Lengths (Leopold, Wolman and Miller, 1964). Whether this is also due to the stream ordering system which was of necessity applied to the Random Walk model for reference, is a point for further arbitration beyond the scope of this study.

The definition of the bifurcation ratio used in this study is slightly wider than Horton's original definition:

$$r_b\ (1:2) = n_1/n_2$$

$$r_b\ (2:3) = n_2/n_3$$

$$\overline{r_b} = \frac{r_b\ (1:2) + r_b\ (2:3)}{2}$$

The numbers of streams of each order calculated when both the Horton and the Strahler ordering schemes were applied to the same hypothetical drainage basins shown in figure 3 may be used to demonstrate the discrepancies between the two systems with respect to the Law of Stream Numbers. If the numbers of stream of successive orders were related by a perfect inverse geometric series, the following situation would arise:

$$r_b\ (1:2) = r_b\ (2:3) = r_b\ (3:4) = \overline{r_b}$$

The application of the Strahler system to the basin gave the following results:

$n_1 = 18$ $\qquad r_b\ (1:2) = 2.57$

$n_2 = 7$ $\qquad r_b\ (2:3) = 2.33$

$n_3 = 3$ $\qquad r_b\ (3:4) = 3.00$

$n_4 = 1$ $\qquad \overline{r_b} = 2.63$

The application of the Horton ordering scheme as demonstrated in Fig. 3, gave the results:

$n_1 = 11$ $\qquad r_b\ (1:2) = 2.75$

$n_2 = 4$ $\qquad r_b\ (2:3) = 2.00$

$n_3 = 2$ $\qquad r_b\ (3:4) = 2.00$

$n_4 = 1$ $\qquad \overline{r_b} = 2.25$

From this example it may be seen that provided a sufficiently large sample is taken, there is no reason to claim that the Strahler segment ordering system is any the less suitable for the application of the Horton Law of Stream Numbers.

The Application Of The Law Of Stream Numbers

The numbers of stream segments of all orders were counted in the chosen third order basins. These results are summarized in Appendix (A). Since it seems to be a generally held opinion that the Horton Law of Stream Numbers is equally applicable to segment ordered data, these values of stream numbers were then plotted on semi-logarithmic paper to test for the fit of the law; stream numbers which form an inverse geometric series with order should lie on a straight line when plotted on this paper. The plot of Stream Numbers for each of the basins studied may be seen in Figs. 7a to 24a. Each of the basins is identified by a subscripted number; the subscript 'c' identifies those basins which lie in areas described by Chapman and Putnam (1952) as clay plain, and basins chosen from the sand plain areas of these authors are indicated by a number subscripted by the letters 's'. Table IV lists these identifying numbers, the corresponding maps from which the drainage nets were drawn, and the scale of the maps. The location of all basins is recorded in Fig. 2.

It is obvious from these plots of the stream numbers against order that many of these small Ontario basins do not give a very close approximation to the first Horton Law. For basins $(1)_c$ and $(3)_s$ in particular the points which mark the number of stream segments of each order are noticeably far removed from a straight line. The testing of the Horton Laws by a plot of the appropriate data on semi-logarithmic paper

TABLE IV

BASINS	MAP LOCATIONS	SCALES
I. CLAY BASINS	MAP[1]	SCALE
1_c	Blackheath	1:25,000
2_c	Smithville	1:25,000
3_c	Caistorville	1:25,000
4_c	Grimsby East	1:50,000
5_c	Simcoe East	1:50,000
6_c	Simcoe East	1:50,000
7_c	Smithville	1:25,000
8_c	Caistorville	1:25,000
9_c	Caistorville	1:25,000
II. SAND BASINS	MAP	SCALE
1_s	Brantford East	1:50,000
2_s	Simcoe East	1:50,000
3_s	Brantford West	1:50,000
4_s	Dunnville East	1:50,000
5_s	Tillsonburg East	1:50,000
6_s	Simcoe West	1:50,000
7_s	Brantford West	1:50,000
8_s	Tillsonburg East	1:50,000
9_s	Brantford East	1:50,000

[1]Maps are the National Topographic series of maps of Canada, and the names stated here are the official names of the sheets.

is in any case a technique which may obscure minor discrepancies: the use of the logarithmic scale on the ordinate is a device which, although it makes graphical representation of this law much simpler, may also reduce the apparent divergence from a perfect geometric series. Table V, which records the values of $r_b(1:2)$, $r_b(2:3)$, and $\bar{\bar{r}}_b$, gives a much clearer and more accurate indication of variation from an inverse geometric series within the individual basins of this study. The table also records directly the deviation of $r_b(1:2)$ and $r_b(2:3)$ from $\bar{\bar{r}}_b$. The values quoted in this table demonstrate that only in basin $(2)_c$ do the numbers of stream segments form a perfect inverse geometric series. Of the remaining basins, only 7 have values of $r_b(1:2)$ and $r_b(2:3)$ which deviate less than 0.26 from $\bar{\bar{r}}_b$. Basins $(1)_c$ and $(3)_s$ deviate furthest from an exponential function: for basin $(1)_c$ the values of $r_b(1:2)$ and $r_b(2:3)$ are respectively 2.17 higher than and lower than the mean bifurcation ratio. In the case of basin$(3)_s$, the value of $r_b(1:2)$ is only 2.00, yet the value of $r_b(2:3)$ is 6.00. The bifurcation ratios of all other basins in the selected groups have a deviation from the mean bifurcation ratio equal to or less than 1.00.

One other fact must be noted from Table V. The mean bifurcation ratio for each basin is, on the whole, a low value. Basins $(1)_c$ and $(3)_s$ have the highest values due primarily to the high figures for $r_b(1:2)$; in all other basins, the mean bifurcation ratio is less than 3.17. This situation agrees fairly well with Horton's (1945, p. 290) statement that "...the bifurcation ratio ranges from about 2 for flat or rolling drainage basins up to 3 or 4 for mountaineous or highly dissected drainage basins. As would be expected, the bifurcation ratio is generally higher for hilly,

TABLE V

BIFURCATION RATIOS

BASIN	$r_b(1:2)$	$r_b(2:3)$	$\bar{r}_b$	DEVIATIONS FROM $\bar{r}_b$
$(1)_c$	5.67	3.00	4.34	2.17
$(2)_c$	2.00	2.00	2.00	0.00
$(3)_c$	3.33	3.00	3.17	0.17
$(4)_c$	3.00	2.00	2.50	0.50
$(5)_c$	4.00	2.00	3.00	1.00
$(6)_c$	3.50	2.00	2.75	0.75
$(7)_c$	2.50	2.00	2.25	0.25
$(8)_c$	2.50	2.00	2.25	0.25
$(9)_c$	2.50	2.00	2.25	0.25
$(1)_s$	3.33	3.00	3.17	0.17
$(2)_s$	3.50	2.00	2.75	0.75
$(3)_s$	6.00	2.00	4.00	2.00
$(4)_s$	3.50	2.00	2.75	0.75
$(5)_s$	2.67	3.00	2.84	0.17
$(6)_s$	3.50	2.00	2.75	0.75
$(7)_s$	2.50	2.00	2.25	0.25
$(8)_s$	3.50	2.00	2.75	0.75
$(9)_s$	3.00	2.00	2.50	0.50

$$r_b(1:2) = \frac{n_1}{n_2}$$

$$r_b(2:3) = \frac{n_2}{n_3}$$

$$\bar{r}_b = \frac{r_b(1:2) + r_b(2:3)}{2}$$

FIG 7 BASIN $(1)_c$

PLOTS OF THE HORTON LAWS

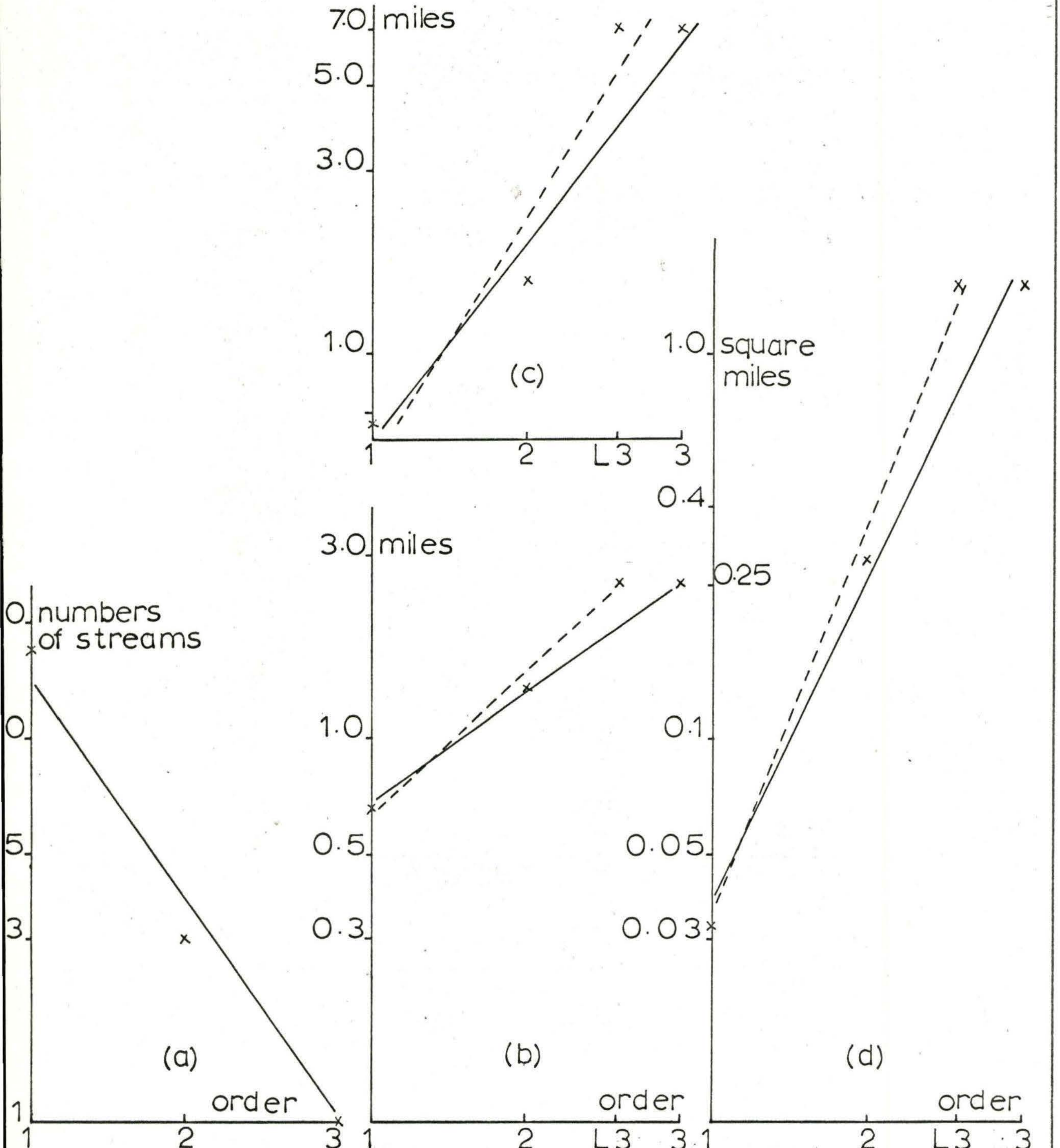


FIG 8 BASIN $(2)_c$
PLOTS OF THE HORTON LAWS

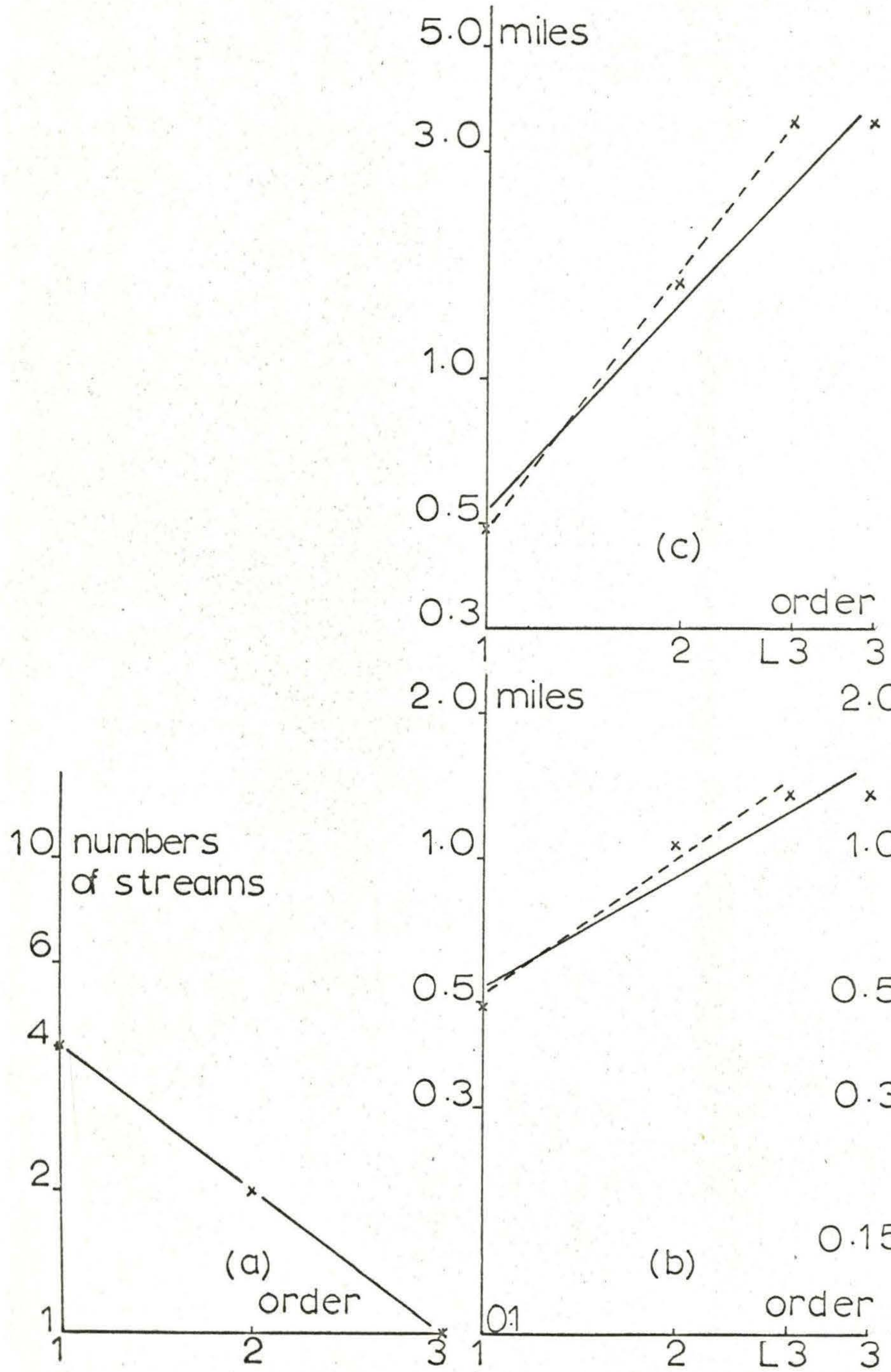
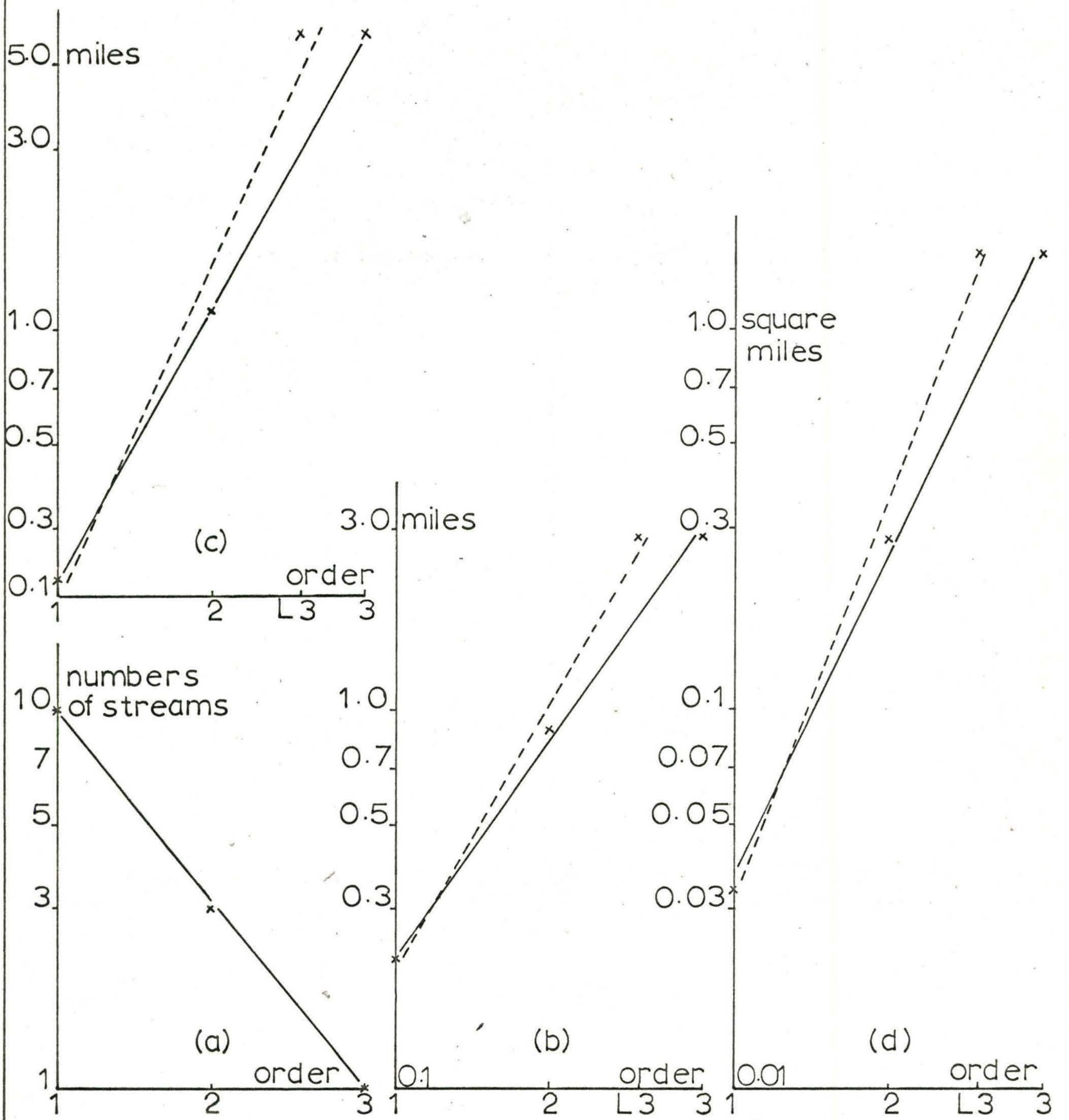


FIG 9 BASIN $(3)_c$

PLOTS OF THE HORTON LAWS



well dissected drainage basins than for rolling basins." The local relief for basins $(1)_c$ and $(3)_s$ is 100 and 165 feet respectively; for the other basins it varies from 27' to 165': although the local relief of basins $(1)_c$ and $(3)_s$ is not outstandingly high, these 2 basins have the highest Relief Ratios of the 18 basins. This would seem to confirm Horton's postulate which was made only on the basis of his own results. The ratios themselves are slightly lower than those cited by Strahler (1957, p. 914 - p. 915, 1952, p. 1134) although Strahler maintains (1957, p. 914) that "....the number (bifurcation ratio) is highly stable and shows a small range of variation from region or environment to environment except where powerful geologic controls dominate."

The Law Of Stream Lengths

Although the application of the Law of Stream Numbers to basins ordered according to the Strahler scheme poses no particular problems, the difficulty of applying the Law of Stream Lengths to segment lengths is much greater. The first and most obvious result of the Strahler scheme is that the lengths of the segments of higher order are considerable shorter since the order number does not apply to the limit of the tributaries as with the Horton system; the segment of highest order is most affected, the actual length of first order segments on the other hand shows no variation from the length of Horton's first order channels. As a result of this variation, it has been claimed that the Law of Stream Lengths as Horton first expressed it has little validity when the Strahler ordering system has been adopted. Horton (1945, p. 286) expressed the Law of Stream

Lengths as follows:

$$\ell_o = \ell_1 \, r_l^{(o-1)},$$

where ℓ_o = average length of streams of given order, ℓ_1 = average length of first order streams, r_ℓ = length ratio = ℓ_2 / ℓ_1.

This law is the reverse of the Law of Stream Numbers because , according to it, stream lengths increase geometrically with order. When the data is plotted on semi-logarithmic paper, the straight line joining the points should trend in the opposite direction to the line joining the plot of stream numbers against order.

In a discussion of the application of the Horton Law to Strahler segment ordering, Bowden and Wallis (1964, p. 769) go as far as to say that "in fact, only by chance will segment length data form a direct geometric series" with order. They cite the work of authors such as Maxwell in California, and Melton who studied stream basins in the south west of the United States to demonstrate how great a deviation from a straight line occurs when segment length data is plotted on semi-logarithmic paper in the form in which Horton expressed his law. Strahler himself recognized this problem, and in fact he broke away entirely from the conventional statement of the Horton Law (Strahler, 1957, p. 915). For basins ordered according to his own scheme, Strahler tested for a relationship between the logarithm of total stream length of each order and the logarithm of the order number by seeking a straight line plot of total stream segment length and order on logarithmic paper (Strahler, 1957, Fig. 4).

Bowden and Wallis (1964, p. 772) and Broscoe (1959) cited by Bowden and Wallis, (1964, p. 771) on the other hand attempted a re-expression of the

FIG 10 BASIN $(4)_C$

PLOTS OF THE HORTON LAWS

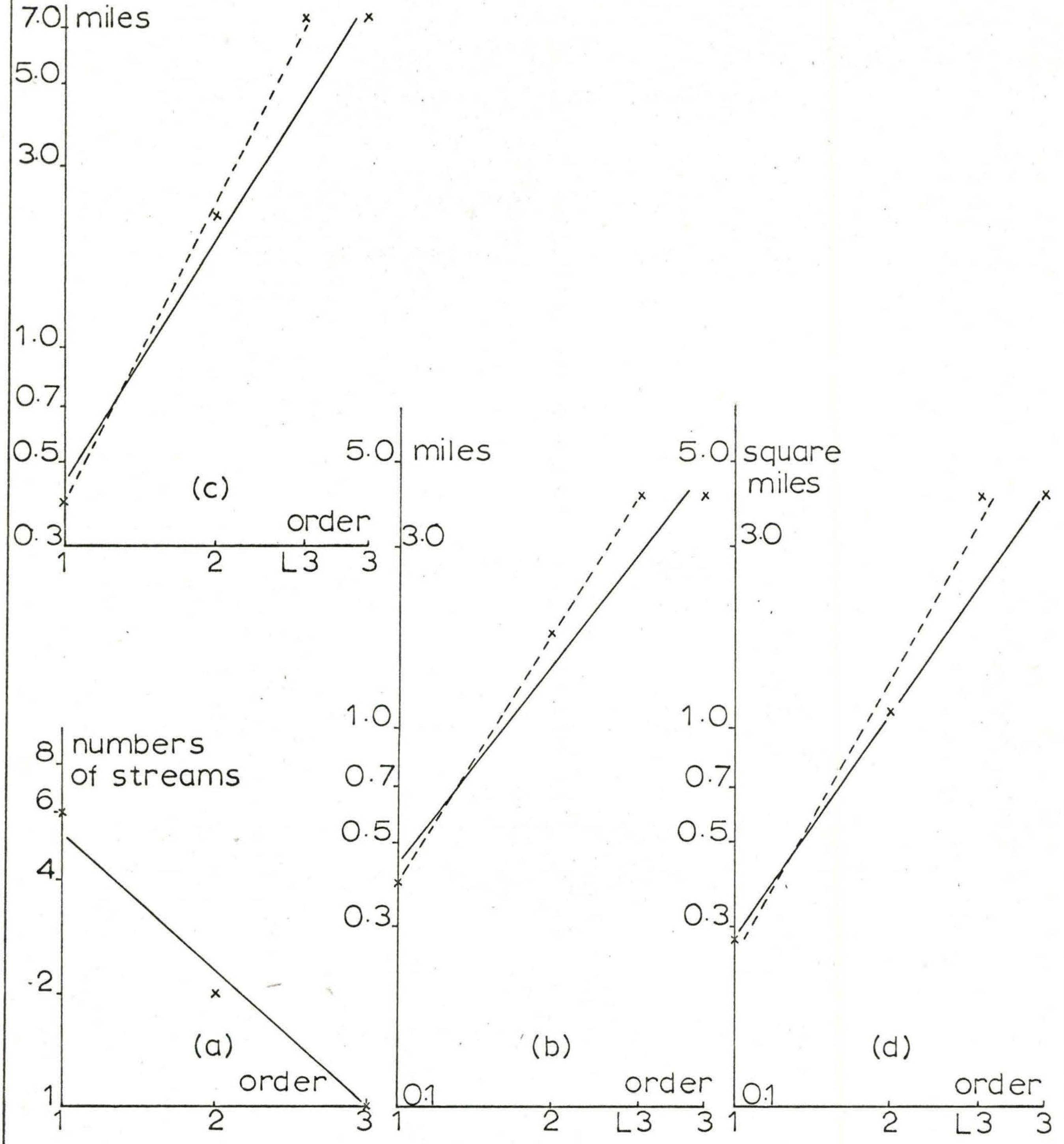


FIG 11 BASIN $(5)_c$

PLOTS OF THE HORTON LAWS

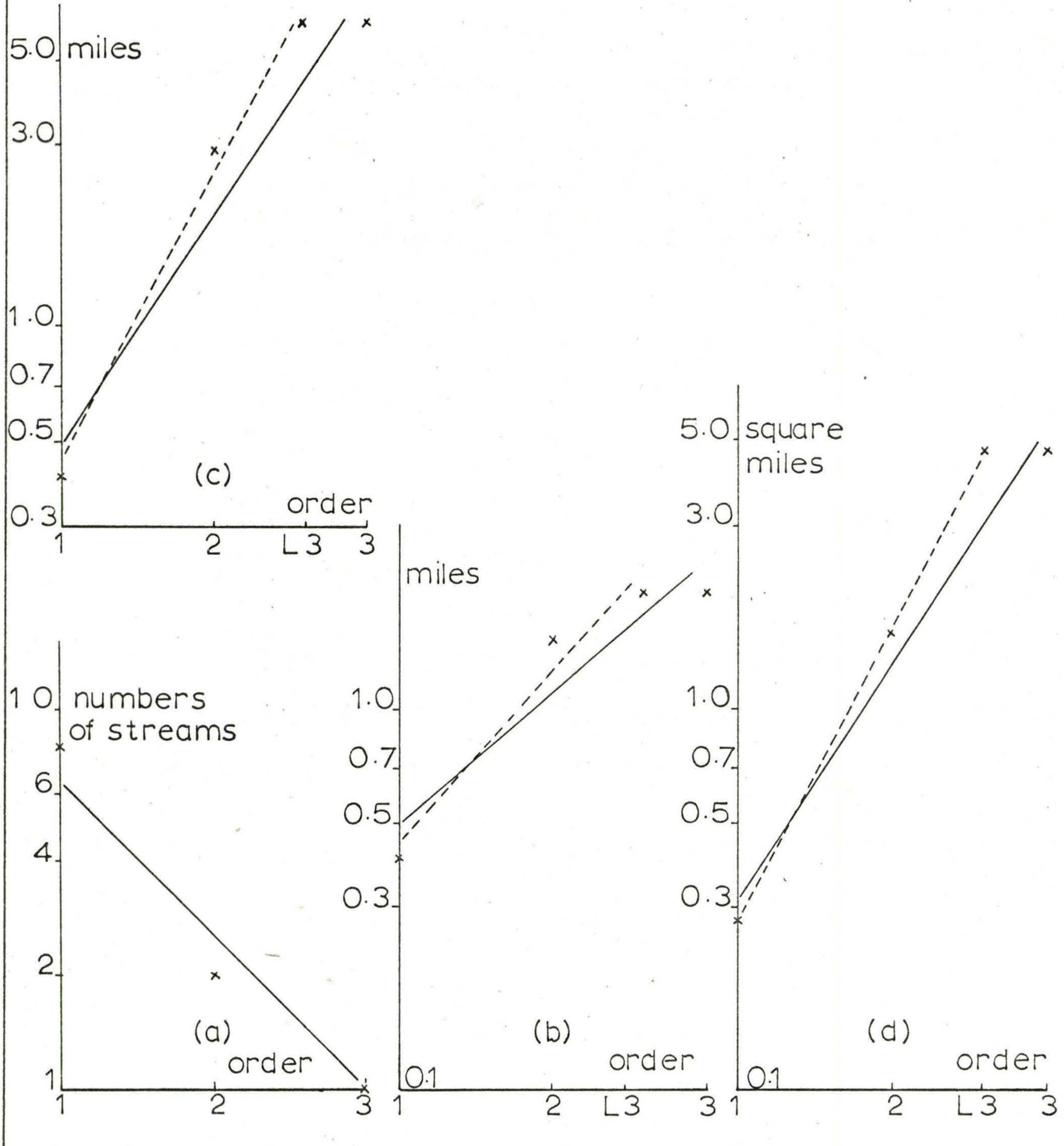
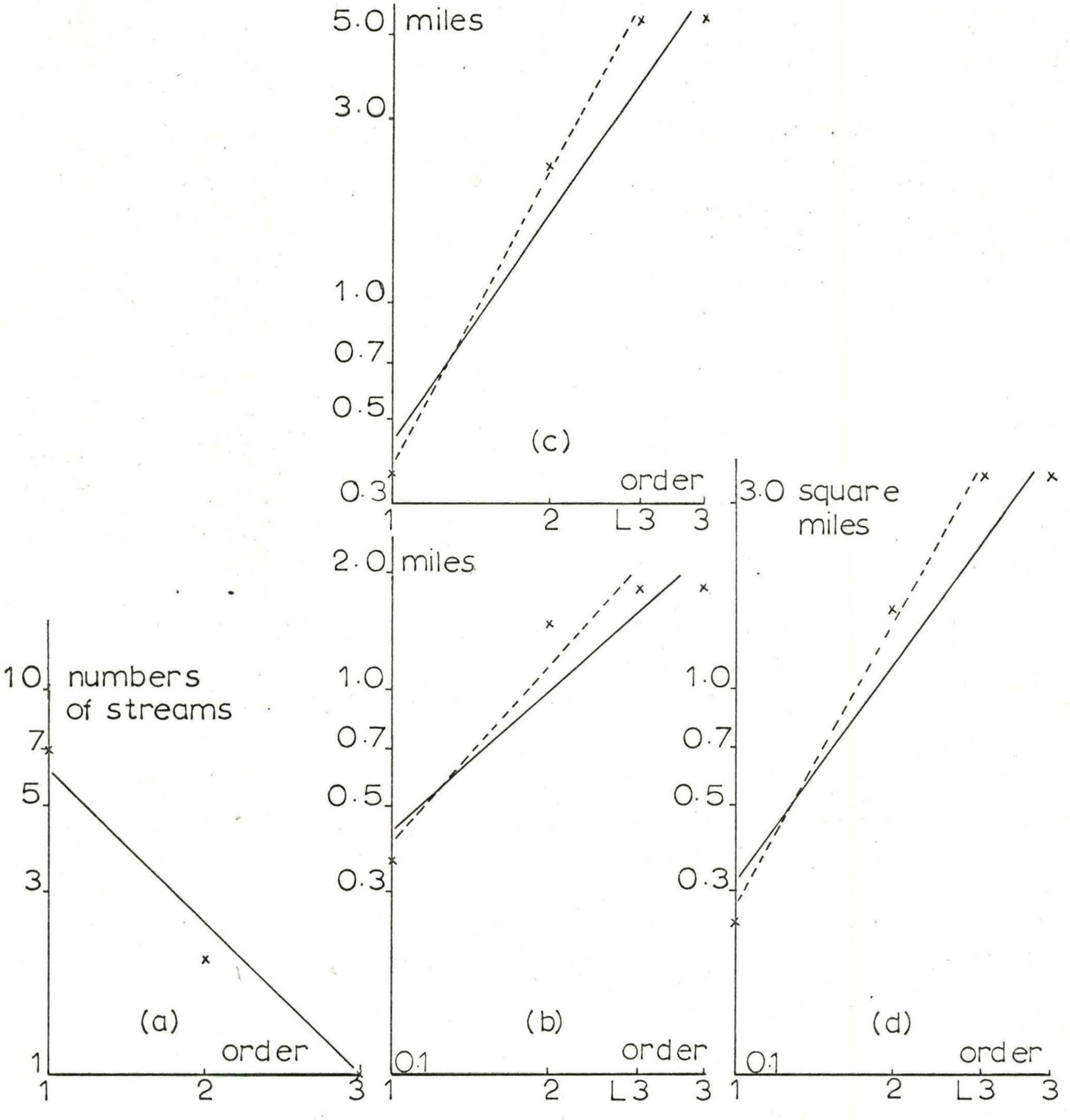


FIG 12 BASIN $(6)_c$

PLOTS OF THE HORTON LAWS



Law of Stream Lengths in a form which recognized the attributes of the Strahler ordering scheme, but which was also closer to Horton's own statement than was Strahler's suggestion. These authors suggested that for segment-ordered basins the Law of Stream Lengths could be applied using cumulative mean segment lengths instead of the average length of streams. Cumulative mean segment length is defined as follows:

$$L' = \sum_{u=1}^{n} \overline{L_u},$$

where u = segment order, $\overline{\overline{L}}_u$ = observed mean length of segments of order u, n = the order under investigation, L' = cumulative mean segment length of order n. The Law of Stream Lengths is rephrased using L' as follows:

$$L' = \ell_1 r_\ell^{(o-1)}$$

where $r_\ell = L'_2/\ell_1$

Bowden and Wallis (1964, p. 771) demonstrated the difference between a semi-logarithmic plot of average stream lengths against order and a plot of cumulative mean segment length and order for some of Melton's results: the former plot did not produce anything like a straight line relationship, whilst, for the same data, the latter definition indicated that an almost perfect geometric series was achieved. A further test in 66 watersheds from 8 different physiographic regions led the authors to state that "our data conformed to Horton's theoretical exponential function in every watershed tested". (Bowden and Wallis, 1964, p. 773).

A test of the Horton statement of the Law of Stream Lengths in the 18 basins of this study also confirmed the fact that the average length of stream segments is not an exponential function of order. In most basins average segment length plotted against order on semi-logarithmic paper resulted in a line which was concave downward; in some cases the length of the third order segment was shorter than the length of the second order segment, and occasionally it was even shorter than the average length of the first order streams. Thus the Horton formula was abandoned for this segment ordered data and an attempt was made to test for an exponential relationship between cumulative mean segment length and order. The results are shown in Fig. 7b to Fig. 24b. It was felt that the Strahler (1957, p. 915) version of the Law was farther from the original Horton definition than was theoretically desirable, and this relationship was not tested.

As with the bifurcation ratio, the length ratio[1] is a much more sensitive test of the closeness of the data to a direct geometric series. Table VI(a) records the length ratios for cumulative mean segment lengths, and the amount of departure from an exponential function is exemplified by the deviation from the mean length ratio. If the data fitted a perfect geometric series $r_{\ell}(2:1)$, $r_{\ell}(3:2)$, and $\bar{\bar{r}}_{\ell}$ would be equal. The actual values of the length ratios are low, and the mean ratios for the 18 basins range between 3.63 and 1.31; similarily, the deviations from the mean ratios are low. Deviations from the mean length ratios are in fact lower than deviations from the mean bifurcation ratios, and these length ratio deviations vary between 0.09 and 1.58 (Table VIa). These low deviations must in part be attributed to the low values of the actual length ratios themselves, and the fact that such low deviations may be

[1]The length ratio is here defined as the ratio of the cumulative mean segment length of any order to the cumulative mean segment length of the next lower order.

TABLE VI (a)

LENGTH RATIOS - CUMULATIVE MEAN SEGMENT LENGTHS

BASIN	$r_\ell(2:1)$	$r_\ell(3:2)$	$\bar{r}_\ell$	DEVIATION FROM $\bar{r}_\ell$
$(1)_c$	2.00	1.88	1.94	0.06
$(2)_c$	2.20	1.28	1.74	0.46
$(3)_c$	4.05	3.21	3.63	0.42
$(4)_c$	4.54	2.34	3.44	1.10
$(5)_c$	3.74	1.24	2.49	1.25
$(6)_c$	4.14	1.21	2.68	1.47
$(7)_c$	1.53	1.09	1.31	0.22
$(8)_c$	2.82	2.18	2.50	0.32
$(9)_c$	3.25	1.64	2.45	0.81
$(1)_s$	2.72	2.09	2.41	0.32
$(2)_s$	2.76	2.59	2.68	0.09
$(3)_s$	4.69	1.53	3.11	1.58
$(4)_s$	3.50	1.46	2.48	1.02
$(5)_s$	4.23	1.62	2.93	1.31
$(6)_s$	2.01	2.18	2.09	0.09
$(7)_s$	2.26	2.08	2.17	0.09
$(8)_s$	2.87	2.53	2.70	0.17
$(9)_s$	2.56	3.22	2.89	0.33

$r_\ell\,(2:1) = \ell_2/\ell_1$

$r_\ell\,(3:2) = \ell_3/\ell_2$

$$\bar{r}_\ell = \frac{r_\ell(2:1) + r_\ell(3:2)}{2}$$

FIG 13 BASIN $(7)_c$

PLOTS OF THE HORTON LAWS

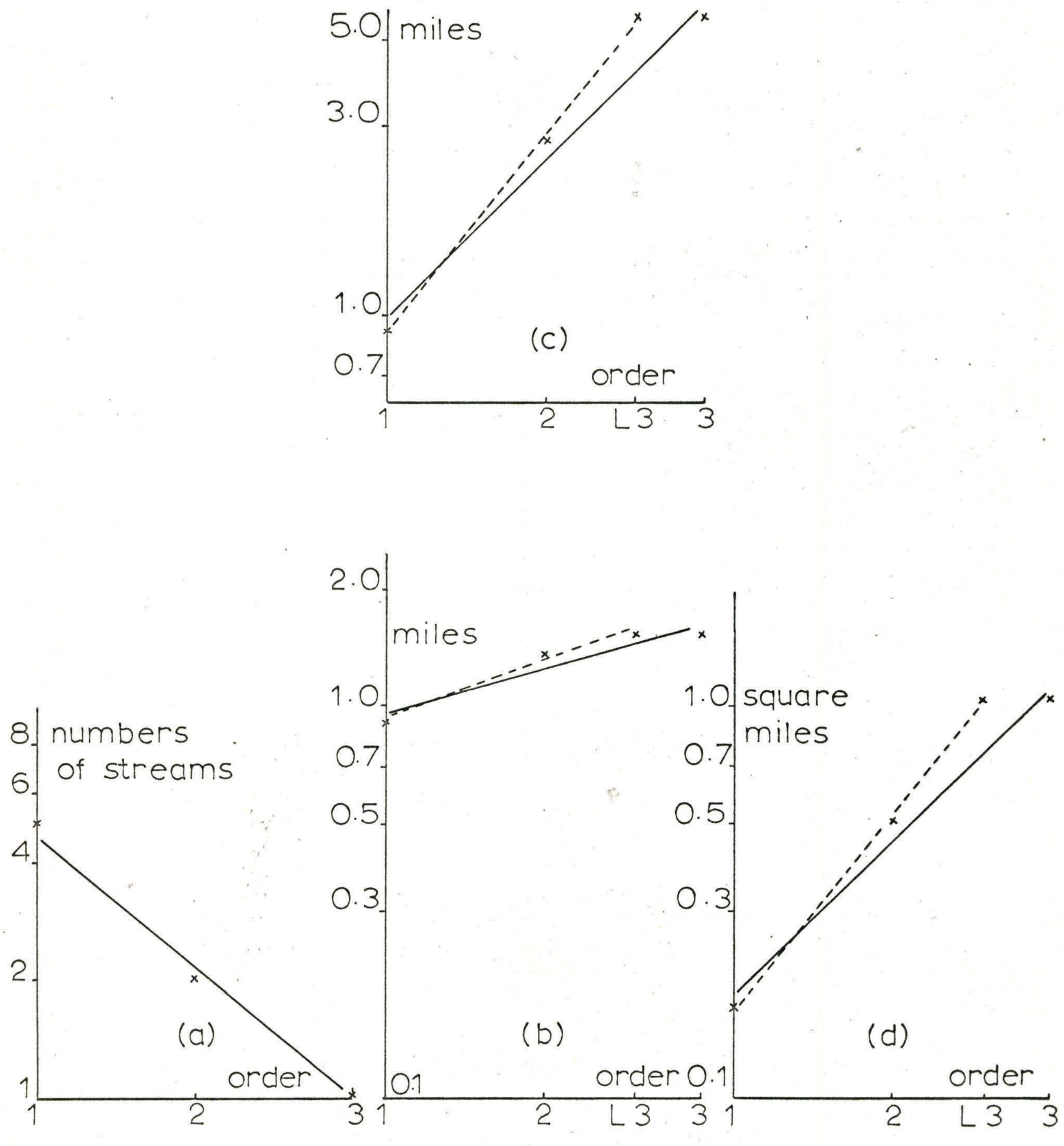


FIG 14 BASIN $(8)_c$

PLOTS OF THE HORTON LAWS

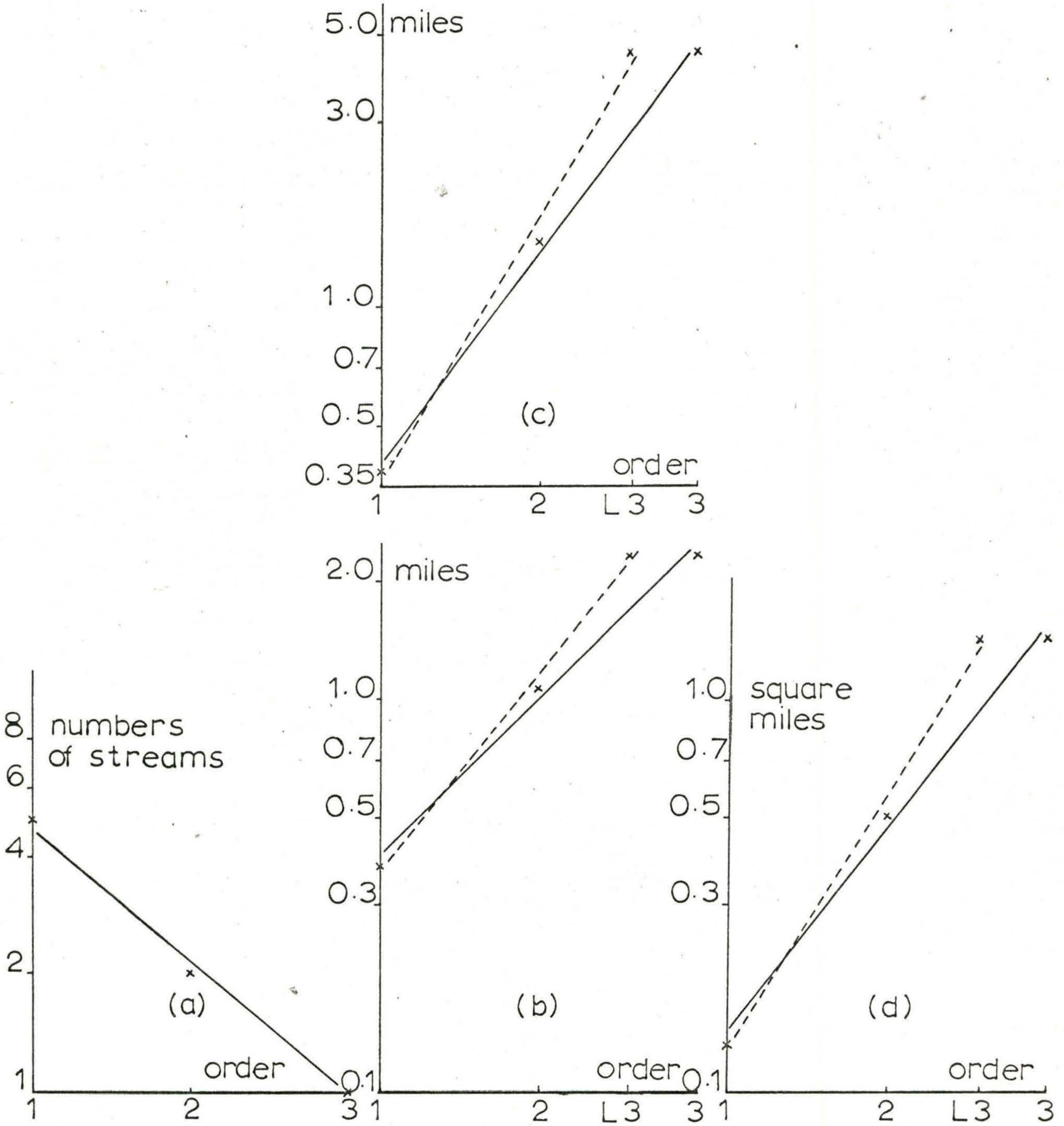
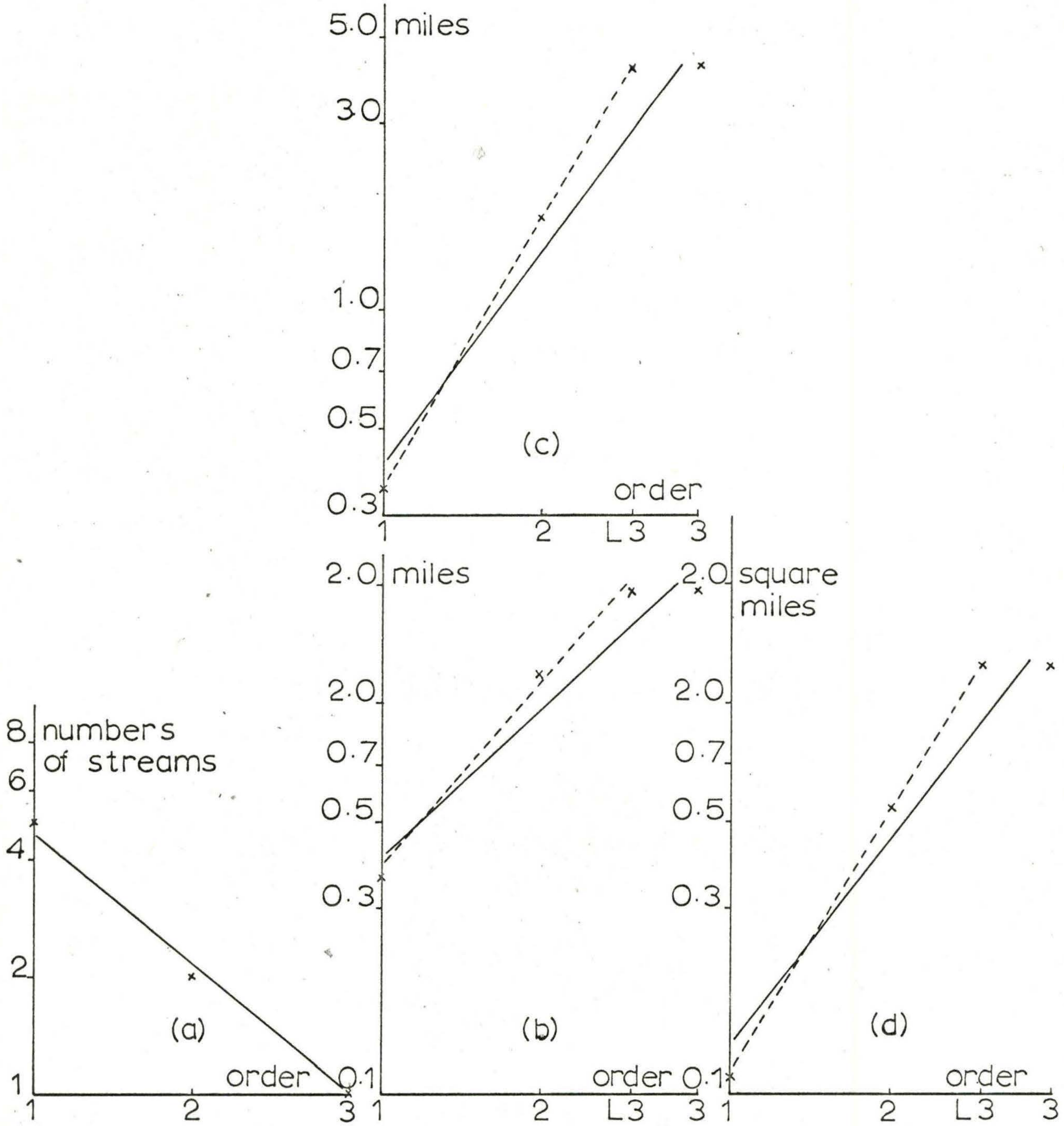


FIG 15 BASIN $(9)_c$
PLOTS OF THE HORTON LAWS



slightly mis-leading is typified by the semi-logarithmic plots of the data. The plots of cumulative mean segment length against order are in many cases very far removed from the 'line of best fit' for each basin.

The most common technique for assessing the fit of the Horton Laws in any basin is an estimation by eye of the closeness to which the appropriate data approaches the 'line of best fit'. In this case, the Bowden and Wallis modification of the Horton Law of Stream Lengths is not particularly satisfactory in the Ontario stream basins: many of the plots indicate that the data is better expressed by a curve which is concave downwards. Since this is the form in which a logarithmic relationship appears when plotted on semi-logarithmic paper, the logarithm of cumulative mean segment length was also plotted against the logarithm of order for each basin; these are superimposed on plots (b), Fig. 7 to 24. Although it is recognized that in any "study" three points is barely sufficient evidence on which to base statistically reasonable conclusions, Table VII records a comparison of the closeness of fit of the cumulative mean segment length data to a semi-logarithmic straight line or to a logarithmic straight line. The closeness of fit in either case was estimated by eye, and the + in the appropriate column signifies which of the alternatives seemed the best expression of a relationship, that is, in which case the data fell closest to a straight line. As the summary indicates (Table VII) in 11 cases out of 18 the visual impression suggested that a plot of the logarithm of cumulative segment length against the logarithm of order gave the better approximation to a straight line; in the remaining seven cases these lengths seemed better expressed as a direct geometric series.

TABLE VII

PROXIMITY TO 'LINE OF BEST FIT' OF PLOTS OF CUMULATIVE MEAN SEGMENT LENGTH AND AVERAGE ACCUMULATED SEGMENT LENGTH WITH ORDER

BASIN	VS. ORDER	VS. LOG. ORDER	VS. ORDER	VS. LOG. ORDER
$(1)_c$	*		+	
$(2)_c$		+		*
$(3)_c$	+		*	
$(4)_c$		*		+
$(5)_c$		+		*
$(6)_c$		+		*
$(7)_c$		+		*
$(8)_c$		*	+	
$(9)_c$		+		*
$(1)_s$		+	*	
$(2)_s$	*		+	
$(3)_s$		+		*
$(4)_s$		+		*
$(5)_s$		+		*
$(6)_s$	*		+	
$(7)_s$	*		+	
$(8)_s$	+			*
$(9)_s$	+		*	

+ - closer fit to straight line, one definition of length

* - closest fit to straight line, both definitions of length

Summary	Closer fit (or 2)	Closest fit (of 4)
Cumulative mean segment length		
vs order	7	4
vs log. order	11	2
Average accumulated segment length		
vs order	8	3
vs log order	10	9
	36	18

A certain amount of dissatisfaction was still felt with the statement of cumulative mean segment lengths either as a function of order or as a function of the logarithm of order, since neither form was suitable for a sufficiently large majority of basins. As a result, the definition of cumulative mean segment length was re-examined; the writer believes that certain inaccuracies and approximations are inherent in it. In the form quoted by Bowden and Wallis (1964, p. 772), the mean cumulative length of second order segments includes the mean length of all first order stream segments in the drainage basins. A re-appraisal of the drainage nets from which the present data was amassed indicated that not all first order tributaries flow into second order segments. It was therefore felt that the total length of all second order segments would be more accurately represented as the sum of the total length of all second order segments and the total length of all first order streams which are tributary to these second order segments; the average length of second order stream segments was then found by dividing this sum by the number of second order segments. The term 'average accumulated segment length' was coined and applied to this definition of stream length:

$$\ell_2 = \frac{\ell_{1-2} + \ell_2}{n_2}$$

The subscript 1--2 indicates that only the length of first order streams flowing directly to second order segments should be included. Similarly the average accumulated length of third order channels includes the length of the limited third order segment, the total accumulated length of second

FIG 16 BASIN $(1)_S$

PLOTS OF THE HORTON LAWS

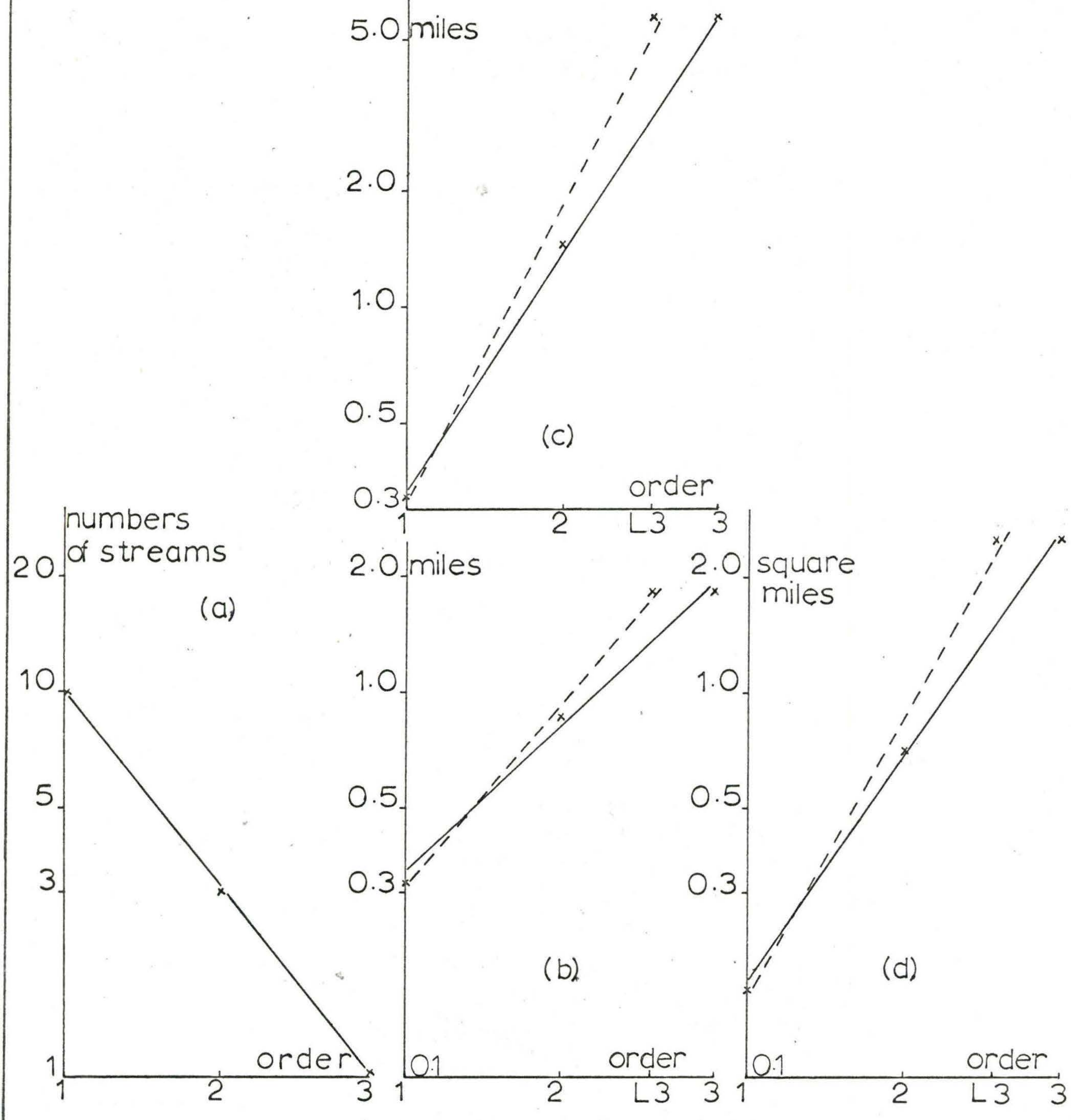


FIG 17 BASIN $(2)_S$

PLOTS OF THE HORTON LAWS

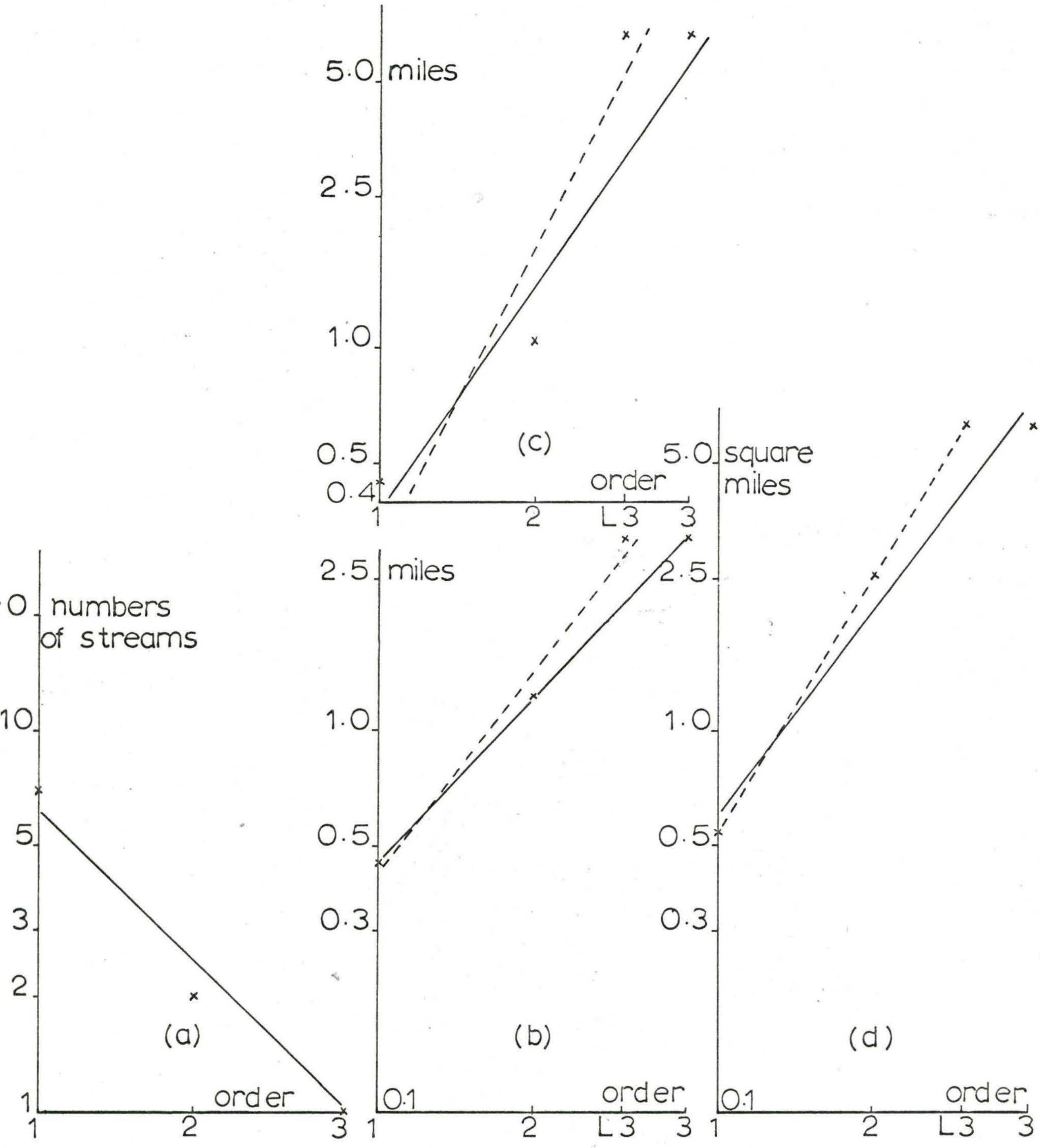
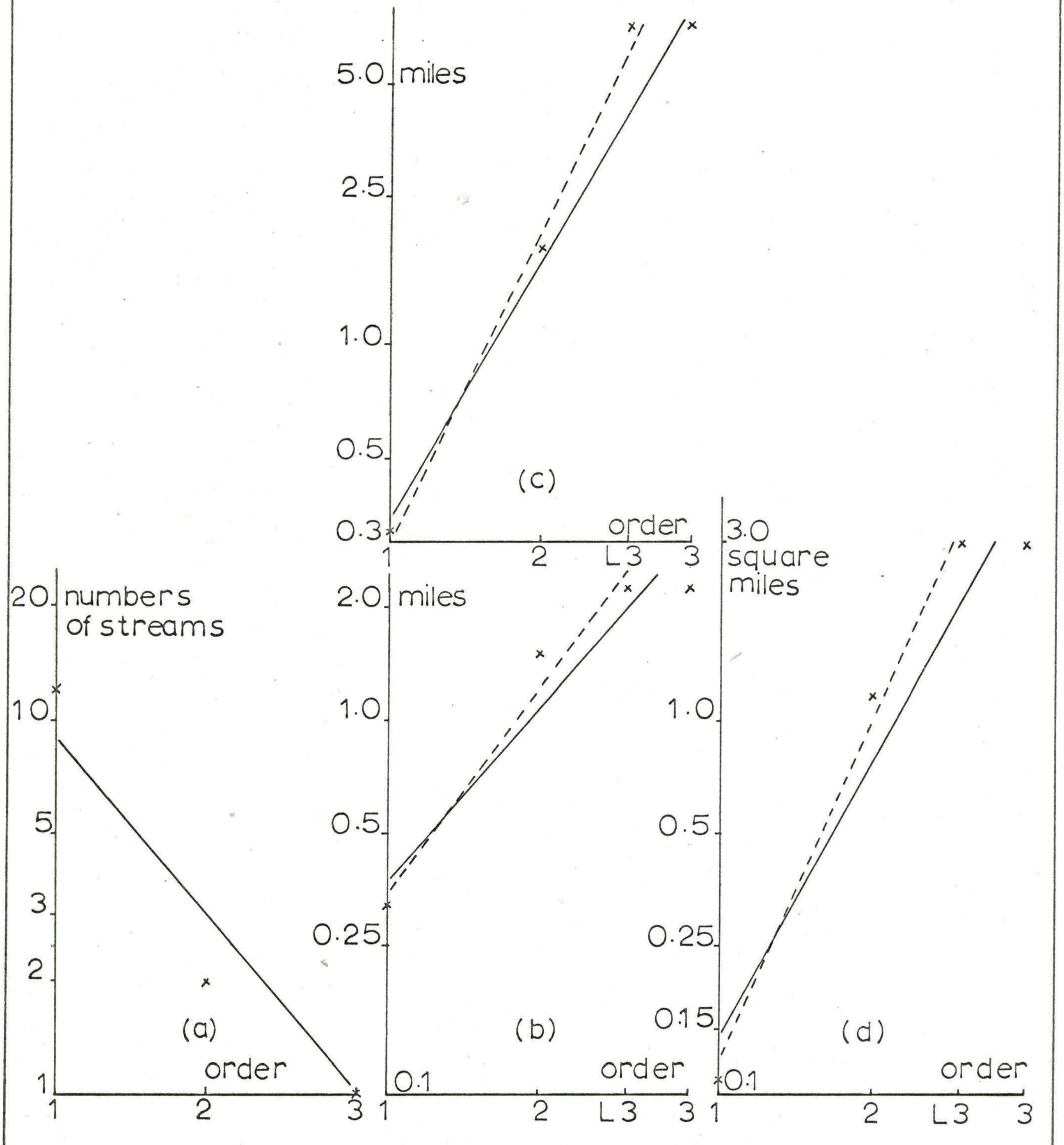


FIG 18 BASIN (3)$_S$

PLOTS OF THE HORTON LAWS



order channels as defined above, and the total length of all first order tributaries which flow directly into the third order channel; where there are streams of higher than third order within the basin, the accumulated length of third order segments includes only the length of those channels which are directly tributary to these third order segments.

i.e. $$\ell_3 = \frac{\ell_{1--3} + \ell_{2--3} + \ell_3}{n}$$

In the case of third order basins such as the chosen Ontario basins, the average accumulated length of third order segment is obviously equal to the total channel length within the basin. Thus in this experimental re-expression of the definition of average stream length in the Horton Law of Stream Lengths,

$$\ell_3 = \Sigma L$$

The definition of the length of first order channels remains unchanged:

$$\ell_1 = \frac{\Sigma \ell_1}{n_1}$$

The above definitions of stream length were then tested for a direct geometric series relationship with order by a plot of the relevant data for each basin on semi-logarithmic paper as convention demands. The resultant plots are recorded in Figs. 7c to 24c. Table VI(b) lists the values of the ratios $r_\ell(2:1)$, $r_\ell(3:2)$, and $\bar{\bar{r}}_\ell$ for the average accumulated segment length data; as with ratios derived from the values of cumulative mean stream lengths, deviations within each basin from the mean length ratio are also recorded in this table. The average accumulated segment length ratios are higher than the corresponding ratios of cumulative mean

TABLE VI(b)

LENGTH RATIOS - AVERAGE ACCUMULATED SEGMENT LENGTHS

BASIN	$r_\ell(2:1)$	$r_\ell(3:2)$	$\bar{r}_\ell$	DEVIATION FROM $\bar{r}_\ell$
$(1)_c$	2.36	4.49	3.43	1.06
$(2)_c$	3.22	2.18	2.70	0.52
$(3)_c$	5.00	5.56	5.28	0.28
$(4)_c$	5.72	3.36	4.54	1.18
$(5)_c$	6.88	2.13	4.51	2.38
$(6)_c$	6.22	2.43	4.33	1.90
$(7)_c$	3.03	2.04	2.54	0.50
$(8)_c$	3.89	3.07	3.48	0.41
$(9)_c$	4.78	2.43	3.61	1.18
$(1)_s$	4.53	3.97	4.25	0.28
$(2)_s$	2.29	6.51	4.40	2.11
$(3)_s$	5.69	3.90	4.79	0.89
$(4)_s$	5.97	2.27	4.12	1.85
$(5)_s$	6.19	3.29	4.74	1.45
$(6)_s$	2.70	4.22	3.46	0.76
$(7)_s$	3.74	2.65	3.19	0.54
$(8)_s$	5.37	2.82	4.09	1.27
$(9)_s$	3.56	4.12	3.84	0.28

$$r_\ell(2:1) = \ell_2/\ell_1$$

$$r_\ell(3:2) = \ell_3/\ell_2$$

$$\bar{r}_\ell = \frac{r_\ell(2:1) + r_\ell(3:2)}{2}$$

segment lengths, and the deviation from the mean ratio are similarly higher. To permit a direct comparison with the two plots of cumulative mean segment lengths, these average accumulated segment lengths were plotted both against order and against the logarithm of order. In this case, the crosses in Table VII indicate that in 10 out of 18 basins, the points lay closer to a straight line when the lengths were plotted with the logarithm of order on the abscissa.

Although the deviations from the mean ratios of average accumulated segment lengths are higher than deviations from the mean ratios of mean cumulative segment lengths in 15 out of 18 basins, a visual comparison of the lines of best fit in plots (b) and (c) Figs. 7 to 24was nevertheless attempted. The stars in Table VII indicate surprisingly that in 12 cases out of 18, a plot of average accumulated segment lengths against either the arithmetic value of the order number, or the logarithm of order, resulted in points which lie closer to a straight line. Of these 12 cases, in 9 basins the approximation to a straight line in the plot of the logarithm of average accumulated segment lengths against the logarithm of order number was the most visibly satisfactory of the four techniques of plotting attempted for the individual basins.

The Law of Stream Areas

Since Schumm (1956) himself adopted the Strahler ordering system, the application of the third 'Horton' Law to the 18 basins of this study posed no particular problems. Schumm's Law of Drainage Areas is expressed as:

$$a_o = a_1 r_a^{(o-1)}$$

FIG 19 BASIN $(4)_S$

PLOTS OF THE HORTON LAWS

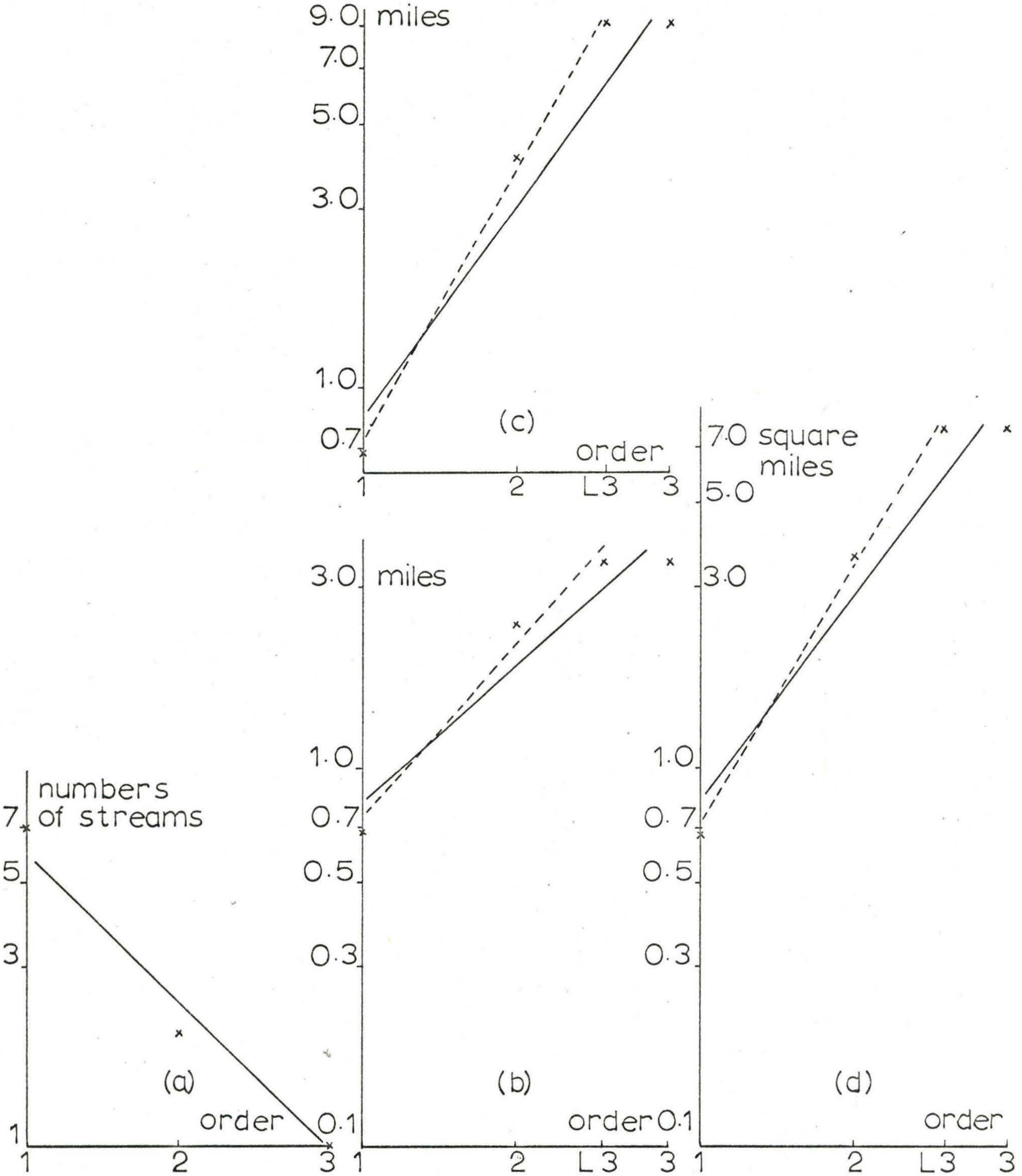


FIG 20 BASIN $(5)_S$
PLOTS OF THE HORTON LAWS

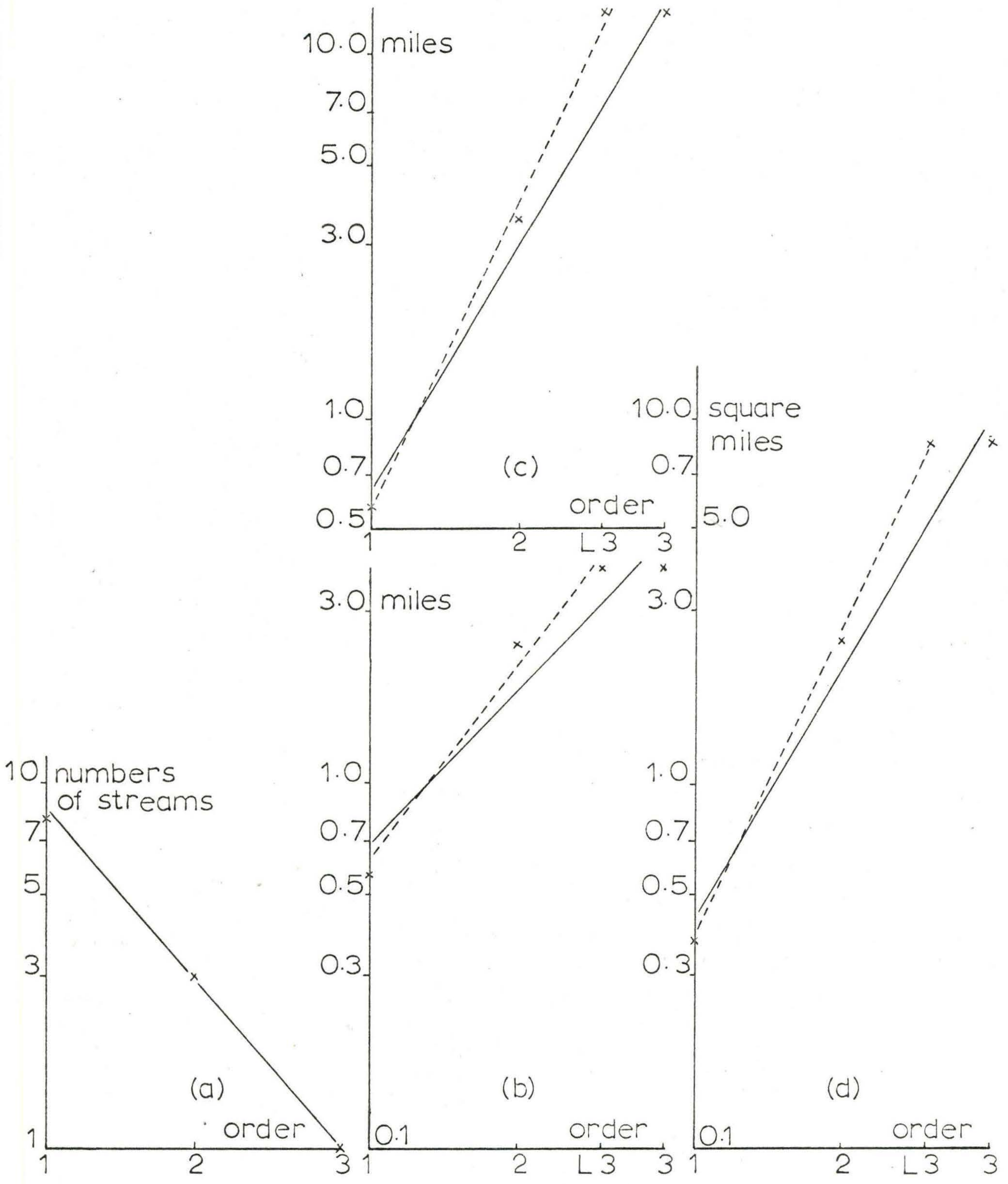
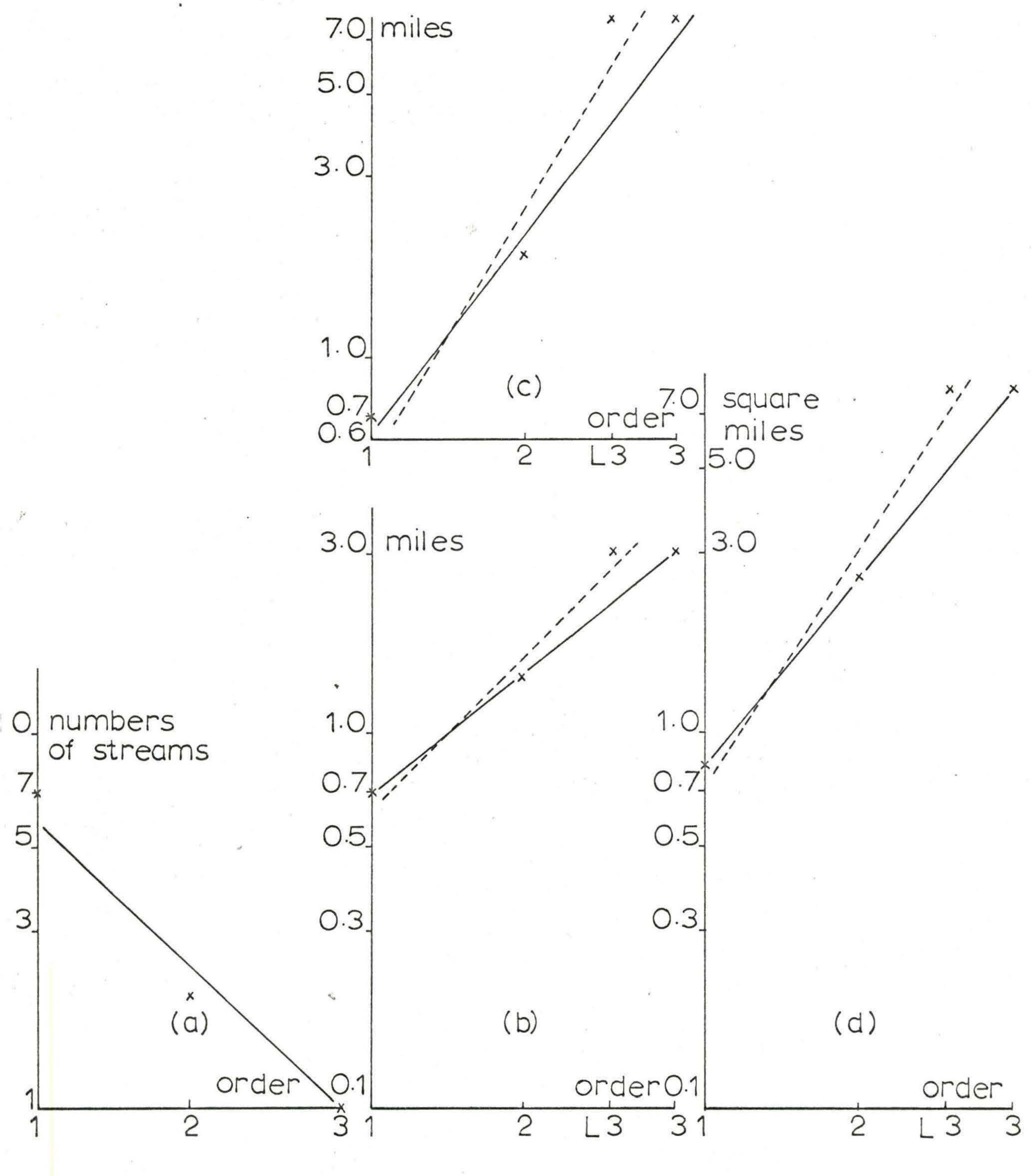


FIG 21 BASIN $(6)_S$

PLOTS OF THE HORTON LAWS



where a_o is the average drainage area of given order, a_1 is the average drainage area of first order streams, r_a is the area ratio.

For each of the basins of this study, the mean drainage area of each order was plotted against the order number on semi-logarithmic paper to test for the existence of Schumm's direct geometric series in the mean drainage areas of successive orders. If the law is applicable, the points should lie on a positively sloping straight line. The results of the plotting of the Ontario stream data can be seen in Figs. 7d to 24d. As these plots demonstrate, the three points for each basin lie reasonably close to the 'line of best fit' which was later inserted by eye; again the difficulty of projecting any line through only three points was noted. However, as with the Laws of Stream Numbers and Stream Lengths, the applicability of the laws is more conclusively demonstrated by the values of the area ratios:

$$r_a(2:1) = a_2/a_1$$

$$r_a(3:2) = a_3/a_2$$

$$\bar{\bar{r}}_a = \frac{r_a(2:1) + r_a(3:2)}{2}$$

Should the data fit a perfect geometric series, these ratios should be of equal value, and Table VIII lists a summary of the area ratios of each basin. The same practice was followed as for the tables of bifurcation ratios and the length ratios (Tables V and VI), and the deviation of the ratios $r_a(2:1)$ and $r_a(3:2)$ from the mean area ratio is also recorded. It will be seen that the values of the area ratios are on the whole higher than either the corresponding bifurcation ratios or the

TABLE VIII

AREA RATIOS - AVERAGE ACCUMULATED AREAS

BASIN	$r_a(2:1)$	$r_a(3:2)$	$\bar{r}_a$	DEVIATIONS FROM $\bar{r}_a$
$(1)_c$	8.81	4.19	6.50	2.31
$(2)_c$	3.14	2.22	2.68	0.46
$(3)_c$	8.18	5.59	6.89	1.30
$(4)_c$	4.26	3.61	3.94	0.33
$(5)_c$	5.57	2.01	3.75	1.74
$(6)_c$	6.41	2.33	4.37	2.04
$(7)_c$	3.11	2.02	2.57	0.55
$(8)_c$	3.98	2.83	3.41	0.58
$(9)_c$	4.96	2.31	3.59	1.28
$(1)_s$	4.32	3.48	3.90	0.42
$(2)_s$	4.79	2.46	3.63	1.17
$(3)_s$	10.87	2.55	6.71	4.16
$(4)_s$	5.36	2.17	3.77	1.60
$(5)_s$	6.60	3.43	5.02	1.59
$(6)_s$	3.12	3.13	3.13	0.01
$(7)_s$	3.92	2.89	3.41	0.52
$(8)_s$	5.12	2.63	3.88	1.25
$(9)_s$	4.39	4.34	4.36	0.03

$$r_a(2:1) = \frac{a_2}{a_1}$$

$$r_a(3:2) = \frac{a_3}{a_2}$$

$$\bar{r}_a = \frac{r_a(2:1) + r_a(3:2)}{2}$$

ratios of average accumulated segment lengths and the ratios of cumulative mean segment lengths. This fact is further demonstrated by the lines of best fit, which, for the plots of average drainage basin area against order, are steeper than the lines of best fit which were inserted in plots (a), (b) and (c) (Figs. 7 to 24). The deviations from the mean area ratios are similarily high, although in basins $(6)_s$ and $(9)_s$ the average drainage area increases with order in an almost perfect geometric progression; as a result the deviations from the mean ratios in these basins are 0.01 and 0.03 respectively. In basin $(3)_s$ the deviation from the mean area ratio is equal to 4.16. It will be remembered that this basin also showed an extremely high deviation of $r_b(1:2)$ and $r_b(2:3)$ from the mean bifurcation ratio. Similarily, in basin $(1)_c$ which recorded the highest deviation from the mean bifurcation ratio, the deviation of $r_a(2:1)$ and $r_a(3:2)$ from the mean area ratio is 2.31, the second highest deviation of all 18 basins. Of the remaining basins, all but two have a deviation from the mean area ratio of 1.60 or less.

Although many of the deviations are not excessively high, the alignment of the three points in many basins suggests a curve which is concave downwards. Because of this, the same device was employed as with the segment lengths; the logarithm of the mean drainage area of each order was again plotted against the logarithm of the order number in each basin (Figs. 7d to 24d). The crosses in the appropriate column of Table IX indicate whether the data lay closer to a straight line when the logarithm of the mean drainage area of each order was plotted against order on a linear scale or on a logarithmic scale on the abscissa. The

TABLE IX

PROXIMITY TO 'LINE OF BEST FIT' OF PLOTS OF AVERAGE ACCUMULATED AREA AGAINST ORDER

AVERAGE ACCUMULATED AREA	VS ORDER	VS LOG. ORDER
$(1)_c$		+
$(2)_c$		+
$(3)_c$		+
$(4)_c$	+	
$(5)_c$		+
$(6)_c$		+
$(7)_c$		+
$(8)_c$		+
$(9)_c$		+
$(1)_s$	+	
$(2)_s$		+
$(3)_s$		+
$(4)_s$		+
$(5)_s$		+
$(6)_s$	+	
$(7)_s$		+
$(8)_s$		+
$(9)_s$	+	

Summary

Average accumulated area	Better Plot
vs order	4
vs log. order	14
	18

summary of this table indicates that in 14 out of 18 basins, the plots of drainage area against the logarithm of the order number lie closer to a straight line. In the remaining four basins, the deviations from the mean area ratios are less than 0.43, and the points lie closer to the straight line which expresses the logarithm of the mean drainage area of each order as a function of the order number. Again, it must be emphasized that the decision as to whether the semi-logarithmic or the logarithmic plot is the better expression of the relationship is a subjective decision based on a visual inspection of the two plots.

The Applicability of the Horton Laws

For each basin, the data on stream numbers, lengths, and drainage areas was plotted on semi-logarithmic paper (Figs. 7 to 24), and the relevant ratios (Tables V, VI, and VIII) were included as a further demonstration. Where the stream segment numbers, lengths, and drainage areas in each basin form a perfect geometric series with order, the points plotted on semi-logarithmic paper lie directly on a straight line, and the relevant ratios obey the equations:

$$r_b(1:2) = r_b(2:3) = \bar{r}_b$$

$$r_\ell(2:1) = r_\ell(3:2) = \bar{r}_\ell$$

$$r_a(2:1) = r_a(3:2) = \bar{r}_a$$

This perfect result was found on only a small proportion of the basins. In basin $(2)_c$ the number of stream segments form a perfect inverse geometric series with order; mean cumulative segment lengths form a

51a.

FIG 22 BASIN $(7)_S$
PLOTS OF THE HORTON LAWS

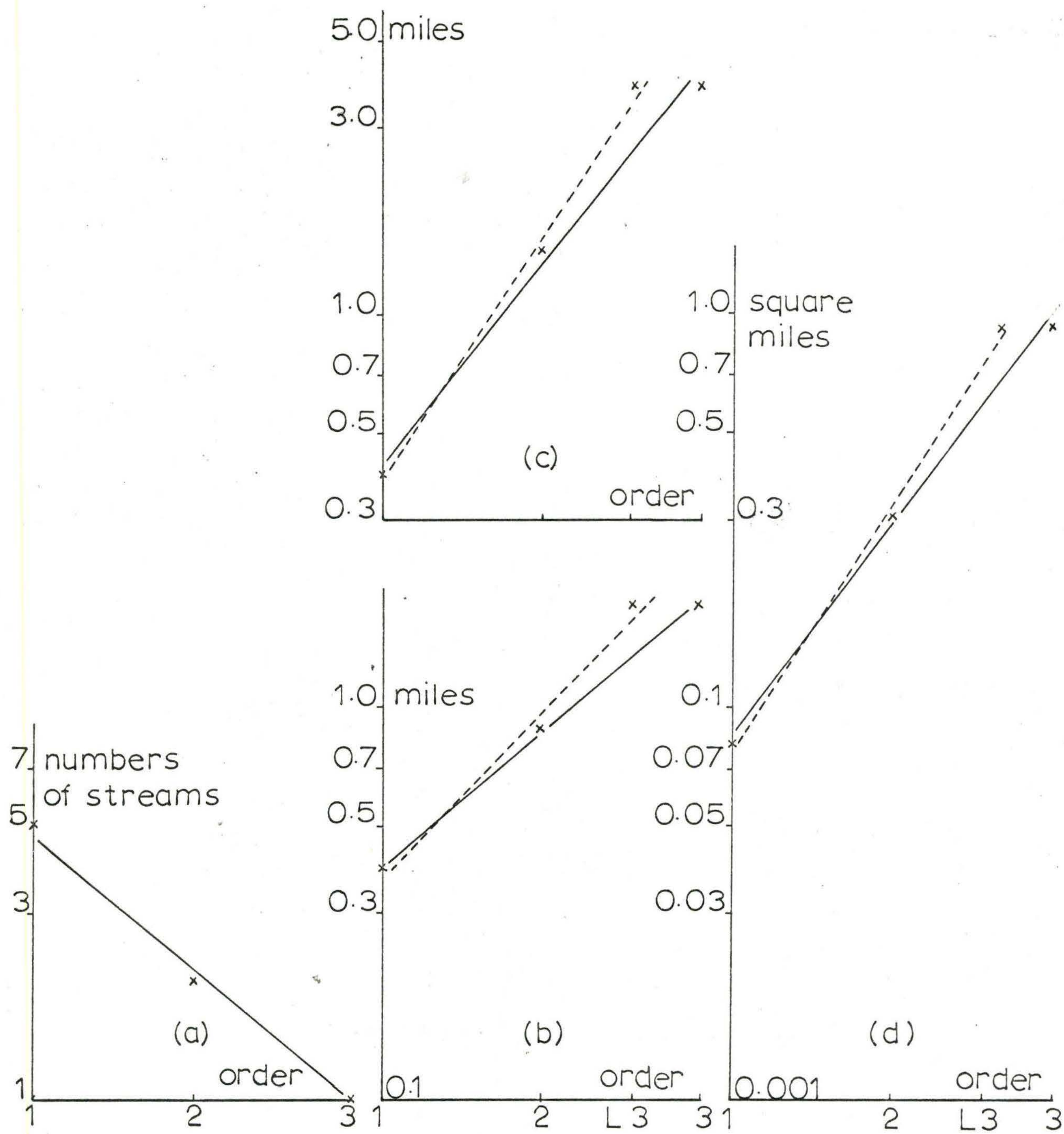


FIG 23 BASIN $(8)_S$
PLOTS OF THE HORTON LAWS

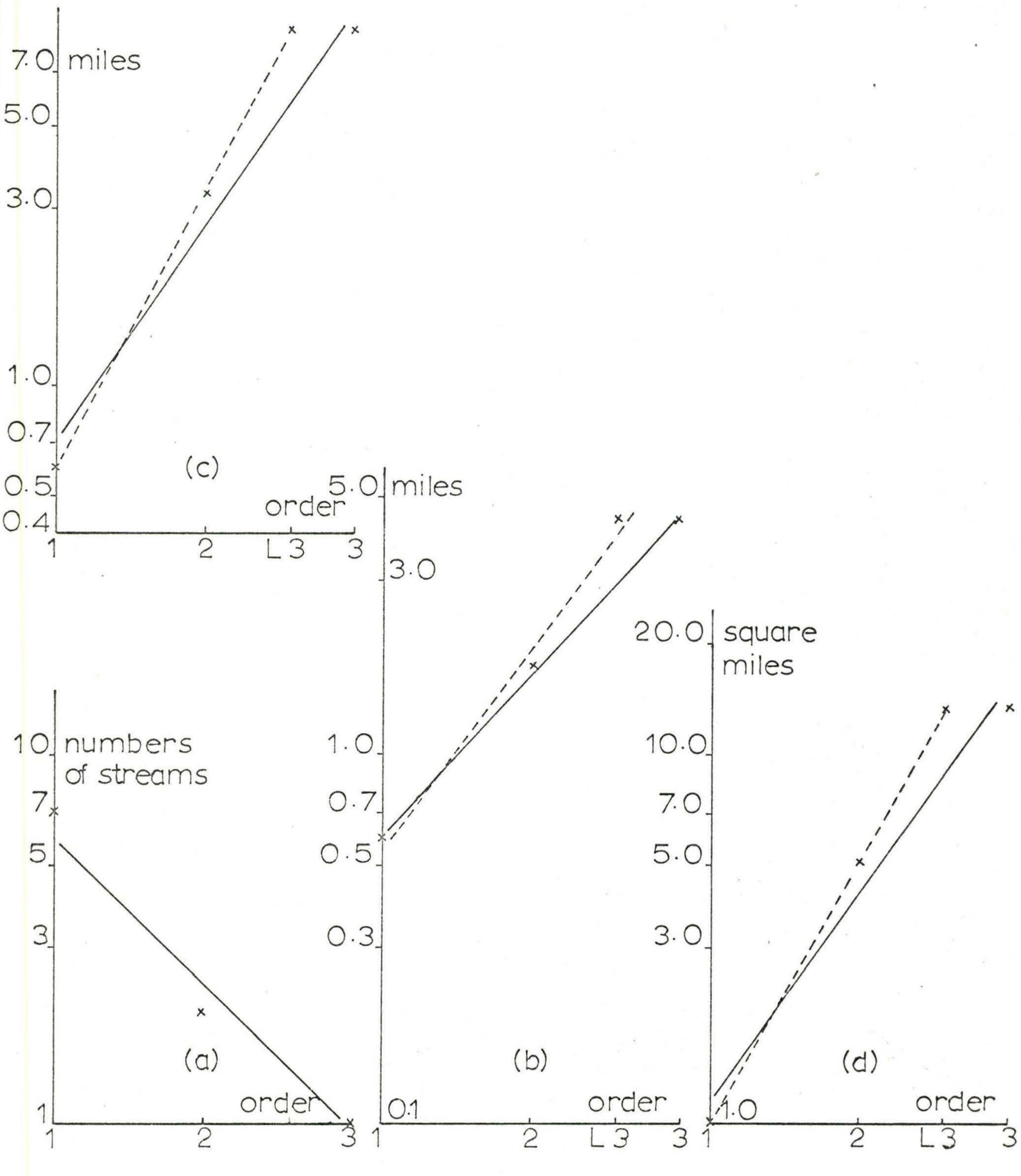
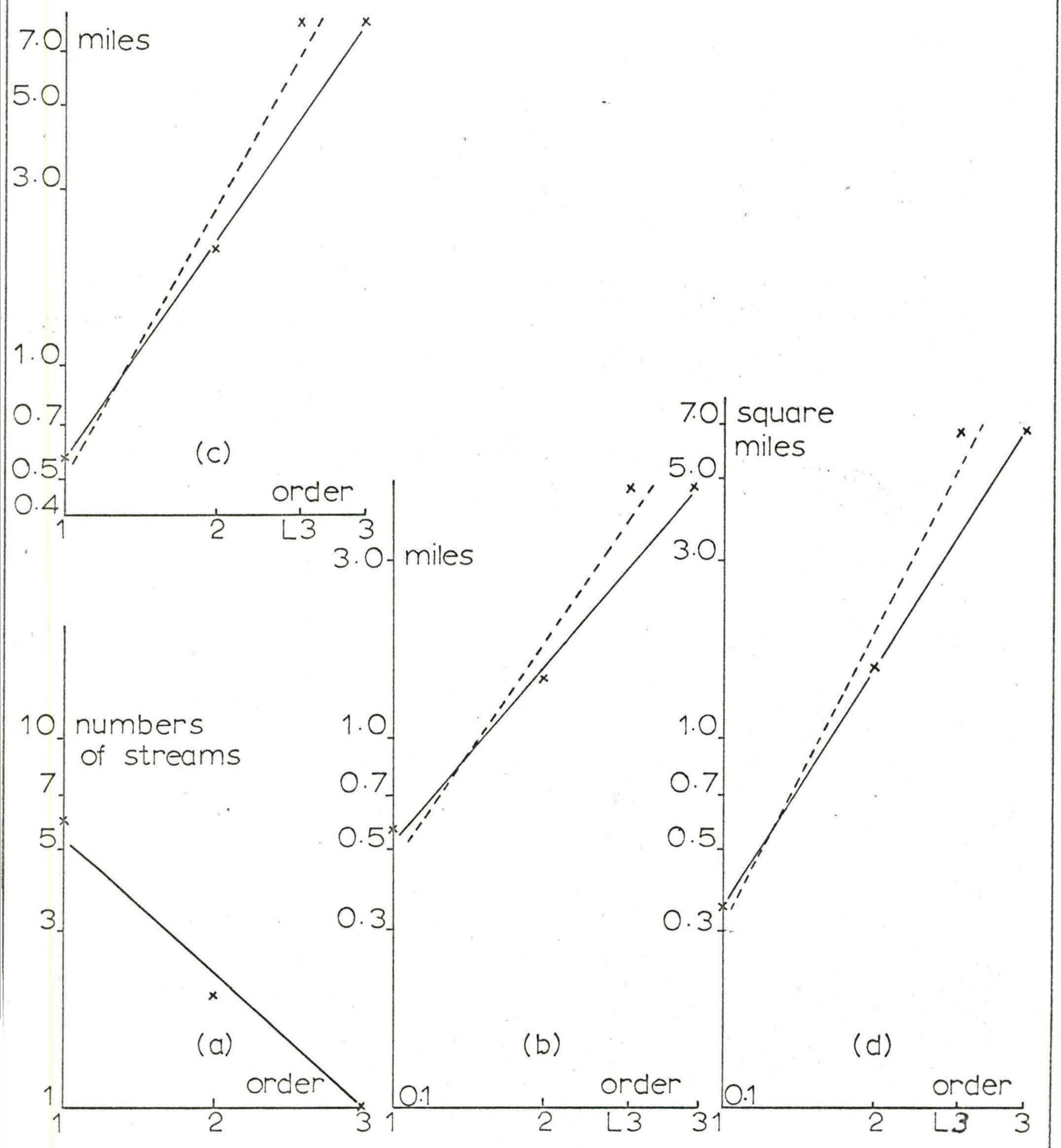


FIG 24 BASIN $(9)_S$

PLOTS OF THE HORTON LAWS



nearly perfect direct geometric series with order in basins $(1)_c$, $(2)_s$, $(6)_s$, and $(7)_s$. A test of the Law of Stream Drainage Areas demonstrated that only in basins $(6)_s$ and $(9)_s$ is there an almost perfect exponential relationship between the mean drainage areas of each order and the order number. In the remaining basins anomalies exist in the application of the Horton Laws: these are reflected in deviations from a straight line plot on semi-logarithmic paper, and in the inequality of the respective ratios.

In the first place, it must be emphasised that these laws are expressions of a situation which is approximated in nature, but which is rarely achieved to perfection. Horton (1945, p. 291) himself stated only that "the numbers of streams.......tend closely to approximate an inverse geometric series", "the average lengths of streams.....tend closely to approximate a direct geometric series", and, of his initial plotting of the data, (Horton, 1945, p. 286), he said that "plotting the data on semi-logarithmic paper it was found that the stream numbers fall close to straight lines, and the same is true of the stream lengths". Thus, low deviations from each of these geometric series laws may be satisfactorily accepted, and the applicability of the laws need not be questioned in such cases.

Generally the laws are tested merely by a plot of the data on semi-logarithmic paper as Horton suggested; the closeness of the data to the 'line of best fit' may be assessed by eye (Horton, 1945), or a regression line may be fitted to the points (Strahler, 1957). The chief difficulty with respect to these Southern Ontario streams was the problem of fitting a straight line by either of these methods to the three points on each plot. It was for this reason that as much emphasis has been laid on the deviation of the respective ratios as on the closeness of the data to a straight line indicative of a geometric relationship.

Numerous reasons may be advanced for the departures from an inverse geometric series law of stream numbers. It was stated in an earlier chapter than an attempt was made to complete the channel net, since it was felt that not all first order channels were inserted on the official topographic map sheets. If this attempt was not entirely successful, then the numbers of streams of each order would be under-estimated and the bifurcation ratios would deviate. The air photograph check of basin $(5)_c$ and $(1)_s$ indicate that the number of first order streams has almost certainly been underestimated in these basins; if these streams had been included, the number of second order segments would also have been raised. The photographic analysis suggested that basins $(5)_c$ and $(1)_s$ are of fifth order, and first order tributaries on the map sheets should correspond to third order segments on the photograph. However, in basin $(5)_c$, there are 15 third order basins on the photograph, as compared with 8 first order streams on the map sheet. Similarily in basin $(1)_s$ 17 third order stream segments were ennumerated on the photograph, yet only 10 first order segments were included in the drainage net from the topographic map. In contrast to the poor fit of the Law of Stream Numbers for map drainage nets, the numbers of each of the five orders of stream segments in the photographic analysis of basin $(5)_c$ and $(1)_s$

lie fairly close to a straight line.

It thus seems reasonably certain that the prime cause of deviation from the Horton Law of Stream Numbers in the map analysis of the basins is the inadequate representation of the drainage net on the map sheet, which could not be satisfactorily rectified by a subjective completion of the drainage net based on contour information.

Considerable dissatisfaction was expressed in Chapter IV at the 'fit' of the Law of Stream Lengths, and as a result four alternative expressions of this law were offered. The logarithms of cumulative mean segment lengths were tested for a straight line relationship with both the order number and the logarithm of the order numbers; similarily, the logarithm values of an alternate definition of stream segments lengths, average accumulated segment lengths, were tested against order number and against the logarithm of order for a straight line plot. In retrospect, it seems likely that the best expression of the behavior of the Ontario streams is given by a direct exponential relationship between the cumulative mean segment lengths of successive orders and the order number. Although the graphical representation of this relationship is on the whole less satisfactory than the plot of average accumulated segment lengths, the deviations of $r_{\ell}(2:1)$ and $r_{\ell}(3:2)$ from $\bar{\bar{r}}_{\ell}$ in the former case are considerably lower. This is considered a much more sensitive test of the approximation to a direct geometric series law than the plot on semi-logarithmic paper, since the log scale on the ordinate of this paper may be used to obscure many discrepancies. In the same manner, the use of a logarithmic scale on the abscissa which was attempted for both definitions of stream length is considered a device which on the whole merely reduces deviations from a straight line due to the scale change rather than being demonstrative of a logarithmic relationship between stream segment lengths and order. The length of stream segments is closely related to the area drained by these segments (Leopold, Wolman, and Miller 1964, p. 134); as a result, possible reasons that plots of both cumulative mean segment length and average accumulated segment lengths against order number in each basin apparently fit curves which are concave downwards[1] may be discussed with respect to the same phenomenon in the plots of the Law of Basin Areas. However, specific mention must be made of the extreme inequality of the length ratios in basins $(5)_c$ and $(6)_c$. The ratios of cumulative mean segment lengths in these basins are:

$(5)_c$	$r_\ell(2:1) = 3.74$	$r_\ell(3:2) = 1.24$
$(6)_c$	$r_\ell(2:1) = 4.14$	$r_\ell(3:2) = 1.14$

Both of these streams flow directly into Lake Erie, and at their mouths, the action of the Lake has produced a high cliff face. As a result, the limited third order segment in each of these basins is extremely short, and in fact the length of the third order segment is less than the average length of first order segments in the respective basin.

The graphical plots of the logarithmic value of the average area drained by segments of each order against order number are reasonably satisfactory. A visual examination of the lines of best fit suggested that

---

[1]This is further demonstrated by the fact that in almost every basin, the ratio $r_\ell(3:2)$ is less than the value of the ratio $r_\ell(2:1)$ for both definitions of segment length. For cumulative mean segment lengths, the value of the ratio $r_\ell(2:1)$ is more than twice as great as the value of $r_\ell(3:2)$ in basins $(9)_c$, $(3)_s$, and $(4)_s$, and $(5)_s$.

for these Southern Ontario basins, average drainage areas do tend to approximate a direct geometric series. The area ratios nevertheless indicate that these approximations to a geometric series are not as close as the visual assessment of the plots indicated. It is true that in basins $(6)_s$ and $(9)_s$ the deviations from the mean area ratios are respectively .01 and .03, demonstrating that a near perfect geometric series relationship is present in these basins; but, on the other hand, in another 10 basins, the deviations of $r_a(2:1)$ and $r_a(3:2)$ from $\bar{\bar{r}}_a$ are more than 1.00. Further, a visual comparison of the closeness of fit of the data to a semi-logarithmic straight line and to a logarithmic straight line which was also plotted, indicated that in 14 basins out of 18 the latter plot was a better expression of a straight line relationship. In most basins, the points of drainage area plotted against order number on semi-logarithmic paper lie on a curve which is concave downwards; this is confirmed by the fact that in all but one of the basins, the ratio $r_a(3:2)$ is less than the ratio $r_a(2:1)$. In seven of the basins the value of $r_a(3:2)$ is less than half of the value of $r_a(2:1)$. There are two possible conclusions to be drawn from this: the area drained by second order segments (and similarily the length of these second order segments) may be too large, or, on the other hand, the area drained by third order segments may be smaller than is necessary for a direct geometric series between the drainage areas of first, second and third order. The evidence of the semi-logarithmic plots of drainage area and order number for the five orders of the drainage nets of basins $(5)_c$ and $(1)_s$ which were found from air photographs suggests that the departure occurs in the third order basins of the map sheets; in these plots of the photograph data, the first four orders approximate closely to a direct geometric series of drainage areas, but the plot of the fifth order drainage area against the order number lies below the straight line projected through the points of the first four orders.

Three possible explanations may be advanced for this phenomenon, whether the discrepancies occur in the second order basins or in the third order basins. It is possible to suggest that the Horton Laws of Stream Lengths and Drainage Areas are inapplicable to these drainage basins of Southern Ontario. This seems an unnecessarily premature decision, particuarly when in some basins the highly sensitive ratios indicate the presence of a nearly perfect exponential relationship between order and drainage areas and order and cumulative segment lengths; the application of the Horton Laws to the expanded data found from the basins studied on aerial photographs was also reasonably satisfactory.

In the second case, it must be remembered that these basins are developed on highly variable glacial materials; although the choice of basins was limited to 'sand plain' and 'clay plain' areas (Chapman and Putnam, 1952), these general terms may well cover a multitude of lithological variations. The possibility of great intra-basin variations in lithology has by no means been eliminated by the initial selection of these areas of glacial materials. Because the time period which has elapsed since the disappearance of glacial ice and glacial lakes from this region has hardly allowed sufficient time for the modification or elimination of such intra-basin variations, it is highly likely indeed that the selected basins show great lithological variations within themselves. Horton (1945, p. 303) himself states that "departures from the two laws (of stream numbers and stream lengths) will, however, be observed, and if other conditions are normal, these departures may in general be ascribed to effects of geological controls". The writer considers it highly possible that intra-basin variations in lithology contribute to the observed departures from the Horton Laws, yet it seems unlikely that such variations can completely account for the extremely common tendency for plotted points of the Horton data to fit a downward concave curve.

The third explanation which is offered here may in many ways be considered an extension of the second theory. As well as possessing a variable lithology, it seems highly credible and even expectable that these sand and clay plain deposits of the glacial lakes had a certain amount of topographic variation on their surfaces even immediately after the retreat of the lake waters. It is quite plausible that this glacial surface was sufficiently irregular that 'basins' may have existed to control and determine the development of drainage nets and stream basins. If this is the case, the drainage basins of Southern Ontario may owe a considerable legacy to the initial configuration of the post-Pleistocene surface, and again the time which has elapsed since the end of the Pleistocene has likely been insufficient for the elimination of these controls. "Stream lengths (and drainage basin areas).....may definitely be limited by geologic controls such as fixed boundaries of the outline of the drainage basins". (Horton, 1945, p. 303). Under these circumstances, one can imagine restrictions of basin outlines which have resulted in the observed situation where the drainage areas of second order segments (and the lengths of these segments) are too large or where the third order basin areas are limited to an areal size smaller than that required to obey a direct geometric series law.

It seems probable that the recorded departures from the Horton Laws in the stream basins of Southern Ontario are the result both of intra-basin variations in lithology and of the basin boundary control imposed by the configuration of the post-Pleistocene surface. The writer does not consider that these discrepancies are sufficient to warrant the statement that the Horton Laws of Stream Composition are inapplicable to these stream basins developed on glacial materials.

CHAPTER V

ANALYSIS OF AERIAL PHOTOGRAPHS

Basin $(1)_s$, Brantford East

This basin, which was initially analysed from the 1:50,000 map of Brantford East, was also located on a set of aerial photographs. By comparison with the map sheet, the scale of these photographs was calculated to be approximately 1:16,000. The basin was outlined and the stream net marked by means of a stereoscopic analysis of successive pairs of photographs. The employment of this technique of three dimensional analysis permitted both the watershed and the drainage net of the basin to be drawn considerably more accurately and in considerably greater detail than had been possible from the information on the contour map. Further, the larger scale of the air photographs naturally allowed the recognition of greater detail than is printed on the map sheet. As a result of the larger scale and increased detail, the first and most obvious discrepancy of the basin as it was drawn from the map sheet became apparent through the stereoscope analysis. Due to insufficient contour information, the actual line of the water-shed lay approximately half a mile further east than it was drawn from the 1:50,000 map sheet; the drainage basin of this stream is actually larger than it appeared from the map analysis. This factor alone was enough to suggest that there would be little correlation between parameters measured from the map basin and those measured from the basin as it appeared on the air photographs. Table X shows this to be the case.

Due primarily to this mistake in the line of the water-shed, but

TABLE X

BASIN $(1)_s$ - PHOTOGRAPH AND MAP PARAMETERS

	n_1	n_2	n_3	n_4	n_5	N	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ΣL
Map			10	3	1	14			.3157	.8681	1.8151	5.76
Photograph	412	100	16	3	1	532	.07	.21	.51	1.21	4.81	52.21

	a_1	a_2	a_3	a_4	a_5
Map			.1673	.72	2.51
Photograph	.005	.02	.12	.49	3.48

	$r_b(1:2)$	$r_b(2:3)$	$r_b(3:4)$	$r_b(4:5)$	$\bar{r}_b$	$r_{\ell}(2:1)$	$r_{\ell}(3:2)$	$r_{\ell}(4:3)$	$r_{\ell}(5:4)$	$\bar{r}_{\ell}$
Map			3.33	3.00	3.17			2.72	2.09	2.41
Photograph	4.12	6.25	5.33	3.00	4.68	3.22	2.42	2.37	4,76	3.19

	$r_a(2:1)$	$r_a(3:2)$	$r_a(4:3)$	$r_a(5:4)$	$\bar{r}_a$
Map			4.32	3.48	3.90
Photograph	5.05	5.03	4.20	7.06	5.34

	D_d	F	R_e	R_c
Map	2.29	5.57	0.64	0.59
Photograph	14.99	152.82	0.52	0.35

also to the scale difference, few of the basin parameters are directly comparable in size. Since the air photograph analysis resulted in a basin which was of fifth order, the third, fourth, and fifth orders of the photograph basin should be comparable with orders one, two and three respectively of the map basin. In this case, the greater detail permitted by the air photograph analysis raised a basin which was enumerated as third order from the map to a fifth order basin. The comparison between each pair of orders as regards stream numbers is reasonable. There are 10 segments of first order in the map basin as compared with 16 third orders basins in the photograph basin, and there are three second order basins on the map, and three fourth order basins on the photograph; there is, of course one third order segment in the map basin, and one segment of fifth order on the photograph. However, the average segment lengths (cumulative mean segment lengths) and average drainage areas of each order do not seem to show any reasonable relationship between the map basin and the photograph basin. Thus, for example, the average segment length of fourth order on the photograph is 1.21 miles, whereas the average length of second order segments on the map is 0.87 miles; similarily, the length of the fifth order segment on the photograph (4.81 miles) is considerably longer than the third order map segment (1.82 miles).

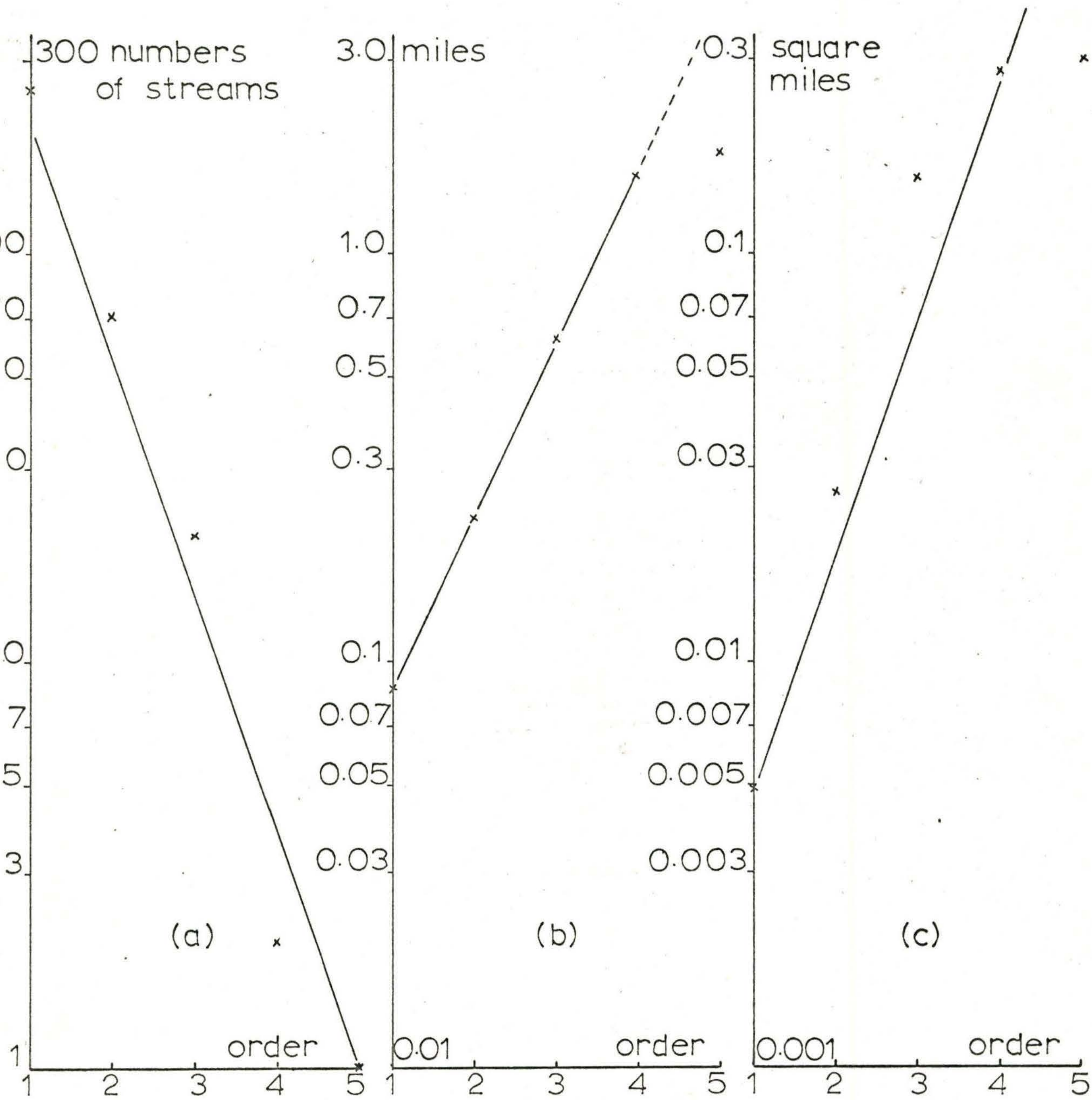
Since the size of the basin was extended on the air photograph when the actual position of the water-shed was located, the discrepancy between the total size of the basin on the photograph as compared with the map is not unexpected, but this same discrepancy is not reflected in the average areas of the lower order basins. In fact, the average drainage areas of first and second order basins on the map are larger than the average drainage areas of third and fourth order from the photograph. Since the contour information of the map sheet has been shown to be inadequate, it may be that this same inadequacy is reflected in the fact that some streams which apparently did not have tributaries from the map information may well actually be of second order magnitude within the map ordering scheme. As a result, these basins are larger than more characteristic first order basins, and have given rise to the situation where the average area of first order map basins is larger than the average area of third order photograph basins, although the size of the entire basin has been underestimated on the map sheet.

Not unexpectedly, figures for drainage density and stream frequency on the photograph and on the map are also at variance. In each case, figures for the photograph basin are much higher, but the greater number of stream segments and stream orders detailed on the photograph account for this discrepancy. More surprisingly perhaps, the values for the Elongation Ratio and the Circularity Ratio are lower when measured from the aerial photographs. From the photographs, these values are respectively 0.52 and 0.39, whereas values of 0.64 and 0.59 respectively were calculated from the map basin. It is suggested that much of this discrepancy is due to the inaccurate marking of the water-shed from the map sheet. The basin as it was outlined from the photographs is considerably longer, but its width is little different from the width of the basin on the map sheet.

Both the map and the photograph basins fit reasonably well to the Horton Laws of Stream Composition. In both cases, there is a close approximation to a straight line when stream numbers are plotted against order (Fig. 16a, and Fig. 25a). From the map, cumulative mean segment lengths

62a

FIG 25 BASIN $(5)_c$
(photograph data)
PLOTS OF THE HORTON LAWS



of each order join to form a curve which is concave downwards rather than perfectly straight (Fig. 16b). The photograph data on the other hand, tends to plot fairly close to a straight line, but with a degree of upward concavity in the higher orders (Fig. 25b). The data for the Law of Stream Areas both from the map analysis and from the photographic analysis gives an extremely good fit to a straight line; deviations from the 'line of best fit' are very low.

A 't' test was set up to determine whether the map and photograph parameters for this basin are at all comparable in statistical terms. The average area of first order basins from the map analysis was compared with the average area of third order basins from the photograph analysis. With 24 degrees of freedom, the calculated t value is only 0.4694, and it is accepted that there is no significant difference between the mean values. On the basis of this result, it is assumed that despite the obvious differences in the values of the parameters, the photograph and map data on the same basin are related, and it is postulated that the observed differences are the result mainly of the scale and detail contrast between the map and the photograph.

Basin $(5)_c$, and Simcoe East

A set of aerial photographs of the area bounding Lake Erie in Norfolk County gave a complete coverage of the area of basin $(5)_c$ which was originally studied from the 1:50,000 map of Simcoe East. By comparison with the printed map sheet, the scale of the photographs was calculated as approximately 1:15,000, and using this information, a stereoscopic analysis of this basin was made. Unlike basin $(1)_s$, this examination of the basin in three dimensions did not reveal any obvious mistakes in the form of the basin as it had been outlined on the topographic map sheet. Nevertheless, scale differences and the greater detail of the air photographs resulted in differences in the values of many of the basin parameters between the map analysis and the photographic analysis.

As with the air photograph examination of basin $(1)_s$, this third order map basin was elevated to fifth order under the greater detail of the air photographs. Thus the first, second, and third orders of the map basin will be compared with the third, fourth, and fifth orders of the photograph basin. There are more third order segments in the photograph basin (15) than there are first order segments in the map basin, (8), but there are the same number of fourth and second order segments (two), and of fifth and third order segments (one). The comparison between the cumulative mean segment lengths of the photograph and mapbasins is, however, much closer than the same comparison in basin $(1)_s$. As can be seen from Table XI, the average length of map first order segments is 0.43 miles, whereas the average length of photograph third order segments is 0.62 miles. Second order map segments and fourth order photographs are even closer in average lengths, being 1.61 and 1.59 miles respectively. Discrepancies between the average drainage areas of each order are somewhat larger, since the average drainage area of first order map basins is 0.28 square miles compared with an average drainage area of 0.15 square miles for the photograph third order basins. The greatest difference occurs between the drainage areas of second order map basins (1.57 square miles) and the fourth order photograph basins

TABLE XI

BASIN $(5)_c$ - PHOTOGRAPH AND MAP PARAMETERS

	n_1	n_2	n_3	n_4	n_5	N	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ΣL
Map			8	2	1	11			0.43	1.61	2.00	6.31
Photograph	263	71	15	2	1	352	0.08	0.23	0.62	1.59	1.79	39.06

	a_1	a_2	a_3	a_4	a_5
Map			.28	1.57	3.15
Photograph	0.005	0.03	0.15	2.87	2.92

	$r_b(1:2)$	$r_b(2:3)$	$r_b(3:4)$	$r_b(4:5)$	$\bar{r}_b$	$r_\ell(2:1)$	$r_\ell(3:2)$	$r_\ell(4:3)$	$r_\ell(5:4)$	$\bar{r}_\ell$
Map			4.00	2.00	3.00			3.74	1.24	2.49
Photograph	3.70	4.73	7.50	2.00	4.48	2.88	2.69	2.56	1.13	2.32

	$r_a(2:1)$	$r_a(3:2)$	$r_a(4:3)$	$r_a(5:4)$	$\bar{r}_a$
Map			5.57	2.01	3.79
Photograph	5.36	5.67	18.91	1.02	7.74

	D_d	F	R_e	R_c
Map	2.00	3.49	0.79	0.88
Photograph	13.38	120.54	0.66	0.56

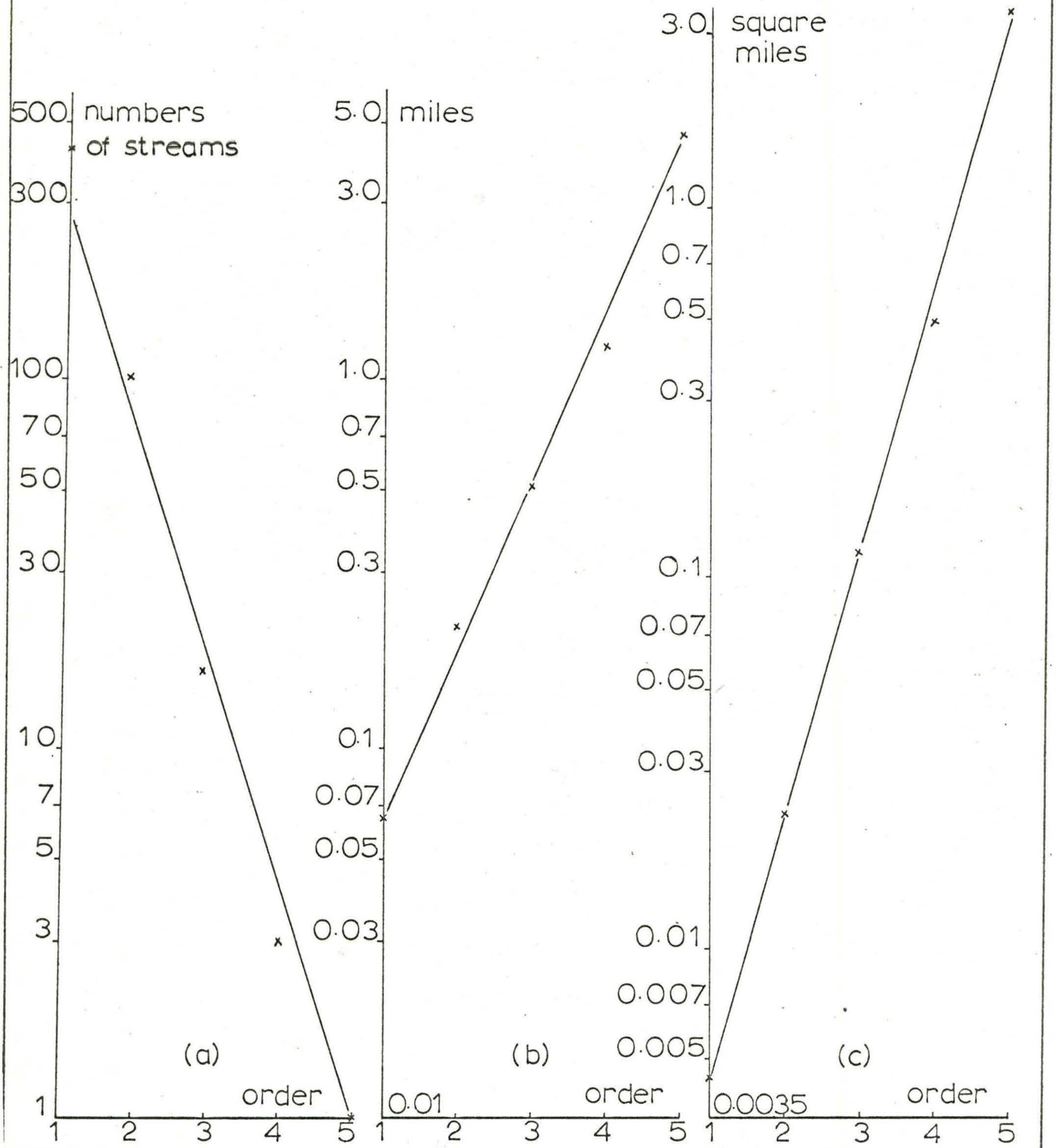
(2.87 square miles). The total drainage area of basin $(5)_c$ as it was measured from the map sheet (3.15 square miles) is very similar to the value of the same unit as it was measured from the air photographs (2.92 square miles). It seems very likely that the discrepancies in the lower orders are again the result of inaccurate ordering of the map basins due to a lack of sufficient contour information on the topographic sheet by means of which the channel net was completed.

As in basin $(1)_s$ from the Brantford East map sheet, values of drainage density and stream frequency are much higher when these parameters are calculated from the greater channel detail of the air photograph. However, in both basins, the differences between the values of these properties in map basins and in photograph basins is of the same order of magnitude. Drainage density is approximately seven times higher on the air photograph of basin $(1)_s$ and of basin $(5)_c$ than it is on the relevant map sheets, and stream frequency as it is calculated from the air photographs is approximately 30 times higher in both examples. Values of the Elongation and Circularity Ratios, on the other hand are higher from the map basin $(5)_c$ than they are from the corresponding photograph basin: the value of the Elongation Ratio from the map basin is 0.79 as compared with a calculated value of 0.66 from the photograph basin, and the value of the Circularity Ratio for the map and photograph basin is 0.88 and 0.56 respectively. It is postulated that this difference may be attributed to the greater accuracy with which it was possible to mark the water-shed from the air photographs.

Neither the map nor the photographic data produces results which exhibit a satisfactory fit to the Horton Laws of Streams Compostion. In both cases even the stream numbers of each order do not join to form a good straight line; it seems that there should be more than two second order map segments and two fourth order photograph segments to produce a reasonable approximation to a logarithmic series of stream numbers. The extremely low length of the third order segment in the map basin has already been discussed in Chapter 4, and this is also reflected in the extremely low length of the fifth order segment of the photograph basin. It can be seen from Fig. 26b that the average lengths of the segments of the first four orders of the photograph basin lie very close to a straight line on semi-logarithmic paper, and this would seem to confirm the suggestion which was advanced in Chapter 4. It was postulated that the erosive action of Lake Erie had removed much of the lower portion of the highest order segment in the basin. The plot of the Law of Stream Areas would also seem to support this suggestion, since in both Fig. 11d (map basin) and Fig. 26c (photograph basin), the points form a line which is markedly concave downward indicating that part of the mouth of the basin has quite probably been removed.

A 't' test was also used to test the relationship between the mean value of the third order photograph drainage areas and the mean value of the first order drainageareas on the map basin. With 21 degrees of freedom, the null hypothesis that there is no significant difference between these means was rejected; the calculated t value of 2.057 is significant between the 95 and 97.5 percentiles. Thus it would seem that the comparison between basin $(5)_c$ as it is represented on the topographic map sheet and as it is drawn from the much larger scale air

FIG 26 BASIN $(1)_S$
(photograph data)
PLOTS OF THE HORTON LAWS



photographs is not statistically valid. It seems that this is in part due to the lack of accurate detail, particularily of contour information, on the topographic map sheets, and it may be that there is a certain amount of tilt distortion of the air photographs.

Basin $(1)_c$, Blackheath

A final air photograph check was chosen to represent the third grouping of drainage basins in this study. Both basins $(1)_s$ and $(5)_c$ were initially studied from maps on a scale of 1:50,000, whereas basin $(1)_c$ was analysed from a topographic map on a scale of 1:25,000. The air photograph coverage of this basin was, however, on a smaller scale than the coverage of either basin $(1)_s$ or basin $(5)_c$. By comparison with the Blackheath map sheet, it was determined that in fact the photographs which covered basin $(1)_c$ are on a scale of approximately 1:28,000, and thus these photographs are actually on a smaller scale than the map sheet.

Since the topographic map and the photograph coverage of basin $(1)_c$ are actually much closer in scale than was the case with the coverage of basins $(1)_s$ amd $(5)_c$, it was anticipated that there might possibly be less divergence between the value of parameters measured from the map basin and from the photograph basin. This is not entirely true, although on the whole differences in the values of the parameters do tend to be less. The greater detail which can be noted during a stereoscopic analysis of the photographs is a primary cause of divergence in this example. Unlike basins $(1)_s$ and $(5)_c$, only four orders of stream were recognized on the photographs of basin $(1)_c$. It is thus suggested that the order of the basin has only been raised by one order as a result of the air photograph analysis, and orders one, two, and three from the map analysis are assumed to be equivalent to orders two, three and four respectively in the photograph basin.

Stream numbers diverge rather more between the air photograph and map coverage of basin $(1)_c$. There are 17 first order, three second order and one third order segment in the map basin, as compared with 28 second order, five third order and one fourth order segment in the photograph basin. The effect of the greater detail which was visible on the aerial photographs is obvious in this instance. This same difference has presumably also resulted in the situation where the average length of first order streams on the map is 0.67 miles, whereas the average length of second order segments on the photograph is only 0.26 miles. A discrepancy of similar magnitude can be seen between the average length of map second order segments and photograph third order segments (Table XII). However, the length of the third order segment on the map (2.52 miles) is very close to the length of the fourth order segment on the photograph (2.68 miles). The fact that the total length of stream marked in the basin through the air photograph analysis is more than twice the total stream length marked in the map basin, must be attributed to the greater detail of the air photographs.

The average drainage areas of each order are much closer in size when the map basin is compared with the photograph basin. Nevertheless, the greater accuracy with which the water-shed could be drawn as a result of the three-dimensional examination of the photographs has given rise to the situation where the total area of the basin was measured as 1.51

TABLE XII

BASIN $(1)_c$ - PHOTOGRAPH AND MAP PARAMETERS

	n_1	n_2	n_3	n_4	N	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ΣL
Map		17	3	1	21		0.67	1.34	2.52	7.10
Photograph	110	28	5	1	144	0.09	0.26	0.72	2.68	19.58

	a_1	a_2	a_3	a_4
Map		0.03	0.29	1.21
Photograph	0.008	0.03	0.15	1.51

	$r_b(1:2)$	$r_b(2:3)$	$r_b(3:4)$	$\bar{r}_b$	$r_\ell(2:1)$	$r_\ell(3:2)$	$r_\ell(4:3)$	$\bar{r}_\ell$	$r_a(2:1)$	$r_a(3:2)$	$r_a(4:3)$	$\bar{r}_a$
Map		5.67	3.00	4.34		2.00	1.88	1.94		8.81	4.19	6.50
Photograph	3.93	5.60	5.00	4.84	2.59	2.79	3.74	3.04	3.41	5.20	10.07	6.23

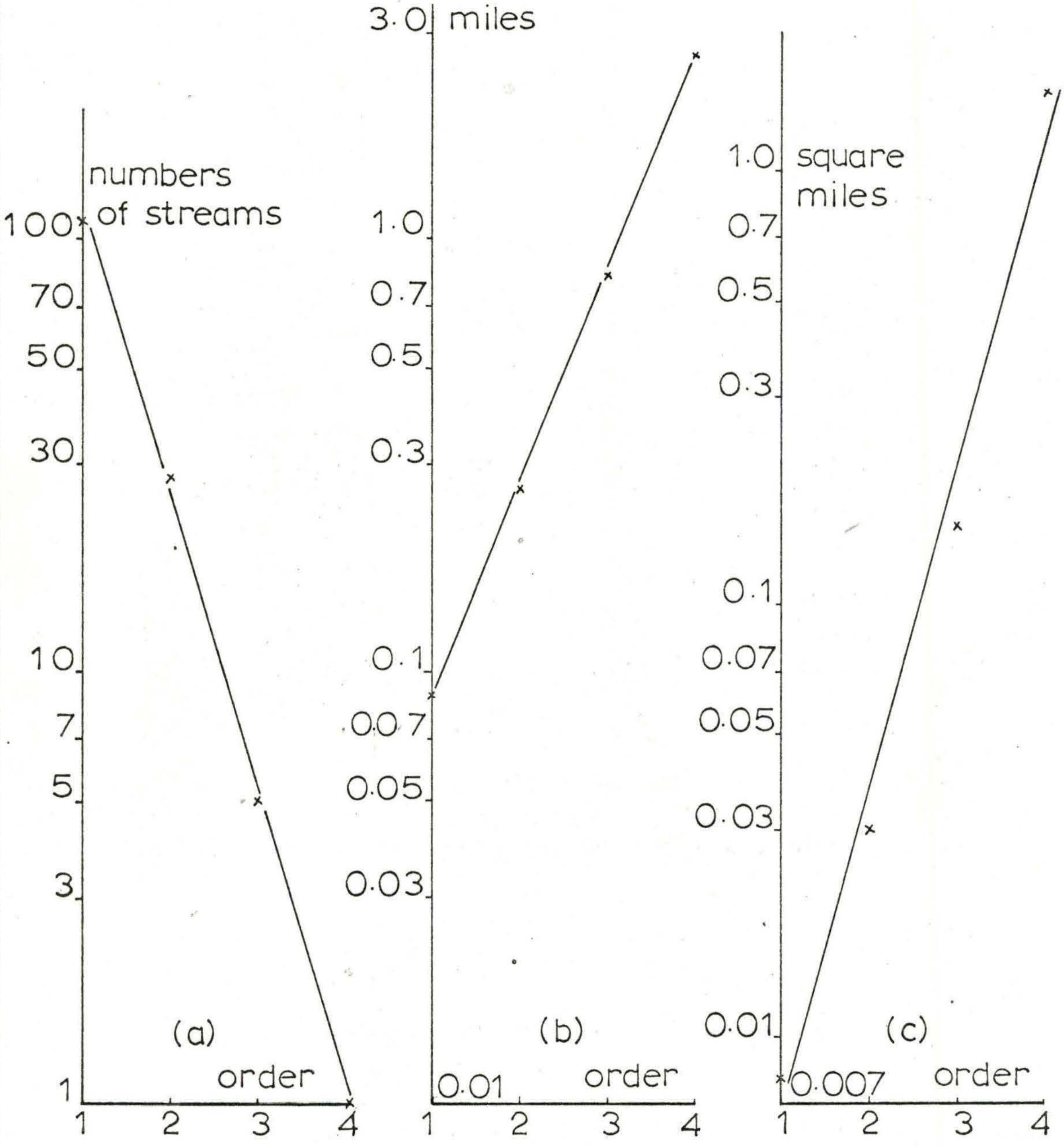
	D_d	F	R_e	R_c
Map	5.84	17.29	0.57	0.54
Photograph	12.95	95.23	0.59	0.50

square miles from the photograph, but only 1.21 square from the map. Although the mean area ratio ($\bar{r}_a$) in the map basin (6.50) is almost precisely the same as the mean area ratio in the photograph (6.23), it should be noticed at this point that considerable divergence exists between the area ratios of each pair of orders; for example the ratio between second and third order drainage areas on the map is 4.19, whereas the corresponding ratio between third and fourth order drainage areas on the photograph is 10.07.

As was noticed in basins $(1)_s$ and $(5)_c$ the greater visible detail of the air photographs allows the recognition of far more stream segments in the lower orders. Since the scale difference between the topographic map and the air photographs of basin is less, however, drainage density as it is calculated from the air photographs is only twice as great as the density as measured from the map in basin $(1)_c$. Similarily, the frequency of streams is only approximately eight times as high in the photograph basin. The close correlation between the values of the Elongation and Circularity Ratios as they were calculated from the photograph and from the map is presumably attributable to the close correlation of scale between the photograph and the map basin. (Table XII).

Data on stream numbers from the photograph basin gives a reasonable approximation to a logarithmic series with order, (Fig. 27a), but figures for stream numbers in the map basin depart farther from the 'line of best fit' on semi-logarithmic paper (Fig. 7a). In both cases, however, the logarithmic series of the Law of Stream Lengths is closely

FIG 27 BASIN $(1)_c$
(photograph data)
PLOTS OF THE HORTON LAWS



approached, as can be seen from Fig. 7b and Fig. 27b. Deviations from the straight line of best fit are negligible. On the other hand, the approximation to the Law of Drainage Areas is less close. Average drainage areas as they were measured from the photograph basin tend to plot as a curve which is markedly concave downwards, and stream areas measured from the topographic map sheet form a curve which is similarily concave downwards. It was suggested in Chapter 4 that this concavity was fairly common in these small Ontario basins, and it was further postulated that this may be due to basin boundary control which resulted from the configuration of the Pleistocene surface. There seems no reason to dispute this theory on the evidence of the air photograph checks of these basins.

In a third 't' test, the mean drainage area of all first order streams in the map basin was tested against the mean value of the mean drainage area of all second order basins in the photograph basin, with the null hypothesis that there is no difference in these mean values. With 43 degrees of freedom, the t value was calculated as only 0.4716, and the null hypothesis was accepted. It seems that the map and air photograph representations of basin $(1)_c$ are statistically comparable.

Basin Comparisons

Despite certain variations in the values of parameters as they were measured from the maps and from the photographs of each of these basins, in two cases out of three, 't' tests suggested that these air photograph checks are statistically comparable to the map analyses. With the exception of basin $(5)_c$ it seems therefore that these checks have reasonably extended the accuracy and detail of the map analyses. In the case of basin $(1)_s$

which was originally studied from a topographic map on the scale of 1:50,000, a third order map basin was found to be of fifth order under the greater accuracy and detail of the air photographs. Despite the fact that the aerial photographs covering basin $(1)_c$ are on a smaller scale than the initial map (1:28,000 as compared with 1:25,000), the order of this basin was raised to fourth as a result of the greater detail which was visible on the air photographs.

In all three basins, the application of the Horton Laws of Stream Composition was reasonably satisfactory, but as was mentioned earlier, the basins tended to maintain the general downward concavity which was noted in Chapter 4 in the plots of the Law of Stream Lengths and of the Law of Stream Areas. This was particularily noticeable in basin $(1)_c$ (Fig. 27c).

These air photograph checks of the map analysis were finally used to test for differences between the basins, since each set of photographs covered a basin in one of the three major groupings of the map basins. In the first place a 't' test was set up to examine the differences between the area of third order basins in the photographic analysis of basins $(5)_c$ and $(1)_s$. It was hoped that this test would demonstrate the statistical validity of the division of these basins according to Chapman and Putnam's label of sand plain and clay plain lithology. In this example, the air photographs were of a very similar scale, and the map basins were both elevated to fifth order. With 29 degrees of freedom, the calculated t value was only 0.9501, and thus the null hypothesis that there is no statistical difference between the mean value of third order drainage areas was accepted.

In the second of these tests, the areas of second order drainage basins in the air photograph analysis of basin $(1)_c$ were compared to the areas of third order drainage basins in the air photograph analysis of basin $(5)_c$. This particular comparison was employed since these orders in the photograph basins were both presumed to be equivalent to the first order basins of the respective map analyses. With 41 degrees of freedom, the calculated t value of 4.8119 is significant above the 97.5% level (Dixon and Massey, 1957, Table A-5). Thus it is concluded that the mean value of second order drainage basins in the photographic analysis of basin $(1)_c$ (original map scale of 1:25,000) is significantly different from the mean drainage area of third order basins from the photograph analysis of basin $(5)_c$ (original map scale of 1:50,000).

It is not suggested that the results of these 't' tests are conclusive, but they do indicate that although sand and clay basins which were originally examined from maps on the scale of 1:50,000 may not be statistically different, there is strong evidence that a comparison of basins which were originally drawn from maps of different scale is not statistically valid. As a reservation, however, it must be remembered that not only were basins $(1)_c$ and $(5)_c$ initially studied from maps of a different scale, but the scale of their air photograph coverage was also quite different. Nevertheless, some allowance for this was made when second order basins from basin $(1)_c$ (air photograph scale of 1:28,000) were compared with third order basins from basin $(5)_c$ (air photograph scale of 1:15,000).

CHAPTER VI

ANALYSIS OF MORPHOMETRIC PROPERTIES - I

Inter-Basin Variations

The values of the measured properties in each of the selected basins are recorded in Appendix A. During the first stage in the investigation of intra-basin variations, these variables were grouped into four sections according to certain characteristics of the basins. The groups, which encompassed respectively basins mapped on the 1:50,000 scale, the 1:25,000 scale, sand basins and clay basins,[1] are to a certain extent overlapping. Of the nine clay basins, three were studied from maps on the 1:50,000 scale, six on the 1:25,000 scale; all nine sand basins were studied on the 1:50,000 map scale. Reference should be made to Table IV for a summary of the location, underlying lithology, and map scale of each basin.

The first step in the statistical investigation was the calculation of the mean, mean square, variance and standard eviation for 15 basin parameters in each of the four groups mentioned above. The formulae used were cited in Chapter III. The results of these primary calculations are summarized in Appendix B and a visual examination of these tables indicated some of the variation which occurs between the four groups. It was apparent, however, that in many cases differences in the means of many parameters in the four groups of basins were low as compared with their standard deviations.

---

[1]The terms 'sand' and 'clay' basins refer to basins developed in areas called sand plain and clay plain by Chapman and Putnam (1952). This shorter terminology has been used throughout the study.

The initial hypothesis was that lithological variations between basins of the sand group and those developed on the clay plains would result in significantly different values of the morphometric properties. Student's t test was used to investigate differences in the mean values of the parameters in clay basins and in sand basins. Fifteen parameters in each of the 18 basins were used. The value of the statistic 't' was calculated for each pair of mean values by the use of a computer programme written specifically for this test, (Table XIII). Where the calculated value of t with sixteen degrees of freedom exceeded the t .05 value given in the appropriate tables (Dixon and Massey, 1957, Table A-5), the null hypothesis that there is no difference between the means was rejected. At the 95% level, only eight of these variables showed significant differences between the means. Sand basin means and clay basin means of total stream length, average drainage areas of each order, average inter-areas of each order and drainage density are the only properties which exhibit a statistically significant difference. It is, however, these very properties which may be expected to vary significantly with lithology (Miller, 1953, Coates, 1958).

The means of all variables measured in basins from the 1:50,000 scale maps and those from basins on the 1:25,000 scale are apparently equally as different as are the means of the sand and clay basin groupings of variables (Appendix B); as a result, the t test was applied to the mean values of the parameters divided according to map scale (Table XIV). This testing indicated that almost precisely the same variables had significantly different mean values at the 95% level in the map scale

TABLE XIII

'**t**' TESTS, CLAY BASINS AGAINST SAND BASINS

VARIABLE	t VALUE
n_1	0.14625
N	0.13371
ℓ_1	0.51627
ℓ_2	0.97615
ℓ_3	1.76794
ΣL	2.48969
a_1	2.67433
a_2	2.76885
a_3	3.23394
ia_2	2.69151
ia_3	2.06308
D_d	2.70442
R_r	0.18062
R_e	0.62565
R_c	0.95503

16 Degrees of Freedom $t_{.95}$ = 1.746

TABLE XIV

'T' TESTS, 1:25,000 BASINS AGAINST 1:50,000 BASINS

VARIABLE	t VALUE
n_1	0.07822
N	0.14318
ℓ_1	0.21427
ℓ_2	3.29670
ℓ_3	1.45655
ΣL	2.77344
a_1	3.58787
a_2	4.19611
a_3	4.44464
ia_2	4.32695
ia_3	2.55195
Dd	4.74468
R_r	0.28084
R_e	1.35761
R_c	0.07245

16 Degrees of Freedom t_{95} = 1.746

groups as in the groups based on lithology; but in each case the calculated t value was higher for the groups divided on the basis of map scale, (compare Table XIII and Table XIV). Certain other variables also exhibit statistically different mean values in 1:25,000 maps as compared with 1:50,000 maps: at the 95% level, the null hypothesis that there is no difference between the mean value of the average length of second order segments in 1:25,000 map basins and in 1:50,000 map basins was rejected. In the comparison of sand and clay basin second order stream segment lengths, the calculated t value for this property equalled only 0.97615.

Another application of the t test was used to test for significant differences between the morphometric properties in clay basins on a scale of 1:50,000 and in sand basins on a scale of 1:50,000. With 12 basins and 10 degrees of freedom, only the average length of first order segments and total basin area exhibit a statistically significant difference at the 95% level between the values of these parameters in sand basins and in clay basins when scale variations had been eliminated, (Table XV). The t values calculated in this test also indicate that total stream length in each basin, and the average area of second and first order basins in these 1:50,000 sand basins and clay basins may also derive from different populations, but with a lesser degree of statistical probability. The value of the statistic t calculated for the means of these parameters is greater than the tabulated value of $t_{.90}$ but less than the value of $t_{.95}$ (Dixon and Massey, 1957, Table A-5). It is possible that with a larger sample these t values would be greater, the degrees of freedom would be larger and there might conceivably be a statistically significant difference

TABLE XV

't' TESTS, CLAY BASINS AGAINST SAND BASINS, 1:50,000 MAPS ONLY

VARIABLE	t VALUE
n_1	0.73030
N	0.92222
ℓ_1	2.23418
ℓ_2	1.12957
ℓ_3	0.84851
ΣL	1.39510
a_1	1.73338
a_2	1.50887
a_3	2.03748
ia_2	1.17088
ia_3	1.00711
D_d	0.09373
R_r	0.01364
R_e	0.26353
R_c	1.27701

10 Degrees of Freedom t.95 = 1.812

between these means at the 95% level which was arbitrarily accepted for this investigation.

In a final effort to elucidate differences in the measured parameters which could be attributed to map scale and to lithology, slightly different t tests were established using measurements of the area of individual first order basins, in basins $(4)_c$, $(5)_c$ and $(6)_c$, basins $(1)_c$, $(2)_c$, and $(3)_c$ and in basins $(1)_s$, $(2)_s$, and $(4)_s$. In the first test, the mean value of all first order drainage basins in basins $(4)_c$, $(5)_c$, and $(6)_c$ was tested against the mean value of all first order drainage areas in basin $(1)_c$, $(2)_c$, and $(3)_c$; all basins in the first group had been selected from maps on the scale of 1:50,000, while the basins in the second were from maps on a scale of 1:25,000. With a calculated t value of 2.46 and 40 degrees of freedom, first order drainage areas in basin $(1)_c$, $(2)_c$, and $(3)_c$ (1:25,000 scale) are significantly different from individual first order areas in $(4)_c$, $(5)_c$, and $(6)_c$ (1:50,000 scale) at the 99% level of probability.

In the second of this particular group of tests, individual first order drainage areas in basins $(4)_c$, $(5)_c$, and $(6)_c$ were tested against values of the same parameter in basins $(1)_s$, $(2)_s$, and $(4)_s$ both these groups being drawn from a map scale of 1:50,000. In this case, the t value was only 1.605, and with 40 degrees of freedom this value of t is only significant at just below the 95% level of probability; the tabulated $t_{.95}$ value is equal to 1.684.

As a final test in this group, the same data on individual first order basin areas in each of these three groups (basin $(1)_c$, $(2)_c$, and $(3)_c$, basins $(4)_c$, $(5)_c$ and $(6)_c$, and basins $(1)_s$, $(2)_s$, and $(4)_s$) was used in an analysis of variance test. The $F_{.95}$ value for two and 60 degrees of freedom is equal to 3.15 (Dixon and Massey, 1957, Table A-7), and the calculated F value is equal to 3.960. Thus, it seems statistically probable that the three groups of basins do not derive from the same parent population.

Summary of Inter-Basin Variations

The initial hypothesis that lithological differences between sand and clay basins would result in statistically significant differences in the mean values of morphometric properties of drainage basins proved to be somewhat tenuous. When map scale differences were eliminated by an analysis of these parameters in basins measured only on a scale of 1:50,000, results of t tests for significant differences in mean values were disappointing. Only the means of the average length of first order segments and total basin area were accepted as being demonstrative of differences which might be attributed to lithology. It was also pointed out that total stream length, and the average basin area of each order might show statistically significant variations between sand and clay basins in an expanded test.

Although the length of first order segments and total drainage area were proved statistically different in sand and clay basins, drainage densities in the two groups have an almost identical mean value, and the calculated t value is only .0937. This can only mean that drainage areas and segment lengths may vary between sand and clay basins, but when these are combined and expressed as miles of segment length per unit drainage area, differences are cancelled. Further, measures of basin shape, relief, and stream numbers exhibit no significant differences between these sand and clay basins from 1:50,000 maps (Table XIII).

One may draw two conclusions from these results. It is possible to state that with respect to the development of stream systems, these glacial areas as defined by Chapman and Putnam (1952) impose very similar restrictions. Since various authors (Miller, 1953, Coates, 1958) have suggested that lithology is an important factor in determining drainage density, the logical conclusion may be that there is little or no difference in lithological effect of sand and clay upon fluvial systems; nevertheless, they were supposedly identified and named on lithological variations. Since deviations from the mean value within the groups are large (Appendix B), however, it is possible to suggest that lithological variations between sand and clay plain areas are masked by variations within these areas. This analysis of the chosen basins and the great variations between basins within the same lithological group would seem to indicate that this is the case. As a result, with a relatively small sample, statistically significant differences between the parameters in the two lithological groups are obscured by the effects of great lithological variations within the groups.

The t test of differences in the mean value of individual first order basin areas (basin $(4)_c$, $(5)_c$, and $(6)_c$ against basins $(1)_s$, $(2)_s$, and $(4)_s$) would also seem to indicate that with a much larger sample it may be possible to ennumerate more accurately differences caused by lithology. In this test, the calculated t value indicated a difference in the means which was significant just below the 95% level of probability, and an even larger sample might demonstrate a statistically conclusive difference which could be attributed to lithology.

In summary, however, it must be said that the results of this testing do not permit statistically sound conclusions to be drawn regarding the difference in stream basin development which may be attributed to the lithology of the sand and clay plain areas.

In contrast, the tests discussed indicate that the scale of the mpa from which the stream nets were drawn has an important effect on the values of the morphometric properties. It seems reasonable that map scale would show little influence on measures of basin shape, and this fact is confirmed in Table XIV. On the other hand, measures of stream length, basin areas, inter-basin areas and drainage density are all statistically different when the values are compared for 1:25,000 maps and 1:50,000 maps, regardless of lithology. In fact, calculated t values for the means of these parameters are very high (Table XIV).

The results indicate that map scale differences are not only more important than the lithological variations between sand and clay plain areas, but that these differences are strikingly important with regard to the morphometric properties of drainage basins in Southern Ontario. The differences in the values of these properties in the map scale groups are so highly significant that basins drawn from 1:50,000 maps and from 1:25,000 maps must be treated as deriving from different populations. It cannot be considered statistically sound to compare and contrast third order stream basins represented on maps of the 1:25,000 scale of Southern Ontario with third order basins represented on maps of the 1:50,000 scale, although the maps are issued by the same authority. This striking difference between map scales with regard to stream lengths, basin areas and drainage density is apparent despite the fact that scale differences have theoretically been eliminated by the conversion of all units of measurements to miles and square miles.

CHAPTER VII

ANALYSIS OF MORPHOMETRIC PROPERTIES - II

Parameter Interrelationships

In chapter six it was concluded from the results of the t tests that not only are map scales more important than lithological differences in causing variations in the values of basin parameters, but when map scale differences are eliminated, lithological differences are still of negligible importance. Only the length of first order segments and the total area of the basins show significant differences between sand basins and clay basins of the same scale. It was concluded for the purposes of this analysis that all basins on the 1:50,000 map scale could be treated as a sample from the same parent populations.

On this basis, correlation analyses were applied to all pairings of nine morphometric properties of each of the twelve 1:50,000 map scale basins. The correlation coefficient 'r' was calculated for each pairing of these parameters; values of r greater than 0.576 were considered probably significant, and values of r greater than 0.497 possibly significant for N = 12. For each pairing of variables, the value of r was calculated for the four combinations:

x with y

x with log y

y with log x

log x with log y

The highest correlation coefficient of the four was assumed to indicate the most significant correlation. Table XVI illustrates all coefficients for

TABLE XVI

CORRELATION COEFFICIENTS[1] FOR 1:50,000 BASINS, NINE VARIABLES

$\bar{r}_b$	$r_b(1:2)$	$r_b(2:3)$	$r_a(2:1)$	$r_a(3:2)$	D_d	F	R_r	R_e	
	0.9008		0.8158				0.6329		$\bar{r}_b$
			0.7939				0.5932		$r_b(1:2)$
									$r_b(2:3)$
							0.5200		$r_a(2:1)$
									$r_a(3:2)$
						0.9701	0.5579	0.7422	D_d
							0.6767	0.6768	F
									R_r
									R_e

[1]Coefficients less than 0.50 are omitted

the pairings of these nine variables which are greater than 0.500, but without any indication as to whether a logarithmic relationship between the variables was most significant. Where the relationship is linear, a coefficient of this magnitude is significant at above the 95% level (Dixon and Massey, 1957, Table A-30a). It must be emphasized, however, that these correlations are not necessarily demonstrative of cause and effect relationships. A similar series of analyses was performed using a larger number of slightly different variables, a total of 15 parameters, and significant results of this analysis are recorded in Table XVII. This division of the variables was purely one of convenience, and in fact, the second correlation analyses were performed when it was seen that the shorter series was insufficiently informative. The shorter analysis, for example, included such parameters as the bifurcation and area ratios, and it was felt that actual stream numbers and drainage areas might also be useful variables to examine for their relationships with other basin parameters.

The first and most obvious correlations are those indicated by the high values of the coefficient for the relationships between the number of first order segments, and the total number of streams in each basin, and between $r_b(1:2)$ and $\bar{\bar{r}}_b$. Since the number of first order streams forms such a large proportion of the total number of streams, both of these correlations are reasonable and expected. On the other hand, the correlation between both the number of first order streams and the total number of streams and the Relief Ratio seems a little unusual; in both cases, this correlation is linear and is significant above the 97.5% level. As the Relief Ratio is a dimensionless expression of overall basin slope and thus of the average value of gravitational acceleration in the basin, it must be postulated that the

TABLE XVII

CORRELATION COEFFICIENTS[1] FOR 1:50,000 BASINS, 15 VARIABLES

	n_1	N	ℓ_1	ℓ_2	ℓ_3	ΣL	a_1	a_2	a_3	ia_2	ia_3	D_d	R_r	R_e	R_c
n_1		0.9875											0.6214		
N													0.5801		
ℓ_1					0.5002	0.6876	0.8213	0.7249	0.7948	0.6479		0.7138	0.6935		
ℓ_2						0.7311		0.6026	0.5442	0.7779		0.4989	0.4999		
ℓ_3						0.5088			0.5990		0.9298	0.5539			
ΣL							0.6917	0.7930	0.8857	0.8555	0.5108	0.7725			0.5135
a_1								0.9195	0.9181	0.7506	0.5276	0.9549	0.6637	0.7365	0.5203
a_2									0.9479	0.9235	0.5212	0.9388	0.5183	0.7276	0.5817
a_3										0.9040	0.7126	0.9612	0.5629	0.6521	0.5491
ia_2											0.5096	0.8569		0.5021	
ia_3												0.6300			
D_d													0.5579	0.7422	0.6357
R_r															
R_e															0.8550
R_c															

[1]Coefficients less than 0.50 omitted

downslope component of gravity is an important factor in determining the number of streams which are initiated. The acceleration of overland downslope flow with gravity must be greater than the resistivity of the soil before the surface is broken and a channel initiated. The Relief Ratio is presumably an expression of the average situation over the basin which determines the frequency with which stream channels are developed. Both the mean bifurcation ratio and the ratio of the number of first order streams to the number of second order segments correlate highly with $r_a(2:1)$ giving correlation coefficients of 0.8518 and 0.7939 respectively, although there is no significant relationship between actual stream numbers and drainage areas. The only common factor is that all three variables are linearly related to the Relief Ratio, with correlation coefficients of approximately the same order of magnitude. It seems possible to suggest that it is the effect of the Relief Ratio on the bifurcation of streams and on the breakdown between first and second order basins which has given rise to the correlation of bifurcation and area ratios. No other correlations exist between stream numbers or bifurcation ratios and segment lengths, basin areas, drainage density, or ratios of basin shape. Perhaps the initiation of streams is a function of the ground slope as it is epitomized by the Relief Ratio, but the number and bifurcation of streams is then a random expression of conditions within the drainage basin. As such it bears no relation to the length of channel necessary to maintain drainage, nor to basin shape.

The average segment lengths[1] of each order and the total length

---

[1]These average segment lengths are the lengths of all first order channels averaged, the lengths of all second order segments, and only second order segments, averaged, and the length of the third order segment of restricted definition. Thus ℓ_2 and ℓ_3 used in this analysis of morphometric properties are shorter than the cumulative mean segment lengths and the average accumulated segment lengths used in the tests of the Horton Law of Stream Lengths in Chapter IV.

of channel in a basin are inter-connected in a fairly complex manner. The average length of first order segments is not correlated with the average length of second order segments; nor is the average length of second order segments correlated with the length of third order segments. Coefficients of correlation in both of these cases are considerably less than 0.50. But the correlation coefficient for the average length of first order segments with the average length of third order segments is 0.5002. It is suggested that this pattern of correlation and non-correlation between the average segment lengths of each order reflects the amount of lithological variation within the basins. As a result, there is no standard relationship between the lengths of successive orders. On the other hand, the average segment lengths of each order correlate with the total length of channel within the basin. The correlation coefficient for the relationship between the restricted length of third order segments (l_3) and total segment length of all orders is only 0.5088. Each of these correlations depends to a large extent on the fact that the component parts of the total segment length are a function of the average segment lengths of each order. Since the length of the restricted third order segment is very short in many basins (Chapter IV, and Appendix A), the low coefficient between this variable and total channel length is not unexpected. Finally, it was demonstrated in Chapter III that the accuracy of linear measurement may well have confused the relationships.

It seems possible that inaccuracies in measurement may also have affected the correlation coefficients between the average segment lengths of each order and drainage density. The definition of drainage density ($D_d = \Sigma L/A$), is such that the average segment lengths of each order should correlate with this variable, yet the correlation of the average length of second order segments with drainage density, the lowest of these three, is only 0.4989. In contrast, the correlation coefficient for the relationship between the average length of second order segments and total stream length (r = .7311) is markedly greater than the value of the coefficient for the correlation between the length of the restricted third order segments and total channel length. Since the correlation coefficient for the length of the third order segment with drainage density is only 0.5539, the explanation must lie in the fact that the correlation coefficient between the length of third order segments and the area of third order basins is 0.5990, whereas the relationships between the average area and the average segment length of second order gives a coefficient of only 0.5442.

The relationships between the average drainage areas of each order can be satisfactorily explained. Coefficients of correlation for the average area of first order basins with the average area of second order basins, for the average area of first order basins with the area of third order basins, and for the average area of second order basins with the area of third order basins are high (r is greater than 0.90 in each case), but it must be remembered that these average drainage areas are cumulative. For example, the drainage area of a second order stream includes the area drained by the first order streams which are tributary to it. Thus, variations in lithology and their effects on the average areas of each of the orders within the basin may be less obvious. These high coefficients between drainage areas of the lower orders and the total area of the basin must be the explanation for the high degree of correlation which exists between the average area of first and second order basins and drainage density.

The inter-areas of orders two and three are highly correlated with the average drainage areas of each order, and with the drainage density. In each case, however, the correlation coefficients for the inter-area of the second order with the average areas of first, second, and third orders are higher than the corresponding values for the relationships between the inter-area of third order and the average drainage areas of the first, second and third orders. Since the inter-area is the area which drains exclusively into the channel of given order, part of the explanation for this situation lies in the fact that the third order segment in most of these basins is rather short; as a result, the inter-area of third order also tends to be small as compared with the inter-area of second order. The inter-relationships between drainage areas and inter-areas reflect the influence of factors which determine the average drainage area per unit channel length over the entire basin. It seems likely, however, that the variations in the value of the correlation coefficient are also due to changes in causative factors over the basin. Thus for example, the correlation between the inter-area of third order and the average drainage area of first order is only 0.5276, and similarily the correlation between the inter-area of third order and the average area of second order is only 0.5212. Nevertheless, the correlation coefficients for the relationships between the average inter-area of second order and the average drainage areas of both second and third orders are greater than 0.9, and for the inter-area of third order with the drainage area of third order is 0.7126. Obviously, the fact that the inter-area of second order is a contributing portion of the average area drained by second order segments and also of the third order drainage area, and that the inter-area of third order is a component part of the third order drainage area, is at least in part responsible for these high coefficients. The lower r value for the correlation between the inter-area of third order and the third order drainage area indicates that the inter-area of third order forms a lesser proportion of the drainage area of the third order than does the inter-area of second order in the drainage area of second order segments.

The inter-areas of both second and third orders are highly correlated with drainage density, demonstrating the importance of areas which drain directly into the higher order channels to the spread of the drainage net over the entire basin.

The high correlations between drainage areas and stream segment lengths in basins of many orders have been demonstrated in the literature (Leopold, Wolman and Miller, 1964, Morisawa, 1962). This analysis has substantiated such a correlation for streams in Southern Ontario. The correlations between the average drainge area, the average inter-area and the average segment length of each order are greater than 0.6. For example, the correlation between total stream length and the average area of third order basins gives an r value 0.8837. Less well demonstrated in the literature are the relationships between the average drainage areas and segment lengths of the lower orders in the basin; variations in the value of the coefficients for the different orders are again attributed to chance variations in the lithological factors over the basins. With the exception of the average inter-area of third order, the average drainage area of each order is highly correlated with the average segment lengths of the other orders. The average inter-area of third order correlates only with the length of the third order segment and with the total length of channel in the basin, with coefficients of 0.9298 and 0.5018 respectively. This may be another example of the peculiar characteristics of the third order basins, and of the suggested boundary limitation on the size of third order basins by the configuration of the post-glacial surface. Confirmation of this suggestion may also be assumed from the fact that the correlation coefficient for the average length of the restricted third order segments with the average drainage area of third order basins is only 0.5990, whereas the relationship between the average length and drainage area of first order segments is 0.8213.

The average area of each order correlates highly with the total stream length in the basins. The highest coefficient is that of the area of third order basins with the total channel length, the lowest correlation is that between the average area of first order streams and the total channel length; this progression seems reasonable in view of the fact that the correlation between total stream length and total basin area has been conclusively demonstrated (Leopold, Wolman and Miller, 1964, Morisawa 1962), and obviously the drainage area of first order streams forms a smaller proportion of the total drainage area than does the drainage area of second order.

The inter-relationships between basin shape and relief factors and drainage areas and segment lengths are much more complex. As well as exhibiting significant correlations with the numbers of streams in the basin, the Relief Ratio also correlates with the average length of first order segments ($r = 0.6935$); the correlation between the Relief Ratio and the average length of second order segments, however, is only 0.4999, and with the length of the third order segment it is even less. Since these are the average lengths of the restricted segments of each order, this decreasing correlation is assumed to indicate the decreasing influence of the Relief Ratio outside the first order basins. Within first order basins the influence of the Relief Ratio is generally assumed to be proportionally greater than over the basins of higher order. This postulation is confirmed by the fact that the correlation between the Relief Ratio and the average areas of first, second and third orders decreases from 0.6637 to 0.5183 with increasing order. Nevertheless, the calculation of coefficients less than 0.60 for the correlations between the Relief Ratio and the average areas of both second and third orders again demonstrates the importance of the first order drainage areas in the size and characteristics of the higher order basins. The correlation coefficients for the relationships between the inter-areas of both second and third orders and the Relief Ratio are in both cases considerably less than 0.50.

It is perhaps surprising that there is no correlation between the Relief Ratio and total channel length in the basins; this occurs despite the high correlation between the average length of first order segments and the Relief Ratio. Over the entire area of these third order basins in Southern Ontario, the slope of the land as it is expressed in the Relief Ratio obviously does not have an important influence on the length of segments of all orders. Nevertheless, the Relief Ratio does affect the length of stream segment per unit area, since the correlation coefficient between the Relief Ratio and drainage density was calculated as 0.5579. This confirms the work of Schumm (1956, p. 612) who found that "within homogeneous areas of similar development the drainage density is a power function of the Relief Ratio". One can only suggest once more that this reflects the effect of the Relief Ratio and the general slope of the land in determining the drainage characteristics of first order streams. As higher order basins are considered, the restricted measures of length and drainage area of the particular order become less affected by the influence of the gradient, which itself is also presumably lessened in the higher order basins. The effect of the Relief Ratio and it's correlations with stream lengths, basin areas and drainage density is felt largely as an extension of its influence on these factors in first order basins and when the value of such parameters in first order basins is included in the value of the parameters of the higher order basins. This is the case with the average drainage area of both second and third orders and with drainage density, but not with the length of the restricted third order segment.

Finally, Schumm (1956) suggested that a correlation might exist between the Relief Ratio and his Elongation Ratio such that "as the Relief Ratio increases the drainage area becomes more elongate". For these Ontario streams the correlation coefficient between the Relief Ratio and the Elongation Ratio was calculated as less than 0.25, and similarily, the correlation between the Relief Ratio and the Circularity Ratio is extremely low. Obviously the Relief Ratio does not have an important effect on basin shape as Schumm's evidence seemed to indicate; in fact it seems possible to present this lack of correlation as one more piece of evidence on the glacial surface control on basin boundaries and hence presumably on basin shape.

Two measures of basin shape were included in this simple correlation analysis. Neither the Elongation Ratio nor the Circularity Ratio correlates to any degree of significance with stream numbers, nor generally with segment lengths. The only exception to this rule is a correlation coefficient of 0.5135 for the relationship between total channel length and the Circularity Ratio; in all other cases the coefficients are considerably less than 0.50. Apparently the outline form or shape of the basin does not affect to any great extent the numbers and lengths of streams developed within the boundary. It would seem that factors such as rainfall, run-off, infiltration and lithology which have an important effect on the initiation and development of the stream nets have a much lesser effect on the outline form of the basin. Again it is possible to suggest that the shape of the drainage basin owes rather more to the configuration of the post-glacial surface, which has already been advanced as an important factor in these drainage basins.

In contrast, both these measures of basin shape correlate with the average drainage areas of each order; the correlations between the Elongation Ratio and the average drainage areas of each order are in each case higher than the correlations between the drainage areas and the Circularity Ratio. If the outlines of these basins were fixed or partially fixed, then these correlations between the measures of basin shape and the average drainage areas would seem a reasonable result. It must be noted, however, that the only correlation between the inter-areas and the Circularity Ratio and the Elongation Ratio is a calculated r value of 0.5021 between the Elongation Ratio and the inter-area of second order. Perhaps one can conclude that on the small scale of the average inter-areas the influence of basin shape is less important, despite its effect on the average drainage areas of each order. Similarly, there is an over-all inter-action between drainage density and both the Elongation Ratio and the Circularity Ratio ($r = 0.7422$ and 0.6357 respectively). This supports the conclusions of Melton (1958 b) who suggested as a 'heuristic explanation' that "....for a given area and channel length, the shape" affects only "their arrangement and manner of connection".

It is noticeable that with the exception of the correlation between the Circularity Ratio and total channel length, correlation coefficients are higher for the relationships between the average drainage areas of each order, drainage density and the Elongation Ratio. Since the Elongation Ratio and the Circularity Ratio are correlated with a coefficient of 0.8550, it seems possible to suggest that in Southern Ontario the Elongation Ratio is a more efficient and more meaningful measure of basin shape than is the Circularity Ratio.

Finally on the basis of these simple correlation analyses it is possible to express the geometric characteristics of these basins of Southern Ontario in terms of a very small number of variables. The total number of streams is almost completely expressed by the number of first order streams, and the correlation between the number of first order streams and the Relief Ratio gives a coefficient of 0.6214. Each of the remaining 11 variables is correlated with the total drainage area of these third order streams; in each case, the coefficient of correlation is greater than 0.54. It is not suggested that all characteristics of these basins can be adequately expressed only in terms of these two parameters, number of first order streams and drainage area of third order basins. It seems possible, however, to agree with Melton ' s (1958 b) suggestion that only four basin parameters are essential in determining the total geometric properties of drainage basins. These parameters were the total length of stream segments, total drainage area, total basin relief and the length of the perimeter. In the present analysis, the total basin relief has been expressed as a relief ratio, and the length of the perimeter is incorporated within the Elongation Ratio, nevertheless, the correlation coefficients in Tables XVI and XVII support Melton's conclusions.

CHAPTER VIII

SUMMARY OF CONCLUSIONS

The application of the Horton Laws to these 18 stream basins in Southern Ontario was outlined in Chapter IV, and graphical illustrations of the exponential relationships between stream numbers, lengths, and drainage areas, and order can be seen in Figures 7 to 24. On the basis of these graphs and on the evidence of the bifurcation, length and area ratios, there seems little reason to doubt the validity of the 'laws' as they apply to the chosen streams.

Although deviations from the Law of Stream Numbers are in some cases quite large, these departures may be plausibly explained in terms of the basin size and map scale. All of the basins are small, and the total number of streams in each basin is low. As a result, many of the departures from a decreasing logarithmic series of stream numbers may be attributed to the small sample size of the stream segments mapped in each basin; in only one basin were more than 20 stream segments included on the drainage net. Although an attempt was made to complete the channel system according to contour information, the evidence of air photograph checks of basins $(1)_s$, $(5)_c$, and $(1)_c$ suggests that the number of first order segments in the map basins were nevertheless underestimated. This is undoubtedly one factor which contributes to deviations from the Law of Stream Numbers.

Four attempts were made to express a relationship between stream segment length and order. On the combined evidence of the graphical plots (Figs. 7b to 24b, and Figs. 7c to 24c) and the length ratios (Table VIa and

VIb) it was felt that the best statement of a Law of Stream Lengths in these basins is an exponential relationship between cumulative mean segment lengths and order. This agrees with the work of Bowden and Wallis (1964) and Shreve (1964).

Schumm's Law of Stream Areas was also accepted as a good expression of the behaviour of these streams on the basis of the appropriate graphs (Figs. 7d to 24d) and of the area ratios (Table VIII). On the whole, average drainage areas increase geometrically with order, deviations from a straight-line plot are low and deviations from equality in the area ratios are also relatively low.

Notable departures from straight line plots of the length and area laws were marked in basins $(5)_c$ and $(6)_c$ (Fig. 11 and Fig. 12); a plot of the data on stream lengths and areas from the air photograph analysis of basin $(5)_c$ revealed similar discrepancies in the application of the Laws of Stream Lengths and Drainage Areas (Fig. 26). In these basins, the tendency for an extremely short third order segment and small third order drainage area can be at least partially explained by the fact that these basins debouch into Lake Erie where a cliff face indicates that coastal erosion has been active for some time. It seems likely that some of the lower part of the basin has been removed as a result of this erosion.

In other basins, the tendency for a degree of downward concavity in the lines joining the plots of cumulative mean segment length and drainage area may possibly be explained in terms of lithological inhomogeneity within the basins; an added factor may be boundary restrictions limiting the size of the third order basins. This, it has been suggested, results from the configuration of the post-glacial surface. Substantial lithological variation within the basins is implied by the absence of many significant differences between the values of morphometric properties in sand basins and clay basins of the same map scale (Table XV). Similarly, the significant correlations found between the Elongation and Circularity Ratios and the average drainage area of each order and the length of the restricted third order segment may be cited as possible evidence of boundary control of basin size (Table XVII). As with the Law of Stream Numbers, the small size of the sample of segments in each basin may also contribute to deviations from a perfect geometric series in the stream lengths and drainage areas of successive orders.

The application of 't' tests to groupings of morphometric properties of each basin based on basin lithology and map scale gave surprising results. Measures of basin shape, relief, and the numbers of streams in each basin are little different in sand basins and in clay basins of either scale. Stream numbers of themselves need not necessarily show any relationship with lithology, nor need there be any significant difference in stream numbers on the different map scales, since all basins are of third order.

Obviously no difference was expected between the Elongation Ratios and the Circularity Ratios of the two maps scales, (Tables XIV), and the fact that these ratios also do not apparently vary significantly with the sand and clay plain lithologies (Table XIII) confirms the work of Miller (1953) and Coates (1958). Since the range in relief between basins was deliberately restricted at the outset and since both sand and clay plain areas are the deposits of glacial lakes, it seems quite plausible that significant differences in the Relief Ratio have been successfully eliminated.

Of the remaining properties, those of stream lengths, drainage areas and drainage density were expected to vary most conclusively with lithology. The work of various authors confirms this assumption (Coates, 1958, Miller 1953, Gray 1961). Nevertheless, calculated 't' values were higher for each of these parameters when the mean values of the parameters in 1:50,000 map basins were tested against their mean values in 1:25,000 map basins, than when the mean values in sand basins were tested against the mean values in clay basins (Tables XIII and XIV). When the parameters measured in clay basins on a scale of 1:50,000 were tested against the same parameters measured in sand basins of the same scale (Table XV), only two properties had significantly different mean values. For the mean values of the length of first order streams in these two groups, the calculated t value was 2.23418, and for the total area of the basins, the t value for the difference in the means was 2.03748. In each case there are 10 degrees of freedom, and these calculated t values are significant above the 95% level. The calculated t value for the means of the remainder of the 15 parameters did not reach the 95% level of significance. It is possible that a larger sample of sand and clay basins would produce significant results in some of these 13 parameters. This postulate is, however, partly nullified by the fact that a test of the differences between the mean values of individual first order basin areas in three sand basins against those in three clay basins did not quite reach the 95% level of significance; in this test there were 40 degrees of freedom.

It is finally suggested that this lack of overall significant differences between sand and clay basins is the result of inherent lithological variations within the individual basins. These variations are apparently so great as to obscure contrasts between the two groups.

Since these 't' tests demonstrated highly significant differences between 1:50,000 basins and 1:25,000 basins for eight of the measured parameters, it was not felt that these groups could be treated as if they derived from the same parent population. The larger group of basins, those on the 1:50,000 scale, were however, assumed to be a sample from one parent population since the differences between sand and clay basins were inconclusive. As a result, the morphometric properties of all basins mapped on the 1:50,000 scale were used in a series of simple correlation analyses. The correlation coefficient 'r' was calculated for each pairing of the parameters, and significant correlations in which the coefficient had a calculated value greater than 0.50 are recorded in Tables XVI and XVII.

On the whole significant correlations correspond well with such correlations quoted in the literature. Average drainage areas and segment lengths of each order correlate highly with each other (most coefficients for these correlations are greater than 0.60). For example, total drainage area and total channel length in each basin correlate with an r value of 0.8857. Similarly, the average drainage area of each order is highly correlated with the average drainage area of the other orders; the average segment lengths of each order are also highly correlated with the average segment lengths of the other orders. These relationships may be attributed to the effect of factors such as lithology, soil resistivity, infiltration, vegetation, and climate, which determine or help to determine stream lengths and drainage areas. The variable value of the correlation coefficients is probably related to differences in the influence of these factors within each basin. Since the value of the average drainage area of each order and the segment lengths of each order obviously contribute to the value of drainage density for the basin, it is not surprising that these parameters are highly correlated with drainage density. Similarily, the very high value of r for the correlation between the number of first order streams and the total number of streams in the basin is a reflection of the fact that the number of streams forms such a large proportion of the total number.

Surprising results were discovered when correlations were sought between parameters of stream lengths, areas and numbers and measures of basin shape and relief. Both the number of first order streams and total number of streams are highly correlated with the Relief Ratio. Since this Relief Ratio is a measure of the overall basin gradient, and must reflect the value of the downslope component of gravity, it seems that this force of gravity must be an important factor in the initiation of stream channels. The Relief Ratio is also highly correlated with the length and drainage area of first order streams; this again must be a reflection of the effect of ground slope on stream development, and on the spread of the drainage network. These relationships substantiate the work of Schumm (1956, p. 613) who stated that "the steeper the slope on which small basins develop, the more closely spaced are the drainage channels". His emphasis on small basins is confirmed in this study by the fact that correlation coefficients between stream lengths and areas and the Relief Ratio are lower for the higher order streams. In these larger basins, the effect of the force of gravity is minimized, and apparently other factors become more important in determining drainage areas and segment lengths. Nevertheless, there is a low correlation between the Relief Ratio and drainage density, emphasizing the continued importance of basin relief in it's influence on the spread of stream channels over the third order basins.

The ratios of basin circularity and elongation apparently do not generally correlate with segment lengths. Only the total length of channel correlates to a low degree with the Circularity Ratio. It seems that the shape of the basins has little influence on the length of channels within the basin. On the other hand, correlations with fairly high coefficients exist between the Elongation Ratio and the average drainage area of each order, and slightly lower coefficients were calculated for the relationships between the Circularity Ratio and the average drainage areas of each order. This may well be a reflection of the postulation of boundary control on the basins by the configuration of the post-glacial surface on which they are developed. Such limitation of basin shape and size would be reflected in high correlations between parameters of basin shape and drainage areas such as were actually found. This conclusion is further substantiated by the fact that neither the Elongation Ratio nor the Circularity Ratio show any tendency to correlate with the Relief Ratio. Schumm (1956), suggested that the Relief Ratio may be a power function of the Elongation Ratio. The lack of such a relationship in this study supports the conclusion that some external factor may have more influence on basin shape. The irregularities of the post-Pleistocene surface may well have offered 'ready-made' basins owing more to retreating lake waters and earlier effects than to factors such as the Relief Ratio, which have apparently affected basin shape in non-glaciated areas.

In conclusion, it is found that Horton's empirical Laws of Drainage Composition are broadly applicable to stream basins in these areas of glacial lake sediments. This agrees with the conclusions of Roberts (1963) who analysed the characteristics of the Humber drainage net near Toronto. Map scale differences are more important than the supposed lithological differences between Chapman and Putnam's (1951) sand and clay plain areas in their influence on the values of morphometric properties. Finally, most parameter inter-relationships are comparable to those found in different areas of the United States. However, the influence of the characteristics of the post-Pleistocene surface on which these small streams were initiated has resulted in certain peculiarities in the application of the Horton Laws, the confusion of the expected lithological influence on the morphological properties, and in certain unusual parameter inter-relationships.

APPENDIX A

CLAY BASINS FROM 1/50,000 MAPS - DATA

BASIN	N 1	N 2	N 3	N	RB1/2	RB2/3	RB
(4)C	6	2	1	9	3.00	2.00	2.50
(5)C	8	2	1	11	4.00	2.00	3.00
(6)C	7	2	1	10	3.50	2.00	2.75

BASIN	L 1	L 2	L 3	L	RL2/1	RL3/I	RL
(4)C	0.39	1.38	2.37	7.50	0.28	1.21	4.37
(5)C	0.43	1.18	0.39	6.31	0.28	1.57	3.15
(6)C	0.36	1.13	0.32	5.44	0.25	1.60	3.57

BASIN	IA2	IA3	DD	F	RR	RE	RC
(4)C	0.56	1.55	1.71	2.06	21.79	0.69	0.83
(5)C	0.44	0.01	2.00	3.49	35.64	0.79	0.88
(6)C	0.79	0.25	1.52	2.80	26.39	0.59	0.55

x

CLAY BASINS FROM 1/25,000 MAPS - DATA

BASIN	N 1	N 2	N 3	N	RB1/2	RB2/3	RB
(1)C	17	3	1	21	5.67	3.00	4.34
(2)C	4	2	1	7	2.00	2.00	2.00
(3)C	10	3	1	14	3.33	3.00	3.17
(7)C	5	2	1	8	2.50	2.00	2.25
(8)C	5	2	1	8	2.50	2.00	2.25
(9)C	5	2	1	8	2.50	2.00	2.25

BASIN	L 1	L 2	L 3	L	RL2/1	RL3/I	RL
(1)C	0.67	0.67	1.18	7.10	0.03	0.29	1.21
(2)C	0.49	0.59	0.30	3.45	0.13	0.41	0.92
(3)C	0.22	0.67	1.97	6.12	0.03	0.28	1.58
(7)C	0.91	0.48	0.12	5.64	0.17	0.51	1.04
(8)C	0.38	0.69	1.26	4.54	0.13	0.51	1.43
(9)C	0.36	0.81	0.75	4.18	0.11	0.54	1.24

BASIN	IA2	IA3	DD	F	RR	RE	RC
(1)C	0.14	0.24	5.84	17.29	46.08	0.57	0.54
(2)C	0.15	0.09	3.76	6.53	24.64	0.76	0.97
(3)C	0.23	0.56	3.87	8.87	16.40	0.58	0.58
(7)C	0.10	0.01	5.43	7.69	24.79	0.63	0.67
(8)C	0.21	0.37	3.16	5.58	19.73	0.59	0.52
(9)C	0.27	0.17	3.37	6.45	26.40	0.55	0.57

SAND BASINS FROM 1/50,000 MAPS - DATA

BASIN	N 1	N 2	N 3	N	RB1/2	RB2/3	RB
(1)S	10	3	1	14	3.33	3.00	3.17
(2)S	7	2	1	10	3.50	2.00	2.75
(3)S	12	2	1	15	6.00	2.00	4.00
(4)S	7	2	1	10	3.50	2.00	2.75
(5)S	8	3	1	12	2.67	3.00	2.84
(6)S	7	2	1	10	3.50	2.00	2.75
(7)S	5	2	1	8	2.50	2.00	2.25
(8)S	7	2	1	10	3.50	2.00	2.75
(9)S	6	2	1	9	3.00	2.00	2.50

BASIN	L 1	L 2	L 3	L	RL2/1	RL3/I	RL
(1)S	0.32	0.55	0.95	5.76	0.17	0.72	2.51
(2)S	0.45	0.79	1.97	6.71	0.54	2.59	6.38
(3)S	0.32	1.18	0.79	7.10	0.11	1.18	2.99
(4)S	0.68	1.70	1.10	9.23	0.67	3.61	7.84
(5)S	0.57	1.84	1.50	11.60	0.38	2.48	8.50
(6)S	0.70	0.71	1.66	7.97	0.83	2.58	8.09
(7)S	0.39	0.49	0.95	3.87	0.08	0.32	0.92
(8)S	0.60	1.12	2.64	9.08	0.99	5.09	13.40
(9)S	0.57	0.89	3.24	8.40	0.35	1.55	6.75

SAND BASINS FROM 1/50,000 MAPS - DATA

BASIN	IA2	IA3	DD	F	RR	RE	RC
(1)S	0.19	0.27	2.29	5.57	37.27	0.64	0.59
(2)S	0.76	1.08	1.05	1.57	21.36	0.85	0.66
(3)S	0.66	0.38	2.37	5.01	53.61	0.63	0.67
(4)S	1.25	0.62	1.18	1.28	5.54	0.70	0.60
(5)S	1.48	1.06	1.36	1.41	21.75	0.62	0.61
(6)S	0.69	0.93	0.98	1.24	24.37	0.82	0.76
(7)S	0.12	0.28	4.20	8.69	36.79	0.44	0.34
(8)S	1.61	3.22	0.68	0.75	30.22	0.76	0.73
(9)S	0.74	3.14	1.25	1.33	19.80	0.58	0.55

APPENDIX B

CLAY BASINS

VARIABLE	MEAN	MEAN SQUARE	VARIANCE	STD. DEVN
NO. 1	7.4444	55.4198	16.2778	4.0346
TOTALN	10.6667	113.7778	19.5000	4.4159
LTH. 1	0.4678	0.2188	0.0421	0.2052
LTH. 2	0.8444	0.7131	0.0956	0.3091
LTH. 3	0.9622	0.9259	0.6333	0.7958
TOTALL	5.5867	31.2108	1.8028	1.3427
AREA 1	0.1567	0.0245	0.0094	0.0968
AREA 2	0.7689	0.5912	0.2888	0.5374
AREA 3	2.0567	4.2299	1.6458	1.2829
I.A. 2	0.3211	0.1031	0.0532	0.2306
I.A. 3	0.3600	0.1296	0.2305	0.4801
DNSITY	3.4067	11.6054	2.3542	1.5343
RRATIO	26.8733	722.1760	80.2618	8.9589
ERATIO	0.6389	0.4082	0.0076	0.0874
CRATIO	0.6789	0.4609	0.0289	0.1699

SAND BASINS

VARIABLE	MEAN	MEAN SQUARE	VARIANCE	STD. DEVN
NO. 1	7.6667	58.7778	4.5000	2.1213
TOTALN	10.8889	118.5679	5.3611	2.3154
LTH. 1	0.5111	0.2612	0.0213	0.1460
LTH. 2	1.0300	1.0609	0.2297	0.4792
LTH. 3	1.6444	2.7042	0.7069	0.8408
TOTALL	7.7467	60.0108	4.9714	2.2297
AREA 1	0.4578	0.2096	0.1047	0.3236
AREA 2	2.2356	4.9977	2.2364	1.4955
AREA 3	6.3756	40.6477	14.4060	3.7955
I.A. 2	0.8333	0.6944	0.2728	0.5223
I.A. 3	1.2200	1.4884	1.3334	1.1547
DNSITY	1.7067	2.9127	1.2021	1.0964
RRATIO	27.8567	775.9938	186.4792	13.6557
ERATIO	0.6711	0.4504	0.0162	0.1274
CRATIO	0.6122	0.3748	0.0150	0.1225

1/50,000 MAP BASINS

VARIABLE	MEAN	MEAN SQUARE	VARIANCE	STD. DEVN
NO. 1	7.5000	56.2500	3.5455	1.8829
TOTALN	10.6667	113.7778	4.2424	2.0597
LTH. 1	0.4817	0.2320	0.0186	0.1362
LTH. 2	1.0800	1.1664	0.1784	0.4224
LTH. 3	1.4900	2.2201	0.8385	0.9157
TOTALL	7.4142	54.9699	4.1718	2.0425
AREA 1	0.4108	0.1688	0.0834	0.2888
AREA 2	2.0417	4.1684	1.7581	1.3259
AREA 3	5.7058	32.5565	12.0148	3.4662
I.A. 2	0.7742	0.5993	0.2156	0.4643
I.A. 3	1.0658	1.1360	1.1724	1.0828
DNSITY	1.7158	2.9441	0.8851	0.9408
RRATIO	27.8775	777.1550	144.6695	12.0279
ERATIO	0.6758	0.4568	0.0137	0.1170
CRATIO	0.6475	0.4193	0.0207	0.1440

1/25,000 MAP BASINS

VARIABLE	MEAN	MEAN SQUARE	VARIANCE	STD. DEVN
NO. 1	7.6667	58.7778	25.4667	5.0465
TOTALN	11.0000	121.0000	30.4000	5.5136
LTH. 1	0.5050	0.2550	0.0619	0.2487
LTH. 2	0.6517	0.4247	0.0121	0.1100
LTH. 3	0.9300	0.8649	0.4677	0.6839
TOTALL	5.1717	26.7461	1.8367	1.3553
AREA 1	0.1000	0.0100	0.0033	0.0576
AREA 2	0.4233	0.1792	0.0134	0.1159
AREA 3	1.2367	1.5293	0.0590	0.2429
I.A. 2	0.1833	0.0336	0.0041	0.0638
I.A. 3	0.2400	0.0576	0.0399	0.1998
DNSITY	4.2383	17.9635	1.2533	1.1195
RRATIO	26.3400	693.7956	107.4919	10.3678
ERATIO	0.6133	0.3762	0.0059	0.0766
CRATIO	0.6417	0.4117	0.0285	0.1689

CORRELATION COEFFICIENTS, 1/50,000 MAP BASINS ONLY

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
NO. 1	TOTALN	1.0769	0.9000	0.9845
NO. 1	LTH. 1	-0.0274	-5.2408	0.3792
NO. 1	LTH. 2	0.0215	0.4281	0.0960
NO. 1	LTH. 3	-0.1990	-0.8414	0.4092
NO. 1	TOTALL	0.0658	0.0559	0.0606
NO. 1	AREA 1	-0.0394	-1.6727	0.2566
NO. 1	AREA 2	-0.0908	-0.1831	0.1289
NO. 1	AREA 3	-0.3350	-0.0989	0.1820
NO. 1	I.A. 2	-0.0101	-0.1665	0.0411
NO. 1	I.A. 3	-0.2017	-0.6099	0.3507
NO. 1	DNSITY	0.0109	0.0437	0.0218
NO. 1	RRATIO	3.9696	0.0973	0.6214
NO. 1	ERATIO	0.0071	1.8249	0.1134
NO. 1	CRATIO	0.0171	2.9163	0.2230
NO. 1	L NO.1	0.1219	8.0340	0.9896
NO. 1	L TOTN	0.0943	10.1239	0.9772
NO. 1	L LTH1	-0.0649	-2.8996	0.4339
NO. 1	L LTH2	0.0261	0.5384	0.1186
NO. 1	L LTH3	-0.1267	-0.8561	0.3294
NO. 1	L TOTL	0.0178	0.7608	0.1165
NO. 1	LAREA1	-0.1124	-0.6643	0.2733
NO. 1	LAREA2	-0.0034	-0.0218	0.0086
NO. 1	LAREA3	-0.0239	-0.1624	0.0622
NO. 1	L I A2	0.0122	0.0730	0.0299
NO. 1	L I A3	-0.2243	-0.3340	0.2737
NO. 1	L DDY	0.0414	0.6260	0.1610
NO. 1	L RRAT	0.1244	1.4089	0.4187
NO. 1	L ERAT	0.0163	1.7211	0.1673
NO. 1	L CRAT	0.0384	2.2169	0.2918
TOTALN	LTH. 1	-0.0245	-5.6000	0.3704
TOTALN	LTH. 2	0.0229	0.5453	0.1118
TOTALN	LTH. 3	-0.1776	-0.8988	0.3996
TOTALN	TOTALL	0.1092	0.1111	0.1101
TOTALN	AREA 1	-0.0387	-1.9687	0.2761
TOTALN	AREA 2	-0.0948	-0.2287	0.1472
TOTALN	AREA 3	-0.2886	-0.1019	0.1715
TOTALN	I.A. 2	-0.0059	-0.1152	0.0260
TOTALN	I.A. 3	-0.1857	-0.6720	0.3533
TOTALN	DNSITY	0.0138	0.0661	0.0302
TOTALN	RRATIO	3.3874	0.0993	0.5801
TOTALN	ERATIO	0.0039	1.2166	0.0691
TOTALN	CRATIO	0.0122	2.4997	0.1747
TOTALN	L NO.1	0.1105	8.7164	0.9816
TOTALN	L TOTN	0.0879	11.2831	0.9956
TOTALN	L LTH1	-0.0579	-3.0914	0.4230
TOTALN	L LTH2	0.0220	0.5427	0.1093
TOTALN	L LTH3	-0.1064	-0.8604	0.3026
TOTALN	L TOTL	0.0207	1.0541	0.1476
TOTALN	LAREA1	-0.1038	-0.7343	0.2761
TOTALN	LAREA2	-0.0117	-0.0904	0.0325

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
TOTALN	LAREA3	-0.0203	-0.1657	0.0580
TOTALN	L I A2	0.0033	0.0239	0.0089
TOTALN	L I A3	-0.1885	-0.3359	0.2517
TOTALN	L DDY	0.0407	0.7361	0.1730
TOTALN	L RRAT	0.1097	1.4869	0.4039
TOTALN	L ERAT	0.0112	1.4192	0.1261
TOTALN	L CRAT	0.0299	2.0677	0.2488
LTH. 1	LTH. 2	0.9370	0.0975	0.3022
LTH. 1	LTH. 3	2.9613	0.0656	0.4406
LTH. 1	TOTALL	10.2378	0.0455	0.6829
LTH. 1	AREA 1	1.7411	0.3874	0.8213
LTH. 1	AREA 2	7.0157	0.0741	0.7209
LTH. 1	AREA 3	19.9875	0.0309	0.7856
LTH. 1	I.A. 2	2.1674	0.1866	0.6359
LTH. 1	I.A. 3	3.6249	0.0574	0.4561
LTH. 1	DNSITY	-4.2006	-0.0881	0.6083
LTH. 1	RRATIO	-59.3870	-0.0076	0.6727
LTH. 1	ERATIO	0.3202	0.4339	0.3728
LTH. 1	CRATIO	0.1447	0.1296	0.1370
LTH. 1	L NO.1	-0.5317	-0.1834	0.3123
LTH. 1	L TOTN	-0.4308	-0.2421	0.3229
LTH. 1	L LTH1	2.0577	0.4809	0.9948
LTH. 1	L LTH2	0.8681	0.0937	0.2852
LTH. 1	L LTH3	2.5563	0.0904	0.4807
LTH. 1	L TOTL	1.4170	0.3164	0.6695
LTH. 1	LAREA1	4.6333	0.1433	0.8149
LTH. 1	LAREA2	3.7995	0.1283	0.6983
LTH. 1	LAREA3	3.9355	0.1403	0.7430
LTH. 1	L I A2	3.3824	0.1058	0.5981
LTH. 1	L I A3	4.8133	0.0375	0.4250
LTH. 1	L DDY	-2.5118	-0.1989	0.7069
LTH. 1	L RRAT	-2.6719	-0.1584	0.6506
LTH. 1	L ERAT	0.4935	0.2734	0.3673
LTH. 1	L CRAT	0.3114	0.0941	0.1712
LTH. 2	LTH. 3	-0.0471	-0.0100	0.0217
LTH. 2	TOTALL	3.5358	0.1512	0.7311
LTH. 2	AREA 1	0.1079	0.2308	0.1578
LTH. 2	AREA 2	1.2608	0.1279	0.4016
LTH. 2	AREA 3	3.0380	0.0451	0.3702
LTH. 2	I.A. 2	0.7613	0.6299	0.6925
LTH. 2	I.A. 3	0.1392	0.0212	0.0543
LTH. 2	DNSITY	-0.9061	-0.1826	0.4068
LTH. 2	RRATIO	-11.4552	-0.0141	0.4022
LTH. 2	ERATIO	0.0281	0.3663	0.1015
LTH. 2	CRATIO	0.1049	0.9030	0.3078
LTH. 2	L NO.1	0.0822	0.2725	0.1497
LTH. 2	L TOTN	0.0664	0.3588	0.1544
LTH. 2	L LTH1	0.2095	0.4707	0.3140
LTH. 2	L LTH2	0.9621	0.9976	0.9796
LTH. 2	L LTH3	-0.0305	-0.0104	0.0178

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
LTH. 2	L TOTL	0.4829	1.0361	0.7073
LTH. 2	LAREA1	0.5571	0.1656	0.3038
LTH. 2	LAREA2	0.9327	0.3027	0.5314
LTH. 2	LAREA3	0.8281	0.2837	0.4847
LTH. 2	L I A2	1.3002	0.3907	0.7127
LTH. 2	L I A3	0.2710	0.0203	0.0742
LTH. 2	L DDY	-0.3448	-0.2624	0.3008
LTH. 2	L RRAT	-0.6623	-0.3773	0.4999
LTH. 2	L ERAT	0.0724	0.3855	0.1671
LTH. 2	L CRAT	0.2080	0.6042	0.3545
LTH. 3	TOTALL	1.0146	0.2039	0.4549
LTH. 3	AREA 1	0.1440	1.4468	0.4564
LTH. 3	AREA 2	0.5359	0.2556	0.3701
LTH. 3	AREA 3	2.2711	0.1585	0.5999
LTH. 3	I.A. 2	0.1747	0.6793	0.3445
LTH. 3	I.A. 3	1.0995	0.7863	0.9298
LTH. 3	DNSITY	-0.4818	-0.4564	0.4689
LTH. 3	RRATIO	-5.0492	-0.0293	0.3844
LTH. 3	ERATIO	0.0210	1.2828	0.1640
LTH. 3	CRATIO	0.0182	0.7359	0.1157
LTH. 3	L NO.1	-0.1018	-1.5870	0.4020
LTH. 3	L TOTN	-0.0786	-1.9957	0.3961
LTH. 3	L LTH1	0.1460	1.5411	0.4743
LTH. 3	L LTH2	0.0114	0.0553	0.0251
LTH. 3	L LTH3	0.7482	1.1953	0.9457
LTH. 3	L TOTL	0.1521	1.5336	0.4829
LTH. 3	LAREA1	0.4053	0.5664	0.4791
LTH. 3	LAREA2	0.2779	0.4240	0.3433
LTH. 3	LAREA3	0.4464	0.7188	0.5665
LTH. 3	L I A2	0.2975	0.4202	0.3536
LTH. 3	L I A3	1.3774	0.4851	0.8175
LTH. 3	L DDY	-0.2929	-1.0477	0.5539
LTH. 3	L RRAT	-0.1319	-0.3531	0.2158
LTH. 3	L ERAT	0.0338	0.8454	0.1690
LTH. 3	L CRAT	0.0395	0.5388	0.1458
TOTALL	AREA 1	0.0764	3.8189	0.5400
TOTALL	AREA 2	0.4155	0.9861	0.6401
TOTALL	AREA 3	1.3139	0.4562	0.7742
TOTALL	I.A. 2	0.1913	3.7022	0.8416
TOTALL	I.A. 3	0.2582	0.9188	0.4871
TOTALL	DNSITY	-0.3154	-1.4867	0.6848
TOTALL	RRATIO	-2.8110	-0.0811	0.4773
TOTALL	ERATIO	0.0167	5.0932	0.2919
TOTALL	CRATIO	0.0214	4.3027	0.3033
TOTALL	L NO.1	0.0148	1.1482	0.1304
TOTALL	L TOTN	0.0142	1.7942	0.1597
TOTALL	L LTH1	0.0949	4.9841	0.6876
TOTALL	L LTH2	0.1446	3.5058	0.7119
TOTALL	L LTH3	0.1777	1.4124	0.5010
TOTALL	L TOTL	0.1388	6.9640	0.9831

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
TOTALL	LAREA1	0.2421	1.6834	0.6384
TOTALL	LAREA2	0.2670	2.0269	0.7356
TOTALL	LAREA3	0.2950	2.3632	0.8349
TOTALL	L I A2	0.3104	2.1815	0.8229
TOTALL	L I A3	0.3748	0.6568	0.4961
TOTALL	L DDY	-0.1559	-2.7750	0.6577
TOTALL	L RRAT	-0.1289	-1.7179	0.4706
TOTALL	L ERAT	0.0310	3.8654	0.3464
TOTALL	L CRAT	0.0470	3.1934	0.3875
AREA 1	AREA 2	4.2209	0.2003	0.9195
AREA 1	AREA 3	10.8628	0.0754	0.9052
AREA 1	I.A. 2	1.1020	0.4264	0.6855
AREA 1	I.A. 3	1.9777	0.1407	0.5276
AREA 1	DNSITY	-2.4581	-0.2317	0.7547
AREA 1	RRATIO	-20.6996	-0.0119	0.4971
AREA 1	ERATIO	0.2647	1.6122	0.6533
AREA 1	CRATIO	0.1760	0.7084	0.3531
AREA 1	L NO.1	-0.1403	-0.2176	0.1747
AREA 1	L TOTN	-0.1385	-0.3498	0.2201
AREA 1	L LTH1	0.7952	0.8354	0.8150
AREA 1	L LTH2	0.2879	0.1396	0.2005
AREA 1	L LTH3	1.1951	0.1900	0.4765
AREA 1	L TOTL	0.5759	0.5779	0.5769
AREA 1	LAREA1	2.5159	0.3498	0.9381
AREA 1	LAREA2	2.1657	0.3288	0.8438
AREA 1	LAREA3	2.0694	0.3315	0.8283
AREA 1	L I A2	1.7397	0.2445	0.6522
AREA 1	L I A3	2.4596	0.0862	0.4604
AREA 1	L DDY	-1.4905	-0.5306	0.8893
AREA 1	L RRAT	-0.8541	-0.2276	0.4409
AREA 1	L ERAT	0.4083	1.0167	0.6442
AREA 1	L CRAT	0.3378	0.4588	0.3937
AREA 2	AREA 3	2.4422	0.3574	0.9342
AREA 2	I.A. 2	0.3047	2.4846	0.8701
AREA 2	I.A. 3	0.4256	0.6383	0.5212
AREA 2	DNSITY	-0.5341	-1.0608	0.7527
AREA 2	RRATIO	-4.1337	-0.0502	0.4557
AREA 2	ERATIO	0.0494	6.3400	0.5596
AREA 2	CRATIO	0.0333	2.8223	0.3065
AREA 2	L NO.1	-0.0081	-0.2653	0.0464
AREA 2	L TOTN	-0.0122	-0.6482	0.0888
AREA 2	L LTH1	0.1541	3.4108	0.7249
AREA 2	L LTH2	0.1378	1.4080	0.4404
AREA 2	L LTH3	0.2038	0.6828	0.3731
AREA 2	L TOTL	0.1434	3.0335	0.6596
AREA 2	LAREA1	0.5098	1.4937	0.8726
AREA 2	LAREA2	0.5100	1.6316	0.9122
AREA 2	LAREA3	0.4620	1.5599	0.8490
AREA 2	L I A2	0.4627	1.3704	0.7963
AREA 2	L I A3	0.4890	0.3611	0.4202

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
AREA 2	L DDY	-0.3176	-2.3826	0.8699
AREA 2	L RRAT	-0.1944	-1.0914	0.4606
AREA 2	L ERAT	0.0786	4.1264	0.5696
AREA 2	L CRAT	0.0683	1.9544	0.3653
AREA 3	I.A. 2	0.1179	6.5721	0.8804
AREA 3	I.A. 3	0.2226	2.2813	0.7126
AREA 3	DNSITY	-0.2164	-2.9372	0.7972
AREA 3	RRATIO	-1.6574	-0.1376	0.4776
AREA 3	ERATIO	0.0162	14.2310	0.4805
AREA 3	CRATIO	0.0119	6.8763	0.2856
AREA 3	L NO.1	-0.0069	-1.5489	0.1036
AREA 3	L TOTN	-0.0061	-2.2347	0.1172
AREA 3	L LTH1	0.0646	9.7760	0.7948
AREA 3	L LTH2	0.0482	3.3671	0.4029
AREA 3	L LTH3	0.1244	2.8469	0.5950
AREA 3	L TOTL	0.0651	9.4122	0.7829
AREA 3	LAREA1	0.1972	3.9497	0.8826
AREA 3	LAREA2	0.1876	4.1025	0.8774
AREA 3	LAREA3	0.1924	4.4400	0.9244
AREA 3	L I A2	0.1802	3.6467	0.8106
AREA 3	L I A3	0.2744	1.3848	0.6164
AREA 3	L DDY	-0.1270	-6.5085	0.9090
AREA 3	L RRAT	-0.0650	-2.4948	0.4027
AREA 3	L ERAT	0.0263	9.4305	0.4980
AREA 3	L CRAT	0.0253	4.9411	0.3533
I.A. 2	I.A. 3	1.1882	0.2185	0.5096
I.A. 2	DNSITY	-1.4406	-0.3509	0.7110
I.A. 2	RRATIO	-11.7642	-0.0175	0.4542
I.A. 2	ERATIO	0.0713	1.1216	0.2827
I.A. 2	CRATIO	0.0537	0.5588	0.1733
I.A. 2	L NO.1	0.0161	0.0644	0.0322
I.A. 2	L TOTN	0.0110	0.0719	0.0281
I.A. 2	L LTH1	0.3932	1.0675	0.6479
I.A. 2	L LTH2	0.6276	0.7865	0.7026
I.A. 2	L LTH3	0.5282	0.2170	0.3386
I.A. 2	L TOTL	0.5080	1.3176	0.8182
I.A. 2	LAREA1	1.1780	0.4233	0.7061
I.A. 2	LAREA2	1.3653	0.5357	0.8552
I.A. 2	LAREA3	1.3092	0.5420	0.8424
I.A. 2	L I A2	1.5288	0.5553	0.9213
I.A. 2	L I A3	1.6394	0.1485	0.4934
I.A. 2	L DDY	-0.7987	-0.7348	0.7661
I.A. 2	L RRAT	-0.5506	-0.3792	0.4569
I.A. 2	L ERAT	0.1271	0.8182	0.3225
I.A. 2	L CRAT	0.1380	0.4844	0.2585
I.A. 3	DNSITY	-0.4549	-0.6026	0.5236
I.A. 3	RRATIO	-3.3643	-0.0273	0.3029
I.A. 3	ERATIO	0.0121	1.0392	0.1123
I.A. 3	CRATIO	0.0137	0.7726	0.1027
I.A. 3	L NO.1	-0.0717	-1.5635	0.3349

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
I.A. 3	L TOTN	-0.0575	-2.0414	0.3427
I.A. 3	L LTH1	0.1274	1.8808	0.4895
I.A. 3	L LTH2	0.0473	0.3222	0.1234
I.A. 3	L LTH3	0.5436	1.2142	0.8124
I.A. 3	L TOTL	0.1360	1.9181	0.5108
I.A. 3	LAREA1	0.3698	0.7226	0.5169
I.A. 3	LAREA2	0.3100	0.6613	0.4528
I.A. 3	LAREA3	0.4195	0.9443	0.6294
I.A. 3	L I A2	0.3410	0.6734	0.4792
I.A. 3	L I A3	1.0877	0.5357	0.7633
I.A. 3	L DDY	-0.2817	-1.4091	0.6300
I.A. 3	L RRAT	-0.0799	-0.2994	0.1547
I.A. 3	L ERAT	0.0219	0.7661	0.1295
I.A. 3	L CRAT	0.0325	0.6201	0.1419
DNSITY	RRATIO	7.1183	0.0436	0.5568
DNSITY	ERATIO	-0.0868	-5.6069	0.6975
DNSITY	CRATIO	-0.0812	-3.4664	0.5305
DNSITY	L NO.1	-0.0221	-0.3639	0.0897
DNSITY	L TOTN	-0.0086	-0.2311	0.0447
DNSITY	L LTH1	-0.1829	-2.0391	0.6108
DNSITY	L LTH2	-0.2199	-1.1315	0.4989
DNSITY	L LTH3	-0.3013	-0.5081	0.3913
DNSITY	L TOTL	-0.2368	-2.5207	0.7725
DNSITY	LAREA1	-0.7408	-1.0928	0.8998
DNSITY	LAREA2	-0.7300	-1.1758	0.9265
DNSITY	LAREA3	-0.7302	-1.2410	0.9519
DNSITY	L I A2	-0.7018	-1.0464	0.8569
DNSITY	L I A3	-0.7286	-0.2709	0.4442
DNSITY	L DDY	0.4927	1.8607	0.9574
DNSITY	L RRAT	0.2744	0.7758	0.4614
DNSITY	L ERAT	-0.1444	-3.8152	0.7422
DNSITY	L CRAT	-0.1675	-2.4134	0.6357
RRATIO	ERATIO	-0.0022	-23.6073	0.2297
RRATIO	CRATIO	-0.0000	-0.2950	0.0035
RRATIO	L NO.1	0.0104	28.0896	0.5417
RRATIO	L TOTN	0.0080	35.0083	0.5290
RRATIO	L LTH1	-0.0162	-29.5998	0.6935
RRATIO	L LTH2	-0.0127	-10.6831	0.3684
RRATIO	L LTH3	-0.0226	-6.2189	0.3746
RRATIO	L TOTL	-0.0113	-19.5897	0.4696
RRATIO	LAREA1	-0.0427	-10.3062	0.6637
RRATIO	LAREA2	-0.0319	-8.4095	0.5183
RRATIO	LAREA3	-0.0338	-9.3828	0.5629
RRATIO	L I A2	-0.0304	-7.4168	0.4751
RRATIO	L I A3	-0.0511	-3.1056	0.3984
RRATIO	L DDY	0.0225	13.8611	0.5579
RRATIO	L RRAT	0.0429	19.8294	0.9225
RRATIO	L ERAT	-0.0036	-15.6719	0.2385
RRATIO	L CRAT	-0.0010	-2.4318	0.0501
ERATIO	CRATIO	0.9733	0.6432	0.7913

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
ERATIO	L NO.1	0.4131	0.1052	0.2084
ERATIO	L TOTN	0.2076	0.0861	0.1337
ERATIO	L LTH1	0.8996	0.1552	0.3736
ERATIO	L LTH2	0.6908	0.0550	0.1949
ERATIO	L LTH3	1.1091	0.0289	0.1792
ERATIO	L TOTL	1.0029	0.1653	0.4071
ERATIO	LAREA1	4.8169	0.1100	0.7278
ERATIO	LAREA2	4.4091	0.1099	0.6962
ERATIO	LAREA3	3.7761	0.0993	0.6125
ERATIO	L I A2	2.9731	0.0686	0.4517
ERATIO	L I A3	-0.2136	-0.0012	0.0162
ERATIO	L DDY	-2.7759	-0.1623	0.6711
ERATIO	L RRAT	-0.8488	-0.0371	0.1776
ERATIO	L ERAT	1.5539	0.6354	0.9937
ERATIO	L CRAT	1.7246	0.3847	0.8145
CRATIO	L NO.1	0.4737	0.1825	0.2941
CRATIO	L TOTN	0.2840	0.1782	0.2250
CRATIO	L LTH1	0.2760	0.0720	0.1410
CRATIO	L LTH2	1.2015	0.1448	0.4171
CRATIO	L LTH3	0.3258	0.0129	0.0648
CRATIO	L TOTL	0.8442	0.2105	0.4216
CRATIO	LAREA1	2.4731	0.0854	0.4597
CRATIO	LAREA2	2.5516	0.0963	0.4956
CRATIO	LAREA3	2.2692	0.0903	0.4527
CRATIO	L I A2	2.0286	0.0708	0.3791
CRATIO	L I A3	-1.8085	-0.0157	0.1688
CRATIO	L DDY	-1.4292	-0.1264	0.4251
CRATIO	L RRAT	0.1659	0.0110	0.0427
CRATIO	L ERAT	1.0421	0.6448	0.8197
CRATIO	L CRAT	1.6958	0.5724	0.9852
L NO.1	L TOTN	0.7738	1.2602	0.9875
L NO.1	L LTH1	-0.4435	-0.3004	0.3650
L NO.1	L LTH2	0.3157	0.0987	0.1766
L NO.1	L LTH3	-1.0344	-0.1060	0.3312
L NO.1	L TOTL	0.2391	0.1548	0.1924
L NO.1	LAREA1	-0.5459	-0.0489	0.1635
L NO.1	LAREA2	0.3084	0.0302	0.0965
L NO.1	LAREA3	0.1124	0.0116	0.0361
L NO.1	L I A2	0.3737	0.0339	0.1125
L NO.1	L I A3	-1.8593	-0.0420	0.2795
L NO.1	L DDY	0.1243	0.0285	0.0595
L NO.1	L RRAT	0.8648	0.1486	0.3585
L NO.1	L ERAT	0.2103	0.3377	0.2665
L NO.1	L CRAT	0.3976	0.3483	0.3721
L TOTN	L LTH1	-0.5795	-0.2410	0.3737
L TOTN	L LTH2	0.3558	0.0683	0.1559
L TOTN	L LTH3	-1.2193	-0.0767	0.3059
L TOTN	L TOTL	0.3189	0.1267	0.2010
L TOTN	LAREA1	-0.8628	-0.0475	0.2024
L TOTN	LAREA2	0.1705	0.0103	0.0418

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
L TOTN	LAREA3	0.0374	0.0024	0.0094
L TOTN	L I A2	0.3015	0.0168	0.0711
L TOTN	L I A3	-2.1802	-0.0303	0.2568
L TOTN	L DDY	0.2777	0.0391	0.1043
L TOTN	L RRAT	1.1215	0.1183	0.3643
L TOTN	L ERAT	0.1945	0.1918	0.1931
L TOTN	L CRAT	0.4152	0.2233	0.3045
L LTH1	L LTH2	0.4405	0.2033	0.2993
L LTH1	L LTH3	1.2859	0.1946	0.5002
L LTH1	L TOTL	0.6851	0.6545	0.6696
L LTH1	LAREA1	2.2565	0.2986	0.8209
L LTH1	LAREA2	1.8465	0.2668	0.7019
L LTH1	LAREA3	1.9149	0.2920	0.7478
L LTH1	L I A2	1.6602	0.2221	0.6072
L LTH1	L I A3	2.3273	0.0776	0.4251
L LTH1	L DDY	-1.2262	-0.4155	0.7138
L LTH1	L RRAT	-1.2882	-0.3267	0.6488
L LTH1	L ERAT	0.2369	0.5615	0.3647
L LTH1	L CRAT	0.1490	0.1926	0.1694
L LTH2	L LTH3	-0.0295	-0.0097	0.0169
L LTH2	L TOTL	0.4988	1.0322	0.7176
L LTH2	LAREA1	0.6657	0.1909	0.3564
L LTH2	LAREA2	1.0770	0.3371	0.6026
L LTH2	LAREA3	0.9469	0.3128	0.5442
L LTH2	L I A2	1.4449	0.4188	0.7779
L LTH2	L I A3	0.2863	0.0207	0.0770
L LTH2	L DDY	-0.4473	-0.3283	0.3832
L LTH2	L RRAT	-0.6114	-0.3359	0.4532
L LTH2	L ERAT	0.1164	0.5977	0.2638
L LTH2	L CRAT	0.2793	0.7825	0.4675
L LTH3	L TOTL	0.2025	1.2785	0.5088
L LTH3	LAREA1	0.4894	0.4281	0.4578
L LTH3	LAREA2	0.3153	0.3011	0.3081
L LTH3	LAREA3	0.5338	0.5380	0.5359
L LTH3	L I A2	0.3163	0.2797	0.2974
L LTH3	L I A3	1.8178	0.4008	0.8535
L LTH3	L DDY	-0.3299	-0.7389	0.4937
L LTH3	L RRAT	-0.1964	-0.3293	0.2543
L LTH3	L ERAT	0.0444	0.6964	0.1759
L LTH3	L CRAT	0.0324	0.2773	0.0948
L TOTL	LAREA1	1.8584	0.2575	0.6917
L TOTL	LAREA2	2.0388	0.3084	0.7930
L TOTL	LAREA3	2.2169	0.3539	0.8857
L TOTL	L I A2	2.2859	0.3201	0.8555
L TOTL	L I A3	2.6266	0.0917	0.4908
L TOTL	L DDY	-1.2145	-0.4308	0.7233
L TOTL	L RRAT	-0.9095	-0.2415	0.4687
L TOTL	L ERAT	0.2968	0.7364	0.4675
L TOTL	L CRAT	0.4413	0.5974	0.5135
LAREA1	LAREA2	0.8771	0.9577	0.9165

VARIABLES		SLOPE Y ON X	SLOPE X ON Y	COEFFICIENT
LAREA1	LAREA3	0.8553	0.9855	0.9181
LAREA1	L I A2	0.7465	0.7546	0.7506
LAREA1	L I A3	0.8453	0.2131	0.4244
LAREA1	L DDY	-0.5968	-1.5279	0.9549
LAREA1	L RRAT	-0.4085	-0.7828	0.5655
LAREA1	L ERAT	0.1740	3.1170	0.7365
LAREA1	L CRAT	0.1665	1.6263	0.5203
LAREA2	LAREA3	0.9227	0.9737	0.9479
LAREA2	L I A2	0.9598	0.8885	0.9235
LAREA2	L I A3	0.7453	0.1720	0.3581
LAREA2	L DDY	-0.6131	-1.4376	0.9388
LAREA2	L RRAT	-0.3785	-0.6644	0.5015
LAREA2	L ERAT	0.1797	2.9469	0.7276
LAREA2	L CRAT	0.1945	1.7400	0.5817
LAREA3	L I A2	0.9652	0.8467	0.9040
LAREA3	L I A3	1.2016	0.2628	0.5620
LAREA3	L DDY	-0.6448	-1.4328	0.9612
LAREA3	L RRAT	-0.3795	-0.6313	0.4895
LAREA3	L ERAT	0.1654	2.5710	0.6521
LAREA3	L CRAT	0.1886	1.5989	0.5491
L I A2	L I A3	0.8956	0.2233	0.4472
L I A2	L DDY	-0.5255	-1.3310	0.8363
L I A2	L RRAT	-0.3409	-0.6464	0.4695
L I A2	L ERAT	0.1193	2.1137	0.5021
L I A2	L CRAT	0.1535	1.4840	0.4773
L I A3	L DDY	-0.1705	-1.7320	0.5434
L I A3	L RRAT	-0.1106	-0.8409	0.3049
L I A3	L ERAT	0.0001	0.0056	0.0007
L I A3	L CRAT	-0.0145	-0.5617	0.0902
L DDY	L RRAT	0.5183	0.3880	0.4484
L DDY	L ERAT	-0.2625	-1.8361	0.6942
L DDY	L CRAT	-0.2630	-1.0035	0.5137
L RRAT	L ERAT	-0.0641	-0.5992	0.1960
L RRAT	L CRAT	-0.0054	-0.0276	0.0122
L ERAT	L CRAT	1.1577	0.6315	0.8550

APPENDIX C

```
      CALCULATION OF MEAN, MEAN SQUARE, VARIANCE, AND STANDARD DEVIATION
      DIMENSION V(25,25),SX1(25),SX2(25),AL(25)
      WRITE(6,30)
   30 FORMAT(1H0,65HMEAN, MEAN SQUARE AND STD. DEVIATION OF DRAINAGE BAS
     1IN PARAMETERS)
      READ(5,18)NA
   18 FORMAT(I2)
      DO 24 N=1,NA
      READ(5,10) JA,KA
   10 FORMAT(2I2)
      READ(5,12) (AL(K),K=1,KA)
   12 FORMAT(13A6)
      DO 20 J=1,JA
   20 READ(5,11) (V(J,K),K=1,KA)
   11 FORMAT(15F5.2)
      READ(5,34)VN
   34 FORMAT(3A6)
      WRITE(6,31)VN
   31 FORMAT(1H0,3A6)
      WRITE(6,32)
   32 FORMAT(1H0,58HVBLES      MEAN          MEAN SQUARE   VARIANCE      STD
     1. DEVN)
      DO 24 K=1,KA
      SX1(K)=0.0
      SX2(K)=0.0
      DO 22 J=1,JA
      SX1(K)=SX1(K)+V(J,K)
   22 SX2(K)=SX2(K)+V(J,K)**2
      A1=SX1(K)/FLOAT(JA)
      A2=A1**2.0
      VA=(SX2(K)-(SX1(K)**2)/FLOAT(JA))/FLOAT(JA-1)
      SD=(ABS(VA))**0.5
  133 FORMAT(1H ,A6,4(4X,F10.4))
      WRITE(7,133) AL(K),A1,A2,VA,SD
   24 WRITE(6,33)AL(K),A1,A2,VA,SD
   33 FORMAT(1H0,A6,4(4X,F10.4))
      STOP
      END
```

```
      T TESTS OF DRAINAGE BASIN PARAMETERS
      CALCULATION OF 'T' VALUES AND MEAN VALUES
      DIMENSION V(20,20),SX1(20),SX2(20),SXX1(20),SXX2(20),AL(20)
      READ(5,10) JC,KA
   10 FORMAT(2I2)
      DO 20 J=1,JC
   20 READ(5,11)(V(J,K),K=1,KA)
   11 FORMAT(15F5.2)
      READ(5,13)(AL(K),K=1,KA)
      WRITE(6,31)
   31 FORMAT(1H0,36HT-TESTS OF DRAINAGE BASIN PARAMETERS)
      DO 24 N=1,2
      DO 24 K=1,KA
      SX1(K)=0.0
      SXX1(K)=0.0
      SX2(K)=0.0
      SXX2(K)=0.0
      READ(5,12)JA
   12 FORMAT(I2)
   13 FORMAT(13A6)
      DO 22 J=1,JA
      SX1(K)=SX1(K)+V(J,K)
   22 SXX1(K)=SXX1(K)+V(J,K)**2
      JB=JA+1
      DO 23 J=JB,JC
      SX2(K)=SX2(K)+V(J,K)
   23 SXX2(K)=SXX2(K)+V(J,K)**2
      WRITE(6,32)
   32 FORMAT (1H0,31HCLAY BASINS AGAINST SAND BASINS)
      WRITE(6,33)
   33 FORMAT(1H0,39H1/25,000 BASINS AGAINST 1/50,000 BASINS)
      M1=SX1(K)/FLOAT(JA)
      M2=SX2(K)/FLOAT(JC-JA)
      T=(M1-M2)/((SXX1(K)-(SX1(K)**2)/FLOAT(JA-1))/FLOAT(JA)+(SXX2(K)-(S
     1X2(K)**2)/FLOAT(JC-JA-1))/FLOAT(JC-JA))
   24 WRITE(6,30) AL(K),T,M1,M2
   30 FORMAT(1H ,A6,3(4X,F9.5))
      STOP
      END
```

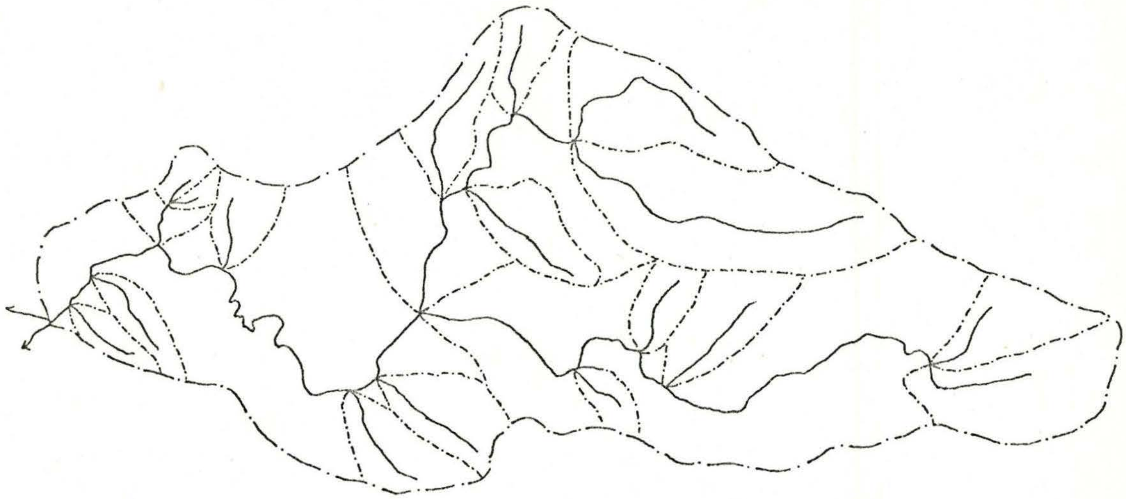
```
      CALCULATION OF CORRELATION COEFFICIENTS, 1/50,000 MAPS ONLY
      15 BASIN PARAMETERS (VARIABLES)

      DIMENSION AL(30),V(40,30),SX1(30),SX2(30),SXY1(40,30),SXY2(40,30)
      READ(5,10)KA,JB,JA
      KC=2*KA
   10 FORMAT(3I2)
      READ(5,11)(AL(K),K=1,KC)
   11 FORMAT(13A6)
      WRITE(6,31)
      WRITE(6,331)
  331 FORMAT(1H0,9HCLAY DATA)
      WRITE(6,332)
  332 FORMAT(1H0,118HNO. 1   TOTALN  LTH. 1  LTH. 2  LTH. 3  TOTALL  ARE
     1A 1  AREA 2  AREA 3  I.A. 2  I.A. 3  DNSITY  RRATIO  ERATIO  CRATI
     2O)
      DO 101 K=1,KC
      SX1(K)=0.0
      SX2(K)=0.0
      DO 101 L=1,KC
      SXY1(K,L)=0.0
  101 SXY2(K,L)=0.0
      DO 23 J=1,JB
      READ(5,12)(V(J,K),K=1,KA)
   12 FORMAT(15F5.2)
      WRITE(6,32)(V(J,K), K=1,KA)
   32 FORMAT(1H ,15(F6.2,2X))
      DO 22 K=1,KA
      KB=K+KA
   22 V(J,KB)=ALOG(V(J,K))
      DO 23 K=1,KC
      SX1(K)=SX1(K)+V(J,K)
      DO 23 L=1,KC
   23 SXY1(K,L)=SXY1(K,L)+V(J,K)*V(J,L)
      JC=JB+1
      WRITE(6,330)
  330 FORMAT(1H0,9HSAND DATA)
      WRITE(6,332)
      DO 26 J=JC,JA
      READ(5,12)(V(J,K),K=1,KA)
      WRITE(6,32)(V(J,K), K=1,KA)
      DO 25 K=1,KA
      KB=K+KA
   25 V(J,KB)=ALOG(V(J,K))
      DO 26 K=1,KC
      SX2(K)=SX2(K)+V(J,K)
      DO 26 L=1,KC
   26 SXY2(K,L)=SXY2(K,L)+V(J,K)*V(J,L)
      KD=KC-1
      WRITE(6,33)
   33 FORMAT(1H0,106HVARIABLES                    SLOPES Y ON X
     1  SLOPES X ON Y                    CORRELATION COEFFICIENTS)
      WRITE(6,34)
   34 FORMAT(1H ,111HX VBLE     Y VBLE          CLAY        SAND       CLAY+SAND
     1  CLAY         SAND         CLAY+SAND  CLAY          SAND        CLAY+SAND)
```

```
      DO 27 K=1,KD
      KE=K+1
      DO 27 L=KE,KC
      A1=(SXY1(K,L)-SX1(K)*SX1(L)/FLOAT(JB))/(SXY1(K,K)-SX1(K)*SX1(K)/FL
     1OAT(JB))
      A2=(SXY2(K,L)-SX2(K)*SX2(L)/FLOAT(JA-JB))/(SXY2(K,K)-SX2(K)*SX2(K)
     1/FLOAT(JA-JB))
      A3=((SXY1(K,L)+SXY2(K,L))-(SX1(K)+SX2(K))*(SX1(L)+SX2(L))/FLOAT(JA
     1))/((SXY1(K,K)+SXY2(K,K))-(SX1(K)+SX2(K))*(SX1(K)+SX2(K))/FLOAT(JA
     2))
      B1=(SXY1(L,K)-SX1(L)*SX1(K)/FLOAT(JB))/(SXY1(L,L)-SX1(L)*SX1(L)/FL
     1OAT(JB))
      B2=(SXY2(L,K)-SX2(L)*SX2(K)/FLOAT(JA-JB))/(SXY2(L,L)-SX2(L)*SX2(L)
     1/FLOAT(JA-JB))
      B3=((SXY1(L,K)+SXY2(L,K))-(SX1(L)+SX2(L))*(SX1(K)+SX2(K))/FLOAT(JA
     1))/((SXY1(L,L)+SXY2(L,L))-(SX1(L)+SX2(L))*(SX1(L)+SX2(L))/FLOAT(JA
     2))
      R1=(ABS(A1*B1))**0.5
      R2=(ABS(A2*B2))**0.5
      R3=(ABS(A3*B3))**0.5
      WRITE (7,530) AL(K) , AL(L),A3,B3,R3
  530 FORMAT(1H ,A6,4X,A6,3(8X,F9.4))
   27 WRITE(6,30)AL(K),AL(L),A1,A2,A3,B1,B2,B3,R1,R2,R3
   30 FORMAT (1H ,A6,4X,A6,4X,9(F9.4,1X))
   31 FORMAT(1H0,72HCORRELATION ANALYSES OF SAND AND CLAY BASIN VARIABLE
     1S FROM 1/50,000 MAPS)
      STOP
      END
```

FIG 28

BASIN $(1)_c$



BASIN $(2)_c$

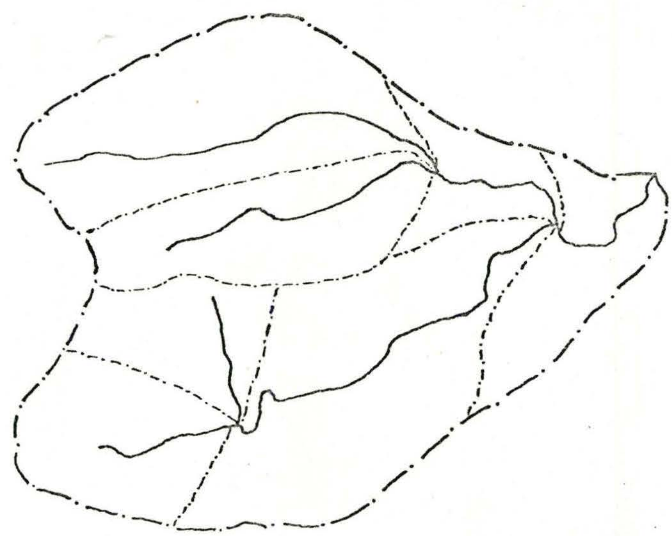
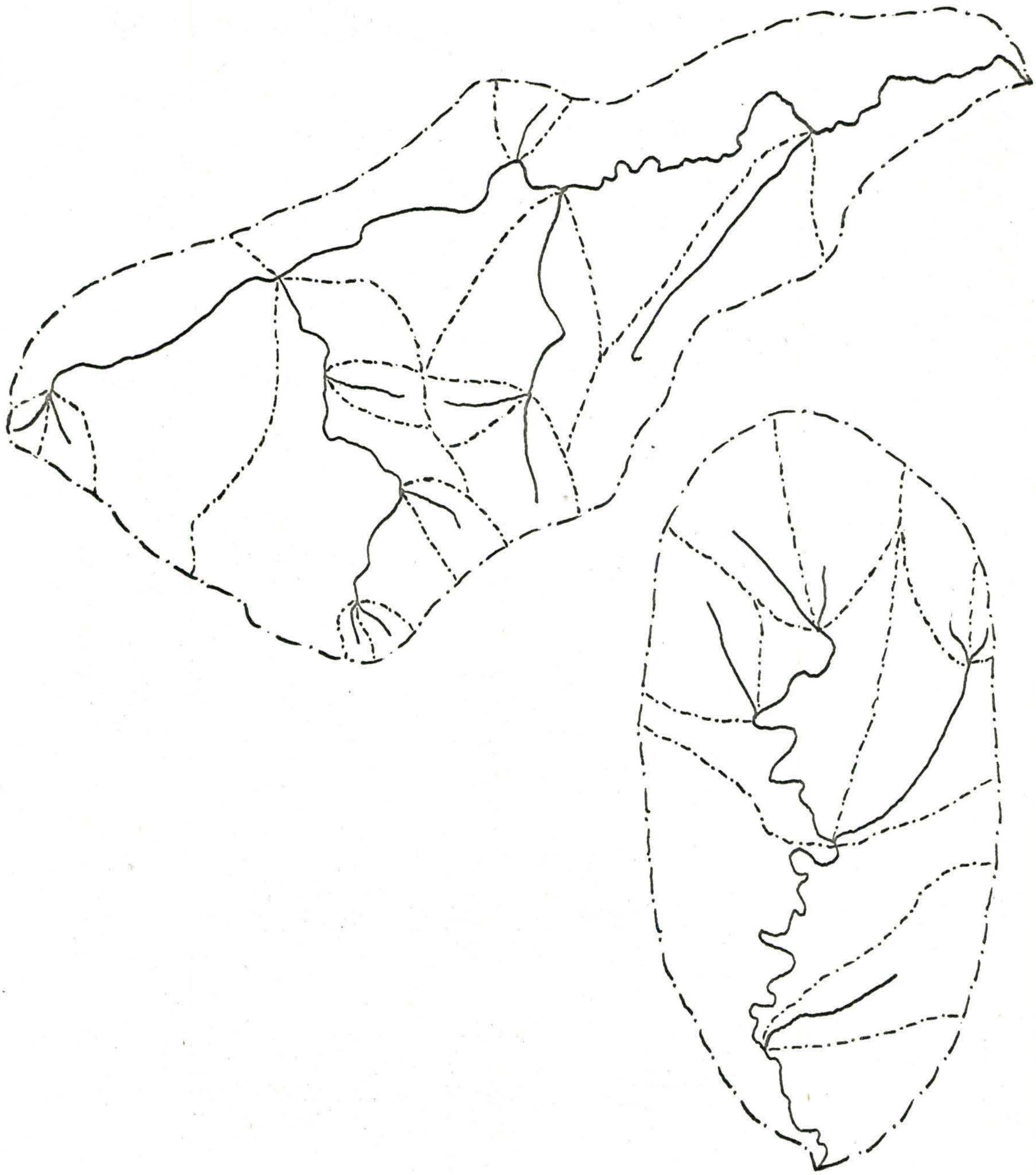


FIG 29

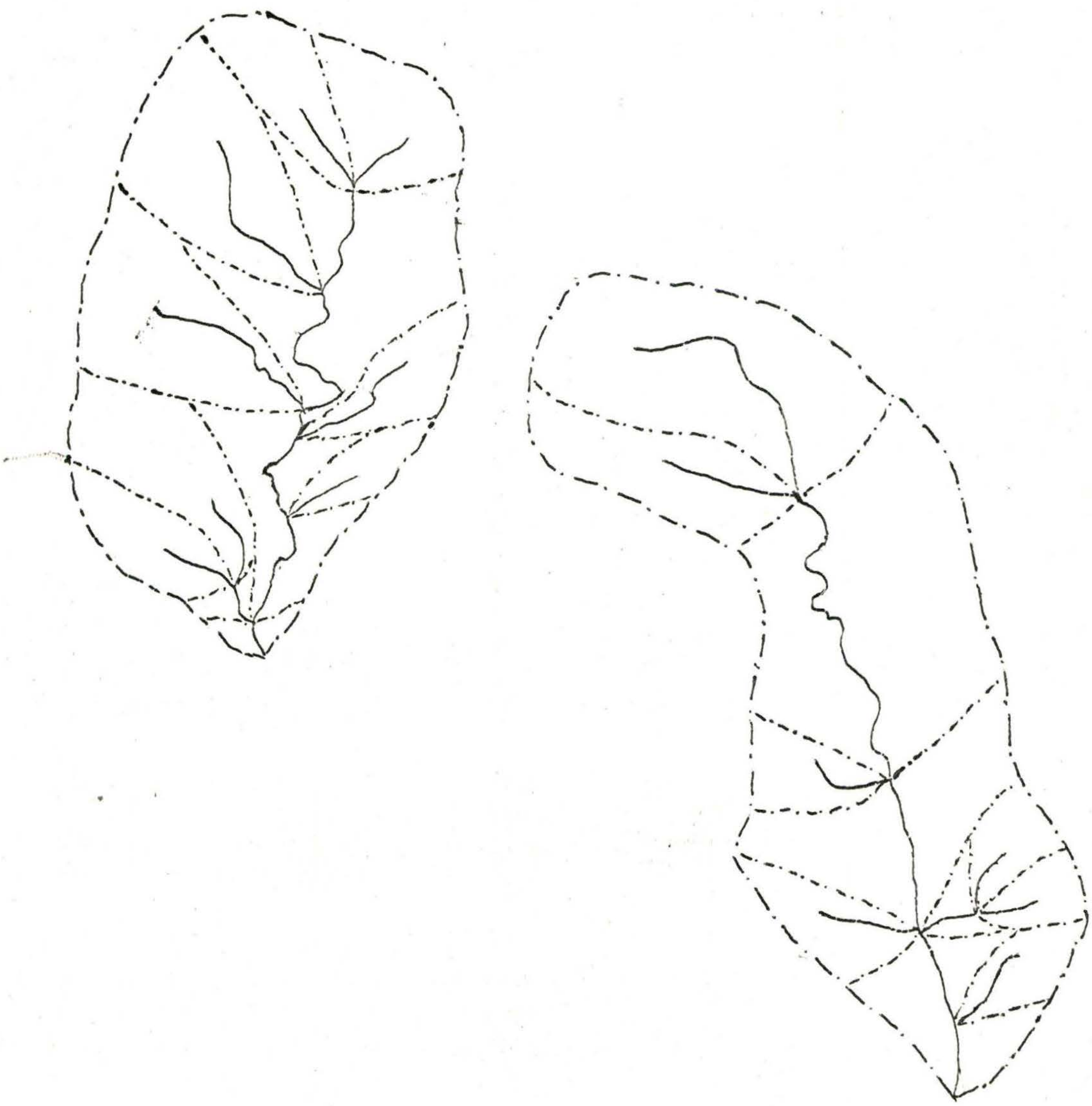
BASIN $(3)_c$



BASIN $(4)_c$

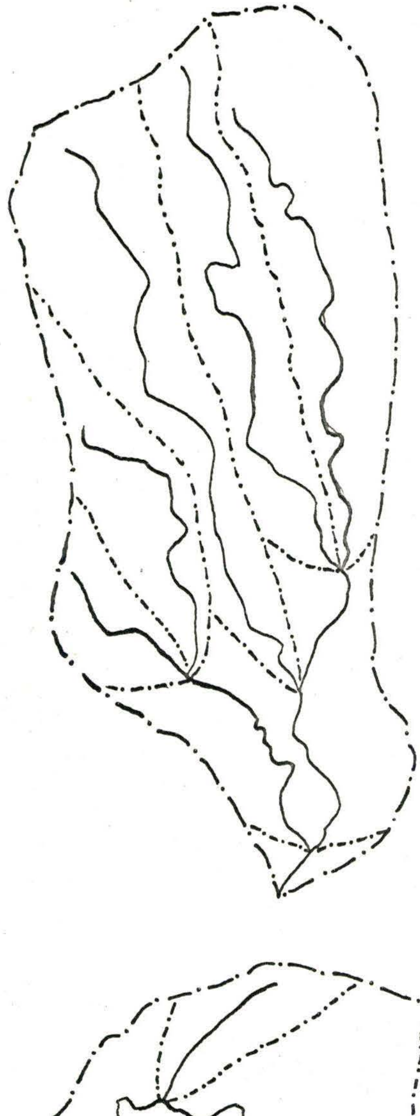
FIG 30

BASIN $(5)_c$

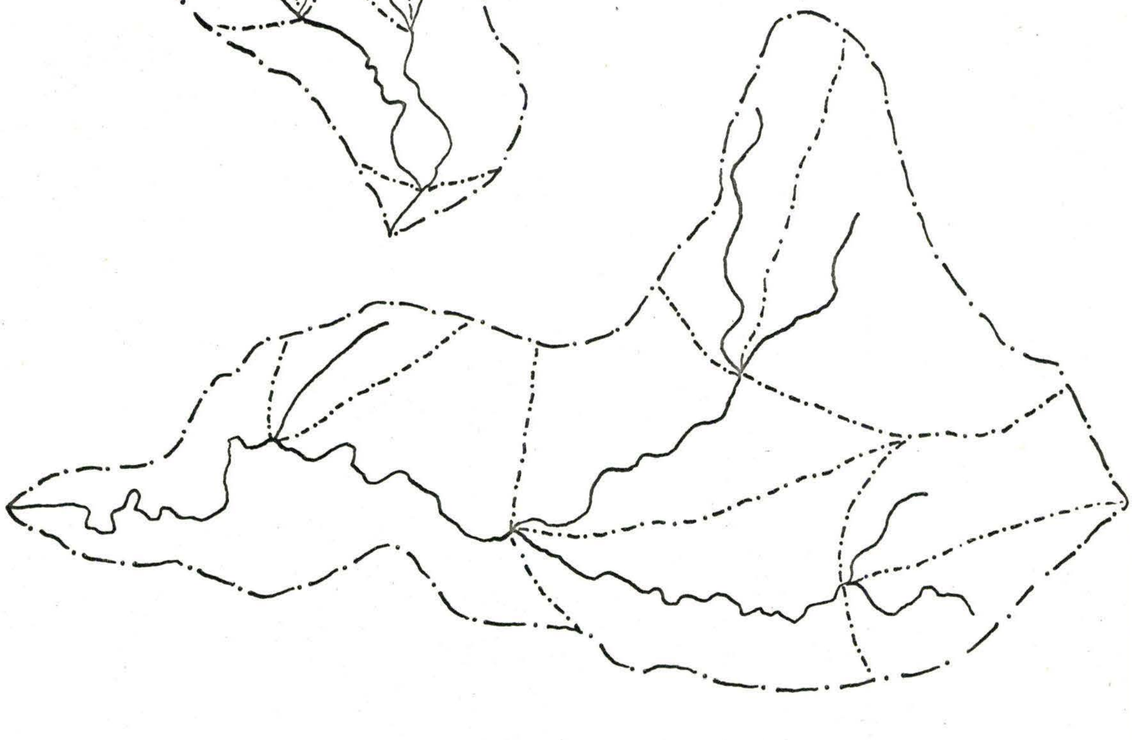


BASIN $(6)_c$

FIG 31



BASIN $(7)_c$



BASIN $(8)_c$

FIG 32

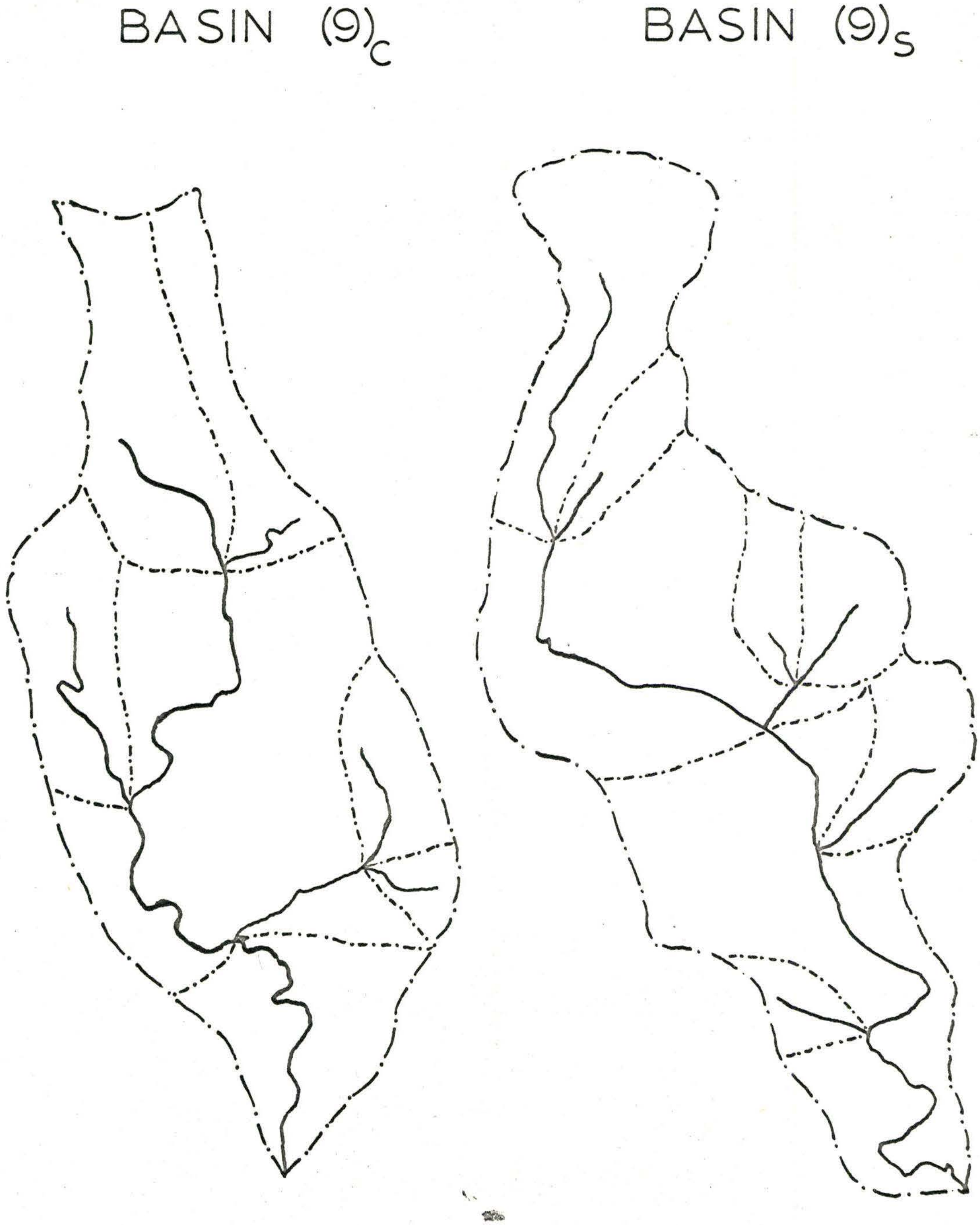
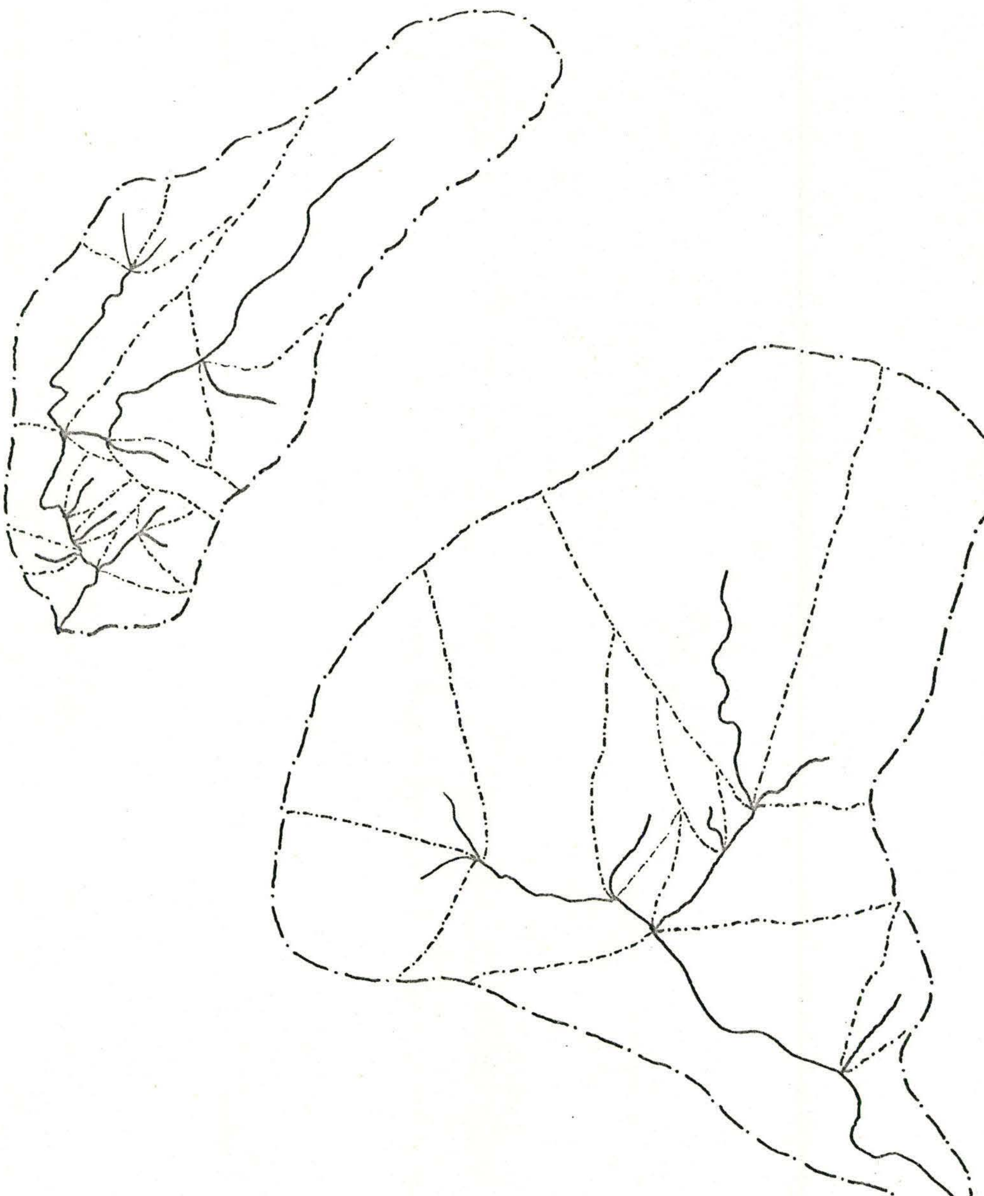


FIG 33

BASIN $(1)_s$



BASIN $(2)_s$

FIG 34

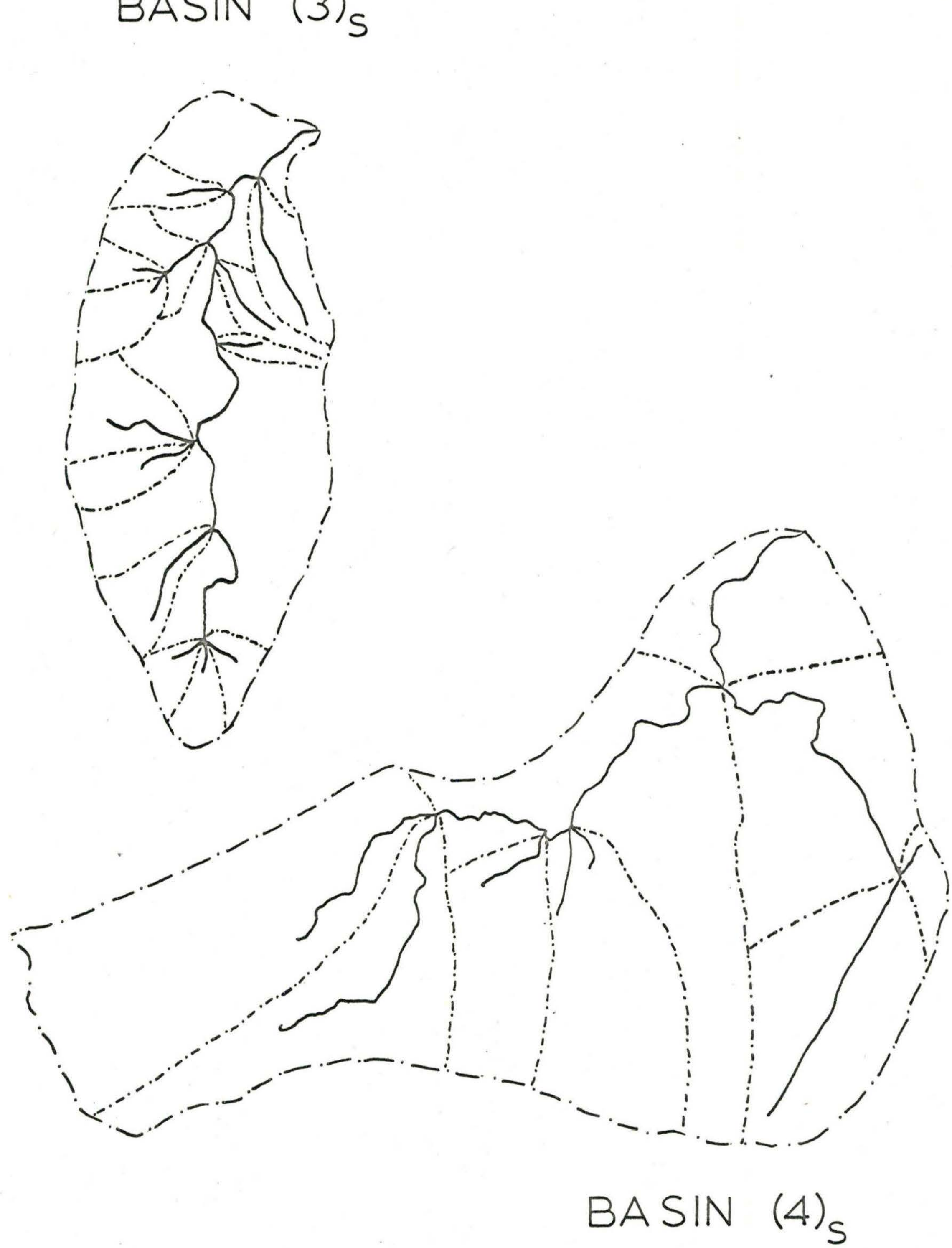


FIG 35

BASIN $(5)_S$

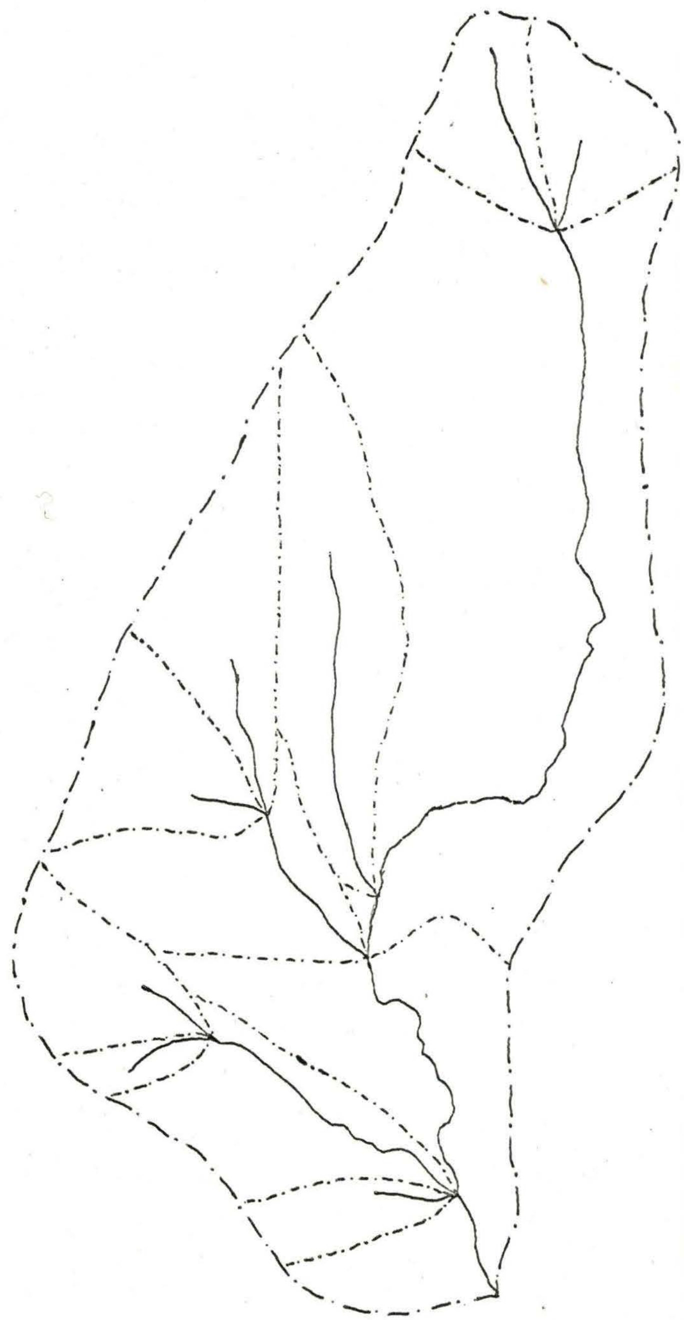
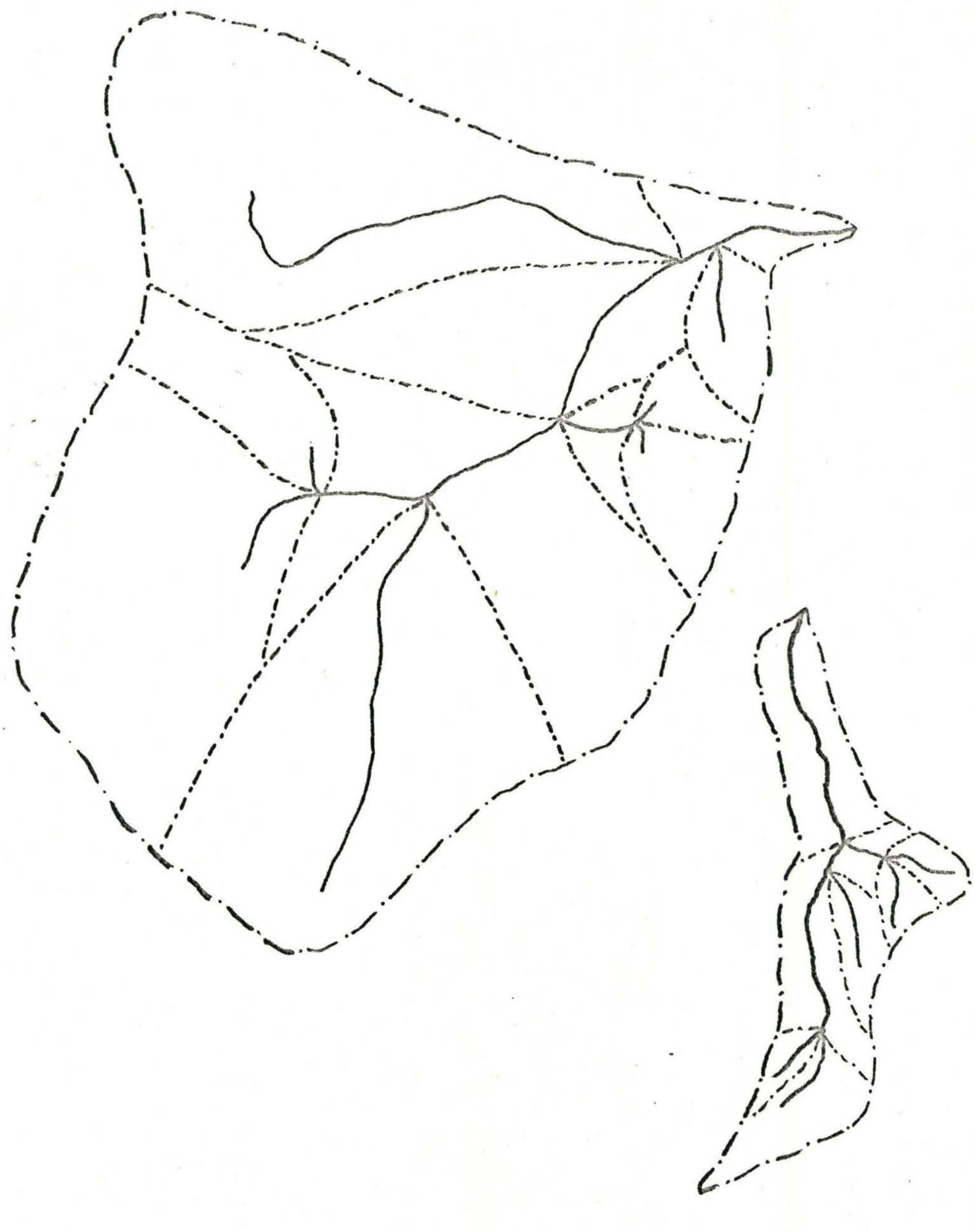


FIG 36

BASIN $(6)_s$



BASIN $(7)_s$

FIG 37

BASIN $(8)_s$

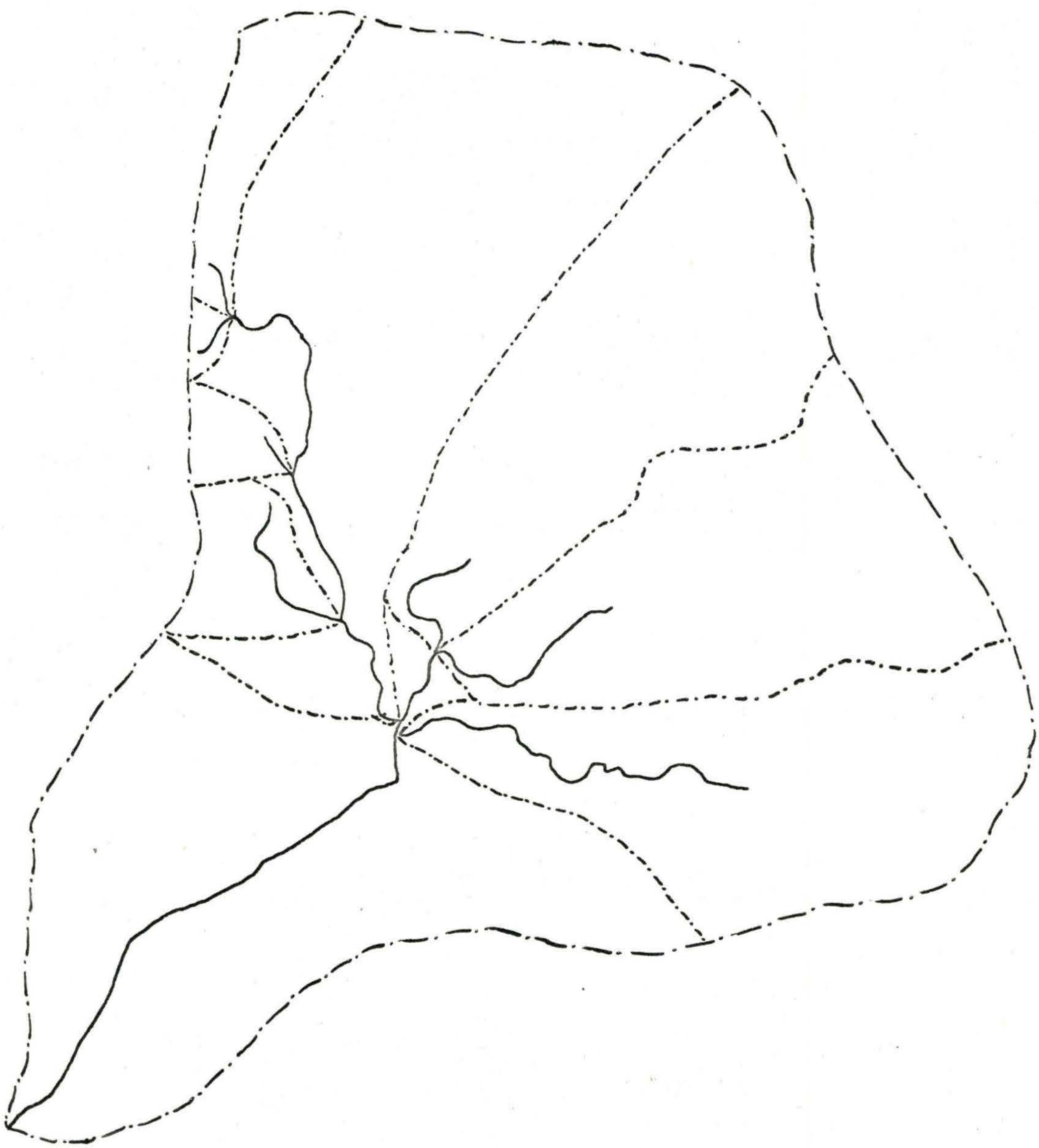


FIG 38
BASIN $(1)_s$
(photograph data)

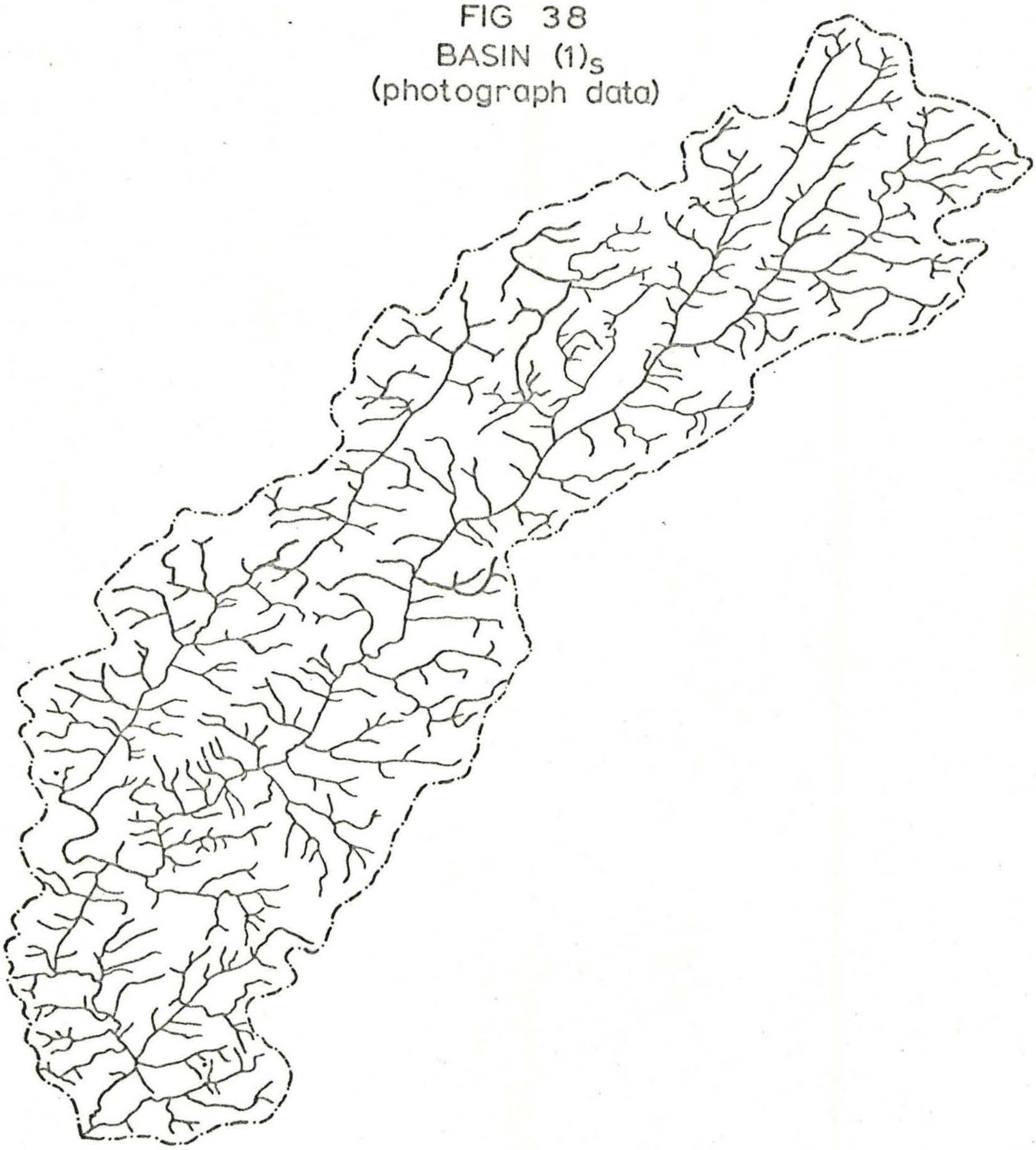


FIG 39
BASIN $(5)_c$
(photograph data)

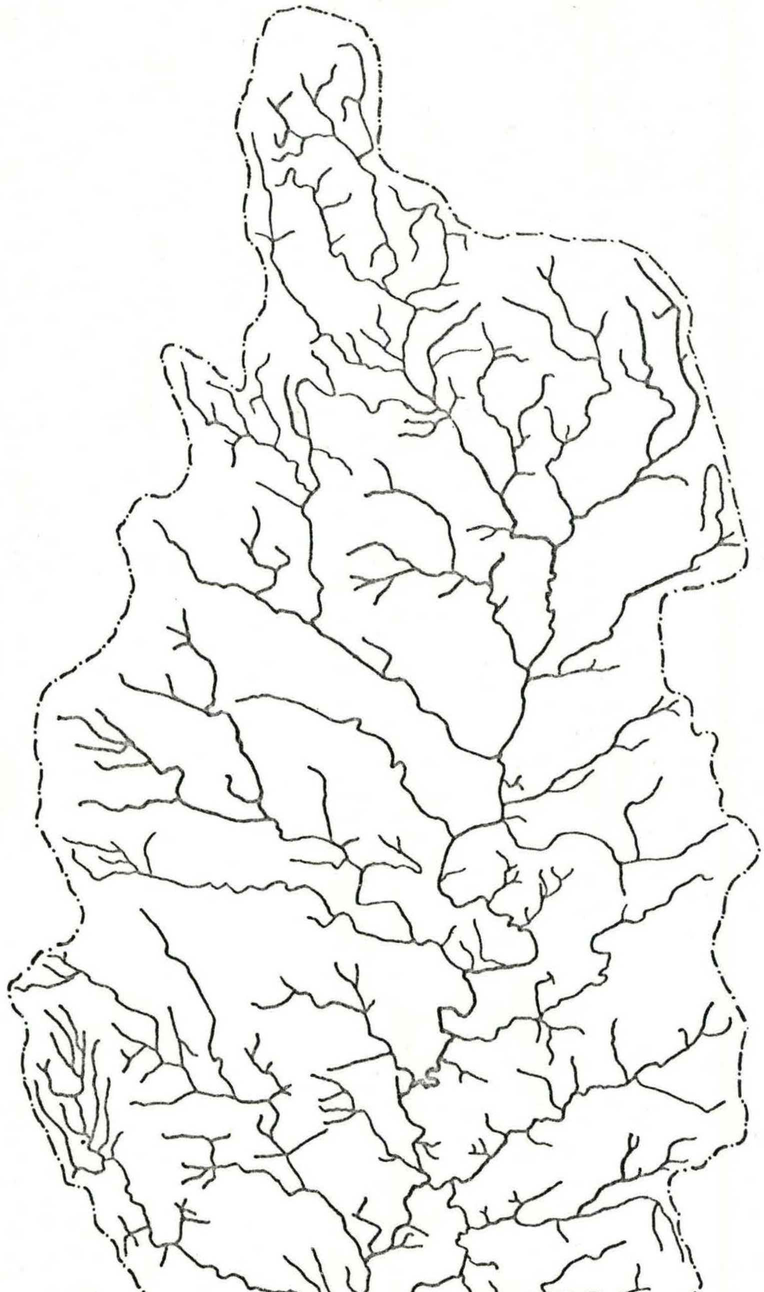
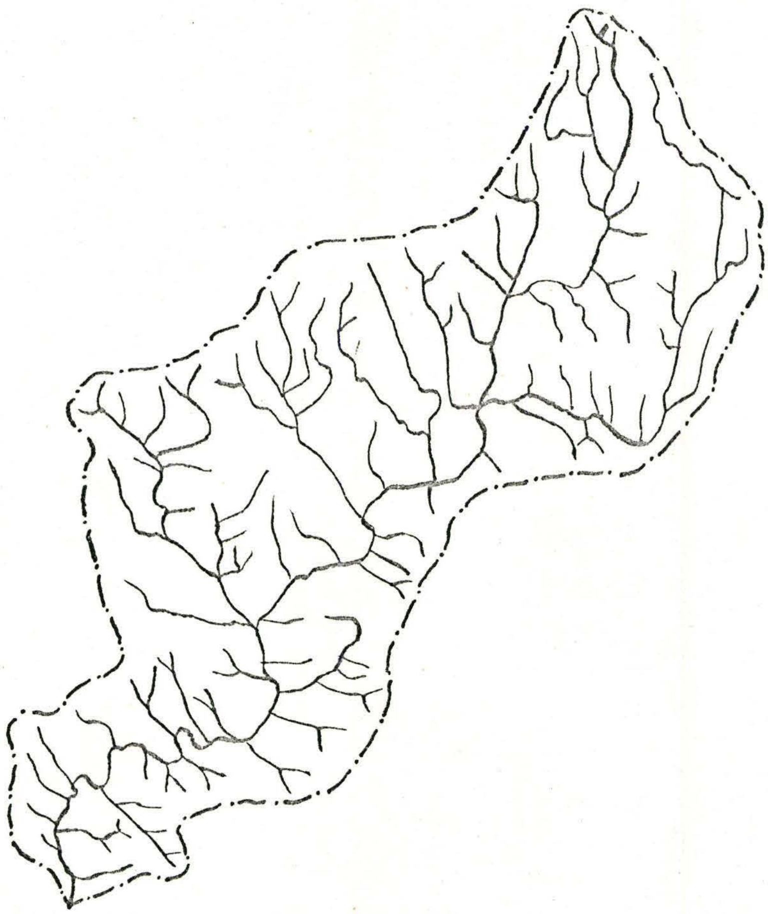


FIG 40
BASIN $(1)_c$
(photograph data)



REFERENCES CITED IN THE TEXT

Bowden, K. L., and Wallis, J. R., 1964, The effect of stream ordering on Horton's Laws of Drainage Composition: Bull. Geol. Soc. America, v. 75, p. 767 - 774.

Chapman, L. J. and Putnam, D. F., 1951, The physiography of Southern Ontario: Toronto, University of Toronto Press.

Chorley, R. J., Malm, D. E. G., and Pogorzelski, H. A., 1957, A new standard for estimating drainage basin shape: Am. Jour. Sci., v. 225, p. 138 - 141.

Coates, D. R., 1958, Quantitative geomorphology of small drainage basins of Southern Indiana: Technical Report No. 10, Project NR 389 - 042, Office of Naval Research, Geography Branch.

Gray, D. M., 1961, Interrelationships of watershed characteristics: Jour. Geophys. Res., v. 66, p. 1215 - 1223.

Horton, R. E., 1945, Erosional development of streams and their drainage basins; hydro-physical approach to quantitative morphology: Bull. Geol. Soc. America, v. 56, p. 275 - 370.

Hough, J. L., 1958, The geology of the Great Lakes: Urbana, University of Illinois Press.

Howard, A. D., 1965, Geomorphological systems - equilibrium and dynamics: Am. Jour. Sci., v. 263, p. 302 - 312.

Langbein, W. B., and Leopold, L. B., 1964, Quasi-equilibrium states in channel morphology: Am. Jour. Sci., v. 262, p. 782 - 795.

Leopold, L. B., and Maddock, T., Jr., 1953, The hydraulic geometry of stream channels and some physiographic implications: U. S. Geol. Survey Prof. Paper 252.

Leopold, L. B., and Miller, J. P., 1956, Ephemeral streams - hydraulic factors and their relation to the drainage net: U. S. Geol. Survey Prof. Paper 282-A.

Leopold, L. B., Wolman, M. G., and Miller, J. P., 1964, Fluvial Processes in geomorphology, San Francisco, W. H. Freeman and Co.

Lubowe, J. K., 1964, Stream junction angles in the dendritic drainage pattern: Am. Jour. Sci., v. 262, p. 325 - 339.

Mackin, J. H., 1948, The concept of the graded river: Bull. Geol. Soc. America, v. 59, p. 463 - 512.

Melton, M. A., 1957, An analysis of the relations among elements of climate, surface properties, and geomorphology: Technical Report No. 11, Project NR 389 - 042, Office of Naval Research, Geography Branch.

Melton, M. A., 1958a, Correlation structure of morphometric properties of drainage systems and their controlling agents: Jour. Geology, v. 66, p. 442 - 460.

----------, 1958b, Geomorphic properties of mature drainage systems and their representation in E_4 space: Jour. Geology, v. 66, p. 35 - 54.

----------, 1958c, List of sample parameters of quantitative properties of land forms: their use in determining the size of geomorphic experiments: Technical Report No. 16, Project NR 389 - 042, Office of Naval Research, Geography Branch.

----------, 1959, A derivation of Strahler's channel ordering system: Jour. Geology, v. 67, p. 345 - 346.

Miller, V. C., 1953, A quantitative geomorphic study of drainage basin characteristics in the Uinch Mountain area, Virginia and Tennessee: Technical Report No. 3, Project NR 389 - 042, Office of Naval Research, Geography Branch.

Morisawa, M. E., 1958, Measurement of drainage basin outline form: Jour. Geology, v. 66, p. 587 - 591.

----------, 1962, The quantitative geomorphology of some watersheds in the Appalachian Plateau: Bull. Geol. Soc. America, v. 73, p. 1025 -

Roberts, M. C., 1963, Some empirical regularities of the Humber drainage net: Iowa Academy of Science, v. 70, p. 321 - 326.

Schumm, S. A., 1956, Evolution of drainage systems and slopes in badlands at Perth Amboy, New Jersey: Bull. Geol. Soc. America, v. 67, p. 597 - 646.

Shreve, R. L., 1964, An analysis of Horton's Law of Stream Numbers: Am. Geophys. Union Trans., v. 45, p. 50 - 51.

Strahler, A. N., 1950a, Davis' concepts of slope development viewed in the light of recent quantitative investigations: Annals Assoc. American Geographers, v. 40, p. 209 - 213.

----------, 1952a, Dynamic basis for geomorphology: Bull. Geol. Soc. America, v. 63, p. 923 - 938.

----------, 1952b, Hypsometric (area - altitude) analysis of erosional topography: Bull. Geol. Soc. America, v. 63, p. 1117 - 1142.

Strahler, A. N., 1954a, Dimensional analysis in quantitative terrain descriptions: Annals Assoc. American Geographers, v. 44, p. 282.

----------, 1954b, Quantitative geomorphology of erosional landscapes: Comptes Rendus, 19th Session, International Geological Congress, Algiers, Sec. XIII, p. 341 - 354.

----------, 1954c, Statistical analysis in geomorphic research: Jour. Geology, v. 62, p. 1 - 25.

----------, 1957a, Objective field sampling of physical terrain properties: Annals Assoc. American Geographers, v. 47, p. 179 - 180.

----------, 1957b, Quantitative analysis of watershed geometry: Am. Geophys. Union Trans., v. 38, p. 913 - 920.

----------, 1958, Dimensional analysis applied to fluvially eroded land forms: Bull. Geol. Soc. America, v. 69, p. 279 - 300.

Wolman, M. G., and Brush, L. M., Jr., 1961, Factors controlling the size and shape of stream Channels in coarse Noncohesive sands: U. S. Geol. Survey Prof. Paper 282-G.